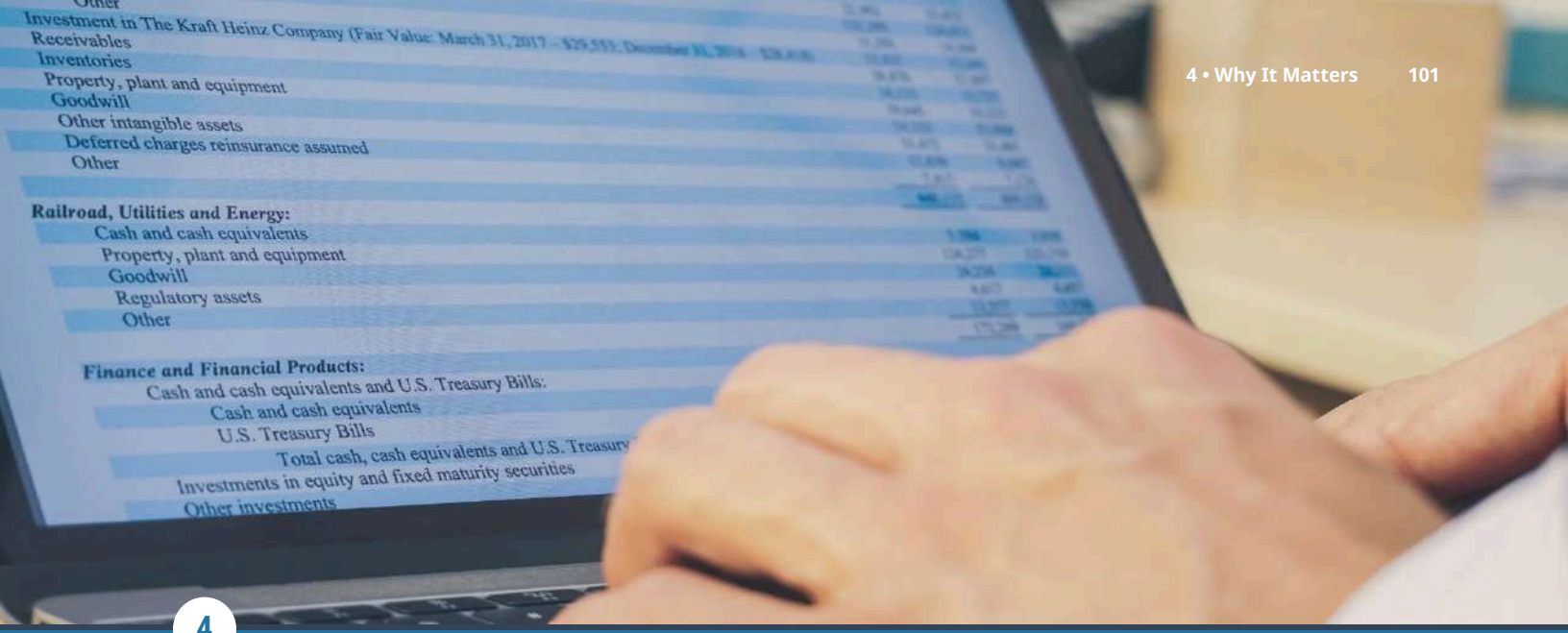
openstax™

Principles of Finance



4

Accrual Accounting Process

Figure 4.1 Accrual accounting reports revenue when goods are delivered or services are performed but are not necessarily paid for. (credit: modification of work "Reviewing Financial Statements" by Instructional Institutions)

Chapter Outline

Why It Matters

Emma, an accounting major, was on her way to her Wednesday morning accounting class with her friend Sam. Sam, a finance major, kept complaining to Emma that he didn’t enjoy their accounting class at all. “Why do we have to spend so much time worried about debits, credits, accounts, and all this other accounting jargon anyway? I’m a finance major. I won’t be spending my days recording entries and counting pennies. I’m dreading class today. I looked at the syllabus, we’re studying the accrual method. So what? Just give me the financial statements so I have the data I need.”

Emma smiled at her friend and thought for a moment before responding. Emma loves the details of debits, credits, and “all that other accounting jargon,” as Sam put it. She knows that while not everyone enjoys the details or understands them, they do play a big role in the timing and accuracy of financial statements. “Yeah, yeah, Sam, accounting isn’t always easy or fun, but it’s important. The financial statements you will use in finance are a part of the foundation for many decisions. They are an important tool. Even though you don’t have to create financial statements in a finance role, it’s still necessary to understand the accounting principles they are based on. We haven’t covered the details yet, but I do know that income or loss in a period can be significantly different if the income statement is prepared under cash versus accrual methods. I would think that would be important to know if you are the one using the income statement.”

4.1 Cash versus Accrual Accounting

Learning Outcomes

By the end of this section, you will be able to:

- Outline the key concepts of the cash-basis accounting method.
- Explain the key characteristics of the accrual-basis accounting method.
- Identify businesses for which cash accounting or accrual accounting is more appropriate.

In this section, we will explore the basic elements of cash and accrual accounting and the businesses that are most likely to use each one. Some private companies may choose to use **cash-basis accounting** rather than **accrual-basis accounting** to report financial information.

Cash-Basis Accounting

In business, cash is certainly important. In fact, it's so important that it dictates one of two ways we can account for our business transactions. The *cash method* is just as the name implies—it records transactions only when cash flows. We track cash inflows and outflows as they occur. This method is most commonly used by small businesses that deal primarily in cash transactions. The other method, called the *accrual method*, records transactions when they occur, rather than waiting for cash to be accumulated. Using the accrual method, we match cash inflows and the outflows required to generate them. We call this the *matching principle*. This method is used by most publicly traded companies. In this chapter, you'll explore both methods, see how each impacts financial statements differently, note the role of timing in each method, and learn how and when to record capital and expense transactions.

Let's look at an example. Chris just finished the first month of her landscaping business operations at the end of August, and she used the cash method of accounting to figure out her **net income**. Most small start-up companies use the cash method of accounting because it is easy to understand, requires no special training, and helps them focus on one big key to their survival—cash. This means that she simply recorded the cash that came in and the cash that went out of her business. She brought in $1,400 in revenue in her first month, which she felt was substantial given that it was her first month. But after deducting her expenses, she had only $250 left, so she worried about the future of her business. Would she have to increase her sales exponentially in order to start bringing in a decent profit each month?

As you move through the chapter, you'll get to see the impact of the two methods of accounting and how these methods impact the insights and decisions Chris made for her new business.

Cash-basis accounting is a method of accounting in which transactions are not recorded in the financial statements until there is an exchange of cash. Cash-basis accounting sometimes impacts the timing of **revenue** and **expense** reporting until cash receipts or outlays occur. For example, as you saw above, Chris measured the performance of her landscaping business for the month of August using cash flows. Cash accounting is far simpler to track than accrual-basis accounting.

Accrual-Basis Accounting

Public companies reporting their financial positions use either US generally accepted accounting principles (GAAP) or International Financial Reporting Standards (IFRS), as allowed under the Securities and Exchange Commission (SEC) regulations. GAAP is a set of accounting standards created by the Financial Accounting Standards Board (FASB) and the Governmental Accounting Standards Board (GASB). It's key to note that though they are similar in many areas, there are still key areas that differ between GAAP and IFRS. Therefore, when using financial statements, it's important to be aware of the standards under which they were prepared. However, public or private companies using GAAP or IFRS must prepare their financial statements using the rules of accrual accounting. Accrual-basis accounting prescribes that revenues and expenses must be recorded in the accounting period in which they were earned or incurred, no matter when cash receipts or payments occur. It is because of accrual accounting that we have the revenue recognition principle and the

expense recognition principle (also known as the matching principle).

The accrual method is considered to better match revenues and expenses and standardizes reporting information for comparability purposes. Having comparable information is important to external users of information trying to make investment or lending decisions and to internal users trying to make decisions about company performance, budgeting, and growth strategies.

Who Uses Each Method?

Cash-basis accounting can be more efficient and well-suited for certain types of businesses, such as farming or professional services provided by lawyers and doctors. However, the accrual basis of accounting is theoretically preferable to the cash basis of accounting because it takes into account the timing of the transactions (when goods and services are provided and when the cash involved in the transactions is received). Cash can often be received a significant amount of time after the initial transaction. Considering this amount allows accountants to provide, in a timely manner, relevant and complete information to stakeholders.

There are several reasons accrual-basis accounting is preferred to cash-basis accounting. Accrual-basis accounting is required by GAAP because it typically provides a better sense of the financial well-being of a company. Accrual-based accounting information allows management to analyze a company's progress, and management can use that information to improve their business. Accrual accounting is also used to assist companies in securing financing because banks will typically require a company to provide accrual-basis financial income statements. The Internal Revenue Service requires businesses to report using accrual-basis information when preparing tax returns. In addition, companies with inventory must use accrual-based accounting for income tax purposes, though there are exceptions to the general rule.

So why might a company use cash-basis accounting? Companies that do not sell stock publicly can use cash-basis instead of accrual-basis accounting for internal management purposes or because they are exempt from such requirements in agreements such as a bank loan. Cash-basis accounting is a simpler accounting system to use than an accrual-basis accounting system when tracking real-time revenues and expenses.

THINK IT THROUGH

Cash- or Accrual-Basis Accounting?

You are a new accountant at a beauty salon. The salon had previously used cash-basis accounting to prepare its financial records but is now considering switching to an accrual-basis method. You have been tasked with determining if this transition is appropriate.

When you go through the records, you notice that this transition will greatly impact how the salon reports revenues and expenses. The salon will now report some revenues and expenses before it receives or pays cash.

How will this change positively impact its business reporting? How will it negatively impact its business reporting? If you were the accountant, would you recommend the salon transition from cash basis to accrual basis?

Solution:

Accrual accounting creates a more accurate picture of profit or loss, so the salon's owner can have a better understanding of its profitability from period to period. However, it can be more work to record under accrual accounting. If the salon is small and the profits and costs are easily understood, it might not be worth the extra effort to the owner to use accrual-basis accounting. If the salon is seeking ways to better understand profits and costs, accrual-basis accounting would be a great choice.

4.2 Economic Basis for Accrual Accounting

Learning Outcomes

By the end of this section, you will be able to:

- Assess the impact of business transactions on cash flow.
- Define double-entry accounting and explain how it supports the accounting equation.

How and when we record our transactions can have a significant impact on financial statements, especially the **income statement (net income)**. In this section you will explore the impact business transactions have on financial statements under each method. In doing so, you'll be introduced to **double-entry accounting** and see how it functions to support the accounting equation.

Timing of Business Activity versus Cash Flow

The first financial statement prepared is the income statement, a statement that shows the organization's financial performance *for a given period of time*. We already saw that Chris, who is a sole proprietor, started a summer landscaping business on August 1, 2020. She categorized her business as a service entity and used the cash-basis method of accounting to record the initial operations for her business. Although Chris was using her family's tractor to get her work done, she was responsible for paying for fuel and any maintenance costs. So, on August 31, Chris realized she had only $250 in her checking account.

This balance was lower than expected because she had spent only slightly less ($1,150 for brakes, fuel, and insurance) than she earned ($1,400)—leaving a net income of $250. While she would like the checking balance to grow each month, she realized that most of the August expenses were infrequent (brakes and insurance) and the insurance, in particular, was an unusually large expense. She knew that the checking account balance would likely grow more in September because she would earn money from some new customers; she also anticipated having fewer expenses.

This simple landscaping example can be used to discuss the elements of the income statement, which are revenues, expenses, **gains**, and **losses** for a particular period of time (see Figure 4.2). Together, these determine whether the organization has net income (where revenues and gains are greater than expenses and losses) or net loss (where expenses and losses are greater than revenues and gains). Revenues, expenses, gains, and losses are further defined in the Income Statement provided.

Chris's Landscaping
Income Statement
For the Month Ended August 31

Revenue	$1,400	
Total revenue		$1,400
Expenses		
Tractor brake repair	100	
Tractor fuel	50	
Business insurance	1,000	
Total expenses		1,150
Net income		$ 250

Figure 4.2 Income Statement for Chris's Landscaping

The income statement can also be visualized by the formula:

$$\text{Revenue} - \text{Expenses} = \text{Net Income or Net Loss}$$

Let's change this example slightly and assume the $1,000 payment to the insurance company will be paid in

September rather than in August. In this case, the ending balance in Chris's checking account would be $1,250, a result of earning $1,400 and only spending $100 for the brakes on the tractor and $50 for fuel. This stream of cash flows is an example of cash-basis accounting because it reflects when payments are received and made, not necessarily the time period that they affect. At the end of this section, you will address accrual accounting, which does reflect the time period that payments affect.

The Accounting Equation

It may be helpful to think of the **accounting equation** from a "sources and claims" perspective. Under this approach, the assets (items owned by the organization) were obtained by incurring liabilities or were provided by owners. Stated differently, every asset has a claim against it—by creditors and/or owners.

$$\text{Assets} = \text{Liabilities} + \text{Owner's Equity}$$

You may recall from mathematics courses that an equation must always be in balance. Therefore, we must ensure that the two sides of the accounting equation are always equal. We will explore the components of the accounting equation in more detail shortly. First, we need to examine several underlying concepts that form the foundation for the accounting equation: the double-entry accounting system, debits and credits, and the "normal" balance for each account that is part of a formal accounting system.

THINK IT THROUGH

The Accounting Equation

On a sheet of paper, use three columns to create your own accounting equation. In the first column, list all the things you own (assets). List only the asset itself; don't worry about any associated liabilities (expenses) in that column. In the second column, list any amounts owed (the liabilities). When you are done, total up all the assets. Then total up all the liabilities.

Now, use the accounting equation to calculate the net amount of the asset (equity). To do so, subtract the total assets from the total liabilities. This figure makes the accounting equation balance and represents equity, or an estimate of your net worth.

Here is something else to consider: Is it possible to have negative equity? It sure is . . . ask any college student who has taken out loans. At first glance there is no asset directly associated with the amount of the loan. But is that, in fact, the case? You might ask yourself why you should make an investment in a college education—what is the benefit (asset) to going to college? The answer lies in the difference in lifetime earnings with a college degree versus without a college degree. This is influenced by many things, including the supply and demand of jobs and employees. It is also influenced by the earnings for the type of college degree pursued.

Solution:

Answers will vary but may include vehicles, clothing, electronics (include cell phones and computer/gaming systems), and sports equipment. They may also include money owed on these assets, most likely vehicles and perhaps cell phones. In the case of a student loan, there may be a liability with no corresponding asset (yet). Responses should be able to evaluate the benefit of investing in college and the wage differential between earnings with and without a college degree.

Let's continue our exploration of the accounting equation, focusing on the equity component in particular. Recall that we defined equity as the net worth of an organization. It is helpful to also think of net worth as the accounting *value* of the organization. Recall, too, that revenues (inflows as a result of providing goods and services) *increase* the accounting value of the organization. So every dollar of revenue an organization generates increases the overall value of the organization.

Likewise, expenses (outflows as a result of generating revenue) *decrease* the value of the organization. So each dollar of expenses an organization incurs decreases the overall value of the organization. The same approach can be taken with the other elements of the financial statements:

- Gains *increase* the value (equity) of the organization.
- Losses *decrease* the value (equity) of the organization.
- Investments by owners *increase* the value (equity) of the organization.
- Distributions to owners *decrease* the value (equity) of the organization.
- Changes in assets and liabilities can *either* increase or decrease the value (equity) of the organization depending on the net result of the transaction.

Let's look at Chris's Landscaping business again and do the same quick exercise you did with your personal finances. If we were to total all of Chris's assets, we would find just one: $250 in cash. She's using the family's tractor, but she doesn't own the tractor, so it is not her asset. Her liabilities are currently $0, as she paid cash for all the expenses she incurred already. If we total her assets, we get $250. Liabilities total $0. Using the accounting equation, we find her equity to currently be $250, or

$$\text{Assets } (\$250) - \text{ Liabilities } (\$0) = \text{Equity } (\$250)$$

Double-Entry Accounting

Accounting is based on a double-entry accounting system, which requires the following:

- Each time we record a transaction, we must record a change in at least two different accounts. Having two or more accounts change will allow us to keep the accounting equation in balance.
- Not only will at least two accounts change, but there must also be at least one **debit** and one **credit** side impacted.
- The sum of the debits must equal the sum of the credits for each transaction.

In order for companies to record the myriad of transactions they have each year, there is a need for a simple but detailed system. Journals are useful tools to meet this need.

Debits and Credits

Each account can be represented visually by splitting the account into left and right sides as shown. The graphic representation of a general ledger account is known as a **T-account**. It is called this because it looks like a "T," as you can see with the T-account shown in Figure 4.3.

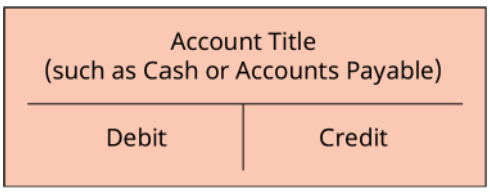


Figure 4.3 T-Account

A debit records financial information on the left side of each account. A credit records financial information on the right side of an account. One side of each account will increase, and the other side will decrease. The ending account balance is found by calculating the difference between debits and credits for each account. You will often see the terms *debit* and *credit* represented in shorthand, written as *DR* or *dr* and *CR* or *cr*, respectively. Depending on the account type, the sides that increase and decrease may vary. We can illustrate each account type and its corresponding debit and credit effects in the form of an *expanded* equation (see Figure 4.4).

Assets		= Liabilities		+ Equity: Common Stock		– Dividends		+ Revenues		– Expenses	
Debit	Credit	Debit	Credit	Debit	Credit	Debit	Credit	Debit	Credit	Debit	Credit
Increase	Decrease	Decrease	Increase	Decrease	Increase	Increase	Decrease	Decrease	Increase	Increase	Decrease

Figure 4.4 Expanded Accounting Equation

As we can see from this expanded accounting equation, Assets accounts increase on the debit side and decrease on the credit side. This is also true of Dividends and Expenses accounts. Liabilities increase on the credit side and decrease on the debit side. This is also true of Common Stock and Revenues accounts. This becomes easier to understand as you become familiar with the *normal balance* of an account.

The balance sheet is a reflection of the accounting equation (see Figure 4.5). It has two sections, assets in one section and liabilities and equity in the other section. It's key to note that both assets and liabilities are broken down on the balance sheet into current and noncurrent classifications in order to provide more detail and transparency. Current assets are those that are consumed within a year. Assets that will be in use longer than a year are considered noncurrent. Current liabilities are those that will be due within a year. Noncurrent liabilities are those that are due more than a year into the future.

Assets				= Liabilities				+ Owner's Equity	
Current		Noncurrent		Current		Noncurrent			
+	–	+	–	–	+	–	+	–	+
								Distribution to Owners Expenses Losses Comprehensive Income	Investments by Owners Revenues Gains Comprehensive Income

Figure 4.5 Graphical Representation of the Accounting Equation Both assets and liabilities are categorized as current and noncurrent. Also highlighted are the various activities that affect the equity (or net worth) of the business.

Notice each account subcategory (Current Assets and Noncurrent Assets, for example) has an "increase" side and a "decrease" side. As you may recall, these are called T-accounts, and they are used to analyze transactions.

The basic components of even the simplest accounting system are *accounts* and a *general ledger*. An account is a record showing increases and decreases to assets, liabilities, and equity—the basic components found in the accounting equation. Each of these categories, in turn, includes many individual accounts, all of which a company maintains in its general ledger. A general ledger is a comprehensive listing of all of a company's accounts with their individual balances.

4.3 How Does a Company Recognize a Sale and an Expense?

Learning Outcomes

By the end of this section, you will be able to:

- Explain the revenue recognition principle and how it relates to current and future sales transactions.
- Explain the expense recognition principle and how it relates to current and future purchase transactions.
- Assess the role of ethics in revenue and expense recognition.

You've learned the basics of each method as well as the accounting equation and double-entry accounting. Next, let's turn our attention to when we record transactions, as timing is key.

Revenue Recognition

Revenue is the value of goods and services the organization sold or provided to customers for a given period of time. In our current example, Chris's landscaping business, the "revenue," or the value of services

performed, for the month of August would be $1,400. It is the value Chris received in exchange for the services provided to her clients. Likewise, when a business provides goods or services to customers for cash at the time of the service or in the future, the business classifies the amount(s) as revenue. Just as the $1,400 revenues from a business made Chris's checking account balance increase, revenues increase the value of a business. In accounting, **revenue recognition** involves recording sales or fees earned within the period earned. Just as earning wages from a business or summer job reflects the number of hours worked for a given rate of pay or payments from clients for services rendered, revenues (and the other terms) are used to indicate the dollar value of goods and services provided to customers for a given period of time.

THINK IT THROUGH

Coffee Shop Products

Think about a coffee shop in your area. Identify items the coffee shop sells that would be classified as revenues. Remember, revenues for the coffee shop are related to its primary purpose: selling coffee and related items. Or, better yet, make a trip to the local coffee shop and get a first-hand experience.

Solution:

Many coffee shops earn revenue through multiple revenue streams, including coffee and other specialty drinks, food items, gift cards, and merchandise.

Short-Term Revenue Recognition Examples

Two brief examples may help illustrate the difference between cash accounting and accrual accounting. Assume that a business sells $200 worth of merchandise. In some businesses, there are two ways the customers pay: cash and credit (also referred to as "on account"). Cash sales include checks and credit cards and are paid at the time of the sale. Credit sales (not to be confused with credit card sales) allow the customer to take the merchandise but pay within a specified period of time, usually up to 45 days.

A cash sale would be recorded in the financial statements under both the cash basis and accrual basis of accounting. It makes sense because the customer received the merchandise and paid the business at the same time. It is considered two events that occur simultaneously (exchange of merchandise for cash).

A credit sale, however, would be treated differently under each of these types of accounting. Under the cash basis of accounting, a credit sale would not be recorded in the financial statements until the cash is received, under terms stipulated by the seller. For example, assume that in the next year of Chris's landscaping business, on April 1, she provides $500 worth of services to one of her customers. The sale is made on account, with the payment due 45 days later. Under the cash basis of accounting, the revenue would not be recorded until May 16, when the cash was received. Under the accrual basis of accounting, this sale would be recorded in the financial statements at the time the services were provided, April 1. The reason the sale would be recorded is that, under accrual accounting, the business reports that it provided $500 worth of services to its customer. The fact that the customers will pay later is viewed as a separate transaction under accrual accounting (see Figure 4.6).

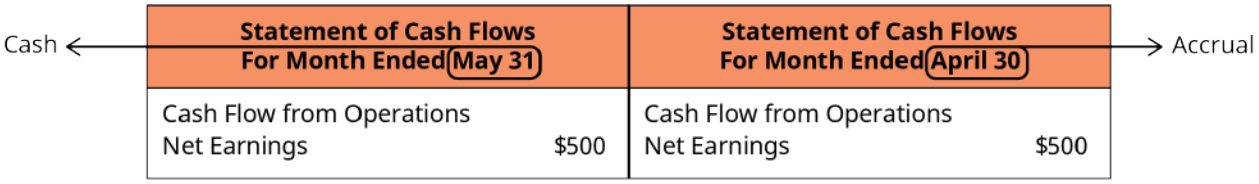


Figure 4.6 Credit versus Cash On the left is a credit sale recorded under the cash basis of accounting. On the right, the same credit sale is recorded under the accrual basis of accounting.

Let's now explore the difference between the cash basis and accrual basis of accounting using an expense.

Assume a business purchases $160 worth of printing supplies from a supplier (vendor). Similar to a sale, a purchase of merchandise can be paid for at the time of sale using cash (also a check or credit card) or at a later date (on account). A purchase paid with cash at the time of the sale would be recorded in the financial statements under both cash basis and accrual basis of accounting. It makes sense because the business received the printing supplies from the supplier and paid the supplier at the same time. It is considered two events that occur simultaneously (exchange of merchandise for cash).

If the purchase was made on account (also called a credit purchase), however, the transaction would be recorded differently under each of these types of accounting. Under the cash basis of accounting, the $160 purchase on account would not be recorded in the financial statements until the cash is paid, as stipulated by the seller's terms. For example, if the printing supplies were received on July 17 and the payment terms were 15 days, no transaction would be recorded until August 1, when the goods were paid for. Under the accrual basis of accounting, this purchase would be recorded in the financial statements at the time the business received the printing supplies from the supplier (July 17). The reason the purchase would be recorded is that the business reports that it bought $160 worth of printing supplies from its vendors. The fact that the business will pay later is viewed as a separate issue under accrual accounting. Table 4.1 summarizes these examples under the different bases of accounting.

Transaction	Under Cash-Basis Accounting	Under Accrual-Basis Accounting
$200 sale for cash	Recorded in financial statements at time of sale	Recorded in financial statements at time of sale
$200 sale on account	*Not* recorded in financial statements until cash is received	Recorded in financial statements at time of sale
$160 purchase for cash	Recorded in financial statements at time of purchase	Recorded in financial statements at time of purchase
$160 purchase on account	*Not* recorded in financial statements until cash is paid	Recorded in financial statements at time of purchase

Table 4.1 How Transactions Are Viewed under Cash and Accrual Accounting

Businesses often sell items for cash as well as on account, where payment terms are extended for a period of time (for example, 30 to 45 days). Likewise, businesses often purchase items from suppliers (also called vendors) for cash or, more likely, on account. Under the cash basis of accounting, these transactions would not be recorded until the cash is exchanged. In contrast, under accrual accounting, the transactions are recorded when the transaction occurs, regardless of when the cash is received or paid.

CONCEPTS IN PRACTICE

Ethics in Revenue Recognition

Because each industry typically has a different method for recognizing income, revenue recognition is one of the most difficult tasks for accountants, as it involves a number of ethical dilemmas related to income reporting. To provide an industry-wide approach, Accounting Standards Update No. 2014-09 and other related updates were implemented to clarify revenue recognition rules. The American Institute of Certified Public Accountants (AICPA) announced that these updates would replace US GAAP's current industry-specific revenue recognition practices with a principle-based approach, potentially affecting both day-to-day business accounting and the execution of business contracts with customers.[1] The AICPA and the International Federation of Accountants (IFAC) require professional accountants to act with due care and to remain abreast of new accounting rules and methods of accounting for different transactions, including revenue recognition.

The IFAC emphasizes the role of professional accountants working within a business in ensuring the quality of financial reporting: "Management is responsible for the financial information produced by the company. As such, professional accountants in businesses therefore have the task of defending the quality of financial reporting right at the source where the numbers and figures are produced!"[2] In accordance with proper revenue recognition, accountants do not recognize revenue before it is earned.

CONCEPTS IN PRACTICE

Gift Card Revenue Recognition

Gift cards have become an essential part of revenue generation and growth for many businesses. Although they are practical for consumers and are a low cost for businesses, navigating revenue recognition guidelines can be difficult. Gift cards with expiration dates require that revenue recognition be delayed until customer use or expiration. However, most gift cards now have no expiration date. So when do you recognize revenue?

Companies may need to provide an estimation of projected (or deferred) gift card revenue and usage during a period based on past experience or industry standards. There are a few rules governing reporting. If a company determines that a portion of all the issued gift cards will never be used, it may write this off to income. In some states, if a gift card remains unused, in part or in full, the unused portion of the card is transferred to the state government. It is considered unclaimed property for the customer, meaning that the company cannot keep these funds as revenue because, in this case, they have reverted to the state government.

Expense Recognition

An expense is a cost associated with providing goods or services to customers. In our opening example, the expenses that Chris incurred totaled $1,150 (consisting of $100 for brakes, $50 for fuel, and $1,000 for insurance). You might think of expenses as the opposite of revenue, in that expenses reduce Chris's checking account balance. Likewise, expenses decrease the value of the business and represent the dollar value of costs incurred to provide goods and services to customers for a given period of time.

1 American Institute of Certified Public Accountants (AICPA). "Revenue Recognition." n.d. https://www.aicpa.org/interestareas/frc/accountingfinancialreporting/revenuerecognition.html
2 Len Jui and Jessie Wong. "Roles and Importance of Professional Accountants in Business." *China Accounting Journal.* October 21, 2013. https://www.ifac.org/news-events/2013-10/roles-and-importance-professional-accountants-business

THINK IT THROUGH

Coffee Shop Expenses

While thinking about or visiting a coffee shop in your area, look around (or visualize) and identify items or activities that are the expenses of the coffee shop. Remember, expenses for the coffee shop are related to resources consumed while generating revenue from selling coffee and related items. Do not forget about any expenses that might not be so obvious—as a general rule, every activity in a business has an associated cost.

Solution:

Costs of the coffee shop that might be readily observed would include rent, wages for the employees, and the cost of the coffee, pastries, and other items/merchandise that may be sold. In addition, costs such as utilities, equipment, and cleaning or other supplies might also be readily observable. More obscure costs of the coffee shop would include insurance, regulatory costs such as health department licensing, point-of-sale/credit card costs, advertising, donations, and payroll costs such as workers' compensation, unemployment, and so on. There are also unseen costs, such as aging of the building (if owned by the coffee shop) and wear and tear or aging of the equipment.

4.4 When Should a Company Capitalize or Expense an Item?

Learning Outcomes

By the end of this section, you will be able to:

- Define the key characteristics of a fixed asset.
- Explain how the cost of a fixed asset is spread throughout its useful life via depreciation.
- Assess the impact to net income of expensing versus capitalizing an item.

Assets are items a business owns. For accounting purposes, assets are categorized as current versus long term and tangible versus intangible. Any asset that is expected to be used by the business for more than one year is considered a **long-term asset**. These assets are not intended for resale and are anticipated to help generate revenue for the business in the future. Some common long-term assets are computers and other office machines, buildings, vehicles, software, computer code, and copyrights. Although these are all considered long-term assets, some are tangible and some are intangible.

To better understand the nature of fixed assets, let's get to know Liam and their new business. Liam is excited to be graduating from their MBA program and looks forward to having more time to pursue their business venture. During one of their courses, Liam came up with the business idea of creating trendy workout attire. For their class project, they started silk-screening vintage album cover designs onto tanks, tees, and yoga pants. They tested the market by selling their wares on campus and were surprised how quickly and how often they sold out. In fact, sales were high enough that they decided to go into business for themselves. One of their first decisions involved whether they should continue to pay someone else to silk-screen their designs or do their own silk-screening. To do their own silk-screening, they would need to invest in a silk screen machine.

Liam will need to analyze the purchase of a silk screen machine to determine the impact on their business in the short term as well as the long term, including the accounting implications related to the expense of this machine. Liam knows that over time, the value of the machine will decrease, but they also know that an asset is supposed to be recorded on the books at its historical cost. They also wonder what costs are considered part of this asset. Additionally, Liam has learned about the matching principle (**expense recognition**) but needs to learn how that relates to a machine that is purchased in one year and used for many years to help generate revenue. Liam has a lot of information to consider before making this decision.

What Is a Fixed Asset?

An asset is considered a **tangible asset** when it is an economic resource that has physical substance—it can be seen and touched. Tangible assets can be either short term, such as inventory and supplies, or long term, such as land, buildings, and equipment. To be considered a long-term tangible asset, the item needs to be used in the normal operation of the business for more than one year, be of material value, and not be near the end of its useful life, and the company must have no plan to sell the item in the near future. The useful life is the time period over which an asset cost is allocated. Long-term tangible assets are known as fixed assets. It's also key to note that companies will **capitalize** a fixed asset if they have material value. A $10 stapler to be used in the office, for example, may last for years, but the value of the item is not significant enough to warrant capitalizing it.

Businesses typically need many different types of these assets to meet their objectives. These assets differ from the company's products. For example, the computers that Apple, Inc. intends to sell are considered inventory (a short-term asset), whereas the computers Apple's employees use for day-to-day operations are long-term assets. In Liam's case, the new silk screen machine would be considered a long-term tangible asset as they plan to use it over many years to help generate revenue for their business. Long-term tangible assets are listed as noncurrent assets on a company's balance sheet. Typically, these assets are listed under the category of Property, Plant, and Equipment (PP&E), but they may be referred to as fixed assets or plant assets.

Apple, Inc. lists a total of $36.766 million in total Property, Plant, and Equipment (net) on its September 2020 consolidated balance sheet (see Figure 4.7). As shown in the figure, this net total includes land and buildings, machinery, equipment and internal-use software, and leasehold improvements, resulting in a gross PP&E of $103.526 million—less accumulated depreciation and amortization of $66.760 million—to arrive at the net amount of $36.766 million.

	2020	2019
Land and Buildings	$ 17,952	$ 17,085
Machinery, Equipment, and Internal-Use Software	75,291	69,797
Leasehold Improvements	10,283	9,075
Gross Property, Plant, and Equipment	**103,526**	**95,957**
Accumulated Depreciation and Amortization	(66,760)	(58,579)
Total Property, Plant, and Equipment, Net	**$ 36,766**	**$ 37,378**

Figure 4.7 Apple's Property, Plant, and Equipment, Net (September 2020, in $ million)[3] This report shows the company's consolidated financial statement details as of September 30, 2020, and September 30, 2019.

THINK IT THROUGH

Classifying Assets and Related Expenditures

You work at a business consulting firm. Your new colleague, Milan, is helping a client company organize its accounting records by types of assets and expenditures. Milan is a bit stumped on how to classify certain assets and related expenditures, such as capitalized costs versus expenses. They have given you the following list and asked for your help to sort through it. Help your colleague classify the expenditures as either capitalized or expensed, and note which assets are property, plant, and equipment.

Expenditures:

- Normal repair and maintenance on the manufacturing facility
- Cost of taxes on new equipment used in business operations
- Shipping costs on new equipment used in business operations
- Cost of a minor repair on existing equipment used in business operations

Assets:

- Land next to the production facility held for use next year as a place to build a warehouse
- Land held for future resale when the value increases
- Equipment used in the production process

Solution:

Expenditures:

- Normal repair and maintenance on the manufacturing facility: expensed
- Cost of taxes on new equipment used in business operations: capitalized
- Shipping costs on new equipment used in business operations: capitalized
- Cost of a minor repair on existing equipment used in business operations: expensed

Assets:

- Land next to the production facility held for use next year as a place to build a warehouse: property, plant, and equipment
- Land held for future resale when the value increases: investment
- Equipment used in the production process: property, plant, and equipment

Why are the costs of putting a long-term asset into service capitalized and written off as expenses (depreciated) over the economic life of the asset? Let's return to Liam's start-up business as an example. Liam plans to buy a silk screen machine to help create clothing that they will sell. The machine is a long-term asset because it will be used in the business's daily operation for many years. If the machine costs Liam $5,000 and it is expected to be used in their business for several years, GAAP require the allocation of the machine's costs over its useful life, which is the period over which it will produce revenues. Overall, in determining a company's financial performance, we would not expect that Liam should have an expense of $5,000 this year and $0 in expenses for this machine for future years in which it is being used. GAAP addressed this through the expense recognition (matching) principle, which states that expenses should be recorded in the same period with the revenues that the expense helped create. In Liam's case, the $5,000 for this machine should be allocated over the years in which it helps to generate revenue for the business. Capitalizing the machine allows this to occur. As stated previously, to capitalize is to record a long-term asset on the balance sheet and expense its allocated costs on the income statement over the asset's economic life. Therefore, when Liam purchases the machine, they will record it as an asset on the financial statements (see journal entry in Figure 4.8).

Journal			
Date	**Account**	**Debit**	**Credit**
Jan. 1	Machine	5,000	
	Cash		5,000

Figure 4.8 Journal Entry for Machine/Cash

When capitalizing an asset, the total cost of acquiring the asset is included in the cost of the asset. This includes additional costs beyond the purchase price, such as shipping costs, taxes, assembly, and legal fees. For example, if a real estate broker is paid $8,000 as part of a transaction to purchase land for $100,000, the land would be recorded at a cost of $108,000.

Over time, as the asset is used to generate revenue, Liam will need to depreciate recognize the cost of the asset.

3 In the Chapter 4 financial statements, a number contained within parentheses is a negative number, such as the "Accumulated depreciation and amortization" line item.

What Is Depreciation?

When a business purchases a long-term asset (used for more than one year), it classifies the asset based on whether the asset is used in the business's operations. If a long-term asset is used in the business's operations, it will belong in property, plant, and equipment or intangible assets. In this situation, the asset is typically capitalized. Capitalization is the process by which a long-term asset is recorded on the balance sheet and its allocated costs are expensed on the income statement over the asset's economic life.

Long-term assets that are not used in daily operations are typically classified as an investment. For example, if a business owns land on which it operates a store, warehouse, factory, or offices, the cost of that land would be included in property, plant, and equipment. However, if a business owns a vacant piece of land on which the business conducts no operations (and assuming no current or intermediate-term plans for development), the land would be considered an investment.

Depreciation is the process of allocating the cost of a tangible asset over its useful life, or the period of time that the business believes it will use the asset to help generate revenue.

Fundamentals of Depreciation

As you have learned, when accounting for a long-term fixed asset, we cannot simply record an expense for the cost of the asset and record the entire outflow of cash in one accounting period. Like all other assets, when you purchase or acquire a long-term asset, it must be recorded at the historical (initial) cost, which includes all costs to acquire the asset and put it into use. The initial recording of an asset has two steps:

1. Record the initial purchase on the date of purchase, which places the asset on the balance sheet (as property, plant, and equipment) at cost, and record the amount as notes payable, accounts payable, or an outflow of cash.
2. At the end of the period, make an adjusting entry to recognize the depreciation expense. Depreciation expense is the amount of the asset's cost to be recognized, or expensed, in the current period. Companies may record depreciation expense incurred annually, quarterly, or monthly.

Following GAAP and the expense recognition principle, the depreciation expense is recognized over the asset's estimated useful life.

Recording the Initial Asset

Assets are recorded on the balance sheet at cost, meaning that all costs to purchase the asset and to prepare the asset for operation should be included. Costs outside of the purchase price may include shipping, taxes, installation, and modifications to the asset.

The journal entry to record the purchase of a fixed asset (assuming that a note payable, not a short-term account payable, is used for financing) is shown in Figure 4.9.

Journal			
Date	**Account**	**Debit**	**Credit**
Jan. 1	Fixed Asset (truck, building, etc.) Cash/Notes Payable *To record purchase of fixed asset*	XXX	 XXX

Figure 4.9 Journal Entry for Fixed Asset

Applying this to Liam's silk-screening business, we learn that they purchased their silk screen machine for $54,000 by paying $10,000 cash and the remainder in a note payable over five years. The journal entry to record the purchase is shown in Figure 4.10.

Journal			
Date	**Account**	**Debit**	**Credit**
Jan. 1	Equipment	54,000	
	Cash		10,000
	Notes Payable		44,000
	To recognize purchase of silk-screening machine		

Figure 4.10 Journal Entry for Equipment/Cash

CONCEPTS IN PRACTICE

Estimating Useful Life and Salvage Value

Useful life and salvage value are estimates made at the time an asset is placed in service. It is common and expected that the estimates are inaccurate due to the uncertainty involved in estimating the future. Sometimes, however, a company may attempt to take advantage of estimating salvage value and useful life to improve earnings. A larger salvage value and longer useful life decrease annual depreciation expense and increase annual net income. An example of this behavior is Waste Management, which was disciplined by the SEC in March 2002 for fraudulently altering its estimates to reduce depreciation expense and overstate net income by $1.7 billion.[4]

Components Used in Calculating Depreciation

The expense recognition principle that requires that the cost of the asset be allocated over the asset's useful life is the process of depreciation. For example, if we buy a delivery truck to use for the next five years, we would allocate the cost and record depreciation expense across the entire five-year period. The calculation of the depreciation expense for a period is not based on anticipated changes in the fair-market value of the asset; instead, the depreciation is based on the allocation of the cost of owning the asset over the period of its useful life.

The following items are important in determining and recording depreciation:

- Book value: the asset's original cost less accumulated depreciation.
- Useful life: the length of time the asset will be productively used within operations.
- Salvage (residual) value: the price the asset will sell for or be worth as a trade-in when its useful life expires. The determination of salvage value can be an inexact science since it requires anticipating what will occur in the future. Often, the salvage value is estimated based on past experiences with similar assets.
- Depreciable base (cost): the depreciation expense over the asset's useful life. For example, if we paid $50,000 for an asset and anticipate a salvage value of $10,000, the depreciable base is $40,000. We expect $40,000 in depreciation over the time period in which the asset was used, and then it would be sold for $10,000.

Depreciation records an expense for the value of an asset consumed and removes that portion of the asset from the balance sheet. The journal entry to record depreciation is shown in Figure 4.11.

4 United States Securities and Exchange Commission. "Waste Management, Inc. Founder and Five Other Former Top Officers Sued for Massive Earnings Management Fraud." March 26, 2002. https://www.sec.gov/litigation/litreleases/lr17435.htm

Journal			
Date	**Account**	**Debit**	**Credit**
Jan. 1	Depreciation Expense	XXX	
	Accumulated Depreciation		XXX
	To record depreciation on asset for period		

Figure 4.11 Journal Entry for Depreciation Expense

Depreciation expense is a common operating expense that appears on an income statement. It represents the amount of expense being recognized in the current period. Accumulated depreciation, on the other hand, represents the sum of all depreciation expense recognized to date, or the total of all prior depreciation expense for the asset. It is a contra account, meaning it is attached to another account and is used to offset the main account balance that records the total depreciation expense for a fixed asset over its life. In this case, the asset account stays recorded at the historical value but is offset on the balance sheet by accumulated depreciation. Accumulated depreciation is subtracted from the historical cost of the asset on the balance sheet to show the asset at book value. Book value is the amount of the asset that has not been allocated to expense through depreciation.

It is important to note, however, that not all long-term assets are depreciated. For example, land is not depreciated because depreciation is the allocating of the expense of an asset over its useful life. How can one determine a useful life for land? It is assumed that land has an unlimited useful life; therefore, it is not depreciated, and it remains on the books at historical cost.

Once it is determined that depreciation should be accounted for, there are three methods that are most commonly used to calculate the allocation of depreciation expense: the *straight-line method*, the *units-of-production method*, and the *double-declining-balance method*. A fourth method, the *sum-of-the-years-digits* method, is another *accelerated* option that has been losing popularity and can be learned in intermediate accounting courses. Note that these methods are for accounting and reporting purposes. The IRS allows firms to use the same or different methods to depreciate assets in calculating taxable income.

THINK IT THROUGH

Fixed Assets

You work for Georgia-Pacific as an accountant in charge of the fixed assets subsidiary ledger at a production and warehouse facility in Pennsylvania. The facility is in the process of updating and replacing several asset categories, including warehouse storage units, fork trucks, and equipment on the production line. It is your job to keep the information in the fixed assets subsidiary ledger up to date and accurate. You need information on original historical cost, estimated useful life, salvage value, depreciation methods, and additional capital expenditures. You are excited about the new purchases and upgrades to the facility and how they will help the company serve its customers better. However, you have been in your current position for only a few years and have never overseen extensive updates, and you realize that you will have to gather a lot of information at once to keep the accounting records accurate. You feel overwhelmed and take a minute to catch your breath and think through what you need. After a few minutes, you realize that you have many people and many resources to work with to tackle this project. Whom will you work with, and how will you go about gathering what you need?

Solution:

Though answers may vary, common resources would likely include purchasing managers (those actually buying the new equipment), maintenance managers (those who will repair and take care of the new equipment), and line managers (those in charge of the departments that will use the new equipment). To

gather the information needed, set up short meetings to visit with the individuals involved, walk around to see the equipment, and ask questions about functionality, life span, common problems or repairs, and more.

Assume that on January 1, Liam bought a silk screen machine for $54,000. Liam pays shipping costs of $1,500 and setup costs of $2,500 and assumes a useful life of five years or 960,000 prints. Based on experience, Liam anticipates a salvage value of $10,000. Recall that determination of the costs to be depreciated requires including all costs that prepare the asset for use by the company. Liam's example would include shipping and setup costs. Any costs for maintaining or repairing the equipment would be treated as regular expenses, so the total cost would be $58,000, and after allowing for an anticipated salvage value of $10,000 in five years, the business could take $48,000 in depreciation over the machine's economic life (see Figure 4.12).

	Total Cost
Purchase Price	$ 54,000
Shipping Costs	1,500
Set-up Costs	2,500
Total Cost	$ 58,000
– Salvage Value	(10,000)
Depreciable Base	$ 48,000

Figure 4.12 Purchase Price and Depreciable Base

Straight-line depreciation is a method of depreciation that evenly splits the depreciable amount across the useful life of the asset. Therefore, we must determine the yearly depreciation expense by dividing the depreciable base of $48,000 by the economic life of five years, giving an annual depreciation expense of $9,600. The journal entries to record the first two years of expenses are shown, along with the balance sheet information. Here are the journal entry and information for year one (Figure 4.13):

Journal			
Date	**Account**	**Debit**	**Credit**
Dec. 31	Depreciation Expense: Silk Screen Machine	9,600	
	Accumulated Depreciation: Silk Screen Machine		9,600
	To record depreciation on asset for period		

Silk Screen Machine	$ 58,000
– Accumulated Depreciation: Silk Screen Machine	(9,600)
Net Book Value	$ 48,400

Figure 4.13 Journal Entry for Silk Screen Machine Depreciation Expense, Year 1

After the journal entry in year one, the machine would have a book value of $48,400. This is the original cost of $58,000 less the accumulated depreciation of $9,600. The journal entry and information for year two are shown in Figure 4.14.

Journal			
Date	**Account**	**Debit**	**Credit**
Dec. 31	Depreciation Expense: Silk Screen Machine	9,600	
	Accumulated Depreciation: Silk Screen Machine		9,600
	To record depreciation on asset for period		

Silk Screen Machine	$ 58,000
– Accumulated Depreciation: Silk Screen Machine	(19,200)
Net Book Value	$ 38,800

Figure 4.14 Journal Entry for Silk Screen Machine Depreciation Expense, Year 2

Liam records an annual depreciation expense of $9,600. Each year, the accumulated depreciation balance increases by $9,600, and the machine's book value decreases by the same $9,600. At the end of five years, the asset will have a book value of $10,000, which is calculated by subtracting the accumulated depreciation of $\$48{,}000\ (5 \times \$9{,}600)$ from the cost of $58,000.

Units-of-Production Depreciation

Straight-line depreciation is efficient accounting for assets used consistently over their lifetime, but what about assets that are used with less regularity? The units-of-production depreciation method bases depreciation on the actual usage of the asset, which is more appropriate when an asset's life is a function of usage instead of time. For example, this method could account for depreciation of a silk screen machine for which the depreciable base is $48,000 (as in the straight-line method), but now the number of prints is important.

In our example, the machine will have total depreciation of $48,000 over its useful life of 960,000 prints. Therefore, we would divide $48,000 by 960,000 prints to get a cost per print of $0.05. If Liam printed 180,000 items in the first year, the depreciation expense would be $180{,}000 \text{ prints} \times \$0.05 \text{ per print}$, or $\$9{,}000$. The journal entry to record this expense would be the same as with straight-line depreciation: only the dollar amount would have changed. The presentation of accumulated depreciation and the calculation of the book value would also be the same. Liam would continue to depreciate the asset until a total of $48,000 in depreciation was taken after running 960,000 total prints.

THINK IT THROUGH

Deciding on a Depreciation Method

Liam is struggling to determine which deprecation method they should use for their new silk screen machine. They expect sales to increase over the next five years. They also expect (hope) that in two years they will need to buy a second silk screen machine to keep up with the demand for products of their growing company. Which depreciation method makes more sense for Liam: higher expenses in the first few years or keeping expenses consistent over time? Or would it be better for them to think not in terms of time, but rather in the usage of the machine?

Double-Declining-Balance Depreciation

The double-declining-balance depreciation method is the most complex of the three methods because it accounts for both time and usage and takes more expense in the first few years of the asset's life. Double declining considers time by determining the percentage of depreciation expense that would exist under straight-line depreciation. To calculate this, divide 100 percent by the estimated life in years. For example, a five-year asset would be 100/5, or 20 percent a year. A four-year asset would be 100/4, or 25 percent a year. Next, because assets are typically more efficient and are used more heavily early in their life span, the double-

declining method takes usage into account by doubling the straight-line percentage. For a four-year asset, multiply 25 percent $(100\%/4\text{-year life}) \times 2$, or 50 percent. For a five-year asset, multiply 20 percent $(100\%/5\text{-year life}) \times 2$, or 40 percent.

One unique feature of the double-declining-balance method is that in the first year, the estimated salvage value is not subtracted from the total asset cost before calculating the first year's depreciation expense. Instead, the total cost is multiplied by the calculated percentage. However, depreciation expense is not permitted to take the book value below the estimated salvage value, as demonstrated in Figure 4.15.

Year	Depreciation Expense	Accumulated Depreciation	Book Value
			$58,000
1: $58,000 × 40% =	$23,200	$23,200	34,800
2: $34,800 × 40% =	13,920	37,120	20,880
3: $20,880 × 40% =	8,352	45,472	12,528
4: $12,528 − $10,000 =	2,528	48,000	10,000
5	0	48,000	10,000
Total	$48,000	$48,000	$10,000

Figure 4.15 Depreciation Expense, Accumulated Depreciation, and Book Value, Years 1–5

Notice that in year four, the remaining book value of $12,528 was not multiplied by 40 percent. This is because the expense would have been $5,011.20, and since we cannot depreciate the asset below the estimated salvage value of $10,000, the expense cannot exceed $2,528, which is the amount left to depreciate (difference between the book value of $12,528 and the salvage value of $10,000). Since the asset has been depreciated to its salvage value at the end of year four, no depreciation can be taken in year five.

In our example, the first year's double-declining-balance depreciation expense would be $\$58{,}000 \times 40\%$, or $\$23{,}200$. For the remaining years, the double-declining percentage is multiplied by the remaining book value of the asset. Liam would continue to depreciate the asset until the book value and the estimated salvage value are the same (in this case, $10,000). The net effect of the differences in straight-line depreciation versus double-declining-balance depreciation is that under the double-declining-balance method, the allowable depreciation expenses are greater in the earlier years than those allowed for straight-line depreciation. However, over the depreciable life of the asset, the total depreciation expense taken will be the same no matter which method the entity chooses. In the current example, both straight-line and double-declining-balance depreciation will provide a total depreciation expense of $48,000 over its five-year depreciable life.

LINK TO LEARNING

McDonald's Corporation

See Form 10-K (https://openstax.org/r/2019-annual-report-pdf) that was filed with the SEC to determine which depreciation method McDonald's Corporation used for its long-term assets in 2019.

Hint: Use the search feature to search for key words in an annual report to find information more quickly. For example, search "depreciation." You should find the information you are looking for on page 40 of the annual report.

Based on the company's business model and the industry in which it operates, why do you think McDonald's chose this method of depreciating assets? Do you agree with this choice? Why or why not?

Summary of Depreciation

Table 4.2 and Figure 4.16 compare the three methods discussed. Note that although each time-based (straight-line and double-declining balance) annual depreciation expense is different, after five years the total amount depreciated (accumulated depreciation) is the same. This occurs because at the end of the asset's useful life, it was expected to be worth $10,000: thus, both methods depreciated the asset's value by $48,000 over that time period.

Depreciation Method	Calculation
Straight line	(Cost − Salvage Value)/Useful Life
Units of production	(Cost − Salvage Value) × (Units Produced in Current Period /Estimated Total Units to Be Produced)
Double-declining balance	Book Value × Straight − Line Annual Depreciation Percentage × 2

Table 4.2 Three Methods of Calculating Depreciation Expense

Period	Straight-Line Depreciation Method	Units of Production Method		Double-Declining-Balance Method
Year 1	$ 9,600	(180,000 units)	$ 9,000	$23,200
Year 2	9,600	(200,000 units)	10,000	13,920
Year 3	9,600	(210,000 units)	10,500	8,352
Year 4	9,600	(190,000 units)	9,500	2,528
Year 5	9,600	(180,000 units)	9,000	0
Total	$48,000		$48,000	$48,000

Figure 4.16 Straight-Line Depreciation, Units of Production, Double-Declining Balance Method, Years 1–5

When analyzing depreciation, accountants are required to make a supportable estimate of an asset's useful life and its salvage value. However, "management teams typically fail to invest either time or attention into making or periodically revisiting and revising reasonably supportable estimates of asset lives or salvage values, or the selection of depreciation methods, as prescribed by GAAP."[5] This failure is not an ethical approach to properly accounting for the use of assets.

Accountants need to analyze depreciation of an asset over the entire useful life of the asset. As an asset supports the cash flow of the organization, expensing its cost needs to be allocated, not just recorded as an arbitrary calculation. An asset's depreciation may change over its life according to its use. If asset depreciation is arbitrarily determined, the recorded "gains or losses on the disposition of depreciable property assets seen in financial statements"[6] are not true best estimates. Due to operational changes, the depreciation expense needs to be periodically reevaluated and adjusted.

Any mischaracterization of asset usage is not proper GAAP and is not proper accrual accounting. Therefore, "financial statement preparers, as well as their accountants and auditors, should pay more attention to the quality of depreciation-related estimates and their possible mischaracterization and losses of credits and charges to operations as disposal gains."[7] An accountant should always follow GAAP guidelines and allocate the expense of an asset according to its usage.

5 Howard B. Levy. "Depreciable Asset Lives: The Forgotten Estimate in GAAP." *The CPA Journal.* September 2016. cpajournal.com/2016/09/08/depreciable-asset-lives/
6 Ibid.

CONCEPTS IN PRACTICE

Ethical Considerations: How WorldCom's Improper Capitalization of Costs Almost Shut Down the Internet

In 2002, telecommunications giant WorldCom filed for the largest Chapter 11 bankruptcy to date, a situation resulting from manipulation of its accounting records.[8] At the time, WorldCom operated nearly a third of the bandwidth of the 20 largest US internet backbone routes, connecting over 3,400 global networks that serviced more than 70,000 businesses in 114 countries.[9]

WorldCom used a number of accounting gimmicks to defraud investors, mainly including capitalizing costs that should have been expensed. Under normal circumstances, this might have been considered just another accounting fiasco leading to the end of a company. However, WorldCom controlled a large percentage of backbone routes, a major component of the hardware supporting the internet, as even the Securities and Exchange Commission recognized. If such an event were to happen today, it could shut down international commerce and would be considered a national emergency.[10] As demonstrated by WorldCom, the unethical behavior of a few accountants could have shut down the world's online businesses and international commerce. An accountant's job is fundamental and important: keep businesses operating in a transparent fashion.

(Sources: "WorldCom (UNNET)." *Cybertelecom*. n.d. http://www.cybertelecom.org/industry/wcom.htm; Dennis R. Beresford, Nicholas DeB. Katzenbach, and C. B. Rogers, Jr. "Report of the Special Investigative Committee of the Board of Directors of WorldCom, Inc." US Securities and Exchange Commission. March 31, 2003. https://www.sec.gov/Archives/edgar/data/723527/000093176303001862/dex991.htm)

4.5 What Is "Profit" versus "Loss" for the Company?

Learning Outcomes

By the end of this section, you will be able to:

- Outline the components necessary to calculate profit or loss.
- Contrast revenue and gains versus expenses and losses.
- Differentiate revenue and expense versus receipt or payment of cash.

It's a common misconception that if a business has cash they are making a profit, and if they are suffering a loss they must not have any cash. While this could be true, it's not necessarily true. In this section you'll explore key differences between cash flow, revenue, expense, profits, and losses.

Calculating Profit and Loss

Net income (net loss) is determined by comparing revenues and expenses. Net income is a result of revenues (inflows) being greater than expenses (outflows). A net loss occurs when expenses (outflows) are greater than revenues (inflows). In accounting it is common to present net income in the following format:

Recall that revenue is the value of goods and services a business provides to its customers and increases the value of the business. Expenses, on the other hand, are the costs of providing the goods and services and decrease the value of the business. When revenues exceed expenses, companies have net income. This means the business has been successful at earning revenues, containing expenses, or a combination of both. If, on the other hand, expenses exceed revenues, companies experience a net loss. This means the business was

7 Ibid.
8 Luisa Beltran. "WorldCom Files Largest Bankruptcy Ever." *CNN Money*. July 22, 2002. https://money.cnn.com/2002/07/19/news/worldcom_bankruptcy/
9 Jeff Keefe. *Monopoly.com: Will the WorldCom–MCI Merger Tangle the Web*? Washington, DC: Economic Policy Institute, 1998. https://www.epi.org/publication/monopoly-will-the-worldcom-mci-merger-tangle-the-web/
10 Dan Schiller. *Bad Deal of the Century: The Worrisome Implications of the WorldCom–MCI Merger*. Washington, DC: Economic Policy Institute, 1998. https://www.epi.org/publication/studies_baddealfull/

unsuccessful in earning adequate revenues, sufficiently containing expenses, or a combination of both. While businesses work hard to avoid net loss situations, it is not uncommon for a company to sustain a net loss from time to time. It is difficult, however, for businesses to remain viable while experiencing net losses over the long term.

Shown as a formula, the net income (loss) function is:

$$\text{Revenues } (R) - \text{Expenses } (E) = \text{Net Income (when } R > E)$$
$$\text{Revenues } (R) - \text{Expenses } (E) = \text{Net Loss (when } E > R)$$

To be complete, we must also consider the impact of gains and losses. While gains and losses are infrequent in a business, it is not uncommon that a business would present a gain and/or loss in its financial statements. Recall that gains are similar to revenue and losses are similar to expenses. Therefore, the traditional accounting format would be as shown in Figure 4.17.

Gains and Losses
Revenue (sometimes called Sales or Fees Earned) + Gains – Expenses – Losses Net Income (or Net Loss)

Figure 4.17 Gains and Losses

Shown as a formula, the net income (loss) function, including gains and losses, is:

$$\text{Revenues } (R) + \text{Gains } (G) - \text{Expenses } (E) - \text{Losses } (L) = \text{Net Income } [\text{when } (R + G) > (E + L)]$$
$$\text{Revenues } (R) + \text{Gains } (G) - \text{Expenses } (E) - \text{Losses } (L) = \text{Net Loss } [\text{when } (E + L) > (R + G)]$$

When assessing a company's net income, it is important to understand the source of the net income. Businesses strive to attain "high quality" net income (earnings). High-quality earnings are based on sustainable earnings, also called permanent earnings, while relying less on infrequent earnings, also called temporary earnings. Recall that revenues represent the ongoing value of goods and services the business provides (sells) to its customers, while gains are infrequent and involve items ancillary to the primary purpose of the business. For example, assume a bakery sells the truck it uses to deliver wedding cakes and experiences a gain on the sale. The bakery is not in the business of buying and selling trucks. It is in the baked goods business. Thus, the gain on the sale of the truck would be ancillary to the primary purpose of the business and represent a gain rather than revenue. We should use caution if a business attains a significant portion of its net income as a result of gains rather than revenues. Likewise, net losses derived as a result of losses should be put into the proper perspective due to the infrequent nature of losses. While net losses are undesirable for any reason, net losses that result from expenses related to ongoing operations, rather than losses that are infrequent, are more concerning for the business.

Profit versus Cash Flow

Knowing the difference between the cash basis and accrual basis of accounting is necessary to understand the need for the statement of cash flows. Stakeholders need to know the financial performance (as measured by the income statement—that is, net income or net loss) and financial position (as measured by the balance sheet—that is, assets, liabilities, and owners' equity) of the business. This information is provided in the income statement, statement of owner's equity, and balance sheet. However, since these financial statements are prepared using accrual accounting, stakeholders do not have a clear picture of the business's cash activities. The statement of cash flows solves this inadequacy by specifically focusing on the cash inflows and cash outflows. It also helps better delineate the difference between revenues and cash flow in versus expenses and cash flow out. As mentioned in prior sections, revenue can occur without cash actually flowing. For

example, a customer may buy a good on account. Revenues would be recorded, but cash would not yet be received. The same is true on the expenses side. An expense can be incurred, such as an electric bill, but cash may not have been paid out yet. Thus, an expense is recorded and recognized on the income statement, but cash has not yet been given up. The statement of cash flows helps reconcile the difference between net income (a result of recorded revenues and expenses) and actual cash flow.

Summary

4.1 Cash versus Accrual Accounting

Cash-basis accounting records revenues and expenses only when cash is received or distributed. Accrual-basis accounting, on the other hand, records revenues and expenses when they are earned or incurred rather than waiting until cash changes hands. Most publicly traded companies are required by the SEC to use accrual-basis accounting. Generally, smaller businesses that deal primarily in cash are the best candidates for using the cash basis since the accrual method more accurately depicts net income or net loss each period.

4.2 Economic Basis for Accrual Accounting

Double-entry accounting means that each time a transaction is recorded, there are at minimum two accounts impacted by the entry. Each entry's debits must total its credits in order to support and maintain the balance in the accounting equation. The accounting equation is expressed as

$$\text{Assets} = \text{Liabilities} + \text{Owner's Equity}.$$

4.3 How Does a Company Recognize a Sale and an Expense?

In accrual accounting, the timing of revenues (when to record them) is governed by the revenue recognition principle. The principle indicates that revenue is only recognized on the income statement once it is earned. This means goods or services must have been delivered or rendered. Expenses, on the other hand, are guided by the expense recognition principle, which dictates that expenses must be recorded in the period in which they are incurred.

4.4 When Should a Company Capitalize or Expense an Item?

Determining when an item is actually an expense depends on whether it is capitalized (a fixed asset) or not. Fixed assets are those that are of significant value and last longer than one year. The cost of fixed assets is capitalized (placed on the balance sheet as an asset) and expensed over the useful life of the asset by recording depreciation. Depreciation can be calculated using straight-line, double-declining-balance, or units-of-production methods.

4.5 What Is "Profit" versus "Loss" for the Company?

Profit or loss for a company is calculated by subtracting expenses from revenue. The result is a profit if revenues are larger than expenses. Profit (or loss) is the money earned from the day-to-day general business operations. Gains and losses, on the other hand, occur when the business does something they don't normally do (like sell a piece of their equipment) and earns or loses money on the transaction. It's key to note that the timing of cash flows can vary from the timing of recording revenues or expenses. Thus a revenue does not necessarily equal cash in, and an expense does not necessarily equal a cash flow out.

Key Terms

accounting equation $\text{Assets} = \text{Liabilities} + \text{Owner's Equity}$

accrual-basis accounting an accounting system in which revenue is recorded or recognized when earned yet not necessarily received, and in which expenses are recorded when legally incurred and not necessarily when paid

capitalize the process in which a long-term asset is recorded on the balance sheet and its allocated costs are expensed on the income statement over the asset's economic life

cash-basis accounting a method of accounting in which transactions are not recorded in the financial statements until there is an exchange of cash

credit a record of financial information on the right side of an account

debit a record of financial information on the left side of each account

depreciation the process of allocating the costs of a tangible asset over the asset's economic life
double-entry accounting an accounting method that requires the sum of the debits to equal the sum of the credits for each transaction
expense a cost associated with providing goods or services
expense recognition (also, matching principle) matches expenses with associated revenues in the period in which the revenues were generated
gains increases in organizational value from activities that are "incidental or peripheral" to the primary purpose of the business
income statement a financial statement that measures the organization's financial performance for a given period of time
long-term asset asset used in the normal, ongoing course of business for more than one year that is not intended to be resold
losses decreases in organizational value from activities that are "incidental or peripheral" to the primary purpose of the business
net income income earned when revenues and gains are greater than expenses and losses
revenue inflows or other enhancements of assets of an entity or settlements of its liabilities (or a combination of both) from delivering or producing goods, rendering services, or other activities that constitute the entity's ongoing major or central operations
revenue recognition principle stating that a company must recognize revenue in the period in which it is earned; it is not considered earned until a product or service has been provided
T-account a graphic representation of a general ledger account in which each account is visually split into left and right sides
tangible asset an asset that has physical substance

Multiple Choice

1. Which method of accounting must be used by publicly traded companies?
   a. cash basis
   b. accrual basis
   c. a hybrid of cash and accrual basis
   d. modified accelerated basis

2. Which two accounting principles directly support the accrual method of accounting?
   a. periodicity, transparency
   b. historical cost, time period
   c. conservativism, going concern
   d. expense recognition, revenue recognition

3. Which of the following account types has a normal debit balance?
   a. cash
   b. notes payable
   c. wages payable
   d. revenue

4. Sara sells $100 worth of inventory to her client on credit on January 15. She delivers the inventory to the client on January 20. The client pays for the inventory of February 26. On what date should Sara recognize the revenue from the sale?
   a. January 15
   b. January 20
   c. January 31

d. February 26

5. How is the book value of a fixed asset calculated?
   a. original cost less accumulated depreciation
   b. original cost divided by useful life
   c. market value less depreciation expense
   d. depreciable base less market value

6. Which of the following balance sheet item is classified as a fixed asset?
   a. inventory
   b. goodwill
   c. equipment
   d. accounts receivable

7. What is the formula to determine net income or net loss?
   a. revenues less expenses
   b. cash less expenses
   c. revenues plus cash
   d. revenues divided by expenses

Review Questions

1. Joe runs a small barbershop. Most of his customers pay with cash. He has only a few monthly expenses including wages for one employee and utilities. Which method of accounting should Joe use?
2. Which method of accounting generally provides the most accurate information on organizational performance (income or loss)?
3. Under the accrual basis of accounting, when is an expense recorded?
4. Describe the basic elements of a T-account.
5. What is double-entry accounting?
6. What is the expense recognition principle?
7. Why is ethics an important concept pertaining to revenue and expense recognition principles?
8. What is the revenue recognition principle?
9. What is the difference between a tangible and an intangible fixed asset? List common examples of each.
10. How is the cost of a fixed asset recorded and recognized over time?

Problems

1. Padma's Pools Inc. paid $500 for office supplies. What was the impact of this transaction on Padma's cash flow?
2. Sally's BigBox Store issued 1,000 shares of common stock with par value of $10 each and market value of $16 each in exchange for a new building. What was the impact of this transaction on Sally's cash flow?
3. Jose sells $500 worth of merchandise to a client on June 1. He delivers the product and invoices the customer on June 10. The customer pays Jose on July 9. What is Jose's revenue for June and July, respectively, under the cash and accrual methods of accounting?

4. Jamal's Car Repair purchases a new piece of equipment with a 10-year useful life for $10,000. What is the impact to Jamal's net income in the year of purchase if he expenses the equipment? If he capitalizes it using straight-line depreciation?
5. Mariela's Shop had revenues of $10,000 and expenses of $6,000 and has cash on hand of $5,000. What is Mariela's net income or net loss?

Video Activity

Depreciation Basics! With Journal Entries

Click to view content (https://openstax.org/r/depreciation-basics)

1. In what ways do depreciation and capitalization impact the income statement? If you were an investor considering investing a large sum of money into a company, what questions would you ask or what accounts would you look at to assess the impact to their income statement specifically? How would the characteristics of capitalization affect your feelings toward investing in the company?
2. If a company spends a large sum of cash to invest in equipment or another fixed asset, resulting in a loss that year on the income statement if they expense it, have they really experienced a loss? Or have they simply traded one asset for another? How does this delineation relate to capitalizing an asset and spreading out the cost throughout its useful life? Do you feel the depreciation process is ethical? Or do you feel it hides the true cost of business from being fully transparent on the financial statements? Explain your answer.

Difference between Cash Flow and Profit

Click to view content (https://openstax.org/r/difference-cashflow-profit)

3. When a business incurs an expense (office supplies, utilities, or wages, for example), is their cash flow the same in both timing and amount as the expense they recognize?
4. What are the key components necessary to calculate profit or loss for a business? Are they the same elements necessary to calculate cash flow?

5

Financial Statements

Figure 5.1 Financial statements are needed to understand a firm's financial position and performance. (credit: modification of work "Drowning by Numbers" by Jorge Franganillo/flickr, CC BY 2.0)

Chapter Outline

Why It Matters

People say that accounting is the "language of business." Using the language of business, accountants are able to communicate the financial performance and health of a firm via four key financial statements. These statements are the income statement, balance sheet, statement of owner's equity, and statement of cash flows. Each statement provides different insights into a firm's performance and financial health. Though some users may favor one or two statements over another, they are best used together to get a full picture.

In this chapter, you'll be introduced to a firm called Clear Lake Sporting Goods. Clear Lake Sporting Goods is a small merchandising company (a company that buys finished goods and sells them to consumers) that sells hunting and fishing gear. It needs financial statements to understand its profitability and current financial position, manage cash flow, and communicate its finances to outside parties like investors, governing bodies, and lenders. We will walk through each financial statement, its components, how they are connected, and how financial statement users understand each one.

5.1 The Income Statement

Learning Outcomes

By the end of this section, you will be able to:

- Outline the purpose and importance of the income statement.
- Identify the structure and key elements of the income statement.
- Discuss the use of EBITDA as a measure of a company's profit.

Financial information flows from one financial statement to the next. Thus, the statements are prepared in a specific order. The first statement prepared is the income statement.

Functionality of the Income Statement

The **income statement** shows a firm's performance over a specific period of time. The statement helps financial statement users understand the sales generated during the period and the **expenses** incurred to generate those sales. If the expenses are smaller than the sales, the net result is profitability, or net income, rather than a net **loss**.

Breaking the income statement down into smaller pieces provides a more transparent view of the firm's performance, allowing users to see more clearly what areas of the business incurred expenses. This is helpful to management in driving improvements and to outside users in assessing performance.

Sales and Gross Profit

The first section of the income statement begins with sales. Though financial statements are required to follow a certain format, account names can differ slightly from one firm to another. You may see the first line, often referred to as *the top line*, called *sales*, *sales revenue*, ***revenue***, *service revenues*, and other similar titles. All of these titles are meant to reflect the sales generated by selling product to customers in the day-to-day business. On Clear Lake's income statement in Figure 5.2, we see its top line referred to as Sales.

Income from items that aren't common to the firm's day-to-day business are reported as **gains** and losses, and they are reported further down in the income statement rather than at the top line with its regular, core business activities. This is to ensure that anomalies like selling a machine or a loss on retiring a bond don't mislead financial statement users as to the general performance of the firm and impact their assumptions of future results.

Firms report their sales and any reductions to sales separately on the income statement. They begin with gross sales, which includes all sales to customers. Clear Lake reported gross sales of $105,000 last year and $126,000 this year. The next line is sales returns and allowances, which is deducted from gross sales in order to find net sales. Clear Lake's sales returns and allowances were $5,000 and $6,000 respectively, leaving the company with net sales of $100,000 and $120,000 respectively

$$(\$105{,}000 - \$5{,}000 \text{ and } \$126{,}000 - \$6{,}000)\,.$$

Next, the cost of goods sold (COGS) is deducted from net sales in order to arrive at gross profit. (It is customary to refer to sales minus COGS as gross profit because gross margin = gross profit/sales.) Cost of goods sold includes the costs directly involved in making the product that was sold during the period. Common examples of costs included in cost of goods sold include direct labor, direct materials, and the overhead assigned to the product in production. For a service business, this would include its direct labor and any materials used to deliver its services. For a retail firm like Clear Lake Sporting Goods, this would include the costs of all the goods it purchased for resale. Clear Lake's COGS is seen at $50,000 and $60,000 for the prior and current years. Note that different types of companies will have different types of costs deducted in their Cost of Goods section. Clear Lake Sporting Goods is a retailer, or merchandiser that buys good to resell. Their cost of goods includes the cost of goods they purchased to resell. In the link to learning, you will explore Apple, a technology manufacturer. Their cost of goods would include the cost to manufacture the goods they

sell. Another type of firm is a service firm. A law office, for example, would include primarily the cost of labor in their cost of services.

Gross profit is a reflection of how profitable the firm's performance was in its core business function. It includes only the core business and direct costs of performing that business. If the company were a shoe company, gross profit would show how profitable the company was in simply making the shoes it sold. If it were a bakery, gross profit would show how profitable the company was in simply baking the goods it sold. Gross profit shows financial statement users how effective the business is at generating top-line profits on their core business function. It does not reflect the performance of other areas of the firm such as other operating costs to support the direct production process, indirect costs, and financing.

For Clear Lake Sporting Goods, we see its gross profit in Figure 5.2. The company earned $50,000 of gross profit ($\$100{,}000 - \$50{,}000$) the prior year and $60,000 in the current year ($\$120{,}000 - \$60{,}000$).

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Current Year
Gross Sales	$105,000	$126,000
Sales Returns & Allowances	5,000	6,000
Net Sales	100,000	120,000
Cost of Goods Sold	50,000	60,000
Gross Profit	$ 50,000	$ 60,000

Figure 5.2 Income Statement through Gross Profit Line

LINK TO LEARNING

Gross Profit

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate the income statement (it begins on page 31). Review gross sales, sales returns and allowances, and net sales for the last few years. What can you learn about the company's recent performance in the area of sales? Is the gross sales line improving? How about the sales returns and allowances line? Has it improved or declined or simply changed incrementally with gross sales?

Review the firm's gross profit for the past few years. Has it improved or declined? Consider the company's improvement (or decline) in gross sales as compared to the improvement or decline in gross profit. Does it reflect any change in performance over time (earning more or less gross profit on its gross sales)?

Income from Operations

Gross profit is a very helpful measure, but it is only the first of several provided by the income statement. After gross profit is calculated, other operating expenses are deducted in order to calculate the firm's income from operations, also commonly called operating income. Common operating costs found in this section include building rent and utilities, property taxes, wages and salaries, and other overhead costs. In Figure 5.3, we can see Clear Lake's operating expenses. To sell its hunting and fishing equipment in the current year, Clear Lake Sporting Goods paid rent for its building ($5,500) and utilities for its retail and warehouse spaces ($2,500); recorded **depreciation** on equipment, buildings, and store furnishings (shelves, racks, etc.) ($3,600); and paid salaries to its indirect employees in accounting, purchasing, and human resources ($5,400). The company's operating expenses are deducted from gross profit to arrive at operating income

$$(\$60{,}000 - 5{,}500 - 3{,}600 - 5{,}400 - 2{,}500 = \$43{,}000)\,.$$

While gross profit reflects only how profitable the firm was in making its core product, operating income

reflects how profitable the firm's daily operations were as a whole. This still does not include other miscellaneous items outside the scope of a firm's normal business. Just as the name implies, it shows income from the core operations of the firm.

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Current Year
Gross Sales	$105,000	$126,000
Sales Returns & Allowances	5,000	6,000
Net Sales	100,000	120,000
Cost of Goods Sold	50,000	60,000
Gross Profit	50,000	60,000
Rent Expense	5,000	5,500
Depreciation Expense	2,500	3,600
Salaries Expense	3,000	5,400
Utility Expense	1,500	2,500
Operating Income	$ 38,000	$ 43,000

Figure 5.3 Income Statement through Income from Operations

We can see that the company was able to generate $\$20{,}000$ ($\$120{,}000 - \$100{,}000$) more in net sales in the current year than the prior year. However, it only generated $\$10{,}000$ ($\$60{,}000 - \$50{,}000$) in gross profit and $\$5{,}000$ ($\$43{,}000 - \$38{,}000$) of additional operating income. Further investigation shows that while net sales increased, so did the direct costs of its goods (COGS) and its operating expenses.

We will further explore how to assess each of these expenses later in the chapter using common-size analysis. Common-size analysis reflects each element of a financial statement as a percentage of the base. In the case of the income statement, the base is net sales. Here we would see that COGS was 50 percent of net sales in both the current ($\$60{,}000/\$120{,}000$) and prior year ($\$50{,}000/\$100{,}000$), indicating there wasn't any significant change in performance we could detect from the information provided in the income statement.

LINK TO LEARNING

Operating Expenses

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate the company's income statement (the income statement begins on page 31). Review Apple's operating income for the last few years. Has it improved or declined? Does this fall in line with your expectations based on your previous review of the company's net sales and gross profit?

Based on the improvement or decline in operating income, review Apple's operating expenses to investigate. Have any of the operating expenses changed significantly? How did that impact the company's operating income? What might it tell you about the company's performance?

Net Income

Finally, we move on to **net income**, or what is commonly referred to as *the bottom line*. Net income (or loss) reflects the net impact of all financial transactions for the firm, including those that are caused by events outside the normal course of business. The most common items deducted from operating income to arrive at net income include interest expense, gains/losses, and income tax expense. Remember, gains and losses are those that result from unusual transactions outside the normal course of business. Examples include selling a piece of old equipment or a loss on retiring debt.

We can see in Figure 5.4 that Clear Lake Sporting Goods has outstanding debt, so it incurred interest expense

of $2,000 in the current year and $3,000 the prior year. Since it recorded net income (not a loss), it must also record income tax expense of $6,000 in the current and $5,000 in the prior year.

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Current Year
Gross Sales	$105,000	$126,000
Sales Returns & Allowances	5,000	6,000
Net Sales	100,000	120,000
Cost of Goods Sold	50,000	60,000
Gross Profit	50,000	60,000
Rent Expense	5,000	5,500
Depreciation Expense	2,500	3,600
Salaries Expense	3,000	5,400
Utility Expense	1,500	2,500
Operating Income	38,000	43,000
Interest Expense	3,000	2,000
Income Tax Expense	5,000	6,000
Net Income	$ 30,000	$ 35,000

Figure 5.4 Comparative Year-End Income Statement

EBITDA (Earnings before Interest, Taxes, Depreciation, and Amortization)

Now that we have a full income statement, we can look at another commonly used measure of financial performance called EBITDA. EBITDA stands for earnings before interest, taxes, depreciation, and amortization. Amortization is similar to depreciation. It is the spreading of the cost of an intangible asset over the course of its useful life. Intangible assets are long-term assets that lack physical substance, such as patents and copyrights.

Since EBITDA removes noncash items from the net income equation, it is considered a useful measure in assessing the cash flows provided by operating activities. We will assess cash flows using the **statement of cash flows** and various other cash flow measures later in this chapter as well.

