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Phys-ics

HIGH SCHOOL

CHAPTER 3
Acceleration

Figure 3.1 A plane slows down as it comes in for landing in St. Maarten. Its acceleration is in the opposite direction of its velocity. (Steve Conry, Flickr)

Chapter Outline

INTRODUCTION You may have heard the term *accelerator*, referring to the gas pedal in a car. When the gas pedal is pushed down, the flow of gasoline to the engine increases, which increases the car's velocity. Pushing on the gas pedal results in acceleration because the velocity of the car increases, and acceleration is defined as a change in velocity. You need two quantities to define velocity: a speed and a direction. Changing either of these quantities, or both together, changes the velocity. You may be surprised to learn that pushing on the brake pedal or turning the steering wheel also causes acceleration. The first reduces the *speed* and so changes the velocity, and the second changes the *direction* and also changes the velocity.

In fact, any change in velocity—whether positive, negative, directional, or any combination of these—is called an acceleration in physics. The plane in the picture is said to be accelerating because its velocity is decreasing as it prepares to land. To begin our study of acceleration, we need to have a clear understanding of what acceleration means.

3.1 Acceleration

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain acceleration and determine the direction and magnitude of acceleration in one dimension
- Analyze motion in one dimension using kinematic equations and graphic representations

Section Key Terms

average acceleration   instantaneous acceleration   negative acceleration

Defining Acceleration

Throughout this chapter we will use the following terms: *time*, *displacement*, *velocity*, and *acceleration*. Recall that each of these terms has a designated variable and SI unit of measurement as follows:

- Time: t, measured in seconds (s)
- Displacement: Δd, measured in meters (m)
- Velocity: v, measured in meters per second (m/s)
- Acceleration: a, measured in meters per second per second (m/s^2, also called meters per second squared)
- Also note the following:
  - Δ means *change in*
  - The subscript 0 refers to an initial value; sometimes subscript i is instead used to refer to initial value.
  - The subscript f refers to final value
  - A bar over a symbol, such as $\overline{a}$, means *average*

Acceleration is the change in velocity divided by a period of time during which the change occurs. The SI units of velocity are m/s and the SI units for time are s, so the SI units for acceleration are m/s^2. **Average acceleration** is given by

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

Average acceleration is distinguished from **instantaneous acceleration**, which is acceleration at a specific instant in time. The magnitude of acceleration is often not constant over time. For example, runners in a race accelerate at a greater rate in the first second of a race than during the following seconds. You do not need to know all the instantaneous accelerations at all times to calculate average acceleration. All you need to know is the change in velocity (i.e., the final velocity minus the initial velocity) and the change in time (i.e., the final time minus the initial time), as shown in the formula. Note that the average acceleration can be positive, negative, or zero. A **negative acceleration** is simply an acceleration in the negative direction.

Keep in mind that although acceleration points in the same direction as the *change* in velocity, it is not always in the direction of the velocity itself. When an object slows down, its acceleration is opposite to the direction of its velocity. In everyday language, this is called deceleration; but in physics, it is acceleration—whose direction happens to be opposite that of the velocity. For now, let us assume that motion to the right along the x-axis is *positive* and motion to the left is *negative*.

Figure 3.2 shows a car with positive acceleration in (a) and negative acceleration in (b). The arrows represent vectors showing both direction and magnitude of velocity and acceleration.

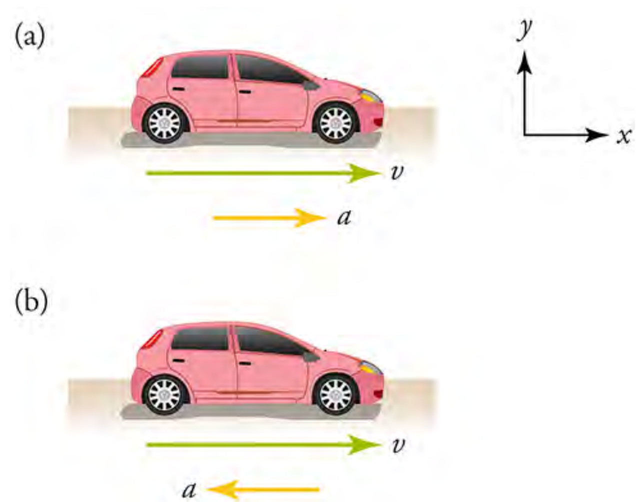


Figure 3.2 The car is speeding up in (a) and slowing down in (b).

Velocity and acceleration are both vector quantities. Recall that vectors have both magnitude and direction. An object traveling at a constant velocity—therefore having no acceleration—does accelerate if it changes direction. So, turning the steering wheel of a moving car makes the car accelerate because the velocity changes direction.

Virtual Physics

The Moving Man

With this animation in , you can produce both variations of acceleration and velocity shown in Figure 3.2, plus a few more variations. Vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Try changing acceleration from positive to negative while the man is moving. We will come back to this animation and look at the *Charts* view when we study graphical representation of motion.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

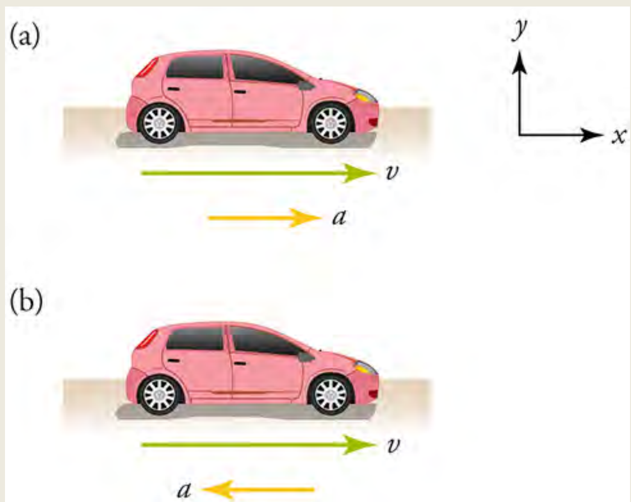


Figure 3.3

Which part, (a) or (b), is represented when the velocity vector is on the positive side of the scale and the acceleration vector is set on the negative side of the scale? What does the car's motion look like for the given scenario?

a. Part (a). The car is slowing down because the acceleration and the velocity vectors are acting in the opposite direction.
b. Part (a). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.
c. Part (b). The car is slowing down because the acceleration and velocity vectors are acting in the opposite directions.
d. Part (b). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.

Calculating Average Acceleration

Look back at the equation for average acceleration. You can see that the calculation of average acceleration involves three values: change in time, (Δt); change in velocity, (Δv); and acceleration (a).

Change in time is often stated as a time interval, and change in velocity can often be calculated by subtracting the initial velocity from the final velocity. Average acceleration is then simply change in velocity divided by change in time. Before you begin calculating, be sure that all distances and times have been converted to meters and seconds. Look at these examples of acceleration of a subway train.

WORKED EXAMPLE

An Accelerating Subway Train

A subway train accelerates from rest to 30.0 km/h in 20.0 s. What is the average acceleration during that time interval?

Strategy

Start by making a simple sketch.

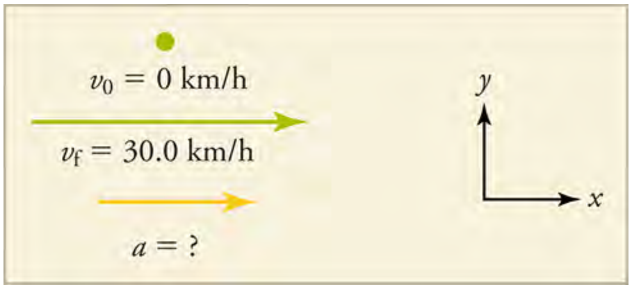


Figure 3.4

This problem involves four steps:

1. Convert to units of meters and seconds.
2. Determine the change in velocity.
3. Determine the change in time.
4. Use these values to calculate the average acceleration.

Solution

1. Identify the knowns. Be sure to read the problem for given information, which may not *look* like numbers. When the problem states that the train starts from rest, you can write down that the initial velocity is 0 m/s. Therefore, $v_0 = 0$; $v_f =$ 30.0 km/h; and $\Delta t = 20.0$ s.
2. Convert the units.

$$\frac{30.0\text{ km}}{\text{h}} \times \frac{10^3\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 8.333\ \frac{\text{m}}{\text{s}} \tag{3.1}$$

3. Calculate change in velocity, $\Delta v = v_f - v_0 = 8.333\text{ m/s} - 0 = +\,8.333\text{ m/s}$, where the plus sign means the change in velocity is to the right.
4. We know Δt, so all we have to do is insert the known values into the formula for average acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8.333\text{ m/s}}{20.00\text{ s}} = +0.417\frac{\text{m}}{\text{s}^2} \tag{3.2}$$

Discussion

The plus sign in the answer means that acceleration is to the right. This is a reasonable conclusion because the train starts from rest and ends up with a velocity directed to the right (i.e., positive). So, acceleration is in the same direction as the *change* in velocity, as it should be.

WORKED EXAMPLE

An Accelerating Subway Train

Now, suppose that at the end of its trip, the train slows to a stop in 8.00 s from a speed of 30.0 km/h. What is its average acceleration during this time?

Strategy

Again, make a simple sketch.

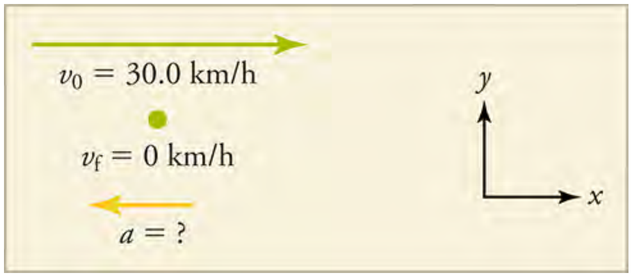


Figure 3.5

In this case, the train is decelerating and its acceleration is negative because it is pointing to the left. As in the previous example, we must find the change in velocity and change in time, then solve for acceleration.

Solution

1. Identify the knowns: $v_0 = 30.0$ km/h; $v_f = 0$; and $\Delta t = 8.00$ s.
2. Convert the units. From the first problem, we know that 30.0 km/h = 8.333 m/s.
3. Calculate change in velocity, $\Delta v = v_f - v_0 = 0 - 8.333 \text{ m/s} = -8.333 \text{ m/s}$, where the minus sign means that the change in velocity points to the left.
4. We know $\Delta t = 8.00$ s, so all we have to do is insert the known values into the equation for average acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-8.333 \text{ m/s}}{8.00 \text{ s}} = -1.04\frac{\text{m}}{\text{s}^2} \quad (3.3)$$

Discussion

The minus sign indicates that acceleration is to the left. This is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would reduce the velocity. Again, acceleration is in the same direction as the *change* in velocity, which is negative in this case. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

TIPS FOR SUCCESS

- It is easier to get plus and minus signs correct if you always assume that motion is away from zero and toward positive values on the *x*-axis. This way *v* always starts off being positive and points to the right. If speed is increasing, then acceleration is positive and also points to the right. If speed is decreasing, then acceleration is negative and points to the left.
- It is a good idea to carry two extra significant figures from step-to-step when making calculations. Do not round off with each step. When you arrive at the final answer, apply the rules of significant figures for the operations you carried out and round to the correct number of digits. Sometimes this will make your answer slightly more accurate.

Practice Problems

1. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

a. -0.23 m/s^2
b. -4.29 m/s^2
c. 0.23 m/s^2
d. 4.29 m/s^2

2. A women backs her car out of her garage with an acceleration of 1.40 m/s^2. How long does it take her to reach a speed of 2.00 m/s?

a. 0.70 s
b. 1.43 s
c. 2.80 s
d. 3.40 s

WATCH PHYSICS

Acceleration

This video shows the basic calculation of acceleration and some useful unit conversions.

Click to view content (https://www.khanacademy.org/embed_video?v=FOkQszg1-j8)

GRASP CHECK

Why is acceleration a vector quantity?

a. It is a vector quantity because it has magnitude as well as direction.
b. It is a vector quantity because it has magnitude but no direction.
c. It is a vector quantity because it is calculated from distance and time.
d. It is a vector quantity because it is calculated from speed and time.

GRASP CHECK

What will be the change in velocity each second if acceleration is 10 m/s/s?

a. An acceleration of $10\ \text{m/s/s}$ means that every second, the velocity increases by $10\ \text{m/s}$.
b. An acceleration of $10\ \text{m/s/s}$ means that every second, the velocity decreases by $10\ \text{m/s}$.
c. An acceleration of $10\ \text{m/s/s}$ means that every $10\ \text{seconds}$, the velocity increases by $10\ \text{m/s}$.
d. An acceleration of $10\ \text{m/s/s}$ means that every $10\ \text{seconds}$, the velocity decreases by $10\ \text{m/s}$.

Snap Lab

Measure the Acceleration of a Bicycle on a Slope

In this lab you will take measurements to determine if the acceleration of a moving bicycle is constant. If the acceleration is constant, then the following relationships hold: $\overline{v} = \frac{\Delta d}{\Delta t} = \frac{v_0 + v_f}{2}$ If $v_0 = 0$, then $v_f = 2\overline{v}$ and $\overline{a} = \frac{v_f}{\Delta t}$

You will work in pairs to measure and record data for a bicycle coasting down an incline on a smooth, gentle slope. The data will consist of distances traveled and elapsed times.

- Find an open area to minimize the risk of injury during this lab.

- stopwatch
- measuring tape
- bicycle

1. Find a gentle, paved slope, such as an incline on a bike path. The more gentle the slope, the more accurate your data will likely be.
2. Mark uniform distances along the slope, such as 5 m, 10 m, etc.
3. Determine the following roles: the bike rider, the timer, and the recorder. The recorder should create a data table to collect the distance and time data.
4. Have the rider at the starting point at rest on the bike. When the timer calls *Start*, the timer starts the stopwatch and the rider begins coasting down the slope on the bike without pedaling.
5. Have the timer call out the elapsed times as the bike passes each marked point. The recorder should record the times in the data table. It may be necessary to repeat the process to practice roles and make necessary adjustments.
6. Once acceptable data has been recorded, switch roles. Repeat Steps 3–5 to collect a second set of data.
7. Switch roles again to collect a third set of data.
8. Calculate average acceleration for each set of distance-time data. If your result for $\overline{a}$ is not the same for different pairs of Δv and Δt, then acceleration is not constant.
9. Interpret your results.

GRASP CHECK

If you graph the average velocity (y-axis) vs. the elapsed time (x-axis), what would the graph look like if acceleration is uniform?

a. a horizontal line on the graph
b. a diagonal line on the graph
c. an upward-facing parabola on the graph
d. a downward-facing parabola on the graph

Check Your Understanding

3. What are three ways an object can accelerate?
   a. By speeding up, maintaining constant velocity, or changing direction
   b. By speeding up, slowing down, or changing direction
   c. By maintaining constant velocity, slowing down, or changing direction
   d. By speeding up, slowing down, or maintaining constant velocity

4. What is the difference between average acceleration and instantaneous acceleration?
   a. Average acceleration is the change in displacement divided by the elapsed time; instantaneous acceleration is the acceleration at a given point in time.
   b. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in displacement divided by the elapsed time.
   c. Average acceleration is the change in velocity divided by the elapsed time; instantaneous acceleration is acceleration at a given point in time.
   d. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in velocity divided by the elapsed time.

5. What is the rate of change of velocity called?
   a. Time
   b. Displacement
   c. Velocity
   d. Acceleration

3.2 Representing Acceleration with Equations and Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the kinematic equations related to acceleration and illustrate them with graphs
- Apply the kinematic equations and related graphs to problems involving acceleration

Section Key Terms

acceleration due to gravity	kinematic equations	uniform acceleration

How the Kinematic Equations are Related to Acceleration

We are studying concepts related to motion: time, displacement, velocity, and especially acceleration. We are only concerned with motion in one dimension. The **kinematic equations** apply to conditions of constant acceleration and show how these concepts are related. **Constant acceleration** is acceleration that does not change over time. The first kinematic equation relates displacement d, average velocity $\overline{v}$, and time t.

$$d = d_0 + \overline{v}t \tag{3.4}$$

The initial displacement d_0 is often 0, in which case the equation can be written as $\overline{v} = \frac{d}{t}$

This equation says that average velocity is displacement per unit time. We will express velocity in meters per second. If we graph displacement versus time, as in Figure 3.6, the slope will be the velocity. Whenever a rate, such as velocity, is represented graphically, time is usually taken to be the independent variable and is plotted along the *x* axis.

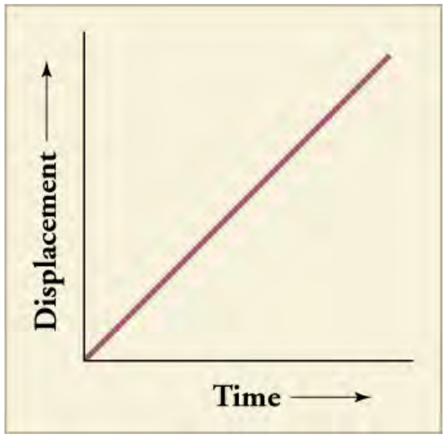


Figure 3.6 The slope of displacement versus time is velocity.

The second kinematic equation, another expression for average velocity $\overline{v}$, is simply the initial velocity plus the final velocity divided by two.

$$\overline{v} = \frac{v_0 + v_\mathrm{f}}{2} \tag{3.5}$$

Now we come to our main focus of this chapter; namely, the kinematic equations that describe motion with constant acceleration. In the third kinematic equation, acceleration is the rate at which velocity increases, so velocity at any point equals initial velocity plus acceleration multiplied by time

$$v = v_0 + at \text{ Also, if we start from rest } (v_0 = 0 \text{), we can write } a = \frac{v}{t} \tag{3.6}$$

Note that this third kinematic equation does not have displacement in it. Therefore, if you do not know the displacement and are not trying to solve for a displacement, this equation might be a good one to use.

The third kinematic equation is also represented by the graph in Figure 3.7.

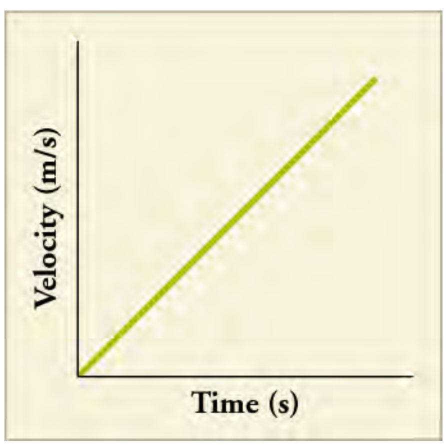


Figure 3.7 The slope of velocity versus time is acceleration.

The fourth kinematic equation shows how displacement is related to acceleration

$$d = d_0 + v_0 t + \frac{1}{2}at^2. \tag{3.7}$$

When starting at the origin, $d_0 = 0$ and, when starting from rest, $v_0 = 0$, in which case the equation can be written as

$$a = \frac{2d}{t^2}.$$

This equation tells us that, for constant acceleration, the slope of a plot of $2d$ versus t^2 is acceleration, as shown in Figure 3.8.

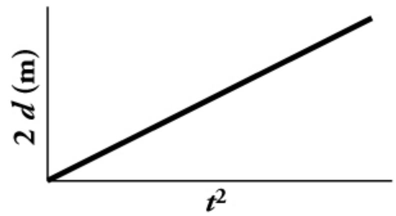


Figure 3.8 When acceleration is constant, the slope of $2d$ versus t^2 gives the acceleration.

The fifth kinematic equation relates velocity, acceleration, and displacement

$$v^2 = v_0^2 + 2a\left(d - d_0\right). \quad (3.8)$$

This equation is useful for when we do not know, or do not need to know, the time.

When starting from rest, the fifth equation simplifies to

$$a = \frac{v^2}{2d}.$$

According to this equation, a graph of velocity squared versus twice the displacement will have a slope equal to acceleration.

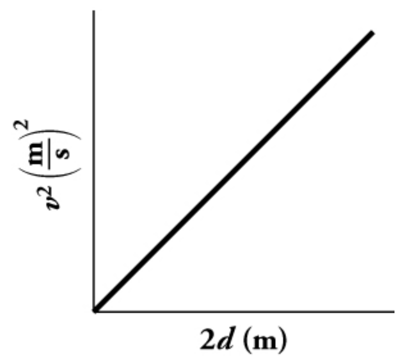


Figure 3.9

Note that, in reality, knowns and unknowns will vary. Sometimes you will want to rearrange a kinematic equation so that the knowns are the values on the axes and the unknown is the slope. Sometimes the intercept will not be at the origin (0,0). This will happen when v_0 or d_0 is not zero. This will be the case when the object of interest is already in motion, or the motion begins at some point other than at the origin of the coordinate system.

Virtual Physics

The Moving Man (Part 2)

Look at the Moving Man simulation again and this time use the *Charts* view. Again, vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Observe how the graphs of position, velocity, and acceleration vary with time. Note which are linear plots and which are not.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

On a velocity versus time plot, what does the slope represent?

a. Acceleration

b. Displacement
c. Distance covered
d. Instantaneous velocity

GRASP CHECK

On a position versus time plot, what does the slope represent?

a. Acceleration
b. Displacement
c. Distance covered
d. Instantaneous velocity

The kinematic equations are applicable when you have constant acceleration.

1. $d = d_0 + \bar{v}t$, or $\bar{v} = \frac{d}{t}$ when $d_0 = 0$
2. $\bar{v} = \frac{v_0 + v_f}{2}$
3. $v = v_0 + at$, or $a = \frac{v}{t}$ when $v_0 = 0$
4. $d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$
5. $v^2 = v_0^2 + 2a(d - d_0)$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$

Applying Kinematic Equations to Situations of Constant Acceleration

Problem-solving skills are essential to success in a science and life in general. The ability to apply broad physical principles, which are often represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Essential analytical skills will be developed by solving problems in this text and will be useful for understanding physics and science in general throughout your life.

Problem-Solving Steps

While no single step-by-step method works for every problem, the following general procedures facilitate problem solving and make the answers more meaningful. A certain amount of creativity and insight are required as well.

1. *Examine the situation to determine which physical principles are involved. It is vital to draw a simple sketch at the outset.* Decide which direction is positive and note that on your sketch.
2. *Identify the knowns: Make a list of what information is given or can be inferred from the problem statement.* Remember, not all given information will be in the form of a number with units in the problem. If something starts *from rest*, we know the initial velocity is zero. If something *stops*, we know the final velocity is zero.
3. *Identify the unknowns: Decide exactly what needs to be determined in the problem.*
4. *Find an equation or set of equations that can help you solve the problem.* Your list of knowns and unknowns can help here. For example, if time is not needed or not given, then the fifth kinematic equation, which does not include time, could be useful.
5. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made.
6. *Check the answer to see if it is reasonable: Does it make sense?* This final step is extremely important because the goal of physics is to accurately describe nature. To see if the answer is reasonable, check its magnitude, its sign, and its units. Are the significant figures correct?

Summary of Problem Solving

- Determine the knowns and unknowns.
- Find an equation that expresses the unknown in terms of the knowns. More than one unknown means more than one equation is needed.
- Solve the equation or equations.

- Be sure units and significant figures are correct.
- Check whether the answer is reasonable.

FUN IN PHYSICS

Drag Racing

Figure 3.10 Smoke rises from the tires of a dragster at the beginning of a drag race. (Lt. Col. William Thurmond. Photo courtesy of U.S. Army.)

The object of the sport of drag racing is acceleration. Period! The races take place from a standing start on a straight one-quarter-mile (402 m) track. Usually two cars race side by side, and the winner is the driver who gets the car past the quarter-mile point first. At the finish line, the cars may be going more than 300 miles per hour (134 m/s). The driver then deploys a parachute to bring the car to a stop because it is unsafe to brake at such high speeds. The cars, called dragsters, are capable of accelerating at 26 m/s^2. By comparison, a typical sports car that is available to the general public can accelerate at about 5 m/s^2.

Several measurements are taken during each drag race:

- Reaction time is the time between the starting signal and when the front of the car crosses the starting line.
- Elapsed time is the time from when the vehicle crosses the starting line to when it crosses the finish line. The record is a little over 3 s.
- Speed is the average speed during the last 20 m before the finish line. The record is a little under 400 mph.

The video shows a race between two dragsters powered by jet engines. The actual race lasts about four seconds and is near the end of the video (https://openstax.org/l/28dragsters) .

GRASP CHECK

A dragster crosses the finish line with a velocity of 140 m/s. Assuming the vehicle maintained a constant acceleration from start to finish, what was its average velocity for the race?

a. 0 m/s
b. 35 m/s
c. 70 m/s
d. 140 m/s

WORKED EXAMPLE

Acceleration of a Dragster

The time it takes for a dragster to cross the finish line is unknown. The dragster accelerates from rest at 26 m/s^2 for a quarter mile (0.250 mi). What is the final velocity of the dragster?

Strategy

The equation $v^2 = v_0^2 + 2a\left(d - d_0\right)$ is ideally suited to this task because it gives the velocity from acceleration and displacement, without involving the time.

Solution

1. Convert miles to meters.

$$(0.250\ \ \text{mi}) \times \frac{1609\ \ \text{m}}{1\ \ \text{mi}} = 402\ \ \text{m} \tag{3.9}$$

2. Identify the known values. We know that $v_0 = 0$ since the dragster starts from rest, and we know that the distance traveled, $d - d_0$ is 402 m. Finally, the acceleration is constant at $a = 26.0\ \text{m/s}^2$.
3. Insert the knowns into the equation $v^2 = v_0^2 + 2a\,(d - d_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0\frac{\text{m}}{\text{s}^2}\right)(402\ \ \text{m}) = 2.09 \times 10^4\,\frac{\text{m}^2}{\text{s}^2} \tag{3.10}$$

Taking the square root gives us $v = \sqrt{2.09 \times 10^4\,\frac{\text{m}^2}{\text{s}^2}} = 145\,\frac{\text{m}}{\text{s}}$.

Discussion

145 m/s is about 522 km/hour or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values. We took the positive value because we know that the velocity must be in the same direction as the acceleration for the answer to make physical sense.

An examination of the equation $v^2 = v_0^2 + 2a\,(d - d_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on the magnitude of the acceleration and the distance over which it applies.
- For a given acceleration, a car that is going twice as fast does not stop in twice the distance—it goes much further before it stops. This is why, for example, we have reduced speed zones near schools.

Practice Problems

6. Dragsters can reach a top speed of 145 m/s in only 4.45 s. Calculate the average acceleration for such a dragster.

a. $-32.6\ \text{m/s}^2$
b. $0\ \text{m/s}^2$
c. $32.6\ \text{m/s}^2$
d. $145\ \text{m/s}^2$

7. An Olympic-class sprinter starts a race with an acceleration of $4.50\ \text{m/s}^2$. Assuming she can maintain that acceleration, what is her speed 2.40 s later?

a. 4.50 m/s
b. 10.8 m/s
c. 19.6 m/s
d. 44.1 m/s

Constant Acceleration

In many cases, acceleration is not uniform because the force acting on the accelerating object is not constant over time. A situation that gives constant acceleration is the acceleration of falling objects. When air resistance is not a factor, all objects near Earth's surface fall with an acceleration of about $9.80\ \text{m/s}^2$. Although this value decreases slightly with increasing altitude, it may be assumed to be essentially constant. The value of $9.80\ \text{m/s}^2$ is labeled g and is referred to as **acceleration due to gravity**. Gravity is the force that causes nonsupported objects to accelerate downward—or, more precisely, toward the center of Earth. The magnitude of this force is called the weight of the object and is given by mg where m is the mass of the object (in kg). In places other than on Earth, such as the Moon or on other planets, g is not $9.80\ \text{m/s}^2$, but takes on other values. When using g for the acceleration a in a kinematic equation, it is usually given a negative sign because the acceleration due to gravity is downward.

WORK IN PHYSICS

Effects of Rapid Acceleration

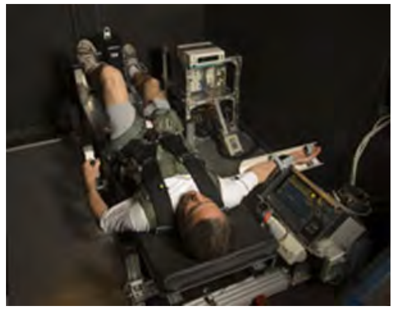


Figure 3.11 Astronauts train using G Force Simulators. (NASA)

When in a vehicle that accelerates rapidly, you experience a force on your entire body that accelerates your body. You feel this force in automobiles and slightly more on amusement park rides. For example, when you ride in a car that turns, the car applies a force on your body to make you accelerate in the direction in which the car is turning. If enough force is applied, you will accelerate at 9.80 m/s^2. This is the same as the acceleration due to gravity, so this force is called one G.

One G is the force required to accelerate an object at the acceleration due to gravity at Earth's surface. Thus, one G for a paper cup is much less than one G for an elephant, because the elephant is much more massive and requires a greater force to make it accelerate at 9.80 m/s^2. For a person, a G of about 4 is so strong that his or her face will distort as the bones accelerate forward through the loose flesh. Other symptoms at extremely high Gs include changes in vision, loss of consciousness, and even death. The space shuttle produces about 3 Gs during takeoff and reentry. Some roller coasters and dragsters produce forces of around 4 Gs for their occupants. A fighter jet can produce up to 12 Gs during a sharp turn.

Astronauts and fighter pilots must undergo G-force training in simulators. The video (https://www.youtube.com/watch?v=n-8QHOUWECU) shows the experience of several people undergoing this training.

People, such as astronauts, who work with G forces must also be trained to experience zero G—also called free fall or weightlessness—which can cause queasiness. NASA has an aircraft that allows it occupants to experience about 25 s of free fall. The aircraft is nicknamed the *Vomit Comet*.

GRASP CHECK

A common way to describe acceleration is to express it in multiples of g, Earth's gravitational acceleration. If a dragster accelerates at a rate of 39.2 m/s^2, how many g's does the driver experience?

a. 1.5 g
b. 4.0 g
c. 10.5 g
d. 24.5 g

WORKED EXAMPLE

Falling Objects

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity v_0 of 13 m/s.
(a) Calculate the position and velocity of the rock at 1.00, 2.00, and 3.00 seconds after it is thrown. Ignore the effect of air resistance.

Strategy

Sketch the initial velocity and acceleration vectors and the axes.

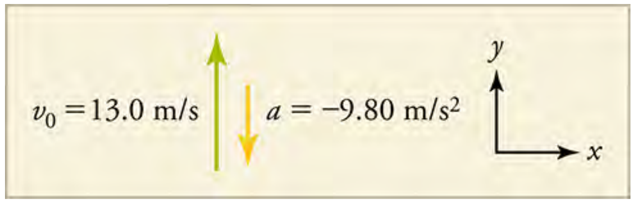


Figure 3.12 Initial conditions for rock thrown straight up.

List the knowns: time $t = 1.00$ s, 2.00 s, and 3.00 s; initial velocity $v_0 = 13$ m/s; acceleration $a = g = -9.80$ m/s^2; and position $y_0 = 0$ m

List the unknowns: y_1, y_2, and y_3; v_1, v_2, and v_3—where 1, 2, 3 refer to times 1.00 s, 2.00 s, and 3.00 s

Choose the equations.

$$d = d_0 + v_0 t + \frac{1}{2}at^2 \text{ becomes } y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{3.11}$$

$$v = v_0 + at \text{ becomes } v = v_0 + -gt \tag{3.12}$$

These equations describe the unknowns in terms of knowns only.

Solution

$$\begin{aligned}
y_1 &= 0 + (13.0 \text{ m/s})(1.00 \text{ s}) + \frac{(-9.80 \text{m/s}^2)(1.00 \text{ s})^2}{2} = 8.10 \text{ m} \\
y_2 &= 0 + (13.0 \text{ m/s})(2.00 \text{ s}) + \frac{(-9.80 \text{m/s}^2)(2.00 \text{ s})^2}{2} = 6.40 \text{ m} \\
y_3 &= 0 + (13.0 \text{ m/s})(3.00 \text{ s}) + \frac{(-9.80 \text{m/s}^2)(3.00 \text{ s})^2}{2} = -5.10 \text{ m} \\
v_1 &= 13.0 \text{ m/s} + \left(-9.80 \text{m/s}^2\right)(1.00 \text{ s}) = 3.20 \text{ m/s} \\
v_2 &= 13.0 \text{ m/s} + \left(-9.80 \text{m/s}^2\right)(2.00 \text{ s}) = -6.60 \text{ m/s} \\
v_3 &= 13.0 \text{ m/s} + \left(-9.80 \text{m/s}^2\right)(3.00 \text{ s}) = -16.4 \text{ m/s}
\end{aligned}$$

Discussion

The first two positive values for y show that the rock is still above the edge of the cliff, and the third negative y value shows that it has passed the starting point and is below the cliff. Remember that we set *up* to be positive. Any position with a positive value is above the cliff, and any velocity with a positive value is an upward velocity. The first value for v is positive, so the rock is still on the way up. The second and third values for v are negative, so the rock is on its way down.

(b) Make graphs of position versus time, velocity versus time, and acceleration versus time. Use increments of 0.5 s in your graphs.

Strategy

Time is customarily plotted on the *x*-axis because it is the independent variable. Position versus time will not be linear, so calculate points for 0.50 s, 1.50 s, and 2.50 s. This will give a curve closer to the true, smooth shape.

Solution

The three graphs are shown in Figure 3.13.

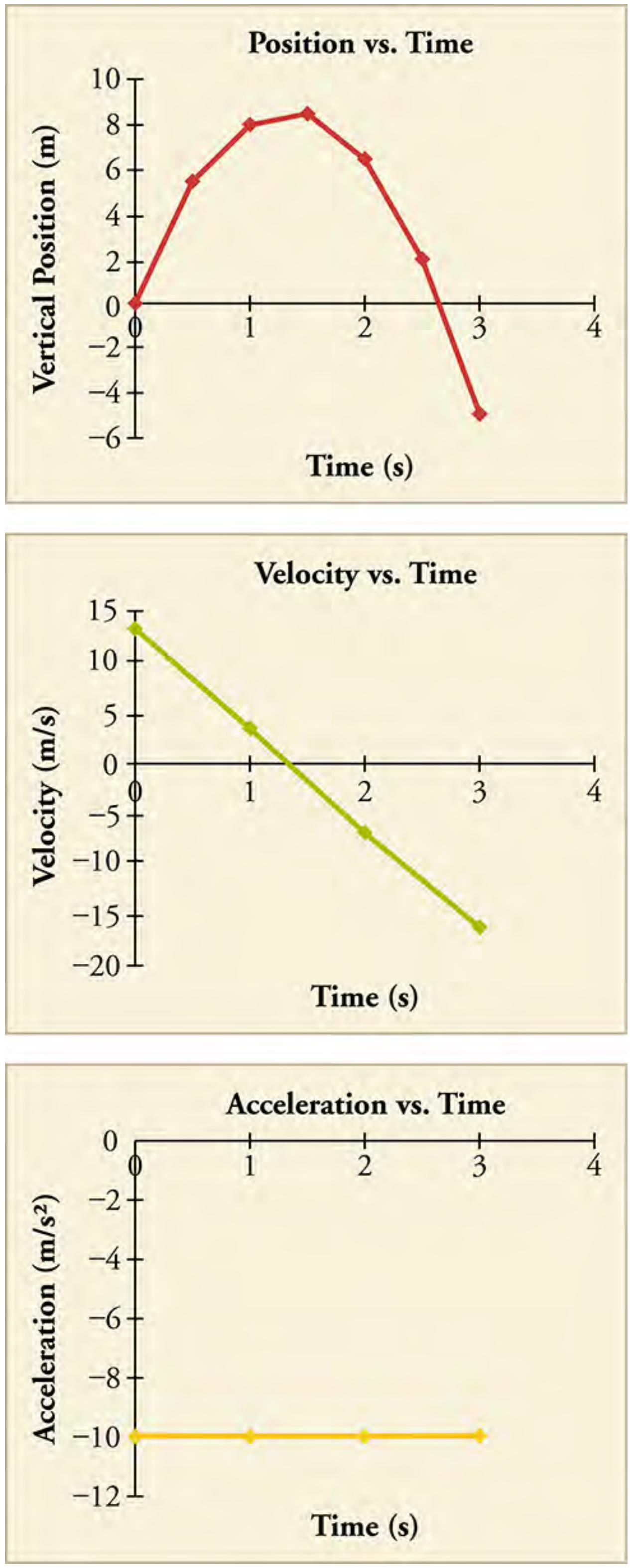


Figure 3.13

Discussion

- *y* vs. *t* does *not* represent the two-dimensional parabolic path of a trajectory. The path of the rock is straight up and straight down. The slope of a line tangent to the curve at any point on the curve equals the velocity at that point—i.e., the instantaneous velocity.

- Note that the v vs. t line crosses the vertical axis at the initial velocity and crosses the horizontal axis at the time when the rock changes direction and begins to fall back to Earth. This plot is linear because acceleration is constant.
- The a vs. t plot also shows that acceleration is constant; that is, it does not change with time.

Practice Problems

8. A cliff diver pushes off horizontally from a cliff and lands in the ocean 2.00 s later. How fast was he going when he entered the water?

a. 0 m/s
b. 19.0 m/s
c. 19.6 m/s
d. 20.0 m/s

9. A girl drops a pebble from a high cliff into a lake far below. She sees the splash of the pebble hitting the water 2.00 s later. How fast was the pebble going when it hit the water?

a. 9.80 m/s
b. 10.0 m/s
c. 19.6 m/s
d. 20.0 m/s

Check Your Understanding

10. Identify the four variables found in the kinematic equations.

a. Displacement, Force, Mass, and Time
b. Acceleration, Displacement, Time, and Velocity
c. Final Velocity, Force, Initial Velocity, and Mass
d. Acceleration, Final Velocity, Force, and Initial Velocity

11. Which of the following steps is always required to solve a kinematics problem?

a. Find the force acting on the body.
b. Find the acceleration of a body.
c. Find the initial velocity of a body.
d. Find a suitable kinematic equation and then solve for the unknown quantity.

12. Which of the following provides a correct answer for a problem that can be solved using the kinematic equations?

a. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 8 m/s.
b. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 16 m/s.
c. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 2 m/s^2.
d. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 0.5 m/s^2.

KEY TERMS

acceleration due to gravity acceleration of an object that is subject only to the force of gravity; near Earth's surface this acceleration is 9.80 m/s^2

average acceleration change in velocity divided by the time interval over which it changed

constant acceleration acceleration that does not change with respect to time

instantaneous acceleration rate of change of velocity at a specific instant in time

kinematic equations the five equations that describe motion in terms of time, displacement, velocity, and acceleration

negative acceleration acceleration in the negative direction

SECTION SUMMARY

3.1 Acceleration

- Acceleration is the rate of change of velocity and may be negative or positive.
- Average acceleration is expressed in m/s^2 and, in one dimension, can be calculated using $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$.

3.2 Representing Acceleration with Equations and Graphs

- The kinematic equations show how time, displacement, velocity, and acceleration are related for objects in motion.
- In general, kinematic problems can be solved by identifying the kinematic equation that expresses the unknown in terms of the knowns.
- Displacement, velocity, and acceleration may be displayed graphically versus time.

KEY EQUATIONS

3.1 Acceleration

Average acceleration	$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$

3.2 Representing Acceleration with Equations and Graphs

Average velocity	$d = d_0 + \overline{v}t$, or $\overline{v} = \frac{d}{t}$ when $d_0 = 0$
Average velocity	$\overline{v} = \frac{v_0 + v_f}{2}$
Velocity	$v = v_0 + at$, or when $v_0 = 0$
Displacement	$d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$
Acceleration	$v^2 = v_0^2 + 2a(d - d_0)$, or $a = \frac{v^2}{2d}$ when $d_0 = 0$ and $v_0 = 0$

CHAPTER REVIEW

Concept Items

3.1 Acceleration

1. How can you use the definition of acceleration to explain the units in which acceleration is measured?

a. Acceleration is the rate of change of velocity. Therefore, its unit is m/s^2.
b. Acceleration is the rate of change of displacement. Therefore, its unit is m/s.
c. Acceleration is the rate of change of velocity. Therefore, its unit is m^2/s.
d. Acceleration is the rate of change of displacement. Therefore, its unit is m^2/s.

2. What are the SI units of acceleration?

a. m^2/s
b. cm^2/s
c. m/s^2
d. cm/s^2

3. Which of the following statements explains why a racecar going around a curve is accelerating, even if the speed is constant?

a. The car is accelerating because the magnitude as well as the direction of velocity is changing.
b. The car is accelerating because the magnitude of velocity is changing.
c. The car is accelerating because the direction of velocity is changing.

d. The car is accelerating because neither the magnitude nor the direction of velocity is changing.

3.2 Representing Acceleration with Equations and Graphs

4. A student calculated the final velocity of a train that decelerated from 30.5 m/s and got an answer of −43.34 m/s. Which of the following might indicate that he made a mistake in his calculation?
   a. The sign of the final velocity is wrong.
   b. The magnitude of the answer is too small.
   c. There are too few significant digits in the answer.
   d. The units in the initial velocity are incorrect.

5. Create your own kinematics problem. Then, create a flow chart showing the steps someone would need to take to solve the problem.
   a. Acceleration
   b. Distance
   c. Displacement
   d. Force

6. Which kinematic equation would you use to find the velocity of a skydiver 2.0 s after she jumps from a plane and before she opens her parachute? Assume the positive direction is downward.
   a. $v = v_0 + at$
   b. $v = v_0 - at$
   c. $v^2 = {v_0}^2 + at$
   d. $v^2 = {v_0}^2 - at$

Critical Thinking Items

3.1 Acceleration

7. Imagine that a car is traveling from your left to your right at a constant velocity. Which two actions could the driver take that may be represented as (a) a velocity vector and an acceleration vector both pointing to the right and then (b) changing so the velocity vector points to the right and the acceleration vector points to the left?
   a. (a) Push down on the accelerator and then (b) push down again on the accelerator a second time.
   b. (a) Push down on the accelerator and then (b) push down on the brakes.
   c. (a) Push down on the brakes and then (b) push down on the brakes a second time.
   d. (a) Push down on the brakes and then (b) push down on the accelerator.

8. A motorcycle moving at a constant velocity suddenly accelerates at a rate of 4.0 m/s^2 to a speed of 35 m/s in 5.0 s. What was the initial speed of the motorcycle?
   a. -34 m/s
   b. -15 m/s
   c. 15 m/s
   d. 34 m/s

3.2 Representing Acceleration with Equations and Graphs

9. A student is asked to solve a problem:
   An object falls from a height for 2.0 s, at which point it is still 60 m above the ground. What will be the velocity of the object when it hits the ground?
   Which of the following provides the correct order of kinematic equations that can be used to solve the problem?
   a. First use $v^2 = {v_0}^2 + 2a(d - d_0)$, then use $v = v_0 + at$.
   b. First use $v = v_0 + at$, then use $v^2 = {v_0}^2 + 2a(d - d_0)$.
   c. First use $d = d_0 + v_0 t + \frac{1}{2}at^2$, then use $v = v_0 + at$.
   d. First use $v = v_0 + at$, then use $d - d_0 = v_0 t + \frac{1}{2}at^2$.

10. Skydivers are affected by acceleration due to gravity and by air resistance. Eventually, they reach a speed where the force of gravity is almost equal to the force of air resistance. As they approach that point, their acceleration decreases in magnitude to near zero.
   Part A. Describe the shape of the graph of the magnitude of the acceleration versus time for a falling skydiver.
   Part B. Describe the shape of the graph of the magnitude of the velocity versus time for a falling skydiver.
   Part C. Describe the shape of the graph of the magnitude of the displacement versus time for a falling skydiver.
   a. Part A. Begins with a nonzero y-intercept with a downward slope that levels off at zero; Part B. Begins at zero with an upward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant
   b. Part A. Begins with a nonzero y-intercept with an upward slope that levels off at zero; Part B. Begins at zero with an upward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant
   c. Part A. Begins with a nonzero y-intercept with a downward slope that levels off at zero; Part B. Begins at zero with a downward slope that

decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant

d. Part A. Begins with a nonzero y-intercept with an upward slope that levels off at zero; Part B. Begins at zero with a downward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant

11. Which graph in the previous problem has a positive slope?

a. Displacement versus time only
b. Acceleration versus time and velocity versus time
c. Velocity versus time and displacement versus time
d. Acceleration versus time and displacement versus time

Problems

3.1 Acceleration

12. The driver of a sports car traveling at 10.0 m/s steps down hard on the accelerator for 5.0 s and the velocity increases to 30.0 m/s. What was the average acceleration of the car during the 5.0 s time interval?

a. $-1.0 \times 102\ m/s^2$
b. $-4.0\ m/s^2$
c. $4.0\ m/s^2$
d. $1.0 \times 102\ m/s^2$

13. A girl rolls a basketball across a basketball court. The ball slowly decelerates at a rate of $-0.20\ m/s^2$. If the initial velocity was 2.0 m/s and the ball rolled to a stop at 5.0 sec after 12:00 p.m., at what time did she start the ball rolling?

a. 0.1 seconds before noon
b. 0.1 seconds after noon
c. 5 seconds before noon
d. 5 seconds after noon

3.2 Representing Acceleration with Equations and Graphs

14. A swan on a lake gets airborne by flapping its wings and running on top of the water. If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ m/s^2$, how far will it travel before becoming airborne?

a. −8.60 m
b. 8.60 m
c. −51.4 m
d. 51.4 m

15. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of $4.00\ m/s$ and her takeoff point is $8\ m$ above the pool. How long are her feet in the air?

a. $0.408\ s$
b. $0.816\ s$
c. $1.34\ s$
d. $1.75\ s$
e. 1.28 s

Performance Task

3.2 Representing Acceleration with Equations and Graphs

16. Design an experiment to measure displacement and elapsed time. Use the data to calculate final velocity, average velocity, acceleration, and acceleration.

Materials

- a small marble or ball bearing
- a garden hose
- a measuring tape
- a stopwatch or stopwatch software download
- a sloping driveway or lawn as long as the garden hose with a level area beyond

(a) How would you use the garden hose, stopwatch, marble, measuring tape, and slope to measure displacement and elapsed time? Hint—The marble is the accelerating object, and the length of the hose is total displacement.
(b) How would you use the displacement and time data to calculate velocity, average velocity, and acceleration? Which kinematic equations would you use?
(c) How would you use the materials, the measured and calculated data, and the flat area below the slope to determine the negative acceleration? What would you measure, and which kinematic equation would you use?

TEST PREP

Multiple Choice

3.1 Acceleration

17. Which variable represents displacement?
 a. *a*
 b. *d*
 c. *t*
 d. *v*

18. If a velocity increases from 0 to 20 m/s in 10 s, what is the average acceleration?
 a. 0.5 m/s²
 b. 2 m/s²
 c. 10 m/s²
 d. 30 m/s²

3.2 Representing Acceleration with Equations and Graphs

19. For the motion of a falling object, which graphs are straight lines?
 a. Acceleration versus time only
 b. Displacement versus time only
 c. Displacement versus time and acceleration versus time
 d. Velocity versus time and acceleration versus time

20. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of 6.30×10^5 m/s² for 8.10×10^{-4} s. What is the bullet's final velocity when it leaves the barrel, commonly known as the muzzle velocity?
 a. 7.79 m/s
 b. 51.0 m/s
 c. 510 m/s
 d. 1020 m/s

Short Answer

3.1 Acceleration

21. True or False—The vector for a negative acceleration points in the opposite direction when compared to the vector for a positive acceleration.
 a. True
 b. False

22. If a car decelerates from 20 m/s to 15 m/s in 5 s, what is Δv?
 a. -5 m/s
 b. -1 m/s
 c. 1 m/s
 d. 5 m/s

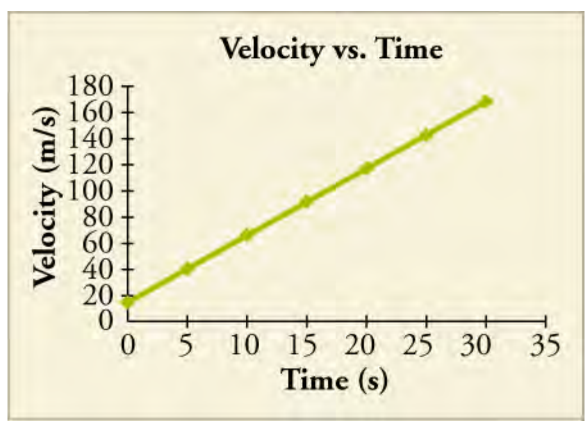
23. How is the vector arrow representing an acceleration of magnitude 3 m/s² different from the vector arrow representing a negative acceleration of magnitude 3 m/s²?
 a. They point in the same direction.
 b. They are perpendicular, forming a 90° angle between each other.
 c. They point in opposite directions.
 d. They are perpendicular, forming a 270° angle between each other.

24. How long does it take to accelerate from 8.0 m/s to 20.0 m/s at a rate of acceleration of 3.0 m/s²?
 a. 0.25 s
 b. 4.0 s
 c. 9.33 s
 d. 36 s

3.2 Representing Acceleration with Equations and Graphs

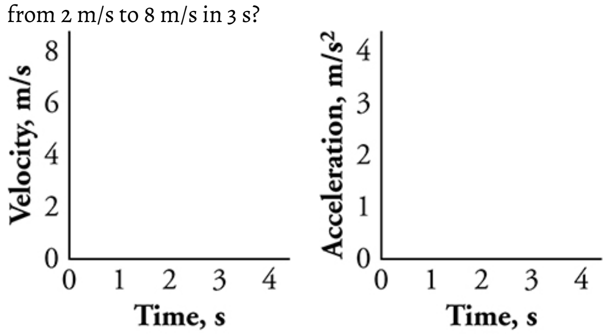
25. If a plot of displacement versus time is linear, what can be said about the acceleration?
 a. Acceleration is 0.
 b. Acceleration is a non-zero constant.
 c. Acceleration is positive.
 d. Acceleration is negative.

26.



True or False: —The image shows a velocity vs. time graph for a jet car. If you take the slope at any point on the graph, the jet car's acceleration will be 5.0 m/s².
 a. True
 b. False

27. When plotted on the blank plots, which answer choice would show the motion of an object that has uniformly accelerated from 2 m/s to 8 m/s in 3 s?



a. The plot on the left shows a line from (0,2) to (3,8) while the plot on the right shows a line from (0,2) to (3,2).
b. The plot on the left shows a line from (0,2) to (3,8) while the plot on the right shows a line from (0,3) to (3,3).
c. The plot on the left shows a line from (0,8) to (3,2) while the plot on the right shows a line from (0,2) to (3,2).
d. The plot on the left shows a line from (0,8) to (3,2) while the plot on the right shows a line from (0,3) to (3,3).

28. When is a plot of velocity versus time a straight line and when is it a curved line?
a. It is a straight line when acceleration is changing and is a curved line when acceleration is constant.
b. It is a straight line when acceleration is constant and is a curved line when acceleration is changing.
c. It is a straight line when velocity is constant and is a curved line when velocity is changing.
d. It is a straight line when velocity is changing and is a curved line when velocity is constant.

Extended Response

3.1 Acceleration

29. A test car carrying a crash test dummy accelerates from 0 to 30 m/s and then crashes into a brick wall. Describe the direction of the initial acceleration vector and compare the initial acceleration vector's magnitude with respect to the acceleration magnitude at the moment of the crash.
a. The direction of the initial acceleration vector will point towards the wall, and its magnitude will be less than the acceleration vector of the crash.
b. The direction of the initial acceleration vector will point away from the wall, and its magnitude will be less than the vector of the crash.
c. The direction of the initial acceleration vector will point towards the wall, and its magnitude will be more than the acceleration vector of the crash.
d. The direction of the initial acceleration vector will point away from the wall, and its magnitude will be more than the acceleration vector of the crash.

30. A car accelerates from rest at a stop sign at a rate of 3.0 m/s^2 to a speed of 21.0 m/s, and then immediately begins to decelerate to a stop at the next stop sign at a rate of 4.0 m/s^2. How long did it take the car to travel from the first stop sign to the second stop sign? Show your work.
a. 1.7 seconds
b. 5.3 seconds
c. 7.0 seconds
d. 12 seconds

3.2 Representing Acceleration with Equations and Graphs

31. True or False: Consider an object moving with constant acceleration. The plot of displacement versus time for such motion is a curved line while the plot of displacement versus time squared is a straight line.
a. True
b. False

32. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?
a. 0.574 s
b. 0.956 s
c. 1.53 s
d. 1.91 s

CHAPTER 4
Forces and Newton's Laws of Motion

Figure 4.1 Newton's laws of motion describe the motion of the dolphin's path. (Credit: Jin Jang)

Chapter Outline

INTRODUCTION Isaac Newton (1642–1727) was a natural philosopher; a great thinker who combined science and philosophy to try to explain the workings of nature on Earth and in the universe. His laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance period of history to the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. Drawing upon earlier work by scientists Galileo Galilei and Johannes Kepler, Newton's laws of motion allowed motion on Earth and in space to be predicted mathematically. In this chapter you will learn about force as well as Newton's first, second, and third laws of motion.

4.1 Force

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Differentiate between force, net force, and dynamics
- Draw a free-body diagram

Section Key Terms

dynamics	external force	force
free-body diagram	net external force	net force

Defining Force and Dynamics

Force is the cause of motion, and motion draws our attention. Motion itself can be beautiful, such as a dolphin jumping out of the water, the flight of a bird, or the orbit of a satellite. The study of motion is called kinematics, but kinematics describes only the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws describe the way objects speed up, slow down, stay in motion, and interact with other objects. They are also universal laws: they apply everywhere on Earth as well as in space.

A force pushes or pulls an object. The object being moved by a force could be an inanimate object, a table, or an animate object, a person. The pushing or pulling may be done by a person, or even the gravitational pull of Earth. Forces have different magnitudes and directions; this means that some forces are stronger than others and can act in different directions. For example, a cannon exerts a strong force on the cannonball that is launched into the air. In contrast, a mosquito landing on your arm exerts only a small force on your arm.

When multiple forces act on an object, the forces combine. Adding together all of the forces acting on an object gives the total force, or **net force**. An **external force** is a force that acts on an object within the system *from outside* the system. This type of force is different than an internal force, which acts between two objects that are both within the system. **The net external force** combines these two definitions; it is the total combined external force. We discuss further details about net force, external force, and net external force in the coming sections.

In mathematical terms, two forces acting in opposite directions have opposite *signs* (positive or negative). By convention, the negative sign is assigned to any movement to the left or downward. If two forces pushing in opposite directions are added together, the larger force will be somewhat canceled out by the smaller force pushing in the opposite direction. It is important to be consistent with your chosen coordinate system within a problem; for example, if negative values are assigned to the downward direction for velocity, then distance, force, and acceleration should also be designated as being negative in the downward direction.

Free-Body Diagrams and Examples of Forces

For our first example of force, consider an object hanging from a rope. This example gives us the opportunity to introduce a useful tool known as a **free-body diagram**. A free-body diagram represents the object being acted upon—that is, the free body—as a single point. Only the forces acting *on* the body (that is, external forces) are shown and are represented by vectors (which are drawn as arrows). These forces are the only ones shown because only external forces acting on the body affect its motion. We can ignore any internal forces within the body because they cancel each other out, as explained in the section on Newton's third law of motion. Free-body diagrams are very useful for analyzing forces acting on an object.

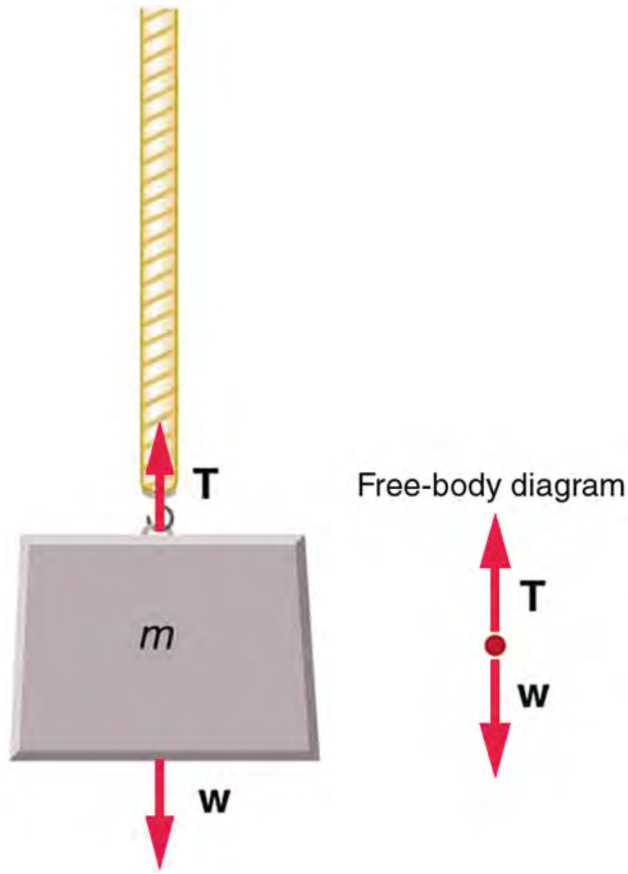


Figure 4.2 An object of mass, *m*, is held up by the force of tension.

Figure 4.2 shows the force of tension in the rope acting in the upward direction, opposite the force of gravity. The forces are indicated in the free-body diagram by an arrow pointing up, representing tension, and another arrow pointing down, representing gravity. In a free-body diagram, the lengths of the arrows show the relative magnitude (or strength) of the forces. Because forces are vectors, they add just like other vectors. Notice that the two arrows have equal lengths in Figure 4.2, which means that the forces of tension and weight are of equal magnitude. Because these forces of equal magnitude act in opposite directions, they are perfectly balanced, so they add together to give a net force of zero.

Not all forces are as noticeable as when you push or pull on an object. Some forces act without physical contact, such as the pull of a magnet (in the case of magnetic force) or the gravitational pull of Earth (in the case of gravitational force).

In the next three sections discussing Newton's laws of motion, we will learn about three specific types of forces: friction, the normal force, and the gravitational force. To analyze situations involving forces, we will create free-body diagrams to organize the framework of the mathematics for each individual situation.

TIPS FOR SUCCESS

Correctly drawing and labeling a free-body diagram is an important first step for solving a problem. It will help you visualize the problem and correctly apply the mathematics to solve the problem.

Check Your Understanding

1. What is kinematics?
   a. Kinematics is the study of motion.
   b. Kinematics is the study of the cause of motion.
   c. Kinematics is the study of dimensions.
   d. Kinematics is the study of atomic structures.

2. Do two bodies have to be in physical contact to exert a force upon one another?

a. No, the gravitational force is a field force and does not require physical contact to exert a force.
b. No, the gravitational force is a contact force and does not require physical contact to exert a force.
c. Yes, the gravitational force is a field force and requires physical contact to exert a force.
d. Yes, the gravitational force is a contact force and requires physical contact to exert a force.

3. What kind of physical quantity is force?
a. Force is a scalar quantity.
b. Force is a vector quantity.
c. Force is both a vector quantity and a scalar quantity.
d. Force is neither a vector nor a scalar quantity.

4. Which forces can be represented in a free-body diagram?
a. Internal forces
b. External forces
c. Both internal and external forces
d. A body that is not influenced by any force

4.2 Newton's First Law of Motion: Inertia

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's first law and friction, and
- Discuss the relationship between mass and inertia.

Section Key Terms

friction	inertia	law of inertia
mass	Newton's first law of motion	system

Newton's First Law and Friction

Newton's first law of motion states the following:

1. A body at rest tends to remain at rest.
2. A body in motion tends to remain in motion at a constant velocity unless acted on by a net external force. (Recall that *constant velocity* means that the body moves in a straight line and at a constant speed.)

At first glance, this law may seem to contradict your everyday experience. You have probably noticed that a moving object will usually slow down and stop unless some effort is made to keep it moving. The key to understanding why, for example, a sliding box slows down (seemingly on its own) is to first understand that a net external force acts on the box to make the box slow down. Without this net external force, the box would continue to slide at a constant velocity (as stated in Newton's first law of motion). What force acts on the box to slow it down? This force is called **friction**. Friction is an external force that acts opposite to the direction of motion (see Figure 4.3). Think of friction as a resistance to motion that slows things down.

Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it lifts the puck slightly, so the puck experiences very little friction as it moves over the surface. With friction almost eliminated, the puck glides along with very little change in speed. On a frictionless surface, the puck would experience no net external force (ignoring air resistance, which is also a form of friction). Additionally, if we know enough about friction, we can accurately predict how quickly objects will slow down.

Now let's think about another example. A man pushes a box across a floor at constant velocity by applying a force of +50 N. (The positive sign indicates that, by convention, the direction of motion is to the right.) What is the force of friction that opposes the motion? The force of friction must be –50 N. Why? According to Newton's first law of motion, any object moving at constant velocity has no net external force acting upon it, which means that the sum of the forces acting on the object must be zero. The mathematical way to say that no net external force acts on an object is $\mathbf{F}_{\text{net}} = 0$ or $\Sigma\mathbf{F} = 0$. So if the man applies +50 N of force, then the force of friction must be –50 N for the two forces to add up to zero (that is, for the two forces to *cancel* each other). Whenever you encounter the phrase *at constant velocity*, Newton's first law tells you that the net external force is zero.

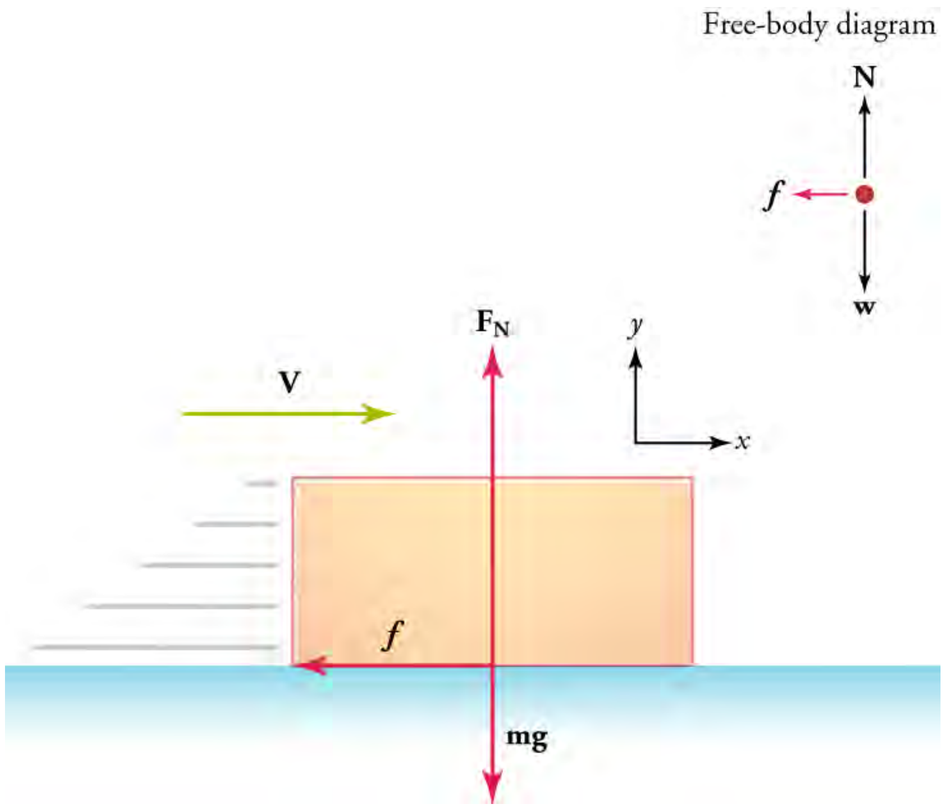


Figure 4.3 For a box sliding across a floor, friction acts in the direction opposite to the velocity.

The force of friction depends on two factors: the coefficient of friction and the normal force. For any two surfaces that are in contact with one another, the coefficient of friction is a constant that depends on the nature of the surfaces. The normal force is the force exerted by a surface that pushes on an object in response to gravity pulling the object down. In equation form, the force of friction is

$$\mathbf{f} = \mu\mathbf{N}, \tag{4.1}$$

where μ is the coefficient of friction and **N** is the normal force. (The coefficient of friction is discussed in more detail in another chapter, and the normal force is discussed in more detail in the section *Newton's Third Law of Motion*.)

Recall from the section on Force that a net external force acts from outside on the object of interest. A more precise definition is that it acts on the **system** of interest. A system is one or more objects that you choose to study. It is important to define the system at the beginning of a problem to figure out which forces are external and need to be considered, and which are internal and can be ignored.

For example, in Figure 4.4 (a), two children push a third child in a wagon at a constant velocity. The system of interest is the wagon plus the small child, as shown in part (b) of the figure. The two children behind the wagon exert external forces on this system ($\mathbf{F}_1$, $\mathbf{F}_2$). Friction *f* acting at the axles of the wheels and at the surface where the wheels touch the ground two other external forces acting on the system. Two more external forces act on the system: the weight **W** of the system pulling down and the normal force **N** of the ground pushing up. Notice that the wagon is not accelerating vertically, so Newton's first law tells us that the normal force balances the weight. Because the wagon is moving forward at a constant velocity, the force of friction must have the same strength as the sum of the forces applied by the two children.

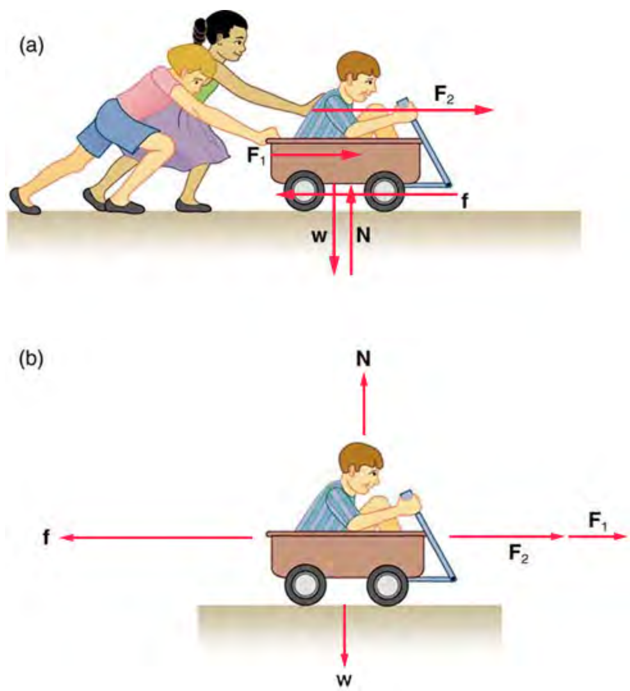


Figure 4.4 (a) The wagon and rider form a *system* that is acted on by external forces. (b) The two children pushing the wagon and child provide two external forces. Friction acting at the wheel axles and on the surface of the tires where they touch the ground provide an external force that act against the direction of motion. The weight **W** and the normal force **N** from the ground are two more external forces acting on the system. All external forces are represented in the figure by arrows. All of the external forces acting on the system add together, but because the wagon moves at a constant velocity, all of the forces must add up to zero.

Mass and Inertia

Inertia is the tendency for an object at rest to remain at rest, or for a moving object to remain in motion in a straight line with constant speed. This key property of objects was first described by Galileo. Later, Newton incorporated the concept of inertia into his first law, which is often referred to as the **law of inertia**.

As we know from experience, some objects have more inertia than others. For example, changing the motion of a large truck is more difficult than changing the motion of a toy truck. In fact, the inertia of an object is proportional to the mass of the object. **Mass** is a measure of the amount of matter (or *stuff*) in an object. The quantity or amount of matter in an object is determined by the number and types of atoms the object contains. Unlike weight (which changes if the gravitational force changes), mass does not depend on gravity. The mass of an object is the same on Earth, in orbit, or on the surface of the moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so mass is usually not determined this way. Instead, the mass of an object is determined by comparing it with the standard kilogram. Mass is therefore expressed in kilograms.

TIPS FOR SUCCESS

In everyday language, people often use the terms *weight* and *mass* interchangeably—but this is not correct. Weight is actually a force. (We cover this topic in more detail in the section *Newton's Second Law of Motion*.)

WATCH PHYSICS

Newton's First Law of Motion

This video contrasts the way we thought about motion and force in the time before Galileo's concept of inertia and Newton's first law of motion with the way we understand force and motion now.

[Click to view content (https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)](https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)

GRASP CHECK

Before we understood that objects have a tendency to maintain their velocity in a straight line unless acted upon by a net force, people thought that objects had a tendency to stop on their own. This happened because a specific force was not yet understood. What was that force?

a. Gravitational force
b. Electrostatic force
c. Nuclear force
d. Frictional force

Virtual Physics

Forces and Motion—Basics

In this simulation, you will first explore net force by placing blue people on the left side of a tug-of-war rope and red people on the right side of the rope (by clicking people and dragging them with your mouse). Experiment with changing the number and size of people on each side to see how it affects the outcome of the match and the net force. Hit the "Go!" button to start the match, and the “reset all” button to start over.

Next, click on the Friction tab. Try selecting different objects for the person to push. Slide the *applied force* button to the right to apply force to the right, and to the left to apply force to the left. The force will continue to be applied as long as you hold down the button. See the arrow representing friction change in magnitude and direction, depending on how much force you apply. Try increasing or decreasing the friction force to see how this change affects the motion.

Click to view content (https://phet.colorado.edu/sims/html/forces-and-motion-basics/latest/forces-and-motion-basics_en.html)

GRASP CHECK

Click on the tab for the *Acceleration Lab and check the Sum of Forces* option. Push the box to the right and then release. Notice which direction the sum of forces arrow points after the person stops pushing the box and lets it continue moving to the right on its own. At this point, in which direction is the net force, the sum of forces, pointing? Why?

a. The net force acts to the right because the applied external force acted to the right.
b. The net force acts to the left because the applied external force acted to the left.
c. The net force acts to the right because the frictional force acts to the right.
d. The net force acts to the left because the frictional force acts to the left.

Check Your Understanding

5. What does Newton’s first law state?
    a. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant acceleration unless acted on by a net external force.
    b. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant velocity unless acted on by a net external force.
    c. The rate of change of momentum of a body is directly proportional to the external force applied to the body.
    d. The rate of change of momentum of a body is inversely proportional to the external force applied to the body.

6. According to Newton’s first law, a body in motion tends to remain in motion at a constant velocity. However, when you slide an object across a surface, the object eventually slows down and stops. Why?
    a. The object experiences a frictional force exerted by the surface, which opposes its motion.
    b. The object experiences the gravitational force exerted by Earth, which opposes its motion
    c. The object experiences an internal force exerted by the body itself, which opposes its motion.
    d. The object experiences a pseudo-force from the body in motion, which opposes its motion.

7. What is inertia?
   a. Inertia is an object's tendency to maintain its mass.
   b. Inertia is an object's tendency to remain at rest.
   c. Inertia is an object's tendency to remain in motion
   d. Inertia is an object's tendency to remain at rest or, if moving, to remain in motion.

8. What is mass? What does it depend on?
   a. Mass is the weight of an object, and it depends on the gravitational force acting on the object.
   b. Mass is the weight of an object, and it depends on the number and types of atoms in the object.
   c. Mass is the quantity of matter contained in an object, and it depends on the gravitational force acting on the object.
   d. Mass is the quantity of matter contained in an object, and it depends on the number and types of atoms in the object.

4.3 Newton's Second Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's second law, both verbally and mathematically
- Use Newton's second law to solve problems

Section Key Terms

freefall	Newton's second law of motion	weight

Describing Newton's Second Law of Motion

Newton's first law considered bodies at rest or bodies in motion at a constant velocity. The other state of motion to consider is when an object is moving with a changing velocity, which means a change in the speed and/or the direction of motion. This type of motion is addressed by **Newton's second law of motion**, which states how force causes changes in motion. Newton's second law of motion is used to calculate what happens in situations involving forces and motion, and it shows the mathematical relationship between force, mass, and *acceleration*. Mathematically, the second law is most often written as

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \text{ or } \Sigma\mathbf{F} = m\mathbf{a}, \tag{4.2}$$

where $\mathbf{F}_{\text{net}}$ (or $\sum\mathbf{F}$) is the net external force, m is the mass of the system, and $\mathbf{a}$ is the acceleration. Note that $\mathbf{F}_{\text{net}}$ and $\sum\mathbf{F}$ are the same because the net external force is the sum of all the external forces acting on the system.

First, what do we mean by *a change in motion*? A change in motion is simply a change in velocity: the speed of an object can become slower or faster, the direction in which the object is moving can change, or both of these variables may change. A change in velocity means, by definition, that an acceleration has occurred. Newton's first law says that only a nonzero net external force can cause a change in motion, so a net external force must cause an acceleration. Note that acceleration can refer to slowing down or to speeding up. Acceleration can also refer to a change in the direction of motion with no change in speed, because acceleration is the change in velocity divided by the time it takes for that change to occur, *and* velocity is defined by speed *and* direction.

From the equation $\mathrm{F}_{\text{net}} = m\mathrm{a}$, we see that force is directly proportional to both mass and acceleration, which makes sense. To accelerate two objects from rest to the same velocity, you would expect more force to be required to accelerate the more massive object. Likewise, for two objects of the same mass, applying a greater force to one would accelerate it to a greater velocity.

Now, let's rearrange Newton's second law to solve for acceleration. We get

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m} \text{ or } \mathbf{a} = \frac{\Sigma\mathbf{F}}{m}. \tag{4.3}$$

In this form, we can see that acceleration is directly proportional to force, which we write as

$$\mathbf{a} \propto \mathbf{F}_{\text{net}}, \tag{4.4}$$

where the symbol $\propto$ means *proportional to*.

This proportionality mathematically states what we just said in words: acceleration is directly proportional to the net external force. When two variables are directly proportional to each other, then if one variable doubles, the other variable must double. Likewise, if one variable is reduced by half, the other variable must also be reduced by half. In general, when one variable is multiplied by a number, the other variable is also multiplied by the same number. It seems reasonable that the acceleration of a system should be directly proportional to and in the same direction as the net external force acting on the system. An object experiences greater acceleration when acted on by a greater force.

It is also clear from the equation $\mathbf{a} = \mathbf{F}_{\text{net}}/m$ that acceleration is inversely proportional to mass, which we write as

$$\mathbf{a} \propto \frac{1}{m}. \tag{4.5}$$

Inversely proportional means that if one variable is multiplied by a number, the other variable must be *divided* by the same number. Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. This relationship is illustrated in Figure 4.5, which shows that a given net external force applied to a basketball produces a much greater acceleration than when applied to a car.

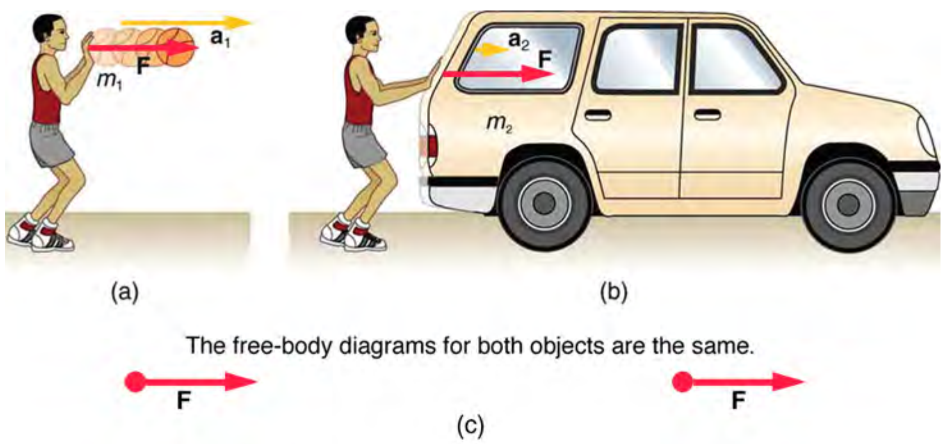


Figure 4.5 The same force exerted on systems of different masses produces different accelerations. (a) A boy pushes a basketball to make a pass. The effect of gravity on the ball is ignored. (b) The same boy pushing with identical force on a stalled car produces a far smaller acceleration (friction is negligible). Note that the free-body diagrams for the ball and for the car are identical, which allows us to compare the two situations.

Applying Newton's Second Law

Before putting Newton's second law into action, it is important to consider units. The equation $\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units of mass, length, and time (recall that acceleration has units of length divided by time squared). The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of 1 m/s². That is, because $\mathbf{F}_{\text{net}} = m\mathbf{a}$, we have

$$1 \text{ N} = 1 \text{ kg} \times 1\text{m/s}^2 = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}. \tag{4.6}$$

One of the most important applications of Newton's second law is to calculate **weight** (also known as the gravitational force), which is usually represented mathematically as $\mathbf{W}$. When people talk about gravity, they don't always realize that it is an acceleration. When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that the net external force acting on an object is responsible for the acceleration of the object. If air resistance is negligible, the net external force on a falling object is only the gravitational force (i.e., the weight of the object).

Weight can be represented by a vector because it has a direction. Down is defined as the direction in which gravity pulls, so weight is normally considered a downward force. By using Newton's second law, we can figure out the equation for weight.

Consider an object with mass m falling toward Earth. It experiences only the force of gravity (i.e., the gravitational force or weight), which is represented by W. Newton's second law states that $\mathbf{F}_{\text{net}} = m\mathbf{a}$. Because the only force acting on the object is the gravitational force, we have $\mathbf{F}_{\text{net}} = \mathbf{W}$. We know that the acceleration of an object due to gravity is $\mathbf{g}$, so we have $\mathbf{a} = \mathbf{g}$. Substituting these two expressions into Newton's second law gives

$$\mathbf{W} = m\mathbf{g}. \tag{4.7}$$

This is the equation for weight—the gravitational force on a mass m. On Earth, $\mathbf{g} = 9.80 \text{ m/s}^2$, so the weight (disregarding for now the direction of the weight) of a 1.0-kg object on Earth is

$$\mathbf{W} = m\mathbf{g} = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}. \tag{4.8}$$

Although most of the world uses newtons as the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Recall that although gravity acts downward, it can be assigned a positive or negative value, depending on what the positive direction is in your chosen coordinate system. Be sure to take this into consideration when solving problems with weight. When the downward direction is taken to be negative, as is often the case, acceleration due to gravity becomes $\mathbf{g} = -9.8 \text{ m/s}^2$.

When the net external force on an object is its weight, we say that it is in **freefall**. In this case, the only force acting on the object is the force of gravity. On the surface of Earth, when objects fall downward toward Earth, they are never truly in freefall because there is always some upward force due to air resistance that acts on the object (and there is also the buoyancy force of air, which is similar to the buoyancy force in water that keeps boats afloat).

Gravity varies slightly over the surface of Earth, so the weight of an object depends very slightly on its location on Earth. Weight varies dramatically away from Earth's surface. On the moon, for example, the acceleration due to gravity is only 1.67 m/s^2. Because weight depends on the force of gravity, a 1.0-kg mass weighs 9.8 N on Earth and only about 1.7 N on the moon.

It is important to remember that weight and mass are very different, although they are closely related. Mass is the quantity of matter (how much *stuff*) in an object and does not vary, but weight is the gravitational force on an object and is proportional to the force of gravity. It is easy to confuse the two, because our experience is confined to Earth, and the weight of an object is essentially the same no matter where you are on Earth. Adding to the confusion, the terms mass and weight are often used interchangeably in everyday language; for example, our medical records often show our weight in kilograms, but never in the correct unit of newtons.

Snap Lab

Mass and Weight

In this activity, you will use a scale to investigate mass and weight.

- 1 bathroom scale
- 1 table

1. What do bathroom scales measure?
2. When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled.
3. The springs provide a measure of your weight (provided you are not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is now divided by 9.80 to give a reading in kilograms, which is a of mass. The scale detects weight but is calibrated to display mass.
4. If you went to the moon and stood on your scale, would it detect the same *mass* as it did on Earth?

GRASP CHECK

While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why?

a. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
b. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction opposite to your weight.
c. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
d. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on

you that acts in the direction opposite to your weight.

TIPS FOR SUCCESS

Only *net external force* impacts the acceleration of an object. If more than one force acts on an object and you calculate the acceleration by using only one of these forces, you will not get the correct acceleration for that object.

WATCH PHYSICS

Newton's Second Law of Motion

This video reviews Newton's second law of motion and how net external force and acceleration relate to one another and to mass. It also covers units of force, mass, and acceleration, and reviews a worked-out example.

Click to view content (https://www.khanacademy.org/embed_video?v=ou9YMWlJgkE)

GRASP CHECK

True or False—If you want to reduce the acceleration of an object to half its original value, then you would need to reduce the net external force by half.

a. True
b. False

WORKED EXAMPLE

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N parallel to the ground. The mass of the mower is 240 kg. What is its acceleration?

Figure 4.6

Strategy

Because $\mathbf{F}_{\mathbf{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law: $\mathbf{F}_{\mathbf{net}} = m\mathbf{a}$.

Solution

Solving Newton's second law for the acceleration, we find that the magnitude of the acceleration, $\mathbf{a}$, is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$. Entering the given values for net external force and mass gives

$$\mathbf{a} = \frac{51 \text{ N}}{240 \text{ kg}} \tag{4.9}$$

Inserting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$\mathbf{a} = \frac{51\ \text{kg} \cdot \text{m/s}^2}{240\ \text{kg}} = 0.21\ \text{m/s}^2. \tag{4.10}$$

Discussion

The acceleration is in the same direction as the net external force, which is parallel to the ground and to the right. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion, because we are given that the net external force is in the direction in which the person pushes. Also, the vertical forces must cancel if there is no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is reasonable for a person pushing a mower; the mower's speed must increase by 0.21 m/s every second, which is possible. The time during which the mower accelerates would not be very long because the person's top speed would soon be reached. At this point, the person could push a little less hard, because he only has to overcome friction.

