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University

Phys-ics

Volume 2

CHAPTER 7
Electric Potential

Figure 7.1 The energy released in a lightning strike is an excellent illustration of the vast quantities of energy that may be stored and released by an electric potential difference. In this chapter, we calculate just how much energy can be released in a lightning strike and how this varies with the height of the clouds from the ground. (credit: modification of work by Anthony Quintano)

Chapter Outline

INTRODUCTION In Electric Charges and Fields, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two terms commonly used to describe electricity are its energy and *voltage*, which we show in this chapter is directly related to the potential energy in a system.

We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country via currents through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at the molecular level, ions cross cell membranes and transfer information.

We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home frequently produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing a much larger car battery, yet each has the same voltage. In this chapter, we examine the relationship between voltage and electrical energy, and begin to explore some of the many applications of electricity.

7.1 Electric Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Define the work done by an electric force
- Define electric potential energy
- Apply work and potential energy in systems with electric charges

When a free positive charge q is accelerated by an electric field, it is given kinetic energy (Figure 7.2). The process is analogous to an object being accelerated by a gravitational field, as if the charge were going down an electrical hill where its electric potential energy is converted into kinetic energy, although of course the sources of the forces are very different. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

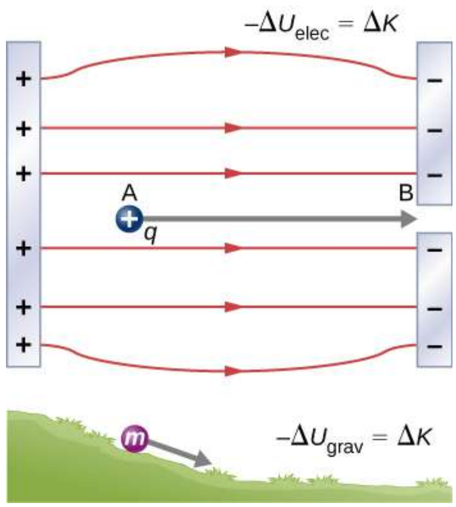


Figure 7.2 A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases, potential energy decreases as kinetic energy increases, $-\Delta U = \Delta K$. Work is done by a force, but since this force is conservative, we can write $W = -\Delta U$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken, as we will demonstrate later. This is exactly analogous to the gravitational force. When a force is conservative, it is possible to define a potential energy associated with the force. It is usually easier to work with the potential energy (because it depends only on position) than to calculate the work directly.

To show this explicitly, consider an electric charge $+q$ fixed at the origin and move another charge $+Q$ toward q in such a manner that, at each instant, the applied force $\vec{\mathbf{F}}$ exactly balances the electric force $\vec{\mathbf{F}}_e$ on Q (Figure 7.3). The work done by the applied force $\vec{\mathbf{F}}$ on the charge Q changes the potential energy of Q. We call this potential energy the **electrical potential energy** of Q.

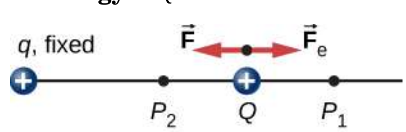


Figure 7.3 Displacement of "test" charge Q in the presence of fixed "source" charge q.

The work W_{12} done by the applied force $\vec{\mathbf{F}}$ when the particle moves from P_1 to P_2 may be calculated by

$$W_{12} = \int_{P_1}^{P_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}.$$

Since the applied force $\vec{\mathbf{F}}$ balances the electric force $\vec{\mathbf{F}}_e$ on Q, the two forces have equal magnitude and opposite directions. Therefore, the applied force is

$$\vec{\mathbf{F}} = -\vec{\mathbf{F}}_e = -\frac{kqQ}{r^2}\hat{\mathbf{r}},$$

where we have defined positive to be pointing away from the origin and r is the distance from the origin. The directions of both the displacement and the applied force in the system in Figure 7.3 are parallel, and thus the work done on the system is positive.

We use the letter U to denote electric potential energy, which has units of joules (J). When a conservative force does negative work, the system gains potential energy. When a conservative force does positive work, the system loses potential energy, $\Delta U = -W$. In the system in Figure 7.3, the Coulomb force acts in the opposite direction to the displacement; therefore, the work is negative. However, we have increased the potential energy in the two-charge system.

EXAMPLE 7.1

Kinetic Energy of a Charged Particle

A +3.0-nC charge Q is initially at rest a distance of 10 cm (r_1) from a +5.0-nC charge q fixed at the origin (Figure 7.4). Naturally, the Coulomb force accelerates Q away from q, eventually reaching 15 cm (r_2).

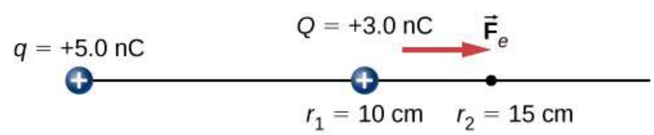


Figure 7.4 The charge Q is repelled by q, thus having work done on it and gaining kinetic energy.

a. What is the work done by the electric field between r_1 and r_2?
b. How much kinetic energy does Q have at r_2?

Strategy

Calculate the work with the usual definition. Since Q started from rest, this is the same as the kinetic energy.

Solution

Integrating force over distance, we obtain

$$\begin{aligned} W_{12} &= \int_{r_1}^{r_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{r_1}^{r_2} \frac{kqQ}{r^2}\,dr = \left[-\frac{kqQ}{r}\right]_{r_1}^{r_2} = kqQ\left[\frac{-1}{r_2} + \frac{1}{r_1}\right] \\ &= \left(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2\right)\left(5.0 \times 10^{-9}\ \text{C}\right)\left(3.0 \times 10^{-9}\ \text{C}\right)\left[\frac{-1}{0.15\ \text{m}} + \frac{1}{0.10\ \text{m}}\right] \\ &= 4.5 \times 10^{-7}\ \text{J}. \end{aligned}$$

This is also the value of the kinetic energy at r_2.

Significance

Charge Q was initially at rest; the electric field of q did work on Q, so now Q has kinetic energy equal to the work done by the electric field.

CHECK YOUR UNDERSTANDING 7.1

If Q has a mass of 4.00 μg, what is the speed of Q at r_2?

In this example, the work W done to accelerate a positive charge from rest is positive and results from a loss in U, or a negative ΔU. A value for U can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Electric Potential Energy

Work W done to accelerate a positive charge from rest is positive and results from a loss in U, or a negative ΔU. Mathematically,

$$W = -\Delta U. \quad (7.1)$$

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of electric potential energy than to deal with the Coulomb force directly in real-world applications.

In polar coordinates with q at the origin and Q located at r, the displacement element vector is $d\vec{\mathbf{l}} = \hat{\mathbf{r}}\, dr$ and thus the work becomes

$$W_{12} = kqQ \int_{r_1}^{r_2} \frac{1}{r^2} \hat{\mathrm{r}} \cdot \hat{\mathrm{r}} dr = kqQ\frac{1}{r_2} - kqQ\frac{1}{r_1}.$$

Notice that this result only depends on the endpoints and is otherwise independent of the path taken. To explore this further, compare path P_1 to P_2 with path $P_1 P_3 P_4 P_2$ in Figure 7.5.

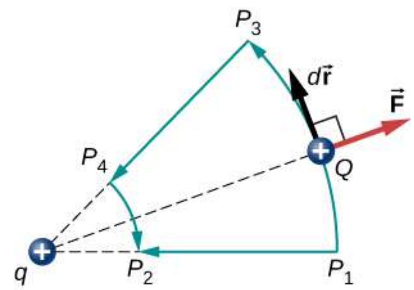


Figure 7.5 Two paths for displacement P_1 to P_2. The work on segments $P_1 P_3$ and $P_4 P_2$ are zero due to the electrical force being perpendicular to the displacement along these paths. Therefore, work on paths $P_1 P_2$ and $P_1 P_3 P_4 P_2$ are equal.

The segments $P_1 P_3$ and $P_4 P_2$ are arcs of circles centered at q. Since the force on Q points either toward or away from q, no work is done by a force balancing the electric force, because it is perpendicular to the displacement along these arcs. Therefore, the only work done is along segment $P_3 P_4$, which is identical to $P_1 P_2$.

One implication of this work calculation is that if we were to go around the path $P_1 P_3 P_4 P_2 P_1$, the net work would be zero (Figure 7.6). Recall that this is how we determine whether a force is conservative or not. Hence, because the electric force is related to the electric field by $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, the electric field is itself conservative. That is,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = 0.$$

Note that Q is a constant.

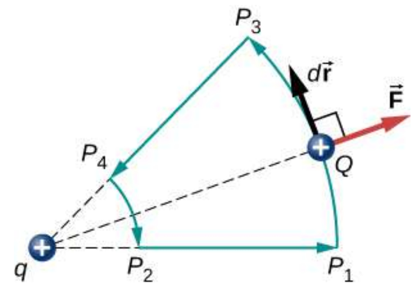


Figure 7.6 A closed path in an electric field. The net work around this path is zero.

Another implication is that we may define an electric potential energy. Recall that the work done by a conservative force is also expressed as the difference in the potential energy corresponding to that force. Therefore, the work W_{ref} to bring a charge from a reference point to a point of interest may be written as

$$W_{\text{ref}} = \int_{r_{\text{ref}}}^{r} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$$

and, by Equation 7.1, the difference in potential energy $(U_2 - U_1)$ of the test charge Q between the two points is

$$\Delta U = -\int_{r_{\text{ref}}}^{r} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}.$$

Therefore, we can write a general expression for the potential energy of two point charges (in spherical coordinates):

$$\Delta U = -\int_{r_{\text{ref}}}^{r} \frac{kqQ}{r^2} dr = -\left[-\frac{kqQ}{r}\right]_{r_{\text{ref}}}^{r} = kqQ\left[\frac{1}{r} - \frac{1}{r_{\text{ref}}}\right].$$

We may take the second term to be an arbitrary constant reference level, which serves as the zero reference:

$$U(r) = k\frac{qQ}{r} - U_{\text{ref}}.$$

A convenient choice of reference that relies on our common sense is that when the two charges are infinitely far apart, there is no interaction between them. (Recall the discussion of reference potential energy in Potential Energy and Conservation of Energy.) Taking the potential energy of this state to be zero removes the term U_{ref} from the equation (just like when we say the ground is zero potential energy in a gravitational potential energy problem), and the potential energy of Q when it is separated from q by a distance r assumes the form

$$U(r) = k\frac{qQ}{r} \left(\text{zero reference at } r = \infty\right). \qquad 7.2$$

This formula is symmetrical with respect to q and Q, so it is best described as the potential energy of the two-charge system.