As shown in Figure 5.5, Clear Lake Sporting Goods' EBITDA in the prior year was

$$\$19{,}500\ (\$30{,}000 - \$3{,}000 - \$5{,}000 - \$2{,}500)$$

and in the current year was

$$\$23{,}400\ (\$35{,}000 - \$2{,}000 - \$6{,}000 - \$3{,}600).$$

Clear Lake Sporting Goods Comparative Year-End Income Statements		
	Prior Year	Current Year
Gross Sales	$105,000	$126,000
Sales Returns & Allowances	5,000	6,000
Net Sales	100,000	120,000
Cost of Goods Sold	50,000	60,000
Gross Profit	50,000	60,000
Rent Expense	5,000	5,500
Depreciation Expense	2,500	3,600
Salaries Expense	3,000	5,400
Utility Expense	1,500	2,500
Operating Income	38,000	43,000
Interest Expense	3,000	2,000
Income Tax Expense	5,000	6,000
Net Income	$ 30,000	$ 35,000

Figure 5.5 EBITDA (Earnings before Interest, Taxes, Depreciation, and Amortization)

THINK IT THROUGH

Net Income

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate the company's income statement (the income statement begins on page 31). Review net income for the last few years. Has it improved or declined? Does this fall in line with your expectations based on your previous review of the company's operating income?

Solution:

Net income improved in the current year over last year but declined from 2018 to 2019 (from $59,531 to $55,256). Apple's EBITDA in 2020 was 108.89. (Amounts are in millions.)

5.2 The Balance Sheet

Learning Outcomes

By the end of this section, you will be able to:

- Outline the purpose and importance of the balance sheet.
- Identify the structure and key elements of the balance sheet.

The Accounting Equation and the Classified Balance Sheet

Recall that the income statement shows the performance of a firm over the course of time. The classified balance sheet shows the financial state of a company as of a specific point in time. It is a key distinction between the two statements. The classified balance sheet is prepared in sections that align with the accounting equation.

Remember, the **accounting equation** reflects the **assets** (items owned by the organization) and how they were obtained (by incurring **liabilities** or provided by owners). Liabilities are debts owed to other parties. Stated differently, every asset has a claim against it—by creditors and/or owners.

$$\text{Assets} = \text{Liabilities} + \text{Owner's Equity}$$

The classified balance sheet is thus broken down into three sections; assets, liabilities, and owner's equity. If

prepared correctly, the total assets on the balance sheet equals the total liabilities and owner's equity sections of the balance sheet.

The classified balance sheet is considered or termed *classified* when the assets and liabilities within the balance sheet are grouped into even smaller sections: current and noncurrent (see Figure 5.6). Both assets and liabilities are broken down on the balance sheet into current and noncurrent classifications in order to provide more detail and transparency as well as abide by the convention of reporting in descending order of liquidity. **Current assets** are those that can be used or converted to cash within a year. Common examples of current assets include cash, inventory, accounts receivable, and short-term investments. Assets that will be in use longer than a year are considered noncurrent. Common examples of **noncurrent assets** include notes receivable, machinery and equipment, buildings, and land.

Just as we noted a few key differences in the income statements based on the type of firm, you may also notice a few slight differences in the balance sheet depending on the firm type. Clear Lake Sporting Goods is a retailer. Thus, you will see that their inventory for resale on their balance sheet is simply called "Inventory." This is the goods they have purchased for resale but have not yet sold. A manufacturer, like Apple, Inc. in the Link to Learning sections, will have a variety of inventory types including raw materials, work in progress, and finished goods inventory. These represent the various states of the inventory (ready to use, partially complete, and fully completed product). A service firm, on the other hand, may not have inventory at all. If it does, it may be simple goods it uses to help deliver its service. For example, a cleaning company may keep an inventory of cleaning supplies.

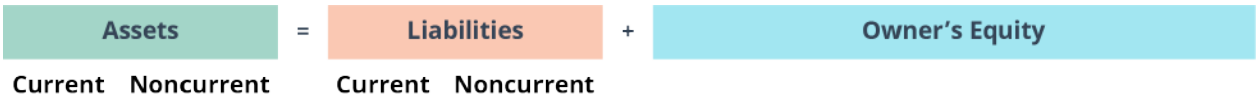


Figure 5.6 Graphical Representation of the Accounting Equation Both assets and liabilities are categorized as current and noncurrent.

Clear Lake Sporting Goods has cash, accounts receivable, inventory, short-term investments, and equipment. It rents its facilities, so it has no buildings on its balance sheets. Of all its assets, only the equipment is long term. The assets section for Clear Lake's classified balance sheet is shown in Figure 5.7.

Clear Lake Sporting Goods
Comparative Year-End Balance Sheets

	Prior Year	Current Year
Assets		
Current Assets:		
Cash	$ 90,000	$110,000
Accounts Receivable	20,000	30,000
Inventory	35,000	40,000
Short-Term Investments	15,000	20,000
Total Current Assets	160,000	200,000
Noncurrent Assets:		
Equipment	40,000	50,000
Total Assets	$200,000	$250,000

Figure 5.7 Assets Section of Classified Balance Sheet

THINK IT THROUGH

Current and Noncurrent Assets

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate the company's balance sheet (the balance sheet begins on page 33). What types of current assets

does the company have? What types of noncurrent assets does it have? How has the total of each type of asset changed over time?

Solution:

Apple reports cash and cash equivalents, marketable securities, accounts receivable, inventories, vendor non-trade receivables, and "other" current assets on its balance sheet. The company's current assets decreased from $162,819 in 2019 to $143,713 in 2020. Apple reports marketable securities, property, plant and equipment, and other noncurrent assets in the noncurrent asset section of its balance sheet. The company's noncurrent assets increased from $175,697 in 2019 to $180,175 in 2020. (Amounts are in millions.)

We've covered the assets side of the accounting equation; now let's turn our attention to the other side of the equation and the other two sections of the balance sheet: liabilities and equity. Just like the assets section, the liabilities section is broken down between current and noncurrent. **Current liabilities** are those that will be due within a year. Common examples of current liabilities include accounts payable, wages payable, and unearned revenue. **Noncurrent liabilities** are those that are due more than a year into the future. Notes payable is a common noncurrent liability.

Clear Lake Sporting Goods has accounts payable and has collected payments from a few customers that it hasn't yet shipped its product to (unearned revenue). It also owes money on a note payable. Its accounts payable and unearned revenue are both current liabilities. The note payable is not due for several years, thus making it a noncurrent liability (see Figure 5.8).

	Prior Year	Current Year
Current Liabilities:		
Accounts Payable	$ 60,000	$ 75,000
Unearned Revenue	10,000	25,000
Total Current Liabilities	70,000	100,000
Noncurrent Liabilities:		
Notes Payable	40,000	50,000
Total Liabilities	$110,000	$150,000

Figure 5.8 Liability Section of Classified Balance Sheet

THINK IT THROUGH

Current and Noncurrent Liabilities

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate its balance sheet (the balance sheet begins on page 33). What types of current and noncurrent liabilities does the company have? How has the total of each type of liability changed over time?

Solution:

Apple has accounts payable, deferred revenue, commercial paper, and term debt listed as current liabilities. Its current liabilities declined by only a small amount from 2019 to 2020 ($105,718 to $105,392).

The company has term debt and "other" listed as noncurrent liabilities, which increased from 2019 to 2020 ($142,310 to $153,157). (Amounts are in millions.)

The stockholders' equity section of the balance sheet for corporations contains two primary categories of accounts. The first is contributed capital, which is funds paid in by owners. The second category is earned

capital, which is funds earned by the corporation as part of business operations. On the balance sheet, **retained earnings** is a key component of the earned capital section, while the stock accounts such as common stock, preferred stock, and additional paid-in capital are the primary components of the contributed capital section.

Clear Lake Sporting Goods has just one contributed capital account—common stock—and one earned capital account—retained earnings. The equity section of its balance sheet is shown in Figure 5.9.

	Prior Year	Current Year
Stockholder Equity		
Common Stock	$ 75,000	$ 80,000
Ending Retained Earnings	15,000	20,000
Total Stockholders' Equity	90,000	100,000
Total Liabilities and Stockholder Equity	$200,000	$250,000

Figure 5.9 Stockholders' Equity Section of Classified Balance Sheet

THINK IT THROUGH

Assets, Liability, and Equity

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate the company's balance sheet (the balance sheet begins on page 33). What is the amount of the company's total assets for the most recent year? What is the amount of its total liabilities and equity?

Solution:

Apple's total assets for 2020 were $323,888, and its total liabilities and equity were also $323,888. (Amounts are in millions.)

Importance of the Balance Sheet

Now that we have gone to all the work to carefully assemble a classified balance sheet, what do we use it for? The answer lies within the accounting equation itself. Think of the accounting equation from a "sources and claims" perspective. Under this approach, the assets (items owned by the organization) were obtained by incurring liabilities or were provided by owners. Stated differently, every asset has a claim against it—by creditors and/or owners. The balance sheet shows us what the firm has (its assets), who owns them (equity), and who the firm owes (its liabilities).

Limitations of the Balance Sheet

The balance sheet is indeed a very helpful financial statement, but it also poses challenges. First, assets on the balance sheet, under generally accepted accounting principles (GAAP), are recorded at historical cost. Historical cost is simply the cost paid for the item at the time it was purchased. Changes in market value of big-ticket items like land or buildings are not reflected in the balance sheet. Land remains at historical cost, and depreciable items like buildings are reflected at their current book value (historical cost less accumulated depreciation). If the asset has appreciated over time, the higher market value of the assets would not be seen on the balance sheet.

Estimates are another limitation of the balance sheet. Items on the balance sheet such as allowance for doubtful accounts and allowance for bad debt are based on estimates. The useful lives for calculating depreciation is another common estimate. If these estimates are incorrect, the net value of the asset can be under- or overstated.

Another key limitation is the fact that a balance sheet reflects balances at only one given point in time. This

means that the account value could have been quite different on the day before or the day after the date of the balance sheet. For example, if a firm were concerned with certain ratios or investor/lender expectations of its cash balance, it could choose to not pay several vendor payments in the last week of December. Thus, on December 31, the firm reflects a high cash balance on its balance sheet. However, by the end of the first week of January, it has caught up on late vendor payments and again shows a low cash balance.

Finally, there are many possible things of value that are not recorded on the balance sheet. Internally generated assets and the firm's human capital are two common examples. Internally generated assets can be anything from a website, a process, to an idea.

5.3 The Relationship between the Balance Sheet and the Income Statement

Learning Outcomes

By the end of this section, you will be able to:

- Identify connected elements between the balance sheet and the income statement.
- Differentiate between expenses and payables.

Net Income and Retained Earnings

As mentioned earlier, the financial statements are linked by certain elements and thus must be prepared in a certain order. The income statement was first since net income (or loss) is a required figure in preparing the balance sheet. During the period close process, all temporary accounts are closed to the income summary account, which is then closed to retained earnings. All revenue and expense accounts are closed since they are temporary. The net result is either net profit or net loss as the balance in the income summary account.

Remember that the retained earnings account reflects all income the firm has earned since its inception less any **dividends** paid out to shareholders. Thus the result (net income) of the income statement feeds the retained earnings account on the balance sheet. Retained earnings is also an element of the statement of stockholders' equity, which we will cover later in this chapter.

In Figure 5.10, we see net income in the current year of $35,000, which was added to the company's prior year retained earnings balance of $15,000. Notice, however, that the prior year balance was $15,000, and the current year balance is only $20,000. It does not total $50,000 as we might have expected.

Remember, retained earnings represents all earnings since inception less any dividends paid out. Clear Lake Sporting Goods must have paid out $30,000 in dividends in the current year. We will see this information laid out in the statement of retained earnings. In the prior year they began with a $10,000 balance in retained earnings. Income of $30,000 increased retained earnings and dividends paid back out to investors reduced retained earnings, leaving an ending balance in the prior year of $15,000. This rolls over and is the beginning balance for the current year. In the current year Clear Lake had net income of $35,000 and paid $30,000 of their earnings out to shareholders, essentially resulting in a $5,000 increase to the retained earnings account.

Clear Lake Sporting Goods
Statement of Retained Earnings

	Prior Year	Current Year
Beginning Retained Earnings Balance Jan 1	$10,000	$15,000
Plus Net Income	30,000	35,000
Less Dividends Paid to Shareholders	25,000	30,000
Ending Retained Earnings Dec 31	$15,000	$20,000

Figure 5.10 Statement of Retained Earnings

Now we can see the full flow of information from the income statement to the statement of retained earnings (Figure 5.10) and finally to the balance sheet. Clear Lake's net income flows from the income statement into

retained earnings, which is reflected on the statement of retained earnings. The balance in retained earnings is then reflected on the balance sheet. This flow is depicted in Figure 5.11.

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Current Year
Gross Sales	$105,000	$126,000
Sales Returns & Allowances	5,000	6,000
Net Sales	100,000	120,000
Cost of Goods Sold	50,000	60,000
Gross Margin	50,000	60,000
Rent Expense	5,000	5,500
Depreciation Expense	2,500	3,600
Salaries Expense	3,000	5,400
Utility Expense	1,500	2,500
Operating Income	38,000	43,000
Interest Expense	3,000	2,000
Income Tax Expense	5,000	6,000
Net Income	$ 30,000	$ 35,000

Clear Lake Sporting Goods
Comparative Year-End Balance Sheets

	Prior Year	Current Year
Assets		
Current Assets:		
Cash	$ 90,000	$110,000
Accounts Receivable	20,000	30,000
Inventory	35,000	40,000
Short-Term Investments	15,000	20,000
Total Current Assets	160,000	200,000
Noncurrent Assets:		
Equipment	40,000	50,000
Total Assets	200,000	250,000
Current Liabilities:		
Accounts Payable	60,000	75,000
Unearned Revenue	10,000	25,000
Total Current Liabilities	70,000	100,000
Noncurrent Liabilities:		
Notes Payable	40,000	50,000
Total Liabilities	110,000	150,000
Stockholder Equity		
Common Stock	75,000	80,000
Ending Retained Earnings	15,000	20,000
Total Stockholders' Equity	90,000	100,000
Total Liabilities and Stockholder Equity	$200,000	$250,000

Clear Lake Sporting Goods
Statement of Retained Earnings

	Prior Year	Current Year
Beginning Retained Earnings Balance Jan 1	$10,000	$15,000
Plus Net Income	30,000	35,000
Less Dividends Paid to Shareholders	25,000	30,000
Ending Retained Earnings Dec 31	$15,000	$20,000

Figure 5.11 Connections between Clear Lake Sporting Goods' Balance Sheet and Income Statement

LINK TO LEARNING

Apple's Income Statement and Balance Sheet

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate the company's income statement and balance sheet (they begin on page 31). What is the company's net income for the most recent year? What is the amount of retained earnings on the balance sheet for the current and prior years? Does it appear the company paid out dividends this year?

Expenses versus Payables

There is another key relationship between the income statement and the balance sheet can often be confusing to non-accountants: an expense versus a payable. The two are often assumed to be the same thing. However, it is important to note that the two are distinctly different.

Let's look at an example to outline the key differences. Clear Lake Sporting Goods incurred utility expenses during the current period (electric and gas). Its utilities totaled $1,500. In the month that followed, the utilities vendor sent an invoice for $1,500. Clear Lake has incurred an expense. It will reflect an expense of $1,500 on the income statement for the utilities expense. This is the income statement impact of the transaction. So is it safe to assume that because Clear Lake has an expense, it also used cash? Not necessarily. Or is it safe to assume that if the company has an expense, it is the same as a payable? Again, the answer is no.

Remember, the accounting equation rests on the foundation of the double-entry accounting system. This means that every transaction has two sides: a debit and a credit. They must be equal. When Clear Lake records an expense of $1,500, it must also record the other half of that transaction. In this case, the company incurred utilities expenses throughout the period "on account," which means it records an increase in their accounts payable. When the invoice comes due, another transaction must then be recorded to reduce accounts payable and reduce cash. Accounts payable is a liability found on the balance sheet, normally a current liability. The expense incurred caused the payable, but it is distinctly separate from the payable (see Figure 5.12).

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Current Year
Gross Sales	$105,000	$126,000
Sales Returns & Allowances	5,000	6,000
Net Sales	100,000	120,000
Cost of Goods Sold	50,000	60,000
Gross Profit	50,000	60,000
Rent Expense	5,000	5,500
Depreciation Expense	2,500	3,600
Salaries Expense	3,000	5,400
Utility Expense	1,500	2,500
Operating Income	38,000	43,000
Interest Expense	3,000	2,000
Income Tax Expense	5,000	6,000
Net Income	$ 30,000	$ 35,000

Clear Lake Sporting Goods
Comparative Year-End Balance Sheets

	Prior Year	Current Year
Assets		
Current Assets:		
Cash	$ 90,000	$110,000
Accounts Receivable	20,000	30,000
Inventory	35,000	40,000
Short-Term Investments	15,000	20,000
Total Current Assets	160,000	200,000
Equipment	40,000	50,000
Total Assets	200,000	250,000
Current Liabilities:		
Accounts Payable	60,000	75,000
Unearned Revenue	10,000	25,000
Total Current Liabilities	70,000	100,000
Noncurrent Liabilities:		
Notes Payable	40,000	50,000
Total Liabilities	110,000	150,000
Stockholder Equity		
Common Stock	75,000	80,000
Ending Retained Earnings	15,000	20,000
Total Stockholders' Equity	90,000	100,000
Total Liabilities and Stockholder Equity	$200,000	$250,000

Figure 5.12 Connections between Expenses and Accounts Payable

5.4 The Statement of Owner's Equity

Learning Outcomes

By the end of this section, you will be able to:

- Explain the concept of owner's equity.
- Outline the purpose and importance of the statement of owner's equity.
- Identify the structure and key elements of the statement of owner's equity.

What Is Equity?

Recall that the accounting equation can help us see what is owned (assets), who is owed (liabilities), and finally who the owners are (equity). The statement of owner's equity addresses the last segment of the accounting equation in detail by laying out the equity elements of the firm and highlighting changes in these elements throughout the period.

Equity represents the ownership of the firm. The stockholders' equity section of the balance sheet for corporations contains two primary categories of accounts. The first is paid-in capital or contributed capital—consisting of amounts paid in by owners. The second category is earned capital, consisting of amounts earned by the corporation as part of business operations. On the balance sheet, retained earnings is a key component of the earned capital section, while the stock accounts such as common stock, preferred stock, and additional paid-in capital are the primary components of the contributed capital section.

Common stock represents ownership in the firm. Common stockholders normally have voting rights. Preferred stock has unique rights that are "preferred," or more advantageous, to shareholders than common stock. Unlike common stockholders, preferred shareholders typically do not have voting rights and do not share in the common stock dividend distributions. Instead, the "preferred" classification entitles shareholders to a dividend that is fixed (assuming sufficient dividends are declared). Treasury stock is shares that were outstanding and have been repurchased by the firm but not retired. Thus they are still issued, but not outstanding. Additional paid-in capital is the difference between the issue price and par value of the common stock. For example, if a firm issued and sold stock at a market price of $20 that had a $5 par value, $5 for each share would be recorded into common stock and the excess $15 per share would be recorded into the additional paid-in capital account.

If we review the balance sheet for Clear Lake Sporting Goods, we see just two elements of equity: common stock and retained earnings. Common stock in the prior year was $75,000 and increased to $80,000 in the current year, indicating Clear Lake issued additional common stock (see Figure 5.13).

Clear Lake Sporting Goods
Comparative Year-End Balance Sheets

	Prior Year	Current Year
Stockholder Equity		
Common Stock	$ 75,000	$ 80,000
Ending Retained Earnings	15,000	20,000
Total Stockholders' Equity	90,000	100,000
Total Liabilities and Stockholder Equity	$200,000	$250,000

Figure 5.13 Stockholder Equity Section of Balance Sheet

THINK IT THROUGH

Equity Accounts

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and

locate the company's balance sheet (it begins on page 33). What types of equity accounts does it report?

Solution:

Apple reports common stock, retained earnings, and accumulated other comprehensive income.

Distributions to Owners

When firms earn a profit, they have two options as to what to do with their earnings. They can keep (retain) them and reinvest them back into the business, or they can pay them out to their shareholders in the form of dividends. Dividends are commonly in the form of cash, but dividends can be paid out in the form of stock or other assets as well.

To pay a cash dividend, the firm must have enough cash on hand and sufficient retained earnings. They cannot pay out a dividend beyond the retained earnings available. Some companies issue shares of stock as a dividend rather than cash or property. This often occurs when the company has insufficient cash but wants to keep its investors happy. When a company issues a stock dividend, it distributes additional shares of stock to existing shareholders.

A property dividend occurs when the firm pays out dividends in the form of something other than stock or cash, often one of their assets or something they hold in inventory. For example, Walt Disney Company may choose to distribute tickets to visit its theme parks. A property dividend may be declared when a company wants to reward its investors but doesn't have the cash to distribute, or if it needs to hold on to its existing cash for other investments.

Remember, the retained earnings account reflects the cumulative earnings of a firm since they began business, less dividends paid out to shareholders. This includes all forms of dividends (cash, stock, and other assets). Note that dividends are distributed or paid only to shares of stock that are outstanding. Treasury shares are not outstanding, so no dividends are declared or distributed for these shares. Regardless of the type of dividend, the declaration always causes a decrease in the retained earnings account.

THINK IT THROUGH

Dividends

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020. Review the notes to the financial statements found on pages 19 and 20. What kind of dividends did the company pay (cash, property, stock)? Review the notes regarding dividends on page 26. What does Apple intend to do regarding dividends in the future, pending board approval?

Solution:

Apple issued cash dividends. On page 26, it notes that the company intends to increase the dividend annually, pending approval by the board.

Elements of the Statement of Owner's Equity

Now that we have covered the basic elements of equity and know what dividends are, we have the basic pieces we need to create the statement of **owner's equity**. The statement is broken out into columns, one for each element of equity: common stock, preferred stock, additional paid-in capital, retained earnings, and treasury stock.

The first line of the statement provides the balance of each segment as of the first day of the period. Each

following line provides information on any events during the period that changed the value of any of the accounts. Common examples of events found on the statement include net income or loss for the period, issuing common or preferred stock, purchasing or selling treasury stock, and declaring a dividend.

Clear Lake Sporting Goods has just common stock and retained earnings to report in their statement of owner's equity. They had just two events to report in their statement that impacted their equity accounts; they reported net income and they issued dividends (see Figure 5.14).

	Common Stock	Additional Paid-in Capital	Retained Earnings	Treasury Stock	Total
Beginning Balance	$75,000	$0	$15,000	$0	$ 90,000
Net Income			35,000		35,000
Dividends Paid to Shareholders			(30,000)		(30,000)
Common Stock Issued	5,000				5,000
Ending Balance	$80,000	$0	$20,000	$0	$100,000

Figure 5.14 Statement of Stockholder Equity for Clear Lake Sporting Goods

LINK TO LEARNING

Cash Flows

Khan Academy (https://openstax.org/l/50CashFlowsVid) explains cash flows in a unique way.

5.5 The Statement of Cash Flows

Learning Outcomes

By the end of this section, you will be able to:

- Outline the purpose and importance of the statement of cash flows.
- Identify the structure and key elements of the statement of cash flows.

The final financial statement is the statement of cash flows. It is a crucial statement, as it shows the sources of and uses of cash for the firm during the accounting period. Remember, under accrual accounting, transactions are recorded when they occur, not necessarily when cash moves. Thus, the income statement does not provide all the insights necessary to understand a firm's cash flows. To fully understand the firm's flow of cash, the statement of cash flows is needed.

Importance of the Statement of Cash Flows

Remember, most firms use accrual accounting. Revenues and expenses are recorded when they occur, not necessarily when cash moves. This can create timing differences between profits and cash flows. A firm can be profitable and still not have an adequate flow of cash. The opposite is also true: it can experience a net loss and still have cash on hand. Earning a profit is wonderful, but it is not the only goal an organization has. It must also have adequate cash flow to support daily operations. To support cash planning and to provide external financial statement users such as lenders and investors information about the firm's cash flow, the statement of cash flows is prepared. Cash flow is also a crucial metric for determining the value of a company.

External financial statement users also rely on the statement of cash flows to help them evaluate the quality of the firm's earnings. Users compare earnings to cash flow to assess the validity of the earnings data. For example, a firm reporting a strong profit but very little cash flow might raise some questions as to what was recorded to drive profits that isn't also driving cash flows.

The statement of cash flows also helps external users determine the driving forces behind the firm's cash flows. They can see if cash is generated primarily by daily operations or if cash is being generated or

consumed by events outside the firm's normal course of business.

There are two key methods of preparing the statement: direct and indirect. FASB (Financial Accounting Standards Board) favors the **direct method**. Despite that, the most common method used by far in general practice is the **indirect method**. The direct method lists cash flows directly from revenues and expenses, whereas the indirect method reconciles income to cash flows. The indirect method begins with net income and reconciles each account in order to arrive at net cash flow. It essentially reconciles **accrual basis** accounting to **cash basis**, or cash flow.

Operating Activities

To provide clear information about what areas of the business generated and used cash, the statement of cash flows is broken down into three key categories: operating, financing, and investing. The operating section reflects cash flows generated by and used by the day-to-day operations of the business. Investing activities include investments in other firms as well as investments in the firm itself (items like machinery, land, or other fixed assets). Finally, financing activities are those used to provide funds to run the business (loans, interest).

As mentioned, operating activities are those that are used or generated by the day-to-day operations of the firm. The operating activities section of the statement of cash flows begins with net income. All lines thereafter, in that section, are then adjustments to reconcile net income to actual cash flows by adding back **noncash expenses** like depreciation and adjusting for changes in asset and liability accounts. For example, depreciation is a noncash expense. Thus, depreciation is added back to net income.

To prepare the statement of cash flows for Clear Lake Sporting Goods, we need the beginning cash balance from the balance sheet, net income and depreciation expense from the income statement, and a set of comparative balance sheets to see the change in asset and liability accounts (see Figure 5.15).

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Current Year
Gross Sales	$105,000	$126,000
Sales Returns & Allowances	5,000	6,000
Net Sales	100,000	120,000
Cost of Goods Sold	50,000	60,000
Gross Margin	50,000	60,000
Rent Expense	5,000	5,500
Depreciation Expense	2,500	3,600
Salaries Expense	3,000	5,400
Utility Expense	1,500	2,500
Operating Income	38,000	43,000
Interest Expense	3,000	2,000
Income Tax Expense	5,000	6,000
Net Income	$ 30,000	$ 35,000

Clear Lake Sporting Goods
Comparative Year-End Balance Sheets

	Prior Year	Current Year
Assets		
Current Assets:		
Cash	$ 90,000	$110,000
Accounts Receivable	20,000	30,000
Inventory	35,000	40,000
Short-Term Investments	15,000	20,000
Total Current Assets	160,000	200,000
Noncurrent Assets:		
Equipment	40,000	50,000
Total Assets	200,000	250,000
Current Liabilities:		
Accounts Payable	60,000	75,000
Unearned Revenue	10,000	25,000
Total Current Liabilities	70,000	100,000
Noncurrent Liabilities:		
Notes Payable	40,000	50,000
Total Liabilities	110,000	150,000
Stockholder Equity		
Common Stock	75,000	80,000
Ending Retained Earnings	15,000	20,000
Total Stockholders' Equity	90,000	100,000
Total Liabilities and Stockholder Equity	$200,000	$250,000

Figure 5.15 Comparative Income Statements and Balance Sheets

Clear Lake's statement of cash flows begins with the current year net income of $35,000 from the income statement. Next, noncash expenses are deducted. Clear Lake's only noncash expense on their current year income statement is depreciation of $3,600. Since deprecation is an expense that reduces income but is not actually paid out in cash in the current period, it must be added back to net income to reconcile net income to cash flow.

Next, changes in operational assets and liabilities are used to continue reconciling net income to actual cash flow. For example, Clear Lake's accounts receivable increased from the prior period to the current period. This means that there were more sales recorded but not yet received in cash in this period than there were in the prior period, making an increase in accounts receivable a reduction on the statement. Inventory increased, which means additional cash was spent to acquire it, making it a use of cash or reduction to net income to move closer to cash. Accounts payable and unearned revenue, both liability accounts, increased. Since these are liabilities, an increase would indicate that the liability was incurred but not as quickly paid out; thus it is an increase to the statement.

Tallying all these adjustments to net income shows Clear Lake's net cash flows provided by operating activities of $53,600 (see Figure 5.16).

Clear Lake Sporting Goods Statement of Cash Flows Indirect Method		
Cash Flow from Operating Activities		
Net Income		$35,000
Adjustment to Reconcile Net Income to Net Cash Flow from Operating Activities:		
Depreciation	$ 3,600	
Accounts Receivable Increase	(10,000)	
Inventory Increase	(5,000)	
Accounts Payable Increase	15,000	
Unearned Revenue Increase	15,000	18,600
Net Cash Flow from Operating Activities		$53,600

Figure 5.16 Operating Activities Section of the Statement of Cash Flows

Investing Activities

As mentioned, investing activities include investments in other firms as well as investments in the firm itself (items like machinery, land, or other fixed assets). These are items that are capitalized (placed on the balance sheet and depreciated over time) and thus did not reduce net income. They did, however, cause an impact to cash flow (see Figure 5.17).

During the current year, Clear Lake purchased an additional $5,000 in short-term investments (see the comparative balances sheets; the balance in that account increased by $5,000 since the prior year). They also purchased additional plant assets in the amount of $13,600. This amount can be figured by comparing the difference in the current and prior plant assets accounts and including the impact of current year depreciation ($50,000 current year balance less $40,000 prior year balance and $3,600 of depreciation = $13,600 of new assets purchased).

Cash Flow from Investing Activities		
Purchase of Short-Term Investments	$(5,000)	
Cost of New Plant Assets	(13,600)	
Net Cash Flow: Investing Activities		$(18,600)

Figure 5.17 Investing Activities Section of the Statement of Cash Flows

Financing Activities

Recall that financing activities are those used to provide funds to run the business. Common items in this section of the statement include the payment of dividends, issuance of common or preferred stock, and issuance or payment of notes payable (see Figure 5.18).

In the current year, Clear Lake took out additional notes payable (a cash inflow). We can see this by the increase in their notes payable account from the prior year to current year ($40,000 to $50,000). This is an inflow of cash and thus an increase on the statement. Dividends of $30,000 were paid to shareholders (found on the statement of retained earnings and the statement of owner's equity). Finally, we see that Clear Lake must have issued additional common stock, as their common stock balance increased from $75,000 to $80,000.

Cash Flow from Financing Activities		
Additional Notes Payable	$10,000	
Issuance of Common Stock	5,000	
Payment of Dividends	(30,000)	
Net Cash Flow from Financing Activities		$(15,000)

Figure 5.18 Financing Activities Section of the Statement of Cash Flows

LINK TO LEARNING

Statement of Cash Flows

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and locate the company's statement of cash flows (it begins on page 35). What is the cash provided by/used by operating activities? Review the key adjustments in the operating section. What are the biggest adjustments? Review the investing section of the statement. Did the company have a net cash inflow or net cash used by investing? What key items appear in this section? What was the net cash provided by or used by financing activities? What key items appear in this section for Apple?

The final task to wrap up the statement of cash flows is to tally net cash generated or used by summing all three sections. This amount is then used to adjust the beginning cash balance from the balance sheet. Assuming the statement was prepared correctly, the sum should equal the ending cash balance on the balance sheet.

In the full statement, we can see that Clear Lake has net cash flow of $20,000. The beginning cash balance was $90,000, making the ending cash balance $110,000 (see Figure 5.19).

Clear Lake Sporting Goods
Statement of Cash Flows
Indirect Method

Cash Flow from Operating Activities		
Net Income		$ 35,000
Adjustment to Reconcile Net Income to Net Cash Flow from Operating Activities:		
Depreciation	$ 3,600	
Accounts Receivable Increase	(10,000)	
Inventory Increase	(5,000)	
Accounts Payable Increase	15,000	
Unearned Revenue Increase	15,000	18,600
Net Cash Flow from Operating Activities		53,600
Cash Flow from Investing Activities		
Purchase of Short-Term Investment	(5,000)	
Cost of New Plant Assets	(13,600)	
Net Cash Flow: Investing Activities		(18,600)
Cash Flow from Financing Activities		
Additional Notes Payable	10,000	
Issuance of Common Stock	5,000	
Payment of Dividends	$(30,000)	
Net Cash Flow from Financing Activities		(15,000)
Total Cash Flow Increase/Decrease		20,000
Beginning Cash Balance		90,000
Ending Cash Balance		$110,000

Figure 5.19 Full Statement of Cash Flows

Analyzing Performance

The statement of cash flows can be used in a number of ways to assess firm performance by both internal and external financial statement users. Internal users can assess sources of and uses of cash in order to aid in adapting, as necessary, to ensure adequate future cash flows. External users also use the statement. Recall that comparing net income to operational cash flows can help assess the quality of earnings. In the next section you'll explore operating cash flow and **free cash flow** to the firm, two key points of analysis in

assessing cash flows.

5.6 Operating Cash Flow and Free Cash Flow to the Firm (FCFF)

Learning Outcomes

By the end of this section, you will be able to:

- Calculate operating cash flow and free cash flow.
- Assess organizational cash management performance.

Operating Cash Flow

Now that we have a statement of cash flows prepared, we can move on to a few key elements of the statement used to assess organizational cash management performance. Operating cash flow, or net cash flow from operating activities, is calculated in the first section of the statement of cash flows. It depicts the cash generated (or used by) the primary business activities. Remember, operating cash flow is calculated under the indirect method by adjusting net income for noncash expenses like depreciation and adjustments for changes in current asset and liability accounts (changes in working capital).

$$\text{Operating Cash Flow} = \text{Operating Income} + \text{Depreciation} - \text{Taxes} + \text{Change in Working Capital}$$

Operating cash flow is helpful in assessing organizational cash management performance as it relates to the core business function—operations. Key management practices in this area can have a profound impact on the firm's cash flow. Practices and policies include customer payment terms, collection policies and practices, and vendor payment terms. Though changing a customer or vendor payment terms will not change the profit or loss for the firm, it will have an impact on the timing to cash flows. This is a key element of managing operational cash flow.

Free Cash Flow

Free cash flow (FCF) is calculated by taking operating cash flows less capital expenditures. Free cash flow is an important measure, as it depicts the cash available to support the business's operations and maintain its fixed assets. It is commonly used by investors as part of their overall evaluation of an investment, as it is a key measure of cash flow management practices and a firm's ability to generate enough cash to cover operations and capital assets and it shows if there is any left over for other considerations such as dividend payments, debt repayment, and contributions to increase working capital for future growth.

$$\text{Free Cash Flow (FCF)} = \text{Operating Cash Flow} - \text{Capital Expenditures}$$

Using the data for Clear Lake Sporting Goods, we can calculate its free cash flow as follows:

$$\text{Free Cash Flow (FCF)} = \$53{,}600 - \$13{,}600 = \$40{,}000$$

This means that Clear Lake Sporting Goods has $40,000 of cash available to repay debt or pay cash dividends after having covered the cash needs of its operations and capital asset investments.

Managing Cash Flow

Managing cash flow is not an easy task. A firm has a myriad of places that its cash flows from or to. However, there are a few key areas to place attention in order to manage or improve cash flows. First, consider where cash is coming from—customers. Managing terms and collection efforts with customers can have a significant impact on a firm's cash flow. For example, a customer with terms of net 10 will likely yield payment quite quickly—10 days, give or take. A customer with terms of net 60, on the other hand, will require roughly 60 days to collect assuming they pay on time. The 50-day difference between these two examples means that the firm will go 50 additional days having expended the resources to provide the customer their good or service, but with no cash flow yet to cover it.

The same theory holds true on the opposite side with accounts payable and the vendors a firm uses. Accepting

net 10 terms requires the firm to give up its cash quickly, while pushing for more favorable terms like net 30 or net 60 allows the firm to wait much longer to give up its cash.

It is important to assess both sides of cash flow and the impact it has on the firm as well as the customer and vendor relationships it maintains. Some customers may not have difficulty negotiating a more favorable payment term in order for the firm to improve its cash flow. Others, however, may not be willing to accept shorter payment terms. In order to be competitive in the industry, the firm must assess the customer relationships, industry standards, and its own ability to support cash flow when considering its customer payment terms.

When there is a gap in cash flow, it is crucial that it is recognized early, with proper cash flow forecasting, so financing can be obtained to bridge the gap. A common tool used to manage the ebb and flow of cash flow for a firm is an open line of credit with a bank. This allows the firm to borrow and repay from month to month as cash flow fluctuates.

Now that you've become more familiar with the four basic financial statements, let's move on to a tool helpful in evaluating them: common-size statements. Common-size financial statements, also termed vertical analysis, restate the financial statement items as a percentage of a base item. Doing so helps reveal relationships between items, aids in assessing performance over time, and makes it easier to compare one company to another, regardless of size (thus the name common-size).

5.7 Common-Size Statements

Learning Outcomes

By the end of this section, you will be able to:

- Prepare common-size statements.
- Assess organizational performance using common-size statements.
- Use industry comparisons to assess organizational performance.

Common-Size Income Statements

A common-size income statement is created by restating each line as a percentage of net sales. Expressing each item on the income statement as a percentage rather than in absolute dollars makes it much easier to make comparisons, particularly to other divisions or competitors of varying sizes. The formula to calculate each item on the income statement is:

$$\text{Common-Size Item} = \frac{\text{Income Statement Line Item}}{\text{Net Sales}}$$

Using Clear Lake Sporting Goods' current year income statement, we can see how each line item in it is divided by net sales in order to assemble a common-size income statement (see Figure 5.20).

Clear Lake Sporting Goods Comparative Year-End Income Statements			
	Current Year	Current Year %	Formula
Gross Sales	$126,000		
Sales Returns & Allowances	6,000		
Net Sales	120,000	100%	=120,000/120,000
Cost of Goods Sold	60,000	50%	=60,000/120,000
Gross Margin	60,000	50%	=60,000/120,000
Rent Expense	5,500	5%	=5,500/120,000
Depreciation Expense	3,600	3%	=3,600/120,000
Salaries Expense	5,400	5%	=5,400/120,000
Utility Expense	2,500	2%	=2,500/120,000
Operating Income	43,000	36%	=43,000/120,000
Interest Expense	2,000	2%	=2,000/120,000
Income Tax Expense	6,000	5%	=6,000/120,000
Net Income	$ 35,000	29%	=35,000/120,000

Figure 5.20 Common-Size Income Statement

It may seem cumbersome to create a common-size statement. However, a simple tool like Microsoft Excel can be quite handy in making the process easier and faster. The same formula can be copied and replicated in each income statement line, making the calculations much faster. In Figure 5.21, you can see the formulas used to create Clear Lake Sporting Goods' common-size income statement in Excel. Notice that the $ can be inserted to anchor a cell reference, making it easier to copy and paste the same formula onto many lines or columns.

	A	B	C	D	E	F	G
1		Prior Year	Prior Year %	Formula	Current Year	Current Year %	Formula
2	Gross Sales	$ 105,000			$ 126,000		
3	Sales Returns	5,000			6,000		
4	Net Sales	100,000	100%	=B4/$B$4	120,000	100%	=E4/$E$4
5	Cost of Goods Sold	50,000	50%	=B5/$B$4	60,000	50%	=E5/$E$4
6	Gross Margin	50,000	50%	=B6/$B$4	60,000	50%	=E6/$E$4
7	Rent Expense	5,000	5%	=B7/$B$4	5,500	5%	=E7/$E$4
8	Depreciation Expense	2,500	3%	=B8/$B$4	3,600	3%	=E8/$E$4
9	Salaries Expense	3,000	3%	=B9/$B$4	5,400	5%	=E9/$E$4
10	Utility Expense	1,500	2%	=B10/$B$4	2,500	2%	=E10/$E$4
11	Operating Income	38,000	38%	=B11/$B$4	43,000	36%	=E11/$E$4
12	Interest Expense	3,000	3%	=B12/$B$4	2,000	2%	=E12/$E$4
13	Income Tax Expense	5,000	5%	=B13/$B$4	6,000	5%	=E13/$E$4
14	Net Income	$ 30,000	30%	=B14/$B$4	$ 35,000	29%	=E14/$E$4

Figure 5.21 Clear Lake Sporting Goods Common-Size Income Statements with Excel Formulas

Common-Size Balance Sheet

The common-size balance sheet functions much like the common-size income statement. Each line item on the balance sheet is restated as a percentage of total assets.

$$\text{Common-Size Item} = \frac{\text{Balance Sheet Line Item}}{\text{Total Assets}}$$

Using Clear Lake Sporting Goods' current balance sheet, we can see how each line item in its statement is divided by total assets in order to assemble a common-size balance sheet (see Figure 5.22).

Clear Lake Sporting Goods Comparative Year-End Balance Sheets			
	Current Year	**Current Year %**	**Formula**
Assets			
Current Assets:			
Cash	$110,000	44%	=110,000/250,000
Accounts Receivable	30,000	12%	=30,000/250,000
Inventory	40,000	16%	=40,000/250,000
Short-Term Investments	20,000	8%	=20,000/250,000
Total Current Assets	200,000	80%	=200,000/250,000
Noncurrent Assets:			
Equipment	50,000	20%	=50,000/250,000
Total Assets	250,000	100%	=250,000/250,000
Current Liabilities:			
Accounts Payable	75,000	30%	=75,000/250,000
Unearned Revenue	25,000	10%	=25,000/250,000
Total Current Liabilities	100,000	40%	=100,000/250,000
Noncurrent Liabilities:			
Notes Payable	50,000	20%	=50,000/250,000
Total Liabilities	150,000	60%	=150,000/250,000
Stockholder Equity			
Common Stock	80,000	32%	=80,000/250,000
Ending Retained Earnings	20,000	8%	=20,000/250,000
Total Stockholders' Equity	100,000	40%	=100,000/250,000
Total Liabilities and Stockholder Equity	$250,000	100%	=250,000/250,000

Figure 5.22 Common-Size Balance Sheet

Excel can also be used to create a common-size balance sheet. Once the formula is created, it can be copied into each line, making the process to create a common-size statement much easier (see Figure 5.23):

	A	B	C	D
1		**Current Year**	**Current Year %**	**Formula**
2	**Assets**			
3	**Current Assets:**			
4	Cash	$ 110,000	44%	=B4/B$11
5	Accounts Receivable	30,000	12%	=B5/B$11
6	Inventory	40,000	16%	=B6/B$11
7	Short-Term Investments	20,000	8%	=B7/B$11
8	**Total Current Assets**	200,000	80%	=B8/B$11
9	**Noncurrent Assets:**			
10	Equipment	50,000	20%	=B10/B$11
11	**Total Assets**	$ 250,000	100%	=B11/B$11
12	**Current Liabilities:**			
13	Accounts Payable	75,000	30%	=B13/B$11
14	Unearned Revenue	25,000	10%	=B14/B$11
15	**Total Current Liabilities**	100,000	40%	=B15/B$11
16	**Noncurrent Liabilities:**			
17	Notes Payable	50,000	20%	=B17/B$11
18	**Total Liabilities**	150,000	60%	=B18/B$11
19	**Stockholder Equity**			
20	Common Stock	80,000	32%	=B20/B$11
21	Ending Retained Earnings	20,000	8%	=B21/B$11
22	Total Stockholders' Equity	100,000	40%	=B22/B$11
23	**Total Liabilities and Stockholder Equity**	$ 250,000	100%	=B23/B$11

Figure 5.23 Common-Size Balance Sheet with Excel Formulas

It is important to note that while we have provided two years of data here to explore the process, when performing analysis for a firm or investment, several years of data are commonly used to provide a better view of historical performance.

Analyzing Organizational Performance

Common-size financial statements facilitate the analysis of financial performance by converting each element of the statements to a percentage. This makes it easier to compare figures from one period to the next, compare departments within an organization, and compare the firm to other companies of any size as well as industry averages. On the income statement, analysts can see how much of sales revenue is spent on each type of expense. They can see this breakdown for each firm and compare how different firms function in terms of expenses, proportionally. They can also look at the percentage for each expense over time to see if they are spending more or less on certain areas of the business, such as research and development. On the balance sheet, analysts commonly look to see the percentage of debt and equity to determine capital structure. They can also quickly see the percentage of current versus noncurrent assets and liabilities.

In Clear Lake Sporting Goods' common-size income statement for the current and prior years, we can see that cost of goods as a percentage of sales remained the same (see Figure 5.24). This means that while sales

increased, so did cost of goods sold, but it increased at the same proportion as sales. No improvement or decline occurred in the company's cost of goods sold. The same is true for rent, depreciation, and utilities expenses. One key item did change slightly as a percentage: salaries expense. The 2 percent decrease in operating income from the prior year's 38 percent to the current year's 36 percent was caused by the increase in salaries expense as a percentage of sales.

Net income, however, only declined by 1 percent from 30 percent in the prior year to 29 percent in the current year because interest expense dropped by 1 percent, offsetting the 2 percent increase in salaries expense.

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Prior Year %	Current Year	Current Year %
Gross Sales	$105,000		$126,000	
Sales Returns & Allowances	5,000		6,000	
Net Sales	100,000	100%	120,000	100%
Cost of Goods Sold	50,000	50%	60,000	50%
Gross Profit	50,000	50%	60,000	50%
Rent Expense	5,000	5%	5,500	5%
Depreciation Expense	2,500	3%	3,600	3%
Salaries Expense	3,000	3%	5,400	5%
Utility Expense	1,500	2%	2,500	2%
Operating Income	38,000	38%	43,000	36%
Interest Expense	3,000	3%	2,000	2%
Income Tax Expense	5,000	5%	6,000	5%
Net Income	$ 30,000	30%	$ 35,000	29%

Figure 5.24 Common-Size Income Statement

LINK TO LEARNING

Common-Size Income Statement

Visit the Apple, Inc. Common-Size Income Statement (https://openstax.org/r/common-size-income-statement) provided by the Stock Analysis on Net website. Review the company's cost of sales, gross profit, operating income, other income, and net income percentages for the current and prior year. What can you learn about the company's recent performance in these areas? Has it improved or declined, or has it simply changed incrementally with gross sales?

On the Clear Lake Sporting Goods' common-size balance sheet, we see that current assets remained at 80 percent of total assets from the prior to current year (see Figure 5.25). The mix of current assets that comprise that 80 percent changed only slightly with a 1 percent decrease in cash, 2 percent increase in accounts receivable, 2 percent decrease in inventory, and no change in short-term investments. Noncurrent assets includes only equipment. While the balance in the equipment account did change as a percentage of total assets, equipment remained the same at 20 percent.

On the debt and equity side of the balance sheet, however, there were a few percentage changes worth noting. In the prior year, the balance sheet reflected 55 percent debt and 45 percent equity. In the current year, that balance shifted to 60 percent debt and 40 percent equity. The firm did issue additional stock and showed an increase in retained earnings, both totaling a $10,000 increase in equity. However, the equity increase was much smaller than the total increase in liabilities of $40,000. Long-term debt increased by only $10,000 by issuing additional notes payable. The remainder of that increase is seen in the 5 percent increase in current liabilities. In that increase, most of it was in unearned revenue.

Clear Lake Sporting Goods Comparative Year-End Balance Sheets				
	Prior Year	Prior Year %	Current Year	Current Year %
Assets				
Current Assets:				
Cash	$ 90,000	45%	$110,000	44%
Accounts Receivable	20,000	10%	30,000	12%
Inventory	35,000	18%	40,000	16%
Short-Term Investments	15,000	8%	20,000	8%
Total Current Assets	160,000	80%	200,000	80%
Noncurrent Assets:				
Equipment	40,000	20%	50,000	20%
Total Assets	200,000	100%	250,000	100%
Current Liabilities:				
Accounts Payable	60,000	30%	75,000	30%
Unearned Revenue	10,000	5%	25,000	10%
Total Current Liabilities	70,000	35%	100,000	40%
Noncurrent Liabilities:				
Notes Payable	40,000	20%	50,000	20%
Total Liabilities	110,000	55%	150,000	60%
Stockholder Equity				
Common Stock	75,000	38%	80,000	32%
Ending Retained Earnings	15,000	8%	20,000	8%
Total Stockholders' Equity	90,000	45%	100,000	40%
Total Liabilities and Stockholder Equity	$200,000	100%	$250,000	100%

Figure 5.25 Common-Size Balance Sheet

LINK TO LEARNING

Common-Size Assets and Common-Size Liabilities and Equity

Visit the Apple, Inc. Common-Size Assets (https://openstax.org/r/common-size-assets) Balance Sheet and Common-Size Liabilities and Equity (https://openstax.org/r/common-size-liabilities) Balance Sheet provided by the Stock Analysis on Net website. Review the composition of the company's assets, liabilities, and equity. How have assets changed? Has capital structure changed? If so, what elements impacted the change?

Industry Comparison

Recall that a key benefit of common-size analysis is comparing the firm's performance to the industry. Expressing the figures on the income statement and balance sheet as percentages rather than raw dollar figures allows for comparison to other companies regardless of size differences.

Clear Lake Sporting Goods, for example, might compare their financial performance on their income statement to a key competitor, Charlie's Camping World. Charlie is a much bigger retailer for outdoor gear, as Charlie has nearly seven times greater sales than Clear Lake. Common-size statements allow Clear Lake to compare their statements in a meaningful way (see Figure 5.26). Notice that Clear Lake spends 50 percent of its sales on cost of goods sold while Charlie spends 59 percent. This is a significant difference that would be an indicator that Clear Lake and Charlie have key differences in their operations, purchasing policies, or general performance in their core products.

We know that Charlie is a bigger retailer, and we see this clearly in the rent expense as a percentage of sales. Charlie spends 11 percent of its sales on building rent, while Clear Lake spends only 5 percent. A clear difference in performance is hinted at here, alluding to Charlie spending more per square foot on rent or using its retail space differently, causing it to rent more space for its product per sales dollar than Clear Lake does. Depreciation expense, though not a large figure, is smaller for Charlie, giving us a hint that Charlie has less capital equipment than Clear Lake, perhaps tied to the higher rent expense. It is possible that Charlie rents some of its equipment, which would help explain the higher rent percentage. Finally, Charlie's salaries percentage is significantly higher at 12 percent than Clear Lake's 5 percent. Though the simple percentage does not tell us why, it does provide us a hint and allow for further questions or investigation. Charlie may pay its employees a much higher wage than Clear Lake. They may also have far more sales associates on the floor in their larger spaces than Clear Lake does in their smaller retail spaces.

Note that although we have compared just two years of data for Charlie and Clear Lake, it is more common to use several years of data to get a more robust view of long-term trends.

	Charlie's Camping World Common-Size Income Statement		Clear Lake Sporting Goods Common-Size Income Statement	
	Prior Year	Prior Year %	Current Year	Current Year %
Gross Sales	$748,000		$126,000	
Sales Returns & Allowances	41,000		6,000	
Net Sales	707,000	100%	120,000	100%
Cost of Goods Sold	418,000	59%	60,000	50%
Gross Profit	289,000	41%	60,000	50%
Rent Expense	75,000	11%	5,500	5%
Depreciation Expense	5,000	1%	3,600	3%
Salaries Expense	85,000	12%	5,400	5%
Utility Expense	25,000	4%	2,500	2%
Operating Income	99,000	14%	43,000	36%
Interest Expense	1,000	0%	2,000	2%
Income Tax Expense	14,000	2%	6,000	5%
Net Income	$ 84,000	12%	$ 35,000	29%

Figure 5.26 Comparison of Common-Size Income Statements

LINK TO LEARNING

Microsoft versus Apple

Locate the sales, cost of goods sold, and gross profit data for Microsoft (https://openstax.org/r/microsoft-financial)'s most current period using Yahoo! Finance. Calculate the common-size percentage for cost of goods sold and gross profit (cost of goods sold divided by net sales, and gross profit divided by net sales). Compare these percentages to those of Apple, Inc (https://openstax.org/r/common-size-income-statement)., a key competitor, on its common-size income statement. What can you learn about the performance of each firm based on these two percentages?

Now that you have covered the basic financial statements and a little bit about how they are used, where do we find them? How often are they prepared? Who gets them? In this next section we will explore the requirements for what needs to be reported, when, and to whom.

5.8 Reporting Financial Activity

Learning Outcomes

By the end of this section, you will be able to:

- Identify the most common types of accounting periods.
- Outline key considerations and accounting principles that dictate the timing of financial reporting.
- Describe the SEC reporting requirements relevant to financial statements.
- Identify the key elements of a company annual report.

Defining Accounting Periods

An accounting period can be any period of time, but the most common accounting periods are months, quarters, and a year. Accounting periods are important, particularly in accrual accounting, so there are clear cutoffs for recording transactions.

It is important to note that Generally Accepted Accounting Principles (GAAP) require companies to provide quarterly financial statements. However, most firms, even those not covered by GAAP, prepare financial statements monthly in order to provide timely data to their financial statement users both internally and externally.

Time Period Principle

Providing information to financial statement users in a timely fashion brings us to our next key topic, which is the time period principle. In order for information to be useful, it must be timely. This means that financial statement users need to get the statements quickly enough to be able to make relevant decisions with them. Providing statements on a timely basis is the foundation of the time period accounting principle.

Fiscal versus Calendar Year

Though a firm may present financial statements monthly, quarterly, and annually, not all time periods are created equal. Firms have the option to choose between a fiscal and a calendar year. The calendar year, which begins January 1 and ends December 31, is the traditional year we are accustomed to. The calendar year, however, doesn't always work well with a firm's business cycle. Due to seasonality or other factors, a firm may choose to adopt a fiscal year with their own beginning and end date. For example, a firm may choose to start its fiscal year on June 1 and end it on May 31. A firm can opt to change from a fiscal to a calendar year or vice versa but must only do so for a justifiable reason.

THINK IT THROUGH

Fiscal versus Calendar Year

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and review the title of the table of contents on page 1. The title reads "Apple Inc. Form 10-K." The next line provides information on the company's chosen time period. Does Apple use a fiscal or calendar year?

Solution:

Apple uses a fiscal year. Their fiscal year ended on September 26, 2020.

The Annual Report

An annual report is part of the 10-K filing that publicly traded firms must provide to the SEC and investors each year. The report details the firm's operations and performance for the year and its current financial conditions. The report contains general company information, the firm's financial statements, notes to the financial

statements, various required disclosures pertaining to accounting policies, the management discussion and analysis (MD&A) statement, a letter from the CEO to the shareholders, and the firm's audit report.

THINK IT THROUGH

Annual Report Key Elements

Visit the Apple, Inc. Annual Report (https://openstax.org/r/2020-doc-financial-annual-report) for 2020 and review the title of the table of contents on page 1. What key elements would you be drawn to review if you were a potential investor? In Part II of the table of contents, find the MD&A report (page 20) and use the table of contents link to navigate to it. What key highlights and challenges did the company report for the year?

Solution:

Answers may vary. A few key items of note might include risk factors, financial statements, the MD&A letter, and disclosures about market risk. Apple's MD&A reported significant updates on the impact of the COVID-19 pandemic, a few financial updates, product updates, segment performance data, a conversation about its debt, contractual obligations, and accounting policies.

SEC Reporting Requirements

The Securities and Exchange Commission (SEC) requires publicly traded firms to regularly provide several reports. The two most common are the 10-K and quarterly report (10-Q). Certain unique events also require an additional filing called an 8-K. There are many events that might require reporting. A few key examples include changes to rights of shareholders, changes in control of the company, and amendments to the company charter or bylaws.

The quarterly report is much like the annual report already discussed. It contains the firm's financial statements, required accounting disclosures, statements on internal control, and a management discussion and analysis (MD&A) letter. It does not contain an auditor's report, as firms are not audited on a quarterly basis. Quarterly reports are helpful to investors because they provide information on a timely enough basis to be relevant.

LINK TO LEARNING

Investor Relations

Visit the Apple, Inc. Investor Relations page, SEC Filings section (https://openstax.org/r/sec-filings). Notice how many different reports Apple files regularly with the SEC. Locate the most recent quarterly (10-Q) and annual reports (10-K) and scan the table of contents. How does the quarterly report differ from the annual report? Do you notice key items in the annual report not provided in the quarterly report?

Summary

5.1 The Income Statement

The income statement reflects a firm's performance over a period of time. Most financial statements are prepared monthly, quarterly, and annually. The income statement reflects sales less cost of goods sold to arrive at gross profit. Operating costs are deducted to arrive at operating income. Finally, other nonoperational costs like interest and taxes are deducted to arrive at net income.

5.2 The Balance Sheet

The balance sheet reflects the financial position of a firm as of a particular point in time. It is laid out to clearly depict and support the accounting equation: $\text{assets} = \text{liabilities} + \text{owner's equity}$. A classified balance sheet breaks down the assets and liabilities sections into current and noncurrent for greater transparency.

5.3 The Relationship between the Balance Sheet and the Income Statement

The financial statements are all tied together. The income statement is generated first, as net income is needed in order to determine the ending balance of retained earnings, a key account in the equity section of the balance sheet.

5.4 The Statement of Owner's Equity

The statement of owner's equity is divided by each type of equity the firm has: common stock, preferred stock, additional paid-in capital, and retained earnings, for example. Beginning balances are provided, and all key transactions impacting equity are provided in order to show how ending balance were derived. Key transactions commonly include recording net income or loss, issuing additional stock, and paying out dividends.

5.5 The Statement of Cash Flows

Under accrual accounting, transactions are recorded when they occur, not necessarily when cash moves. This creates a timing difference. Net profit, therefore, does not necessarily mean a firm has cash, and a net loss doesn't mean they don't have any cash. To reconcile net income to actual cash flow and see how a firm generates and uses its funds, a statement of cash flows is prepared. The statement reflects cash flow from operating activities, financing activities, and investing activities.

5.6 Operating Cash Flow and Free Cash Flow to the Firm (FCFF)

Operating cash flow reflects the cash generated by (or used by) the core business function. Free cash flow to the firm (FCFF) or simply free cash flow (FCF) is calculated by deducting capital expenditures from operating cash flow. FCF reflects the cash available to repay debts, pay dividends to shareholders, and contribute to cash needs for growth.

5.7 Common-Size Statements

Common-size statements are a restatement of the financial statements with all dollar figures restated as a percentage. On the income statement, each line is restated as a percentage of net sales. On the balance sheet, each line is restated as a percentage of total assets. Common-size statements are useful for analysis and are particularly helpful in comparing firms of different sizes.

5.8 Reporting Financial Activity

Publicly traded firms must file company and financial data with the Securities and Exchange Commission (SEC) on a regular basis. Key reports include the quarterly report, called a 10-Q, and the annual report, called a 10-K.

Key Terms

accounting equation $\text{assets} = \text{liabilities} + \text{owner's equity}$
accrual basis accounting system in which revenue is recorded or recognized when earned yet not necessarily received, and in which expenses are recorded when legally incurred and not necessarily when paid
assets tangible or intangible resources owned or controlled by a company, individual, or other entity with the intent that they will provide economic value
cash basis method of accounting in which transactions are not recorded in the financial statements until there is an exchange of cash
current assets asset typically used up, sold, or converted to cash in one year or less
current liabilities debt or obligation due within one year or, in rare cases, a company's standard operating cycle, whichever is greater
depreciation process of allocating the costs of a tangible asset over the asset's economic life
direct method approach used to determine net cash flows from operating activities, whereby accrual basis revenue and expenses are converted to cash basis collections and payments
dividends portion of the net worth (equity) that is returned to owners of a corporation as a reward for their investment
expenses costs associated with providing goods or services
free cash flow operating cash, reduced by expected capital expenditures
gains increases in organizational value from activities that are "incidental or peripheral" to the primary purpose of the business
income statement financial statement that measures the organization's financial performance for a given period of time
indirect method approach used to determine net cash flows from operating activities, starting with net income and adjusting for items that impact new income but do not require outlay of cash
liabilities probable future sacrifice of economic benefits arising from present obligations of a particular entity to transfer assets or provide services to other entities in the future as a result of past transactions or events
loss decrease in organizational value from activities that are "incidental or peripheral" to the primary purpose of the business
net income revenues and gains that are greater than expenses and losses
noncash expenses expenses that reduce net income but are not associated with a cash flow; most common example is depreciation expense
noncurrent assets assets used in the normal course of business for more than one year that are not intended to be resold
noncurrent liabilities liabilities that are expected to be settled in more than one year
owner's equity residual interest in the assets of an entity that remains after deducting its liabilities
retained earnings cumulative, undistributed net income or net loss for the business since its inception
revenue inflows or other enhancements of assets of an entity or settlements of liabilities from delivering or producing goods, rendering services, or other activities that constitute the entity's ongoing major or central operations
statement of cash flows financial statement listing the cash inflows and cash outflows for the business for a period of time

CFA Institute

This chapter supports some of the Learning Outcome Statements (LOS) in this CFA® Level I Study Session (https://openstax.org/r/media-document-study-session). Reference with permission of CFA Institute.

Multiple Choice

1. Which of the following is a measure of the performance of a firm's daily operations?
   a. gross profit
   b. cost of goods sold
   c. operating income
   d. net income

2. In which section of the classified balance sheet would you find a note payable due in six months?
   a. current assets
   b. current liabilities
   c. noncurrent liabilities
   d. common stock

3. Which financial statement must be prepared first?
   a. statement of retained earnings
   b. balance sheet
   c. statement of cash flows
   d. income statement

4. Which of the following represents earned capital on the statement of owner's equity?
   a. retained earnings
   b. common stock
   c. preferred stock
   d. additional paid-in capital

5. Which section of the statement of cash flows reflects the cash generated from or used by a company's day-to-day operations?
   a. investing activities
   b. financing activities
   c. operating activities
   d. noncash activities

6. How do you calculate free cash flow (FCF)?
   a. net income less dividends
   b. operating income less capital expenditures
   c. gross profit less depreciation
   d. net income plus interest

7. How do you calculate common-size analysis on the income statement?
   a. income statement line item/gross profit
   b. income statement line item/net income
   c. net sales/income statement line item
   d. income statement line item/net sales

8. Which of the following does not represent a filing commonly required by the SEC?
   a. annual report, 10-K
   b. quarterly report, 10-Q
   c. Form 8-K
   d. 1040

Review Questions

1. What is the difference between gross profit and net income?
2. If a classified balance sheet has total assets of $900,000 and total owner's equity of $350,000, what must the company's total liabilities be?
3. What key element of the income statement flows through to the balance sheet?
4. What key columns are commonly found on the statement of owner's equity?
5. Ted's firm reported net income for the current period of $65,750. Is it safe to assume that because Ted's firm reported such a large net income, it has plenty of cash to fund its operations? Why or why not?
6. What useful insights does free cash flow (FCF) provide in financial analysis?
7. Describe how common-size statements are useful.
8. What is the difference between a calendar year and a fiscal year?

Problems

1. Rickey's Retail has the following financial information for the most recent accounting period. Prepare an income statement.

Rickey's Retail Income Statement	
Gross Sales	$864,740
Sales Returns & Allowances	47,399
Cost of Goods Sold	483,237
Rent Expense	86,705
Interest Expense	1,156
Income Tax Expense	16,185
Depreciation Expense	5,780
Salaries Expense	98,266
Utility Expense	$ 28,902

2. Big Box has the following accounts. In which section of its classified balance sheet does each belong?
   Cash
   Wages payable
   Taxes payable
   Accounts receivable
   Retained earnings
   Common stock
   Land
   Note payable due in 10 years
   Prepaid insurance
3. Big Box Outlet has $10,350 of supplies expense on its income statement. Does this mean that there must also be a supplies payable account balance of $10,350 on its balance sheet? Why or why not?
4. What are the three key types of dividends a firm might distribute to their shareholders? Describe each.
5. Big Box Outlet had an increase in its accounts payable account this period and a decrease in its accounts receivable, took out a long-term note payable, paid dividends to its shareholders, had depreciation on its

equipment, bought new equipment, increased its inventory account, and repaid a bond. In which section of the statement of cash flows would each of these items appear?

6. Kokoya's Firm calculates its free cash flow at only $2,000, which the company feels is quite low based on its historical performance and compared to others in its industry. What actions might Kokoya's Firm take to improve its overall cash flow?

7. Complete the common-size income statement for Big Box Outlet using the information below:

Big Box Outlet Common-Size Income Statement		
	Current Year	**Current Year %**
Gross Sales	$1,089,836	_____
Sales Returns & Allowances	59,737	_____
Net Sales	1,030,099	_____
Cost of Goods Sold	609,026	_____
Gross Profit	421,073	_____
Rent Expense	109,275	_____
Depreciation Expense	7,285	_____
Salaries Expense	123,845	_____
Utility Expense	36,425	_____
Operating Income	144,243	_____
Interest Expense	1,457	_____
Income Tax Expense	20,398	_____
Net Income	$ 122,388	

8. List at least eight items commonly found in a firm's annual report filed with the SEC.

Video Activity

The Income Statement Explained

Click to view content (https://openstax.org/r/income-statement-explain)

1. In the video, you see two years of data for Imaginary Ltd. The company had the same sales and same operating profit in both years. However, its performance really wasn't the same from one year to the next. How do the details of the income statement help you see this? What key performance measures were you able to glean from the income statement?

2. Though the key elements of the income statement can be summarized as simply all the firm's revenues and expenses, how would you describe the elements of the income statement in more detail? Assume you were consulting for a friend who owns a small dairy farm. What items would you expect to see on their income statement? Make a list and be as detailed as you can. Curious if your list is accurate? Consider doing an online search to find an income statement for a dairy farm or similar business. Compare its income statement to your list to see if you had the right idea.

How the Balance Sheet Works

Click to view content (https://openstax.org/r/balance-sheet-works)

3. In the video, before the example started, a fair amount of discussion time was devoted to the accounting equation, double-entry accounting, and the past versus the present data. How do all of these concepts tie together in the balance sheet and the type of data we can hope to glean from a balance sheet?

4. Three key account types are represented on the balance sheet: assets, liabilities, and equity. Much like the income statement, however, there is a great deal more detail to a balance sheet than simply these three

figures. What additional accounts and detail can you find on the balance sheet? If you were to look up the balance sheet for the company that sells your favorite thing (e.g., coffee, your laptop, your phone, candy), what types of accounts do you think you will find on its balance sheet? Consider doing an online search for the company's balance sheet to see if your guess is correct.

6

Measures of Financial Health

Figure 6.1 Organizations must continually measure their financial health in order to remain successful. (credit: "Money" by Pictures of Money/flickr, CC BY 2.0)

Chapter Outline

Why It Matters

Financial analysis is a crucial element of business, but it can be used in personal finance as well. It differs depending on the role and perspective of those performing the analysis. For example, your personal accountant will have different goals and needs in making recommendations to you about your personal finances. All accounting professionals use financial analysis to check for validity, accurate data, compliance in reporting, and more.

Some tactics for managing your personal finances can be the same as for managing business finances. For example, reducing expenses and maximizing returns on long-term investments are always good practices. Debt can also be a beneficial tool in both personal and professional finances when used appropriately. Debt is neither inherently good nor bad; it simply needs to be properly managed in order to achieve a reasonable return in exchange for the cost and risk it poses.

Though the process and tools may be similar, financial analysis from a business perspective has different goals and needs. Investors are looking to identify firm performance, financial health, and profitability. Financial analysts closely review information found on financial statements so they can make informed business decisions. The income statement, statement of retained earnings, balance sheet, and statement of cash flows, among other financial information, are analyzed for internal and external stakeholders and provide a company with valuable information about its overall performance and specific areas for improvement. The analysis can

help with budgeting and making decisions about where the company could cut costs, how it might increase revenues, and what capital investment opportunities it should pursue.

LINK TO LEARNING

Financial Analyst

Lots of individuals and companies perform financial analysis. One of these roles is that of a financial analyst. The skills and qualifications of a financial analyst vary widely from one industry to another, but there are a number of similarities in individuals who hold these roles. As you watch the video about financial analysts (https://openstax.org/r/video-about-financial-analysts), consider your own career path and how your skills, abilities, and interests may fit this role.

6.1 Ratios: Condensing Information into Smaller Pieces

By the end of this section, you will be able to:

- Explain the importance of financial statement analysis in making informed decisions about business opportunities.
- Outline the limitations of financial statement analysis in making investment decisions.

When considering the outcomes from analysis, it is important for a company to understand that the data generated needs to be compared to similar data within the industry at large as well as that of close competitors. The company should also consider its past experience and how it corresponds to current and future performance expectations.

Importance of Ratios and Analysis

Financial ratios help internal and external stakeholders make informed decisions about actions like investing, becoming a supplier, making a loan, or altering internal operations, among other things. The information resulting from ratio analysis can be used to examine trends in performance, establish benchmarks for success, set budget expectations, and compare industry competitors. There are four main types of ratios: liquidity, solvency, efficiency, and profitability. While outcomes for some ratios may seem more ideal, the industry in which the business operates can change the influence of these outcomes on stakeholder decisions.

There are several benefits to analyzing financial statements. The information can show trends over time, which can help in making future business decisions. Converting information to percentages or ratios eliminates some of the disparities between competitors' sizes and operating abilities, making it easier for stakeholders to make informed decisions. It can assist with understanding the makeup of current operations within the business and which shifts need to occur internally to increase productivity.

Limitation of Financial Statement Analysis

Though useful, it's important to note that there are limitations to financial statement analysis as well. Stakeholders need to remember that past performance does not always predict future performance. Economic influences, such as inflation or a recession, could skew the data being analyzed. Additionally, the way a company reports information may change over time. For example, there could be changes in where and when certain transactions are recorded, and this may not be immediately evident to financial statement users. It is also key to note that though all publicly traded companies in the United States are required to follow Generally Accepted Accounting Principles (GAAP), there are many estimates and flexibility in how some standards are applied. This means that firms can still follow accounting standards appropriately but present some information differently from other firms.

It makes good sense for a company to use financial statement analysis to guide future operations so it can

budget properly, control costs, increase revenues, and make long-term expenditure decisions. As long as stakeholders understand the limitations of financial statement analysis, it is a useful way to predict growth and financial strength.

Despite limitations, ratios are still a valuable tool if used appropriately. The next section discusses several operating efficiency ratios including accounts receivable turnover, total asset turnover, inventory turnover, and days' sales in inventory. Operating efficiency ratios help users see how well management is using the financial assets of the firm.

6.2 Operating Efficiency Ratios

By the end of this section, you will be able to:

- Calculate accounts receivable turnover to assess a firm's performance in managing customer receivables.
- Evaluate management's use of assets using total asset turnover and inventory turnover.
- Assess organizational performance using days' sales in inventory calculations.

Efficiency ratios show how well a company uses and manages its assets, one key element of financial health. Important areas of efficiency are the management of sales, accounts receivable, and inventory. A company that is efficient will usually be able to generate revenues quickly using the assets it has acquired. Let's examine four **efficiency ratios**: accounts receivable turnover, total asset turnover, inventory turnover, and **days' sales in inventory**.

Accounts Receivable Turnover

For our discussion of financial statement analysis, we will look at Clear Lake Sporting Goods. Clear Lake Sporting Goods is a small merchandising company (a company that buys finished goods and sells them to consumers) that sells hunting and fishing gear. Figure 6.2 shows the comparative income statements and balance sheets for the past two years.

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Prior Year	Current Year
Net Sales	$100,000	$120,000
Cost of Goods Sold	50,000	60,000
Gross Profit	50,000	60,000
Rent Expense	5,000	5,500
Depreciation Expense	2,500	3,600
Salaries Expense	3,000	5,400
Utility Expense	1,500	2,500
Operating Income	38,000	43,000
Interest Expense	3,000	2,000
Income Tax Expense	5,000	6,000
Net Income	$ 30,000	$ 35,000

Clear Lake Sporting Goods
Comparative Year-End Balance Sheets

	Prior Year	Current Year
Assets:		
Cash	$ 90,000	$110,000
Accounts Receivable	20,000	30,000
Inventory	35,000	40,000
Short-Term Investments	15,000	20,000
Total Current Assets	160,000	200,000
Equipment	40,000	50,000
Total Assets	$200,000	$250,000
Liabilities:		
Accounts Payable	$ 60,000	$ 75,000
Unearned Revenue	10,000	25,000
Total Current Liabilities	70,000	100,000
Notes Payable	40,000	50,000
Total Liabilities	$110,000	$150,000
Stockholder Equity		
Common Stock	75,000	80,000
Ending Retained Earnings	15,000	20,000
Total Stockholders' Equity	90,000	100,000
Total Liabilities and Stockholder Equity	$200,000	$250,000

Figure 6.2 Comparative Income Statements and Year-End Balance Sheets Note that the comparative income statements and

balance sheets have been simplified here and do not fully reflect all possible company accounts.

To begin an analysis of receivables, it's important to first understand the cycles and periods used in the calculations.

Operating Cycle

A period is one **operating cycle** of a business. The operating cycle includes the cash conversion cycle plus the accounts receivable cycle (discussed below). Essentially it is the time it takes a business to purchase or make inventory and then sell it. For example, assume Clear Lake Sporting Goods orders and receives a shipment of fishing lures on June 1. It stocks the shelves with lures and tracks its inventory and sales. By July 15, all the lures from that shipment are gone. In this example, Clear Lake's operating cycle is 45 days.

Cash Conversion Cycle

Cash, however, doesn't necessarily flow linearly with accounting periods or operating cycles. The cash conversion cycle is the time it takes to spend cash to purchase inventory, produce the product, sell it, and then collect cash from the customer. Accounts receivable is one section of that cycle. Referring to Clear Lake's June 1 shipment of lures that sold by July 15, assume that some of the customers were fishing guides that keep an open account with Clear Lake. This company did not pay for its lures until August 15 when it settled its account. In this example, Clear Lake's cash cycle is 75 days.

Let's take a look at the accounts receivable turnover ratio, which helps assess that element of the cash conversion cycle.

Accounts Receivable Turnover Ratio

Receivables ratios show company performance in relation to current receivables (what is due from customers), as well as credit policy effect on sales growth. One receivables ratio is called the **accounts receivable turnover ratio**.This ratio determines how many times (i.e., how often) accounts receivable are collected during a year and converted to cash. A higher number of times indicates that receivables are collected quickly. This quick cash collection may be viewed as a positive occurrence because liquidity improves, and the company may reinvest in its business sooner when the value of the dollar has more buying power (time value of money). The higher number of times may also be a negative occurrence, signaling that credit extension terms are too tight, and it may exclude qualified consumers from purchasing. Excluding these customers means that they may take their business to a competitor, thus reducing potential sales.

In contrast, a lower number of times indicates that receivables are collected at a slower rate. A slower collection rate could signal that lending terms are too lenient; management might consider tightening lending opportunities and more aggressively pursuing payment from its customers. The lower turnover also shows that the company has cash tied up in receivables longer, thus hindering its ability to reinvest this cash in other current projects. The lower turnover rate may signal a high level of bad debt accounts. The determination of a high or low turnover rate really depends on the standards of the company's industry. It's key to note the tradeoff in adjusting credit terms. Loose credit terms may attract more customers but may also increase bad debt expense. Tighter credit terms may attract fewer customers but may also reduce bad debt expense.

The formula for accounts receivable turnover is

$$\text{Accounts Receivable Turnover} = \frac{\text{Net Credit Sales}}{\text{Average Accounts Receivable}}$$

$$\text{Average Accounts Receivable} = \frac{\text{Beginning Accounts Receivable} + \text{Ending Accounts Receivable}}{2}$$

Net credit sales are sales made on credit only; cash sales are not included because they do not produce receivables. However, many companies do not report credit sales separately from cash sales, so "net sales" may be substituted for "net credit sales" in this case. Beginning and ending accounts receivable refer to the

beginning and ending balances in accounts receivable for the period. The beginning accounts receivable balance is the same figure as the ending accounts receivable balance from the prior period.

When computing the accounts receivable turnover for Clear Lake Sporting Goods, let's assume net credit sales make up $100,000 of the $120,000 of the net sales found on the income statement in the current year.

$$\text{Average Accounts Receivable} = \frac{\$20{,}000 + \$30{,}000}{2} = \$25{,}000$$

$$\text{Accounts Receivable Turnover} = \frac{\$100{,}000}{\$25{,}000} = 4 \text{ times}$$

To gain a better understanding of its ratio performance, Clear Lake Sporting Goods can compare its turnover to industry averages, key competitors, and its own historical ratios. Given this outcome, the managers may want to consider stricter credit lending practices to make sure credit customers are of a higher quality. They may also need to be more aggressive with collecting any outstanding accounts.

THINK IT THROUGH

Accounts Receivable Turnover

You are a consultant assessing cash management practices for two firms, Company A and Company B (see Figure 6.3).

	Company A	Company B
Beginning Accounts Receivable	$ 50,000	$ 60,000
Ending Accounts Receivable	80,000	90,000
Net Credit Sales	$550,000	$460,000

Figure 6.3 Financial Information for Company A and Company B

Based on the information provided, do the following:

- Compute the accounts receivable turnover ratio.
- Interpret the outcomes, indicating how each company is performing

Solution:

Company A: $\text{ART} = 8.46$ times, Company B: $\text{ART} = 6.13$ times. Upon initial review of this limited information, Company A seems to be performing better since its turnover ratio is higher. Accounts receivable turnover has a significant impact on cash flows. One might want more information on trends for each company with these ratios and a comparison to others in the same industry. More information is helpful in assessing performance.

LINK TO LEARNING

American Superconductor Corporation

American Superconductor Corporation specializes in the production and service of energy-efficient wind turbine systems, as well as energy grid construction solutions. On the company's 2019 financial statement (https://openstax.org/r/financial-statement), the accounts receivable turnover ratio is approximately 6.32 times.