WORKED EXAMPLE

What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on humans at high accelerations. Rocket sleds consisted of a platform mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust, **T**, for the four-rocket propulsion system shown below. The sled's initial acceleration is $49\ \text{m/s}^2$, the mass of the system is 2,100 kg, and the force of friction opposing the motion is 650 N.

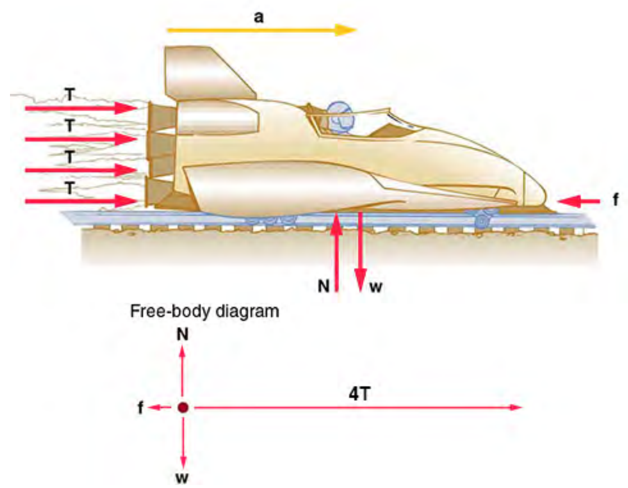


Figure 4.7

Strategy

The system of interest is the rocket sled. Although forces act vertically on the system, they must cancel because the system does not accelerate vertically. This leaves us with only horizontal forces to consider. We'll assign the direction to the right as the positive direction. See the free-body diagram in Figure 4.8.

Solution

We start with Newton's second law and look for ways to find the thrust **T** of the engines. Because all forces and acceleration are along a line, we need only consider the magnitudes of these quantities in the calculations. We begin with

$$\mathbf{F}_{\text{net}} = m\mathbf{a}, \tag{4.11}$$

where $\mathbf{F}_{\text{net}}$ is the net external force in the horizontal direction. We can see from Figure 4.8 that the engine thrusts are in the same direction (which we call the positive direction), whereas friction opposes the thrust. In equation form, the net external force is

$$\mathbf{F}_{\text{net}} = 4\mathbf{T} - \mathbf{f}. \quad (4.12)$$

Newton's second law tells us that $\mathbf{F}_{\text{net}} = m\mathbf{a}$, so we get

$$m\mathbf{a} = 4\mathbf{T} - \mathbf{f}. \quad (4.13)$$

After a little algebra, we solve for the total thrust 4**T**:

$$4\mathbf{T} = m\mathbf{a} + \mathbf{f}, \quad (4.14)$$

which means that the individual thrust is

$$\mathbf{T} = \frac{m\mathbf{a} + \mathbf{f}}{4}. \quad (4.15)$$

Inserting the known values yields

$$\mathbf{T} = \frac{(2100\text{ kg})(49\text{ m/s}^2) + 650\text{ N}}{4} = 2.6 \times 10^4\text{ N}. \quad (4.16)$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and to test the apparatus designed to protect fighter pilots in emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 **g**. (Recall that g, the acceleration due to gravity, is 9.80 m/s^2. An acceleration of 45 **g** is $45 \times 9.80\text{ m/s}^2$, which is approximately 440 m/s^2.) Living subjects are no longer used, and land speeds of 10,000 km/h have now been obtained with rocket sleds. In this example, as in the preceding example, the system of interest is clear. We will see in later examples that choosing the system of interest is crucial—and that the choice is not always obvious.

Practice Problems

9. If 1 N is equal to 0.225 lb, how many pounds is 5 N of force?
   a. 0.045 lb
   b. 1.125 lb
   c. 2.025 lb
   d. 5.000 lb

10. How much force needs to be applied to a 5-kg object for it to accelerate at 20 m/s^2?
    a. 1 N
    b. 10 N
    c. 100 N
    d. 1,000 N

Check Your Understanding

11. What is the mathematical statement for Newton's second law of motion?
    a. $F = ma$
    b. $F = 2ma$
    c. $F = \frac{m}{a}$
    d. $F = ma^2$

12. Newton's second law describes the relationship between which quantities?
    a. Force, mass, and time
    b. Force, mass, and displacement
    c. Force, mass, and velocity
    d. Force, mass, and acceleration

13. What is acceleration?
    a. Acceleration is the rate at which displacement changes.
    b. Acceleration is the rate at which force changes.
    c. Acceleration is the rate at which velocity changes.

d. Acceleration is the rate at which mass changes.

4.4 Newton's Third Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's third law, both verbally and mathematically
- Use Newton's third law to solve problems

Section Key Terms

Newton's third law of motion	normal force	tension	thrust

Describing Newton's Third Law of Motion

If you have ever stubbed your toe, you have noticed that although your toe initiates the impact, the surface that you stub it on exerts a force back on your toe. Although the first thought that crosses your mind is probably "ouch, that hurt" rather than "this is a great example of Newton's third law," both statements are true.

This is exactly what happens whenever one object exerts a force on another—each object experiences a force that is the same strength as the force acting on the other object but that acts in the opposite direction. Everyday experiences, such as stubbing a toe or throwing a ball, are all perfect examples of Newton's third law in action.

Newton's third law of motion states that whenever a first object exerts a force on a second object, the first object experiences a force equal in magnitude but opposite in direction to the force that it exerts.

Newton's third law of motion tells us that forces always occur in pairs, and one object cannot exert a force on another without experiencing the same strength force in return. We sometimes refer to these force pairs as *action-reaction* pairs, where the force exerted is the action, and the force experienced in return is the reaction (although which is which depends on your point of view).

Newton's third law is useful for figuring out which forces are external to a system. Recall that identifying external forces is important when setting up a problem, because the external forces must be added together to find the net force.

We can see Newton's third law at work by looking at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 4.8. She pushes against the pool wall with her feet and accelerates in the direction opposite to her push. The wall has thus exerted on the swimmer a force of equal magnitude but in the direction opposite that of her push. You might think that two forces of equal magnitude but that act in opposite directions would cancel, *but they do not because they act on different systems.*

In this case, there are two different systems that we could choose to investigate: the swimmer or the wall. If we choose the swimmer to be the system of interest, as in the figure, then $\mathrm{F}_{\text{wall on feet}}$ is an external force on the swimmer and affects her motion. Because acceleration is in the same direction as the net external force, the swimmer moves in the direction of $\mathrm{F}_{\text{wall on feet}}$. Because the swimmer is our system (or object of interest) and not the wall, we do not need to consider the force $\mathrm{F}_{\text{feet on wall}}$ because it originates *from* the swimmer rather than *acting on* the swimmer. Therefore, $\mathrm{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathrm{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to the direction in which she wants to move.

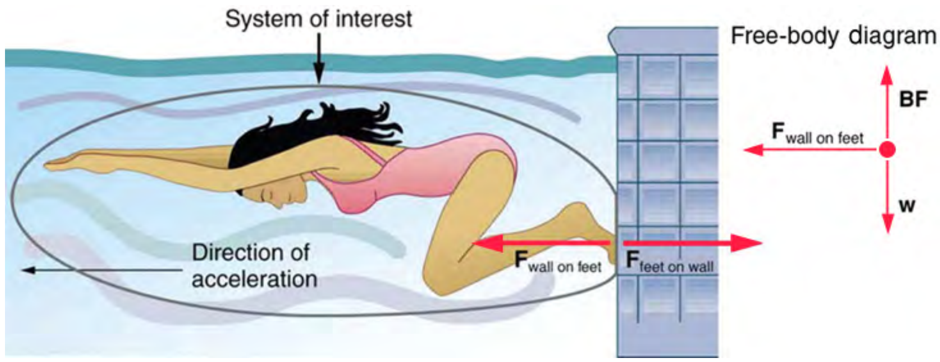


Figure 4.8 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means that the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition is the result of Newton's third law of motion, which dictates that the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on the swimmer that is equal in magnitude but that acts in the direction opposite to the force that the swimmer exerts on the wall.

Other examples of Newton's third law are easy to find. As a teacher paces in front of a whiteboard, he exerts a force backward on the floor. The floor exerts a reaction force in the forward direction on the teacher that causes him to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the car's wheels in reaction to the car's wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward.

Another example is the force of a baseball as it makes contact with the bat. Helicopters create lift by pushing air down, creating an upward reaction force. Birds fly by exerting force on air in the direction opposite that in which they wish to fly. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself forward in the water by ejecting water backward through a funnel in its body, which is similar to how a jet ski is propelled. In these examples, the octopus or jet ski push the water backward, and the water, in turn, pushes the octopus or jet ski forward.

Applying Newton's Third Law

Forces are classified and given names based on their source, how they are transmitted, or their effects. In previous sections, we discussed the forces called *push*, *weight*, and *friction*. In this section, applying Newton's third law of motion will allow us to explore three more forces: the **normal force**, **tension**, and **thrust**. However, because we haven't yet covered vectors in depth, we'll only consider one-dimensional situations in this chapter. Another chapter will consider forces acting in two dimensions.

The gravitational force (or weight) acts on objects at all times and everywhere on Earth. We know from Newton's second law that a net force produces an acceleration; so, why is everything not in a constant state of freefall toward the center of Earth? The answer is the normal force. The normal force is the force that a surface applies to an object to support the weight of that object; it acts perpendicular to the surface upon which the object rests. If an object on a flat surface is not accelerating, the net external force is zero, and the normal force has the same magnitude as the weight of the system but acts in the opposite direction. In equation form, we write that

$$\mathbf{N} = m\mathbf{g}. \quad (4.17)$$

Note that this equation is only true for a horizontal surface.

The word *tension* comes from the Latin word meaning *to stretch*. Tension is the force along the length of a flexible connector, such as a string, rope, chain, or cable. Regardless of the type of connector attached to the object of interest, one must remember that the connector can only pull (or *exert tension*) in the direction *parallel* to its length. Tension is a pull that acts parallel to the connector, and that acts in opposite directions at the two ends of the connector. This is possible because a flexible connector is simply a long series of action-reaction forces, except at the two ends where outside objects provide one member of the action-reaction forces.

Consider a person holding a mass on a rope, as shown in Figure 4.9.

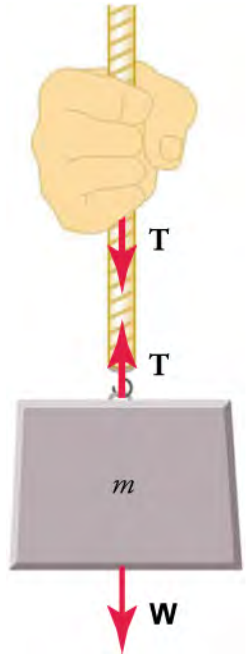


Figure 4.9 When a perfectly flexible connector (one requiring no force to bend it) such as a rope transmits a force **T**, this force must be parallel to the length of the rope, as shown. The pull that such a flexible connector exerts is a tension. Note that the rope pulls with equal magnitude force but in opposite directions to the hand and to the mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that transmits forces of equal magnitude between the two objects but that act in opposite directions.

Tension in the rope must equal the weight of the supported mass, as we can prove by using Newton's second law. If the 5.00 kg mass in the figure is stationary, then its acceleration is zero, so $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight **W** and the tension **T** supplied by the rope. Summing the external forces to find the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{T} - \mathbf{W} = 0, \tag{4.18}$$

where **T** and **W** are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. By substituting $m\mathbf{g}$ for $\mathbf{F}_{\text{net}}$ and rearranging the equation, the tension equals the weight of the supported mass, just as you would expect

$$\mathbf{T} = \mathbf{W} = m\mathbf{g}. \tag{4.19}$$

For a 5.00-kg mass (neglecting the mass of the rope), we see that

$$\mathbf{T} = m\mathbf{g} = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}. \tag{4.20}$$

Another example of Newton's third law in action is thrust. Rockets move forward by expelling gas backward at a high velocity. This means that the rocket exerts a large force backward on the gas in the rocket combustion chamber, and the gas, in turn, exerts a large force forward on the rocket in response. This reaction force is called *thrust*.

TIPS FOR SUCCESS

A common misconception is that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can expel exhaust gases more easily.

LINKS TO PHYSICS

Math: Problem-Solving Strategy for Newton's Laws of Motion

The basics of problem solving, presented earlier in this text, are followed here with specific strategies for applying Newton's laws of motion. These techniques also reinforce concepts that are useful in many other areas of physics.

First, identify the physical principles involved. If the problem involves forces, then Newton's laws of motion are involved, and it is important to draw a careful sketch of the situation. An example of a sketch is shown in Figure 4.10. Next, as in Figure 4.10, use vectors to represent all forces. Label the forces carefully, and make sure that their lengths are proportional to the magnitude of the forces and that the arrows point in the direction in which the forces act.

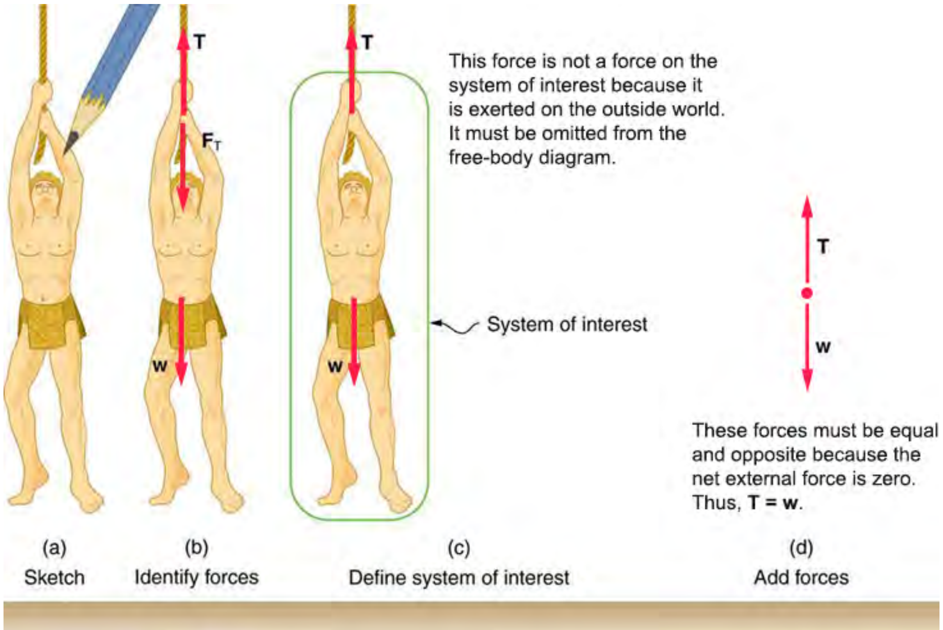


Figure 4.10 (a) A sketch of Tarzan hanging motionless from a vine. (b) Arrows are used to represent all forces. **T** is the tension exerted on Tarzan by the vine, $\mathbf{F}_\mathrm{T}$ is the force exerted on the vine by Tarzan, and **W** is Tarzan's weight (i.e., the force exerted on Tarzan by Earth's gravity). All other forces, such as a nudge of a breeze, are assumed to be negligible. (c) Suppose we are given Tarzan's mass and asked to find the tension in the vine. We define the system of interest as shown and draw a free-body diagram, as shown in (d). $\mathbf{F}_\mathrm{T}$ is no longer shown because it does not act on the system of interest; rather, $\mathbf{F}_\mathrm{T}$ acts on the outside world. (d) The free-body diagram shows only the external forces acting on Tarzan. For these to sum to zero, we must have $\mathbf{T} = \mathbf{W}$.

Next, make a list of knowns and unknowns and assign variable names to the quantities given in the problem. Figure out which variables need to be calculated; these are the unknowns. Now carefully define the system: which objects are of interest for the problem. This decision is important, because Newton's second law involves only external forces. Once the system is identified, it's possible to see which forces are external and which are internal (see Figure 4.10).

If the system acts on an object outside the system, then you know that the outside object exerts a force of equal magnitude but in the opposite direction on the system.

A diagram showing the system of interest and all the external forces acting on it is called a free-body diagram. Only external forces are shown on free-body diagrams, not acceleration or velocity. Figure 4.10 shows a free-body diagram for the system of interest.

After drawing a free-body diagram, apply Newton's second law to solve the problem. This is done in Figure 4.10 for the case of Tarzan hanging from a vine. When external forces are clearly identified in the free-body diagram, translate the forces into equation form and solve for the unknowns. Note that forces acting in opposite directions have opposite signs. By convention, forces acting downward or to the left are usually negative.

GRASP CHECK

If a problem has more than one system of interest, more than one free-body diagram is required to describe the external forces acting on the different systems.

a. True
b. False

WATCH PHYSICS

Newton's Third Law of Motion

This video explains Newton's third law of motion through examples involving push, normal force, and thrust (the force that propels a rocket or a jet).

Click to view content (https://www.openstax.org/l/astronaut)

GRASP CHECK

If the astronaut in the video wanted to move upward, in which direction should he throw the object? Why?

a. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., upward).
b. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., downward).
c. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., upward).
d. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., downward).

WORKED EXAMPLE

An Accelerating Subway Train

A physics teacher pushes a cart of demonstration equipment to a classroom, as in Figure 4.11. Her mass is 65.0 kg, the cart's mass is 12.0 kg, and the equipment's mass is 7.0 kg. To push the cart forward, the teacher's foot applies a force of 150 N in the opposite direction (backward) on the floor. Calculate the acceleration produced by the teacher. The force of friction, which opposes the motion, is 24.0 N.

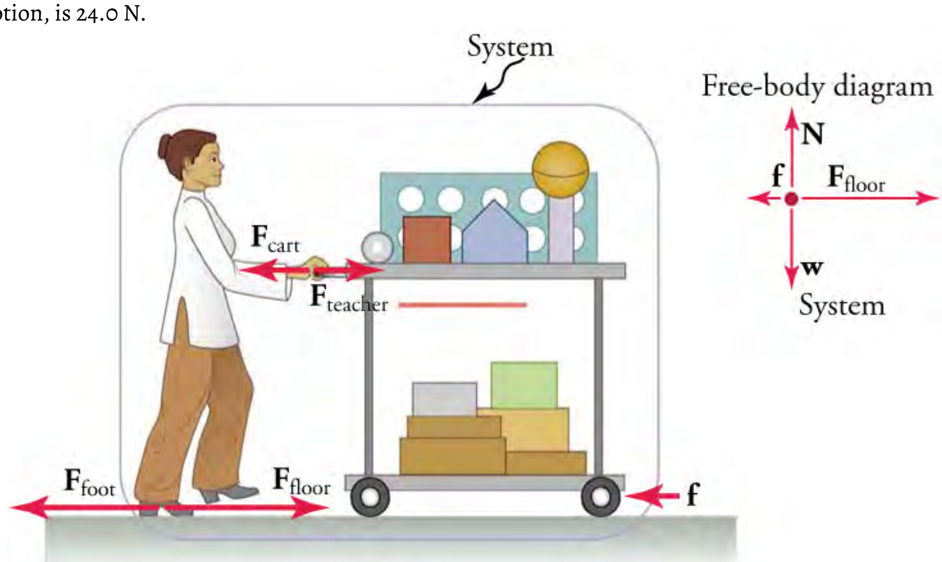


Figure 4.11

Strategy

Because they accelerate together, we define the system to be the teacher, the cart, and the equipment. The teacher pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward force $\mathbf{F}_{\text{floor}}$ of 150 N on the system. Because all motion is horizontal, we can assume that no net force acts in the vertical direction, and the problem becomes one dimensional. As noted in the figure, the friction f opposes the motion and therefore acts opposite the direction of $\mathbf{F}_{\text{floor}}$.

We should not include the forces $\mathbf{F}_{\text{teacher}}$, $\mathbf{F}_{\text{cart}}$, or $\mathbf{F}_{\text{foot}}$ because these are exerted *by* the system, not *on* the system. We find the net external force by adding together the external forces acting on the system (see the free-body diagram in the figure) and then use Newton's second law to find the acceleration.

Solution

Newton's second law is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}. \quad (4.21)$$

The net external force on the system is the sum of the external forces: the force of the floor acting on the teacher, cart, and equipment (in the horizontal direction) and the force of friction. Because friction acts in the opposite direction, we assign it a negative value. Thus, for the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{F}_{\text{floor}} - \mathbf{f} = 150 \text{ N} - 24.0 \text{ N} = 126 \text{ N}. \quad (4.22)$$

The mass of the system is the sum of the mass of the teacher, cart, and equipment.

$$m = (65.0 + 12.0 + 7.0) \text{ kg} = 84 \text{ kg} \quad (4.23)$$

Insert these values of net F and *m* into Newton's second law to obtain the acceleration of the system.

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$$
$$a = \frac{126 \text{ N}}{84 \text{ kg}} = 1.5 \text{ m/s}^2 \quad (4.24)$$

$$F_1 < F_2 \quad (4.25)$$

Discussion

None of the forces between components of the system, such as between the teacher's hands and the cart, contribute to the net external force because they are internal to the system. Another way to look at this is to note that the forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the teacher on the cart is of equal magnitude but in the opposite direction of the force exerted by the cart on the teacher. In this case, both forces act on the same system, so they cancel. Defining the system was crucial to solving this problem.

Practice Problems

14. What is the equation for the normal force for a body with mass *m* that is at rest on a horizontal surface?
    a. N = *m*
    b. N = *mg*
    c. N = *mv*
    d. N = *g*

15. An object with mass *m* is at rest on the floor. What is the magnitude and direction of the normal force acting on it?
    a. N = *mv* in upward direction
    b. N = *mg* in upward direction
    c. N = *mv* in downward direction
    d. N = *mg* in downward direction

Check Your Understanding

16. What is Newton's third law of motion?
    a. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude and acts in the direction of the applied force.
    b. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude and acts in the direction of the applied force.
    c. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude but acts in the direction opposite the direction of the applied force.
    d. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude but

acts in the direction opposite the direction of the applied force.

17. Considering Newton's third law, why don't two equal and opposite forces cancel out each other?

a. Because the two forces act in the same direction
b. Because the two forces have different magnitudes
c. Because the two forces act on different systems
d. Because the two forces act in perpendicular directions

KEY TERMS

dynamics the study of how forces affect the motion of objects and systems

external force a force acting on an object or system that originates outside of the object or system

force a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

free-body diagram a diagram showing all external forces acting on a body

freefall a situation in which the only force acting on an object is the force of gravity

friction an external force that acts in the direction opposite to the direction of motion

inertia the tendency of an object at rest to remain at rest, or for a moving object to remain in motion in a straight line and at a constant speed

law of inertia Newton's first law of motion: a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

mass the quantity of matter in a substance; measured in kilograms

net external force the sum of all external forces acting on an object or system

net force the sum of all forces acting on an object or system

Newton's first law of motion a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion the net external force, $\mathbf{F}_{\text{net}}$, on an object is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and also proportional to the object's mass, m; defined mathematically as $\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$.

Newton's third law of motion when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts

normal force the force that a surface applies to an object; acts perpendicular and away from the surface with which the object is in contact

system one or more objects of interest for which only the forces acting on them from the outside are considered, but not the forces acting between them or inside them

tension a pulling force that acts along a connecting medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force exerted on the object by the rope is called tension

thrust a force that pushes an object forward in response to the backward ejection of mass by the object; rockets and airplanes are pushed forward by a thrust reaction force in response to ejecting gases backward

weight the force of gravity, $\mathbf{W}$, acting on an object of mass m; defined mathematically as $\mathbf{W} = m\mathbf{g}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

SECTION SUMMARY

4.1 Force

- Dynamics is the study of how forces affect the motion of objects and systems.
- Force is a push or pull that can be defined in terms of various standards. It is a vector and so has both magnitude and direction.
- External forces are any forces outside of a body that act on the body. A free-body diagram is a drawing of all external forces acting on a body.

4.2 Newton's First Law of Motion: Inertia

- Newton's first law states that a body at rest remains at rest or, if moving, remains in motion in a straight line at a constant speed, unless acted on by a net external force. This law is also known as the law of inertia.
- Inertia is the tendency of an object at rest to remain at rest or, if moving, to remain in motion at constant velocity. Inertia is related to an object's mass.
- Friction is a force that opposes motion and causes an object or system to slow down.
- Mass is the quantity of matter in a substance.

4.3 Newton's Second Law of Motion

- Acceleration is a change in velocity, meaning a change in speed, direction, or both.
- An external force acts on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to the system's mass.
- In equation form, Newton's second law of motion is $\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$. This is sometimes written as $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$.
- The weight of an object of mass m is the force of gravity that acts on it. From Newton's second law, weight is

given by $\mathbf{W} = m\mathbf{g}$.
- If the only force acting on an object is its weight, then the object is in freefall.

4.4 Newton's Third Law of Motion

- Newton's third law of motion states that when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.
- When an object rests on a surface, the surface applies a force on the object that opposes the weight of the object. This force acts perpendicular to the surface and is called the normal force.
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension. When a rope supports the weight of an object at rest, the tension in the rope is equal to the weight of the object.
- Thrust is a force that pushes an object forward in response to the backward ejection of mass by the object. Rockets and airplanes are pushed forward by thrust.

KEY EQUATIONS

4.2 Newton's First Law of Motion: Inertia

Newton's first law of motion	$\mathbf{F}_{\text{net}} = 0$ or $\Sigma\mathbf{F} = 0$

4.3 Newton's Second Law of Motion

Newton's second law of motion	$\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$
Newton's second law of motion to solve acceleration	$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$
Newton's second law of motion to solve weight	$\mathbf{W} = m\mathbf{g}$

4.4 Newton's Third Law of Motion

normal force for a nonaccelerating horizontal surface	$\mathbf{N} = m\mathbf{g}$
tension for an object at rest	$\mathbf{T} = m\mathbf{g}$

CHAPTER REVIEW

Concept Items

4.1 Force

1. What is dynamics?
   a. Dynamics is the study of internal forces.
   b. Dynamics is the study of forces and their effect on motion.
   c. Dynamics describes the motion of points, bodies, and systems without consideration of the cause of motion.
   d. Dynamics describes the effect of forces on each other.

2. Two forces acting on an object are perpendicular to one another. How would you draw these in a free-body diagram?
   a. The two force arrows will be drawn at a right angle to one another.
   b. The two force arrows will be pointing in opposite directions.
   c. The two force arrows will be at a 45° angle to one another.
   d. The two force arrows will be at a 180° angle to one another.

3. A free-body diagram shows the forces acting on an object. How is that object represented in the diagram?
   a. A single point
   b. A square box
   c. A unit circle
   d. The object as it is

4.2 Newton's First Law of Motion: Inertia

4. A ball rolls along the ground, moving from north to south. What direction is the frictional force that acts on the ball?
   a. North to south
   b. South to north
   c. West to east
   d. East to west

5. The tires you choose to drive over icy roads will create more friction with the road than your summer tires. Give another example where more friction is desirable.

a. Children's slide
b. Air hockey table
c. Ice-skating rink
d. Jogging track

6. How do you express, mathematically, that no external force is acting on a body?
   a. $F_{net} = -1$
   b. $F_{net} = 0$
   c. $F_{net} = 1$
   d. $F_{net} = \infty$

4.3 Newton's Second Law of Motion

7. What does it mean for two quantities to be inversely proportional to each other?
   a. When one variable increases, the other variable decreases by a greater amount.
   b. When one variable increases, the other variable also increases.
   c. When one variable increases, the other variable decreases by the same factor.
   d. When one variable increases, the other variable also increases by the same factor.

8. True or False: Newton's second law can be interpreted based on Newton's first law.
   a. True
   b. False

4.4 Newton's Third Law of Motion

9. Which forces cause changes in the motion of a system?
   a. internal forces
   b. external forces
   c. both internal and external forces
   d. neither internal nor external forces

10. True or False—Newton's third law applies to the external forces acting on a system of interest.
    a. True
    b. False

11. A ball is dropped and hits the floor. What is the direction of the force exerted by the floor on the ball?
    a. Upward
    b. Downward
    c. Right
    d. Left

Critical Thinking Items

4.1 Force

12. Only two forces are acting on an object: force A to the left and force B to the right. If force B is greater than force A, in which direction will the object move?
    a. To the right
    b. To the left
    c. Upward
    d. The object does not move

13. In a free-body diagram, the arrows representing tension and weight have the same length but point away from one another. What does this indicate?
    a. They are equal in magnitude and act in the same direction.
    b. They are equal in magnitude and act in opposite directions.
    c. They are unequal in magnitude and act in the same direction.
    d. They are unequal in magnitude and act in opposite directions.

14. An object is at rest. Two forces, X and Y, are acting on it. Force X has a magnitude of *x* and acts in the downward direction. What is the magnitude and direction of Y?
    a. The magnitude is *x* and points in the upward direction.
    b. The magnitude is 2*x* and points in the upward direction.
    c. The magnitude is *x* and points in the downward direction.
    d. The magnitude is 2*x* and points in the downward direction.

15. Three forces, A, B, and C, are acting on the same object with magnitudes *a*, *b*, and *c*, respectively. Force A acts to the right, force B acts to the left, and force C acts downward. What is a necessary condition for the object to move straight down?
    a. The magnitude of force A must be greater than the magnitude of force B, so $a > b$.
    b. The magnitude of force A must be equal to the magnitude of force B, so $a = b$.
    c. The magnitude of force A must be greater than the magnitude of force C, so $A > C$.
    d. The magnitude of force C must be greater than the magnitude of forces A or B, so $A < C > B$.

4.2 Newton's First Law of Motion: Inertia

16. Two people push a cart on a horizontal surface by applying forces F_1 and F_2 in the same direction. Is the magnitude of the net force acting on the cart, F_{net}, equal to, greater than, or less than $F_1 + F_2$? Why?
    a. $F_{net} < F_1 + F_2$ because the net force will not include the frictional force.
    b. $F_{net} = F_1 + F_2$ because the net force will not include

the frictional force

c. $F_{net} < F_1 + F_2$ because the net force will include the component of frictional force

d. $F_{net} = F_1 + F_2$ because the net force will include the frictional force

17. True or False: A book placed on a balance scale is balanced by a standard 1-kg iron weight placed on the opposite side of the balance. If these objects are taken to the moon and a similar exercise is performed, the balance is still level because gravity is uniform on the moon's surface as it is on Earth's surface.

a. True

b. False

4.3 Newton's Second Law of Motion

18. From the equation for Newton's second law, we see that F_{net} is directly proportional to **a** and that the constant of proportionality is ***m***. What does this mean in a practical sense?

a. An increase in applied force will cause an increase in acceleration if the mass is constant.

b. An increase in applied force will cause a decrease in acceleration if the mass is constant.

c. An increase in applied force will cause an increase in acceleration, even if the mass varies.

d. An increase in applied force will cause an increase in acceleration and mass.

4.4 Newton's Third Law of Motion

19. True or False: A person accelerates while walking on the ground by exerting force. The ground in turn exerts force F_2 on the person. F_1 and F_2 are equal in magnitude but act in opposite directions. The person is able to walk because the two forces act on the different systems and the net force acting on the person is nonzero.

a. True

b. False

20. A helicopter pushes air down, which, in turn, pushes the helicopter up. Which force affects the helicopter's motion? Why?

a. Air pushing upward affects the helicopter's motion because it is an internal force that acts on the helicopter.

b. Air pushing upward affects the helicopter's motion because it is an external force that acts on the helicopter.

c. The downward force applied by the blades of the helicopter affects its motion because it is an internal force that acts on the helicopter.

d. The downward force applied by the blades of the helicopter affects its motion because it is an external force that acts on the helicopter.

Problems

4.3 Newton's Second Law of Motion

21. An object has a mass of 1 kg on Earth. What is its weight on the moon?

a. 1 N

b. 1.67 N

c. 9.8 N

d. 10 N

22. A bathroom scale shows your mass as 55 kg. What will it read on the moon?

a. 9.4 kg

b. 10.5 kg

c. 55.0 kg

d. 91.9 kg

4.4 Newton's Third Law of Motion

23. A person pushes an object of mass 5.0 kg along the floor by applying a force. If the object experiences a friction force of 10 N and accelerates at 18 m/s^2, what is the magnitude of the force exerted by the person?

a. −90 N

b. −80 N

c. 90 N

d. 100 N

Performance Task

4.4 Newton's Third Law of Motion

24. A car weighs 2,000 kg. It moves along a road by applying a force on the road with a parallel component of 560 N. There are two passengers in the car, each weighing 55 kg. If the magnitude of the force of friction experienced by the car is 45 N, what is the acceleration of the car?

a. 0.244 m/s^2

b. 0.265 m/s^2

c. 4.00 m/s^2

d. 4.10 m/s^2

TEST PREP

Multiple Choice

4.1 Force

25. Which of the following is a physical quantity that can be described by dynamics but not by kinematics?
a. Velocity
b. Acceleration
c. Force

26. Which of the following is used to represent an object in a free-body diagram?
a. A point
b. A line
c. A vector

4.2 Newton's First Law of Motion: Inertia

27. What kind of force is friction?
a. External force
b. Internal force
c. Net force

28. What is another name for Newton's first law?
a. Law of infinite motion
b. Law of inertia
c. Law of friction

29. True or False—A rocket is launched into space and escapes Earth's gravitational pull. It will continue to travel in a straight line until it is acted on by another force.
a. True
b. False

30. A 2,000-kg car is sitting at rest in a parking lot. A bike and rider with a total mass of 60 kg are traveling along a road at 10 km/h. Which system has more inertia? Why?
a. The car has more inertia, as its mass is greater than the mass of the bike.
b. The bike has more inertia, as its mass is greater than the mass of the car.
c. The car has more inertia, as its mass is less than the mass of the bike.
d. The bike has more inertia, as its mass is less than the mass of the car.

4.3 Newton's Second Law of Motion

31. In the equation for Newton's second law, what does F_{net} stand for?
a. Internal force
b. Net external force
c. Frictional force

32. What is the SI unit of force?
a. Kg
b. dyn
c. N

33. What is the net external force on an object in freefall on Earth if you were to neglect the effect of air?
a. The net force is zero.
b. The net force is upward with magnitude *mg*.
c. The net force is downward with magnitude *mg*.
d. The net force is downward with magnitude 9.8 N.

34. Two people push a 2,000-kg car to get it started. An acceleration of at least 5.0 m/s^2 is required to start the car. Assuming both people apply the same magnitude force, how much force will each need to apply if friction between the car and the road is 300 N?
a. 4850 N
b. 5150 N
c. 97000 N
d. 10300 N

4.4 Newton's Third Law of Motion

35. One object exerts a force of magnitude F_1 on another object and experiences a force of magnitude F_2 in return. What is true for F_1 and F_2?
a. $F_1 > F_2$
b. $F_1 < F_2$
c. $F_1 = F_2$

36. A weight is suspended with a rope and hangs freely. In what direction is the tension on the rope?
a. parallel to the rope
b. perpendicular to the rope

37. A person weighing 55 kg walks by applying 160 N of force on the ground, while pushing a 10-kg object. If the person accelerates at 2 m/s^2, what is the force of friction experienced by the system consisting of the person and the object?
a. 30 N
b. 50 N
c. 270 N
d. 290 N

38. A 65-kg swimmer pushes on the pool wall and accelerates at 6 m/s^2. The friction experienced by the swimmer is 100 N. What is the magnitude of the force that the swimmer applies on the wall?
a. −490 N
b. −290 N
c. 290 N
d. 490 N

Short Answer

4.1 Force

39. True or False—An external force is defined as a force generated outside the system of interest that acts on an object inside the system.
   a. True
   b. False

40. By convention, which sign is assigned to an object moving downward?
   a. A positive sign (+)
   b. A negative sign (−)
   c. Either a positive or negative sign (±)
   d. No sign is assigned

41. A body is pushed downward by a force of 5 units and upward by a force of 2 units. How would you draw a free-body diagram to represent this?
   a. Two force vectors acting at a point, both pointing up with lengths of 5 units and 2 units
   b. Two force vectors acting at a point, both pointing down with lengths of 5 units and 2 units
   c. Two force vectors acting at a point, one pointing up with a length of 5 units and the other pointing down with a length of 2 units
   d. Two force vectors acting at a point, one pointing down with a length of 5 units and the other pointing up with a length of 2 units

42. A body is pushed eastward by a force of four units and southward by a force of three units. How would you draw a free-body diagram to represent this?
   a. Two force vectors acting at a point, one pointing left with a length of 4 units and the other pointing down with a length of 3 units
   b. Two force vectors acting at a point, one pointing left with a length of 4 units and the other pointing up with a length of 3 units
   c. Two force vectors acting at a point, one pointing right with a length of 4 units and the other pointing down with a length of 3 units
   d. Two force vectors acting at a point, one pointing right with a length of 4 units and the other pointing up with a length of 3 units

4.2 Newton's First Law of Motion: Inertia

43. A body with mass m is pushed along a horizontal surface by a force F and is opposed by a frictional force *f*. How would you draw a free-body diagram to represent this situation?
   a. A dot with an arrow pointing right, labeled F, and an arrow pointing left, labeled *f*, that is of equal length or shorter than F
   b. A dot with an arrow pointing right, labeled F, and an arrow pointing right, labeled *f*, that is of equal length or shorter than F
   c. A dot with an arrow pointing right, labeled F, and a smaller arrow pointing up, labeled *f*, that is of equal length or longer than F
   d. A dot with an arrow pointing right, labeled F, and a smaller arrow pointing down, labeled *f*, that is of equal length or longer than F

44. Two objects rest on a uniform surface. A person pushes both with equal force. If the first object starts to move faster than the second, what can be said about their masses?
   a. The mass of the first object is less than that of the second object.
   b. The mass of the first object is equal to the mass of the second object.
   c. The mass of the first object is greater than that of the second object.
   d. No inference can be made because mass and force are not related to each other.

45. Two similar boxes rest on a table. One is empty and the other is filled with pebbles. Without opening or lifting either, how can you tell which box is full? Why?
   a. By applying an internal force; whichever box accelerates faster is lighter and so must be empty
   b. By applying an internal force; whichever box accelerates faster is heavier and so the other box must be empty
   c. By applying an external force; whichever box accelerates faster is lighter and so must be empty
   d. By applying an external force; whichever box accelerates faster is heavier and so the other box must be empty

46. True or False—An external force is required to set a stationary object in motion in outer space away from all gravitational influences and atmospheric friction.
   a. True
   b. False

4.3 Newton's Second Law of Motion

47. A steadily rolling ball is pushed in the direction from east to west, which causes the ball to move faster in the same direction. What is the direction of the acceleration?
   a. North to south
   b. South to north
   c. East to west
   d. West to east

48. A ball travels from north to south at 60 km/h. After being hit by a bat, it travels from west to east at 60 km/

h. Is there a change in velocity?

a. Yes, because velocity is a scalar.
b. Yes, because velocity is a vector.
c. No, because velocity is a scalar.
d. No, because velocity is a vector

49. What is the weight of a 5-kg object on Earth and on the moon?

a. On Earth the weight is 1.67 N, and on the moon the weight is 1.67 N.
b. On Earth the weight is 5 N, and on the moon the weight is 5 N.
c. On Earth the weight is 49 N, and on the moon the weight is 8.35 N.
d. On Earth the weight is 8.35 N, and on the moon the weight is 49 N.

50. An object weighs 294 N on Earth. What is its weight on the moon?

a. 50.1 N
b. 30.0 N
c. 249 N
d. 1461 N

4.4 Newton's Third Law of Motion

51. A large truck with mass 30 m crashes into a small sedan with mass *m*. If the truck exerts a force F on the sedan, what force will the sedan exert on the truck?

a. $\frac{F}{30}$
b. F
c. 2F
d. 30F

52. A fish pushes water backward with its fins. How does this propel the fish forward?

a. The water exerts an internal force on the fish in the opposite direction, pushing the fish forward.
b. The water exerts an external force on the fish in the opposite direction, pushing the fish forward.
c. The water exerts an internal force on the fish in the same direction, pushing the fish forward.
d. The water exerts an external force on the fish in the same direction, pushing the fish forward.

53. True or False—Tension is the result of opposite forces in a connector, such as a string, rope, chain or cable, that pulls each point of the connector apart in the direction parallel to the length of the connector. At the ends of the connector, the tension pulls toward the center of the connector.

a. True
b. False

54. True or False—Normal reaction is the force that opposes the force of gravity and acts in the direction of the force of gravity.

a. True
b. False

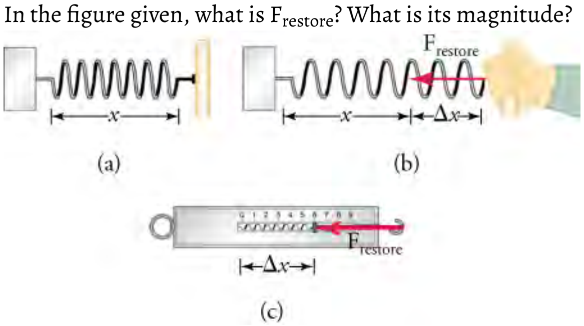
Extended Response

4.1 Force

55. True or False—When two unequal forces act on a body, the body will not move in the direction of the weaker force.

a. True
b. False

56. In the figure given, what is $F_{restore}$? What is its magnitude?



a. $F_{restore}$ is the force exerted by the hand on the spring, and it pulls to the right.
b. $F_{restore}$ is the force exerted by the spring on the hand, and it pulls to the left.
c. $F_{restore}$ is the force exerted by the hand on the spring, and it pulls to the left.
d. $F_{restore}$ is the force exerted by the spring on the hand, and it pulls to the right.

4.2 Newton's First Law of Motion: Inertia

57. Two people apply the same force to throw two identical balls in the air. Will the balls necessarily travel the same distance? Why or why not?

a. No, the balls will not necessarily travel the same distance because the gravitational force acting on them is different.
b. No, the balls will not necessarily travel the same distance because the angle at which they are thrown may differ.
c. Yes, the balls will travel the same distance because the gravitational force acting on them is the same.
d. Yes, the balls will travel the same distance because the angle at which they are thrown may differ.

58. A person pushes a box from left to right and then lets the box slide freely across the floor. The box slows down as it slides across the floor. When the box is sliding freely, what is the direction of the net external force?

a. The net external force acts from left to right.
b. The net external force acts from right to left.
c. The net external force acts upward.
d. The net external force acts downward.

4.3 Newton's Second Law of Motion

59. A 55-kg lady stands on a bathroom scale inside an elevator. The scale reads 70 kg. What do you know about the motion of the elevator?

a. The elevator must be accelerating upward.
b. The elevator must be accelerating downward.
c. The elevator must be moving upward with a constant velocity.
d. The elevator must be moving downward with a constant velocity.

60. True or False—A skydiver initially accelerates in his jump. Later, he achieves a state of constant velocity called terminal velocity. Does this mean the skydiver becomes weightless?

a. Yes
b. No

4.4 Newton's Third Law of Motion

61. How do rockets propel themselves in space?

a. Rockets expel gas in the forward direction at high velocity, and the gas, which provides an internal force, pushes the rockets forward.
b. Rockets expel gas in the forward direction at high velocity, and the gas, which provides an external force, pushes the rockets forward.
c. Rockets expel gas in the backward direction at high velocity, and the gas, which is an internal force, pushes the rockets forward.
d. Rockets expel gas in the backward direction at high velocity, and the gas, which provides an external force, pushes the rockets forward.

62. Are rockets more efficient in Earth's atmosphere or in outer space? Why?

a. Rockets are more efficient in Earth's atmosphere than in outer space because the air in Earth's atmosphere helps to provide thrust for the rocket, and Earth has more air friction than outer space.
b. Rockets are more efficient in Earth's atmosphere than in outer space because the air in Earth's atmosphere helps to provide thrust to the rocket, and Earth has less air friction than the outer space.
c. Rockets are more efficient in outer space than in Earth's atmosphere because the air in Earth's atmosphere does not provide thrust but does create more air friction than in outer space.
d. Rockets are more efficient in outer space than in Earth's atmosphere because the air in Earth's atmosphere does not provide thrust but does create less air friction than in outer space.

CHAPTER 5
Motion in Two Dimensions

Figure 5.1 Billiard balls on a pool table are in motion after being hit with a cue stick. (Popperipopp, Wikimedia Commons)

Chapter Outline

INTRODUCTION In Chapter 2, we learned to distinguish between vectors and scalars; the difference being that a vector has magnitude and direction, whereas a scalar has only magnitude. We learned how to deal with vectors in physics by working straightforward one-dimensional vector problems, which may be treated mathematically in the same as scalars. In this chapter, we'll use vectors to expand our understanding of forces and motion into two dimensions. Most real-world physics problems (such as with the game of pool pictured here) are, after all, either two- or three-dimensional problems and physics is most useful when applied to real physical scenarios. We start by learning the practical skills of graphically adding and subtracting vectors (by using drawings) and analytically (with math). Once we're able to work with two-dimensional vectors, we apply these skills to problems of projectile motion, inclined planes, and harmonic motion.

5.1 Vector Addition and Subtraction: Graphical Methods

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the graphical method of vector addition and subtraction
- Use the graphical method of vector addition and subtraction to solve physics problems

Section Key Terms

graphical method	head (of a vector)	head-to-tail method	resultant
resultant vector	tail	vector addition	vector subtraction

The Graphical Method of Vector Addition and Subtraction

Recall that a vector is a quantity that has magnitude and direction. For example, displacement, velocity, acceleration, and force are all vectors. In one-dimensional or straight-line motion, the direction of a vector can be given simply by a plus or minus sign. Motion that is forward, to the right, or upward is usually considered to be *positive* (+); and motion that is backward, to the left, or downward is usually considered to be *negative* (–).

In two dimensions, a vector describes motion in two perpendicular directions, such as vertical and horizontal. For vertical and horizontal motion, each vector is made up of vertical and horizontal components. In a one-dimensional problem, one of the components simply has a value of zero. For two-dimensional vectors, we work with vectors by using a frame of reference such as a coordinate system. Just as with one-dimensional vectors, we graphically represent vectors with an arrow having a length proportional to the vector's magnitude and pointing in the direction that the vector points.

Figure 5.2 shows a graphical representation of a vector; the total displacement for a person walking in a city. The person first walks nine blocks east and then five blocks north. Her total displacement does not match her path to her final destination. The displacement simply connects her starting point with her ending point using a straight line, which is the shortest distance. We use the notation that a boldface symbol, such as **D**, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction is given by an angle represented by the symbol θ. Note that her displacement would be the same if she had begun by first walking five blocks north and then walking nine blocks east.

TIPS FOR SUCCESS

In this text, we represent a vector with a boldface variable. For example, we represent a force with the vector **F**, which has both magnitude and direction. The magnitude of the vector is represented by the variable in italics, F, and the direction of the variable is given by the angle θ.

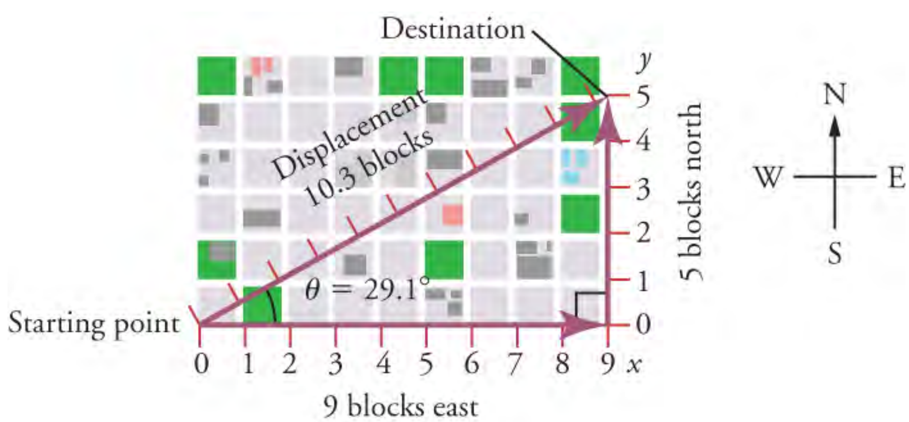


Figure 5.2 A person walks nine blocks east and five blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.

The **head-to-tail method** is a **graphical** way to add vectors. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the pointed end of the arrow. The following steps describe how to use the head-to-tail method for graphical **vector addition**.

1. Let the x-axis represent the east-west direction. Using a ruler and protractor, draw an arrow to represent the first vector (nine blocks to the east), as shown in Figure 5.3(a).

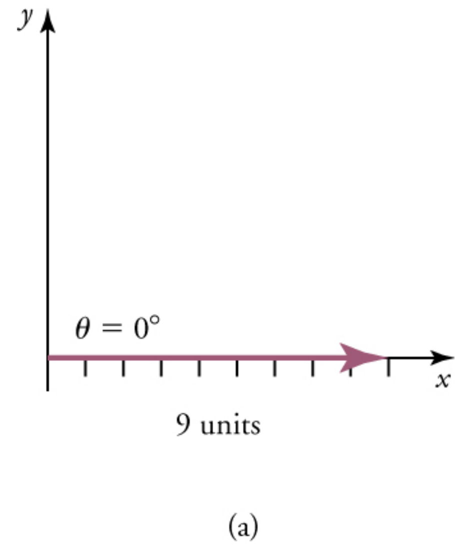


Figure 5.3 The diagram shows a vector with a magnitude of nine units and a direction of 0°.

2. Let the y-axis represent the north-south direction. Draw an arrow to represent the second vector (five blocks to the north). Place the tail of the second vector at the head of the first vector, as shown in Figure 5.4(b).

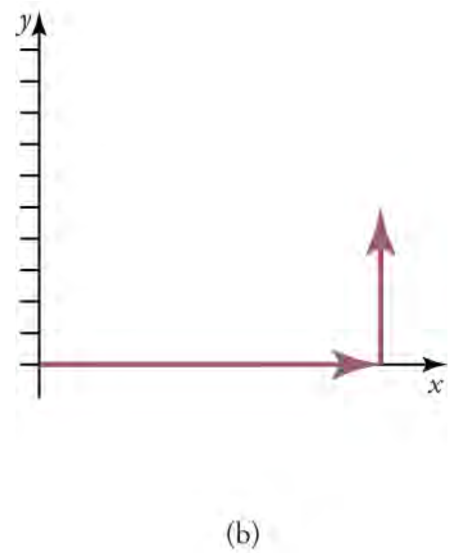


Figure 5.4 A vertical vector is added.

3. If there are more than two vectors, continue to add the vectors head-to-tail as described in step 2. In this example, we have only two vectors, so we have finished placing arrows tip to tail.
4. Draw an arrow from the tail of the first vector to the head of the last vector, as shown in Figure 5.5(c). This is the **resultant**, or the sum, of the vectors.

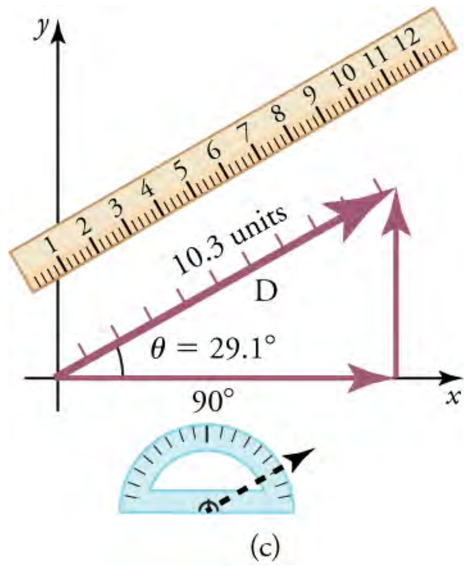


Figure 5.5 The diagram shows the resultant vector, a ruler, and protractor.

5. To find the magnitude of the resultant, measure its length with a ruler. When we deal with vectors analytically in the next section, the magnitude will be calculated by using the Pythagorean theorem.
6. To find the direction of the resultant, use a protractor to measure the angle it makes with the reference direction (in this case, the *x*-axis). When we deal with vectors analytically in the next section, the direction will be calculated by using trigonometry to find the angle.

WATCH PHYSICS

Visualizing Vector Addition Examples

This video shows four graphical representations of vector addition and matches them to the correct vector addition formula.

Click to view content (https://openstax.org/l/02addvector)

GRASP CHECK

There are two vectors $\vec{a}$ and $\vec{b}$. The head of vector $\vec{a}$ touches the tail of vector $\vec{b}$. The addition of vectors $\vec{a}$ and $\vec{b}$ gives a resultant vector $\vec{c}$. Can the addition of these two vectors can be represented by the following two equations? $\vec{a} + \vec{b} = \vec{c}$; $\vec{b} + \vec{a} = \vec{c}$

a. Yes, if we add the same two vectors in a different order it will still give the same resultant vector.
b. No, the resultant vector will change if we add the same vectors in a different order.

Vector subtraction is done in the same way as vector addition with one small change. We add the first vector to the negative of the vector that needs to be subtracted. A negative vector has the same magnitude as the original vector, but points in the opposite direction (as shown in Figure 5.6). Subtracting the vector **B** from the vector **A**, which is written as **A** – **B**, is the same as **A** + (–**B**). Since it does not matter in what order vectors are added, **A** – **B** is also equal to (–**B**) + **A**. This is true for scalars as well as vectors. For example, 5 – 2 = 5 + (–2) = (–2) + 5.

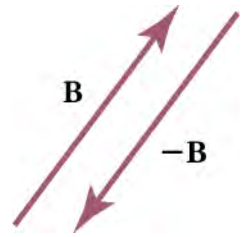


Figure 5.6 The diagram shows a vector, B, and the negative of this vector, –B.

Global angles are calculated in the counterclockwise direction. The clockwise direction is considered negative. For example, an angle of 30° south of west is the same as the global angle 210°, which can also be expressed as -150° from the positive *x*-axis.

Using the Graphical Method of Vector Addition and Subtraction to Solve Physics Problems

Now that we have the skills to work with vectors in two dimensions, we can apply vector addition to graphically determine the **resultant vector**, which represents the total force. Consider an example of force involving two ice skaters pushing a third as seen in Figure 5.7.

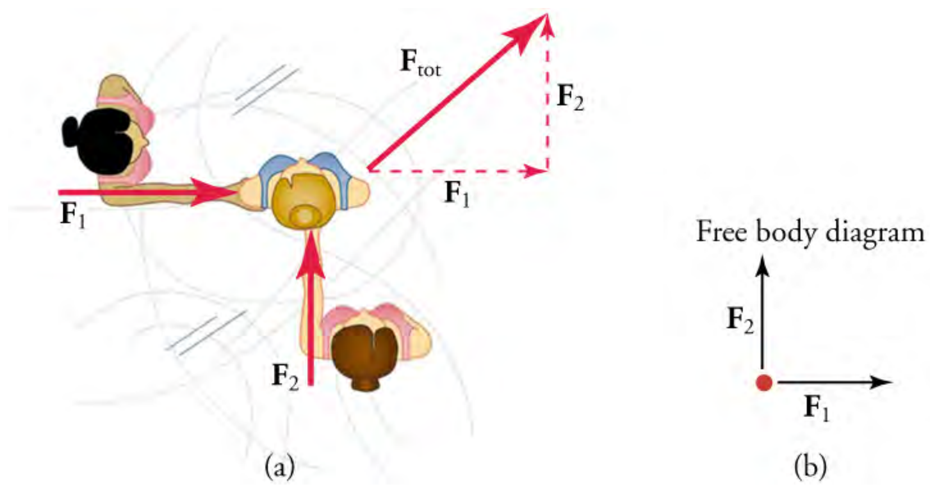


Figure 5.7 Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

In problems where variables such as force are already known, the forces can be represented by making the length of the vectors proportional to the magnitudes of the forces. For this, you need to create a scale. For example, each centimeter of vector length could represent 50 N worth of force. Once you have the initial vectors drawn to scale, you can then use the head-to-tail method to draw the resultant vector. The length of the resultant can then be measured and converted back to the original units using the scale you created.

You can tell by looking at the vectors in the free-body diagram in Figure 5.7 that the two skaters are pushing on the third skater with equal-magnitude forces, since the length of their force vectors are the same. Note, however, that the forces are not equal because they act in different directions. If, for example, each force had a magnitude of 400 N, then we would find the magnitude of the total external force acting on the third skater by finding the magnitude of the resultant vector. Since the forces act at a right angle to one another, we can use the Pythagorean theorem. For a triangle with sides a, b, and c, the Pythagorean theorem tells us that

$$a^2 + b^2 = c^2$$
$$c = \sqrt{a^2 + b^2}.$$

Applying this theorem to the triangle made by $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_{tot}$ in Figure 5.7, we get

$$\mathbf{F}_{tot}^2 = \sqrt{\mathbf{F}_1^2 + \mathbf{F}_1^2},$$

or

$$\mathbf{F}_{\text{tot}} = \sqrt{(400\text{ N})^2 + (400\text{ N})^2} = 566\text{ N}.$$

Note that, if the vectors were not at a right angle to each other (90° to one another), we would not be able to use the Pythagorean theorem to find the magnitude of the resultant vector. Another scenario where adding two-dimensional vectors is necessary is for velocity, where the direction may not be purely east-west or north-south, but some combination of these two directions. In the next section, we cover how to solve this type of problem analytically. For now let's consider the problem graphically.

WORKED EXAMPLE

Adding Vectors Graphically by Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, he walks 25 m in a direction 49° north of east. Then, he walks 23 m heading 15° north of east. Finally, he turns and walks 32 m in a direction 68° south of east.

Strategy

Graphically represent each displacement vector with an arrow, labeling the first **A**, the second **B**, and the third **C**. Make the lengths proportional to the distance of the given displacement and orient the arrows as specified relative to an east-west line. Use the head-to-tail method outlined above to determine the magnitude and direction of the resultant displacement, which we'll call **R**.

Solution

(1) Draw the three displacement vectors, creating a convenient scale (such as 1 cm of vector length on paper equals 1 m in the problem), as shown in Figure 5.8.

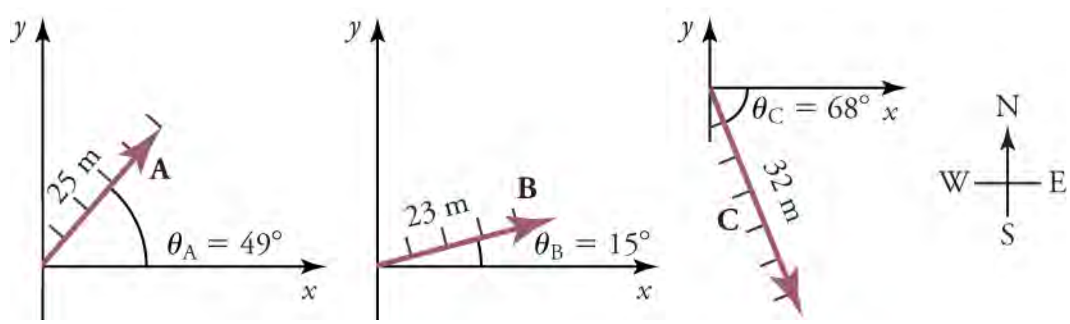


Figure 5.8 The three displacement vectors are drawn first.

(2) Place the vectors head to tail, making sure not to change their magnitude or direction, as shown in Figure 5.9.

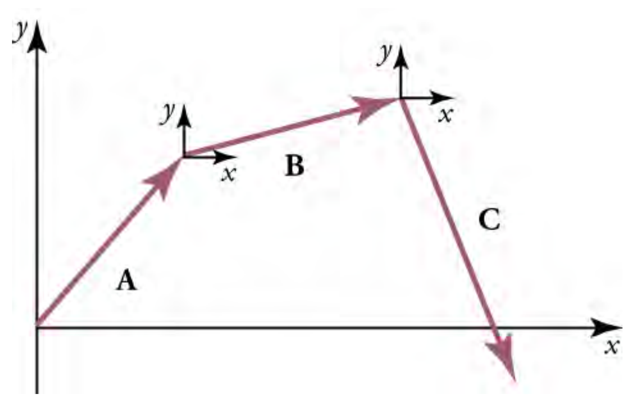


Figure 5.9 Next, the vectors are placed head to tail.

(3) Draw the **resultant vector R** from the tail of the first vector to the head of the last vector, as shown in Figure 5.10.

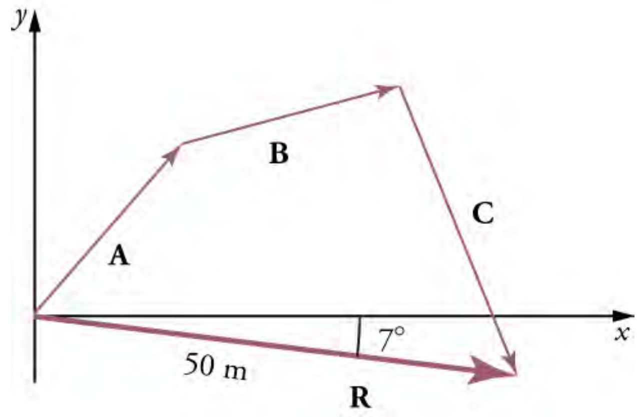


Figure 5.10 The resultant vector is drawn .

(4) Use a ruler to measure the magnitude of **R**, remembering to convert back to the units of meters using the scale. Use a protractor to measure the direction of **R**. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since **R** is south of the eastward pointing axis (the *x*-axis), we flip the protractor upside down and measure the angle between the eastward axis and the vector, as illustrated in Figure 5.11.

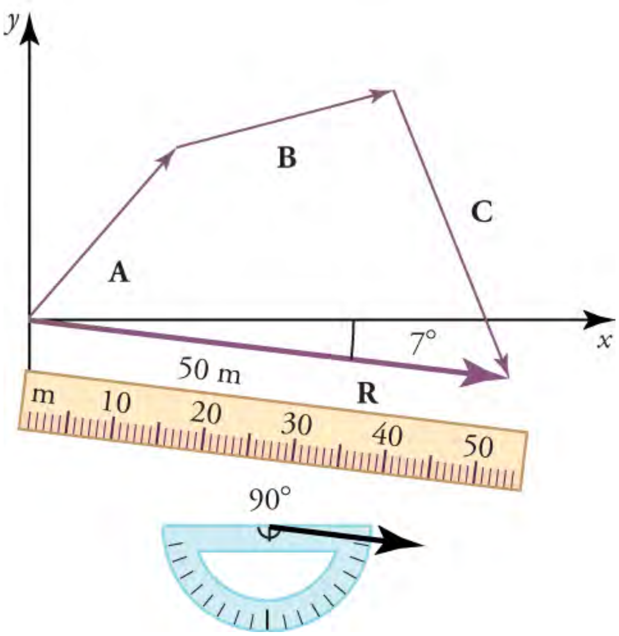


Figure 5.11 A ruler is used to measure the magnitude of **R**, and a protractor is used to measure the direction of **R**.

In this case, the total displacement **R** has a magnitude of 50 m and points 7° south of east. Using its magnitude and direction, this vector can be expressed as

$$R = 50 \text{ m} \tag{5.1}$$

and

$$\theta = 7^\circ \text{south of east.} \tag{5.2}$$

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that it does not matter in what order the vectors are added. Changing the order does not change the resultant. For example, we could add the vectors as shown in Figure 5.12, and we would still get the same solution.

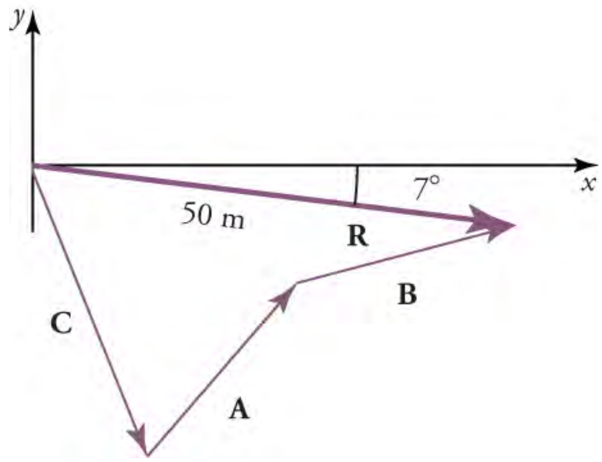


Figure 5.12 Vectors can be added in any order to get the same result.

WORKED EXAMPLE

Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? The two legs of the woman's trip are illustrated in Figure 5.13.

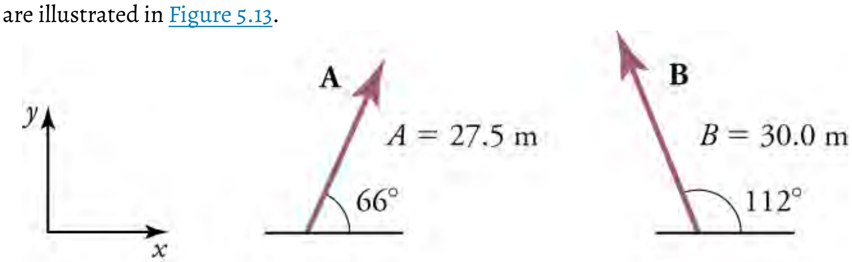


Figure 5.13 In the diagram, the first leg of the trip is represented by vector A and the second leg is represented by vector B.

Strategy

We can represent the first leg of the trip with a vector **A**, and the second leg of the trip that she was *supposed to* take with a vector **B**. Since the woman mistakenly travels in the opposite direction for the second leg of the journey, the vector for second leg of the trip she *actually* takes is –**B**. Therefore, she will end up at a location **A** + (–**B**), or **A** – **B**. Note that –**B** has the same magnitude as **B** (30.0 m), but is in the opposite direction, $68^\circ\,(180^\circ - 112^\circ)$ south of east, as illustrated in Figure 5.14.

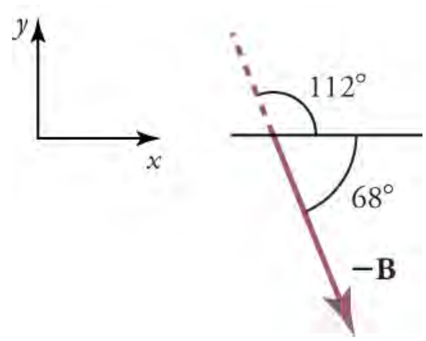


Figure 5.14 Vector –B represents traveling in the opposite direction of vector B.

We use graphical vector addition to find where the woman arrives **A** + (–**B**).

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors **A** and –**B**.

(2) Place the vectors head to tail.

(3) Draw the resultant vector **R**.

(4) Use a ruler and protractor to measure the magnitude and direction of **R**.

These steps are demonstrated in Figure 5.15.

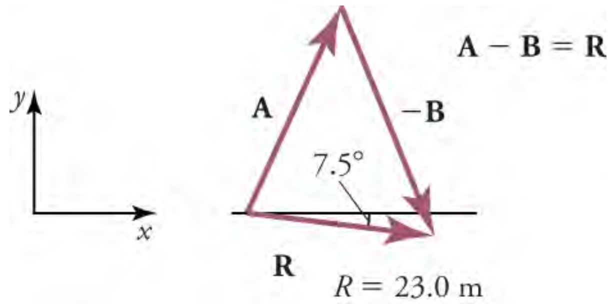


Figure 5.15 The vectors are placed head to tail.

In this case

$$R = 23.0\text{ m} \tag{5.3}$$

and

$$\theta = 7.5°\text{south of east.} \tag{5.4}$$

Discussion

Because subtraction of a vector is the same as addition of the same vector with the opposite direction, the graphical method for subtracting vectors works the same as for adding vectors.

WORKED EXAMPLE

Adding Velocities: A Boat on a River

A boat attempts to travel straight across a river at a speed of 3.8 m/s. The river current flows at a speed v_{river} of 6.1 m/s to the right. What is the total velocity and direction of the boat? You can represent each meter per second of velocity as one centimeter of vector length in your drawing.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river. Because the boat is directed straight toward the other shore, its velocity is perpendicular to the velocity of the river. We draw the two vectors, $\mathbf{v}_{\text{boat}}$ and $\mathbf{v}_{\text{river}}$, as shown in Figure 5.16.

Using the head-to-tail method, we draw the resulting total velocity vector from the tail of $\mathbf{v}_{\text{boat}}$ to the head of $\mathbf{v}_{\text{river}}$.

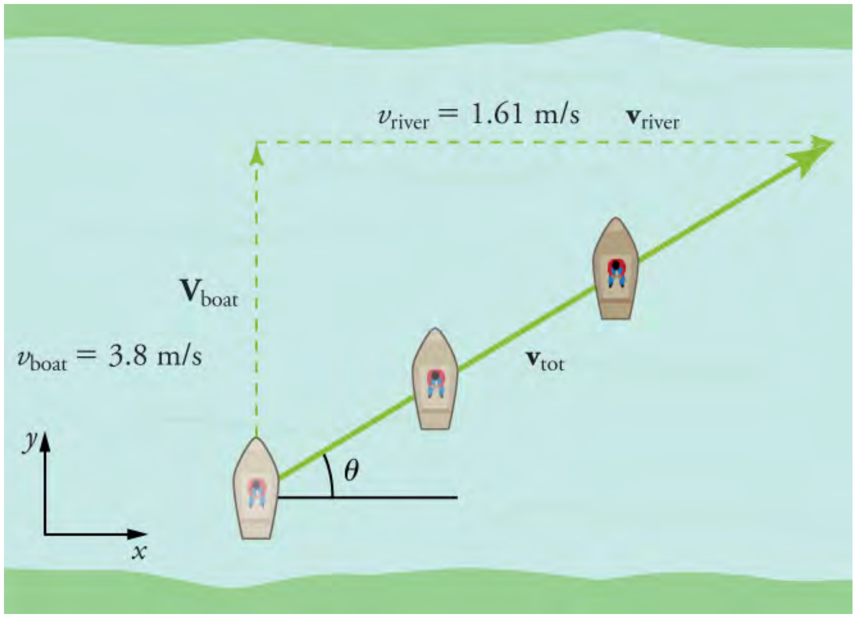


Figure 5.16 A boat attempts to travel across a river. What is the total velocity and direction of the boat?

Solution

By using a ruler, we find that the length of the resultant vector is 7.2 cm, which means that the magnitude of the total velocity is

$$v_{tot} = 7.2 \text{ m/s.} \tag{5.5}$$

By using a protractor to measure the angle, we find $\theta = 32.0^{\circ}$.

Discussion

If the velocity of the boat and river were equal, then the direction of the total velocity would have been 45°. However, since the velocity of the river is greater than that of the boat, the direction is less than 45° with respect to the shore, or *x* axis.

Practice Problems

1. Vector $\vec{A}$, having magnitude 2.5 m, pointing 37° south of east and vector $\vec{B}$ having magnitude 3.5 m, pointing 20° north of east are added. What is the magnitude of the resultant vector?
   a. 1.0 m
   b. 5.3 m
   c. 5.9 m
   d. 6.0 m

2. A person walks 32° north of west for 94 m and 35° east of south for 122 m. What is the magnitude of his displacement?
   a. 28 m
   b. 51 m
   c. 180 m
   d. 216 m

Virtual Physics

Vector Addition

In this simulation (https://archive.cnx.org/specials/d218bf9b-e50e-4d50-9a6c-b3db4dad0816/vector-addition/), you will experiment with adding vectors graphically. Click and drag the red vectors from the Grab One basket onto the graph in the middle of the screen. These red vectors can be rotated, stretched, or repositioned by clicking and dragging with your mouse. Check the Show Sum box to display the resultant vector (in green), which is the sum of all of the red vectors placed on the graph. To remove a red vector, drag it to the trash or click the Clear All button if you wish to start over. Notice that, if you click on any of the vectors, the $|\mathbf{R}|$ is its magnitude, θ is its direction with respect to the positive x-axis, $\mathbf{R_x}$ is its horizontal component, and R_y is its vertical component. You can check the resultant by lining up the vectors so that the head of the first vector touches the tail of the second. Continue until all of the vectors are aligned together head-to-tail. You will see that the resultant magnitude and angle is the same as the arrow drawn from the tail of the first vector to the head of the last vector. Rearrange the vectors in any order head-to-tail and compare. The resultant will always be the same.

Click to view content (https://archive.cnx.org/specials/d218bf9b-e50e-4d50-9a6c-b3db4dad0816/vector-addition/)

GRASP CHECK

True or False—The more long, red vectors you put on the graph, rotated in any direction, the greater the magnitude of the resultant green vector.

a. True
b. False

Check Your Understanding

3. While there is no single correct choice for the sign of axes, which of the following are conventionally considered positive?
   a. backward and to the left
   b. backward and to the right
   c. forward and to the right
   d. forward and to the left

4. True or False—A person walks 2 blocks east and 5 blocks north. Another person walks 5 blocks north and then two blocks east. The displacement of the first person will be more than the displacement of the second person.
   a. True
   b. False

5.2 Vector Addition and Subtraction: Analytical Methods

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define components of vectors
- Describe the analytical method of vector addition and subtraction
- Use the analytical method of vector addition and subtraction to solve problems

Section Key Terms

analytical method	component (of a two-dimensional vector)

Components of Vectors

For the **analytical method** of vector addition and subtraction, we use some simple geometry and trigonometry, instead of using a ruler and protractor as we did for graphical methods. However, the graphical method will still come in handy to visualize the problem by drawing vectors using the head-to-tail method. The analytical method is more accurate than the graphical method, which is limited by the precision of the drawing. For a refresher on the definitions of the sine, cosine, and tangent of an angle, see Figure 5.17.

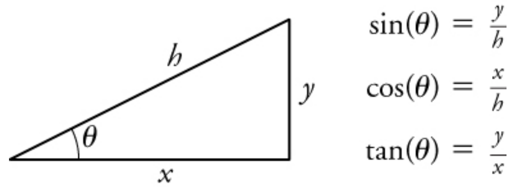


Figure 5.17 For a right triangle, the sine, cosine, and tangent of θ are defined in terms of the adjacent side, the opposite side, or the hypotenuse. In this figure, x is the adjacent side, y is the opposite side, and h is the hypotenuse.

Since, by definition, $\cos\theta = x/h$, we can find the length x if we know h and θ by using $x = h\cos\theta$. Similarly, we can find the length of y by using $y = h\sin\theta$. These trigonometric relationships are useful for adding vectors.

When a vector acts in more than one dimension, it is useful to break it down into its x and y components. For a two-dimensional vector, a **component** is a piece of a vector that points in either the x- or y-direction. Every 2-d vector can be expressed as a sum of its x and y components.

For example, given a vector like $\mathbf{A}$ in Figure 5.18, we may want to find what two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it. In this example, $\mathbf{A}_x$ and $\mathbf{A}_y$ form a right triangle, meaning that the angle between them is 90 degrees. This is a common situation in physics and happens to be the least complicated situation trigonometrically.

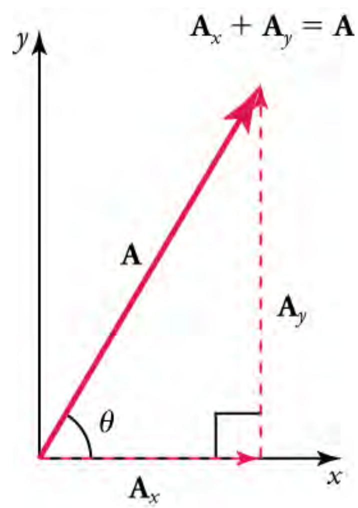


Figure 5.18 The vector $\mathbf{A}$, with its tail at the origin of an x- y-coordinate system, is shown together with its x- and y-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the x- and y-axes. The three vectors, $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$, form a right triangle.

$$\mathbf{A_x} + \mathbf{A_y} = \mathbf{A}$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its x- and y-components, we use the following relationships for a right triangle:

$$A_x = A\cos\theta$$

and

$$A_y = A\sin\theta,$$

where A_x is the magnitude of $\mathbf{A}$ in the x-direction, A_y is the magnitude of $\mathbf{A}$ in the y-direction, and θ is the angle of the resultant with respect to the x-axis, as shown in Figure 5.19.

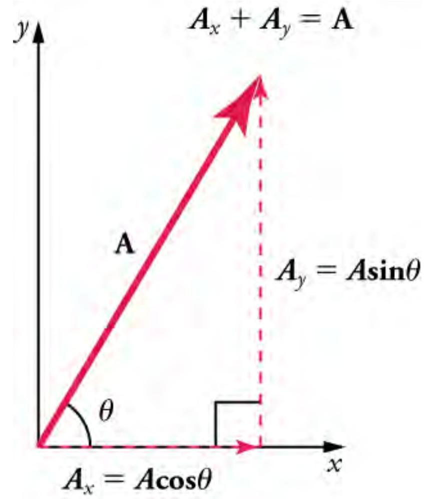


Figure 5.19 The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city, as illustrated in Figure 5.20.

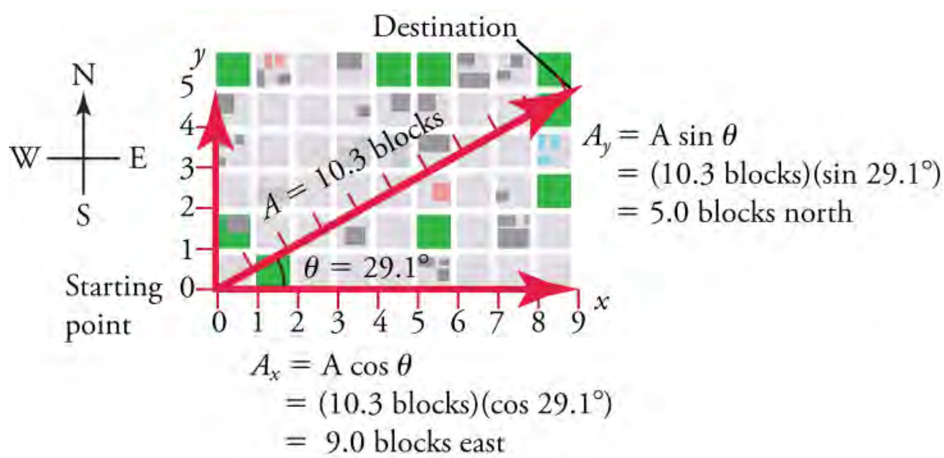


Figure 5.20 We can use the relationships $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then A = 10.3 blocks and $\theta = 29.1^\circ$, so that

$$\begin{aligned} A_x &= A\cos\theta \\ &= (10.3 \text{ blocks})(\cos 29.1^\circ) \\ &= (10.3 \text{ blocks})(0.874) \\ &= 9.0 \text{ blocks.} \end{aligned} \tag{5.6}$$

This magnitude indicates that the walker has traveled 9 blocks to the east—in other words, a 9-block eastward displacement. Similarly,

$$\begin{aligned} A_y &= A\sin\theta \\ &= (10.3 \text{ blocks})(\sin 29.1^\circ) \\ &= (10.3 \text{ blocks})(0.846) \\ &= 5.0 \text{ blocks,} \end{aligned} \tag{5.7}$$

indicating that the walker has traveled 5 blocks to the north—a 5-block northward displacement.

Analytical Method of Vector Addition and Subtraction

Calculating a resultant vector (or vector addition) is the reverse of breaking the resultant down into its components. If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then we can find $\mathbf{A}$ analytically. How do we do this? Since, by definition,

$$\tan\theta = y/x \text{ (or in this case } \tan\theta = A_y/A_x),$$

we solve for θ to find the direction of the resultant.

$$\theta = \tan^{-1}(A_y/A_x)$$

Since this is a right triangle, the Pythagorean theorem ($x^2 + y^2 = h^2$) for finding the hypotenuse applies. In this case, it becomes

$$A^2 = A_x^2 + A_y^2.$$

Solving for A gives

$$A = \sqrt{{A_x}^2 + {A_y}^2}.$$

In summary, to find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, as illustrated in Figure 5.21, we use the following relationships:

$$A = \sqrt{{A_x}^2 + {A_y}^2}$$
$$\theta = \tan^{-1}(A_y/A_x)$$

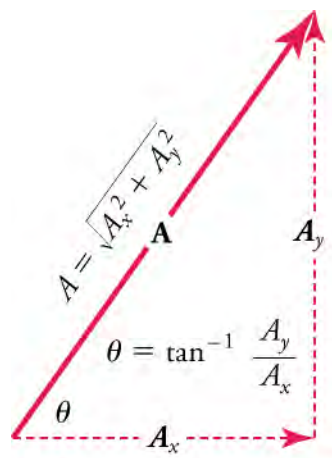


Figure 5.21 The magnitude and direction of the resultant vector $\mathbf{A}$ can be determined once the horizontal components $\mathbf{A}_x$ and $\mathbf{A}_y$ have been determined.

Sometimes, the vectors added are not perfectly perpendicular to one another. An example of this is the case below, where the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$, as illustrated in Figure 5.22.