EXAMPLE 7.2

Potential Energy of a Charged Particle

A $+3.0$-nC charge Q is initially at rest a distance of 10 cm (r_1) from a $+5.0$-nC charge q fixed at the origin (Figure 7.7). Naturally, the Coulomb force accelerates Q away from q, eventually reaching 15 cm (r_2).

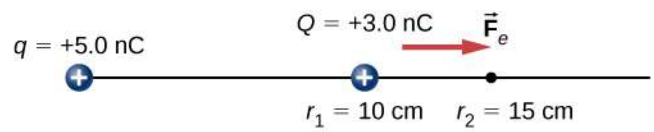


Figure 7.7 The charge Q is repelled by q, thus having work done on it and losing potential energy.

What is the change in the potential energy of the two-charge system from r_1 to r_2?

Strategy

Calculate the potential energy with the definition given above: $\Delta U_{12} = -\int_{r_1}^{r_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$. Since Q started from rest, this is the same as the kinetic energy.

Solution

We have

$$\begin{aligned}\Delta U_{12} &= -\int_{r_1}^{r_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = -\int_{r_1}^{r_2} \frac{kqQ}{r^2} dr = -\left[-\frac{kqQ}{r}\right]_{r_1}^{r_2} = kqQ\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \\ &= \left(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\right)\left(5.0 \times 10^{-9} \text{ C}\right)\left(3.0 \times 10^{-9} \text{ C}\right)\left[\frac{1}{0.15 \text{ m}} - \frac{1}{0.10 \text{ m}}\right] \\ &= -4.5 \times 10^{-7} \text{ J}.\end{aligned}$$

Significance

The change in the potential energy is negative, as expected, and equal in magnitude to the change in kinetic energy in this system. Recall from Example 7.1 that the change in kinetic energy was positive.

CHECK YOUR UNDERSTANDING 7.2

What is the potential energy of Q relative to the zero reference at infinity at r_2 in the above example?

Due to Coulomb's law, the forces due to multiple charges on a test charge Q superimpose; they may be calculated individually and then added. This implies that the work integrals and hence the resulting potential energies exhibit the same behavior. To demonstrate this, we consider an example of assembling a system of four charges.

EXAMPLE 7.3

Assembling Four Positive Charges

Find the amount of work an external agent must do in assembling four charges $+2.0\ \mu$C,$+3.0\ \mu$C, $+4.0\ \mu$C, and $+5.0\ \mu$C at the vertices of a square of side 1.0 cm, starting each charge from infinity (Figure 7.8).

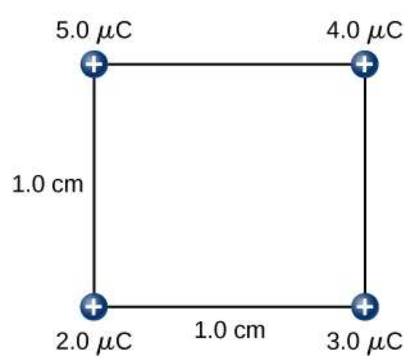


Figure 7.8 How much work is needed to assemble this charge configuration?

Strategy

We bring in the charges one at a time, giving them starting locations at infinity and calculating the work to bring them in from infinity to their final location. We do this in order of increasing charge.

Solution

Step 1. First bring the $+2.0\text{-}\mu\text{C}$ charge to the origin. Since there are no other charges at a finite distance from this charge yet, no work is done in bringing it from infinity,

$$W_1 = 0.$$

Step 2. While keeping the $+2.0\text{-}\mu\text{C}$ charge fixed at the origin, bring the $+3.0\text{-}\mu\text{C}$ charge to $(x, y, z) = (1.0\,\text{cm}, 0, 0)$ (Figure 7.9). Now, the applied force must do work against the force exerted by the $+2.0\text{-}\mu\text{C}$ charge fixed at the origin. The work done equals the change in the potential energy of the $+3.0\text{-}\mu\text{C}$ charge:

$$W_2 = k\frac{q_1 q_2}{r_{12}} = \left(9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(2.0 \times 10^{-6}\,\text{C}\right)\left(3.0 \times 10^{-6}\,\text{C}\right)}{1.0 \times 10^{-2}\,\text{m}} = 5.4\,\text{J}.$$

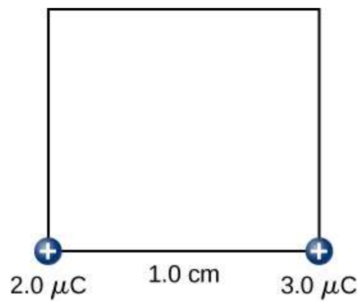


Figure 7.9 Step 2. Work W_2 to bring the $+3.0\text{-}\mu\text{C}$ charge from infinity.

Step 3. While keeping the charges of $+2.0\,\mu\text{C}$ and $+3.0\,\mu\text{C}$ fixed in their places, bring in the $+4.0\text{-}\mu\text{C}$ charge to $(x, y, z) = (1.0\,\text{cm}, 1.0\,\text{cm}, 0)$ (Figure 7.10). The work done in this step is

$$\begin{aligned} W_3 &= k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}} \\ &= \left(9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{\left(2.0 \times 10^{-6}\,\text{C}\right)\left(4.0 \times 10^{-6}\,\text{C}\right)}{\sqrt{2} \times 10^{-2}\,\text{m}} + \frac{\left(3.0 \times 10^{-6}\,\text{C}\right)\left(4.0 \times 10^{-6}\,\text{C}\right)}{1.0 \times 10^{-2}\,\text{m}}\right] = 15.9\,\text{J}. \end{aligned}$$

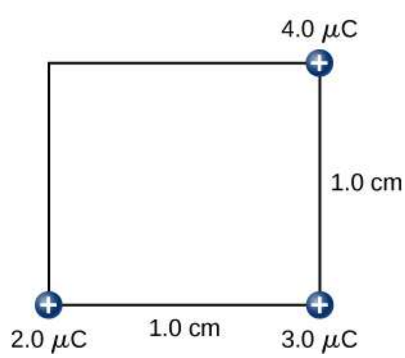


Figure 7.10 Step 3. The work W_3 to bring the $+4.0\text{-}\mu\text{C}$ charge from infinity.

Step 4. Finally, while keeping the first three charges in their places, bring the $+5.0\text{-}\mu\text{C}$ charge to $(x, y, z) = (0,\ 1.0\,\text{cm},\ 0)$ (Figure 7.11). The work done here is

$$\begin{aligned} W_4 &= kq_4\left[\frac{q_1}{r_{14}}+\frac{q_2}{r_{24}}+\frac{q_3}{r_{34}}\right], \\ &= \left(9.0\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(5.0\times10^{-6}\text{ C}\right)\left[\frac{\left(2.0\times10^{-6}\text{ C}\right)}{1.0\times10^{-2}\text{ m}}+\frac{\left(3.0\times10^{-6}\text{ C}\right)}{\sqrt{2}\times10^{-2}\text{ m}}+\frac{\left(4.0\times10^{-6}\text{ C}\right)}{1.0\times10^{-2}\text{ m}}\right]=36.5\text{ J}. \end{aligned}$$

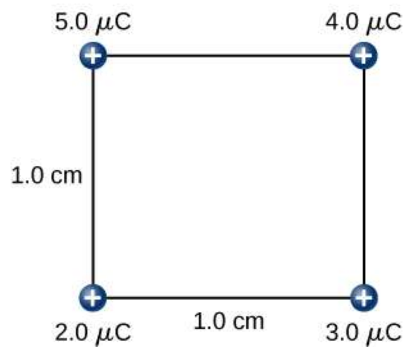


Figure 7.11 Step 4. The work W_4 to bring the $+5.0$-μC charge from infinity.

Hence, the total work done by the applied force in assembling the four charges is equal to the sum of the work in bringing each charge from infinity to its final position:

$$W_{\text{T}} = W_1 + W_2 + W_3 + W_4 = 0 + 5.4\text{ J} + 15.9\text{ J} + 36.5\text{ J} = 57.8\text{ J}.$$

Significance

The work on each charge depends only on its pairwise interactions with the other charges. No more complicated interactions need to be considered; the work on the third charge only depends on its interaction with the first and second charges, the interaction between the first and second charge does not affect the third.

CHECK YOUR UNDERSTANDING 7.3

Is the electrical potential energy of two point charges positive or negative if the charges are of the same sign? Opposite signs? How does this relate to the work necessary to bring the charges into proximity from infinity?

Note that the electrical potential energy is positive if the two charges are of the same type, either positive or negative, and negative if the two charges are of opposite types. This makes sense if you think of the change in the potential energy ΔU as you bring the two charges closer or move them farther apart. Depending on the relative types of charges, you may have to work on the system or the system would do work on you, that is, your work is either positive or negative. If you have to do positive work on the system (actually push the charges closer), then the energy of the system should increase. If you bring two positive charges or two negative charges closer, you have to do positive work on the system, which raises their potential energy. Since potential energy is proportional to $1/r$, the potential energy goes up when r goes down between two positive or two negative charges.

On the other hand, if you bring a positive and a negative charge nearer, you have to do negative work on the system (the charges are pulling you), which means that you take energy away from the system. This reduces the potential energy. Since potential energy is negative in the case of a positive and a negative charge pair, the increase in $1/r$ makes the potential energy more negative, which is the same as a reduction in potential energy.

The result from Example 7.1 may be extended to systems with any arbitrary number of charges. In this case, it is most convenient to write the formula as

$$W_{12\cdots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}}\text{ for } i \neq j. \quad (7.3)$$

The factor of 1/2 accounts for adding each pair of charges twice.