Total Asset Turnover

Total asset turnover measures the ability of a company to use its assets to generate revenues. A company would like to use as few assets as possible to generate the most net sales. Therefore, a higher total asset turnover means the company is using their assets very efficiently to produce net sales. The formula for total asset turnover is

$$\text{Total Asset Turnover} = \frac{\text{Net Sales}}{\text{Average Total Assets}}$$

$$\text{Average Total Assets} = \frac{\text{Beginning Total Assets} + \text{Ending Total Assets}}{2}$$

Average total assets are found by dividing the sum of beginning and ending total assets balances found on the balance sheet. The beginning total assets balance in the current year is taken from the ending total assets balance in the prior year.

Clear Lake Sporting Goods' total asset turnover is

$$\text{Average Total Assets} = \frac{\$200{,}000 + \$250{,}000}{2} = \$225{,}000$$

$$\text{Total Asset Turnover} = \frac{\$120{,}000}{\$225{,}000} = 0.53 \text{ times (rounded)}$$

The outcome of 0.53 means that for every $1 of assets, $0.53 of net sales are generated. Over time, Clear Lake Sporting Goods would like to see this turnover ratio increase.

Inventory Turnover

Inventory turnover measures how many times during the year a company has sold and replaced inventory. This can tell a company how well inventory is managed. A higher ratio is preferable; however, an extremely high turnover may mean that the company does not have enough inventory available to meet demand. A low turnover may mean the company has too much supply of inventory on hand. The formula for inventory turnover is

$$\text{Inventory Turnover} = \frac{\text{Cost of Goods Sold}}{\text{Average Inventory}}$$

$$\text{Average Inventory} = \frac{\text{Beginning Inventory} + \text{Ending Inventory}}{2}$$

Cost of goods sold for the current year is found on the income statement. Average inventory is found by dividing the sum of beginning and ending inventory balances found on the balance sheet. The beginning inventory balance in the current year is taken from the ending inventory balance in the prior year.

Clear Lake Sporting Goods' inventory turnover is

$$\text{Average Inventory} = \frac{\$35{,}000 + \$40{,}000}{2} = \$37{,}500$$

$$\text{Inventory Turnover} = \frac{\$60{,}000}{\$37{,}500} = 1.6 \text{ times}$$

A ratio of 1.6 times seems to be a very low turnover rate for Clear Lake Sporting Goods. This may mean the company is maintaining too high an inventory supply to meet a low demand from customers. Managers may want to decrease their on-hand inventory to free up more liquid assets to use in other ways. Keep in mind, ratios should not be taken out of context. One ratio alone can't tell the whole story. Ratios should be used with caution and in conjunction with other ratios and additional financial and contextual information.

As with accounts receivable, there is a trade-off to consider in managing inventory. Low turnover will usually

mean a low risk of stockouts and the ability to carry more of what customers are looking for. But high inventory levels will mean that more cash is tied up in inventory. High turnover will mean carrying less inventory and the higher risk of stockouts, causing customers to go elsewhere to find what they need.

Figure 6.4 Inventory turnover can help determine how well a company manages its inventory. (credit: "Untitled" by Marcin Wichary/ flickr, CC BY 2.0)

LINK TO LEARNING

Target Corporation

As we have learned, the inventory turnover ratio shows how well a company manages its inventory. Look through the financial statements in the 2019 Annual Report for Target (https://openstax.org/r/annual-report-for-target) and calculate the inventory turnover ratio. What does the outcome mean for Target?

Days' Sales in Inventory

Days' sales in inventory expresses the number of days it takes a company to turn inventory into sales. The fewer the number of days, the more quickly the company can sell its inventory. The greater the number of days, the longer it takes to sell its inventory. The formula for days' sales in inventory is

$$\text{Days' Sales in Inventory} = \frac{\text{Ending Inventory}}{\text{Cost of Goods Sold}} \times 365$$

Clear Lake Sporting Goods' days' sales in inventory is

$$\text{Days' Sales in Inventory} = \frac{\$40{,}000}{\$60{,}000} \times 365 = 243 \text{ days (rounded)}$$

Depending on the industry, 243 days may be a long time to sell inventory. While industry dictates what is an acceptable number of days to sell inventory, 243 days is likely to be unsustainable long-term. Remember, it's important to not take one ratio out of context. Review the ratio in conjunction with other ratios and other financial data. For example, we might review the days' sales in inventory along with accounts receivable turnover for Clear Lake Sporting Goods relative to the industry average to get a better picture of Clear Lake's performance in this area.

6.3 Liquidity Ratios

By the end of this section, you will be able to:

- Calculate current, quick, and cash ratios to assess a firm's liquidity and make informed business decisions.
- Assess organizational performance using liquidity ratios.

Liquidity refers to the business's ability to manage current assets or convert assets into cash in order to meet short-term cash needs, another aspect of a firm's financial health. Examples of the most liquid assets include cash, accounts receivable, and inventory for merchandising or manufacturing businesses. The reason these are among the most liquid assets is that these assets will be turned into cash more quickly than land or buildings, for example. Accounts receivable represents goods or services that have already been sold and will typically be paid/collected within 30 to 45 days.

Inventory is less liquid than accounts receivable because the product must first be sold before it generates cash (either through a cash sale or sale on account). Inventory is, however, more liquid than land or buildings because, under most circumstances, it is easier and quicker for a business to find someone to purchase its goods than it is to find a buyer for land or buildings.

Current Ratio

The **current ratio** is closely related to working capital; it represents the current assets divided by current liabilities. The current ratio utilizes the same amounts as working capital (current assets and current liabilities) but presents the amount in ratio, rather than dollar, form. That is, the current ratio is defined as current assets/current liabilities. The interpretation of the current ratio is similar to working capital. A ratio of greater than one indicates that the firm has the ability to meet short-term obligations with a buffer, while a ratio of less than one indicates that the firm should pay close attention to the composition of its current assets as well as the timing of the current liabilities.

$$\text{Current Ratio} = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

The current ratio in the current year for Clear Lake Sporting Goods is

$$\text{Current Ratio} = \frac{\$200{,}000}{\$100{,}000} = 2 \text{ or } 2{:}1$$

A 2:1 ratio means the company has twice as many current assets as current liabilities; typically, this would be plenty to cover obligations. A 2:1 ratio is actually quite high for most companies and most industries. Again, it's recommended that ratios be used in conjunction with one another. An analyst would likely look at the high current ratio and low accounts receivable turnover to begin asking questions about management performance, as this might indicate a trouble area (high inventory and slow collections).

LINK TO LEARNING

Target Corporation

As we have learned, the current ratio shows how well a company can cover short-term liabilities with short-term assets. Look through the balance sheet in the 2019 Annual Report for Target (https://openstax.org/r/annual-report-for-target) and calculate the current ratio. What does the outcome mean for Target?

Quick Ratio

The **quick ratio**, also known as the *acid-test ratio*, is similar to the current ratio except current assets are more narrowly defined as the most liquid assets, which exclude inventory and prepaid expenses. The conversion of

inventory and prepaid expenses to cash can sometimes take more time than the liquidation of other current assets. A company will want to know what it has on hand and can use quickly if an immediate obligation is due. The formula for the quick ratio is

$$\text{Quick Ratio} = \frac{\text{Cash} + \text{Short-Term Investments} + \text{Accounts Receivable}}{\text{Current Liabilities}}$$

The quick ratio for Clear Lake Sporting Goods in the current year is

$$\text{Quick Ratio} = \frac{\$110{,}000 + \$20{,}000 + \$30{,}000}{\$100{,}000} = 1.6 \text{ or } 1.6{:}1$$

A 1.6:1 ratio means the company has enough quick assets to cover current liabilities. It's again key to note that a single ratio shouldn't be used out of context. A 1.6 ratio is difficult to interpret on its own. Industry averages and trend analysis for Clear Lake Sporting Goods would also be helpful in giving the ratio more meaning.

LINK TO LEARNING

Target Corporation

As we have learned, the quick ratio shows how quickly a company can liquidate current assets to cover current liabilities. Look through the financial statements in the 2019 Annual Report for Target (https://openstax.org/r/annual-report-for-target) and calculate the quick ratio. What does the outcome mean for Target?

Cash Ratio

Cash is the most liquid asset a company has, and **cash ratio** is often used by investors and lenders to asses an organization's liquidity. It represents the firm's cash and cash equivalents divided by current liabilities and is a more conservative look at a firm's liquidity than the current or quick ratios. The ratio is reflected as a number, not a percentage. A cash ratio of 1.0 means the firm has enough cash to cover all current liabilities if something happened and it was required to pay all current debts immediately. A ratio of less than 1.0 means the firm has more current liabilities than it has cash on hand. A ratio of more than 1.0 means it has enough cash on hand to pay all current liabilities and still have cash left over. While a ratio greater than 1.0 may sound ideal, it's important to consider the specifics of the company. Sitting on idle cash is not ideal, as the cash could be used to earn a return. And having a ratio less than 1.0 isn't always bad, as many firms operate quite successfully with a ratio of less than 1.0. Comparing the company ratio with trend analysis and with industry averages will help provide more insight.

$$\text{Cash Ratio} = \frac{\text{Cash and Cash Equivalents}}{\text{Current Liabilities}}$$

The cash ratio for Clear Lake Sporting Goods in the current year is:

$$\text{Cash Ratio} = \frac{\$110{,}000}{\$100{,}000} = 1.1$$

A 1.1 ratio means the company has enough cash to cover current liabilities.

Figure 6.5 Cash is the most liquid asset a company has and is often used by investors and lenders to assess an organization's liquidity. (credit: "20 US Dollar" by Jack Sem/flickr CC BY 2.0)

6.4 Solvency Ratios

By the end of this section, you will be able to:

- Evaluate organizational solvency using the debt-to-assets and debt-to-equity ratios.
- Calculate the times interest earned ratio to assess a firm's ability to cover interest expense on debt as it comes due.

Solvency implies that a company can meet its long-term obligations and will likely stay in business in the future. Meeting long-term obligations includes the ability to pay any interest incurred on long-term debt. Two main solvency ratios are the debt-to-equity ratio and the times interest earned ratio.

Debt-to-Assets Ratio

The **debt-to-assets ratio** shows the relationship between debt and assets. It reflects how much of the assets of the business was financed through debt. It reflects the company's leverage and is helpful to analysts in comparing how leveraged one company is compared to another.

Debts normally carry interest expense and must be repaid. The debt-to-assets ratio includes all debt—both long-term debt and current liabilities. The formula for the debt-to-assets ratio is

$$\text{Debt-to-Assets Ratio} = \frac{\text{Current Liabilities} + \text{Long-Term Liabilities}}{\text{Total Assets}}$$

The information needed to compute the debt-to-assets ratio for Clear Lake Sporting Goods in the current year can be found on the balance sheet. The debt-to-assets ratio for Clear Lake Sporting Goods in the current year is

$$\text{Debt-to-Assets Ratio} = \frac{\$100{,}000 + \$50{,}000}{\$250{,}000} = 0.6 \text{ or } 60\%$$

This means that 60 percent of Clear Lake's assets are financed by debt. We can also then infer that the other 40 percent is financed by equity. A ratio higher than 1.0 means the company has more debts than assets, which means it has negative equity. In Clear Lake's case, a 60 percent debt-to-assets ratio indicates some risk, but perhaps not a high risk. Comparing Clear Lake's ratio to industry averages would provide better insight.

LINK TO LEARNING

Target Corporation

As we have learned, the debt-to-assets ratio shows the relationship between a firm's debt and assets. Look through the financial statements in the 2019 Annual Report for Target (https://openstax.org/r/annual-report-for-target) and calculate the debt-to-assets ratio. What does the outcome mean for Target?

Debt-to-Equity Ratio

The **debt-to-equity ratio** shows the relationship between debt and equity as it relates to business financing. A company can take out loans, issue stock, and retain earnings to be used in future periods to keep operations running. A key difference in debt and equity is the interest expense repayment that a loan carries as opposed to equity, which does not have this requirement. Therefore, a company wants to know how much debt and equity contribute to its financing. The formula for the debt-to-equity ratio is

$$\text{Debt-to-Equity Ratio} = \frac{\text{Total Liabilities}}{\text{Total Stockholder Equity}}$$

The information needed to compute the debt-to-equity ratio for Clear Lake Sporting Goods in the current year can be found on the balance sheet.

$$\text{Debt-to-Equity Ratio} = \frac{\$150{,}000 + \$50{,}000}{\$100{,}000} = 1.5 \text{ or } 1.5{:}1$$

This means that for every one dollar of equity contributed toward financing, $1.50 is contributed from lenders. Recall that total assets equal total liabilities plus total equity. Both the debt-to-assets and debt-to-equity ratio have total liabilities in the numerator. The difference in the two ratios is the denominator. The denominator for the debt-to-equity ratio is total stockholder equity. The denominator for the debt-to-assets ratio is total assets, or total liabilities plus total equity. Thus, the two ratios contain the same information, making calculating both ratios redundant. A financial analyst may prefer to calculate one ratio over the other because of the format of readily available industry data to use for comparison purposes or for consistency with other calculations the analyst is performing.

THINK IT THROUGH

Financing a Business Expansion

You are the CFO of a small corporation. The president, who is one of five shareholders, has created an innovative new product that is testing well with substantial demand. To begin manufacturing, $400,000 is needed to acquire the equipment. The corporation's balance sheet shows total assets of $2,400,000 and total liabilities of $600,000. Most of the liabilities relate to debt that carries a covenant requiring that the company maintain a debt-to-equity ratio not exceeding 0.50. Determine the effect that each of the two options of obtaining additional capital will have on the debt covenant.

Solution:

We know the total liabilities for the firm to be $600,000. Using the accounting equation, we can find that the firm has $1,800,000 in equity. $\$600{,}000/\$1{,}800{,}000 =$ current debt-to-equity ratio of 0.33, which is well below the requirement for the debt covenant. If the firm issues debt, the ratio changes to $\$1{,}000{,}000/\$1{,}800{,}000$, which is 0.55 and would violate the debt covenant. If the firm chooses to issue additional stock, the new debt-to-equity ratio would be $\$600{,}000/\$2{,}200{,}000$, which is 0.27. This is well below the requirements in the debt covenant.

Times Interest Earned (TIE) Ratio

The **times interest earned (TIE) ratio** measures the company's ability to pay interest expense on all debt incurred. This ability to pay is determined by the available earnings before interest and taxes (EBIT) are deducted. These earnings are considered the operating income. Lenders will pay attention to this ratio before extending credit. The more times over a company can cover interest, the more likely a lender will extend long-term credit. The formula for times interest earned is

$$\text{Times Interest Earned} = \frac{\text{Earnings Before Interest and Taxes (EBIT)}}{\text{Interest Expense}}$$

The information needed to compute times interest earned for Clear Lake Sporting Goods in the current year can be found on the income statement.

$$\text{Times Interest Earned} = \frac{\$43{,}000}{\$2{,}000} = 21.5 \text{ times}$$

The $43,000 is the operating income, representing earnings before interest and taxes. The 21.5 times outcome suggests that Clear Lake Sporting Goods can easily repay interest on an outstanding loan and creditors would have little risk that Clear Lake Sporting Goods would be unable to pay.

LINK TO LEARNING

Times Interest Earned

This video about times interest earned (https://openstax.org/r/video-about-times) explains how to calculate it and why the ratio is useful, and it provides an example.

6.5 Market Value Ratios

By the end of this section, you will be able to:

- Calculate earnings per share to determine the portion of profit allocated to each outstanding share of common stock.
- Evaluate firm value using the price/earnings ratio and book value per share.

In this section we will turn our attention to **market value ratios**, measures used to assess a firm's overall market price. Common ratios used include earnings per share, the price/earnings ratio, and book value per share.

Earnings per Share (EPS)

Earnings per share (EPS) measures the portion of a corporation's profit allocated to each outstanding share of common stock. An increasing earnings per share can drive up a stock price. Conversely, falling earnings per share can lower a stock's market price. Earnings per share is also a component in calculating the price-to-earnings ratio (the market price of the stock divided by its earnings per share), which many investors find to be a key indicator of the value of a company's stock.

It's key to note, however, that EPS, like any ratio, should be used with caution and in tandem with other ratios and contextual data. Many financial professionals choose not to rely on income statement data and, similarly, EPS because they feel the cash flow statement provides more reliable and insightful information.

CONCEPTS IN PRACTICE

Alibaba Group Earnings Announcements Continue to Exceed Market Expectations

Alibaba, a Chinese-based company traded in the United States, exceeded market expectations in 2020 quarterly earnings releases. In the November 2020 earnings release, Alibaba reported earnings per share of 17.97 yuan versus market estimates of 14.33. Despite many companies struggling due to the pandemic, Alibaba reported strong earnings as a result of the surge in online shopping and remote work.

(sources: "Alibaba Beats Estimates as Pandemic Fuels Online, Cloud Computing Demand." *CNBC.* August 20, 2020. https://www.cnbc.com/2020/08/20/alibaba-beats-quarterly-revenue-estimates.html; Emily Bary. "Alibaba Earnings Top Expectations as Pandemic Drives Increased Digital Purchases. *Market Watch.* August 20, 2020. https://www.marketwatch.com/story/alibaba-earnings-top-expectations-as-pandemic-drives-increased-digital-purchases-2020-08-20; Matthew Johnston. "Alibaba Earnings: What Happened." *Investopedia*. November 5, 2020. https://www.investopedia.com/alibaba-q2-2021-earnings-5085444; Chris Versace. "Why S&P 500 EPS Expectations Showcase the Need for Thematic Investing." *Tematica Research.* June 3, 2020. https://www.tematicaresearch.com/why-sp-500-eps-expectations-showcase-the-need-for-thematic-investing)

Calculating Earnings per Share

Earnings per share is the profit a company earns for each of its outstanding common shares. Both the balance sheet and income statement are needed to calculate earnings per share. The balance sheet provides details on the preferred dividend rate, the total par value of the preferred stock, and the number of common shares outstanding. The income statement indicates the net income for the period. The formula to calculate basic earnings per share is

$$\text{Earnings per Share} = \frac{\text{Net Income} - \text{Preferred Dividends}}{\text{Weighted Average Common Shares Outstanding}}$$

By removing the preferred dividends from net income, the numerator represents the profit available to common shareholders. Because preferred dividends represent the amount of net income to be distributed to preferred shareholders, this portion of the income is obviously not available for common shareholders. While a number of variations of measuring a company's profit, such as NOPAT (net operating profit after taxes) and EBITDA (earnings before interest, taxes, depreciation, and amortization), are used in the financial world, GAAP requires companies to calculate earnings per share based on a corporation's net income, as this amount appears directly on a company's income statement, which for public companies must be audited.

In the denominator, only common shares are used to determine earnings per share because earnings per share is a measure of earnings for each common share of stock. The denominator can fluctuate throughout the year as a company issues and buys back shares of its own stock. The weighted average number of shares is used on the denominator because of this fluctuation. To illustrate, assume that a corporation began the year with 600 shares of common stock outstanding and then on April 1 issued 1,000 more shares. During the period January 1 to March 31, the company had the original 600 shares outstanding. Once the new shares were issued, the company had the original 600 plus the new 1,000 shares, for a total of 1,600 shares for each of the next nine months—from April 1 to December 31. To determine the weighted average shares, apply these fractional weights to both of the stock amounts (see Figure 6.6).

Number of Shares	×	Portion of Year	=	Weighted Shares
600	×	3/12	=	150
1,600	×	9/12	=	1,200
Weighted Average Shares				1,350

Figure 6.6 Weighted Shares

If the shares were not weighted, the calculation would not consider the time period during which the shares were outstanding.

To illustrate how earnings per share is calculated, assume Clear Lake Sporting Goods earns $35,000 in net income during the current year. During the year, the company also declared a $5,000 dividend on preferred stock and a $6,000 dividend on common stock. The company had 8,000 common shares outstanding the entire year. Clear Lake Sporting Goods has generated $3.75 of earnings ($35,000 less the $5,000 of preferred dividends) for each of the 8,000 common shares of stock it has outstanding.

$$\text{Earnings per Share} = \frac{\$35{,}000 - \$5{,}000}{8{,}000} = \$3.75$$

Measuring Performance with Earnings per Share

Earnings per share is a key profitability measure that both current and potential common stockholders monitor. Its importance is accentuated by the fact that GAAP requires public companies to report earnings per share on the face of a company's income statement. This is the only ratio that requires such prominent reporting. If fact, public companies are required to report two different earnings per share amounts on their income statements—basic and diluted. We've illustrated the calculation of basic earnings per share. Diluted earnings per share, which is not demonstrated here, involves the consideration of all securities, such as stocks and bonds, that could potentially dilute, or reduce, the basic earnings per share.

LINK TO LEARNING

Finding Earning per Share for Public Companies

Where can you find earnings per share information on public companies? Use the Yahoo! Finance (https://openstax.org/r/yahoo-finance) website to look up stock and earnings per share data for Tellurian Inc. (TELL), Amazon (AMZN), or CVS Pharmacy (CVS). Or use the search function to search for earnings per share data for your favorite corporation. Enter the ticker for the company you are looking up, and a basic chart and graph will display with stock price data along with several commonly used ratios (including earnings per share).

As you review data, keep in mind that a company can manipulate or impact its earnings per share by issuing new shares or buying back issued shares. What are the ethical implications of earnings per share calculations?

Common stock shares are normally purchased by investors to generate income through dividends or to sell at a profit in the future. Investors realize that inadequate earnings per share can result in poor or inconsistent dividend payments and fluctuating stock prices. As such, companies seek to produce earnings per share amounts that rise each period. However, an increase in earnings per share may not always reflect favorable performance, as there are multiple reasons that earnings per share may increase. One way earnings per share can increase is through increased net income. On the other hand, it can also increase when a company buys back its own shares of stock.

For example, assume that Clear Lake Sporting Goods generated net income of $30,000 and paid out $3,000 in

preferred shareholder dividends last year. In addition, 10,550 shares of common stock were outstanding throughout the entire year. In January of the current year, the company buys back shares of its common stock and holds them as treasury shares, making its current weighted average shares outstanding for this year 8,000. Net income for the current year is $35,000, $5,000 of which was paid to preferred shareholders in dividends. In the prior year, the company's earnings per share was

$$\text{Earnings per Share} = \frac{\$30{,}000 - \$3{,}000}{10{,}550} = \$2.56$$

Clear Lake Sporting Goods' current year earnings per share is

$$\text{Earnings per Share} = \frac{\$35{,}000 - \$5{,}000}{8{,}000} = \$3.75$$

The purchase of treasury stock in the current year reduces the common shares outstanding to 8,000 because treasury shares are considered issued but not outstanding. Earnings per share for the current year is now $3.75 per share even though earnings only increased by $5,000. It's key to note the impact of purchasing treasury stock and the intentions in doing so. Treasury stock is commonly purchased for a variety of reasons, but doing so to intentionally manipulate earnings per should not be a primary reason.

This increase in earnings per share occurred because the net income is now spread over fewer shares of stock. Similarly, earnings per share can decline even when a company's net income increases if the number of shares increases at a higher degree than net income.

CONCEPTS IN PRACTICE

Stock Buybacks Can Drive Up Earnings per Share: Ethical?

As many companies struggled to make ends meet or meet their cash flow needs amid the COVID-19 pandemic, some companies continued to thrive. Apple continued to have a healthy financial position with ample cash supply. It repurchased $18.5 billion of its own stock in the second quarter of 2020.[1] The total stock buyback over the preceding five years was $282.87 billion, which is 3.5 times higher than any other company. Since the earnings per share calculation is earnings divided by average outstanding shares, the fewer shares there are outstanding, the higher earnings per share goes without the firm having to actually raise earnings.

What do you think? Did Apple act ethically in repurchasing large quantities of its own shares? Is it ethical for any company to do so? If you were an investor or analyst, what questions would you ask or what cautions would you take in assessing and comparing earnings per share data?

(sources: Wayne Duggan. "7 S&P 500 Companies with Stock Buybacks." *US News & World Report.* December 14, 2020. https://money.usnews.com/investing/stock-market-news/slideshows/s-p-500-companies-with-stock-buybacks?slide=2; "Apple's $460 Billion Stock Buyback." *Above Avalon.* April 23, 2020. https://www.aboveavalon.com/notes/2020/4/23/apples-460-billion-stock-buyback; "Apple Stock Buybacks (Quarterly)." *Ycharts.* n.d. https://ycharts.com/companies/AAPL/stock_buyback)

LINK TO LEARNING

Stock Buybacks

This *Wall Street Journal* video about stock buybacks (https://openstax.org/r/video-about-stock-buybacks)

1 Bill Maurer. "Apple: New Highs Seem Likely." Seeking Alpha. May 11, 2020. https://seekingalpha.com/article/4346246-apple-new-highs-seem-likely

explains the various perspectives on the subject. It walks through the basic concepts of how buybacks work and explores some viewpoints on whether buybacks are good, bad, or otherwise.

To put a firm's earnings per share into perspective and allow for a more meaningful analysis, earnings per share is often tracked over a number of years, such as when presented in the comparative income statements for Clear Lake Sporting Goods (see Figure 6.7).

Clear Lake Sporting Goods
Comparative Year-End Income Statements

	Current Year	Prior Year	2 Years Prior
Net Sales	$120,000	$100,000	$90,000
Cost of Goods Sold	60,000	50,000	45,000
Gross Profit	60,000	50,000	45,000
Rent Expense	5,500	5,000	5,000
Depreciation Expense	3,600	2,500	2,000
Salaries Expense	5,400	3,000	2,750
Utility Expense	2,500	1,500	1,250
Operating Income	43,000	38,000	34,000
Interest Expense	2,000	3,000	2,000
Income Tax Expense	6,000	5,000	5,000
Net Income	$ 35,000	$ 30,000	$27,000
Basic Weighted Shares Outstanding	8,000	10,550	11,100
Basic Net Income per Share (EPS)	$ 3.75	$ 2.56	$ 2.21
Common Dividends	$ 6,000	$ 4,000	$ 3,500
Preferred Dividends	$ 5,000	$ 3,000	$ 2,500

Figure 6.7 Comparative Year-End Income Statements Earnings per share year after year can be a good indication of a company's financial health.

Most analysts believe that a consistent improvement in earnings per share year after year is an indication of continuous improvement in the earning power of a company. This is what is seen in Clear Lake Sporting Goods' earnings per share amounts over each of the three years reported, moving from $2.21 to $2.56 to $3.75. However, it is important to remember that earnings per share is calculated on historical data, which is not always predictive of the future.

THINK IT THROUGH

Would You Have Invested?

What if, in 1997, you invested $5,000 in Amazon? Today, your investment would be worth nearly $6 million. Potential investors viewing Amazon's income statement in 1997 would have seen earnings per share of negative $1.27. In other words, Amazon lost $1.27 for each share of common stock outstanding. Would you have invested?

Solution:

Answers will vary. A strong response would include the idea that a negative or small earnings per share reflects upon the historical operations of a company. Earnings per share does not predict the future. Investors in 1997 looked beyond Amazon's profitability and saw its business model having strong future potential.

Price/Earnings (P/E) Ratio

The **price/earnings (P/E) ratio** measures the current market share price of a company's stock relative to its earnings per share (EPS). The ratio is helpful in comparing performance and stock price of a company to other companies. It's also helpful in evaluating how much investors are willing to pay for earnings performance. Investors, in particular, use this ratio and rely on two key characteristics: past performance (trailing) and future estimates (forward). Trailing data can be calculated but is also easily found online, as it's a common measure reported on financial sites. Investors will often look for P/E TTM, which is the price/earnings ratio for the trailing 12 months (last year worth of earnings data). This helps investors assess one day's stock price relative to the earnings per share over the past 12 months. P/E ratio is widely used by investors to determine if a stock is over- or undervalued. It also helps them compare one firm to that of the industry average or index, such as the S&P 500.

$$\text{Price/Earnings Ratio} = \frac{\text{Market Value per Share}}{\text{Earnings per Share}}$$

In the prior section we saw earnings per share data for Clear Lake Sporting Goods. Using its current year earnings per share of $3.75 and the current stock price of $69.41, we can calculate price/earnings ratio for Clear Lake Sporting Goods:

$$\text{Price Earnings Ratio} = \frac{\$69.41}{\$3.75} = 18.51$$

An 18.51 ratio means an investor would expect to invest $18.51 to gain $1 of earnings.

Book Value per Share

Book value per share is often used hand in hand with market value per share. Investors compare the two in order to see if the stock is possibly over- or undervalued. Book value is derived from accounting practices and shows the value of the firm on paper. Market value, on the other hand, is determined by supply and demand, based on what investors are willing to pay for the stock. If the market value per share is higher than the book value, the stock is considered overvalued. If the market value is lower than the book value, it's considered undervalued.

In theory, book value per share represents the total value common shareholders would receive if the firm were liquidated. It is total equity less preferred equity, spread across the total shares outstanding. The formula to calculate book value per share is

$$\text{Book Value per Share} = \frac{\text{Total Equity} - \text{Preferred Equity}}{\text{Total Shares Outstanding}}$$

The book value per share for Clear Lake Sporting Goods is

$$\text{Book Value per Share} = \frac{\$100{,}000 - \$20{,}000}{8{,}000} = \$10$$

If investors compared the book value per share of $10.00 for Clear Lake Sporting Goods to the P/E ratio of $18.51, they would likely conclude that the stock was undervalued in the year of analysis.

LINK TO LEARNING

Book Value versus Market Value of Shares

This video about book value and market value (https://openstax.org/r/video-about-book-value) explains the basic concepts and discusses how the two differ. Samples of the concept are then explored using Apple Inc. as an example.

6.6 Profitability Ratios and the DuPont Method

By the end of this section, you will be able to:

- Calculate profit margin to determine how much sales revenues the firm has translated into income.
- Evaluate firm performance by calculating return on total assets and return on equity.
- Analyze organizational performance using DuPont method calculations.

Profitability considers how well a company produces returns given its operational performance. The company needs to use its assets and operations efficiently to increase profit. To assist with profit goal attainment, company revenues need to outweigh expenses. Let's consider three profitability measurements and ratios: profit margin, return on total assets, and return on equity.

Profit Margin

Profit margin represents how much of sales revenue has translated into income. This ratio shows how much of each $1 of sales is returned as profit. The larger the ratio figure (the closer it gets to 1), the more of each sales dollar is returned as profit. The portion of the sales dollar not returned as profit goes toward expenses. The formula for profit margin is

$$\text{Profit Margin} = \frac{\text{Net Income}}{\text{Net Sales}}$$

For Clear Lake Sporting Goods, the profit margin in the current year is

$$\text{Profit Margin} = \frac{\$35{,}000}{\$120{,}000} = 0.29 \text{ (rounded) or } 29\%$$

This means that for every dollar of sales, $0.29 returns as profit. If Clear Lake Sporting Goods thinks this is too low, the company would try to find ways to reduce expenses and increase sales.

Return on Total Assets

The **return on total assets** measures the company's ability to use its assets successfully to generate a profit. The higher the return (ratio outcome), the more profit is created from asset use. Average total assets are found by dividing the sum of beginning and ending total assets balances found on the balance sheet. The beginning total assets balance in the current year is taken from the ending total assets balance in the prior year. The formula for return on total assets is

$$\text{Return on Total Assets} = \frac{\text{Net Income}}{\text{Average Total Assets}}$$

$$\text{Average Total Assets} = \frac{(\text{Beginning Total Assets} + \text{Ending Total Assets})}{2}$$

For Clear Lake Sporting Goods, the return on total assets for the current year is

$$\text{Average Total Assets} = \frac{(\$200{,}000 + \$250{,}000)}{2} = \$225{,}000$$

$$\text{Return on Total Assets} = \frac{\$35{,}000}{\$225{,}000} = 0.16 \text{ (rounded) or } 16\%$$

The higher the figure, the better the company is using its assets to create a profit. Industry standards can dictate what an acceptable return is.

LINK TO LEARNING

Return on Assets

This video explains how to calculate return on assets (https://openstax.org/r/video-explains-how-to-calculate) and how to interpret the results. The video provides the formula, a discussion of the concept, and the importance of the ratio.

Return on Equity

Return on equity measures the company's ability to use its invested capital to generate income. The invested capital comes from stockholders' investments in the company's stock and its retained earnings and is leveraged to create profit. The higher the return, the better the company is doing at using its investments to yield a profit. The formula for return on equity is

$$\text{Return on Equity} = \frac{\text{Net Income}}{\text{Average Stockholder Equity}}$$

$$\text{Average Stockholder Equity} = \frac{\text{Beginning Stockholder Equity} + \text{Ending Stockholder Equity}}{2}$$

Average stockholders' equity is found by dividing the sum of beginning and ending stockholders' equity balances found on the balance sheet. The beginning stockholders' equity balance in the current year is taken from the ending stockholders' equity balance in the prior year. Keep in mind that the net income is calculated after preferred dividends have been paid.

For Clear Lake Sporting Goods, we will use the net income figure and deduct the preferred dividends that have been paid. The return on equity for the current year is

$$\text{Average Stockholder Equity} = \frac{90{,}000 + 100{,}000}{2} = 95{,}000$$

$$\text{Return on Equity} = \frac{\$35{,}000 - \$5{,}000}{\$95{,}000} = 0.32 \text{ (rounded) or } 32\%$$

The higher the figure, the better the company is using its investments to create a profit. Industry standards can dictate what an acceptable return is.

The DuPont Method

ROE in its basic form is useful; however, there are really three components of ROE: operating efficiency (profit margin), asset usage (total asset turnover), and leverage (equity ratio). This is known as the **DuPont method**. It originated in 1919 when the DuPont company implemented it for internal measurement purposes.[2] The DuPont method can be expressed using this formula:

$$\text{ROE} = \text{Profit Margin} \times \text{Total Asset Turnover} \times \text{Equity Multiplier}$$

Profit margin indicates how much profit is generated by each dollar of sales and is computed as shown:

$$\text{Profit Margin} = \frac{\text{Net Income}}{\text{Net Sales}}$$

Total asset turnover indicates the number of sales dollars produced by every dollar invested in capital assets—in other words, how efficiently the company is using its capital assets to generate sales. It is computed as shown:

2 Joshua Kennon. "What Is the DuPont Method Return on Equity, or ROE, Formula?" The Balance. December 16, 2020. https://thebalance.com/the-dupont-model-return-on-equity-formula-for-beginners-357494

$$\text{Equity Multiplier} = \frac{\text{Average Total Assets}}{\text{Average Stockholders' Equity}}$$

The equity multiplier measures leverage. It is computed as shown:

$$\text{Equity Multiplier} = \frac{\text{Average Total Assets}}{\text{Average Stockholders' Equity}}$$

Using DuPont analysis, investors can see overall performance broken down into smaller pieces, which helps them better understand what is driving ROE. We already have the computations for Clear Lake Sporting Goods' profit margin and total asset turnover:

$$\text{Profit Margin} = \frac{\$35{,}000}{\$120{,}000} = 0.29 \text{ (rounded) or } 29\%$$

$$\text{Total Asset Turnover} = \frac{\$120{,}000}{\$225{,}000} = 0.53 \text{ times (rounded)}$$

We can calculate the equity multiplier using the equity multiplier equation and prior calculations for Clear Lake's average total assets and average stockholder equity:

$$\text{Equity Multiplier} = \frac{\text{Average Total Assets}}{\text{Average Stockholders' Equity}}$$

$$\text{Average Total Assets} = \frac{\$200{,}000 + \$250{,}000}{2} = \$225{,}000$$

$$\text{Average Stockholder Equity} = \frac{\$90{,}000 + \$100{,}000}{2} = \$95{,}000$$

$$\text{Equity Multiplier} = \frac{\$225{,}000}{\$95{,}000} = 2.37$$

Now that we have all three elements, we can complete the DuPont analysis for Clear Lake Sporting Goods:

$$\text{ROE} = \text{Profit Margin} \times \text{Total Asset Turnover} \times \text{Equity Multiplier}$$

$$\text{ROE} = 29\% \times 0.53 \times 2.37 = 0.36 \text{ or } 36.4\%$$

LINK TO LEARNING

The DuPont Method

This video about the DuPont method (https://openstax.org/r/video-about-DuPont-method) walks through its history, discusses its basic components, and shows how to calculate and interpret each measurement.

Performance Analysis

ROE captures the nuances of all three elements. A good sales margin and a proper asset turnover are both needed for a successful operation. Like all ratios, assessing performance is relative. It's important to look at the ratio in context of the organization, its history, and the industry. If we compare Clear Lake's ROE of 26.4% to the recreational products industry average of 12.56% for the same year, it would appear as though Clear Lake Sporting Goods is outperforming the general industry. However, recreational products can include a wide variety of businesses beyond just the outdoor gear in which Clear Lake Sporting Goods specializes. An analyst could look at other key competitors such as Cabela's or Bass Pro Shops to get even more relevant comparisons.

Clear Lake Sporting Goods is also technically a retail store, albeit a specialized one. An analyst might also consider the industry averages for general or online retail of 20.64% and 27.05%, respectively. Compared to the broader retail industry, Clear Lake Sporting Goods is still performing well, but its performance is not as

disparate to industry average as when compared to recreational products (see Table 6.1).

Industry	ROE (%)
Advertising	2.93
Air Transportation	−47.03
Computer Services	13.50
Banking	8.22
Financial Services (nonbanking)	64.28
Food Processing	10.12
Renewable Energy	−20.59
Hospitals/Health Care Facilities	70.64
Hotels/Gaming	−30.40
Publishers	−14.18
Recreational Products	12.56
Real Estate (general)	2.00
Retail:	0.00
Automotive	36.28
Building Supply	0.27
General	20.64
Grocery	30.63
Online	27.05
Rubber and Tires	−26.69
Shoes	23.70
Software (systems and applications)	28.09
Transportation	21.47
Total Market Average	8.25

Table 6.1 Return on Equity by Industry in 2020 It's important to look at any ratio in context of the organization, its history, and the industry. (data source: Aswath Damodaran Online)

Summary

6.1 Ratios: Condensing Information into Smaller Pieces

Ratios used in financial analysis can help investors and other analysts identify trends over time, compare companies to one another, and make informed decisions about a company. There are, however, limitations to analysis, so it should be used wisely and in conjunction with other contextual information available.

6.2 Operating Efficiency Ratios

Efficiency ratios measure how well management uses the assets of the organization to earn a profit. Common efficiency ratios include accounts receivable turnover, total asset turnover, inventory turnover, and days' sales in inventory.

6.3 Liquidity Ratios

Liquidity ratios help analysts measure how well an organization can meet its short-term obligations (liabilities) as they come due. Common ratios to measure liquidity include the current ratio, the quick ratio, and the cash ratio. Each of these three ratios includes more (cash ratio) or less liquid (current ratio) current assets in its measure of liquidity.

6.4 Solvency Ratios

Solvency ratios measure how well an organization can meet its long-term obligations (liabilities) as they come due, or in more general terms, its ability to stay in business. Common solvency ratios include the debt-to-assets ratio and the debt-to-equity ratio.

6.5 Market Value Ratios

Market value ratios help analysts assess the value of publicly traded firms in the market. The most commonly used market value ratios include earnings per share (EPS), the price/earnings ratio, and book value per share.

6.6 Profitability Ratios and the DuPont Method

Profitability ratios help measure how effectively the organization earns a profit. Common profitability ratios include profit margin, return on total assets, and return on equity. The DuPont method breaks down return on equity into three smaller components for a more thorough assessment of performance: profit margin, total asset turnover, and an equity multiplier.

Key Terms

accounts receivable turnover ratio measures how many times in a period (usually a year) a company will collect cash from accounts receivable

book value per share total book value (assets – liabilities) of a firm expressed on a per-share basis

cash ratio represents the firm's cash and cash equivalents divided by current liabilities; often used by investors and lender to asses an organization's liquidity

current ratio current assets divided by current liabilities; used to determine a company's liquidity (ability to meet short-term obligations)

days' sales in inventory the number of days it takes a company to turn inventory into sales

debt-to-assets ratio measures the portion of debt used by a company relative to the amount of assets

debt-to-equity ratio measures the portion of debt used by a company relative to the amount of stockholders' equity

DuPont method framework for financial analysis that breaks return on equity down into smaller elements

earnings per share (EPS) measures the portion of a corporation's profit allocated to each outstanding share of common stock

efficiency ratios ratios that show how well a company uses and manages its assets

inventory turnover measures the number of times an average quantity of inventory was bought and sold during the period
liquidity ability to convert assets into cash in order to meet primarily short-term cash needs or emergencies
market value ratios measures used to assess a firm's overall market price
operating cycle amount of time it takes a company to use its cash to provide a product or service and collect payment from the customer
price/earnings (P/E) ratio company's stock price divided by the company's earnings per share; indicates the amount investors are willing to pay for one dollar of earnings
profit margin represents how much of sales revenue has translated into income
quick ratio also known as the *acid test ratio*; ratio used to determine a firm's ability to pay short-term debts using its most liquid assets
return on equity measures the company's ability to use its invested capital to generate income
return on total assets measures the company's ability to use its assets successfully to generate a profit
solvency implies that a company can meet its long-term obligations and will likely stay in business in the future
times interest earned (TIE) ratio measures the company's ability to pay interest expense on long-term debt incurred
total asset turnover measures the ability of a company to use its assets to generate revenues

CFA Institute

This chapter supports some of the Learning Outcome Statements (LOS) in this CFA® Level I Study Session (https://openstax.org/r/cfa-study-session11). Reference with permission of CFA Institute.

Multiple Choice

1. Which of the following is financial statement analysis not used for?
   a. identifying trends over time
   b. benchmarking against other firms
   c. complying with SEC (Securities and Exchange Commission) regulations
   d. setting budget expectations

2. How is converting financial data to percentages helpful in financial analysis?
   a. It makes the figures easier to calculate.
   b. It masks actual financial data so the competition can't see it.
   c. It saves time.
   d. It makes comparisons to companies of varying sizes possible.

3. What is a common economic influence that has the potential to skew financial analysis figures?
   a. past performance
   b. inflation
   c. Generally Accepted Accounting Principles
   d. benchmark data

4. What is the formula for accounts receivable turnover?
   a. net credit sales / average accounts receivable
   b. total sales / average accounts receivable
   c. net credit sales / beginning accounts receivable
   d. net cash sales / ending accounts receivable

5. What is the formula for the times interest earned ratio?

a. net income / interest expense
b. earnings before interest and taxes / interest expense
c. interest expense / earnings before interest and taxes
d. earnings before interest and taxes – interest expense

6. What is the formula for the calculation of earnings per share?
a. (net income + preferred dividends) / weighted average common shares outstanding
b. net income / weighted average common shares outstanding
c. (net income – preferred dividends) / weighted average common shares outstanding
d. (net income – preferred dividends) / treasury shares outstanding

7. Most analysts believe which of the following is true about earnings per share?
a. Consistent improvement in earnings per share year after year is an indication of continuous improvement in the company's earning power.
b. Consistent improvement in earnings per share year after year is an indication of continuous decline in the company's earning power.
c. Consistent improvement in earnings per share year after year is an indication of fraud within the company.
d. Consistent improvement in earnings per share year after year is an indication that the company will never suffer a year of net loss rather than net income.

8. What is the formula for profit margin?
a. net sales / net income
b. cost of goods sold / net sales
c. sales / cost of goods sold
d. net income / net sales

Review Questions

1. Is past performance considered a good indicator of future performance?

2. The Pony Parts Tack Shop had inventory turnover of 12.8, 12.2, and 9.9 over the last three years, respectively. What can you learn about how well the Pony Parts management team is managing their inventory using this information?

3. The Pony Parts Tack Shop had total asset turnover of 1.8., 2.1, and 2.4 over the last three years, respectively. What can you learn about how well the Pony Parts management team is managing their total assets using this information?

4. Big Box Store Inc. had days' sales in inventory of 30, 32, and 34 for the last three years. Small Box Store Inc. had days' sales in inventory of 40, 38, and 36 for the last three years. What can you infer about the inventory management for the two companies based on this information? Which company is performing better?

5. Jackson's Beef Jerky Shop has a current ratio of 2.35. What does that mean? Is 2.35 a good or bad current ratio?

6. Jackson's Beef Jerky Shop has a quick ratio of 1.9. What does that mean? Is 1.9 a good or bad quick ratio?

7. Jackson's Beef Jerky Shop has a cash ratio of 1.75. What does that mean? Is 1.75 a good or bad cash ratio?

8. You have some funds that you would like to invest. Do some internet research to find two publicly traded companies in the same industry and compare their earnings per share. Would the earnings per share

reported by each company influence your decision in selecting which company to invest in?

9. Company A has a market value per share of 19.55 and book value per share of 12.79. If you were an investor, what would you conclude about the current value of Company A stock?
10. Company B has a price/earnings ratio of 21.2. The current industry average price/earnings ratio is 20.75. What might an investor conclude about investing in Company B?
11. What are the key elements of the DuPont formula, and how do these components function to help analysts assess an organization?

Problems

1. Sarah's Toy Shop has total sales of $100,000, net credit sales of $70,000, beginning accounts receivable of $20,000, and ending accounts receivable of $30,000. What is Sarah's accounts receivable turnover? Assume industry average is 2.9 times. How would you interpret Sarah's turnover?
2. Fantastic Foods has total assets of $150,000, current assets of $80,000 (current assets includes $30,000 of cash, $10,000 of short term investments, $20,000 of accounts receivable, and $20,000 of inventory), total liabilities of $120,000, and current liabilities of $70,000. What is Fantastic Foods' current ratio?
3. The Big Club has total assets of $150,000, current assets of $80,000 (current assets includes $30,000 of cash, $10,000 of short-term investments, $20,000 of accounts receivable, and $20,000 of inventory), total liabilities of $120,000, and current liabilities of $70,000. What is The Big Club's quick ratio?
4. Giant Sales has total assets of $150,000, current assets of $80,000 (current assets includes $30,000 of cash, $10,000 of short-term investments, $20,000 of accounts receivable, and $20,000 of inventory), total liabilities of $120,000, and current liabilities of $70,000. What is Giant Sales' cash ratio?
5. Bonita's Bread Company has total debt of $250,000 and total assets of $150,000. What is Bonita's debt-to-assets ratio, and what can we infer about Bonita's company using the ratio?
6. Jai Company has total liabilities of $200,000 and total stockholder equity of $300,000. What is the debt-to-equity ratio for Jai Company, and what can we infer about the firm using this ratio?
7. Jamilah's Manufacturing Company has earnings before interest and taxes of $29,000 and interest expense of $4,000 for the most current period. What is Jamilah's times interest earned ratio?
8. Sarai's Sandy Beach Gear has net sales of $100,000, cost of goods sold of $60,000, and net income of $25,000. What is Sarai's profit margin?
9. Bob's Tires Inc. has sales of $100,000, net income of $50,000, beginning asset balance of $200,000, ending asset balance of $220,000, beginning stockholder equity of $160,000, and ending stockholder equity of $200,000. What is Bob's return on total assets?
10. Bob's Tires Inc. has sales of $100,000, net income of $50,000, beginning asset balance of $200,000, ending asset balance of $220,000, beginning stockholder equity of $160,000, and ending stockholder equity of $200,000. What is Bob's return on equity?

Video Activity

Ratio Analysis—Limitations of Ratios

Click to view content (https://openstax.org/r/ratio-analysis)

1. In the video, Jim walks through several key limitations in the areas of reliability, comparability, relying on only one data source, and using information based in the past. Which limitation do you feel is the most worrisome? What might you do to compensate for the limitation you identified?

2. Outside of the four key areas of limitations, Jim also explores a number of key elements that ratios aren't able to convey. What characteristics of a firm would you most want to know about if you were to invest that you would not be able to glean from ratios? How would you go about gathering that information if you cannot get it through financial statements and ratio analysis?

The Problem with Earnings per Share (EPS)

Click to view content (https://openstax.org/r/problem-with-earnings)

3. What does Zach DeGregorio cite as the most beneficial characteristics of using EPS in financial analysis? And the most problematic?
4. After watching the video and listing the key benefits and problems with EPS, how do you feel about the EPS calculation? Should investors use it? Why or why not? If you were going to invest a large sum of money in a company, would you use EPS data in your decision-making process? Why or why not?

7

Time Value of Money I: Single Payment Value

Figure 7.1 Time has an impact on the value of money. (credit: modification of "Time is money" by Marco Verch/flickr, CC BY 2.0)

Chapter Outline

Why It Matters

One of the single most important concepts in the study of finance is the time value of money (TVM). This concept puts forward the idea that a dollar received today is worth more than, and therefore preferable to, a dollar received at some point in the future. The three primary reasons for this are that (1) money received now can be saved or invested now and earn interest or a return, resulting in more money in the future; (2) any promise of future payments of cash will always carry the risk of default; and (3) it is simple human nature for people to prefer making their purchases of goods and services in the present rather than waiting to make them at some future time.

For this reason, it is important to incentivize people to give up their present consumption patterns by offering them greater value in the future. Based on the concept of TVM, it can be said that a dollar was worth more to us, and thus carried more value, yesterday than it is to us today. It also then follows that a dollar in our possession right now carries a greater value for us than a dollar we might receive tomorrow or at some other point in the future.

The entire concept of the time value of money is particularly important because it allows savers and investors to make better-informed decisions about what to do with their money. TVM can help a person understand which option may be best based on the critical factors of overall risk, rates of interest, inflation, and return. TVM can also be used to help a person understand how much money they'll need to save in an interest-bearing account in order to reach a desired financial goal, such as saving $50,000 in 10 years in an account that earns 4% compound interest each year. TVM is the key underlying principle of such important financial analytical activities as retirement planning, corporate capital project evaluation, and even deciding on your

own personal investments and bank accounts.

If the main concept behind the TVM is that a specific amount of money in hand now is worth more today than that same amount of money will be worth tomorrow, you might think that a person would be better off spending their money now rather than saving it for later use. However, we know that this is not always the case. Sometimes it is simply a better idea to save your money. While inflation can have the effect of making a dollar worth less tomorrow than it is worth today, the positive effect of compound interest works in favor of savers and investors.

7.1 Now versus Later Concepts

Learning Outcomes

By the end of this section, you will be able to:

- Explain why time has an impact on the value of money.
- Explain the concepts of future value and present value.
- Explain why lump sum cash flow is the basis for all other cash flows.

How and Why the Passage of Time Affects the Value of Money

The concept of the **time value of money (TVM)** is predicated on the fact that it is possible to earn **interest** income on cash that you decide to deposit in an **investment** or interest-bearing account. As times goes by, interest is earned on amounts you have invested (*present value*), which effectively means that time will add value (*future value*) to your savings. The longer the period of time you have your money invested, the more interest income will accrue. Also, the higher the rate of interest your account or investment is earning, again, the more your money will grow.

Understanding how to calculate values of money in the present and at different points in the future is a key component of understanding the material presented in this chapter—and of making important personal financial decisions in your future (see Figure 7.2).

Figure 7.2 The Time Value of Money It is better to be paid today, or you will lose out on the money you would have earned in interest.

The Lump Sum Payment or Receipt

The most basic type of financial transaction involves a simple, one-time amount of cash, which can be either a receipt (inflow) or a payment (outflow). Such a one-time transaction is typically referred to as a lump sum. A lump sum consists of a one-off cash flow that occurs at any single point in time, present or future. Because it is always possible to dissect more complex transactions into smaller parts, the lump sum cash flow is the basis on which all other types of cash flow are treated. According to the general principle of adding value, every type of cash flow stream can be divided into a series of lump sums. For this reason, it is critical to understand the math associated with lump sums if you wish to have a greater appreciation, and a complete understanding, of more complicated forms of cash flow that may be associated with an investment or a capital purchase by a

company.

7.2 Time Value of Money (TVM) Basics

Learning Outcomes

By the end of this section, you will be able to:

- Define *future value* and provide examples.
- Explain how future dollar amounts are calculated using a single-period scenario.
- Describe the impact of compounding.

Because we can invest our money in interest-bearing accounts and investments, its value can grow over time as interest income accrues or returns are realized on our investments. This concept is referred to as **future value (FV)**. In short, future value refers to how a specific amount of money today can have greater value tomorrow.

Single-Period Scenario

Let us start with the following example. Your friend is considering putting money in a bank account that will pay 4% interest per year and is particularly interested in knowing how much money they will have one year from now if they deposit $1,000 in this account. Your friend understands that you are studying finance and turns to you for help. By using the TVM principle of future value (FV), you can tell your friend that the answer is $1,040. The additional $40 that will be in the account after one year will be due to interest earned over that time. You can calculate this amount relatively easily by taking the original deposit (also referred to as the principal) of $1,000 and multiplying it by the annual interest rate of 4% for one period (in this case, one year).

$$\text{Interest Earned} = \$1{,}000 \times 0.04 = \$40.00$$

By taking the interest earned amount of $40 and adding it to the original principal of $1,000, you will arrive at a total value of $1,040 in the bank account at the end of the year. So, the $1,040 one year from today is equal to $1,000 today when working with a 4% earning rate. Therefore, based on the concept of TVM, we can say that $1,040 represents the future value of $1,000 one year from today and at a 4% rate of interest. We will discuss interest rates and their importance in TVM decisions in more detail later in this chapter; for now, we can consider interest rate as a percentage of the principal amount that is earned by the original lender of funds and/or charged to the borrower of these same funds. Following are a few more examples of the single-period scenario.

If a person deposits $300 in an account that pays 5% per year, at the end of one year, they will have

$$\text{FV} = \$300 + (\$300 \times 0.05) = \$315$$

If a company has earnings of $2.50 per share and experiences a 10% increase in the following year, the earnings per share in year two are

$$\$2.50 + \$2.50 \times 0.10 = \$2.75 \text{ per share}$$

If a retail store decides on a 3% price increase for the following year on an item that is currently selling for $50, the new price in the following year will be

$$\$50 + \$50 \times 0.03 = \$51.50$$

The Impact of Compounding

What would happen if your friend were willing to wait one more year to receive their lump sum payment? What would the future dollar value in their account be after a two-year period? Returning to our earlier example, assume that during the second year, your friend leaves the principal ($1,000) and the earned interest ($40) in the account, thereby reinvesting the entire account balance for another year. The quoted interest rate of 4% reflects the interest the account would earn each year, not over the entire two-year savings period. So, during the second year of savings, the $1,000 deposit and the $40 interest earned during the first year would

both earn 4%:

$$\$1{,}000 \times 0.04 + 40 \times 0.04 = \$41.60$$

The additional $1.60 is interest on the first year's interest and reflects the compounding of interest. *Compound interest* is the term we use to refer to interest income earned in subsequent periods that is based on interest income earned in prior periods. To put it simply, compound interest refers to interest that is earned on interest. Here, it refers to the $1.60 of interest earned in the second year on the $40.00 of interest earned in the first year. Therefore, at the end of two years, the account would have a total value of $1,081.60. This consists of the original principal of $1,000 plus the $40.00 interest income earned in year one and the $41.60 interest income earned in year two.

The amount of money your friend would have in the account at the end of two years, $1,081.60, is referred to as the future value of the original $1,000 amount deposited today in an account that will earn 4% interest every year.

Simple interest applies to year 1 while compound interest or "interest on interest" applies to year 2. This is calculated using the following method:

$$\begin{aligned} \text{Year 1:} \quad & 1{,}000 \times 0.04 = 40.00 \\ \text{Year 2:} \quad & 1{,}040 \times 0.04 = 41.60 \end{aligned}$$

So, the total amount that would be in the account after two years, at 4% annual interest, would be $\$1{,}000 + \$40.00 + \$41.60 = \$1{,}081.60$.

To determine any future value of money in an interest-bearing account, we multiply the principal amount by 1 plus the interest rate for each year the money remains in the account. From this, we can develop the future value formula:

$$\text{Future Value} = \text{Original Deposit} \times (1 + r) \times (1 + r)$$

In this formula, the number of times we multiply by $(1 + r)$ depends entirely on the number of years the money will remain in the bank account, earning interest, before it is withdrawn in a final lump sum distribution paid out from the account at the end of the chosen savings period. The 1 in the formula represents the principal amount, or the original $1,000 deposit, which will be included in the final total lump sum payment when the account is closed and all money is withdrawn at the end of the predetermined savings period.

We can write the above equation in a more condensed mathematical form using time value of money notation, as follows:

$$\begin{aligned} \text{FV} &= \text{Future Value} \\ \text{PV} &= \text{Present Value} \\ r &= \text{Interest Rate} \\ n &= \text{Number of Periods} \end{aligned}$$

Using these inputs, we have the following formula:

$$\text{FV} = \text{PV} \times (1 + r)^n$$

With this equation, we can calculate the value of the savings account after any number of years. For example, suppose we are considering 3, 10, and 50 years from the original deposit date at the annual 4% interest rate:

$$\begin{aligned} \text{3 years:} \quad & \text{FV} = \$1{,}000 \times (1.04)^3 = \$1{,}000 \times 1.12486 = \$1{,}124.86 \\ \text{10 years:} \quad & \text{FV} = \$1{,}000 \times (1.04)^{10} = \$1{,}000 \times 1.48024 = \$1{,}480.24 \\ \text{50 years:} \quad & \text{FV} = \$1{,}000 \times (1.04)^{50} = \$1{,}000 \times 7.106683 = \$7{,}106.68 \end{aligned}$$

How can this savings account have grown to be so large after 50 years? This question is answered by the impact of compounding interest. Every year, the interest earned in previous years will also earn interest along

with the initial deposit. This will have the effect of accelerating the growth of the total dollar value of the account.

This is the important effect of the compounding of interest: money grows in larger and larger increments the longer you leave it in an interest-bearing account. In effect, the compounding of interest over time accelerates the growth of money.

In order to determine the FV of any amount of money, it will always be necessary to know the following pieces of information: (1) the principal, initial deposit, or **present value (PV)**; (2) the rate of interest, usually expressed on an annual basis as *r*; and (3) the number of time periods that the money will remain in the account (*n*). The interest rate is often referred to as the **growth rate**, or the annual percentage increase on savings or on an investment. When the rate is raised to the power of the number of periods, the formula $(1 + r)^n$ will yield a number that is commonly referred to as the future value interest factor (FVIF). As a result of this process, as *n* (time, or the number of periods) increases, the future value interest factor will increase. Also, as *r* (interest rate) increases, the FVIF will increases. For these reasons, the future value calculation is directly determined by both the interest rate being used and the total amount of time—specifically, the number of periods—being considered.

THINK IT THROUGH

Calculating Future Values

Here's another example of calculating future values in multiple-period scenarios.

On a recent drive, you spotted your dream home, which is currently listed at $400,000. Unfortunately, you are not in a position to buy it right away and will have to wait at least another six years before you can afford it. If house values are appreciating at an annual rate of inflation of 4%, how much will a similar house cost after six years?

Solution:

In this case, PV is the current cost of the house, or $400,000; *n* is six years; *r* is the average annual inflation rate, or 4%; and we have to solve for FV.

$$\begin{aligned} \text{FV} &= \$400{,}000 \times 1.046 \\ &= \$400{,}000 \times (1.265319) = \$506{,}127.61 \end{aligned}$$

Therefore, under these circumstances, the house will cost $506,127.60 after six years.

How Time Impacts Compounding

We have just seen that time will lead to the growth of our money. As long as the prevailing growth or interest rate of any account we have our money in is positive, the passage of time will have the effect of growing the value of our money. The longer the period of time, the greater the growth and the larger the future value of the money will be. This can be reinforced very clearly with the following example.

Melvin is saving money in an account at a local bank that earns 5% per year. He begins with a deposit in his account of $100 and decides to save his money for exactly one year. He will not be making any further deposits into the account during the year. Melvin will earn $5\% \times \$100,$ or $5, in interest income. Adding this to the original deposit balance of $100 will give him a total of $\$100 + \$5,$ or $105, in the account at the end of one year.

Melvin likes this idea and believes he may be able to keep his money in the account for a longer period of time. How much money will he have in his account, without any further deposits, at the end of years two, three, four, and five?

Using the future value formula, the calculation is as follows:

$$\begin{aligned}
&\text{FV} = \text{PV} \times (1+r)^n \\
\text{Year 2: } &\text{FV} = \$100 \times (1+0.05)^2 = \$110.25 \\
\text{Year 3: } &\text{FV} = \$100 \times (1+0.05)^3 = \$115.76 \\
\text{Year 4: } &\text{FV} = \$100 \times (1+0.05)^4 = \$121.55 \\
\text{Year 5: } &\text{FV} = \$100 \times (1+0.05)^5 = \$127.63
\end{aligned}$$

How the Interest Rate Impacts Compounding

Melvin likes the idea of earning more money over time, but he also believes that what he would earn in interest may not be enough for some of the things he plans to buy in the future. His friend suggests finding an account or some form of investment with a greater interest rate than the 5% he can get at his local bank.

Melvin thinks he can leave his money in an account or investment for a total of five years. He found investments that will provide annual returns of 6%, 7%, 10%, and 12%. Using the $\text{FV} = \text{PV} \times (1+r)^n$ formula, we can complete the following calculations for him:

$$\begin{aligned}
5\%: &\text{FV} = \$100 \times (1+0.05)^5 = \$127.63 \\
6\%: &\text{FV} = \$100 \times (1+0.06)^5 = \$133.82 \\
7\%: &\text{FV} = \$100 \times (1+0.07)^5 = \$140.26 \\
10\%: &\text{FV} = \$100 \times (1+0.10)^5 = \$161.05 \\
12\%: &\text{FV} = \$100 \times (1+0.12)^5 = \$176.23
\end{aligned}$$

Again, Melvin likes this information, and he states that he will try to find the highest interest rate available. This makes sense, but it's important to remember that investments are usually not guaranteed to earn you specific interest rates, or rates of return. Most investments, other than **Treasury investments** such as Treasury bonds, carry some form of **financial risk**, either small or large, and the greater the rate of return, the more likely it is that the risk associated with the investment will also be greater. This risk does not have any effect on the future calculations we have just completed, but it an important factor to bear in mind and consider well before moving ahead and putting your money in any investment or **financial instrument**.

7.3 Methods for Solving Time Value of Money Problems

Learning Outcomes

By the end of this section, you will be able to:

- Explain how future dollar amounts are calculated.
- Explain how present dollar amounts are calculated.
- Describe how discount rates are calculated.
- Describe how growth rates are calculated.
- Illustrate how periods of time for specified growth are calculated.
- Use a financial calculator and Excel to solve TVM problems.

We can determine future value by using any of four methods: (1) mathematical equations, (2) calculators with financial functions, (3) spreadsheets, and (4) FVIF tables. With the advent and wide acceptance and use of financial calculators and spreadsheet software, FVIF (and other such time value of money tables and factors) have become obsolete, and we will not discuss them in this text. Nevertheless, they are often still published in other finance textbooks and are also available on the internet to use if you so choose.

Using Timelines to Organize TVM Information

A useful tool for conceptualizing present value and future value problems is a timeline. A timeline is a visual,

linear representation of periods and cash flows over a set amount of time. Each timeline shows today at the left and a desired ending, or future point (maturity date), at the right.

Now, let us take an example of a future value problem that has a time frame of five years. Before we begin to solve for any answers, it would be a good approach to lay out a timeline like that shown in Table 7.1:

Year	0 (Today)	1	2	3	4	5

Table 7.1

The timeline provides a visual reference for us and puts the problem into perspective.

Now, let's say that we are interested in knowing what today's balance of $100 in our saving account, earning 5% annually, will be worth at the end of each of the next five years. Using the future value formula

$$\text{FV} = \text{PV} \times (1 + r)^n$$

that we covered earlier, we would arrive at the following values: $105 at the end of year one, $110.25 at the end of year two, $115.76 at the end of year three, $121.55 at the end of year four, and $127.63 at the end of year five.

With the numerical information, the timeline (at a 5% interest or growth rate) would look like Table 7.2:

Year	0	1	2	3	4	5
	$100.00	$105.00	$110.25	$115.76	$121.55	$127.63

Table 7.2

Using timelines to lay out TVM problems becomes more and more valuable as problems become more complex. You should get into the habit of using a timeline to set up these problems prior to using the equation, a calculator, or a spreadsheet to help minimize input errors. Now we will move on to the different methods available that will help you solve specific TVM problems. These are the financial calculator and the Excel spreadsheet.

Using a Financial Calculator to Solve TVM Problems

An extremely popular method of solving TVM problems is through the use of a financial calculator. Financial calculators such as the Texas Instruments BAII Plus™ Professional (https://openstax.org/r/baii-plus-professional) will typically have five keys that represent the critical variables used in most common TVM problems: `N`, `I/Y`, `PV`, `FV`, and `PMT`. These represent the following:

N: Number of Periods
I/Y: Interest Rate (Interest per Year)
PV: Present Value of a Lump Sum
V: Future Value of a Lump Sum
PMT: Payment

These are the only keys on a financial calculator that are necessary to solve TVM problems involving a **single payment or lump sum**.

Example 1: Future Value of a Single Payment or Lump Sum

Let's start with a simple example that will provide you with most of the skills needed to perform TVM functions involving a single lump sum payment with a financial calculator.

Suppose that you have $1,000 and that you deposit this in a savings account earning 3% annually for a period of four years. You will naturally be interested in knowing how much money you will have in your account at the end of this four-year time period (assuming you make no other deposits and withdraw no cash).

To answer this question, you will need to work with factors of $1,000, the present value (PV); four periods or years, represented by N; and the 3% interest rate, or I/Y. Make sure that the calculator register information is cleared, or you may end up with numbers from previous uses that will interfere with the solution. The register-clearing process will depend on what type of calculator you are using, but for the TI BA II Plus™ Professional calculator, clearing can be accomplished by pressing the keys 2ND and FV [CLR TVM].

Once you have cleared any old data, you can enter the values in the appropriate key areas: 4 for N, 3 for I/Y, and 1000 for PV. Now you have entered enough information to calculate the future value. Continue by pressing the CPT (compute) key, followed by the FV key. The answer you end up with should be displayed as 1,125.51 (see Table 7.3).

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.00
2	Enter present value (as a negative integer)	1000 +|– PV	PV =	-1,000.00
3	Enter interest rate	3 I/Y	I/Y =	3.00
4	Enter time periods	4 N	N =	4.00
5	Indicate no payments or deposits	0 PMT	PMT =	0.00
6	Compute future value	CPT FV	FV =	1,125.51

Table 7.3 Calculator Steps for Finding the Future Value of a Single Payment or Lump Sum[1]

Important Notes for Using a Calculator and the Cash Flow Sign Convention

Please note that the PV was entered as negative $1,000 (or -$1000). This is because most financial calculators (and spreadsheets) follow something called the cash flow sign convention, which is a way for calculators and spreadsheets to keep the relative direction of the cash flow straight. Positive numbers are used to represent cash inflows, and negative numbers should always be used for cash outflows.

In this example, the $1,000 is an investment that requires a cash outflow. For this reason, -1000 is entered as the present value, as you will be essentially handing this $1,000 to a bank or to someone else to initiate the transaction. Conversely, the future value represents a cash inflow in four years' time. This is why the calculator generates a positive 1,125.51 as the end result of this calculation.

Had you entered the present value of $1,000 as a positive number, there would have been no real concern, but the ending future value answer would have been returned expressed as a negative number. This would be correct had you borrowed $1,000 today (cash inflow) and agreed to repay $1,125.51 (cash outflow) four years from now. Also, it is important that you do not change the sign of any input value by using the – (minus) key). For example, on the TI BA II Plus™ Professional, you must use the +|– key instead of the minus key. If you enter 1000 and then hit the +|– key, you will get a negative 1,000 amount showing in the calculator display.

An important feature of most financial calculators is that it is possible to change any of the variables in a problem without needing to reenter all of the other data. For example, suppose that we wanted to find out the future value in our bank account if we left the money from our previous example invested for 20 years instead

1 The specific financial calculator in these examples is the Texas Instruments BA II Plus™ Professional model, but you can use other financial calculators for these types of calculations.

of 4. Before clearing any of the data, simply enter 20 for N and then press the CPT key and then the FV key. After this is done, all other inputs will remain the same, and you will arrive at an answer of $1,806.11.

THINK IT THROUGH

How to Determine Future Value When Other Variables Are Known

Here's an example of using a financial calculator to solve a common time value of money problem. You have $2,000 invested in a money market account that is expected to earn 4% annually. What will be the total value in the account after five years?

Solution:

Follow the recommended financial calculator steps in Table 7.4.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.00
2	Enter present value (as a negative integer)	2000 +|– PV	PV =	-2,000.00
3	Enter interest rate	4 I/Y	I/Y =	4.00
4	Enter time periods	5 N	N =	5.00
5	Indicate no payments or deposits	0 PMT	PMT =	0.00
6	Compute future value	CPT FV	FV =	2,433.31

Table 7.4 Calculator Steps for Determining Future Value

The result of this future value calculation of the invested money is $2,433.31.

Example 2: Present Value of Lump Sums

Solving for the present value (discounted value) of a lump sum is the exact opposite of solving for a future value. Once again, if we enter a negative value for the FV, then the calculated PV will be a positive amount.

Taking the reverse of what we did in our example of future value above, we can enter -1,125.51 for FV, 3 for I/Y, and 4 for N. Hit the CPT and PV keys in succession, and you should arrive at a displayed answer of 1,000.

An important constant within the time value of money framework is that the present value will always be less than the future value unless the interest rate is negative. It is important to keep this in mind because it can help you spot incorrect answers that may arise from errors with your input.

THINK IT THROUGH

How to Determine Present Value When Other Variables Are Known

Here is another example of using a financial calculator to solve a common time value of money problem. You have just won a second-prize lottery jackpot that will pay a single total lump sum of $50,000 five years from now. How much value would this have in today's dollars, assuming a 5% interest rate?

Solution:

Follow the recommended financial calculator steps in Table 7.5.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.00
2	Enter future value (as a negative integer)	50000 +|− FV	FV =	-50,000.00
3	Enter interest rate	5 I/Y	I/Y =	5.00
4	Enter time periods	5 N	N =	5.00
5	Indicate no payments or deposits	0 PMT	PMT =	0.00
6	Compute present value	CPT PV	PV =	39,176.31

Table 7.5 Calculator Steps for Determining Present Value

The present value of the lottery jackpot is $39,176.31.

Example 3: Calculating the Number of Periods

There will be times when you will know both the value of the money you have now and how much money you will need to have at some unknown point in the future. If you also know the interest rate your money will be earning for the foreseeable future, then you can solve for N, or the exact amount of time periods that it will take for the present value of your money to grow into the future value that you will require for your eventual use.

Now, suppose that you have $100 today and you would like to know how long it will take for you to be able to purchase a product that costs $133.82.

After making sure your calculator is clear, you will enter 5 for I/Y, -100 for PV, and 133.82 for FV. Now press CPT N, and you will see that it will take 5.97 years for your money to grow to the desired amount of $133.82.

Again, an important thing to note when using a financial calculator to solve TVM problems is that you must enter your numbers according to the cash flow sign convention discussed above. If you do not make either the PV or the FV a negative number (with the other being a positive number), then you will end up getting an error message on the screen instead of the answer to the problem. The reason for this is that if both numbers you enter for the PV and FV are positive, the calculator will operate under the assumption that you are receiving a financial benefit without making any cash outlay as an initial investment. If you get such an error message in your calculations, you can simply press the CE/C key. This will clear the error, and you can reenter your data correctly by changing the sign of either PV or FV (but not both of these, of course).

THINK IT THROUGH

Determining Periods of Time

Here is an additional example of using a financial calculator to solve a common time value of money problem. You want to be able to contribute $25,000 to your child's first year of college tuition and related expenses. You currently have $15,000 in a tuition savings account that is earning 6% interest every year. How long will it take for this account grow into the targeted amount of $25,000, assuming no additional deposits or withdrawals will be made?

Solution:

Table 7.6 shows the steps you will take.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter present value (as a negative integer)	15000 +|− PV	PV =	-15,000.0000
3	Enter interest rate	6 I/Y	I/Y =	6.0000
4	Enter future value	25000 FV	FV =	25,000.0000
5	Indicate no payments or deposits	0 PMT	PMT =	0.0000
6	Compute time periods	CPT N	N =	8.7667

Table 7.6 Calculator Steps for Determining Period of Time

The result of this calculation is a time period of 8.7667 years for the account to reach the targeted amount.

Example 4: Solving for the Interest Rate

Solving for an interest rate is a common TVM problem that can be easily addressed with a financial calculator. Let's return to our earlier example, but in this case, we know that we have $1,000 at the present time and that we will need to have a total of $1,125.51 four years from now. Let's also say that the only way we can add to the current value of our savings is through interest income. We will not be able to make any further deposits in addition to our initial $1,000 account balance.

What interest rate should we be sure to get on our savings account in order to have a total savings account value of $1,125.51 four years from now?

Once again, clear the calculator, and then enter 4 for N, -1,000 for PV, and 1,125.51 for FV. Then, press the CPT and I/Y keys and you will find that you need to earn an average 3% interest per year in order to grow your savings balance to the desired amount of $1,125.51. Again, if you end up with an error message, you probably failed to follow the sign convention relating to cash inflow and outflow that we discussed earlier. To correct this, you will need to clear the calculator and reenter the information correctly.

After you believe you are done and have arrived at a final answer, always make sure you give it a quick review. You can ask yourself questions such as "Does this make any sense?" "How does this compare to other answers I have arrived at?" or "Is this logical based on everything I know about the scenario?" Knowing how to go about such a review will require you to understand the concepts you are attempting to apply and what you are trying to make the calculator do. Further, it is critical to understand the relationships among the different inputs and variables of the problem. If you do not fully understand these relationships, you may end up with an incorrect answer. In the end, it is important to realize that any calculator is simply a tool. It will only do what you direct it to do and has no idea what your objective is or what it is that you really wish to accomplish.

THINK IT THROUGH

Determining Interest or Growth Rate

Here is another example of using a financial calculator to solve a common time value of money problem. Let's use a similar example to the one we used when calculating periods of time to determine an interest or growth rate. You still want to help your child with their first year of college tuition and related expenses. You also still have a starting amount of $15,000, but you have not yet decided on a savings plan to use.

Instead, the information you now have is that your child is just under 10 years old and will begin college at age 18. For simplicity's sake, let's say that you have eight and a half years before you will need to meet your total savings target of $25,000. What rate of interest will you need to grow your saved money from $15,000 to $25,000 in this time period, again with no other deposits or withdrawals?

Solution:

Follow the steps shown in Table 7.7.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter present value (as a negative integer)	15000 +|− PV	PV =	-15,000.0000
3	Enter time periods	8.5 N	N =	8.5000
4	Enter future value	25000 FV	FV =	25,000.0000
5	Indicate no payments or deposits	0 PMT	PMT =	0.0000
6	Compute interest rate	CPT I/Y	I/Y =	6.1940

Table 7.7 Calculator Steps for Determining Interest Rate

The result of this calculation is a necessary interest rate of 6.194%.

Using Excel to Solve TVM Problems

Excel spreadsheets can be excellent tools to use when solving time value of money problems. There are dozens of financial functions available in Excel, but a student who can use a few of these functions can solve almost any TVM problem. Special functions that relate to TVM calculations are as follows:

Future Value (FV)

Present Value (PV)

Number of Periods (NPER)

Interest Rate (RATE)

Excel also includes a function called Payment (PMT) that is used in calculations involving multiple payments or deposits (annuities). These will be covered in Time Value of Money II: Equal Multiple Payments.

Future Value (FV)

The Future Value function in Excel is also referred to as FV and can be used to calculate the value of a single lump sum amount carried to any point in the future. The FV function syntax is similar to that of the other four basic time-value functions and has the following inputs (referred to as arguments), similar to the functions listed above:

Rate: Interest Rate

Nper: Number of Periods

Pmt: Payment

PV: Present Value

Lump sum problems do not involve payments, so the value of Pmt in such calculations is 0. Another argument,

Type, refers to the timing of a payment and carries a default value of the end of the period, which is the most common timing (as opposed to the beginning of a period). This may be ignored in our current example, which means the default value of the end of the period will be used.

The spreadsheet in Figure 7.3 shows two examples of using the FV function in Excel to calculate the future value of $100 in five years at 5% interest.

In cell E1, the FV function references the values in cells B1 through B4 for each of the arguments. When a user begins to type a function into a spreadsheet, Excel provides helpful information in the form of on-screen tips showing the argument inputs that are required to complete the function. In our spreadsheet example, as the FV formula is being typed into cell E2, a banner showing the arguments necessary to complete the function appears directly below, hovering over cell E3.

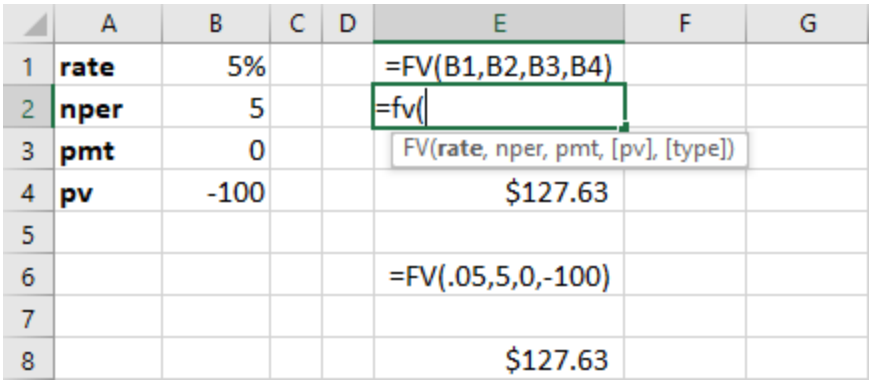


Figure 7.3 Using the FV Function in Excel

Cells E1 and E2 show how the FV function appears in the spreadsheet as it is typed in with the required arguments. Cell E4 shows the calculated answer for cell E1 after hitting the enter key. Once the enter key is pressed, the hint banner hovering over cell E3 will disappear. The second example of the FV function in our example spreadsheet is in cell E6. Here, the actual numerical values are used in the FV function equation rather than cell references. The method in cell E8 is referred to as *hard coding*. In general, it is preferable to use the cell reference method, as this allows for copying formulas and provides the user with increased flexibility in accounting for changes to input data. This ability to accept cell references in formulas is one of the greatest strengths of Excel as a spreadsheet tool.