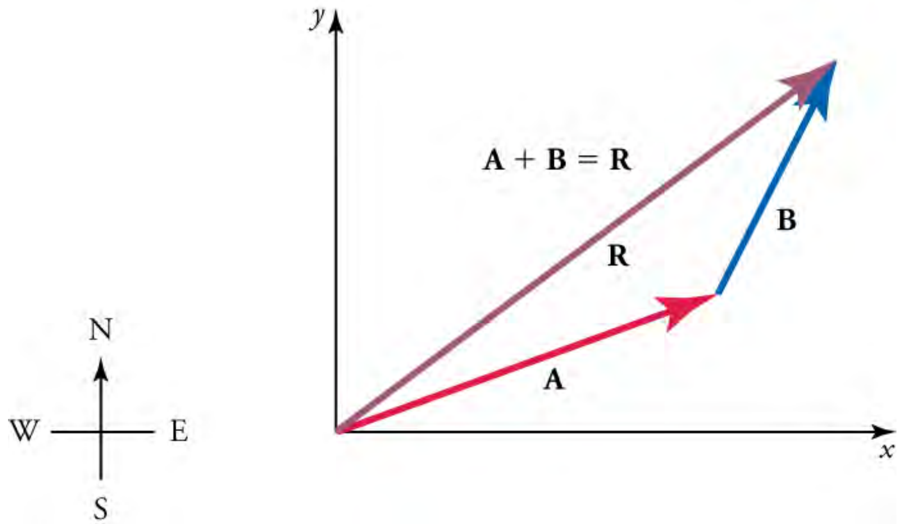


Figure 5.22 Vectors **A** and **B** are two legs of a walk, and **R** is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of **R** .

If **A** and **B** represent two legs of a walk (two displacements), then **R** is the total displacement. The person taking the walk ends up at the tip of **R** . There are many ways to arrive at the same point. The person could have walked straight ahead first in the *x*-direction and then in the *y*-direction. Those paths are the *x*- and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $R = \sqrt{R_x{}^2 + R_y{}^2}$ and $\theta = tan^{-1}(R_y/R_x)$.

1. Draw in the *x* and *y* components of each vector (including the resultant) with a dashed line. Use the equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to find the components. In Figure 5.23, these components are A_x, A_y, B_x, and B_y. Vector **A** makes an angle of θ_A with the *x*-axis, and vector **B** makes and angle of θ_B with its own *x*-axis (which is slightly above the *x*-axis used by vector **A**).

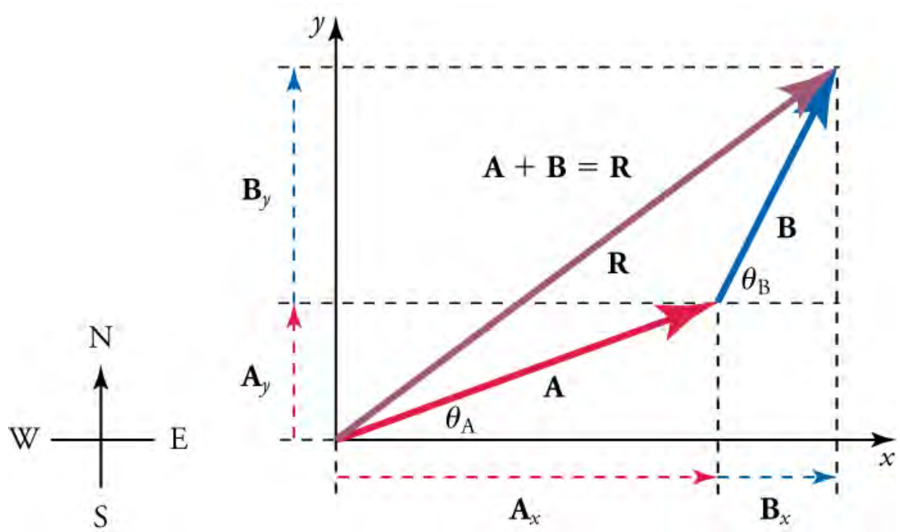


Figure 5.23 To add vectors **A** and **B**, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$ $\mathbf{B}_y$ shown in the image.

2. Find the *x* component of the resultant by adding the *x* component of the vectors

$$R_x = A_x + B_x$$

and find the *y* component of the resultant (as illustrated in Figure 5.24) by adding the *y* component of the vectors.

$$R_y = A_y + B_y.$$

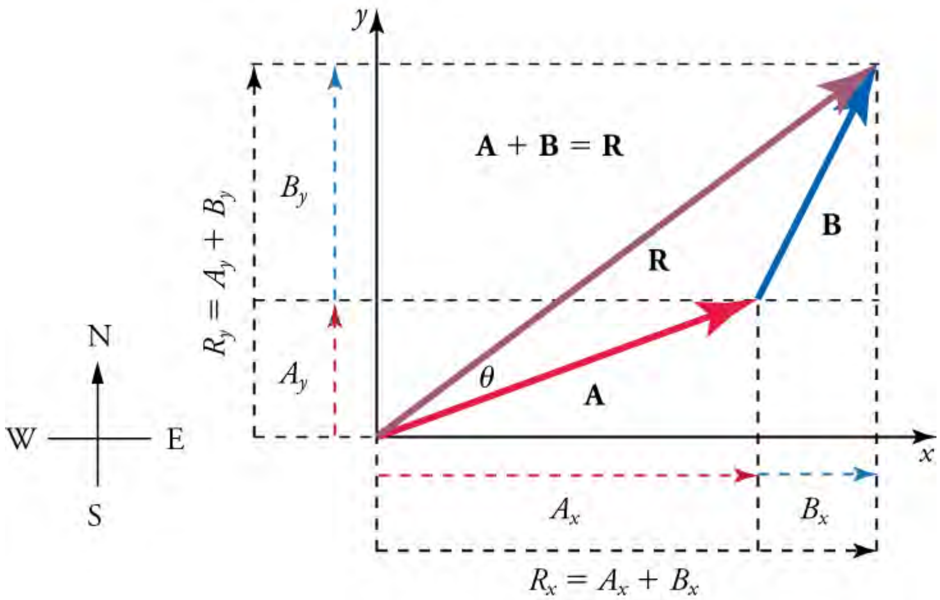


Figure 5.24 The vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude of the resultant vector in the horizontal direction, R_x. Similarly, the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude of the resultant vector in the vertical direction, R_y.

Now that we know the components of **R**, we can find its magnitude and direction.

3. To get the magnitude of the resultant R, use the Pythagorean theorem.

$$R = \sqrt{R_x^2 + R_y^2}$$

4. To get the direction of the resultant

$$\theta = \tan^{-1}(R_y/R_x).$$

WATCH PHYSICS

Classifying Vectors and Quantities Example

This video contrasts and compares three vectors in terms of their magnitudes, positions, and directions.

[Click to view content (https://www.youtube.com/embed/YpoEhcVBxNU)](https://www.youtube.com/embed/YpoEhcVBxNU)

GRASP CHECK

Three vectors, $\vec{u}$, $\vec{v}$, and $\vec{w}$, have the same magnitude of 5 units. Vector $\vec{v}$ points to the northeast. Vector $\vec{w}$ points to the southwest exactly opposite to vector $\vec{u}$. Vector $\vec{u}$ points in the northwest. If the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ were added together, what would be the magnitude of the resultant vector? Why?

a. 0 units. All of them will cancel each other out.
b. 5 units. Two of them will cancel each other out.
c. 10 units. Two of them will add together to give the resultant.
d. 15 units. All of them will add together to give the resultant.

TIPS FOR SUCCESS

In the video, the vectors were represented with an arrow above them rather than in bold. This is a common notation in math classes.

Using the Analytical Method of Vector Addition and Subtraction to Solve Problems

Figure 5.25 uses the analytical method to add vectors.

WORKED EXAMPLE

An Accelerating Subway Train

Add the vector **A** to the vector **B** shown in Figure 5.25, using the steps above. The *x*-axis is along the east–west direction, and the *y*-axis is along the north–south directions. A person first walks 53.0 m in a direction 20.0° north of east, represented by vector **A**. The person then walks 34.0 m in a direction 63.0° north of east, represented by vector **B**.

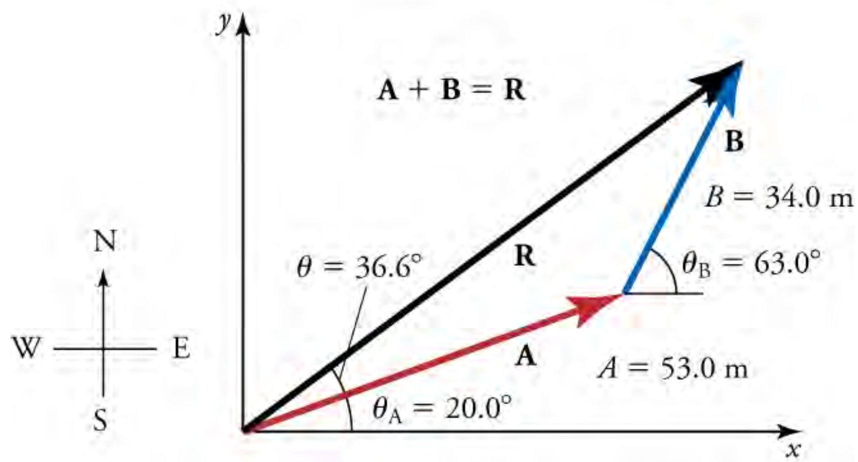


Figure 5.25 You can use analytical models to add vectors.

Strategy

The components of **A** and **B** along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. We will solve for these components and then add them in the x-direction and y-direction to find the resultant.

Solution

First, we find the components of **A** and **B** along the *x*- and *y*-axes. From the problem, we know that $A = 53.0\ \text{m}, \theta_{\text{A}} = 20.0°$, $B = 34.0$ m, and $\theta_{\text{B}} = 63.0°$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

$$\begin{aligned} A_x &= A\cos\theta_A = (53.0\ \text{m})(\cos 20.0°) \\ &= (53.0\ \text{m})(0.940) = 49.8\ \text{m} \end{aligned}$$

and

$$\begin{aligned} B_x &= B\cos\theta_B = (34.0\ \text{m})(\cos 63.0°) \\ &= (34.0\ \text{m})(0.454) = 15.4\ \text{m}. \end{aligned}$$

Similarly, the *y*-components are found using $A_y = A\sin\theta_A$

$$\begin{aligned} A_y &= A\sin\theta_A = (53.0\ \text{m})(\sin 20.0°) \\ &= (53.0\ \text{m})(0.342) = 18.1\ \text{m} \end{aligned}$$

and

$$\begin{aligned} B_y &= B\sin\theta_B = (34.0\ \text{m})(\sin 63.0°) \\ &= (34.0\ \text{m})(0.891) = 30.3\ \text{m}. \end{aligned}$$

The *x*- and *y*-components of the resultant are

$$R_x = A_x + B_x = 49.8\ \text{m} + 15.4\ \text{m} = 65.2\ \text{m}$$

and

$$R_y = A_y + B_y = 18.1 \text{ m} + 30.3 \text{ m} = 48.4 \text{ m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2} \text{ m} \tag{5.8}$$

so that

$$R = \sqrt{6601 \text{ m}} = 81.2 \text{ m}.$$

Finally, we find the direction of the resultant

$$\theta = \tan^{-1}(R_y/R_x) = +\tan^{-1}(48.4/65.2).$$

This is

$$\theta = \tan^{-1}(0.742) = 36.6^\circ.$$

Discussion

This example shows vector addition using the analytical method. Vector subtraction using the analytical method is very similar. It is just the addition of a negative vector. That is, $\mathrm{A} - \mathrm{B} \equiv \mathrm{A} + (-\mathrm{B})$. The components of $-\mathrm{B}$ are the negatives of the components of B. Therefore, the *x*- and *y*-components of the resultant $\mathrm{A} - \mathrm{B} = \mathrm{R}$ are

$$R_x = A_x + -B_x$$

and

$$R_y = A_y + -B_y$$

and the rest of the method outlined above is identical to that for addition.

Practice Problems

5. What is the magnitude of a vector whose *x*-component is 4 cm and whose *y*-component is 3 cm?

a. 1 cm
b. 5 cm
c. 7 cm
d. 25 cm

6. What is the magnitude of a vector that makes an angle of 30° to the horizontal and whose *x*-component is 3 units?

a. 2.61 units
b. 3.00 units
c. 3.46 units
d. 6.00 units

LINKS TO PHYSICS

Atmospheric Science

Figure 5.26 This picture shows Bert Foord during a television Weather Forecast from the Meteorological Office in 1963. (BBC TV)

Atmospheric science is a physical science, meaning that it is a science based heavily on physics. Atmospheric science includes meteorology (the study of weather) and climatology (the study of climate). Climate is basically the average weather over a longer time scale. Weather changes quickly over time, whereas the climate changes more gradually.

The movement of air, water and heat is vitally important to climatology and meteorology. Since motion is such a major factor in weather and climate, this field uses vectors for much of its math.

Vectors are used to represent currents in the ocean, wind velocity and forces acting on a parcel of air. You have probably seen a weather map using vectors to show the strength (magnitude) and direction of the wind.

Vectors used in atmospheric science are often three-dimensional. We won't cover three-dimensional motion in this text, but to go from two-dimensions to three-dimensions, you simply add a third vector component. Three-dimensional motion is represented as a combination of *x*-, *y*- and *z* components, where *z* is the altitude.

Vector calculus combines vector math with calculus, and is often used to find the rates of change in temperature, pressure or wind speed over time or distance. This is useful information, since atmospheric motion is driven by changes in pressure or temperature. The greater the variation in pressure over a given distance, the stronger the wind to try to correct that imbalance. Cold air tends to be more dense and therefore has higher pressure than warm air. Higher pressure air rushes into a region of lower pressure and gets deflected by the spinning of the Earth, and friction slows the wind at Earth's surface.

Finding how wind changes over distance and multiplying vectors lets meteorologists, like the one shown in Figure 5.26, figure out how much rotation (spin) there is in the atmosphere at any given time and location. This is an important tool for tornado prediction. Conditions with greater rotation are more likely to produce tornadoes.

GRASP CHECK

Why are vectors used so frequently in atmospheric science?

a. Vectors have magnitude as well as direction and can be quickly solved through scalar algebraic operations.
b. Vectors have magnitude but no direction, so it becomes easy to express physical quantities involved in the atmospheric science.
c. Vectors can be solved very accurately through geometry, which helps to make better predictions in atmospheric science.
d. Vectors have magnitude as well as direction and are used in equations that describe the three dimensional motion of the atmosphere.

Check Your Understanding

7. Between the analytical and graphical methods of vector additions, which is more accurate? Why?
   a. The analytical method is less accurate than the graphical method, because the former involves geometry and

trigonometry.
b. The analytical method is more accurate than the graphical method, because the latter involves some extensive calculations.
c. The analytical method is less accurate than the graphical method, because the former includes drawing all figures to the right scale.
d. The analytical method is more accurate than the graphical method, because the latter is limited by the precision of the drawing.

8. What is a component of a two dimensional vector?
a. A component is a piece of a vector that points in either the *x* or *y* direction.
b. A component is a piece of a vector that has half of the magnitude of the original vector.
c. A component is a piece of a vector that points in the direction opposite to the original vector.
d. A component is a piece of a vector that points in the same direction as original vector but with double of its magnitude.

9. How can we determine the global angle θ (measured counter-clockwise from positive *x*) if we know A_x and A_y?
a. $\theta = \cos^{-1} \frac{A_y}{A_x}$
b. $\theta = \cot^{-1} \frac{A_y}{A_x}$
c. $\theta = \sin^{-1} \frac{A_y}{A_x}$
d. $\theta = \tan^{-1} \frac{A_y}{A_x}$

10. How can we determine the magnitude of a vector if we know the magnitudes of its components?
a. $\left|\overrightarrow{\mathrm{A}}\right| = A_x + A_y$
b. $\left|\overrightarrow{\mathrm{A}}\right| = {A_x}^2 + {A_y}^2$
c. $\left|\overrightarrow{\mathrm{A}}\right| = \sqrt{{A_x}^2 + {A_y}^2}$
d. $\left|\overrightarrow{\mathrm{A}}\right| = \left({A_x}^2 + {A_y}^2\right)^2$

5.3 Projectile Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the properties of projectile motion
- Apply kinematic equations and vectors to solve problems involving projectile motion

Section Key Terms

air resistance	maximum height (of a projectile)	projectile
projectile motion	range	trajectory

Properties of Projectile Motion

Projectile motion is the motion of an object thrown (projected) into the air. After the initial force that launches the object, it only experiences the force of gravity. The object is called a **projectile**, and its path is called its **trajectory**. As an object travels through the air, it encounters a frictional force that slows its motion called **air resistance**. Air resistance does significantly alter trajectory motion, but due to the difficulty in calculation, it is ignored in introductory physics.

The most important concept in projectile motion is that *horizontal and vertical motions are independent*, meaning that they don't influence one another. Figure 5.27 compares a cannonball in free fall (in blue) to a cannonball launched horizontally in projectile motion (in red). You can see that the cannonball in free fall falls at the same rate as the cannonball in projectile motion. Keep in mind that if the cannon launched the ball with any vertical component to the velocity, the vertical displacements would not line up perfectly.

Since vertical and horizontal motions are independent, we can analyze them separately, along perpendicular axes. To do this, we separate projectile motion into the two components of its motion, one along the horizontal axis and the other along the vertical.

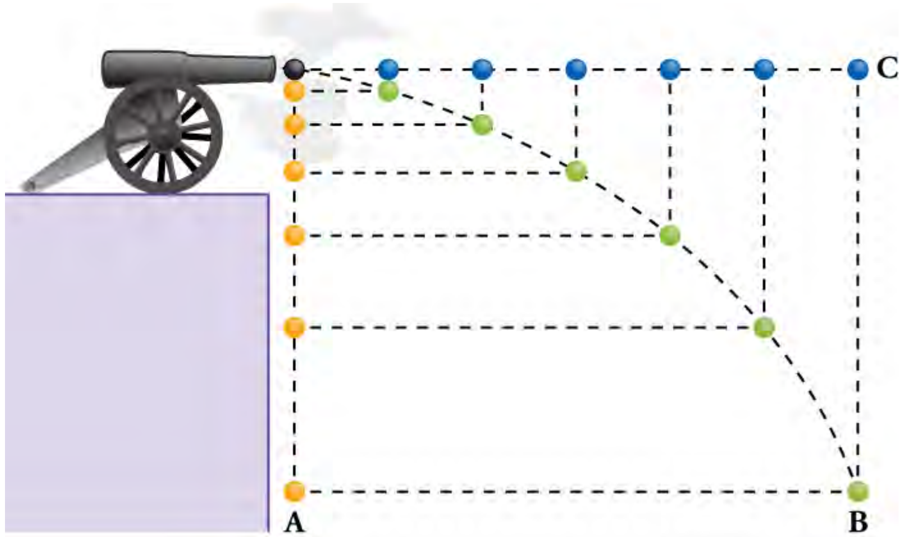


Figure 5.27 The diagram shows the projectile motion of a cannonball shot at a horizontal angle versus one dropped with no horizontal velocity. Note that both cannonballs have the same vertical position over time.

We'll call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. For notation, **d** is the total displacement, and **x** and **y** are its components along the horizontal and vertical axes. The magnitudes of these vectors are *x* and *y*, as illustrated in Figure 5.28.

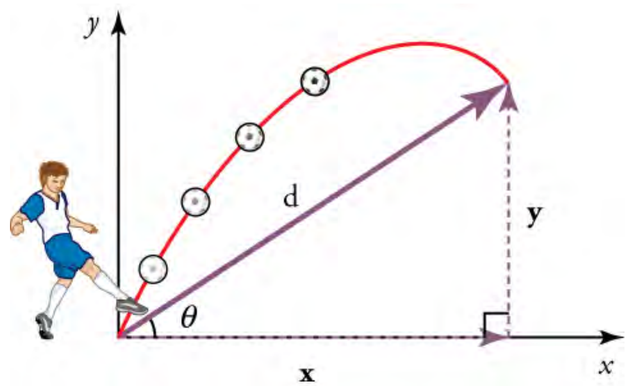


Figure 5.28 A boy kicks a ball at angle θ, and it is displaced a distance of **s** along its trajectory.

As usual, we use velocity, acceleration, and displacement to describe motion. We must also find the components of these variables along the *x*- and *y*-axes. The components of acceleration are then very simple $\mathbf{a_y} = -\mathbf{g} = -9.80\text{ m/s}^2$. Note that this definition defines the upwards direction as positive. Because gravity is vertical, $\mathbf{a_x} = 0$. Both accelerations are constant, so we can use the kinematic equations. For review, the kinematic equations from a previous chapter are summarized in Table 5.1.

$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_{\mathbf{avg}} t$ (when $\mathbf{a}=0$)
$\mathbf{v}_{\mathbf{avg}} = \frac{\mathbf{v}_0+\mathbf{v}}{2}$ (when $\mathbf{a}=0$)
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$
$\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_0)$

Table 5.1 Summary of Kinematic Equations (constant a)

Where **x** is position, $\mathbf{x}_0$ is initial position, **v** is velocity, $\mathbf{v}_{avg}$ is average velocity, *t* is time and a is acceleration.

Solve Problems Involving Projectile Motion

The following steps are used to analyze projectile motion:

1. Separate the motion into horizontal and vertical components along the x- and y-axes. These axes are perpendicular, so $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used. The magnitudes of the displacement $\mathbf{s}$ along x- and y-axes are called x and y. The magnitudes of the components of the velocity $\mathbf{v}$ are $v_x = v \cos\theta$ and $v_y = v \sin\theta$, where v is the magnitude of the velocity and θ is its direction. Initial values are denoted with a subscript 0.
2. Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical. The kinematic equations for horizontal and vertical motion take the following forms

$$\text{Horizontal Motion}(\mathbf{a}_x = 0)$$
$$x = x_0 + v_x t$$
$$v_x = v_{0x} = \mathbf{v}_\text{x} = \text{velocity is a constant.}$$

Vertical motion (assuming positive is up $\mathbf{a}_y = -\mathbf{g} = -9.80 \text{ m/s}^2$)

$$\begin{aligned} y &= y_0 + \tfrac{1}{2}(v_{0y} + v_y)t \\ v_y &= v_{0y} - \mathbf{g}t \\ y &= y_0 + v_{0y}t - \tfrac{1}{2}\mathbf{g}t^2 \\ v_y^2 &= v_{0y}^2 - 2g(y - y_0) \end{aligned}$$

3. Solve for the unknowns in the two separate motions (one horizontal and one vertical). Note that the only common variable between the motions is time t . The problem solving procedures here are the same as for one-dimensional kinematics.
4. Recombine the two motions to find the total displacement $\mathbf{s}$ and velocity $\mathbf{v}$. We can use the analytical method of vector addition, which uses $A = \sqrt{{A_x}^2 + {A_y}^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ to find the magnitude and direction of the total displacement and velocity.

$$\text{Displacement}$$
$$\mathbf{d} = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$
$$\text{Velocity}$$
$$\mathbf{v} = \sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2}$$
$$\theta_v = \tan^{-1}(\mathbf{v}_y/\mathbf{v}_x)$$

θ is the direction of the displacement $\mathbf{d}$, and θ_v is the direction of the velocity $\mathbf{v}$. (See Figure 5.29

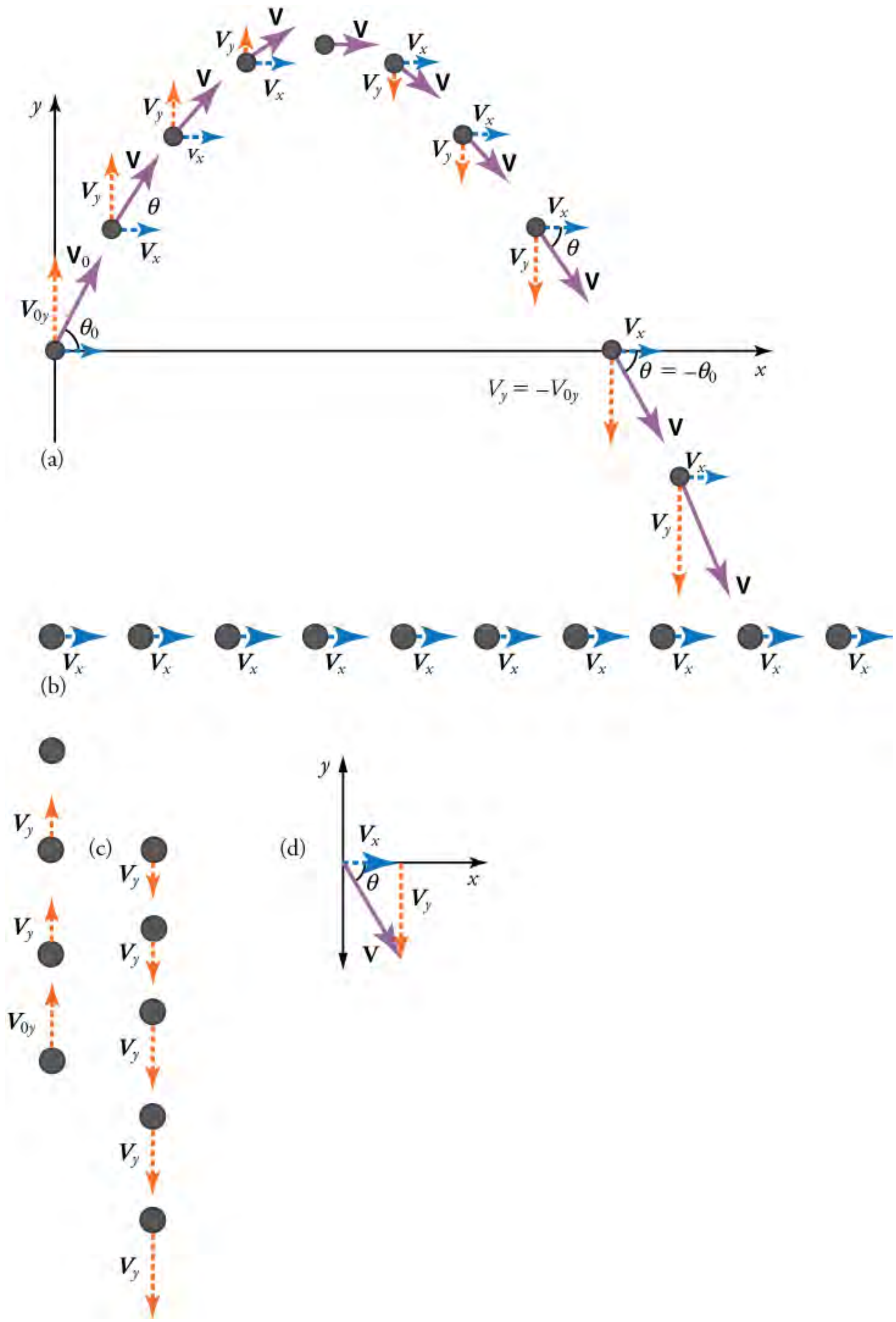


Figure 5.29 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $\mathbf{a}_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The *x*- and *y*-motions are recombined to give the total velocity at any given point on the trajectory.

TIPS FOR SUCCESS

For problems of projectile motion, it is important to set up a coordinate system. The first step is to choose an initial position for $\mathbf{x}$ and $\mathbf{y}$. Usually, it is simplest to set the initial position of the object so that $\mathbf{x}_0 = 0$ and $\mathbf{y}_0 = 0$.

WATCH PHYSICS

Projectile at an Angle

This video presents an example of finding the displacement (or range) of a projectile launched at an angle. It also reviews basic trigonometry for finding the sine, cosine and tangent of an angle.

Click to view content (https://www.khanacademy.org/embed_video?v=ZZ39o1rAZWY)

GRASP CHECK

Assume the ground is uniformly level. If the horizontal component a projectile's velocity is doubled, but the vertical component is unchanged, what is the effect on the time of flight?

a. The time to reach the ground would remain the same since the vertical component is unchanged.
b. The time to reach the ground would remain the same since the vertical component of the velocity also gets doubled.
c. The time to reach the ground would be halved since the horizontal component of the velocity is doubled.
d. The time to reach the ground would be doubled since the horizontal component of the velocity is doubled.

WORKED EXAMPLE

A Fireworks Projectile Explodes High and Away

During a fireworks display like the one illustrated in Figure 5.30, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75° above the horizontal. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

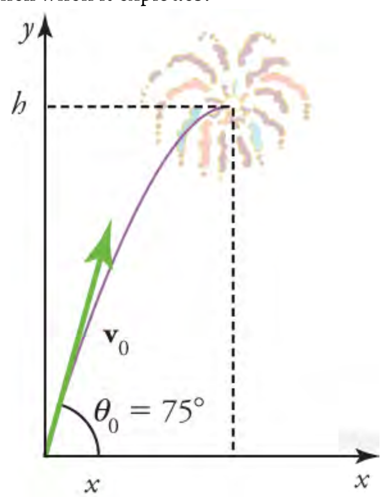


Figure 5.30 The diagram shows the trajectory of a fireworks shell.

Strategy

The motion can be broken into horizontal and vertical motions in which $\mathbf{a}_x = 0$ and $\mathbf{a}_y = \mathbf{g}$. We can then define $\mathbf{x}_0$ and $\mathbf{y}_0$ to be zero and solve for the **maximum height**.

Solution for (a)

By height we mean the altitude or vertical position **y** above the starting point. The highest point in any trajectory, the maximum height, is reached when $\mathbf{v}_y = 0$; this is the moment when the vertical velocity switches from positive (upwards) to negative (downwards). Since we know the initial velocity, initial position, and the value of $\mathbf{v}_y$ when the firework reaches its maximum height, we use the following equation to find **y**

$$\mathbf{v}_y^2 = \mathbf{v}_{0y}^2 - 2\mathbf{g}(\mathbf{y} - y_0).$$

Because $\mathbf{y}_0$ and $\mathbf{v}_y$ are both zero, the equation simplifies to

$$0 = \mathbf{v}_{0y}^2 - 2\mathbf{g}\mathbf{y}.$$

Solving for $\mathbf{y}$ gives

$$\mathbf{y} = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}.$$

Now we must find $\mathbf{v}_{0y}$, the component of the initial velocity in the *y*-direction. It is given by $\mathbf{v}_{0y} = \mathbf{v}_0 \sin\theta$, where $\mathbf{v}_{0y}$ is the initial velocity of 70.0 m/s, and $\theta = 75^\circ$ is the initial angle. Thus,

$$\mathbf{v}_{0y} = \mathbf{v}_0 \sin\theta_0 = (70.0 \text{ m/s})(\sin 75^\circ) = 67.6 \text{ m/s}$$

and $\mathbf{y}$ is

$$\mathbf{y} = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2),}$$

so that

$$\mathbf{y} = 233 \text{ m}.$$

Discussion for (a)

Since up is positive, the initial velocity and maximum height are positive, but the acceleration due to gravity is negative. The maximum height depends only on the vertical component of the initial velocity. The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding.

Solution for (b)

There is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $\mathbf{y} = \mathbf{y}_0 + \frac{1}{2}(\mathbf{v}_{0y} + \mathbf{v}_y)t$. Because y_0 is zero, this equation reduces to

$$\mathbf{y} = \frac{1}{2}(\mathbf{v}_{0y} + \mathbf{v}_y)t.$$

Note that the final vertical velocity, $\mathbf{v}_y$, at the highest point is zero. Therefore,

$$\begin{aligned} t &= \frac{2\mathbf{y}}{(\mathbf{v}_{0y}+\mathbf{v}_y)} = \frac{2(233 \text{ m})}{(67.6 \text{ m/s})} \\ &= 6.90 \text{ s}. \end{aligned}$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. Another way of finding the time is by using $\mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0\mathbf{y}}t - \frac{1}{2}\mathbf{g}t^2$, and solving the quadratic equation for t .

Solution for (c)

Because air resistance is negligible, $\mathbf{a}_x = 0$ and the horizontal velocity is constant. The horizontal displacement is horizontal velocity multiplied by time as given by $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_x t$, where $\mathbf{x}_0$ is equal to zero

$$\mathbf{x} = \mathbf{v}_x t,$$

where $\mathbf{v}_x$ is the *x*-component of the velocity, which is given by $\mathbf{v}_x = \mathbf{v}_0 \cos\theta_0$. Now,

$$\mathbf{v}_x = \mathbf{v}_0 \cos\theta_0 = (70.0 \text{ m/s})(\cos 75^\circ) = 18.1 \text{ m/s}.$$

The time *t* for both motions is the same, and so $\mathbf{x}$ is

$$\mathbf{x} = (18.1 \text{ m/s})(6.90 \text{ s}) = 125 \text{ m}.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below, while some of the fragments may now have a velocity in the –x direction due to the forces of the explosion.

The expression we found for $\mathbf{y}$ while solving part (a) of the previous problem works for any projectile motion problem where air resistance is negligible. Call the maximum height $\mathbf{y} = h$; then,

$$h = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}.$$

This equation defines the **maximum height of a projectile**. The maximum height depends only on the vertical component of the initial velocity.

WORKED EXAMPLE

Calculating Projectile Motion: Hot Rock Projectile

Suppose a large rock is ejected from a volcano, as illustrated in Figure 5.31, with a speed of 25.0 m/s and at an angle $35°$ above the horizontal. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path.

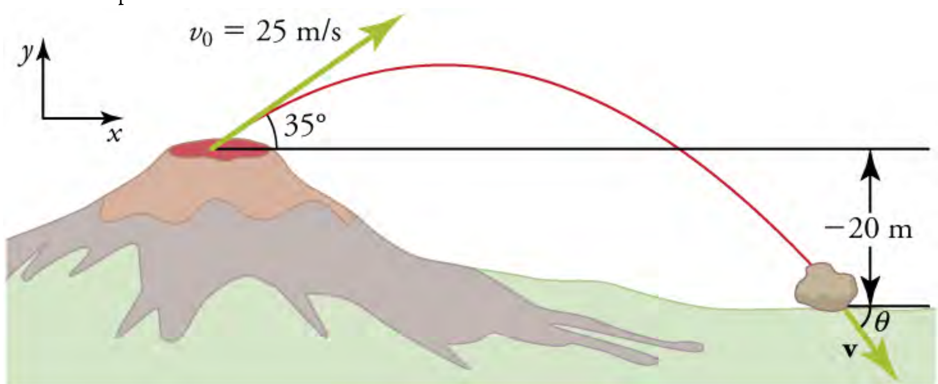


Figure 5.31 The diagram shows the projectile motion of a large rock from a volcano.

Strategy

Breaking this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the time. The time a projectile is in the air depends only on its vertical motion.

Solution

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$\mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0\mathbf{y}}t - \frac{1}{2}\mathbf{g}t^2.$$

If we take the initial position $\mathbf{y}_0$ to be zero, then the final position is $\mathbf{y} = -20.0 \ \text{ m}$. Now the initial vertical velocity is the vertical component of the initial velocity, found from

$$\mathbf{v}_{0y} = \mathbf{v}_0 \sin\theta_0 = (25.0 \ \text{ m/s})(\sin 35^\circ) = 14.3 \ \text{ m/s}. \quad (5.9)$$

Substituting known values yields

$$-20.0 \ \text{ m} = (14.3 \ \text{ m/s})t - \left(4.90 \text{ m/s}^2\right)t^2.$$

Rearranging terms gives a quadratic equation in t

$$\left(4.90 \text{ m/s}^2\right)t^2 - (14.3 \text{ m/s})\,t - (20.0 \ \text{ m}) = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90$, $b = -14.3$, and $c = -20.0$. Its solutions are given by the quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This equation yields two solutions $t = 3.96$ and $t = -1.03$. You may verify these solutions as an exercise. The time is t = 3.96 s or −1.03 s. The negative value of time implies an event before the start of motion, so we discard it. Therefore,

$$t = 3.96\ \text{s}.$$

Discussion

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Practice Problems

11. If an object is thrown horizontally, travels with an average x-component of its velocity equal to $5\ \text{m/s}$, and does not hit the ground, what will be the x-component of the displacement after $20\ \text{s}$?

a. $-100\ \text{m}$
b. $-4\ \text{m}$
c. $4\ \text{m}$
d. $100\ \text{m}$

12. If a ball is thrown straight up with an initial velocity of $20\ \text{m/s}$ upward, what is the maximum height it will reach?

a. $-20.4\ \text{m}$
b. $-1.02\ \text{m}$
c. $1.02\ \text{m}$
d. $20.4\ \text{m}$

The fact that vertical and horizontal motions are independent of each other lets us predict the range of a projectile. The **range** is the horizontal distance **R** traveled by a projectile on level ground, as illustrated in Figure 5.32. Throughout history, people have been interested in finding the range of projectiles for practical purposes, such as aiming cannons.

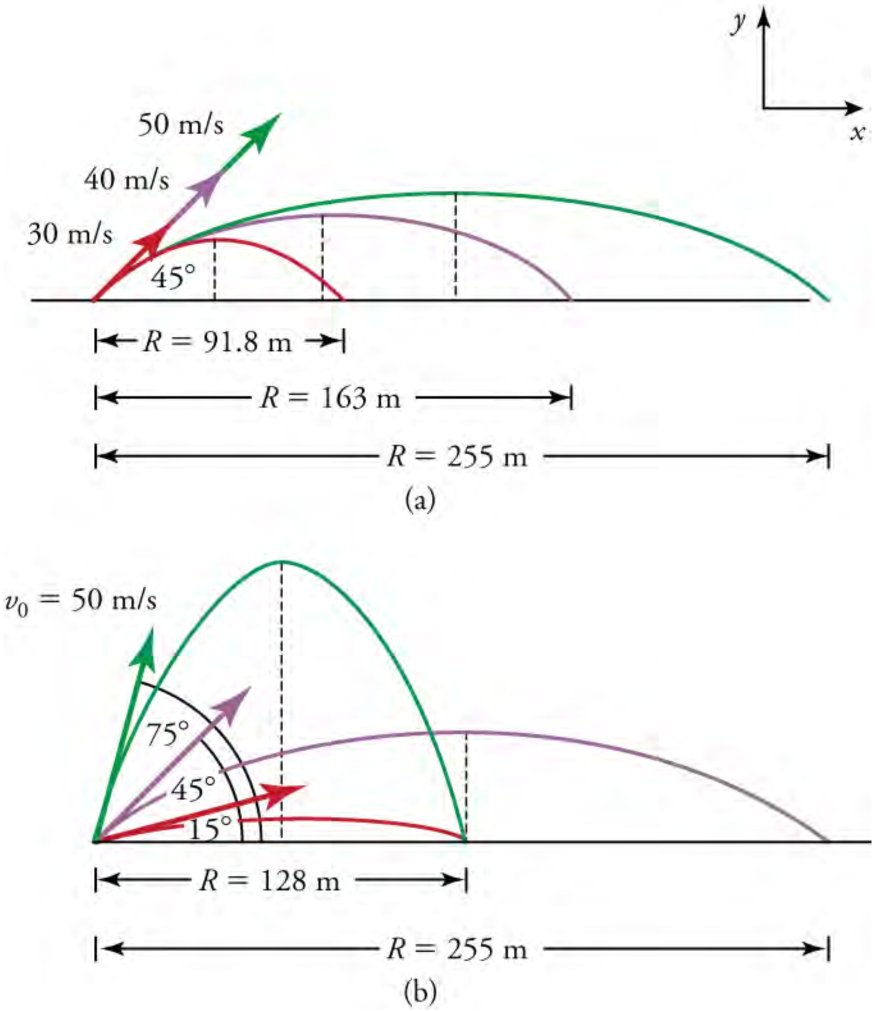


Figure 5.32 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that any combination of trajectories that add to 90 degrees will have the same range in the absence of air resistance, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in the figure above. The initial angle θ_0 also has a dramatic effect on the range. When air resistance is negligible, the range R of a projectile on *level ground* is

$$R = \frac{v_0^2 \sin 2\theta_0}{\mathbf{g}},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. It is important to note that the range doesn't apply to problems where the initial and final y position are different, or to cases where the object is launched perfectly horizontally.

Virtual Physics

Projectile Motion

In this simulation you will learn about projectile motion by blasting objects out of a cannon. You can choose between objects such as a tank shell, a golf ball or even a Buick. Experiment with changing the angle, initial speed, and mass, and adding in air resistance. Make a game out of this simulation by trying to hit the target.

Click to view content (https://archive.cnx.org/specials/317dbd00-8e61-4065-b3eb-f2b80db9b7ed/projectile-motion/)

GRASP CHECK

If a projectile is launched on level ground, what launch angle maximizes the range of the projectile?

a. 0°
b. 30°
c. 45°
d. 60°

Check Your Understanding

13. What is projectile motion?

a. Projectile motion is the motion of an object projected into the air, which moves under the influence of gravity.
b. Projectile motion is the motion of an object projected into the air which moves independently of gravity.
c. Projectile motion is the motion of an object projected vertically upward into the air which moves under the influence of gravity.
d. Projectile motion is the motion of an object projected horizontally into the air which moves independently of gravity.

14. What is the force experienced by a projectile after the initial force that launched it into the air in the absence of air resistance?

a. The nuclear force
b. The gravitational force
c. The electromagnetic force
d. The contact force

5.4 Inclined Planes

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Distinguish between static friction and kinetic friction
- Solve problems involving inclined planes

Section Key Terms

kinetic friction   static friction

Static Friction and Kinetic Friction

Recall from the previous chapter that friction is a force that opposes motion, and is around us all the time. Friction allows us to move, which you have discovered if you have ever tried to walk on ice.

There are different types of friction—kinetic and static. **Kinetic friction** acts on an object in motion, while **static friction** acts on an object or system at rest. The maximum static friction is usually greater than the kinetic friction between the objects.

Imagine, for example, trying to slide a heavy crate across a concrete floor. You may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion, it is easier to keep it in motion than it was to get it started because the kinetic friction force is less than the static friction force. If you were to add mass to the crate, (for example, by placing a box on top of it) you would need to push even harder to get it started and also to keep it moving. If, on the other hand, you oiled the concrete you would find it easier to get the crate started and keep it going.

Figure 5.33 shows how friction occurs at the interface between two objects. Magnifying these surfaces shows that they are rough on the microscopic level. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them.

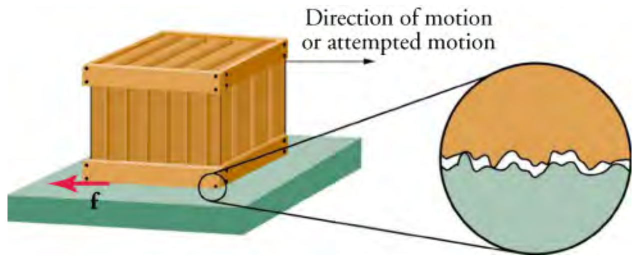


Figure 5.33 Frictional forces, such as **f**, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view.

The magnitude of the frictional force has two forms: one for static friction, the other for kinetic friction. When there is no motion between the objects, the magnitude of static friction $\mathbf{f_s}$ is

$$\mathbf{f}_\mathrm{s} \leq \mu_\mathrm{s}\mathbf{N}_\mathrm{s},$$

where μ_s is the coefficient of static friction and **N** is the magnitude of the normal force. Recall that the normal force opposes the force of gravity and acts perpendicular to the surface in this example, but not always.

Since the symbol $\leq$ means less than or equal to, this equation says that static friction can have a maximum value of $\mu_\mathrm{s}\mathbf{N}$. That is,

$$\mathbf{f}_\mathrm{s}(\max) = \mu_\mathrm{s}\mathbf{N}.$$

Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds **f**s(max), the object will move. Once an object is moving, the magnitude of kinetic friction $\mathbf{f}_\mathrm{k}$ is given by

$$\mathbf{f}_\mathrm{k} = \mu_\mathrm{k}\mathbf{N}.$$

where μ_k is the coefficient of kinetic friction.

Friction varies from surface to surface because different substances are rougher than others. Table 5.2 compares values of static and kinetic friction for different surfaces. The coefficient of the friction depends on the two surfaces that are in contact.

System	Static Friction μ_s	Kinetic Friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

Table 5.2 Coefficients of Static and Kinetic Friction

System	Static Friction μ_s	Kinetic Friction μ_k
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Table 5.2 Coefficients of Static and Kinetic Friction

Since the direction of friction is always opposite to the direction of motion, friction runs parallel to the surface between objects and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight

$$\mathbf{W} = m\mathbf{g} = (100\ \text{kg})(9.80\text{m/s}^2) = 980\ \ \text{N},$$

perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

$$\mathbf{f}_\text{s}(\text{max}) = \mu_\text{s}\mathbf{N} = (0.45)(980\ \ \text{N}) = 440\ \ \text{N}$$

to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N

$$\mathbf{f}_\text{k} = \mu_\text{k}\mathbf{N} = (0.30)(980\ \ \text{N}) = 290\ \ \text{N}$$

would keep it moving at a constant speed. If the floor were lubricated, both coefficients would be much smaller than they would be without lubrication. The coefficient of friction is unitless and is a number usually between 0 and 1.0.

Working with Inclined Planes

We discussed previously that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Up until now, we dealt only with normal force in one dimension, with gravity and normal force acting perpendicular to the surface in opposing directions (gravity downward, and normal force upward). Now that you have the skills to work with forces in two dimensions, we can explore what happens to weight and the normal force on a tilted surface such as an inclined plane. For inclined plane problems, it is easier breaking down the forces into their components if we rotate the coordinate system, as illustrated in Figure 5.34. The first step when setting up the problem is to break down the force of weight into components.

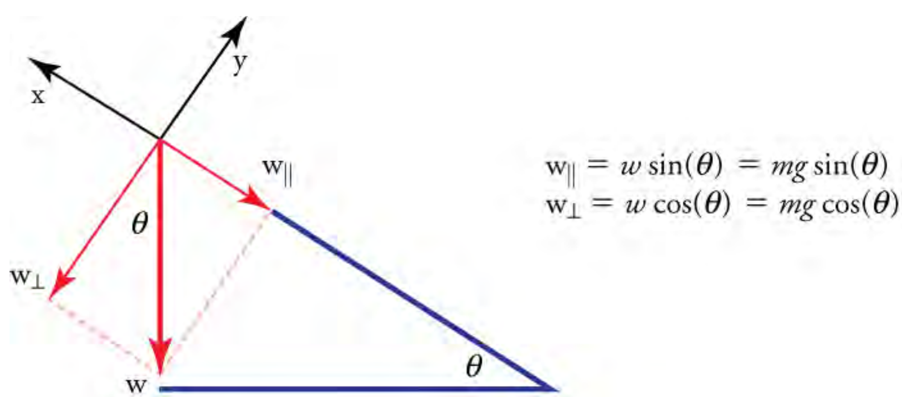


Figure 5.34 The diagram shows perpendicular and horizontal components of weight on an inclined plane.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: A force acting perpendicular to the plane, $\mathbf{w}_\perp$, and a force acting parallel to the plane, $\mathbf{w}_{||}$. The perpendicular force of weight, $\mathbf{w}_\perp$, is typically equal in magnitude and opposite in direction to the normal force, $\mathbf{N}$. The force acting parallel to the plane, $\mathbf{w}_{||}$, causes the object to accelerate down the incline. The force of friction, $\mathbf{f}$, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$\mathbf{w}_{||} = \mathbf{w}sin(\theta) = m\mathbf{g}sin(\theta) \text{ and}$$
$$\mathbf{w}_{\perp} = \mathbf{w}cos(\theta) = m\mathbf{g}cos(\theta).$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components

$$cos(\theta) = \frac{\mathbf{w}_{\perp}}{\mathbf{w}}$$
$$\mathbf{w}_{\perp} = \mathbf{w}cos(\theta) = m\mathbf{g}cos(\theta)$$
$$sin(\theta) = \frac{\mathbf{w}_{||}}{\mathbf{w}}$$
$$\mathbf{w}_{||} = \mathbf{w}sin(\theta) = m\mathbf{g}sin(\theta).$$

WATCH PHYSICS

Inclined Plane Force Components

This video (https://www.khanacademy.org/embed_video?v=TC23wD34C7k) shows how the weight of an object on an inclined plane is broken down into components perpendicular and parallel to the surface of the plane. It explains the geometry for finding the angle in more detail.

GRASP CHECK

Click to view content (https://www.youtube.com/embed/TC23wD34C7k)

This video shows how the weight of an object on an inclined plane is broken down into components perpendicular and parallel to the surface of the plane. It explains the geometry for finding the angle in more detail.

When the surface is flat, you could say that one of the components of the gravitational force is zero; Which one? As the angle of the incline gets larger, what happens to the magnitudes of the perpendicular and parallel components of gravitational force?

a. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component decreases and the perpendicular component increases. This is because the cosine of the angle shrinks while the sine of the angle increases.
b. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component decreases and the perpendicular component increases. This is because the cosine of the angle increases while the sine of the angle shrinks.
c. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component increases and the perpendicular component decreases. This is because the cosine of the angle shrinks while the sine of the angle increases.
d. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component increases and the perpendicular component decreases. This is because the cosine of the angle increases while the sine of the angle shrinks.

TIPS FOR SUCCESS

Normal force is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. It is important to tell apart these symbols, especially since the units for normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force, $\mathbf{N}$, that the floor exerts on a chair might be $\mathbf{N} = 100 \text{ N}$. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations!

To review, the process for solving inclined plane problems is as follows:

1. Draw a sketch of the problem.
2. Identify known and unknown quantities, and identify the system of interest.
3. Draw a free-body diagram (which is a sketch showing all of the forces acting on an object) with the coordinate system rotated at the same angle as the inclined plane. Resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
4. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x -direction) then **F**net $x = 0$. If the object does accelerate in that direction, **F**net $x = m\mathbf{a}$.
5. Check your answer. Is the answer reasonable? Are the units correct?

WORKED EXAMPLE

Finding the Coefficient of Kinetic Friction on an Inclined Plane

A skier, illustrated in Figure 5.35(a), with a mass of 62 kg is sliding down a snowy slope at an angle of 25 degrees. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

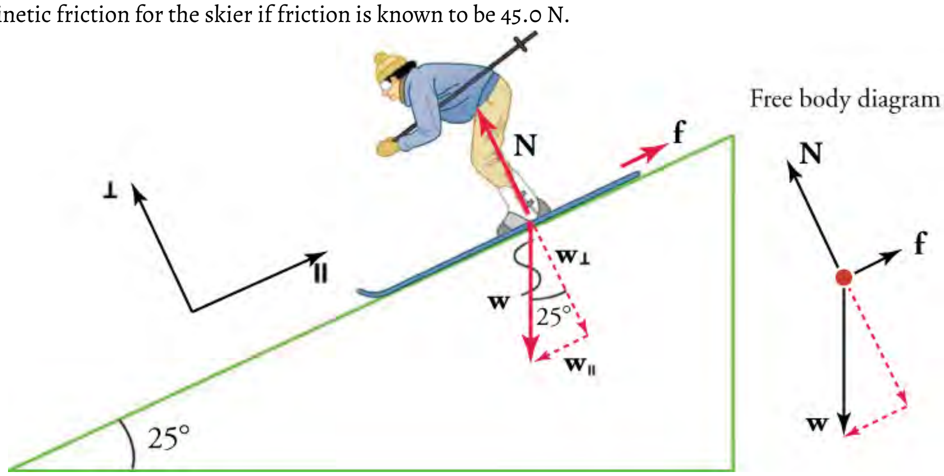


Figure 5.35 Use the diagram to help find the coefficient of kinetic friction for the skier.

Strategy

The magnitude of kinetic friction was given as 45.0 N. Kinetic friction is related to the normal force **N** as $\mathbf{f}_k = \mu_k \mathbf{N}$. Therefore, we can find the coefficient of kinetic friction by first finding the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope.

That is,

$$\mathbf{N} = \mathbf{w}_\perp = \mathbf{w}\cos(25^\circ) = m\mathbf{g}\cos(25^\circ).$$

Substituting this into our expression for kinetic friction, we get

$$\mathbf{f}_k = \mu_k m\mathbf{g}\cos 25^\circ,$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{\mathbf{f}_k}{\mathbf{w}\cos 25^\circ} = \frac{\mathbf{f}_k}{m\mathbf{g}\cos 25^\circ}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0\ \text{N}}{(62\ \text{kg})(9.80\ \text{m/s}^2)(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in Table 5.2 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $\mathbf{f}_{\mathrm{k}} = \mu_{\mathrm{k}} m\mathbf{g}\cos\theta$.

WORKED EXAMPLE

Weight on an Incline, a Two-Dimensional Problem

The skier's mass, including equipment, is 60.0 kg. (See Figure 5.36(b).) (a) What is her acceleration if friction is negligible? (b) What is her acceleration if the frictional force is 45.0 N?

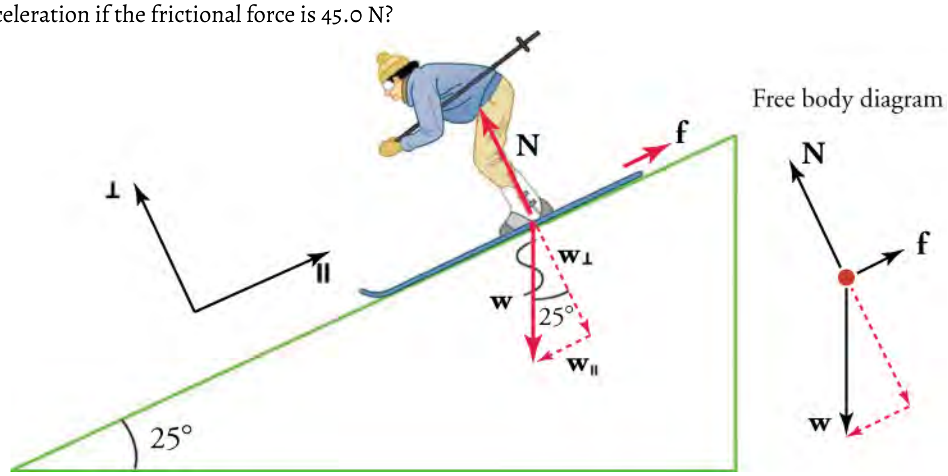


Figure 5.36 Now use the diagram to help find the skier's acceleration if friction is negligible and if the frictional force is 45.0 N.

Strategy

The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. Remember that motions along perpendicular axes are independent. We use the symbol $\perp$ to mean perpendicular, and $||$ to mean parallel.

The only external forces acting on the system are the skier's weight, friction, and the normal force exerted by the ski slope, labeled $\mathbf{w}$, $\mathbf{f}$, and $\mathbf{N}$ in the free-body diagram. $\mathbf{N}$ is always perpendicular to the slope and $\mathbf{f}$ is parallel to it. But $\mathbf{w}$ is not in the direction of either axis, so we must break it down into components along the chosen axes. We define $\mathbf{w}_{||}$ to be the component of weight parallel to the slope and $\mathbf{w}_{\perp}$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $\mathbf{w}_{||} = \mathbf{w}\sin(25°) = m\mathbf{g}\sin(25°)$, and the magnitude of the component of the weight perpendicular to the slope is $\mathbf{w}_{\perp} = \mathbf{w}\cos(25°) = m\mathbf{g}\cos(25°\ )$.

(a) Neglecting friction: Since the acceleration is parallel to the slope, we only need to consider forces parallel to the slope. Forces perpendicular to the slope add to zero, since there is no acceleration in that direction. The forces parallel to the slope are the amount of the skier's weight parallel to the slope $\mathbf{w}_{||}$ and friction $\mathbf{f}$. Assuming no friction, by Newton's second law the acceleration parallel to the slope is

$$\mathbf{a}_{||} = \frac{\mathbf{F}_{\text{net }||}}{m},$$

Where the net force parallel to the slope $\mathbf{F}_{\text{net }||} = \mathbf{w}_{||} = m\mathbf{g}\sin(25°)$, so that

$$\begin{aligned}\mathbf{a}_{||} &= \frac{\mathbf{F}_{\text{net }||}}{m} = \frac{m\mathbf{g}\sin(25°)}{m} = \mathbf{g}\sin(25°) \\ &= (9.80\ \text{m/s}^2)(0.423) = 4.14\ \text{m/s}^2\end{aligned}$$

is the acceleration.

(b) Including friction: Here we now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$\mathbf{F}_{\text{net } ||} = \mathbf{w}_{||} - \mathbf{f},$$

and substituting this into Newton's second law, $a_{||} = \frac{\mathbf{F}_{\text{net } ||}}{m}$ gives

$$\mathbf{a}_{||} = \frac{\mathbf{F}_{\text{net } ||}}{m} = \frac{\mathbf{w}_{||} - \mathbf{f}}{m} = \frac{m\mathbf{g}\sin(25°) - \mathbf{f}}{m}.$$

We substitute known values to get

$$\mathbf{a}_{||} = \frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(0.423) - 45.0 \text{ N}}{60.0 \text{ kg}},$$

or

$$\mathbf{a}_{||} = 3.39 \text{ m/s}^2,$$

which is the acceleration parallel to the incline when there is 45 N opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is not.

Practice Problems

15. When an object sits on an inclined plane that makes an angle θ with the horizontal, what is the expression for the component of the objects weight force that is parallel to the incline?

a. $w_{||} = w\cos\theta$
b. $w_{||} = w\sin\theta$
c. $w_{||} = w\sin\theta - \cos\theta$
d. $w_{||} = w\cos\theta - \sin\theta$

16. An object with a mass of 5 kg rests on a plane inclined $30°$ from horizontal. What is the component of the weight force that is parallel to the incline?

a. 4.33 N
b. 5.0 N
c. 24.5 N
d. 42.43 N

Snap Lab

Friction at an Angle: Sliding a Coin

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in the first Worked Example, the kinetic friction on a slope $\mathbf{f}_k = \mu_k m\mathbf{g}\cos\theta$, and the component of the weight down the slope is equal to $m\mathbf{g}\sin\theta$. These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out

$$\begin{aligned} \mathbf{f}_k &= \mathbf{Fg}_x \\ \mu_k m\mathbf{g}\cos\theta &= m\mathbf{g}\sin\theta. \end{aligned}$$

Solving for μ_k, since $\tan\theta = \sin\theta/\cos\theta$ we find that

$$\mu_k = \frac{m\mathbf{g}\sin\theta}{m\mathbf{g}\cos\theta} = \tan\theta. \quad (5.10)$$

- 1 coin
- 1 book
- 1 protractor

1. Put a coin flat on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move.

2. Measure the angle of tilt relative to the horizontal and find μ_k .

GRASP CHECK

True or False—If only the angles of two vectors are known, we can find the angle of their resultant addition vector.

a. True
b. False

Check Your Understanding

17. What is friction?

a. Friction is an internal force that opposes the relative motion of an object.
b. Friction is an internal force that accelerates an object's relative motion.
c. Friction is an external force that opposes the relative motion of an object.
d. Friction is an external force that increases the velocity of the relative motion of an object.

18. What are the two varieties of friction? What does each one act upon?

a. Kinetic and static friction both act on an object in motion.
b. Kinetic friction acts on an object in motion, while static friction acts on an object at rest.
c. Kinetic friction acts on an object at rest, while static friction acts on an object in motion.
d. Kinetic and static friction both act on an object at rest.

19. Between static and kinetic friction between two surfaces, which has a greater value? Why?

a. The kinetic friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
b. The static friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
c. The kinetic friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.
d. The static friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.

5.5 Simple Harmonic Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Hooke's law and Simple Harmonic Motion
- Describe periodic motion, oscillations, amplitude, frequency, and period
- Solve problems in simple harmonic motion involving springs and pendulums

Section Key Terms

amplitude	deformation	equilibrium position	frequency
Hooke's law	oscillate	period	periodic motion
restoring force	simple harmonic motion	simple pendulum	

Hooke's Law and Simple Harmonic Motion

Imagine a car parked against a wall. If a bulldozer pushes the car into the wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important things can happen. First, unlike the car and bulldozer example, the object returns to its original shape when the force is removed. Second, the size of the deformation is proportional to the force. This second property is known as **Hooke's law**. In equation form, Hooke's law is

$$\mathbf{F} = -\mathbf{kx},$$

where **x** is the amount of deformation (the change in length, for example) produced by the restoring force **F**, and **k** is a constant that depends on the shape and composition of the object. The restoring force is the force that brings the object back to its equilibrium position; the minus sign is there because the restoring force acts in the direction opposite to the displacement. Note that the restoring force is proportional to the deformation **x**. The deformation can also be thought of as a displacement from equilibrium. It is a change in position due to a force. In the absence of force, the object would rest at its equilibrium position. The force constant **k** is related to the stiffness of a system. The larger the force constant, the stiffer the system. A stiffer system is more difficult to deform and requires a greater restoring force. The units of **k** are newtons per meter (N/m). One of the most common uses of Hooke's law is solving problems involving springs and pendulums, which we will cover at the end of this section.

Oscillations and Periodic Motion

What do an ocean buoy, a child in a swing, a guitar, and the beating of hearts all have in common? They all **oscillate**. That is, they move back and forth between two points, like the ruler illustrated in Figure 5.37. All oscillations involve force. For example, you push a child in a swing to get the motion started.

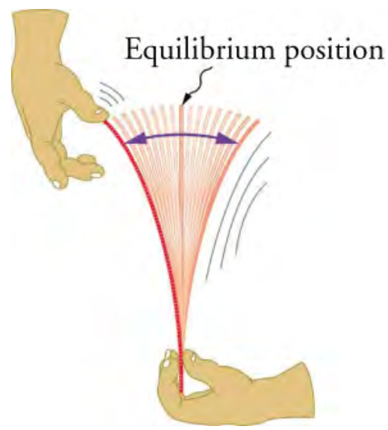


Figure 5.37 A ruler is displaced from its equilibrium position.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in Figure 5.38. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until it gradually loses all of its energy. The simplest oscillations occur when the restoring force is directly proportional to displacement. Recall that Hooke's law describes this situation with the equation $\mathbf{F} = -\mathbf{kx}$. Therefore, Hooke's law describes and applies to the simplest case of oscillation, known as **simple harmonic motion**.

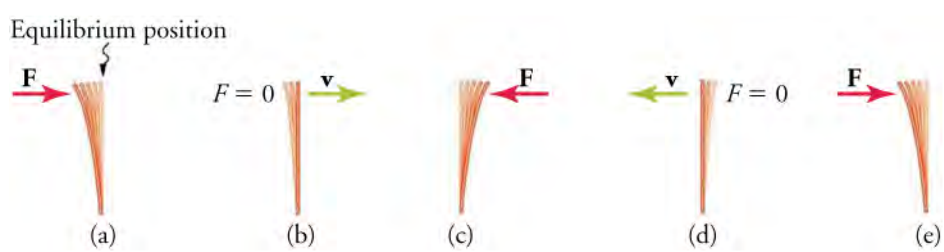


Figure 5.38 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each vibration of the string takes the same time as the previous one. **Periodic motion** is a motion that repeats itself at regular time intervals, such as with an object bobbing up and down on a spring or a pendulum swinging back and forth. The time to complete one oscillation (a complete cycle of motion) remains constant and is called the **period** *T*. Its units are usually seconds.

Frequency *f* is the number of oscillations per unit time. The SI unit for frequency is the hertz (Hz), defined as the number of oscillations per second. The relationship between frequency and period is

$$f = 1/T.$$

As you can see from the equation, frequency and period are different ways of expressing the same concept. For example, if you get a paycheck twice a month, you could say that the frequency of payment is two per month, or that the period between checks is half a month.

If there is no friction to slow it down, then an object in simple motion will oscillate forever with equal displacement on either side of the equilibrium position. The **equilibrium position** is where the object would naturally rest in the absence of force. The maximum displacement from equilibrium is called the **amplitude X**. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, shown in Figure 5.39, the units of amplitude and displacement are meters.

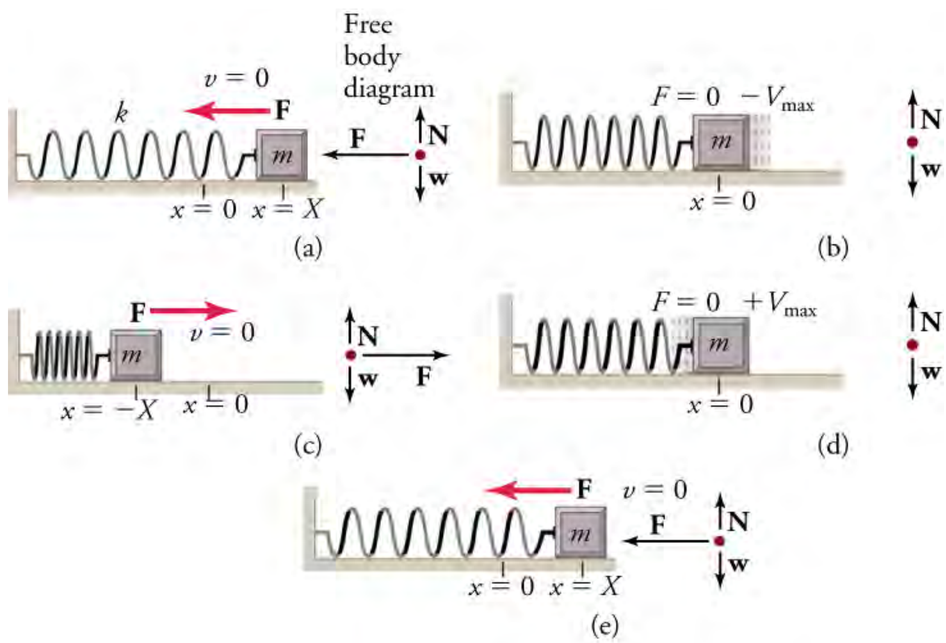


Figure 5.39 An object attached to a spring sliding on a frictionless surface is a simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude **X** and a period *T*. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period *T*. The greater the mass of the object is, the greater the period *T*.

The mass *m* and the force constant **k** are the *only* factors that affect the period and frequency of simple harmonic motion. The period of a simple harmonic oscillator is given by

$$T = 2\pi\sqrt{\frac{m}{\mathbf{k}}}$$

and, because $f = 1/T$, the frequency of a simple harmonic oscillator is

$$f = \frac{1}{2\pi}\sqrt{\frac{\mathbf{k}}{m}}.$$

WATCH PHYSICS

Introduction to Harmonic Motion

This video shows how to graph the displacement of a spring in the x-direction over time, based on the period. Watch the first 10 minutes of the video (you can stop when the narrator begins to cover calculus).

Click to view content (https://www.khanacademy.org/embed_video?v=Nk2q-_jkJVs)

GRASP CHECK

If the amplitude of the displacement of a spring were larger, how would this affect the graph of displacement over time? What would happen to the graph if the period was longer?

a. Larger amplitude would result in taller peaks and troughs and a longer period would result in greater separation in time between peaks.
b. Larger amplitude would result in smaller peaks and troughs and a longer period would result in greater distance between peaks.
c. Larger amplitude would result in taller peaks and troughs and a longer period would result in shorter distance between peaks.
d. Larger amplitude would result in smaller peaks and troughs and a longer period would result in shorter distance between peaks.

Solving Spring and Pendulum Problems with Simple Harmonic Motion

Before solving problems with springs and pendulums, it is important to first get an understanding of how a pendulum works. Figure 5.40 provides a useful illustration of a simple pendulum.

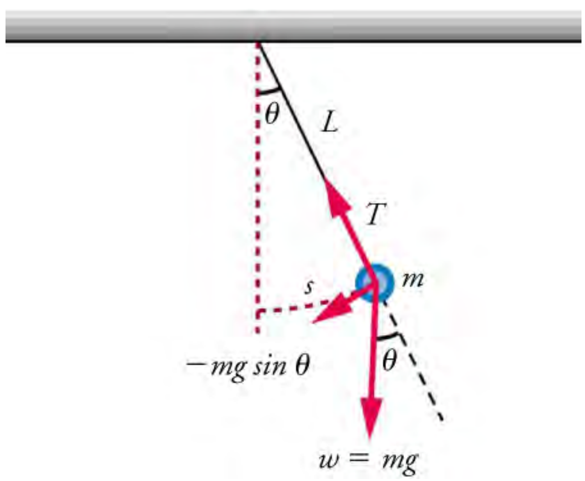


Figure 5.40 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg\sin\theta$ toward the equilibrium position—that is, a restoring force.

Everyday examples of pendulums include old-fashioned clocks, a child's swing, or the sinker on a fishing line. For small displacements of less than 15 degrees, a pendulum experiences simple harmonic oscillation, meaning that its restoring force is directly proportional to its displacement. A pendulum in simple harmonic motion is called a **simple pendulum**. A pendulum has an object with a small mass, also known as the pendulum bob, which hangs from a light wire or string. The equilibrium position for a pendulum is where the angle θ is zero (that is, when the pendulum is hanging straight down). It makes sense that without any force applied, this is where the pendulum bob would rest.

The displacement of the pendulum bob is the arc length s. The weight $m\mathbf{g}$ has components $m\mathbf{g}\cos\theta$ along the string and $m\mathbf{g}\sin\theta$ tangent to the arc. Tension in the string exactly cancels the component $m\mathbf{g}\cos\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position that runs tangent to the arc and equals $-m\mathbf{g}\sin\theta$.

For a simple pendulum, The period is $T=2\pi\sqrt{\frac{L}{\mathbf{g}}}$.

The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass or amplitude. However, note that T does depend on **g**. This means that if we know the length of a pendulum, we can actually use it to measure gravity! This will come in useful in Figure 5.40.

TIPS FOR SUCCESS

Tension is represented by the variable **T**, and period is represented by the variable T. It is important not to confuse the two, since tension is a force and period is a length of time.

WORKED EXAMPLE

Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find **g** given the period T and the length L of a pendulum. We can solve $T=2\pi\sqrt{\frac{L}{\mathbf{g}}}$ for **g**, assuming that the angle of deflection is less than 15 degrees. Recall that when the angle of deflection is less than 15 degrees, the pendulum is considered to be in simple harmonic motion, allowing us to use this equation.

Solution

1. Square $T=2\pi\sqrt{\frac{L}{\mathbf{g}}}$ and solve for g.

$$\mathbf{g}=4\pi^2\frac{L}{T^2}$$

2. Substitute known values into the new equation.

$$\mathbf{g}=4\pi^2\frac{0.75000\ \text{m}}{(1.7357\ \text{s})^2}$$

3. Calculate to find **g**.

$$\mathbf{g}=9.8281\ \text{m/s}^2$$

Discussion

This method for determining **g** can be very accurate. This is why length and period are given to five digits in this example.

WORKED EXAMPLE

Hooke's Law: How Stiff Are Car Springs?

What is the force constant for the suspension system of a car, like that shown in Figure 5.41, that settles 1.20 cm when an 80.0-kg person gets in?

Figure 5.41 A car in a parking lot. (exfordy, Flickr)

Strategy

Consider the car to be in its equilibrium position $\mathbf{x} = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $\mathbf{x} = -1.20 \times 10^{-2}$ m.

At that point, the springs supply a restoring force **F** equal to the person's weight

$\mathbf{w} = m\mathbf{g} = (80.0\ \text{kg})(9.80\ \text{m/s}^2) = 784$ N. We take this force to be **F** in Hooke's law.

Knowing **F** and **x**, we can then solve for the force constant **k**.

Solution

Solve Hooke's law, $\mathbf{F} = -\mathbf{kx}$, for **k**.

$$\mathbf{k} = \frac{\mathbf{F}}{\mathbf{x}}$$

Substitute known values and solve for **k**.

$$\begin{aligned}\mathbf{k} &= \frac{-784\ \ \text{N}}{-1.20\times10^{-2}\ \ \text{m}} \\ &= 6.53 \times 10^{4}\ \text{N/m}\end{aligned}$$

Discussion

Note that **F** and **x** have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in, if it were not for the shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Practice Problems

20. A force of $70\ \text{N}$ applied to a spring causes it to be displaced by $0.3\ \text{m}$. What is the force constant of the spring?

a. $-233\ \text{N/m}$
b. $-21\ \text{N/m}$
c. $21\ \text{N/m}$
d. $233\ \text{N/m}$

21. What is the force constant for the suspension system of a car that settles $3.3\ \text{cm}$ when a $65\ \text{kg}$ person gets in?

a. $1.93 \times 10^4\ \text{N/m}$
b. $1.97 \times 10^3\ \text{N/m}$

c. 1.93×10^{2} N/m
d. 1.97×10^{1} N/m

Snap Lab

Finding Gravity Using a Simple Pendulum

Use a simple pendulum to find the acceleration due to gravity **g** in your home or classroom.

- 1 string
- 1 stopwatch
- 1 small dense object

1. Cut a piece of a string or dental floss so that it is about 1 m long.
2. Attach a small object of high density to the end of the string (for example, a metal nut or a car key).
3. Starting at an angle of less than 10 degrees, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch.
4. Calculate **g**.

GRASP CHECK

How accurate is this measurement for *g*? How might it be improved?

a. Accuracy for value of *g* will increase with an increase in the mass of a dense object.
b. Accuracy for the value of *g* will increase with increase in the length of the pendulum.
c. The value of *g* will be more accurate if the angle of deflection is more than 15°.
d. The value of *g* will be more accurate if it maintains simple harmonic motion.

Check Your Understanding

22. What is deformation?
a. Deformation is the magnitude of the restoring force.
b. Deformation is the change in shape due to the application of force.
c. Deformation is the maximum force that can be applied on a spring.
d. Deformation is regaining the original shape upon the removal of an external force.

23. According to Hooke's law, what is deformation proportional to?
a. Force
b. Velocity
c. Displacement
d. Force constant

24. What are oscillations?
a. Motion resulting in small displacements
b. Motion which repeats itself periodically
c. Periodic, repetitive motion between two points
d. motion that is the opposite to the direction of the restoring force

25. True or False—Oscillations can occur without force.
a. True
b. False

KEY TERMS

air resistance a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

amplitude the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

analytical method the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

component (of a 2-dimensional vector) a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

deformation displacement from equilibrium, or change in shape due to the application of force

equilibrium position where an object would naturally rest in the absence of force

frequency number of events per unit of time

graphical method drawing vectors on a graph to add them using the head-to-tail method

head (of a vector) the end point of a vector; the location of the vector's arrow; also referred to as the tip

head-to-tail method a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

Hooke's law proportional relationship between the force $\mathbf{F}$ on a material and the deformation ΔL it causes, $\mathbf{F} = \mathbf{k}\Delta L$

kinetic friction a force that opposes the motion of two systems that are in contact and moving relative to one another

maximum height (of a projectile) the highest altitude, or maximum displacement in the vertical position reached in the path of a projectile

oscillate moving back and forth regularly between two points

period time it takes to complete one oscillation

periodic motion motion that repeats itself at regular time intervals

projectile an object that travels through the air and experiences only acceleration due to gravity

projectile motion the motion of an object that is subject only to the acceleration of gravity

range the maximum horizontal distance that a projectile travels

restoring force force acting in opposition to the force caused by a deformation

resultant the sum of the a collection of vectors

resultant vector the vector sum of two or more vectors

simple harmonic motion the oscillatory motion in a system where the net force can be described by Hooke's law

simple pendulum an object with a small mass suspended from a light wire or string

static friction a force that opposes the motion of two systems that are in contact and are not moving relative to one another

tail the starting point of a vector; the point opposite to the head or tip of the arrow

trajectory the path of a projectile through the air

vector addition adding together two or more vectors

SECTION SUMMARY

5.1 Vector Addition and Subtraction: Graphical Methods

- The graphical method of adding vectors $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them by using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A} + \mathbf{B} = \mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor.
- The graphical method of subtracting vectors $\mathbf{A}$ and $\mathbf{B}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) = \mathbf{R}$. Next, use the head-to-tail method as for vector addition to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is independent of the order in which they are added; $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
- The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- Variables in physics problems, such as force or velocity, can be represented with vectors by making the length of the vector proportional to the magnitude of the force or velocity.
- Problems involving displacement, force, or velocity may be solved graphically by measuring the resultant vector's magnitude with a ruler and measuring the direction with a protractor.

5.2 Vector Addition and Subtraction: Analytical Methods

- The analytical method of vector addition and subtraction uses the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.

- The steps to add vectors A and B using the analytical method are as follows:

1. Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations
$$A_x = A\cos\theta$$
$$B_x = B\cos\theta$$
and
$$A_y = A\sin\theta$$
$$B_y = B\sin\theta.$$
2. Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, R.
$$R_x = A_x + B_x$$
and
$$R_y = A_y + B_y.$$
3. Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector R.
$$R = \sqrt{R_x^2 + R_y^2}$$
4. Use a trigonometric identity to determine the direction, θ, of R.
$$\theta = \tan^{-1}(R_y/R_x)$$

5.3 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- Projectile motion in the horizontal and vertical directions are independent of one another.
- The maximum height of an projectile is the highest altitude, or maximum displacement in the vertical position reached in the path of a projectile.
- The range is the maximum horizontal distance traveled by a projectile.
- To solve projectile problems: choose a coordinate system; analyze the motion in the vertical and horizontal direction separately; then, recombine the horizontal and vertical components using vector addition equations.

5.4 Inclined Planes

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force **N** pushing the systems together. A normal force is always perpendicular to the contact surface between systems. Friction depends on both of the materials involved.
- μ_s is the coefficient of static friction, which depends on both of the materials.
- μ_k is the coefficient of kinetic friction, which also depends on both materials.
- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be broken into components that act perpendicular ($\mathbf{w}_\perp$) and parallel ($\mathbf{w}_{||}$) to the surface of the plane.

5.5 Simple Harmonic Motion

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations are related to systems that can be described by Hooke's law.
- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency
- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15 degrees.

KEY EQUATIONS

5.2 Vector Addition and Subtraction: Analytical Methods

resultant magnitude	$R = \sqrt{R_x^2 + R_y^2}$
resultant direction	$\theta = \tan^{-1}(R_y/R_x)$
x-component of a vector A (when an angle is given relative to the horizontal)	$A_x = A\cos\theta$
y-component of a vector A (when an angle is given relative to the horizontal)	$A_y = A\sin\theta$
addition of vectors	$\mathbf{A_x} + \mathbf{A_y} = \mathbf{A}$

5.3 Projectile Motion

angle of displacement	$\theta = \tan^{-1}(\mathbf{y}/\mathbf{x})$

velocity	$\mathbf{v} = \sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2}$
angle of velocity	$\theta_v = \tan^{-1}(\mathbf{v}_y/\mathbf{v}_x)$
maximum height	$h = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}$
range	$R = \frac{\mathbf{v}_0^2 \sin 2\theta_0}{\mathbf{g}}$

5.4 Inclined Planes

force of static friction	$\mathbf{f}_\mathrm{s} \leq \mu_\mathrm{s}\mathbf{N}$
force of kinetic friction	$\mathbf{f}_\mathrm{k} = \mu_\mathrm{k}\mathbf{N}$
perpendicular component of weight on an inclined plane	$\mathbf{w}_\perp = \mathbf{w}\cos(\theta) = m\mathbf{g}\cos(\theta)$
parallel component of weight on an inclined plane	$\mathbf{w}_{\|\|} = \mathbf{w}\sin(\theta) = m\mathbf{g}\sin(\theta)$

5.5 Simple Harmonic Motion

Hooke's law	$\mathbf{F} = -\mathbf{kx}$
period in simple harmonic motion	$T = 2\pi\sqrt{\frac{m}{\mathbf{k}}}$
frequency in simple harmonic motion	$f = \frac{1}{2\pi}\sqrt{\frac{\mathbf{k}}{m}}$
period of a simple pendulum	$T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$

CHAPTER REVIEW

Concept Items

5.1 Vector Addition and Subtraction: Graphical Methods

1. There is a vector $\overrightarrow{\mathrm{A}}$, with magnitude 5 units pointing towards west and vector $\overrightarrow{\mathrm{B}}$, with magnitude 3 units, pointing towards south. Using vector addition, calculate the magnitude of the resultant vector.

a. 4.0
b. 5.8
c. 6.3
d. 8.0

2. If you draw two vectors using the head-to-tail method, how can you then draw the resultant vector?

a. By joining the head of the first vector to the head of the last
b. By joining the head of the first vector with the tail of the last
c. By joining the tail of the first vector to the head of the last
d. By joining the tail of the first vector with the tail of the last

3. What is the global angle of 20° south of west?

a. 110°
b. 160°
c. 200°
d. 290°

5.2 Vector Addition and Subtraction: Analytical Methods

4. What is the angle between the x and y components of a vector?

a. 0°
b. 45°
c. 90°
d. 180°

5. Two vectors are equal in magnitude and opposite in direction. What is the magnitude of their resultant vector?

a. The magnitude of the resultant vector will be zero.
b. The magnitude of resultant vector will be twice the magnitude of the original vector.
c. The magnitude of resultant vector will be same as magnitude of the original vector.
d. The magnitude of resultant vector will be half the magnitude of the original vector.

6. How can we express the x and y-components of a vector in terms of its magnitude, A, and direction, global angle θ?

a. $A_x = A\cos\theta\, A_y = A\sin\theta$
b. $A_x = A\cos\theta\, A_y = A\cos\theta$
c. $A_x = A\sin\theta\, A_y = A\cos\theta$

d. $A_x = A \sin \theta A_y = A \sin \theta$

7. True or False—Every 2-D vector can be expressed as the product of its x and y-components.
   a. True
   b. False

5.3 Projectile Motion

8. Horizontal and vertical motions of a projectile are independent of each other. What is meant by this?
   a. Any object in projectile motion falls at the same rate as an object in freefall, regardless of its horizontal velocity.
   b. All objects in projectile motion fall at different rates, regardless of their initial horizontal velocities.
   c. Any object in projectile motion falls at the same rate as its initial vertical velocity, regardless of its initial horizontal velocity.
   d. All objects in projectile motion fall at different rates and the rate of fall of the object is independent of the initial velocity.