7.2 Electric Potential and Potential Difference

Learning Objectives

By the end of this section, you will be able to:

- Define electric potential, voltage, and potential difference
- Define the electron-volt
- Calculate electric potential and potential difference from potential energy and electric field
- Describe systems in which the electron-volt is a useful unit
- Apply conservation of energy to electric systems

Recall that earlier we defined electric field to be a quantity independent of the test charge in a given system, which would nonetheless allow us to calculate the force that would result on an arbitrary test charge. (The default assumption in the absence of other information is that the test charge is positive.) We briefly defined a field for gravity, but gravity is always attractive, whereas the electric force can be either attractive or repulsive. Therefore, although potential energy is perfectly adequate in a gravitational system, it is convenient to define a quantity that allows us to calculate the work on a charge independent of the magnitude of the charge. Calculating the work directly may be difficult, since $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$ and the direction and magnitude of $\vec{\mathbf{F}}$ can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that because $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, the work, and hence ΔU, is proportional to the test charge q. To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

Electric Potential

The electric potential energy per unit charge is

$$V = \frac{U}{q}. \tag{7.4}$$

Since U is proportional to q, the dependence on q cancels. Thus, V does not depend on q. The change in potential energy ΔU is crucial, so we are concerned with the difference in potential or potential difference ΔV between two points, where

$$\Delta V = V_B - V_A = \frac{\Delta U}{q}.$$

Electric Potential Difference

The **electric potential difference** between points A and B, $V_B - V_A$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1 \text{ V} = 1 \text{ J/C}$$

The familiar term **voltage** is the common name for electric potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor. It is worthwhile to emphasize the distinction between potential difference and electrical potential energy.

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta U}{q} \text{ or } \Delta U = q\Delta V. \tag{7.5}$$

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus, a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other because $\Delta U = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12-V batteries.

EXAMPLE 7.4

Calculating Energy

You have a 12.0-V motorcycle battery that can move 5000 C of charge, and a 12.0-V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0-V battery means that its terminals have a 12.0-V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta U = q\Delta V$. To find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000 \text{ C}$ and $\Delta V = 12.0 \text{ V}$. The total energy delivered by the motorcycle battery is

$$\Delta U_{\text{cycle}} = (5000 \text{ C})(12.0 \text{ V}) = (5000 \text{ C})(12.0 \text{ J/C}) = 6.00 \times 10^4 \text{ J}.$$

Similarly, for the car battery, $q = 60{,}000 \text{ C}$ and

$$\Delta U_{\text{car}} = (60{,}000 \text{ C})(12.0 \text{ V}) = 7.20 \times 10^5 \text{ J}.$$

Significance

Voltage and energy are related, but they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. A car battery has a much larger engine to start than a motorcycle. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a depleted car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

CHECK YOUR UNDERSTANDING 7.4

How much energy does a 1.5-V AAA battery have that can move 100 C?

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B), as shown in Figure 7.12. The change in potential is $\Delta V = V_B - V_A = +12 \text{ V}$ and the charge q is negative, so that $\Delta U = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

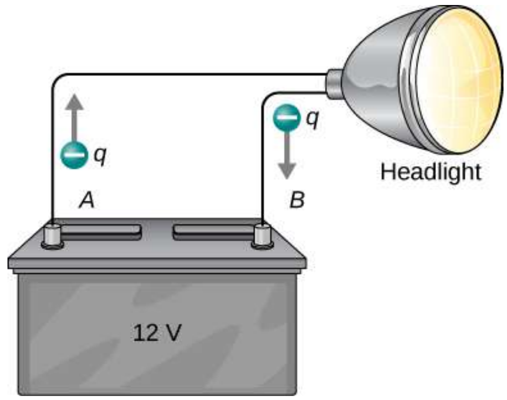


Figure 7.12 A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative terminal. Inside the battery, both positive and negative charges move.

EXAMPLE 7.5

How Many Electrons Move through a Headlight Each Second?

When a 12.0-V car battery powers a single 30.0-W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moves in 1.00 s. The charge moved is related to voltage and energy through the equations $\Delta U = q\Delta V$. A 30.0-W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta U = -30\ \text{J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\ \text{V}$.

Solution

To find the charge q moved, we solve the equation $\Delta U = q\Delta V$:

$$q = \frac{\Delta U}{\Delta V}.$$

Entering the values for ΔU and ΔV, we get

$$q = \frac{-30.0\ \text{J}}{+12.0\ \text{V}} = \frac{-30.0\ \text{J}}{+12.0\ \text{J/C}} = -2.50\ \text{C}.$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

$$n_e = \frac{-2.50\ \text{C}}{-1.60 \times 10^{-19}\ \text{C/e}^-} = 1.56 \times 10^{19}\ \text{electrons}.$$

Significance

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

CHECK YOUR UNDERSTANDING 7.5

How many electrons would go through a 24.0-W lamp?

The Electron-Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful X-rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects.

Figure 7.13 shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates, as it might be in an old-model television tube or oscilloscope. The electron gains kinetic energy that is later converted into another form—light in the television tube, for example. (Note that in terms of energy, "downhill" for the electron is "uphill" for a positive charge.) Since energy is related to voltage by $\Delta U = q\Delta V$, we can think of the joule as a coulomb-volt.

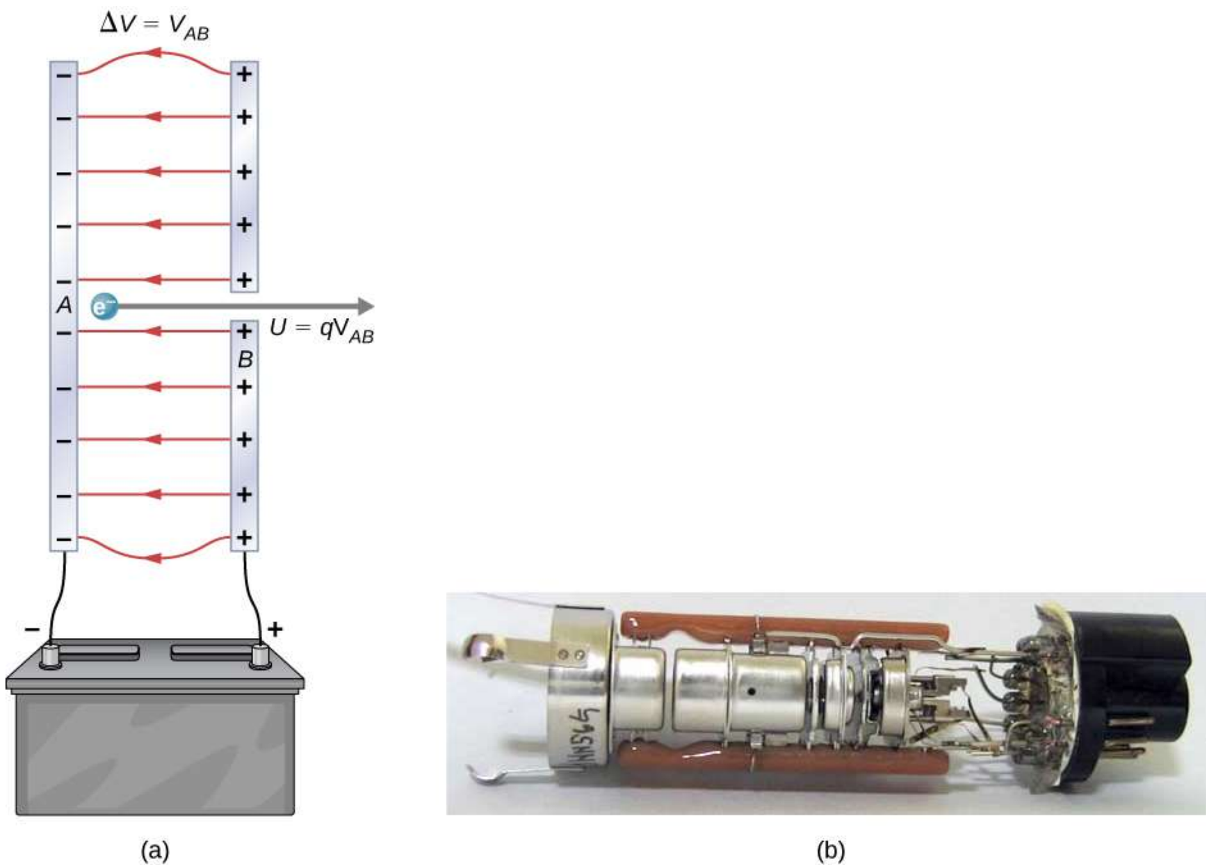


Figure 7.13 A typical electron gun accelerates electrons using a potential difference between two separated metal plates. By conservation of energy, the kinetic energy has to equal the change in potential energy, so $KE = qV$. The energy of the electron in electron-volts is numerically the same as the voltage between the plates. For example, a 5000-V potential difference produces 5000-eV electrons. The conceptual construct, namely two parallel plates with a hole in one, is shown in (a), while a real electron gun is shown in (b).

Electron-Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron-volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$1\ \text{eV} = (1.60 \times 10^{-19}\ \text{C})(1\ \text{V}) = (1.60 \times 10^{-19}\ \text{C})(1\ \text{J/C}) = 1.60 \times 10^{-19}\ \text{J}.$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V gains 50 eV. A potential difference of 100,000 V (100 kV) gives an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V gains 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron-volt a simple and convenient energy unit in such circumstances.

The electron-volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron-volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it acquires an energy of 30 keV (30,000 eV) and can break up as many as 6000 of these molecules ($30{,}000\ \text{eV} \div 5\ \text{eV per molecule} = 6000\ \text{molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can thus produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) due to work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $K + U = \text{constant}$. A loss of U for a charged particle becomes an increase in its K. Conservation of energy is stated in equation form as

$$K + U = \text{constant}$$

or

$$K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

EXAMPLE 7.6

Electrical Potential Energy Converted into Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $K_\text{i} = 0, K_\text{f} = \frac{1}{2}mv^2, U_\text{i} = qV, U_\text{f} = 0.$

Solution

Conservation of energy states that

$$K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}.$$

Entering the forms identified above, we obtain

$$qV = \frac{mv^2}{2}.$$

We solve this for v:

$$v = \sqrt{\frac{2qV}{m}}.$$

Entering values for q, V, and m gives

$$v = \sqrt{\frac{2(-1.60 \times 10^{-19}\ \text{C})(-100\ \text{J/C})}{9.11 \times 10^{-31}\ \text{kg}}} = 5.93 \times 10^{6}\ \text{m/s}.$$

Significance

Note that both the charge and the initial voltage are negative, as in Figure 7.13. From the discussion of electric charge and electric field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. These higher voltages produce electron speeds so great that effects from special relativity must be taken into account and hence are reserved for a later chapter (Relativity). That is why we consider a low voltage (accurately) in this example.

CHECK YOUR UNDERSTANDING 7.6

How would this example change with a positron? A positron is identical to an electron except the charge is positive.