Download the spreadsheet file (https://openstax.org/r/docs.google_uc_export) containing key Chapter 7 Excel exhibits.

Determining Future Value When Other Variables Are Known. You have $2,000 invested in a money market account that is expected to earn 4% annually. What will be the total value in the account in five years?

$$\text{Present Value (PV)} = (\$2{,}000.00)$$

Note: Be sure to follow the sign conventions. In this case, the PV should be entered as a negative value.

$$\text{Interest Rate (Rate or I/Y)} = 4\%$$

Note: In Excel, interest and growth rates must be entered as percentages, not as whole integers. So, 4 percent must be entered as 4% or 0.04—not 4, as you would enter in a financial calculator.

$$\text{Number of Periods (Nper or N)} = 5.00$$

Note: It is always assumed that if not specifically stated, the compounding period of any given interest rate is annual, or based on years.

$$\text{Future Value (FV)} = \$2{,}433.31$$

Note: The Excel command used to calculate future value is as follows:

```
=FV(rate, nper, pmt, [pv], [type])
```

You may simply type the values for the arguments in the above formula. Another option is to use the Excel insert function option. If you decide on this second method, below are several screenshots of dialog boxes you will encounter and will be required to complete.

1. First, go to Formulas in the upper menu bar, and select the Insert Function option. When you do so, a dialog box will appear that looks like what you see in Figure 7.4.

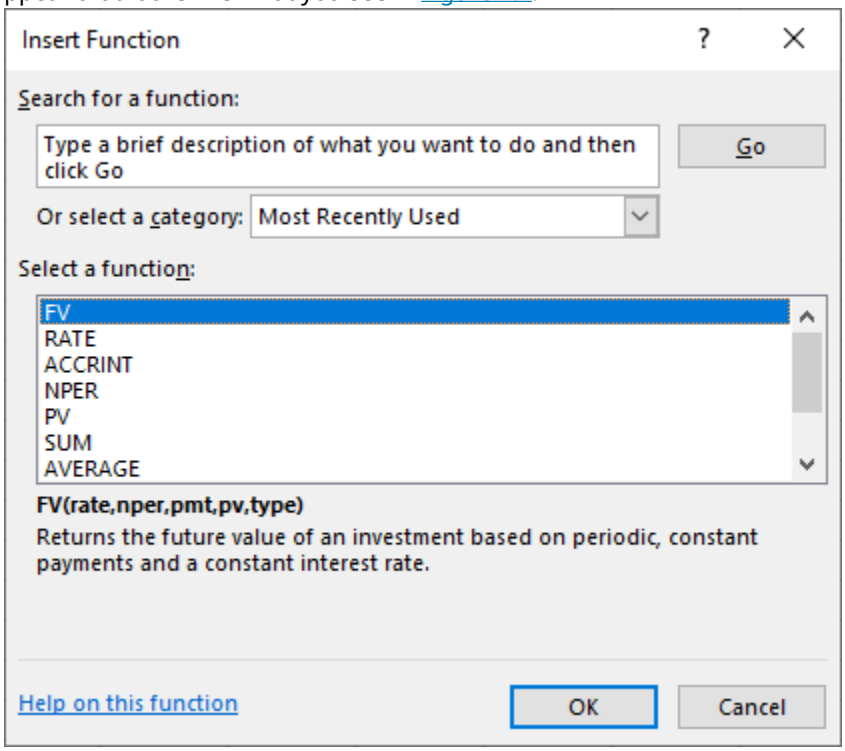


Figure 7.4 Dialog Box to Insert FV Function

   This dialog box allows you to either search for a function or select a function that has been used recently. In this example, you can search for FV by typing this in the search box and selecting Go, or you can simply choose FV from the list of most recently used functions (as shown here with the highlighted FV option).

2. Once you select FV and click the OK button, a new dialog box will appear for you to enter the necessary details. See Figure 7.5.

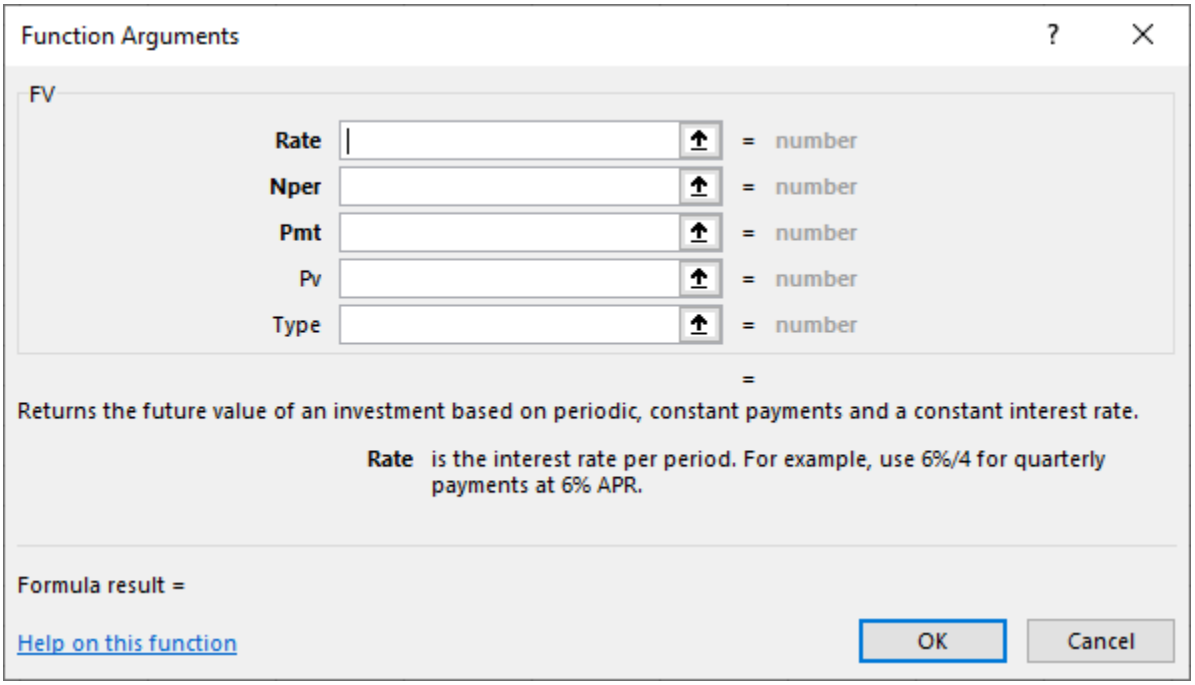


Figure 7.5 New Dialog Box for FV Function Arguments

Figure 7.6 shows the completed data input for the variables, referred to here as "function arguments." Note that cell addresses are used in this example. This allows the spreadsheet to still be useful if you decide to change any of the variables. You may also type values directly into the Function Arguments dialog box, but if you do this and you have to change any of your inputs later, you will have to reenter the new information. Using cell addresses is always a preferable method of entering the function argument data.

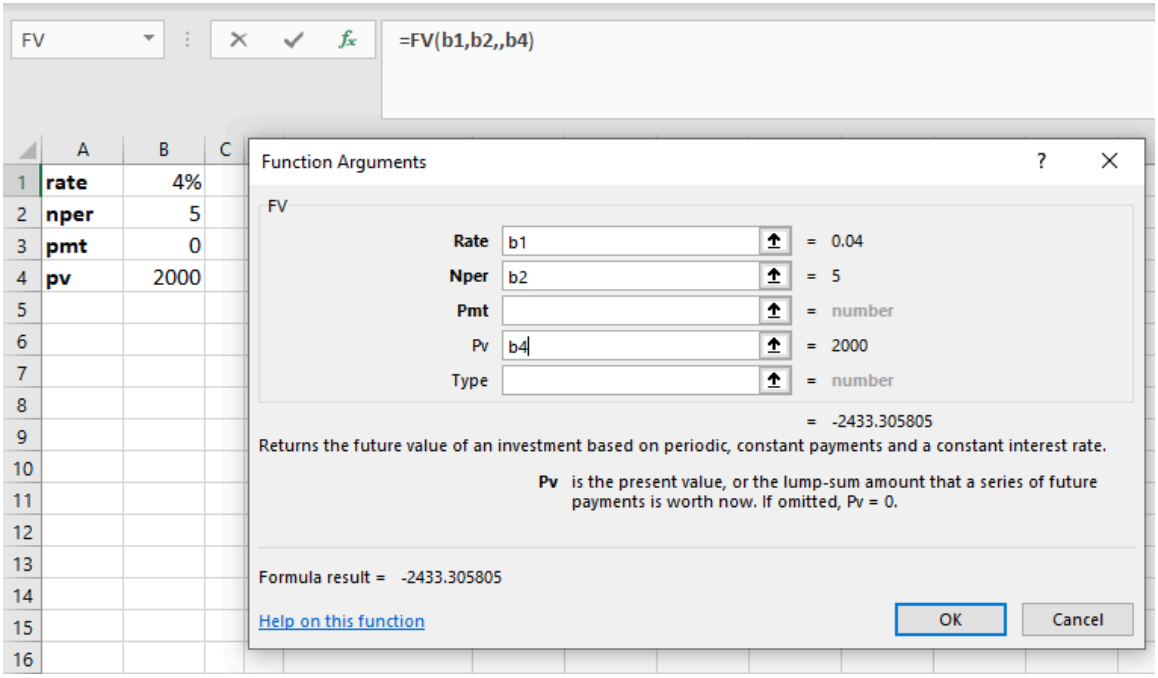


Figure 7.6 Completed Data Entry Menu for FV Function Arguments

Additional notes:

1. The Pmt argument or variable can be ignored in this instance, or you can enter a placeholder value of

zero. This example shows a blank or ignored entry, but either option may be used in problems such as this where the information is not relevant.

2. The Type argument does not apply to this problem. Type refers to the timing of cash flows and is usually used in multiple payment or annuity problems to indicate whether payments or deposits are made at the beginning of periods or at the end. In single lump sum problems, this is not relevant information, and the Type argument box is left empty.
3. When you use cell addresses as function argument inputs, the numerical values within the cells are displayed off to the right. This helps you ensure that you are identifying the correct cells in your function. The final answer generated by the function is also displayed for your preliminary review.

Once you are satisfied with the result, hit the OK button, and the dialog box will disappear, with only the final numerical result appearing in the cell where you have set up the function.

The FV of this present value has been calculated as approximately $2,433.31.

Present Value (PV)

We have covered the idea that present value is the opposite of future value. As an example, in the spreadsheet shown in Figure 7.3, we calculated that the future value of $100 five years from now at a 5% interest rate would be $127.63. By reversing this process, we can safely state that $127.63 received five years from now with a 5% interest (or discount) rate would have a value of just $100 today. Thus, $100 is its present value. In Excel, the PV function is used to determine present value (see Figure 7.7).

	A	B	C	D	E	F	G
1	**rate**	5%			=PV(B1,B2,B3,B4)		
2	**nper**	5			=pv(		
3	**pmt**	0			PV(**rate**, nper, pmt, [fv], [type])		
4	**fv**	-127.6			$100.00		
5							
6					=PV(.05,5,0,-100)		
7							
8					$100.00		

Figure 7.7 Using the PV Function in Excel

The formula in cell E1 uses cell references in a similar fashion to our FV example spreadsheet above. Also similar to our earlier example is the hard-coded formula for this calculation, which is shown in cell E6. In both cases, the answers we arrive at using the PV function are identical, but once again, using cell references is preferred over hard coding if possible.

THINK IT THROUGH

Determining Present Value When Other Variables Are Known

You have just won a second-prize lottery jackpot that will pay a single total lump sum of $50,000 five years from now. You are interested in knowing how much value this would have in today's dollars, assuming a 5% interest rate.

$$\text{Future Value (FV)} = (\$50{,}000.00)$$
$$\text{Interest Rate (Rate or I/Y)} = 5\%$$
$$\text{Number of Periods (Nper or N)} = 5.00$$
$$\text{Present Value (PV)} = \$39{,}176.31$$

Notes:

1. If you wish for the present value amount to be positive, the future value you enter here should be a negative value.
2. In Excel, interest and growth rates must be entered as percentages, not as whole integers. So, 5 percent must be entered as 5% or 0.05—not 5, as you would enter in a financial calculator.
3. It is always assumed that if not specifically stated, the compounding period of any given interest rate is annual, or based on years.
4. The Excel command used to calculate present value is as shown here:

```
=PV(rate, nper, pmt, [fv], [type])
```

Solution:

As with the FV formula covered in the first tab of this workbook, you may simply type the values for the arguments in the above formula. Another option is to again use the Insert Function option in Excel. Figure 7.8, Figure 7.9, and Figure 7.10 provide several screenshots that demonstrate the steps you'll need to follow if you decide to enter the PV function from the Insert Function menu.

1. First, go to Formulas in the upper menu bar, and select Insert Function. When you do so, the Insert Function dialog box will appear (see Figure 7.8).

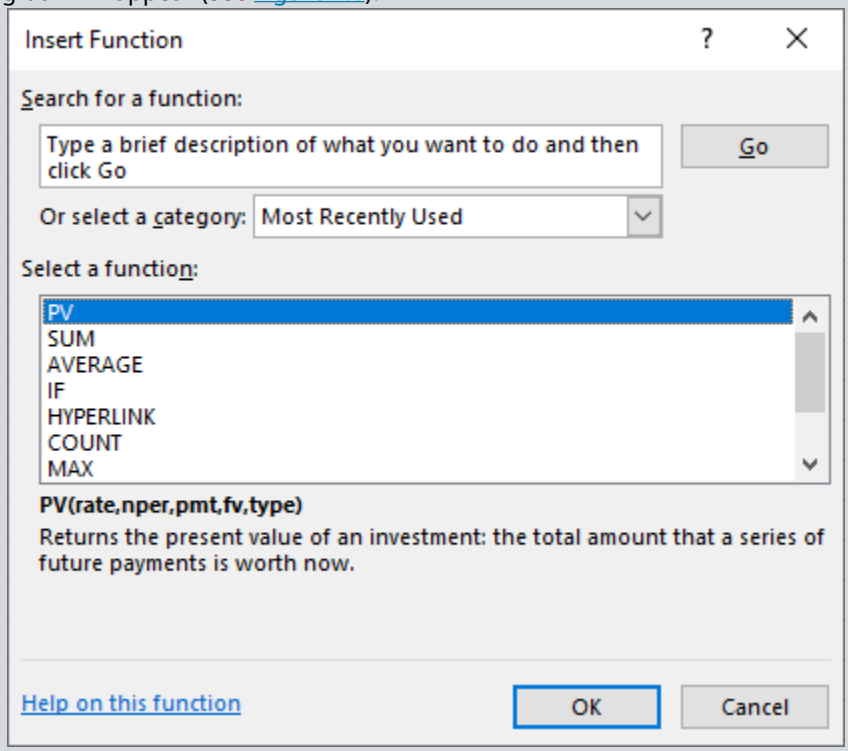


Figure 7.8 Dialog Box to Insert PV Function

   As discussed in the FV function example above, this dialog box allows you to either search for a function or select a function that has been used recently. In this example, you can search for PV by typing this into the search box and selecting Go, or you can simply choose PV from the list of the most recently used functions.

2. Once you have highlighted PV, click the OK button, and a new dialog box will appear for you to enter the necessary details. Similar to our FV function example, it will look like Figure 7.9.

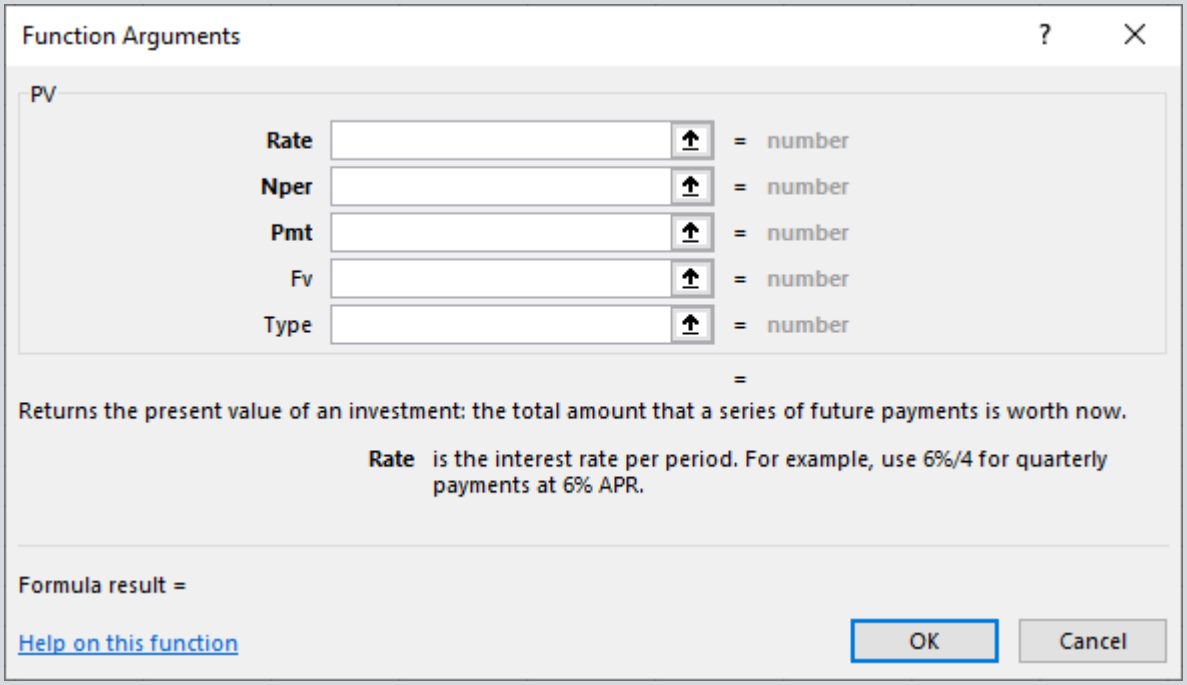


Figure 7.9 New Dialog Box for PV Function Arguments

Figure 7.10 shows the completed data input for the function arguments. Note that once again, cell addresses are used in this example. This allows the spreadsheet to still be useful if you decide to change any of the variables. As in the FV function example, you may also type values directly in the Function Arguments dialog box, but if you do this and you have to change any of your input later, you will have to reenter the new information. Remember that using cell addresses is always a preferable method of entering the function argument data.

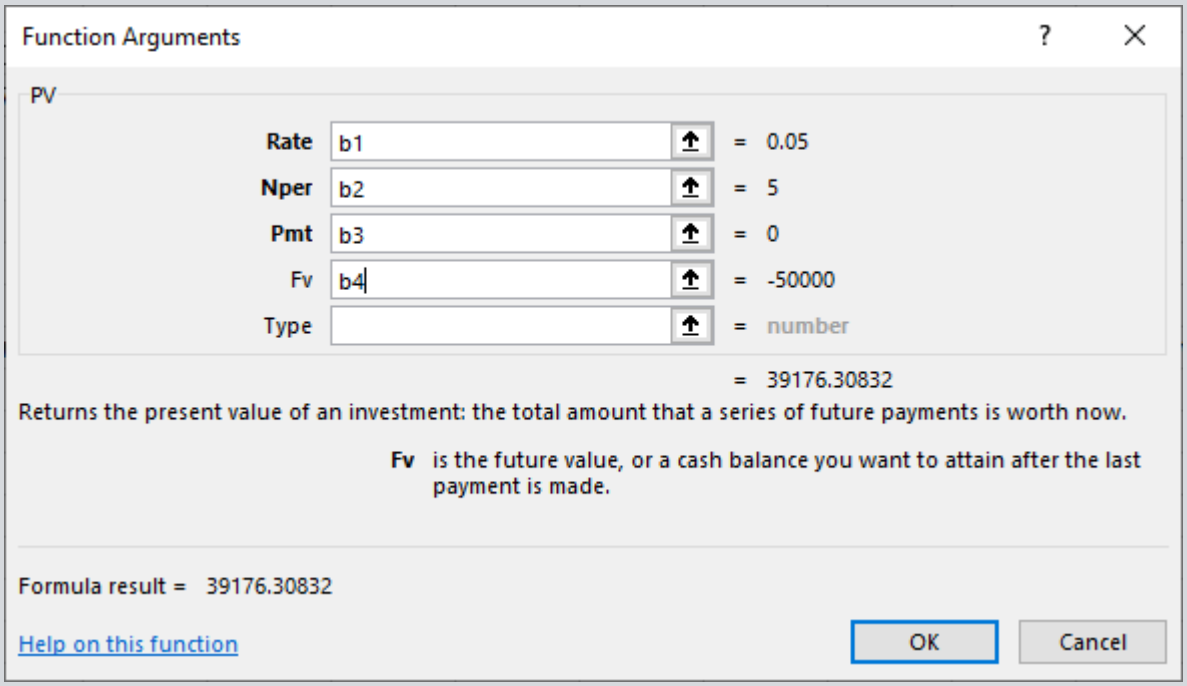


Figure 7.10 Completed Dialog Box for PV Function Arguments

Again, similar to our FV function example, the Function Arguments dialog box shows values off to the

right of the data entry area, including our final answer. The Pmt and Type boxes are again not relevant to this single lump sum example, for reasons we covered in the FV example.

Review your answer. Once you are satisfied with the result, click the OK button, and the dialog box will disappear, with only the final numerical result appearing in the cell where you have set up the function. The PV of this future value has been calculated as approximately $39,176.31.

Periods of Time

The following discussion will show you how to use Excel to determine the amount of time a given present value will need to grow into a specified future value when the interest or growth rate is known.

You want to be able to contribute $25,000 to your child's first year of college tuition and related expenses. You currently have $15,000 in a tuition savings account that is earning 6% interest every year. How long will it take for this account grow into the targeted amount of $25,000, assuming no additional deposits or withdrawals are made?

$$\text{Future Value (FV)} = \$25{,}000.00$$
$$\text{Interest Rate (Rate or I/Y)} = 6\%$$
$$\text{Present Value (PV)} = (\$15{,}000.00)$$
$$\text{Number of Periods (NPER)} = 8.7667$$

Notes:

1. As with our other examples, interest and growth rates must be entered as percentages, not as whole integers. So, 6 percent must be entered as 6% or 0.06—not 6, as you would enter in a financial calculator.
2. The present value needs to be entered as a negative value in accordance with the sign convention covered earlier.
3. The Excel command used to calculate the amount of time, or number of periods, is this:

```
=NPER (rate, pmt, pv, [fv], [type])
```

As with our FV and PV examples, you may simply type the values of the arguments in the above formula, or we can again use the Insert Function option in Excel. If you do so, you will need to work with the various dialog boxes after you select Insert Function.

1. First, go to Formulas in the upper menu bar, and select the Insert Function option. When you do so, the Insert Function dialog box will appear (see Figure 7.11).

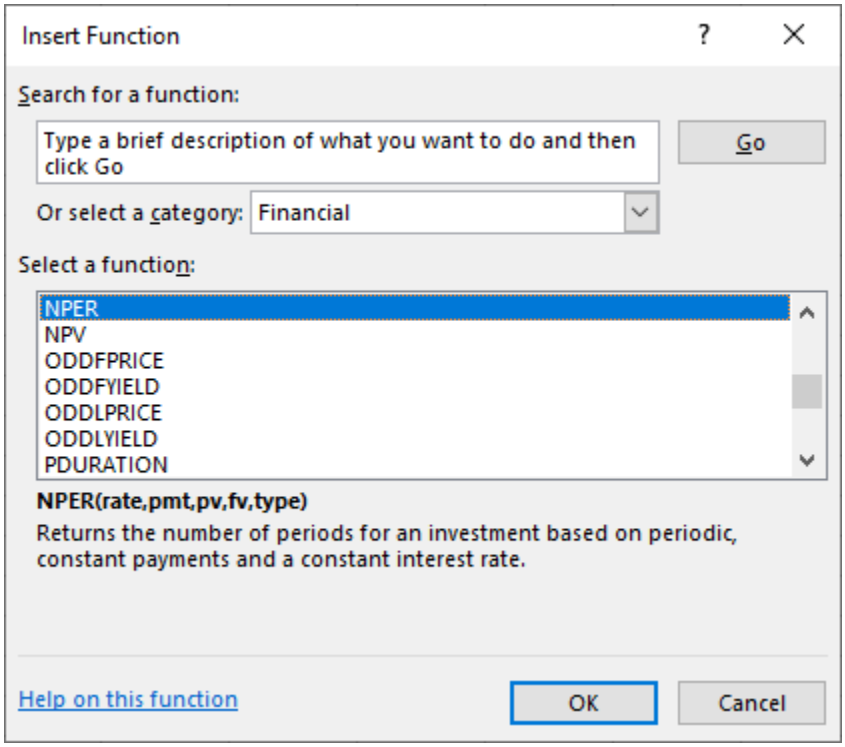


Figure 7.11 Dialog Box to Insert NPER Function

As discussed in our previous examples on FV and PV, this menu allows you to either search for a function or select a function that has been used recently. In this example, you can search for NPER by typing this into the search box and selecting Go, or you can simply choose NPER from the list of most recently used functions.

2. Once you have highlighted NPER, click the OK button, and a new dialog box will appear for you to enter the necessary details. As in our previous examples, it will look like Figure 7.12.

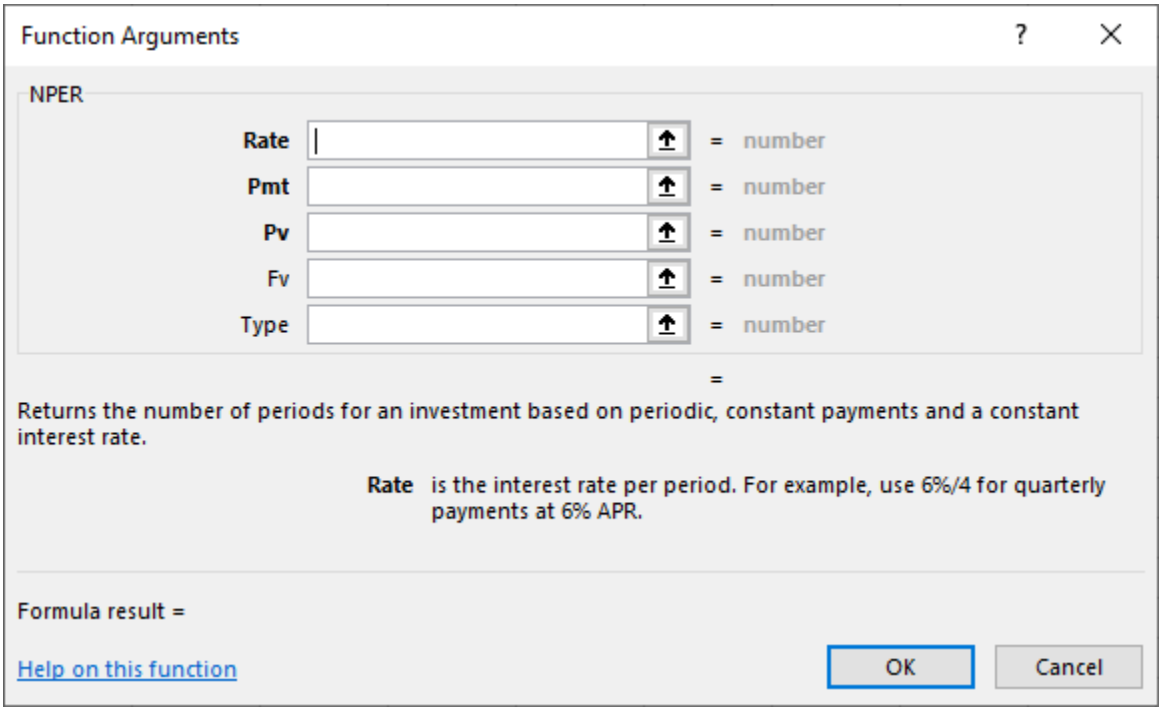


Figure 7.12 New Dialog Box for NPER Function Arguments

Figure 7.13 shows the completed Function Arguments dialog box. Note that once again, we are using cell addresses in this example.

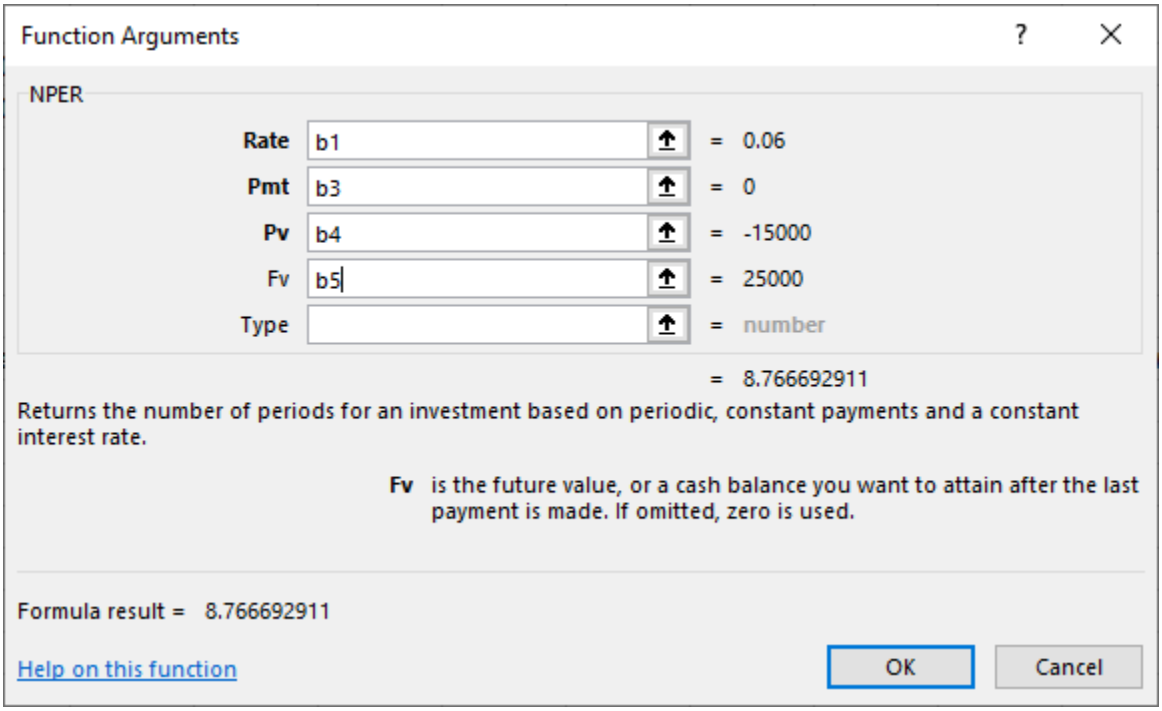


Figure 7.13 Completed Dialog Box for NPER Function Arguments

As in the previous function examples, values are shown off to the right of the data input area, and our final answer of approximately 8.77 is displayed at the bottom. Also, once again, the Pmt and Type boxes are not relevant to this single lump sum example.

Review your answer, and once you are satisfied with the result, click the OK button. The dialog box will disappear, with only the final numerical result appearing in the cell where you have set up the function.

The amount of time required for the desired growth to occur is calculated as approximately 8.77 years.

Interest or Growth Rate

You can also use Excel to determine the required growth rate when the present value, future value, and total number of required periods are known.

Let's discuss a similar example to the one we used to calculate periods of time. You still want to help your child with their first year of college tuition and related expenses, and you still have a starting amount of $15,000, but you have not yet decided which savings plan to use.

Instead, the information you now have is that your child is just under 10 years old and will begin college at age 18. For simplicity's sake, let's say that you have eight and a half years until you will need to meet your total savings target of $25,000. What rate of interest will you need to grow your saved money from $15,000 to $25,000 in this time, again with no other deposits or withdrawals?

$$\text{Future Value (FV)} = \$25{,}000.00$$
$$\text{Number of Periods (Nper or N)} = 8.50$$
$$\text{Present/Value (PV)} = (\$15{,}000.00)$$

Note: The present value needs to be entered as a negative value.

`Interest Rate (RATE)`

Note: The Excel command used to calculate interest or growth rate is as follows:

```
=RATE(nper, pmt, pv, [fv], [type], [guess])
```

As with our other TVM function examples, you may simply type the values for the arguments into the above formula. We also again have the same alternative to use the Insert Function option in Excel. If you choose this option, you will again see the Insert Function dialog box after you click the Insert Function button.

1. First, go to Formulas in the upper menu bar, and select the Insert Function option. When you do so, the Insert Function dialog box will appear (see Figure 7.14).

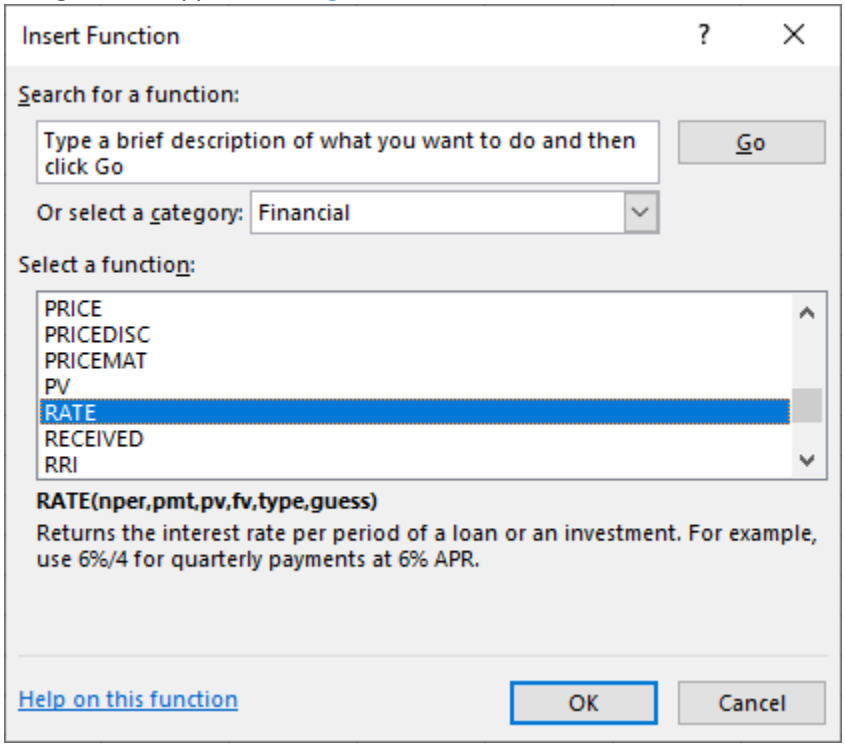


Figure 7.14 Dialog Box to Insert RATE Function

2. This time, find and highlight RATE, and click the OK button once you have done so. The Function Arguments dialog box will look like Figure 7.15.

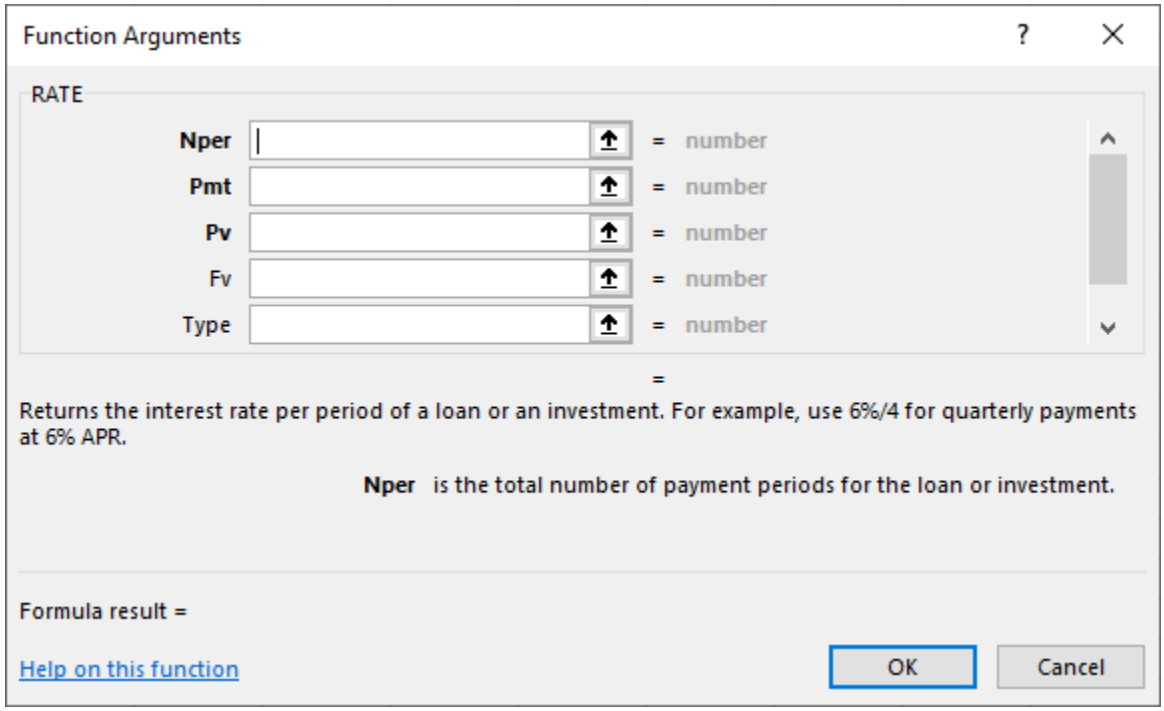


Figure 7.15 New Dialog Box for RATE Function Arguments

Once we complete the input, again using cell addresses for the required argument values, we will see what is shown in Figure 7.16.

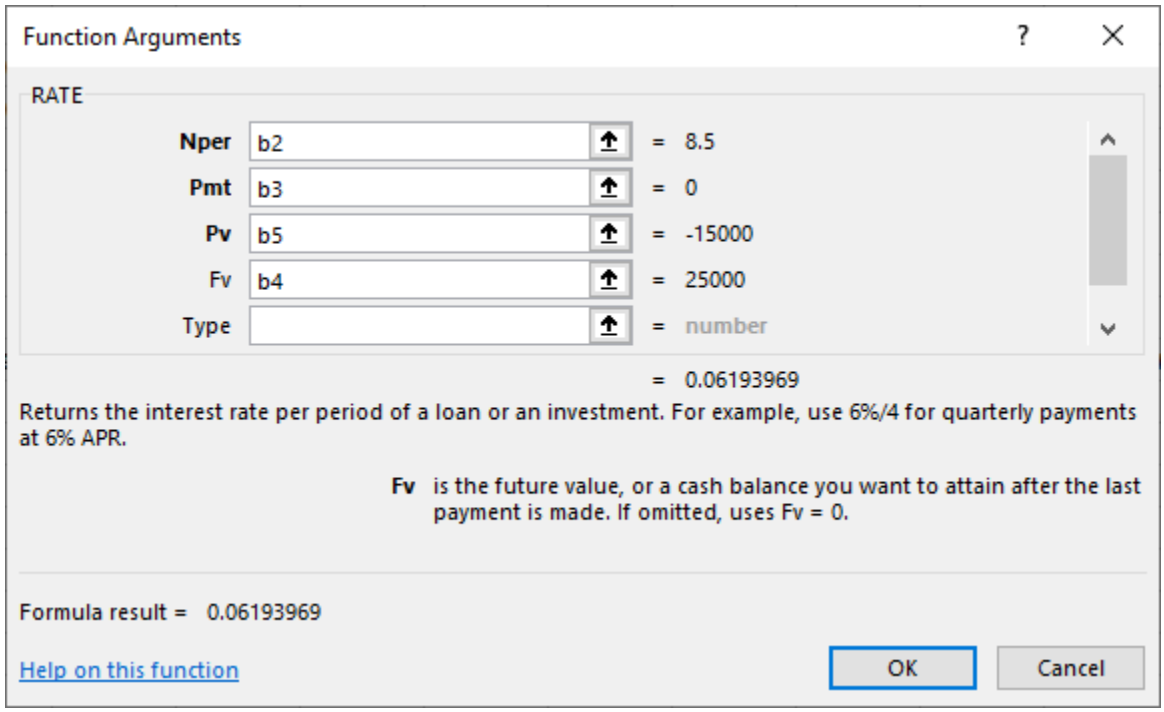


Figure 7.16 Completed Dialog Box for RATE Function Arguments

As in our other examples, cell values are shown as numerical values off to the right, and our answer of approximately 0.0619, or 6.19%, is shown at the bottom of the dialog box.

This answer also can be checked from a logic point of view because of the similar example we worked through when calculating periods of time. Our present value and future value are the same as in that example, and our time period is now 8.5 years, which is just under the result we arrived at (8.77 years) in

the periods example.

So, if we are now working with a slightly shorter time frame for the savings to grow from $15,000 into $25,000, then we would expect to have a slightly greater growth rate. That is exactly how the answer turns out, as the calculated required interest rate of approximately 6.19% is just slightly greater than the growth rate of 6% used in the previous example. So, based on this, it looks like our answer here passes a simple "sanity check" review.

7.4 Applications of TVM in Finance

Learning Outcomes

By the end of this section, you will be able to:

- Explain how the time value of money can impact your personal financial goals.
- Explain how the time value of money is related to inflation.
- Explain how the time value of money is related to financial risk.
- Explain how compounding period frequency affects the time value of money.

Single-Period Scenario

Let's say you want to buy a new car next year, and the one you have your eye on should be selling for $20,000 a year from now. How much will you need to put away today at 5% interest to have $20,000 a year from now? Essentially, you are trying to determine how much $20,000 one year from now is worth today at 5% interest over the year. To find a present value, we reverse the growth concept and lower or *discount* the future value back to the current period.

The interest rate that we use to determine the present value of a future cash flow is referred to as the **discount rate** because it is bringing the money back in time in terms of its value. The discount rate refers to the annual rate of reduction on a future value and is the inverse of the growth rate. Once we know this discount rate, we can solve for the present value (PV), the value today of tomorrow's cash flow. By changing the FV equation, we can turn

$$\text{FV} = \text{PV} \times (1+r)^n$$

into

$$\text{PV} = \text{FV} \times \frac{1}{(1+r)^n}$$

which is the present value equation. The fraction shown above is referred to as the present value interest factor (PVIF). The PVIF is simply the reciprocal of the FVIF, which makes sense because these factors are doing exactly opposite things. Therefore, the amount you need to deposit today to earn $20,000 in one year $(n = 1)$ at 5% interest is

$$\$20{,}000 \times \frac{1}{(1.05)^1} = \$20{,}000 \times 0.95328 = \$19{,}047.62$$

The Multiple-Period Scenario

There will often be situations when you need to determine the present value of a cash flow that is scheduled to occur several years in the future (see Figure 7.17). We can again use the formula for present value to calculate a value today of future cash flows over multiple time periods.

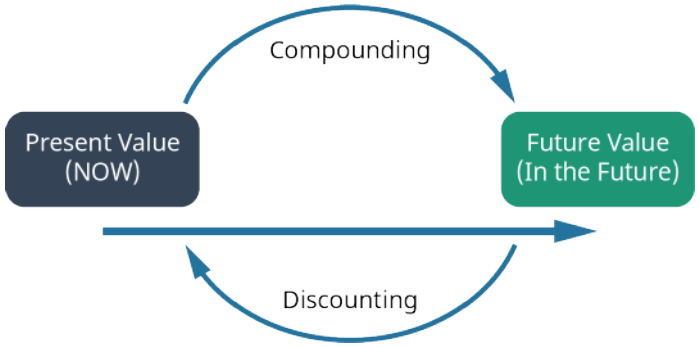


Figure 7.17 Determining Future Cash Flow

An example of this would be if you wanted to buy a savings bond for Charlotte, the daughter of a close friend. The face value of the savings bond you have in mind is $1,000, which is the amount Charlotte would receive in 30 years (the future value). If the government is currently paying 5% per year on savings bonds, how much will it cost you today to buy this savings bond?

The $1,000 face value of the bond is the future value, and the number of years *n* that Charlotte must wait to get this face value is 30 years. The interest rate *r* is 5.0% and is the discount rate for the savings bond. Applying the present value equation, we calculate the current price of this savings bond as follows:

$$\text{PV} = \$1{,}000 \times \frac{1}{(0.05)^{30}} = \$1{,}000 \times 0.231377 = \$231.38$$

So, it would cost you $231.38 to purchase this 30-year, 5%, $1,000 face-valued bond.

What we have done in the above example is reduce, or discount, the future value of the bond to arrive at a value expressed in today's dollars. Effectively, this discounting process is the exact opposite of compounding interest that we covered earlier in our discussion of future value.

An important concept to remember is that *compounding* is the process that takes a present valuation of money to some point in the future, while *discounting* takes a future value of money and equates it to present dollar value terms.

Common applications in which you might use the present value formula include determining how much money you would need to invest in an interest-bearing account today in order to finance a college education for your oldest child and how much you would need to invest today to meet your retirement plans 30 years from now.

TVM, Inflation, Compounding Interest, Investing, Opportunity Costs, and Risk

The time value of money (TVM) is a critical concept in understanding the value of money relative to the amount of time it is held, saved, or invested. The TVM concept and its specific applications are frequently used by individuals and organizations that might wish to better understand the values of financial assets and to improve investing and saving strategies, whether these are personal or within business environments.

As we have discussed, the key element behind the concept of TVM is that a given amount of money is worth more today than that same amount of money will be at any point in the future. Again, this is because money can be saved or invested in interest-bearing accounts or investments that will generate interest income over time, thus resulting in increased savings and dollar values as time passes.

Inflation

The entire concept of TVM exists largely due to the presence of inflation. Inflation is defined as a general increase in the prices of goods and services and/or a drop in the value of money and its purchasing power.

The purchasing power of the consumer dollar is a statistic tracked by the part of the US Bureau of Labor

Statistics and is part of the consumer price index (CPI) data that is periodically published by that government agency. In a way, purchasing power can be viewed as a mirror image or exact opposite of inflation or increases in consumer prices, as measured by the CPI. Figure 7.18 demonstrates the decline in the purchasing power of the consumer dollar over the 13-year period from 2007 to 2020.

With this in mind, we can work with the TVM formula and use it to help determine the present value of money you have in hand today, as well as how this same amount of money may be valued at any specific point in the future and at any specific rate of interest.

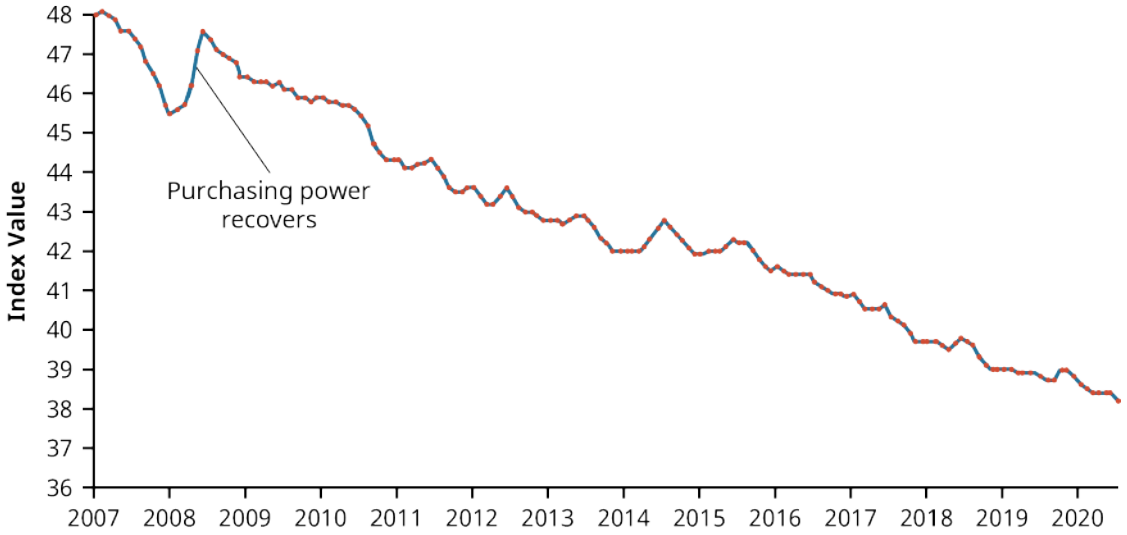


Figure 7.18 Historical Purchasing Power Decline as a Result of Inflation, 2007–2020 (data source: Bureau of Labor Statistics)

The Relationship between TVM and Inflation

As we have seen, the future value formula can be very helpful in calculating the value of a sum of cash (or any **liquid asset**) at some future point in time. One of the important ideas relating to the concept of TVM is that it is preferable to spend money today instead of at some point in the future (all other things being equal) when inflation is positive. However, in very rare instances in our economy when inflation is negative, spending money later is preferable to spending it now. This is because in cases of negative inflation, the purchasing power of a dollar is actually greater in the future, as the costs of goods and services are declining as we move into the future.

Most investors would be inclined to take a payment of money today rather than wait five years to receive a payment in the same amount. This is because inflation is almost always positive, which means that general prices of goods and services tend to increase over the passage of time. This is a direct function and result of normal economic growth. The crux of the concept of TVM is directly related to maintaining the present value of financial assets or increasing the value of these financial assets at different points in the future when they may be needed to obtain goods and services. If a consumer's monetary assets grow at a greater rate than inflation over any period of time, then the consumer will realize an increase in their overall purchasing power. Conversely, if inflation exceeds savings or investment growth, then the consumer will lose purchasing power as time goes by under such conditions.

The Impact of Inflation on TVM

The difference between present and future values of money can be easily seen when considered under the effects of inflation. As we discussed above, inflation is defined as a state of continuously rising prices for goods and services within an economy. In the study of economics, the laws of supply and demand state that increasing the amount of money within an economy without increasing the amount of goods and services available will give consumers and businesses more money to spend on those goods and services. When more money is created and made available to the consuming public, the value of each unit of currency will diminish.

This will then have the effect of incentivizing consumers to spend their money now, or in the very near future, instead of saving cash for later use. Another concept in economics states that this relationship between **money supply** and monetary value is one of the primary reasons why the **Federal Reserve** might at times take steps to inject money into a stagnant, lethargic economy. Increasing the money supply will lead to increased economic activity and consumer spending, but it can also have the negative effect of increasing the costs of goods and services, furthering an increase in the rate of inflation.

Consumers who decide to save their money now and for the foreseeable future, as opposed to spending it now, are simply making the economic choice to have their cash on hand and available. So, this ends up being a decision that is made despite the risk of potential inflation and perhaps losing purchasing power. When inflationary risk is low, most people will save their money to have it available to spend later. Conversely, in times when inflationary risk is high, people are more likely to spend their money now, before its purchasing power erodes. This idea of inflationary risk is the primary reason why savers and investors who decide to save now in order to have their money available at some point in the future will insist they are paid, through interest or return on investment, for the future value of any savings or financial instrument.

Lower interest rates will usually lead to higher inflation. This is because, in a way, interest rates can be viewed as the cost of money. This allows for the idea that interest rates can be further viewed as a tax on holding on to sums of money instead of using it. If an economy is experiencing lower interest rates, this will make money less expensive to hold, thus incentivizing consumers to spend their money more frequently on the goods and services they may require. We have also seen that the more quickly inflation rates rise, the more quickly the general purchasing power of money will be eroded. Rational investors who set money aside for the future will demand higher interest rates to compensate them for such periods of inflation. However, investors who save for future consumption but leave their money **uninvested** or **underinvested** in low-interest-bearing accounts will essentially lose value from their financial assets because each of their future dollars will be worth less, carrying less purchasing power when they end up needing it for use. This relationship of saving and planning for the future is one of the most important reasons to understand the concept of the time value of money.

Nominal versus Real Interest Rates

One of the main problems of allowing inflation to determine interest rates is that current interest rates are actually nominal interest rates. Nominal rates are "stated," not adjusted for the effects of inflation. In order to determine more practical **real interest rates**, the original nominal rate must be adjusted using an inflation rate, such as those that are calculated and published by the **Bureau of Labor Statistics** within the **consumer price index (CPI)**.

A concept referred to as the **Fisher effect**, named for economist Irving Fisher, describes the relationship between inflation and the nominal and real interest rates and is expressed using the following formula:

$$(1 + i) = (1 + R) \times (1 + h)$$

where i is the nominal interest rate, R is the real interest rate, and h is the expected inflation rate.

An example of the Fisher effect would be seen in the case of a bond investor who is expecting a real interest rate of return of 6% on the bond, in an economy that is experiencing an expected inflation rate of 2%. Using the above formula, we have

$$\begin{aligned} i &= (1 + R) \times (1 + h) - 1 \\ i &= (1 + 6\%) \times (1 + 2\%) - 1 \\ i &= 8.12\% \end{aligned}$$

So, the nominal interest rate on the bond amounts to 8.12%, with a real interest rate of 6% within an economy that is experiencing a 2% inflation rate. This is a logical result because in a scenario of positive inflation, a real rate of return would always be expected to amount to less than the stated or nominal rate.

Interest and Savings

Savings are adversely affected by negative real interest rates. A person who holds money in the form of cash is actually losing future purchasing value when real interest rates are negative. A saver who decides to hold $1,000 in the form of cash for one year at a negative real interest rate of −3.65% per year will lose $\$1{,}000 \times -0.0365$ or $36.50, in purchasing power by the end of that year.

Ordinarily, interest rates would rise to compensate for negative real rates, but this might not happen if the Federal Reserve takes steps to maintain low interest rates to help stimulate and stabilize the economy. When interest rates are at such low levels, investors are forced out of Treasury and **money market investments** due to their extremely poor returns.

It soon becomes obvious that the time value of money is a critical concept because of its tremendous and direct impact on the daily spending, saving, and investment decisions of the people in our society. It is therefore extremely important that we understand how TVM and government fiscal policy can affect our savings, investments, purchasing behavior, and our overall personal financial health.

Compounding Interest

As we discussed earlier, compound interest can be defined as interest that is being earned on interest. In cases of **compounding interest**, the amount of money that is being accrued on previous amounts of earned interest income will continue to grow with each **compounding period**. So, for example, if you have $1,000 in a savings account and it is earning interest at a 10% annual rate and is compounded every year for a period of five years, the compounding will allow for growth after one year to an amount of $1,100. This comprises the original principal of $1,000 plus $100 in interest. In year two, you would actually be earning interest on the total amount from the previous compounding period—the $1,100 amount.

So, to continue with this example, by the end of year two, you would have earned $1,210 ($1,100 plus $110 in interest). If you continue on until the end of year five, that $1,000 will have grown to approximately $1,610. Now, if we consider that the highest annual inflation rate over the last 20 years has been 3%, then in this scenario, choosing to invest your present money in an account where interest is being compounded leaves you in a much better position than you would be in if you did not invest your money at all. The concept of the time value of money puts this entire idea into context for us, leading to more informed decisions on personal saving and investing.

It is important to understand that interest does not always compound annually, as assumed in the examples we have already covered. In some cases, interest can be compounded quarterly, monthly, daily, or even continuously. The general rule to apply is that the more frequent the compounding period, the greater the future value of a savings amount, a bond, or any other financial instrument. This is, of course, assuming that all other variables are constant.

The math for this remains the same, but it is important that you be careful with your treatment and usage of rate (*r*) and number of periods (*n*) in your calculations.

For example, $1,000 invested at 6% for a year compounded annually would be worth $\$1{,}000 \times (1.06)^1 = \$1{,}060.00$. But that same $1,000 invested for that same period of time—one year—and earning interest at the same annual rate but compounded monthly would grow to $\$1{,}000 \times (1.005)^{12} = \$1{,}061.67$, because the interest paid each month is earning interest on interest at a 6% rate. Note that we represent *r* as the interest paid per period $\left(\frac{0.06 \text{ annual interest}}{12 \text{ months in a year}} = 0.005\right)$ and *n* as the number of periods (12 months in a year; $12 \times 1 = 12$) rather than the number of years, which is only one.

Continuing with our example, that same $1,000 in an account with interest compounded quarterly, or four times a year, would grow to $\$1{,}000 \times (1.05)^4 = \$1{,}061.36$ in one year. Note that this final amount ends up being greater than the annually compounded future value of $1,060.00 and slightly less than the monthly

compounded future value of $1.061.67, which would appear to make logical sense.

The total differences in future values among annual, monthly, and quarterly compounding in these examples are insignificant, amounting to less than $1.70 in total. However, when working with larger amounts, higher interest rates, more frequent compounding periods, and longer terms, compounding periods and frequency become far more important and can generate some exceptionally large differences in future values.

Ten million dollars at 12% growth for one year and compounded annually amounts to $\$10{,}000 \times (1.12) = \$11{,}200{,}000$, while 10 million dollars on the same terms but compounded quarterly will produce $\$10{,}000{,}000 \times (1.03)^4 = \$11{,}255{,}088.10$. Most wealthy and rational investors and savers would be very pleased to earn that additional $55,088.10 by simply having their funds in an account that features quarterly compounding.

In another example, $200 at 60% interest, compounded annually for six years, becomes $\$200 \times (1.6)^6 = \$3{,}355.44$, while this same amount compounded quarterly grows to $\$200 \times (1.15)^{24} = \$5{,}725.04$.

An amount of $1 at 3%, compounded annually for 100 years, will be worth $\$1 \times (1.03)^{100} = \$19.22$. The same dollar at the same interest rate, compounded monthly over the course of a century, will grow to $\$1 \times (1.0025)^{1{,}200} = \$397.44$.

This would all seem to make sense due to the fact that in situations when compounding increases in frequency, interest income is being received during the year as opposed to at the end of the year and thus grows more rapidly to become a larger and more valuable sum of money. This is important because we know through the concept of TVM that having money now is more useful to us than having that same amount of money at some later point in time.

The Rule of 72

The rule of 72 is a simple and often very useful mathematical shortcut that can help you estimate the impact of any interest or growth rate and can be used in situations ranging from financial calculations to projections of population growth. The formula for the rule of 72 is expressed as the unknown (the required amount of time to double a value) calculated by taking the number 72 and dividing it by the known interest rate or growth rate. When using this formula, it is important to note that the rate should be expressed as a whole integer, not as a percentage. So, as a result, we have

$$\text{Years for an Amount to Double} = \frac{72}{\text{Interest or Growth Rate}}$$

This formula can be extremely practical when working with financial estimates or projections and for understanding how compound interest can have a dramatic effect on an original amount or monetary balance.

Following are just a few examples of how the rule of 72 can help you solve problems very quickly and very easily, often enabling you to solve them “in your head,” without the need for a calculator or spreadsheet.

Let’s say you are interested in knowing how long it will take your savings account balance to double. If your account earns an interest rate of 9%, your money will take $72/9$ or 8, years to double. However, if you are earning only 6% on this same investment, your money will take $72/6$, or 12, years to double.

Now let’s say you have a specific future purchasing need and you know that you will need to double your money in five years. In this case, you would be required to invest it at an interest rate of $72/5$, or 14.4%. Through these sample examples, it is easy to see how relatively small changes in a growth or interest rate can have significant impact on the time required for a balance to double in size.

To further illustrate some uses of the rule of 72, let’s say we have a scenario in which we know that a country’s **gross domestic product** is growing at 4% a year. By using the rule of 72 formula, we can determine that it will

take the economy 72/4, or 18, years to effectively double.

Now, if the economic growth slips to 2%, the economy will double in 72/2, or 36, years. However, if the rate of growth increases to 11%, the economy will effectively double in 72/11, or 6.55, years. By performing such calculations, it becomes obvious that reducing the time it takes to grow an economy, or increasing its rate of growth, could end up being very important to a population, given its current level of technological innovation and development.

It is also very easy to use the rule of 72 to express future costs being impacted by inflation or future savings amounts that are earning interest.

To apply another example, if the inflation rate in an economy were to increase from 2% to 3%, consumers would lose half of the purchasing power of their money. This is calculated as the value of their money doubling in 72/3, or 24, years as compared to 72/2, or 36, years—quite a substantial difference.

Now, let's say that tuition costs at a certain college are increasing at a rate of 7% per year, which happens to be greater than current inflation rates. In this case, tuition costs would end up doubling in 72/7, or about 10.3, years.

In an example related to personal finance, we can say that if you happen to have an annual percentage rate of 24% interest on your credit card and you do not make any payments to reduce your balance, the total amount you owe to the credit card company will double in only 72/24, or 3, years.

So, as we have seen, the rule of 72 can clearly demonstrate how a relatively small difference of 1 percentage point in GDP growth or inflation rates can have significant effects on any short- or long-term economic forecasting models.

It is important to understand that the rule of 72 can be applied in any scenario where we have a quantity or an amount that is in the process of growing or is expected to grow for any period of time into the future. A good nonfinancial use of the rule of 72 might be to apply it to some population projections. For example, an increase in a country's population growth rate from 2% to 3% could present a serious problem for the planning of facilities and infrastructure in that country. Instead of needing to double overall economic capacity in 72/2 or 36, years, capacity would have to be expanded in only 72/3, or 24, years. It is easy to see how dramatic an effect this would be when we consider that the entire schedule for growth or infrastructure would be reduced by 12 years due to a simple and relatively small 1% increase in population growth.

Investing and Risk

Investing is usually a sound financial strategy if you have the money to do so. When investing, however, there are certain risks you should always consider first when applying the concepts of the time value of money. For example, making the decision to take $1,000 and invest it in your favorite company, even if it is expected to provide a 5% return each year, is not a guarantee that you will earn that return—or any return at all, for that matter. Instead, as with any investment, you will be accepting the risk of losing some or even all of your money in exchange for the opportunity to beat inflation and increase your future overall wealth. Essentially, it is risk and return that are responsible for the entire idea of the time value of money.

Risk and return are the factors that will cause a rational person to believe that a dollar risked should end up earning more than that single dollar.

To summarize, the concept of the time value of money and the related TVM formulas are extremely important because they can be used in different circumstances to help investors and savers understand the value of their money today relative to its earning potential in the future. TVM is critical to understanding the effect that inflation has on your money and why saving your money early can help increase the value of your savings dollars by giving them time to grow and outpace the effects of inflation.

Opportunity Costs

The concept of **opportunity cost** arises from the idea that there will always be possible options that are sacrificed with every option we decide on or for every choice that we make. For example, let's consider the decision to go to college after you graduate from high school. This decision, as with just about any other, will involve evaluating opportunity costs. If you choose to go to college, this will result in your sacrificing four years of potential earnings that you could have had if you had decided to take a job instead of attending school. Also, in addition to the lost salary, you would be losing out on four years of work experience that could have had a positive impact on your résumé or your future earnings prospects.

Of course, the entire idea behind furthering one's education is that you are hopeful that by choosing to go to college, you will increase the likelihood of earning a greater salary over the course of your lifetime than you would have if you had chosen to join the workforce directly out of high school. So, this ends up being a bit of a risk, but one that you have considered. The idea is that you are hoping for a more significant payoff down the road than if you had made the decision not to continue with your studies. When it comes to opportunity costs and the time value of money, it is obvious that there will always be costs associated with every forgone financial opportunity we pass on when we make a different choice. The logical individual can only hope that these choices produce a better end result than if we had made different choices and pursued any of our forgone alternatives. This also applies in situations where we may sit idly by and decide to take no action at all.

For example, if you are putting $1,000 in a savings account to save for a house, you may be giving up an opportunity to grow that money in an investment account that would earn a greater rate of return. In another example, being able to calculate the future value of your money will tell you that instead of investing, you probably should be paying down your 24% APR credit card debt that is costing you hundreds of dollars a month—hundreds of dollars more than you might earn from an investment account.

Summary

7.1 Now versus Later Concepts

This section discussed the underlying concepts of the time value of money (TVM). Because it is possible to earn interest income on cash that you decide to deposit in an investment or an interest-bearing account, money that you have now or receive sooner will be more valuable to you than the same amount of money received later.

7.2 Time Value of Money (TVM) Basics

Future value refers to the value that a current amount will eventually grow into at a given interest rate over a specific period of time. The single-period scenario is one way in which future amounts are calculated. Compounding, which is interest earned on interest, also affects the future value of money.

7.3 Methods for Solving Time Value of Money Problems

Calculations can be used to determine future and present dollar amounts, discount and growth rates, and periods of time required for specific growth. Time value of money problems can be solved using mathematical equations, calculators with financial functions, and spreadsheets. A useful tool for conceptualizing present value and future value problems is a timeline. A timeline is a visual, linear representation of the timing of periods and cash flows over a set amount of time.

7.4 Applications of TVM in Finance

The idea of the time value of money is often considered to be the cornerstone concept of the study of finance. TVM can help investors and savers understand the value of money today relative to its earning potential in the future. TVM is critical to understanding the effect that inflation has on money and why saving your money early can help increase the value of your savings dollars by giving them time to grow and outpace the effects of inflation. Of course, it is important to remember that there will always be possible options that are sacrificed with every option you decide on and every choice you make.

Key Terms

Bureau of Labor Statistics a group within the United States Department of Labor that is the primary fact-finding agency for the US government in the fields of labor, statistics, and economics; serves as the principal agency of the US Federal Statistical System

compounding interest the continual addition of interest to the original principal sum of a loan or deposit, often referred to as interest on interest

compounding period the period between points in time when interest is paid or added to the principal

consumer price index (CPI) a measure that examines the weighted average of prices of a basket of consumer goods and services such as transportation, food, and medical care

discount rate the interest rate used to determine the present value of future cash inflows

Federal Reserve the central bank system of the United States

financial instrument an asset or bundle of assets, including monetary contracts between parties, that can be bought, sold, or traded for financial gain

financial risk the possibility of losing money or purchasing power on an investment, business transaction, or venture or simply due to inflation

Fisher effect an economic theory created by economist Irving Fisher that describes the relationship between inflation and both real and nominal interest rates

future value (FV) the value that a current amount will grow to at a given interest rate over a given period of time

gross domestic product (GDP) the total value of goods produced and services provided in a country during one year

growth rate the percentage increase of a specific variable within a specific time period; synonymous with *interest rate* in the context of the time value of money

interest the amount of money that is paid by a borrower to a lender for the use of their money, typically calculated from an annualized rate

investment an asset or item acquired with the goal of generating financial gain through increased income or appreciation in value

liquid asset an asset that can be readily converted into cash within a short period of time

money market investments low-risk financial instruments such as T-bills, federal notes, commercial paper, certificates of deposit (CDs), repurchase agreements (repos), and bankers' acceptances, among others

money supply the total dollar value of legal tender that is available to consumers within an economy at any single point in time

opportunity cost the loss of potential gain from other alternatives when a single alternative is chosen

present value (PV) the current value of a future amount, calculated by discounting the future value back at a known discount or interest rate for a specified period of time

real interest rate a rate of interest that has been adjusted to account for the effects of inflation

single payment or lump sum a single payment or deposit made at one time, as opposed to a number of smaller payments or deposits made in installments

time value of money (TMV) the concept that an amount of money is worth more today than the exact same amount of money at some point in the future

Treasury investments debt obligations such as T-bills (Treasury bills), bonds, and notes issued by the US Department of the Treasury

underinvested describes an insufficient amount of investment or an investment that is earning an insufficient rate of interest

uninvested describes cash that is being held in reserve, is not invested in an account or financial instrument, and is not earning interest or any return

CFA Institute

This chapter supports some of the Learning Outcome Statements (LOS) in this CFA® Level I Study Session (https://openstax.org/r/cfainstitute_link). Reference with permission of CFA Institute.

Multiple Choice

1. The most basic type of financial transaction involves _____.
   a. an amount of money that is not invested
   b. a series of equal installment amounts paid or received over a period of time
   c. a simple, one-time amount of cash that can be either a receipt or a payment
   d. None of the above

2. If a discount (or interest) rate has a positive value, then the future value of any amount deposited in an interest-bearing account will _____.
   a. be less than the present value
   b. be equal to the present value
   c. be greater than the present value
   d. decline over time

3. If the discount (or interest) rate used to calculate the present value of a future payment increases, the calculated present value will do which of the following?
   a. Increase
   b. Decrease

c. Remain the same
d. Increase as the period of time shortens

4. The discount rate that is required to equate a future payment of $500 in three years to a present value of $400 is _____.
   a. 4.7%
   b. 6.5%
   c. 7.7%
   d. 8.8%

5. If compounding periods increase in frequency and all else remains the same, the dollar values of any resulting future value calculations will _____.
   a. increase
   b. remain the same
   c. decrease
   d. None of the above

Review Questions

1. All other things being the same, would you prefer a bank account that compounds interest quarterly or one that compounds interest semiannually?
2. Briefly describe the concept of future value within the context of the larger overall concept of the time value of money.
3. Which of the following two options will give you the greatest future value: (A) an initial deposit of $100 earning 20% per year, compounded annually and left to grow for 10 years, or (B) an initial deposit of $75 earning 12% per year, compounded monthly and left to grow for 15 years?
4. If a savings account pays interest on a quarterly basis and you are performing future value calculations on deposited amounts, how can you calculate the rate?
5. Briefly describe the relationship among consumer savings, purchasing power, and inflation.

Problems

1. Find the future value of $100 in five years at 5% interest.
2. Find the future value of $1,800 in 3 years at 8% interest.
3. How much would you have to deposit now to have $15,000 in eight years if interest is 7%?
4. What is the present value of $5,000 that will be paid to you eight years from today at 8% interest?
5. How many years will it take a $700 balance to grow into $900 in an account earning 5%?
6. If you borrow $1,000 and pay back $1,728 in three years, what annual rate of interest are you paying?
7. How long will it take you to triple your money at 8%?
8. A company's sales were $250 million in 2019. If sales grow at 6% per year, how large will they be 10 years later, in 2029 (as expressed in millions)?
9. A US government bond in the amount of $1,000 will mature in six years, has no coupon payments, and carries an interest rate of 8%. What is the value of this bond today?
10. You spend $725.00 to purchase a $1,000 bond that will have no coupon payments and matures in 12 years.

What interest rate will you be earning on this bond?

11. At what interest rate would you be ambivalent about receiving either $50,000 10 years from now or $35,000 today?

12. Vance Corporation had earnings last year of $3.25 per share. The company has experienced 8% annual growth over the last several years, and management expects that growth rate to continue. Based on this information, after how long will earnings per share double?

13. What is the amount of total interest dollars earned on a $5,000 deposit earning 6% for 20 years?

14. You have decided that you will sell your $300,000 house when it appreciates in value to $500,000. If houses are appreciating at an average annual rate of 5% in your neighborhood, for approximately how long will you be staying in your house?

15. You just won some money in the lottery and would like to save a portion of it so that you will have $50,000 to put a down payment on a house in five years. Your bank pays a 5% rate of interest. How much money will you have to set aside from the lottery winnings?

16. Bauer Bookstore sells books before they are published. Today, they offered the book *Journeys in Finance* for $14.20, but the book will not be published for another two years. Upon publishing, the price of the book will be $24.00. What is the discount rate Bauer Bookstore is offering its customers for this book?

17. One of your professional goals is to one day earn a six-figure salary ($100,000). You hope to accomplish this objective within the next 30 years. Salaries grow at 3.75% per year in your field of work. What beginning salary will you need in order to reach this 30-year goal?

18. How much will $25,000 grow to in five years at a 5% annual rate that is compounded quarterly?

19. If Eisenberg Industries revenues have increased from $30 million to $90 million over a 10-year period, what has been their annual rate of growth?

20. If you are scheduled to receive $4,000 six years from today and the discount rate is 8.5%, what is the present value of this payment?

21. If you were to open a savings account that earns 3% interest and is compounded quarterly, what would be the total amount in your account after 10 years if you made an opening deposit of $9,500?

22. You are considering four possible options for your new savings account. You plan to deposit $13,000 and leave this amount in the account for 20 years with no additional deposits or withdrawals. All of these account options would earn 6% interest, but each one has a different compounding frequency, listed below. What would be the value of each account at the end of the 20-year period?

a. Annually
b. Semiannually
c. Quarterly
d. Monthly

Video Activity

Time Value of Money Calculations with Financial Calculator

Click to view content (https://openstax.org/r/Time-Value-of-Money-Calculations)

1. Name and describe the five primary keys on a financial calculator that are used to solve time value of money problems.

2. Following the instructions laid out in this video, practice calculating the following TVM variables using these inputs:

Find a Future Value (FV): Calculate FV when

$$\text{PV} = -1{,}000,\ \text{N} = 5,\ \text{I/Y} = 4, \text{ and } \text{PMT} = 0.$$

Find a Present Value (PV): Calculate PV when

$$\text{FV} = 2{,}000,\ \text{N} = 7,\ \text{I/Y} = 5, \text{ and } \text{PMT} = 0.$$

Find the Number of Periods (N) when

$$\text{FV} = 5{,}000,\ \text{PV} = -3{,}000,\ \text{I/Y} = 3, \text{ and } \text{PMT} = 0.$$

Find the Interest Rate (I/Y) when

$$\text{FV} = 6{,}000,\ \text{PV} = -1{,}500,\ \text{N} = 10, \text{ and } \text{PMT} = 2.$$

Find the Payment (PMT) when

$$\text{FV} = 15{,}000,\ \text{PV} = -8{,}000,\ \text{N} = 12, \text{ and } \text{I/Y} = 4.$$

Continue to practice using a financial calculator to solve various time value of money problems using different input factors until you feel comfortable with the process of using a calculator to solve TVM problems.

Time Value of Money Using Excel with 10 Examples

Click to view content (https://openstax.org/r/Money_Using_Excel)

3. Where within the Excel spreadsheet and its different menus (under what menu button option) can you find the different functions that will enable you to solve for different time value of money factors?
4. Following the instructions laid out in this video, practice calculating the following TVM variables in Excel.

   Find a Future Value (FV): Calculate FV when

   $$\text{PV} = -1{,}000,\ \text{N} = 5,\ \text{I/Y} = 4, \text{ and } \text{PMT} = 0.$$

   Find a Present Value (PV): Calculate PV when

   $$\text{FV} = 2{,}000,\ \text{N} = 7,\ \text{I/Y} = 5, \text{ and } \text{PMT} = 0.$$

   Find the Number of Periods (N) when

   $$\text{FV} = 5{,}000,\ \text{PV} = -3{,}000,\ \text{I/Y} = 3, \text{ and } \text{PMT} = 0.$$

   Find the Interest Rate (I/Y) when

   $$\text{FV} = 6{,}000,\ \text{PV} = -1{,}500,\ \text{N} = 10, \text{ and } \text{PMT} = 2.$$

   Find the Payment (PMT) when

   $$\text{FV} = 15{,}000,\ \text{PV} = -8{,}000,\ \text{N} = 12, \text{ and } \text{I/Y} = 4.$$

   Go back and check the answers to these questions that you ended up with under the video *Time Value of Money Calculations with Financial Calculator*. You should get the exact same answers when using both methods (calculator and Excel).

   Continue to practice using Excel to solve various time value of money problems using different input factors until you feel comfortable with the process of using spreadsheets to solve TVM problems.

8

Time Value of Money II: Equal Multiple Payments

Figure 8.1 The value of an investment generally represents our expectations of all future cash flows from that investment, once discounted. (credit: modification of "Money" by Ervins Strauhmanis/flickr CC BY 2.0)

Chapter Outline

Why It Matters

Although this text is directed at business finance students, our daily decisions as consumers are largely based on money and finance to just as great an extent. An old adage in finance claims, “If you aren’t in control of your money, your money is controlling you.” Fortunately, learning to manage your money is not difficult if you’re disciplined and understand some simple techniques. For example, several years ago, the author was negotiating for a three-year auto loan from a well-known regional dealer, who was offering an interest rate of 2%. When the manager left the room for a few minutes, we pulled out a financial calculator and proved in less than a minute that the actual interest rate in the payments he was proposing was nearly double the quoted and advertised rate.

In addition to understanding how the loan process works, which improves your negotiation skills when borrowing, businesses and individuals can better control their investments by understanding basic rules of finance, particularly as seen in this and the preceding chapter. Assume you pledge to invest $1,000 per year at 5% return per year and are curious about how much you will have accumulated by age 60. If you begin at age 30, you will have $69,760; if you begin at age 20, you will have $126,840. Can the extra 10 years make that much of a difference? We’ll see that indeed they can, and the calculations required to prove it can take less than two minutes.

As another example, many professionals confuse *income* and *wealth* during their career growth. In their popular book *The Millionaire Next Door: The Surprising Secrets of America’s Wealthy*, authors Thomas Stanley

and William Danko illustrate these terms with a flowing river. A river is in constant movement, and as the flow or depth increases, this is comparable to one's income increasing through the promotions, salary increases, and bonuses one receives. Unfortunately, many individuals then increase their spending habits in response, justifying a better car, a second home, or more lavish vacations. Wealth, in contrast, is comparable to taking a bucket of water from the river and holding it aside in a tank for oneself. Financial professionals often call this "paying yourself first." Stanley and Danko list this among the "secrets" referenced in the title of their book.

8.1 Perpetuities

Learning Outcomes

By the end of this section, you will be able to:

- Define perpetuity.
- Explain how perpetuities are valued.