9. Using the conventional choice for positive and negative axes described in the text, what is the y-component of the acceleration of an object experiencing projectile motion?
   a. -9.8 m/s
   b. -9.8 m/s^2
   c. 9.8 m/s
   d. 9.8 m/s^2

5.4 Inclined Planes

10. True or False—Kinetic friction is less than the limiting static friction because once an object is moving, there are fewer points of contact, and the friction is reduced. For this reason, more force is needed to start moving an object than to keep it in motion.
   a. True
   b. False

11. When there is no motion between objects, what is the relationship between the magnitude of the static friction f_s and the normal force N?
   a. $f_s \leq N$
   b. $f_s \leq \mu_s N$
   c. $f_s \geq N$
   d. $f_s \geq \mu_s N$

12. What equation gives the magnitude of kinetic friction?
   a. $f_k = \mu_s N$
   b. $f_k = \mu_k N$
   c. $f_k \leq \mu_s N$
   d. $f_k \leq \mu_k N$

5.5 Simple Harmonic Motion

13. Why is there a negative sign in the equation for Hooke's law?
   a. The negative sign indicates that displacement decreases with increasing force.
   b. The negative sign indicates that the direction of the applied force is opposite to that of displacement.
   c. The negative sign indicates that the direction of the restoring force is opposite to that of displacement.
   d. The negative sign indicates that the force constant must be negative.

14. With reference to simple harmonic motion, what is the equilibrium position?
   a. The position where velocity is the minimum
   b. The position where the displacement is maximum
   c. The position where the restoring force is the maximum
   d. The position where the object rests in the absence of force

15. What is Hooke's law?
   a. Restoring force is directly proportional to the displacement from the mean position and acts in the the opposite direction of the displacement.
   b. Restoring force is directly proportional to the displacement from the mean position and acts in the same direction as the displacement.
   c. Restoring force is directly proportional to the square of the displacement from the mean position and acts in the opposite direction of the displacement.
   d. Restoring force is directly proportional to the square of the displacement from the mean position and acts in the same direction as the displacement.

Critical Thinking Items

5.1 Vector Addition and Subtraction: Graphical Methods

16. True or False—A person is following a set of directions. He has to walk 2 km east and then 1 km north. He takes a wrong turn and walks in the opposite direction for the second leg of the trip. The magnitude of his total displacement will be the same as it would have been had he followed directions correctly.
   a. True
   b. False

5.2 Vector Addition and Subtraction: Analytical Methods

17. What is the magnitude of a vector whose x-component is 2 units and whose angle is $60°$?
a. 1.0 units
b. 2.0 units
c. 2.3 units
d. 4.0 units

18. Vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude and opposite in direction. Does $\vec{A} - \vec{B}$ have the same direction as vector $\vec{A}$ or $\vec{B}$?
a. $\vec{A}$
b. $\vec{B}$

5.3 Projectile Motion

19. Two identical items, object 1 and object 2, are dropped from the top of a 50.0 m building. Object 1 is dropped with an initial velocity of 0 m/s, while object 2 is thrown straight downward with an initial velocity of 13.0 m/s. What is the difference in time, in seconds rounded to the nearest tenth, between when the two objects hit the ground?
a. Object 1 will hit the ground 3.2 s after object 2.
b. Object 1 will hit the ground 2.1 s after object 2.
c. Object 1 will hit the ground at the same time as object 2.
d. Object 1 will hit the ground 1.1 s after object 2.

20. An object is launched into the air. If the y-component of its acceleration is 9.8 m/s^2, which direction is defined as positive?
a. Vertically upward in the coordinate system
b. Vertically downward in the coordinate system
c. Horizontally to the right side of the coordinate system
d. Horizontally to the left side of the coordinate system

5.4 Inclined Planes

21. A box weighing 500 N is at rest on the floor. A person pushes against it and it starts moving when 100 N force is applied to it. What can be said about the coefficient of kinetic friction between the box and the floor?
a. $\mu_k = 0$
b. $\mu_k = 0.2$
c. $\mu_k < 0.2$
d. $\mu_k > 0.2$

22. The component of the weight parallel to an inclined plane of an object resting on an incline that makes an angle of $70.0°$ with the horizontal is 100.0 N. What is the object's mass?
a. 10.9 kg
b. 29.8 kg
c. 106 kg
d. 292 kg

5.5 Simple Harmonic Motion

23. Two springs are attached to two hooks. Spring A has a greater force constant than spring B. Equal weights are suspended from both. Which of the following statements is true?
a. Spring A will have more extension than spring B.
b. Spring B will have more extension than spring A.
c. Both springs will have equal extension.
d. Both springs are equally stiff.

24. Two simple harmonic oscillators are constructed by attaching similar objects to two different springs. The force constant of the spring on the left is 5 N/m and that of the spring on the right is 4 N/m. If the same force is applied to both, which of the following statements is true?
a. The spring on the left will oscillate faster than spring on the right.
b. The spring on the right will oscillate faster than the spring on the left.
c. Both the springs will oscillate at the same rate.
d. The rate of oscillation is independent of the force constant.

Problems

5.1 Vector Addition and Subtraction: Graphical Methods

25. A person attempts to cross a river in a straight line by navigating a boat at 15 m/s. If the river flows at 5.0 m/s from his left to right, what would be the magnitude of the boat's resultant velocity? In what direction would the boat go, relative to the straight line across it?
a. The resultant velocity of the boat will be 10.0 m/s. The boat will go toward his right at an angle of $26.6°$ to a line drawn across the river.
b. The resultant velocity of the boat will be 10.0 m/s. The boat will go toward his left at an angle of $26.6°$ to a line drawn across the river.
c. The resultant velocity of the boat will be 15.8 m/s. The boat will go toward his right at an angle of

$18.4°$ to a line drawn across the river.

d. The resultant velocity of the boat will be 15.8 m/s. The boat will go toward his left at an angle of $18.4°$ to a line drawn across the river.

26. A river flows in a direction from south west to north east at a velocity of 7.1 m/s. A boat captain wants to cross this river to reach a point on the opposite shore due east of the boat's current position. The boat moves at 13 m/s . Which direction should it head towards if the resultant velocity is 19.74 m/s?

a. It should head in a direction $22.6°$ east of south.
b. It should head in a direction $22.6°$ south of east.
c. It should head in a direction $45.0°$ east of south.
d. It should head in a direction $45.0°$ south of east.

5.2 Vector Addition and Subtraction: Analytical Methods

27. A person walks 10.0 m north and then 2.00 m east. Solving analytically, what is the resultant displacement of the person?

a. $\vec{R}$ = 10.2 m, θ = 78.7° east of north
b. $\vec{R}$ = 10.2 m, θ = 78.7° north of east
c. $\vec{R}$ = 12.0 m, θ = 78.7° east of north
d. $\vec{R}$ = 12.0 m, θ = 78.7° north of east

28. A person walks $12.0°$ north of west for 55.0 m and $63.0°$ south of west for 25.0 m. What is the magnitude of his displacement? Solve analytically.

a. 10.84 m
b. 65.1 m
c. 66.04 m
d. 80.00 m

5.3 Projectile Motion

29. A water balloon cannon is fired at 30 m/s at an angle of $50°$ above the horizontal. How far away will it fall?

a. 2.35 m
b. 3.01 m
c. 70.35 m
d. 90.44 m

30. A person wants to fire a water balloon cannon such that it hits a target 100 m away. If the cannon can only be launched at 45° above the horizontal, what should be the initial speed at which it is launched?

a. 31.3 m/s
b. 37.2 m/s
c. 980.0 m/s
d. 1,385.9 m/s

5.4 Inclined Planes

31. A coin is sliding down an inclined plane at constant velocity. If the angle of the plane is $10°$ to the horizontal, what is the coefficient of kinetic friction?

a. $\mu_k = 0$
b. $\mu_k = 0.18$
c. $\mu_k = 5.88$
d. $\mu_k = \infty$

32. A skier with a mass of 55 kg is skiing down a snowy slope that has an incline of 30°. Find the coefficient of kinetic friction for the skier if friction is known to be 25 N .

a. $\mu k = 0$
b. $\mu k = 0.05$
c. $\mu k = 0.09$
d. $\mu k = \infty$

5.5 Simple Harmonic Motion

33. What is the time period of a 6 cm long pendulum on earth?

a. 0.08 s
b. 0.49 s
c. 4.9 s
d. 80 s

34. A simple harmonic oscillator has time period 4 s. If the mass of the system is 2 kg, what is the force constant of the spring used?

a. 0.125 N/m
b. 0.202 N/m
c. 0.81 N/m
d. 4.93 N/m

Performance Task

5.5 Simple Harmonic Motion

35. Construct a seconds pendulum (pendulum with time period 2 seconds).

TEST PREP

Multiple Choice

5.1 Vector Addition and Subtraction: Graphical Methods

36. True or False—We can use Pythagorean theorem to calculate the length of the resultant vector obtained from the addition of two vectors which are at right angles to each other.
  a. True
  b. False

37. True or False—The direction of the resultant vector depends on both the magnitude and direction of added vectors.
  a. True
  b. False

38. A plane flies north at 200 m/s with a headwind blowing from the north at 70 m/s. What is the resultant velocity of the plane?
  a. 130 m/s north
  b. 130 m/s south
  c. 270 m/s north
  d. 270 m/s south

39. Two hikers take different routes to reach the same spot. The first one goes 255 m southeast, then turns and goes 82 m at 14° south of east. The second hiker goes 200 m south. How far and in which direction must the second hiker travel now, in order to reach the first hiker's location destination?
  a. 200 m east
  b. 200 m south
  c. 260 m east
  d. 260 m south

5.2 Vector Addition and Subtraction: Analytical Methods

40. When will the x-component of a vector with angle θ be greater than its y-component?
  a. $0^\circ < \theta < 45^\circ$
  b. $\theta = 45^\circ$
  c. $45^\circ < \theta < 60^\circ$
  d. $60^\circ < \theta < 90^\circ$

41. The resultant vector of the addition of vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}$. The magnitudes of $\vec{a}$, $\vec{b}$, and $\vec{r}$ are A, B, and R, respectively. Which of the following is true?
  a. $R_x + R_y = 0$
  b. $A_x + A_y = \vec{A}$
  c. $A_x + B_y = B_x + A_y$
  d. $A_x + B_x = R_x$

42. What is the dimensionality of vectors used in the study of atmospheric sciences?
  a. One-dimensional
  b. Two-dimensional
  c. Three-dimensional

5.3 Projectile Motion

43. After a projectile is launched in the air, in which direction does it experience constant, non-zero acceleration, ignoring air resistance?
  a. The x direction
  b. The y direction
  c. Both the x and y directions
  d. Neither direction

44. Which is true when the height of a projectile is at its maximum?
  a. $v_y = 0$
  b. $v_y = \text{maximum}$
  c. $v_x = \text{maximum}$

45. A ball is thrown in the air at an angle of 40°. If the maximum height it reaches is 10 m, what must be its initial speed?
  a. 17.46 m/s
  b. 21.78 m/s
  c. 304.92 m/s
  d. 474.37 m/s

46. A large rock is ejected from a volcano with a speed of 30 m/s and at an angle 60° above the horizontal. The rock strikes the side of the volcano at an altitude of 10.0 m lower than its starting point. Calculate the horizontal displacement of the rock.
  a. 84.90 m
  b. 96.59 m
  c. 169.80 m
  d. 193.20 m

5.4 Inclined Planes

47. For objects of identical masses but made of different materials, which of the following experiences the most static friction?
  a. Shoes on ice
  b. Metal on wood
  c. Teflon on steel

48. If an object sits on an inclined plane and no other object makes contact with the object, what is typically equal in magnitude to the component of the weight perpendicular to the plane?

a. The normal force
b. The total weight
c. The parallel force of weight

49. A 5 kg box is at rest on the floor. The coefficient of static friction between the box and the floor is 0.4. A horizontal force of 50 N is applied to the box. Will it move?
a. No, because the applied force is less than the maximum limiting static friction.
b. No, because the applied force is more than the maximum limiting static friction.
c. Yes, because the applied force is less than the maximum limiting static friction.
d. Yes, because the applied force is more than the maximum limiting static friction.

50. A skier with a mass of 67 kg is skiing down a snowy slope with an incline of 37°. Find the friction if the coefficient of kinetic friction is 0.07.
a. 27.66 N
b. 34.70 N
c. 36.71 N
d. 45.96 N

5.5 Simple Harmonic Motion

51. A change in which of the following is an example of deformation?
a. Velocity
b. Length
c. Mass
d. Weight

52. The units of amplitude are the same as those for which of the following measurements?
a. Speed
b. Displacement
c. Acceleration
d. Force

53. Up to approximately what angle is simple harmonic motion a good model for a pendulum?
a. $15°$
b. $45°$
c. $75°$
d. $90°$

54. How would simple harmonic motion be different in the absence of friction?
a. Oscillation will not happen in the absence of friction.
b. Oscillation will continue forever in the absence of friction.
c. Oscillation will have changing amplitude in the absence of friction.
d. Oscillation will cease after a certain amount of time in the absence of friction.

55. What mass needs to be attached to a spring with a force constant of $7\ \text{N/m}$ in order to make a simple harmonic oscillator oscillate with a time period of $3\ \text{s}$?
a. $0.03\ \text{kg}$
b. $1.60\ \text{kg}$
c. $30.7\ \text{kg}$
d. $63.0\ \text{kg}$

Short Answer

5.1 Vector Addition and Subtraction: Graphical Methods

56. Find $\vec{A} - \vec{B}$ for the following vectors:
$\vec{A} = (122\ \text{cm}, \angle 145°)\ \vec{B} = (110\ \text{cm}, \angle 270°)$
a. 108 cm, $\theta = 119.0°$
b. 108 cm, $\theta = 125.0°$
c. 206 cm, $\theta = 119.0°$
d. 206 cm, $\theta = 125.0°$

57. Find $\vec{A} + \vec{B}$ for the following vectors:
$\vec{A} = (122\ \text{cm}, \angle 145°)\ \vec{B} = (110\ \text{cm}, \angle 270°)$
a. 108 cm, $\theta = 119.1°$
b. 108 cm, $\theta = 201.8°$
c. 232 cm, $\theta = 119.1°$
d. 232 cm, $\theta = 201.8°$

58. Consider six vectors of 2 cm each, joined from head to tail making a hexagon. What would be the magnitude of the addition of these vectors?
a. Zero
b. Six
c. Eight
d. Twelve

59. Two people pull on ropes tied to a trolley, each applying 44 N of force. The angle the ropes form with each other is 39.5°. What is the magnitude of the net force exerted on the trolley?
a. 0.0 N
b. 79.6 N
c. 82.8 N
d. 88.0 N

5.2 Vector Addition and Subtraction: Analytical Methods

60. True or False—A vector can form the shape of a right angle triangle with its x and y components.
a. True

b. False

61. True or False—All vectors have positive x and y components.
   a. True
   b. False

62. Consider $\vec{A} - \vec{B} = \vec{R}$. What is R_x in terms of A_x and B_x?
   a. $R_x = \frac{A_x}{B_x}$
   b. $R_x = \frac{B_x}{A_x}$
   c. $R_x = A_x + B_x$
   d. $R_x = A_x - B_x$

63. Consider $\vec{A} - \vec{B} = \vec{R}$. What is R_y in terms of A_y and B_y?
   a. $R_y = \frac{A_y}{B_y}$
   b. $R_y = \frac{B_y}{A_y}$
   c. $R_y = A_y + B_y$
   d. $R_y = A_y - B_y$

64. When a three dimensional vector is used in the study of atmospheric sciences, what is z?
   a. Altitude
   b. Heat
   c. Temperature
   d. Wind speed

65. Which method is not an application of vector calculus?
   a. To find the rate of change in atmospheric temperature
   b. To study changes in wind speed and direction
   c. To predict changes in atmospheric pressure
   d. To measure changes in average rainfall

5.3 Projectile Motion

66. How can you express the velocity, $\vec{v}$, of a projectile in terms of its initial velocity, $\vec{v_0}$, acceleration, $\vec{a}$, and time, *t*?
   a. $\vec{v} = \vec{a}t$
   b. $\vec{v} = \vec{v_0} + \vec{a}t$
   c. $\vec{v} + \vec{v_0} = \vec{a}t$
   d. $\vec{v_0} + \vec{v} + \vec{a}t$

67. In the equation for the maximum height of a projectile, what does v_{0y} stand for? $h = \frac{{v_{0y}}^2}{2g}$
   a. Initial velocity in the x direction
   b. Initial velocity in the y direction
   c. Final velocity in the x direction
   d. Final velocity in the y direction

68. True or False—Range is defined as the maximum vertical distance travelled by a projectile.
   a. True
   b. False

69. For what angle of a projectile is its range equal to zero?
   a. 0° or 30°
   b. 0° or 45°
   c. 90° or 0°
   d. 90° or 45°

5.4 Inclined Planes

70. What are the units of the coefficient of friction?
   a. N
   b. m/s
   c. m/s^2
   d. unitless

71. Two surfaces in contact are moving slowly past each other. As the relative speed between the two surfaces in contact increases, what happens to the magnitude of their coefficient of kinetic friction?
   a. It increases with the increase in the relative motion.
   b. It decreases with the increase in the relative motion.
   c. It remains constant and is independent of the relative motion.

72. When will an object slide down an inclined plane at constant velocity?
   a. When the magnitude of the component of the weight along the slope is equal to the magnitude of the frictional force.
   b. When the magnitude of the component of the weight along the slope is greater than the magnitude of the frictional force.
   c. When the magnitude of the component of the weight perpendicular to the slope is less than the magnitude of the frictional force.
   d. When the magnitude of the component of the weight perpendicular to the slope is equal to the magnitude of the frictional force.

73. A box is sitting on an inclined plane. At what angle of incline is the perpendicular component of the box's weight at its maximum?
   a. 0°
   b. 30°
   c. 60°
   d. 90°

5.5 Simple Harmonic Motion

74. What is the term used for changes in shape due to the application of force?
   a. Amplitude

b. Deformation
c. Displacement
d. Restoring force

75. What is the restoring force?
a. The normal force on the surface of an object
b. The weight of a mass attached to an object
c. Force which is applied to deform an object from its original shape
d. Force which brings an object back to its equilibrium position

76. For a given oscillator, what are the factors that affect its period and frequency?
a. Mass only
b. Force constant only
c. Applied force and mass
d. Mass and force constant

77. For an object in simple harmonic motion, when does the maximum speed occur?
a. At the extreme positions
b. At the equilibrium position
c. At the moment when the applied force is removed
d. Midway between the extreme and equilibrium positions

78. What is the equilibrium position of a pendulum?
a. When the tension in the string is zero
b. When the pendulum is hanging straight down
c. When the tension in the string is maximum
d. When the weight of the mass attached is minimum

79. If a pendulum is displaced by an angle θ, what is the net restoring force it experiences?
a. $mg\sin\theta$
b. $mg\cos\theta$
c. $-mg\sin\theta$
d. $-mg\cos\theta$

Extended Response

5.1 Vector Addition and Subtraction: Graphical Methods

80. True or False—For vectors the order of addition is important.
a. True
b. False

81. Consider five vectors *a, b, c, d,* and *e.* Is it true or false that their addition always results in a vector with a greater magnitude than if only two of the vectors were added?
a. True
b. False

5.2 Vector Addition and Subtraction: Analytical Methods

82. For what angle of a vector is it possible that its magnitude will be equal to its y-component?
a. $\theta = 0°$
b. $\theta = 45°$
c. $\theta = 60°$
d. $\theta = 90°$

83. True or False—If only the angles of two vectors are known, we can find the angle of their resultant addition vector.
a. True
b. False

84. True or false—We can find the magnitude and direction of the resultant vector if we know the angles of two vectors and the magnitude of one.
a. True
b. False

5.3 Projectile Motion

85. Ignoring drag, what is the x-component of the acceleration of a projectile? Why?
a. The x-component of the acceleration of a projectile is 0 because acceleration of a projectile is due to gravity, which acts in the y direction.
b. The x component of the acceleration of a projectile is g because acceleration of a projectile is due to gravity, which acts in the x direction.
c. The x-component of the acceleration of a projectile is 0 because acceleration of a projectile is due to gravity, which acts in the x direction.
d. The x-component of the acceleration of a projectile is g because acceleration of a projectile is due to gravity, which acts in the y direction.

86. What is the optimum angle at which a projectile should be launched in order to cover the maximum distance?
a. 0°
b. 45°
c. 60°
d. 90°

5.4 Inclined Planes

87. True or False—Friction varies from surface to surface because different substances have different degrees of roughness or smoothness.
a. True
b. False

88. As the angle of the incline gets larger, what happens to the magnitudes of the perpendicular and parallel components of gravitational force?

a. Both the perpendicular and the parallel component will decrease.
b. The perpendicular component will decrease and the parallel component will increase.
c. The perpendicular component will increase and the parallel component will decrease.
d. Both the perpendicular and the parallel component will increase.

5.5 Simple Harmonic Motion

89. What physical characteristic of a system is its force constant related to?

a. The force constant *k* is related to the stiffness of a system: The larger the force constant, the stiffer the system.
b. The force constant *k* is related to the stiffness of a system: The larger the force constant, the looser the system.
c. The force constant *k* is related to the friction in the system: The larger the force constant, the greater the friction in the system.
d. The force constant *k* is related to the friction in the system: The larger the force constant, the lower the friction in the system.

90. How or why does a pendulum oscillate?

a. A pendulum oscillates due to applied force.
b. A pendulum oscillates due to the elastic nature of the string.
c. A pendulum oscillates due to restoring force arising from gravity.
d. A pendulum oscillates due to restoring force arising from tension in the string.

91. If a pendulum from earth is taken to the moon, will its frequency increase or decrease? Why?

a. It will increase because *g* on the Moon is less than *g* on Earth.
b. It will decrease because *g* on the Moon is less than *g* on Earth.
c. It will increase because *g* on the Moon is greater than *g* on Earth.
d. It will decrease because *g* on the Moon is greater than *g* on Earth.

CHAPTER 6
Circular and Rotational Motion

Figure 6.1 This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly. The same physical principles are involved in both of these motions. (Richard Munckton).

Chapter Outline

INTRODUCTION You may recall learning about various aspects of motion along a straight line: kinematics (where we learned about displacement, velocity, and acceleration), projectile motion (a special case of two-dimensional kinematics), force, and Newton's laws of motion. In some ways, this chapter is a continuation of Newton's laws of motion. Recall that Newton's first law tells us that objects move along a straight line at constant speed unless a net external force acts on them. Therefore, if an object moves along a circular path, such as the car in the photo, it must be experiencing an external force. In this chapter, we explore both circular motion and rotational motion.

6.1 Angle of Rotation and Angular Velocity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the angle of rotation and relate it to its linear counterpart
- Describe angular velocity and relate it to its linear counterpart
- Solve problems involving angle of rotation and angular velocity

Section Key Terms

angle of rotation	angular velocity	arc length	circular motion
radius of curvature	rotational motion	spin	tangential velocity

Angle of Rotation

What exactly do we mean by *circular motion* or *rotation*? **Rotational motion** is the circular motion of an object about an axis of rotation. We will discuss specifically circular motion and spin. **Circular motion** is when an object moves in a circular path. Examples of circular motion include a race car speeding around a circular curve, a toy attached to a string swinging in a circle around your head, or the circular *loop-the-loop* on a roller coaster. **Spin** is rotation about an axis that goes through the center of mass of the object, such as Earth rotating on its axis, a wheel turning on its axle, the spin of a tornado on its path of destruction, or a figure skater spinning during a performance at the Olympics. Sometimes, objects will be spinning while in circular motion, like the Earth spinning on its axis while revolving around the Sun, but we will focus on these two motions separately.

When solving problems involving rotational motion, we use variables that are similar to linear variables (distance, velocity, acceleration, and force) but take into account the curvature or rotation of the motion. Here, we define the **angle of rotation**, which is the angular equivalence of distance; and **angular velocity**, which is the angular equivalence of linear velocity.

When objects rotate about some axis—for example, when the CD in Figure 6.2 rotates about its center—each point in the object follows a circular path.

Figure 6.2 All points on a CD travel in circular paths. The pits (dots) along a line from the center to the edge all move through the same angle $\Delta\theta$ in time Δt .

The **arc length,** , is the distance traveled along a circular path. The **radius of curvature**, **r**, is the radius of the circular path. Both are shown in Figure 6.3.

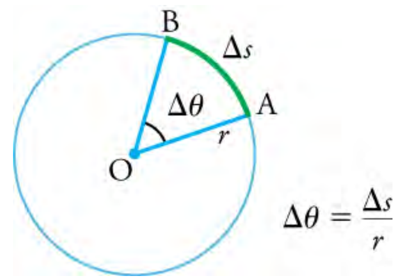


Figure 6.3 The radius (r) of a circle is rotated through an angle $\Delta\theta$. The arc length, Δs, is the distance covered along the circumference.

Consider a line from the center of the CD to its edge. In a given time, each *pit* (used to record information) on this line moves through the same angle. The angle of rotation is the amount of rotation and is the angular analog of distance. The angle of rotation $\Delta\theta$ is the arc length divided by the radius of curvature.

$$\Delta\theta = \frac{\Delta s}{r}$$

The angle of rotation is often measured by using a unit called the radian. (Radians are actually dimensionless, because a radian is defined as the ratio of two distances, radius and arc length.) A revolution is one complete rotation, where every point on the circle returns to its original position. One revolution covers 2π radians (or 360 degrees), and therefore has an angle of rotation of 2π radians, and an arc length that is the same as the circumference of the circle. We can convert between radians, revolutions, and degrees using the relationship

1 revolution = 2π rad = 360°. See Table 6.1 for the conversion of degrees to radians for some common angles.

$$\begin{aligned} 2\pi \text{ rad} &= 360^\circ \\ 1 \text{ rad} &= \frac{360^\circ}{2\ \pi} \approx 57.3^\circ \end{aligned} \quad (6.1)$$

Degree Measures	Radian Measures
30°	$\frac{\pi}{6}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	π

Table 6.1 Commonly Used Angles in Terms of Degrees and Radians

Angular Velocity

How fast is an object rotating? We can answer this question by using the concept of angular velocity. Consider first the angular speed (ω) is the rate at which the angle of rotation changes. In equation form, the angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t}, \quad (6.2)$$

which means that an angular rotation ($\Delta\theta$) occurs in a time, Δt . If an object rotates through a greater angle of rotation in a given time, it has a greater angular speed. The units for angular speed are radians per second (rad/s).

Now let's consider the direction of the angular speed, which means we now must call it the angular velocity. The direction of the angular velocity is along the axis of rotation. For an object rotating clockwise, the angular velocity points away from you along the axis of rotation. For an object rotating counterclockwise, the angular velocity points toward you along the axis of rotation.

Angular velocity (ω) is the angular version of linear velocity **v**. **Tangential velocity** is the instantaneous linear velocity of an object in rotational motion**.** To get the precise relationship between angular velocity and tangential velocity, consider again a pit on the rotating CD. This pit moves through an arc length (Δs) in a *short* time (Δt) so its tangential *speed* is

$$v = \frac{\Delta s}{\Delta t}. \tag{6.3}$$

From the definition of the angle of rotation, $\Delta\theta = \frac{\Delta s}{r}$, we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

$$v = \frac{\mathrm{r}\Delta\theta}{\Delta t} = r\omega.$$

The equation $v = r\omega$ says that the tangential speed v is proportional to the distance r from the center of rotation. Consequently, tangential speed is greater for a point on the outer edge of the CD (with larger r) than for a point closer to the center of the CD (with smaller r). This makes sense because a point farther out from the center has to cover a longer arc length in the same amount of time as a point closer to the center. Note that both points will still have the same angular speed, regardless of their distance from the center of rotation. See Figure 6.4.

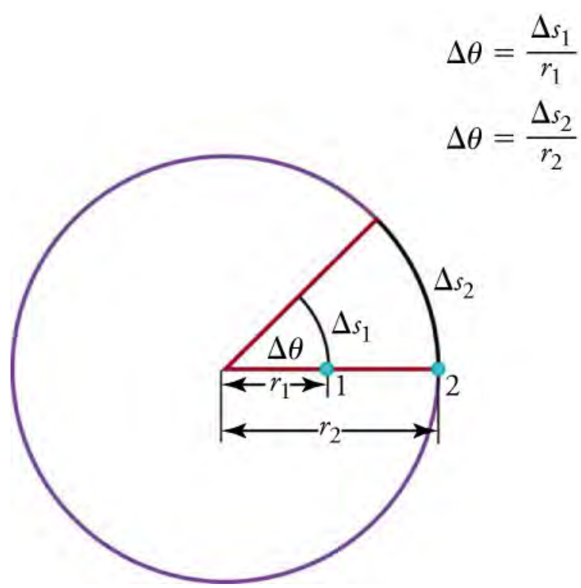


Figure 6.4 Points 1 and 2 rotate through the same angle ($\Delta\theta$), but point 2 moves through a greater arc length (Δs_2) because it is farther from the center of rotation.

Now, consider another example: the tire of a moving car (see Figure 6.5). The faster the tire spins, the faster the car moves—large ω means large v because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity, **ω**, will produce a greater linear (tangential) velocity, **v,** for the car. This is because a larger radius means a longer arc length must contact the road, so the car must move farther in the same amount of time.

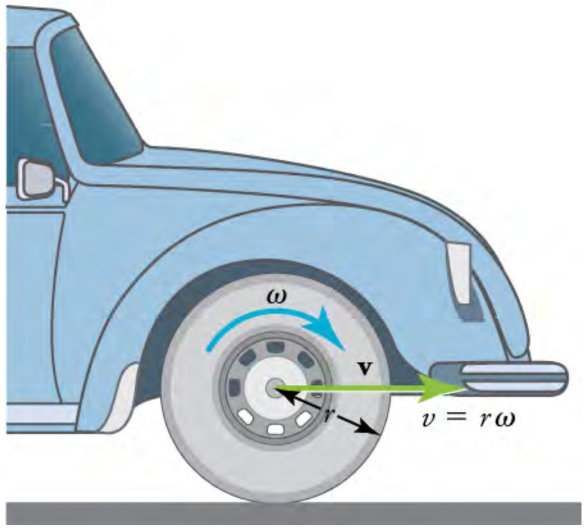


Figure 6.5 A car moving at a velocity, **v,** to the right has a tire rotating with angular velocity $\boldsymbol{\omega}$. The speed of the tread of the tire relative to the axle is *v*, the same as if the car were jacked up and the wheels spinning without touching the road. Directly below the axle, where the tire touches the road, the tire tread moves backward with respect to the axle with tangential velocity $v = r\omega$, where *r* is the tire radius. Because the road is stationary with respect to this point of the tire, the car must move forward at the linear velocity **v**. A larger angular velocity for the tire means a greater linear velocity for the car.

However, there are cases where linear velocity and tangential velocity are not equivalent, such as a car spinning its tires on ice. In this case, the linear velocity will be less than the tangential velocity. Due to the lack of friction under the tires of a car on ice, the arc length through which the tire treads move is greater than the linear distance through which the car moves. It's similar to running on a treadmill or pedaling a stationary bike; you are literally going nowhere fast.

TIPS FOR SUCCESS

Angular velocity $\boldsymbol{\omega}$ and tangential velocity **v** are vectors, so we must include magnitude and direction. The direction of the angular velocity is along the axis of rotation, and points away from you for an object rotating clockwise, and toward you for an object rotating counterclockwise. In mathematics this is described by the right-hand rule. Tangential velocity is usually described as up, down, left, right, north, south, east, or west, as shown in Figure 6.6.

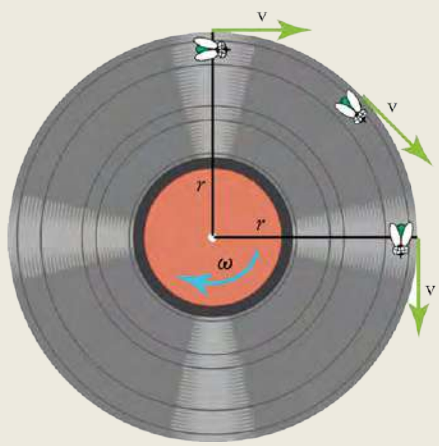


Figure 6.6 As the fly on the edge of an old-fashioned vinyl record moves in a circle, its instantaneous velocity is always at a tangent to the circle. The direction of the angular velocity is into the page this case.

WATCH PHYSICS

Relationship between Angular Velocity and Speed

This video reviews the definition and units of angular velocity and relates it to linear speed. It also shows how to convert between revolutions and radians.

Click to view content (https://www.youtube.com/embed/zAx61CO5mDw)

GRASP CHECK

For an object traveling in a circular path at a constant angular speed, would the linear speed of the object change if the radius of the path increases?

a. Yes, because tangential speed is independent of the radius.
b. Yes, because tangential speed depends on the radius.
c. No, because tangential speed is independent of the radius.
d. No, because tangential speed depends on the radius.

Solving Problems Involving Angle of Rotation and Angular Velocity

Snap Lab

Measuring Angular Speed

In this activity, you will create and measure uniform circular motion and then contrast it with circular motions with different radii.

- One string (1 m long)
- One object (two-hole rubber stopper) to tie to the end
- One timer

Procedure

1. Tie an object to the end of a string.
2. Swing the object around in a horizontal circle above your head (swing from your wrist). It is important that the circle be horizontal!
3. Maintain the object at uniform speed as it swings.
4. Measure the angular speed of the object in this manner. Measure the time it takes in seconds for the object to travel 10 revolutions. Divide that time by 10 to get the angular speed in revolutions per second, which you can convert to radians per second.
5. What is the approximate linear speed of the object?
6. Move your hand up the string so that the length of the string is 90 cm. Repeat steps 2–5.
7. Move your hand up the string so that its length is 80 cm. Repeat steps 2–5.
8. Move your hand up the string so that its length is 70 cm. Repeat steps 2–5.
9. Move your hand up the string so that its length is 60 cm. Repeat steps 2–5
10. Move your hand up the string so that its length is 50 cm. Repeat steps 2–5
11. Make graphs of angular speed vs. radius (i.e. string length) and linear speed vs. radius. Describe what each graph looks like.

GRASP CHECK

If you swing an object slowly, it may rotate at less than one revolution per second. What would be the revolutions per second for an object that makes one revolution in five seconds? What would be its angular speed in radians per second?

a. The object would spin at $\frac{1}{5}$ rev/s. The angular speed of the object would be $\frac{2\pi}{5}$ rad/s.
b. The object would spin at $\frac{1}{5}$ rev/s. The angular speed of the object would be $\frac{\pi}{5}$ rad/s.

c. The object would spin at 5 rev/s. The angular speed of the object would be 10π rad/s.
d. The object would spin at 5 rev/s. The angular speed of the object would be 5π rad/s.

Now that we have an understanding of the concepts of angle of rotation and angular velocity, we'll apply them to the real-world situations of a clock tower and a spinning tire.

WORKED EXAMPLE

Angle of rotation at a Clock Tower

The clock on a clock tower has a radius of 1.0 m. (a) What angle of rotation does the hour hand of the clock travel through when it moves from 12 p.m. to 3 p.m.? (b) What's the arc length along the outermost edge of the clock between the hour hand at these two times?

Strategy

We can figure out the angle of rotation by multiplying a full revolution (2π radians) by the fraction of the 12 hours covered by the hour hand in going from 12 to 3. Once we have the angle of rotation, we can solve for the arc length by rearranging the equation $\Delta\theta = \frac{\Delta s}{r}$ since the radius is given.

Solution to (a)

In going from 12 to 3, the hour hand covers 1/4 of the 12 hours needed to make a complete revolution. Therefore, the angle between the hour hand at 12 and at 3 is $\frac{1}{4} \times 2\ \pi\text{rad} = \frac{\pi}{2}$ (i.e., 90 degrees).

Solution to (b)

Rearranging the equation

$$\Delta\theta = \frac{\Delta s}{r}, \tag{6.4}$$

we get

$$\Delta s = r\Delta\theta. \tag{6.5}$$

Inserting the known values gives an arc length of

$$\begin{aligned} \Delta s &= (1.0\text{ m})\left(\tfrac{\pi}{2}\text{rad}\right) \\ &= 1.6\text{ m} \end{aligned} \tag{6.6}$$

Discussion

We were able to drop the radians from the final solution to part (b) because radians are actually dimensionless. This is because the radian is defined as the ratio of two distances (radius and arc length). Thus, the formula gives an answer in units of meters, as expected for an arc length.

WORKED EXAMPLE

How Fast Does a Car Tire Spin?

Calculate the angular speed of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h). See Figure 6.5.

Strategy

In this case, the speed of the tire tread with respect to the tire axle is the same as the speed of the car with respect to the road, so we have $v = 15.0$ m/s. The radius of the tire is $r = 0.300$ m. Since we know v and r, we can rearrange the equation $v = r\omega$, to get $\omega = \frac{v}{r}$ and find the angular speed.

Solution

To find the angular speed, we use the relationship: $\omega = \frac{v}{r}$.

Inserting the known quantities gives

$$\begin{aligned}\omega &= \frac{15.0 \text{ m/s}}{0.300 \text{ m}} \\ &= 50.0 \text{ rad/s}.\end{aligned} \tag{6.7}$$

Discussion

When we cancel units in the above calculation, we get 50.0/s (i.e., 50.0 per second, which is usually written as 50.0 s^{-1}). But the angular speed must have units of rad/s. Because radians are dimensionless, we can insert them into the answer for the angular speed because we know that the motion is circular. Also note that, if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular speed of

$$\begin{aligned}\omega &= \frac{15.0 \text{ m/s}}{1.20 \text{ m}} \\ &= 12.5 \text{ rad/s}\end{aligned} \tag{6.8}$$

Practice Problems

1. What is the angle in degrees between the hour hand and the minute hand of a clock showing 9:00 a.m.?

a. 0°
b. 90°
c. 180°
d. 360°

2. What is the approximate value of the arc length between the hour hand and the minute hand of a clock showing 10:00 a.m if the radius of the clock is 0.2 m?

a. 0.1 m
b. 0.2 m
c. 0.3 m
d. 0.6 m

Check Your Understanding

3. What is circular motion?

a. Circular motion is the motion of an object when it follows a linear path.
b. Circular motion is the motion of an object when it follows a zigzag path.
c. Circular motion is the motion of an object when it follows a circular path.
d. Circular motion is the movement of an object along the circumference of a circle or rotation along a circular path.

4. What is meant by radius of curvature when describing rotational motion?

a. The radius of curvature is the radius of a circular path.
b. The radius of curvature is the diameter of a circular path.
c. The radius of curvature is the circumference of a circular path.
d. The radius of curvature is the area of a circular path.

5. What is angular velocity?

a. Angular velocity is the rate of change of the diameter of the circular path.
b. Angular velocity is the rate of change of the angle subtended by the circular path.
c. Angular velocity is the rate of change of the area of the circular path.
d. Angular velocity is the rate of change of the radius of the circular path.

6. What equation defines angular velocity, ω? Take that r is the radius of curvature, θ is the angle, and t is time.

a. $\omega = \frac{\Delta\theta}{\Delta t}$
b. $\omega = \frac{\Delta t}{\Delta\theta}$
c. $\omega = \frac{\Delta r}{\Delta t}$
d. $\omega = \frac{\Delta t}{\Delta r}$

7. Identify three examples of an object in circular motion.

a. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a top spinning on its axis
b. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
c. Earth spinning on its own axis, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
d. Earth spinning on its own axis, blades of a working ceiling fan, and a top spinning on its own axis

8. What is the relative orientation of the radius and tangential velocity vectors of an object in uniform circular motion?
a. Tangential velocity vector is always parallel to the radius of the circular path along which the object moves.
b. Tangential velocity vector is always perpendicular to the radius of the circular path along which the object moves.
c. Tangential velocity vector is always at an acute angle to the radius of the circular path along which the object moves.
d. Tangential velocity vector is always at an obtuse angle to the radius of the circular path along which the object moves.

6.2 Uniform Circular Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe centripetal acceleration and relate it to linear acceleration
- Describe centripetal force and relate it to linear force
- Solve problems involving centripetal acceleration and centripetal force

Section Key Terms

centrifugal force	centripetal acceleration	centripetal force	uniform circular motion

Centripetal Acceleration

In the previous section, we defined circular motion. The simplest case of circular motion is **uniform circular motion**, where an object travels a circular path *at a constant speed*. Note that, unlike speed, the linear velocity of an object in circular motion is constantly changing because it is always changing direction. We know from kinematics that acceleration is a change in velocity, either in magnitude or in direction or both. Therefore, an object undergoing uniform circular motion is always accelerating, even though the magnitude of its velocity is constant.

You experience this acceleration yourself every time you ride in a car while it turns a corner. If you hold the steering wheel steady during the turn and move at a constant speed, you are executing uniform circular motion. What you notice is a feeling of sliding (or being flung, depending on the speed) away from the center of the turn. This isn't an actual force that is acting on you—it only happens because your body wants to continue moving in a straight line (as per Newton's first law) whereas the car is turning off this straight-line path. Inside the car it appears as if you are forced away from the center of the turn. This fictitious force is known as the **centrifugal force**. The sharper the curve and the greater your speed, the more noticeable this effect becomes.

Figure 6.7 shows an object moving in a circular path at constant speed. The direction of the instantaneous tangential velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity; in this case it points roughly toward the center of rotation. (The center of rotation is at the center of the circular path). If we imagine Δs becoming smaller and smaller, then the acceleration would point *exactly* toward the center of rotation, but this case is hard to draw. We call the acceleration of an object moving in uniform circular motion the **centripetal acceleration** $\mathbf{a}_c$ because centripetal means *center seeking*.

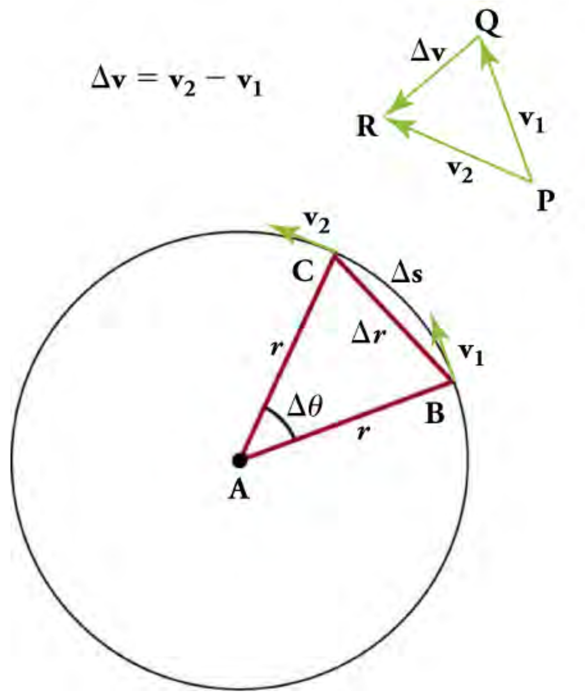


Figure 6.7 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta\mathbf{v}$ is seen to point approximately toward the center of curvature (see small inset). For an extremely small value of Δs, $\Delta\mathbf{v}$ points exactly toward the center of the circle (but this is hard to draw). Because $\mathbf{a}_\mathrm{c} = \Delta\mathbf{v}/\Delta t$, the acceleration is also toward the center, so $\mathbf{a}_c$ is called centripetal acceleration.

Now that we know that the direction of centripetal acceleration is toward the center of rotation, let's discuss the magnitude of centripetal acceleration. For an object traveling at speed v in a circular path with radius r, the magnitude of centripetal acceleration is

$$\mathbf{a}_\mathrm{c} = \frac{v^2}{r}.$$

Centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you may have noticed when driving a car, because the car actually pushes you toward the center of the turn. But it is a bit surprising that $\mathbf{a}_\mathrm{c}$ is proportional to the speed squared. This means, for example, that the acceleration is four times greater when you take a curve at 100 km/h than at 50 km/h.

We can also express $\mathbf{a}_\mathrm{c}$ in terms of the magnitude of angular velocity. Substituting $v = r\omega$ into the equation above, we get $a_c = \frac{(r\omega)^2}{r} = r\omega^2$. Therefore, the magnitude of centripetal acceleration in terms of the magnitude of angular velocity is

$$\mathbf{a}_c = r\omega^2. \tag{6.9}$$

TIPS FOR SUCCESS

The equation expressed in the form $a_c = r\omega^2$ is useful for solving problems where you know the angular velocity rather than the tangential velocity.

Virtual Physics

Ladybug Motion in 2D

In this simulation, you experiment with the position, velocity, and acceleration of a ladybug in circular and elliptical motion. Switch the type of motion from linear to circular and observe the velocity and acceleration vectors. Next, try elliptical motion and notice how the velocity and acceleration vectors differ from those in circular motion.

Click to view content (https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/)

GRASP CHECK

In uniform circular motion, what is the angle between the acceleration and the velocity? What type of acceleration does a body experience in the uniform circular motion?

a. The angle between acceleration and velocity is 0°, and the body experiences linear acceleration.
b. The angle between acceleration and velocity is 0°, and the body experiences centripetal acceleration.
c. The angle between acceleration and velocity is 90°, and the body experiences linear acceleration.
d. The angle between acceleration and velocity is 90°, and the body experiences centripetal acceleration.

Centripetal Force

Because an object in uniform circular motion undergoes constant acceleration (by changing direction), we know from Newton's second law of motion that there must be a constant net external force acting on the object.

Any force or combination of forces can cause a centripetal acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, the friction between a road and the tires of a car as it goes around a curve, or the normal force of a roller coaster track on the cart during a loop-the-loop.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of rotation, the same as for centripetal acceleration. According to Newton's second law of motion, a net force causes the acceleration of mass according to $\mathbf{F}_{\text{net}} = m\mathbf{a}$. For uniform circular motion, the acceleration is centripetal acceleration: $\mathbf{a} = \mathbf{a}_\text{c}$. Therefore, the magnitude of centripetal force, $\mathbf{F}_\text{c}$, is $\mathbf{F}_\text{c} = m\mathbf{a}_\text{c}$.

By using the two different forms of the equation for the magnitude of centripetal acceleration, $\mathbf{a}_\text{c} = v^2/r$ and $\mathbf{a}_c = r\omega^2$, we get two expressions involving the magnitude of the centripetal force $\mathbf{F}_\text{c}$. The first expression is in terms of tangential speed, the second is in terms of angular speed: $\mathbf{F}_\text{c} = m\frac{v^2}{r}$ and $\mathbf{F}_\text{c} = mr\omega^2$.

Both forms of the equation depend on mass, velocity, and the radius of the circular path. You may use whichever expression for centripetal force is more convenient. Newton's second law also states that the object will accelerate in the same direction as the net force. By definition, the centripetal force is directed towards the center of rotation, so the object will also accelerate towards the center. A straight line drawn from the circular path to the center of the circle will always be perpendicular to the tangential velocity. Note that, if you solve the first expression for r, you get

$$r = \frac{mv^2}{\mathbf{F}_\text{c}}.$$

From this expression, we see that, for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

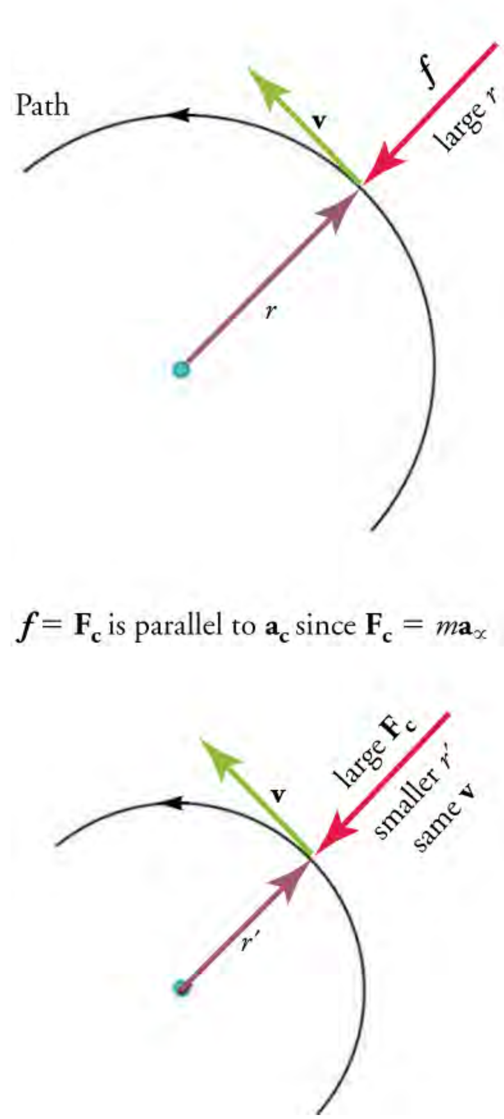


Figure 6.8 In this figure, the frictional force f serves as the centripetal force $\mathbf{F}_c$. Centripetal force is perpendicular to tangential velocity and causes uniform circular motion. The larger the centripetal force $\mathbf{F}_c$, the smaller is the radius of curvature r and the sharper is the curve. The lower curve has the same velocity $\mathbf{v}$, but a larger centripetal force $\mathbf{F}_c$ produces a smaller radius r' .

WATCH PHYSICS

Centripetal Force and Acceleration Intuition

This video explains why a centripetal force creates centripetal acceleration and uniform circular motion. It also covers the difference between speed and velocity and shows examples of uniform circular motion.

Click to view content (https://www.youtube.com/embed/vZOk8NnjILg)

GRASP CHECK

Imagine that you are swinging a yoyo in a vertical clockwise circle in front of you, perpendicular to the direction you are facing. Now, imagine that the string breaks just as the yoyo reaches its bottommost position, nearest the floor. Which of the following describes the path of the yoyo after the string breaks?

a. The yoyo will fly upward in the direction of the centripetal force.
b. The yoyo will fly downward in the direction of the centripetal force.

c. The yoyo will fly to the left in the direction of the tangential velocity.
d. The yoyo will fly to the right in the direction of the tangential velocity.

Solving Centripetal Acceleration and Centripetal Force Problems

To get a feel for the typical magnitudes of centripetal acceleration, we'll do a lab estimating the centripetal acceleration of a tennis racket and then, in our first Worked Example, compare the centripetal acceleration of a car rounding a curve to gravitational acceleration. For the second Worked Example, we'll calculate the force required to make a car round a curve.

Snap Lab

Estimating Centripetal Acceleration

In this activity, you will measure the swing of a golf club or tennis racket to estimate the centripetal acceleration of the end of the club or racket. You may choose to do this in slow motion. Recall that the equation for centripetal acceleration is $\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$.

- One tennis racket or golf club
- One timer
- One ruler or tape measure

Procedure

1. Work with a partner. Stand a safe distance away from your partner as he or she swings the golf club or tennis racket.
2. Describe the motion of the swing—is this uniform circular motion? Why or why not?
3. Try to get the swing as close to uniform circular motion as possible. What adjustments did your partner need to make?
4. Measure the radius of curvature. What did you physically measure?
5. By using the timer, find either the linear or angular velocity, depending on which equation you decide to use.
6. What is the approximate centripetal acceleration based on these measurements? How accurate do you think they are? Why? How might you and your partner make these measurements more accurate?

GRASP CHECK

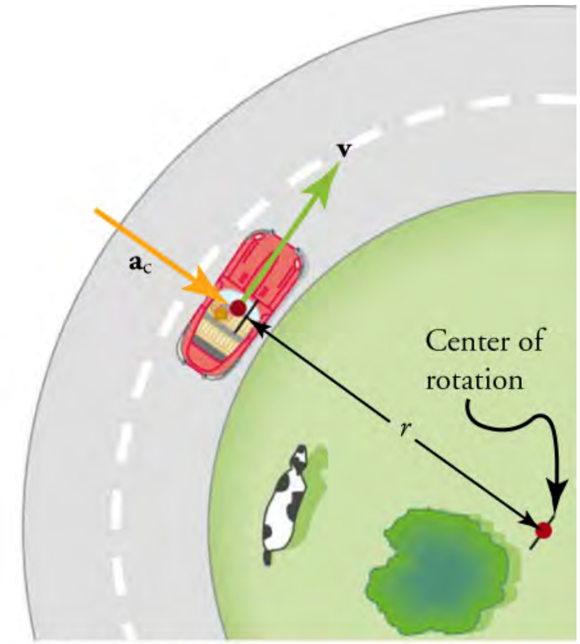
Was it more useful to use the equation $a_c = \frac{v^2}{r}$ or $a_c = r\omega^2$ in this activity? Why?

a. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be easier.
b. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be easier.
c. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be difficult.
d. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be difficult.

WORKED EXAMPLE

Comparing Centripetal Acceleration of a Car Rounding a Curve with Acceleration Due to Gravity

A car follows a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h). What is the magnitude of the car's centripetal acceleration? Compare the centripetal acceleration for this fairly gentle curve taken at highway speed with acceleration due to gravity (g).



Strategy

Because linear rather than angular speed is given, it is most convenient to use the expression $\mathbf{a}_c = \frac{v^2}{r}$ to find the magnitude of the centripetal acceleration.

Solution

Entering the given values of $v = 25.0$ m/s and $r = 500$ m into the expression for $\mathbf{a}_c$ gives

$$\begin{aligned}\mathbf{a}_c &= \frac{v^2}{r} \\ &= \frac{(25.0\text{ m/s})^2}{500\text{ m}} \\ &= 1.25\text{m/s}^2.\end{aligned}$$

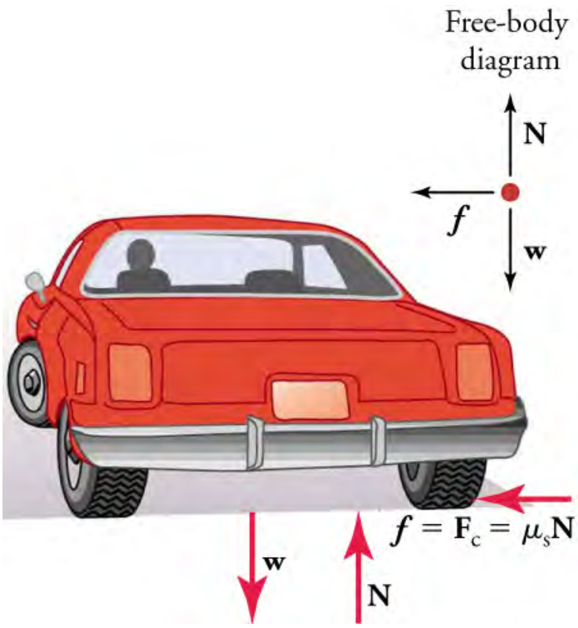
Discussion

To compare this with the acceleration due to gravity ($g = 9.80$ m/s^2), we take the ratio $\mathbf{a}_c/g = (1.25\text{ m/s}^2)/(9.80\text{m/s}^2) = 0.128$. Therefore, $\mathbf{a}_c = 0.128g$, which means that the centripetal acceleration is about one tenth the acceleration due to gravity.

WORKED EXAMPLE

Frictional Force on Car Tires Rounding a Curve

a. Calculate the centripetal force exerted on a 900 kg car that rounds a 600-m-radius curve on horizontal ground at 25.0 m/s.
b. Static friction prevents the car from slipping. Find the magnitude of the frictional force between the tires and the road that allows the car to round the curve without sliding off in a straight line.



Strategy and Solution for (a)

We know that $\mathbf{F}_c = m\frac{v^2}{r}$. Therefore,

$$\begin{aligned}\mathbf{F}_c &= m\frac{v^2}{r} \\ &= \frac{(900 \text{ kg})(25.0 \text{ m/s})^2}{600 \text{ m}} \\ &= 938 \text{ N}.\end{aligned}$$

Strategy and Solution for (b)

The image above shows the forces acting on the car while rounding the curve. In this diagram, the car is traveling into the page as shown and is turning to the left. Friction acts toward the left, accelerating the car toward the center of the curve. Because friction is the only horizontal force acting on the car, it provides all of the centripetal force in this case. Therefore, the force of friction is the centripetal force in this situation and points toward the center of the curve.

$$f = \mathbf{F}_c = 938 \text{ N}$$

Discussion

Since we found the force of friction in part (b), we could also solve for the coefficient of friction, since $f = \mu_s \text{N} = \mu_s mg$.

Practice Problems

9. What is the centripetal acceleration of an object with speed 12 m/s going along a path of radius 2.0 m?
   a. 6 m/s
   b. 72 m/s
   c. 6 m/s^2
   d. 72 m/s^2

10. Calculate the centripetal acceleration of an object following a path with a radius of a curvature of 0.2 m and at an angular velocity of 5 rad/s.
    a. 1 m/s
    b. 5 m/s
    c. 1 m/s^2
    d. 5 m/s^2

Check Your Understanding

11. What is uniform circular motion?

a. Uniform circular motion is when an object accelerates on a circular path at a constantly increasing velocity.
b. Uniform circular motion is when an object travels on a circular path at a variable acceleration.
c. Uniform circular motion is when an object travels on a circular path at a constant speed.
d. Uniform circular motion is when an object travels on a circular path at a variable speed.

12. What is centripetal acceleration?
a. The acceleration of an object moving in a circular path and directed radially toward the center of the circular orbit
b. The acceleration of an object moving in a circular path and directed tangentially along the circular path
c. The acceleration of an object moving in a linear path and directed in the direction of motion of the object
d. The acceleration of an object moving in a linear path and directed in the direction opposite to the motion of the object

13. Is there a net force acting on an object in uniform circular motion?
a. Yes, the object is accelerating, so a net force must be acting on it.
b. Yes, because there is no acceleration.
c. No, because there is acceleration.
d. No, because there is no acceleration.

14. Identify two examples of forces that can cause centripetal acceleration.
a. The force of Earth's gravity on the moon and the normal force
b. The force of Earth's gravity on the moon and the tension in the rope on an orbiting tetherball
c. The normal force and the force of friction acting on a moving car
d. The normal force and the tension in the rope on a tetherball

6.3 Rotational Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe rotational kinematic variables and equations and relate them to their linear counterparts
- Describe torque and lever arm
- Solve problems involving torque and rotational kinematics

Section Key Terms

angular acceleration	kinematics of rotational motion	lever arm
tangential acceleration	torque	

Rotational Kinematics

In the section on uniform circular motion, we discussed motion in a circle at constant speed and, therefore, constant angular velocity. However, there are times when angular velocity is not constant—rotational motion can speed up, slow down, or reverse directions. Angular velocity is not constant when a spinning skater pulls in her arms, when a child pushes a merry-go-round to make it rotate, or when a CD slows to a halt when switched off. In all these cases, **angular acceleration** occurs because the angular velocity $\boldsymbol{\omega}$ changes. The faster the change occurs, the greater is the angular acceleration. Angular acceleration $\boldsymbol{\alpha}$ is the rate of change of angular velocity. In equation form, angular acceleration is

$$\boldsymbol{\alpha} = \frac{\Delta\boldsymbol{\omega}}{\Delta t},$$

where $\Delta\boldsymbol{\omega}$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2. If $\boldsymbol{\omega}$ increases, then $\boldsymbol{\alpha}$ is positive. If $\boldsymbol{\omega}$ decreases, then $\boldsymbol{\alpha}$ is negative. Keep in mind that, by convention, counterclockwise is the positive direction and clockwise is the negative direction. For example, the skater in Figure 6.9 is rotating counterclockwise as seen from above, so her angular velocity is positive. Acceleration would be negative, for example, when an object that is rotating counterclockwise slows down. It would be positive when an object that is rotating counterclockwise speeds up.

Figure 6.9 A figure skater spins in the counterclockwise direction, so her angular velocity is normally considered to be positive. (Luu, Wikimedia Commons)

The relationship between the magnitudes of **tangential acceleration**, **a**, and angular acceleration,

$$\boldsymbol{\alpha}, \text{is} \mathbf{a} = r\boldsymbol{\alpha} \text{or} \boldsymbol{\alpha} = \frac{\mathbf{a}}{r}. \tag{6.10}$$

These equations mean that the magnitudes of tangential acceleration and angular acceleration are directly proportional to each other. The greater the angular acceleration, the larger the change in tangential acceleration, and vice versa. For example, consider riders in their pods on a Ferris wheel at rest. A Ferris wheel with greater angular acceleration will give the riders greater tangential acceleration because, as the Ferris wheel increases its rate of spinning, it also increases its tangential velocity. Note that the radius of the spinning object also matters. For example, for a given angular acceleration $\boldsymbol{\alpha}$, a smaller Ferris wheel leads to a smaller tangential acceleration for the riders.

TIPS FOR SUCCESS

Tangential acceleration is sometimes denoted $\mathbf{a}_t$. It is a linear acceleration in a direction tangent to the circle at the point of interest in circular or rotational motion. Remember that tangential acceleration is parallel to the tangential velocity (either in the same direction or in the opposite direction.) Centripetal acceleration is always perpendicular to the tangential velocity.

So far, we have defined three rotational variables: θ, $\boldsymbol{\omega}$, and $\boldsymbol{\alpha}$. These are the angular versions of the linear variables x, $\mathbf{v}$, and $\mathbf{a}$. Table 6.2 shows how they are related.

Rotational	Linear	Relationship
θ	x	$\theta = \frac{x}{r}$

Table 6.2 Rotational and Linear Variables

Rotational	Linear	Relationship
$\boldsymbol{\omega}$	$\mathbf{v}$	$\boldsymbol{\omega} = \frac{\mathbf{v}}{r}$
$\boldsymbol{\alpha}$	$\mathbf{a}$	$\boldsymbol{\alpha} = \frac{\mathbf{a}}{r}$

Table 6.2 Rotational and Linear Variables

We can now begin to see how rotational quantities like θ, $\boldsymbol{\omega}$, and $\boldsymbol{\alpha}$ are related to each other. For example, if a motorcycle wheel that starts at rest has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. Putting this in terms of the variables, if the wheel's angular acceleration $\boldsymbol{\alpha}$ is large for a long period of time t, then the final angular velocity $\boldsymbol{\omega}$ and angle of rotation θ are large. In the case of linear motion, if an object starts at rest and undergoes a large linear acceleration, then it has a large final velocity and will have traveled a large distance.

The **kinematics of rotational motion** describes the relationships between the angle of rotation, angular velocity, angular acceleration, and time. It only *describes* motion—it does not include any forces or masses that may affect rotation (these are part of dynamics). Recall the kinematics equation for linear motion: $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ (constant $\mathbf{a}$).

As in linear kinematics, we assume $\mathbf{a}$ is constant, which means that angular acceleration $\boldsymbol{\alpha}$ is also a constant, because $\mathbf{a} = r\boldsymbol{\alpha}$. The equation for the kinematics relationship between $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, and t is

$$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t (\text{constant} \alpha),$$

where $\boldsymbol{\omega}_0$ is the initial angular velocity. Notice that the equation is identical to the linear version, except with angular analogs of the linear variables. In fact, all of the linear kinematics equations have rotational analogs, which are given in Table 6.3. These equations can be used to solve rotational or linear kinematics problem in which $\mathbf{a}$ and $\boldsymbol{\alpha}$ are constant.

Rotational	Linear	
$\theta = \bar{\boldsymbol{\omega}}t$	$x = \bar{\mathbf{v}}t$	
$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$	$\mathbf{v} = \mathbf{v}_0 + \boldsymbol{\alpha}t$	constant $\boldsymbol{\alpha}$, $\mathbf{a}$
$\theta = \boldsymbol{\omega}_0 t + \frac{1}{2}\boldsymbol{\alpha}t^2$	$x = \mathbf{v}_0 t + \frac{1}{2}\boldsymbol{\alpha}t^2$	constant $\boldsymbol{\alpha}$, $\mathbf{a}$
$\boldsymbol{\omega}^2 = {\boldsymbol{\omega}_0}^2 + 2\boldsymbol{\alpha}\theta$	$\mathbf{v}^2 = {\mathbf{v}_0}^2 + 2\boldsymbol{\alpha}x$	constant $\boldsymbol{\alpha}$, $\mathbf{a}$

Table 6.3 Equations for Rotational Kinematics

In these equations, $\boldsymbol{\omega}_0$ and $\mathbf{v}_0$ are initial values, t_0 is zero, and the average angular velocity $\bar{\boldsymbol{\omega}}$ and average velocity $\bar{\mathbf{v}}$ are

$$\bar{\boldsymbol{\omega}} = \frac{\boldsymbol{\omega}_0 + \boldsymbol{\omega}}{2} \text{and} \bar{\mathbf{v}} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}. \tag{6.11}$$

FUN IN PHYSICS

Storm Chasing

Figure 6.10 Tornadoes descend from clouds in funnel-like shapes that spin violently. (Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

Storm chasers tend to fall into one of three groups: Amateurs chasing tornadoes as a hobby, atmospheric scientists gathering data for research, weather watchers for news media, or scientists having fun under the guise of work. Storm chasing is a dangerous pastime because tornadoes can change course rapidly with little warning. Since storm chasers follow in the wake of the destruction left by tornadoes, changing flat tires due to debris left on the highway is common. The most active part of the world for tornadoes, called *tornado alley*, is in the central United States, between the Rocky Mountains and Appalachian Mountains.

Tornadoes are perfect examples of rotational motion in action in nature. They come out of severe thunderstorms called supercells, which have a column of air rotating around a horizontal axis, usually about four miles across. The difference in wind speeds between the strong cold winds higher up in the atmosphere in the jet stream and weaker winds traveling north from the Gulf of Mexico causes the column of rotating air to shift so that it spins around a vertical axis, creating a tornado.

Tornadoes produce wind speeds as high as 500 km/h (approximately 300 miles/h), particularly at the bottom where the funnel is narrowest because the rate of rotation increases as the radius decreases. They blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw.

GRASP CHECK

What is the physics term for the eye of the storm? Why would winds be weaker at the eye of the tornado than at its outermost edge?

a. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to radius of curvature.
b. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to radius of curvature.
c. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to the square of the radius of curvature.
d. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to the square of the radius of curvature.

Torque

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity. The farther the force is applied from the pivot point (or fulcrum), the greater the angular acceleration. For example, a door opens slowly if you push too close to its hinge, but opens easily if you push far from the hinges. Furthermore, we know that the more massive the door is, the more slowly it opens; this is because angular acceleration is inversely proportional to mass. These relationships are very similar to the relationships between force, mass, and acceleration from Newton's second law of motion. Since we have already covered the angular versions of distance, velocity and time, you may wonder what the angular version of force is, and how it relates to linear force.

The angular version of force is **torque $\boldsymbol{\tau}$**, which is the turning effectiveness of a force. See Figure 6.11. The equation for the magnitude of torque is

$$\boldsymbol{\tau} = r\mathbf{F}\sin\theta,$$

where r is the magnitude of the **lever arm**, **F** is the magnitude of the linear force, and θ is the angle between the lever arm and the force. The lever arm is the vector from the point of rotation (pivot point or fulcrum) to the location where force is applied. Since the magnitude of the lever arm is a distance, its units are in meters, and torque has units of N·m. Torque is a vector quantity and has the same direction as the angular acceleration that it produces.

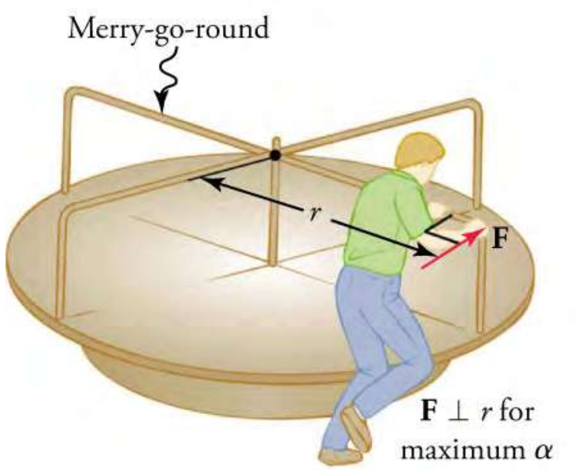


Figure 6.11 A man pushes a merry-go-round at its edge and perpendicular to the lever arm to achieve maximum torque.

Applying a stronger torque will produce a greater angular acceleration. For example, the harder the man pushes the merry-go-round in Figure 6.11, the faster it accelerates. Furthermore, the more massive the merry-go-round is, the slower it accelerates for the same torque. If the man wants to maximize the effect of his force on the merry-go-round, he should push as far from the center as possible to get the largest lever arm and, therefore, the greatest torque and angular acceleration. Torque is also maximized when the force is applied perpendicular to the lever arm.

Solving Rotational Kinematics and Torque Problems

Just as linear forces can balance to produce zero net force and no linear acceleration, the same is true of rotational motion. When two torques of equal magnitude act in opposing directions, there is no net torque and no angular acceleration, as you can see in the following video. If zero net torque acts on a system spinning at a constant angular velocity, the system will continue to spin at the same angular velocity.

WATCH PHYSICS

Introduction to Torque

This video (https://www.khanacademy.org/science/physics/torque-angular-momentum/torque-tutorial/v/introduction-to-torque) defines torque in terms of moment arm (which is the same as lever arm). It also covers a problem with forces acting in opposing directions about a pivot point. (At this stage, you can ignore Sal's references to work and mechanical advantage.)

GRASP CHECK

Click to view content (https://www.openstax.org/l/28torque)

If the net torque acting on the ruler from the example was positive instead of zero, what would this say about the angular acceleration? What would happen to the ruler over time?

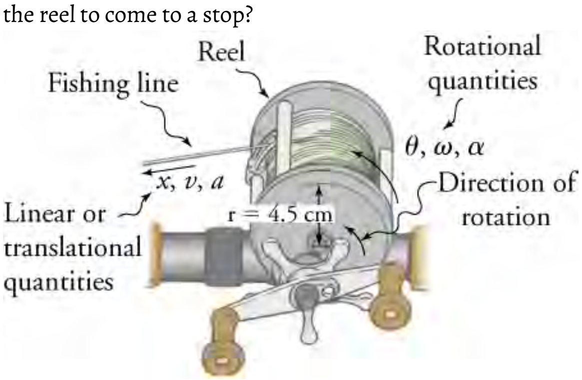
a. The ruler is in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.
b. The ruler is not in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.
c. The ruler is not in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.
d. The ruler is in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.

Now let's look at examples applying rotational kinematics to a fishing reel and the concept of torque to a merry-go-round.

WORKED EXAMPLE

Calculating the Time for a Fishing Reel to Stop Spinning

A deep-sea fisherman uses a fishing rod with a reel of radius 4.50 cm. A big fish takes the bait and swims away from the boat, pulling the fishing line from his fishing reel. As the fishing line unwinds from the reel, the reel spins at an angular velocity of 220 rad/s. The fisherman applies a brake to the spinning reel, creating an angular acceleration of –300 rad/s^2. How long does it take the reel to come to a stop?



Strategy

We are asked to find the time *t* for the reel to come to a stop. The magnitude of the initial angular velocity is $\boldsymbol{\omega}_0 = 220$ rad/s, and the magnitude of the final angular velocity $\boldsymbol{\omega} = 0$. The signed magnitude of the angular acceleration is $\boldsymbol{\alpha} = -300$ rad/s^2, where the minus sign indicates that it acts in the direction opposite to the angular velocity. Looking at the rotational kinematic equations, we see all quantities but *t* are known in the equation $\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$, making it the easiest equation to use for this problem.

Solution

The equation to use is $\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$.

We solve the equation algebraically for *t*, and then insert the known values.

$$\begin{aligned} t &= \frac{\omega - \omega_0}{\alpha} \\ &= \frac{0 - 220 \text{ rad/s}}{-300 \text{rad/s}^2} \\ &= 0.733 \text{ s} \end{aligned} \quad (6.12)$$

Discussion

The time to stop the reel is fairly small because the acceleration is fairly large. Fishing lines sometimes snap because of the forces involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration and therefore a smaller force.

WORKED EXAMPLE

Calculating the Torque on a Merry-Go-Round

Consider the man pushing the playground merry-go-round in Figure 6.11. He exerts a force of 250 N at the edge of the merry-go-round and perpendicular to the radius, which is 1.50 m. How much torque does he produce? Assume that friction acting on the merry-go-round is negligible.

Strategy

To find the torque, note that the applied force is perpendicular to the radius and that friction is negligible.

Solution

$$\begin{aligned} \tau &= r\mathbf{F}\sin\theta \\ &= (1.50\text{ m})(250\text{ N})\sin\left(\frac{\pi}{2}\right). \\ &= 375\text{ N}\cdot\text{m} \end{aligned} \qquad 6.13$$

Discussion

The man maximizes the torque by applying force perpendicular to the lever arm, so that $\theta = \frac{\pi}{2}$ and $\sin\theta = 1$. The man also maximizes his torque by pushing at the outer edge of the merry-go-round, so that he gets the largest-possible lever arm.

Practice Problems

15. How much torque does a person produce if he applies a 12 N force 1.0 m away from the pivot point, perpendicularly to the lever arm?
   a. $\frac{1}{144}$ N-m
   b. $\frac{1}{12}$ N-m
   c. 12 N-m
   d. 144 N-m

16. An object's angular velocity changes from 3 rad/s clockwise to 8 rad/s clockwise in 5 s. What is its angular acceleration?
   a. 0.6 rad/s^2
   b. 1.6 rad/s^2
   c. 1 rad/s^2
   d. 5 rad/s^2

Check Your Understanding

17. What is angular acceleration?
   a. Angular acceleration is the rate of change of the angular displacement.
   b. Angular acceleration is the rate of change of the angular velocity.
   c. Angular acceleration is the rate of change of the linear displacement.
   d. Angular acceleration is the rate of change of the linear velocity.

18. What is the equation for angular acceleration, α? Assume θ is the angle, ω is the angular velocity, and t is time.
   a. $\alpha = \frac{\Delta\omega}{\Delta t}$
   b. $\alpha = \Delta\omega\Delta t$
   c. $\alpha = \frac{\Delta\theta}{\Delta t}$
   d. $\alpha = \Delta\theta\Delta t$

19. Which of the following best describes torque?
   a. It is the rotational equivalent of a force.
   b. It is the force that affects linear motion.
   c. It is the rotational equivalent of acceleration.
   d. It is the acceleration that affects linear motion.

20. What is the equation for torque?

a. $\tau = F\,cos\theta\,r$
b. $\tau = \frac{F \sin \theta}{r}$
c. $\tau = rF\cos\theta$
d. $\tau = rF\sin\theta$

KEY TERMS

angle of rotation the ratio of the arc length to the radius of curvature of a circular path

angular acceleration the rate of change of angular velocity with time

angular velocity ($\boldsymbol{\omega}$) the rate of change in the angular position of an object following a circular path

arc length (Δs) the distance traveled by an object along a circular path

centrifugal force a fictitious force that acts in the direction opposite the centripetal acceleration

centripetal acceleration the acceleration of an object moving in a circle, directed toward the center of the circle

centripetal force any force causing uniform circular motion

circular motion the motion of an object along a circular path

kinematics of rotational motion the relationships between rotation angle, angular velocity, angular acceleration, and time

lever arm the distance between the point of rotation (pivot point) and the location where force is applied

radius of curvature the distance between the center of a circular path and the path

rotational motion the circular motion of an object about an axis of rotation

spin rotation about an axis that goes through the center of mass of the object

tangential acceleration the acceleration in a direction tangent to the circular path of motion and in the same direction or opposite direction as the tangential velocity

tangential velocity the instantaneous linear velocity of an object in circular or rotational motion

torque the effectiveness of a force to change the rotational speed of an object

uniform circular motion the motion of an object in a circular path at constant speed

SECTION SUMMARY

6.1 Angle of Rotation and Angular Velocity

- Circular motion is motion in a circular path.
- The angle of rotation $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature.
- The arc length Δs is the distance traveled along a circular path and r is the radius of curvature of the circular path.
- The angle of rotation $\Delta\theta$ is measured in units of radians (rad), where $2\ \pi\text{rad} = 360° = 1$ revolution.
- Angular velocity $\boldsymbol{\omega}$ is the rate of change of an angle, where a rotation $\Delta\theta$ occurs in a time Δt .
- The units of angular velocity are radians per second (rad/s).
- Tangential speed v and angular speed ω are related by $v = r\omega$, and tangential velocity has units of m/s.
- The direction of angular velocity is along the axis of rotation, toward (away) from you for clockwise (counterclockwise) motion.

6.2 Uniform Circular Motion

- Centripetal acceleration $\mathbf{a}_c$ is the acceleration experienced while in uniform circular motion.
- Centripetal acceleration force is a *center-seeking* force that always points toward the center of rotation, perpendicular to the linear velocity, in the same direction as the net force, and in the direction opposite that of the radius vector.
- The standard unit for centripetal acceleration is m/s^2.
- Centripetal force $\mathbf{F}_c$ is any net force causing uniform circular motion.

6.3 Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.
- Torque is the effectiveness of a force to change the rotational speed of an object. Torque is the rotational analog of force.
- The lever arm is the distance between the point of rotation (pivot point) and the location where force is applied.
- Torque is maximized by applying force perpendicular to the lever arm and at a point as far as possible from the pivot point or fulcrum. If torque is zero, angular acceleration is zero.

KEY EQUATIONS

6.1 Angle of Rotation and Angular Velocity

Angle of rotation	$\Delta\theta = \frac{\Delta s}{r}$
Angular speed:	$\omega = \frac{\Delta\theta}{\Delta t}$
Tangential speed:	$v = r\omega$

6.2 Uniform Circular Motion

Centripetal acceleration	$\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$
Centripetal force	$\mathbf{F}_c = m\mathbf{a}_c, \mathbf{F}_c = m\frac{v^2}{r}$, $\mathbf{F}_c = mr\omega^2$

6.3 Rotational Motion

Angular acceleration	$\boldsymbol{\alpha} = \frac{\Delta\boldsymbol{\omega}}{\Delta t}$
Rotational kinematic equations	$\theta = \boldsymbol{\omega}t, \boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$, $\theta = \boldsymbol{\omega}_0 t + \frac{1}{2}\boldsymbol{\alpha}t^2$, $\boldsymbol{\omega}^2 = {\boldsymbol{\omega}_0}^2 + 2\boldsymbol{\alpha}\theta$
Tangential (linear) acceleration	$\mathbf{a} = r\boldsymbol{\alpha}$
Torque	$\boldsymbol{\tau} = r\mathbf{F}\sin\theta$

CHAPTER REVIEW

Concept Items

6.1 Angle of Rotation and Angular Velocity

1. One revolution is equal to how many radians? Degrees?
  a. $1\text{ rev} = \pi\text{ rad} = 180°$
  b. $1\text{ rev} = \pi\text{ rad} = 360°$
  c. $1\text{ rev} = 2\pi\text{ rad} = 180°$
  d. $1\text{ rev} = 2\pi\text{ rad} = 360°$

2. What is tangential velocity?
  a. Tangential velocity is the average linear velocity of an object in a circular motion.
  b. Tangential velocity is the instantaneous linear velocity of an object undergoing rotational motion.
  c. Tangential velocity is the average angular velocity of an object in a circular motion.
  d. Tangential velocity is the instantaneous angular velocity of an object in a circular motion.

3. What kind of motion is called *spin*?
  a. Spin is rotational motion of an object about an axis parallel to the axis of the object.
  b. Spin is translational motion of an object about an axis parallel to the axis of the object.
  c. Spin is the rotational motion of an object about its center of mass.
  d. Spin is translational motion of an object about its own axis.

6.2 Uniform Circular Motion

4. What is the equation for centripetal acceleration in terms of angular velocity and the radius?
  a. $a_c = \frac{\omega^2}{r}$
  b. $a_c = \frac{\omega}{r}$
  c. $a_c = r\omega^2$
  d. $a_c = r\omega$

5. How can you express centripetal force in terms of centripetal acceleration?
  a. $F_c = \frac{a_c^2}{m}$
  b. $F_c = \frac{a_c}{m}$
  c. $F_c = ma_c^2$
  d. $F_c = ma_c$

6. What is meant by the word centripetal?
  a. center-seeking
  b. center-avoiding
  c. central force
  d. central acceleration

6.3 Rotational Motion

7. Conventionally, for which direction of rotation of an object is angular acceleration considered positive?
  a. the positive *x* direction of the coordinate system
  b. the negative *x* direction of the coordinate system
  c. the counterclockwise direction
  d. the clockwise direction

8. When you push a door closer to the hinges, why does it open more slowly?
   a. It opens slowly, because the lever arm is shorter so the torque is large.
   b. It opens slowly because the lever arm is longer so the torque is large.
   c. It opens slowly, because the lever arm is shorter so the torque is less.
   d. It opens slowly, because the lever arm is longer so the torque is less.

9. When is angular acceleration negative?
   a. Angular acceleration is the rate of change of the displacement and is negative when ω increases.
   b. Angular acceleration is the rate of change of the displacement and is negative when ω decreases.
   c. Angular acceleration is the rate of change of angular velocity and is negative when ω increases.
   d. Angular acceleration is the rate of change of angular velocity and is negative when ω decreases.

Critical Thinking Items

6.1 Angle of Rotation and Angular Velocity

10. When the radius of the circular path of rotational motion increases, what happens to the arc length for a given angle of rotation?
   a. The arc length is directly proportional to the radius of the circular path, and it increases with the radius.
   b. The arc length is inversely proportional to the radius of the circular path, and it decreases with the radius.
   c. The arc length is directly proportional to the radius of the circular path, and it decreases with the radius.
   d. The arc length is inversely proportional to the radius of the circular path, and it increases with the radius.