Voltage and Electric Field

So far, we have explored the relationship between voltage and energy. Now we want to explore the relationship between voltage and electric field. We will start with the general case for a non-uniform $\vec{\mathbf{E}}$ field. Recall that our general formula for the potential energy of a test charge q at point P relative to reference point R is

$$U_P = -\int_R^P \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}.$$

When we substitute in the definition of electric field ($\vec{\mathbf{E}} = \vec{\mathbf{F}}/q$), this becomes

$$U_P = -q\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}.$$

Applying our definition of potential ($V = U/q$) to this potential energy, we find that, in general,

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}. \tag{7.6}$$

From our previous discussion of the potential energy of a charge in an electric field, the result is independent of the path chosen, and hence we can pick the integral path that is most convenient.

Consider the special case of a positive point charge q at the origin. To calculate the potential caused by q at a distance r from the origin relative to a reference of 0 at infinity (recall that we did the same for potential energy), let $P = r$ and $R = \infty$, with $d\vec{\mathbf{l}} = d\vec{\mathbf{r}} = \hat{\mathbf{r}}dr$ and use $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. When we evaluate the integral

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

for this system, we have

$$V_r = -\int_\infty^r \frac{kq}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}dr,$$

which simplifies to

$$V_r = -\int_{\infty}^{r} \frac{kq}{r^2}\, dr = \frac{kq}{r} - \frac{kq}{\infty} = \frac{kq}{r}.$$

This result,

$$V_r = \frac{kq}{r}$$

is the standard form of the potential of a point charge. This will be explored further in the next section.

To examine another interesting special case, suppose a uniform electric field $\vec{\mathbf{E}}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B (Figure 7.14). Examining this situation will tell us what voltage is needed to produce a certain electric field strength. It will also reveal a more fundamental relationship between electric potential and electric field.

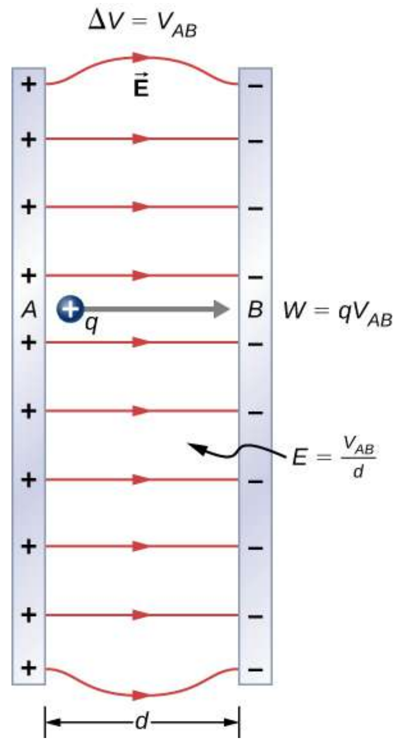


Figure 7.14 The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{AB}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_A - V_B = V_{AB}$.)

From a physicist's point of view, either ΔV or $\vec{\mathbf{E}}$ can be used to describe any interaction between charges. However, ΔV is a scalar quantity and has no direction, whereas $\vec{\mathbf{E}}$ is a vector quantity, having both magnitude and direction. (Note that the magnitude of the electric field, a scalar quantity, is represented by E.) The relationship between ΔV and $\vec{\mathbf{E}}$ is revealed by calculating the work done by the electric force in moving a charge from point A to point B. But, as noted earlier, arbitrary charge distributions require calculus. We therefore look at a uniform electric field as an interesting special case.

The work done by the electric field in Figure 7.14 to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta U = -q\Delta V.$$

The potential difference between points A and B is

$$-\Delta V = -(V_B - V_A) = V_A - V_B = V_{AB}.$$

Entering this into the expression for work yields

$$W = qV_{AB}.$$

Work is $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta$; here $\cos\theta = 1$, since the path is parallel to the field. Thus, $W = Fd$. Since $F = qE$, we see that $W = qEd$.

Substituting this expression for work into the previous equation gives

$$qEd = qV_{AB}.$$

The charge cancels, so we obtain for the voltage between points A and B

$$\left.\begin{array}{l} V_{AB} = Ed \\ E = \dfrac{V_{AB}}{d} \end{array}\right\}\text{(uniform } E\text{-field only)}$$

where d is the distance from A to B, or the distance between the plates in Figure 7.14. Note that this equation implies that the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus, the following relation among units is valid:

$$1\text{ N}/\text{C} = 1\text{ V}/\text{m}.$$

Furthermore, we may extend this to the integral form. Substituting Equation 7.5 into our definition for the potential difference between points A and B, we obtain

$$V_{BA} = V_B - V_A = -\int_R^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} + \int_R^A \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

which simplifies to

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}.$$

As a demonstration, from this we may calculate the potential difference between two points (A and B) equidistant from a point charge q at the origin, as shown in Figure 7.15.

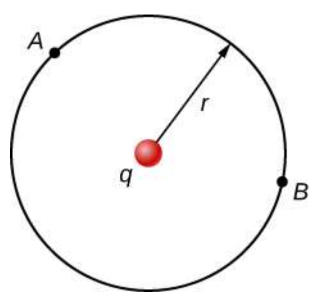


Figure 7.15 The arc for calculating the potential difference between two points that are equidistant from a point charge at the origin.

To do this, we integrate around an arc of the circle of constant radius r between A and B, which means we let $d\vec{\mathbf{l}} = r\hat{\varphi}\,d\varphi$, while using $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. Thus,

$$\Delta V_{BA} = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} \qquad 7.7$$

for this system becomes

$$V_B - V_A = -\int_A^B \frac{kq}{r^2}\hat{\mathbf{r}} \cdot r\hat{\boldsymbol{\varphi}}\,d\varphi.$$

However, $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\varphi}} = 0$ and therefore

$$V_B - V_A = 0.$$

This result, that there is no difference in potential along a constant radius from a point charge, will come in handy when we map potentials.

EXAMPLE 7.7

What Is the Highest Voltage Possible between Two Plates?

Dry air can support a maximum electric field strength of about 3.0×10^6 V/m. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. We can use the equation $V_{AB} = Ed$ to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

$$V_{AB} = Ed.$$

Entering the given values for E and d gives

$$V_{AB} = (3.0 \times 10^6 \text{ V/m})(0.025 \text{ m}) = 7.5 \times 10^4 \text{ V}$$

or

$$V_{AB} = 75 \text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Significance

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5-cm (1-in.) gap, or 150 kV for a 5-cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage can cause a spark if there are spines on the surface, since sharp points have larger field strengths than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up with static electricity on dry days (Figure 7.16).

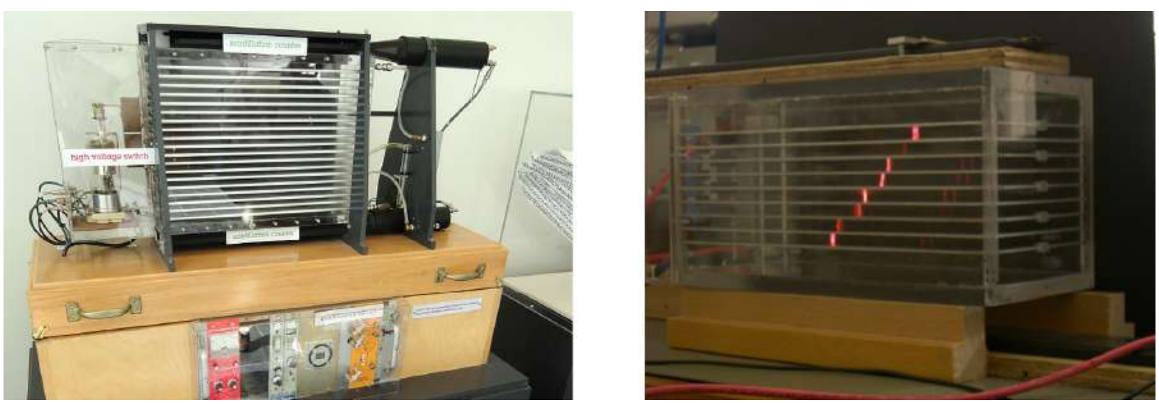


Figure 7.16 A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). This form of detector is now archaic and no longer in use except for demonstration purposes. (credit b: modification of work by Jack Collins)

EXAMPLE 7.8

Field and Force inside an Electron Gun

An electron gun (Figure 7.13) has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. (a) What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\text{-}\mu\text{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{AB}}{d}$. Once we know the electric field strength, we can find the force on a charge by using $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = qE$.

Solution

a. The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{AB}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\,\text{kV}}{0.0400\,\text{m}} = 6.25 \times 10^5\ \text{V/m}.$$

b. The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE.$$

Substituting known values gives

$$F = (0.500 \times 10^{-6}\ \text{C})(6.25 \times 10^5\ \text{V/m}) = 0.313\,\text{N}.$$

Significance

Note that the units are newtons, since $1\ \text{V/m} = 1\ \text{N/C}$. Because the electric field is uniform between the plates, the force on the charge is the same no matter where the charge is located between the plates.

EXAMPLE 7.9

Calculating Potential of a Point Charge

Given a point charge $q = +2.0\,\text{nC}$ at the origin, calculate the potential difference between point P_1 a distance $a = 4.0\,\text{cm}$ from q, and P_2 a distance $b = 12.0\,\text{cm}$ from q, where the two points have an angle of $\varphi = 24°$ between them (Figure 7.17).

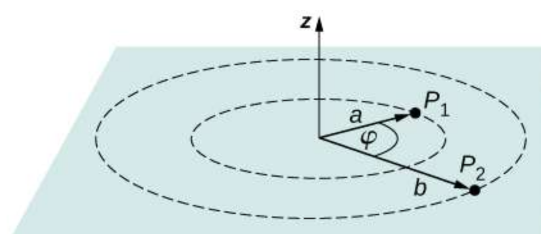


Figure 7.17 Find the difference in potential between P_1 and P_2.

Strategy

Do this in two steps. The first step is to use $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ and let $A = a = 4.0\,\text{cm}$ and

$B = b = 12.0$ cm, with $d\vec{\mathbf{l}} = d\vec{\mathbf{r}} = \hat{\mathbf{r}}dr$ and $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. Then perform the integral. The second step is to integrate $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ around an arc of constant radius r, which means we let $d\vec{\mathbf{l}} = r\hat{\varphi}d\varphi$ with limits $0 \le \varphi \le 24°$, still using $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. Then add the two results together.