In Time Value of Money I, we learned that the value of money changes with the passage of time. Decision-makers consider how investments, projects, and even opportunity costs gain value as we move forward into the future. They similarly consider how value in the future can be reduced to a value in present or past periods. We saw that these value projections are called determination of future value (compounding, moving forward on a timeline) or present value (discounting, moving backward on a timeline). The easiest way to visualize this movement through time, whether forward or backward, is by use of a timeline.

Throughout the first chapter on the time value of money, we were analyzing a single amount. In this chapter, we deal with a stream of payments made periodically—in other words, payments made or received regularly over a span of time. We begin with the illustration of a perpetuity.

What Is a Perpetuity?

A **perpetuity** is a series of payments or receipts that continues forever, or perpetually. One of the best ways to analyze the basics of an *annuity* (the stream of payments to be paid or received in the future) is by starting with a perpetuity. The most common examples of perpetuities in the author's experience are college chair endowments and preferred stock.

If you gift $1,500,000 to a college to name a professor's chair for your family, you might specify that the money must be held in perpetuity and invested by the college to yield a fixed 3%. The college will take those proceeds of the investment, leaving your original $1,500,000 intact, and use the annual interest of $45,000 to fund a portion of the professor's salary.

Another common example is **preferred stock**. Most preferred stock issues carry a fixed and predetermined rate of dividend. If we assume that the dividend will not change in future years, then preferred dividend shareholders will receive a fixed amount of money in future years—assuming, of course, that the company's board of directors declares the dividends sufficient to fund these requirements. If we assume that the dividend is declared and paid and that it remains constant, this represents a perpetuity.

For example, Shaw Inc. has issued 100,000 shares of preferred stock with a stated value of $50 and a 4% dividend. Therefore, if they can fund and decide to declare dividends for the full amount, they will pay out $200,000, or $2.00 per preferred share. Because shares such as these are created with the intention of continuity, the owner of this preferred stock can theoretically expect this dividend income stream in perpetuity.

To place a current market value on this stream of future income, how much should an investor pay for one share of this preferred stock? The calculation is a present value. The amount the investor pays today for that one share is equal to the annual dividend (assuming it is declared and paid) divided by the rate of return. But be careful—it is not the rate on the face of the preferred stock but the **required rate of return**, the "market rate" that investors expect from a stock of this level of risk. We must also note an important fact affecting all

investment valuations: the value of an investment generally represents our expectations of all future cash flows from that investment, discounted to today's dollars.

Because a perpetuity is a stream of payments continuing indefinitely, determining the future value isn't possible. Determining the present value, however, *is* possible, although one might wonder how. As we learned earlier, the greater the amount of time used in a present value calculation, the smaller the amount of dollars needed at the beginning, regardless of the interest rate involved. Therefore, when we discount each payment in an infinite series, remembering that we would then add them together once we discount them to the present, the infinite payments become negligible at some point and will no longer have a significant impact on today's value. To grow to one dollar 70 years from now, even at a growth rate of 5% per year, we would only need $0.0329—not even four cents. Keeping all other facts the same, if we had 100 years to grow an investment to one dollar, we would only need 0.76 cents—not even a whole penny! There is no question that the effect of time is substantial and dramatic.

The study of perpetuities in corporate finance is a first step to understanding valuation models of certain investments, such as the dividend discount model and the constant growth model, to be addressed in other chapters. Our ability to discount future cash flows, even infinite cash flows, to a present value is a clue to the price at which a company's stock might trade. From a personal financial planning perspective, the individual investor is also better able to be certain that they are paying a fair price for holdings in their portfolio. For purposes of long-term or retirement planning, the investor must consider that a fixed and unchanging dividend, such as from preferred stock, might not adequately protect the holder from inflation in times of rising prices.

How to Value a Perpetuity

Given these facts, how do we place a value on a perpetuity? Let's keep the preferred stock example for Shaw Inc. in mind. The holder of one share will expect to receive a $2.00 dividend for every share owned. Although a perpetuity may allow for growth of that dividend, we will hold that constant now. We must know one additional fact: the required rate of return. This is our random variable, which can cause fluctuation in the price of the preferred stock. Let's assume that the required rate of return, which we'll call R_S, is 7%. This is the rate of return that the market expects in order to take on the risk of an investment such as Shaw.

Determination of the price of Shaw's preferred stock becomes quite simple because the expected annual cash flow should not change, making it a **constant perpetuity**. The constant perpetuity formula is

$$\text{PV} = \frac{C}{R_S}$$

where PV is the price of the preferred stock, C is the constant dividend, and R_s is the required rate of return.

By substitution,

$$\text{PV} = \frac{\$2.00}{0.07} = \$28.57$$

The price one should pay for a share of Shaw's preferred stock is $28.57.

Here's another constant perpetuity to try. The preferred stock of Rooney Corporation pays an annual dividend of $1.75 per share. If the required rate of return in the market for shares such as Rooney's is 5.8%, at what price should these preferred shares be trading? The answer is $\frac{\$1.75}{0.058}$, or $30.17.

Some investments might involve a **growing perpetuity**. In this case, some degree of change in the amount of the dividend is expected. The formula is altered slightly to include a rate of growth in the denominator, noted as G, making the growing perpetuity formula

$$\text{PV} = \frac{C}{R_S - G}$$

To illustrate a growing perpetuity, let's revisit Rooney Corp.'s stock, with its annual dividend of $1.75 and a required rate of return in the market of 5.8%. If the expected dividend growth rate *G* is 1.2%, then the value changes to $\frac{\$1.75}{0.058 - 0.021}$, or $38.04. The expectation of growth in the dividend provides incentive for the investor to pay a higher price.

THINK IT THROUGH

A Growing Perpetuity

Savo Corporation. While we think of perpetuities as being static, with a constant benefit or dividend, as seen above, they might have the possibility of growth. Let's assume that our preferred stock in Savo Corporation is expected to grow at a rate of 0.2% per year. Its annual dividend per share is $4.00, and its required rate of return in the market is 3%. If this is a constant dividend stock, like most preferred stock, its price would be expected to approximate

$$\text{PV} = \frac{C}{R_s}$$

or

$$\frac{\$4.00}{0.03} = \$133.33$$

When we factor in the 0.2% annual growth in the dividend, what does the price per share become?

Solution:

$$\text{PV} = \frac{4}{(0.03 - 0.002)} = \$142.86.86$$

THINK IT THROUGH

College Endowment

In the case of an endowment for a college chair, as noted in the beginning of this section, instead of a dividend amount for a preferred stock, we would use the desired amount of the distribution that the chair would receive as part of their compensation package. Assume the college can invest this money at a fixed 3.5% annual rate. If you wish to gift the college enough funds to be held in perpetuity to produce $75,000 each year for that professor, how would you calculate this?

Solution:

We modify the constant perpetuity formula:

$$\text{Gift} = \frac{\text{Annual Distribution}}{R_s}$$

$$\text{Gift} = \frac{\$75{,}000}{0.035} = \$2{,}142{,}857$$

In one year, $2,142,857 grows to

$$\$2{,}142{,}857 \times 1.035 = \$2{,}217{,}857$$

The earnings (gift) of $75,000 are withdrawn to compensate the professor, and we are left with the amount originally endowed, $2,142,857:

$$\$2{,}217{,}857 - \$75{,}000 = \$2{,}142{,}857$$

8.2 Annuities

Learning Outcomes

By the end of this section, you will be able to:

- Define annuity.
- Distinguish between an ordinary annuity and an annuity due.
- Calculate the present value of an ordinary annuity and an annuity due.
- Explain how annuities may be used in lotteries and structured settlements.
- Explain how annuities might be used in retirement planning.

Calculating the Present Value of an Annuity

An **annuity** is a stream of fixed periodic payments to be paid or received in the future. Present or future values of these streams of payments can be calculated by applying time value of money formulas to each of these payments. We'll begin with determining the present value.

Before exploring present value, it's helpful to analyze the behavior of a stream of payments over time. Assume that we commit to a program of investing $1,000 at the *end* of each year for five years, earning 7% compounded annually throughout. The high rate is locked in based partly on our commitment beginning today, even though we will invest no money until the end of the first year. Refer to the timeline shown in Table 8.1.

Year	0	1	2	3	4	5
Balance Forward ($)	0.00	0.00	1,000.00	2,070.00	3,214.90	4,439.94
Interest Earned ($)		0.00	70.00	144.90	225.04	310.80
Principal Added ($)		1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
New Balance ($)		**1,000.00**	**2,070.00**	**3,214.90**	**4,439.94**	**5,750.74**

Table 8.1

At the end of the first year, we deposit the first $1,000 in our fund. Therefore, it has not yet had an opportunity to earn us any interest. The "new balance" number beneath is the cumulative amount in our fund, which then carries to the top of the column for the next year. In year 2, that first amount will earn 7% interest, and at the end of year 2, we add our second $1,000. Our cumulative balance is therefore $2,070, which then carries up to the top of year 3 and becomes the basis of the interest calculation for that year. At the end of the fifth year, our investing arrangement ends, and we've accumulated $5,750.74, of which $5,000 represents the money we invested and the other $750.74 represents accrued interest on both our invested funds and the accumulated interest from past periods.

Notice two important aspects that might appear counterintuitive: (1) we've "wasted" the first year because we deposited no funds at the beginning of this plan, and our first $1,000 begins working for us only at the beginning of the second year; and (2) our fifth and final investment earns no interest because it's deposited at the end of the last year. We will address these two issues from a practical application point of view shortly.

Keeping this illustration in mind, we will first focus on finding the present value of an annuity. Assume that you wish to receive $25,000 each year from an existing fund for five years, beginning one year from now. This stream of annual $25,000 payments represents an annuity. Because the first payment will be received one year from now, we specifically call this an **ordinary annuity**. We will look at an alternative to ordinary annuities later. How much money do we need in our fund today to accomplish this stream of payments if our remaining balance will always be earning 8% annually? Although we'll gradually deplete the fund as we withdraw periodic payments of the same amount, whatever funds remain in the account will always be earning interest.

Before we investigate a formula to calculate this amount, we can illustrate the objective: determining the present value of this future stream of payments, either manually or using Microsoft Excel. We can take each of the five payments of $25,000 and discount them to today's value using the simple present value formula:

$$\text{PV} = \frac{\text{FV}}{(1+r)^n}$$

where FV is the future value, PV is the present value, *r* is the interest rate, and *n* is the number of periods.

For example, the first $25,000 is discounted by the equation as follows:

$$\text{PV} = \frac{\$25{,}000}{(1+0.08)^1}$$

$$\text{PV} = \frac{\$25{,}000}{1.08} = \$23{,}148.15$$

Proof that $23,148.15 will grow to $25,000 in one year at 8% interest:

$$\$23{,}148.15 \times 1.08 = \$25{,}000$$

If we use this same method for each of the five years, increasing the exponent *n* for each year, we see the result in Table 8.2.

Year	Calculation	Result
1	$\$25{,}000 \div (1.08)^1$	$23,148.15
2	$\$25{,}000 \div (1.08)^2$	$21,433.47
3	$\$25{,}000 \div (1.08)^3$	$19,846.81
4	$\$25{,}000 \div (1.08)^4$	$18,375.75
5	$\$25{,}000 \div (1.08)^5$	$17,014.58
TOTAL		**$99,818.76**

Table 8.2 Present Value of Future Payments

We begin with the amount calculated in our table, $99,818.76. Before any money is withdrawn, a year's worth of interest at 8% is compounded and added to our balance. Then our first $25,000 is withdrawn, leaving us with $82,804.26. This process continues until the end of five years, when, aside from a minor rounding difference, the fund has "done its job" and is equal to zero. However, we can make this simpler. Because each payment withdrawn (or added, as we will see later) is the same, we can calculate the present value of an annuity in one step using an equation. Rather than the multiple steps above, we will use the following equation:

$$\text{PVa} = \text{PYMT} \times \frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r}$$

where PVa is the present value of the annuity and PYMT is the amount of one payment.

In this example, PYMT is $25,000 at the end of each of five years. Note that the greater the number of periods and/or the size of the amount borrowed, the greater the chances of large rounding errors. We have used six decimal places in our calculations, though the actual time value of money factor, combining interest and time, can be much longer. Therefore, our solutions will often use ≅ rather than the equal sign.

By substitution, and then following the proper order of operations:

$$\begin{aligned}
\text{PVa} &= \$25{,}000 \times \frac{\left[1 - \frac{1}{(1 + 0.08)^5}\right]}{0.08} \\
\text{PVa} &= \$25{,}000 \times \frac{\left(1 - \frac{1}{1.469328}\right)}{0.08} \\
\text{PVa} &= \$25{,}000 \times \frac{1 - 0.680583}{0.08} \\
\text{PVa} &= \$25{,}000 \times \frac{0.319417}{0.08} \\
\text{PVa} &= \$25{,}000 \times 3.9927125 \\
\text{PVa} &\approx \$99{,}817.81
\end{aligned}$$

In both cases, barring a rounding difference caused by decimal expansion, we come to the same result using the equation as when we calculate each of multiple years. It's important to note that rounding differences can become significant when dealing with larger multipliers, as in the financing of a multimillion-dollar machine or facility. In this text, we will ignore them.

In conclusion, five payments of $25,000, or $125,000 in total, can be funded today with $99,817.81, with the difference being obtained from interest always accumulating on the remaining balance at 8%. The running balance is obtained by calculating the year's interest on the previous balance, adding it to that balance, and subtracting the $25,000 that is withdrawn on the last day of the year. In the last (fifth) year, just enough interest will accrue to bring the balance to the $25,000 needed to complete the fifth payment.

A common use of the PVa is with large-money lotteries. Let's assume you win the North Dakota Lottery for $1.2 million, and they offer you $120,000 per year for 10 years, beginning one year from today. We will ignore taxes and other nonmathematical considerations throughout these discussions and problems. The Lottery Commission will likely contact you with an alternative: would you like to accept that stream of payments … or would you like to accept a **lump sum** of $787,000 right now instead? Can you complete a money-based analysis of these alternatives? Based purely on the dollars, no, you cannot. The reason is that you can't compare future amounts to present amounts without considering the effect of time—that is, the time value of money. Therefore, we need an interest rate that we can use as a discounting factor to place these alternatives on the same playing field by expressing them in terms of today's dollars, the present value. Let's use 9%. If we discount the future stream of fixed payments (an ordinary annuity, as the payments are identical and they begin one year from now), we can then compare that result to the cash lump sum that the Lottery Commission is offering you instead.

By substitution, and following the proper order of operations:

$$\begin{aligned}
\text{PVa} &= \$120{,}000 \times \frac{\left[1 - \frac{1}{(1 + 0.09)^{10}}\right]}{0.09} \\
\text{PVa} &= \$120{,}000 \times \frac{\left(1 - \frac{1}{2.3673637}\right)}{0.09} \\
\text{PVa} &= \$120{,}000 \times \frac{1 - 0.4224108}{0.09} \\
\text{PVa} &= \$120{,}000 \times \frac{0.5775892}{0.09} \\
\text{PVa} &= \$120{,}000 \times 6.4176578 \\
\text{PVa} &\approx \$770{,}119
\end{aligned}$$

All things being equal, that expected future stream of ten $120,000 payments is worth approximately $770,119 today. Now you can compare like numbers, and the $787,000 cash lump sum is worth more than the discounted future payments. That is the choice one would accept without considering such aspects as taxation, desire, need, confidence in receiving the future payments, or other variables.

Calculating the Present Value of an Annuity Due

Earlier, we defined an ordinary annuity. A variation is the **annuity due**. The difference between the two is one period. That's all—just one additional period of interest. An ordinary annuity assumes that there is a one-period lag between the start of a stream of payments and the actual first payment. In contrast, an annuity due assumes that payments begin immediately, as in the lottery example above. We would assume that you would receive the first annual lottery check of $120,000 immediately, not a year from now. In summary, whether calculating future value (covered in the next section) or present value of an annuity due, the one-year lag is eliminated, and we begin immediately.

Since the difference is simply one additional period of time, we can adjust for this easily by taking the formula for an ordinary annuity and multiplying by one additional period. One more period, of course, is $(1 + i)$. Recall from Time Value of Money I that the formula for compounding is $(1 + i)^N$, where i is the interest rate and N is the number of periods. The superscript N does not apply because it represents 1, for one additional period, and the power of 1 can be ignored. Therefore, faced with an annuity due problem, we solve as if it were an ordinary annuity, but we multiply by $(1 + i)$ one more time.

In our original example from this section, we wished to withdraw $25,000 each year for five years from a fund that we would establish now. We determined how much that fund should be worth today if we intend to receive our first payment one year from now. Throughout this fund's life, it will earn 8% annually. This time, let's assume we'll withdraw our first payment immediately, at point zero, making this an annuity due. Because we're trying to determine how much our starting balance should be, it makes sense that we must begin with a larger number. Why? Because we're pulling our first payment out immediately, so less money will remain to start compounding to the amount we need to fund all five of our planned payments! Our rule can be stated as follows:

Whether one is calculating present value or future value, the result of an annuity due must always be larger than that of an ordinary annuity, all other facts remaining constant. Here is the stream of solutions for the example above, but please notice that we will multiply by $(1 + i)$, one additional period, following the same order of operations:

$$\begin{aligned}
\text{PVa} &= \$25{,}000 \times \frac{\left[1 - \frac{1}{(1 + 0.08)^5}\right]}{0.08} \times (1 + 0.08) \\
\text{PVa} &= \$25{,}000 \times \frac{\left(1 - \frac{1}{1.469328}\right)}{0.08} \times 1.08 \\
\text{PVa} &= \$25{,}000 \times \frac{1 - 0.680583}{0.08} \times 1.08 \\
\text{PVa} &= \$25{,}000 \times \frac{0.319417}{0.08} \times 1.08 \\
\text{PVa} &= \$25{,}000 \times 3.9927125 \times 1.08 \\
\text{PVa} &\approx \$107{,}803.24
\end{aligned}$$

That's how much we must start our fund with today, before we earn any interest or draw out any money. Note that it's larger than the $99,817.81 that would be required for an ordinary annuity. It must be, because we're about to diminish our compounding power with an immediate withdrawal, so we have to begin with a larger amount.

We notice several things:

1. The formula must change because the annual payment is subtracted first, prior to the calculation of annual interest.
2. We accomplish the same result, aside from an insignificant rounding difference: the fund is depleted once the last payment is withdrawn.
3. The last payment is withdrawn on the first day of the final year, not the last. Therefore, no interest is earned during the fifth and final year.

To reinforce this, let's use the same approach for our lottery example above. Reviewing the facts, you have a choice of receiving 10 annual payments of your $1.2 million winnings, each worth $120,000, and you discount at a rate of 9%. The only difference is that this time, you can receive your first $120,000 right away; you don't have to wait a year. This is now an annuity due. We solve it just as before, except that we multiply by one additional period of interest, $(1 + i)$:

$$\begin{aligned}
\text{PVa} &= \$120{,}000 \times \frac{\left[1 - \frac{1}{(1 + 0.09)^{10}}\right]}{0.09} \times 1.09 \\
\text{PVa} &= \$120{,}000 \times \frac{\left(1 - \frac{1}{2.3673637}\right)}{0.09} \times 1.09 \\
\text{PVa} &= \$120{,}000 \times \frac{1 - 0.4224108}{0.09} \times 1.09 \\
\text{PVa} &= \$120{,}000 \times \frac{0.5775892}{0.09} \times 1.09 \\
\text{PVa} &= \$120{,}000 \times 6.4176578 \times 1.09 \\
\text{PVa} &\approx \$839{,}429
\end{aligned}$$

Again, this result must be larger than the amount we determined when this was calculated as an ordinary annuity.

The calculations above, representing the present values of ordinary annuities and annuities due, have been presented on an annual basis. In Time Value of Money I, we saw that compounding and discounting calculations can be based on non-annual periods as well, such as quarterly or monthly compounding and discounting. This aspect, quite common in periodic payment calculations, will be explored in a later section of this chapter.

Calculating Annuities Used in Structured Settlements

In addition to lottery payouts, annuity calculations are often used in **structured settlements** by attorneys at law. If you win a $450,000 settlement for an insurance claim, the opposing party may ask you to accept an annuity so that they can pay you in installments rather than a lump sum of cash. What would a fair cash distribution by year mean? If you have a preferred **discount rate** (the percentage we all must know to calculate the time value of money) of 6% and you expect equal distributions of $45,000 over 10 years, beginning one year from now, you can use the present value of an annuity formula to compare the alternatives:

$$\text{PVa} = \text{PYMT} \times \frac{\left[1 - \frac{1}{(1 + r)^n}\right]}{r}$$

By substitution:

$$\begin{aligned}
\text{PVa} &= \$45{,}000 \times \frac{\left[1 - \frac{1}{(1 + 0.06)^{10}}\right]}{0.06} \\
\text{PVa} &= \$45{,}000 \times \frac{\left(1 - \frac{1}{1.790848}\right)}{0.06} \\
\text{PVa} &= \$45{,}000 \times \frac{0.4416053}{0.06} \\
\text{PVa} &= \$45{,}000 \times 7.360089 \\
\text{PVa} &\approx \$331{,}204
\end{aligned}$$

If the opposing attorney offered you a lump sum of cash less than that, all things equal, you would refuse it; if

the lump sum were greater than that, you would likely accept it.

What if you negotiate the first payment to be made to you immediately, turning this ordinary annuity into an annuity due? As noted above, we simply multiply by one additional period of interest, (1 + 0.06). Repeating the last step of the solution above and then multiplying by (1 + 0.06), we determine that

$$\text{PVa} = \$45{,}000 \times 7.360089 \times 1.06$$
$$\text{PVa} \approx \$351{,}076$$

You would insist on that number as an absolute minimum before you would consider accepting the offered stream of payments.

To further verify that ordinary annuity can be converted into an annuity due by multiplying the solution by one additional period's worth of interest before applying the annuity factor to the payment, we can divide the difference between the two results by the value of the original annuity. When the result is expressed as a percent, it must be the same as the rate of interest used in the annuity calculations. Using our example of an annuity with five payments of $25,000 at 8%, we compare the present values of the ordinary annuity of $99,817.81 and the annuity due of $107,803.24.

$$\frac{\$107{,}803.24 - \$99{,}817.81}{\$99{,}817.81} = 0.08 = 8\%$$

The result shows that the present value of the annuity due is 8% higher than the present value of the ordinary annuity.

Calculating the Future Value of an Annuity

In the previous section, we addressed discounting a periodic stream of payments from the future to the present. We are also interested in how to project the future value of a series of payments. In this case, an investment may be made periodically. Keeping with the definition of an annuity, if the amount of periodic investment is always the same, we may take a one-step shortcut to calculate the future value of that stream by using the formula presented below:

$$\text{FVa} = \text{PYMT} \times \frac{(1+r)^N - 1}{r}$$

where FVa is the future value of the annuity, PYMT is a one-time payment or receipt in the series, r is the interest rate, and n is the number of periods.

As we did in our section on present values of annuities, we will begin with an ordinary annuity and then proceed to an annuity due.

Let's assume that you lock in a contract for an investment opportunity at 4% per year, but you cannot make the first investment until one year from now. This is counterintuitive for an investor, perhaps, but because it is the basis of the formula and procedures for ordinary annuities, we will accept this assumption. You plan to invest $3,000 at the end of each year. How much money will you have at the end of five years?

Let's start by placing this on a timeline like the one appearing earlier in this chapter (see Table 8.3):

Year	0	1	2	3	4	5
Balance Forward ($)	0.00	0.00	3,000.00	6,120.00	9,364.80	12,739.39
Interest Earned ($)		0.00	120.00	244.80	374.59	509.58
Principal Added ($)		3,000.00	3,000.00	3,000.00	3,000.00	3,000.00
New Balance ($)		**3,000.00**	**6,120.00**	**9,364.80**	**12,739.39**	**16,248.97**

Table 8.3

As we explained earlier when describing ordinary annuities, the payment for year 1 is not invested until the last day of that year, so year 1 is wasted as a compounding opportunity. Therefore, the amount only compounds for four years rather than five. Also, our fifth payment is not made until the last day of our contract in year 5, so it has no chance to earn a compounded future value. The investor has lost on both ends. In the table above, we have made five calculations, and for a longer-term contract such as 10, 25, or 40 years, this would be tedious. Fortunately, as with present values, this ordinary annuity can be solved in one step because all payments are identical.

Repeating the formula, and then by substitution:

$$\begin{aligned}
\text{FVa} &= \text{PYMT} \times \frac{(1+r)^N - 1}{r} \\
\text{FVa} &= \$3{,}000 \times \frac{(1+0.04)^5 - 1}{0.04} \\
\text{FVa} &= \$3{,}000 \times \frac{1.216653 - 1}{0.04} \\
\text{FVa} &= \$3{,}000 \times 5.416325 \\
\text{FVa} &= \$16{,}248.98
\end{aligned}$$

This proof emphasizes that year 1 is wasted, with no compounding because the payment is made on the last day of year 1 rather than immediately. We lose compounding through this ordinary annuity in another way: year 5's investment is made on the last day of this five-year contract and has no chance to accumulate interest. A more intuitive method would be to enter a contract for an annuity due so that our first payment can be made immediately. In this way, we don't waste the first year, and all five payments work in year 5 as well. As stated previously, this means that annuities due will yield larger results than ordinary annuities, whether one is discounting (PVa) or compounding (FVa).

Let's hold all facts constant with the previous example, except that we will invest at the beginning of each year, starting immediately upon locking in this five-year contract. We follow the same technique as in the present value section: we multiply by one additional period to convert this ordinary annuity factor into a factor for an annuity due. Whether one is solving for a future value or a present value, the result of an annuity due must always be larger than an ordinary annuity. With future value, we begin investing immediately, so the result will be larger than if we waited for a period to elapse. With present value, we begin extracting funds immediately rather than letting them work for us during the first year, so logically we would have to start with more.

Continuing our example but converting it to an annuity due, we will multiply by one additional period, $(1 + i)$. All else remains the same:

$$\begin{aligned}
\text{FVa} &= \$3{,}000 \times \frac{(1+0.04)^5 - 1}{0.04} \times 1.04 \\
\text{FVa} &= \$3{,}000 \times \frac{1.216653 - 1}{0.04} \times 1.04 \\
\text{FVa} &= \$3{,}000 \times 5.416325 \times 1.04 \\
\text{FVa} &\approx \$16{,}898.93
\end{aligned}$$

Let's provide one additional example of each. Assume that you have a chance to invest $15,000 per year for 10 years, earning 8% compounded annually. What amount would you have after the 10 years? If we can only make our first payment at the end of each year, our ending value will be

$$\begin{aligned}
\text{FVa} &= \$15{,}000 \times \frac{(1+0.08)^{10} - 1}{0.08} \\
\text{FVa} &= \$15{,}000 \times \frac{2.158925 - 1}{0.08} \\
\text{FVa} &= \$15{,}000 \times 14.486563 \\
\text{FVa} &\approx \$217{,}298
\end{aligned}$$

However, if we can make our first payment immediately and then make subsequent payments at the start of

each following year, we modify the formula above by multiplying the annual payment by one additional period:

$$\text{FVa} = \$15{,}000 \times 14.486563 \times 1.08$$
$$\text{FVa} \approx \$234{,}682$$

THINK IT THROUGH

Begin an Investing Program at Age 20 or 30?

In this chapter's introductory section, Why It Matters, we posed a question about pledging to invest $1,000 each year until you reach age 60. If you can earn a 5% annual rate of interest, how much will you have if you begin at age 20? What if you delay this program until age 30? The additional 10 years can make a surprisingly large difference. How can you calculate that difference?

Solution:

Perform two separate calculations comparable to the chapter examples above, using the formula for the future value of an ordinary annuity. You plan to make the first investment immediately, making this an annuity due, so you will multiply by one additional period, (1 + 0.05). Notice that the only difference between the two calculations is the exponent *N*, representing the number of periods.

Thirty years (starting at age 30):

$$\text{FVa} = \$1{,}000 \times \frac{(1 + 0.05)^{30} - 1}{0.05} \times 1.05$$
$$\text{FVa} = \$1{,}000 \times 66.438848 \times 1.05$$
$$\text{FVa} \approx \$69{,}761$$

Forty years (starting at age 20):

$$\text{FVa} = \$1{,}000 \times \frac{(1 + 0.05)^{40} - 1}{0.05} \times 1.05$$
$$\text{FVa} = \$1{,}000 \times 120.799774 \times 1.05$$
$$\text{FVa} \approx \$126{,}840$$

Waiting 10 years before committing to this program comes with a surprisingly high cost—a loss of almost 82% of the potential value.

How Annuities Are Used for Retirement Planning

On a final note, how might annuities be used for **retirement planning**? A person might receive a lump-sum windfall from an investment, and rather than choosing to accept the proceeds, they might decide to invest the sum (ignoring taxes) in an annuity. Their intention is to let this invested sum produce annual distributions to supplement Social Security payments. Assume the recipient just received $75,000, again ignoring tax effects. They have the chance to invest in an annuity that will provide a distribution at the end of each of the next five years, and that annuity contract provides interest at 3% annually. Their first receipt will be one year from now. This is an ordinary annuity.

We can also solve for the payment given the other variables, an important aspect of financial analysis. If the person with the $75,000 windfall wants this fund to last five years and they can earn 3%, then how much can they withdraw from this fund each year? To solve this question, we can apply the present value of an annuity formula. This time, the payment (PYMT) is the unknown, and we know that the PVa, or the present value that they have at this moment, is $75,000:

$$\begin{aligned}
\text{PVa} &= \text{PYMT} \times \frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r} \\
\$75{,}000 &= \text{PYMT} \times \frac{\left[1 - \frac{1}{(1+0.03)^5}\right]}{0.03} \\
\$75{,}000 &= \text{PYMT} \times \frac{\left(1 - \frac{1}{1.159274}\right)}{0.03} \\
\$75{,}000 &= \text{PYMT} \times 4.579705 \\
\text{PYMT} &= \$75{,}000 \div 4.579705 \\
\text{PYMT} &\approx \$16{,}376.60
\end{aligned}$$

The person can withdraw this amount every year beginning one year from now, and when the final payment is withdrawn, the fund will be depleted. Interest accrues each year on the beginning balance, and then $16,376.60 is withdrawn at the end of each year.

LINK TO LEARNING

Another View of Annuities

Many examples of annuities are available, with presentations as varied as the opinions as to how appropriate they are for investors, especially retirees. Math Is Fun (https://openstax.org/r/Math_Is_Fun) is particularly interesting and potentially helpful for understanding how to apply this knowledge.

8.3 Loan Amortization

Learning Outcomes

By the end of this section, you will be able to:

- Distinguish between different types of loans.
- Explain how amortization works.
- Create an amortization schedule.
- Calculate the cost of borrowing.

Types of Loans

Funds can be loaned to businesses of any type, including corporations, partnerships, limited liability companies, and proprietorships. Bankers often refer to these lending structures as *facilities*, and they can be tailored to the specific needs of the borrower in a number of ways. Similarly, lenders develop loans and lines of credit for individuals. Whether for a business or an individual, the purpose of the loan, method of repayment, interest rate, specific terms, and time involved must all be tailored to the goals of the borrower and the lender. In this chapter, we will focus on fixed-rate loans, although other alternatives exist.

Typical business loans include the following:

- Term loans generally bear a maturity date and a set rate of interest and are typically used to finance investments in assets such as equipment, buildings, and possibly other acquired firms. The length of the term loan is generally designed to match the useful life of the asset being financed, and it will usually be repaid on a monthly schedule. It's common for a term loan to be backed by collateral, such as the asset itself or other assets of the business.
- Revolving lines of credit (revolvers), are used to finance the short-term working capital needs of a business. Revolvers will have a specific maximum but no set schedule of monthly payments. Interest accrues on the amount of cash that a company has drawn down from the facility. These credit lines may be secured by accounts receivable, inventory, other assets of the business, or sometimes simply the good

faith and credit of the company if the firm is strong, creditworthy, and established with the lender. Revolvers must often be fully repaid and unused for a short period of time to assure the lender that the borrower is not using this facility for longer-term needs.

Personal loans also come in several types, designed for the purpose the borrower (consumer) has in mind, with assistance from the lender in determining the appropriate structure:

- Personal lines of credit are similar to lines of credit on bank cards, with interest being charged on the outstanding balance of the credit line. These are available on the basis of personal credit scores, with data being supplied by the three best-known credit reporting firms: Experian, Equifax, and TransUnion. Individuals should check their scores with each of these companies at least once per year, which they can do for no charge. Additional requests from the same company require a small fee.
- An unsecured personal loan is an installment loan, initially drawn for a fixed amount and repaid on a periodic schedule with interest, as we have seen in our annuity examples. Unsecured means that the loan is not secured by collateral but is instead based on the strong credit history of the borrower.
- In contrast, a secured personal loan has an asset backing up the unpaid amount, and if the consumer defaults on the debt, the asset can be seized by the lender to satisfy their claim. A common example is an auto loan, which is secured by the car being purchased; nonpayment or default on the loan can lead to the borrower's car being repossessed.
- A mortgage loan is another type of secured personal loan, but for a longer period, such as 20, 25, or even 30 years. The home being purchased or built is the collateral, and the home may be foreclosed upon if the borrower defaults. Full title to the home typically remains with the lender as long as an unpaid balance remains on the debt.
- Student loans are borrowings intended to fund college or career education, and they can come from a financial institution or the federal government. Interest rates on these loans are generally low and advantageous, and repayment does not begin until after the borrower's education is complete (or if they drop below a certain level of time status, such as becoming a half-time student).

Calculating Loan Payments Using Simple Amortization

Loan amortization refers to a schedule of how and when a debt will be repaid with interest. As noted, we will focus on fixed-rate debts, such as auto loans, personal loans with installment payments, or mortgages. Before entering into a borrowing agreement, the borrower can use any of a number of tools to verify the terms being offered, such as the monthly payment on a car loan financed by the dealer. In many cases, this is accomplished by using the present value of an annuity formula:

$$\text{PVa} = \text{PYMT} \times \frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r}$$

We've already reviewed the present value of ordinary annuities in several examples. Before we look into business or consumer loans and their repayment, we must review an area of Time Value of Money I.

We're not likely to make annual payments on a home mortgage or auto loan, as these are commonly paid on a monthly basis. Fortunately, our formulas are easily adjusted from annual to non-annual periods. You will recall that we solve for non-annual periods in the same way, with two adjustments: (1) we divide the annual interest rate by the number of periods in the year, and (2) we multiply the time periods by the number of those periods within a year. Therefore, in the case of monthly debt service, including interest and principal, we use 12 periods.

Given a three-year car loan at 6%, rather than using 6% and 3 periods in our formula, we would instead use 0.5% (6% ÷ 12) and 36 periods (3 years × 12), and then apply the present value of an annuity formula in the same way. Let's say the three-year, 6% auto loan is for $32,000. You need to know if you can squeeze the monthly payment into your budget. For our examples, we will ignore any other charges, fees, taxes, or extras that your lender might include in these payments, and we will focus only on interest and principal repayment.

You will make the first payment one month from now, making this an ordinary annuity. What is the amount of your monthly debt service? In this case, you would be solving for a different unknown: the payment amount.

By substitution into the present value of an annuity formula, adjusting for monthly payments as noted:

$$\$32{,}000 = \text{PYMT} \times \frac{\left[1 - \frac{1}{(1 + 0.005)^{36}}\right]}{0.005}$$
$$\$32{,}000 = \text{PYMT} \times \frac{\left(1 - \frac{1}{1.196681}\right)}{0.005}$$
$$\$32{,}000 = \text{PYMT} \times \frac{1 - 0.835645}{0.005}$$
$$\$32{,}000 = \text{PYMT} \times \frac{0.164355}{0.005}$$
$$\$32{,}000 = \text{PYMT} \times 32.871$$

Dividing both sides by 32.781 to isolate the payment amount (PYMT) gives us

$$\text{PYMT} = \frac{\$32{,}000}{32.871}$$
$$\text{PYMT} \approx \$973.50$$

Solving for the payment, we find that it's approximately $973.50 per month. You consult your monthly budget and find that you can cover this monthly payment, so you conclude the deal. Ask the salesperson for the amortization table on this debt to show how your 36 payments of $973.50 will cover your interest plus repayment of the principal amount of the debt. At this point, you know how to complete your own table. Using a financial calculator or Microsoft Excel simplifies the operation above to a few keystrokes, as presented later in this chapter.

Two extracts from an amortization table are shown in Table 8.4.

Month	Payment	Interest	Principal	Remaining Balance
1	973.50	160.00	813.50	31,186.50
2	973.50	155.93	817.57	30,368.93
3	973.50	151.84	821.66	29,547.27
4	973.50	147.74	825.77	28,721.51
5	973.50	143.61	829.89	27,891.61
...continued...				
33	973.50	19.23	954.27	2,891.54
34	973.50	14.46	959.04	1,932.50
35	973.50	9.66	963.84	968.66
36	973.50	4.84	968.66	0.00
Total	**35,046.00**	**3,046.08**	**32,000.00**	

Table 8.4 Extracts from an Amortization Table ($)

This table resembles proofs we have seen of annuities, but let's focus on some details:

1. Each fixed payment contains both interest and principal repayment.
2. Because the payments are fixed and the amount of remaining debt is decreasing, the monthly interest portion is always decreasing, and the amount of principal payment therefore must be increasing.

We can conclude that the lender is making more of their revenue (interest) in the early months than in the later months. In addition, the debt is decreasing slowly in the early months and more rapidly in the later

months. We can all agree that lenders are compensated for the risks they take earlier rather than later. Of the 36 payments of $973.50, $32,000 has been repaid as the principal borrowed. The remaining $3,046.08 is the lender's revenue, the cost of credit.

For an additional example, one that drives home the point that more interest is paid in the early months of a long-term loan, we will consider a 20-year home mortgage. Home mortgage payments are typically made monthly, and again, we will ignore additional charges by the lender, such as real estate tax and homeowner's insurance. Let's assume you buy a $200,000 home, pay $60,000 as a cash deposit, and will finance the remaining $140,000 over 20 years. The bank offers you a 3.6% annual interest rate. What will the amount of your monthly payment be for the interest and principal repayment? The bank will tell you, of course, but let's prove it for ourselves. We'll do it in exactly the same fashion as the car loan above, using the present value of an annuity formula. Remember that you are not financing the entire $200,000 purchase; you pay $60,000 in cash, so you are only financing the remaining $140,000.

We modify the periods from years to months by multiplying by 12, and we modify the annual rate to a monthly rate by dividing by 12, resulting in

$$n = 240 \text{ months}$$
$$r = 0.3\%$$

By substitution into the present value of an annuity formula:

$$\$140{,}000 = \text{PYMT} \times \frac{\left[1 - \frac{1}{(1 + 0.003)^{240}}\right]}{0.003}$$
$$\$140{,}000 = \text{PYMT} \times \frac{\left(1 - \frac{1}{2.052220}\right)}{0.003}$$
$$\$140{,}000 = \text{PYMT} \times \frac{1 - 0.487277}{0.003}$$
$$\$140{,}000 = \text{PYMT} \times \frac{0.512723}{0.003}$$
$$\$140{,}000 = \text{PYMT} \times 170.907667$$

We divide both sides by 170.907667 to isolate the payment amount (PYMT):

$$\text{PYMT} = \frac{\$140{,}000}{170.907667}$$
$$\text{PYMT} \approx \$819.16$$

Your monthly mortgage payment is $819.16. As in our auto loan example, we'll complete an amortization table of our own—though, of course, you'll remember to ask your lender for their version. Extracts from a full 240-month table are shown in Table 8.5 below. The front-end packing of interest revenue is more obvious here because of the longer time period.

Month	Payment	Interest	Principal	Remaining Balance
				140,000.00
1	819.16	420.00	399.16	139,600.84
2	819.16	418.80	400.36	139,200.48
3	819.16	417.60	401.56	138,798.92
4	819.16	416.40	402.76	138,396.16
5	819.16	415.19	403.97	137,992.19
6	819.16	413.98	405.18	137,587.01
7	819.16	412.76	406.40	137,180.61

Table 8.5 Amortization Table for a Mortgage ($)

Month	Payment	Interest	Principal	Remaining Balance
8	819.16	411.54	407.62	136,772.99
9	819.16	410.32	408.84	136,364.15
...continued....				
236	819.16	12.17	806.99	3,250.84
237	819.16	9.75	809.41	2,441.44
238	819.16	7.32	811.84	1,629.60
239	819.16	4.89	814.27	815.33
240	819.16	2.45	816.71	(1.38)
Total	**196,598.40**	**56,597.02**	**140,001.38**	**(Rounding)**

Table 8.5 Amortization Table for a Mortgage ($)

As with your car loan, earlier payments contain more interest than loan repayment, so the lender's revenue is at a significant peak in the early years. The length of the loan, coupled with the frequent compounding, emphasizes this. In month 10, the interest and principal amounts "pass" each other, and now the loan balance is dropping at a quicker rate. Finally, note that you will pay more than $56,000 to finance this $140,000 borrowing. If you pay off this mortgage over 240 months as planned, the interest cost represents an additional 28% of the full cost of the home!

If the borrower has the means to make an accelerated payment against this debt—for example, due to a bonus or other windfall—doing so can make a significant difference in the total cost of financing over the life of the loan. Assume that after three years (month 36), you receive a bonus of $2,000 and decide to apply the entire amount to prepay the remaining balance. Your loan agreement allows you to apply the entire amount to the remaining unpaid balance of the mortgage. While this might seem equal to just 2.5 months' worth of payment, the debt is fully paid off almost 6 months ahead of schedule, and total interest is reduced from over $56,000 to $55,000. The ability to prepay long-term debts such as this is clearly worth negotiating initially.

8.4 Stated versus Effective Rates

Learning Outcomes

By the end of this section, you will be able to:

- Explain the difference between stated and effective rates.
- Calculate the true cost of borrowing.

The Difference between Stated and Effective Rates

If you look at the bottom of your monthly credit card statement, you could see language such as "The interest rate on unpaid balances is 1.5% per month." You might think to yourself, "So, that's 12 months times 1.5%, or 18% per year." This is a fine example of the difference between *stated* and *effective* annual interest rates. The **effective interest rate** reflects compounding within a one-year period, an important distinction because we tend to focus on annual interest rates. Because compounding occurs more than once per year, the true annual rate is higher than appears. Please remember that if interest is calculated and compounded annually, the stated and effective interest rates will be the same. Keep in mind that the following principles work whether you are the debtor paying off an obligation or an investor hoping for more frequent compounding. The dynamics of the time value of money apply in either direction.

Effective Rates and Period of Compounding

Let's remain with our example of a credit card statement that indicates an interest rate of 1.5% per month on

unpaid balances. If you use this card only once, to make a $1,000 purchase in January, and then fail to pay the bill when it comes due, the issuer will bill you $15. Now you owe them $1,015. Assume you completely ignore this bill and never pay it throughout the rest of the year. The monthly calculation of interest starts to compound on past interest assessments in addition to the $1,000 initial purchase (see Table 8.6).

Month	Interest	Balance
		1,000.00
1	15.00	1,015.00
2	15.23	1,030.23
3	15.45	1,045.69
4	15.69	1,061.36
5	15.92	1.077.28
6	16.16	1,093.44
7	16.40	1,109.84
8	16.65	1,126.49
9	16.90	1,143.39
10	17.15	1,160.54
11	17.41	1,177.95
12	17.67	1,195.62

Table 8.6 Compounded Interest on a Credit Card Statement ($)

Because interest compounds monthly rather than annually, the effective annual rate is 19.56%, not the intuitive rate of the stated 1.5% times 12 months, or 18%. Our basic compounding formula of *(1+i)^n* by substitution shows:

$$(1 + 0.015)^{12} = 1.19562$$

To isolate the effective annual rate, we then deduct 1 because our interest calculations are based on the value of $1:

$$(1 + 0.015)^{12} - 1 = 1.19562 - 1 = 0.19562 = 19.562\%$$

Therefore, it falls to the consumer/borrower to understand the true cost of borrowing, especially when larger dollar amounts are involved. If we had been dealing with $10,000 rather than $1,000, the annual difference would be more than $156.

LINK TO LEARNING

A Helpful Demonstration . . .

From the Corporate Finance Institute (https://openstax.org/r/Corporate_Finance_Institute) comes a fine visual of a similar example. Here, we see the effective annual rate that results from taking a nominal annual rate of 12%, with a benefit to an investor if they have the benefit of monthly compounding.

One example of the importance of understanding effective interest rates is an invention from the early 1990s:

the payday advance loan (PAL). The practice of offering such loans can be controversial because it can lead to very high rates of interest, perhaps even illegally high, in an act known as usury. Although some states have outlawed PALs and others place limits on them, some do not. A PAL is a short-term loan in anticipation of a person's next paycheck. A person in need of money for short-term needs will write a check on Thursday but date the check *next* Thursday, which is their normal payday; assume this transaction is for $200. The lender, typically operating from a storefront, will advance the $200 cash and hold the postdated check. The lender charges a fee—let's say $14—as their compensation. The following Thursday, the borrower is expected to pay off the advance, and if they do not, the lender can deposit the postdated check. If that check has insufficient funds, more fees and penalties will likely be assessed.

One primary reason that arrangements such as these are controversial is the excessively high nominal (stated) interest rate that they can represent. For a one-week loan of $200, the borrower is paying $14, or 7% of the borrowed amount. If this is annualized, with 52 seven-day periods in a year, the stated rate is 364%! While a PAL might seem to be an effective immediate solution to a cash shortfall, the mathematics behind the true cost of borrowing simply do not make sense, and a person who uses such arrangements regularly is placing themselves at a dreadful financial disadvantage.

THINK IT THROUGH

How Tempting Is That Refund Anticipation?

Refund anticipation loans (RALs) began in 1987, and they are still available (though not from banks) and used by millions of people.[1] Now, RALs come from private lending chains. These loans allow you to determine your April 15 personal income tax liability through a preparer and receive an advance against your expected refund.[2] But beware: your ability to analyze the true cost of money is always critical. Like all loans, RALs bear a rate of interest. Let's assume that the firm that prepared your tax return determines that you're entitled to an $800 refund. Once they advance that amount to you, it will bear interest at a certain rate; we'll assume 0.5% per week. You might expect a tax refund in four weeks. Half a percent of $800 doesn't sound like much, but what happens when you annualize it into an effective rate, assuming your tax refund arrives exactly four weeks from when you accept the loan? Assume no compounding during those four weeks.

Solution:

A weekly rate of 0.5% on the $800 advance is $4 per week, so for four full weeks, you've paid $16 for the use of $800. Of course, that totals 2% of the amount advanced. There are 13 four-week periods in a year, so even though the interest rate appears to be small, it amounts to 26% when annualized! We assumed no compounding to keep the illustration simple, but we further assume that you are not using this advance throughout the year. If you were, then periodic compounding would drive the effective rate even higher, to just over 29.3%.

1 Michelle Singletary. "Another Reason Not to Opt for a Tax Refund Loan: It May Delay Your Next Stimulus Payment." *Washington Post*, February 16, 2021. https://www.washingtonpost.com/business/2021/02/16/tax-refund-loan-problems/
2 Amelia Josephson. "What Is a Refund Anticipation Loan?" SmartAsset. March 18, 2021. https://smartasset.com/taxes/what-is-a-refund-anticipation-loan

8.5 Equal Payments with a Financial Calculator and Excel

Learning Outcomes

By the end of this section, you will be able to:

- Use a financial calculator and Excel to solve perpetuity problems.
- Use a financial calculator and Excel to solve annuity problems.
- Calculate an effective rate of interest.
- Schedule the amortization of a loan repayment.

Solving Time Value of Money Problems Using a Financial Calculator

Since the 1980s, many convenient and inexpensive tools have become available to simplify business and personal calculations, including personal computers with financial applications and handheld/desktop or online calculators with many of the functions we've studied already. This section will explore examples of both, beginning with financial calculators. While understanding and mastery of the use of time value of money equations are part of a solid foundation in the study of business and personal finance, calculators are rapid and efficient.

We'll begin with the constant perpetuity that we used to illustrate the constant perpetuity formula. A share of preferred stock of Shaw Inc., pays an annual $2.00 dividend, and the required rate of return that investors in this stock expect is 7%. The simple technique to solve this problem using the calculator is shown in Table 8.7.

Step	Description	Enter	Display	
1	Set all variables to defaults	2ND [RESET] ENTER	RST	0.00
2	Enter formula	2 ÷ 7 % =		28.57

Table 8.7 Calculator Steps to Find the Required Rate of Return[3]

Earlier we solved for the present value of a 5-year ordinary annuity of $25,000 earning 8% annually. We then solved for an annuity due, all other facts remaining the same. The two solutions were $99,817.50 and $107,802.50, respectively. We enter our variables as shown in Table 8.8 to solve for an ordinary annuity:

Step	Description	Enter	Display	
1	Set all variables to defaults	2ND [RESET] ENTER	RST	0.00
2	Enter number of payments	5 N	N =	5.00
3	Enter interest rate per payment period	8 I/Y	I/Y =	8.00
4	Enter payment amount	25000 +/− PMT	PMT =	-25,000.00
5	Compute present value	CPT PV	PV =	99,817.75

Table 8.8 Calculator Steps to Solve for an Ordinary Annuity

Note that the default setting on the financial calculator is END to indicate that payment is made at the end of a period, as in our ordinary annuity. In addition, we follow the payment amount of $25,000 with the +/- keystroke—an optional step to see the final present value result as a positive value.

To perform the same calculation as an annuity due, we can perform the same procedures as above, but with two additional steps after Step 1 to change the default from payments at the end of each period to payments at the beginning of each period (see Table 8.9).

3 The specific financial calculator in these examples is the Texas Instruments BA II Plus™ Professional model, but you can use other financial calculators for these types of calculations.

Step	Description	Enter	Display	
1	Set all variables to defaults	2ND [RESET] ENTER	RST	0.00
2	Change default to payment at end of period	2ND [BGN] 2ND [SET]	BGN	0.00
3	Return to calculator mode	2ND [QUIT]		0.00
4	Enter number of payments	5 N	N =	5.00
5	Enter interest rate per payment period	8 I/Y	I/Y =	8.00
6	Enter payment amount	25000 +/− PMT	PMT =	-25,000.00
7	Compute present value	CPT PV	PV =	107,803.17

Table 8.9 Calculator Steps to Solve for an Annuity Due

The procedures to find future values of both ordinary annuities and annuities due are comparable to the two procedures above. We begin with the ordinary annuity, with reminders that this is the default for the financial calculator and that entering the payment as a negative number produces a positive result (see Table 8.10).

Step	Description	Enter	Display	
1	Set all variables to defaults	2ND [RESET] ENTER	RST	0.00
2	Enter number of payments	5 N	N =	5.00
3	Enter interest rate per payment period	4 I/Y	I/Y =	4.00
4	Enter payment amount	3000 +/− PMT	PMT =	-3,000.00
5	Compute future value	CPT FV	FV =	16,248.97

Table 8.10 Calculator Steps to Find the Future Value of an Ordinary Annuity

Solving for an annuity due with the same details requires the keystrokes listed in Table 8.11.

Step	Description	Enter	Display	
1	Set all variables to defaults	2ND [RESET] ENTER	RST	0.00
2	Change default to payment at end of period	2ND [BGN] 2ND [SET]	BGN	0.00
3	Return to calculator mode	2ND [QUIT]		0.00
4	Enter number of payments	5 N	N =	5.00
5	Enter interest rate per payment period	4 I/Y	I/Y =	4.00
6	Enter payment amount	3000 +/− PMT	PMT =	-3,000.00
7	Compute future value	CPT FV	FV =	16,898.93

Table 8.11 Calculator Steps to Find the Future Value of an Annuity Due

Earlier in the chapter, we explored the effect of interannual compounding on the true cost of money, recalling the basic compounding formula:

$$(1+i)^N$$

We saw that when modified for monthly compounding at a stated rate of 1.5%, the actual (effective) rate of interest per year was 19.56%. One simple way to prove this is by using the calculator keystrokes listed in Table

8.12.

Step	Description	Enter	Display	
1	Set all variables to defaults	2ND [RESET] ENTER	RST	0.00
2	Set the display to four decimal places	2ND [FORMAT] 4 ENTER	DEC =	4.0000
3	Return to calculator mode	2ND [QUIT]		0.0000
4	Enter (1 + the monthly interest rate)	1.015 Y^X		1.0150
5	Enter the number of months	12 Y^X		1.1956

Table 8.12 Calculator Steps to Prove the Actual Rate of Interest per Year

Had we assumed that the stated monthly interest rate of 1.5% could be simply multiplied by 12 months for an annual rate of 18%, we would be ignoring the effect of more frequent compounding. As indicated above, the annual interest on the money that we spent initially, accumulating at a rate of 1.5% *per month*, is 19.56%, not 18%:

$$1.1956 - \$1 \text{ spent intially} = 0.1956 = 19.56\%$$

The final example in this chapter will represent the amortization of a loan. Using a 36-month auto loan for $32,000 at 6% per year compounded monthly, we can easily find the monthly payment and the amortization of this loan on our calculator using the following procedures and keystrokes.

First, we find the monthly payment (see Table 8.13).

Step	Description	Enter	Display	
1	Set all variables to defaults	2ND [RESET] ENTER	RST	0.00
2	Set payments per year to 12	2ND [P/Y] 12 ENTER	P/Y =	12.00
3	Return to calculator mode	2ND [QUIT]		0.00
4	Enter number of payments with the payment multiplier	3 2ND [xP/Y] N	N =	36.00
5	Enter annual interest rate	6 I/Y	I/Y =	6.00
6	Enter loan amount	32000 PV	PV =	32,000.00
7	Compute the monthly payment	CPT PMT	PMT =	-973.50

Table 8.13 Calculator Steps to Find the Monthly Payment of a Loan

We've verified the amount of our monthly debt service, including both the interest and repayment of the principal, as $973.50. The next step with our calculator is to verify our amortization at any point (see Table 8.14).

Step	Description	Enter	Display	
1	Set previous work as an amortization worksheet	2ND [AMORT]	P1 =	1.00
2	Set beginning period to 1	1 ENTER	P1 =	1.00
3	Set ending period to 12	↓ 12 ENTER	P2 =	12.00
4	Display amortization data at the end of month 12	↓	BAL =	21,965.02
5		↓	PRN =	-10,034.98

Table 8.14 Calculator Steps to Verify Amortization at the End of One Year

Step	Description	Enter	Display	
6		↓	INT =	-1,647.02

Table 8.14 Calculator Steps to Verify Amortization at the End of One Year

Without resetting the calculator, we will try a second example, this time reviewing the second full year of amortization at the end of 24 months (see Table 8.15).

Step	Description	Enter	Display	
1	Set previous work as an amortization worksheet	2ND [AMORT]	P1 =	1.00
2	Change beginning period to month 13	13 ENTER	P1 =	13.00
3	Change ending period to month 24	↓ 24 ENTER	P2 =	24.00
4	Display amortization data at the end of month 24	↓	BAL =	11,311.13
5		↓	PRN =	-10,653.89
6		↓	INT =	-1,028.11

Table 8.15 Calculator Steps to Verify Amortization at the End of Two Years

Solving Time Value of Money Problems Using Excel

Microsoft's popular spreadsheet program Excel is arguably one of the most common and powerful numeric and data analysis products available. Yet while mastery of Excel requires extensive study and practice, enough basics can be learned in two or three hours to provide the user with the ability to solve problems quickly and conveniently, including extensive financial capability. Most of the calculations in this chapter were prepared with Excel.

The boxes in the Excel gridwork, known individually as cells (located at the intersection of a column and a row), can contain numbers, text, and very powerful formulas (or functions) for calculations and data analytics. Cells, rows, columns, and groups of cells (ranges) are easily moved, formatted, and replicated. In the mortgage amortization table for 240 months seen in Section 8.3.2, only the formulas for month 1 were typed in. With one simple command, that row of formulas was replicated 239 more times, with each line updating itself with relevant number adjustments automatically. With some practice, a long table such as that can be constructed by even a relatively new user in less than 10 minutes.

In this section, we will illustrate how to use Excel to solve problems from earlier in the chapter, including perpetuities, ordinary annuities, effective interest rates, and loan amortization. We will omit the basic dynamics of an Excel spreadsheet because they were presented sufficiently in preceding chapters.

Revisiting the constant perpetuity from Section 8.1, in which our shares of Shaw Inc., preferred stock pay an annual fixed dividend of $2.00 and the required rate of return is 7%, we do not use an Excel function for this simple operation. The two values are entered in cells B3 and B4, respectively.

We enter a formula in cell B6 to perform the division and display the result in that cell. The actual contents of cell B6 are typed below it for your reference, in cell B8 (see Figure 8.2).

	A	B
1		
2		
3	**Indefinite Dividend per Share ($)**	2.00
4	**Required Rate of Return (%)**	0.07
5		
6	**Price per Share**	$ 28.57
7		=B3/B4

Figure 8.2 Excel Spreadsheet for Valuing a Perpetuity

Download the spreadsheet file (https://openstax.org/r/spreadsheet_file_Chapter08_finance) containing key Chapter 8 Excel exhibits.

To find the present value of an ordinary annuity, we revisit Section 8.2.1. You will draw $25,000 at the end of each year for five years from a fund earning 8% annually, and you want to know how much you need in that fund today to accomplish this. We accomplish this in Excel easily with the PV function. The format of the PV command is

```
=PV(rate,periods,payment,0,0)
```

Only the first three arguments inside the parentheses are used. We'll place them in cells and refer to those cells in our PV function. As an option, you could also type the numbers into the parentheses directly. Notice the slight rounding error because of decimal expansion. Also, the payment must be entered as a negative number for your result to be positive; this can be accomplished either by making the $25,000 in cell B5 a negative amount or by placing a minus sign in front of the B5 in the formula's arguments. In cell B3, you must enter the percent either as 0.08 or as 8% (with the percent sign). We repeated the formula syntax and the actual formula inputs in column A near the result, for your reference (see Figure 8.3).

	A	B
1		
2		
3	**Rate (%)**	0.08
4	**Periods**	5
5	**Payment**	(25,000)
6		
7	**=PV(rate,periods,payment,0,0)**	$ 99,817.75
8	**=PV(B3,B4,B5,0,0)**	
9		

Figure 8.3 Excel Spreadsheet Showing the Present Value of an Ordinary Annuity

We also found the present value of an annuity due. We use the same information from the ordinary annuity problem above, but you will recall that the first of five payments happens immediately at the start of year 1, not at the end. We follow the same procedures and inputs as in the previous example, but with one change to the PV function: the last argument in the parentheses will change from 0 to 1. This is a toggle switch that commands the PV function to treat this as an annuity due instead of an ordinary annuity (see Figure 8.4).

	A	B
1		
2		
3	**Rate (%)**	0.08
4	**Periods**	5
5	**Payment**	(25,000)
6		
7	**=PV(rate,periods,payment,0,1)**	$ 107,803.17
8	**=PV(B3,B4,B5,0,1)**	

Figure 8.4 Excel Spreadsheet Showing the Present Value of an Annuity Due

Section 8.2 introduced us to future values. Comparable to the PV function above, Excel provides the FV function. Using the same information—$3,000 invested annually for five years, starting one year from now, at 4%—we'll solve using Excel (see Figure 8.5). The format of the command is

```
=FV(rate,periods,payment,0,0)
```

	A	B
1		
2		
3	**Rate (%)**	0.04
4	**Periods**	5
5	**Payment**	(3,000)
6		
7	**=FV(rate,periods,payment,0,0)**	$ 16,248.97
8	**=FV(B3,B4,B5,0,0)**	

Figure 8.5 Excel Spreadsheet Showing the Future Value of an Ordinary Annuity

As with present values, using the same data but solving for an annuity due requires the fifth argument inside the parentheses to be changed from 0 to 1; all other values remain the same (see Figure 8.6).

	A	B
1		
2		
3	**Rate (%)**	0.04
4	**Periods**	5
5	**Payment**	(3,000)
6		
7	**=FV(rate,periods,payment,0,1)**	$ 16,898.93
8	**=FV(B3,B4,B5,0,1)**	

Figure 8.6 Excel Spreadsheet Showing the Future Value of an Annuity Due

In Section 8.4, we explained the difference between stated and effective rates of interest to show the true cost of borrowing, in this case for a one-year period, if interest is compounded for periods within a year. The syntax for the Excel effect function to calculate this rate is

```
=EFFECT(rate,periods)
```

where *rate* is the nominal rate and *periods* represents the number of periods within a year.

Earlier, our example showed that 1.5% compounded monthly results in not 18% per year but actually over 19.56% (see Figure 8.7).

	A	B	C
1			
2			
3	**Nominal Annual Rate**	18.00%	
4			
5			
6	**Period Type**	**Periods per Year**	**Effective Interest**
7	Annual	1	18.00%
8	Quarterly	4	19.25%
9	Monthly	12	19.56%

Figure 8.7 Excel Spreadsheet Showing Effective Interest Rate

Note several things: First, the nominal interest rate is entered as a percent. Second, the actual effect function in C7 is typed as =EFFECT(rate,B7); we use the word *rate* because we actually assigned a name to cell B3, so Excel can use it in a function and replicate it without it changing. When cell C7 is replicated to C8 and C9, *rate* remains the same, but the formulas automatically adjust to use B8 and B9 for the periods.

To assign a name to a cell, keep in mind that every cell has column-row coordinates. We want cell B3 to be the anchor of our effective rate calculations. Rather than referring to cell B3, we can name it, and in this case, we use the name *rate*, which we can then use in formulas like any other Excel cell letter-number reference. Place the cursor in cell B3. Now, look at cell A1 on the grid: right above that cell, you see a box displaying B3, the current cursor location. If you click in that box and type “rate” (without the quotation marks), as we did, then hit the enter key, the value in that box will change to *rate*. Now, if you type “rate” (again, without quotation marks) into a formula, Excel knows to use the contents of cell B3.

Excel provides convenient tools for figuring out amortization. We’ll revisit our 36-month auto loan for $32,000 at 6% per year, compounded monthly. A loan amortization table for a fixed interest rate debt is usually formatted as follows, with the Interest and Principal columns interchangeable:

```
Period     Payment     Interest     Principal     Balance
```

In Excel, a table is completed by using the function PMT. The individual steps follow.

1. List the information about the loan in the upper left of the worksheet, and create the column headings for the schedule of amortization. Type “B5” (without the quotation marks) in cell E9 to begin the schedule. Then enter 1 for the first month under the Payment # (or Month) column, in cell A10 (see Figure 8.8).

	A	B	C	D	E
1		**Actual**	**Use**		
2	**Annual Interest Rate**	6%	0.50%		
3	**Years**	3	36		
4	**Payments per Year**	12			
5	**Loan Amount**	$ 32,000			
6					
7					Remaining
8	Payment # (or Month)	Payment	Interest	Principal	Balance
9					$32,000.00

Figure 8.8 Step 1 of Creating an Amortization Table

2. Next, in cell B10, the payment is derived from the formula =PMT(rate,periods,pv), with PV representing the present value, or the loan amount. Because we are compounding monthly, enter C$2 and C$3 for the rate and periods, respectively. Cell B5 is used for the loan amount, but notice the optional minus sign placed in front of the entry B$5; this causes the results in the schedule to be displayed as positive numbers. The dollar sign ($) inserted in the cell references forces Excel to "freeze" those locations so that they don't attempt to update when we replicate them later; this is known in spreadsheet programs as an *absolute reference* (see Figure 8.9).

	A	B	C	D	E
1		**Actual**	**Use**		
2	**Annual Interest Rate**	6%	0.50%		
3	**Years**	3	36		
4	**Payments per Year**	12			
5	**Loan Amount**	$ 32,000			
6					
7					Remaining
8	Payment # (or Month)	**Payment**	Interest	Principal	Balance
9					32,000.00
10	1	**973.50**			
11		**=PMT(C$2,C$3,-B$5)**			

Figure 8.9 Step 2 of Creating an Amortization Table

3. The next step is to calculate the interest. We take the remaining balance from the previous line, in this case cell E9, and multiply it by the monthly interest rate in cell C2, typing C$2 to lock in the reference. The remaining balance of the loan should always be multiplied by this monthly percentage (see Figure 8.10).

	A	B	C	D	E
1		**Actual**	**Use**		
2	**Annual Interest Rate**	6%	0.50%		
3	**Years**	3	36		
4	**Payments per Year**	12			
5	**Loan Amount**	$ 32,000			
6					
7					Remaining
8	Payment # (or Month)	Payment	**Interest**	Principal	Balance
9					32,000.00
10	1	973.50	**160.00**		
11			**=E9*C$2**		

Figure 8.10 Step 3 of Creating an Amortization Table

4. Because this is a fixed-rate loan, whatever is left from each payment after first deducting the interest represents principal, the amount by which the balance of the outstanding loan balance is reduced. Therefore, the contents of cell D10 represent B10, the total payment, minus C10, the interest portion (see Figure 8.11). No dollar signs are included because this cell reference can adjust to each row into which this formula is replicated, as will be seen in the following examples.

	A	B	C	D	E
1		**Actual**	**Use**		
2	**Annual Interest Rate**	6%	0.50%		
3	**Years**	3	36		
4	**Payments per Year**	12			
5	**Loan Amount**	$ 32,000			
6					
7					Remaining
8	Payment # (or Month)	Payment	Interest	**Principal**	Balance
9					32,000.00
10	1	973.50	160.00	**813.50**	
11				**=B10-C10**	

Figure 8.11 Step 4 of Creating an Amortization Table

5. Because our principal portion of the last payment has reduced our outstanding balance, it is subtracted from the preceding balance in cell E9 (see Figure 8.12). The command therefore is `=E9-D10`.

	A	B	C	D	E
1		**Actual**	**Use**		
2	**Annual Interest Rate**	6%	0.50%		
3	**Years**	3	36		
4	**Payments per Year**	12			
5	**Loan Amount**	$ 32,000			
6					
7					**Remaining**
8	Payment # (or Month)	Payment	Interest	Principal	**Balance**
9					32,000.00
10	1	973.50	160.00	813.50	**31,186.50**
11					**=E9-D10**

Figure 8.12 Step 5 of Creating an Amortization Table

Now that the first full row is defined, an amortization schedule is easily developed by Excel's replication abilities. Place the cursor on cell A10, hold down the left mouse button, and drag the cursor to cell E10. Cells A10 through E10 in row 10 should now be highlighted. Release the mouse button. Then "grab" the tiny square symbol at the bottom right of cell E10 and drag it downward as far as you need; in this case, you'll need 35 more rows because this is a 36-month loan, so it will end at row 45. We added a line for totals.

This is now a complete loan amortization schedule (see Figure 8.13). The first several periods display, followed by the last few periods, to prove that the schedule is complete (data rows for month 4 to month 22 are hidden).

	A	B	C	D	E
1		**Actual**	**Use**		
2	**Annual Interest Rate**	6%	0.50%		
3	**Years**	3	36		
4	**Payments per Year**	12			
5	**Loan Amount**	$ 32,000			
6					
7					**Remaining**
8	**Payment # (or Month)**	**Payment**	**Interest**	**Principal**	**Balance**
9					32,000.00
10	1	973.50	160.00	813.50	31,186.50
11	2	973.50	155.93	817.57	30,368.93
12	3	973.50	151.84	821.66	29,547.27
42	(continued)				
43	33	973.50	19.23	954.27	2,891.54
44	34	973.50	14.46	959.04	1,932.50
45	35	973.50	9.66	963.84	968.66
46	36	973.50	4.84	968.66	0.00
47	**Totals**	**$ 35,046.07**	**$ 3,046.07**	**$ 32,000.00**	

Figure 8.13 Completed Amortization Schedule

This will look familiar; it's the same amortization table used as a proof in Section 8.3 (see Table 8.4). There is no rounding error because Excel uses the full decimal expansion in its calculations.

This chapter has explored the time value of money by expanding on the concepts discussed in Time Value of Money I with additional funds being periodically added to or subtracted from our investment, either compounding or discounting them according to the situation. In all cases, the payments in the stream were identical. If they had not been identical, a separate set of operators would be required, and these will be addressed in the next chapter.

Summary

8.1 Perpetuities

A perpetuity is an investment that is intended to provide an expected return indefinitely, either remaining constant or growing by an incremental amount. Preferred stock is a common example with a preestablished dividend formula. An indefinite stream of payments cannot be compounded into a future value, but it can be discounted to a present value, providing an opportunity to determine the amount an investor should be willing to pay for a share of that stock.

8.2 Annuities

An annuity is a stream of fixed periodic payments that is expected to be paid or received. Calculations of future value or present value are commonly performed on these payment streams for a wide number of reasons in business and personal financial analysis, as seen in the chapter focusing on single amounts, particularly in loan repayment. Annuities may be ordinary annuities, in which the first cash flow of a series occurs at the end of the first period, or annuities due, if the first cash flow occurs at the beginning point of the first period.

8.3 Loan Amortization

Loans are contracts between a lender and a borrower. Failure to observe the rules of that contract, such as payment of interest or repayment of the amount owed, can subject the borrower to substantial penalties as well as damage to their credit. Loan agreements bearing a fixed rate of interest have a scheduled amortization, or rate and time of repayments with interest. Several types of business and personal loans were described.

8.4 Stated versus Effective Rates

For a borrower to understand the true cost of financing, they must be familiar with interannual compounding, which can cause a stated interest rate that appears to be annual to actually be higher. The effective rate of interest was demonstrated to understand that true cost.

8.5 Equal Payments with a Financial Calculator and Excel

The use of two tools for managing and understanding the time value of money and its many applications was discussed: a professional financial calculator and the popular Microsoft Office Suite spreadsheet application Excel.

Key Terms

annuity a stream of regular, periodic payments to be received or paid

annuity due a stream of periodic payments in which the payment or receipt occurs at the beginning of each period

constant perpetuity a stream of periodic payments that is expected to continue indefinitely with no change in the amount paid or received

discount rate an interest rate used in time value of money calculations to determine present value; may derive from several sources, such as stated contract rates, costs to borrow, or expected rates of return on investments

effective interest rate the interest rate that results when compounding occurs multiple times within a year; the true cost of borrowing

growing perpetuity a stream of periodic payments that is expected to continue indefinitely with growth of the amount paid or received in the future, usually by a fixed percentage

loan amortization the scheduling of periodic repayment of a debt, typically involving regular payments or receipts of amounts that include both interest payment and repayment of the principal of the amount owed

lump sum a single cash payment made in lieu of a series of future payments, such as a lottery payout or a

legal settlement

ordinary annuity a stream of periodic payments in which the payment or receipt occurs at the end of each period

perpetuity a stream of periodic payments that is expected to continue indefinitely

preferred stock shares of ownership in a corporation that typically entitle the holder to a fixed dividend per share, if declared by the corporation, with priority over holders of that corporation's common stock

required rate of return the minimum amount of return that an investor will accept on an investment given the level of risk involved

retirement planning the process of determining one's objectives for retirement, including one's finances, and developing strategies and tactics to achieve them

structured settlements monetary legal settlements that are paid out in installments, such as an annuity, rather than a lump sum cash amount

CFA Institute

This chapter supports some of the Learning Outcome Statements (LOS) in this CFA® Level I Study Session (https://openstax.org/r/CFA_Level_I_Study_Session2). Reference with permission of CFA Institute.

Multiple Choice

1. The best example of a constant perpetuity would most likely be _____.
   a. an annuity due
   b. dividends from common stock
   c. preferred stock
   d. an ordinary annuity

2. You wish to endow a university chair of accounting for a salary of $100,000 per year to the recipient. The university will withdraw $100,000 each year for the recipient's salary. The amount of your gift will remain untouched indefinitely, in perpetuity. The university can lock in a fixed rate for your investment of 2.8% per year. In order to achieve this, what is the approximate amount of the gift you would have to make now?
   a. $3,103,569
   b. $3,571,429
   c. $4,101,218
   d. $4,227,827

3. Preferred stock in Blue Agate Inc. is issued for dividends of $3.00 per share. The dividends will increase each year at 0.178%, a growing perpetuity. The required rate of return on a stock such as this is 2.5%. At what approximate price will this preferred stock most likely sell today?
   a. $117.21
   b. $119.87
   c. $120.00
   d. $129.20

4. Julio's attorney negotiates a structured settlement after an injury, consisting of seven equal payments to Barry of $150,000 each. The first payment is due today, and the remaining payments will be received in annual amounts, with the second payment occurring one year from now. What is the approximate value of this settlement in today's dollars if Barry uses a discount rate of 5%?
   a. $911,352
   b. $867,960
   c. $746,235

d. $1,050,000

5. What is the approximate present value of an ordinary annuity (beginning one year from now) of a stream of 12 annual payments of $87,000 if you use a discount rate of 6%?
a. $773,154.04
b. $747,278.92
c. $729,394.95
d. $718,974.58

6. If Maria invests $2,700 at the end of each six-month period for six years at an annual rate of 4%, what is the approximate future value of her ordinary annuity? Review Chapter 7 for the techniques of interannual compounding.
a. $17,909.10
b. $20,248.23
c. $31,755.54
d. $36,212.67

7. Assume all of the same facts as in exercise 6 above, except that Maria begins immediately and makes each of her payments at the beginning of each 6-month period instead of the end. What is the approximate future value of her annuity due at the end of the six years?
a. $17,909.10
b. $36,936.92
c. $32,707.24
d. $22,997.88

8. Rather than spending her $48,000 in casino winnings, Christy places the money in an investment that will earn her 5% per year, compounded annually. She will withdraw the money in four equal annual installments beginning one year from today. What must the approximate amount of each annual withdrawal be for this investment to be fully depleted in four years?
a. $11,136.38
b. $12,892.56
c. $12,243.47
d. $13,536.61

9. Your friend Jamal borrows $5,000 from you, agreeing to pay you back with 8% annual interest, with the first payment due to you one year from today. You ask that you be fully repaid over the next four years. However, to lower his annual payment, Jamal asks you to extend the period over five full years instead. What will be the approximate **difference** in his total payments to you, including interest and principal, if the debt is amortized over five years rather than four?
a. Jamal will pay $544 less.
b. Jamal will pay $544 more.
c. Jamal will pay $223 less.
d. Jamal will pay $223 more.

10. Your new El Supremo credit card arrangement indicates that you will owe interest on unpaid balances at a nominal (stated) rate of 1.2% per month. If the interest rate is compounded monthly, what is the approximate effective annual rate of interest?
a. 15.39%
b. 12.00%
c. 14.02%

d. 14.40%

Problems

Use four decimal places on time value of money factors unless otherwise specified. Approximations and minor differences because of rounding are acceptable. Ignore the effect of taxes. Assume that all percentages are annual rates and that compounding occurs annually unless indicated otherwise.

1. Steve purchases preferred stock in Berklee Corporation, with each share paying a $2.50 dividend. This dividend will remain constant. If the public's required rate of return for Berklee stock is 8%, at what price should this company's stock sell?
2. Donna enters into an investment contract that will guarantee her 4% per year if she deposits $3,500 each year for the next 10 years. She must make the first deposit one year from today, the day she signs the agreement. How much will she have when she makes her last payment 10 years from now?
3. Assume the same facts as in problem 2 above, except that Donna negotiates the chance to make her first payment now and continue to pay at the beginning of each year for the 10-year period. How much will she have accumulated?
4. Bill will receive a royalty payment of $18,000 per year for the next 25 years, beginning one year from now, as a result of a book he has written. If a discount rate of 10 percent is applied, should he be willing to sell out his future rights now for $160,000? How about $162,500? $165,000?
5. Debbie won the $60 million lottery. She is to receive $1 million a year for the next 50 years beginning one year from now, plus an additional lump sum payment of $10 million after 50 years. The discount rate is 10 percent. How much cash would she need to be offered today to tempt her to take a lump-sum cash offer instead, all things equal?
6. Kim started a paper route on January 1, 2016. Every three months, she deposited $300 in her new bank account, which earned 4 percent annually but was compounded quarterly. On December 31, 2019, she placed the entire balance in her bank account in an investment that earned 5 percent annually. How much will she have on December 31, 2022?
7. You hire Thomas to work for you for five years, and you agree to put away enough money as a lump sum now to fund an annuity for him. At the end of those five years, he will retire and may begin drawing out $20,000 per year for five years, starting on the last day of each year (in this case, the end of year 6, from when this arrangement began, through year 10). How much must you invest today if your guaranteed interest rate is 3% compounded annually for all 10 years?
8. Your new boss doesn't have a pension or 401(k) plan for your retirement, but she agrees to place aside $12,000 every year once a year for four years. She gives you the option of either starting immediately on your first day of work or starting one year from now. That makes this the difference between an ordinary annuity and an annuity due. If the plan earns 5% per year, compounded annually, what will be the difference between the two approaches after the four years / four payments?
9. Jada is borrowing $40,000 from you today. She agrees to pay you back in annual installments beginning a year from now over eight years, with interest at 3%. What would her annual payment amount be, including both interest and principal?
10. You agree to finance your new SUV with an auto loan of $38,000. This loan will be repaid over three years with monthly payments (and compounding) at a 4% annual interest rate (0.33% per month). What will your monthly loan payment be?

Video Activity

Future Value of Ordinary Annuities

Click to view content (https://openstax.org/r/Future_Value_of_an_Annuity)

1. What is the primary difference between this demonstration and our chapter examples, keeping the chapter "Time Value of Money I" in mind?
2. Explain the significance of Dr. van Biezen's comment at 4:32 regarding a difference when payments are made at the beginning of each pay period rather than the end.

Practical Example of Annuities

Click to view content (https://openstax.org/r/What_is_an_annuity?)

3. What is the primary difference between a fixed annuity and a variable annuity?
4. Annuities are often recommended to retirees and seniors. Why would a fixed annuity be more attractive to such a person than a variable annuity?