11. Consider a CD spinning clockwise. What is the sum of the instantaneous velocities of two points on both ends of its diameter?
   a. $2v$
   b. $\frac{v}{2}$
   c. $-v$
   d. 0

6.2 Uniform Circular Motion

12. What are the directions of the velocity and acceleration of an object in uniform circular motion?
   a. Velocity is tangential, and acceleration is radially outward.
   b. Velocity is tangential, and acceleration is radially inward.
   c. Velocity is radially outward, and acceleration is tangential.
   d. Velocity is radially inward, and acceleration is tangential.

13. Suppose you have an object tied to a rope and are rotating it over your head in uniform circular motion. If you increase the length of the rope, would you have to apply more or less force to maintain the same speed?
   a. More force is required, because the force is inversely proportional to the radius of the circular orbit.
   b. More force is required because the force is directly proportional to the radius of the circular orbit.
   c. Less force is required because the force is inversely proportional to the radius of the circular orbit.
   d. Less force is required because the force is directly proportional to the radius of the circular orbit.

6.3 Rotational Motion

14. Consider two spinning tops with different radii. Both have the same linear instantaneous velocities at their edges. Which top has a higher angular velocity?
   a. the top with the smaller radius because the radius of curvature is inversely proportional to the angular velocity
   b. the top with the smaller radius because the radius of curvature is directly proportional to the angular velocity
   c. the top with the larger radius because the radius of curvature is inversely proportional to the angular velocity
   d. The top with the larger radius because the radius of curvature is directly proportional to the angular velocity

15. A person tries to lift a stone by using a lever. If the lever arm is constant and the mass of the stone increases, what is true of the torque necessary to lift it?
   a. It increases, because the torque is directly proportional to the mass of the body.
   b. It increases because the torque is inversely proportional to the mass of the body.
   c. It decreases because the torque is directly proportional to the mass of the body.
   d. It decreases, because the torque is inversely proportional to the mass of the body.

Problems

6.1 Angle of Rotation and Angular Velocity

16. What is the angle of rotation (in degrees) between two hands of a clock, if the radius of the clock is 0.70 m and the arc length separating the two hands is 1.0 m?
  a. $40°$
  b. $80°$
  c. $81°$
  d. $163°$

17. A clock has radius of 0.5 m. The outermost point on its minute hand travels along the edge. What is its tangential speed?
  a. 9×10^{-4} m/s
  b. 3.4×10^{-3} m/s
  c. 8.5×10^{-4} m/s
  d. 1.3×10^{1} m/s

6.2 Uniform Circular Motion

18. What is the centripetal force exerted on a 1,600 kg car that rounds a 100 m radius curve at 12 m/s?
  a. 192 N
  b. 1, 111 N
  c. 2, 300 N
  d. 13, 333 N

19. Find the frictional force between the tires and the road that allows a 1,000 kg car traveling at 30 m/s to round a 20 m radius curve.
  a. 22 N
  b. 667 N
  c. 1, 500 N
  d. 45, 000 N

6.3 Rotational Motion

20. An object's angular acceleration is 36 rad/s^2. If it were initially spinning with a velocity of 6.0 m/s, what would its angular velocity be after 5.0 s?
  a. 186 rad/s
  b. 190 rad/s^2
  c. −174 rad/s
  d. −174 rad/s^2

21. When a fan is switched on, it undergoes an angular acceleration of 150 rad/s^2. How long will it take to achieve its maximum angular velocity of 50 rad/s?
  a. −0.3 s
  b. 0.3 s
  c. 3.0 s

Performance Task

6.3 Rotational Motion

22. Design a lever arm capable of lifting a 0.5 kg object such as a stone. The force for lifting should be provided by placing coins on the other end of the lever. How many coins would you need? What happens if you shorten or lengthen the lever arm? What does this say about torque?

TEST PREP

Multiple Choice

6.1 Angle of Rotation and Angular Velocity

23. What is 1 radian approximately in degrees?
  a. 57.3°
  b. 360°
  c. $\pi°$
  d. $2\pi°$

24. If the following objects are spinning at the same angular velocities, the edge of which one would have the highest speed?
  a. Mini CD
  b. Regular CD
  c. Vinyl record

25. What are possible units for tangential velocity?
  a. m/s
  b. rad/s
  c. °/s

26. What is $30°$ in radians?
  a. $\frac{\pi}{12}$
  b. $\frac{\pi}{9}$
  c. $\frac{\pi}{6}$
  d. $\frac{\pi}{3}$

27. For a given object, what happens to the arc length as the angle of rotation increases?
  a. The arc length is directly proportional to the angle of rotation, so it increases with the angle of rotation.
  b. The arc length is inversely proportional to the angle of rotation, so it decreases with the angle of rotation.
  c. The arc length is directly proportional to the angle of rotation, so it decreases with the angle of rotation.

d. The arc length is inversely proportional to the angle of rotation, so it increases with the angle of rotation.

6.2 Uniform Circular Motion

28. Which of these quantities is constant in uniform circular motion?
a. Speed
b. Velocity
c. Acceleration
d. Displacement

29. Which of these quantities impact centripetal force?
a. Mass and speed only
b. Mass and radius only
c. Speed and radius only
d. Mass, speed, and radius all impact centripetal force

30. An increase in the magnitude of which of these quantities causes a reduction in centripetal force?
a. Mass
b. Radius of curvature
c. Speed

31. What happens to centripetal acceleration as the radius of curvature decreases and the speed is constant, and why?
a. It increases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
b. It increases, because the centripetal acceleration is directly proportional to the radius of curvature.
c. It decreases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
d. It decreases, because the centripetal acceleration is directly proportional to the radius of the curvature.

32. Why do we experience more sideways acceleration while driving around sharper curves?
a. Centripetal acceleration is inversely proportional to the radius of curvature, so it increases as the radius of curvature decreases.
b. Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature decreases.
c. Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature increases.
d. Centripetal acceleration is directly proportional to the radius of curvature, so it increases as the radius of curvature increases.

6.3 Rotational Motion

33. Which of these quantities is not described by the kinematics of rotational motion?
a. Rotation angle
b. Angular acceleration
c. Centripetal force
d. Angular velocity

34. In the equation $\tau = rF\sin\theta$, what is *F*?
a. Linear force
b. Centripetal force
c. Angular force

35. What happens when two torques act equally in opposite directions?
a. Angular velocity is zero.
b. Angular acceleration is zero.

36. What is the mathematical relationship between angular and linear accelerations?
a. $a = r\alpha$
b. $a = \frac{\alpha}{r}$
c. $a = r^2\alpha$
d. $a = \frac{\alpha}{r^2}$

Short Answer

6.1 Angle of Rotation and Angular Velocity

37. What is the rotational analog of linear velocity?
a. Angular displacement
b. Angular velocity
c. Angular acceleration
d. Angular momentum

38. What is the rotational analog of distance?
a. Rotational angle
b. Torque
c. Angular velocity
d. Angular momentum

39. What is the equation that relates the linear speed of a point on a rotating object with the object's angular quantities?
a. $v = \frac{\omega}{r}$
b. $v = r\omega$
c. $v = \frac{\alpha}{r}$
d. $v = r\alpha$

40. As the angular velocity of an object increases, what happens to the linear velocity of a point on that object?
a. It increases, because linear velocity is directly proportional to angular velocity.
b. It increases, because linear velocity is inversely proportional to angular velocity.

c. It decreases because linear velocity is directly proportional to angular velocity.
d. It decreases because linear velocity is inversely proportional to angular velocity.

41. What is angular speed in terms of tangential speed and the radius?
a. $\omega = \frac{v^2}{r}$
b. $\omega = \frac{v}{r}$
c. $\omega = rv$
d. $\omega = rv^2$

42. Why are radians dimensionless?
a. Radians are dimensionless, because they are defined as a ratio of distances. They are defined as the ratio of the arc length to the radius of the circle.
b. Radians are dimensionless because they are defined as a ratio of distances. They are defined as the ratio of the area to the radius of the circle.
c. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the arc length to the radius of the circle.
d. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the area to the radius of the circle.

6.2 Uniform Circular Motion

43. What type of quantity is centripetal acceleration?
a. Scalar quantity; centripetal acceleration has magnitude only but no direction
b. Scalar quantity; centripetal acceleration has magnitude as well as direction
c. Vector quantity; centripetal acceleration has magnitude only but no direction
d. Vector quantity; centripetal acceleration has magnitude as well as direction

44. What are the standard units for centripetal acceleration?
a. m/s
b. m/s^2
c. m^2/s
d. m^2/s^2

45. What is the angle formed between the vectors of tangential velocity and centripetal force?
a. 0°
b. 30°
c. 90°
d. 180°

46. What is the angle formed between the vectors of centripetal acceleration and centripetal force?
a. 0°
b. 30°
c. 90°
d. 180°

47. What are the standard units for centripetal force?
a. m
b. m/s
c. m/s^2
d. newtons

48. As the mass of an object in uniform circular motion increases, what happens to the centripetal force required to keep it moving at the same speed?
a. It increases, because the centripetal force is directly proportional to the mass of the rotating body.
b. It increases, because the centripetal force is inversely proportional to the mass of the rotating body.
c. It decreases, because the centripetal force is directly proportional to the mass of the rotating body.
d. It decreases, because the centripetal force is inversely proportional to the mass of the rotating body.

6.3 Rotational Motion

49. The relationships between which variables are described by the kinematics of rotational motion?
a. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, and angular acceleration.
b. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and angular momentum.
c. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.
d. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, torque, and time.

50. What is the kinematics relationship between ω, α, and t ?
a. $\omega = \alpha t$
b. $\omega = \omega_0 - \alpha t$
c. $\omega = \omega_0 + \alpha t$
d. $\omega = \omega_0 + \frac{1}{2}\alpha t$

51. What kind of quantity is torque?
a. Scalar
b. Vector

c. Dimensionless
d. Fundamental quantity

52. If a linear force is applied to a lever arm farther away from the pivot point, what happens to the resultant torque?
a. It decreases.
b. It increases.
c. It remains the same.
d. It changes the direction.

53. How can the same force applied to a lever produce different torques?
a. By applying the force at different points of the lever arm along the length of the lever or by changing the angle between the lever arm and the applied force.
b. By applying the force at the same point of the lever arm along the length of the lever or by changing the angle between the lever arm and the applied force.
c. By applying the force at different points of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.
d. By applying the force at the same point of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.

Extended Response

6.1 Angle of Rotation and Angular Velocity

54. Consider two pits on a CD, one close to the center and one close to the outer edge. When the CD makes one full rotation, which pit would have gone through a greater angle of rotation? Which one would have covered a greater arc length?
a. The one close to the center would go through the greater angle of rotation. The one near the outer edge would trace a greater arc length.
b. The one close to the center would go through the greater angle of rotation. The one near the center would trace a greater arc length.
c. Both would go through the same angle of rotation. The one near the outer edge would trace a greater arc length.
d. Both would go through the same angle of rotation. The one near the center would trace a greater arc length.

55. Consider two pits on a CD, one close to the center and one close to the outer edge. For a given angular velocity of the CD, which pit has a higher angular velocity? Which has a higher tangential velocity?
a. The point near the center would have the greater angular velocity and the point near the outer edge would have the higher linear velocity.
b. The point near the edge would have the greater angular velocity and the point near the center would have the higher linear velocity.
c. Both have the same angular velocity and the point near the outer edge would have the higher linear velocity.
d. Both have the same angular velocity and the point near the center would have the higher linear velocity.

56. What happens to tangential velocity as the radius of an object increases provided the angular velocity remains the same?
a. It increases because tangential velocity is directly proportional to the radius.
b. It increases because tangential velocity is inversely proportional to the radius.
c. It decreases because tangential velocity is directly proportional to the radius.
d. It decreases because tangential velocity is inversely proportional to the radius.

6.2 Uniform Circular Motion

57. Is an object in uniform circular motion accelerating? Why or why not?
a. Yes, because the velocity is not constant.
b. No, because the velocity is not constant.
c. Yes, because the velocity is constant.
d. No, because the velocity is constant.

58. An object is in uniform circular motion. Suppose the centripetal force was removed. In which direction would the object now travel?
a. In the direction of the centripetal force
b. In the direction opposite to the direction of the centripetal force
c. In the direction of the tangential velocity
d. In the direction opposite to the direction of the tangential velocity

59. An object undergoes uniform circular motion. If the radius of curvature and mass of the object are constant, what is the centripetal force proportional to?
a. $F_c \propto \frac{1}{v}$
b. $F_c \propto \frac{1}{v^2}$
c. $F_c \propto v$
d. $F_c \propto v^2$

6.3 Rotational Motion

60. Why do tornadoes produce more wind speed at the bottom of the funnel?

a. Wind speed is greater at the bottom because rate of rotation increases as the radius increases.
b. Wind speed is greater at the bottom because rate of rotation increases as the radius decreases.
c. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.
d. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.

61. How can you maximize the torque applied to a given lever arm without applying more force?

a. The force should be applied perpendicularly to the lever arm as close as possible from the pivot point.
b. The force should be applied perpendicularly to the lever arm as far as possible from the pivot point.
c. The force should be applied parallel to the lever arm as far as possible from the pivot point.
d. The force should be applied parallel to the lever arm as close as possible from the pivot point.

62. When will an object continue spinning at the same angular velocity?

a. When net torque acting on it is zero
b. When net torque acting on it is non zero
c. When angular acceleration is positive
d. When angular acceleration is negative

CHAPTER 7
Newton's Law of Gravitation

Figure 7.1 Johannes Kepler (left) showed how the planets move, and Isaac Newton (right) discovered that gravitational force caused them to move that way. ((left) unknown, Public Domain; (right) Sir Godfrey Kneller, Public Domain)

Chapter Outline

INTRODUCTION What do a falling apple and the orbit of the moon have in common? You will learn in this chapter that each is caused by gravitational force. The motion of all celestial objects, in fact, is determined by the gravitational force, which depends on their mass and separation.

Johannes Kepler discovered three laws of planetary motion that all orbiting planets and moons follow. Years later, Isaac Newton found these laws useful in developing his law of universal gravitation. This law relates gravitational force to the masses of objects and the distance between them. Many years later still, Albert Einstein showed there was a little more to the gravitation story when he published his theory of general relativity.

7.1 Kepler's Laws of Planetary Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain Kepler's three laws of planetary motion
- Apply Kepler's laws to calculate characteristics of orbits

Section Key Terms

aphelion	Copernican model	eccentricity
Kepler’s laws of planetary motion	perihelion	Ptolemaic model

Concepts Related to Kepler’s Laws of Planetary Motion

Examples of orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The moon’s orbit around Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets around the sun are no less interesting. If we look farther, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force. The orbital motions of objects in our own solar system are simple enough to describe with a few fairly simple laws. The orbits of planets and moons satisfy the following two conditions:

- The mass of the orbiting object, m, is small compared to the mass of the object it orbits, M.
- The system is isolated from other massive objects.

Based on the motion of the planets about the sun, Kepler devised a set of three classical laws, called **Kepler’s laws of planetary motion**, that describe the orbits of all bodies satisfying these two conditions:

1. The orbit of each planet around the sun is an ellipse with the sun at one focus.
2. Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times.
3. The ratio of the squares of the periods of any two planets about the sun is equal to the ratio of the cubes of their average distances from the sun.

These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630). He devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed. Let’s look closer at each of these laws.

Kepler’s First Law

The orbit of each planet about the sun is an ellipse with the sun at one focus, as shown in Figure 7.2. The planet’s closest approach to the sun is called **aphelion** and its farthest distance from the sun is called **perihelion**.

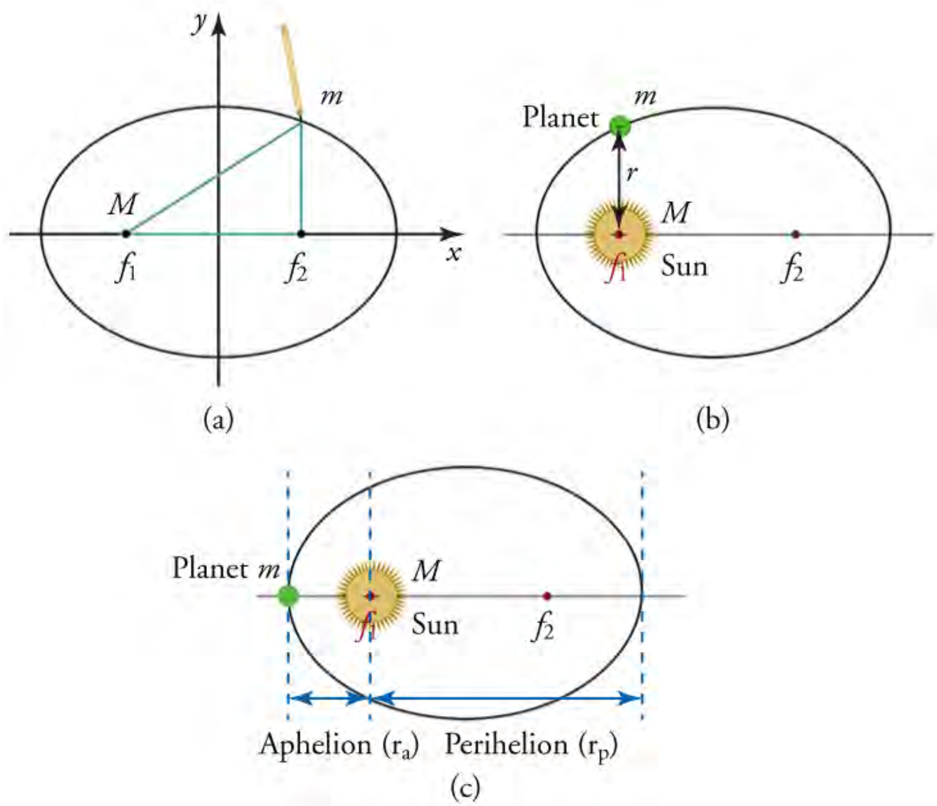


Figure 7.2 (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is constant. (b) For any closed orbit, *m* follows an elliptical path with *M* at one focus. (c) The aphelion (*r*a) is the closest distance between the planet and the sun, while the perihelion (*r*p) is the farthest distance from the sun.

If you know the aphelion (r_a) and perihelion (r_p) distances, then you can calculate the semi-major axis (*a*) and semi-minor axis (*b*).

$$a = \frac{(r_a + r_p)}{2}$$
$$b = \sqrt{r_a r_p}$$

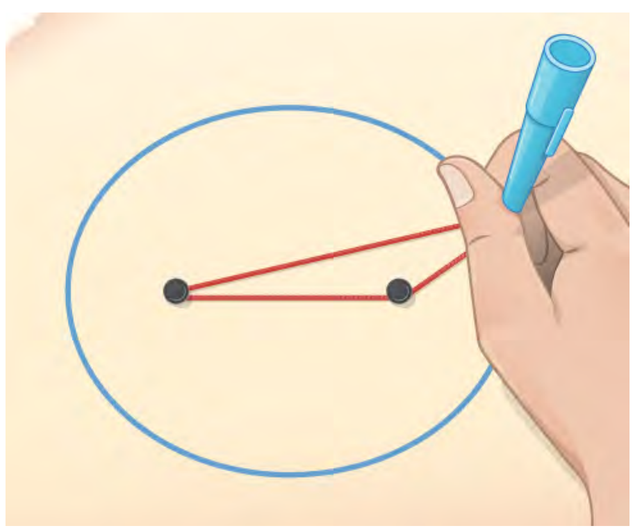


Figure 7.3 You can draw an ellipse as shown by putting a pin at each focus, and then placing a loop of string around a pen and the pins and tracing a line on the paper.

Kepler's Second Law

Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times, as shown in Figure 7.4.

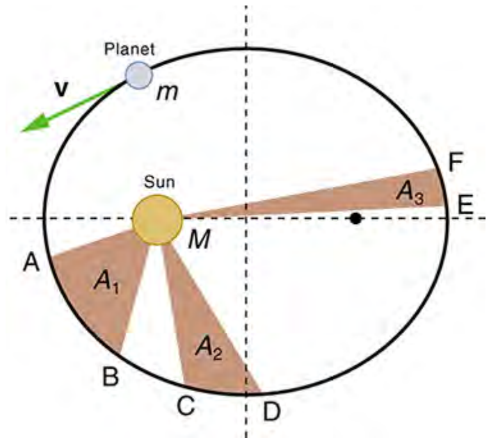


Figure 7.4 The shaded regions have equal areas. The time for *m* to go from A to B is the same as the time to go from C to D and from E to F. The mass *m* moves fastest when it is closest to *M*. Kepler's second law was originally devised for planets orbiting the sun, but it has broader validity.

TIPS FOR SUCCESS

Note that while, for historical reasons, Kepler's laws are stated for planets orbiting the sun, they are actually valid for all bodies satisfying the two previously stated conditions.

Kepler's Third Law

The ratio of the periods squared of any two planets around the sun is equal to the ratio of their average distances from the sun cubed. In equation form, this is

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

where T is the period (time for one orbit) and r is the average distance (also called orbital radius). This equation is valid only for comparing two small masses orbiting a single large mass. Most importantly, this is only a descriptive equation; it gives no information about the cause of the equality.

LINKS TO PHYSICS

History: Ptolemy vs. Copernicus

Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in Figure 7.5 **(a)**. This is called the **Ptolemaic model**, named for the Greek philosopher Ptolemy who lived in the second century AD. The Ptolemaic model is characterized by a list of facts for the motions of planets, with no explanation of cause and effect. There tended to be a different rule for each heavenly body and a general lack of simplicity.

Figure 7.5 **(b)** represents the modern or **Copernican model**. In this model, a small set of rules and a single underlying force explain not only all planetary motion in the solar system, but also all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling.

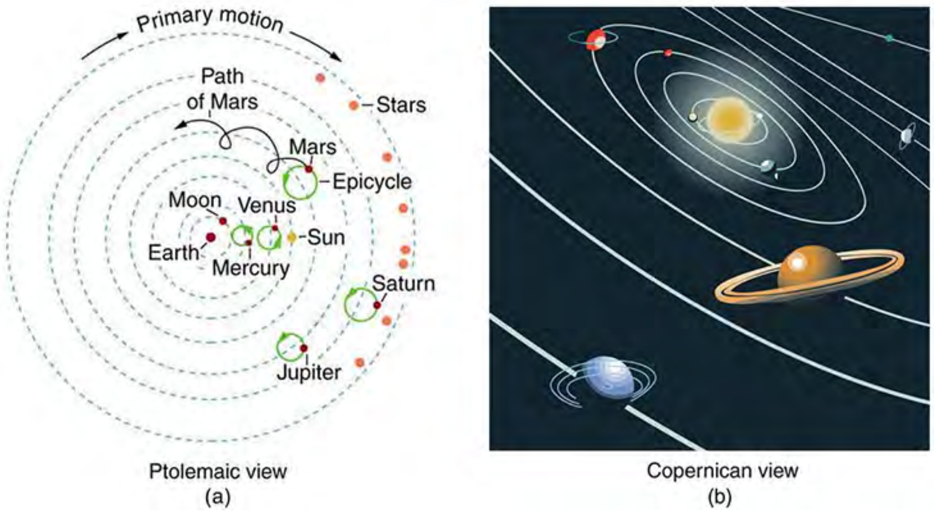


Figure 7.5 (a) The Ptolemaic model of the universe has Earth at the center with the moon, the planets, the sun, and the stars revolving about it in complex circular paths. This geocentric (Earth-centered) model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints about the causes of these motions. (b) The Copernican heliocentric (sun-centered) model is a simpler and more accurate model.

Nicolaus Copernicus (1473–1543) first had the idea that the planets circle the sun, in about 1514. It took him almost 20 years to work out the mathematical details for his model. He waited another 10 years or so to publish his work. It is thought he hesitated because he was afraid people would make fun of his theory. Actually, the reaction of many people was more one of fear and anger. Many people felt the Copernican model threatened their basic belief system. About 100 years later, the astronomer Galileo was put under house arrest for providing evidence that planets, including Earth, orbited the sun. In all, it took almost 300 years for everyone to admit that Copernicus had been right all along.

GRASP CHECK

Explain why Earth does actually appear to be the center of the solar system.

a. Earth appears to be the center of the solar system because Earth is at the center of the universe, and everything revolves around it in a circular orbit.
b. Earth appears to be the center of the solar system because, in the reference frame of Earth, the sun, moon, and planets all appear to move across the sky as if they were circling Earth.
c. Earth appears to be at the center of the solar system because Earth is at the center of the solar system and all the heavenly bodies revolve around it.
d. Earth appears to be at the center of the solar system because Earth is located at one of the foci of the elliptical orbit of the sun, moon, and other planets.

Virtual Physics

Acceleration

This simulation allows you to create your own solar system so that you can see how changing distances and masses determines the orbits of planets. Click *Help* for instructions.

Click to view content (https://archive.cnx.org/specials/ee816dff-0b5f-4e6f-8250-f9fb9e39d716/my-solar-system/)

GRASP CHECK

When the central object is off center, how does the speed of the orbiting object vary?

a. The orbiting object moves fastest when it is closest to the central object and slowest when it is farthest away.
b. The orbiting object moves slowest when it is closest to the central object and fastest when it is farthest away.

c. The orbiting object moves with the same speed at every point on the circumference of the elliptical orbit.
d. There is no relationship between the speed of the object and the location of the planet on the circumference of the orbit.

Calculations Related to Kepler’s Laws of Planetary Motion

Kepler’s First Law

Refer back to Figure 7.2 **(a)**. Notice which distances are constant. The foci are fixed, so distance $\overline{f_1 f_2}$ is a constant. The definition of an ellipse states that the sum of the distances $\overline{f_1 m} + \overline{mf_2}$ is also constant. These two facts taken together mean that the perimeter of triangle $\Delta f_1 mf_2$ must also be constant. Knowledge of these constants will help you determine positions and distances of objects in a system that includes one object orbiting another.

Kepler’s Second Law

Refer back to Figure 7.4. The second law says that the segments have equal area and that it takes equal time to sweep through each segment. That is, the time it takes to travel from A to B equals the time it takes to travel from C to D, and so forth. Velocity **v** equals distance *d* divided by time *t*: $\mathbf{v} = d/t$. Then, $t = d/\mathbf{v}$, so distance divided by velocity is also a constant. For example, if we know the average velocity of Earth on June 21 and December 21, we can compare the distance Earth travels on those days.

The degree of elongation of an elliptical orbit is called its **eccentricity** (*e*). Eccentricity is calculated by dividing the distance *f* from the center of an ellipse to one of the foci by half the long axis *a*.

$$(e) = f/a \tag{7.1}$$

When $e = 0$, the ellipse is a circle.

The area of an ellipse is given by $A = \pi ab$, where *b* is half the short axis. If you know the axes of Earth’s orbit and the area Earth sweeps out in a given period of time, you can calculate the fraction of the year that has elapsed.

WORKED EXAMPLE

Kepler’s First Law

At its closest approach, a moon comes within 200,000 km of the planet it orbits. At that point, the moon is 300,000 km from the other focus of its orbit, f_2. The planet is focus f_1 of the moon’s elliptical orbit. How far is the moon from the planet when it is 260,000 km from f_2?

Strategy

Show and label the ellipse that is the orbit in your solution. Picture the triangle $f_1 \mathrm{m} f_2$ collapsed along the major axis and add up the lengths of the three sides. Find the length of the unknown side of the triangle when the moon is 260,000 km from f_2.

Solution

Perimeter of $f_1 mf_2 = 200,000 \text{ km} + 100{,}000 \text{ km} + 300{,}000 \text{ km} = 600{,}000 \text{ km}.$

$$mf_1 = 600{,}000 \text{ km} - (100{,}000 \text{ km} + 200{,}000 \text{ km}) = 240{,}000 \text{ km}.$$

Discussion

The perimeter of triangle $f_1 mf_2$ must be constant because the distance between the foci does not change and Kepler’s first law says the orbit is an ellipse. For any ellipse, the sum of the two sides of the triangle, which are $f_1 m$ and mf_2, is constant.

WORKED EXAMPLE

Kepler’s Second Law

Figure 7.6 shows the major and minor axes of an ellipse. The semi-major and semi-minor axes are half of these, respectively.

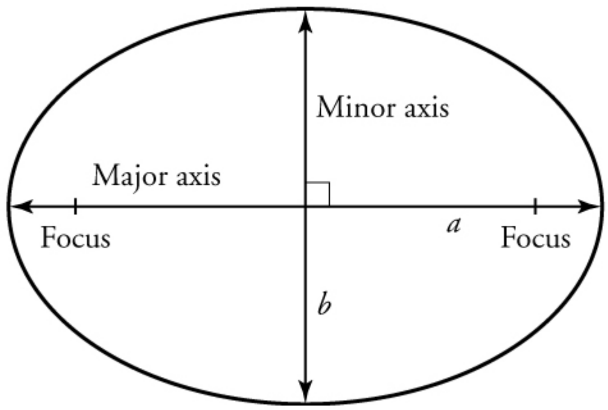


Figure 7.6 The major axis is the length of the ellipse, and the minor axis is the width of the ellipse. The semi-major axis is half the major axis, and the semi-minor axis is half the minor axis.

Earth's orbit is slightly elliptical, with a semi-major axis of 152 million km and a semi-minor axis of 147 million km. If Earth's period is 365.26 days, what area does an Earth-to-sun line sweep past in one day?

Strategy

Each day, Earth sweeps past an equal-sized area, so we divide the total area by the number of days in a year to find the area swept past in one day. For total area use $A = \pi ab$. Calculate A, the area inside Earth's orbit and divide by the number of days in a year (i.e., its period).

Solution

$$\begin{aligned}\text{area per day} &= \frac{\text{total area}}{\text{total number of days}} \\ &= \frac{\pi ab}{365\ \text{d}} \\ &= \frac{\pi\left(1.47\times10^{8}\ \text{km}\right)\left(1.52\times10^{3}\ \text{km}\right)}{365\ \text{d}} \\ &= 1.92\times10^{14}\ \text{km}^2/\text{d}\end{aligned} \tag{7.2}$$

The area swept out in one day is thus $1.92 \times 10^{14} \text{km}^2$.

Discussion

The answer is based on Kepler's law, which states that a line from a planet to the sun sweeps out equal areas in equal times.

Kepler's Third Law

Kepler's third law states that the ratio of the squares of the periods of any two planets (T_1, T_2) is equal to the ratio of the cubes of their average orbital distance from the sun (r_1, r_2). Mathematically, this is represented by

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}.$$

From this equation, it follows that the ratio r^3/T^2 is the same for all planets in the solar system. Later we will see how the work of Newton leads to a value for this constant.

WORKED EXAMPLE

Kepler's Third Law

Given that the moon orbits Earth each 27.3 days and that it is an average distance of 3.84×10^8 m from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1,500 km above Earth's surface.

Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form by

$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$. Let us use the subscript 1 for the moon and the subscript 2 for the satellite. We are asked to find T_2. The given information tells us that the orbital radius of the moon is $r_1 = 3.84 \times 10^8\,\text{m}$, and that the period of the moon is $T_1 = 27.3\ \text{days}$. The height of the artificial satellite above Earth's surface is given, so to get the distance r_2 from the center of Earth we must add the height to the radius of Earth (6380 km). This gives $r_2 = 1500\ \text{km} + 6380\ \text{km} = 7880\ \text{km}$. Now all quantities are known, so T_2 can be found.

Solution

To solve for T_2, we cross-multiply and take the square root, yielding

$$T_2^2 = T_1^2\left(\frac{r_2}{r_1}\right)^3;\ T_2 = T_1\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$$

$$T_2 = (27.3\ \text{d})\left(\frac{24.0\ \text{h}}{\text{d}}\right)\left(\frac{7880\ \text{km}}{3.84\times10^5\ \text{km}}\right)^{\frac{3}{2}} = 1.93\ \text{h}. \tag{7.3}$$

Discussion

This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will complete one orbit in the same amount of time.

Practice Problems

1. A planet with no axial tilt is located in another solar system. It circles its sun in a very elliptical orbit so that the temperature varies greatly throughout the year. If the year there has 612 days and the inhabitants celebrate the coldest day on day 1 of their calendar, when is the warmest day?
   a. Day 1
   b. Day 153
   c. Day 306
   d. Day 459

2. A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). The ratio $\frac{r^3}{T^2}$ for the moon is $1.01 \times 10^{18}\ \frac{\text{km}^3}{y^2}$. Calculate the radius of the orbit of such a satellite.
   a. $2.75 \times 10^3\ \text{km}$
   b. $1.96 \times 10^4\ \text{km}$
   c. $1.40 \times 10^5\ \text{km}$
   d. $1.00 \times 10^6\ \text{km}$

Check Your Understanding

3. Are Kepler's laws purely descriptive, or do they contain causal information?
   a. Kepler's laws are purely descriptive.
   b. Kepler's laws are purely causal.
   c. Kepler's laws are descriptive as well as causal.
   d. Kepler's laws are neither descriptive nor causal.

4. True or false—According to Kepler's laws of planetary motion, a satellite increases its speed as it approaches its parent body and decreases its speed as it moves away from the parent body.
   a. True
   b. False

5. Identify the locations of the foci of an elliptical orbit.
   a. One focus is the parent body, and the other is located at the opposite end of the ellipse, at the same distance from the center as the parent body.
   b. One focus is the parent body, and the other is located at the opposite end of the ellipse, at half the distance from the center as the parent body.

c. One focus is the parent body and the other is located outside of the elliptical orbit, on the line on which is the semi-major axis of the ellipse.
d. One focus is on the line containing the semi-major axis of the ellipse, and the other is located anywhere on the elliptical orbit of the satellite.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain Newton's law of universal gravitation and compare it to Einstein's theory of general relativity
- Perform calculations using Newton's law of universal gravitation

Section Key Terms

Einstein's theory of general relativity	gravitational constant	Newton's universal law of gravitation

Concepts Related to Newton's Law of Universal Gravitation

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure 7.7. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner, Galileo Galilei, had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph. It had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose an explanation of the mechanism that caused them to follow these paths and not others.

Figure 7.7 The popular legend that Newton suddenly discovered the law of universal gravitation when an apple fell from a tree and hit him on the head has an element of truth in it. A more probable account is that he was walking through an orchard and wondered why all the apples fell in the same direction with the same acceleration. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Expressed in modern language, **Newton’s universal law of gravitation** states that every object in the universe attracts every other object with a force that is directed along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This attraction is illustrated by Figure 7.8.

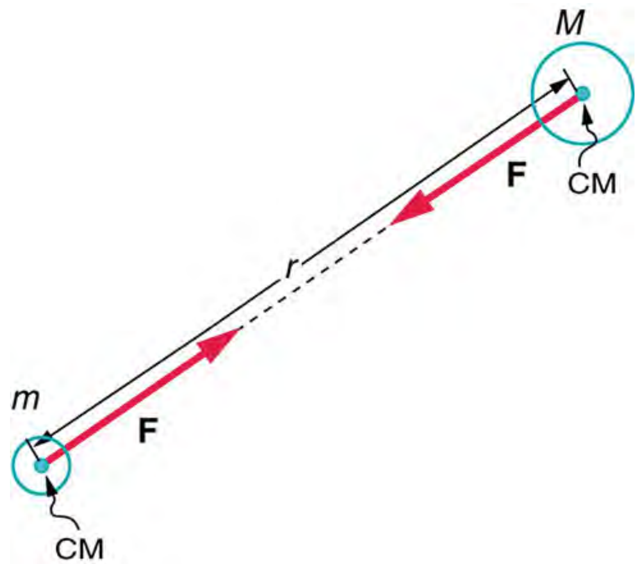


Figure 7.8 Gravitational attraction is along a line joining the centers of mass (CM) of the two bodies. The magnitude of the force on each body is the same, consistent with Newton’s third law (action-reaction).

For two bodies having masses *m* and *M* with a distance *r* between their centers of mass, the equation for Newton’s universal law of gravitation is

$$\mathbf{F} = G\frac{mM}{r^2}$$

where **F** is the magnitude of the gravitational force and *G* is a proportionality factor called the **gravitational constant**. *G* is a universal constant, meaning that it is thought to be the same everywhere in the universe. It has been measured experimentally to be $G = 6.673 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$.

If a person has a mass of 60.0 kg, what would be the force of gravitational attraction on him at Earth’s surface? *G* is given above, Earth’s mass *M* is 5.97×10^{24} kg, and the radius *r* of Earth is 6.38×10^6 m. Putting these values into Newton’s universal law of gravitation gives

$$\mathbf{F} = G\frac{mM}{r^2} = \left(6.673 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)\left(\frac{(60.0\ \mathrm{kg})\left(5.97 \times 10^{24}\ \mathrm{kg}\right)}{\left(6.38 \times 10^6\ \mathrm{m}\right)^2}\right) = 584\ \ \mathrm{N}$$

We can check this result with the relationship: $\mathbf{F} = m\mathbf{g} = (60\ \mathrm{kg})(9.8\mathrm{m/s^2}) = 588\ \mathrm{N}$

You may remember that **g**, the acceleration due to gravity, is another important constant related to gravity. By substituting **g** for **a** in the equation for Newton’s second law of motion we get $\mathbf{F} = m\mathbf{g}$. Combining this with the equation for universal gravitation gives

$$m\mathbf{g} = G\frac{mM}{r^2}$$

Cancelling the mass *m* on both sides of the equation and filling in the values for the gravitational constant and mass and radius of the Earth, gives the value of *g*, which may look familiar.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)\left(\frac{5.98 \times 10^{24}\mathrm{kg}}{\left(6.38 \times 10^6\mathrm{m}\right)^2}\right) = 9.80\mathrm{m/s^2}$$

This is a good point to recall the difference between mass and weight. Mass is the amount of matter in an object; weight is the force of attraction between the mass within two objects. Weight can change because g is different on every moon and planet. An object's mass m does not change but its weight $m\mathbf{g}$ can.

Virtual Physics

Gravity and Orbits

Move the sun, Earth, moon and space station in this simulation to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies. Turn off gravity to see what would happen without it!

[Click to view content (https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)](https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)

GRASP CHECK

Why doesn't the Moon travel in a smooth circle around the Sun?

a. The Moon is not affected by the gravitational field of the Sun.
b. The Moon is not affected by the gravitational field of the Earth.
c. The Moon is affected by the gravitational fields of both the Earth and the Sun, which are always additive.
d. The moon is affected by the gravitational fields of both the Earth and the Sun, which are sometimes additive and sometimes opposite.

Snap Lab

Take-Home Experiment: Falling Objects

In this activity you will study the effects of mass and air resistance on the acceleration of falling objects. Make predictions (hypotheses) about the outcome of this experiment. Write them down to compare later with results.

- Four sheets of $8\ \text{-}1/2 \times 11$ -inch paper

Procedure

- Take four identical pieces of paper.
  - Crumple one up into a small ball.
  - Leave one uncrumpled.
  - Take the other two and crumple them up together, so that they make a ball of exactly twice the mass of the other crumpled ball.
  - Now compare which ball of paper lands first when dropped simultaneously from the same height.
    1. Compare crumpled one-paper ball with crumpled two-paper ball.
    2. Compare crumpled one-paper ball with uncrumpled paper.

GRASP CHECK

Why do some objects fall faster than others near the surface of the earth if all mass is attracted equally by the force of gravity?

a. Some objects fall faster because of air resistance, which acts in the direction of the motion of the object and exerts more force on objects with less surface area.
b. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with less surface area.
c. Some objects fall faster because of air resistance, which acts in the direction of motion of the object and exerts more force on objects with more surface area.
d. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with more surface area.

It is possible to derive Kepler's third law from Newton's law of universal gravitation. Applying Newton's second law of motion to angular motion gives an expression for centripetal force, which can be equated to the expression for force in the universal gravitation equation. This expression can be manipulated to produce the equation for Kepler's third law. We saw earlier that the expression r^3/T^2 is a constant for satellites orbiting the same massive object. The derivation of Kepler's third law from Newton's law of universal gravitation and Newton's second law of motion yields that constant:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

where *M* is the mass of the central body about which the satellites orbit (for example, the sun in our solar system). The usefulness of this equation will be seen later.

The universal gravitational constant *G* is determined experimentally. This definition was first done accurately in 1798 by English scientist Henry Cavendish (1731–1810), more than 100 years after Newton published his universal law of gravitation. The measurement of *G* is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most) by using an apparatus like that in Figure 7.9. Remarkably, his value for *G* differs by less than 1% from the modern value.

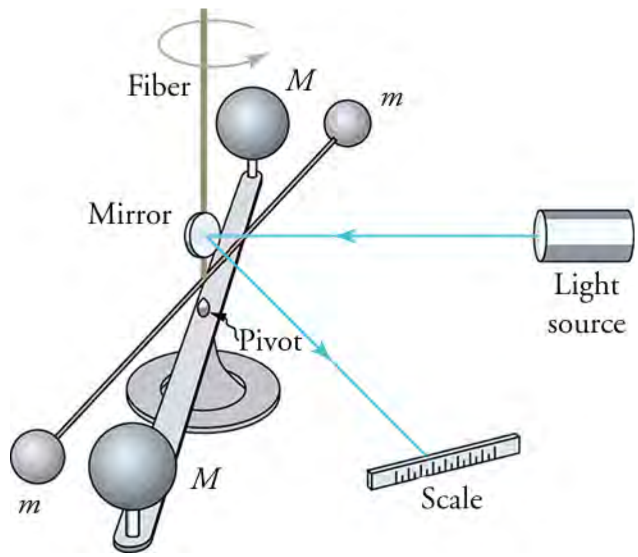


Figure 7.9 Cavendish used an apparatus like this to measure the gravitational attraction between two suspended spheres (*m*) and two spheres on a stand (*M*) by observing the amount of torsion (twisting) created in the fiber. The distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Einstein's Theory of General Relativity

Einstein's theory of general relativity explained some interesting properties of gravity not covered by Newton's theory. Einstein based his theory on the postulate that acceleration and gravity have the same effect and cannot be distinguished from each other. He concluded that light must fall in both a gravitational field and in an accelerating reference frame. Figure 7.10 shows this effect (greatly exaggerated) in an accelerating elevator. In Figure 7.10**(a)**, the elevator accelerates upward in zero gravity. In Figure 7.10**(b)**, the room is not accelerating but is subject to gravity. The effect on light is the same: it "falls" downward in both situations. The person in the elevator cannot tell whether the elevator is accelerating in zero gravity or is stationary and subject to gravity. Thus, gravity affects the path of light, even though we think of gravity as acting between masses, while photons are massless.

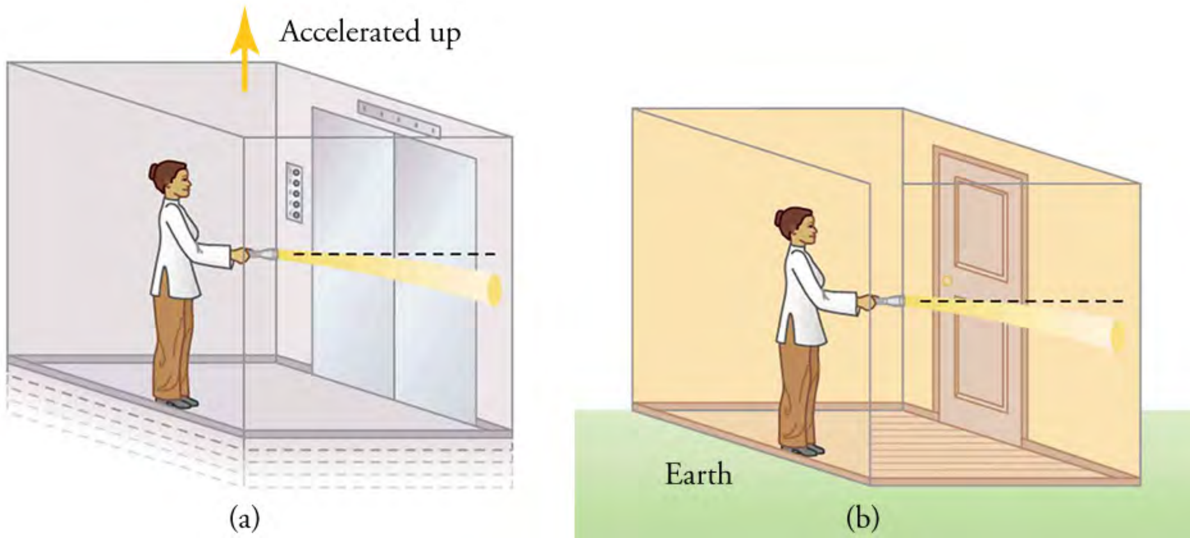


Figure 7.10 (a) A beam of light emerges from a flashlight in an upward-accelerating elevator. Since the elevator moves up during the time the light takes to reach the wall, the beam strikes lower than it would if the elevator were not accelerated. (b) Gravity must have the same effect on light, since it is not possible to tell whether the elevator is accelerating upward or is stationary and acted upon by gravity.

Einstein's theory of general relativity got its first verification in 1919 when starlight passing near the sun was observed during a solar eclipse. (See Figure 7.11.) During an eclipse, the sky is darkened and we can briefly see stars. Those on a line of sight nearest the sun should have a shift in their apparent positions. Not only was this shift observed, but it agreed with Einstein's predictions well within experimental uncertainties. This discovery created a scientific and public sensation. Einstein was now a folk hero as well as a very great scientist. The bending of light by matter is equivalent to a bending of space itself, with light following the curve. This is another radical change in our concept of space and time. It is also another connection that any particle with mass *or* energy (e.g., massless photons) is affected by gravity.

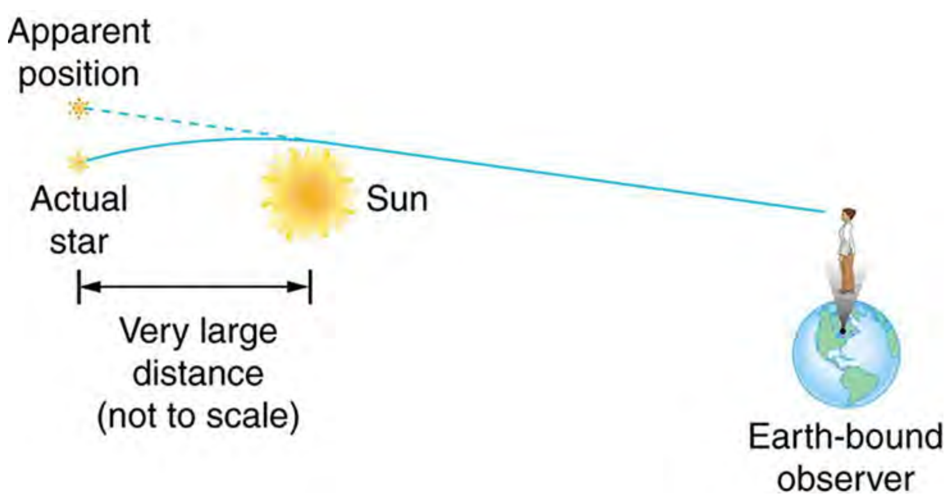


Figure 7.11 This schematic shows how light passing near a massive body like the sun is curved toward it. The light that reaches the Earth then seems to be coming from different locations than the known positions of the originating stars. Not only was this effect observed, but the amount of bending was precisely what Einstein predicted in his general theory of relativity.

To summarize the two views of gravity, Newton envisioned gravity as a tug of war along the line connecting any two objects in the universe. In contrast, Einstein envisioned gravity as a bending of space-time by mass.

BOUNDLESS PHYSICS

NASA gravity probe B

NASA's Gravity Probe B (GP-B) mission has confirmed two key predictions derived from Albert Einstein's general theory of relativity. The probe, shown in Figure 7.12 was launched in 2004. It carried four ultra-precise gyroscopes designed to measure two effects hypothesized by Einstein's theory:

- The geodetic effect, which is the warping of space and time by the gravitational field of a massive body (in this case, Earth)
- The frame-dragging effect, which is the amount by which a spinning object pulls space and time with it as it rotates

Figure 7.12 Artist concept of Gravity Probe B spacecraft in orbit around the Earth. (credit: NASA/MSFC)

Both effects were measured with unprecedented precision. This was done by pointing the gyroscopes at a single star while orbiting Earth in a polar orbit. As predicted by relativity theory, the gyroscopes experienced very small, but measureable, changes in the direction of their spin caused by the pull of Earth's gravity.

The principle investigator suggested imagining Earth spinning in honey. As Earth rotates it drags space and time with it as it would a surrounding sea of honey.

GRASP CHECK

According to the general theory of relativity, a gravitational field bends light. What does this have to do with time and space?

a. Gravity has no effect on the space-time continuum, and gravity only affects the motion of light.
b. The space-time continuum is distorted by gravity, and gravity has no effect on the motion of light.
c. Gravity has no effect on either the space-time continuum or on the motion of light.
d. The space-time continuum is distorted by gravity, and gravity affects the motion of light.

Calculations Based on Newton's Law of Universal Gravitation

TIPS FOR SUCCESS

When performing calculations using the equations in this chapter, use units of kilograms for mass, meters for distances, newtons for force, and seconds for time.

The mass of an object is constant, but its weight varies with the strength of the gravitational field. This means the value of **g** varies from place to place in the universe. The relationship between force, mass, and acceleration from the second law of motion can be written in terms of **g**.

$$\mathbf{F} = m\mathbf{a} = m\mathbf{g}$$

In this case, the force is the weight of the object, which is caused by the gravitational attraction of the planet or moon on which the object is located. We can use this expression to compare weights of an object on different moons and planets.

WATCH PHYSICS

Mass and Weight Clarification

This video shows the mathematical basis of the relationship between mass and weight. The distinction between mass and weight are clearly explained. The mathematical relationship between mass and weight are shown mathematically in terms of the equation for Newton's law of universal gravitation and in terms of his second law of motion.

Click to view content (https://www.khanacademy.org/embed_video?v=IuBoeDihLUc)

GRASP CHECK

Would you have the same mass on the moon as you do on Earth? Would you have the same weight?

a. You would weigh more on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
b. You would weigh less on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.
c. You would weigh less on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
d. You would weigh more on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.

Two equations involving the gravitational constant, G, are often useful. The first is Newton's equation, $\mathbf{F} = G\frac{mM}{r^2}$. Several of the values in this equation are either constants or easily obtainable. **F** is often the weight of an object on the surface of a large object with mass M, which is usually known. The mass of the smaller object, m, is often known, and G is a universal constant with the same value anywhere in the universe. This equation can be used to solve problems involving an object on or orbiting Earth or other massive celestial object. Sometimes it is helpful to equate the right-hand side of the equation to $m\mathbf{g}$ and cancel the m on both sides.

The equation $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ is also useful for problems involving objects in orbit. Note that there is no need to know the mass of the object. Often, we know the radius r or the period T and want to find the other. If these are both known, we can use the equation to calculate the mass of a planet or star.

WATCH PHYSICS

Mass and Weight Clarification

This video demonstrates calculations involving Newton's universal law of gravitation.

Click to view content (https://www.khanacademy.org/embed_video?v=391txUI76gM)

GRASP CHECK

Identify the constants g and G.

a. g and G are both the acceleration due to gravity
b. g is acceleration due to gravity on Earth and G is the universal gravitational constant.
c. g is the gravitational constant and G is the acceleration due to gravity on Earth.
d. g and G are both the universal gravitational constant.

WORKED EXAMPLE

Change in g

The value of **g** on the planet Mars is 3.71 m/s². If you have a mass of 60.0 kg on Earth, what would be your mass on Mars? What would be your weight on Mars?

Strategy

Weight equals acceleration due to gravity times mass: $\mathbf{W} = m\mathbf{g}$. An object's mass is constant. Call acceleration due to gravity on Mars $\mathbf{g}_M$ and weight on Mars $\mathbf{W}_M$.

Solution

Mass on Mars would be the same, 60 kg.

$$\mathbf{W}_M = m\mathbf{g}_M = (60.0\ \text{kg})\left(3.71\text{m/s}^2\right) = 223\ \text{N} \tag{7.4}$$

Discussion

The value of **g** on any planet depends on the mass of the planet and the distance from its center. If the material below the surface varies from point to point, the value of **g** will also vary slightly.

WORKED EXAMPLE

Earth's g at the Moon

Find the acceleration due to Earth's gravity at the distance of the moon.

$$\begin{aligned} G &= 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2 \\ \text{Earth-moon distance} &= 3.84 \times 10^8\ \text{m} \\ \text{Earth's mass} &= 5.98 \times 10^{24}\ \text{kg} \end{aligned} \tag{7.5}$$

Express the force of gravity in terms of g.

$$\text{F} = \text{W} = ma = mg \tag{7.6}$$

Combine with the equation for universal gravitation.

$$m\mathbf{g} = mG\frac{M}{r^2} \tag{7.7}$$

Solution

Cancel m and substitute.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\left(\frac{5.98 \times 10^{24}\ \text{kg}}{\left(3.84 \times 10^8\ \text{m}\right)^2}\right) = 2.70 \times 10^{-3}\,\text{m/s}^2 \tag{7.8}$$

Discussion

The value of **g** for the moon is 1.62 m/s^2. Comparing this value to the answer, we see that Earth's gravitational influence on an object on the moon's surface would be insignificant.

Practice Problems

6. What is the mass of a person who weighs 600 N?
   a. 6.00 kg
   b. 61.2 kg
   c. 600 kg
   d. 610 kg

7. Calculate Earth's mass given that the acceleration due to gravity at the North Pole is 9.830m/s^2 and the radius of the Earth is 6371 km from pole to center.
   a. 5.94×10^{17} kg
   b. 5.94×10^{24} kg
   c. 9.36×10^{17} kg
   d. 9.36×10^{24} kg

Check Your Understanding

8. Some of Newton's predecessors and contemporaries also studied gravity and proposed theories. What important advance did Newton make in the study of gravity that the other scientists had failed to do?
   a. He gave an exact mathematical form for the theory.

b. He added a correction term to a previously existing formula.
c. Newton found the value of the universal gravitational constant.
d. Newton showed that gravitational force is always attractive.

9. State the law of universal gravitation in words only.
a. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the square of the distance between them.
b. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
c. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the distance between them.
d. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the distance between them.

10. Newton's law of universal gravitation explains the paths of what?
a. A charged particle
b. A ball rolling on a plane surface
c. A planet moving around the sun
d. A stone tied to a string and whirled at constant speed in a horizontal circle

KEY TERMS

aphelion closest distance between a planet and the sun (called apoapsis for other celestial bodies)

Copernican model the model of the solar system where the sun is at the center of the solar system and all the planets orbit around it; this is also called the heliocentric model

eccentricity a measure of the separation of the foci of an ellipse

Einstein's theory of general relativity the theory that gravitational force results from the bending of spacetime by an object's mass

gravitational constant the proportionality constant in Newton's law of universal gravitation

Kepler's laws of planetary motion three laws derived by Johannes Kepler that describe the properties of all orbiting satellites

Newton's universal law of gravitation states that gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

perihelion farthest distance between a planet and the sun (called periapsis for other celestial bodies)

Ptolemaic model the model of the solar system where Earth is at the center of the solar system and the sun and all the planets orbit around it; this is also called the geocentric model

SECTION SUMMARY

7.1 Kepler's Laws of Planetary Motion

- All satellites follow elliptical orbits.
- The line from the satellite to the parent body sweeps out equal areas in equal time.
- The radius cubed divided by the period squared is a constant for all satellites orbiting the same parent body.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

- Newton's law of universal gravitation provides a mathematical basis for gravitational force and Kepler's laws of planetary motion.
- Einstein's theory of general relativity shows that gravitational fields change the path of light and warp space and time.
- An object's mass is constant, but its weight changes when acceleration due to gravity, **g**, changes.

KEY EQUATIONS

7.1 Kepler's Laws of Planetary Motion

Kepler's third law	$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$
eccentricity	$e = \frac{f}{a}$
area of an ellipse	$A = \pi ab$
semi-major axis of an ellipse	$a = \left(r_a + r_p\right)/2$
semi-minor axis of an ellipse	$b = \sqrt{r_a r_p}$

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

Newton's second law of motion	$\mathbf{F} = m\mathbf{a} = m\mathbf{g}$
Newton's universal law of gravitation	$\mathbf{F} = G\frac{mM}{r^2}$
acceleration due to gravity	$\mathbf{g} = G\frac{M}{r^2}$
constant for satellites orbiting the same massive object	$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

CHAPTER REVIEW

Concept Items

7.1 Kepler's Laws of Planetary Motion

1. A circle is a special case of an ellipse. Explain how a circle is different from other ellipses.
   a. The foci of a circle are at the same point and are located at the center of the circle.
   b. The foci of a circle are at the same point and are

located at the circumference of the circle.
   c. The foci of a circle are at the same point and are located outside of the circle.
   d. The foci of a circle are at the same point and are located anywhere on the diameter, except on its midpoint.

2. Comets have very elongated elliptical orbits with the sun at one focus. Using Kepler's Law, explain why a comet travels much faster near the sun than it does at the other end of the orbit.
   a. Because the satellite sweeps out equal areas in equal times
   b. Because the satellite sweeps out unequal areas in equal times
   c. Because the satellite is at the other focus of the ellipse
   d. Because the square of the period of the satellite is proportional to the cube of its average distance from the sun

3. True or False—A planet-satellite system must be isolated from other massive objects to follow Kepler’s laws of planetary motion.
   a. True
   b. False

4. Explain why the string, pins, and pencil method works for drawing an ellipse.
   a. The string, pins, and pencil method works because the length of the two sides of the triangle remains constant as you are drawing the ellipse.
   b. The string, pins, and pencil method works because the area of the triangle remains constant as you are drawing the ellipse.
   c. The string, pins, and pencil method works because the perimeter of the triangle remains constant as you are drawing the ellipse.
   d. The string, pins, and pencil method works because the volume of the triangle remains constant as you are drawing the ellipse.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

5. Describe the postulate on which Einstein based the theory of general relativity and describe an everyday experience that illustrates this postulate.
   a. Gravity and velocity have the same effect and cannot be distinguished from each other. An acceptable illustration of this is any description of the feeling of constant velocity in a situation where no outside frame of reference is considered.
   b. Gravity and velocity have different effects and can be distinguished from each other. An acceptable illustration of this is any description of the feeling of constant velocity in a situation where no outside frame of reference is considered.
   c. Gravity and acceleration have the same effect and cannot be distinguished from each other. An acceptable illustration of this is any description of the feeling of acceleration in a situation where no outside frame of reference is considered.
   d. Gravity and acceleration have different effects and can be distinguished from each other. An acceptable illustration of this is any description of the feeling of acceleration in a situation where no outside frame of reference is considered.

6. Titan, with a radius of 2.58×10^6 m, is the largest moon of the planet Saturn. If the mass of Titan is 1.35×10^{23} kg, what is the acceleration due to gravity on the surface of this moon?
   a. $1.35\ m/s^2$
   b. $3.49\ m/s^2$
   c. $3.49 \times 10^6\ m/s^2$
   d. $1.35 \times 10^6\ m/s^2$

7. Saturn’s moon Titan has an orbital period of 15.9 days. If Saturn has a mass of 5.68×10^{23} kg, what is the average distance from Titan to the center of Saturn?
   a. 1.22×10^6 m
   b. 4.26×10^7 m
   c. 5.25×10^4 km
   d. 4.26×10^{10} km

8. Explain why doubling the mass of an object doubles its weight, but doubling its distance from the center of Earth reduces its weight fourfold.
   a. The weight is two times the gravitational force between the object and Earth.
   b. The weight is half the gravitational force between the object and Earth.
   c. The weight is equal to the gravitational force between the object and Earth, and the gravitational force is inversely proportional to the distance squared between the object and Earth.
   d. The weight is directly proportional to the square of the gravitational force between the object and Earth.

9. Explain why a star on the other side of the Sun might appear to be in a location that is not its true location.
   a. It can be explained by using the concept of atmospheric refraction.
   b. It can be explained by using the concept of the special theory of relativity.
   c. It can be explained by using the concept of the general theory of relativity.
   d. It can be explained by using the concept of light

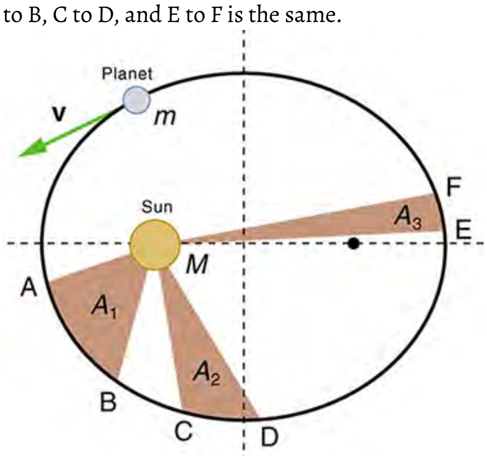
scattering in the atmosphere.

10. The Cavendish experiment marked a milestone in the study of gravity.
Part A. What important value did the experiment determine?
Part B. Why was this so difficult in terms of the masses used in the apparatus and the strength of the gravitational force?
    a. Part A. The experiment measured the acceleration due to gravity, g. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between very massive objects.
    b. Part A. The experiment measured the gravitational constant, G. Part B. Gravity is a very weak force but, despite this limitation, Cavendish was able to measure the attraction between very massive objects.
    c. Part A. The experiment measured the acceleration due to gravity, g. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between less massive objects.
    d. Part A. The experiment measured the gravitational constant, G. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between less massive objects.

Critical Thinking Items

7.1 Kepler's Laws of Planetary Motion

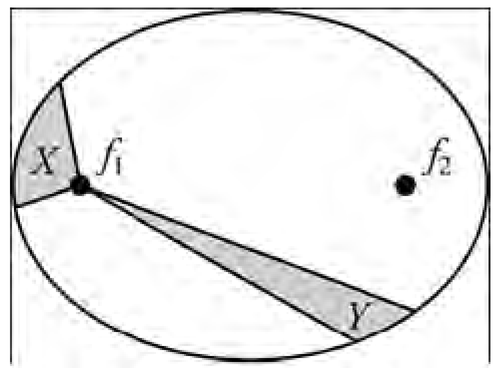
11. In the figure, the time it takes for the planet to go from A to B, C to D, and E to F is the same.



Compare the areas A_1, A_2, and A_3 in terms of size.
    a. $A_1 \neq A_2 \neq A_3$
    b. $A_1 = A_2 = A_3$
    c. $A_1 = A_2 > A_3$
    d. $A_1 > A_2 = A_3$

12. A moon orbits a planet in an elliptical orbit. The foci of the ellipse are 50, 000 km apart. The closest approach of the moon to the planet is 400, 000 km. What is the length of the major axis of the orbit?
    a. 400, 000 km
    b. 450, 000, km
    c. 800, 000 km
    d. 850, 000 km

13. In this figure, if f_1 represents the parent body, which set of statements holds true?



    a. Area X < Area Y; the speed is greater for area X.
    b. Area X > Area Y; the speed is greater for area Y.
    c. Area X = Area Y; the speed is greater for area X.
    d. Area X = Area Y; the speed is greater for area Y.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

14. Rhea, with a radius of 7.63×10^5 m, is the second-largest moon of the planet Saturn. If the mass of Rhea is 2.31×10^{21} kg, what is the acceleration due to gravity on the surface of this moon?
    a. 2.65×10^{-1} m/s
    b. 2.02×10^5 m/s
    c. 2.65×10^{-1} m/s^2
    d. 2.02×10^5 m/s^2

15. Earth has a mass of 5.971×10^{24} kg and a radius of 6.371×10^6 m. Use the data to check the value of the gravitational constant.
    a. $6.66 \times 10^{-11}\ \frac{N \cdot m}{kg^2}$, it matches the value of the gravitational constant G.
    b. $1.05 \times 10^{-17}\ \frac{N \cdot m}{kg^2}$, it matches the value of the gravitational constant G.
    c. $6.66 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$, it matches the value of the gravitational constant G.

d. $1.05 \times 10^{-17}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$, it matches the value of the gravitational constant G.

16. The orbit of the planet Mercury has a period of 88.0 days and an average radius of 5.791×10^{10} m. What is the mass of the sun?

a. 3.43×10^{19} kg
b. 1.99×10^{30} kg
c. 2.56×10^{29} kg
d. 1.48×10^{40} kg

Problems

7.1 Kepler's Laws of Planetary Motion

17. The closest Earth comes to the sun is 1.47×10^8 km, and Earth's farthest distance from the sun is 1.52×10^8 km. What is the area inside Earth's orbit?

a. 2.23×10^{16} km²
b. 6.79×10^{16} km²
c. 7.02×10^{16} km²
d. 7.26×10^{16} km²

18. Earth is 1.496×10^8 km from the sun, and Neptune is 4.490×10^9 km from the sun. What best represents the number of Earth years it takes for Neptune to complete one orbit around the sun?

a. 10 years
b. 30 years
c. 160 years
d. 900 years

Performance Task

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

19. Design an experiment to test whether magnetic force is inversely proportional to the square of distance. Gravitational, magnetic, and electrical fields all act at a distance, but do they all follow the inverse square law? One difference in the forces related to these fields is that gravity is only attractive, but the other two can repel as well. In general, the inverse square law says that force *F* equals a constant *C* divided by the distance between objects, *d*, squared: $F = C/d^2$.
Incorporate these materials into your design:

- Two strong, permanent bar magnets
- A spring scale that can measure small forces
- A short ruler calibrated in millimeters

Use the magnets to study the relationship between attractive force and distance.

a. What will be the independent variable?
b. What will be the dependent variable?
c. How will you measure each of these variables?
d. If you plot the independent variable versus the dependent variable and the inverse square law is upheld, will the plot be a straight line? Explain.
e. Which plot would be a straight line if the inverse square law were upheld?

TEST PREP

Multiple Choice

7.1 Kepler's Laws of Planetary Motion

20. A planet of mass m circles a sun of mass M. Which distance changes throughout the planet's orbit?

a. $\overline{f_1 f_2}$
b. $\overline{mM}$
c. $\overline{Mf_2}$
d. $\overline{Mf_1}$

21. The focal point of the elliptical orbit of a moon is $50,000$ km from the center of the orbit. If the eccentricity of the orbit is 0.25, what is the length of the semi-major axis?

a. $12,500$ km
b. $100,000$ km
c. $200,000$ km
d. $400,000$ km

22. An artificial satellite orbits the Earth at a distance of 1.45×10^4 km from Earth's center. The moon orbits the Earth at a distance of 3.84×10^5 km once every 27.3 days. How long does it take the satellite to orbit the Earth?

a. 0.200 days
b. 3.07 days
c. 243 days
d. 3721 days

23. Earth is 1.496×10^8 km from the sun, and Venus is 1.08×10^8 km from the sun. One day on Venus is 243 Earth days long. What best represents the number of Venusian days in a Venusian year?

a. 0.78 days
b. 0.92 days
c. 1.08 days
d. 1.21 days

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

24. What did the Cavendish experiment measure?
   a. The mass of Earth
   b. The gravitational constant
   c. Acceleration due to gravity
   d. The eccentricity of Earth's orbit

25. You have a mass of 55 kg and you have just landed on one of the moons of Jupiter where you have a weight of 67.9 N. What is the acceleration due to gravity, g, on the moon you are visiting?
   a. $.810 m/s^2$
   b. $1.23 m/s^2$
   c. $539 m/s^2$
   d. $3735 m/s^2$

26. A person is in an elevator that suddenly begins to descend. The person knows, intuitively, that the feeling of suddenly becoming lighter is because the elevator is accelerating downward. What other change would produce the same feeling? How does this demonstrate Einstein's postulate on which he based the theory of general relativity?
   a. It would feel the same if the force of gravity suddenly became weaker. This illustrates Einstein's postulates that gravity and acceleration are indistinguishable.
   b. It would feel the same if the force of gravity suddenly became stronger. This illustrates Einstein's postulates that gravity and acceleration are indistinguishable.
   c. It would feel the same if the force of gravity suddenly became weaker. This illustrates Einstein's postulates that gravity and acceleration are distinguishable.
   d. It would feel the same if the force of gravity suddenly became stronger. This illustrates Einstein's postulates that gravity and acceleration are distinguishable.

Short Answer

7.1 Kepler's Laws of Planetary Motion

27. Explain how the masses of a satellite and its parent body must compare in order to apply Kepler's laws of planetary motion.
   a. The mass of the parent body must be much less than that of the satellite.
   b. The mass of the parent body must be much greater than that of the satellite.
   c. The mass of the parent body must be equal to the mass of the satellite.
   d. There is no specific relationship between the masses for applying Kepler's laws of planetary motion.

28. Hyperion is a moon of the planet Saturn. Its orbit has an eccentricity of 0.123 and a semi-major axis of 1.48×10^6 km. How far is the center of the orbit from the center of Saturn?
   a. 1.82×10^5 km
   b. 3.64×10^5 km
   c. 1.20×10^7 km
   d. 2.41×10^7 km

29. The orbits of satellites are elliptical. Define an ellipse.
   a. An ellipse is an open curve wherein the sum of the distance from the foci to any point on the curve is constant.
   b. An ellipse is a closed curve wherein the sum of the distance from the foci to any point on the curve is constant.
   c. An ellipse is an open curve wherein the distances from the two foci to any point on the curve are equal.
   d. An ellipse is a closed curve wherein the distances from the two foci to any point on the curve are equal.

30. Mars has two moons, Deimos and Phobos. The orbit of Deimos has a period of 1.26 days and an average radius of 2.35×10^3 km. The average radius of the orbit of Phobos is 9.374×10^3 km. According to Kepler's third law of planetary motion, what is the period of Phobos?
   a. 0.16 d
   b. 0.50 d
   c. 3.17 d
   d. 10.0 d

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

31. Newton's third law of motion says that, for every action force, there is a reaction force equal in magnitude but that acts in the opposite direction. Apply this law to gravitational forces acting between the Washington Monument and Earth.
   a. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with a force equal to Earth's weight. The situation can be represented with two force vectors of unequal magnitude and pointing in the same direction.

b. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with a force equal to Earth's weight. The situation can be represented with two force vectors of unequal magnitude but pointing in opposite directions.
c. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with an equal force. The situation can be represented with two force vectors of equal magnitude and pointing in the same direction.
d. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with an equal force. The situation can be represented with two force vectors of equal magnitude but pointing in opposite directions.

32. True or false—Gravitational force is the attraction of the mass of one object to the mass of another. Light, either as a particle or a wave, has no rest mass. Despite this fact gravity bends a beam of light.
a. True
b. False

33. The average radius of Earth is 6.37×10^6 m. What is Earth's mass?
a. 9.35×10^{17} kg
b. 5.96×10^{24} kg
c. 3.79×10^{31} kg
d. 2.42×10^{38} kg

34. What is the gravitational force between two 60.0 kg people sitting 100 m apart?
a. 2.4×10^{-11} N
b. 2.4×10^{-9} N
c. 3.6×10^{-1} N
d. 3.6×10^{1} N

Extended Response

7.1 Kepler's Laws of Planetary Motion

35. The orbit of Halley"s Comet has an eccentricity of 0.967 and stretches to the edge of the solar system.
Part A. Describe the shape of the comet's orbit.
Part B. Compare the distance traveled per day when it is near the sun to the distance traveled per day when it is at the edge of the solar system.
Part C. Describe variations in the comet's speed as it completes an orbit. Explain the variations in terms of Kepler's second law of planetary motion.
a. Part A. The orbit is circular, with the sun at the center. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet decelerates as it approaches the sun and accelerates as it leaves the sun.
b. Part A. The orbit is circular, with the sun at the center. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet accelerates as it approaches the sun and decelerates as it leaves the sun.
c. Part A. The orbit is very elongated, with the sun near one end. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet decelerates as it approaches the sun and accelerates as it moves away from the sun.

36. For convenience, astronomers often use astronomical units (AU) to measure distances within the solar system. One AU equals the average distance from Earth to the sun. Halley's Comet returns once every 75.3 years. What is the average radius of the orbit of Halley's Comet in AU?
a. 0.002 AU
b. 0.056 AU
c. 17.8 AU
d. 653 AU

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

37. It took scientists a long time to arrive at the understanding of gravity as explained by Galileo and Newton. They were hindered by two ideas that seemed like common sense but were serious misconceptions. First was the fact that heavier things fall faster than light things. Second, it was believed impossible that forces could act at a distance. Explain why these ideas persisted and why they prevented advances.
a. Heavier things fall faster than light things if they have less surface area and greater mass density. In the Renaissance and before, forces that acted at a distance were considered impossible, so people were skeptical about scientific theories that invoked such forces.
b. Heavier things fall faster than light things because they have greater surface area and less mass density. In the Renaissance and before, forces that act at a distance were considered impossible, so people were skeptical about scientific theories that invoked such forces.
c. Heavier things fall faster than light things because they have less surface area and greater mass density. In the Renaissance and before, forces that

act at a distance were considered impossible, so people were quick to accept scientific theories that invoked such forces.

d. Heavier things fall faster than light things because they have larger surface area and less mass density. In the Renaissance and before, forces that act at a distance were considered impossible because of people's faith in scientific theories.

38. The masses of Earth and the moon are 5.97×10^{24} kg and 7.35×10^{22} kg, respectively. The distance from Earth to the moon is 3.80×10^{5} km. At what point between the Earth and the moon are the opposing gravitational forces equal? (Use subscripts e and m to represent Earth and moon.)

a. 3.42×10^{5} km from the center of Earth
b. 3.80×10^{5} km from the center of Earth
c. 3.42×10^{6} km from the center of Earth
d. 3.10×10^{7} km from the center of Earth

CHAPTER 8
Momentum

Figure 8.1 NFC defensive backs Ronde Barber and Roy Williams along with linebacker Jeremiah Trotter gang tackle AFC running back LaDainian Tomlinson during the 2006 Pro Bowl in Hawaii. (United States Marine Corps)

Chapter Outline

INTRODUCTION We know from everyday use of the word *momentum* that it is a tendency to continue on course in the same direction. Newscasters speak of sports teams or politicians gaining, losing, or maintaining the momentum to win. As we learned when studying about inertia, which is Newton's first law of motion, every object or system has inertia—that is, a tendency for an object in motion to remain in motion or an object at rest to remain at rest. Mass is a useful variable that lets us quantify inertia. Momentum is mass in motion.

Momentum is important because it is conserved in isolated systems; this fact is convenient for solving problems where objects collide. The magnitude of momentum grows with greater mass and/or speed. For example, look at the football players in the photograph (Figure 8.1). They collide and fall to the ground. During their collisions, momentum will play a large part. In this chapter, we will learn about momentum, the different types of collisions, and how to use momentum equations to solve collision problems.

8.1 Linear Momentum, Force, and Impulse

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe momentum, what can change momentum, impulse, and the impulse-momentum theorem
- Describe Newton's second law in terms of momentum
- Solve problems using the impulse-momentum theorem

Section Key Terms

change in momentum	impulse	impulse–momentum theorem	linear momentum

Momentum, Impulse, and the Impulse-Momentum Theorem

Linear momentum is the product of a system's mass and its velocity. In equation form, linear momentum $\mathbf{p}$ is

$$\mathbf{p} = m\mathbf{v}.$$

You can see from the equation that momentum is directly proportional to the object's mass (m) and velocity ($\mathbf{v}$). Therefore, the greater an object's mass or the greater its velocity, the greater its momentum. A large, fast-moving object has greater momentum than a smaller, slower object.

Momentum is a vector and has the same direction as velocity $\mathbf{v}$. Since mass is a scalar, when velocity is in a negative direction (i.e., opposite the direction of motion), the momentum will also be in a negative direction; and when velocity is in a positive direction, momentum will likewise be in a positive direction. The SI unit for momentum is kg m/s.

Momentum is so important for understanding motion that it was called the *quantity of motion* by physicists such as Newton. Force influences momentum, and we can rearrange Newton's second law of motion to show the relationship between force and momentum.

Recall our study of Newton's second law of motion ($\mathbf{F}_{\text{net}} = m\mathbf{a}$). Newton actually stated his second law of motion in terms of momentum: The net external force equals the **change in momentum** of a system divided by the time over which it changes. The change in momentum is the difference between the final and initial values of momentum.

In equation form, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

We can solve for $\Delta \mathbf{p}$ by rearranging the equation

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

to be

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t.$$

$\mathbf{F}_{\text{net}} \Delta t$ is known as **impulse** and this equation is known as the **impulse-momentum theorem**. From the equation, we see that the impulse equals the average net external force multiplied by the time this force acts. It is equal to the change in momentum. *The effect of a force on an object depends on how long it acts, as well as the strength of the force.* Impulse is a useful concept because it quantifies the effect of a force. A very large force acting for a short time can have a great effect on the momentum of an object, such as the force of a racket hitting a tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time.

Newton's Second Law in Terms of Momentum

When Newton's second law is expressed in terms of momentum, it can be used for solving problems where mass varies, since $\Delta \mathbf{p} = \Delta(m\mathbf{v})$. In the more traditional form of the law that you are used to working with, mass is assumed to be constant. In fact, this traditional form is a special case of the law, where mass is constant. $\mathbf{F}_{\text{net}} = m\mathbf{a}$ is actually derived from the equation:

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

For the sake of understanding the relationship between Newton's second law in its two forms, let's recreate the derivation of $\mathbf{F}_{\text{net}} = m\mathbf{a}$ from

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

by substituting the definitions of acceleration and momentum.

The change in momentum $\Delta \mathbf{p}$ is given by

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}).$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta \mathbf{v}.$$

By substituting $m\Delta \mathbf{v}$ for $\Delta \mathbf{p}$, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}$$

for a constant mass.

Because

$$\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a},$$

we can substitute to get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a}$$

when the mass of the system is constant.

TIPS FOR SUCCESS

We just showed how $\mathbf{F}_{\text{net}} = m\mathbf{a}$ applies only when the mass of the system is constant. An example of when this formula would not apply would be a moving rocket that burns enough fuel to significantly change the mass of the rocket. In this case, you would need to use Newton's second law expressed in terms of momentum to account for the changing mass.

Snap Lab

Hand Movement and Impulse

In this activity you will experiment with different types of hand motions to gain an intuitive understanding of the relationship between force, time, and impulse.

- one ball
- one tub filled with water

Procedure:

1. Try catching a ball while *giving* with the ball, pulling your hands toward your body.
2. Next, try catching a ball while keeping your hands still.
3. Hit water in a tub with your full palm. Your full palm represents a swimmer doing a belly flop.
4. After the water has settled, hit the water again by diving your hand with your fingers first into the water. Your diving hand represents a swimmer doing a dive.
5. Explain what happens in each case and why.

GRASP CHECK

What are some other examples of motions that impulse affects?

a. a football player colliding with another, or a car moving at a constant velocity
b. a car moving at a constant velocity, or an object moving in the projectile motion
c. a car moving at a constant velocity, or a racket hitting a ball
d. a football player colliding with another, or a racket hitting a ball

LINKS TO PHYSICS

Engineering: Saving Lives Using the Concept of Impulse

Cars during the past several decades have gotten much safer. Seat belts play a major role in automobile safety by preventing people from flying into the windshield in the event of a crash. Other safety features, such as airbags, are less visible or obvious, but are also effective at making auto crashes less deadly (see Figure 8.2). Many of these safety features make use of the concept of impulse from physics. Recall that impulse is the net force multiplied by the duration of time of the impact. This was expressed mathematically as $\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$.

Figure 8.2 Vehicles have safety features like airbags and seat belts installed.

Airbags allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant whether an airbag is deployed or not. But the force that brings the occupant to a stop will be much less if it acts over a larger time. By rearranging the equation for impulse to solve for force $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$, you can see how increasing Δt while $\Delta \mathbf{p}$ stays the same will decrease $\mathbf{F}_{\text{net}}$. This is another example of an inverse relationship. Similarly, a padded dashboard increases the time over which the force of impact acts, thereby reducing the force of impact.

Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the occupants of the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

GRASP CHECK

You may have heard the advice to bend your knees when jumping. In this example, a friend dares you to jump off of a park bench onto the ground without bending your knees. You, of course, refuse. Explain to your friend why this would be a foolish thing. Show it using the impulse-momentum theorem.

a. Bending your knees increases the time of the impact, thus decreasing the force.
b. Bending your knees decreases the time of the impact, thus decreasing the force.
c. Bending your knees increases the time of the impact, thus increasing the force.
d. Bending your knees decreases the time of the impact, thus increasing the force.

Solving Problems Using the Impulse-Momentum Theorem

WORKED EXAMPLE

Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110 kg football player running at 8 m/s. (b) Compare the player's momentum with the momentum of a 0.410 kg football thrown hard at a speed of 25 m/s.

Strategy

No information is given about the direction of the football player or the football, so we can calculate only the magnitude of the momentum, *p*. (A symbol in italics represents magnitude.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum:

$$\mathbf{p} = m\mathbf{v}$$

Solution for (a)

To find the player's momentum, substitute the known values for the player's mass and speed into the equation.

$$\mathbf{p}_{\text{player}} = (110\ \text{kg})(8\ \text{m/s}) = 880\ \text{kg} \cdot \text{m/s}$$

Solution for (b)

To find the ball's momentum, substitute the known values for the ball's mass and speed into the equation.

$$\mathbf{p}_{\text{ball}} = (0.410\ \text{kg})(25\ \text{m/s}) = 10.25\ \text{kg} \cdot \text{m/s}$$

The ratio of the player's momentum to the ball's momentum is

$$\frac{\mathbf{p}_{\text{player}}}{\mathbf{p}_{\text{ball}}} = \frac{880}{10.3} = 85.9\ .$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Therefore, the momentum of the player is about 86 times greater than the momentum of the football.