Solution

For the first part, $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ for this system becomes $V_b - V_a = -\int_a^b \frac{kq}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}dr$ which computes to

$$\begin{aligned}\Delta V &= -\int_a^b \frac{kq}{r^2}dr = kq\left[\tfrac{1}{a} - \tfrac{1}{b}\right] \\ &= \left(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\right)\left(2.0 \times 10^{-9} \text{ C}\right)\left[\tfrac{1}{0.040 \text{ m}} - \tfrac{1}{0.12 \text{ m}}\right] = 300 \text{ V}.\end{aligned}$$

For the second step, $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ becomes $\Delta V = -\int_0^{24°} \frac{kq}{r^2}\hat{\mathbf{r}} \cdot r\hat{\boldsymbol{\varphi}}d\varphi$, but $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\varphi}} = 0$ and therefore $\Delta V = 0$. Adding the two parts together, we get 300 V.

Significance

We have demonstrated the use of the integral form of the potential difference to obtain a numerical result. Notice that, in this particular system, we could have also used the formula for the potential due to a point charge at the two points and simply taken the difference.

CHECK YOUR UNDERSTANDING 7.7

From the examples, how does the energy of a lightning strike vary with the height of the clouds from the ground? Consider the cloud-ground system to be two parallel plates.

Before presenting problems involving electrostatics, we suggest a problem-solving strategy to follow for this topic.

PROBLEM-SOLVING STRATEGY

Electrostatics

1. Examine the situation to determine if static electricity is involved; this may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E, for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

7.3 Calculations of Electric Potential

Learning Objectives

By the end of this section, you will be able to:

- Calculate the potential due to a point charge
- Calculate the potential of a system of multiple point charges
- Describe an electric dipole
- Define dipole moment
- Calculate the potential of a continuous charge distribution

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (such as charge on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider.

We can use calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge q. Noting the connection between work and potential $W = -q\Delta V$, as in the last section, we can obtain the following result.

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kq}{r} \text{(point charge)} \tag{7.8}$$

where k is a constant equal to $8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

The potential at infinity is chosen to be zero. Thus, V for a point charge decreases with distance, whereas $\vec{\mathbf{E}}$ for a point charge decreases with distance squared:

$$E = \frac{F}{q_t} = \frac{kq}{r^2}.$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\vec{\mathbf{E}}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as vectors, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\vec{\mathbf{E}}$ is closely associated with force, a vector.

EXAMPLE 7.10

What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm-diameter solid metal sphere that has a –3.00-nC static charge?

Strategy

As we discussed in Electric Charges and Fields, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus, we can find the voltage using the equation $V = \frac{kq}{r}$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

$$V = k\frac{q}{r} = \left(8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}\right)\left(\frac{-3.00 \times 10^{-9}\ \mathrm{C}}{5.00 \times 10^{-2}\ \mathrm{m}}\right) = -539\ \mathrm{V}.$$

Significance

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

EXAMPLE 7.11

What Is the Excess Charge on a Van de Graaff Generator?

A demonstration Van de Graaff generator has a 25.0-cm-diameter metal sphere that produces a voltage of 100 kV near its surface (Figure 7.18). What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

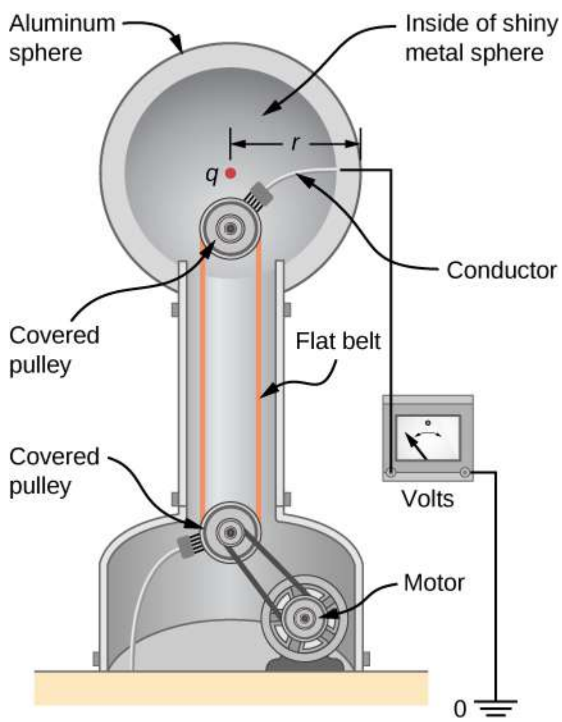


Figure 7.18 The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface is the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

$$V = \frac{kq}{r}.$$

Solution

Solving for q and entering known values gives

$$q = \frac{rV}{k} = \frac{(0.125\text{ m})\left(100 \times 10^3\text{ V}\right)}{8.99 \times 10^9\text{ N} \cdot \text{m}^2/\text{C}^2} = 1.39 \times 10^{-6}\text{ C} = 1.39\ \mu\text{C}.$$

Significance

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

CHECK YOUR UNDERSTANDING 7.8

What is the potential inside the metal sphere in Example 7.10?

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted earlier, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy $U_g = mgh$.

Systems of Multiple Point Charges

Just as the electric field obeys a superposition principle, so does the electric potential. Consider a system consisting of N charges $q_1, q_2, \ldots, q_N$. What is the net electric potential V at a space point P from these charges? Each of these charges is a source charge that produces its own electric potential at point P, independent of whatever other changes may be doing. Let $V_1, V_2, \ldots, V_N$ be the electric potentials at P produced by the charges $q_1, q_2, \ldots, q_N$, respectively. Then, the net electric potential V_P at that point is equal to the sum of these individual electric potentials. You can easily show this by calculating the potential energy of a test charge when you bring the test charge from the reference point at infinity to point P:

$$V_P = V_1 + V_2 + \cdots + V_N = \sum_1^N V_i.$$

Note that electric potential follows the same principle of superposition as electric field and electric potential energy. To show this more explicitly, note that a test charge q_i at the point P in space has distances of $r_1, r_2, \ldots, r_N$ from the N charges fixed in space above, as shown in Figure 7.19. Using our formula for the potential of a point charge for each of these (assumed to be point) charges, we find that

$$V_P = \sum_1^N k\frac{q_i}{r_i} = k\sum_1^N \frac{q_i}{r_i}. \tag{7.9}$$

Therefore, the electric potential energy of the test charge is

$$U_P = q_t V_P = q_t k \sum_1^N \frac{q_i}{r_i},$$

which is the same as the work to bring the test charge into the system, as found in the first section of the chapter.

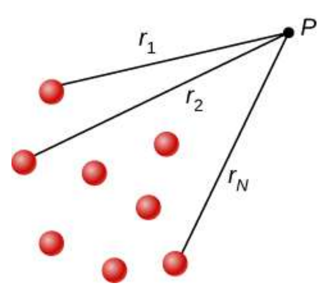


Figure 7.19 Notation for direct distances from charges to a space point P.

The Electric Dipole

An **electric dipole** is a system of two equal but opposite charges a fixed distance apart. This system is used to model many real-world systems, including atomic and molecular interactions. One of these systems is the water molecule, under certain circumstances. These circumstances are met inside a microwave oven, where electric fields with alternating directions make the water molecules change orientation. This vibration is the same as heat at the molecular level.

EXAMPLE 7.12

Electric Potential of a Dipole

Consider the dipole in Figure 7.20 with the charge magnitude of $q = 3.0\,\text{nC}$ and separation distance $d = 4.0\,\text{cm}$. What is the potential at the following locations in space? (a) (0, 0, 1.0 cm); (b) (0, 0, −5.0 cm); (c) (3.0 cm, 0, 2.0 cm).

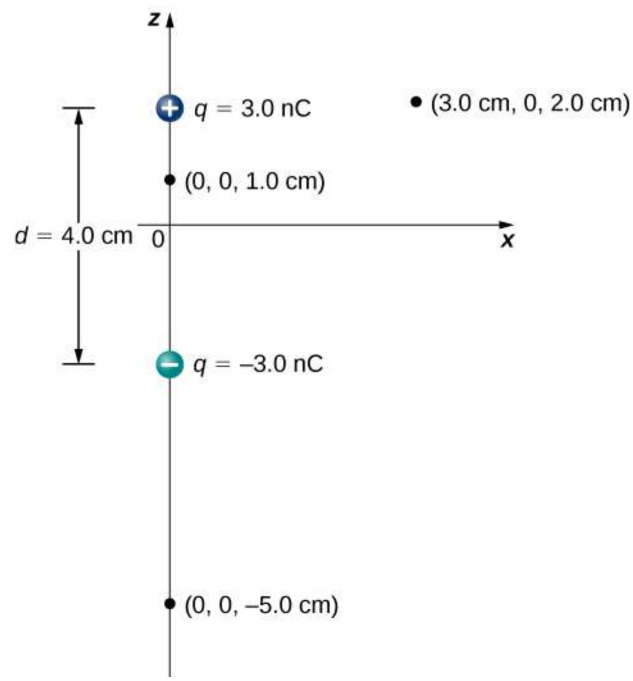


Figure 7.20 A general diagram of an electric dipole, and the notation for the distances from the individual charges to a point *P* in space.

Strategy

Apply $V_P = k\sum_{1}^{N}\frac{q_i}{r_i}$ to each of these three points.

Solution

a. $$V_P = k\sum_{1}^{N}\frac{q_i}{r_i} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{3.0\ \text{nC}}{0.010\ \text{m}} - \frac{3.0\ \text{nC}}{0.030\ \text{m}}\right) = 1.8 \times 10^3\ \text{V}$$

b. $$V_P = k\sum_{1}^{N}\frac{q_i}{r_i} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{3.0\ \text{nC}}{0.070\ \text{m}} - \frac{3.0\ \text{nC}}{0.030\ \text{m}}\right) = -5.1 \times 10^2\ \text{V}$$

c. $$V_P = k\sum_{1}^{N}\frac{q_i}{r_i} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{3.0\ \text{nC}}{0.030\ \text{m}} - \frac{3.0\ \text{nC}}{0.050\ \text{m}}\right) = 3.6 \times 10^2\ \text{V}$$

Significance

Note that evaluating potential is significantly simpler than electric field, due to potential being a scalar instead of a vector.