9

Time Value of Money III: Unequal Multiple Payment Values

Figure 9.1 Capital investment in production equipment such as this robotic arm requires extensive analysis of the benefits the investment is likely to produce over time. (credit: modification of "Webb Telescope Crew Flexes Robotic Arm at NASA" by Chris Gunn/NASA/flickr, CC BY 2.0)

Chapter Outline

Why It Matters

Baseball legend Ted Williams once said, "Baseball is the only field of endeavor where a man can succeed three times out of ten and be considered a good performer."[1] On routine or unimportant decisions, business managers might aspire to do as well as Ted Williams, making the right decisions only 30% of the time. But a professional decision maker must "hit it out of the park" when making major capital investment choices and recommendations.

As a student in a course of business studies and career development, it is highly likely that you will be a decision maker about projects that are likely to generate future cash flows but will also require a large initial expense. When you ask your manager to invest $500,000 or more in a new piece of equipment that could help your department meet or exceed its goals, you must be prepared to defend your request. Competing managers and departments will be asking for similar funding, and there simply might not be enough for everyone. This decision process requires financial analysis.

Mark Cuban of *Shark Tank* fame enjoys citing the series' catchphrase: "Know thy numbers." As a business professional, you must be able to assess potential profit against expenditures to be successful. In most cases, this is based on our understanding of *cash flow*. A major capital investment might seem initially like a gamble, but it is a gamble that can be hedged in your favor with understanding, analysis, and knowledge of your numbers.

The purpose of this chapter is to give you information and instruction on how this is done. The techniques we will discuss in this chapter will clarify decisions that must be made in the process of investing in a business. We

1 Pete Palmer and Gary Gillette, eds. *The 2006 ESPN Baseball Encyclopedia*. New York: Sterling Publishing, 2006, 5.

focus first on decisions we make about our own money as investors if uneven cash receipts or payments are involved.

9.1 Timing of Cash Flows

Learning Outcomes

By the end of this section, you will be able to:

- Describe how multiple payments of unequal value are present in everyday situations.
- Calculate the future value of a series of multiple payments of unequal value.
- Calculate the present value of a series of multiple payments of unequal value.

Multiple Payments or Receipts of Unequal Value: The Mixed Stream

At this point, you are familiar with the time value of money of single amounts and annuities and how they must be managed and controlled for business as well as personal purposes. If a stream of payments occurs in which the amount of the payments changes at any point, the techniques for solving for annuities must be modified. Shortcuts that we have seen in earlier chapters cannot be taken. Fortunately, with tools such as financial or online calculators and Microsoft Excel, the method can be quite simple.

The ability to analyze and understand **cash flow** is essential. From a personal point of view, assume that you have an opportunity to invest $2,000 every year, beginning next year, to save for a down payment on the purchase of your first home seven years from now. In the third year, you also inherit $10,000 and put it all toward this goal. In the fifth year, you receive a large bonus of $3,000 and also dedicate this to your ongoing investment.

The stream of regular payments has been interrupted—which is, of course, good news for you. However, it does add a new complexity to the math involved in finding values related to time, whether compounding into the future or discounting to the present value. Analysts refer to such a series of payments as a **mixed stream**. If you make the first payment on the first day of next year and continue to do so on the first day of each following year, and if your investment will always be earning 7% interest, how much cash will you have accumulated—principal plus earned interest—at the end of the seven years?

This is a future value question, but because the stream of payments is mixed, we cannot use annuity formulas or approaches and the shortcuts they provide. As noted in previous chapters, when solving a problem involving the time value of money, a timeline and/or table is helpful. The cash flows described above are shown in Table 9.1 . Remember that all money is assumed to be deposited in your investment at the beginning of each year. The cumulative cash flows do not yet consider interest.

Year	0	1	2	3	4	5	6	7
Cash Invested	$0.00	$2,000	$2,000	$2,000	$12,000	$2,000	$5,000	$2,000
Cumulative Cash Flows		$2,000	$4,000	$6,000	$18,000	$20,000	$25,000	$27,000

Table 9.1

By the end of seven years, you have invested $27,000 of your own money before we consider interest:

- Seven years times $2,000 each year, or $14,000
- The extra $10,000 you received in year 3 (which is invested at the start of year 4)
- The extra $3,000 you received in year 5 (which is invested at the start of year 6)

These funds were invested at different times, and time and interest rate will work for you on all accumulated balances as you proceed. Therefore, focus on the line in your table with the cumulative cash flows. How much cash will you have accumulated at the end of this investment program if you're earning 7% compounded

annually? You could use the *future value of a single amount* equation, but not for an annuity. Because the amount invested changes, you must calculate the future value of each amount invested and add them together for your result.

Recall that the formula for finding the future value of a single amount is $\text{FV} = \text{PV} \times (1 + i)^n$, where FV is the future value we are trying to determine, PV is the value invested at the start of each period, *i* is the interest rate, and *n* is the number of periods remaining for compounding to take effect.

Let us repeat the table with your cash flows above. Table 9.2 includes a line to show for how many periods (years, in this case) each investment will compound at 7%.

Year	0	1	2	3	4	5	6	7
Cash Invested	$0.00	$2,000	$2,000	$2,000	$12,000	$2,000	$5,000	$2,000
Cumulative Cash Flows		$2,000	$4,000	$6,000	$18,000	$20,000	$25,000	$27,000
Years to Compound		7	6	5	4	3	2	1

Table 9.2

The $2,000 that you deposit at the start of year 1 will earn 7% interest for the entire seven years. When you make your second investment at the start of year 2, you will now have spent $4,000. However, the interest from your first $2,000 investment will have earned you $\$2{,}000 \times 0.07 = \$140$, so you will begin year 2 with $4,140 rather than $4,000.

Before we complicate the problem with a schedule that ties everything together, let's focus on years 1 and 2 with the original formula for the future value of a single amount. What will your year 1 investment be worth at the end of seven years?

$$\text{FV}_1 = \$2{,}000 \times (1 + 0.07)^7 \approx \$3{,}211.56$$

You need to address the year 2 investment separately at this point because you've calculated the year 1 investment and its compounding on its own. Now you need to know what your year 2 investment will be worth in the future, but it will only compound for six years. What will it be worth?

$$\text{FV}_2 = \$2{,}000 \times (1 + 0.07)^6 \approx \$3{,}001.46$$

You can perform the same operation on each of the remaining five invested amounts, remembering that you invest $12,000 at the start of year 4 and $5,000 at the start of year 6, as per the table. Here are the five remaining calculations:

$$\begin{aligned}
\text{FV}_3 &= \$2{,}000 \times (1 + 0.07)^5 \approx \$2{,}805.10 \\
\text{FV}_4 &= \$12{,}000 \times (1 + 0.07)^4 \approx \$15{,}729.55 \\
\text{FV}_5 &= \$2{,}000 \times (1 + 0.07)^3 \approx \$2{,}450.09 \\
\text{FV}_6 &= \$5{,}000 \times (1 + 0.07)^2 \approx \$5{,}724.50 \\
\text{FV}_7 &= \$2{,}000 \times (1 + 0.07)^1 \approx \$2{,}140.00
\end{aligned}$$

Notice how the exponent representing *n* decreases each year to reflect the decreasing number of years that each invested amount will compound until the end of your seven-year stream. For clarity, let us insert each of these amounts in a row of Table 9.3 :

Year	0	1	2	3	4	5	6	7
Cash Invested	$0.00	$2,000	$2,000	$2,000	$12,000	$2,000	$5,000	$2,000
Cumulative Cash Flows		$2,000	$4,000	$6,000	$18,000	$20,000	$25,000	$27,000

Table 9.3

Year	0	1	2	3	4	5	6	7
Years to Compound		7	6	5	4	3	2	1
Compounded Value at End of Year 7		$3,211.56	$3,001.46	$2,805.10	$15,729.55	$2,450.09	$5,724.50	$2,140.00

Table 9.3

The solution to the original question—the value of your seven different investments at the end of the seven-year period—is the total of each individual investment compounded over the remaining years. Adding the compounded values in the bottom row provides the answer: $35,062.26. This includes the $27,000 that you invested plus $8,062.26 in interest earned by compounding.

It's important to note that throughout these sections on the time value of money and compounded or discounted values of mixed streams and their analysis, we are placing the valuation at the end or beginning of a period for simplicity in the examples. In reality, businesses might consider valuations happening within the period to allow for a degree of regularity in the revenue streams provided by the asset being considered. However, because this is a technique of forecasting, which is inherently uncertain, we will continue with analysis by period.

THINK IT THROUGH

Future Value of a Mixed Stream

Assume that you can invest five annual payments of $10,000, beginning immediately, but you believe you will be able to invest additional amounts of $5,000 at the beginning of years 4 and 5. This investment is expected to earn 4% each year. What is the anticipated future value of this investment after the full five years?

Solution:

Year	0	1	2	3	4	5
Cash Invested	$0.00	$10,000	$10,000	$10,000	$15,000	$15,000
Cumulative Cash Flows		$10,000	$20,000	$30,000	$45,000	$60,000
Years to Compound		5	4	3	2	1
Compounded Value at End of Year 5		$12,166.53	$11,698.59	$11,248.64	$16,224.00	$15,600.00

Table 9.4

The equations to calculate each individual year's compounded value at the end of the five years are as follows:

$$\begin{aligned} \text{FV}_1 &= \$10{,}000 \times (1+0.04)^5 \approx \$12{,}166.53 \\ \text{FV}_2 &= \$10{,}000 \times (1+0.04)^4 \approx \$11{,}698.59 \\ \text{FV}_3 &= \$10{,}000 \times (1+0.04)^3 \approx \$11{,}248.64 \\ \text{FV}_4 &= \$15{,}000 \times (1+0.04)^2 \approx \$16{,}224.00 \\ \text{FV}_5 &= \$15{,}000 \times (1+0.04)^5 \approx \$15{,}600.00 \end{aligned}$$

The sum of these individual calculations is $66,937.76, which is the total value of this stream of invested amounts plus compounded interest.

Let's take the example above and review it from a different angle. Keeping in mind that we have not yet explored the use of Excel, is there another way to view our solution? The problem above takes each annual investment and compounds it into the future, then adds the results of each calculation to find the total future value of the stream of payments.

But when you break the problem down, another way to look at the problem is as a five-year annuity of $10,000 per year *plus* added payments in years 4 and 5. Can we solve for the future value of an annuity first and then perform two separate calculations on the additional amounts ($5,000 each in years 4 and 5)? Yes, we can.

Let's summarize:

- Future value of a $10,000 annuity due, 4%, 5 years, *plus*
- Future value of a single payment of $5,000, 4%, 2 years, *plus*
- Future value of a single payment of $5,000, 4%, 1 year

This must give us the same result. The formula for the future value of an annuity due is

$$\text{FVa} = \text{PYMT} \times \frac{(1+i)^n - 1}{i} \times (1 + \text{i})$$

This problem can be solved in the three steps of the summary above.

Step 1:

$$\text{FVa} = \$10{,}000 \times \frac{(1+0.04)^5 - 1}{0.04} \times (1 + 0.04)$$

$$\text{FVa} = \$10{,}000 \times 5.416323 \times 1.04 \approx \$56{,}329.76$$

Step 2:

$$\text{FV}_{\text{Year 4}} = \$5{,}000 \times (1 + 0.04)^2 = \$5{,}408.00$$

Step 3:

$$\text{FV}_{\text{Year 5}} = \$5{,}000 \times (1 + 0.04)^1 = \$5{,}200.00$$

Combining the results from each of the three steps gives us

$$56{,}329.76 + 5{,}408.00 + 5{,}200.00 = \$66{,}937.76$$

It works. Whether you view this problem as five separate periods that can be compounded separately and then combined or as a combination of one or more annuities and/or single payment problems, we always arrive at the same solution if we are diligent about the time, the interest, and the stream of payments.

The Present Value of a Mixed Stream

Now that we've seen the calculation of a **future value**, consider a **present value**. We will begin with a personal example. You win a cash windfall through your state's lottery. You would like to take a portion of the funds and place them in a fixed investment so that you can draw $17,000 per year starting one year from now and continue to do so for the next two years. At the end of year 4, you want to withdraw $17,500, and at the end of year 5, you will withdraw the last $18,000 to close the account. When you take your last payment of $18,000, your fund will be totally depleted. You will always be earning 6% annually. How much of your cash windfall should you set aside today to accomplish this?

Let us break down the problem, remembering that we are thinking in reverse from the earlier problems that involved future values. In this case, we're bringing future values back in time to find their present values. You will recall that this process is called *discounting* rather than *compounding*.

Regardless of how we solve this, the question remains the same: How much money must we invest today (present value) to achieve this? And remember that we will always be earning 6% compounded annually on any invested balances.

We are calculating present values as we did in previous chapters, given a known future value "target," in order to determine how much money you need today to achieve that goal. Let us break this down by first reviewing the relevant equations from previous chapters.

Present value of an ordinary annuity:

$$\text{PVa} = \text{PYMT} \times \frac{\left[1 - \frac{1}{(1+i)^n}\right]}{i}$$

Present value of a single amount:

$$\text{PV} = \text{FV} \times \frac{1}{(1+i)^n}$$

where PVa is the present value of an annuity, PYMT is one payment in a consistent stream (an annuity), *i* is the interest rate (annual unless otherwise specified), *n* is the number of periods, PV is the present value of a single amount, and FV is the future value of a single amount.

You want to find out how much money you need to set aside today to accomplish your goal. You can also find out how much money you need to set aside in each period to accomplish this goal. Therefore, we can address this problem in increments. Let us look at potential solutions.

First, we will break this down into the cash flows of each year. Table 9.5 shows the timing of the future cash flows you're expecting:

Year	0	1	2	3	4	5
Expected Amount to Be Withdrawn at End of Year	$0.00	$17,000	$17,000	$17,000	$17,500	$18,000

Table 9.5

One method is to take each year's cash flows, which happen at the end of the year, and discount them to today using the present value formula for a single amount:

$$\text{PV} = \text{FV} \times \frac{1}{(1+i)^n}$$

$$\text{PV}_1 = \$17{,}000 \times \frac{1}{(1+0.06)^1} \approx \$16{,}037.74$$

Because year 1's withdrawal from your fund only has one year to earn interest, we discounted it for one year. The second amount is discounted for two years:

$$\text{PV}_2 = \$17{,}000 \times \frac{1}{(1+0.06)^2} \approx \$15{,}129.94$$

The next three years are discounted in the same way, for three, four, and five years, respectively:

$$\text{PV}_3 = \$17{,}000 \times \frac{1}{(1+0.06)^3} \approx \$14{,}273.53$$

$$\text{PV}_4 = \$17{,}500 \times \frac{1}{(1+0.06)^4} \approx \$13{,}861.64$$

$$\text{PV}_5 = \$18{,}000 \times \frac{1}{(1+0.06)^5} \approx \$13{,}450.65$$

Notice how we reverse our thinking on the exponent *n* from our approach to future value. This time, it increases each period because we discount each future amount for a longer period to arrive at the value in today's dollars.

When we add all five discounted present value amounts from above, we derive today's value of $72,753.49. Expressed more simply, if you wanted to extract the specified stream of cash flows at the end of each year

($17,000 for three years, then $17,500, then $18,000), you would have to begin with $72,753.49. The thing to remember is that any amounts remaining in this fund, regardless of how you deplete it, will always be earning 6% annually. See Table 9.6.

Year	0	1	2	3	4	5
Withdrawn at End of Year		$17,000.00	$17,000.00	$17,000.00	$17,500.00	$18,000.00
Interest on Balance		$4,365.21	$3,607.12	$2,803.55	$1,951.76	$1,018.87
Remaining Balance	$72,753.49	$60,118.70	$46,725.82	$32,529.37	$16,981.13	$0.00

Table 9.6

Let us try another approach. Because the amount of cash withdrawn in the first three years remains constant at $17,000, it can be viewed as an annuity—specifically, a three-period annuity of $17,000 and two single payments of $17,500 and $18,000. Therefore, we could also discount (bring to present value) an annuity of $17,000 for three years (the first three) and then combine it with the year 4 discounted amount and the year 5 discounted amount. We can try it using the formulas for PVa and PVused above. In Step 1, we will discount the first three years as an annuity (ordinary, as the first withdrawal is not made until one year from now); in Step 2, we will discount the year 4 single payment amount; and in Step 3, we will do the same for the year 5 single payment amount. Then we can add them together.

Step 1: Find the present value of the annuity using the PVa formula:

$$\text{PVa} = \$17{,}000 \times \frac{\left[1 - \frac{1}{(1 + 0.06)^3}\right]}{0.06}$$

$$\text{PVa} = \$17{,}000 \times \frac{1 - 0.839619}{0.06}$$

$$\text{PVa} = \$17{,}000 \times 2.673017 \approx \$45{,}441.29$$

Step 2: Discount the year 4 amount using the formula for the present value of a single amount:

$$\text{PV}_{(\text{Year } 4)} = \$17{,}500 \times \frac{1}{(1 + 0.06)^4} \approx \$13{,}861.64$$

Step 3: Perform the same operation as in Step 2 for the year 5 amount:

$$\text{PV}_{(\text{Year } 5)} = \$18{,}000 \times \frac{1}{(1 + 0.06)^5} \approx \$13{,}450.65$$

Now that all three amounts have been discounted to today's value, we can add them:

$$45{,}441.20 + 13{,}861.64 + 13{,}450.65 = \$72{,}753.49$$

Calculating the present value of cash flows is very common and critical in the analysis of **capital investments** in business for two compelling reasons: first, the investment is likely quite significant, and second, the risk will usually encompass a longer time frame. When the author of this chapter would purchase a large machine, it would likely take several years for that machine to justify its purchase with the revenues it would generate. This is one of the primary reasons that accountants require us to depreciate the cost of an asset over time: to assess the cost against the time it will take for that asset to produce profits and cash flow.

THINK IT THROUGH

Present Value of a Mixed Stream

Assume that you decide to invest $450,000. All cash flows are discounted at 4%. You are told by your financial advisor to expect cash inflows from your investment of $100,000 in year 1, $125,000 in year 2, $175,000 in year 3, $90,000 in year 4, and $50,000 in year 5. Would you agree to this plan based only on the numbers? Each amount will be withdrawn at the end of every year, and interest will be compounded annually.

Solution:

Year	0	1	2	3	4	5
Expected Amount to Be Withdrawn at End of Year	$0.00	$100,000	$125,000	$175,000	$90,000	$50,000

Table 9.7

Applying the formula for the present value of a single amount, we discount each amount and then add the discounted amounts. We will simplify this approach with Excel shortly, but we must understand the reasoning behind discounting uneven cash flow streams with a direct solution.

$$\begin{aligned}
PV_1 &= \$100{,}000 \times \frac{1}{(1+0.04)^1} \approx \$96{,}153.85 \\
PV_2 &= \$125{,}000 \times \frac{1}{(1+0.04)^2} \approx \$115{,}569.53 \\
PV_3 &= \$175{,}000 \times \frac{1}{(1+0.04)^3} \approx \$155{,}574.36 \\
PV_4 &= \$90{,}000 \times \frac{1}{(1+0.04)^4} \approx \$76{,}932.38 \\
PV_5 &= \$50{,}000 \times \frac{1}{(1+0.04)^5} \approx \$41{,}096.36
\end{aligned}$$

By combining the five discounted amounts above, we get a total present value of $485,326.48. This amount represents the value today of the five expected cash inflows for as long as our remaining balance is earning 4%.

CONCEPTS IN PRACTICE

Thoughts on Cash Flow from Irina Simmons

In 2013, the author interviewed Irina Simmons, senior vice president, chief risk officer, and former treasurer of EMC Corporation. The importance and understanding of cash flow analysis is fundamental to this text, and several of her insights are highly relevant to our content and procedures here.

AA: Ms. Simmons, why is cash management so important to an existing or start-up firm, and how does it compare to the more basic and traditional focus on profitability?

Simmons: While profitability is very useful for analysis by investors to measure performance, an organization's cash flow provides superior measurement. Cash flow is easy to understand, provides a transparent way of assessing a firm's health, and is not subject to any qualifications. By focusing upon cash flow, any firm—whether it is mature or a start-up organization—can have a clear picture of its health and success.

AA: In your bio, you mention liquidity management. Can you elaborate on this and why liquidity management is so important to a firm?

Simmons: Just as effective forecasting can provide superior cash management, the same holds true for liquidity management. For example, if you are able to confidently predict levels and timing of cash, then based on that forecast, you can make effective short- and long-term borrowing decisions. A disciplined approach to projecting one's cash position means that instead of investing cash in the money market to maximize day-to-day liquidity, you can look into longer-term investments that can provide a significantly higher return. This is essential to the effective matching of cash inflows and outflows for the firm.

AA: In summary, do you have any words of advice to students who might have an eye to entrepreneurial ventures?

Simmons: "Cash is king," don't forget that. Understand how cash moves through a business. It is also very important to implement and retain a cash management discipline. Never put that off until later. Many times, start-ups will say, "Well, I have all this venture money, and we can start making things happen and worry about being good cash managers later." But what I've seen is that the longer companies wait, the harder it is to break bad habits. Making cash management a priority now will serve entrepreneurs in perfect stead as their business starts to gain traction.

We closed this excellent interview with agreement that we were "kindred spirits" regarding the importance of cash flow analysis, including capital decisions such as those mentioned in this chapter. We confirmed with each other the core belief that "cash flow is the axis upon which the world of business spins."

(source: *Business Finance: A Clear View*, 3rd edition, by Alan S. Adams. LAD Publishing, 2015.)

LINK TO LEARNING

Analyst Training Materials

These materials, developed to help professionals prepare for an analyst certification exam, describe the sources of return from investing in a bond (https://openstax.org/r/return-investing-bond).

9.2 Unequal Payments Using a Financial Calculator or Microsoft Excel

Learning Outcomes

By the end of this section, you will be able to:

- Calculate unequal payments using a financial calculator.
- Calculate unequal payments using Microsoft Excel.

Using a Financial Calculator

A financial calculator provides utilities to simplify the analysis of uneven mixed cash streams (see Table 9.8).

Earlier, we explored the future value of a seven-year mixed stream, with $2,000 being saved each year, plus an additional $10,000 in year 4 and an additional $3,000 in year 6. All cash flows and balances earn 7% per year compounded annually, and the payments are made at the start of each year. We proved that this result totals approximately $35,062.26. We begin by clearing all memory functions and then entering each cash flow as follows:

Step	Description	Enter	Display	
1	Clear cash flow memory	CF 2ND [CLR WORK]	CF0	0.00
2	Enter 0 for cash flow at Time 0	ENTER ↓	CF0	0.00
3	Move to next entry	↓	C01	0.00
4	Enter first cash flow	2000 ENTER	C01 =	2000.00
5	Move to next entry	↓ ↓	C02	0.00
6	Enter second cash flow	2000 ENTER ↓	C02 =	2000.00
7	Move to next entry	↓ ↓	C03	0.00
8	Enter third cash flow	2000 ENTER	C03 =	2000.00
9	Move to next entry	↓ ↓	C04	0.00
10	Enter fourth cash flow	12000 ENTER	C04 =	12000.00
11	Move to next entry	↓ ↓	C05	0.00
12	Enter fifth cash flow	2000 ENTER	C05 =	2000.00
13	Move to next entry	↓ ↓	C06	0.00
14	Enter sixth cash flow	5000 ENTER	C06 =	5000.00
15	Move to next entry	↓ ↓	C07	0.00
16	Enter seventh cash flow	2000 ENTER	C07 =	2000.00
17	Press NPV	NPV	I	0.00
18	Enter interest rate	7 ENTER	I =	7.00
19	Press down arrow to show current NPV rate	↓	NPV	0.00
20	Press CPT to find net present value	CPT	NPV	20,406.56

Table 9.8 Steps for Calculating Uneven Mixed Cash Flows[2]

At this point, we have found the net present value of this uneven stream of payments. You will recall, however, that we are not trying to calculate present values; we are looking for future values. The TI BA II Plus™ Professional calculator does not have a similar function for future value. This means that either we can find the future value for each payment in the stream and combine them, or we can take the net present value we just calculated and easily project it forward using the following keystrokes. Note the net present value solution in Step 20 above. We will use that and then use the simpler of the two approaches to calculate future value (see Table 9.9).

Step	Description	Enter	Display	
21	Enter NPV from Step 20	20406.56	PV =	20,406.56
22	Enter number of compounding periods	8 N	N =	8.00
23	Enter interest rate	7 I/Y	I/Y =	7.00
24	Calculate future value	CPT FV	FV =	-35,062.27

Table 9.9 Steps for Calculating Uneven Mixed Cash Flows, Continued

2 The specific financial calculator in these examples is the Texas Instruments BA II Plus™ Professional model, but you can use other financial calculators for these types of calculations.

This is consistent with the solution we found earlier, with a difference of one cent due to rounding error.

We may also use the calculator to solve for the present value of a mixed cash stream. Earlier in this chapter, we asked how much money you would need today to fund the following five annual withdrawals, with each withdrawal made at the end of the year, beginning one year from now, and all remaining money earning 6% compounded annually:

Year	1	2	3	4	5
	$17,000	$17,000	$17,000	$17,500	$18,000

Table 9.10

We determined these withdrawals to have a total present value of $72,753.30. Here is an approach to a solution using a financial calculator. In this example, we will store all cash flows in the calculator and perform an operation on them as a whole (see Table 9.11). Because we will use the NPV function (to be explored in more detail in a later chapter), we enter our starting point as 0 because we do not withdraw any cash until one year after we begin.

Step	Description	Enter	Display	
1	Clear cash flow memory	CF 2ND [CLR WORK]	CF0	0.00
2	Enter 0 for cash flow at Time 0	ENTER ↓	CF0	0.00
3	Move to next entry	↓	C01	0.00
4	Enter first cash flow	17000 ENTER	C01 =	17000.00
5	Move to next entry	↓ ↓	C02	0.00
6	Enter second cash flow	17000 ENTER	C02 =	17000.00
7	Move to next entry	↓ ↓	C03	0.00
8	Enter third cash flow	17000 ENTER	C03 =	17000.00
9	Move to next entry	↓ ↓	C04	0.00
10	Enter fourth cash flow	17500 ENTER	C04 =	17500.00
11	Move to next entry	↓ ↓	C05	0.00
12	Enter fifth cash flow	18000 ENTER	C05 =	18000.00
13	Press NPV	NPV	I	0.00
14	Enter interest rate	6 ENTER	I =	6
15	Press down arrow to show current NPV rate	↓	NPV	0.00
16	Press CPT to find net present value	CPT	NPV =	72,753.49

Table 9.11 Steps for Calculating the Present Value of a Mixed Cash Stream

This result, you will remember, was calculated earlier in the chapter by the formula approach.

Using Microsoft Excel

Several of the exhibits already in this chapter have been prepared with Microsoft Excel. While full mastery of Excel requires extensive study and practice, enough basics can be learned in two or three hours to provide the user with the ability to quickly and conveniently solve problems, including extensive financial applications. Potential employers and internship hosts have come to expect basic Excel knowledge, something to which you

are exposed in college.

We will demonstrate the same two problems using Excel rather than a calculator:

1. The future value of a mixed cash stream for a seven-year investment
2. The present value of a mixed cash stream of five withdrawals that you wish to make from a fund to be established today

Beginning with the future value problem, we created a simple matrix to lay out the mixed stream of future cash flows, starting on the first day of each year, with all funds earning 7% throughout. Our goal is to determine how much money you will have saved at the end of this seven-year period.

Table 9.12 repeats the data from earlier in the chaper for your convenience.

Year	0	1	2	3	4	5	6	7
Cash Invested	$0.00	$2,000	$2,000	$2,000	$12,000	$2,000	$5,000	$2,000
Cumulative Cash Flows		$2,000	$4,000	$6,000	$18,000	$20,000	$25,000	$27,000
Years to Compound		7	6	5	4	3	2	1

Table 9.12

Figure 9.2 is an Excel matrix that parallels Table 9.12 above.

	A	B	C	D	E	F	G	H	I	J
1	Assumed Interest Rate		7.00%							
2	**Year**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
3	Amount Invested at Start of Year		2,000.00	2,000.00	2,000.00	12,000.00	2,000.00	5,000.00	2,000.00	**27,000.00**
4	Interest on Balance		140.00	289.80	450.09	1,321.59	1,554.10	2,012.89	2,293.79	**8,062.27**
5	Cumulative Cash Flow		2,140.00	4,429.80	6,879.89	20,201.48	23,755.58	30,768.47	35,062.27	**35,062.27**

Figure 9.2 Compound Interest Example

Download the spreadsheet file (https://openstax.org/r/chapter9-excel-exhibits) containing key Chapter 9 Excel exhibits.

We begin by entering the cash flow as shown in Figure 9.2. The assumed interest rate is 7%. The interest on the balance is calculated as the amount invested at the start of the year multiplied by the assumed interest rate. The cumulative cash flows of each year are calculated as follows: for year 1, the amount invested plus the interest on the balance; for years 2 through 7, the amount invested plus the interest on the balance plus the previous year's running balance. By adding up the amount invested and the interest on the balance, you should arrive at a total of $35,062.27.

We can use Excel formulas to solve time value of money problems. For example, if we wanted to find the present value of the amount invested at 7% over the seven-year time period, we could use the NPV function in Excel. The dialog box for this function (Rate, Value 1, Value2) is shown in Figure 9.3.

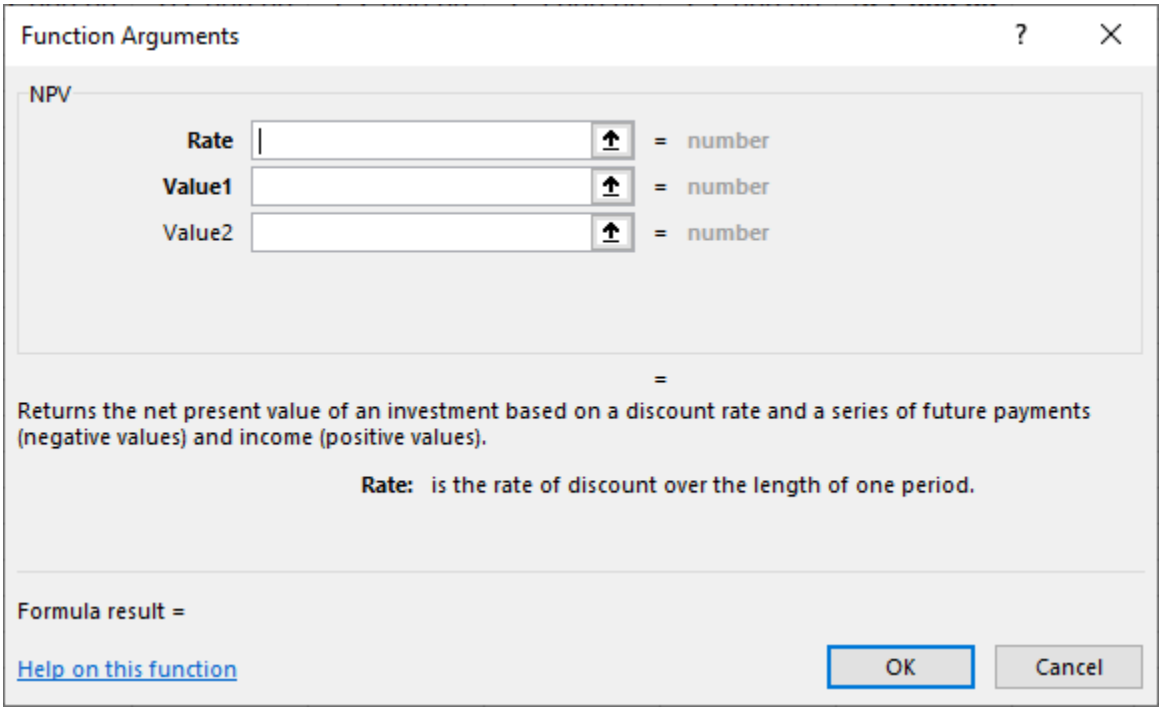


Figure 9.3 Dialog Box for NPV Function, Problem 1

The function argument Rate is the interest rate; Value1, Value2, and so on are the cash flows; and "Formula result" is the answer.

We can apply the NPV function to our problem as shown in Figure 9.4.

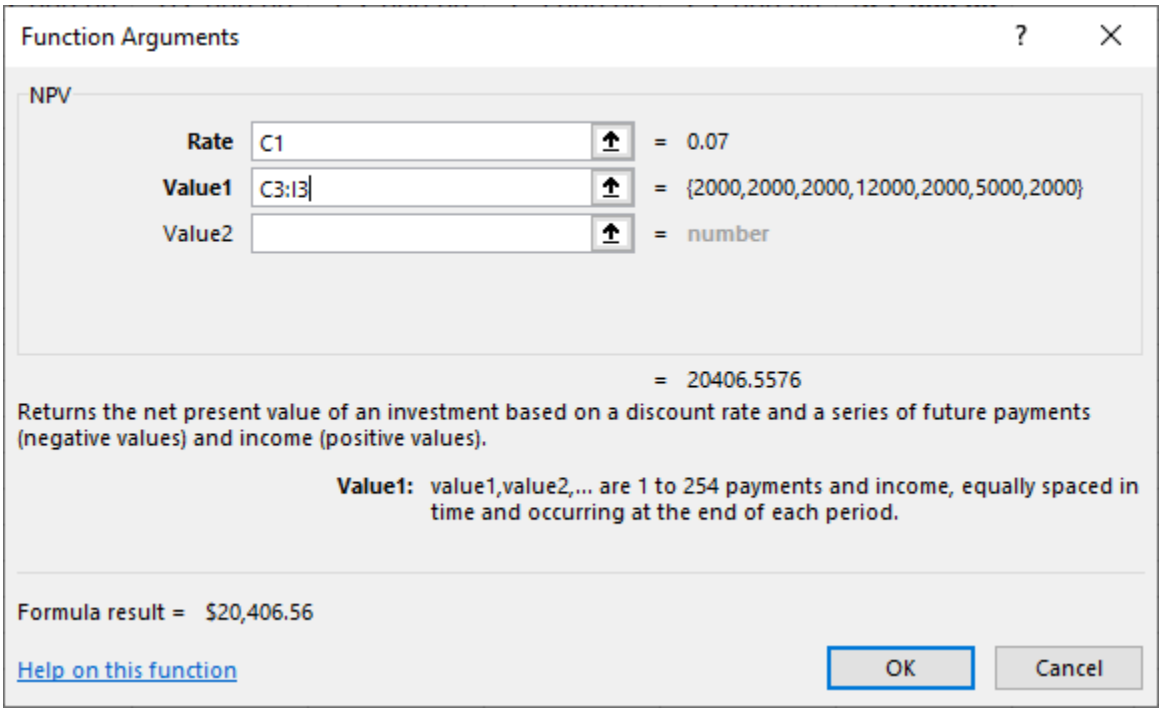


Figure 9.4 Applying the NPV Function, Problem 1

Please note that the Rate cell value (C1) and the Value1 cell range (C3:I3) will vary depending on how you set up your spreadsheet.

The non-Excel version of the problem, using an assumed interest rate of 7%, produces the same result.

Year	1	2	3	4	5	6	7
Amount Invested at Start	$2,000	$2,000	$2,000	$12,000	$2,000	$5,000	$2,000
NPV	$20,406.56						

Table 9.13

We conclude with the second problem addressed earlier in this chapter: finding the present value of an uneven stream of payments. We can use Excel's NPV function to solve this problem as well (see Figure 9.5).

Year	0	1	2	3	4	5
Expected Amount to Be Withdrawn at End of Year	$0.00	$17,000	$17,000	$17,000	$17,500	$18,000

Table 9.14

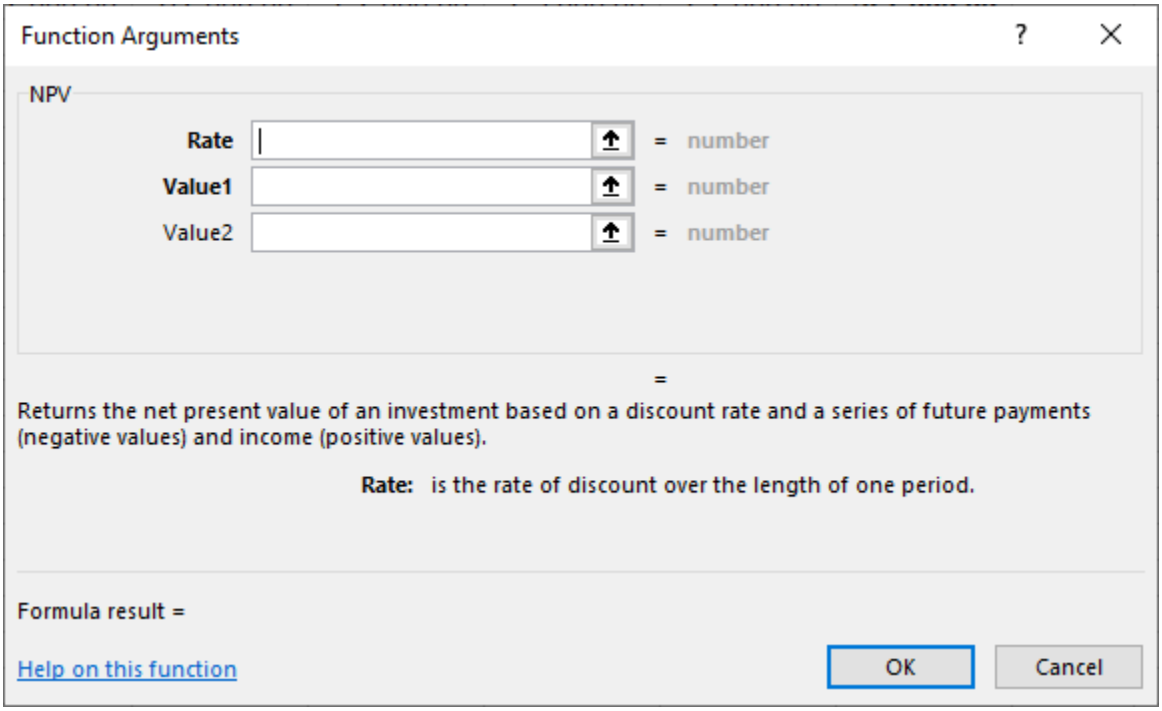


Figure 9.5 Dialog Box for NPV Function, Problem 2

Again, Rate is the interest rate; Value1, Value 2, and so on are the cash flows; and "Formula result" is the answer.

Let us apply the NPV function to our problem, as shown in Figure 9.6 and Figure 9.7.

	A	B	C	D	E	F	G
1	**Year**	**0**	**1**	**2**	**3**	**4**	**5**
2	Desired Withdrawals Each Year		17,000.00	17,000.00	17,000.00	17,500.00	18,000.00
3	Interest Rate	6.00%					
4		**NPV**	**72,753.49**				

Figure 9.6 Applying the NPV Function, Problem 2: Excel Data

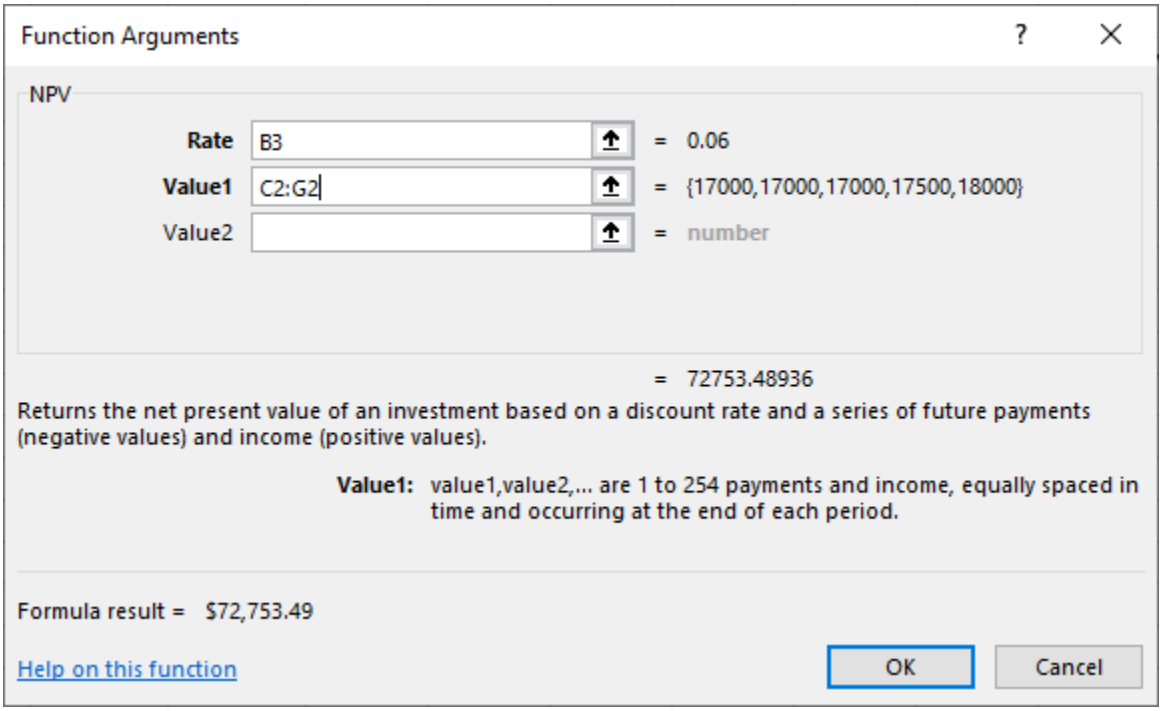


Figure 9.7 Applying the NPV Function, Problem 2: Function Argument

Please note that the Rate cell value (B3) and the Value1 cell range (C2:G2) will vary depending on how you set up your spreadsheet.

The non-Excel version of the problem produces the same result: an NPV of $72,753.49.

Year	0	1	2	3	4	5
Desired Withdrawals Each Year	$17,000	$17,000	$17,000	$17,500	$18,000	
Interest Rate	0.06					
NPV						**$72,753.49**

Table 9.15

Summary

9.1 Timing of Cash Flows

To understand the true value and strength of cash, it is necessary to consider its timing. This is relevant for investments for the future and for analyses of the value of projects that require investment today to produce expected flows of cash later. These future cash flows could involve inflows or outflows of cash in unequal amounts. This section analyzed the determination of present and future value of these uneven or mixed cash flows.

9.2 Unequal Payments Using a Financial Calculator or Microsoft Excel

This section discussed the use of two tools for managing and understanding the time value of money and its many applications when the flows of cash are unequal: the TI BA II Plus™ Professional financial calculator and the spreadsheet application Excel.

Key Terms

capital investment a major expenditure that requires a large up-front investment and is expected to generate substantial cash inflow in return

cash flow the amount of cash actually flowing into and out of a business, as opposed to *income* (which is based on accounting rules, accruals, and reports)

future value the value of an asset or holding at a point in the future based on expectations of that asset's growth at a certain rate of return

mixed stream a set of cash flows over a period of time that can vary in amount from one period to the next

present value the value of an asset in today's dollars based on future growth expectations at an assumed interest rate

CFA Institute

This chapter supports some of the Learning Outcome Statements (LOS) in this CFA® Level I Study Session (https://openstax.org/r/cfa-level1-study-session). Reference with permission of CFA Institute.

Multiple Choice

General instructions: Approximations and minor differences because of rounding are acceptable. Ignore the effect of taxes. Please assume that all percentages are annual rates and compounding occurs annually unless otherwise indicated.

1. For which of the following situations would you NOT be able to use the one-step shortcut provided by an annuity calculation?
   a. Planning an amortization table for a three-year car loan at 2%
   b. Investing $2,000 per year for 10 years at 5%
   c. Calculating the monthly payment on a 20-year home mortgage of $150,000 at 3%
   d. Finding the present value of a fund that will pay $1,000 the first year, $2,000 in the second year, and $3,500 in the third year

2. If you invest $3,000 today, $5,000 one year from now, and $3,000 two years from now, approximately how much money will you have at the end of the third year if you invest in a fund paying 7%?
   a. $11,770.24
   b. $12,609.63
   c. $13,187.79
   d. $14,274.50

3. You intend to invest four annual payments, starting immediately: $10,000 now, another $10,000 at the end of year 1, $12,000 at the end of year 2, and $15,000 at the end of year 3. How much will you have at the end of year 4 if the fund is always earning 6% compounded annually?
   a. $53,918.13
   b. $54,417.52
   c. $57,685.30
   d. $57,894.77

4. You can invest a windfall of $40,000 today at 4% compounded annually. You don't believe you can make another investment one year from now, but you believe that you can save enough that two years from now, you can deposit another $10,000 in the same investment, which will still be earning 4% compounded annually. How much will you have accumulated in this investment three years from now?
   a. $50,400.00
   b. $54,875.32
   c. $55,394.56
   d. $57,123.33

5. Donna can invest today in a fund that will guarantee her a 5% return compounded annually for five years. She can invest $1,200 today, $1,400 one year from now, $1,800 two years from now, and $2,100 three years from now. If she can make these investments as scheduled, how much will she have accumulated five years from now?
   a. $6,878.97
   b. $7,632.23
   c. $8,142.52
   d. $8,799.49

6. Jane wants to create a fund today that will provide her a 4% guaranteed compounded annual rate. She wants to withdraw $15,000 one year from now and $27,000 two years from now, after which the fund will be depleted. How much must she invest today to achieve this goal?
   a. $42,400.00
   b. $41,912.43
   c. $40,775.89
   d. $39,386.10

7. You wish to draw from a fund you're creating today with a 3% guaranteed compounded annual rate. You hope to withdraw $40,000 one year from now, $35,000 two years from now, and $25,000 three years from now, at which point the fund will be depleted. How much must you invest today to achieve this?
   a. $94,704.35
   b. $95,346.98
   c. $95,255.12
   d. $95,945.78

Review Questions

1. Refer to Question 3 above. Solve using the financial calculator, documenting your steps/keystrokes.
2. Refer to Question 7 above. Solve using the financial calculator, documenting your steps/keystrokes.
3. You agree to repay a loan over five years with the following stream of cash payments: $1,000; $1,100; $1,250; $1,280; and $1,300. If you wish to discount these payments to their present value today using 4%, why can you not use one annuity calculation, as seen in previous chapters?

Problems

1. Your aunt promises to gift you $1,500 now, $1,700 one year from now, $1,900 two years from now, and $2,500 three years from now. You will deposit all four amounts in an account that bears 3% interest compounded annually. How much will be in the account at the end of the fourth year?
2. You believe that you can set aside $1,200 each year for the next four years, starting immediately, in order to buy a small fishing boat for your retirement. Your friend Luis promises that he'll pay you back $4,900 that he owes you three years from now, so you will add that to the payment you make at the start of year 4. Then, at the start of year 5, you will increase your payment to $1,400; in year 6, to $1,500; and in year 7, to $1,600. Every payment will be deposited in a fund bearing 4% interest compounded annually. How much will you have set aside for your boat at the end of the seventh year?
3. Assume you win a lottery that will pay you $10,000 immediately, plus $20,000 one year from now, $30,000 two years from now, $40,000 three years from now, and $75,000 four years from now. You're curious about how much the lottery commission might offer you as an immediate lump sum instead. What is the minimum amount you should consider accepting today instead? Use a 5% annual discount rate. Please remember that we are ignoring taxation and other considerations and basing this only on the math itself.
4. Assume you win a lottery, and you are offered the following stream of payments by the lottery commission: $25,000 today, $32,000 one year from now, another $32,000 two years from now, and a final payment of $55,000 three years from now. You accept the offer. If you invest all of these proceeds at 6% compounded annually and extract nothing from the investment, how much will you have at the end of the fourth year?
5. You are offered a business partnership that guarantees you cash returns of $150,000 one year from now, nothing at the end of year 2, and $350,000 at the end of year 3. After year 3, the partnership will be dissolved, and there will be no more expected returns on your investment. If you analyze this plan expecting 7% compounded annually, what is this potential deal worth to you today?
6. Tony owns a small business that he is attempting to sell. A potential buyer offers him $500,000 today, plus $1,500,000 two years from now and a balance of $1,700,000 three years from now. Tony always analyzes cash flows using a rate of 4% compounded annually. To compare this to other offers, which would be paid in cash immediately, he wants to know the present value of these future cash flows. Show the calculation and solution that you would present to Tony to use in his decision.
7. Continuing Problem 6 above, assume that as Tony receives each payment from the buyer, he can immediately invest it in a fund that returns a guaranteed 3.5% compounded annually. If he accepts the terms of this offer, how much will he have accumulated in this investment four years from now?
8. You hire Susan for a three-year term. She has no retirement plan, so you agree to invest money immediately to allow her a stream of three payments beginning one year after her employment term ends. Draw a timeline! The money you invest for the full six years of this arrangement (three years of employment and three years of withdrawals) always earns 4% compounded annually. When Susan receives her third and last payment, the fund will be depleted and equal zero. The three payments she will receive are as follows:
   End of year 4: $25,000
   End of year 5: $30,000
   End of year 6: $37,000

   Therefore, your goal is to have enough money in this account at the end of Susan's three-year employment term to assure her of receiving these payments.

   How much money must you invest today to accomplish this strategy?

9. Abby owns a business that projects two years of strong cash flow, followed by a two-year hiatus while she pursues a graduate degree. She then plans to resume her business and is confident about the following two years of cash flow. At the end of each year, she will invest her profits in a fund that returns 3% compounded annually. Each investment will be made at the end of the year. Her expected investments are as follows:

   Year 1: $50,000
   Year 2: $78,000
   Year 3: 0
   Year 4: 0
   Year 5: $84,000
   Year 6: $89,000

   If Abby meets her expectations, how much money will she have in this fund at the end of year 7?

Video Activity

Calculate the Present Value for Multiple Cash Flows

Click to view content (https://openstax.org/r/calculate-the-present-value)

1. Provide a practical example from your own personal financial management of why an understanding of the present value of future cash flows would be important.
2. You expect to receive $10,800 one year from now, $17,400 two years from now, nothing at the end of the third year, and $24,000 four years from now. If you discount all of your cash flows at 7%, then with annual compounding, what are these future cash amounts worth to you in total today?

Future Value of Uneven Cash Flows

Click to view content (https://openstax.org/r/uneven-cash-flows)

3. Using Excel, determine the future value of this series of expected unequal receipts five years from now if each payment is received at the end of each year, beginning one year from now, and the interest rate is 6% compounded annually.

   End of year 1: $3,800
   End of year 2: $4,400
   End of year 3: $5,100
   End of year 4: $5,800

4. When using Excel's built-in Future Value function, why does Dr. Konners enter dollar amounts as negative numbers?

10

Bonds and Bond Valuation

Figure 10.1 Bonds are very useful investments to add to a portfolio. (credit: modification of "US Treasury Checks - 3D Illustration" by DonkeyHotey/flickr, CC BY 2.0)

Chapter Outline

Why It Matters

When an investor purchases a bond, that person is, for all intents and purposes, making a loan to the bond issuer. Bonds issues are used to raise funds and can be issued by corporations, governments, or even subagencies of governments (including local municipalities).

As with any type of loan, the borrowing party is expected to offer something to the lender in exchange for their time and trouble. In this case, the bond-issuing entity will agree not only to repay the original face value of the loan on a specific date (the maturity of the bond) but also to pay the lender interest—or, in bond terminology, coupon payments.

Coupon payments are designed to make a bond purchase more acceptable for investors by helping compensate them for the time value of money. Because investors are parting with money that they have right now in order to make the initial bond purchase but will not see repayment of principal until the maturity date of the bond, they will experience the negative impact of time value over the bond term. When a bond issuer offers periodic coupon payments, this helps offset the negative effect of the delayed receipt of the principal amount for the investor. Also, because coupon payments will be coming to the investor throughout the term of the bond, essentially in installment payments (an annuity), the time value of money plays a critical role in bond transactions and in calculating bond valuation.

Bonds, along with stocks and mutual funds, are considered to be one of the most basic financial instruments available to any investor. It is quite common for investors to round out their portfolios by purchasing bonds,

adding a degree of safety and diversity to their investment mix.

10.1 Characteristics of Bonds

Learning Outcomes

By the end of this section, you will be able to:

- List and define the basic characteristics of bonds.
- List and describe the various types of bonds available.
- Explain how a bond price is inversely related to its return (yield).

Bonds as Investments

One way to look at bond investments is to consider the fact that any investor who purchases a bond is essentially buying a future cash flow stream that the bond issuer (or borrower) promises to make as per agreement.

Because bonds provide a set amount of cash inflow to their owners, they are often called fixed-income securities. Thus, future cash flows from the bond are clearly stated per agreement and fixed when the bond sale is completed.

Bonds are a basic form of investment that typically include a straightforward financial agreement between issuer and purchaser. Nevertheless, the terminology surrounding bonds is unique and rather extensive. Much of the specialized vocabulary surrounding bonds is designed to convey the concept that a bond is similar to other financial instruments in that it is an investment that can be bought and sold. Much of this unique terminology will be covered later in this chapter, but we can set out some of the basics here with an example.

Let's say that you buy a $1,000 bond that was issued by Apple Inc. at 5% interest, paid annually, for 20 years. Here, you are the lender, and Apple Inc. is the borrower.

LINK TO LEARNING

Informational Basics on Bonds

This video (https://openstax.org/r/the-basics-of-bonds) from *MoneyWeek* editor Tim Bennett provides information about the basics of bonds—what they are, how they work, why companies and governments issue them, and why investors might buy them.

Basic Terminology

We need to know the following basic bond terms and pricing in order to apply the necessary time value of money equation to value this Apple, Inc. bond issue:

- **Par value**: A bond will always clearly state its par value, also called face amount or face value. This is equal to the principal amount that the issuer will repay at the end of the bond term or maturity date. In our example, the par value of the bond is $1,000.
- **Coupon rate**: This is the interest rate that is used to calculate periodic interest, or coupon payments, on the bond. It is important to note that coupon rates are always expressed in annual terms, even if coupon payments are scheduled for different periods of time. The most common periods for coupon payments other than annual are semiannual and quarterly. Coupon rates will typically remain unchanged for the entire life of the bond. In our example, the coupon rate is the 5% interest rate.
- **Coupon payment**: This refers to the regular interest payment on the bond. The coupon or periodic interest payment is determined by multiplying the par value of the bond by the coupon rate. It is important to note that no adjustment needs to be made to the coupon rate if the bond pays interest

annually. However, if a bond pays interest on a semiannual or quarterly basis, the coupon rate will have to be divided by 2 or 4, respectively, to convert the stated annual rate to the correct periodic rate. In our example, coupons are paid annually, so the periodic or annual interest that is paid is equal to $\$1{,}000 \times 0.05 = \$50$. You may notice that because these payments are the same amount and made at regular intervals, they constitute an annuity stream (refer to Time Value of Money II: Equal Multiple Payments.

- **Maturity date**: The maturity date is the expiration date of the bond, or the point in time when the term of a bond comes to an end. On the maturity date, the issuer will make the final interest or coupon payment on the bond and will also pay off its principal, or face value. In our example, the maturity date is at the end of the 20-year period.
- **Yield to maturity (YTM)**: The YTM is essentially the discount rate used to bring the future cash flows of a bond into present value terms. It also equals the return that the investor will receive if the bond is held to maturity. The YTM helps quantify the overall investment value of a bond. We will explore how to compute this rate later in the chapter.

Table 10.1 displays a selected listing of bonds available for purchase or sale. First, let's review the columns so you can learn how to read this table.

Issuer	Bond Type	Current Price %	Callable?	Coupon Rate %	Maturity Date	Yield%	Rating
3M Co.	Corp	105.120	Yes	2.25	9/19/2026	1.240	A+
Alteryx	Corp	98.818	No	1.00	8/1/2026	1.206	None
Anheuser-Busch Cos. LLC	Corp	125.319	Yes	5.95	1/25/2033	3.340	BBB+
City of Chicago	Muni	103.164	Yes	5.00	1/1/2033	1.030	BBB+
Coca-Cola Co.	Corp	95.206	Yes	1.00	3/15/2028	1.731	A+
DuPont De Nemours Inc.	Corp	100.815	Yes	2.17	5/1/2023	1.775	BBB+
Exxon Mobil Corp.	Corp	107.325	Yes	3.18	3/15/2024	0.483	AA-
Ford Motor Co.	Corp	114.880	No	7.13	11/15/2025	3.623	BB+
Nordstrom Inc.	Corp	112.905	No	6.95	3/15/2028	4.758	BB+
Tennessee Energy Acquisition Corp.	Muni	102.168	No	5.25	9/1/2021	0.451	BBB+

Table 10.1 Bond Information, March 2021 (source: FINRA-Markets.Morningstar.com)

- **Column 1: Issuer.** The first column shows the company, city, or state issuing the bond. This bond listing includes two municipal issuers (City of Chicago and Tennessee Energy) as well as several corporate issuers.
- **Column 2: Bond Type.** This describes the issue of the bond and indicates whether it is a corporation or a municipality.
- **Column 3: Current Price.** The third column shows the price as a percent of par value. It is the price someone is willing to pay for the bond in today's market. We quote the price in relation to $100. For example, the Nordstrom bond is selling for 112.905% of its par value, or $112.905 per $100.00 of par value. If this bond has a $1,000.00 par value, it will sell for $\$1{,}129.05$ ($\$1{,}000 \times 1.12905$). Note: Throughout this chapter, we use $1,000 as the par value of a bond because it is the most common par value for corporate bonds.
- **Column 4: Callable?** This column states whether or not the bond has a call feature (if it can be retired or ended before its normal maturity date).
- **Column 5: Coupon Rate.** The fifth column states the coupon rate, or annual interest rate, of each bond.
- **Column 6: Maturity Date.** This column shows the maturity date of the issue—the date on which the

corporation will pay the final interest installment and repay the principal.

- **Column 7: Yield.** The seventh column indicates each bond's yield to maturity—the yield or investment return that you would receive if you purchased the bond today at the price listed in column 3 and held the bond to maturity. We will use the YTM as the discount rate in the bond pricing formula.
- **Column 8: Rating.** The final column gives the bond rating, a grade that indicates credit quality. As we progress through this chapter, we will examine prices, coupon rates, yields, and bond ratings in more detail

Types of Bonds

There are three primary categories of bonds, though the specifics of these different types of bond can vary depending on their issuer, length until maturity, interest rate, and risk.

Government Bonds

The safest category of bonds are short-term **US Treasury bills (T-bills)**. These investments are considered safe because they have the full backing of the US government and the likelihood of **default** (nonpayment) is remote. However, T-bills also pay the least interest due to their safety and the economics of risk and return, which state that investors must be compensated for the assumption of risk. As risk increases, so should return on investment. Treasury notes are a form of government security that have maturities ranging from one to 10 years, while Treasury bonds are long-term investments that have maturities of 10 to 30 years from their date of issue.

Savings bonds are debt securities that investors purchase to pay for certain government programs. Essentially, the purchase of a US savings bond involves the buyer loaning money to the government with a guaranteed promise that they will earn back the face value of the bond plus a certain amount of interest in the future. Savings bonds are backed by the US government, meaning that there is virtually no possibility of the buyer losing their investment. For this reason, the return on savings bonds is relatively low compared to other forms of bonds and investments.

LINK TO LEARNING

Government Bonds

Review this introductory video (https://openstax.org/r/treasury-bonds-prices) about government bonds.

Municipal bonds ("munis") are issued by cities, states, and localities or their agencies. Munis typically will return a little more than Treasury bills while being just a bit riskier.

Corporate Bonds

Corporate bonds are issued by companies. They carry more risk than government bonds because corporations can't raise taxes to pay for their bond issues. The risk and return of a corporate bond will depend on how creditworthy the company is. The highest-paying and highest-risk corporate bonds are often referred to as non-investment grade or, more commonly, **junk bonds**.

Corporate bonds that do not make regular coupon payments to their owners are referred to as **zero-coupon bonds**. These bonds are issued at a deep discount from their par values and will repay the full par value at their maturity date. The difference between what the investor spends on them in original purchase price and the par value paid at maturity will represent the investor's total dollar value return.

Convertible bonds are similar to other types of corporate bonds but have a feature that allows for their conversion into a predetermined number of common stock shares. The conversion from the bond to stock can be done at certain times during the bond's life, usually at the discretion of the bondholder.

The Global Bond Market versus the Global Stock Market

Bonds have long been a trusted investment vehicle for many investors. Though the global fixed-income debt market remains considerably larger than the global stock market, this is not an entirely fair comparison. Bond markets include sovereign bonds, or bonds that are issues by governments, while stock markets do not. Some experts believe that a more relevant comparison is between the value of corporate bond markets only (excluding sovereign bonds) and total stock market value.

The chart in Figure 10.2 provides global market value information by category so that we may make our own conclusions about these markets.

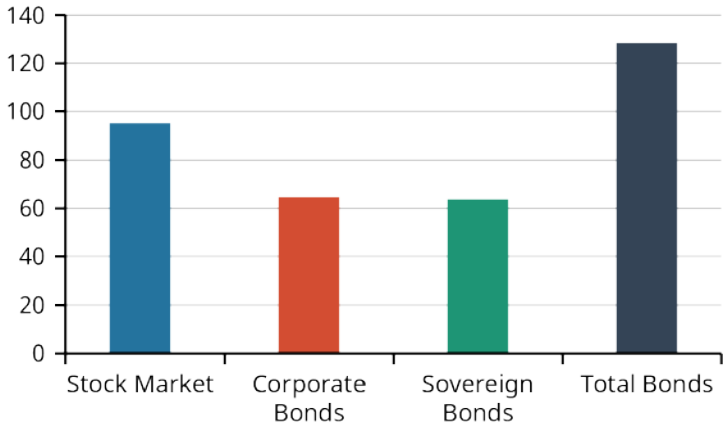


Figure 10.2 Global Bonds versus Stocks: Total Market Values ($Trillions), November 2020 (data source: Nasdaq)

While the total value of bond markets continues to exceed that of stocks, the prevailing trend over the past several years has been that stock markets are gaining in terms of total market size. The primary reason for this is that stocks have traditionally outperformed bonds in terms of return on investment over extended periods of time and are likely to continue to do so. This makes them more attractive to investors, despite the higher risk associated with stock.

The Two Sides of a Bond Investment

There are essentially two sides to a bond investment, meaning the bondholder will receive two types of cash inflow from the bond investment over its term. These are the payment of the par value at maturity (often referred to as payment of the face value of the bond at term end), and the periodic coupon payments (also called **interest income**) from the bond. These coupon payments are contractually determined and clearly indicated in the bond issue documentation received by the bondholder upon purchase.

As a result of these two types of inflow, bond valuation requires two different time value of money techniques—specifically, present value calculations—to be computed separately and then added together.

The Relationship between Bond Prices and Interest Rates

Bond price and interest rate have an inverse relationship. When interest rates fall, bond prices rise, and vice versa (see Figure 10.3). If interest rates increase, the value of bonds sold at lower interest rates will decline. Similarly, if interest rates decline, the value of fixed-rate bonds will increase. An exception to this general rule is **floating-rate bonds** (often referred to as “floaters,” floating-rate notes, or FRNs).

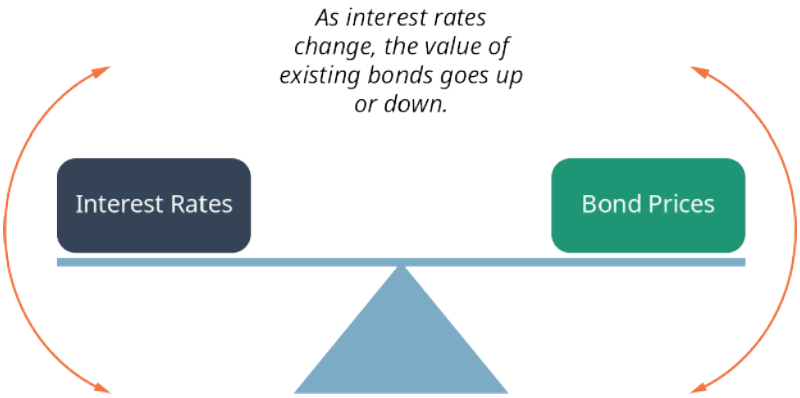


Figure 10.3 The Seesaw Effect of Bond Prices and Interest Rates

A floating-rate note is a form of debt instrument that is similar to a standard fixed-rate bond but has a variable interest rate. Rates for floating-rate bonds are typically tied to a benchmark interest rate that exists in the economy. Common benchmark rates include the **US Treasury note rate**, the **Federal Reserve funds rate (federal funds rate)**, the **London Interbank Offered Rate (LIBOR)**, and the **prime rate**.

So, investors who decide to purchase normal (fixed rate) bonds may not be thrilled to hear that the economy is signaling inflation and that interest rates are forecasted to rise. These bond investors are aware that when interest rates rise, their bond investments will lose value. This is not the case with floating- or variable-rate bonds.

Bonds with very low coupon rates are referred to as **deep discount bonds**. Of course, the bond that has the greatest discount is the zero-coupon bond, with a coupon rate of zero. The smaller the coupon rate, the greater the change in price when interest rates move.

LINK TO LEARNING

Relationship between Bond Prices and Interest Rates

Review this video (https://openstax.org/r/relationship-bond-prices) explaining the relationship between bond prices and interest rates within financial and capital markets.

10.2 Bond Valuation

Learning Outcomes

By the end of this section, you will be able to:

- Determine the value (price) of a bond.
- Understand the characteristics of and differences between discount and premium bonds.
- Draw a timeline indicating bond cash flows.
- Differentiate between fixed-rate and variable-rate bonds.
- Determine bond yields.

Pricing a Bond in Steps

Why do we want to learn how to price a bond? The answer goes to the heart of finance: the valuation of assets. We need to ascertain what a given bond is worth to a willing buyer and a willing seller. What is its value to these interested parties? Remember that a bond is a financial asset that a company sells to raise money from willing investors. Whether you are the company selling the bond or the investor buying the bond, you want to make sure that you are selling or buying at the best available price.

Let's begin our pricing examples with the 3M Company corporate bond listed in Table 10.1 above. The table

information tells us that 3M issued a series of corporate bonds that promise to pay coupons annually on September 19 and to pay back the principal, or face value, on the maturity date of September 19, 2026. While this is not specified in the table, let's say these are 15-year corporate bonds. In that case, we know that they were issued on September 20, 2011.

The 3M bonds have an annual coupon rate of 2.25%, which indicates that the annual interest payment on the bond will be the face value (assumed to be $1,000.00 multiplied by 2.25%), or $22.50. The appropriate discount rate to apply to these future payments is the yield to bond maturity, 1.24%.

Note that the 3M bond is selling at a premium (above par or face value) due to the fact that its coupon rate is greater than the YTM percentage. This means that the bond earns more value in interest than it loses due to discounting its cash flows to allow for the time value of money principle.

Finally, the table tells us some of the bond's features. For example, Standard & Poor's, an international rating agency, rates 3M Co. as A+ (high credit quality). Additionally, the bonds are designated as callable, meaning that 3M has the option of redeeming them before their maturity on September 19, 2026.

We can price a bond using the same methods from earlier chapters: an equation, a calculator, and a spreadsheet. Let's start with the equation method (see Figure 10.4).

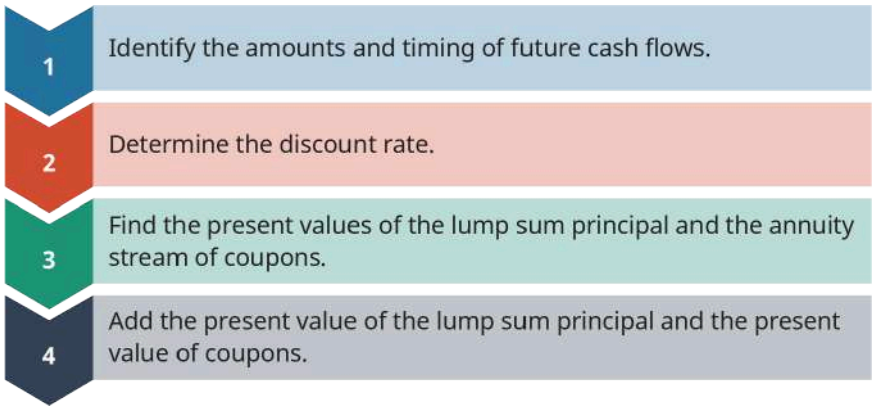


Figure 10.4 Steps in Pricing a Bond

The first step is to identify the amounts and the timing of the two types of future cash flows to be received on the bond. Any bond that pays interest or coupon payments (coupon bonds) will have two sources of future cash flow to its bondholder/investor: the periodic coupon payments, which are a form of annuity, and the final lump sum payment of the face value amount at maturity.

As discussed above, the principal or face value is paid in a one-time lump sum payment at bond maturity. In our example with 3M Co., this is the $1,000 par value of the bond that will be paid on the maturity date of September 19, 2026. Step 1 is to lay out the timing and amount of the future cash flows. The first future cash flow we need to determine is the annual interest payment. Here, it is the coupon rate of 2.25% times the par value of the bond. As mentioned above, we will use $1,000 as the par value of this bond, so the annual coupon or interest payment will equal $22.50:

$$\text{Par Value} \times \text{Annual Coupon Rate} = \text{Annual Coupon Payment}$$

$$\$1{,}000 \times 2.25\% = \$22.50$$

The next future cash flow that we need to determine is the payment of the par value or principal—in this case, the $1,000 par value of the bond—at the maturity date of September 19, 2026. We can set out the future cash flows for the bond as shown in Table 10.2:

Year (Period)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Coupons	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50	$22.50
Principal															$1,000

Table 10.2

Note that annual coupon payments are made each year on September 19, and the first annual coupon payment date is September 19, 2012. The annual payments continue for 15 years, with the last payment being made on September 19, 2026. At this point, we can apply previously learned concepts: the coupon payments constitute an annuity stream, or payments of the same amount at regular intervals.

The principal of $1,000 is also paid out at maturity. Here, we recognize another key concept: the final amount is a lump sum payment. So, we now have the promised set of future cash flows for the 3M Co. bond.

In Step 2, we will need to decide on a discount rate to use on these future bond cash payments. For now, we will jump to the answer and simply use the YTM of 1.24% from the bond data in Table 10.1. Later in the chapter, we will develop the concepts behind how an appropriate discount rate is determined.

For Step 3, we now apply two equations to the set of future cash flows from the bond. This will then provide us with the present values of these cash flows, or the expected present-day value of the bond. Because we know that the coupon payments constitute an annuity stream, we can use the equation for the present value of an annuity (discussed in Time Value of Money II: Equal Multiple Payments. To value the one-time par value payment, we use the equation for the present value of a lump sum payment. So, by combining these, we will have the present value of the coupon payment stream, or

$$\text{Coupon Payment Amount} \times \frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r} = \text{PV}_{\text{Coupon}}$$

So, for our example above, this becomes

$$\$22.50 \times \frac{\left[1 - \frac{1}{(1+0.0124)^{15}}\right]}{0.0124} = \text{PV}_{\text{Coupon}}$$
$$\$22.50 \times 13.61099 \approx \$306.25$$

Next, we need to determine the present value of the payment of the par or face value of the bond at maturity. This is calculated as follows:

$$\text{Par Value} \times \frac{1}{(1+r)^n} = \text{PV}_{\text{Principal}}$$

Inserting our values into this formula gives us

$$\$1{,}000 \times \frac{1}{(1+0.0124)^{15}} = \text{PV}_{\text{Principal}}$$
$$\$1{,}000 \times 0.831224 \approx \$831.22$$

Adding the present values of the two payment streams gives us

$$\$306.25 + \$831.22 = \$1{,}137.47$$

Our bond price is $1,137.47. This bond price represents the value of the financial asset to both a willing buyer and a willing seller.

In this example, the willing seller is 3M Company. The willing buyer is an investor who is demanding a 1.24% yield on the investment. As per Table 10.1 above, the 3M bond sold for $1,051.20 in March 2021. However, we

display the price as a percentage of the par value, so we have the displayed price as

$$\frac{\$1{,}051.20}{\$1{,}000} = 1.05120 \text{ or } 105.120\%$$

Because we round the percent of par, we do not see the cents digit in the quoted price.

Pricing a Bond Using a Financial Calculator

A financial calculator can also be used to solve common types of bond valuations. For example, what would be the current price (value) of a 4% coupon bond, paid semiannually, with a face value of $1,000 and a remaining term to maturity of 15 years, assuming a required YTM rate of 5%? The steps to solve this problem are shown in Table 10.3 below.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter future or par value as a negative amount	1000 +|− FV	FV =	-1,000.0000
3	Enter interest rate $\left(\frac{5\% \text{ annual rate}}{2} = 2.5\% \text{ semiannual rate}\right)$	2.5 I/Y	I/Y =	2.5000
4	Enter periods Enter periods $(15 \text{ years} \times 2 = 30 \text{ semiannual periods})$	30 N	N =	30.0000
5	Enter coupon payment $\frac{\$1{,}000 \times 4\%}{2} = \$20$ as a negative amount	20 +|− PMT	PMT =	-20.0000
6	Compute present value or price	CPT PV	PV =	895.3485

Table 10.3 Calculator Steps for Bond Valuations[1]

The current price is $895.35.

Time Value Connection

As we have briefly discussed, bond valuation is determined by time value of money techniques, most notably present value calculations. This makes logical sense when one considers that an investment in a bond involves a series of future cash inflows, or payments from the bond issuer to the bondholder over the term of the bond's maturity.

To determine the value of a bond today, the two-step time value of money calculation we discussed earlier must be used, and the present value of a series of coupon payments (or an annuity) must be determined. This present value amount will then be added to the present value of a single lump sum payment (the principal or face value) that will come to the bondholder at the end of the bond's term (maturity).

Fixed Income

Because standard fixed-rate bonds have their coupon payments and maturity amounts locked in, they are often referred to as fixed-income investments. This is because their values are relatively straightforward to calculate. Bonds are generally viewed as stable investments that offer income and a lower amount of volatility compared to stocks.

1 The specific financial calculator in these examples is the Texas Instruments BA II Plus™ Professional model, but you can use other financial calculators for these types of calculations.

While yields provided by corporate and government bonds such as US T-bills and municipal bonds are currently low because the **Federal Reserve System (the Fed)** has kept interest rates low for several years, investors may still consider adding bonds to their portfolios.[2] This is especially true as investors enter their retirement years and seek to generate income while avoiding the volatility of the stock market. Such investors can add a mix of individual bonds, mutual funds, or exchange-traded funds to their portfolios, thus generating potential return while keeping risks at a minimum. Fixed-income investments such as intermediate- or longer-term bond funds are still providing good yields despite the low-interest-rate state of the economy.

It is important to note, however, that even though bonds are generally thought of as safer investments, they still are subject to a number of risks. Because income from most bonds is fixed, such instruments can have their values eroded by external factors such as interest rates and inflation. We will discuss some of these risks after the next section.

Period	1	2	3	4	5	6	7	8	9	10
End Date	6/30/20	12/31/20	6/30/21	12/31/21	6/30/22	12/31/22	6/30/23	12/31/23	6/30/24	12/31/24
Coupon	$4,500	$4,500	$4,500	$4,500	$4,500	$4,500	$4,500	$4,500	$4,500	$4,500
Principal										$100,000

Table 10.4

Table 10.4 shows the cash inflow of a five-year, 9%, $100,000 corporate bond dated January 1, 2020. The bond will have coupon (interest) payment dates of June 30 and December 31 for each of the following five years. Because the bond was issued on January 1, 2020, the year 2020 is the first full year of the bond, followed by the years 2021, 2022, 2023, and 2024, with the bond maturing in December of the latter year.

Cash inflows will be (1) the coupon or interest payments of $\frac{9\% \times \$100{,}000}{2} = \$4{,}500$, paid to the bondholder every six months, and (2) the one-time principal or face-value payment of $100,000 upon maturity on December 31, 2024.

Yields and Coupon Rates

The two interest rates that we associate with a bond are often confusing to students when they first begin to work with bonds. The coupon rate is the interest rate printed on the bond; this is only used to determine the interest or coupon payments. The yield to maturity (YTM) is an interest rate that is used to discount the bond's future cash flow. The YTM is derived from the marketplace and is based on the riskiness of future cash flows.

As we have seen when pricing bonds, a bond's YTM is the rate of return that the bondholder will receive at the current price if the investor holds the bond to maturity.

Yield to Maturity

As noted above, the market sets this discount rate, or the yield to maturity. The YTM reflects the going rate in the bond market for this type of bond and the bond issuer's perceived ability to make the future payments. Hence, we base the yield on a mutually agreeable price between seller and buyer. The bond market determines the YTM and the available supply of competing financial assets. By competing against other available financial assets, the YTM reflects the risk-free rate and inflation, plus such premiums as maturity and default specific to the issued bond.

The YTM is the expected return rate on the bond held to maturity. How do we determine the bond's YTM? We can use our same three trusty methods: equations, a financial calculator, and Microsoft Excel (as shown at the end of the chapter).

2 Adam Hayes. "What Do Constantly Low Bond Yields Mean for the Stock Market?" *Investopedia*. June 15, 2021. https://www.investopedia.com/ask/answers/061715/how-can-bond-yield-influence-stock-market.asp

Determining Bond Yield Using an Equation

The solution, when solving for discount rates, requires us to revisit the bond pricing formula, which is

$$\text{Bond Price} = \text{Par Value} \times \frac{1}{(1+r)^n} + \text{Coupon Payment Amount} \times \frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r}$$

Of course, with one equation, we can solve for only one unknown, and here the variable of concern is *r*, which is the YTM. Unfortunately, it is difficult to isolate *r* on the left-hand side of the equation. Therefore, we need to use a calculator or spreadsheet to solve for the bond's YTM.