WORKED EXAMPLE

Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams (Figure 8.3) hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What was the average force exerted on the 0.057 kg tennis ball by Williams' racquet? Assume that the ball's speed just after impact was 58 m/s, the horizontal velocity before impact is negligible, and that the ball remained in contact with the racquet for 5 ms (milliseconds).

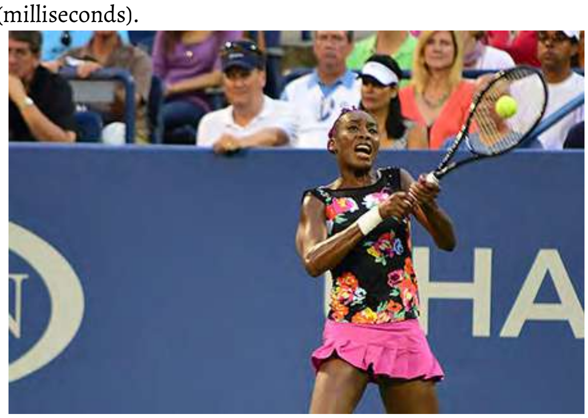


Figure 8.3 Venus Williams playing in the 2013 US Open (Edwin Martinez, Flickr)

Strategy

Recall that Newton's second law stated in terms of momentum is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta \mathbf{p} = m\Delta \mathbf{v} = m(\mathbf{v}_{\text{f}} - \mathbf{v}_{\text{i}}),$$

where $\mathbf{v}_{\text{f}}$ is the final velocity and $\mathbf{v}_{\text{i}}$ is the initial velocity. In this example, the velocity just after impact and the change in time are given, so after we solve for $\Delta \mathbf{p}$, we can use $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$ to find the force.

Solution

To determine the change in momentum, substitute the values for mass and the initial and final velocities into the equation above.

$$\begin{aligned} \Delta \mathbf{p} &= m(\mathbf{v}_f - \mathbf{v}_i) \\ &= (0.057 \text{ kg})\,(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.306 \text{ kg·m/s} \approx 3.3 \text{ kg·m/s} \end{aligned} \qquad (8.1)$$

Now we can find the magnitude of the net external force using $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$

$$\begin{aligned} \mathbf{F}_{\text{net}} &= \frac{\Delta \mathbf{p}}{\Delta t} = \frac{3.306}{5\times10^{-3}} \\ &= 661 \text{ N} \\ &\approx 660 \text{ N}. \end{aligned} \qquad (8.2)$$

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact. This problem could also be solved by first finding the acceleration and then using $\mathbf{F}_{\text{net}} = m\mathbf{a}$, but we would have had to do one more step. In this case, using momentum was a shortcut.

Practice Problems

1. What is the momentum of a bowling ball with mass 5 kg and velocity 10 m/s?
   a. 0.5 kg · m/s
   b. 2 kg · m/s
   c. 15 kg · m/s
   d. 50 kg · m/s

2. What will be the change in momentum caused by a net force of 120 N acting on an object for 2 seconds?
   a. 60 kg · m/s
   b. 118 kg · m/s
   c. 122 kg · m/s
   d. 240 kg · m/s

Check Your Understanding

3. What is linear momentum?
   a. the sum of a system's mass and its velocity
   b. the ratio of a system's mass to its velocity
   c. the product of a system's mass and its velocity
   d. the product of a system's moment of inertia and its velocity

4. If an object's mass is constant, what is its momentum proportional to?
   a. Its velocity
   b. Its weight
   c. Its displacement
   d. Its moment of inertia

5. What is the equation for Newton's second law of motion, in terms of mass, velocity, and time, when the mass of the system is

constant?

a. $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{v}}{\Delta m \Delta t}$
b. $\mathbf{F}_{\text{net}} = \frac{m \Delta t}{\Delta \mathbf{v}}$
c. $\mathbf{F}_{\text{net}} = \frac{m \Delta \mathbf{v}}{\Delta t}$
d. $\mathbf{F}_{\text{net}} = \frac{\Delta m \Delta \mathbf{v}}{\Delta t}$

6. Give an example of a system whose mass is not constant.

a. A spinning top
b. A baseball flying through the air
c. A rocket launched from Earth
d. A block sliding on a frictionless inclined plane

8.2 Conservation of Momentum

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the law of conservation of momentum verbally and mathematically

Section Key Terms

angular momentum	isolated system	law of conservation of momentum

Conservation of Momentum

It is important we realize that momentum is conserved during collisions, explosions, and other events involving objects in motion. To say that a quantity is conserved means that it is constant throughout the event. In the case of conservation of momentum, the total momentum in the system remains the same before and after the collision.

You may have noticed that momentum was *not* conserved in some of the examples previously presented in this chapter. where forces acting on the objects produced large changes in momentum. Why is this? The systems of interest considered in those problems were not inclusive enough. If the systems were expanded to include more objects, then momentum would in fact be conserved in those sample problems. It is always possible to find a larger system where momentum is conserved, even though momentum changes for individual objects within the system.

For example, if a football player runs into the goalpost in the end zone, a force will cause him to bounce backward. His momentum is obviously greatly changed, and considering only the football player, we would find that momentum is not conserved. However, the system can be expanded to contain the entire Earth. Surprisingly, Earth also recoils—conserving momentum—because of the force applied to it through the goalpost. The effect on Earth is not noticeable because it is so much more massive than the player, but the effect is real.

Next, consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—in the example shown in Figure 8.4 of one car bumping into another. Both cars are coasting in the same direction when the lead car, labeled m_2, is bumped by the trailing car, labeled m_1. The only unbalanced force on each car is the force of the collision, assuming that the effects due to friction are negligible. Car *m*1 slows down as a result of the collision, losing some momentum, while car *m*2 speeds up and gains some momentum. If we choose the system to include both cars and assume that friction is negligible, then the momentum of the two-car system should remain constant. Now we will prove that the total momentum of the two-car system does in fact remain constant, and is therefore conserved.

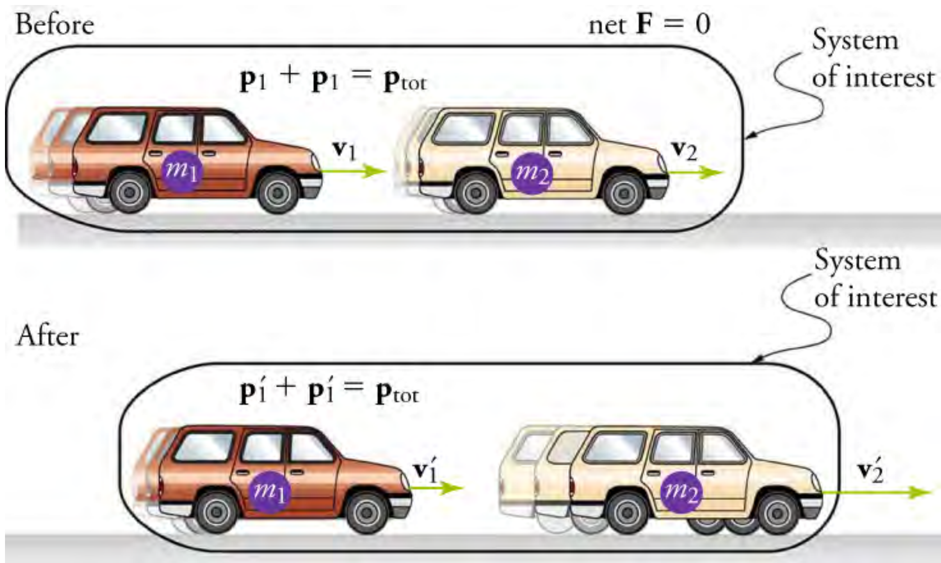


Figure 8.4 Car of mass m_1 moving with a velocity of $\mathbf{v}_1$ bumps into another car of mass m_2 and velocity $\mathbf{v}_2$. As a result, the first car slows down to a velocity of $\mathbf{v'}_1$ and the second speeds up to a velocity of $\mathbf{v'}_2$. The momentum of each car is changed, but the total momentum $\mathbf{p}_{tot}$ of the two cars is the same before and after the collision if you assume friction is negligible.

Using the impulse-momentum theorem, the change in momentum of car 1 is given by

$$\Delta\mathbf{p}_1 = \mathbf{F}_1 \Delta t,$$

where $\mathbf{F}_1$ is the force on car 1 due to car 2, and Δt is the time the force acts, or the duration of the collision.

Similarly, the change in momentum of car 2 is $\Delta\mathbf{p}_2 = \mathbf{F}_2 \Delta t$ where $\mathbf{F}_2$ is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law of motion that $\mathbf{F}_2 = -\mathbf{F}_1$, and so $\Delta\mathbf{p}_2 = -\mathbf{F}_1 \Delta t = -\Delta\mathbf{p}_1$.

Therefore, the changes in momentum are equal and opposite, and $\Delta\mathbf{p}_1 + \Delta\mathbf{p}_2 = 0$.

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$$

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2,$$

where $\mathbf{p}'_1$ and $\mathbf{p}'_2$ are the momenta of cars 1 and 2 after the collision.

This result that momentum is conserved is true not only for this example involving the two cars, but for any system where the net external force is zero, which is known as an **isolated system**. The **law of conservation of momentum** states that for an isolated system with any number of objects in it, the total momentum is conserved. In equation form, the law of conservation of momentum for an isolated system is written as

$$\mathbf{p}_{tot} = \text{constant}$$

or

$$\mathbf{p}_{tot} = \mathbf{p}'_{tot},$$

where $\mathbf{p}_{tot}$ is the total momentum, or the sum of the momenta of the individual objects in the system at a given time, and $\mathbf{p}'_{tot}$ is the total momentum some time later.

The conservation of momentum principle can be applied to systems as diverse as a comet striking the Earth or a gas containing huge numbers of atoms and molecules. Conservation of momentum appears to be violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

TIPS FOR SUCCESS

Momenta is the plural form of the word momentum. One object is said to have momentum, but two or more objects are said to have *momenta*.

FUN IN PHYSICS

Angular Momentum in Figure Skating

So far we have covered linear momentum, which describes the inertia of objects traveling in a straight line. But we know that many objects in nature have a curved or circular path. Just as linear motion has linear momentum to describe its tendency to move forward, circular motion has the equivalent **angular momentum** to describe how rotational motion is carried forward.

This is similar to how torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass or inertia. You may recall learning that the quantity mr^2 is called the rotational inertia or moment of inertia of a point mass m at a distance r from the center of rotation.

We already know the equation for linear momentum, $\mathbf{p} = m\mathbf{v}$. Since angular momentum is analogous to linear momentum, the moment of inertia (I) is analogous to mass, and angular velocity ($\boldsymbol{\omega}$) is analogous to linear velocity, it makes sense that angular momentum ($\mathbf{L}$) is defined as

$$\mathbf{L} = I\boldsymbol{\omega}$$

Angular momentum is conserved when the net external torque ($\boldsymbol{\tau}$) is zero, just as linear momentum is conserved when the net external force is zero.

Figure skaters take advantage of the conservation of angular momentum, likely without even realizing it. In Figure 8.5, a figure skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice, and because the friction is exerted very close to the pivot point. Both $\mathbf{F}$ and r are small, and so $\boldsymbol{\tau}$ is negligibly small.

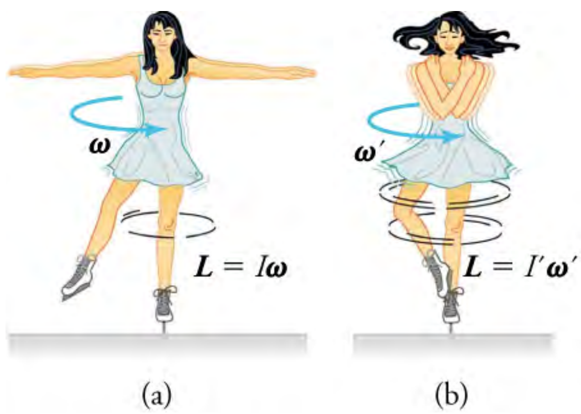


Figure 8.5 (a) An ice skater is spinning on the tip of her skate with her arms extended. In the next image, (b), her rate of spin increases greatly when she pulls in her arms.

Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that $\mathbf{L} = \mathbf{L}'$.

Expressing this equation in terms of the moment of inertia,

$$I\boldsymbol{\omega} = I'\boldsymbol{\omega}',$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity $\boldsymbol{\omega}'$ must increase to keep the angular momentum constant. This allows her to spin much faster without exerting any extra torque.

A video (http://openstax.org/l/28figureskater) is also available that shows a real figure skater executing a spin. It discusses the physics of spins in figure skating.

GRASP CHECK

Based on the equation $\mathbf{L} = I\omega$, how would you expect the moment of inertia of an object to affect angular momentum? How would angular velocity affect angular momentum?

a. Large moment of inertia implies large angular momentum, and large angular velocity implies large angular momentum.
b. Large moment of inertia implies small angular momentum, and large angular velocity implies small angular momentum.
c. Large moment of inertia implies large angular momentum, and large angular velocity implies small angular momentum.
d. Large moment of inertia implies small angular momentum, and large angular velocity implies large angular momentum.

Check Your Understanding

7. When is momentum said to be conserved?

a. When momentum is changing during an event
b. When momentum is increasing during an event
c. When momentum is decreasing during an event
d. When momentum is constant throughout an event

8. A ball is hit by a racket and its momentum changes. How is momentum conserved in this case?

a. Momentum of the system can never be conserved in this case.
b. Momentum of the system is conserved if the momentum of the racket is not considered.
c. Momentum of the system is conserved if the momentum of the racket is also considered.
d. Momentum of the system is conserved if the momenta of the racket and the player are also considered.

9. State the law of conservation of momentum.

a. Momentum is conserved for an isolated system with any number of objects in it.
b. Momentum is conserved for an isolated system with an even number of objects in it.
c. Momentum is conserved for an interacting system with any number of objects in it.
d. Momentum is conserved for an interacting system with an even number of objects in it.

8.3 Elastic and Inelastic Collisions

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Distinguish between elastic and inelastic collisions
- Solve collision problems by applying the law of conservation of momentum

Section Key Terms

elastic collision	inelastic collision	point masses	recoil

Elastic and Inelastic Collisions

When objects collide, they can either stick together or bounce off one another, remaining separate. In this section, we'll cover these two different types of collisions, first in one dimension and then in two dimensions.

In an **elastic collision**, the objects separate after impact and don't lose any of their kinetic energy. Kinetic energy is the energy of motion and is covered in detail elsewhere. The law of conservation of momentum is very useful here, and it can be used whenever the net external force on a system is zero. Figure 8.6 shows an elastic collision where momentum is conserved.

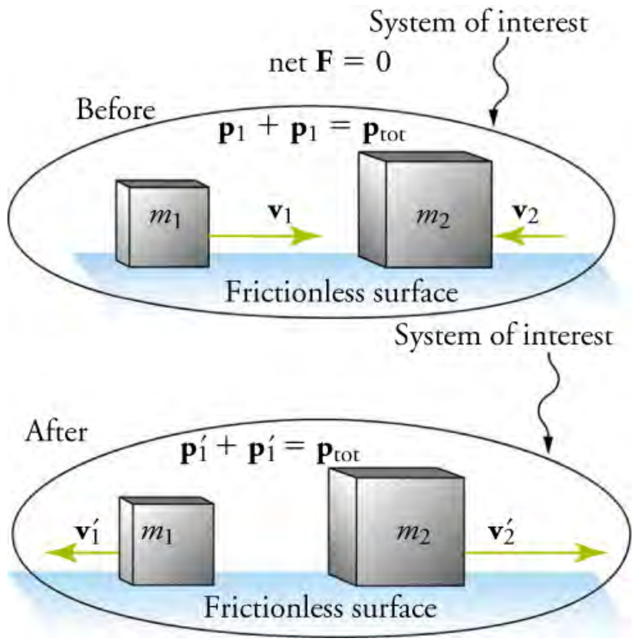


Figure 8.6 The diagram shows a one-dimensional elastic collision between two objects.

An animation of an elastic collision between balls can be seen by watching this [video (http://openstax.org/l/28elasticball)](http://openstax.org/l/28elasticball) . It replicates the elastic collisions between balls of varying masses.

Perfectly elastic collisions can happen only with subatomic particles. Everyday observable examples of perfectly elastic collisions don't exist—some kinetic energy is always lost, as it is converted into heat transfer due to friction. However, collisions between everyday objects are almost perfectly elastic when they occur with objects and surfaces that are nearly frictionless, such as with two steel blocks on ice.

Now, to solve problems involving one-dimensional elastic collisions between two objects, we can use the equation for conservation of momentum. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p'}_1 + \mathbf{p'}_2 (\mathbf{F}_{\text{net}} = 0).$$

Substituting the definition of momentum $\mathbf{p} = m\mathbf{v}$ for each initial and final momentum, we get

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v'}_1 + m_2\mathbf{v'}_2,$$

where the primes (') indicate values after the collision; In some texts, you may see *i* for initial (before collision) and *f* for final (after collision). The equation assumes that the mass of each object does not change during the collision.

WATCH PHYSICS

Momentum: Ice Skater Throws a Ball

This video covers an elastic collision problem in which we find the ***recoil*** *velocity* of an ice skater who throws a ball straight forward. To clarify, Sal is using the equation

$$m_{\text{ball}}\mathbf{V}_{\text{ball}} + m_{\text{skater}}\mathbf{V}_{\text{skater}} = m_{\text{ball}}\mathbf{v'}_{\text{ball}} + m_{\text{skater}}\mathbf{v'}_{\text{skater}}.$$

[Click to view content (https://www.khanacademy.org/embed_video?v=vPkkCOlGND4)](https://www.khanacademy.org/embed_video?v=vPkkCOlGND4)

GRASP CHECK

The resultant vector of the addition of vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}$. The magnitudes of $\vec{a}$, $\vec{b}$, and $\vec{r}$ are A, B, and R, respectively. Which of the following is true?

a. $R_x + R_y = 0$

b. $A_x + A_y = \vec{A}$

c. $A_x + B_y = B_x + A_y$
d. $A_x + B_x = R_x$

Now, let us turn to the second type of collision. An **inelastic collision** is one in which objects stick together after impact, and kinetic energy is *not* conserved. This lack of conservation means that the forces between colliding objects may convert kinetic energy to other forms of energy, such as potential energy or thermal energy. The concepts of energy are discussed more thoroughly elsewhere. For inelastic collisions, kinetic energy may be lost in the form of heat. Figure 8.7 shows an example of an inelastic collision. Two objects that have equal masses head toward each other at equal speeds and then stick together. The two objects come to rest after sticking together, conserving momentum but not kinetic energy after they collide. Some of the energy of motion gets converted to thermal energy, or heat.

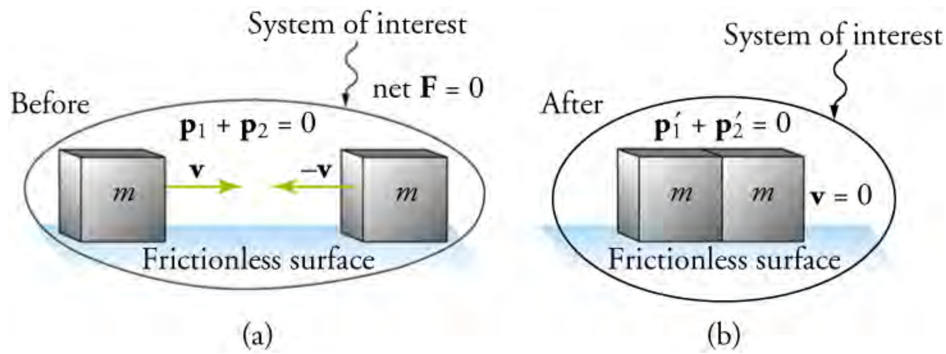


Figure 8.7 A one-dimensional inelastic collision between two objects. Momentum is conserved, but kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward each other at the same speed. (b) The objects stick together, creating a perfectly inelastic collision. In the case shown in this figure, the combined objects stop; This is not true for all inelastic collisions.

Since the two objects stick together after colliding, they move together at the same speed. This lets us simplify the conservation of momentum equation from

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v'}_1 + m_2 \mathbf{v'}_2$$

to

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2) \mathbf{v}'$$

for inelastic collisions, where **v**' is the final velocity for both objects as they are stuck together, either in motion or at rest.

WATCH PHYSICS

Introduction to Momentum

This video reviews the definitions of momentum and impulse. It also covers an example of using conservation of momentum to solve a problem involving an inelastic collision between a car with constant velocity and a stationary truck. Note that Sal accidentally gives the unit for impulse as Joules; it is actually N · s or k · gm/s.

Click to view content (https://www.khanacademy.org/embed_video?v=XFhntPxow0U)

GRASP CHECK

How would the final velocity of the car-plus-truck system change if the truck had some initial velocity moving in the same direction as the car? What if the truck were moving in the opposite direction of the car initially? Why?

a. If the truck was initially moving in the same direction as the car, the final velocity would be greater. If the truck was initially moving in the opposite direction of the car, the final velocity would be smaller.
b. If the truck was initially moving in the same direction as the car, the final velocity would be smaller. If the truck was initially moving in the opposite direction of the car, the final velocity would be greater.
c. The direction in which the truck was initially moving would not matter. If the truck was initially moving in either

direction, the final velocity would be smaller.

d. The direction in which the truck was initially moving would not matter. If the truck was initially moving in either direction, the final velocity would be greater.

Snap Lab

Ice Cubes and Elastic Collisions

In this activity, you will observe an elastic collision by sliding an ice cube into another ice cube on a smooth surface, so that a negligible amount of energy is converted to heat.

- Several ice cubes (The ice must be in the form of cubes.)
- A smooth surface

Procedure

1. Find a few ice cubes that are about the same size and a smooth kitchen tabletop or a table with a glass top.
2. Place the ice cubes on the surface several centimeters away from each other.
3. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes.
4. Explain the speeds and directions of the ice cubes using momentum.

GRASP CHECK

Was the collision elastic or inelastic?

a. perfectly elastic
b. perfectly inelastic
c. Nearly perfect elastic
d. Nearly perfect inelastic

TIPS FOR SUCCESS

Here's a trick for remembering which collisions are elastic and which are inelastic: Elastic is a bouncy material, so when objects *bounce* off one another in the collision and separate, it is an elastic collision. When they don't, the collision is inelastic.

Solving Collision Problems

The Khan Academy videos referenced in this section show examples of elastic and inelastic collisions in one dimension. In one-dimensional collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and just as we did with two-dimensional forces, we will solve these problems by first choosing a coordinate system and separating the motion into its x and y components.

One complication with two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass each other, they will spin in circles. We will not consider such rotation until later, and so for now, we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{\text{net}} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in Figure 8.8. Because momentum is conserved, the components of momentum along the x- and y-axes, displayed as $\mathbf{p}_x$ and $\mathbf{p}_y$, will also be conserved. With the chosen coordinate system, $\mathbf{p}_y$ is initially zero and $\mathbf{p}_x$ is the momentum of the incoming particle.

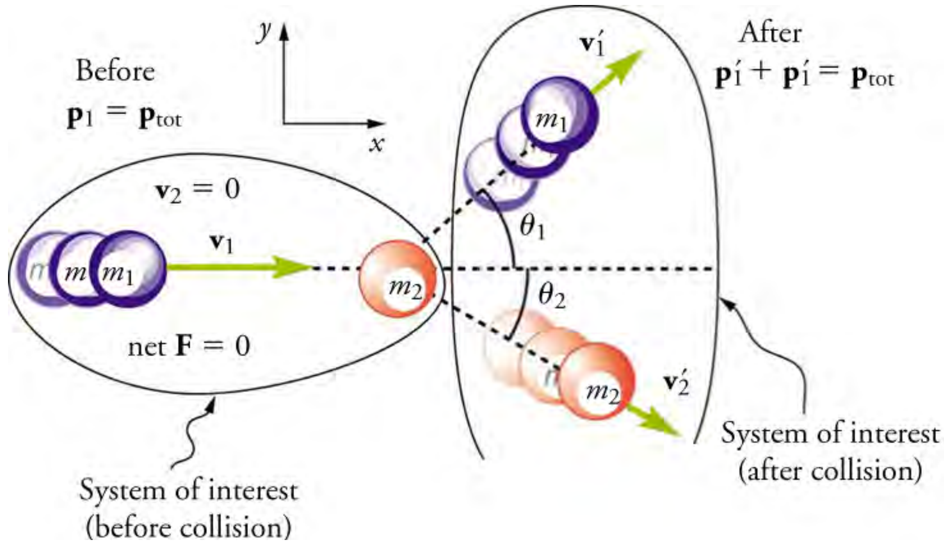


Figure 8.8 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and $\mathbf{v}_1$ is parallel to the x-axis.

Now, we will take the conservation of momentum equation, $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$ and break it into its x and y components.

Along the x-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1x} + \mathbf{p}_{2x} = \mathbf{p}'_{1x} + \mathbf{p}'_{2x}.$$

In terms of masses and velocities, this equation is

$$m_1 \mathbf{v}_{1\,x} + m_2 \mathbf{v}_{2\,x} = m_1 \mathbf{v}'_{1\,x} + m_2 \mathbf{v}'_{2\,x}. \tag{8.3}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 \mathbf{v}_{1\,x} = m_1 \mathbf{v}'_{1\,x} + m_2 \mathbf{v}'_{2\,x}. \tag{8.4}$$

The components of the velocities along the x-axis have the form $\mathbf{v}\cos\theta$. Because particle 1 initially moves along the x-axis, we find $\mathbf{v}_{1x} = \mathbf{v}_1$. Conservation of momentum along the x-axis gives the equation

$$m_1 \mathbf{v}_1 = m_1 \mathbf{v}'_1 \cos\theta_1 + m_2 \mathbf{v}'_2 \cos\theta_2,$$

where θ_1 and θ_2 are as shown in Figure 8.8.

Along the y-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1\,y} + \mathbf{p}_{2\,y} = \mathbf{p}'_{1\,y} + \mathbf{p}'_{2\,y}, \tag{8.5}$$

or

$$m_1 \mathbf{v}_{1\,y} + m_2 \mathbf{v}_{2\,y} = m_1 \mathbf{v}'_{1\,y} + m_2 \mathbf{v}'_{2\,y}. \tag{8.6}$$

But $\mathbf{v}_1 y$ is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, $\mathbf{v}_2 y$ is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1 \mathbf{v}'_1 y + m_2 \mathbf{v}'_2 y. \tag{8.7}$$

The components of the velocities along the y-axis have the form $\mathbf{v}\sin\theta$. Therefore, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1 \mathbf{v}'_1 \sin\theta_1 + m_2 \mathbf{v}'_2 \sin\theta_2$$

Virtual Physics

Collision Lab

In this simulation, you will investigate collisions on an air hockey table. Place checkmarks next to the momentum vectors and momenta diagram options. Experiment with changing the masses of the balls and the initial speed of ball 1. How does this affect the momentum of each ball? What about the total momentum? Next, experiment with changing the elasticity of the collision. You will notice that collisions have varying degrees of elasticity, ranging from perfectly elastic to perfectly inelastic.

Click to view content (https://archive.cnx.org/specials/2c7acb3c-2fbd-11e5-b2d9-e7f92291703c/collision-lab/)

GRASP CHECK

If you wanted to maximize the velocity of ball 2 after impact, how would you change the settings for the masses of the balls, the initial speed of ball 1, and the elasticity setting? Why? Hint—Placing a checkmark next to the velocity vectors and removing the momentum vectors will help you visualize the velocity of ball 2, and pressing the More Data button will let you take readings.

a. Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 50 percent.
b. Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 100 percent.
c. Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 100 percent.
d. Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 50 percent.

WORKED EXAMPLE

Calculating Velocity: Inelastic Collision of a Puck and a Goalie

Find the recoil velocity of a 70 kg ice hockey goalie who catches a 0.150-kg hockey puck slapped at him at a velocity of 35 m/s. Assume that the goalie is at rest before catching the puck, and friction between the ice and the puck-goalie system is negligible (see Figure 8.9).

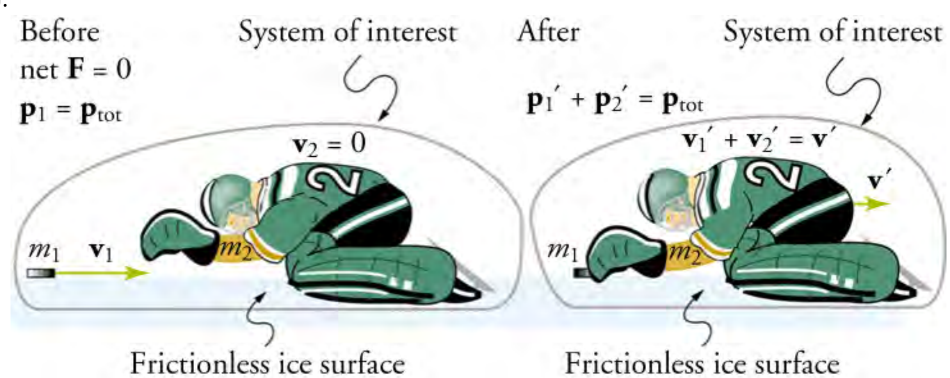


Figure 8.9 An ice hockey goalie catches a hockey puck and recoils backward in an inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. Therefore, we can use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same.

Solution

For an inelastic collision, conservation of momentum is

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2)\mathbf{v}', \quad (8.8)$$

where $\mathbf{v}'$ is the velocity of both the goalie and the puck after impact. Because the goalie is initially at rest, we know $\mathbf{v}_2 = 0$. This simplifies the equation to

$$m_1 \mathbf{v}_1 = (m_1 + m_2)\mathbf{v}'. \quad (8.9)$$

Solving for $\mathbf{v}'$ yields

$$\mathbf{v}' = \left(\frac{m_1}{m_1 + m_2}\right)\mathbf{v}_1. \quad (8.10)$$

Entering known values in this equation, we get

$$\begin{aligned}\mathbf{v}' &= \left(\frac{0.150\text{ kg}}{70.0\text{ kg}+0.150\text{ kg}}\right)(35\text{ m/s}) \\ &= 7.48 \times 10^{-2}\text{m/s.}\end{aligned} \quad (8.11)$$

Discussion

This recoil velocity is small and in the same direction as the puck's original velocity.

WORKED EXAMPLE

Calculating Final Velocity: Elastic Collision of Two Carts

Two hard, steel carts collide head-on and then ricochet off each other in opposite directions on a frictionless surface (see Figure 8.10). Cart 1 has a mass of 0.350 kg and an initial velocity of 2 m/s. Cart 2 has a mass of 0.500 kg and an initial velocity of −0.500 m/s. After the collision, cart 1 recoils with a velocity of −4 m/s. What is the final velocity of cart 2?

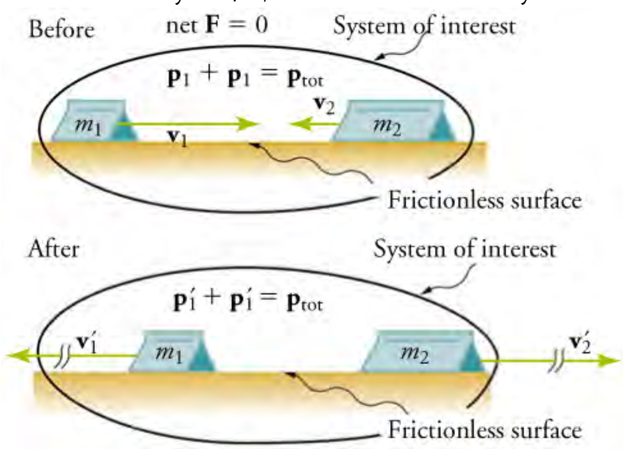


Figure 8.10 Two carts collide with each other in an elastic collision.

Strategy

Since the track is frictionless, $\mathbf{F}_{\text{net}} = 0$ and we can use conservation of momentum to find the final velocity of cart 2.

Solution

As before, the equation for conservation of momentum for a one-dimensional elastic collision in a two-object system is

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2. \quad (8.12)$$

The only unknown in this equation is $\mathbf{v}'_2$. Solving for $\mathbf{v}'_2$ and substituting known values into the previous equation yields

$$\begin{aligned}\mathbf{v}'_2 &= \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 - m_1\mathbf{v}'_1}{m_2} \\ &= \frac{(0.350\text{kg})(2.00\text{m/s})+(0.500\text{ kg})(-0.500\text{ m/s})-(0.350\text{kg})(-4.00\text{m/s})}{0.500\text{kg}} \\ &= 3.70\text{m/s.}\end{aligned} \quad (8.13)$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision.

WORKED EXAMPLE

Calculating Final Velocity in a Two-Dimensional Collision

Suppose the following experiment is performed (Figure 8.11). An object of mass 0.250 kg (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object of mass 0.400 kg (m_2). The 0.250 kg object emerges from the room at an angle of 45° with its incoming direction. The speed of the 0.250 kg object is originally 2 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (v'_2 and θ_2) of the 0.400 kg object after the collision.

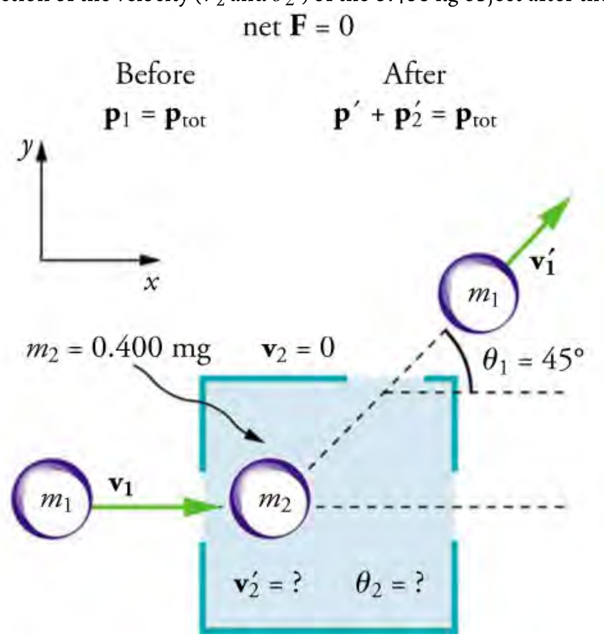


Figure 8.11 The incoming object of mass m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which the object of mass m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Strategy

Momentum is conserved because the surface is frictionless. We chose the coordinate system so that the initial velocity is parallel to the *x*-axis, and conservation of momentum along the *x*- and *y*-axes applies.

Everything is known in these equations except $\mathbf{v}'_2$ and θ_2, which we need to find. We can find two unknowns because we have two independent equations—the equations describing the conservation of momentum in the *x* and *y* directions.

Solution

First, we'll solve both conservation of momentum equations ($m_1 \mathbf{v}_1 = m_1 \mathbf{v'}_1 \cos\theta_1 + m_2 \mathbf{v'}_2 \cos\theta_2$ and $0 = m_1 \mathbf{v'}_1 \sin\theta_1 + m_2 \mathbf{v'}_2 \sin\theta_2$) for $\mathbf{v}'_2 \sin\theta_2$.

For conservation of momentum along x-axis, let's substitute $\sin\theta_2$ /$\tan\theta_2$ for $\cos\theta_2$ so that terms may cancel out later on. This comes from rearranging the definition of the trigonometric identity $\tan\theta = \sin\theta / \cos\theta$. This gives us

$$m_1 \mathbf{v}_1 = m_1 \mathbf{v'}_1 \cos\theta_1 + m_2 \mathbf{v'}_2 \frac{\sin\theta_2}{\tan\theta_2}. \quad (8.14)$$

Solving for $\mathbf{v}'_2 \sin\theta_2$ yields

$$\mathbf{v'}_2 \sin\theta_2 = \frac{(m_1 \mathbf{v}_1 - m_1 \mathbf{v'}_1 \cos\theta_1)(\tan\theta_2)}{m_2}. \quad (8.15)$$

For conservation of momentum along *y*-axis, solving for $\mathbf{v}'_2 \sin\theta_2$ yields

$$\mathbf{v'}_2 \sin\theta_2 = \frac{-(m_1 \mathbf{v'}_1 \sin\theta_1)}{m_2}. \quad (8.16)$$

Since both equations equal $\mathbf{v}'_2 \sin\theta_2$, we can set them equal to one another, yielding

$$\frac{(m_1 \mathbf{v}_1 - m_1 \mathbf{v'}_1 \cos\theta_1)(\tan\theta_2)}{m_2} = \frac{-(m_1 \mathbf{v'}_1 \sin\theta_1)}{m_2}. \tag{8.17}$$

Solving this equation for $\tan\theta_2$, we get

$$\tan\theta_2 = \frac{\mathbf{v'}_1 \sin\theta_1}{\mathbf{v'}_1 \cos\theta_1 - \mathbf{v}_1}. \tag{8.18}$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50)(0.707)}{(1.50)(0.707) - 2.00} = -1.129 . \tag{8.19}$$

Therefore,

$$\theta_2 = \tan^{-1}(-1.129) = 312^0. \tag{8.20}$$

Since angles are defined as positive in the counterclockwise direction, m_2 is scattered to the right.

We'll use the conservation of momentum along the y-axis equation to solve for $\mathbf{v}'_2$.

$$\mathbf{v'}_2 = -\frac{m_1}{m_2}\mathbf{v'}_1 \frac{\sin\theta_1}{\sin\theta_2} \tag{8.21}$$

Entering known values into this equation gives

$$\mathbf{v'}_2 = -\frac{(0.250)}{(0.400)}(1.50)\left(\frac{0.7071}{-0.7485}\right). \tag{8.22}$$

Therefore,

$$\mathbf{v'}_2 = 0.886 \text{ m/s}. \tag{8.23}$$

Discussion

Either equation for the *x*- or *y*-axis could have been used to solve for $\mathbf{v}'_2$, but the equation for the *y*-axis is easier because it has fewer terms.

Practice Problems

10. In an elastic collision, an object with momentum $25 \text{ kg} \cdot \text{m/s}$ collides with another object moving to the right that has a momentum $35 \text{ kg} \cdot \text{m/s}$. After the collision, both objects are still moving to the right, but the first object's momentum changes to $10 \text{ kg} \cdot \text{m/s}$. What is the final momentum of the second object?

a. $10 \text{ kg} \cdot \text{m/s}$
b. $20 \text{ kg} \cdot \text{m/s}$
c. $35 \text{ kg} \cdot \text{m/s}$
d. $50 \text{ kg} \cdot \text{m/s}$

11. In an elastic collision, an object with momentum 25 kg · m/s collides with another that has a momentum 35 kg · m/s. The first object's momentum changes to 10 kg · m/s. What is the final momentum of the second object?

a. 10 kg · m/s
b. 20 kg · m/s
c. 35 kg · m/s
d. 50 kg · m/s

Check Your Understanding

12. What is an elastic collision?

a. An elastic collision is one in which the objects after impact are deformed permanently.
b. An elastic collision is one in which the objects after impact lose some of their internal kinetic energy.

c. An elastic collision is one in which the objects after impact do not lose any of their internal kinetic energy.
d. An elastic collision is one in which the objects after impact become stuck together and move with a common velocity.

13. Are perfectly elastic collisions possible?
a. Perfectly elastic collisions are not possible.
b. Perfectly elastic collisions are possible only with subatomic particles.
c. Perfectly elastic collisions are possible only when the objects stick together after impact.
d. Perfectly elastic collisions are possible if the objects and surfaces are nearly frictionless.

14. What is the equation for conservation of momentum for two objects in a one-dimensional collision?
a. $\mathbf{p}_1 + \mathbf{p}_1' = \mathbf{p}_2 + \mathbf{p}_2'$
b. $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_1' + \mathbf{p}_2'$
c. $\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{p}_1' - \mathbf{p}_2'$
d. $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_1' + \mathbf{p}_2' = 0$

KEY TERMS

angular momentum the product of the moment of inertia and angular velocity

change in momentum the difference between the final and initial values of momentum; the mass times the change in velocity

elastic collision collision in which objects separate after impact and kinetic energy is conserved

impulse average net external force multiplied by the time the force acts; equal to the change in momentum

impulse–momentum theorem the impulse, or change in momentum, is the product of the net external force and the time over which the force acts

inelastic collision collision in which objects stick together after impact and kinetic energy is not conserved

isolated system system in which the net external force is zero

law of conservation of momentum when the net external force is zero, the total momentum of the system is conserved or constant

linear momentum the product of a system's mass and velocity

point masses structureless particles that cannot rotate or spin

recoil backward movement of an object caused by the transfer of momentum from another object in a collision

SECTION SUMMARY

8.1 Linear Momentum, Force, and Impulse

- Linear momentum, often referenced as *momentum* for short, is defined as the product of a system's mass multiplied by its velocity, $\mathbf{p} = m\mathbf{v}$.
- The SI unit for momentum is kg m/s.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes, $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$.
- Impulse is the average net external force multiplied by the time this force acts, and impulse equals the change in momentum, $\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$.
- Forces are usually not constant over a period of time, so we use the average of the force over the time it acts.

8.2 Conservation of Momentum

- The law of conservation of momentum is written $\mathbf{p}_{\text{tot}}$ = constant or $\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}$ (isolated system), where $\mathbf{p}_{\text{tot}}$ is the initial total momentum and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later.
- In an isolated system, the net external force is zero.
- Conservation of momentum applies only when the net external force is zero, within the defined system.

8.3 Elastic and Inelastic Collisions

- If objects separate after impact, the collision is elastic; If they stick together, the collision is inelastic.
- Kinetic energy is conserved in an elastic collision, but not in an inelastic collision.
- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses, where mass 2 is initially at rest, conserve momentum along the initial direction of mass 1, or the x-axis, and along the direction perpendicular to the initial direction, or the y-axis.
- Point masses are structureless particles that cannot spin.

KEY EQUATIONS

8.1 Linear Momentum, Force, and Impulse

impulse	$\mathbf{F}_{\text{net}} \Delta t$
impulse–momentum theorem	$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$
linear momentum	$\mathbf{p} = m\mathbf{v}$
Newton's second law in terms of momentum	$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$

8.2 Conservation of Momentum

law of conservation of momentum	$\mathbf{p}_{\text{tot}}$ = constant, or $\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}$

conservation of momentum for two objects	$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$, or $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$
angular momentum	$\mathbf{L} = I\boldsymbol{\omega}$

8.3 Elastic and Inelastic Collisions

conservation of momentum in an elastic collision	$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2,$
conservation of momentum in an inelastic collision	$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}'$
conservation of momentum along *x*-axis for 2D collisions	$m_1\mathbf{v}_1 = m_1\mathbf{v}'_1\cos\theta_1 + m_2\mathbf{v}'_2\cos\theta_2$
conservation of momentum along *y*-axis for 2D collisions	$0 = m_1\mathbf{v}'_1\sin\theta_1 + m_2\mathbf{v}'_2\sin\theta_2$

CHAPTER REVIEW

Concept Items

8.1 Linear Momentum, Force, and Impulse

1. What is impulse?
  a. Change in velocity
  b. Change in momentum
  c. Rate of change of velocity
  d. Rate of change of momentum

2. In which equation of Newton's second law is mass assumed to be constant?
  a. $\mathbf{F} = ma$
  b. $\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t}$
  c. $\mathbf{F} = \Delta\mathbf{p}\Delta t$
  d. $\mathbf{F} = \frac{\Delta m}{\Delta a}$

3. What is the SI unit of momentum?
  a. N
  b. $\text{kg} \cdot \text{m}$
  c. $\text{kg} \cdot \text{m/s}$
  d. $\text{kg} \cdot \text{m/s}^2$

4. What is the equation for linear momentum?
  a. $\mathbf{p} = m\mathbf{v}$
  b. $\mathbf{p} = m/\mathbf{v}$
  c. $\mathbf{p} = \text{m}\mathbf{v}^2$
  d. $\mathbf{p} = \frac{1}{2}\text{m}\mathbf{v}^2$

8.2 Conservation of Momentum

5. What is angular momentum?
  a. The sum of moment of inertia and angular velocity
  b. The ratio of moment of inertia to angular velocity
  c. The product of moment of inertia and angular velocity
  d. Half the product of moment of inertia and square of angular velocity

6. What is an isolated system?
  a. A system in which the net internal force is zero
  b. A system in which the net external force is zero
  c. A system in which the net internal force is a nonzero constant
  d. A system in which the net external force is a nonzero constant

8.3 Elastic and Inelastic Collisions

7. In the equation $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$ for the collision of two objects, what is the assumption made regarding the friction acting on the objects?
  a. Friction is zero.
  b. Friction is nearly zero.
  c. Friction acts constantly.
  d. Friction before and after the impact remains the same.

8. What is an inelastic collision?

a. when objects stick together after impact, and their internal energy is not conserved
b. when objects stick together after impact, and their internal energy is conserved
c. when objects stick together after impact, and always come to rest instantaneously after collision
d. when objects stick together after impact, and their internal energy increases

Critical Thinking Items

8.1 Linear Momentum, Force, and Impulse

9. Consider two objects of the same mass. If a force of 100 N acts on the first for a duration of 1 s and on the other for a duration of 2 s, which of the following statements is true?
a. The first object will acquire more momentum.
b. The second object will acquire more momentum.
c. Both objects will acquire the same momentum.
d. Neither object will experience a change in momentum.

10. Cars these days have parts that can crumple or collapse in the event of an accident. How does this help protect the passengers?
a. It reduces injury to the passengers by increasing the time of impact.
b. It reduces injury to the passengers by decreasing the time of impact.
c. It reduces injury to the passengers by increasing the change in momentum.
d. It reduces injury to the passengers by decreasing the change in momentum.

11. How much force would be needed to cause a 17 kg · m/s change in the momentum of an object, if the force acted for 5 seconds?
a. 3.4 N
b. 12 N
c. 22 N
d. 85 N

8.2 Conservation of Momentum

12. A billiards ball rolling on the table has momentum $\mathbf{p}_1$. It hits another stationary ball, which then starts rolling. Considering friction to be negligible, what will happen to the momentum of the first ball?
a. It will decrease.
b. It will increase.
c. It will become zero.
d. It will remain the same.

13. A ball rolling on the floor with momentum $\mathbf{p}_1$ collides with a stationary ball and sets it in motion. The momentum of the first ball becomes $\mathbf{p'}_1$, and that of the second becomes $\mathbf{p'}_2$. Compare the magnitudes of $\mathbf{p}_1$ and $\mathbf{p'}_2$.
a. Momenta $\mathbf{p}_1$ and $\mathbf{p'}_2$ are the same in magnitude.
b. The sum of the magnitudes of $\mathbf{p}_1$ and $\mathbf{p'}_2$ is zero.
c. The magnitude of $\mathbf{p}_1$ is greater than that of $\mathbf{p'}_2$.
d. The magnitude of $\mathbf{p'}_2$ is greater than that of $\mathbf{p}_1$.

14. Two cars are moving in the same direction. One car with momentum $\mathbf{p}_1$ collides with another, which has momentum $\mathbf{p}_2$. Their momenta become $\mathbf{p'}_1$ and $\mathbf{p'}_2$ respectively. Considering frictional losses, compare $(\mathbf{p'}_1 + \mathbf{p'}_2)$ with $(\mathbf{p}_1 + \mathbf{p}_2)$.
a. The value of $(\mathbf{p'}_1 + \mathbf{p'}_2)$ is zero.
b. The values of $(\mathbf{p}_1 + \mathbf{p}_2)$ and $(\mathbf{p'}_1 + \mathbf{p'}_2)$ are equal.
c. The value of $(\mathbf{p}_1 + \mathbf{p}_2)$ will be greater than $(\mathbf{p'}_1 + \mathbf{p'}_2)$.
d. The value of $(\mathbf{p'}_1 + \mathbf{p'}_2)$ will be greater than $(\mathbf{p}_1 + \mathbf{p}_2)$.

8.3 Elastic and Inelastic Collisions

15. Two people, who have the same mass, throw two different objects at the same velocity. If the first object is heavier than the second, compare the velocities gained by the two people as a result of recoil.
a. The first person will gain more velocity as a result of recoil.
b. The second person will gain more velocity as a result of recoil.
c. Both people will gain the same velocity as a result of recoil.
d. The velocity of both people will be zero as a result of recoil.

Problems

8.1 Linear Momentum, Force, and Impulse

16. If a force of 50 N is applied to an object for 0.2 s, and it changes its velocity by 10 m/s, what could be the mass of the object?
a. 1 kg
b. 2 kg
c. 5 kg
d. 250 kg

17. For how long should a force of 130 N be applied to an object of mass 50 kg to change its speed from 20 m/s to 60 m/s?
a. 0.031 s
b. 0.065 s
c. 15.4 s

d. 40 s

8.3 Elastic and Inelastic Collisions

18. If a man with mass 70 kg, standing still, throws an object with mass 5 kg at 50 m/s, what will be the recoil velocity of the man, assuming he is standing on a frictionless surface?

a. −3.6 m/s
b. 0 m/s
c. 3.6 m/s
d. 50.0 m/s

19. Find the recoil velocity of a 65 kg ice hockey goalie who catches a 0.15 kg hockey puck slapped at him at a velocity of 50 m/s. Assume that the goalie is at rest before catching the puck, and friction between the ice and the puck-goalie system is negligible.

a. -0.12 m/s
b. 0 m/s
c. 0.12 m/s
d. 7.5 m/s

Performance Task

8.3 Elastic and Inelastic Collisions

20. You will need the following:

- balls of different weights
- a ruler or wooden strip
- some books
- a paper cup

Make an inclined plane by resting one end of a ruler on a stack of books. Place a paper cup on the other end. Roll a ball from the top of the ruler so that it hits the paper cup. Measure the displacement of the paper cup due to the collision. Now use increasingly heavier balls for this activity and see how that affects the displacement of the cup. Plot a graph of mass vs. displacement. Now repeat the same activity, but this time, instead of using different balls, change the incline of the ruler by varying the height of the stack of books. This will give you different velocities of the ball. See how this affects the displacement of the paper cup.

TEST PREP

Multiple Choice

8.1 Linear Momentum, Force, and Impulse

21. What kind of quantity is momentum?

a. Scalar
b. Vector

22. When does the net force on an object increase?

a. When $\Delta\mathbf{p}$ decreases
b. When Δt increases
c. When Δt decreases

23. In the equation $\Delta\mathbf{p} = m(\mathbf{v}_f - \mathbf{v}_i)$, which quantity is considered to be constant?

a. Initial velocity
b. Final velocity
c. Mass
d. Momentum

24. For how long should a force of 50 N be applied to change the momentum of an object by $12\text{ kg}\cdot\text{m/s}$?

a. 0.24 s
b. 4.15 s
c. 62 s
d. 600 s

8.2 Conservation of Momentum

25. In the equation $\mathbf{L} = I\boldsymbol{\omega}$, what is I?

a. Linear momentum
b. Angular momentum
c. Torque
d. Moment of inertia

26. Give an example of an isolated system.

a. A cyclist moving along a rough road
b. A figure skater gliding in a straight line on an ice rink
c. A baseball player hitting a home run
d. A man drawing water from a well

8.3 Elastic and Inelastic Collisions

27. In which type of collision is kinetic energy conserved?

a. Elastic
b. Inelastic

28. In physics, what are structureless particles that cannot rotate or spin called?

a. Elastic particles
b. Point masses
c. Rigid masses

29. Two objects having equal masses and velocities collide with each other and come to a rest. What type of a collision is this and why?

a. Elastic collision, because internal kinetic energy is conserved

b. Inelastic collision, because internal kinetic energy is not conserved
c. Elastic collision, because internal kinetic energy is not conserved
d. Inelastic collision, because internal kinetic energy is conserved

30. Two objects having equal masses and velocities collide with each other and come to a rest. Is momentum conserved in this case?
a. Yes
b. No

Short Answer

8.1 Linear Momentum, Force, and Impulse

31. If an object's velocity is constant, what is its momentum proportional to?
a. Its shape
b. Its mass
c. Its length
d. Its breadth

32. If both mass and velocity of an object are constant, what can you tell about its impulse?
a. Its impulse would be constant.
b. Its impulse would be zero.
c. Its impulse would be increasing.
d. Its impulse would be decreasing.

33. When the momentum of an object increases with respect to time, what is true of the net force acting on it?
a. It is zero, because the net force is equal to the rate of change of the momentum.
b. It is zero, because the net force is equal to the product of the momentum and the time interval.
c. It is nonzero, because the net force is equal to the rate of change of the momentum.
d. It is nonzero, because the net force is equal to the product of the momentum and the time interval.

34. How can you express impulse in terms of mass and velocity when neither of those are constant?
a. $\Delta \mathbf{p} = \Delta(m\mathbf{v})$
b. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{\Delta(m\mathbf{v})}{\Delta t}$
c. $\Delta \mathbf{p} = \Delta\left(\frac{m}{\mathbf{v}}\right)$
d. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{1}{\Delta t} \cdot \Delta(m\mathbf{v})$

35. How can you express impulse in terms of mass and initial and final velocities?
a. $\Delta \mathbf{p} = m(\mathbf{v}_\mathrm{f} - \mathbf{v}_\mathrm{i})$
b. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{m(\mathbf{v}_\mathrm{f} - \mathbf{v}_\mathrm{i})}{\Delta t}$
c. $\Delta \mathbf{p} = \frac{(\mathbf{v}_\mathrm{f} - \mathbf{v}_\mathrm{i})}{m}$
d. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{1}{m} \frac{(\mathbf{v}_\mathrm{f} - \mathbf{v}_\mathrm{i})}{\Delta t}$

36. Why do we use average force while solving momentum problems? How is net force related to the momentum of the object?
a. Forces are usually constant over a period of time, and net force acting on the object is equal to the rate of change of the momentum.
b. Forces are usually not constant over a period of time, and net force acting on the object is equal to the product of the momentum and the time interval.
c. Forces are usually constant over a period of time, and net force acting on the object is equal to the product of the momentum and the time interval.
d. Forces are usually not constant over a period of time, and net force acting on the object is equal to the rate of change of the momentum.

8.2 Conservation of Momentum

37. Under what condition(s) is the angular momentum of a system conserved?
a. When net torque is zero
b. When net torque is not zero
c. When moment of inertia is constant
d. When both moment of inertia and angular momentum are constant

38. If the moment of inertia of an isolated system increases, what happens to its angular velocity?
a. It increases.
b. It decreases.
c. It stays constant.
d. It becomes zero.

39. If both the moment of inertia and the angular velocity of a system increase, what must be true of the force acting on the system?
a. Force is zero.
b. Force is not zero.
c. Force is constant.
d. Force is decreasing.

8.3 Elastic and Inelastic Collisions

40. Two objects collide with each other and come to a rest. How can you use the equation of conservation of momentum to describe this situation?
a. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$
b. $m_1\mathbf{v}_1 - m_2\mathbf{v}_2 = 0$
c. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1'$
d. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_2$

41. What is the difference between momentum and impulse?

a. Momentum is the sum of mass and velocity. Impulse is the change in momentum.
b. Momentum is the sum of mass and velocity. Impulse is the rate of change in momentum.
c. Momentum is the product of mass and velocity. Impulse is the change in momentum.
d. Momentum is the product of mass and velocity. Impulse is the rate of change in momentum.

42. What is the equation for conservation of momentum along the x-axis for 2D collisions in terms of mass and velocity, where one of the particles is initially at rest?

a. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1$
b. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1 + m_2\mathbf{v}_2'\cos\theta_2$
c. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1 - m_2\mathbf{v}_2'\cos\theta_2$
d. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\sin\theta_1 + m_2\mathbf{v}_2'\sin\theta_2$

43. What is the equation for conservation of momentum along the y-axis for 2D collisions in terms of mass and velocity, where one of the particles is initially at rest?

a. $0 = m_1\mathbf{v}_1'\sin\theta_1$
b. $0 = m_1\mathbf{v}_1'\sin\theta_1 + m_2\mathbf{v}_2'\sin\theta_2$
c. $0 = m_1\mathbf{v}_1'\sin\theta_1 - m_2\mathbf{v}_2'\sin\theta_2$
d. $0 = m_1\mathbf{v}_1'\cos\theta_1 + m_2\mathbf{v}_2'\cos\theta_2$

Extended Response

8.1 Linear Momentum, Force, and Impulse

44. Can a lighter object have more momentum than a heavier one? How?

a. No, because momentum is independent of the velocity of the object.
b. No, because momentum is independent of the mass of the object.
c. Yes, if the lighter object's velocity is considerably high.
d. Yes, if the lighter object's velocity is considerably low.

45. Why does it hurt less when you fall on a softer surface?

a. The softer surface increases the duration of the impact, thereby reducing the effect of the force.
b. The softer surface decreases the duration of the impact, thereby reducing the effect of the force.
c. The softer surface increases the duration of the impact, thereby increasing the effect of the force.
d. The softer surface decreases the duration of the impact, thereby increasing the effect of the force.

46. Can we use the equation $F_{net} = \frac{\Delta p}{\Delta t}$ when the mass is constant?

a. No, because the given equation is applicable for the variable mass only.
b. No, because the given equation is not applicable for the constant mass.
c. Yes, and the resultant equation is $F = m\mathbf{v}$
d. Yes, and the resultant equation is $F = ma$

8.2 Conservation of Momentum

47. Why does a figure skater spin faster if he pulls his arms and legs in?

a. Due to an increase in moment of inertia
b. Due to an increase in angular momentum
c. Due to conservation of linear momentum
d. Due to conservation of angular momentum

8.3 Elastic and Inelastic Collisions

48. A driver sees another car approaching him from behind. He fears it is going to collide with his car. Should he speed up or slow down in order to reduce damage?

a. He should speed up.
b. He should slow down.
c. He should speed up and then slow down just before the collision.
d. He should slow down and then speed up just before the collision.

49. What approach would you use to solve problems involving 2D collisions?

a. Break the momenta into components and then choose a coordinate system.
b. Choose a coordinate system and then break the momenta into components.
c. Find the total momenta in the x and y directions, and then equate them to solve for the unknown.
d. Find the sum of the momenta in the x and y directions, and then equate it to zero to solve for the unknown.

CHAPTER 9
Work, Energy, and Simple Machines

Figure 9.1 People on a roller coaster experience thrills caused by changes in types of energy. (Jonrev, Wikimedia Commons)

Chapter Outline

INTRODUCTION Roller coasters have provided thrills for daring riders around the world since the nineteenth century. Inventors of roller coasters used simple physics to build the earliest examples using railroad tracks on mountainsides and old mines. Modern roller coaster designers use the same basic laws of physics to create the latest amusement park favorites. Physics principles are used to engineer the machines that do the work to lift a roller coaster car up its first big incline before it is set loose to roll. Engineers also have to understand the changes in the car's energy that keep it speeding over hills, through twists, turns, and even loops.

What exactly is energy? How can changes in force, energy, and simple machines move objects like roller coaster cars? How can machines help us do work? In this chapter, you will discover the answer to this question and many more, as you learn about work, energy, and simple machines.

9.1 Work, Power, and the Work–Energy Theorem

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe and apply the work–energy theorem
- Describe and calculate work and power

Section Key Terms

energy	gravitational potential energy	joule	kinetic energy	mechanical energy
potential energy	power	watt	work	work–energy theorem

The Work–Energy Theorem

In physics, the term **work** has a very specific definition. Work is application of force, $\mathbf{f}$, to move an object over a distance, d, in the direction that the force is applied. Work, W, is described by the equation

$$W = \mathbf{f}d.$$

Some things that we typically consider to be work are not work in the scientific sense of the term. Let's consider a few examples. Think about why each of the following statements is true.

- Homework *is not* work.
- Lifting a rock upwards off the ground *is* work.
- Carrying a rock in a straight path across the lawn at a constant speed *is not* work.

The first two examples are fairly simple. Homework is not work because objects are not being moved over a distance. Lifting a rock up off the ground is work because the rock is moving in the direction that force is applied. The last example is less obvious. Recall from the laws of motion that force is *not* required to move an object at constant velocity. Therefore, while some force may be applied to keep the rock up off the ground, no net force is applied to keep the rock moving forward at constant velocity.

Work and **energy** are closely related. When you do work to move an object, you change the object's energy. You (or an object) also expend energy to do work. In fact, energy can be defined as the ability to do work. Energy can take a variety of different forms, and one form of energy can transform to another. In this chapter we will be concerned with **mechanical energy**, which comes in two forms: **kinetic energy** and **potential energy**.

- Kinetic energy is also called energy of motion. A moving object has kinetic energy.
- Potential energy, sometimes called stored energy, comes in several forms. **Gravitational potential energy** is the stored energy an object has as a result of its position above Earth's surface (or another object in space). A roller coaster car at the top of a hill has gravitational potential energy.

Let's examine how doing work on an object changes the object's energy. If we apply force to lift a rock off the ground, we increase the rock's potential energy, PE. If we drop the rock, the force of gravity increases the rock's kinetic energy as the rock moves downward until it hits the ground.

The force we exert to lift the rock is equal to its weight, w, which is equal to its mass, m, multiplied by acceleration due to gravity, $\mathbf{g}$.

$$\mathbf{f} = w = m\mathbf{g}$$

The work we do on the rock equals the force we exert multiplied by the distance, d, that we lift the rock. The work we do on the rock also equals the rock's gain in gravitational potential energy, PE_e.

$$W = PE_e = \mathbf{f}m\mathbf{g}$$

Kinetic energy depends on the mass of an object and its velocity, $\mathbf{v}$.

$$KE = \frac{1}{2}m\mathbf{v}^2$$

When we drop the rock the force of gravity causes the rock to fall, giving the rock kinetic energy. When work done on an object increases only its kinetic energy, then the net work equals the change in the value of the quantity $\frac{1}{2}m\mathbf{v}^2$. This is a statement of the **work–energy theorem**, which is expressed mathematically as

$$W = \Delta KE = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2.$$

The subscripts $_2$ and $_1$ indicate the final and initial velocity, respectively. This theorem was proposed and successfully tested by James Joule, shown in Figure 9.2.

Does the name Joule sound familiar? The **joule** (J) is the metric unit of measurement for both work and energy. The measurement of work and energy with the same unit reinforces the idea that work and energy are related and can be converted into one another. 1.0 J = 1.0 N•m, the units of force multiplied by distance. 1.0 N = 1.0 k•m/s^2, so 1.0 J = 1.0 k•m^2/s^2. Analyzing the units of the term (1/2)$m\mathbf{v}^2$ will produce the same units for joules.

Figure 9.2 The joule is named after physicist James Joule (1818–1889). (C. H. Jeens, Wikimedia Commons)

WATCH PHYSICS

Work and Energy

This video explains the work energy theorem and discusses how work done on an object increases the object's KE.

Click to view content (https://www.khanacademy.org/embed_video?v=2WS1sG9fhOk)

GRASP CHECK

True or false—The energy increase of an object acted on only by a gravitational force is equal to the product of the object's weight and the distance the object falls.

a. True
b. False

Calculations Involving Work and Power

In applications that involve work, we are often interested in how fast the work is done. For example, in roller coaster design, the amount of time it takes to lift a roller coaster car to the top of the first hill is an important consideration. Taking a half hour on the ascent will surely irritate riders and decrease ticket sales. Let's take a look at how to calculate the time it takes to do work.

Recall that a rate can be used to describe a quantity, such as work, over a period of time. **Power** is the rate at which work is done. In this case, rate means *per unit of time*. Power is calculated by dividing the work done by the time it took to do the work.

$$P = \frac{W}{t}$$

Let's consider an example that can help illustrate the differences among work, force, and power. Suppose the woman in Figure 9.3 lifting the TV with a pulley gets the TV to the fourth floor in two minutes, and the man carrying the TV up the stairs takes five minutes to arrive at the same place. They have done the same amount of work $(\mathbf{f}d)$ on the TV, because they have moved the same mass over the same vertical distance, which requires the same amount of upward force. However, the woman using the pulley has generated more power. This is because she did the work in a shorter amount of time, so the denominator of the power formula, t, is smaller. (For simplicity's sake, we will leave aside for now the fact that the man climbing the stairs has also done work on himself.)

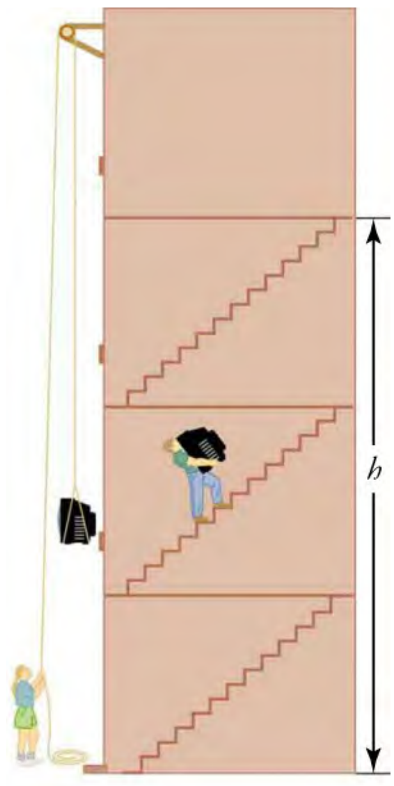


Figure 9.3 No matter how you move a TV to the fourth floor, the amount of work performed and the potential energy gain are the same.

Power can be expressed in units of **watts** (W). This unit can be used to measure power related to any form of energy or work. You have most likely heard the term used in relation to electrical devices, especially light bulbs. Multiplying power by time gives the amount of energy. Electricity is sold in kilowatt-hours because that equals the amount of electrical energy consumed.

The watt unit was named after James Watt (1736–1819) (see Figure 9.4). He was a Scottish engineer and inventor who discovered how to coax more power out of steam engines.

Figure 9.4 Is James Watt thinking about watts? (Carl Frederik von Breda, Wikimedia Commons)

LINKS TO PHYSICS

Watt’s Steam Engine

James Watt did not invent the steam engine, but by the time he was finished tinkering with it, it was more useful. The first steam engines were not only inefficient, they only produced a back and forth, or reciprocal, motion. This was natural because pistons move in and out as the pressure in the chamber changes. This limitation was okay for simple tasks like pumping water or mashing potatoes, but did not work so well for moving a train. Watt was able build a steam engine that converted reciprocal motion to circular motion. With that one innovation, the industrial revolution was off and running. The world would never be the same. One of Watt's steam engines is shown in Figure 9.5. The video that follows the figure explains the importance of the steam engine in the industrial revolution.

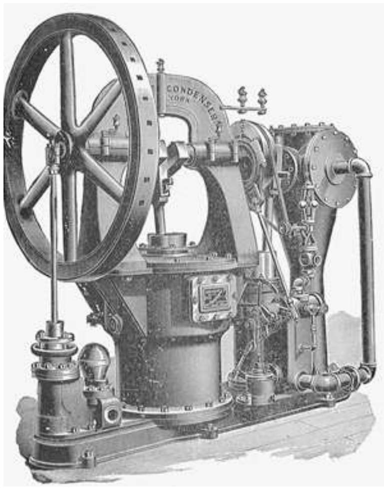


Figure 9.5 A late version of the Watt steam engine. (Nehemiah Hawkins, Wikimedia Commons)

WATCH PHYSICS

Watt's Role in the Industrial Revolution

This video demonstrates how the watts that resulted from Watt's inventions helped make the industrial revolution possible and allowed England to enter a new historical era.

Click to view content (https://www.youtube.com/embed/zhL5DCizj5c)

GRASP CHECK

Which form of mechanical energy does the steam engine generate?

a. Potential energy
b. Kinetic energy
c. Nuclear energy
d. Solar energy

Before proceeding, be sure you understand the distinctions among force, work, energy, and power. Force exerted on an object over a distance does work. Work can increase energy, and energy can do work. Power is the rate at which work is done.

WORKED EXAMPLE

Applying the Work–Energy Theorem

An ice skater with a mass of 50 kg is gliding across the ice at a speed of 8 m/s when her friend comes up from behind and gives her a push, causing her speed to increase to 12 m/s. How much work did the friend do on the skater?

Strategy

The work–energy theorem can be applied to the problem. Write the equation for the theorem and simplify it if possible.

$$W = \Delta \text{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$$

$$\text{Simplify to } W = \frac{1}{2}m(\mathbf{v}_2^2 - \mathbf{v}_1^2)$$

Solution

Identify the variables. $m = 50$ kg,

$$\mathbf{v}_2 = 12\frac{\text{m}}{\text{s}}, \text{ and } \mathbf{v}_1 = 8\frac{\text{m}}{\text{s}} \tag{9.1}$$

Substitute.

$$W = \frac{1}{2}50(12^2 - 8^2) = 2,000 \text{ J} \tag{9.2}$$

Discussion

Work done on an object or system increases its energy. In this case, the increase is to the skater's kinetic energy. It follows that the increase in energy must be the difference in KE before and after the push.

TIPS FOR SUCCESS

This problem illustrates a general technique for approaching problems that require you to apply formulas: Identify the unknown and the known variables, express the unknown variables in terms of the known variables, and then enter all the known values.

Practice Problems

1. How much work is done when a weightlifter lifts a 200 N barbell from the floor to a height of 2 m?
   a. 0 J
   b. 100 J
   c. 200 J
   d. 400 J

2. Identify which of the following actions generates more power. Show your work.
   - carrying a 100 N TV to the second floor in 50 s or
   - carrying a 24 N watermelon to the second floor in 10 s?

   a. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as work done times the time interval.
   b. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as the ratio of work done to the time interval.
   c. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as work done times the time interval.
   d. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as the ratio of work done and the time interval.

Check Your Understanding

3. Identify two properties that are expressed in units of joules.
   a. work and force
   b. energy and weight
   c. work and energy
   d. weight and force

4. When a coconut falls from a tree, work W is done on it as it falls to the beach. This work is described by the equation

$$W = Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2. \tag{9.3}$$

Identify the quantities F, d, m, v_1, and v_2 in this event.

a. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
b. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
c. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.
d. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.

9.2 Mechanical Energy and Conservation of Energy

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the law of conservation of energy in terms of kinetic and potential energy
- Perform calculations related to kinetic and potential energy. Apply the law of conservation of energy

Section Key Terms

law of conservation of energy

Mechanical Energy and Conservation of Energy

We saw earlier that mechanical energy can be either potential or kinetic. In this section we will see how energy is transformed from one of these forms to the other. We will also see that, in a closed system, the sum of these forms of energy remains constant.

Quite a bit of potential energy is gained by a roller coaster car and its passengers when they are raised to the top of the first hill. Remember that the *potential* part of the term means that energy has been stored and can be used at another time. You will see that this stored energy can either be used to do work or can be transformed into kinetic energy. For example, when an object that has gravitational potential energy falls, its energy is converted to kinetic energy. Remember that both work and energy are expressed in joules.

Refer back to . The amount of work required to raise the TV from point A to point B is equal to the amount of gravitational potential energy the TV gains from its height above the ground. This is generally true for any object raised above the ground. If all the work done on an object is used to raise the object above the ground, the amount work equals the object's gain in gravitational potential energy. However, note that because of the work done by friction, these energy–work transformations are never perfect. Friction causes the loss of some useful energy. In the discussions to follow, we will use the approximation that transformations are frictionless.

Now, let's look at the roller coaster in Figure 9.6. Work was done on the roller coaster to get it to the top of the first rise; at this point, the roller coaster has gravitational potential energy. It is moving slowly, so it also has a small amount of kinetic energy. As the car descends the first slope, its PE is converted to KE. At the low point much of the original PE has been transformed to KE, and speed is at a maximum. As the car moves up the next slope, some of the KE is transformed back into PE and the car slows down.

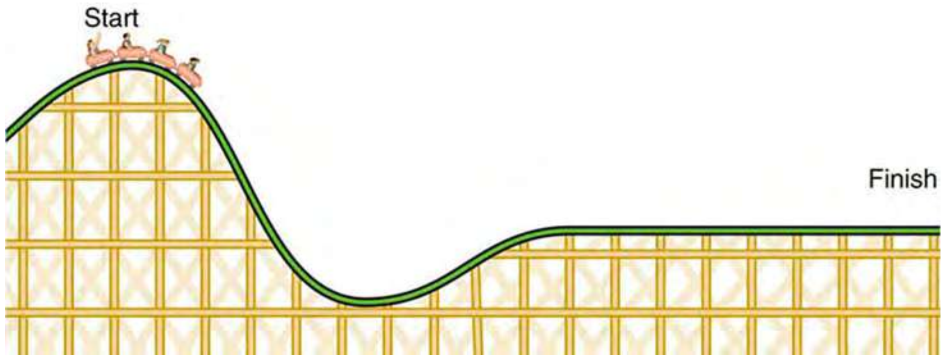


Figure 9.6 During this roller coaster ride, there are conversions between potential and kinetic energy.

Virtual Physics

Energy Skate Park Basics

This simulation shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in *KE* and *PE* by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation.

Click to view content (http://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html)

GRASP CHECK

This simulation (http://phet.colorado.edu/en/simulation/energy-skate-park-basics (http://phet.colorado.edu/en/simulation/energy-skate-park-basics)) shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in KE and PE by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation. The bar graphs show how KE and PE are transformed back and forth. Which statement best explains what happens to the mechanical energy of the system as speed is increasing?

a. The mechanical energy of the system increases, provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
b. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
c. The mechanical energy of the system increases provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.
d. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.

On an actual roller coaster, there are many ups and downs, and each of these is accompanied by transitions between kinetic and potential energy. Assume that no energy is lost to friction. At any point in the ride, the total mechanical energy is the same, and it is equal to the energy the car had at the top of the first rise. This is a result of the **law of conservation of energy**, which says that, in a closed system, total energy is conserved—that is, it is constant. Using subscripts 1 and 2 to represent initial and final energy, this law is expressed as

$$KE_1 + PE_1 = KE_2 + PE_2.$$

Either side equals the total mechanical energy. The phrase *in a closed system* means we are assuming no energy is lost to the surroundings due to friction and air resistance. If we are making calculations on dense falling objects, this is a good assumption. For the roller coaster, this assumption introduces some inaccuracy to the calculation.

Calculations involving Mechanical Energy and Conservation of Energy

TIPS FOR SUCCESS

When calculating work or energy, use units of meters for distance, newtons for force, kilograms for mass, and seconds for time. This will assure that the result is expressed in joules.

WATCH PHYSICS

Conservation of Energy

This video discusses conversion of *PE* to *KE* and conservation of energy. The scenario is very similar to the roller coaster and the skate park. It is also a good explanation of the energy changes studied in the snap lab.

[Click to view content (https://www.khanacademy.org/embed_video?v=kw_4Loo1HR4)](https://www.khanacademy.org/embed_video?v=kw_4Loo1HR4)

GRASP CHECK

Did you expect the speed at the bottom of the slope to be the same as when the object fell straight down? Which statement best explains why this is not exactly the case in real-life situations?

a. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
b. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
c. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.
d. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.

WORKED EXAMPLE

Applying the Law of Conservation of Energy

A 10 kg rock falls from a 20 m cliff. What is the kinetic and potential energy when the rock has fallen 10 m?

Strategy

Choose the equation.

$$KE_1 + PE_1 = KE_2 + PE_2 \tag{9.4}$$

$$KE = \frac{1}{2}m\mathbf{v}^2; \quad PE = m\mathbf{g}h \tag{9.5}$$

$$\frac{1}{2}m\mathbf{v}_1^2 + m\mathbf{g}h_1 = \frac{1}{2}m\mathbf{v}_2^2 + m\mathbf{g}h_2 \tag{9.6}$$

List the knowns.

$m = 10\ \text{kg}$, $\mathbf{v}_1 = 0$, $\mathbf{g} = 9.80$

$$\frac{\text{m}}{\text{s}^2}, \tag{9.7}$$

$h_1 = 20\ \text{m}$, $h_2 = 10\ \text{m}$

Identify the unknowns.

KE_2 and PE_2

Substitute the known values into the equation and solve for the unknown variables.

Solution

$$PE_2 = mgh_2 = 10\,(9.80)\,10 = 980\ \text{J} \qquad 9.8$$

$$KE_2 = PE_2 - (KE_1 + PE_1) = 980 - \{[0 - [10\,(9.80)\,20]]\} = 980\ \text{J} \qquad 9.9$$

Discussion

Alternatively, conservation of energy equation could be solved for $\mathbf{v}_2$ and KE_2 could be calculated. Note that m could also be eliminated.

TIPS FOR SUCCESS

Note that we can solve many problems involving conversion between *KE* and *PE* without knowing the mass of the object in question. This is because kinetic and potential energy are both proportional to the mass of the object. In a situation where $KE = PE$, we know that $m\mathbf{g}h = (1/2)m\mathbf{v}^2$.

Dividing both sides by m and rearranging, we have the relationship

$2\mathbf{g}h = \mathbf{v}^2$.

Practice Problems

5. A child slides down a playground slide. If the slide is 3 m high and the child weighs 300 N, how much potential energy does the child have at the top of the slide? (Round g to $10\ m/s^2$.)
   a. 0 J
   b. 100 J
   c. 300 J
   d. 900 J

6. A 0.2 kg apple on an apple tree has a potential energy of 10 J. It falls to the ground, converting all of its PE to kinetic energy. What is the velocity of the apple just before it hits the ground?
   a. 0 m/s
   b. 2 m/s
   c. 10 m/s
   d. 50 m/s

Snap Lab

Converting Potential Energy to Kinetic Energy

In this activity, you will calculate the potential energy of an object and predict the object's speed when all that potential energy has been converted to kinetic energy. You will then check your prediction.

You will be dropping objects from a height. Be sure to stay a safe distance from the edge. Don't lean over the railing too far. Make sure that you do not drop objects into an area where people or vehicles pass by. Make sure that dropping objects will not cause damage.

You will need the following:

Materials for each pair of students:

- Four marbles (or similar small, dense objects)
- Stopwatch

Materials for class:

- Metric measuring tape long enough to measure the chosen height
- A scale

Instructions

Procedure

1. Work with a partner. Find and record the mass of four small, dense objects per group.
2. Choose a location where the objects can be safely dropped from a height of at least 15 meters. A bridge over water with a safe pedestrian walkway will work well.
3. Measure the distance the object will fall.
4. Calculate the potential energy of the object before you drop it using $PE = m\mathbf{g}h = (9.80)mh$.
5. Predict the kinetic energy and velocity of the object when it lands using $PE = KE$ and so, $m\mathbf{g}h = \frac{m\mathbf{v}^2}{2};\ \mathbf{v} = \sqrt{2(9.80)h} = 4.43\sqrt{h}$.
6. One partner drops the object while the other measures the time it takes to fall.
7. Take turns being the dropper and the timer until you have made four measurements.
8. Average your drop multiplied by and calculate the velocity of the object when it landed using $\mathbf{v} = \mathbf{a}t = \mathbf{g}t = (9.80)t$.
9. Compare your results to your prediction.

GRASP CHECK

Galileo's experiments proved that, contrary to popular belief, heavy objects do not fall faster than light objects. How do the equations you used support this fact?

a. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
b. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
c. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.
d. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.

Check Your Understanding

7. Describe the transformation between forms of mechanical energy that is happening to a falling skydiver before his parachute opens.

a. Kinetic energy is being transformed into potential energy.
b. Potential energy is being transformed into kinetic energy.
c. Work is being transformed into kinetic energy.
d. Kinetic energy is being transformed into work.

8. True or false—If a rock is thrown into the air, the increase in the height would increase the rock's kinetic energy, and then the increase in the velocity as it falls to the ground would increase its potential energy.

a. True
b. False

9. Identify equivalent terms for *stored energy* and *energy of motion*.

a. Stored energy is potential energy, and energy of motion is kinetic energy.
b. Energy of motion is potential energy, and stored energy is kinetic energy.
c. Stored energy is the potential as well as the kinetic energy of the system.
d. Energy of motion is the potential as well as the kinetic energy of the system.

9.3 Simple Machines

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe simple and complex machines
- Calculate mechanical advantage and efficiency of simple and complex machines

Section Key Terms

complex machine	efficiency output	ideal mechanical advantage	inclined plane	input work
lever	mechanical advantage	output work	pulley	screw
simple machine	wedge	wheel and axle		

Simple Machines

Simple machines make work easier, but they do not decrease the amount of work you have to do. Why can't simple machines change the amount of work that you do? Recall that in closed systems the total amount of energy is conserved. A machine cannot increase the amount of energy you put into it. So, why is a simple machine useful? Although it cannot change the amount of work you do, a simple machine can change the amount of force you must apply to an object, and the distance over which you apply the force. In most cases, a simple machine is used to reduce the amount of force you must exert to do work. The down side is that you must exert the force over a greater distance, because the product of force and distance, $\mathbf{f}d$, (which equals work) does not change.

Let's examine how this works in practice. In Figure 9.7(a), the worker uses a type of **lever** to exert a small force over a large distance, while the pry bar pulls up on the nail with a large force over a small distance. Figure 9.7(b) shows the how a lever works mathematically. The effort force, applied at $\mathbf{F_e}$, lifts the load (the resistance force) which is pushing down at $\mathbf{F}_r$. The triangular pivot is called the fulcrum; the part of the lever between the fulcrum and $\mathbf{F}_e$ is the effort arm, L_e; and the part to the left is the resistance arm, L_r. The **mechanical advantage** is a number that tells us how many times a simple machine multiplies the effort force. The **ideal mechanical advantage**, *IMA*, is the mechanical advantage of a perfect machine with no loss of useful work caused by friction between moving parts. The equation for *IMA* is shown in Figure 9.7(b).