CHECK YOUR UNDERSTANDING 7.9

What is the potential on the x-axis? The z-axis?

Now let us consider the special case when the distance of the point P from the dipole is much greater than the distance between the charges in the dipole, $r \gg d$; for example, when we are interested in the electric potential due to a polarized molecule such as a water molecule. This is not so far (infinity) that we can simply treat the potential as zero, but the distance is great enough that we can simplify our calculations relative to the previous example.

We start by noting that in Figure 7.21 the potential is given by

$$V_P = V_+ + V_- = k\left(\frac{q}{r_+} - \frac{q}{r_-}\right)$$

where

$$r_\pm = \sqrt{x^2 + \left(z \mp \frac{d}{2}\right)^2}.$$

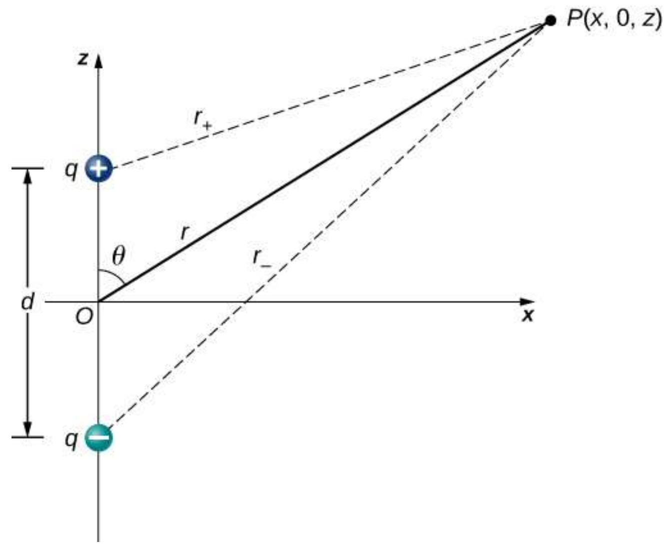


Figure 7.21 A general diagram of an electric dipole, and the notation for the distances from the individual charges to a point P in space.

This is still the exact formula. To take advantage of the fact that $r \gg d$, we rewrite the radii in terms of polar coordinates, with $x = r\sin\theta$ and $z = r\cos\theta$. This gives us

$$r_\pm = \sqrt{r^2\sin^2\theta + \left(r\cos\theta \mp \frac{d}{2}\right)^2}.$$

We can simplify this expression by pulling r out of the root,

$$r_\pm = r\sqrt{\sin^2\theta + \left(\cos\theta \mp \frac{d}{2r}\right)^2}$$

and then multiplying out the parentheses

$$r_\pm = r\sqrt{\sin^2\theta + \cos^2\theta \mp \cos\theta\frac{d}{r} + \left(\frac{d}{2r}\right)^2} = r\sqrt{1 \mp \cos\theta\frac{d}{r} + \left(\frac{d}{2r}\right)^2}.$$

The last term in the root is small enough to be negligible (remember $r \gg d$, and hence $(d/r)^2$ is extremely small, effectively zero to the level we will probably be measuring), leaving us with

$$r_{\pm} = r\sqrt{1 \mp \cos\theta \frac{d}{r}}.$$

Using the binomial approximation (a standard result from the mathematics of series, when α is small)

$$\frac{1}{\sqrt{1 \mp \alpha}} \approx 1 \pm \frac{\alpha}{2}$$

and substituting this into our formula for V_P , we get

$$V_P = k\left[\frac{q}{r}\left(1 + \frac{d\cos\theta}{2r}\right) - \frac{q}{r}\left(1 - \frac{d\cos\theta}{2r}\right)\right] = k\frac{qd\cos\theta}{r^2}.$$

This may be written more conveniently if we define a new quantity, the **electric dipole moment**,

$$\vec{\mathbf{p}} = q\vec{\mathbf{d}}, \tag{7.10}$$

where these vectors point from the negative to the positive charge. Note that this has magnitude qd. This quantity allows us to write the potential at point P due to a dipole at the origin as

$$V_P = k\frac{\vec{\mathbf{p}} \cdot \hat{\mathbf{r}}}{r^2}. \tag{7.11}$$

A diagram of the application of this formula is shown in Figure 7.22.

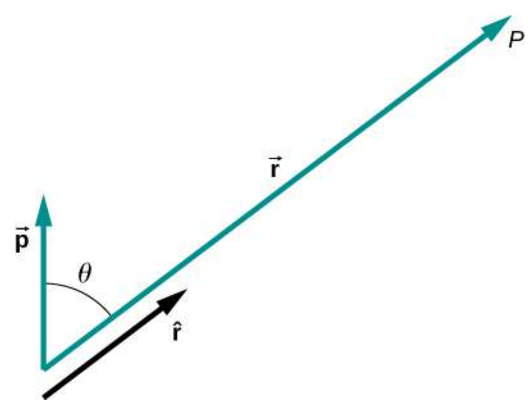


Figure 7.22 The geometry for the application of the potential of a dipole.

There are also higher-order moments, for quadrupoles, octupoles, and so on. You will see these in future classes.

Potential of Continuous Charge Distributions

We have been working with point charges a great deal, but what about continuous charge distributions? Recall from Equation 7.9 that

$$V_P = k\sum \frac{q_i}{r_i}.$$

We may treat a continuous charge distribution as a collection of infinitesimally separated individual points. This yields the integral

$$V_P = k\int \frac{dq}{r} \tag{7.12}$$

for the potential at a point P. Note that r is the distance from each individual point in the charge distribution to the point P. As we saw in Electric Charges and Fields, the infinitesimal charges are given by

$$dq = \begin{cases} \lambda\, dl & \text{(one dimension)} \\ \sigma\, dA & \text{(two dimensions)} \\ \rho\, dV & \text{(three dimensions)} \end{cases}$$

where λ is linear charge density, σ is the charge per unit area, and ρ is the charge per unit volume.

EXAMPLE 7.13

Potential of a Line of Charge

Find the electric potential of a uniformly charged, nonconducting wire with linear density λ (coulomb/meter) and length L at a point that lies on a line that divides the wire into two equal parts.

Strategy

To set up the problem, we choose Cartesian coordinates in such a way as to exploit the symmetry in the problem as much as possible. We place the origin at the center of the wire and orient the y-axis along the wire so that the ends of the wire are at $y = \pm L/2$. The field point P is in the xy-plane and since the choice of axes is up to us, we choose the x-axis to pass through the field point P, as shown in Figure 7.23.

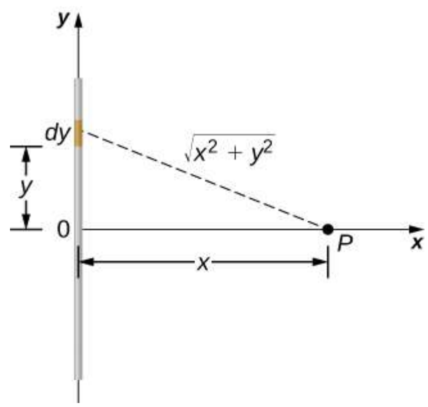


Figure 7.23 We want to calculate the electric potential due to a line of charge.

Solution

Consider a small element of the charge distribution between y and $y + dy$. The charge in this cell is $dq = \lambda\, dy$ and the distance from the cell to the field point P is $\sqrt{x^2 + y^2}$. Therefore, the potential becomes

$$\begin{aligned} V_P &= k\int \frac{dq}{r} = k\int_{-L/2}^{L/2} \frac{\lambda dy}{\sqrt{x^2+y^2}} = k\lambda\left[\ln\left(y + \sqrt{y^2+x^2}\right)\right]_{-L/2}^{L/2} \\ &= k\lambda\left[\ln\left(\left(\tfrac{L}{2}\right) + \sqrt{\left(\tfrac{L}{2}\right)^2 + x^2}\right) - \ln\left(\left(-\tfrac{L}{2}\right) + \sqrt{\left(-\tfrac{L}{2}\right)^2 + x^2}\right)\right] \\ &= k\lambda \ln\left[\frac{L+\sqrt{L^2+4x^2}}{-L+\sqrt{L^2+4x^2}}\right]. \end{aligned}$$

Significance

Note that this was simpler than the equivalent problem for electric field, due to the use of scalar quantities. Recall that we expect the zero level of the potential to be at infinity, when we have a finite charge. To examine this, we take the limit of the above potential as x approaches infinity; in this case, the terms inside the natural log approach one, and hence the potential approaches zero in this limit. Note that we could have done this problem equivalently in cylindrical coordinates; the only effect would be to substitute r for x and z for y.

EXAMPLE 7.14

Potential Due to a Ring of Charge

A ring has a uniform charge density λ, with units of coulomb per unit meter of arc. Find the electric potential at a point on the axis passing through the center of the ring.

Strategy

We use the same procedure as for the charged wire. The difference here is that the charge is distributed on a circle. We divide the circle into infinitesimal elements shaped as arcs on the circle and use cylindrical coordinates shown in Figure 7.24.

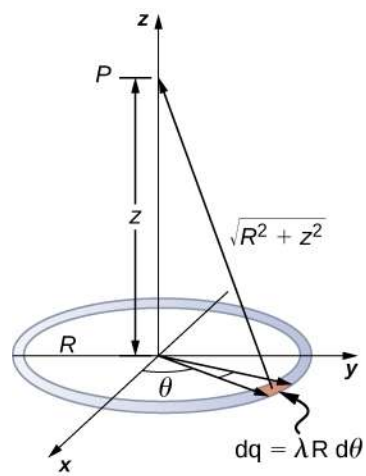


Figure 7.24 We want to calculate the electric potential due to a ring of charge.

Solution

A general element of the arc between θ and $\theta + d\theta$ is of length $Rd\theta$ and therefore contains a charge equal to $\lambda Rd\theta$. The element is at a distance of $\sqrt{z^2 + R^2}$ from P, and therefore the potential is

$$V_P = k\int \frac{dq}{r} = k\int_0^{2\pi} \frac{\lambda Rd\theta}{\sqrt{z^2+R^2}} = \frac{k\lambda R}{\sqrt{z^2+R^2}}\int_0^{2\pi} d\theta = \frac{2\pi k\lambda R}{\sqrt{z^2+R^2}} = k\frac{q_{\text{tot}}}{\sqrt{z^2+R^2}}.$$

Significance

This result is expected because every element of the ring is at the same distance from point P. The net potential at P is that of the total charge placed at the common distance, $\sqrt{z^2 + R^2}$.