Let's take another bond, the Coca-Cola bond, from Table 10.1 above and again back up our time to March 2021. If the Coca-Cola bond has just been issued in March 2021, then it would be a seven-year, semiannual bond with a coupon rate of 1.0% and an original price of $952.06 at the time of issue (Table 10.5).

Bond Characteristic	Details
Price (% of par)	95.20
Coupon rate	1%
Maturity date	March 15, 2028
Standard & Poor's Rating	A+
Coupon payment frequency	Semiannual
First coupon date	September 1, 2021
Type	Corporate
Callable?	Yes

Table 10.5 **Overview of Coca-Cola Bond (as of March 2021)** (credit: FINRA-Markets.Morningstar)

Determining Bond Yield Using a Calculator

For the Coca-Cola bond above, what was the bond's YTM at its issue date? This is not an easy problem to solve with a mathematical formula. It is far more practical, not to mention easier, to use a financial calculator or an Excel spreadsheet to solve for bond prices, yields, and maturity periods.

We will cover Excel applications later, but we can jump into some calculator examples right now. So, to calculate the yield on the Coca-Cola bond, we'll start by entering the values we have for this bond into a calculator. The values we know are as follows:

$$\$1{,}000 \text{ par value}$$

$$14 \text{ payments } (n = 7 \text{ years} \times 2 \text{ payments per year})$$

$$\$5.00 \text{ coupon per period} \left(\frac{\$1{,}000 \times 1\%}{2} = \$5.00\right)$$

If the bond's selling price was $952.06 at issue, we have all the information we need to determine the bond's YTM at issue. Table 10.6 shows the steps for using a calculator to come to an answer.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter present value or price as a negative amount	952.06 +|− PV	PV =	-952.0600

Table 10.6 **Calculator Steps for Bond's Yield to Maturity at Issue**

Step	Description	Enter	Display	
3	Enter future or par value	1000 FV	FV =	1,000.0000
4	Enter periods $7 \text{ years} \times 2 = 14 \text{ semiannual periods}$	14 N	N =	14.0000
5	Enter coupon payment $\left(\frac{\$1{,}000 \times 1\%}{2} = \$5.00\right)$	5 PMT	PMT =	5.0000
6	Compute interest rate	CPT I/Y	I/Y =	0.8651

Table 10.6 Calculator Steps for Bond's Yield to Maturity at Issue

The calculated I/Y (interest rate or YTM) of 0.8651 is a semiannual figure because the periods and coupon payments we entered for the calculation are semiannual values. To covert the semiannual value into an annual rate, we will need multiply the calculated I/Y by 2. This gives us an amount of 1.73%.

So, the YTM of the Coca-Cola bond at issue date was 1.73%. It is important to know that unless otherwise indicated, bond yields are expressed in annual percentage terms.

We have just demonstrated how a calculator can be used to determine the YTM or interest rate of a bond. Let's look at a few more examples that cover the most common types of bond problems. These are determining a YTM, calculating a bond's current price (or value), and determining a bond's maturity period.

First, let's work through another example of calculating a YTM, but this time with a bond that has annual interest payments instead of semiannual coupons.

Let's say you are considering buying a bond, but you want to calculate the YTM to determine if it will meet your overall return requirements. Some facts you have on the bond are that it has a $1,000 face value and that it matures in 12 years. Assume that the current price of the bond is $675 and it pays coupons annually at 3.5%. See Table 10.7 for the steps to calculate the YTM.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter present value or price as a negative amount	675 +|− PV	PV =	-675.0000
3	Enter future or par value	1000 FV	FV =	1,000.0000
4	Enter periods (12 years)	12 N	N =	12.0000
5	Enter coupon payment $(\$1{,}000 \times 3.5\% = \$35.00)$	35 PMT	PMT =	35.0000
6	Compute interest rate	CPT I/Y	I/Y =	7.7589

Table 10.7 Calculator Steps for Computing Yield to Maturity

By following the steps in the table above, you will arrive at a YTM of 7.76%.

Using a calculator is fast and accurate for finding bond yields. Thus, if you know the bond's current price and all of the future cash flows, you can find the YTM, or the return rate that the bond buyer is receiving on the funds loaned to the bond issuer. As mentioned, Excel spreadsheets are as easy and accurate as a financial calculator for determining bond rates, and we will cover these later in the chapter.

Determining Bond Price or Value Using a Calculator

Let's say a friend recommends a 20-year bond that has a face value of $1,000 and a 6% annual coupon rate. If similar bonds are yielding 4% annually, what would be a fair price for this bond today? Table 10.8 shows the steps to make this determination.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter future or par value as a negative amount	1000 +|− FV	FV =	-1,000.0000
3	Enter interest rate (4% annual rate)	4 I/Y	I/Y =	4.0000
4	Enter periods (20 years)	20 N	N =	20.0000
5	Enter coupon payment ($\$1{,}000 \times 6\% = \$60$) as a negative amount	60 +|− PMT	PMT =	-60.0000
6	Compute present value or price	CPT PV	PV =	1,271.8065

Table 10.8 Calculator Steps for Computing Present Value

So, the bond should be priced today at $1,271.81.

Determining Bond Maturity Using a Calculator

Imagine you are considering investing in a bond that is selling for $820, has a face value of $1,000, and has an annual coupon rate of 3%. If the YTM is 10%, how long would it take for the bond to mature? See Table 10.9 for the steps to calculate the time to maturity.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter present value or price as a negative amount	820 +|− PV	PV =	-820.0000
3	Enter interest rate (10% annual rate)	10 I/Y	I/Y =	10.0000
4	Enter future or par value	1000 FV	FV =	1,000.0000
5	Enter coupon payment ($\$1{,}000 \times 3\% = \$30.00$)	30 PMT	PMT =	30.0000
6	Compute periods until maturity	CPT N	N =	3.1188

Table 10.9 Calculator Steps for Computing Time to Maturity

So, the bond's time to maturity would be 3.12 years.

THINK IT THROUGH

Using a Calculator

If a $1,000 face value bond is selling for $595, has 20 years until it matures, and has a YTM of 6.5%, what are the coupon rate and the periodic coupon payment of the bond? Follow the steps in Table 10.10.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter present value or price as a negative amount	595 +|− PV	PV =	-595.0000
3	Enter periods	20 N	N =	20.0000
4	Enter future or par value	1000 FV	FV =	1,000.0000

Step	Description	Enter	Display	
5	Enter interest rate or YTM	6.5 I/Y	I/Y =	6.5000
6	Compute coupon payment	CPT PMT	PMT =	28.2437

Table 10.10 Calculator Steps for Computing Coupon Payment

Solution:

The annual coupon payment amount is \$28.24. This means the coupon rate on the bond is $\frac{28.24}{1{,}000} = 2.824\%$.

The Coupon Rate

The coupon rate is the rate that we use to determine the amount of a bond's coupon payments. The issuer states the rate as an annual rate, even though payments may be made more frequently. Thus, for semiannual bonds, the most common type of corporate and government bond, the coupon payment is the par value of the bond multiplied by the annual coupon rate and then divided by the number of payments per year, 2.

We have already seen the coupon rate. The first bond we reviewed, the 3M Co. bond, was an annual coupon bond with a coupon rate of 2.25%. Using a par value of \$1,000, we determined that the annual coupon payments would be $\$1{,}000 \times 0.0225 = \$22.50$.

For the Coca-Cola bond, we note from Table 10.5 that it has a coupon rate of 1% and is paid semiannually. Using a par value of \$1,000, we can determine that the coupon payments would be $\frac{\$1{,}000 \times 1\%}{2} = \$5.00$.

The Relationship of Yield to Maturity and Coupon Rate to Bond Prices

The value or price of any bond has a direct relationship with the YTM and the coupon rate.

- When the coupon rate of a bond exceeds the YTM, the bond sells at a premium compared to its par value. That is, market demand will push the price of the bond to an amount greater that than its face or par value. We call this kind of bond a **premium bond**.
- When the coupon rate is less than the YTM, the bond sells at a discounted amount, or less than its par value. We refer to such a bond as a **discount bond**.
- When the coupon rate and YTM are identical, a bond will sell at its par value. Bonds that experience this scenario in the market are referred to as par value bonds.

The interest or coupon payments of a bond are determined by its coupon rate and are calculated by multiplying the face value of the bond by this coupon rate.

The inverse relationship of interest rates and bond prices is an important concept for investors to know. Because interest rates fluctuate and can change significantly over time, it is important to understand how these changes will impact bond values.

10.3 Using the Yield Curve

Learning Outcomes

By the end of this section, you will be able to:

- Use the yield curve to show the term structure of interest rates.
- Describe and define changes in the yield curve shape.
- Explain the importance of the yield curve shape.

Term Structure of Interest Rates

The expected yields of various bonds across different maturity periods are referred to as the *term structure of interest rates*. This is because they represent interest rates for different periods of time, maturities, or terms.

When interest rate yields are plotted against their respective maturity periods and these plotted points are connected, the resulting line is called a **yield curve**. Essentially, the yield curve is a result of this plotting process and becomes a graphical representation of the term structure of interest rates. A yield curve will always be constructed by showing the value of yields (rates) on the *y*-axis and maturities or time periods on the *x*-axis (see Figure 10.5).

To create a useful graph of the yield curve, interest rate yields should be computed for all government bonds at all remaining times to maturity. For example, the yields on all government bonds with a single year remaining until maturity should be calculated. This value is then plotted on the *y*-axis against the one-year term on the *x*-axis. Similarly, yields on government bonds with two years remaining until maturity are calculated and plotted on the *y*-axis against two years on the *x*-axis, and so on, until a point of critical mass of information is reached and the resulting graph displays useful information.

The yield curve for government bonds is also known as the *risk-free yield curve* because these securities are thought of as safe investments that are not expected to fail or default and will in all likelihood repay or otherwise meet all financial obligations made through the bond issuance.

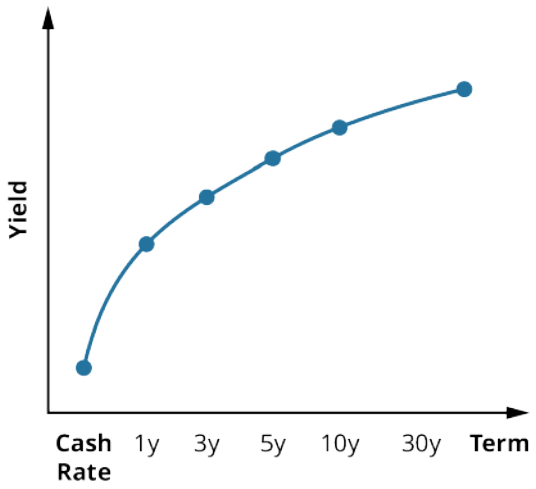


Figure 10.5 A Normal Yield Curve: Long-Term Rates Are Higher Than Short-Term Rates

A normal yield curve slopes upward, with yield increasing as the term increases. This is because yields on fixed-income investments such as bonds will rise as maturity periods increase and produce greater levels of risk.

Corporate issuers of bonds will usually offer bond issues at higher yields that the government, which is understandable because they are potentially riskier for investors. Government securities are guaranteed by governments and have little to no chance of default or nonpayment. This is not the case for corporate bonds, where there is always a chance of default, though the likelihood of this occurring will vary by individual company or issuer as well as by bond type and term. We will discuss bond default and default risk next.

LINK TO LEARNING

The Yield Curve

Review this video (https://openstax.org/r/introduction-to-the-yield-curve) that introduces the concept of the yield curve.

Different Shapes of the Yield Curve

There are two important elements to any yield curve that will define its shape: its level and its slope. The level of a yield curve directly relates to the yield rates depicted on the *y*-axis of the graph (see Figure 10.6). The slope of the yield curve indicates the difference between yields on short-term and longer-term investments. The difference in yields is primarily due to investors' expectations of the direction of interest rates in the economy and how the federal funds rate (referred to as **cash rate** in many countries) is uncertain and may differ significantly over time. As an example, yields on three-year bonds incorporate the expectations of investors on how bank rates might move over the next three years, combined with the uncertainty of those rates over the three-year period.

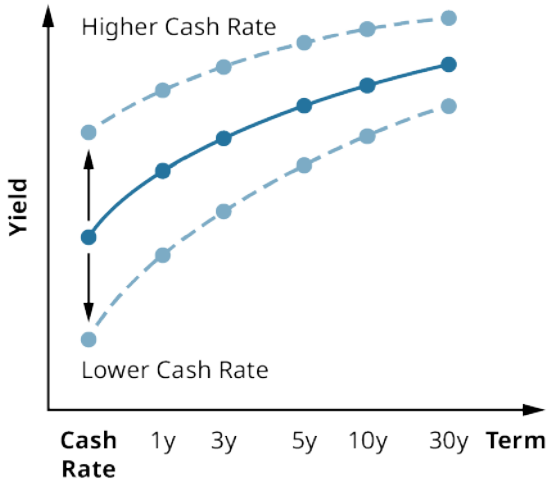


Figure 10.6 Changes in Yield Curve Depending on Cash Rate

As we briefly discussed above, a positive or normal yield curve is indicative of the investment community's requirements for higher rates of return as financial consideration for assuming the risk of entering into fixed-income investments, such as the purchase of bond issues. Typically, as a bond term increases, so will the potential interest rate risk to the bondholder. Therefore, bonds with longer terms will usually carry higher coupon rates to make returns greater for investors. Additionally, economists have come to believe that a steep positive yield curve is a sign that investors anticipate relatively high inflation in the future and thus higher interest rates accompanied by higher investment yields over shorter (inflationary) periods of time.

Normal yield curves are generally observed during periods of economic expansion, when growth and inflation are increasing. In any expansionary economy, there is a greater likelihood that future interest rates will be higher than current rates. This tends to occur because investors will anticipate the Fed or the central bank raising its short-term rates in response to higher inflation rates within the economy.

CONCEPTS IN PRACTICE

How COVID-19 Impacted the Yield Curve

Figure 10.7 shows the relatively normal-shaped yield curve effective in February 2021.

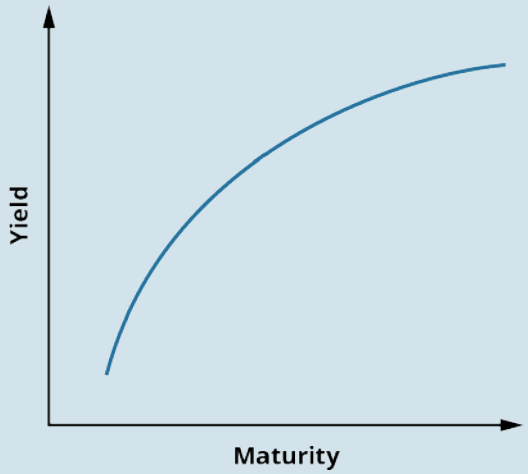


Figure 10.7 Yield Curve in February 2021

A yield curve with an inverted (downward-sloping) shape is considered unusual and will occur when long-term rates are lower than short-term rates. This causes the yield curve to assume an inverted shape with a negative slope. An inverted yield curve has historically been observed as a prelude to a general decline in economic activity and interest rate levels. In some countries, such as the United States, an inverted yield curve has been associated with upcoming recession and economic contraction.

This may occur because central banks, such as the Federal Reserve in the United States, will often attempt to stimulate a stagnant economy by reducing interest rates. Essentially, the potential actions of the central bank to improve the economy have the effect of lowering overall money rates with the economy, which is exactly what investors anticipated would happen and why the yield curve was inverted to begin with.

The yield curve was considered normal with an upward slope in August 2018, as shown in Figure 10.8, but the curve inverted in March 2019 as yields on short-term bonds exceeded those of longer-term bonds, resulting in concerns surrounding impending recession and other economic problems. This inverted shape to the yield curve continued into 2020, as evidenced in Figure 10.9.

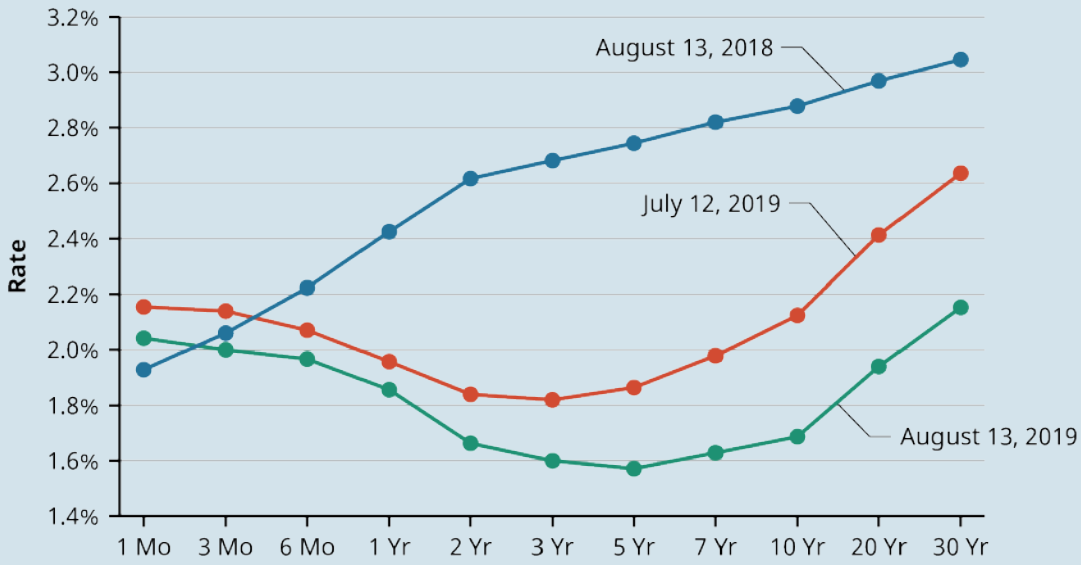


Figure 10.8 The Economy Shifts to an Inverted Yield Curve (data source: US Department of the Treasury, Resource Center, Daily Treasury Yield Curve Rates)

Yield curves constructed on different days in early 2020 appeared similar to the examples below. Again, these are obviously not normal yield structures. As a specific example, note on the February 21, 2020, curve

that rates on five-year securities are lower than those of one-year and even three-month securities.

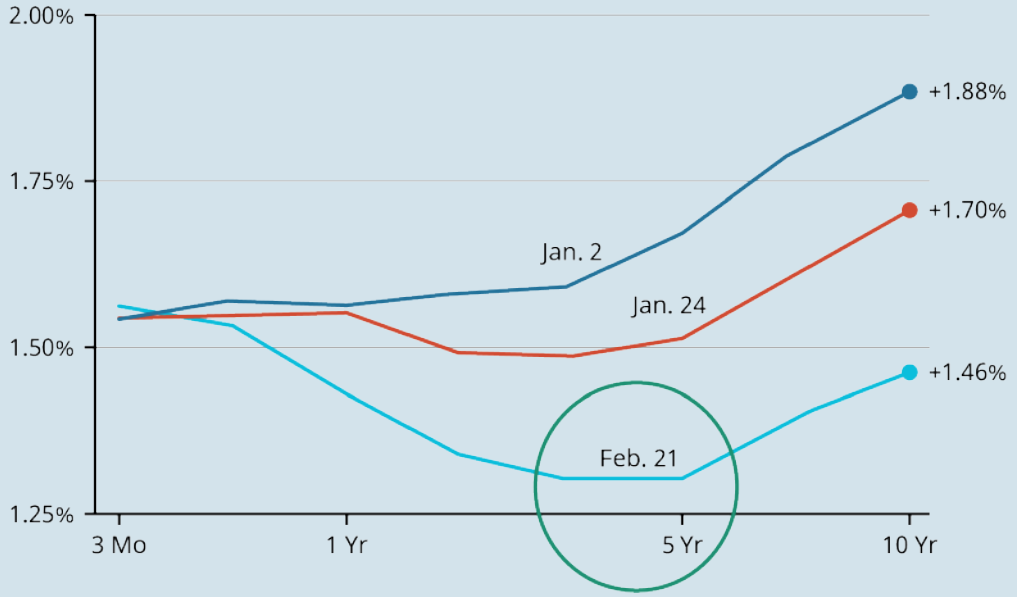


Figure 10.9 Elements of Inversion in Recent Yield Curves (data source: US Department of the Treasury, Resource Center, Daily Treasury Yield Curve Rates)

This inverted yield curve signaled the beginning of a recessionary period in the United States, which was compounded by the COVID-19 pandemic and the closing of many restaurants and businesses.

In March and April 2020, the US economy experienced a significant decline. Most economic indicators dropped so badly that the National Bureau of Economic Research's **Business Cycle Dating Committee**, the US agency that officially declares recessions, was required to intervene.[3]

The recession declaration process by the committee is completed over the course of four months, but in this instance, it only took a total of 15 weeks for the committee to make its declaration. This remains the fastest declaration by the committee on record since the founding of the National Bureau of Economic Research (NBER) in 1978.[4]

In July 2021, the committee declared that the economy had reached a trough in April 2020, marking the end of the recession of the early 2020s and making it the shortest US recession on record as well as the most quickly identified one.[5]

(sources: www.nytimes.com/2019/11/08/business/yield-curve-recession-indicator.html; www.nber.org/news/business-cycle-dating-committee-announcement-june-8-2020; fredblog.stlouisfed.org/2020/11/are-we-still-in-a-recession/)

A flat shape for the yield curve occurs when there is not a great deal of difference between short-term and long-term yields (see Figure 10.10). A flat curve is usually not long lasting and is often observed when the curve is transitioning between a normal and an inverted shape, or vice versa.

A flat yield curve has also been observed as a result of low interest rate levels or some types of unconventional monetary policy.

3 National Bureau of Economic Research. "Business Cycle Dating Committee Announcement June 8, 2020." *NBER News.*
4 Jeffrey Frankel. "The US Is Officially in Recession Thanks to the Corona Virus Crisis." *The Belfer Center for Science and International Affairs.* Harvard Kennedy School, June 16, 2020. https://www.belfercenter.org/publication/us-officially-recession-thanks-corona-virus-crisis
5 National Bureau of Economic Research. "Business Cycle Dating Committee Announcement July 19, 2021." *NBER News.* July 19, 2021. https://www.nber.org/news/business-cycle-dating-committee-announcement-july-19-2021

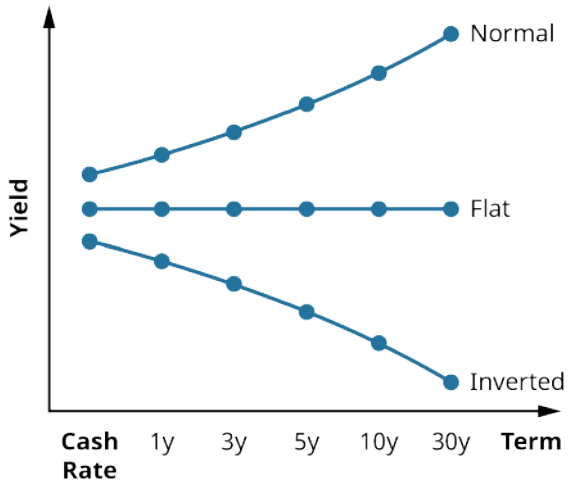


Figure 10.10 Graph Depicting Normal, Flat, and Inverted Yield Curves

Why Is the Yield Curve Important?

Market technicians, brokers, and investment analysts will study the yield curve in great detail by keeping track of its many changes and movements. This is because of the overall importance of the yield curve as an economic indicator and how it can be representative of the ideas, attitudes, and bond market expectations of individuals as well as large institutional investors that exert significant influence on investment markets and the economy as a whole.

10.4 Risks of Interest Rates and Default

Learning Outcomes

By the end of this section, you will be able to:

- Define interest rate, default, and other common forms of bond risk.
- Calculate the primary indicator of interest rate risk.
- Determine factors impacting default risk.
- Understand bond laddering as an investment strategy.
- List major rating agencies and their indications of default.
- Define and calculate the yield to maturity (YTM) on a bond.

Bond Risks

As we touched on earlier, bonds are fixed-income investments, and because of this, they are subject to a number of risks that could have negative effects on their market value. The most common and best-known risks are **interest rate risk** and **default risk**, but other risks exist that should be understood. Among these risks are the following:

- **Credit risk**. If investors believe that a bond issuer is unlikely to meet its payment commitments, they may demand a higher yield to purchase the bond issue in the first place. Due to the relative stability of governments compared to corporations, government bonds are considered to have low credit risk.
- **Liquidity risk**. If investors believe that a bond may be difficult to sell, it will likely have a higher yield. This has the effect of compensating the bondholder for the lack of liquidity (the ability to cash out of the bond). Government bonds usually have the lowest yields of all investments available and are typically among the most liquid in any country where they are traded. Government securities will only face significant liquidity risk in times of great economic distress.
- **Duration risk**. Duration risk is the risk associated with the sensitivity of a bond's price to a single 1% change in interest rates. A bond's **duration** is expressed in numerical measurements. The higher the duration number, the more sensitive a bond investment will be to changes in interest rates.

- **Call risk and reinvestment risk. Call risk** is the risk of bonds being redeemed or called by the issuing firm before their maturity dates. Corporations may elect to call a bond issue (provided the bond issue has a call feature) when interest rates drop and companies are in a position to save a great deal of money by issuing new bonds with lower coupon rates. To investors, this is a risk in and of itself, but call risk also has the effect of potentially causing reinvestment risk. **Reinvestment risk** is defined as the risk to investors when they find themselves facing unfavorable alternatives for investing the proceeds from their called bonds in new, lower-paying investments. This can potentially lead to substantial financial loss for the original bond investors.
- **Term risk**. Investors will generally demand higher returns for lending funds at fixed interest rates. This is because doing so exposes them to the risks presented by rising interest rates and the negative impact of these higher rates on their bond holdings. In a scenario of rising interest rates, investors will find that their return from lending money through a bond purchase just once, at a fixed interest rate, will be lower than the return they might have realized from making several different investments for much shorter periods of time. Term risk is usually measured by a special indicator referred to as the *term premium*.

As mentioned above, however, the most common forms of bond risk are interest rate risk and default risk.

Interest Rate Risk

As we have discussed, when interest rates rise, bond values will fall. This is the general concept behind interest rate risk. Any investor in **fixed-income securities** (such as bonds) will have to contend with interest rate risk at one time or another. Interest rate risk is also referred to as *market risk* and usually increases the longer an investor maintains a bond investment.

Default Risk

Any time a bond is purchased, the investor is taking a risk that the bond issuer may be late in making scheduled payments on a bond issue—or, in the worst case, may not be able to make payments at all. This is the underlying idea behind the concept of default risk.

Because US Treasury securities have the full backing of the government, they are generally considered free of default risk. However, most corporate bonds will face some possibility of default. Obviously, some bonds and their issuing companies are riskier in this respect than others.

To assist potential bond investors in understanding some of these risks, **bond ratings** are regularly published by a number of organizations to express their assessment of the risk quality of various bond issues. We will discuss these bond ratings and the companies that issue them next.

Bond Ratings and Rating Providers

It is important for investors to know the risks they are assuming when investing in bonds. Many investors will take advantage of information provided by bond rating services to assess the likelihood of borrowers (bond issuers) defaulting on the financial obligations of their bond issues.

To help investors evaluate the default risks of bonds, **rating agencies (bond rating services)** were established to evaluate bonds and other fixed-income investments, taking into consideration and then analyzing any information that has been published or otherwise made available to the investing public. These services then apply a rating system that has been developed for measuring the quality of bonds and assign individual grades to each bond and its issuing company.

The three largest and best-known bond rating providers are Fitch Ratings, Moody's Investors Service, and Standard & Poor's (S&P) Global Ratings. The rating system used by these services identifies the very highest-quality bonds (the least likely to default) as triple-A (AAA or Aaa), followed next in quality level by double-A bonds (AA or Aa), and so on. Any bond that is rated BBB (S&P, Fitch) / Baa (Moody's) or higher is referred to as **investment grade** and is considered strong and stable by the investment community (see Table 10.11).

It is important to note that investment-grade bonds are among the most popular due to the fact that many commercial banks, as well as several pension funds, are only allowed to trade bonds that are investment grade.

Any bond that is below investment grade, or rated lower than BBB (S&P, Fitch) or Baa (Moody's), is referred to as a high-yield bond or a junk bond. Junk bonds have had mixed levels of success for companies wishing to issue them to raise capital. In the early 1990s, the market for junk bonds collapsed, due in part to a political movement involving influential people who had been dominating corporate debt markets. This movement, combined with illegal insider trading activities conducted by investments banks, ultimately resulted in the bankruptcy of former financial giant Drexel Burnham Lambert.[6]

S&P / Fitch	Moody's	Grade	Meaning
AAA	Aaa	Investment	Risk almost zero
AA	Aa	Investment	Low risk
A	A	Investment	Risky if economy declines
BBB	Baa	Investment	Some risk; more if economy declines
BB	Ba	Speculative	Risky
B	B	Speculative	Risky; expected to get worse
CCC	Caa	Speculative	Probably bankruptcy
CC	Ca	Speculative	Probably bankruptcy
C	C	Speculative	In bankruptcy or default
D		Speculative	In bankruptcy or default

Table 10.11 Bond Ratings (sources: S&P Global Ratings; Moody's)

The market for junk bonds enjoyed a brief resurgence in popularity when the economy improved later in the 1990s. However, in 2001, the junk bond market shrank once again, resulting in 11% of US junk bond issues defaulting.

In general, it is important to understand that bond ratings are only judgments on corporations' future ability to repay debt obligation and their growth prospects. There is no fixed methodology or basis for calculating a bond rating. However, some financial analysts can get a strong indication of how a bond will be rated by examining certain financial ratios of the issuing firm, such as company debt ratio, earnings-to-interest ratio, and their return on assets.

CONCEPTS IN PRACTICE

The Collapse of Enron: Bond and Credit Ratings and How They Change

The bond rating system is not infallible. A perfect example of this is when energy giant Enron failed in 2001. After the collapse, investors correctly pointed out that a mere two months before this occurred, the company's bonds were rated as investment grade and considered relatively safe, having little risk for investors.

Background on Enron

From its formation in 1985 through the late 1990s, Enron grew to become an energy mega-conglomerate,

6 Lawrence Delevingne. "The Drexel Collapse, 25 Years Later." *CNBC*. February 13, 2015. https://www.cnbc.com/2015/02/13/the-drexel-collpase-25-years-later.html

expanding into areas such as trading of futures contracts, paper products, electricity, water, pipelines, and broadband services. Reported revenues grew at an exceptional pace, Enron's stock price continued to rise, and business was proceeding exceptionally well.

However, things were not as they appeared on the surface. Enron's financial statements were often very confusing to shareholders and analysts. Additionally, Enron's unscrupulous business practices included revenue misstatements and other questionable accounting practices to indicate favorable financial performance. On top of this, some of Enron's speculative business ventures proved to be disastrous, resulting in substantial financial losses.

Initial allegations against Enron also focused on the role of their public accountants, Arthur Andersen. Andersen was one of the Big Five accounting firms in the United States at that time and had served as Enron's auditing firm for over 16 years. According to court documents, Enron and Arthur Andersen had improperly categorized hundreds of millions of dollars of transactions as increases to the company's shareholder equity. It was also later discovered that Andersen failed to follow generally accepted accounting principles (GAAP) when considering Enron's dealings with related partnerships. As a result, Enron was able to conceal some of its losses from the investing public. After investigation by the United States Justice Department, the firm was indicted on obstruction of justice charges in March 2002. The combination of all of these irregularities and issues resulted in the December 2, 2001, bankruptcy of the corporation.[7] It was later determined that the majority of these unethical issues had been perpetuated with the indirect knowledge or even, in some cases, by the direct actions of the board of directors or senior operational management of the company.

Specifics and Enron's Bond Ratings

On October 27, 2001, the company began buying back all its commercial paper, valued at around $3.3 billion, in an effort to calm investor fears about Enron's supply of cash. On November 8, Enron announced that restatements to its financial statements for the years 1997–2000 were necessary to correct several accounting violations. However, by November 28, 2001, credit rating agencies had reduced Enron's bond rating to junk status.[8]

Other Examples of Significant Bond Ratings Downgrades

Some companies that have recently experienced downgrades (or potential downgrades) to their credit and bond ratings include Delta Airlines, Ford, Occidental Petroleum, Carnival Cruises, and T-Mobil. Some of these businesses, such as Delta and Carnival, are suffering the effects of the COVID-19 pandemic, but the hope is that they don't experience the same disastrous fate as Enron.

(sources: www.britannica.com/event/Enron-scandal; www.journalofaccountancy.com/issues/2002/apr/theriseandfallofenron.html; corporatefinanceinstitute.com/resources/knowledge/other/enron-scandal/; www.wsj.com/articles/corporate-bond-downgrades-grow-as-coronavirus-spreads-11585849497)

Concepts of Bond Returns

Bond investors earn profits through two different means: collecting interest income and generating **capital gains**. These are important concepts for any investor who considers putting their money in fixed-income securities such as bonds.

7 Douglas O. Linder. "Enron (Lay & Skilling) Trial (2006)." *Famous Trials*. Accessed November 24, 2021. https://famous-trials.com/enron

8 Paul M. Healy and Krishna G. Palepu. "The Fall of Enron." *Journal of Economic Perspectives* 17, no. 2 (Spring 2003): 3–26. https://doi.org/10.1257/089533003765888403

Collecting Interest Income

As we have covered, when investors buy bonds, they are lending money to bond issuers. The coupon rate of a bond is determined by the issuer and is generally tied to the overall level of interest rates in the economy at the time of issue as well as the maturity period of the bond and the credit rating of the issuer. The established coupon rate then governs how much periodic interest is paid to bondholders. For example, if an investor purchases a 5%, $1,000 bond with a 20-year maturity and annual coupon payments, that investor will receive 20 coupon payments equal to $\$1{,}000 \times 5\%$ or $20 \times \$50$ for a total of $1,000.

Depending on interest and inflation rates over the 20-year period, this could be a very favorable situation resulting in significant **realized return** for the investor. However, if interest rates and inflation over the investment period are at high levels, the investment is not nearly as attractive.

Generating Capital Gains

Many bonds are not held until their maturity dates. Should an investor require funds before maturity, they have the option to sell them through a broker in the secondary market. When this situation occurs, the investor may earn a capital gain or experience a capital loss, depending on whether the bond ends up being sold at a premium (above face value) or at a discount (below face value).

For example, if an investor bought a corporate bond yielding 7% and then the economy changed so that comparable bonds yielded 10%, the investor would have to lower their price on the original 7% bond until it also yielded the 10% market rate. Potential investors would not be very likely to buy the bond if they could simply buy a newly issued bond from an alternate issuer and receive a higher coupon rate.

It is equally possible that prevailing bond rates could fall and an investor could end up selling their bond at a higher price, thus earning a capital gain.

Bond Laddering as an Investment Strategy

There are several successful strategies for successful bond investments, but perhaps one of the most common yet ingenious of these strategies is called *bond laddering*. Bond ladders help investors achieve diversity in their portfolios and reduce risk while helping maintain regular cash inflows in the form of coupon payments or interest. In a bond ladder, an investor will divide their total investment dollars among various bonds that mature at regular intervals, thereby balancing risk and return. An example of a bond ladder would be to purchase 10 different bonds that have maturities of one year, two years, three years, and so on, all the way through to 10 years.

When the first bond matures, the investor will purchase a new bond that matures in 10 years to take its place in the ladder and continue the overall laddering strategy.

This strategy has several benefits. First, the shorter-term bonds in the ladder provide stability because they are less sensitive to risk than longer-term bonds. The longer-term bonds within the ladder will generally provide higher returns but with higher risk due to such factors as rising interest rates. So, by investing in bonds with different maturities and creating a bond ladder, investors can realize superior financial returns to what they would earn by only investing in short-term bonds. Also, the general level of risk from a bond ladder is reduced by the shorter-term component of the investment mix, making the bond ladder less risky than an investment that only included long-term bonds.

It is easy to see why bond laddering has become such a highly adopted bond investment strategy with investors ranging from novice to the most well-seasoned and experienced.

Interest Rate Movements and Bond Prices

We now know that when investors buy bonds, either directly or through mutual funds, they are lending money to bond-issuing firms or governments. In turn, issuers promise to pay back the principal (par or face value) when the loan is due at the bond's maturity date.

Issuers also promise to pay bondholders periodic interest or coupon payments to compensate them for the use of their money over the term of the bond. The rate at which issuers pay investors, or the bond's stated coupon rate, is typically fixed at the time of issuance.

We have also covered the concept that bond values have an inverse relationship with interest rates. As interest rates rise, bond prices fall, and when interest rates fall, bond values increase. Movement of interest rates can have a dramatic effect on a bond's value and presents the typical bondholder with a number of different financial risks that we have described in detail.

Also in this chapter, we have discussed how bond values can be estimated through the use of several different factors. Prevailing interest rates are among the most critical of these, but also important are factors such as maturity periods, the taxability of bond interest, the credit standing of bond issuers, and the likelihood of **bond call**, or issuers paying off their debt early.

When considering purchasing bonds or any such fixed-income investment, investors should remain aware that interest rates are always in a state of flux and can change at any time. The movements of bond values and bond yields will be significantly affected by these changes and can be favorable or unfavorable for any investor.

10.5 Using Spreadsheets to Solve Bond Problems

Learning Outcomes

By the end of this section, you will be able to:

- Demonstrate bond valuations using Excel.
- Demonstrate bond yield calculations using Excel.

Calculating the Price (Present Value) of a Bond

The following examples illustrate how Microsoft Excel can be used to calculate common bond problems. Please be sure to refer to the chapters on the time value of money for examples of using spreadsheets to solve present value problems, as these same concepts are also used in solving bond problems.

You can use the following steps in Excel to determine the price or present value of a coupon bond. Suppose that a bond has a par or face value of $1,000, pays coupons semiannually at a 4% annual rate, and matures in 15 years. We can assume a YTM rate of 5%.

1. First, select Formulas from the Excel upper menu bar, and from the dialog box, select PV (see Figure 10.11).

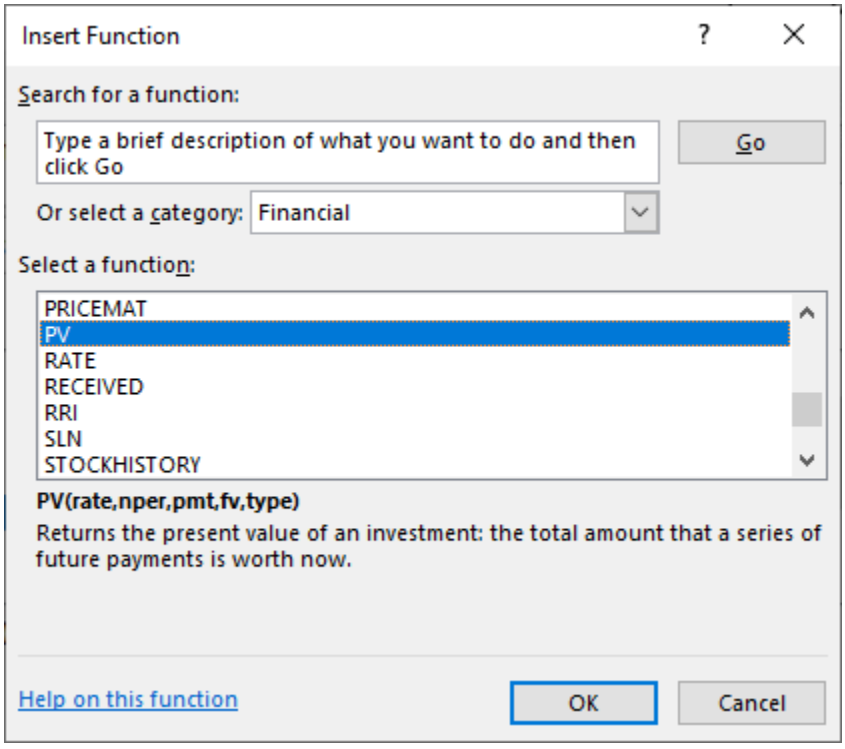


Figure 10.11 Using Excel to Enter a PV (Present Value) Function

2. When the PV function is selected, another dialog box will appear (see Figure 10.12). It is here that the function variables, or arguments, will be entered. It is preferable to use cell addresses to refer to these arguments so that the spreadsheet can be easily used again if inputs/arguments change.

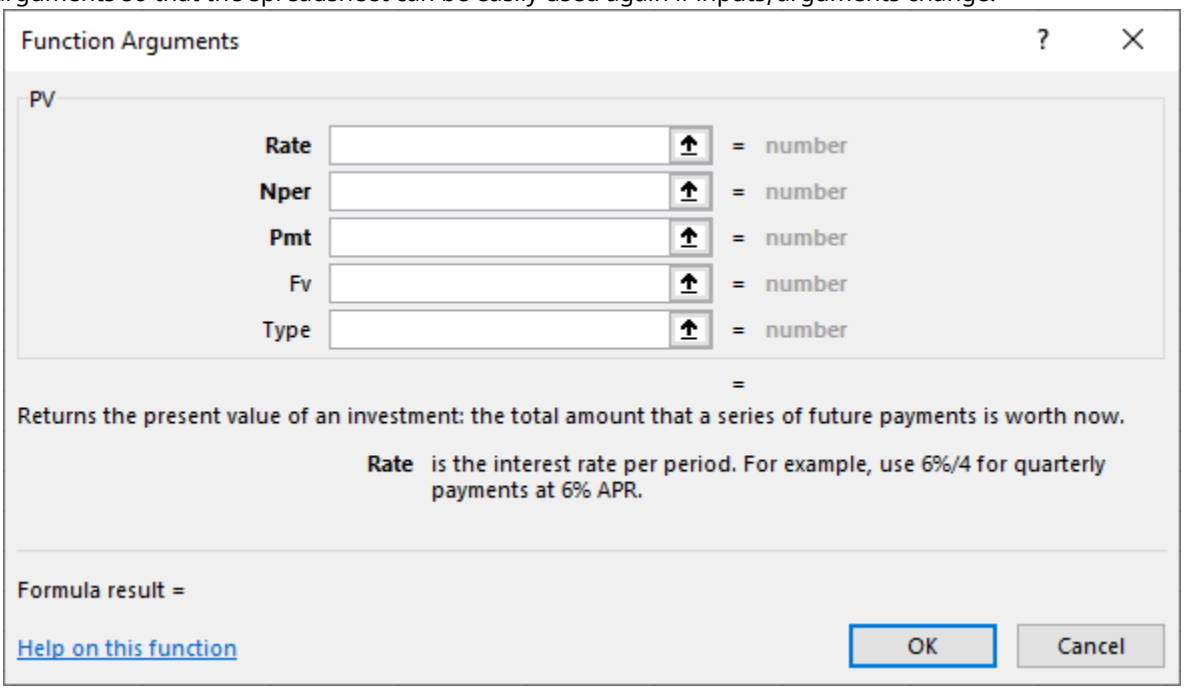


Figure 10.12 Function Arguments Dialog Box

3. Enter the function inputs or arguments (see Figure 10.13). We refer to the cell addresses as per our example spreadsheet.

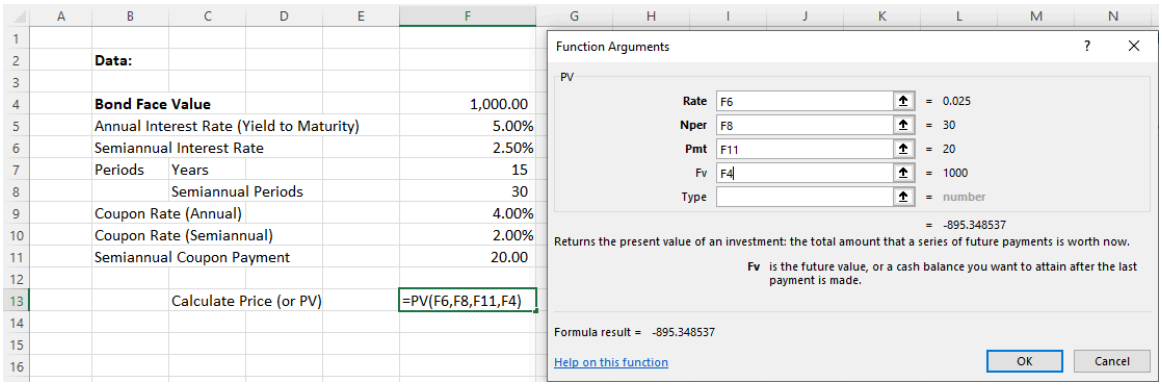


Figure 10.13 Completed Data Entry Menu

Note that the result, the price or present value, will appear in the bottom left section of the Function Arguments box once the arguments are entered. It will appear as a negative value because of the sign convention and because the bond face value in cell F4 was entered as a positive value.

Calculating the Yield to Maturity (Interest Rate) of a Bond

Use the following steps in Excel to determine the YTM (interest rate) of a bond. Assume that you want to find the YTM of a $1,000, 3.5% bond with annual coupon payments that is selling for $675.00 and will mature in 12 years.

1. First, select Formulas from the Excel upper menu bar, and from the dialog box, select Rate (see Figure 10.14).

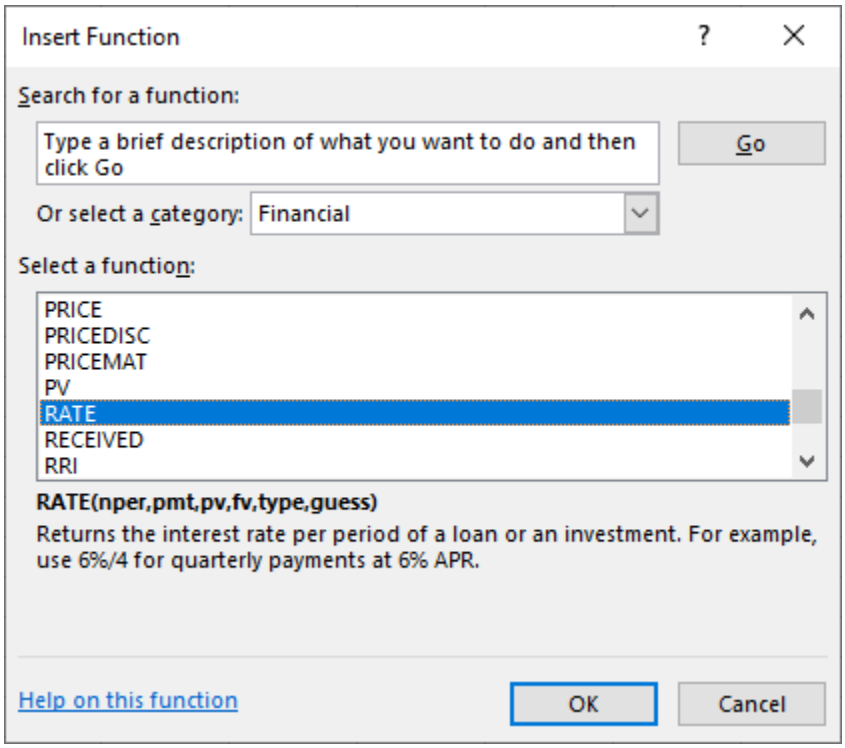


Figure 10.14 Using Excel to Enter a Rate Function

2. After the dialog box appears, enter the variables or arguments. As with our earlier example, we will use the preferred method of identifying the arguments with cell addresses (see Figure 10.15).

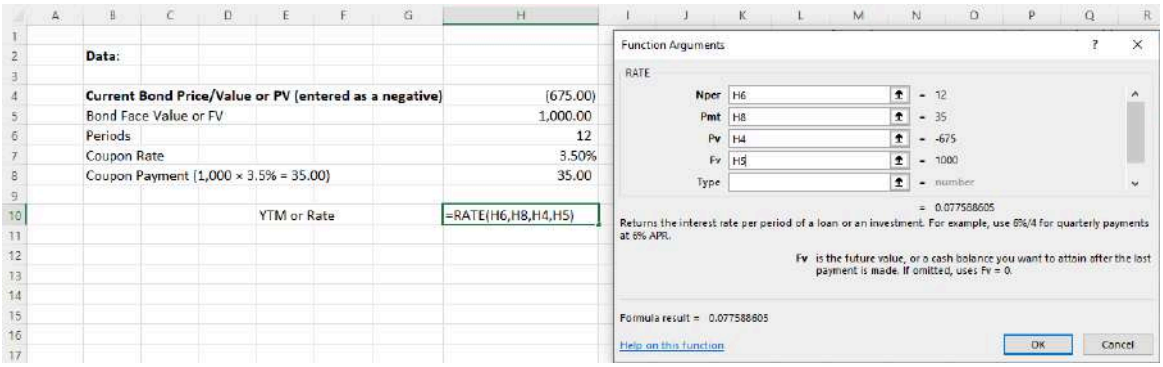


Figure 10.15 Completed Data Entry Menu

3. Again, after all arguments are entered through their correct cell references, the answer will appear in the lower left corner of the box. Once satisfied with the result, you can hit Enter to insert this final calculated value in your spreadsheet. This has been set up in this sheet in cell H10.

THINK IT THROUGH

Calculating a Coca-Cola Bond to Maturity

Earlier, we covered how a financial calculator could be used to determine the YTM of our Coca-Cola bond example. If we wanted to use an Excel spreadsheet to perform this calculation instead of a calculator, we would set up our spreadsheet as shown in the steps below. The current bond price, entered as a negative, is ($952.06). The bond face value of FV is $1,000; the time period is $7 \text{ years} \times 2$, or 14 semiannual periods; the coupon rate is $\frac{1\%}{2}$, or 0.05%; and the coupon payment is $5.00.

1. First, select Formulas from the Excel upper menu bar, and from the dialog box, select Rate (see Figure 10.16).

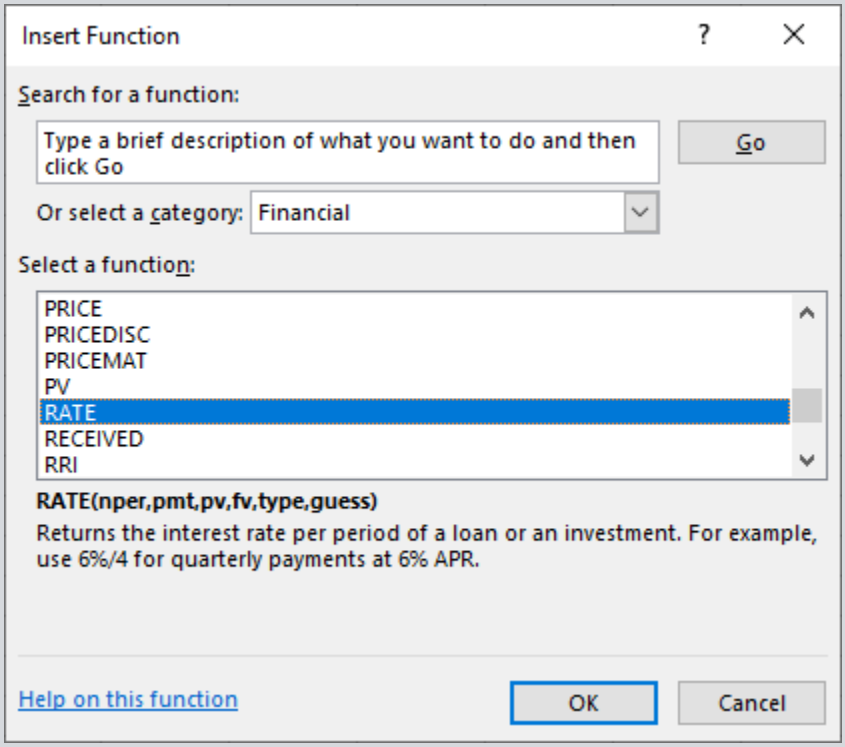


Figure 10.16 Using Excel to Enter a Rate Function

2. After the dialog box appears, enter the variables or arguments. As with our earlier examples, we will use the preferred method of identifying arguments with cell addresses (see Figure 10.17).

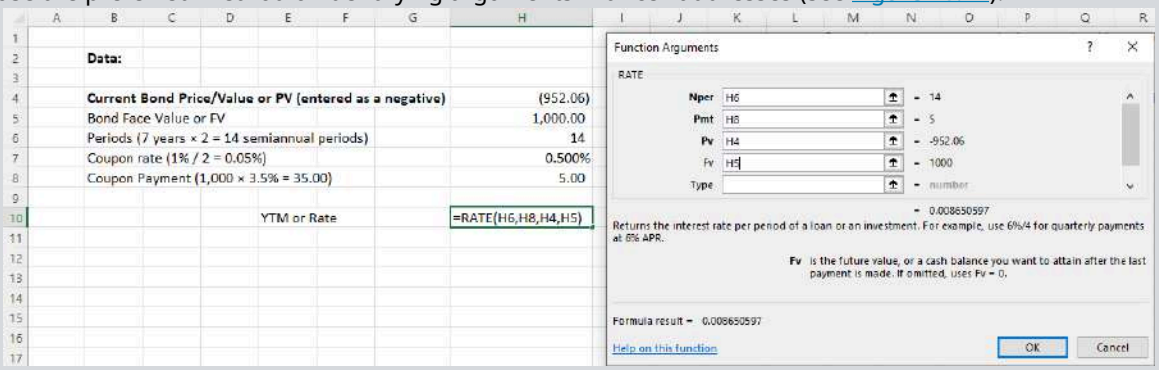


Figure 10.17 Completed Data Entry Menu

3. Again, after all arguments are entered through their correct cell references, the answer will appear in the lower left corner of the box. Once satisfied with the result, you can hit Enter to insert this final calculated value into your spreadsheet. This has been set up in this sheet in cell H10.

As noted above, remember that this is a semiannual rate because it was calculated using semiannual coupon payments and periods. To express it as an annual YTM rate, you must multiply it by 2.

Calculating the Maturity Period (Term) of a Bond

You can use the following steps in Excel to determine the maturity period or term of a bond. Assume that you are considering investing in a bond that is selling for $820.00, has a face value of $1,000, and has an annual coupon rate of 3%. If the YTM is 10%, how long will it be until the bond matures?

1. First, select Formulas from the Excel upper menu bar, and from the dialog box, select Nper (see Figure 10.18).

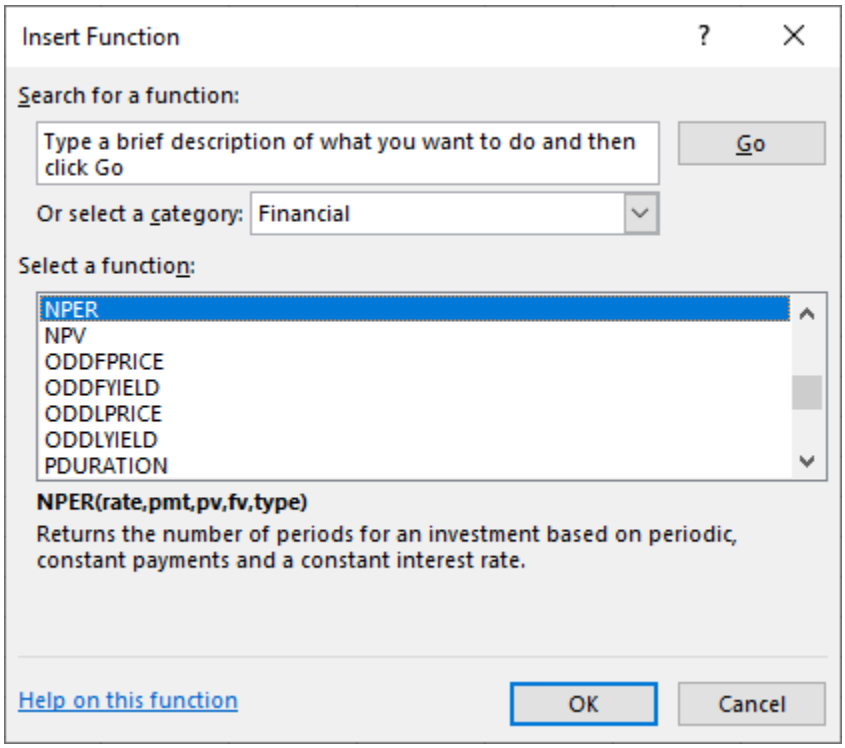


Figure 10.18 Using Excel to Calculate Bond Time to Maturity

2. When the dialog box appears, enter function arguments (see Figure 10.19). Once again, we will use the preferred method of using cell addresses as reference points.

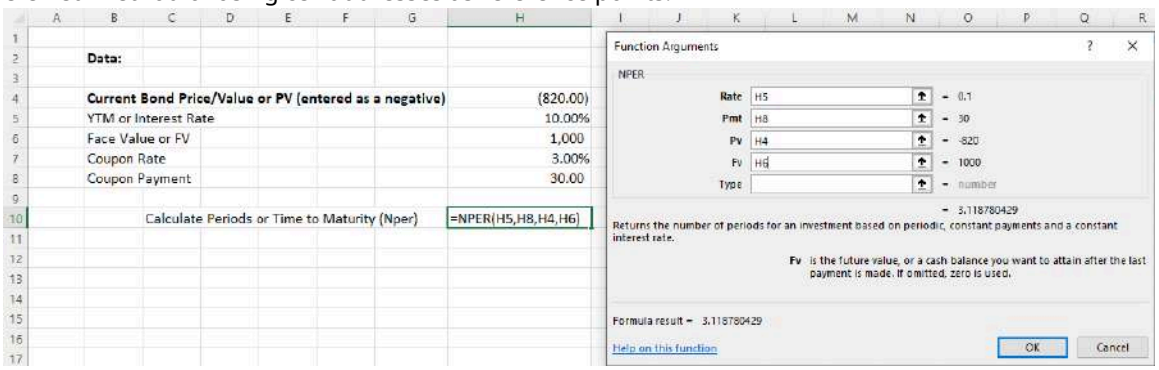


Figure 10.19 Completed Data Entry Menu

3. When arguments have all been entered, the answer will appear in the lower left of the Function Arguments box, as per the above. We arrive at a final answer of 3.12 years until this bond matures.

Calculating Coupon Rate and Interest (Coupon) Payments

Here is how you would determine the coupon or interest rate and coupon payment using Excel. Assume a $1,000 face value bond is selling for $595, has 20 years until it matures, and has a YTM of 6.5%. What are the coupon rate and the periodic coupon payment amount of the bond?

1. First, select Formulas from the Excel upper menu bar, and from the dialog box, select PMT (see Figure 10.20).

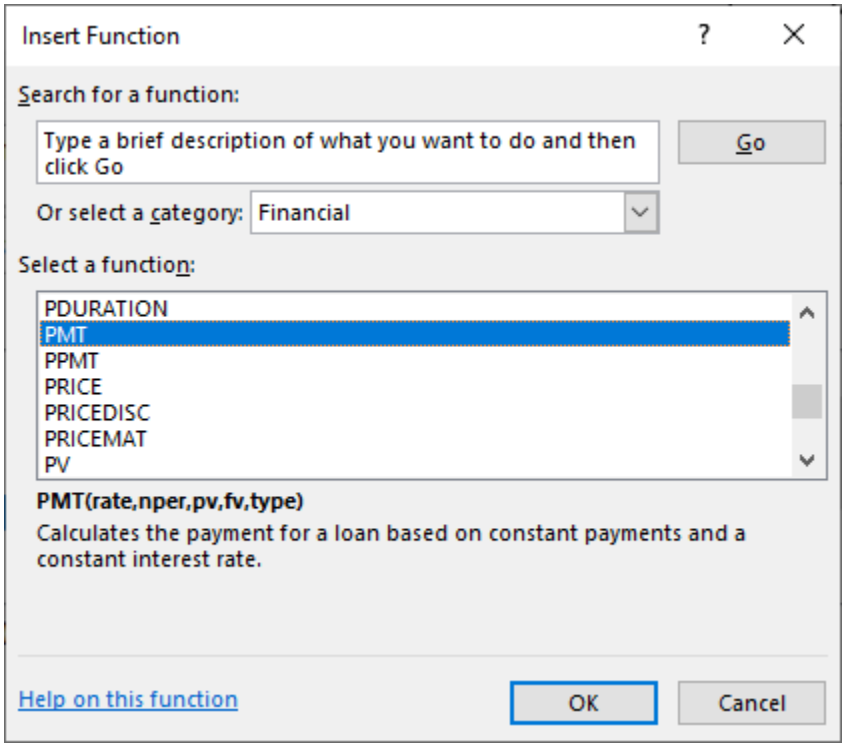


Figure 10.20 Using Excel to Enter a PMT or Payment Function

2. When the dialog box appears, enter function arguments. Once again, we will use the preferred method of using cell addresses as reference points (see Figure 10.21).

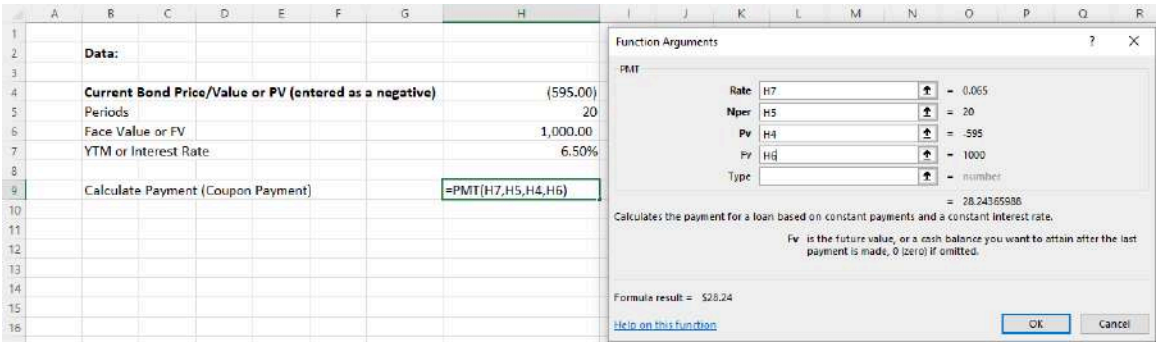


Figure 10.21 Completed Data Entry Menu

3. When arguments have all been entered, the answer will appear in the lower left of the Function Arguments box, as per the above. We arrive at a final answer of $28.24 as the coupon payment.

The coupon rate can be calculated by taking this coupon payment amount and dividing it by the face value:

$$\frac{\$28.24}{\$1{,}000} = 2.824\%$$

So, the coupon rate is 2.824%.

Summary

10.1 Characteristics of Bonds

Bonds are typically a basic form of investment that entails a straightforward financial agreement between issuer and purchaser. There are three primary categories of bonds: government bonds, corporate bonds, and convertible bonds. These different types of bond vary depending on their issuer, length until maturity, interest rate, and risk.

10.2 Bond Valuation

It is important to ascertain what a given bond is worth to a willing buyer and a willing seller. We can price a bond using an equation, a calculator, or a spreadsheet. The essential steps are (1) identify the amount and timing of the future cash flow; (2) determine the discount rate; (3) find the present values of the lump sum principal and the annuity stream of coupons; and (4) add the present value of the lump sum principal and the present value of the coupons.

10.3 Using the Yield Curve

When interest rate yields are plotted against their respective maturity periods and these plotted points are connected, the resulting line is called the yield curve. The yield curve is a graphical representation of the term structure of interest rates. A yield curve always shows the value of yields (rates) on the *y*-axis and maturities or time periods on the *x*-axis.

10.4 Risks of Interest Rates and Default

Because bonds are fixed-income investments, they are subject to a number of risks that could have negative effects on their market value. The most common and best-known risks are interest rate risk and default risk, but there are some others risks that should be understood, such as credit risk, liquidity risk, duration risk, call risk, investment risk, and term risk. To assist potential bond investors in understanding some of these risks, bond ratings have been developed and are regularly published by a number of organizations to express their assessment of the risk quality of various bond issues.

10.5 Using Spreadsheets to Solve Bond Problems

Microsoft Excel can be used to solve common bond problems. It can be used to calculate the value of a coupon bond, the yield to maturity (interest rate) of a bond, the maturity period of a bond, and the coupon rate and interest (coupon) payments of a bond.

Key Terms

bond call a feature of certain bonds or other fixed-income instruments that allows the issuer to repurchase and retire these instruments before maturity

bond price the present, discounted value of the future cash stream generated by a bond; the sum of the present values of all likely coupon payments and the present value of the par value at maturity

bond ratings grades assigned to bonds by rating services that indicate their overall credit quality

Business Cycle Dating Committee a subdivision of the National Bureau of Economic Research (NBER), the US government agency that maintains a chronology of US business cycles

call risk the risk that a bond issuer will redeem a callable bond prior to maturity

capital gains the increase in a capital asset's value that is realized when the asset is sold

cash rate the interest rate that a central bank, such as the Reserve Bank of Australia or the US Federal Reserve System, will charge commercial banks for loans; also known as the bank rate or the base interest rate

convertible bonds fixed-income corporate debt securities that yield interest payments but can be converted into a predetermined number of common stock or equity shares

coupon payment the periodic dollar value of interest that is paid to a bondholder by the bond issuer
coupon rate the amount of annual interest paid by the bond issuer; is multiplied by the face value of a bond to determine annual interest or coupon payment amounts
credit risk the risk taken by a bond investor that the bond issuer will default by failing to pay interest and repay the principal on schedule
deep discount bonds bonds that sell at significantly lower values than their par values
default when an issuer fails to make scheduled interest or principal payments on its bonds
default risk the risk taken by investors that payments will be delayed or will not occur
discount bond a bond currently trading for less than its par value in the secondary market; offers a coupon rate that is lower than prevailing interest rates
duration a measure of how much bond prices are likely to change if and when interest rates move
duration risk the risk associated with the sensitivity of a bond's price to a 1% change in interest rates
Federal Reserve funds rate (federal funds rate) the target interest rate, set by the Federal Reserve, at which commercial banks borrow and lend their excess reserves to each other
Federal Reserve System (the Fed) the central banking system of the United States, responsible for administering fiscal policy for the country
fixed-income securities investments that provide a return in the form of fixed, periodic interest payments and the eventual return of principal at maturity; the most common forms are bonds
floating-rate bonds bonds with variable interest rates that allow investors to benefit from rising interest rates
interest income annual interest amounts paid, or coupon payments made, on a bond between its issue date and the date of maturity
interest rate risk the risk of investment losses that result from changes in interest rates
investment grade describes a municipal or corporate bond with a rating that indicates it presents a low risk of default
junk bonds bonds that have been given a low credit rating, below investment grade; riskier than other bonds due to a greater chance that the issuer will default or experience a credit event
liquidity risk risk that stems from the lack of marketability of an investment, meaning that it cannot be bought or sold quickly enough to prevent or minimize a loss
London Interbank Offered Rate (LIBOR) a benchmark interest rate at which major global banks lend to one another in the international interbank market for short-term loans
maturity date the date on which a bondholder ceases to receive interest payments on a bond investment and instead is repaid its par, or face, value
municipal bonds ("munis") debt securities issued by state and local governments; can be thought of as loans that investors make to local governments to fund infrastructure
par value also called the face amount or face value; the value written on the front of the bond, which is the amount of money that bond issuers promise to be paid at maturity
premium bond a bond that is trading above its par value in the secondary market; offers a coupon rate that is higher than the current prevailing interest rates being offered
prime rate the interest rate that banks charge creditworthy corporate customers; among the most widely used benchmarks for setting home equity lines of credit and credit card rates, based on the federal funds rate set by the Federal Reserve
rating agencies (bond rating services) independent service agencies, such as Fitch, Moody's, or Standard & Poor's, that perform the isolated function of credit risk evaluation
realized return the actual return that an investor earns over a given time period through the buying and selling of a security
reinvestment risk the risk that an investor will be unable to reinvest cash flows received from an investment (e.g., coupon payments or interest) at a rate comparable to their current rate of return
savings bonds debt securities purchased by investors, as a personal investments or as gifts, that the US government issues to pay for certain public or government programs

term risk the risk of potentially earning lower returns on longer-term bond holdings compared to those potentially available when making several shorter-term investments over the same period of time

US Treasury bills (T-bills) short-term US government debt obligations backed by the Treasury Department with a maturity of one year or less

US Treasury note rate the interest rate that the US government pays to borrow money for different lengths of time; notes are issued in terms of two, three, five, seven, and 10 years

yield curve a line that plots yields (interest rates) of bonds having equal credit quality but differing maturity dates; gives an idea of future interest rate changes and economic activity

yield to maturity (YTM) the total return anticipated on a bond if the investment is held until maturity

zero-coupon bonds bonds that are issued at a deep discount from face value and offer no interest or coupon payments

CFA Institute

This chapter supports some of the Learning Outcome Statements (LOS) in this CFA® Level I Study Session (https://openstax.org/r/study-session-14). Reference with permission of CFA Institute.

Multiple Choice

1. When solving bond problems relating to a bond that pays interest on a quarterly basis, the _____ before being applied.
   a. quoted annual yield to maturity should be multiplied by 4
   b. quoted number of years until maturity should be divided by 4
   c. quoted annual coupon payments should be divided by 4
   d. stated face value should be divided by 4

2. Which of the following is NOT an adjustment that must be made when interest is paid semiannually instead of annually?
   a. Dividing the annual coupon payment by 2
   b. Dividing the annual interest rate by 2
   c. Dividing the total number of years by 2
   d. Dividing the annual yield to maturity by 2

3. Which of the following is NOT considered a factor that influences a bondholder's required rate of return?
   a. Financial risk
   b. Other investments by the bondholder
   c. Risk premium
   d. Business risk

4. How might an investment in a bond fund be affected by a decline in interest rates?
   a. The fund investment would not be affected.
   b. The fund investment would likely decrease in value.
   c. The fund investment would likely increase in value.
   d. Coupon payments from bonds in the fund would decline.

5. Interest rates and bond prices _____.
   a. are unrelated
   b. have an inverse relationship
   c. have a direct relationship
   d. are both economic factors set by central banks

6. The coupon rate of a bond is typically _____.
   a. fixed at the time of bond issuance
   b. subject to change based on the federal funds rate
   c. zero in the case of zero-coupon bonds
   d. Both A and C

7. A zero-coupon bond is a bond that _____.
   a. has no value
   b. has no periodic coupon payments
   c. has been rated below investment grade
   d. Both A and C

8. A bond that has a coupon rate less than prevailing interest rates will _____.
   a. sell at par value
   b. sell at a discount
   c. sell at a premium
   d. be overpriced

9. Determining bond prices often involves using which two TVM (time value of money) equations?
   a. The future value of a lump sum and the present value of a lump sum
   b. The present value of an annuity and the future value of a lump sum
   c. The future value of an annuity and the present value of a lump sum
   d. None of the above

10. A normal yield curve will _____.
   a. slope downward as it moves along its *x*-axis (term).
   b. slope upward as it moves along its *x*-axis (yield).
   c. fluctuate depending on the federal funds rate
   d. slope upward as it moves along its *x*-axis (term).

11. An inverted yield curve is an indication that _____.
   a. long-term yields and interest rates are higher than short-term rates
   b. the economy is in the process of a significant recovery
   c. short-term yields and interest rates are higher than long-term rates
   d. the yields to maturity on all bonds are less than market interest rates

12. Bond laddering is _____.
   a. a risky bond investment strategy that may yield tremendous returns
   b. a strategy in which bonds with several different maturity periods are added to a portfolio
   c. a strategy that involves replacing equity investments with bonds in a portfolio
   d. a bond strategy that sacrifices diversity for potential capital gains

13. A call feature _____.
   a. is desirable to an investor
   b. may cause additional risk for the bond issuer
   c. may cause additional risk for an investor
   d. Both A and B

14. The duration of a bond is _____.
   a. a measurement of the bond’s overall risk

b. synonymous with the bond's term
c. a measurement of how long an investor holds the bond
d. a measurement of the bond's sensitivity to interest rate changes

Review Questions

1. What is a junk bond?
2. Briefly describe interest income within the context of bond investments.
3. Briefly describe the two types of cash flow that a bondholder will receive from the issuer.
4. What is an inverted yield curve, and what is its significance?
5. Describe reinvestment risk for a bondholder.

Problems

1. A $1,000 Expo Corp. bond has a coupon rate of 5%, pays interest semiannually, and matures in six years. If the yield to maturity is 7%, what is the bond's value today?
2. A $1,000 Omega Corp. bond has an 8% coupon rate that is paid semiannually. The bond matures in three years. If the current price of the bond is $1,125, what is the yield to maturity?
3. You are considering buying a bond that is currently priced at $830, has a face value of $1,000, and matures in seven years. If interest is paid semiannually and the bond has a yield to maturity of 6%, what is the bond's annual coupon rate?
4. A $1,000 Noah Corp. bond has a coupon rate of 5% with semiannual payments, matures in 10 years, and has a yield to maturity of 6.5%. What is the bond's current price?
5. Chronowerx Inc. has issued a bond that has a face value of $1,000, a 3% coupon rate (with semiannual interest), and a maturity date four years from now. If the bond's current price is $895, what is its yield to maturity?
6. You are considering adding a $1,000, 25-year bond to your portfolio. It has a coupon rate of 8%, which is paid annually, and your required return is 10%. What is the current price of the investment?
7. A Cameron Corp. bond has a $1,000 par value, a 5 percent coupon rate paid semiannually, and nine years until maturity. If similar investments yield 6%, what is the current value of Cameron Corp. bonds?
8. McLaren Motors just issued a series of $1,000.00 bonds with a 10-year maturity and an 8% coupon rate, paid quarterly. If you purchase a McLaren bond at a price of $920.00, what is your required rate of return?
9. Three years ago, Petty Partners Inc. issued 15-year, $1,000 bonds that are currently priced at $911.37. If the prevailing rate of return on similar investments is 5%, what is the coupon rate on Petty Partners bonds, and what is the annual interest payment?
10. A $1,000 Riker Corp. bond has a 20-year maturity and a 6% coupon rate, with interest paid annually. If similar bonds from Riker Corp. are yielding 4%, what is the current market value of the Riker issue?
11. A $1,000 bond that matures in eight years, has quarterly coupon payments of $25, and is currently priced at $962.00 will have a yield to maturity of _____.

Video Activity

Using TI BA II+ to Price a Bond

Click to view content (https://openstax.org/r/ti-ba-pricing-bond)

1. If you are provided with annual information in a bond problem, which bond variables must be adjusted to accommodate semiannual compounding periods, and how must they be adjusted? Review the video to follow along as this information is provided.
2. Following the instructions laid out in the video, practice using a calculator to find the price of a bond under two scenarios. The first scenario should be when compounding periods are annual, or once a year, and the second scenario should be when compounding periods are semiannual, or once every six months.

   Start out this exercise with the bond factor input variables that are used in the video: par value of $1,000, annual coupon rate of 4%, time to maturity of 10 years, and yield to maturity of 8%. Calculate the current value or price of the bond under both of the different compounding period scenarios.

   Once you have arrived at the same results demonstrated in the video, practice calculating prices using different bond factor variables for inputs, changing time or years to maturity, coupon rate (and coupon payments), and yield to maturity until you are completely confident using a financial calculator to find the price or value of any bond.

Bond Pricing, Valuation, Formulas, and Functions in Excel

Click to view content (https://openstax.org/r/bond-pricing-valuation)

Review the examples included in this video, and practice setting up spreadsheets that solve for each of the five primary bond variables using the values in the videos (maturity 10 years, coupon rate 10%, coupon payment $100, yield to maturity 8%, and par value $1,000). Parallel the spreadsheets that are set up in the video, ensuring that you arrive at the same results for each bond variable amount.

3. Name and describe the five primary bond variables that can be solved using Excel spreadsheets.
4. What are the Excel functions that allow you to perform these calculations, and where in the standard Excel spreadsheet are they located?

11

Stocks and Stock Valuation

Figure 11.1 In addition to bonds, corporations will often issue common stock as a means of raising necessary capital to finance future operations. (credit: modification of "New York Stock Exchange Huge US Flag Photo i018" by Grant Wickes/flickr, CC BY 2.0)

Chapter Outline

Why It Matters

Similar to bonds, shares of **common stock** entitle investor owners to a portion of a company's future earnings and cash flows. However, stocks differ significantly from bonds in how they are issued and managed by companies, the methodology used to calculate their values in public markets, and how they can generate income and eventual value for individual investors.

With common stock, there is no specific promise of how much cash investors will receive or when they will receive it. This differs from bond investments, which are valued entirely on the basis of their guaranteed timing of future cash flows to bondholders.

This means that with stocks, there are no maturity dates, face values, or coupon payment guarantees. It also means that stocks do not promise any specified cash flows in the form of coupons or a face value payment at some point in the future. Instead, stocks (only some, not all) may pay dividends. These dividends are declared after shares of stock have been issued by a company and then purchased by the investing public. Following a dividend declaration, the designated per-share amounts are paid to shareholders of record on a specified date, also determined by a company's board.

Because stock investments carry no guarantee of payments to investors, they are far riskier than bonds and other forms of fixed-income investments.

While there are many reasons for an investor to choose to purchase common stock, three of the most

common reasons are

- to use stocks as instruments or repositories for maintaining value;
- to accumulate wealth over the term of the stock investment; and
- to earn income through capital gains and dividend payments.

As with any financial instrument, common stock purchases offer advantages and disadvantages to investors. Important advantages include the following:

- Returns through dividends and price appreciation of shares can be substantial.
- Stocks are a liquid form of investment and can be bought or sold within **secondary markets** relatively easily.
- Information about companies, markets, and important trends are widely published and readily available to the investing public.