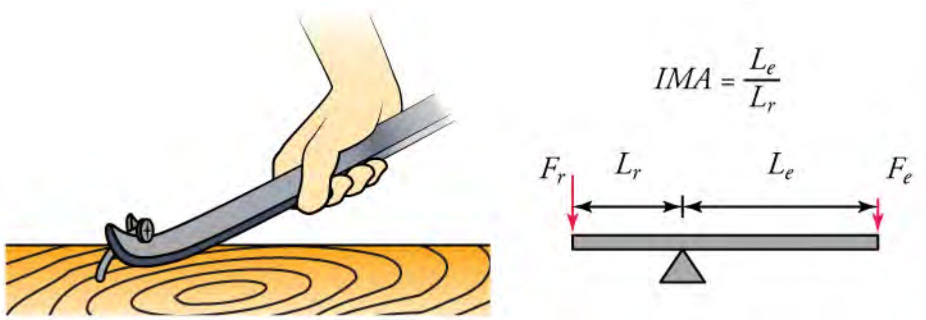


Figure 9.7 (a) A pry bar is a type of lever. (b) The ideal mechanical advantage equals the length of the effort arm divided by the length of the resistance arm of a lever.

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Getting back to conservation of energy, for any simple machine, the work put into the machine, W_i, equals the work the machine puts out, W_o. Combining this with the information in the paragraphs above, we can write

$$W_i = W_o$$
$$\mathbf{F}_e d_e = \mathbf{F}_r d_r$$
$$\text{If } \mathbf{F}_e < \mathbf{F}_r, \text{ then } d_e > d_r.$$

The equations show how a simple machine can output the same amount of work while reducing the amount of effort force by increasing the distance over which the effort force is applied.

WATCH PHYSICS

Introduction to Mechanical Advantage

This video shows how to calculate the *IMA* of a lever by three different methods: (1) from effort force and resistance force; (2) from the lengths of the lever arms, and; (3) from the distance over which the force is applied and the distance the load moves.

Click to view content (https://www.youtube.com/embed/pfzJ-z5Ij48)

GRASP CHECK

Two children of different weights are riding a seesaw. How do they position themselves with respect to the pivot point (the fulcrum) so that they are balanced?

a. The heavier child sits closer to the fulcrum.
b. The heavier child sits farther from the fulcrum.
c. Both children sit at equal distance from the fulcrum.
d. Since both have different weights, they will never be in balance.

Some levers exert a large force to a short effort arm. This results in a smaller force acting over a greater distance at the end of the resistance arm. Examples of this type of lever are baseball bats, hammers, and golf clubs. In another type of lever, the fulcrum is at the end of the lever and the load is in the middle, as in the design of a wheelbarrow.

The simple machine shown in Figure 9.8 is called a **wheel and axle.** It is actually a form of lever. The difference is that the effort arm can rotate in a complete circle around the fulcrum, which is the center of the axle. Force applied to the outside of the wheel causes a greater force to be applied to the rope that is wrapped around the axle. As shown in the figure, the ideal mechanical advantage is calculated by dividing the radius of the wheel by the radius of the axle. Any crank-operated device is an example of a wheel and axle.

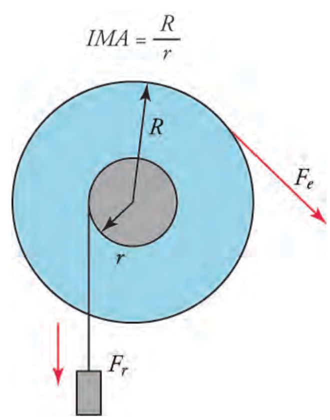


Figure 9.8 Force applied to a wheel exerts a force on its axle.

An **inclined plane** and a **wedge** are two forms of the same simple machine. A wedge is simply two inclined planes back to back. Figure 9.9 shows the simple formulas for calculating the *IMA*s of these machines. All sloping, paved surfaces for walking or driving are inclined planes. Knives and axe heads are examples of wedges.

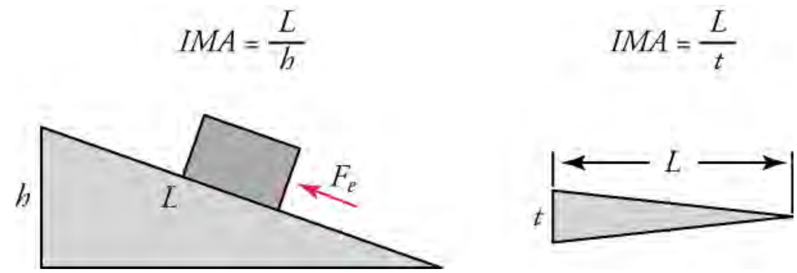


Figure 9.9 An inclined plane is shown on the left, and a wedge is shown on the right.

The **screw** shown in Figure 9.10 is actually a lever attached to a circular inclined plane. Wood screws (of course) are also examples of screws. The lever part of these screws is a screw driver. In the formula for *IMA*, the distance between screw threads is called *pitch* and has the symbol *P*.

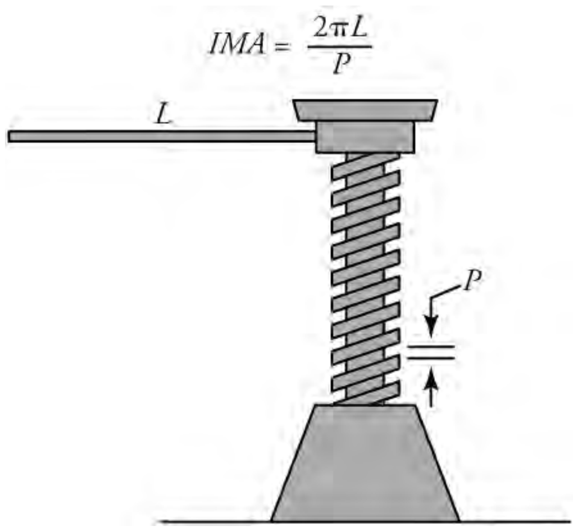


Figure 9.10 The screw shown here is used to lift very heavy objects, like the corner of a car or a house a short distance.

Figure 9.11 shows three different **pulley** systems. Of all simple machines, mechanical advantage is easiest to calculate for pulleys. Simply count the number of ropes supporting the load. That is the *IMA*. Once again we have to exert force over a longer distance to multiply force. To raise a load 1 meter with a pulley system you have to pull *N* meters of rope. Pulley systems are often used to raise flags and window blinds and are part of the mechanism of construction cranes.

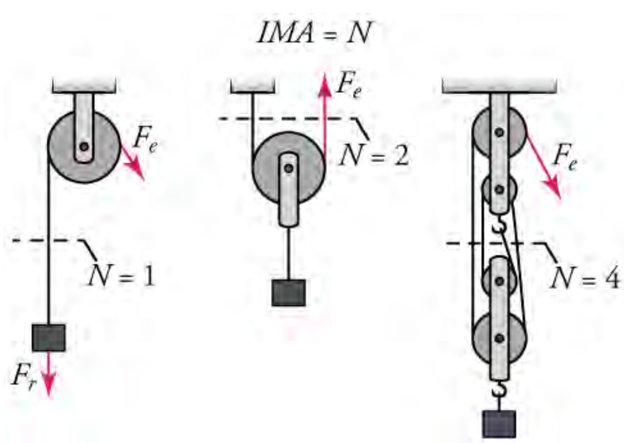


Figure 9.11 Three pulley systems are shown here.

WATCH PHYSICS

Mechanical Advantage of Inclined Planes and Pulleys

The first part of this video shows how to calculate the *IMA* of pulley systems. The last part shows how to calculate the *IMA* of an inclined plane.

Click to view content (https://www.khanacademy.org/embed_video?v=vSsK7Rfa3yA)

GRASP CHECK

How could you use a pulley system to lift a light load to great height?

a. Reduce the radius of the pulley.
b. Increase the number of pulleys.
c. Decrease the number of ropes supporting the load.
d. Increase the number of ropes supporting the load.

A **complex machine** is a combination of two or more simple machines. The wire cutters in Figure 9.12 combine two levers and two wedges. Bicycles include wheel and axles, levers, screws, and pulleys. Cars and other vehicles are combinations of many machines.

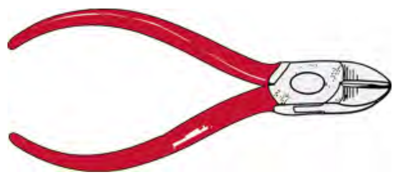


Figure 9.12 Wire cutters are a common complex machine.

Calculating Mechanical Advantage and Efficiency of Simple Machines

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Refer back to the discussions of each simple machine for the specific equations for the *IMA* for each type of machine.

No simple or complex machines have the actual mechanical advantages calculated by the *IMA* equations. In real life, some of the applied work always ends up as wasted heat due to friction between moving parts. Both the **input work** (W_i) and **output work** (W_o) are the result of a force, **F**, acting over a distance, *d*.

$$W_i = \mathbf{F}_i d_i \text{and} W_o = \mathbf{F}_o d_o$$

The **efficiency output** of a machine is simply the output work divided by the input work, and is usually multiplied by 100 so that it is expressed as a percent.

$$\%\text{ efficiency} = \frac{W_o}{W_i} \times 100$$

Look back at the pictures of the simple machines and think about which would have the highest efficiency. Efficiency is related to friction, and friction depends on the smoothness of surfaces and on the area of the surfaces in contact. How would lubrication affect the efficiency of a simple machine?

WORKED EXAMPLE

Efficiency of a Lever

The input force of 11 N acting on the effort arm of a lever moves 0.4 m, which lifts a 40 N weight resting on the resistance arm a distance of 0.1 m. What is the efficiency of the machine?

Strategy

State the equation for efficiency of a simple machine, $\%\ \text{efficiency} = \frac{W_o}{W_i} \times 100$, and calculate W_o and W_i. Both work values are the product *Fd*.

Solution

$W_i = \mathbf{F}_i d_i = (11)(0.4) = 4.4$ J and $W_o = \mathbf{F}_o d_o = (40)(0.1) = 4.0$ J, then $\%\ \text{efficiency} = \frac{W_o}{W_i} \times 100 = \frac{4.0}{4.4} \times 100 = 91\%$

Discussion

Efficiency in real machines will always be less than 100 percent because of work that is converted to unavailable heat by friction and air resistance. W_o and W_i can always be calculated as a force multiplied by a distance, although these quantities are not always as obvious as they are in the case of a lever.

Practice Problems

10. What is the IMA of an inclined plane that is 5 m long and 2 m high?

a. 0.4
b. 2.5
c. 0.4 m
d. 2.5 m

11. If a pulley system can lift a 200N load with an effort force of 52 N and has an efficiency of almost 100 percent, how many ropes are supporting the load?

a. 1 rope is required because the actual mechanical advantage is 0.26.
b. 1 rope is required because the actual mechanical advantage is 3.80.
c. 4 ropes are required because the actual mechanical advantage is 0.26.
d. 4 ropes are required because the actual mechanical advantage is 3.80.

Check Your Understanding

12. True or false—The efficiency of a simple machine is always less than 100 percent because some small fraction of the input work is always converted to heat energy due to friction.

a. True
b. False

13. The circular handle of a faucet is attached to a rod that opens and closes a valve when the handle is turned. If the rod has a diameter of 1 cm and the IMA of the machine is 6, what is the radius of the handle?

A. 0.08 cm
B. 0.17 cm
C. 3.0 cm
D. 6.0 cm

KEY TERMS

complex machine a machine that combines two or more simple machines

efficiency output work divided by input work

energy the ability to do work

gravitational potential energy energy acquired by doing work against gravity

ideal mechanical advantage the mechanical advantage of an idealized machine that loses no energy to friction

inclined plane a simple machine consisting of a slope

input work effort force multiplied by the distance over which it is applied

joule the metric unit for work and energy; equal to 1 newton meter (N•m)

kinetic energy energy of motion

law of conservation of energy states that energy is neither created nor destroyed

lever a simple machine consisting of a rigid arm that pivots on a fulcrum

mechanical advantage the number of times the input force is multiplied

mechanical energy kinetic or potential energy

output work output force multiplied by the distance over which it acts

potential energy stored energy

power the rate at which work is done

pulley a simple machine consisting of a rope that passes over one or more grooved wheels

screw a simple machine consisting of a spiral inclined plane

simple machine a machine that makes work easier by changing the amount or direction of force required to move an object

watt the metric unit of power; equivalent to joules per second

wedge a simple machine consisting of two back-to-back inclined planes

wheel and axle a simple machine consisting of a rod fixed to the center of a wheel

work force multiplied by distance

work–energy theorem states that the net work done on a system equals the change in kinetic energy

SECTION SUMMARY

9.1 Work, Power, and the Work–Energy Theorem

- Doing work on a system or object changes its energy.
- The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.
- Power is the rate at which work is done.

9.2 Mechanical Energy and Conservation of Energy

- Mechanical energy may be either kinetic (energy of motion) or potential (stored energy).
- Doing work on an object or system changes its energy.
- Total energy in a closed, isolated system is constant.

9.3 Simple Machines

- The six types of simple machines make work easier by changing the $\mathbf{f}d$ term so that force is reduced at the expense of increased distance.
- The ratio of output force to input force is a machine's mechanical advantage.
- Combinations of two or more simple machines are called complex machines.
- The ratio of output work to input work is a machine's efficiency.

KEY EQUATIONS

9.1 Work, Power, and the Work–Energy Theorem

equation for work	$W = \mathbf{f}d$
force	$\mathbf{f} = w = m\mathbf{g}$
work equivalencies	$W = PE_e = \mathbf{f}m\mathbf{g}$
kinetic energy	$KE = \frac{1}{2}m\mathbf{v}^2$
work–energy theorem	$W = \Delta\text{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$
power	$P = \frac{W}{t}$

9.2 Mechanical Energy and Conservation of Energy

conservation of energy	$KE_1 + PE_1 = KE_2 + PE_2$

9.3 Simple Machines

ideal mechanical advantage (general)	$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$
ideal mechanical advantage (lever)	$IMA = \frac{L_e}{L_r}$
ideal mechanical advantage (wheel and axle)	$IMA = \frac{R}{r}$
ideal mechanical advantage (inclined plane)	$IMA = \frac{L}{h}$
ideal mechanical advantage (wedge)	$IMA = \frac{L}{t}$
ideal mechanical advantage (pulley)	$IMA = N$
ideal mechanical advantage (screw)	$IMA = \frac{2\pi L}{P}$
input work	$W_i = \mathbf{F}_i d_i$
output work	$W_o = \mathbf{F}_o d_o$
efficiency output	$\%\text{ efficiency} = \frac{W_o}{W_i} \times 100$

CHAPTER REVIEW

Concept Items

9.1 Work, Power, and the Work–Energy Theorem

1. Is it possible for the sum of kinetic energy and potential energy of an object to change without work having been done on the object? Explain.

a. No, because the work-energy theorem states that work done on an object is equal to the change in kinetic energy, and change in KE requires a change in velocity. It is assumed that mass is constant.
b. No, because the work-energy theorem states that work done on an object is equal to the sum of kinetic energy, and the change in KE requires a change in displacement. It is assumed that mass is constant.
c. Yes, because the work-energy theorem states that work done on an object is equal to the change in kinetic energy, and change in KE requires a change in velocity. It is assumed that mass is constant.
d. Yes, because the work-energy theorem states that work done on an object is equal to the sum of kinetic energy, and the change in KE requires a change in displacement. It is assumed that mass is constant.

2. Define work for one-dimensional motion.

a. Work is defined as the ratio of the force over the distance.
b. Work is defined as the sum of the force and the distance.
c. Work is defined as the square of the force over the distance.
d. Work is defined as the product of the force and the distance.

3. A book with a mass of 0.30 kg falls 2 m from a shelf to the floor. This event is described by the work–energy theorem: $W = fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ Explain why this is enough information to calculate the speed with which the book hits the floor.

a. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the initial velocity and v_2 is the final velocity. The final velocity is the only unknown quantity.
b. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the final velocity and v_2 is the initial velocity. The final velocity is the only unknown quantity.
c. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the initial velocity and v_2 is the final velocity. The final velocity and the initial velocities are the only unknown quantities.
d. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the final velocity and v_2 is the initial velocity. The final velocity and the initial velocities are the only unknown quantities.

9.2 Mechanical Energy and Conservation of Energy

4. Describe the changes in KE and PE of a person jumping up and down on a trampoline.
   a. While going up, the person's KE would change to PE. While coming down, the person's PE would change to KE.
   b. While going up, the person's PE would change to KE. While coming down, the person's KE would change to PE.
   c. While going up, the person's KE would not change, but while coming down, the person's PE would change to KE.
   d. While going up, the person's PE would change to KE, but while coming down, the person's KE would not change.

5. You know the height from which an object is dropped. Which equation could you use to calculate the velocity as the object hits the ground?
   a. $v = h$
   b. $v = \sqrt{2h}$
   c. $v = gh$
   d. $v = \sqrt{2gh}$

6. The starting line of a cross country foot race is at the bottom of a hill. Which form(s) of mechanical energy of the runners will change when the starting gun is fired?
   a. Kinetic energy only
   b. Potential energy only
   c. Both kinetic and potential energy
   d. Neither kinetic nor potential energy

9.3 Simple Machines

7. How does a simple machine make work easier?
   a. It reduces the input force and the output force.
   b. It reduces the input force and increases the output force.
   c. It increases the input force and reduces the output force.
   d. It increases the input force and the output force.

8. Which type of simple machine is a knife?
   a. A ramp
   b. A wedge
   c. A pulley
   d. A screw

Critical Thinking Items

9.1 Work, Power, and the Work–Energy Theorem

9. Which activity requires a person to exert force on an object that causes the object to move but does not change the kinetic or potential energy of the object?
   a. Moving an object to a greater height with acceleration
   b. Moving an object to a greater height without acceleration
   c. Carrying an object with acceleration at the same height
   d. Carrying an object without acceleration at the same height

10. Which statement explains how it is possible to carry books to school without changing the kinetic or potential energy of the books or doing any work?
   a. By moving the book without acceleration and keeping the height of the book constant
   b. By moving the book with acceleration and keeping the height of the book constant
   c. By moving the book without acceleration and changing the height of the book
   d. By moving the book with acceleration and changing the height of the book

9.2 Mechanical Energy and Conservation of Energy

11. True or false—A cyclist coasts down one hill and up another hill until she comes to a stop. The point at which the bicycle stops is lower than the point at which it started coasting because part of the original potential energy has been converted to a quantity of heat and this makes the tires of the bicycle warm.
   a. True
   b. False

9.3 Simple Machines

12. We think of levers being used to decrease effort force. Which of the following describes a lever that requires a large effort force which causes a smaller force to act over a large distance and explains how it works?
   a. Anything that is swung by a handle, such as a hammer or racket. Force is applied near the fulcrum over a short distance, which makes the other end move rapidly over a long distance.
   b. Anything that is swung by a handle, such as a hammer or racket. Force is applied far from the fulcrum over a large distance, which makes the other end move rapidly over a long distance.
   c. A lever used to lift a heavy stone. Force is applied near the fulcrum over a short distance, which

makes the other end lift a heavy object easily.

d. A lever used to lift a heavy stone. Force is applied far from the fulcrum over a large distance, which makes the other end lift a heavy object easily

13. A baseball bat is a lever. Which of the following explains how a baseball bat differs from a lever like a pry bar?
    a. In a baseball bat, effort force is smaller and is applied over a large distance, while the resistance force is smaller and is applied over a long distance.
    b. In a baseball bat, effort force is smaller and is applied over a large distance, while the resistance force is smaller and is applied over a short distance.
    c. In a baseball bat, effort force is larger and is applied over a short distance, while the resistance force is smaller and is applied over a long distance.
    d. In a baseball bat, effort force is larger and is applied over a short distance, while the resistance force is smaller and is applied over a short distance.

Problems

9.1 Work, Power, and the Work–Energy Theorem

14. A baseball player exerts a force of 100 N on a ball for a distance of 0.5 m as he throws it. If the ball has a mass of 0.15 kg, what is its velocity as it leaves his hand?
    a. -36.5 m/s
    b. -25.8 m/s
    c. 25.8 m/s
    d. 36.5 m/s

15. A boy pushes his little sister on a sled. The sled accelerates from 0 to 3.2 m/s . If the combined mass of his sister and the sled is 40.0 kg and 18 W of power were generated, how long did the boy push the sled?
    a. 205 s
    b. 128 s
    c. 23 s
    d. 11 s

9.2 Mechanical Energy and Conservation of Energy

16. What is the kinetic energy of a 0.01 kg bullet traveling at a velocity of 700 m/s?
    a. 3.5 J
    b. 7 J
    c. $2.45 \times 10^3 \text{ J}$
    d. $2.45 \times 10^5 \text{ J}$

17. A marble rolling across a flat, hard surface at 2 m/s rolls up a ramp. Assuming that $g = 10 \text{ m/s}^2$ and no energy is lost to friction, what will be the vertical height of the marble when it comes to a stop before rolling back down? Ignore effects due to the rotational kinetic energy.
    a. 0.1 m
    b. 0.2 m
    c. 0.4 m
    d. 2 m

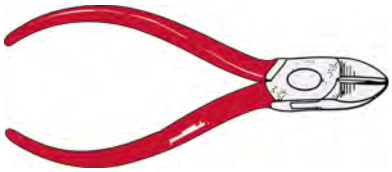
18. The potential energy stored in a compressed spring is $U = \frac{1}{2}kx^2$, where k is the force constant and x is the distance the spring is compressed from the equilibrium position. Four experimental setups described below can be used to determine the force constant of a spring. Which one(s) require measurement of the fewest number of variables to determine k? Assume the acceleration due to gravity is known.
    I. An object is propelled vertically by a compressed spring.
    II. An object is propelled horizontally on a frictionless surface by a compressed spring.
    III. An object is statically suspended from a spring.
    IV. An object suspended from a spring is set into oscillatory motion.

    a. I only
    b. III only
    c. I and II only
    d. III and IV only

9.3 Simple Machines

19. A man is using a wedge to split a block of wood by hitting the wedge with a hammer. This drives the wedge into the wood creating a crack in the wood. When he hits the wedge with a force of 400 N it travels 4 cm into the wood. This caused the wedge to exert a force of 1,400 N sideways increasing the width of the crack by 1 cm . What is the efficiency of the wedge?
    a. 0.875 percent
    b. 0.14
    c. 0.751
    d. 87.5 percent

20. An electrician grips the handles of a wire cutter, like the one shown, 10 cm from the pivot and places a wire between the jaws 2 cm from the pivot. If the cutter blades are 2 cm wide and 0.3 cm thick, what is the overall IMA of this complex machine?

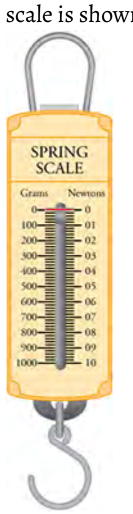


a. 1.34
b. 1.53
c. 33.3
d. 33.5

Performance Task

9.3 Simple Machines

21. Conservation of Energy and Energy Transfer; Cause and Effect; and S&EP, Planning and Carrying Out Investigations
Plan an investigation to measure the mechanical advantage of simple machines and compare to the *IMA* of the machine. Also measure the efficiency of each machine studied. Design an investigation to make these measurements for these simple machines: lever, inclined plane, wheel and axle and a pulley system. In addition to these machines, include a spring scale, a tape measure, and a weight with a loop on top that can be attached to the hook on the spring scale. A spring scale is shown in the image.



A spring scale measures weight, not mass.

LEVER: Beginning with the lever, explain how you would measure input force, output force, effort arm, and resistance arm. Also explain how you would find the distance the load travels and the distance over which the effort force is applied. Explain how you would use this data to determine *IMA* and efficiency.
INCLINED PLANE: Make measurements to determine *IMA* and efficiency of an inclined plane. Explain how you would use the data to calculate these values. Which property do you already know? Note that there are no effort and resistance arm measurements, but there are height and length measurements.
WHEEL AND AXLE: Again, you will need two force measurements and four distance measurements. Explain how you would use these to calculate *IMA* and efficiency.
SCREW: You will need two force measurements, two distance traveled measurements, and two length measurements. You may describe a screw like the one shown in Figure 9.10 or you could use a screw and screw driver. (Measurements would be easier for the former). Explain how you would use these to calculate *IMA* and efficiency.
PULLEY SYSTEM: Explain how you would determine the *IMA* and efficiency of the four-pulley system shown in Figure 9.11. Why do you only need two distance measurements for this machine?
Design a table that compares the efficiency of the five simple machines. Make predictions as to the most and least efficient machines.

TEST PREP

Multiple Choice

9.1 Work, Power, and the Work–Energy Theorem

22. Which expression represents power?
a. fd
b. mgh
c. $\frac{mv^2}{2}$
d. $\frac{W}{t}$

23. The work–energy theorem states that the change in the kinetic energy of an object is equal to what?
a. The work done on the object
b. The force applied to the object
c. The loss of the object’s potential energy
d. The object’s total mechanical energy minus its kinetic energy

24. A runner at the start of a race generates 250 W of power as he accelerates to 5 m/s . If the runner has a mass of 60 kg, how long did it take him to reach that speed?
a. 0.33 s
b. 0.83 s
c. 1.2 s
d. 3.0 s

25. A car's engine generates 100,000 W of power as it exerts a force of 10,000 N. How long does it take the car to travel 100 m?
    a. 0.001 s
    b. 0.01 s
    c. 10 s
    d. 1,000 s

9.2 Mechanical Energy and Conservation of Energy

26. Why is this expression for kinetic energy incorrect? $\mathrm{KE} = (m)(v)^2$.
    a. The constant g is missing.
    b. The term v should not be squared.
    c. The expression should be divided by 2.
    d. The energy lost to friction has not been subtracted.

27. What is the kinetic energy of a $10\,\mathrm{kg}$ object moving at $2.0\,\mathrm{m/s}$?
    a. $10\,\mathrm{J}$
    b. $20\,\mathrm{J}$
    c. $40\,\mathrm{J}$
    d. $100\,\mathrm{J}$

28. Which statement best describes the PE-KE transformations for a javelin, starting from the instant the javelin leaves the thrower's hand until it hits the ground.
    a. Initial PE is transformed to KE until the javelin reaches the high point of its arc. On the way back down, KE is transformed into PE. At every point in the flight, mechanical energy is being transformed into heat energy.
    b. Initial KE is transformed to PE until the javelin reaches the high point of its arc. On the way back down, PE is transformed into KE. At every point in the flight, mechanical energy is being transformed into heat energy.
    c. Initial PE is transformed to KE until the javelin reaches the high point of its arc. On the way back down, there is no transformation of mechanical energy. At every point in the flight, mechanical energy is being transformed into heat energy.
    d. Initial KE is transformed to PE until the javelin reaches the high point of its arc. On the way back down, there is no transformation of mechanical energy. At every point in the flight, mechanical energy is being transformed into heat energy.

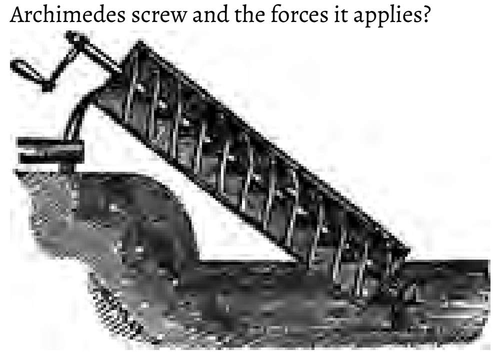
29. At the beginning of a roller coaster ride, the roller coaster car has an initial energy mostly in the form of PE. Which statement explains why the fastest speeds of the car will be at the lowest points in the ride?
    a. At the bottom of the slope kinetic energy is at its maximum value and potential energy is at its minimum value.
    b. At the bottom of the slope potential energy is at its maximum value and kinetic energy is at its minimum value.
    c. At the bottom of the slope both kinetic and potential energy reach their maximum values
    d. At the bottom of the slope both kinetic and potential energy reach their minimum values.

9.3 Simple Machines

30. A large radius divided by a small radius is the expression used to calculate the IMA of what?
    a. A screw
    b. A pulley
    c. A wheel and axle
    d. An inclined plane.

31. What is the IMA of a wedge that is $12\,\mathrm{cm}$ long and $3\,\mathrm{cm}$ thick?
    a. 2
    b. 3
    c. 4
    d. 9

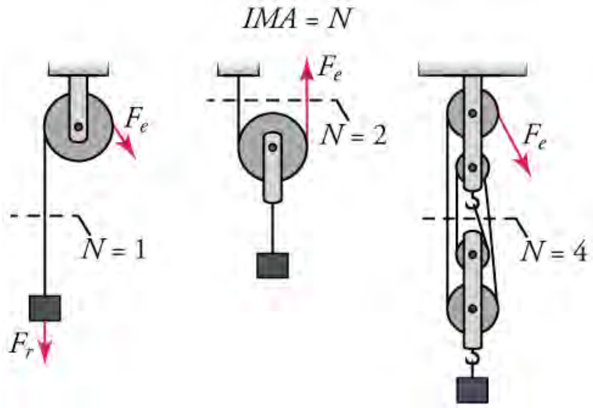
32. Which statement correctly describes the simple machines, like the crank in the image, that make up an Archimedes screw and the forces it applies?



    a. The crank is a wedge in which the IMA is the length of the tube divided by the radius of the tube. The applied force is the effort force and the weight of the water is the resistance force.
    b. The crank is an inclined plane in which the IMA is the length of the tube divided by the radius of the tube. The applied force is the effort force and the weight of the water is the resistance force.
    c. The crank is a wheel and axle. The effort force of the crank becomes the resistance force of the screw.
    d. The crank is a wheel and axle. The resistance force of the crank becomes the effort force of the screw.

33. Refer to the pulley system on right in the image. Assume this pulley system is an ideal machine.
How hard would you have to pull on the rope to lift a 120 N

load?
How many meters of rope would you have to pull out of the system to lift the load 1 m?



a. 480 N
4 m
b. 480 N
$\frac{1}{4}$ m
c. 30 N
4 m
d. 30 N
$\frac{1}{4}$ m

Short Answer

9.1 Work, Power, and the Work–Energy Theorem

34. Describe two ways in which doing work on an object can increase its mechanical energy.
   a. Raising an object to a higher elevation does work as it increases its PE; increasing the speed of an object does work as it increases its KE.
   b. Raising an object to a higher elevation does work as it increases its KE; increasing the speed of an object does work as it increases its PE.
   c. Raising an object to a higher elevation does work as it increases its PE; decreasing the speed of an object does work as it increases its KE.
   d. Raising an object to a higher elevation does work as it increases its KE; decreasing the speed of an object does work as it increases its PE.

35. True or false—While riding a bicycle up a gentle hill, it is fairly easy to increase your potential energy, but to increase your kinetic energy would make you feel exhausted.
   a. True
   b. False

36. Which statement best explains why running on a track with constant speed at 3 m/s is not work, but climbing a mountain at 1 m/s is work?
   a. At constant speed, change in the kinetic energy is zero but climbing a mountain produces change in the potential energy.
   b. At constant speed, change in the potential energy is zero, but climbing a mountain produces change in the kinetic energy.
   c. At constant speed, change in the kinetic energy is finite, but climbing a mountain produces no change in the potential energy.
   d. At constant speed, change in the potential energy is finite, but climbing a mountain produces no change in the kinetic energy.

37. You start at the top of a hill on a bicycle and coast to the bottom without applying the brakes. By the time you reach the bottom of the hill, work has been done on you and your bicycle, according to the equation: $W = \frac{1}{2}m(v_2^2 - v_1^2)$ If m is the mass of you and your bike, what are v_1 and v_2?
   a. v_1 is your speed at the top of the hill, and v_2 is your speed at the bottom.
   b. v_1 is your speed at the bottom of the hill, and v_2 is your speed at the top.
   c. v_1 is your displacement at the top of the hill, and v_2 is your displacement at the bottom.
   d. v_1 is your displacement at the bottom of the hill, and v_2 is your displacement at the top.

9.2 Mechanical Energy and Conservation of Energy

38. True or false—The formula for gravitational potential energy can be used to explain why joules, J, are equivalent to kg × mg² / s² . Show your work.
   a. True
   b. False

39. Which statement best explains why accelerating a car from 20 mph to 40 mph quadruples its kinetic energy?
   a. Because kinetic energy is directly proportional to the square of the velocity.
   b. Because kinetic energy is inversely proportional to the square of the velocity.
   c. Because kinetic energy is directly proportional to the fourth power of the velocity.
   d. Because kinetic energy is inversely proportional to

the fourth power of the velocity.

40. A coin falling through a vacuum loses no energy to friction, and yet, after it hits the ground, it has lost all its potential and kinetic energy. Which statement best explains why the law of conservation of energy is still valid in this case?

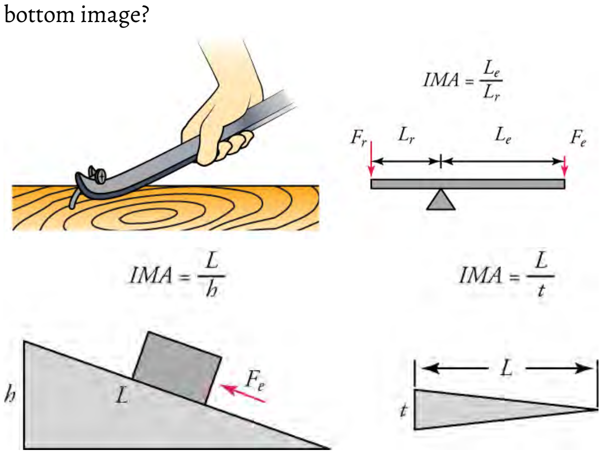
a. When the coin hits the ground, the ground gains potential energy that quickly changes to thermal energy.
b. When the coin hits the ground, the ground gains kinetic energy that quickly changes to thermal energy.
c. When the coin hits the ground, the ground gains thermal energy that quickly changes to kinetic energy.
d. When the coin hits the ground, the ground gains thermal energy that quickly changes to potential energy.

41. True or false—A marble rolls down a slope from height h_1 and up another slope to height h_2, where ($h_2 < h_1$). The difference $mg(h_1 - h_2)$ is equal to the heat lost due to the friction.

a. True
b. False

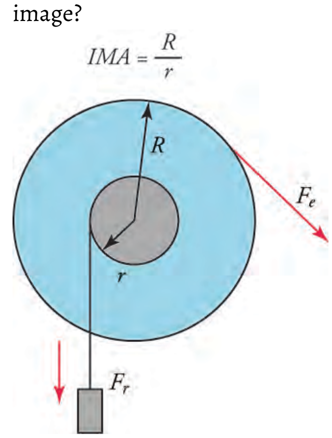
9.3 Simple Machines

42. Why would you expect the lever shown in the top image to have a greater efficiency than the inclined plane shown in the bottom image?



a. The resistance arm is shorter in case of the inclined plane.
b. The effort arm is shorter in case of the inclined plane.
c. The area of contact is greater in case of the inclined plane.

43. Why is the wheel on a wheelbarrow **not** a simple machine in the same sense as the simple machine in the image?



a. The wheel on the wheelbarrow has no fulcrum.
b. The center of the axle is not the fulcrum for the wheels of a wheelbarrow.
c. The wheelbarrow differs in the way in which load is attached to the axle.
d. The wheelbarrow has less resistance force than a wheel and axle design.

44. A worker pulls down on one end of the rope of a pulley system with a force of 75 N to raise a hay bale tied to the other end of the rope. If she pulls the rope down 2.0 m and the bale raises 1.0 m, what else would you have to know to calculate the efficiency of the pulley system?

a. the weight of the worker
b. the weight of the hay bale
c. the radius of the pulley
d. the height of the pulley from ground

45. True or false—A boy pushed a box with a weight of 300 N up a ramp. He said that, because the ramp was 1.0 m high and 3.0 m long, he must have been pushing with force of exactly 100 N.

a. True
b. False

Extended Response

9.1 Work, Power, and the Work–Energy Theorem

46. Work can be negative as well as positive because an object or system can do work on its surroundings as well as have work done on it. Which of the following statements describes:
a situation in which an object does work on its surroundings by decreasing its velocity and
a situation in which an object can do work on its surroundings by decreasing its altitude?

a. A gasoline engine burns less fuel at a slower speed. Solar cells capture sunlight to generate electricity.

b. A hybrid car charges its batteries as it decelerates.
Falling water turns a turbine to generate electricity.
c. Airplane flaps use air resistance to slow down for landing.
Rising steam turns a turbine to generate electricity.
d. An electric train requires less electrical energy as it decelerates.
A parachute captures air to slow a skydiver's fall.

47. A boy is pulling a girl in a child's wagon at a constant speed. He begins to pull harder, which increases the speed of the wagon. Which of the following describes two ways you could calculate the change in energy of the wagon and girl if you had all the information you needed?
a. Calculate work done from the force and the velocity.
Calculate work done from the change in the potential energy of the system.
b. Calculate work done from the force and the displacement.
Calculate work done from the change in the potential energy of the system.
c. Calculate work done from the force and the velocity.
Calculate work done from the change in the kinetic energy of the system.
d. Calculate work done from the force and the displacement.
Calculate work done from the change in the kinetic energy of the system.

9.2 Mechanical Energy and Conservation of Energy

48. Acceleration due to gravity on the moon is 1.6 m/s^2 or about 16% of the value of g on Earth.
If an astronaut on the moon threw a moon rock to a height of 7.8 m, what would be its velocity as it struck the moon's surface?
How would the fact that the moon has no atmosphere affect the velocity of the falling moon rock? Explain your answer.
a. The velocity of the rock as it hits the ground would be 5.0 m/s. Due to the lack of air friction, there would be complete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.
b. The velocity of the rock as it hits the ground would be 5.0 m/s. Due to the lack of air friction, there would be incomplete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.
c. The velocity of the rock as it hits the ground would be 12 m/s. Due to the lack of air friction, there would be complete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.
d. The velocity of the rock as it hits the ground would be 12 m/s. Due to the lack of air friction, there would be incomplete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.

49. A boulder rolls from the top of a mountain, travels across a valley below, and rolls part way up the ridge on the opposite side. Describe all the energy transformations taking place during these events and identify when they happen.
a. As the boulder rolls down the mountainside, KE is converted to PE. As the boulder rolls up the opposite slope, PE is converted to KE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.
b. As the boulder rolls down the mountainside, KE is converted to PE. As the boulder rolls up the opposite slope, KE is converted to PE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.
c. As the boulder rolls down the mountainside, PE is converted to KE. As the boulder rolls up the opposite slope, PE is converted to KE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.
d. As the boulder rolls down the mountainside, PE is converted to KE. As the boulder rolls up the opposite slope, KE is converted to PE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.

9.3 Simple Machines

50. To dig a hole, one holds the handles together and thrusts the blades of a posthole digger, like the one in the image, into the ground. Next, the handles are pulled apart, which squeezes the dirt between them, making it possible to remove the dirt from the hole. This complex machine is composed of two pairs of two different simple machines. Identify and describe the parts that are simple machines and explain how you would find the IMA of each type of simple machine.

a. Each handle and its attached blade is a lever with the

fulcrum at the hinge. Each blade is a wedge.
The IMA of a lever would be the length of the handle divided by the length of the blade. The IMA of the wedges would be the length of the blade divided by its width.

b. Each handle and its attached to blade is a lever with the fulcrum at the end. Each blade is a wedge.
The IMA of a lever would be the length of the handle divided by the length of the blade. The IMA of the wedges would be the length of the blade divided by its width.

c. Each handle and its attached blade is a lever with the fulcrum at the hinge. Each blade is a wedge.
The IMA of a lever would be the length of the handle multiplied by the length of the blade. The IMA of the wedges would be the length of the blade multiplied by its width.

d. Each handle and its attached blade is a lever with the fulcrum at the end. Each blade is a wedge.
The IMA of a lever would be the length of the handle multiplied by the length of the blade. The IMA of the wedges would be the length of the blade multiplied by its width.

51. A wooden crate is pulled up a ramp that is 1.0 m high and 6.0 m long. The crate is attached to a rope that is wound around an axle with a radius of 0.020 m . The axle is turned by a 0.20 m long handle. What is the overall IMA of the complex machine?

A. 6
B. 10
C. 16
D. 60

CHAPTER 10
Special Relativity

Figure 10.1 Special relativity explains why travel to other star systems, such as these in the Orion Nebula, is unlikely using our current level of technology. (s58y, Flickr)

Chapter Outline

INTRODUCTION Have you ever dreamed of traveling to other planets in faraway star systems? The trip might seem possible by traveling fast enough, but you will read in this chapter why it is not. In 1905, Albert Einstein developed the theory of **special relativity**. Einstein developed the theory to help explain inconsistencies between the equations describing electromagnetism and Newtonian mechanics, and to explain why the ether did not exist. This theory explains the limit on an object's speed among other implications.

Relativity is the study of how different observers moving with respect to one another measure the same events. Galileo and Newton developed the first correct version of classical relativity. Einstein developed the modern theory of relativity. Modern relativity is divided into two parts. Special relativity deals with observers moving at constant velocity. **General relativity** deals with observers moving at constant acceleration. Einstein's theories of relativity made revolutionary predictions. Most importantly, his predictions have been verified by experiments.

In this chapter, you learn how experiments and puzzling contradictions in existing theories led to the development of the theory of special relativity. You will also learn the simple postulates on which the theory was based; a postulate is a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument.

10.1 Postulates of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the experiments and scientific problems that led Albert Einstein to develop the special theory of relativity
- Understand the postulates on which the special theory of relativity was based

Section Key Terms

ether	frame of reference	inertial reference frame
general relativity	postulate	relativity
simultaneity	special relativity	

Scientific Experiments and Problems

Relativity is not new. Way back around the year 1600, Galileo explained that motion is relative. Wherever you happen to be, it seems like you are at a fixed point and that everything moves with respect to you. Everyone else feels the same way. Motion is always measured with respect to a fixed point. This is called establishing a **frame of reference**. But the choice of the point is arbitrary, and all frames of reference are equally valid. A passenger in a moving car is not moving with respect to the driver, but they are both moving from the point of view of a person on the sidewalk waiting for a bus. They are moving even faster as seen by a person in a car coming toward them. It is all relative.

TIPS FOR SUCCESS

A frame of reference is not a complicated concept. It is just something you decide is a fixed point or group of connected points. It is completely up to you. For example, when you look up at celestial objects in the sky, you choose the earth as your frame of reference, and the sun, moon, etc., seem to move across the sky.

Light is involved in the discussion of relativity because theories related to electromagnetism are inconsistent with Galileo's and Newton's explanation of relativity. The true nature of light was a hot topic of discussion and controversy in the late 19th century. At the time, it was not generally believed that light could travel across empty space. It was known to travel as waves, and all other types of energy that propagated as waves needed to travel though a material medium. It was believed that space was filled with an invisible medium that light waves traveled through. This imaginary (as it turned out) material was called the **ether** (also spelled aether). It was thought that everything moved through this mysterious fluid. In other words, ether was the one fixed frame of reference. The Michelson–Morley experiment proved it was not.

In 1887, Albert Michelson and Edward Morley designed the interferometer shown in Figure 10.2 to measure the speed of Earth through the ether. A light beam is split into two perpendicular paths and then recombined. Recombining the waves produces an inference pattern, with a bright fringe at the locations where the two waves arrive in phase; that is, with the crests of both waves arriving together and the troughs arriving together. A dark fringe appears where the crest of one wave coincides with a trough of the other, so that the two cancel. If Earth is traveling through the ether as it orbits the sun, the peaks in one arm would take longer than in the other to reach the same location. The places where the two waves arrive in phase would change, and the interference pattern would shift. But, using the interferometer, there was no shift seen! This result led to two conclusions: that there is no ether and that the speed of light is the same regardless of the relative motion of source and observer. The Michelson–Morley investigation has been called the most famous failed experiment in history.

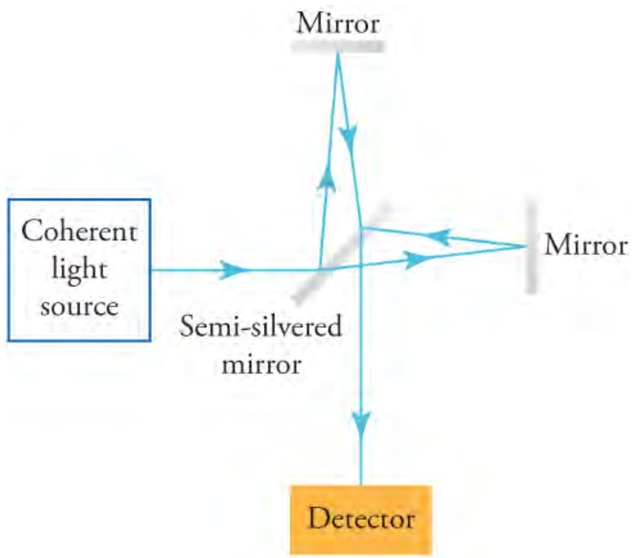


Figure 10.2 This is a diagram of the instrument used in the Michelson–Morley experiment.

To see what Michelson and Morley expected to find when they measured the speed of light in two directions, watch this animation (http://openstax.org/l/28MMexperiment) . In the video, two people swimming in a lake are represented as an analogy to light beams leaving Earth as it moves through the ether (if there were any ether). The swimmers swim away from and back to a platform that is moving through the water. The swimmers swim in different directions with respect to the motion of the platform. Even though they swim equal distances at the same speed, the motion of the platform causes them to arrive at different times.

Einstein's Postulates

The results described above left physicists with some puzzling and unsettling questions such as, why doesn't light emitted by a fast-moving object travel faster than light from a street lamp? A radical new theory was needed, and Albert Einstein, shown in Figure 10.3, was about to become everyone's favorite genius. Einstein began with two simple **postulates** based on the two things we have discussed so far in this chapter.

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light is the same in all inertial reference frames and is not affected by the speed of its source.

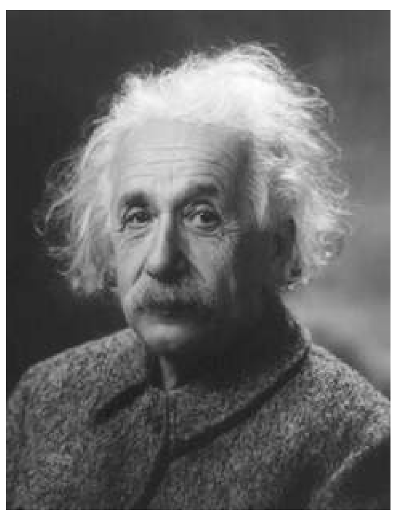


Figure 10.3 Albert Einstein (1879–1955) developed modern relativity and also made fundamental contributions to the foundations of quantum mechanics. (The Library of Congress)

The speed of light is given the symbol *c* and is equal to exactly 299,792,458 m/s. This is the speed of light in vacuum; that is, in the absence of air. For most purposes, we round this number off to 3.00×10^8 m/s The term **inertial reference frame** simply refers to a frame of reference where all objects follow Newton's first law of motion: Objects at rest remain at rest, and objects in motion remain in motion at a constant velocity in a straight line, unless acted upon by an external force. The inside of a car moving along a road at constant velocity and the inside of a stationary house are inertial reference frames.

WATCH PHYSICS

The Speed of Light

This lecture on light summarizes the most important facts about the speed of light. If you are interested, you can watch the whole video, but the parts relevant to this chapter are found between 3:25 and 5:10, which you find by running your cursor along the bottom of the video.

Click to view content (https://www.youtube.com/embed/rLNM8zI4Q_M)

GRASP CHECK

An airliner traveling at 200 m/s emits light from the front of the plane. Which statement describes the speed of the light?

a. It travels at a speed of $c + 200$ m/s.
b. It travels at a speed of $c - 200$ m/s.
c. It travels at a speed c, like all light.
d. It travels at a speed slightly less than c.

Snap Lab

Measure the Speed of Light

In this experiment, you will measure the speed of light using a microwave oven and a slice of bread. The waves generated by a microwave oven are not part of the visible spectrum, but they are still electromagnetic radiation, so they travel at the speed of light. If we know the wavelength, λ, and frequency, f, of a wave, we can calculate its speed, v, using the equation $v = \lambda f$. You can measure the wavelength. You will find the frequency on a label on the back of a microwave oven. The wave in a microwave is a standing wave with areas of high and low intensity. The high intensity sections are one-half wavelength apart.

- High temperature: Very hot temperatures are encountered in this lab. These can cause burns.

- a microwave oven
- one slice of plain white bread
- a centimeter ruler
- a calculator

1. Work with a partner.
2. Turn off the revolving feature of the microwave oven or remove the wheels under the microwave dish that make it turn. It is important that the dish does not turn.
3. Place the slice of bread on the dish, set the microwave on high, close the door, run the microwave for about 15 seconds.
4. A row of brown or black marks should appear on the bread. Stop the microwave as soon as they appear. Measure the distance between two adjacent burn marks and multiply the result by 2. This is the wavelength.
5. The frequency of the waves is written on the back of the microwave. Look for something like "2,450 MHz." Hz is the unit hertz, which means *per second*. The M represents mega, which stands for million, so multiply the number by 10^6.
6. Express the wavelength in meters and multiply it times the frequency. If you did everything correctly, you will get a number very close to the speed of light. Do not eat the bread. It is a general laboratory safety rule never to eat anything in the lab.

GRASP CHECK

How does your measured value of the speed of light compare to the accepted value (% error)?

a. The measured value of speed will be equal to *c*.
b. The measured value of speed will be slightly less than *c*.
c. The measured value of speed will be slightly greater than *c*.
d. The measured value of speed will depend on the frequency of the microwave.

Einstein's postulates were carefully chosen, and they both seemed very likely to be true. Einstein proceeded despite realizing that these two ideas taken together and applied to extreme conditions led to results that contradict Newtonian mechanics. He just took the ball and ran with it.

In the traditional view, velocities are additive. If you are running at 3 m/s and you throw a ball forward at a speed of 10 m/s, the ball should have a net speed of 13 m/s. However, according to relativity theory, the speed of a moving light source is not added to the speed of the emitted light.

In addition, Einstein's theory shows that if you were moving forward relative to Earth at nearly *c* (the speed of light) and could throw a ball forward at *c*, an observer at rest on the earth would not see the ball moving at nearly twice the speed of light. The observer would see it moving at a speed that is still less than *c*. This result conforms to both of Einstein's postulates: The speed of light has a fixed maximum and neither reference frame is privileged.

Consider how we measure elapsed time. If we use a stopwatch, for example, how do we know when to start and stop the watch? One method is to use the arrival of light from the event, such as observing a light turn green to start a drag race. The timing will be more accurate if some sort of electronic detection is used, avoiding human reaction times and other complications.

Now suppose we use this method to measure the time interval between two flashes of light produced by flash lamps on a moving train. (See Figure 10.4)

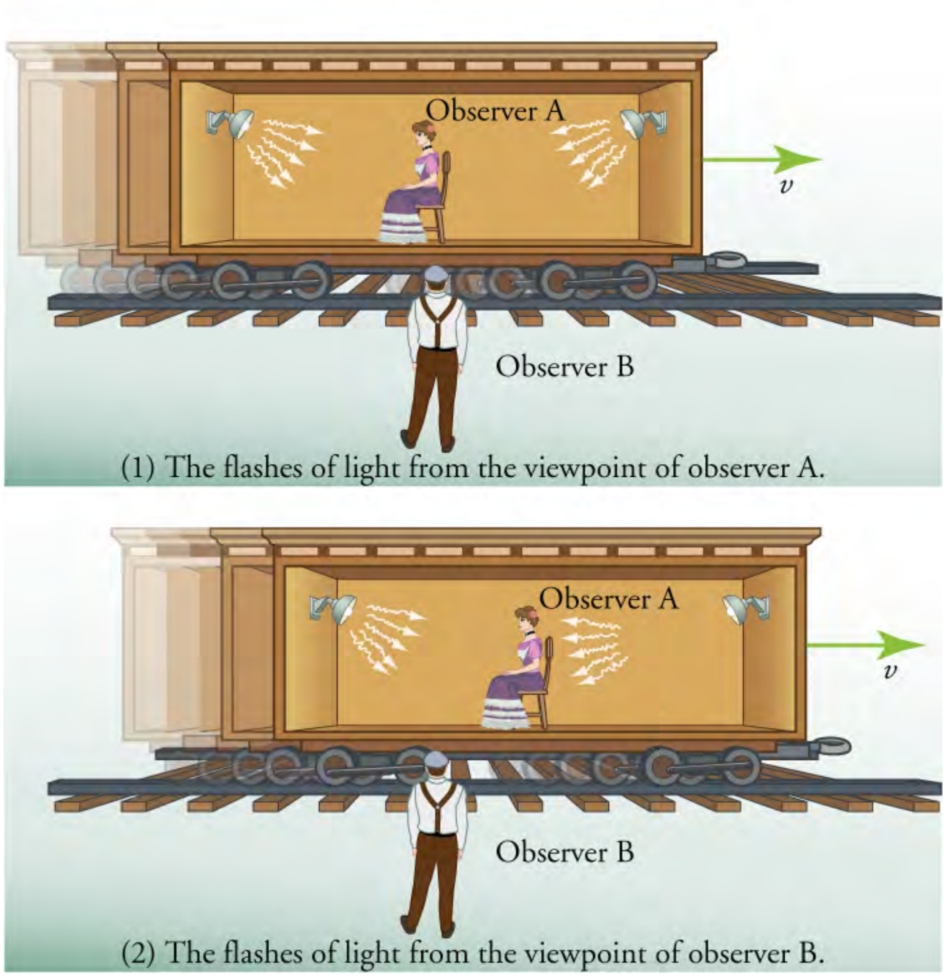


Figure 10.4 Light arriving to observer A as seen by two different observers.

A woman (observer A) is seated in the center of a rail car, with two flash lamps at opposite sides equidistant from her. Multiple light rays that are emitted from the flash lamps move towards observer A, as shown with arrows. A velocity vector arrow for the rail car is shown towards the right. A man (observer B) standing on the platform is facing the woman and also observes the flashes of light.

Observer A moves with the lamps on the rail car as the rail car moves towards the right of observer B. Observer B receives the light flashes simultaneously, and sees the bulbs as both having flashed at the same time. However, he sees observer A receive the flash from the right first. Because the pulse from the right reaches her first, in her frame of reference she sees the bulbs as not having flashed simultaneously. Here, a relative velocity between observers affects whether two events at well-separated locations are observed to be simultaneous. **Simultaneity**, or whether different events occur at the same instant, depends on the frame of reference of the observer. Remember that velocity equals distance divided by time, so $t = d/\mathbf{v}$. If velocity appears to be different, then duration of time appears to be different.

This illustrates the power of clear thinking. We might have guessed incorrectly that, if light is emitted simultaneously, then two observers halfway between the sources would see the flashes simultaneously. But careful analysis shows this not to be the case. Einstein was brilliant at this type of thought experiment (in German, *Gedankenexperiment*). He very carefully considered how an observation is made and disregarded what might seem obvious. The validity of thought experiments, of course, is determined by actual observation. The genius of Einstein is evidenced by the fact that experiments have repeatedly confirmed his theory of relativity. No experiments after that of Michelson and Morley were able to detect any ether medium. We will describe later how experiments also confirmed other predictions of **special relativity**, such as the distance between two objects and the time interval of two events being different for two observers moving with respect to each other.

In summary: Two events are defined to be simultaneous if an observer measures them as occurring at the same time (such as by receiving light from the events). Two events are not necessarily simultaneous to all observers.

The discrepancies between Newtonian mechanics and relativity theory illustrate an important point about how science advances. Einstein's theory did not replace Newton's but rather extended it. It is not unusual that a new theory must be developed to account for new information. In most cases, the new theory is built on the foundation of older theory. It is rare that old theories are completely replaced.

In this chapter, you will learn about the theory of special relativity, but, as mentioned in the introduction, Einstein developed two relativity theories: special and general. Table 10.1 summarizes the differences between the two theories.

Special Relativity	General Relativity
Published in 1905	Final form published in 1916
A theory of space-time	A theory of gravity
Applies to observers moving at constant speed	Applies to observers that are accelerating
Most useful in the field of nuclear physics	Most useful in the field of astrophysics
Accepted quickly and put to practical use by nuclear physicists and quantum chemists	Largely ignored until 1960 when new mathematical techniques made the theory more accessible and astronomers found some important applications

Also note that the theory of general relativity includes the theory of special relativity.

Table 10.1 Comparing Special Relativity and General Relativity

WORKED EXAMPLE

Calculating the Time it Takes Light to Travel a Given Distance

The sun is 1.50×10^8 km from Earth. How long does it take light to travel from the sun to Earth in minutes and seconds?

Strategy

Identify knowns.

$$\begin{aligned} \text{Distance} &= 1.50\ \times\ 10^8\ \text{km} \\ \text{Speed} &= 3.00\ \times\ 10^8\ \text{km} \end{aligned} \tag{10.1}$$

Identify unknowns.

Time

Find the equation that relates knowns and unknowns.

$$v = \frac{d}{t};\quad t = \frac{d}{v} \tag{10.2}$$

Be sure to use consistent units.

Solution

$$t = \frac{d}{v} = \frac{(1.50\ \times\ 10^8 km)\ \times\ \frac{10^3 \text{m}}{\text{km}}}{3.00\ \times\ 10^8\ \frac{\text{m}}{\text{s}}} = 5.00 \times 10^2 s$$

$$500\ \text{s} = 8\ \text{min and}\ 20\ \text{s}$$

Discussion

The answer is written as 5.00×10^2 rather than 500 in order to show that there are three significant figures. When astronomers witness an event on the sun, such as a sunspot, it actually happened minutes earlier. Compare 8 light *minutes* to the distance to stars, which are light *years* away. Any events on other stars happened years ago.

Practice Problems

1. Light travels through 1.00 m of water in 4.42×10^{-9} s. What is the speed of light in water?
   a. 4.42×10^{-9} m/s
   b. 4.42×10^{9} m/s
   c. 2.26×10^{8} m/s
   d. 226×10^{8} m/s

2. An astronaut on the moon receives a message from mission control on Earth. The signal is sent by a form of electromagnetic radiation and takes 1.28 s to travel the distance between Earth and the moon. What is the distance from Earth to the moon?
   a. 2.34×10^{5} km
   b. 2.34×10^{8} km
   c. 3.84×10^{5} km
   d. 3.84×10^{8} km

Check Your Understanding

3. Explain what is meant by a frame of reference.
   a. A frame of reference is a graph plotted between distance and time.
   b. A frame of reference is a graph plotted between speed and time.
   c. A frame of reference is the velocity of an object through empty space without regard to its surroundings.
   d. A frame of reference is an arbitrarily fixed point with respect to which motion of other points is measured.

4. Two people swim away from a raft that is floating downstream. One swims upstream and returns, and the other swims across the current and back. If this scenario represents the Michelson–Morley experiment, what do (i) the water, (ii) the swimmers, and (iii) the raft represent?
   a. the ether rays of light Earth
   b. rays of light the ether Earth
   c. the ether Earth rays of light
   d. Earth rays of light the ether

5. If Michelson and Morley had observed the interference pattern shift in their interferometer, what would that have indicated?
   a. The speed of light is the same in all frames of reference.
   b. The speed of light depends on the motion relative to the ether.
   c. The speed of light changes upon reflection from a surface.
   d. The speed of light in vacuum is less than 3.00×10^{8} m/s.

6. If you designate a point as being fixed and use that point to measure the motion of surrounding objects, what is the point called?
   a. An origin
   b. A frame of reference
   c. A moving frame
   d. A coordinate system

10.2 Consequences of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the relativistic effects seen in time dilation, length contraction, and conservation of relativistic momentum
- Explain and perform calculations involving mass-energy equivalence

Section Key Terms

binding energy	length contraction	mass defect	time dilation

proper length	relativistic	relativistic momentum
relativistic energy	relativistic factor	rest mass

Relativistic Effects on Time, Distance, and Momentum

Consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect. **Time dilation** is the phenomenon of time passing more slowly for an observer who is moving relative to another observer.

For example, suppose an astronaut measures the time it takes for light to travel from the light source, cross her ship, bounce off a mirror, and return. (See Figure 10.5.) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the earth bound observer. The passage of time is different for the two observers because the distance the light travels in the astronaut's frame is smaller than in the earth bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the earth bound frame.

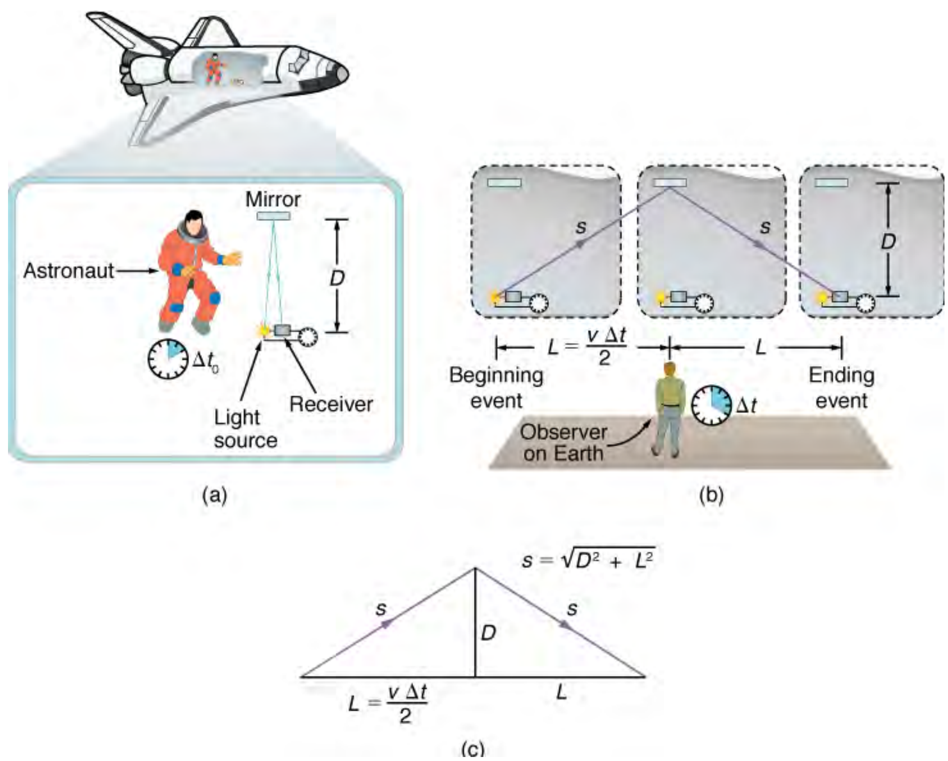


Figure 10.5 (a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut's frame. (b) A person on the earth sees the light follow the longer path $2s$ and take a longer time Δt.

The relationship between Δt and Δt_0 is given by

$$\Delta t = \gamma \Delta t_0,$$

where γ is the **relativistic factor** given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and v and c are the speeds of the moving observer and light, respectively.

TIPS FOR SUCCESS

Try putting some values for *v* into the expression for the relativistic factor (γ). Observe at which speeds this factor will make a difference and when γ is so close to 1 that it can be ignored. Try 225 m/s, the speed of an airliner; 2.98×10^4 m/s, the speed of Earth in its orbit; and 2.990×10^8 m/s, the speed of a particle in an accelerator.

Notice that when the velocity *v* is small compared to the speed of light *c*, then *v/c* becomes small, and γ becomes close to 1. When this happens, time measurements are the same in both frames of reference. **Relativistic** effects, meaning those that have to do with special relativity, usually become significant when speeds become comparable to the speed of light. This is seen to be the case for time dilation.

You may have seen science fiction movies in which space travelers return to Earth after a long trip to find that the planet and everyone on it has aged much more than they have. This type of scenario is a based on a thought experiment, known as the twin paradox, which imagines a pair of twins, one of whom goes on a trip into space while the other stays home. When the space traveler returns, she finds her twin has aged much more than she. This happens because the traveling twin has been in two frames of reference, one leaving Earth and one returning.

Time dilation has been confirmed by comparing the time recorded by an atomic clock sent into orbit to the time recorded by a clock that remained on Earth. GPS satellites must also be adjusted to compensate for time dilation in order to give accurate positioning.

Have you ever driven on a road, like that shown in Figure 10.6, that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it is about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers moving with respect to one other.

Figure 10.6 People might describe distances differently, but at relativistic speeds, the distances really are different. (Corey Leopold, Flickr)

One thing all observers agree upon is their relative speed. When one observer is traveling away from another, they both see the other receding at the same speed, regardless of whose frame of reference is chosen. Remember that speed equals distance divided by time: $v = d/t$. If the observers experience a difference in elapsed time, they must also observe a difference in distance traversed. This is because the ratio d/t must be the same for both observers.

The shortening of distance experienced by an observer moving with respect to the points whose distance apart is measured is called **length contraction**. **Proper length**, L_0, is the distance between two points measured in the reference frame where the observer and the points are at rest. The observer in motion with respect to the points measures *L*. These two lengths are related by the equation

$$L = \frac{L_0}{\gamma}.$$

Because γ is the same expression used in the time dilation equation above, the equation becomes

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}.$$

To see how length contraction is seen by a moving observer, go to this simulation (http://openstax.org/l/28simultaneity) . Here you can also see that simultaneity, time dilation, and length contraction are interrelated phenomena.

This link is to a simulation that illustrates the relativity of simultaneous events.

In classical physics, momentum is a simple product of mass and velocity. When special relativity is taken into account, objects that have mass have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds; i.e., speeds close to the speed of light?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved in classical mechanics whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. We will see that momentum has the same importance in modern physics. **Relativistic momentum** is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

One of the postulates of special relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic momentum, $\mathbf{p}$, is classical momentum multiplied by the relativistic factor γ.

$$\mathbf{p} = \gamma m \mathbf{u}, \tag{10.3}$$

where m is the **rest mass** of the object (that is, the mass measured at rest, without any γ factor involved), $\mathbf{u}$ is its velocity relative to an observer, and γ, as before, is the relativistic factor. We use the mass of the object as measured at rest because we cannot determine its mass while it is moving.

Note that we use $\mathbf{u}$ for velocity here to distinguish it from relative velocity $\mathbf{v}$ between observers. Only one observer is being considered here. With $\mathbf{p}$ defined in this way, $\mathbf{p}_{\text{tot}}$ is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum $\gamma m\mathbf{u}$ becomes the classical $m\mathbf{u}$ at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities. Because of the factor γ, however, relativistic momentum behaves differently from classical momentum by approaching infinity as $\mathbf{u}$ approaches c. (See Figure 10.7.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, which is an unreasonable value.

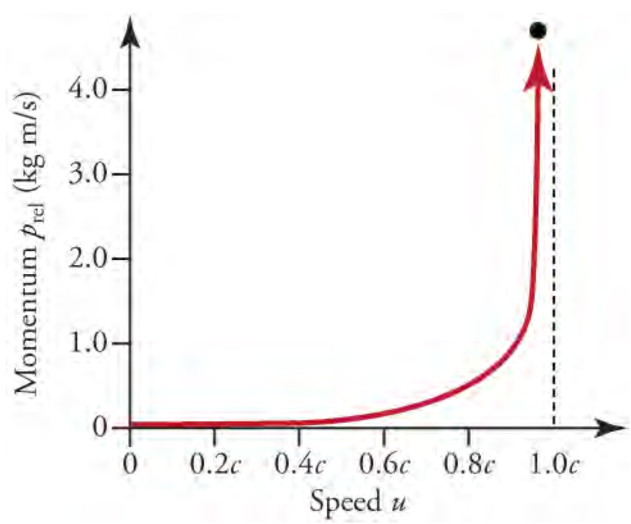


Figure 10.7 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

Mass-Energy Equivalence

Let us summarize the calculation of relativistic effects on objects moving at speeds near the speed of light. In each case we will need to calculate the relativistic factor, given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}},$$

where **v** and *c* are as defined earlier. We use **u** as the velocity of a particle or an object in one frame of reference, and **v** for the velocity of one frame of reference with respect to another.

Time Dilation

Elapsed time on a moving object, Δt_0, as seen by a stationary observer is given by $\Delta t = \gamma \Delta t_0$, where Δt_0 is the time observed on the moving object when it is taken to be the frame or reference.

Length Contraction

Length measured by a person at rest with respect to a moving object, L, is given by

$$L = \frac{L_0}{\gamma},$$

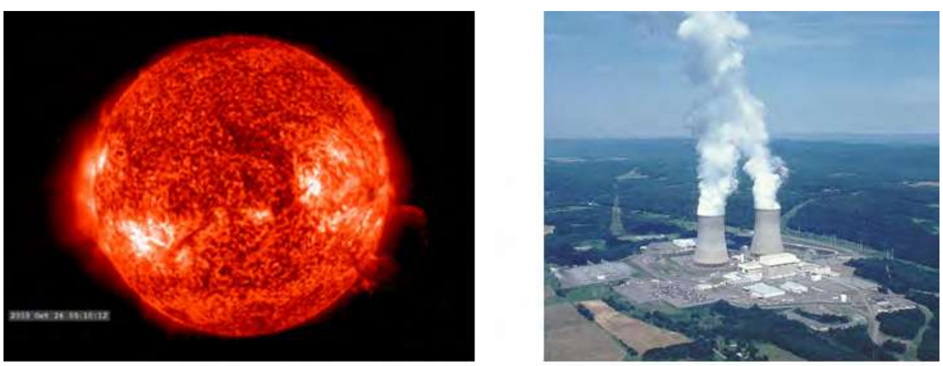
where L_0 is the length measured on the moving object.

Relativistic Momentum

Momentum, $\mathbf{p}$, of an object of mass, m, traveling at relativistic speeds is given by $\mathbf{p} = \gamma m \mathbf{u}$, where **u** is velocity of a moving object as seen by a stationary observer.

Relativistic Energy

The original source of all the energy we use is the conversion of mass into energy. Most of this energy is generated by nuclear reactions in the sun and radiated to Earth in the form of electromagnetic radiation, where it is then transformed into all the forms with which we are familiar. The remaining energy from nuclear reactions is produced in nuclear power plants and in Earth's interior. In each of these cases, the source of the energy is the conversion of a small amount of mass into a large amount of energy. These sources are shown in Figure 10.8.



(a) (b)

Figure 10.8 The sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy. ((a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor. The result of his analysis is that a particle or object of mass m moving at velocity **u** has **relativistic energy** given by

$$E = \gamma m c^2.$$

This is the expression for the total energy of an object of mass m at any speed **u** and includes both kinetic and potential energy. Look back at the equation for γ and you will see that it is equal to 1 when **u** is 0; that is, when an object is at rest. Then the rest energy, E_0, is simply

$$E_0 = mc^2.$$

This is the correct form of Einstein's famous equation.

This equation is very useful to nuclear physicists because it can be used to calculate the energy released by a nuclear reaction. This is done simply by subtracting the mass of the products of such a reaction from the mass of the reactants. The difference is the m in $E_0 = mc^2$. Here is a simple example:

A positron is a type of antimatter that is just like an electron, except that it has a positive charge. When a positron and an electron collide, their masses are completely annihilated and converted to energy in the form of gamma rays. Because both particles have a rest mass of 9.11×10^{-31} kg, we multiply the mc^2 term by 2. So the energy of the gamma rays is

$$\begin{aligned} E_0 &= 2(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8\ \tfrac{\text{m}}{\text{s}}\right)^2 \\ &= 1.64 \times 10^{-13}\ \tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \\ &= 1.64 \times 10^{-13}\,\text{J} \end{aligned} \tag{10.4}$$

where we have the expression for the joule (J) in terms of its SI base units of kg, m, and s. In general, the nuclei of stable isotopes have less mass then their constituent subatomic particles. The energy equivalent of this difference is called the **binding energy** of the nucleus. This energy is released during the formation of the isotope from its constituent particles because the product is more stable than the reactants. Expressed as mass, it is called the **mass defect**. For example, a helium nucleus is made of two neutrons and two protons and has a mass of 4.0003 atomic mass units (u). The sum of the masses of two protons and two neutrons is 4.0330 u. The mass defect then is 0.0327 u. Converted to kg, the mass defect is 5.0442×10^{-30} kg. Multiplying this mass times c^2 gives a binding energy of 4.540×10^{-12} J. This does not sound like much because it is only one atom. If you were to make one gram of helium out of neutrons and protons, it would release 683,000,000,000 J. By comparison, burning one gram of coal releases about 24 J.

BOUNDLESS PHYSICS

The RHIC Collider

Figure 10.9 shows the Brookhaven National Laboratory in Upton, NY. The circular structure houses a particle accelerator called the RHIC, which stands for Relativistic Heavy Ion Collider. The heavy ions in the name are gold nuclei that have been stripped of their electrons. Streams of ions are accelerated in several stages before entering the big ring seen in the figure. Here, they are accelerated to their final speed, which is about 99.7 percent the speed of light. Such high speeds are called relativistic. All the relativistic phenomena we have been discussing in this chapter are very pronounced in this case. At this speed $\gamma = 12.9$, so that relativistic time dilates by a factor of about 13, and relativistic length contracts by the same factor.

Figure 10.9 Brookhaven National Laboratory. The circular structure houses the RHIC. (energy.gov, Wikimedia Commons)

Two ion beams circle the 2.4-mile long track around the big ring in opposite directions. The paths can then be made to cross, thereby causing ions to collide. The collision event is very short-lived but amazingly intense. The temperatures and pressures produced are greater than those in the hottest suns. At 4 trillion degrees Celsius, this is the hottest material ever created in a laboratory

But what is the point of creating such an extreme event? Under these conditions, the neutrons and protons that make up the gold nuclei are smashed apart into their components, which are called quarks and gluons. The goal is to recreate the conditions that theorists believe existed at the very beginning of the universe. It is thought that, at that time, matter was a sort of soup of quarks and gluons. When things cooled down after the initial bang, these particles condensed to form protons and neutrons.

Some of the results have been surprising and unexpected. It was thought the quark-gluon soup would resemble a gas or plasma. Instead, it behaves more like a liquid. It has been called a *perfect* liquid because it has virtually no viscosity, meaning that it has no resistance to flow.

GRASP CHECK

Calculate the relativistic factor γ, for a particle traveling at 99.7 percent of the speed of light.

a. 0.08
b. 0.71
c. 1.41
d. 12.9

WORKED EXAMPLE

The Speed of Light

One night you are out looking up at the stars and an extraterrestrial spaceship flashes across the sky. The ship is 50 meters long and is travelling at 95 percent of the speed of light. What would the ship's length be when measured from your earthbound frame of reference?

Strategy

List the knowns and unknowns.

Knowns: proper length of the ship, $L_0 = 50$ m; velocity, $\mathbf{v}$, $= 0.95c$

Unknowns: observed length of the ship accounting for relativistic length contraction, L.

Choose the relevant equation.

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{\mathbf{u}^2}{c^2}}$$

Solution

$$L = 50 \text{ m}\sqrt{1 - \frac{(0.95)^2 c^2}{c^2}} = 50 \text{ m}\sqrt{1 - (0.95)^2} = 16 \text{ m}$$

Discussion

Calculations of γ can usually be simplified in this way when v is expressed as a percentage of c because the c^2 terms cancel. Be sure to also square the decimal representing the percentage before subtracting from 1. Note that the aliens will still see the length as L_0 because they are moving with the frame of reference that is the ship.

Practice Problems

7. Calculate the relativistic factor, γ, for an object traveling at 2.00×10^8 m/s.
   a. 0.74
   b. 0.83
   c. 1.2
   d. 1.34

8. The distance between two points, called the proper length, L0, is 1.00 km. An observer in motion with respect to the frame of

reference of the two points measures 0.800 km, which is L. What is the relative speed of the frame of reference with respect to the observer?

a. 1.80×10^8 m/s
b. 2.34×10^8 m/s
c. 3.84×10^8 m/s
d. 5.00×10^8 m/s

9. Consider the nuclear fission reaction $n + {}^{235}_{92}U \rightarrow {}^{137}_{55}Cs + {}^{97}_{37}Rb + 2n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{137}_{55}Cs$ has rest mass of 136.907u, and ${}^{97}_{37}Rb$ has a rest mass of 96.937u, what is the value of E in joules?
   a. 1.8×10^{-11} J
   b. 2.9×10^{-11} J
   c. 1.8×10^{-10} J
   d. 2.9×10^{-10} J

Solution

The correct answer is (b). The mass deficit in the reaction is $235.044\ \text{u} - (136.907 + 96.937 + 1.009)\ \text{u}$, or 0.191u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$$(0.191\ \text{u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^8\text{m/s}\right)^2 \cong 2.85 \times 10^{-11}\text{J}.$$

10. Consider the nuclear fusion reaction ${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H + E$. If ${}^2_1H$ has a rest mass of 2.014u, ${}^3_1H$ has a rest mass of 3.016u, and ${}^1_1H$ has a rest mass of 1.008u, what is the value of E in joules?
   a. 6×10^{-13} J
   b. 6×10^{-12} J
   c. 6×10^{-11} J
   d. 6×10^{-10} J

Solution

The correct answer is (a). The mass deficit in the reaction is $2\,(2.014\ \text{u}) - (3.016 + 1.008)\ \text{u}$, or 0.004u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$$(0.004\ \text{u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^8\text{m/s}\right)^2 \cong 5.98 \times 10^{-13}\text{J}.$$

Check Your Understanding

11. Describe time dilation and state under what conditions it becomes significant.
    a. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
    b. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
    c. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.
    d. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.

12. The equation used to calculate relativistic momentum is $p = \gamma \cdot m \cdot u$. Define the terms to the right of the equal sign and state how m and u are measured.
    a. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity of the frame.
    b. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity relative to an observer.
    c. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity of the frame.

d. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity relative to an observer.

13. Describe length contraction and state when it occurs.

a. When the speed of an object becomes the speed of light, its length appears to shorten when viewed by a stationary observer.
b. When the speed of an object approaches the speed of light, its length appears to shorten when viewed by a stationary observer.
c. When the speed of an object becomes the speed of light, its length appears to increase when viewed by a stationary observer.
d. When the speed of an object approaches the speed of light, its length appears to increase when viewed by a stationary observer.

KEY TERMS

binding energy the energy equivalent of the difference between the mass of a nucleus and the masses of its nucleons

ether scientists once believed there was a medium that carried light waves; eventually, experiments proved that ether does not exist

frame of reference the point or collection of points arbitrarily chosen, which motion is measured in relation to

general relativity the theory proposed to explain gravity and acceleration

inertial reference frame a frame of reference where all objects follow Newton's first law of motion

length contraction the shortening of an object as seen by an observer who is moving relative to the frame of reference of the object

mass defect the difference between the mass of a nucleus and the masses of its nucleons

postulate a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument

proper length the length of an object within its own frame of reference, as opposed to the length observed by an observer moving relative to that frame of reference

relativistic having to do with modern relativity, such as the effects that become significant only when an object is moving close enough to the speed of light for γ to be significantly greater than 1

relativistic energy the total energy of a moving object or particle $E = \gamma mc^2$, which includes both its rest energy mc^2 and its kinetic energy

relativistic factor $\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{u}^2}{c^2}}}$, where **u** is the velocity of a moving object and c is the speed of light

relativistic momentum $\mathbf{p} = \gamma m\mathbf{u}$, where γ is the relativistic factor, m is rest mass of an object, and **u** is the velocity relative to an observer

relativity the explanation of how objects move relative to one another

rest mass the mass of an object that is motionless with respect to its frame of reference

simultaneity the property of events that occur at the same time

special relativity the theory proposed to explain the consequences of requiring the speed of light and the laws of physics to be the same in all inertial frames

time dilation the contraction of time as seen by an observer in a frame of reference that is moving relative to the observer

SECTION SUMMARY

10.1 Postulates of Special Relativity

- One postulate of special relativity theory is that the laws of physics are the same in all inertial frames of reference.
- The other postulate is that the speed of light in a vacuum is the same in all inertial frames.
- Einstein showed that simultaneity, or lack of it, depends on the frame of reference of the observer.

10.2 Consequences of Special Relativity

- Time dilates, length contracts, and momentum increases as an object approaches the speed of light.
- Energy and mass are interchangeable, according to the relationship $E = mc2$. The laws of conservation of mass and energy are combined into the law of conservation of mass-energy.

KEY EQUATIONS

10.1 Postulates of Special Relativity

speed of light	$v = \lambda f$
constant value for the speed of light	$c = 3.00 \times 10^8\,\text{m/s}$

10.2 Consequences of Special Relativity

elapsed time	$\Delta t = \gamma \Delta t_0$
relativistic factor	$\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{u}^2}{c^2}}}$
length contraction	$L = \frac{L_0}{\gamma}$
relativistic momentum	$\mathbf{p} = \gamma m\mathbf{u}$

relativistic energy	$E = \gamma mc^2$	rest energy	$E_0 = mc^2$

CHAPTER REVIEW

Concept Items

10.1 Postulates of Special Relativity

1. Why was it once believed that light must travel through a medium and could not propagate across empty space?
  a. The longitudinal nature of light waves implies this.
  b. Light shows the phenomenon of diffraction.
  c. The speed of light is the maximum possible speed.
  d. All other wave energy needs a medium to travel.

2. Describe the relative motion of Earth and the sun:
  1. if Earth is taken as the inertial frame of reference and
  2. if the sun is taken as the inertial frame of reference.
  a. 1. Earth is at rest and the sun orbits Earth.
     2. The sun is at rest and Earth orbits the sun.
  b. 1. The sun is at rest and Earth orbits the sun.
     2. Earth is at rest and the sun orbits Earth.
  c. 1. The sun is at rest and Earth orbits the sun.
     2. The sun is at rest and Earth orbits the sun.
  d. 1. Earth is at rest and the sun orbits Earth.
     2. Earth is at rest and the sun orbits Earth.