EXAMPLE 7.15

Potential Due to a Uniform Disk of Charge

A disk of radius R has a uniform charge density σ, with units of coulomb meter squared. Find the electric potential at any point on the axis passing through the center of the disk.

Strategy

We divide the disk into ring-shaped cells, and make use of the result for a ring worked out in the previous example, then integrate over r in addition to θ. This is shown in Figure 7.25.

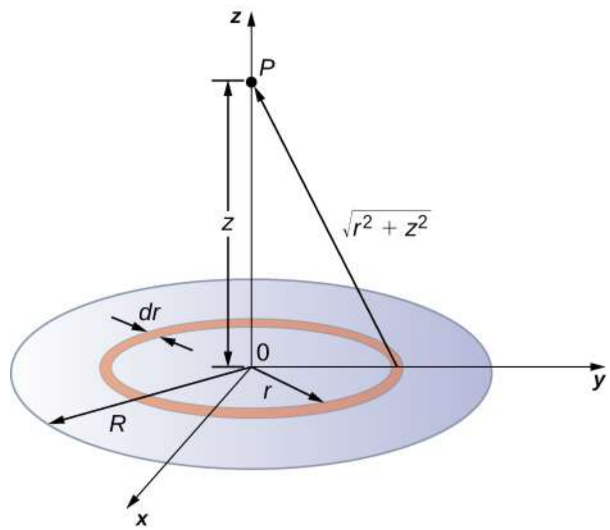


Figure 7.25 We want to calculate the electric potential due to a disk of charge.

Solution

An infinitesimal width cell between cylindrical coordinates r and $r + dr$ shown in Figure 7.25 will be a ring of charges whose electric potential dV_P at the field point has the following expression

$$dV_P = k\frac{dq}{\sqrt{z^2 + r^2}}$$

where

$$dq = \sigma 2\pi r dr.$$

The superposition of potential of all the infinitesimal rings that make up the disk gives the net potential at point P. This is accomplished by integrating from $r = 0$ to $r = R$:

$$\begin{aligned} V_P &= \int dV_P = k2\pi\sigma \int_0^R \frac{r\,dr}{\sqrt{z^2 + r^2}}, \\ &= k2\pi\sigma\left(\sqrt{z^2 + R^2} - \sqrt{z^2}\right). \end{aligned}$$

Significance

The basic procedure for a disk is to first integrate around θ and then over r. This has been demonstrated for uniform (constant) charge density. Often, the charge density will vary with r, and then the last integral will give different results.

EXAMPLE 7.16

Potential Due to an Infinite Charged Wire

Find the electric potential due to an infinitely long uniformly charged wire.

Strategy

Since we have already worked out the potential of a finite wire of length L in Example 7.7, we might wonder if taking $L \to \infty$ in our previous result will work:

$$V_P = \lim_{L \to \infty} k\lambda \ln\left(\frac{L + \sqrt{L^2 + 4x^2}}{-L + \sqrt{L^2 + 4x^2}}\right).$$

However, this limit does not exist because the argument of the logarithm becomes [2/0] as $L \to \infty$, so this way of finding V of an infinite wire does not work. The reason for this problem may be traced to the fact that the charges are not localized in some space but continue to infinity in the direction of the wire. Hence, our (unspoken) assumption that zero potential must be an infinite distance from the wire is no longer valid.

To avoid this difficulty in calculating limits, let us use the definition of potential by integrating over the electric field from the previous section, and the value of the electric field from this charge configuration from the previous chapter.

Solution

We use the integral

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

where R is a finite distance from the line of charge, as shown in Figure 7.26.

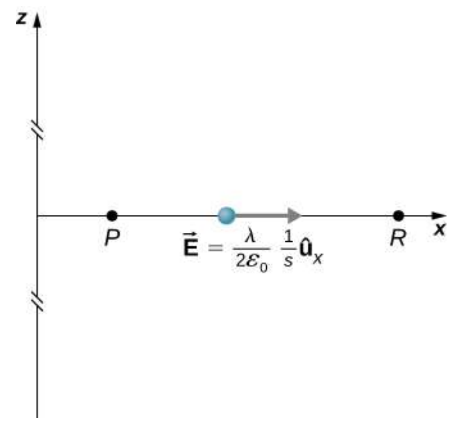


Figure 7.26 Points of interest for calculating the potential of an infinite line of charge.

With this setup, we use $\vec{\mathbf{E}}_P = 2k\lambda \frac{1}{s} \hat{\mathbf{s}}$ and $d\vec{\mathbf{l}} = d\vec{\mathbf{s}}$ to obtain

$$V_P - V_R = -\int_R^P 2k\lambda \frac{1}{s} ds = -2k\lambda \ln \frac{s_P}{s_R}.$$

Now, if we define the reference potential $V_R = 0$ at $s_R = 1$ m, this simplifies to

$$V_P = -2k\lambda \ln s_P.$$

Note that this form of the potential is quite usable; it is 0 at 1 m and is undefined at infinity, which is why we could not use the latter as a reference.

Significance

Although calculating potential directly can be quite convenient, we just found a system for which this strategy does not work well. In such cases, going back to the definition of potential in terms of the electric field may offer a way forward.

CHECK YOUR UNDERSTANDING 7.10

What is the potential on the axis of a nonuniform ring of charge, where the charge density is $\lambda(\theta) = \lambda \cos \theta$?

7.4 Determining Field from Potential

Learning Objectives

By the end of this section, you will be able to:

- Explain how to calculate the electric field in a system from the given potential
- Calculate the electric field in a given direction from a given potential
- Calculate the electric field throughout space from a given potential

Recall that we were able, in certain systems, to calculate the potential by integrating over the electric field. As you may already suspect, this means that we may calculate the electric field by taking derivatives of the potential, although going from a scalar to a vector quantity introduces some interesting wrinkles. We frequently need $\vec{\mathbf{E}}$ to calculate the force in a system; since it is often simpler to calculate the potential directly, there are systems in which it is useful to calculate V and then derive $\vec{\mathbf{E}}$ from it.

In general, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\vec{\mathbf{E}}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\vec{\mathbf{E}}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the greater the electric field. This gives us the following result.

Relationship between Voltage and Uniform Electric Field

In equation form, the relationship between voltage and uniform electric field is

$$E = -\frac{\Delta V}{\Delta s}$$

where Δs is the distance over which the change in potential ΔV takes place. The minus sign tells us that E points in the direction of decreasing potential. The electric field is said to be the gradient (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals, and we need differential calculus to determine the electric field. As shown in Figure 7.27, if we treat the distance Δs as very small so that the electric field is essentially constant over it, we find that

$$E_s = -\frac{dV}{ds}.$$

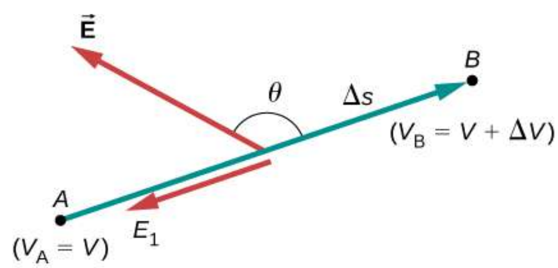


Figure 7.27 The electric field component along the displacement Δs is given by $E = -\frac{\Delta V}{\Delta s}$. Note that *A* and *B* are assumed to be so close together that the field is constant along Δs.

Therefore, the electric field components in the Cartesian directions are given by

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}. \tag{7.13}$$

This allows us to define the “grad” or “del” vector operator, which allows us to compute the gradient in one step. In Cartesian coordinates, it takes the form

$$\vec{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}. \tag{7.14}$$

With this notation, we can calculate the electric field from the potential with

$$\vec{\mathbf{E}} = -\vec{\nabla}V, \tag{7.15}$$

a process we call calculating the gradient of the potential.

If we have a system with either cylindrical or spherical symmetry, we only need to use the del operator in the appropriate coordinates:

$$\text{Cylindrical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial}{\partial z} \tag{7.16}$$

$$\text{Spherical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi} \tag{7.17}$$

EXAMPLE 7.17

Electric Field of a Point Charge

Calculate the electric field of a point charge from the potential.

Strategy

The potential is known to be $V = k\frac{q}{r}$, which has a spherical symmetry. Therefore, we use the spherical del operator in the formula $\vec{\mathbf{E}} = -\vec{\nabla}V$.

Solution

Performing this calculation gives us

$$\vec{\mathbf{E}} = -\left(\hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\right)k\frac{q}{r} = -kq\left(\hat{\mathbf{r}}\frac{\partial}{\partial r}\frac{1}{r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}\frac{1}{r} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\frac{1}{r}\right).$$

This equation simplifies to

$$\vec{\mathbf{E}} = -kq\left(\hat{\mathbf{r}}\frac{-1}{r^2} + \hat{\theta}0 + \hat{\varphi}0\right) = k\frac{q}{r^2}\hat{\mathbf{r}}$$

as expected.

Significance

We not only obtained the equation for the electric field of a point particle that we've seen before, we also have a demonstration that $\vec{\mathbf{E}}$ points in the direction of decreasing potential, as shown in Figure 7.28.

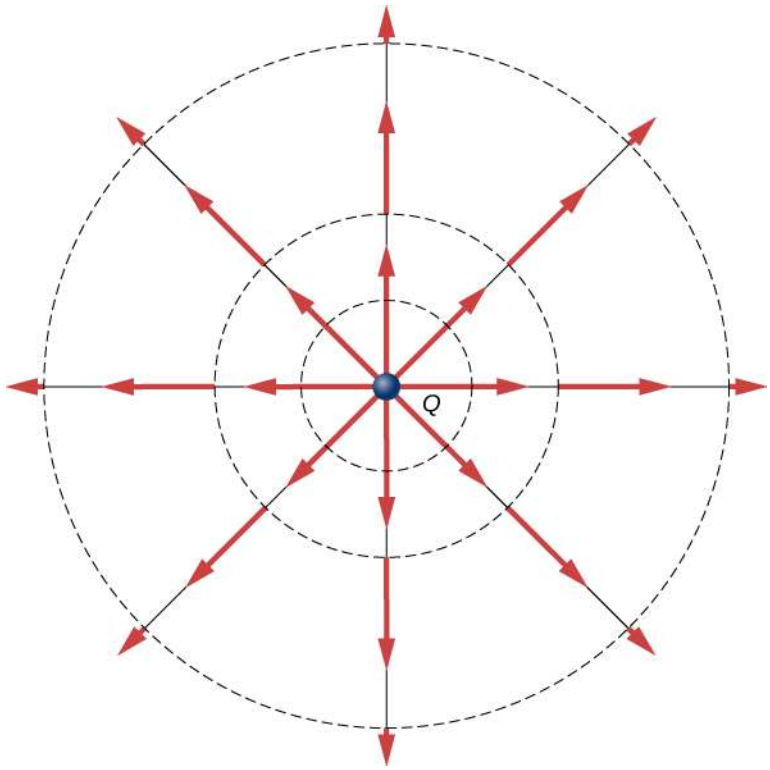


Figure 7.28 Electric field vectors inside and outside a uniformly charged sphere.