These advantages are significant and lead many individuals to move into stock investments. Yet it is important to realize that stock has some significant disadvantages, which can include the following:

- General risk levels are greater than with bonds or other fixed-income investments.
- Timing the buy-and-sell transactions of stock can be tricky and may lead to losses or not taking full advantage of share price opportunities.
- Dividends (provided that the stock does indeed pay them, as not all do) are uncertain and subject to change based on decisions of company management.

We will discuss these topics in this chapter and cover many of the details regarding why corporations issue common stock and why investors purchase that stock.

11.1 Multiple Approaches to Stock Valuation

Learning Outcomes

By the end of this section, you will be able to:

- Define and calculate a P/E (price-to-earnings) ratio given company data.
- Determine relative under- or overvaluation indicated by a P/E (price-to-earnings) ratio.
- Define and calculate a P/B (price-to-book) ratio given company data.
- Determine relative under- or overvaluation indicated by a P/B (price-to-book) ratio.
- Define and detail alternative valuation multipliers, including P/S (price-to-sales) ratio, P/CF (price-to-cash-flow) ratio, and dividend yield.

The Price-to Earnings (P/E) Ratio

Experienced investors use a number of different methods to evaluate information on companies and their common stock before deciding on any potential purchase. One of the most popular techniques used by investors and analysts is to study a company's financial statements in order to uncover basic fundamental information on the company. This involves calculating a number of financial ratios that help identify trends, bringing elements of operational performance to light and allowing for clearer analysis and evaluation.

A well-proven analytical approach for investors to use in evaluating common stock is to review the overall market value of the company that issues a stock. One of the most consistently used calculations in this analysis, which has important applications in company and common stock evaluation, is the **price-to-earnings (P/E) ratio**.

The P/E ratio is computed using the following formula:

$$\text{P/E Ratio} = \frac{\text{Price per Share}}{\text{Earnings per Share}}$$

The P/E ratio is extremely useful to analysts in that it shows the expectations of the market. Essentially, the P/E ratio is representative of the price an investor must pay for every unit of current (or future) corporate earnings.

Bottom-line earnings are a critical factor in valuing common stock. Investors will always want to know how profitable a company is now as well as how profitable it will be in the future. When a company's bottom line remains relatively flat over a period of time, leaving **earnings per share (EPS)** relatively unchanged, the P/E ratio can be interpreted as the payback period for the original amount paid for each share of common stock.

For example, the common stock price of Cameo Corp. is currently at $24.00 a share, and its EPS for the year is $4.00. Cameo's P/E ratio is calculated as

$$\text{P/E}_{\text{Cameo}} = \frac{\$24.00}{\$4.00} = 6$$

This ratio would typically be expressed in the form $6\times$. Essentially, this means that investors are willing to pay up to six dollars for every one dollar of earnings. It can also be stated that Cameo stock is currently trading at a multiple of six.

The P/E ratio is typically expressed in two primary ways. The first is as a metric listed by most finance websites and often carries the notation *P/E (ttm)*. This refers to the Wall Street acronym for "trailing 12 months" and signals the company's operating performance over the past 12 months.

Another form of the P/E ratio is known as the forward (or leading) P/E. This uses future earnings projections rather than actual trailing amounts. The leading P/E, sometimes called the *estimated price to earnings*, is useful for comparing current earnings to future earnings and helps provide a clearer picture of what earnings may look like, assuming there are no major changes in the company's operations or accounting treatments.

Referring back to our calculation for Cameo Corp. above, because the current EPS was used in the calculation, this ratio would be classified as a trailing P/E ratio. If we had used an estimated or projected EPS as the denominator in the calculation, it would then be considered a leading P/E ratio.

Analyzing a company's P/E ratio alone or within a vacuum will actually tell an analyst very little. It is only when a company's P/E is compared to historical P/E ratios or the P/E ratios of other companies in the same industry that it becomes a useful tool for analysis. One of the most important benefits of using comparative P/E ratios is that they can standardize stocks with different prices and various earnings levels.

Generally speaking, it is very difficult to make any conclusions about a stand-alone **stock value**, such as whether a stock that has a ratio of $8\times$ is a good buy at its current price or if a stock with a P/E ratio of $35\times$ is too expensive, without performing any relevant comparisons or further analysis.

Analysts have many different ways to interpret P/E ratio data. One of the most common interpretations is that firms with high P/E ratios should be growth companies. Also, a high P/E ratio could mean that a stock's price is high relative to earnings and possibly overvalued. This could signal a possible undesired downward adjustment in market price in the future.

We can extrapolate from the argument above to put forward the idea that stocks with low P/E ratios should be stabler, more mature organizations. A low P/E might indicate that the current stock price is low relative to earnings and that there may be an opportunity to take advantage of upward price movements and potential investment gains through stock price appreciation.

While this information is often very useful for evaluating stocks and making investment decisions, caution must always be used, as a current stock price may simply be out of line with the company's earning potential, which would mean that price adjustments are likely to occur in the short term. This is why experienced analysts and investors will use multiple evaluation techniques when conducting stock analysis and evaluation and not rely solely on insights provided by a single set of facts or one form of statistical measurement.

The Price-to-Book (P/B) Ratio

Another financial ratio commonly used by investors and analysts is the **price-to-book (P/B) ratio**, also called the market-to-book (M/B) ratio. This is a financial metric used to evaluate a company's current market value relative to its book value.

The market value, or **market capitalization**, of a company is defined as the current price of all its outstanding shares of common stock. This is essentially equal to the total value of the company as perceived by the market. For all intents and purposes, the book value is representative of the residual of a company after it has liquidated all assets and paid off all of its liabilities.

Book value can be determined by performing some financial analysis on a company's balance sheet. Essentially, analysts will use the P/B ratio to compare a business's available net assets relative to the current sales price of its stock. The price-to-book-value ratio formula is

$$\text{P/B Ratio} = \frac{\text{Market Price per Share}}{\text{Book Value per Share}}$$

One of the primary uses of the P/B ratio is to understand market perceptions of a particular stock's value. It is often the metric of choice for evaluating financial services firms such as real estate firms, insurance companies, and investment trusts. The P/B ratio has a notable shortcoming, however, in that it does not evaluate companies that have a high level of **intangible assets** such as patents, trademarks, and copyrights.

Ultimately, this ratio will tell an analyst exactly how much potential investors are willing to pay for each dollar of asset value. The PB(M/B) ratio is computed by dividing the current closing price of the stock by the company's current book value per share, which is calculated by either of the following two equations:

$$\text{P/B Ratio} = \frac{\text{Market Capitalization}}{\text{Net Book Value}}$$

$$\text{P/B Ratio} = \frac{\text{Price per Share}}{\text{Net Book Value per Share}}$$

Net book value is equal to net assets, or the total assets minus the total liabilities of the company.

Analysts often consider a low P/B ratio (less than 1) to indicate that a stock is undervalued and a higher ratio (greater than 1) to mean that a stock is overvalued. A low ratio may be an indication that something is wrong with the company or that an investor may be paying too much for any residual value should the company be liquidated.

However, many market experts will argue the exact opposite of the above interpretations. Because of these discrepancies in interpretation and overall variance of opinion, the use of alternate stock valuation metrics, either in addition to or in place of the P/B ratio, is always worth exploring.

In conclusion, the P/B ratio can help a company understand if its net assets are comparable to the market price of its stock. However, as with the P/E ratio, it is always a good idea to compare P/B ratios of companies within the same industry and use them in conjunction with other metrics and analytical methodologies.

LINK TO LEARNING

Price-to-Earnings Ratio and Price-to-Book Ratio

Two videos cover the price-to-earnings (P/E) ratio (https://openstax.org/r/price-to-earnings) and the price-to-book (P/B) ratio (https://openstax.org/r/price-to-book_ratio). These are excellent introductory videos that will provide you with helpful information on what these ratios represent and how they can be used in stock valuation.

Alternative Multipliers

There are two main types of valuation metrics multiples used to value common stock. These are **equity multiples** and **enterprise value (EV) multiples**. Additionally, there are two primary methods by which to perform analysis using these multiples. These methods are **comparable company analysis (comps)** and **precedent transaction analysis (precedents)**.

Experienced financial analysts advocate the use of multiples in valuation analysis for a number of reasons, the most important being that they help generate realistic and sound judgments of enterprise values (total company values), they are relatively easy to use and interpret, and they can provide helpful information on a company's overall financial condition when used appropriately.

However, it should be noted that simplicity may have some important disadvantages. When such complex information is reduced to a single equation or final value, it can easily be misunderstood, and the influence of important factors may be masked or lost in the evaluation process.

What's more, the calculation of multiples represents a snapshot in time for a firm and cannot easily show how a company grows or progresses. Thus, these calculations are only applicable to short-term analysis, not to long-term scenarios.

Equity Multiples

Equity multiples are especially useful for investment decisions when an investor aspires to minority positions in companies. Below are some common equity multiples used in valuation analyses.

The price-to-earnings (P/E) ratio, which we discussed earlier, is probably the most common equity multiple used in stock valuation because it is relatively simple to calculate and all necessary data are easily accessible by analysts and investors. The market-to-book (M/B), or price-to-book (P/B), ratio is also useful if assets primarily drive a company's earnings. Again, it is computed as the proportion of share price to book value per share.

Dividend yield is another form of equity multiple and is primarily used when conducting comparisons between cash returns and investment types. Dividend yield is computed as the proportion of **dividend** per share to share price. The **price-to-sales (P/S) ratio** is an additional metric used for firms that are experiencing financial losses. The P/S ratio is often used for quick estimates and is computed as the proportion of share price to sales (or revenue) per share.

Another useful metric is the **price-to-cash-flow (P/CF) ratio**. The P/CF ratio is used to compare a company's market value to its operating cash flow (or the company's stock price per share to its operating cash flow per share). This measurement is suitable only in certain cases, such as when a company has substantial noncash expenses (e.g., **depreciation** or **amortization**). In some situations, companies may have positive cash flows but still show a bottom-line loss due to large noncash expenses. The P/CF ratio is helpful for arriving at a less distorted view of such a company's value.

While these various metrics are important, a financial analyst must always consider that companies often operate under their own unique sets of circumstances that ultimately will influence many of these equity multiples.

Enterprise Value (EV) Multiples

The following are some common EV multiples used in valuation analyses:

Gordon growth model is as follows

$$\frac{\text{Enterprise Value}}{\text{Revenue}}$$

$$\frac{\text{Enterprise Value}}{\text{EBIT}}$$

$$\frac{\text{Enterprise Value}}{\text{EBITDA}}$$

$$\frac{\text{Enterprise Value}}{\text{EBITDAR}}$$

EV multiples take an increasingly important role when value decisions surround recent mergers and acquisitions. **Enterprise value (EV)** is a measurement of the total value of a company. Companies often believe that EV offers a more accurate representation of a firm's total value than a basic market capitalization method. Generally, EV is perceived to offer an aggregate value of the firm as an enterprise, which is a more comprehensive measurement (see Figure 11.2).

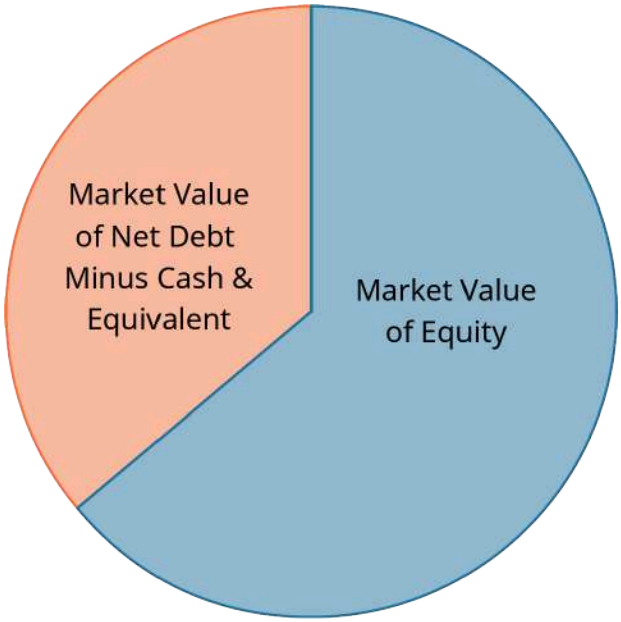


Figure 11.2 Enterprise Value: The Complete Picture

Following are two of the most common enterprise values metrics used in valuing companies and their common stock:

EV/Revenue (EV/R). Also called EV/Sales, EV/R is a valuation metric used to understand a company's total valuation compared to its annual sales levels. EV/R can help provide an analyst with an idea of exactly how much investors pay for every dollar of a company's sales revenue.

EV/R is considered a relatively crude metric but can be useful when analyzing companies that have different methods of revenue recognition. P/E ratios, for example, can be significantly affected by changes in the accounting policies of companies being evaluated. This is another reason why multiple metrics should be used in valuations.

Additionally, EV/R is a useful measure for companies that are consuming cash or experiencing financial losses. Such companies may be start-ups or emerging technology firms that have not fully matured and are still in a growth stage of development.

EV/EBIT. A firm's earnings before interest and taxes (**EBIT**) is an indicator of its profitability before the effects of interest or taxes. EBIT is also referred to as operating earnings, operating profit, and profit before interest and tax.

- **EV/EBITDA.** EV/EBITDA is a ratio that compares a company's enterprise value (EV) to its earnings before interest, taxes, depreciation, and amortization (EBITDA). EBITDA is often used by analysts as a substitute for cash flow and can be applied to capital analysis using tools such as net present value and internal rate of return. It is relatively easy to calculate, as all information required to compete the calculation is

available from any publicly traded company's financial statements. Because of this, the EV/EBITDA ratio is a commonly used metric to compare the relative values of different businesses.

- **EV/EBITDAR.** Another form of valuation based on enterprise value is EV/EBITDAR. This metric divides enterprise value by earnings before interest, tax, depreciation, amortization, and rental costs (EBITDAR). This multiple is used in businesses that have substantial rental and lease expenses, such as hotel chains and airlines. Capital investment can differ significantly for these firms, and when assets are leased, these companies tend to have artificially lower debt and operating income compared to firms that actually own their assets.

EV/Capital Employed. The EV-to-**capital employed** ratio is a measure of enterprise value compared to the level of capital used by a business. For example, a business with a large capital basis is bound to carry a large enterprise value simply due to its large capital holdings.

Final Thoughts on Valuation Ratios and Multiples

There are many equity and enterprise value multiples used in company valuation, but the discussions above cover those that are most commonly used. In any case, gaining a thorough understanding of each multiple and its related concepts can help analysts make better use of these metrics in their stock analysis and valuation efforts. Also, as discussed, it is important that analysts and technicians use multiple ratios and alternate measures for any evaluation of a company and its common stock. Not doing so will limit the ultimate interpretation of the results, can lead to incorrect conclusions, and may cause fundamental mistakes in overall investment strategy.

LINK TO LEARNING

Graph of Historical P/E for S&P 500

Look at the 90-year historical average P/E ratio of the S&P 500 (https://openstax.org/r/90-year_historical_average_P/E). When viewing this information on historical P/E ratios, think about the following:

- Note that the general trend for historical P/E ratios has been one of growth.
- Note that in May 2009, the P/E ratio reached a staggering $123.73\times$, the highest ratio in US history. This was primarily due to depressed earnings during the Great Recession.

11.2 Dividend Discount Models (DDMs)

Learning Outcomes

By the end of this section, you will be able to:

- Identify and use DDMs (dividend discount models).
- Define the constant growth DDM.
- List the assumptions and limitations of the Gordon growth model.
- Understand and be able to use the various forms of DDM.
- Explain the advantages and limitations of DDMs.

The **dividend discount model (DDM)** is a method used to value a stock based on the concept that its worth is the present value of all of its future dividends. Using the stock's price, a required rate of return, and the value of the next year's dividend, investors can determine a stock's value based on the total present value of future dividends.

This means that if an investor is buying a stock primarily based on its dividend, the DDM can be a useful tool to determine exactly how much of the stock's price is supported by future dividends. However, it is important to

understand that the DDM is not without flaws and that using it requires assumptions to be made that, in the end, may not prove to be true.

The Gordon Growth Model

The most common DDM is the **Gordon growth model**, which uses the dividend for the next year (D_1), the **required return** (r), and the estimated future dividend **growth rate** (g) to arrive at a final price or value of the stock. The formula for the Gordon growth model is as follows:

$$\text{Stock Value} = \frac{D_1}{r - g}$$

This calculation values the stock entirely on expected future dividends. You can then compare the calculated price to the actual market price in order to determine whether purchasing the stock at market will meet your requirements.

LINK TO LEARNING

Dividend Discount Model

Watch this short video on the dividend discount model (https://openstax.org/r/short_video_on_the_dividend) and how it is used it in stock valuation and analysis.

The Gordon growth model equation is presented and then applied to a sample problem to demonstrate how the DDM yields an estimated share price for the stock of any company.

Now that we have been introduced to the basic idea behind the dividend discount model, we can move on to cover other forms of DDM.

Zero Growth Dividend Discount Model

The zero growth DDM assumes that all future dividends of a stock will be fixed at essentially the same dollar value forever, or at least for as long as an individual investor holds the shares of stock. In such a case, the stock's **intrinsic value** is determined by dividing the annual dividend amount by the required rate of return:

$$\text{Stock Value} = \frac{\text{Annual Dividends}}{\text{Required Rate of Return}}$$

When examined closely, it can be seen that this is the exact same formula that is used to calculate the present value of a perpetuity, which is

$$\text{Present Value} = \frac{\text{Dividend}}{\text{Discount Rate}}$$

For the purpose of using this formula in stock valuation, we can express this as

$$\text{PV} = \frac{D}{r}$$

where PV is equal to the price or value of the stock, D represents the dividend payment, and r represents the required rate of return.

This makes perfect sense because a stock that pays the exact same dividend amount forever is no different from a **perpetuity**—a continuous, never-ending annuity—and for this reason, the same formula can be used to price preferred stock. The only factor that might alter the value of a stock based on the zero-growth model would be a change in the required rate of return due to fluctuations in perceived risk levels.

Example:

What is the intrinsic value of a stock that pays $2.00 in dividends every year if the required rate of return on similar investments in the market is 6%?

Solution:

We can apply the zero growth DDM formula to get

$$\text{Stock Value} = \frac{\$2.00}{0.6} = \$33.33$$

While this model is relatively easy to understand and to calculate, it has one significant flaw: it is highly unlikely that a firm's stock would pay the exact same dollar amount in dividends forever, or even for an extended period of time. As companies change and grow, dividend policies will change, and it naturally follows that the payout of dividends will also change. This is why it is important to become familiar with other DDMs that may be more practical in their use.

Constant Growth Dividend Discount Model

As indicated by its name, the constant growth DDM assumes that a stock's dividend payments will grow at a fixed annual percentage that will remain the same throughout the period of time they are held by an investor. While the constant growth DDM may be more realistic than the zero growth DDM in allowing for dividend growth, it assumes that dividends grow by the same specific percentage each year. This is also an unrealistic assumption that can present problems when attempting to evaluate companies such as Amazon, Facebook, Google, or other organizations that do not pay dividends. Constant growth models are most often used to value mature companies whose dividend payments have steadily increased over a significant period of time. When applied, the constant growth DDM will generate the present value of an infinite stream of dividends that are growing at a constant rate.

The constant growth DDM formula is

$$\text{Stock Value} = \frac{D_0(1 + g)}{r - g} = \frac{D_1}{r - g}$$

where D_0 is the value of the dividend received this year, D_1 is the value of the dividend to be received next year, g is the growth rate of the dividend, and r is the required rate of return.

As can be seen above, after simplification, the constant growth DDM formula becomes the Gordon growth model formula and works in the same way. Let's look at some examples.

THINK IT THROUGH

Constant Growth DDM: Example 1

If a stock is paying a dividend of $5.00 this year and the dividend has been steadily growing at 4% annually, what is the intrinsic value of the stock, assuming an investor's required rate of return of 8%?

Solution:

Apply the constant growth DDM formula:

$$\text{Stock Value} = \frac{D_0\ (1 + g)}{r - g} = \frac{\$5.00(1 + 0.04)}{0.08 - 0.04}$$

Simplify to the Gordon growth model:

$$D_1 = D_0(1 + g) = \$5.00 \times 1.04 = \$5.20$$

$$\text{Stock Value} = \frac{D_1}{r - g} = \frac{\$5.20}{0.08 - 0.04} = \$130.00$$

THINK IT THROUGH

Constant Growth DDM: Example 2

If a stock is selling at $250 with a current dividend of $10, what would be the dividend growth rate of this stock, assuming a required rate of return of 12%?

Solution:

Apply the constant growth DDM formula:

$$\$250 = \frac{\$10 \times (1 + g)}{0.12 - g}$$

Simplify and continue the calculation:

$$\begin{aligned} 30 - 250g &= 10 + 10g \\ 0 &= 10 + 260g \\ 20 &= 260g \\ 7.69\% &= g \end{aligned}$$

So, the growth rate is 7.69%.

LINK TO LEARNING

Dividend Discount Model: A Complete Animated Guide

In this video for the Investing for Beginners course and podcast, Andrew Sather introduces the DDM, (https://openstax.org/r/Andrew_Sather_introduces_the_DDM) demonstrating both the constant growth DDM (Gordon growth model) and the two-stage DDM.

Variable or Nonconstant Growth Dividend Discount Model

Many experienced analysts prefer to use the variable (nonconstant) growth DDM because it is a much closer approximation of businesses' actual dividend payment policies, making it much closer to reality than other forms of DDM. The variable growth model is based on the real-life assumption that a company and its stock value will progress through different stages of growth.

The variable growth model is estimated by extending the constant growth model to include a separate calculation for each growth period. Determine present values for each of these periods, and then add them all together to arrive at the intrinsic value of the stock. The variable growth model is more involved than other DDM methods, but it is not overly complex and will often provide a more realistic and accurate picture of a stock's true value.

As an example of the variable growth model, let's say that Maddox Inc. paid $2.00 per share in common stock dividends last year. The company's policy is to increase its dividends at a rate of 5% for four years, and then the growth rate will change to 3% per year from the fifth year forward. What is the present value of the stock if the required rate of return is 8%? The calculation is shown in Table 11.1.

Year	Growth %	Dividend ($)	Value after Year 4 ($)	PV Discount Factor at 8%	Present Value of Dividend ($)
0	5%	2.00			
1	5%	2.10		1.0800	1.9444
2	5%	2.21		1.1664	1.8904
3	5%	2.32		1.2597	1.8379
4	5%	2.43		1.3605	1.7869
5	3%	2.50	50.07886	1.4693	35.7870
					Total: $43.2466

Table 11.1 Value of Stock with 8% Required Rate of Return

Note:

$$\text{Value after Year 4} = \frac{2.4310125 \times 1.03}{0.08 - 0.03} = 50.078858$$

The value of Maddox stock in this example would be $43.25 per share.

Two-Stage Dividend Discount Model

The two-stage DDM is a methodology used to value a dividend-paying stock and is based on the assumption of two primary stages of dividend growth: an initial period of higher growth and a subsequent period of lower, more stable growth.

The two-stage DDM is often used with mature companies that have an established track record of making residual cash dividend payments while experiencing moderate rates of growth. Many analysts like to use the two-stage model because it is reasonably grounded in reality. For example, it is probably a more reasonable assumption that a firm that had an initial growth rate of 10% might see its growth drop to a more modest level of, say, 5% as the company becomes more established and mature, rather than assuming that the firm will maintain the initial growth rate of 10%. Experts tend to agree that firms that have higher payout ratios of dividends may be well suited to the two-stage DDM.

As we have seen, the assumptions of the two-stage model are as follows:

- The first period analyzed will be one of high initial growth.
- This stage of higher growth will eventually transition into a period of more mature, stable, and sustainable growth at a lower rate than the initial high-growth period.
- The dividend payout ratio will be based on company performance and the expected growth rate of its operations.

Let's use an example. Lore Ltd. estimates that its dividend growth will be 13% per year for the next five years. It will then settle to a sustainable, constant, and continuing rate of 5%. Let's say that the current year's dividend is $14 and the required rate of return (or discount rate) is 12%. What is the current value of Lore Ltd. stock?

Step 1:

First, we will need to calculate the dividends for each year until the second, stable growth rate phase is reached. Based on the current dividend value of $14 and the anticipated growth rate of 13%, the values of dividends (D_1, D_2, D_3, D_4, D_5) can be determined for each year of the first phase. Because the stable growth rate is achieved in the second phase, after five years have passed, if we assume that the current year is 2021, we can lay out the profile for this stock's dividends through the year 2026, as per Figure 11.3.

	A	B	C	D	E	F	G
1	Step 1						
2		Current					
3		2021	2022	2023	2024	2025	2026
4							
5	Dividend at 13% Growth	14.00	15.82	17.88	20.20	22.83	25.79
6							
7	Growth Rate of Dividends	N/A	13%	13%	13%	13%	13%

Figure 11.3 Profile of Stock Dividend through 2026

Step 2:

Next, we apply the DDM to determine the terminal value, or the value of the stock at the end of the five-year high-growth phase and the beginning of the second, lower growth-phase.

We can apply the DDM formula at any point in time, but in this example, we are working with a stock that has constant growth in dividends for five years and then decreases to a lower growth rate in its secondary phase. Because of this timing and dividend structure, we calculate the value of the stock five years from now, or the terminal value. Again, this is calculated at the end of the high-growth phase, in 2026. By applying the constant growth DDM formula, we arrive at the following:

$$\text{Stock Value}_N = \frac{D_N(1+g)}{r-g} = \frac{D_{N+1}}{r-g}$$

The terminal value can be calculated by applying the DDM formula in Excel, as seen in Figure 11.4 and Figure 11.5. The terminal value, or the value at the end of 2026, is $386.91.

	A	B	C	D	E	F	G	H
9	Step 2A							
10		Current						
11		2021	2022	2023	2024	2025	2026	2027
12								
13	Dividend at 13% Growth	14.00	15.82	17.88	20.20	22.83	25.79	**27.08**
14								
15	Growth Rate of Dividends	N/A	13%	13%	13%	13%	13%	5%
16	Terminal Value						**=H13/(B17-H15)**	
17	Required Rate of Return	**0.12**						

Figure 11.4 Terminal Value at the End of 2026 (Showing Formula)

	A	B	C	D	E	F	G	H
19	Step 2B							
20		Current						
21		2021	2022	2023	2024	2025	2026	2027
22								
23	Dividend at 13% Growth	14.00	15.82	17.88	20.20	22.83	25.79	27.08
24								
25	Growth Rate of Dividends	N/A	13%	13%	13%	13%	13%	5%
26	**Terminal Value**						**386.91**	
27	**Required Rate of Return**	**0.12**						

Figure 11.5 Terminal Value at the End of 2026

Step 3:

Next, we find the PV of all paid dividends that occur during the high-growth period of 2022–2026. This is shown in Figure 11.6. Our required rate of return (discount rate) is 12%.

	A	B	C	D	E	F	G	H
29	Step 3							
30		Current						
31		2021	2022	2023	2024	2025	2026	2027
32								
33	Dividend at 13% Growth	14.00	15.82	17.88	20.20	22.83	25.79	27.08
34								
35	Growth Rate of Dividends	N/A	13%	13%	13%	13%	13%	5%
36	Terminal Value						386.91	
37	Required Rate of Return	0.12						
38								
39	**Present Value of Dividends**	**N/A**	**14.13**	**14.25**	**14.38**	**14.51**	**14.64**	

Figure 11.6 Present Value of All Paid Dividends, 2022–2026

Step 4:

Next, we calculate the PV of the single lump-sum terminal value:

$$\begin{aligned}&\text{Future Value (FV)} = 386.91\\&\text{Interest Rate (I/Y)} = 12\\&\text{Number of Periods (N)} = 5\\&\text{Payment (PMT)} = 0\\&\text{Compute Present Value (CPTPV)} = 219.54\end{aligned}$$

Remember that due to the sign convention, either the FV must be entered as a negative value or, if entered as a positive value, the resulting PV will be negative. This example shows the former.

Step 5:

Our next step is to find the current fair (intrinsic) value of the stock, which comprises the PV of all future dividends plus the PV of the terminal value. This is represented in the following formula, with all factors shown in Figure 11.7:

$$\text{Fair Value} = \text{PV}_{\text{Projected Dividends}} + \text{PV}_{\text{Terminal Value}}$$

	A	B	C	D	E	F	G	H	I
42	Step 5								
43		Current							
44		2021	2022	2023	2024	2025	2026	2027	
45									
46	Dividend at 13% Growth	14.00	15.82	17.88	20.20	22.83	25.79	27.08	
47									
48	Growth Rate of Dividends	N/A	13%	13%	13%	13%	13%	5%	
49	Terminal Value						386.91		
50	Required Rate of Return	0.12							
51									
52	**Present Value of Dividends**	**N/A**	**14.13**	**14.25**	**14.38**	**14.51**	**14.64**		**71.90**
53									
54	**Present Value of Terminal Value**						**219.54**		**219.54**
55									
56						**Fair Value**			**291.44**

Figure 11.7 Fair Value of the Stock

So, we end up with a total current fair value of Lore Ltd. stock of $291.44 (due to Excel's rounding), although the sum can also be calculated as shown below:

$$\text{Fair Value} = 14.13 + 14.25 + 14.38 + 14.51 + 14.64 + 219.54 = 291.45$$

LINK TO LEARNING

Determining Stock Value

Take a few minutes to review this video, which covers methods used to determine stock value (https://openstax.org/r/covers_methods_used_to_determine_stock_value) when dividend growth is nonconstant.

Advantages and Limitations of DDMs

Some of the primary advantages of DDMs are their basis in the sound logic of present value concepts, their consistency, and the implication that companies that pay dividends tend to be mature and stable entities. Also, because the model is essentially a mathematical formula, there is little room for misinterpretation or subjectivity. As a result of these advantages, DDMs are a very popular form of stock evaluation that most analysts show faith in.

Because dividends are paid in cash, companies may keep making their dividend payments even when doing so is not in their best long-term interests. They may not want to manipulate dividend payments, as this can directly lead to stock price volatility. Rather, they may manipulate dividend payments in the interest of buoying up their stock price.

To further illustrate limitations of DDMs, let's examine the Concepts in Practice case.

CONCEPTS IN PRACTICE

Limitations of DDMs

A major limitation of the dividend discount model is that it cannot be used to value companies that do not pay dividends. This is becoming a growing trend, particularly for young high-tech companies. Warren Buffett, CEO of Berkshire Hathaway, has stated that companies are usually better off if they take their excess funds and reinvest them into infrastructure, evolving technologies, and other profitable ventures. The payment of dividends to shareholders is "almost a last resort for corporate management,"[1] says Buffett, and cash balances should be invested in "projects to become more efficient, expand territorially, extend and improve product lines or . . . otherwise widen the economic moat separating the company from its competitors."[2] Berkshire follows this practice of reinvesting cash rather than paying dividends, as do tech companies such as Amazon, Google, and Biogen.[3] So, rather than receiving cash dividends, stockholders of these companies are rewarded by seeing stock price appreciation in their investments and ultimately large capital gains when they finally decide to sell their shares.

The sensitivity of assumptions is also a drawback of using DDMs. The fair price of a stock can be highly sensitive to growth rates and the required rates of return demanded by investors. A single percentage point change in either of these two factors can have a dramatic impact on a company's stock, potentially changing it by as much as 10 to 20%.

1 Dan Caplinger. "Why Don't These Winning Stocks Pay Dividends?" The Motley Fool. Updated October 3, 2018. https://www.fool.com/investing/general/2015/03/01/why-dont-these-winning-stocks-pay-dividends.aspx
2 The Motley Fool. "Why Warren Buffet's Berkshire Hathaway Won't Pay a Dividend in 2015." Nasdaq. November 16, 2014. https://www.nasdaq.com/articles/why-warren-buffetts-berkshire-hathaway-wont-pay-dividend-2015-2014-11-16.
3 Caplinger, "Why Don't These Winning Stocks Pay Dividends?" The Motley Fool.

Finally, the results obtained using DDMs may not be related to the results of a company's operations or its profitability. Dividend payments should theoretically be tied to a company's profitability, but in some instances, companies will make misguided efforts to maintain a stable dividend payout even through the use of increased borrowing and debt, which is not beneficial to an organization's long-term financial health.

(sources: www.wallstreetmojo.com/dividend-discount-model/; pages.stern.nyu.edu/~adamodar/pdfiles/valn2ed/ch13d.pdf; www.managementstudyguide.com/disadvantages-of-dividend-discount-model.htm)

Stock Valuation with Changing Growth Rates and Time Horizons

Before we move on from our discussion of dividend discount models, let's work through some more examples of how the DDM can be used with a number of different scenarios, changing growth rates, and time horizons.

As we have seen, the value or price of a financial asset is equal to the present value of the expected future cash flows received while maintaining ownership of the asset. In the case of stock, investors receive cash flows in the form of dividends from the company, plus a final payout when they decide to relinquish their ownership rights or sell the stock.

Let's look at a simple illustration of the price of a single share of common stock when we know the future dividends and final selling price.

Problem:

Steve wants to purchase shares of Old Peak Construction Company and hold these common shares for five years. The company will pay $5.00 annual cash dividends per share for the next five years.

At the end of the five years, Steve will sell the stock. He believes that he will be able to sell the stock for $25.00 per share. If Steve wants to earn 10% on this investment, what price should he pay today for this stock?

Solution:

The current price of the stock is the discounted cash flow that Steve will receive over the next five years while holding the stock. If we let the final price represent a lump-sum future value and treat the dividend payments as an annuity stream over the next five years, we can apply the time value of money concepts we covered in earlier chapters.

Method 1: Using an Equation

$$\text{Price} = \text{Future Price} \times \frac{1}{(1+r)^n} + \text{Dividend Stream} \times \frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r}$$

$$= \$25.00 \times \frac{1}{(1+0.10)^5} + \$5.00 \times \frac{\left[1 - \frac{1}{(1+0.10)^5}\right]}{0.10}$$

$$= \$25.00 \times 0.6209 + \$5.00 \times 3.7908$$

$$= \$15.52 + \$18.95 = \$34.47$$

Method 2: Using a Financial Calculator

We can also use a calculator or spreadsheet to find the price of the stock (see Table 11.2).

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.00
2	Enter number of periods (5)	5 N	N =	5.00
3	Enter rate of return or interest rate (10%)	10 I/Y	I/Y =	10.00
4	Enter eventual sales price ($25)	25 FV	FV =	25.00
5	Enter dividend amount ($5)	5 PMT	PMT =	5.00
6	Compute present value	CPT PV	PV =	– 34.47

Table 11.2 Calculator Steps for Finding the Price of the Stock[4]

The stock price is calculated as $34.47.

Note that the value given is expressed as a negative value due to the sign convention used by financial calculators. We know the actual stock value is not negative, so we can just ignore the minus sign.

In cases such as the above, we find the present value of a dividend stream and the present value of the lump-sum future price. So, if we know the dividend stream, the future price of the stock, the future selling date of the stock, and the required return, it is possible to price stocks in the same manner that we price bonds.

Method 3: Using Excel

Figure 11.8 shows a spreadsheet setup in Excel to reach a solution to this problem.

	A	B	C	D	E	F
1						
2	Nper (N) or Number of Periods					5
3	Rate of Return or Interest Rate					10%
4	Future Value or Stock Sale Price					$ 25.00
5	Payment (Dividend Amount)					$ 5.00
6						$ (34.48)

Figure 11.8 Excel Solution for Finding the Price of the Stock

Due to the sign convention in Excel, we can ignore the parentheses around the solution, which indicate a negative value. Therefore, the price is $34.48. The Excel command used in cell F6 to calculate present value is as follows:

```
=PV(rate,nper,pmt,[fv],[type])
```

Finding Stock Price with Constant Dividends

Example 1:

Four Seasons Resorts pays a $0.25 dividend every quarter and will maintain this policy forever. What price should you pay for one share of common stock if you want an annual return of 10% on your investment?

Solution:

You can restate your annual required rate of 10% as a quarterly rate of 2.5% $\left(\frac{10\%}{4}\right)$. Apply the quarterly dividend amount and the quarterly rate of return to determine the price:

4 The specific financial calculator in these examples is the Texas Instruments BA II Plus™ Professional model, but you can use other financial calculators for these types of calculations.

$$\text{Price} = \frac{\text{Dividend}}{r}$$

$$\text{Price} = \frac{\$0.25}{0.025} = \$10.00$$

Even though we anticipate that companies will be in business "forever," we are not going to own a company's stock forever. Therefore, the dividend stream to which we would have legal claim is only for that period of the company's life during which we own the stock. We need to modify the dividend model to account for a finite period when we will sell the stock at some future time. This modification brings us from an infinite to a finite dividend pricing model, which we will use to price a finite amount of dividends and the future selling price of the stock. We will maintain a constant dividend assumption. Let's assume we will hold a share in a company that pays a $1 dividend for 20 years and then sell the stock.

Method 1: Using an Equation

The dividend pricing model under a finite horizon is a concept we have seen earlier. It is a simple present value annuity stream application:

$$\text{Value of Future Dividends for Specific Periods} = \text{Dividend} \times \text{PVIFA (Present Value Interest Factor of an Annuity)}$$

$$\text{Dividend Stream} \times \frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r} = \$1.00 \times \frac{\left[1 - \frac{1}{(1+0.10)^{20}}\right]}{0.10}$$

$$= \$1.00 \times 8.5136 = \$8.51$$

We now need to determine the selling price that we would get in 20 years if we were to sell the stock to someone else at that time. What would a willing buyer give us for the stock 20 years from now? This price is difficult to estimate, so for the sake of this exercise, we will assume that the price in 20 years will be $30. So, what is the present value of the price in 20 years with a 10% discount rate? Again, this is just a simple application of the PV formula we covered earlier in the text:

$$\text{PV} = \frac{\text{Price}_{20}}{(1+r)^{20}} = \frac{\$30}{(1.10)^{20}} = \$4.46$$

We can now price the stock as if it were a bond with a dividend stream of 20 years, a sales price in 20 years, and a required return of 10%:

- The dividend stream is analogous to the coupon payments.
- The sales price is analogous to the bond's principal.
- The 20-year investment horizon is analogous to the bond's maturity date.
- The required return is analogous to the bond's yield.

Carrying on with the PV calculations, we have

$$\text{Price} = \$30 \times \frac{1}{(1+0.10)^{20}} + \frac{\left[\$1.00 \times 1 - \frac{1}{(1+0.10)^{20}}\right]}{0.10}$$

$$= \$4.46 + \$8.51 = \$12.97$$

Method 2: Using a Financial Calculator

We can also use a calculator or spreadsheet to find the price of the stock using constant dividends (see Table 11.3).

Step	Description	Enter	Display
1	Clear calculator register	CE/C	0.00

Table 11.3 Calculator Steps for Finding the Price of Stock Using Constant Dividends

Step	Description	Enter	Display	
2	Enter number of periods (20)	20 N	N =	20.00
3	Enter rate of return or interest rate (10%)	10 I/Y	I/Y =	10.00
4	Enter eventual sales price (30)	30 FV	FV =	30.00
5	Enter dividend amount ($1)	1 PMT	PMT =	1.00
6	Compute present value	CPT PV	PV =	−12.97

Table 11.3 Calculator Steps for Finding the Price of Stock Using Constant Dividends

The stock price resulting from the calculation is $12.97.

Method 3: Using Excel

This same problem can be solved using Excel with a setup similar to that shown in Figure 11.9.

	A	B	C	D	E	F
8						
9	Nper (N) or Number of Periods					20
10	Rate of Return or Interest Rate					10%
11	Payment (Dividend Amount)					$ 1.00
12	Future Value or Stock Sale Price					$ 30.00
13						$ (12.97)

Figure 11.9 Excel Solution for Finding the Price of Stock Using Constant Dividends

Once again, we can ignore the negative indicator that is generated by the Excel sign convention because we know that the stock will not have a negative value 20 years from now. Therefore, the price is $12.97. The Excel command used in cell F13 to calculate present value is as follows:

```
=PV(rate,nper,pmt,[fv],[type])
```

Example 2:

Let's look at an example and estimate current stock price given a 10.44% constant growth rate of dividends forever and a desired return on the stock of 13.5%. We will assume that the current stock owner has just received the most recent dividend, D_0, and the new buyer will receive all future cash dividends, beginning with D_1. This part of the setup of the model is important because the price reflects all future dividends, starting with D_1, discounted back to today. (Price_0 refers to the price at time zero, or today.) The first dividend the buyer would receive is one full period away. Using the discounted cash flow approach, we have

$$\text{Price}_0 = \frac{D_0 \times (1+g)^1}{(1+r)^1} + \frac{D_0 \times (1+g)^2}{(1+r)^2} + \frac{D_0 \times (1+g)^3}{(1+r)^3} + \frac{D_0 \times (1+g)^4}{(1+r)^4} \ldots$$

where g is the annual growth rate of the dividends and r is the required rate of return on the stock. We can simplify the equation above into the following:

$$\begin{aligned} \text{Price}_0 &= \frac{D_0 \times (1+g)}{r-g} \\ D_1 &= D_0 \times (1+g) \\ \text{Price}_0 &= \frac{D_1}{r-g} \end{aligned}$$

As we discussed above, this classic model of constant dividend growth, known as the Gordon growth model, is

a fundamental method of stock pricing. The Gordon growth model determines a stock's value based on a future stream of dividends that grows at a constant rate. Again, we assume that this constantly growing dividend stream will pay forever. To see how the constant growth model works, let's use our example from above once again as a test case. The most recent dividend (D_0) is \$1.76, the growth rate (g) is 10.44%, and the required rate of return (r) is 13.5%, so applying our PV equation, we have

$$\text{Price}_0 = \frac{\$1.76 \times (1 + 0.1044)}{0.135 - 0.1044} = \frac{\$1.943774}{0.0306} = \$63.52$$

Our estimated price for this example is \$63.52. Notice that the formula requires the return rate r to be greater than the growth rate g of the dividend stream. If g were greater than r, we would be dividing by a negative number and producing a negative price, which would be meaningless.

Let's pick another company and see if we can apply the dividend growth model and price the company's stock with a different dividend history. In addition, our earlier example will provide a shortcut method to estimate g, although you could still calculate each year's percentage change and then average the changes over the 10 years.

Estimating a Stock Price from a Past Dividend Pattern

Problem:

Phased Solutions Inc. has paid the following dividends per share from 2011 to 2020:

2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
\$0.070	\$0.080	\$0.925	\$1.095	\$1.275	\$1.455	\$1.590	\$1.795	\$1.930	\$2.110

Table 11.4

If you plan to hold this stock for 10 years, believe Phased Solutions will continue this dividend pattern forever, and you want to earn 17% on your investment, what would you be willing to pay per share of Phased Solutions stock as of January 1, 2021?

Solution:

First, we need to estimate the annual growth rate of this dividend stream. We can use a shortcut to determine the average growth rate by using the first and last dividends in the stream and the time value of money equation. We want to find the average growth rate given an initial dividend (present value) of \$0.70, the most recent dividend (future value) of \$2.11, and the number of years (n) between the two dividends, or the number of dividend changes, which is 9. So, we calculate the average growth rate as follows:

$$\begin{aligned} g &= \left(\frac{\text{FV}}{\text{PV}}\right)^{\frac{1}{n}} - 1 \\ &= \left(\frac{\$2.11}{\$0.70}\right)^{\frac{1}{9}} - 1 = 3.0142857^{\frac{1}{9}} - 1 \\ &= 0.1304, \text{ or } 13.04\% \end{aligned}$$

Table 11.5 shows the step-by-step process of using a financial calculator to solve for the growth rate.

Step	Description	Enter	Display	
1	Clear calculator register	CE/C		0.0000
2	Enter number of periods (9)	9 N	N =	9.0000
3	Enter present value or initial dividend (\$0.70) as a negative value	0.7 +|− PV	PV =	-0.7000

Table 11.5 Calculator Steps for Solving the Growth Rate

Step	Description	Enter	Display	
4	Enter future value or the most recent dividend ($2.11)	2.11 FV	FV =	2.1100
5	Enter a zero value for payment as a placeholder, as this factor is not used here	0 PMT	PMT	0.0000
6	Compute annual growth rate	CPT I/Y	I/Y =	13.0427

Table 11.5 Calculator Steps for Solving the Growth Rate

The calculated growth rate is 13.04%.

We can also use Excel to set up a spreadsheet similar to the one in Figure 11.10 that will calculate this growth rate.

	A	B	C	D	E	F	G
15							
16							
17	Present Value or Current Dividend Amount (enter as a negative value)						$ (0.70)
18	Nper (N) or Number of Periods						9
19	Payment or PMT (unused here, so enter zero as a placeholder)						-
20	Future Value (dividend amount in nine years)					13.04%	$ 2.11
21							
22	Growth Rate						13.0427%

Figure 11.10 Excel Solution for Growth Rate

The Excel command used in cell G22 to calculate the growth rate is as follows:

```
=RATE(nper,pmt,pv,[fv],[type],[guess])
```

We now have two methods to estimate *g*, the growth rate of the dividends. The first method, calculating the change in dividend each year and then averaging these changes, is the arithmetic approach. The second method, using the first and last dividends only, is the geometric approach. The arithmetic approach is equivalent to a simple interest approach, and the geometric approach is equivalent to a compound interest approach.

To apply our PV formula above, we had to assume that the company would pay dividends forever and that we would hold on to our stock forever. If we assume that we will sell the stock at some point in the future, however, can we use this formula to estimate the value of a stock held for a finite period of time? The answer is a qualified yes. We can adjust this model for a finite horizon to estimate the present value of the dividend stream that we will receive while holding the stock. We will still have a problem estimating the stock's selling price at the end of this finite dividend stream, and we will address this issue shortly. For the finite growing dividend stream, we adjust the infinite stream in our earlier equation to the following:

$$\text{Price}_0 = D_0 \times \frac{1+g}{r-g} \times \left[1 - \left(\frac{1+g}{1+r}\right)^n\right]$$

where *n* is the number of future dividends.

This equation may look very complicated, but just focus on the far right part of the model. This part calculates the percentage of the finite dividend stream that you will receive if you sell the stock at the end of the *n*th year. Say you will sell Johnson & Johnson after 10 years. What percentage of the $60.23 (the finite dividend stream) will you get? Begin with the following:

$$10 \text{ Years: Percent} = 1 - \left(\frac{1 + 0.1304}{1 + 0.170}\right)^{10}$$
$$= 1 - 0.966154^{10} = 1 - 0.7087 = 0.2913$$

Now, multiply the result by the price for your portion of the infinite stream:

$$\text{Price} = \$60.23 \times 0.2913 = \$17.55$$

The next step is to discount the selling price of Johnson & Johnson in 10 years at 17% and then add the two values to get the stock's price. So, how do we estimate the stock's price at the end of 10 years? If we elect to sell the stock after 10 years and the company will continue to pay dividends at the same growth rate, what would a buyer be willing to pay? How could we estimate the selling price (value) of the stock at that time?

We need to estimate the dividend in 10 years and assume a growth rate and the required return of the new owner at that point in time. Let's assume that the new owner also wants a 17% return and that the dividend growth rate will remain at 13.04%. We calculate the dividend in 10 years by taking the current growth rate plus one raised to the tenth power times the current dividend:

$$D_{10} = \$2.11 \times (1.1304)^{10} = \$2.11 \times 3.4066 = \$7.1879$$

We then use the dividend growth model with infinite horizon to determine the price in 10 years as follows:

Your price for the stock today—given that you will receive the growing dividend stream for 10 years and sell for $258.10 in 10 years, and also given that you want a 17% return over the 10 years—is as shown below:

$$\text{Price} = \frac{\$205.18}{(1.1700)^{10}} + \frac{\$2.11 \times (1 + 0.1304)}{(0.1700 - 0.1304)} \times \left[1 - \left(\frac{1 + 0.1304}{1 + 0.1700}\right)^{10}\right]$$
$$= \$42.68 + \$17.55 = \$60.23$$

Why did you get the same price of $60.23 for your stock with both the infinite growth model and the finite model? The reason is that the required rate of return of the stock remained at 17% (your rate) and the growth rate of the dividends remained at 13.04%. The infinite growth model gives the same price as the finite model with a future selling price as long as the required return and the growth rate are the same for all future sales of the stock.

Although this point may be subtle, what we have just shown is that a stock's price is the present value of its future dividend stream. When you sell the stock, the buyer purchases the remaining dividend stream. If that individual should sell the stock in the future, the new owner would buy the remaining dividends. That will always be the case; a stock's buyer is always buying the future dividend stream.

LINK TO LEARNING

Determining Stock Value Using Different Scenarios

This video explains methods for determining stock value (https://openstax.org/r/explains_methods_for_determining_stock_value) using scenarios of constant dividends and scenarios of constant dividend growth.

THINK IT THROUGH

Nonconstant Growth Dividends

One final issue to address in this section is how we price a stock when dividends are neither constant nor growing at a constant rate. This can make things a bit more complicated. When a future pattern is not an annuity or the modified annuity stream of constant growth, there is no shortcut. You have to estimate every

future dividend and then discount each individual dividend back to the present. All is not lost, however. Sometimes you can see patterns in the dividends. For example, a firm might shift into a dividend stream pattern that will allow you to use one of the dividend models to take a shortcut for pricing the stock. Let's look at an example.

JM and Company is a small start-up firm that will institute a dividend payment—a $0.25 dividend—for the first time at the end of this year. The company expects rapid growth over the next four years and will increase its dividend to $0.50, then to $1.50, and then to $3.00 before settling into a constant growth dividend pattern with dividends growing at 5% every year (see Table 11.6). If you believe that JM and Company will deliver this dividend pattern and you desire a 13% return on your investment, what price should you pay for this stock?

T0	T1	T2	T3	T4	T5
$0.25	$0.50	$1.50	$1.275	$3.00	$3.00 × 1.05

Table 11.6

Solution: To price this stock, we will need to discount the first four dividends at 13% and then discount the constant growth portion of the dividends, the first payment of which will be received at the end of year 5. Let's calculate the first four dividends:

$$\begin{aligned} \text{PV} &= \frac{\$0.25}{(1.13)^1} + \frac{\$0.50}{(1.13)^2} + \frac{\$1.50}{(1.13)^3} + \frac{\$3.00}{(1.13)^4} \\ &= \$0.22 + \$0.39 + \$1.04 + \$1.84 = \$3.49 \end{aligned}$$

We now turn to the constant growth dividend pattern, where we can use our infinite horizon constant growth model as follows:

$$\text{Price}_4 = \frac{\$3.00 \times (1 + 0.05)}{0.13 - 0.05} = \frac{\$3.15}{0.08} = \$39.375$$

This figure is the price of the constant growth portion at the end of the fourth period, so we still need to discount it back to the present at the 13% required rate of return:

$$\text{Price} = \frac{\$39.375}{(1.13)^4} = \$24.15$$

So, the price of this stock with a nonconstant dividend pattern is

$$\text{Price} = \$3.49 + \$24.15 = \$27.64$$

Some Final Thoughts on Dividend Discount Models

The dividend used to calculate a price is the expected future payout and expected future dividend growth. This means the DDM is most useful when valuing companies that have long, consistent dividend records.

If the DDM formula is applied to a company with a limited dividend history, or in an industry exposed to significant risks that could affect a company's ability to maintain its payout, the resulting derived value may not be entirely accurate.

In most cases, dividend models, whether constant growth or constant dividend, appeal to a fundamental concept of asset pricing: the future cash flow to which the owner is entitled while holding the asset and the required rate of return for that cash flow determine the value of a financial asset. However, problems can arise when using these models because the timing and amounts of future cash flows may be difficult to predict.

11.3 Discounted Cash Flow (DCF) Model

Learning Outcomes

By the end of this section, you will be able to:

- Explain how the DCF model differs from DDMs.
- Apply the DCF model.
- Explain the advantages and disadvantages of the DCF model.

When investors buy stock, they do so in order to receive cash inflows at different points in time in the future. These inflows come in the form of cash dividends (provided the stock does indeed pay dividends, because not all do) and also in the form of the final cash inflow that will occur when the investor decides to sell the stock.

The investor hopes that the final sale price of the stock will be higher than the purchase price, resulting in a capital gain. The hope for capital gains is even stronger in the case of stocks that do not pay dividends. When securities have been held for at least one year, the seller is eligible for long-term capital gains tax rates, which are lower than short-term rates for most investors. This makes non-dividend-paying stocks even more attractive, provided that they do indeed appreciate in value over the investor's holding period. Meanwhile, short-term gains, or gains made on securities held for less than one year, are taxed at ordinary income tax rates, which are usually higher and offer no particular advantage to an investor in terms of reducing their taxes.

Understanding How the DCF Model Differs from DDMs

The valuation of an asset is typically based on the present value of future cash flows that are generated by the asset. It is no different with common stock, which brings us to another form of stock valuation: the **discounted cash flow (DCF)** model. The DCF model is usually used to evaluate firms that are relatively young and do not pay dividends to their shareholders. Examples of such companies include Facebook, Amazon, Google, Biogen, and Monster Beverage. The DCF model differs from the dividend discount models we covered earlier, as DDM methodologies are almost entirely based on a stock's periodic dividends.

The DCF model is an absolute valuation model, meaning that it does not involve comparisons with other firms within any specific industry but instead uses objective data to evaluate a company on a stand-alone basis. The DCF model focuses on a company's cash flows, determining the present value of the entire organization and then working this down to the share-value level based on total shares outstanding of the subject organization. This highly regarded methodology is the evaluation tool of choice for experienced financial analysts when evaluating companies and their common stock. Many analysts prefer DCF methods of valuation because these are based on a company's cash flows, which are far less easily manipulated through accounting treatments than revenues or bottom-line earnings.

The DCF model formula in its mathematical form is presented below:

$$\text{Stock Value} = \frac{\text{CF}_1}{(1+r)^1} + \frac{\text{CF}_2}{(1+r)^2} + \ldots + \frac{\left(\frac{\text{TCF}}{r-g}\right)}{(1\ +\ r)^{n-1}}$$

where CF_1 is the estimated cash flow in year 1, CF_2 is the estimated cash flow in year 2, and so on; TCF is the terminal cash flow, or expected cash flow from the ending asset sale; r is the discount rate or required rate of return; g is the anticipated growth rate of the cash flow; and n is the number of years covered in the model.

Applying the DCF Model

We can apply the DCF model to an example to demonstrate this methodology and how the formula works. Calculate the value of Mayweather Inc. and its common stock based on the next six years of cash flow results. Assume that the discount rate (required rate of return) is 8%, Mayweather's growth rate is 3%, and the terminal value (TCF) will be two and one-half times the discounted value of the cash flow in year 6.

Mayweather has a cash flow of $2.0 million in year 1, so its discounted cash flow after one year (CF_1) is $1,851,851.85. We arrive at this amount by applying the discount rate of 8% for a one-year period to determine the present value.

In subsequent years, Mayweather's cash flow will be increasing by 3%. These future cash flows also must be discounted back to present values at an 8% rate, so the discounted cash flow amounts over the next six years will be as follows:

Year 1: $1,851,852

Year 2: $1,766,118

Year 3: $1,684,353

Year 4: $1,606,374

Year 5: $1,532,005

Year 6: $1,461,079

Our earlier assumption that the terminal value will be 2.5 times the value in the sixth year gives us a total terminal cash flow (TCF) of $\$1{,}461{,}079 \times 2.5$, or $3,652,697. Now, if we take all these future discounted cash flows and add them together, we arrive at a grand total of $13,554,477. So, based on our DCF model analysis, the total value of Mayweather Inc. is just over $13.5 million.

At this point, we have the estimated value of the entire company, but we need to work this down to the level of per-share value of common stock.

Let's say that Mayweather is currently trading at $12 per share, and it has 1,000,000 common shares outstanding. This tells us that the market capitalization of the company is $\$12 \times 1{,}000{,}000$, or $12 million, and that a $12 share price may be considered relatively low. The reason for this is that based on our DCF model analysis, investors would theoretically be willing to pay $13,554,477 divided by 1,000,000 shares, or $13.55 per share, for Mayweather. The overall conclusion would be that at $12.00 per share, Mayweather common stock would be a good buy at the present time. Figure 11.11 shows the Excel spreadsheet approach for arriving at the total value of Mayweather.

	A	B	C	D	E
1					
2	Discount Rate =		8.00%		
3					
4		Year	Cash Flow	Present Value	
5					
6	CF	1	2,000,000	**1,851,852**	**=-PV($C$2,B6,0,C6)**
7	CF	2	2,060,000	1,766,118	
8	CF	3	2,121,800	1,684,353	
9	CF	4	2,185,454	1,606,374	
10	CF	5	2,251,018	1,532,005	
11	CF	6	2,318,548	1,461,079	
12					
13	Totals		$ 12,936,820	9,901,780	
14	TCF			3,652,697	
15	Grand Total			**$13,554,477**	

Figure 11.11 Excel Solution for Total Value

Cell E6 displays the present value formula that is active in cell D6.

Advantages and Limitations of the DCF Model

Due to several corporate accounting scandals in recent years, many analysts have given increasing credence to the use of cash flow as a metric for determining accurate corporate valuations. However, it should be noted that cash flow is not always the best means of measuring financial health. A company can always sell a large portion of its assets to generate a positive cash flow, even if it is operating at a loss or experiencing other financial difficulties. Additionally, investors prefer to see companies reinvesting their cash back into their businesses rather than sitting on excessive balances of idle cash.

Similar to other models, the discounted cash flow model is only as good as the information entered. As the common expression goes, "garbage in, garbage out." This can often be the case if reasonably accurate cash flow estimates are not available or if an unrealistic discount rate or required rate of return is used in the calculations. It is always best to use several different methods when valuing companies and their common stock.

LINK TO LEARNING

Discounted Cash Flow

Please view this short video on discounted cash flow (https://openstax.org/r/short_video_on_discounted) and how this method can be applied in stock valuation.

11.4 Preferred Stock

Learning Outcomes

By the end of this section, you will be able to:

- Define preferred stock.
- Calculate the intrinsic value of preferred stock.
- Understand the difference between common stock and preferred stock.

Features of Preferred Stock

Preferred stock is a unique form of equity sold by some firms that offers preferential claims in ownership. Preferred stock will often feature a dividend that a company is obligated to pay out before it makes any dividend payments to common stockholders. In cases of bankruptcy and liquidation of the issuing company, preferred stockholders have priority claim to assets before common stockholders. Additionally, preferred stockholders are usually entitled to a set (or constant) dividend every period.

Preferred stock carries a stated par value, but unlike bonds, they have no maturity date, and consequently, there is no final payment of the par value. The only time a company would pay this par value to the shareholder would be if the company ceased operations or retired the preferred stock. Many preferred stock issues are cumulative in nature, meaning that if a company skips or is otherwise unable to pay a cash dividend, it becomes a liability to the company and must eventually be paid out to preferred shareholders at some point in the future. Other preferred stocks may be noncumulative, in which case if the company skips dividends, they are forever lost to the shareholder.

The term *preferred* comes from preferred shareholders receiving all past (if cumulative) and present dividends before common shareholders receive any cash dividends. In other words, preferred shareholders' dividend claims are given preferential treatment over those of common shareholders. Preferred stock is usually a form of permanent funding, but there are circumstances or covenants that could alter the payoff stream.

For example, a company may convert preferred stock into common stock at a preset point in the future. It is not uncommon for companies to issue preferred stock that has a conversion feature. Such conversion features give preferred shareholders the right to convert to common shares after a predetermined period.

A review of the characteristics of preferred stock will lead to the conclusion that the constant growth dividend model is an excellent approach for valuing such stock. Because shares of preferred stock provide a constant cash dividend based on original par value and the stated dividend rate, these may be considered a form of perpetuity.

It is this constant, preferred dividend stream that makes preferred stock seem more like bonds or another form of debt than like stock. In addition, the constant dividend stream leads nicely to the pricing of preferred stock with the four dividend models we presented earlier in this chapter.

Determining the Intrinsic Value of Preferred Stock

We can apply a version of the present value of a perpetuity formula to value preferred stock, as in the following example. Oh-Well Heath Services Inc. has issued preferred stock that has a par value of $1,000 and pays an annual dividend rate of 5%. If the market considers the risk of Oh-Well to warrant a 10% discount rate, what would be a fair market price for Oh-Well preferred stock?

First, we find the dividend value of Oh-Well:

$$\begin{aligned}\text{Dividend Dollar Value} &= \text{Par Value} \times \text{Annual Dividend Rate} \\ &= \$1{,}000 \times 0.05 \\ &= \$50\end{aligned}$$

We then use the constant dividend model with infinite horizon because we have g equal to zero and n equal to infinity:

$$\begin{aligned}\text{Stock Value} &= \frac{\text{Dividend}}{\text{Discount Rate}} \\ &= \frac{\$50}{.10} \\ &= \$500\end{aligned}$$

We can also rearrange the formula to determine the required return on this stock, given its annual dividend and current price.

THINK IT THROUGH

Calculating the Return on Preferred Stock

Data Forge Inc. has just issued preferred stock (cumulative) with a par value of $100.00 and an annual dividend rate of 7%. The preferred stock is currently selling for $35.00 per share. What is the yield or return on this preferred stock?

Solution:

The first step is to determine the annual dividend by multiplying the dividend rate by the par value:

$$\$100 \times 0.07 = \$7.00$$

Now, using this $7.00 annual dividend, the $35.00 current price, and the equation above, we calculate the rate of return as follows:

$$r = \frac{\$7.00}{\$35.00} = 0.20 \text{ or } 20\%$$

We have introduced the concept of return here, which should be thought of as both the anticipated return for the preferred stockholder and the company's cost of borrowing money for this particular type of capital.

Differences between Preferred and Common Stock

As we have discussed, preferred stock has important differences from common stock that apply to issuing firms and to investors. Some of the most important of these differences are listed in Table 11.7.

Feature	Common Stock	Preferred Stock
Dividends	Paid only after preferred stockholders are paid	Highest priority, paid first
Dividends	Variable and may increase or decrease	Predetermined rates, so constant dividend amounts
Growth	High potential but tied to company performance	
Liquidation	Paid out last, after all creditors and preferred stockholders are paid	Given preference in terms of payments, similar to bonds
Voting Rights	Yes	No
Arrears	No accrual of missed dividends	If cumulative, unpaid dividends become liability that must be paid out eventually
Certainty	Dividends potentially not paid if company earns no profits	Dividends paid even when company experiences financial losses

Table 11.7 Differences between Common Stock and Preferred Stock

11.5 Efficient Markets

Learning Outcomes

By the end of this section, you will be able to:

- Understand what is meant by the term *efficient markets*.
- Understand the term *operational efficiency* when referring to markets.
- Understand the term *informational efficiency* when referring to markets.
- Distinguish between strong, semi-strong, and weak levels of efficiency in markets.

Efficient Markets

For the public, the real concern when buying and selling of stock through the stock market is the question, "How do I know if I'm getting the best available price for my transaction?" We might ask an even broader question: Do these markets provide the best prices and the quickest possible execution of a trade? In other words, we want to know whether markets are efficient. By **efficient markets**, we mean markets in which costs are minimal and prices are current and fair to all traders. To answer our questions, we will look at two forms of efficiency: operational efficiency and informational efficiency.

Operational Efficiency

Operational efficiency concerns the speed and accuracy of processing a buy or sell order at the best available price. Through the years, the competitive nature of the market has promoted operational efficiency.

In the past, the **NYSE (New York Stock Exchange)** used a designated-order turnaround computer system known as SuperDOT to manage orders. SuperDOT was designed to match buyers and sellers and execute trades with confirmation to both parties in a matter of seconds, giving both buyers and sellers the best

available prices. SuperDOT was replaced by a system known as the Super Display Book (SDBK) in 2009 and subsequently replaced by the Universal Trading Platform in 2012.

NASDAQ used a process referred to as the small-order execution system (SOES) to process orders. The practice for registered dealers had been for SOES to publicly display all limit orders (orders awaiting execution at specified price), the best dealer quotes, and the best customer limit order sizes. The SOES system has now been largely phased out with the emergence of all-electronic trading that increased transaction speed at ever higher trading volumes.

Public access to the best available prices promotes operational efficiency. This speed in matching buyers and sellers at the best available price is strong evidence that the stock markets are operationally efficient.

Informational Efficiency

A second measure of efficiency is informational efficiency, or how quickly a source reflects comprehensive information in the available trading prices. A price is efficient if the market has used all available information to set it, which implies that stocks always trade at their fair value (see Figure 11.12). If an investor does not receive the most current information, the prices are "stale"; therefore, they are at a trading disadvantage.

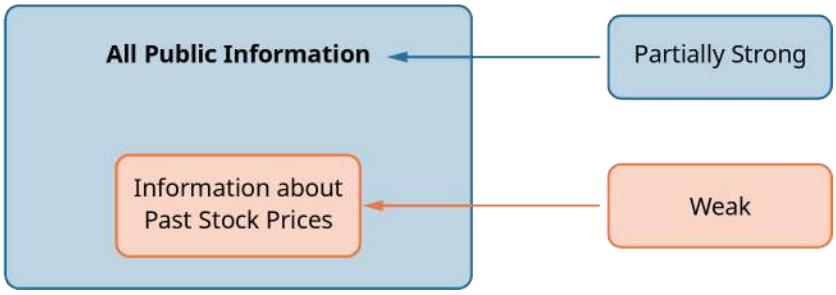


Figure 11.12 Forms of Market Efficiency A market is efficient if it provides all information that is available.

Forms of Market Efficiency

Financial economists have devised three forms of market efficiency from an information perspective: weak form, semi-strong form, and strong form. These three forms constitute the efficient market hypothesis. Believers in these three forms of efficient markets maintain, in varying degrees, that it is pointless to search for undervalued stocks, sell stocks at inflated prices, or predict market trends.

In weak form efficient markets, current prices reflect the stock's price history and trading volume. It is useless to chart historical stock prices to predict future stock prices such that you can identify mispriced stocks and routinely outperform the market. In other words, technical analysis cannot beat the market. The market itself is the best technical analyst out there.

Summary

11.1 Multiple Approaches to Stock Valuation

This section introduced common stock and some of the models and calculation methods used by investors and financial analysts to determine the prices or values of common shares. The most evaluative ratios that can be computed from a company's financial statements include the price-to-earnings (P/E), price-to book (P/B), price-to-sales (P/S), and price-to cash-flow (P/CF) ratios.

11.2 Dividend Discount Models (DDMs)

The dividend discount model, or DDM, is a method used to value a stock based on the concept that its worth is the present value of all of its future dividends. The most common DDM is the Gordon growth model, which values stock entirely on expected future dividends. Other techniques include the zero growth DDM, which depends on fixed dividends; the constant growth DDM, which assumes that dividends will grow at a constant rate; and the variable growth or nonconstant growth DDM, which is based on the assumption that stock value will progress through different stages of growth. There is also the two-stage DDM, which is based on the assumption of two stages of dividend growth: an initial period of higher growth and a subsequent period of lower, more stable growth.

11.3 Discounted Cash Flow (DCF) Model

Investors buy stock to receive cash inflows at different points in the future. These inflows may come in the form of dividends or a final cash inflow. If the investor chooses to wait for a final cash flow, the hope is that capital gains will be even stronger. The DCF model is usually used to evaluate firms that are relatively young and do not pay dividends to their shareholders. The DCF model focuses on a company's cash flows, determining the present value of an entire organization using objective data and then working this down to the share-value level based on total shares outstanding of the subject organization.

11.4 Preferred Stock

Preferred stock is a unique form of equity sold by some firms that offers preferential claims in ownership. Preferred stock carries a stated par value, but unlike bonds, there is no maturity date, and consequently, there is no final payment of the par value. The term *preferred* comes from preferred shareholders receiving all past (if cumulative) and present dividends before common shareholders receive any cash dividends.

11.5 Efficient Markets

Efficient markets are markets in which costs are minimal and prices are current and fair to all traders. There are two forms of efficiency: operational efficiency and informational efficiency. Operational efficiency concerns the speed and accuracy of processing a buy or sell order at the best available price. Informational efficiency concerns how quickly a source reflects comprehensive information in the available trading prices. Financial economists have devised three forms of efficient markets from an information perspective: weak form, semi-strong form, and strong form.

Key Terms

amortization the process of spreading business costs in accounting; similar to depreciation, but differs in that it generally refers to intangible assets such as patents or copyrights

capital employed also known as funds employed; the total amount of capital used for the acquisition of profits by a firm or on a project

common stock a security that represents partial ownership of a corporation

comparable company analysis (comps) a method for valuating a company using the metrics of other businesses of similar size in the same industry

depreciation the process of spreading business costs in accounting; similar to amortization, but differs in

that it generally refers to fixed, tangible assets such as buildings, machinery, furniture, fixtures, and equipment

discounted cash flow (DCF) a method for estimating the value of an investment based on the present value of its expected future cash flows

dividend a sum of money paid regularly (typically quarterly) by a company to its shareholders out of its profits or reserves

dividend discount model (DDM) a quantitative method for predicting the price of a company's stock based on the theory that its present-day price is worth the sum of all of its future dividend payments when discounted back to their present value

dividend yield a financial ratio (dividend/price) that shows how much a company pays out in dividends each year relative to its stock price, expressed as a percentage

earnings per share (EPS) the ratio of a company's profits to the outstanding shares of its common stock; serves as an indicator of a company's profitability

EBIT short for earnings before interest and taxes; an indicator of a firm's profitability before the effects of interest or taxes; also referred to as operating earnings, operating profit, and profit before interest and tax

efficient markets markets in which costs are minimal and prices are current, fair, and reflective of all available relevant information

enterprise value (EV) a company's total value; often used as a more comprehensive alternative to equity market capitalization

enterprise value (EV) multiples also known as company value multiples; ratios used to determine the overall value of a company and, by extension, the value of its common stock

equity multiples metrics that calculate the expected or achieved total return on an initial investment

Gordon growth model a methodology used to determine the intrinsic value of a stock based on a future series of dividends that grow at a constant rate

growth rate the rate at which the dollar amount of dividends paid on a specific stock holding increases

intangible assets assets that are not physical in nature, such as goodwill, brand recognition, and intellectual property (i.e., patents, trademarks, and copyrights)

intrinsic value the value of a firm's stock based entirely on internal factors, such as products, management, and the strength of company brands in the marketplace

market capitalization the value of a company traded on the stock market, calculated by multiplying the total number of shares by the current share price

NASDAQ (National Association of Securities Dealers Automated Quotations) short for National Association of Securities Dealers Automated Quotations; an American stock exchange based in New York City, ranked second behind the New York Stock Exchange in terms of total market capitalization of shares traded

net book value also called net asset value (NAV); the total assets of a company minus its total liabilities

NYSE (New York Stock Exchange) the world's largest stock exchange by market capitalization of its listed companies, based in New York City; often referred to as the "Big Board"

perpetuity in terms of investments, an annuity with no end date

precedent transaction analysis (precedents) a method for valuating a company in which the price paid for similar companies in the past is considered an indicator of the company's current value

preferred stock stock that entitles the holder to a fixed dividend, the payment of which takes priority over payment of common stock dividends

price-to-book (P/B) ratio also called the market-to-book (M/B) ratio; a metric used to compare a company's market capitalization, or market value, to its book value

price-to-cash-flow (P/CF) ratio a stock valuation indicator or multiple that measures the value of a stock's price relative to its operating cash flow per share

price-to-earnings (P/E) ratio a ratio that indicates the dollar amount an investor can expect to invest in a company in order to receive one dollar of that company's earnings

price-to-sales (P/S) ratio a ratio that indicates how much investors are willing to pay for a company's stock

per dollar of that company's sales

required return the minimum return an investor expects to achieve by investing in a project

secondary markets markets where investors buy and sell securities they already own

stock value also called intrinsic value; the fundamental, objective value of a share of stock

CFA Institute

This chapter supports some of the Learning Outcome Statements (LOS) in this CFA® Level I Study Session (https://openstax.org/r/CFA_Level_I_Study_Session). Reference with permission of CFA Institute.

Multiple Choice

1. The price-to-earnings (P/E) ratio is _____.
   a. used to calculate a company's book value
   b. a multiplier used in enterprise valuation
   c. only applied when valuing preferred stock
   d. a metric for showing the expectations of the market

2. The price-to-book (P/B) ratio is also called the _____ ratio.
   a. market-to-book
   b. enterprise-to-book
   c. asset-to-price
   d. liability-to-book

3. The two primary types of stock valuation multiples are _____.
   a. enterprise value multiples and comparable company value multiples
   b. equity multiples and enterprise value multiples
   c. asset value multiples and liability value multiples
   d. None of the above

4. Dividend yield is a form of equity multiple that is primarily used when _____.
   a. the subject company is operating at a loss
   b. conducting comparisons between companies in different industries
   c. conducting comparisons between cash returns and investment types
   d. analyzing mature companies that have been paying dividends for several years

5. Dividend yield is computed as the proportion of dividend per share to _____.
   a. share price
   b. earnings per share
   c. market value per share
   d. book value per share

6. Which of the following statements about enterprise value metrics is NOT true?
   a. EV to revenue can be used to assess companies with negative cash flows or firms that are currently experiencing financial losses.
   b. ROCE is a profitability ratio that measures the return on the equity in a company in comparison to its financing over a short period of time.
   c. EBIT allows investors to assess the core operations of the business without worrying about the costs of the capital structure.
   d. EBITDAR is a metric that is used in businesses that have substantial rental and least expenses.

7. If an investor's required rate of return increases and all other characteristics of a stock remain the same, the value of the stock will _____.
   a. remain the same
   b. increase
   c. decrease
   d. None of the above

8. A major limitation of the dividend discount model (DDM) is that _____.
   a. it is extremely complicated to use
   b. it cannot be used with companies that do not pay dividends
   c. it must always be used in conjunction with another metric
   d. None of the above

9. In which of the following ways does preferred stock differ from common stock?
   a. Preferred stock carries voting rights for its ownership.
   b. Preferred stock must be purchased through a broker dealer.
   c. Preferred stock may have a cumulative dividend feature.
   d. In the event of corporate liquidation, preferred stockholders are paid last.

10. The term efficient markets refers to the idea that _____.
    a. publicly traded companies always file their financial reports on time
    b. investors are able to identify underpriced stocks for purchase
    c. stocks trade with minimal costs and prices are current and fair to all traders
    d. financial information on companies and their stock is only available to efficient traders

Review Questions

1. Briefly discuss one of the primary benefits of using comparative P/E ratios.
2. Name an important characteristic of companies for which the price-to-book (P/B) ratio does not work well.
3. Briefly describe the main type of scenario in which the two-stage DDM approach might be used to value a firm and its stock.
4. Briefly describe a major shortcoming of the zero growth DDM model.
5. Briefly describe the required inputs for the discounted cash flow (DCF) model.
6. Briefly describe preferred stock and some of its ownership advantages compared to common stock.
7. Briefly explain what is meant by the terms *cumulative* and *noncumulative* as they relate to preferred stocks.
8. What were SuperDOT and SOES, and what were they designed to do?
9. What are operational efficiency and informational efficiency, and how do they differ in terms of trading markets?
10. What is meant by informational efficiency, and how does it affect the price of a stock?

Problems

1. Today, Sysco Enterprises paid dividends on its common stock of $1.25 per share. If dividends per share are expected to increase to $3.50 per share six years from now, what is the percentage dividend growth rate?

2. Let's say you want to purchase shares of Fontaine Ltd. and then hold this stock for six years. The company has a stated dividend policy of $2.00 annually per share for the next six years, at the end of which time you will sell the stock. You expect to be able to sell the stock for $35.00 at that time. If you want to earn an 8% return on this investment, what price should you pay today for this stock?
3. Damian Painting Systems has established a dividend policy of $3.00 per share per year. If the company plans to be in business forever, what is the value of this stock if an investor wants a 10% return?
4. Wilk Productions wants its shareholders to earn a 12% return on their investment in the company. At what value would Wilk stock be priced if the company paid $2.75 per share in constant annual dividends forever?
5. Dax Industrial Systems has stock currently priced at $50.00 per share. If investors are earning a 7% return on Dax Industrial, what is the company's annual dividend payment per share?
6. If a stock is selling at $400 with a current dividend of $40 and a potential investor's required rate of return is 15%, what would be the anticipated dividend growth rate?
7. Mind Max Inc. has a dividend policy that increases annual dividends by 3% each year. If last year's dividend was $2.00, the company intends to stay in business for 50 years, and an investor wants a 9% return, what would be the price of Mind Max stock?
8. Odon Corp. paid dividends today in the amount of $1.50 per share. If Odon will pay dividends of $5.00 10 years from today, what is the annual dividend growth rate over this 10-year period?
9. The Kirkson Distributors common stock is currently selling at $52.00 per share, pays dividends annually at $2.50 per share, and has an annual dividend growth rate of 2%. What is the required return?
10. If a preferred share of stock pays dividends of $2.50 per year and the required rate of return for the stock is 6%, what is its intrinsic value?

Video Activity

Efficient Markets

Click to view content (https://openstax.org/r/Efficient_Markets)

1. In the efficient market hypothesis (EMH), describe what is meant by the terms *weak form efficiency, semi-strong form efficiency*, and *strong form efficiency*. How do these forms of market efficiency differ from each other, and what are their characteristics?
2. What is meant by the term *random walk*, and how does this concept relate to the EMH?

What Is Preferred Stock?

Click to view content (https://openstax.org/r/Preferred_Stock)

3. Discuss the relative risks of the following financial instruments and how they compare to each other: bonds, common stocks, and preferred stocks. How and why will these three investment types typically carry different levels of risk to an investor?
4. Discuss some of the important differences between preferred stocks and common stocks.