10.2 Consequences of Special Relativity

3. A β particle (a free electron) is speeding around the track in a cyclotron, rapidly gaining speed. How will the particle's momentum change as its speed approaches the speed of light? Explain.
  a. The particle's momentum will rapidly decrease.
  b. The particle's momentum will rapidly increase.
  c. The particle's momentum will remain constant.
  d. The particle's momentum will approach zero.

4. An astronaut goes on a long space voyage at near the speed of light. When she returns home, how will her age compare to the age of her twin who stayed on Earth?
  a. Both of them will be the same age.
  b. This is a paradox and hence the ages cannot be compared.
  c. The age of the twin who traveled will be less than the age of her twin.
  d. The age of the twin who traveled will be greater than the age of her twin.

5. A comet reaches its greatest speed as it travels near the sun. True or false— Relativistic effects make the comet's tail look longer to an observer on Earth.
  a. True
  b. False

Critical Thinking Items

10.1 Postulates of Special Relativity

6. Explain how the two postulates of Einstein's theory of special relativity, when taken together, could lead to a situation that seems to contradict the mechanics and laws of motion as described by Newton.
  a. In Newtonian mechanics, velocities are multiplicative but the speed of a moving light source cannot be multiplied to the speed of light because, according to special relativity, the speed of light is the maximum speed possible.
  b. In Newtonian mechanics, velocities are additive but the speed of a moving light source cannot be added to the speed of light because the speed of light is the maximum speed possible.
  c. An object that is at rest in one frame of reference may appear to be in motion in another frame of reference, while in Newtonian mechanics such a situation is not possible.
  d. The postulates of Einstein's theory of special relativity do not contradict any situation that Newtonian mechanics explains.

7. It takes light 6.0 minutes to travel from the sun to the planet Venus. How far is Venus from the sun?
  a. 18×10^6 km
  b. 18×10^8 km
  c. 1.08×10^{11} km
  d. 1.08×10^8 km

8. In 2003, Earth and Mars were the closest they had been in 50,000 years. The two planets were aligned so that Earth was between Mars and the sun. At that time it took light from the sun 500 s to reach Earth and 687 s to get to Mars. What was the distance from Mars to Earth?
  a. 5.6×10^7 km
  b. 5.6×10^{10} km
  c. 6.2×10^6 km
  d. 6.2×10^{12} km

9. Describe two ways in which light differs from all other forms of wave energy.

a. 1. Light travels as a longitudinal wave.
   2. Light travels through a medium that fills up the empty space in the universe.

b. 1. Light travels as a transverse wave.
   2. Light travels through a medium that fills up the empty space in the universe.

c. 1. Light travels at the maximum possible speed in the universe.
   2. Light travels through a medium that fills up the empty space in the universe.

d. 1. Light travels at the maximum possible speed in the universe.
   2. Light does not require any material medium to travel.

10. Use the postulates of the special relativity theory to explain why the speed of light emitted from a fast-moving light source cannot exceed 3.00×10^8 m/s.

a. The speed of light is maximum in the frame of reference of the moving object.
b. The speed of light is minimum in the frame of reference of the moving object.
c. The speed of light is the same in all frames of reference, including in the rest frame of its source.
d. Light always travels in a vacuum with a speed less than 3.00×10^8 m/s, regardless of the speed of the source.

10.2 Consequences of Special Relativity

11. Halley's Comet comes near Earth every 75 years as it travels around its 22 billion km orbit at a speed of up to 700, 000 m/s. If it were possible to put a clock on the comet and read it each time the comet passed, which part of special relativity theory could be tested? What would be the expected result? Explain.

a. It would test time dilation. The clock would appear to be slightly slower.
b. It would test time dilation. The clock would appear to be slightly faster.
c. It would test length contraction. The length of the orbit would appear to be shortened from Earth's frame of reference.
d. It would test length contraction. The length of the orbit would appear to be shortened from the comet's frame of reference.

12. The nucleus of the isotope fluorine-18 (^{18} F) has mass defect of 2.44×10^{-28} kg. What is the binding energy of ^{18}F?

a. 2.2×10^{-11} J
b. 7.3×10^{-20} J
c. 2.2×10^{-20} J
d. 2.4×10^{-28} J

Problems

10.2 Consequences of Special Relativity

13. Deuterium (2 H) is an isotope of hydrogen that has one proton and one neutron in its nucleus. The binding energy of deuterium is 3.56×10^{-13} J. What is the mass defect of deuterium?

a. 3.20×10^{-4} kg
b. 1.68×10^{-6} kg
c. 1.19×10^{-21} kg
d. 3.96×10^{-30} kg

14. The sun orbits the center of the galaxy at a speed of 2.3×10^5 m/s. The diameter of the sun is 1.391684×109 m. An observer is in a frame of reference that is stationary with respect to the center of the galaxy. True or false—The sun is moving fast enough for the observer to notice length contraction of the sun's diameter.

a. True
b. False

15. Consider the nuclear fission reaction $n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{144}_{56}Ba$ has rest mass of 143.923u, and ${}^{89}_{36}Kr$ has a rest mass of 88.918u, what is the value of E in joules?

a. 1.8×10^{-11} J
b. 2.8×10^{-11} J
c. 1.8×10^{-10} J
d. 3.3×10^{-10} J

16. Consider the nuclear fusion reaction ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n + E$. If ${}^{2}_{1}H$ has a rest mass of 2.014u, ${}^{3}_{1}H$ has a rest mass of 3.016u, ${}^{4}_{2}He$ has a rest mass of 4.003u, and a neutron has a rest mass of 1.009u, what is the value of E in joules?

a. 2.7×10^{-14} J
b. 2.7×10^{-13} J
c. 2.7×10^{-12} J
d. 2.7×10^{-11} J

Performance Task

10.2 Consequences of Special Relativity

17. People are fascinated by the possibility of traveling across the universe to discover intelligent life on other planets. To do this, we would have to travel enormous distances. Suppose we could somehow travel at up to 90 percent of the speed of light. The closest star is Alpha Centauri, which is 4.37 light years away. (A light year is the distance light travels in one year.)

a. How long, from the point of view of people on Earth, would it take a space ship to travel to Alpha Centauri and back at 0.9*c*?

b. How much would the astronauts on the spaceship have aged by the time they got back to Earth?

c. Discuss the problems related to travel to stars that are 20 or 30 light years away. Assume travel speeds near the speed of light.

TEST PREP

Multiple Choice

10.1 Postulates of Special Relativity

18. What was the purpose of the Michelson–Morley experiment?

a. To determine the exact speed of light
b. To analyze the electromagnetic spectrum
c. To establish that Earth is the true frame of reference
d. To learn how the ether affected the propagation of light

19. What is the speed of light in a vacuum to three significant figures?

a. 1.86×10^5 m/s
b. 3.00×10^8 m/s
c. 6.71×10^8 m/s
d. 1.50×10^{11} m/s

20. How far does light travel in 1.00 min?

a. 1.80×10^7 km
b. 1.80×10^{13} km
c. 5.00×10^6 m
d. 5.00×10^8 m

21. Describe what is meant by the sentence, "Simultaneity is not absolute."

a. Events may appear simultaneous in all frames of reference.
b. Events may not appear simultaneous in all frames of reference.
c. The speed of light is not the same in all frames of reference.
d. The laws of physics may be different in different inertial frames of reference.

22. In 2003, Earth and Mars were aligned so that Earth was between Mars and the sun. Earth and Mars were 5.6×10^7 km from each other, which was the closest they had been in 50,000 years. People looking up saw Mars as a very bright red light on the horizon. If Mars was 2.06×10^8 km from the sun, how long did the reflected light people saw take to travel from the sun to Earth?

a. 14 min and 33 s
b. 12 min and 15 s
c. 11 min and 27 s
d. 3 min and 7 s

10.2 Consequences of Special Relativity

23. What does this expression represent: $\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$

a. time dilation
b. relativistic factor
c. relativistic energy
d. length contraction

24. What is the rest energy, E_0, of an object with a mass of 1.00 g ?

a. 3.00×10^5 J
b. 3.00×10^{11} J
c. 9.00×10^{13} J
d. 9.00×10^{16} J

25. The fuel rods in a nuclear reactor must be replaced from time to time because so much of the radioactive material has reacted that they can no longer produce energy. How would the mass of the spent fuel rods compare to their mass when they were new? Explain your answer.

a. The mass of the spent fuel rods would decrease.
b. The mass of the spent fuel rods would increase.
c. The mass of the spent fuel rods would remain the same.
d. The mass of the spent fuel rods would become close to zero.

Short Answer

10.1 Postulates of Special Relativity

26. What is the postulate having to do with the speed of light on which the theory of special relativity is based?
   a. The speed of light remains the same in all inertial frames of reference.
   b. The speed of light depends on the speed of the source emitting the light.
   c. The speed of light changes with change in medium through which it travels.
   d. The speed of light does not change with change in medium through which it travels.

27. What is the postulate having to do with reference frames on which the theory of special relativity is based?
   a. The frame of reference chosen is arbitrary as long as it is inertial.
   b. The frame of reference is chosen to have constant nonzero acceleration.
   c. The frame of reference is chosen in such a way that the object under observation is at rest.
   d. The frame of reference is chosen in such a way that the object under observation is moving with a constant speed.

28. If you look out the window of a moving car at houses going past, you sense that you are moving. What have you chosen as your frame of reference?
   a. the car
   b. the sun
   c. a house

29. Why did Michelson and Morley orient light beams at right angles to each other?
   a. To observe the particle nature of light
   b. To observe the effect of the passing ether on the speed of light
   c. To obtain a diffraction pattern by combination of light
   d. To obtain a constant path difference for interference of light

10.2 Consequences of Special Relativity

30. What is the relationship between the binding energy and the mass defect of an atomic nucleus?
   a. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = mc$.
   b. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = mc^2$.
   c. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = \frac{m}{c}$
   d. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = \frac{m}{c^2}$.

31. True or false—It is possible to just use the relationships $F = ma$ and $E = Fd$ to show that both sides of the equation $E_0 = mc^2$ have the same units.
   a. True
   b. False

32. Explain why the special theory of relativity caused the law of conservation of energy to be modified.
   a. The law of conservation of energy is not valid in relativistic mechanics.
   b. The law of conservation of energy has to be modified because of time dilation.
   c. The law of conservation of energy has to be modified because of length contraction.
   d. The law of conservation of energy has to be modified because of mass-energy equivalence.

33. The sun loses about 4×10^9 kg of mass every second. Explain in terms of special relativity why this is happening.
   a. The sun loses mass because of its high temperature.
   b. The sun loses mass because it is continuously releasing energy.
   c. The Sun loses mass because the diameter of the sun is contracted.
   d. The sun loses mass because the speed of the sun is very high and close to the speed of light.

Extended Response

10.1 Postulates of Special Relativity

34. Explain how Einstein's conclusion that nothing can travel faster than the speed of light contradicts an older concept about the speed of an object propelled from another, already moving, object.
   a. The older concept is that speeds are subtractive. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running minus the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves faster than the speed of light emitted from a stationary source.
   b. The older concept is that speeds are additive. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running plus the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves no

faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

c. The older concept is that speeds are multiplicative. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running multiplied by the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves no faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

d. The older concept is that speeds are frame independent. For example, if a person throws a ball while running, the speed of the ball relative to the ground has nothing to do with the speed at which the person was running. A relativistic example is when light is emitted from car headlights, it moves no faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

35. A rowboat is drifting downstream. One person swims 20 m toward the shore and back, and another, leaving at the same time, swims upstream 20 m and back to the boat. The swimmer who swam toward the shore gets back first. Explain how this outcome is similar to the outcome expected in the Michelson–Morley experiment.

a. The rowboat represents Earth, the swimmers are beams of light, and the water is acting as the ether. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

b. The rowboat represents the beam of light, the swimmers are the ether, and water is acting as Earth. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

c. The rowboat represents the ether, the swimmers are ray of light, and the water is acting as the earth. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

d. The rowboat represents the Earth, the swimmers are the ether, and the water is acting as the rays of light. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

10.2 Consequences of Special Relativity

36. A helium-4 nucleus is made up of two neutrons and two protons. The binding energy of helium-4 is 4.53×10^{-12} J. What is the difference in the mass of this helium nucleus and the sum of the masses of two neutrons and two protons? Which weighs more, the nucleus or its constituents?

a. 1.51×10^{-20} kg; the constituents weigh more

b. 5.03×10^{-29} kg; the constituents weigh more

c. 1.51×10^{-29} kg; the nucleus weighs more

d. 5.03×10^{-29} kg; the nucleus weighs more

37. Use the equation for length contraction to explain the relationship between the length of an object perceived by a stationary observer who sees the object as moving, and the proper length of the object as measured in the frame of reference where it is at rest.

a. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches zero. For other speeds, the length perceived is always less than the proper length.

b. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches zero. For other speeds, the length perceived is always greater than the proper length.

c. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches infinity. For other speeds, the length perceived is always less than the proper length.

d. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches infinity. For other speeds, the length perceived is always greater than the proper length.

CHAPTER 11
Thermal Energy, Heat, and Work

Figure 11.1 The welder's gloves and helmet protect the welder from the electric arc, which transfers enough thermal energy to melt the rod, spray sparks, and emit high-energy electromagnetic radiation that can burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers (Kevin S. O'Brien, U.S. Navy)

Chapter Outline

INTRODUCTION Heat is something familiar to all of us. We feel the warmth of the summer sun, the hot vapor rising up out of a cup of hot cocoa, and the cooling effect of our sweat. When we feel warmth, it means that heat is transferring energy *to* our bodies; when we feel cold, that means heat is transferring energy *away from* our bodies. Heat transfer is the movement of thermal energy from one place or material to another, and is caused by temperature differences. For example, much of our weather is caused by Earth evening out the temperature across the planet through wind and violent storms, which are driven by heat transferring energy away from the equator towards the cold poles. In this chapter, we'll explore the precise meaning of heat, how it relates to temperature as well as to other forms of energy, and its connection to work.

11.1 Temperature and Thermal Energy

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain that temperature is a measure of internal kinetic energy
- Interconvert temperatures between Celsius, Kelvin, and Fahrenheit scales

Section Key Terms

absolute zero	Celsius scale	degree Celsius ($^\circ\text{C}$)	thermal energy
degree Fahrenheit ($^\circ\text{F}$)	Fahrenheit scale	heat	
kelvin (K)	Kelvin scale	temperature	

Temperature

What is **temperature**? It's one of those concepts so ingrained in our everyday lives that, although we know what it means intuitively, it can be hard to define. It is tempting to say that temperature measures heat, but this is not strictly true. **Heat** is the transfer of energy due to a temperature difference. Temperature is defined in terms of the instrument we use to tell us how hot or cold an object is, based on a mechanism and scale invented by people. Temperature is literally defined as what we measure on a thermometer.

Heat is often confused with temperature. For example, we may say that the heat was unbearable, when we actually mean that the temperature was high. This is because we are sensitive to the flow of energy by heat, rather than the temperature. Since heat, like work, transfers energy, it has the SI unit of joule (J).

Atoms and molecules are constantly in motion, bouncing off one another in random directions. Recall that kinetic energy is the energy of motion, and that it increases in proportion to velocity squared. Without going into mathematical detail, we can say that **thermal energy**—the energy associated with heat—is the average kinetic energy of the particles (molecules or atoms) in a substance. Faster moving molecules have greater kinetic energies, and so the substance has greater thermal energy, and thus a higher temperature. The total internal energy of a system is the sum of the kinetic and potential energies of its atoms and molecules. Thermal energy is one of the subcategories of internal energy, as is chemical energy.

To measure temperature, some scale must be used as a standard of measurement. The three most commonly used temperature scales are the Fahrenheit, Celsius, and Kelvin scales. Both the **Fahrenheit scale** and **Celsius scale** are relative temperature scales, meaning that they are made around a reference point. For example, the Celsius scale uses the freezing point of water as its reference point; all measurements are either lower than the freezing point of water by a given number of degrees (and have a negative sign), or higher than the freezing point of water by a given number of degrees (and have a positive sign). The boiling point of water is 100 $^\circ\text{C}$ for the Celsius scale, and its unit is the degree Celsius ($^\circ\text{C}$).

On the Fahrenheit scale, the freezing point of water is at 32 $^\circ\text{F}$, and the boiling point is at 212 $^\circ\text{F}$. The unit of temperature on this scale is the degree Fahrenheit ($^\circ\text{F}$). Note that the difference in degrees between the freezing and boiling points is greater for the Fahrenheit scale than for the Celsius scale. Therefore, a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Since 100 Celsius degrees span the same range as 180 Fahrenheit degrees, one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale (because $\frac{180}{100} = \frac{9}{5} = 1.8$). This relationship can be used to convert between temperatures in Fahrenheit and Celsius (see Figure 11.2).

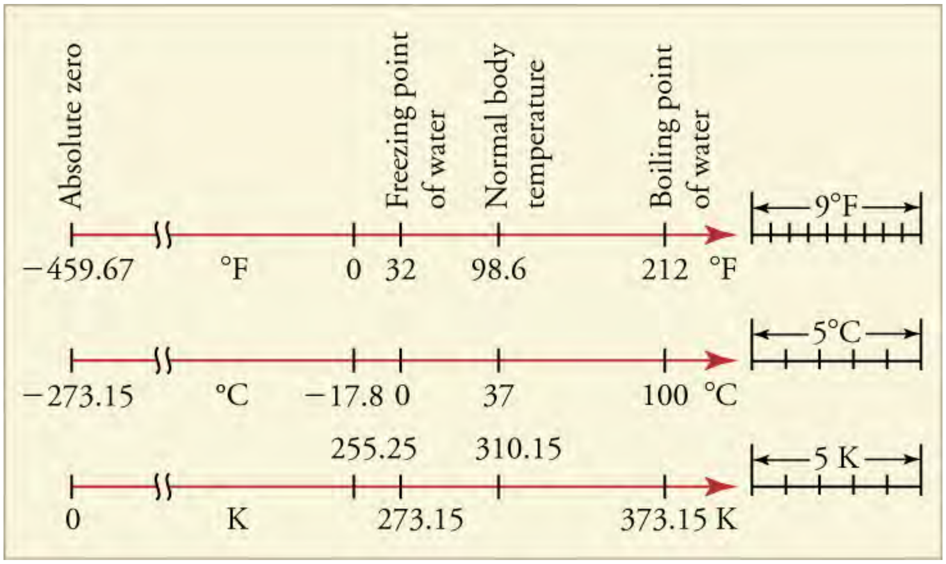


Figure 11.2 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The **Kelvin scale** is the temperature scale that is commonly used in science because it is an absolute temperature scale. This means that the theoretically lowest-possible temperature is assigned the value of zero. Zero degrees on the Kelvin scale is known as **absolute zero**; it is theoretically the point at which there is no molecular motion to produce thermal energy. On the original Kelvin scale first created by Lord Kelvin, all temperatures have positive values, making it useful for scientific work. The official temperature unit on this scale is the kelvin, which is abbreviated as K. The freezing point of water is 273.15 K, and the boiling point of water is 373.15 K.

Although absolute zero is possible in theory, it cannot be reached in practice. The lowest temperature ever created and measured during a laboratory experiment was 1.0×10^{-10} K, at Helsinki University of Technology in Finland. In comparison, the coldest recorded temperature for a place on Earth's surface was 183 K (–89 °C), at Vostok, Antarctica, and the coldest known place (outside the lab) in the universe is the Boomerang Nebula, with a temperature of 1 K. Luckily, most of us humans will never have to experience such extremes.

The average normal body temperature is 98.6 °F (37.0 °C), but people have been known to survive with body temperatures ranging from 75 °F to 111 °F (24 °C to 44 °C).

WATCH PHYSICS

Comparing Celsius and Fahrenheit Temperature Scales

This video shows how the Fahrenheit and Celsius temperature scales compare to one another.

Click to view content (https://www.openstax.org/l/02celfahtemp)

GRASP CHECK

Even without the number labels on the thermometer, you could tell which side is marked Fahrenheit and which is Celsius by how the degree marks are spaced. Why?

a. The separation between two consecutive divisions on the Fahrenheit scale is greater than a similar separation on the Celsius scale, because each degree Fahrenheit is equal to 1.8 degrees Celsius.
b. The separation between two consecutive divisions on the Fahrenheit scale is smaller than the similar separation on the Celsius scale, because each degree Celsius is equal to 1.8 degrees Fahrenheit.
c. The separation between two consecutive divisions on the Fahrenheit scale is greater than a similar separation on the Celsius scale, because each degree Fahrenheit is equal to 3.6 degrees Celsius.
d. The separation between two consecutive divisions on the Fahrenheit scale is smaller than a similar separation on the Celsius scale, because each degree Celsius is equal to 3.6 degrees Fahrenheit.

Converting Between Celsius, Kelvin, and Fahrenheit Scales

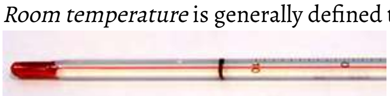
While the Fahrenheit scale is still the most commonly used scale in the United States, the majority of the world uses Celsius, and scientists prefer Kelvin. It's often necessary to convert between these scales. For instance, if the TV meteorologist gave the local weather report in kelvins, there would likely be some confused viewers! Table 11.1 gives the equations for conversion between the three temperature scales.

To Convert From...	Use This Equation
Celsius to Fahrenheit	$T_{°F} = \frac{9}{5}T_{°C} + 32$
Fahrenheit to Celsius	$T_{°C} = \frac{5}{9}(T_{°F}-32)$
Celsius to Kelvin	$T_K = T_{°C} + 273.15$
Kelvin to Celsius	$T_{°C} = T_K - 273.15$
Fahrenheit to Kelvin	$T_K = \frac{5}{9}(T_{°F} - 32) + 273.15$
Kelvin to Fahrenheit	$T_{°F} = \frac{9}{5}(T_K - 273.15) + 32$

Table 11.1 Temperature Conversions

WORKED EXAMPLE

Room temperature is generally defined to be 25 °C. (a) What is room temperature in °F? (b) What is it in K?



STRATEGY

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from °C to °F, use the equation

$$T_{°F} = \frac{9}{5}T_{°C} + 32. \tag{11.1}$$

2. Plug the known value into the equation and solve.

$$T_{°F} = \frac{9}{5}25\ °\text{C} + 32 = 77\ °\text{F} \tag{11.2}$$

Solution for (b)

1. Choose the right equation. To convert from °C to K, use the equation

$$T_K = T_{°C} + 273.15. \tag{11.3}$$

2. Plug the known value into the equation and solve.

$$T_K = 25\ °\text{C} + 273.15 = 298\text{K} \tag{11.4}$$

Discussion

Living in the United States, you are likely to have more of a sense of what the temperature feels like if it's described as 77 °F than as 25 °C (or 298 K, for that matter).

WORKED EXAMPLE

Converting Between Temperature Scales: The Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18^{th} and 19^{th} centuries. On the Reaumur temperature scale, the freezing point of water is 0 °R and the boiling temperature is 80 °R. If "room temperature" is 25 °C on the Celsius scale, what is it on the Reaumur scale?

STRATEGY

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is 80 °R . On the Celsius scale, it is 100 °C . Therefore, 100 °C = 80 °R . Both scales start at 0 ° for freezing, so we can create a simple formula to convert between temperatures on the two scales.

Solution

1. Derive a formula to convert from one scale to the other.

$$T_{°R} = \frac{0.80°R}{°C} \times T_{°C} \tag{11.5}$$

2. Plug the known value into the equation and solve.

$$T_{°R} = \frac{0.80°R}{°C} \times 25\ °C = 20°R \tag{11.6}$$

Discussion

As this example shows, relative temperature scales are somewhat arbitrary. If you wanted, you could create your own temperature scale!

Practice Problems

1. What is 12.0 °C in kelvins?

a. 112.0 K
b. 273.2 K
c. 12.0 K
d. 285.2 K

2. What is 32.0 °C in degrees Fahrenheit?

a. 57.6 °F
b. 25.6 °F
c. 305.2 °F
d. 89.6 °F

TIPS FOR SUCCESS

Sometimes it is not so easy to guess the temperature of the air accurately. Why is this? Factors such as humidity and wind speed affect how hot or cold we feel. Wind removes thermal energy from our bodies at a faster rate than usual, making us feel colder than we otherwise would; on a cold day, you may have heard the TV weather person refer to the *wind chill*.
On humid summer days, people tend to feel hotter because sweat doesn't evaporate from the skin as efficiently as it does on dry days, when the evaporation of sweat cools us off.

Check Your Understanding

3. What is thermal energy?

a. The thermal energy is the average potential energy of the particles in a system.
b. The thermal energy is the total sum of the potential energies of the particles in a system.
c. The thermal energy is the average kinetic energy of the particles due to the interaction among the particles in a system.
d. The thermal energy is the average kinetic energy of the particles in a system.

4. What is used to measure temperature?

a. a galvanometer
b. a manometer
c. a thermometer
d. a voltmeter

11.2 Heat, Specific Heat, and Heat Transfer

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain heat, heat capacity, and specific heat
- Distinguish between conduction, convection, and radiation
- Solve problems involving specific heat and heat transfer

Section Key Terms

conduction	convection	heat capacity	radiation	specific heat

Heat Transfer, Specific Heat, and Heat Capacity

We learned in the previous section that temperature is proportional to the average kinetic energy of atoms and molecules in a substance, and that the average internal kinetic energy of a substance is higher when the substance's temperature is higher.

If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter object (that is, the object with the greater temperature) to the colder (lower temperature) object, until both objects are at the same temperature. There is no net heat transfer once the temperatures are equal because the amount of heat transferred from one object to the other is the same as the amount of heat returned. One of the major effects of heat transfer is temperature change: Heating increases the temperature while cooling decreases it. Experiments show that the heat transferred to or from a substance depends on three factors—the change in the substance's temperature, the mass of the substance, and certain physical properties related to the phase of the substance.

The equation for heat transfer Q is

$$Q = mc\Delta T, \tag{11.7}$$

where m is the mass of the substance and ΔT is the change in its temperature, in units of Celsius or Kelvin. The symbol c stands for **specific heat**, and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00 °C. The specific heat c is a property of the substance; its SI unit is J/(kg ⋅ K) or J/(kg ⋅ $°\text{C}$). The temperature change (ΔT) is the same in units of kelvins and degrees Celsius (but not degrees Fahrenheit). Specific heat is closely related to the concept of **heat capacity**. Heat capacity is the amount of heat necessary to change the temperature of a substance by 1.00 $°\text{C}$. In equation form, heat capacity C is $C = mc$, where m is mass and c is specific heat. Note that heat capacity is the same as specific heat, but without any dependence on mass. Consequently, two objects made up of the same material but with different masses will have different heat capacities. This is because the heat capacity is a property of an object, but specific heat is a property of *any* object made of the same material.

Values of specific heat must be looked up in tables, because there is no simple way to calculate them. Table 11.2 gives the values of specific heat for a few substances as a handy reference. We see from this table that the specific heat of water is five times that of glass, which means that it takes five times as much heat to raise the temperature of 1 kg of water than to raise the temperature of 1 kg of glass by the same number of degrees.

Substances	Specific Heat (*c*)
Solids	J/(kg ⋅ $°\text{C}$)
Aluminum	900

Table 11.2 Specific Heats of Various Substances.

Substances	Specific Heat (*c*)
Asbestos	800
Concrete, granite (average)	840
Copper	387
Glass	840
Gold	129
Human body (average)	3500
Ice (average)	2090
Iron, steel	452
Lead	128
Silver	235
Wood	1700
Liquids	
Benzene	1740
Ethanol	2450
Glycerin	2410
Mercury	139
Water	4186
Gases (at 1 atm constant pressure)	
Air (dry)	1015
Ammonia	2190
Carbon dioxide	833
Nitrogen	1040
Oxygen	913
Steam	2020

Table 11.2 Specific Heats of Various Substances.

Snap Lab

Temperature Change of Land and Water

What heats faster, land or water? You will answer this question by taking measurements to study differences in specific heat capacity.

- Open flame—Tie back all loose hair and clothing before igniting an open flame. Follow all of your teacher's instructions on how to ignite the flame. Never leave an open flame unattended. Know the location of fire safety equipment in the laboratory.

- Sand or soil
- Water
- Oven or heat lamp
- Two small jars
- Two thermometers

Instructions

Procedure

1. Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can get equal masses by using 50 percent more water by volume.)
2. Heat both substances (using an oven or a heat lamp) for the same amount of time.
3. Record the final temperatures of the two masses.
4. Now bring both jars to the same temperature by heating for a longer period of time.
5. Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

GRASP CHECK

Did it take longer to heat the water or the sand/soil to the same temperature? Which sample took longer to cool? What does this experiment tell us about how the specific heat of water compared to the specific heat of land?

a. The sand/soil will take longer to heat as well as to cool. This tells us that the specific heat of land is greater than that of water.
b. The sand/soil will take longer to heat as well as to cool. This tells us that the specific heat of water is greater than that of land.
c. The water will take longer to heat as well as to cool. This tells us that the specific heat of land is greater than that of water.
d. The water will take longer to heat as well as to cool. This tells us that the specific heat of water is greater than that of land.

Conduction, Convection, and Radiation

Whenever there is a temperature difference, heat transfer occurs. Heat transfer may happen rapidly, such as through a cooking pan, or slowly, such as through the walls of an insulated cooler.

There are three different heat transfer methods: **conduction**, **convection**, and **radiation**. At times, all three may happen simultaneously. See Figure 11.3.

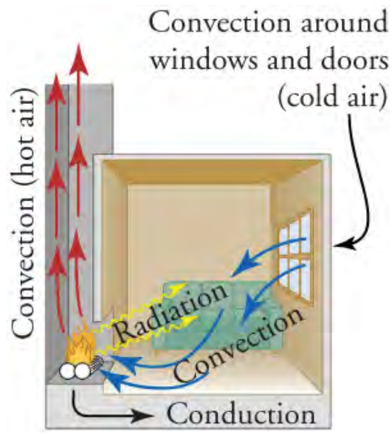


Figure 11.3 In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

Conduction is heat transfer through direct physical contact. Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction. Sometimes, we try to control the conduction of heat to make ourselves more comfortable. Since the rate of heat transfer is different for different materials, we choose fabrics, such as a thick wool sweater, that slow down the transfer of heat away from our bodies in winter.

As you walk barefoot across the living room carpet, your feet feel relatively comfortable…until you step onto the kitchen's tile floor. Since the carpet and tile floor are both at the same temperature, why does one feel colder than the other? This is explained by different rates of heat transfer: The tile material removes heat from your skin at a greater rate than the carpeting, which makes it *feel* colder.

Some materials simply conduct thermal energy faster than others. In general, metals (like copper, aluminum, gold, and silver) are good heat conductors, whereas materials like wood, plastic, and rubber are poor heat conductors.

Figure 11.4 shows particles (either atoms or molecules) in two bodies at different temperatures. The (average) kinetic energy of a particle in the hot body is higher than in the colder body. If two particles collide, energy transfers from the particle with greater kinetic energy to the particle with less kinetic energy. When two bodies are in contact, many particle collisions occur, resulting in a net flux of heat from the higher-temperature body to the lower-temperature body. The heat flux depends on the temperature difference $\Delta T = T_{\text{hot}} - T_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water.

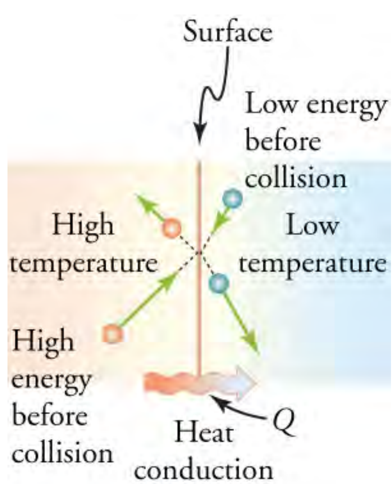


Figure 11.4 The particles in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a particle in the lower-temperature region (right side) has low kinetic energy before collision, but its kinetic energy increases after colliding with the contact

surface. In contrast, a particle in the higher-temperature region (left side) has more kinetic energy before collision, but its energy decreases after colliding with the contact surface.

Convection is heat transfer by the movement of a fluid. This type of heat transfer happens, for example, in a pot boiling on the stove, or in thunderstorms, where hot air rises up to the base of the clouds.

TIPS FOR SUCCESS

In everyday language, the term *fluid* is usually taken to mean liquid. For example, when you are sick and the doctor tells you to "push fluids," that only means to drink more beverages—not to breath more air. However, in physics, fluid means a liquid *or a gas*. Fluids move differently than solid material, and even have their own branch of physics, known as *fluid dynamics*, that studies how they move.

As the temperature of fluids increase, they expand and become less dense. For example, Figure 11.4 could represent the wall of a balloon with different temperature gases inside the balloon than outside in the environment. The hotter and thus faster moving gas particles inside the balloon strike the surface with more force than the cooler air outside, causing the balloon to expand. This decrease in density relative to its environment creates buoyancy (the tendency to rise). Convection is driven by buoyancy—hot air rises because it is less dense than the surrounding air.

Sometimes, we control the temperature of our homes or ourselves by controlling air movement. Sealing leaks around doors with weather stripping keeps out the cold wind in winter. The house in Figure 11.5 and the pot of water on the stove in Figure 11.6 are both examples of convection and buoyancy by human design. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another, and are examples of natural convection.

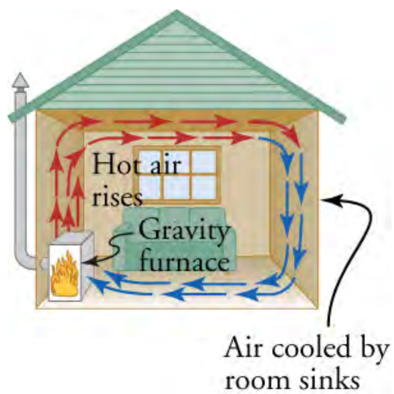


Figure 11.5 Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system like this one, which uses natural convection, can be quite efficient in uniformly heating a home.

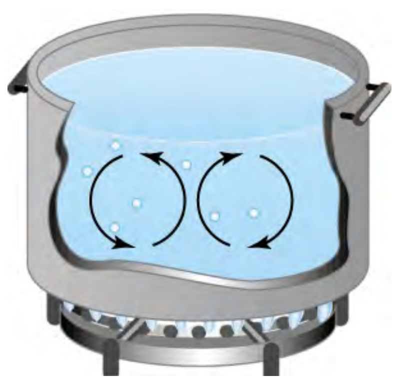


Figure 11.6 Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside fluid, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process repeats as long as there is water in the pot.

Radiation is a form of heat transfer that occurs when electromagnetic radiation is emitted or absorbed. Electromagnetic radiation includes radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays, all of which have different wavelengths and amounts of energy (shorter wavelengths have higher frequency and more energy).

You can feel the heat transfer from a fire and from the sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. Another example is thermal radiation from the human body; people are constantly emitting infrared radiation, which is not visible to the human eye, but is felt as heat.

Radiation is the only method of heat transfer where no medium is required, meaning that the heat doesn't need to come into direct contact with or be transported by any matter. The space between Earth and the sun is largely empty, without any possibility of heat transfer by convection or conduction. Instead, heat is transferred by radiation, and Earth is warmed as it absorbs electromagnetic radiation emitted by the sun.

Figure 11.7 Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light transfers relatively little thermal energy. Since skin is very sensitive to infrared radiation, you can sense the presence of a fire without looking at it directly. (Daniel X. O'Neil)

All objects absorb and emit electromagnetic radiation (see Figure 11.7). The rate of heat transfer by radiation depends mainly on the color of the object. Black is the most effective absorber and radiator, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance. Similarly, black asphalt in a parking lot will be hotter than adjacent patches of grass on a summer day, because black absorbs better than green. The reverse is also true—black radiates better than green. On a clear summer night, the black asphalt will be colder than the green patch of grass, because black radiates energy faster than green. In contrast, white is a poor absorber and also a poor radiator. A white object reflects nearly all radiation, like a mirror.

Virtual Physics

Energy Forms and Changes

Click to view content (http://www.openstax.org/l/28energyForms)

In this animation, you will explore heat transfer with different materials. Experiment with heating and cooling the iron, brick, and water. This is done by dragging and dropping the object onto the pedestal and then holding the lever either to Heat or Cool. Drag a thermometer beside each object to measure its temperature—you can watch how quickly it heats or cools in real time.

Now let's try transferring heat between objects. Heat the brick and then place it in the cool water. Now heat the brick again, but then place it on top of the iron. What do you notice?

Selecting the fast forward option lets you speed up the heat transfers, to save time.

GRASP CHECK

Compare how quickly the different materials are heated or cooled. Based on these results, what material do you think has the greatest specific heat? Why? Which has the smallest specific heat? Can you think of a real-world situation where you would want to use an object with large specific heat?

a. Water will take the longest, and iron will take the shortest time to heat, as well as to cool. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

b. Water will take the shortest, and iron will take the longest time to heat, as well as to cool. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.
c. Brick will take shortest and iron will take longest time to heat up as well as to cool down. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.
d. Water will take shortest and brick will take longest time to heat up as well as to cool down. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

Solving Heat Transfer Problems

WORKED EXAMPLE

Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 L of water from 20.0 °C to 80.0 °C . (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

STRATEGY

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for heat transfer for the given temperature change and masses of water and aluminum. The specific heat values for water and aluminum are given in the previous table.

Solution to (a)

Because the water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference.

$$\Delta T = T_f - T_i = 60.0\ °\text{C} \tag{11.8}$$

2. Calculate the mass of water using the relationship between density, mass, and volume. Density is mass per unit volume, or $\rho = \frac{m}{V}$. Rearranging this equation, solve for the mass of water.

$$m_w = \rho \cdot V = 1000\ \text{kg/m}^3 \times \left(0.250\ \text{L} \times \frac{0.001\ \text{m}^3}{1\ \text{L}}\right) = 0.250\ \text{kg} \tag{11.9}$$

3. Calculate the heat transferred to the water. Use the specific heat of water in the previous table.

$$Q_w = m_w c_w \Delta T = (0.250\ \text{kg})(4186\ \text{J/kg°C})(60.0°\text{C}) = 62.8\ \text{kJ} \tag{11.10}$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in the previous table.

$$Q_{Al} = m_{Al} c_{Al} \Delta T = (0.500\ \text{kg})(900\ \text{J/kg°C})(60.0°\text{C}) = 27.0 \times 10^3\ \text{J} = 27.0\ \text{kJ} \tag{11.11}$$

5. Find the total transferred heat.

$$Q_{Total} = Q_w + Q_{Al} = 62.8\ \text{kJ} + 27.0\ \text{kJ} = 89.8\ \text{kJ} \tag{11.12}$$

Solution to (b)

The percentage of heat going into heating the pan is

$$\frac{27.0\ \text{kJ}}{89.8\ \text{kJ}} \times 100\% = 30.1\% \tag{11.13}$$

Solution to (c)

The percentage of heat going into heating the water is

$$\frac{62.8\ \text{kJ}}{89.8\ \text{kJ}} \times 100\% = 69.9\% \tag{11.14}$$

Discussion

In this example, most of the total heat transferred is used to heat the water, even though the pan has twice as much mass. This is because the specific heat of water is over four times greater than the specific heat of aluminum. Therefore, it takes a bit more than twice as much heat to achieve the given temperature change for the water than for the aluminum pan.

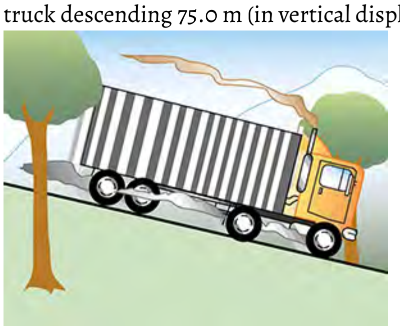
Water can absorb a tremendous amount of energy with very little resulting temperature change. This property of water allows for life on Earth because it stabilizes temperatures. Other planets are less habitable because wild temperature swings make for a harsh environment. You may have noticed that climates closer to large bodies of water, such as oceans, are milder than climates landlocked in the middle of a large continent. This is due to the climate-moderating effect of water's large heat capacity—water stores large amounts of heat during hot weather and releases heat gradually when it's cold outside.

WORKED EXAMPLE

Calculating Temperature Increase: Truck Brakes Overheat on Downhill Runs

When a truck headed downhill brakes, the brakes must do work to convert the gravitational potential energy of the truck to internal energy of the brakes. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck, and keeps the truck from speeding up and losing control. The increased internal energy of the brakes raises their temperature. When the hill is especially steep, the temperature increase may happen too quickly and cause the brakes to overheat.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of 800 J/kg $\cdot$°C from a 10,000 kg truck descending 75.0 m (in vertical displacement) at a constant speed.



STRATEGY

We first calculate the gravitational potential energy (*Mgh*) of the truck, and then find the temperature increase produced in the brakes.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill.

$$Mgh = (10,000\ \text{kg})\,(9.80\ \text{m/s}^2)\,(75.0\ \text{m}) = 7.35 \times 10^6\,\text{J} \tag{11.15}$$

2. Calculate the temperature change from the heat transferred by rearranging the equation $Q = mc\Delta T$ to solve for ΔT.

$$\Delta T = \frac{Q}{mc}, \tag{11.16}$$

where m is the mass of the brake material (not the entire truck). Insert the values $Q = 7.35 \times 10^6$ J (since the heat transfer is equal to the change in gravitational potential energy), $m = 100$ kg, and $c = 800$ J/kg $\cdot$ °C to find

$$\Delta T = \frac{7.35 \times 10^6\,\text{J}}{(100\ \text{kg})(800\ \text{J/kg}\ \cdot\ ^\circ\text{C})} = 91.9\ ^\circ\text{C}. \tag{11.17}$$

Discussion

This temperature is close to the boiling point of water. If the truck had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, which would be hard on the brakes. This is why truck drivers sometimes use a different technique for called "engine braking" to avoid burning their brakes during steep descents. Engine braking is using the slowing forces of an engine in low gear rather than brakes to slow down.

Practice Problems

5. How much heat does it take to raise the temperature of 10.0 kg of water by 1.0 °C ?

a. 84 J
b. 42 J
c. 84 kJ
d. 42 kJ

6. Calculate the change in temperature of 1.0 kg of water that is initially at room temperature if 3.0 kJ of heat is added.

a. 358 °C
b. 716 °C
c. 0.36 °C
d. 0.72 °C

Check Your Understanding

7. What causes heat transfer?

a. The mass difference between two objects causes heat transfer.
b. The density difference between two objects causes heat transfer.
c. The temperature difference between two systems causes heat transfer.
d. The pressure difference between two objects causes heat transfer.

8. When two bodies of different temperatures are in contact, what is the overall direction of heat transfer?

a. The overall direction of heat transfer is from the higher-temperature object to the lower-temperature object.
b. The overall direction of heat transfer is from the lower-temperature object to the higher-temperature object.
c. The direction of heat transfer is first from the lower-temperature object to the higher-temperature object, then back again to the lower-temperature object, and so-forth, until the objects are in thermal equilibrium.
d. The direction of heat transfer is first from the higher-temperature object to the lower-temperature object, then back again to the higher-temperature object, and so-forth, until the objects are in thermal equilibrium.

9. What are the different methods of heat transfer?

a. conduction, radiation, and reflection
b. conduction, reflection, and convection
c. convection, radiation, and reflection
d. conduction, radiation, and convection

10. True or false—Conduction and convection cannot happen simultaneously

a. True
b. False

11.3 Phase Change and Latent Heat

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain changes in heat during changes of state, and describe latent heats of fusion and vaporization
- Solve problems involving thermal energy changes when heating and cooling substances with phase changes

Section Key Terms

condensation	freezing	latent heat	sublimation
latent heat of fusion	latent heat of vaporization	melting	vaporization
phase change	phase diagram	plasma	

Phase Changes

So far, we have learned that adding thermal energy by heat increases the temperature of a substance. But surprisingly, there are situations where adding energy does not change the temperature of a substance at all! Instead, the additional thermal energy acts to loosen bonds between molecules or atoms and causes a **phase change**. Because this energy enters or leaves a system during a phase change without causing a temperature change in the system, it is known as **latent heat** (latent means *hidden*).

The three phases of matter that you frequently encounter are solid, liquid and gas (see Figure 11.8). Solid has the least energetic state; atoms in solids are in close contact, with forces between them that allow the particles to vibrate but not change position with neighboring particles. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.)

Liquid has a more energetic state, in which particles can slide smoothly past one another and change neighbors, although they are still held together by their mutual attraction.

Gas has a more energetic state than liquid, in which particles are broken free of their bonds. Particles in gases are separated by distances that are large compared with the size of the particles.

The most energetic state of all is **plasma**. Although you may not have heard much about plasma, it is actually the most common state of matter in the universe—stars are made up of plasma, as is lightning. The plasma state is reached by heating a gas to the point where particles are pulled apart, separating the electrons from the rest of the particle. This produces an ionized gas that is a combination of the negatively charged free electrons and positively charged ions, known as plasma.

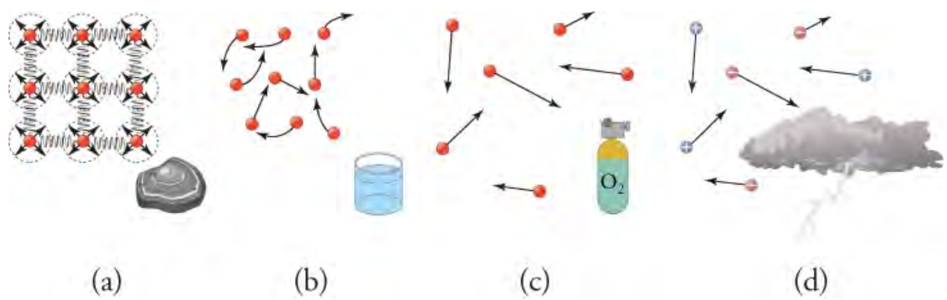


Figure 11.8 (a) Particles in a solid always have the same neighbors, held close by forces represented here by springs. These particles are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms or molecules together. (b) Particles in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its molecules. (c) Particles in a gas are separated by distances that are considerably larger than the size of the particles themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out into its surroundings. (d) The atmosphere is ionized in the extreme heat of a lightning strike.

During a phase change, matter changes from one phase to another, either through the addition of energy by heat and the transition to a more energetic state, or from the removal of energy by heat and the transition to a less energetic state.

Phase changes to a more energetic state include the following:

- **Melting**—Solid to liquid
- **Vaporization**—Liquid to gas (included boiling and evaporation)
- **Sublimation**—Solid to gas

Phase changes to a less energetic state are as follows:

- **Condensation**—Gas to liquid
- **Freezing**—Liquid to solid

Energy is required to melt a solid because the bonds between the particles in the solid must be broken. Since the energy involved in a phase changes is used to break bonds, there is no increase in the kinetic energies of the particles, and therefore no rise in temperature. Similarly, energy is needed to vaporize a liquid to overcome the attractive forces between particles in the liquid. There is no temperature change until a phase change is completed. The temperature of a cup of soda and ice that is initially at 0 °C stays at 0 °C until all of the ice has melted. In the reverse of these processes—freezing and condensation—energy is released from the latent heat (see Figure 11.9).

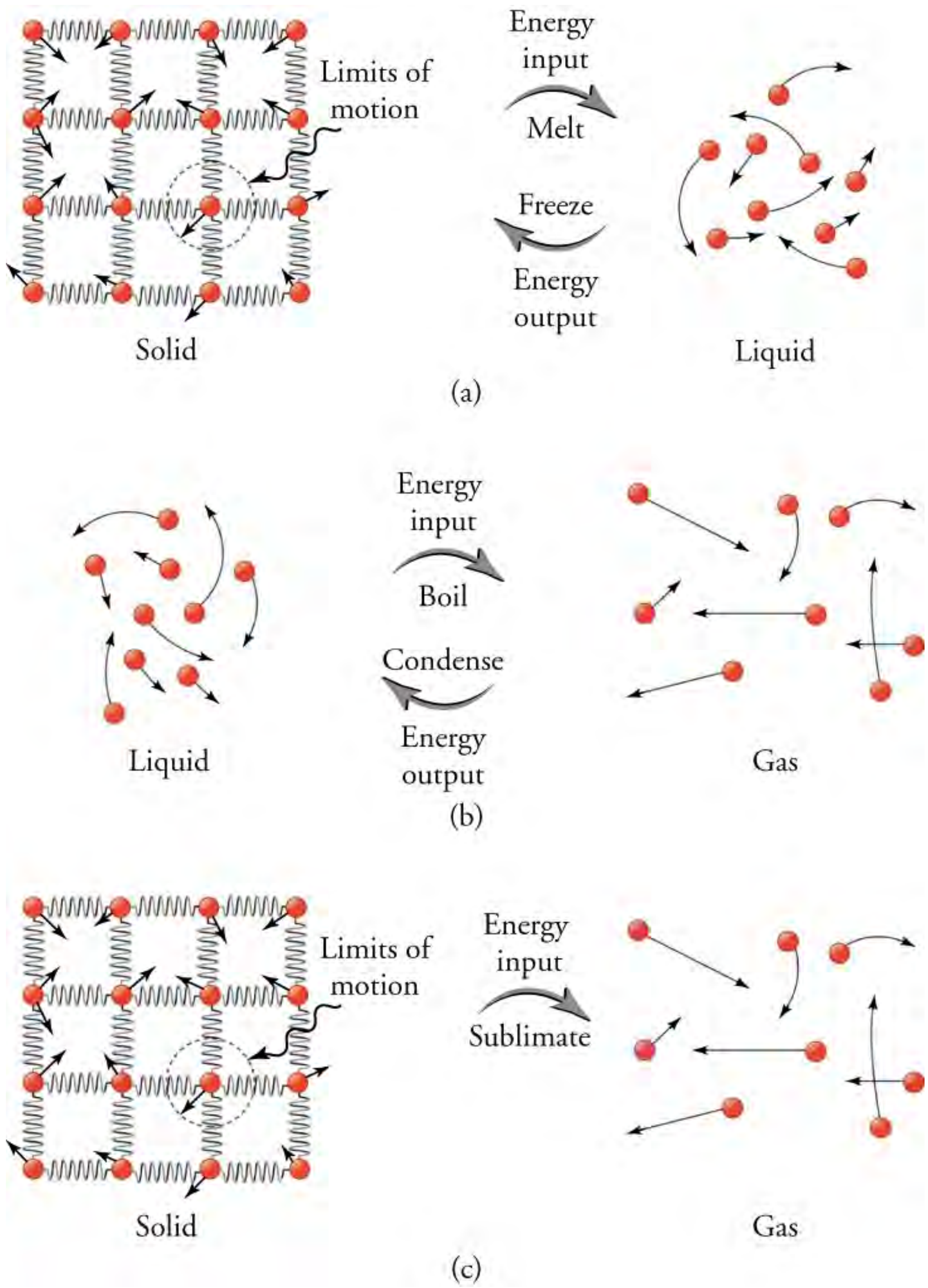


Figure 11.9 (a) Energy is required to partially overcome the attractive forces between particles in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Particles are separated by large distances when changing from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is completed. (c) Enough energy is added that the liquid state is skipped over completely as a substance undergoes sublimation.

The heat, Q, required to change the phase of a sample of mass m is

$Q = mL_f$ (for melting/freezing),

$Q = mL_v$ (for vaporization/condensation),

where L_f is the **latent heat of fusion**, and L_v is the **latent heat of vaporization**. The latent heat of fusion is the amount of heat needed to cause a phase change between solid and liquid. The latent heat of vaporization is the amount of heat needed to cause a phase change between liquid and gas. L_f and L_v are coefficients that vary from substance to substance, depending on the strength of intermolecular forces, and both have standard units of J/kg. See Table 11.3 for values of L_f and L_v of different substances.

Substance	Melting Point (°C)	*Lf* (kJ/kg)	Boiling Point (°C)	*Lv* (kJ/kg)
Helium	−269.7	5.23	−268.9	20.9
Hydrogen	−259.3	58.6	−252.9	452
Nitrogen	−210.0	25.5	−195.8	201
Oxygen	−218.8	13.8	−183.0	213
Ethanol	−114	104	78.3	854
Ammonia	−78	332	−33.4	1370
Mercury	−38.9	11.8	357	272
Water	0.00	334	100.0	2256
Sulfur	119	38.1	444.6	326
Lead	327	24.5	1750	871
Antimony	631	165	1440	561
Aluminum	660	380	2520	11400
Silver	961	88.3	2193	2336
Gold	1063	64.5	2660	1578
Copper	1083	134	2595	5069
Uranium	1133	84	3900	1900
Tungsten	3410	184	5900	4810

Table 11.3 Latent Heats of Fusion and Vaporization, along with Melting and Boiling Points

Let's consider the example of adding heat to ice to examine its transitions through all three phases—solid to liquid to gas. A phase diagram indicating the temperature changes of water as energy is added is shown in Figure 11.10. The ice starts out at −20 °C, and its temperature rises linearly, absorbing heat at a constant rate until it reaches 0 °. Once at this temperature, the ice gradually melts, absorbing 334 kJ/kg. The temperature remains constant at 0 °C during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate. At 100 °C, the water begins to boil and the temperature again remains constant while the water absorbs 2256 kJ/kg during this phase change. When all the liquid has become steam, the temperature rises again at a constant rate.

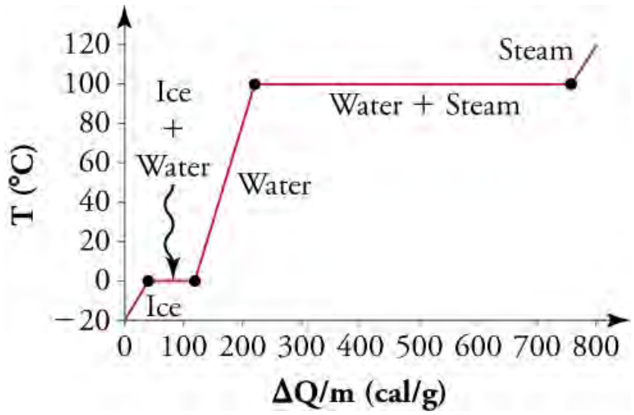


Figure 11.10 A graph of temperature versus added energy. The system is constructed so that no vapor forms while ice warms to become liquid water, and so when vaporization occurs, the vapor remains in the system. The long stretches of constant temperature values at 0 °C and 100 °C reflect the large latent heats of melting and vaporization, respectively.

We have seen that vaporization requires heat transfer to a substance from its surroundings. Condensation is the reverse process, where heat in transferred *away from* a substance *to* its surroundings. This release of latent heat increases the temperature of the surroundings. Energy must be removed from the condensing particles to make a vapor condense. This is why condensation occurs on cold surfaces: the heat transfers energy away from the warm vapor to the cold surface. The energy is exactly the same as that required to cause the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_v$. Latent heat is also released into the environment when a liquid freezes, and can be calculated from $Q = mL_f$.

FUN IN PHYSICS

Making Ice Cream

Figure 11.11 With the proper ingredients, some ice and a couple of plastic bags, you could make your own ice cream in five minutes. (ElinorD, Wikimedia Commons)

Ice cream is certainly easy enough to buy at the supermarket, but for the hardcore ice cream enthusiast, that may not be satisfying enough. Going through the process of making your own ice cream lets you invent your own flavors and marvel at the physics firsthand (Figure 11.11).

The first step to making homemade ice cream is to mix heavy cream, whole milk, sugar, and your flavor of choice; it could be as simple as cocoa powder or vanilla extract, or as fancy as pomegranates or pistachios.

The next step is to pour the mixture into a container that is deep enough that you will be able to churn the mixture without it spilling over, and that is also freezer-safe. After placing it in the freezer, the ice cream has to be stirred vigorously every 45 minutes for four to five hours. This slows the freezing process and prevents the ice cream from turning into a solid block of ice. Most people prefer a soft creamy texture instead of one giant popsicle.

As it freezes, the cream undergoes a phase change from liquid to solid. By now, we're experienced enough to know that this means that the cream must experience a loss of heat. Where does that heat go? Due to the temperature difference between the freezer and the ice cream mixture, heat transfers thermal energy from the ice cream to the air in the freezer. Once the temperature in the freezer rises enough, the freezer is cooled by pumping excess heat outside into the kitchen.

A faster way to make ice cream is to chill it by placing the mixture in a plastic bag, surrounded by another plastic bag half full of ice. (You can also add a teaspoon of salt to the outer bag to lower the temperature of the ice/salt mixture.) Shaking the bag for five minutes churns the ice cream while cooling it evenly. In this case, the heat transfers energy out of the ice cream mixture and into the ice during the phase change.

This video (http://www.openstax.org/l/28icecream) gives a demonstration of how to make home-made ice cream using ice and plastic bags.

GRASP CHECK

Why does the ice bag method work so much faster than the freezer method for making ice cream?

a. Ice has a smaller specific heat than the surrounding air in a freezer. Hence, it absorbs more energy from the ice-cream mixture.
b. Ice has a smaller specific heat than the surrounding air in a freezer. Hence, it absorbs less energy from the ice-cream mixture.
c. Ice has a greater specific heat than the surrounding air in a freezer. Hence, it absorbs more energy from the ice-cream mixture.
d. Ice has a greater specific heat than the surrounding air in a freezer. Hence, it absorbs less energy from the ice-cream mixture.

Solving Thermal Energy Problems with Phase Changes

WORKED EXAMPLE

Calculating Heat Required for a Phase Change

Calculate a) how much energy is needed to melt 1.000 kg of ice at 0 $^\circ\text{C}$ (freezing point), and b) how much energy is required to vaporize 1.000 kg of water at 100 $^\circ\text{C}$ (boiling point).

STRATEGY FOR (A)

Using the equation for the heat required for melting, and the value of the latent heat of fusion of water from the previous table, we can solve for part (a).

Solution to (a)

The energy to melt 1.000 kg of ice is

$$Q = mL_f = (1.000 \text{ kg})(334 \text{ kJ/kg}) = 334 \text{ kJ}. \tag{11.18}$$

STRATEGY FOR (B)

To solve part (b), we use the equation for heat required for vaporization, along with the latent heat of vaporization of water from the previous table.

Solution to (b)

The energy to vaporize 1.000 kg of liquid water is

$$Q = mL_v = (1.000 \text{ kg})(2256 \text{ kJ/kg}) = 2256 \text{ kJ}. \tag{11.19}$$

Discussion

The amount of energy need to melt a kilogram of ice (334 kJ) is the same amount of energy needed to raise the temperature of 1.000 kg of liquid water from 0 °C to 79.8 °C . This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes. It also demonstrates that the amount of energy needed for vaporization is even greater.

WORKED EXAMPLE

Calculating Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Ice cubes are used to chill a soda at 20 °C and with a mass of $m_{soda} = 0.25 \text{ kg}$. The ice is at 0 °C and the total mass of the ice cubes is 0.018 kg. Assume that the soda is kept in a foam container so that heat loss can be ignored, and that the soda has the same specific heat as water. Find the final temperature when all of the ice has melted.

STRATEGY

The ice cubes are at the melting temperature of 0 °C . Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first, the phase change occurs and solid (ice) transforms into liquid water at the melting temperature; then, the temperature of this water rises. Melting yields water at 0 °C, so more heat is transferred from the soda to this water until they are the same temperature. Since the amount of heat leaving the soda is the same as the amount of heat transferred to the ice.

$$Q_{ice} = -Q_{soda} \tag{11.20}$$

The heat transferred to the ice goes partly toward the phase change (melting), and partly toward raising the temperature after melting. Recall from the last section that the relationship between heat and temperature change is $Q = mc\Delta T$. For the ice, the temperature change is $T_f - 0\ °\text{C}$. The total heat transferred to the ice is therefore

$$Q_{ice} = m_{ice}L_f + m_{ice}c_w(T_f - 0\ °\text{C}). \tag{11.21}$$

Since the soda doesn't change phase, but only temperature, the heat given off by the soda is

$$Q_{soda} = m_{soda}c_w(T_f - 20\ °\text{C}). \tag{11.22}$$

Since $Q_{ice} = -Q_{soda}$,

$$m_{ice}L_f + m_{ice}c_w(T_f - 0\ °\text{C}) = -m_{soda}c_w(T_f - 20\ °\text{C}). \tag{11.23}$$

Bringing all terms involving T_f to the left-hand-side of the equation, and all other terms to the right-hand-side, we can solve for T_f .

$$T_f = \frac{m_{soda}c_w(20\ °\text{C}) - m_{ice}L_f}{(m_{soda} + m_{ice})c_w} \tag{11.24}$$

Substituting the known quantities

$$T_f = \frac{(0.25 \text{ kg})(4186 \text{ J/kg} \cdot °\text{C})(20\ °\text{C}) - (0.018 \text{ kg})(334{,}000 \text{ J/kg})}{(0.25 \text{ kg} + 0.018 \text{ kg})(4186 \text{ K/kg} \cdot °\text{C})} = 13\ °\text{C} \tag{11.25}$$

Discussion

This example shows the enormous energies involved during a phase change. The mass of the ice is about 7 percent the mass of the soda, yet it causes a noticeable change in the soda's temperature.

TIPS FOR SUCCESS

If the ice were not already at the freezing point, we would also have to factor in how much energy would go into raising its temperature up to 0 °C, before the phase change occurs. This would be a realistic scenario, because the temperature of ice is often below 0 °C .

Practice Problems

11. How much energy is needed to melt 2.00 kg of ice at 0 °C ?

a. 334 kJ
b. 336 kJ
c. 167 kJ
d. 668 kJ

12. If $2500\ \text{kJ}$ of energy is just enough to melt $3.0\ \text{kg}$ of a substance, what is the substance's latent heat of fusion?

a. $7500\ \text{kJ} \cdot \text{kg}$
b. $7500\ \text{kJ/kg}$
c. $830\ \text{kJ} \cdot \text{kg}$
d. $830\ \text{kJ/kg}$

Check Your Understanding

13. What is latent heat?

a. It is the heat that must transfer energy to or from a system in order to cause a mass change with a slight change in the temperature of the system.
b. It is the heat that must transfer energy to or from a system in order to cause a mass change without a temperature change in the system.
c. It is the heat that must transfer energy to or from a system in order to cause a phase change with a slight change in the temperature of the system.
d. It is the heat that must transfer energy to or from a system in order to cause a phase change without a temperature change in the system.

14. In which phases of matter are molecules capable of changing their positions?

a. gas, liquid, solid
b. liquid, plasma, solid
c. liquid, gas, plasma
d. plasma, gas, solid

KEY TERMS

absolute zero lowest possible temperature; the temperature at which all molecular motion ceases

Celsius scale temperature scale in which the freezing point of water is 0 °C and the boiling point of water is 100 °C at 1 atm of pressure

condensation phase change from gas to liquid

conduction heat transfer through stationary matter by physical contact

convection heat transfer by the movement of fluid

degree Celsius unit on the Celsius temperature scale

degree Fahrenheit unit on the Fahrenheit temperature scale

Fahrenheit scale temperature scale in which the freezing point of water is 32 °F and the boiling point of water is 212 °F

freezing phase change from liquid to solid

heat transfer of thermal (or internal) energy due to a temperature difference

heat capacity amount of heat necessary to change the temperature of a substance by 1.00 °C

Kelvin unit on the Kelvin temperature scale; note that it is never referred to in terms of "degrees" Kelvin

Kelvin scale temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

latent heat heat related to the phase change of a substance rather than a change of temperature

latent heat of fusion amount of heat needed to cause a phase change between solid and liquid

latent heat of vaporization amount of heat needed to cause a phase change between liquid and gas

melting phase change from solid to liquid

phase change transition between solid, liquid, or gas states of a substance

plasma ionized gas that is a combination of the negatively charged free electrons and positively charged ions

radiation energy transferred by electromagnetic waves

specific heat amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 °C

sublimation phase change from solid to gas

temperature quantity measured by a thermometer

thermal energy average random kinetic energy of a molecule or an atom

vaporization phase change from liquid to gas

SECTION SUMMARY

11.1 Temperature and Thermal Energy

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted into temperatures on another scale.

11.2 Heat, Specific Heat, and Heat Transfer

- Heat is thermal (internal) energy transferred due to a temperature difference.
- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material.
- Heat is transferred by three different methods: conduction, convection, and radiation.
- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- Convection is heat transfer by the movement of mass.
- Radiation is heat transfer by electromagnetic waves.

11.3 Phase Change and Latent Heat

- Most substances have four distinct phases: solid, liquid, gas, and plasma.
- Gas is the most energetic state and solid is the least.
- During a phase change, a substance undergoes transition to a higher energy state when heat is added, or to a lower energy state when heat is removed.
- Heat is added to a substance during melting and vaporization.
- Latent heat is released by a substance during condensation and freezing.
- Phase changes occur at fixed temperatures called boiling and freezing (or melting) points for a given substance.

KEY EQUATIONS

11.1 Temperature and Thermal Energy

Celsius to Fahrenheit conversion	$T_{°F} = \frac{9}{5} T_{°C} + 32$
Fahrenheit to Celsius conversion	$T_{°C} = \frac{5}{9} (T_{°F} - 32)$
Celsius to Kelvin conversion	$T_K = T_{°C} + 273.15$
Kelvin to Celsius conversion	$T_{°C} = T_K - 273.15$
Fahrenheit to Kelvin conversion	$T_K = \frac{5}{9} (T_{°F} - 32) + 273.15$
Kelvin to Fahrenheit conversion	$T_{°F} = \frac{9}{5} (T_K - 273.15) + 32$

11.2 Heat, Specific Heat, and Heat Transfer

heat transfer	$Q = mc\Delta T$
density	$\rho = \frac{m}{V}$

11.3 Phase Change and Latent Heat

heat transfer for melting/freezing phase change	$Q = mL_f$
heat transfer for vaporization/ condensation phase change	$Q = mL_v$

CHAPTER REVIEW

Concept Items

11.1 Temperature and Thermal Energy

1. A glass of water has a temperature of 31 degrees Celsius. What state of matter is it in?
 a. solid
 b. liquid
 c. gas
 d. plasma

2. What is the difference between thermal energy and internal energy?
 a. The thermal energy of the system is the average kinetic energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies and the potential energies of its constituent particles.
 b. The thermal energy of the system is the average potential energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies and the potential energies of its constituent particles.
 c. The thermal energy of the system is the average kinetic energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies of its constituent particles.
 d. The thermal energy of the system is the average potential energy of the systems' constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies of its constituent particles.

3. What does the Celsius scale use as a reference point?
 a. The boiling point of mercury
 b. The boiling point of wax
 c. The freezing point of water
 d. The freezing point of wax

11.2 Heat, Specific Heat, and Heat Transfer

4. What are the SI units of specific heat?
 a. $J/kg^2 \cdot °C$
 b. $J \cdot kg^2/°C$
 c. $J \cdot kg/°C$
 d. $J/kg \cdot° C$

5. What is radiation?
 a. The transfer of energy through emission and absorption of the electromagnetic waves is known as radiation.
 b. The transfer of energy without any direct physical

contact between any two substances.

c. The transfer of energy through direct physical contact between any two substances.

d. The transfer of energy by means of the motion of fluids at different temperatures and with different densities.

11.3 Phase Change and Latent Heat

6. Why is there no change in temperature during a phase change, even if energy is absorbed by the system?

a. The energy is used to break bonds between particles, and so does not increase the potential energy of the system's particles.

b. The energy is used to break bonds between particles, and so increases the potential energy of the system's particles.

c. The energy is used to break bonds between particles, and so does not increase the kinetic energy of the system's particles.

d. The energy is used to break bonds between particles, and so increases the kinetic energy of the system's particles.

7. In which two phases of matter do atoms and molecules have the most distance between them?

a. gas and solid

b. gas and liquid

c. gas and plasma

d. liquid and plasma

Critical Thinking Items

11.1 Temperature and Thermal Energy

8. The temperature of two equal quantities of water needs to be raised - the first container by 5 degrees Celsius and the second by 5 degrees Fahrenheit. Which one would require more heat?

a. The heat required by the first container is more than the second because each degree Celsius is equal to 1.8 degrees Fahrenheit.

b. The heat required by the first container is less than the second because each degree Fahrenheit is equal to 1.8 degrees Celsius.

c. The heat required by the first container is more than the second because each degree Celsius is equal to 3.6 degrees Fahrenheit.

d. The heat required by the first container is less than the second because each degree Fahrenheit is equal to 3.6 degrees Celsius.

9. What is 100.00 °C in kelvins?

a. 212.00 K

b. 100.00 K

c. 473.15 K

d. 373.15 K

11.2 Heat, Specific Heat, and Heat Transfer

10. The value of specific heat is the same whether the units are J/kg·K or J/kg·°C. How?

a. Temperature difference is dependent on the chosen temperature scale.

b. Temperature change is different in units of kelvins and degrees Celsius.

c. Reading of temperatures in kelvins and degree Celsius are the same.

d. The temperature change is the same in units of kelvins and degrees Celsius.

11. If the thermal energy of a perfectly black object is increased by conduction, will the object remain black in appearance? Why or why not?

a. No, the energy of the radiation increases as the temperature increases, and the radiation becomes visible at certain temperatures.

b. Yes, the energy of the radiation decreases as the temperature increases, and the radiation remains invisible at those energies.

c. No, the energy of the radiation decreases as the temperature increases, until the frequencies of the radiation are the same as those of visible light.

d. Yes, as the temperature increases, and the energy is transferred from the object by other mechanisms besides radiation, so that the energy of the radiation does not increase.

12. What is the specific heat of a substance that requires 5.00 kJ of heat to raise the temperature of 3.00 kg by 5.00 °F?

a. 3.33×103 J/kg ·° C

b. 6.00×103 J/kg ·° C

c. 3.33×102 J/kg · ° C

d. 6.00×102 J/kg · ° C

11.3 Phase Change and Latent Heat

13. Assume 1.0 kg of ice at 0 °C starts to melt. It absorbs 300 kJ of energy by heat. What is the temperature of the water afterwards?

a. 10 °C

b. 20 °C

c. 5 °C

d. 0 °C

Problems

11.1 Temperature and Thermal Energy

14. What is 35.0 °F in kelvins?
   a. 1.67 K
   b. 35.0 K
   c. -271.5 K
   d. 274.8 K

15. Design a temperature scale where the freezing point of water is 0 degrees and its boiling point is 70 degrees. What would be the room temperature on this scale?
   a. If room temperature is 25.0 °C, the temperature on the new scale will be 17.5 °.
   b. If room temperature is 25.0 °C, the temperature on the new scale will be 25.0°.
   c. If the room temperature is 25.0 °C, the temperature on the new scale will be 35.7°.
   d. If the room temperature is 25.0 °C, the temperature on the new scale will be 50.0°.

11.2 Heat, Specific Heat, and Heat Transfer

16. A certain quantity of water is given 4.0 kJ of heat. This raises its temperature by 30.0 °F. What is the mass of the water in grams?
   a. 5.7 g
   b. 570 g
   c. 5700 g
   d. 57 g

17. 5290 J of heat is given to 0.500 kg water at 15.00 °C. What will its final temperature be?
   a. 15.25° C
   b. 12.47 ° C
   c. 40.3° C
   d. 17.53° C

11.3 Phase Change and Latent Heat

18. How much energy would it take to heat 1.00 kg of ice at 0 °C to water at 15.0 °C?
   a. 271 kJ
   b. 334 kJ
   c. 62.8 kJ
   d. 397 kJ

19. Ice cubes are used to chill a soda with a mass m_{soda} = 0.300 kg at 15.0 °C. The ice is at 0 °C, and the total mass of the ice cubes is 0.020 kg. Assume that the soda is kept in a foam container so that heat loss can be ignored, and that the soda has the same specific heat as water. Find the final temperature when all ice has melted.
   a. 19.02 °C
   b. 90.3 °C
   c. 0.11 °C
   d. 9.03 °C

Performance Task

11.3 Phase Change and Latent Heat

20. You have been tasked with designing a baking pan that will bake batter the fastest. There are four materials available for you to test.

- Four pans of similar design, consisting of aluminum, iron (steel), copper, and glass
- Oven or similar heating source
- Device for measuring high temperatures
- Balance for measuring mass

Instructions

Procedure

1. Design a safe experiment to test the specific heat of each material (i.e., no extreme temperatures should be used)
2. Write down the materials needed for your experiment and the procedure you will follow. Make sure that you include every detail, so that the experiment can be repeated by others.
3. Carry out the experiment and record any data collected.
4. Review your results and make a recommendation as to which metal should be used for the pan.
   a. What physical quantities do you need to measure to determine the specific heats for the different materials?
   b. How does the glass differ from the metals in terms of thermal properties?
   c. What are your sources of error?

TEST PREP

Multiple Choice

11.1 Temperature and Thermal Energy

21. The temperature difference of 1 K is the same as
   a. 1 degree Celsius
   b. 1 degree Fahrenheit
   c. 273.15 degrees Celsius
   d. 273.15 degrees Fahrenheit

22. What is the preferred temperature scale used in scientific laboratories?
   a. celsius
   b. fahrenheit
   c. kelvin
   d. rankine

11.2 Heat, Specific Heat, and Heat Transfer

23. Which phase of water has the largest specific heat?
   a. solid
   b. liquid
   c. gas

24. What kind of heat transfer requires no medium?
   a. conduction
   b. convection
   c. reflection
   d. radiation

25. Which of these substances has the greatest specific heat?
   a. copper
   b. mercury
   c. aluminum
   d. wood

26. Give an example of heat transfer through convection.
   a. The energy emitted from the filament of an electric bulb
   b. The energy coming from the sun
   c. A pan on a hot burner
   d. Water boiling in a pot

11.3 Phase Change and Latent Heat

27. What are the SI units of latent heat?
   a. J/kg
   b. J.kg
   c. J/cal
   d. cal/kg

28. Which substance has the largest latent heat of fusion?
   a. gold
   b. water
   c. mercury
   d. tungsten

29. In which phase changes does matter undergo a transition to a more energetic state?
   a. freezing and vaporization
   b. melting and sublimation
   c. melting and vaporization
   d. melting and freezing

30. A room has a window made from thin glass. The room is colder than the air outside. There is some condensation on the glass window. On which side of the glass would the condensation most likely be found?
   a. Condensation is on the outside of the glass when the cool, dry air outside the room comes in contact with the cold pane of glass.
   b. Condensation is on the outside of the glass when the warm, moist air outside the room comes in contact with the cold pane of glass.
   c. Condensation is on the inside of the glass when the cool, dry air inside the room comes in contact with the cold pane of glass.
   d. Condensation is on the inside of the glass when the warm, moist air inside the room comes in contact with the cold pane of glass.

Short Answer

11.1 Temperature and Thermal Energy

31. What is *absolute zero* on the Fahrenheit scale?
   a. 0 °F
   b. 32 °F
   c. -273.15 °F
   d. -459.67 °F

32. What is *absolute zero* on the Celsius scale?
   a. 0 °C
   b. 273.15 °C
   c. -459.67 °C
   d. -273.15 °C

33. A planet's atmospheric pressure is such that water there boils at a lower temperature than it does at sea level on Earth. If a Celsius scale is derived on this planet, will it be the same as that on Earth?
   a. The Celsius scale derived on the planet will be the same as that on Earth, because the Celsius scale is independent of the freezing and boiling points of water.
   b. The Celsius scale derived on that planet will not be the same as that on Earth, because the Celsius scale is dependent and derived by using the freezing and boiling points of water.
   c. The Celsius scale derived on the planet will be the same as that on Earth, because the Celsius scale is an absolute temperature scale based on molecular motion, which is independent of pressure.
   d. The Celsius scale derived on the planet will not be the same as that on Earth, but the Fahrenheit scale

will be the same, because its reference temperatures are not based on the freezing and boiling points of water.

34. What is the difference between the freezing point and boiling point of water on the Reaumur scale?
    a. The boiling point of water is 80° on the Reaumur scale.
    b. Reaumur scale is less than 120°.
    c. 100°
    d. 80°

11.2 Heat, Specific Heat, and Heat Transfer

35. In the specific heat equation what does *c* stand for?
    a. Total heat
    b. Specific heat
    c. Specific temperature
    d. Specific mass

36. Specific heat may be measured in J/kg · K, J/kg · °C. What other units can it be measured in?
    a. kg/kcal · °C
    b. kcal · °C/kg
    c. kg · °C/kcal
    d. kcal/kg · °C

37. What is buoyancy?
    a. Buoyancy is a downward force exerted by a solid that opposes the weight of an object.
    b. Buoyancy is a downward force exerted by a fluid that opposes the weight of an immersed object.
    c. Buoyancy is an upward force exerted by a solid that opposes the weight an object.
    d. Buoyancy is an upward force exerted by a fluid that opposes the weight of an immersed object.

38. Give an example of convection found in nature.
    a. heat transfer through metallic rod
    b. heat transfer from the sun to Earth
    c. heat transfer through ocean currents
    d. heat emitted by a light bulb into its environment

39. Calculate the temperature change in a substance with specific heat 735 J/kg · °C when 14 kJ of heat is given to a 3.0-kg sample of that substance.
    a. 57 °C
    b. 63 °C
    c. 1.8×10^{-2} °C
    d. 6.3 °C

40. Aluminum has a specific heat of 900 J/kg·°C. How much energy would it take to change the temperature of 2 kg aluminum by 3 °C?
    a. 1.3 kJ
    b. 0.60 kJ
    c. 54 kJ
    d. 5.4 kJ

11.3 Phase Change and Latent Heat

41. Upon what does the required amount of heat removed to freeze a sample of a substance depend?
    a. The mass of the substance and its latent heat of vaporization
    b. The mass of the substance and its latent heat of fusion
    c. The mass of the substance and its latent heat of sublimation
    d. The mass of the substance only

42. What do latent heats, L_f and L_v, depend on?
    a. L_f and L_v depend on the forces between the particles in the substance.
    b. L_f and L_v depend on the mass of the substance.
    c. L_f and L_v depend on the volume of the substance.
    d. L_f and L_v depend on the temperature of the substance.

43. How much energy is required to melt 7.00 kg a block of aluminum that is at its melting point? (Latent heat of fusion of aluminum is 380 kJ/kg.)
    a. 54.3 kJ
    b. 2.66 kJ
    c. 0.0184 kJ
    d. 2.66×10^{3} kJ

44. A 3.00 kg sample of a substance is at its boiling point. If 5,360 kJ of energy are enough to boil away the entire substance, what is its latent heat of vaporization?
    a. 2,685 kJ/kg
    b. 3,580 kJ/kg
    c. 895 kJ/kg
    d. 1,790 kJ/kg

Extended Response

11.1 Temperature and Thermal Energy

45. What is the meaning of absolute zero?
    a. It is the temperature at which the internal energy of the system is maximum, because the speed of its constituent particles increases to maximum at this point.
    b. It is the temperature at which the internal energy of the system is maximum, because the speed of its constituent particles decreases to zero at this point.

c. It is the temperature at which the internal energy of the system approaches zero, because the speed of its constituent particles increases to a maximum at this point.
d. It is that temperature at which the internal energy of the system approaches zero, because the speed of its constituent particles decreases to zero at this point.

46. Why does it feel hotter on more humid days, even though there is no difference in temperature?
a. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is lower, which reduces the evaporation rate from the skin's surface.
b. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is higher, which reduces the evaporation rate from the skin's surface.
c. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is lower, which increases the evaporation rate from the skin's surface.
d. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is higher, which increases the evaporation rate from the skin's surface.

11.2 Heat, Specific Heat, and Heat Transfer

47. A hot piece of metal needs to be cooled. If you were to put the metal in ice or in cold water, such that the ice did not melt and the temperature of either changed by the same amount, which would reduce the metal's temperature more? Why?
a. Water would reduce the metal's temperature more, because water has a greater specific heat than ice.
b. Water would reduce the metal's temperature more, because water has a smaller specific heat than ice.
c. Ice would reduce the metal's temperature more, because ice has a smaller specific heat than water.
d. Ice would reduce the metal's temperature more, because ice has a greater specific heat than water.

48. On a summer night, why does a black object seem colder than a white one?
a. The black object radiates energy faster than the white one, and hence reaches a lower temperature in less time.
b. The black object radiates energy slower than the white one, and hence reaches a lower temperature in less time.
c. The black object absorbs energy faster than the white one, and hence reaches a lower temperature in less time.
d. The black object absorbs energy slower than the white one, and hence reaches a lower temperature in less time.

49. Calculate the difference in heat required to raise the temperatures of 1.00 kg of gold and 1.00 kg of aluminum by 1.00 °C. (The specific heat of aluminum equals 900 J/kg · °C; the specific heat of gold equals 129 J/kg · °C.)
a. 771 J
b. 129 J
c. 90 J
d. 900 J

11.3 Phase Change and Latent Heat

50. True or false—You have an ice cube floating in a glass of water with a thin thread resting across the cube. If you cover the ice cube and thread with a layer of salt, they will stick together, so that you are able to lift the ice-cube when you pick up the thread.
a. True
b. False

51. Suppose the energy required to freeze 0.250 kg of water were added to the same mass of water at an initial temperature of 1.0 °C. What would be the final temperature of the water?
a. -69.8 °C
b. 79.8 °C
c. -78.8 °C
d. 80.8 °C