EXAMPLE 7.18

Electric Field of a Ring of Charge

Use the potential found in Example 7.8 to calculate the electric field along the axis of a ring of charge (Figure 7.29).

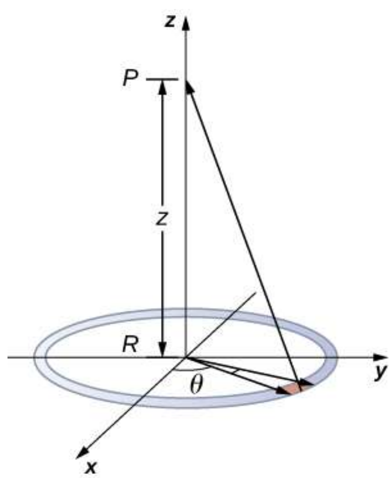


Figure 7.29 We want to calculate the electric field from the electric potential due to a ring charge.

Strategy

In this case, we are only interested in one dimension, the z-axis. Therefore, we use $E_z = -\frac{\partial V}{\partial z}$ with the potential $V = k\frac{q_{\text{tot}}}{\sqrt{z^2+R^2}}$ found previously.

Solution

Taking the derivative of the potential yields

$$E_z = -\frac{\partial}{\partial z} \frac{kq_{\text{tot}}}{\sqrt{z^2 + R^2}} = k\frac{q_{\text{tot}}\, z}{\left(z^2 + R^2\right)^{3/2}}.$$

Significance

Again, this matches the equation for the electric field found previously. It also demonstrates a system in which using the full del operator is not necessary.

CHECK YOUR UNDERSTANDING 7.11

Which coordinate system would you use to calculate the electric field of a dipole?

7.5 Equipotential Surfaces and Conductors

Learning Objectives

By the end of this section, you will be able to:

- Define equipotential surfaces and equipotential lines
- Explain the relationship between equipotential lines and electric field lines
- Map equipotential lines for one or two point charges
- Describe the potential of a conductor
- Compare and contrast equipotential lines and elevation lines on topographic maps

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. This is not surprising, since the two concepts are related. Consider Figure 7.30, which shows an isolated positive point charge and its electric field lines, which radiate out from a positive charge and terminate on negative charges. We use red arrows to represent the magnitude and direction of the electric field, and we use black lines to represent places where the electric potential is constant. These are called **equipotential surfaces** in three dimensions, or **equipotential lines** in two dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true because the potential for a point charge is given by $V = kq/r$ and thus has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of Figure 7.30. Because the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

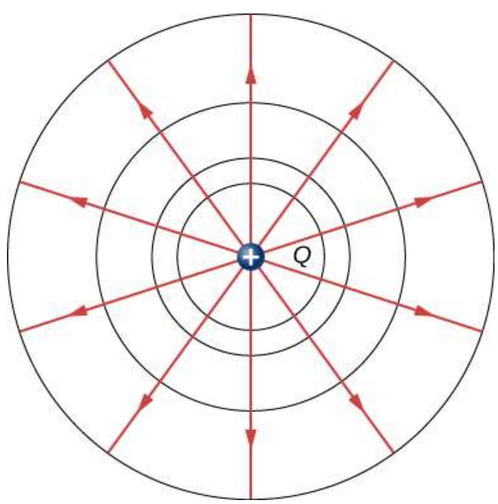


Figure 7.30 An isolated point charge Q with its electric field lines in red and equipotential lines in black. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case. For a three-

dimensional version, explore the first media link.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus, the work is

$$W = -\Delta U = -q\Delta V = 0.$$

Work is zero if the direction of the force is perpendicular to the displacement. Force is in the same direction as E, so motion along an equipotential must be perpendicular to E. More precisely, work is related to the electric field by

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = q\vec{\mathbf{E}} \cdot \vec{\mathbf{d}} = qEd\cos\theta = 0.$$

Note that in this equation, E and F symbolize the magnitudes of the electric field and force, respectively. Neither q nor E is zero; d is also not zero. So $\cos\theta$ must be 0, meaning θ must be 90º. In other words, motion along an equipotential is perpendicular to E.

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at what we consider zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to Earth.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in Figure 7.30, a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

Figure 7.31 shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in Figure 7.32(a), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in Figure 7.32(b).

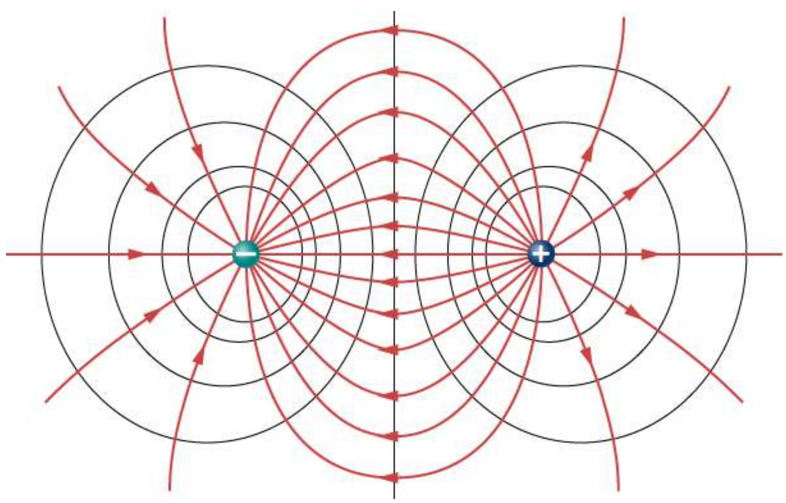


Figure 7.31 The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge. For a three-dimensional version, explore the first media link.

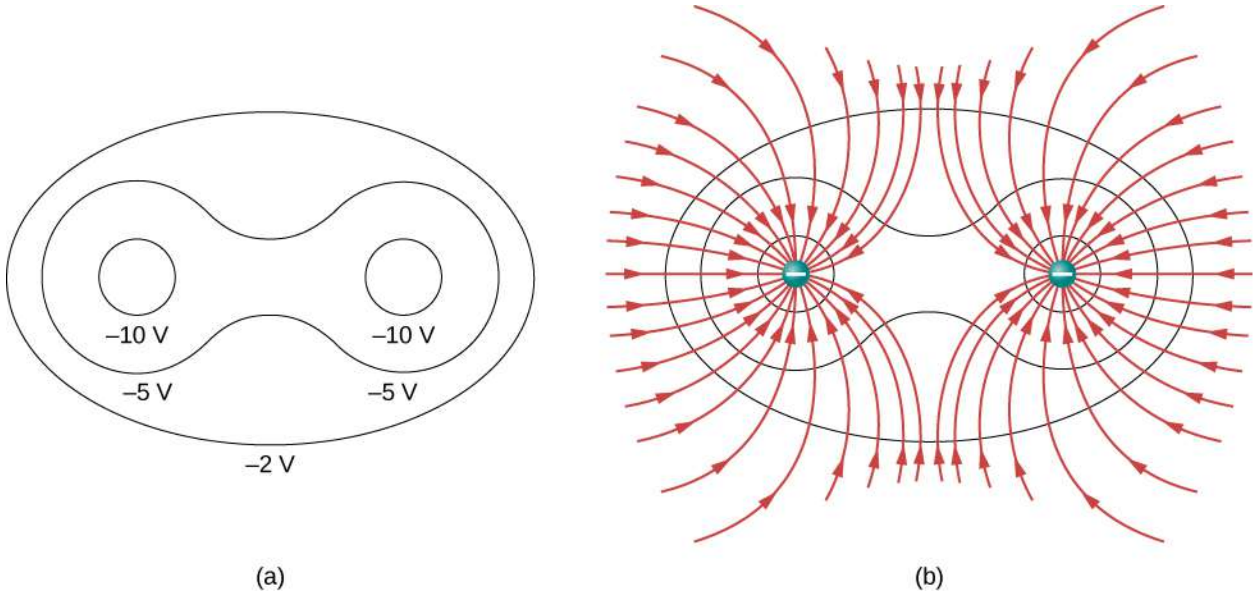


Figure 7.32 (a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges. For a three-dimensional version, play with the first media link.

To improve your intuition, we show a three-dimensional variant of the potential in a system with two opposing charges. Figure 7.33 displays a three-dimensional map of electric potential, where lines on the map are for equipotential surfaces. The hill is at the positive charge, and the trough is at the negative charge. The potential is zero far away from the charges. Note that the cut off at a particular potential implies that the charges are on conducting spheres with a finite radius.

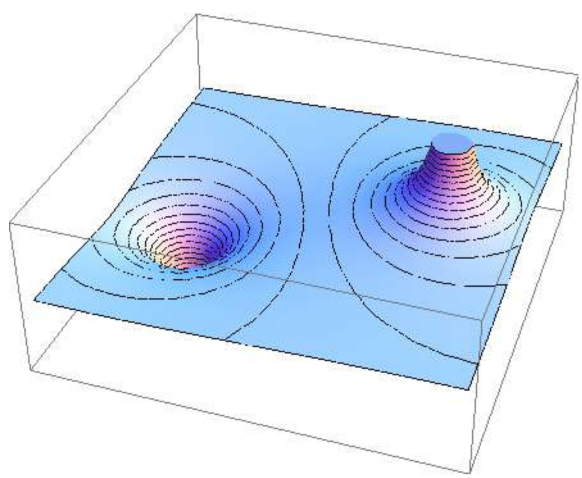


Figure 7.33 Electric potential map of two opposite charges of equal magnitude on conducting spheres. The potential is negative near the negative charge and positive near the positive charge.

A two-dimensional map of the cross-sectional plane that contains both charges is shown in Figure 7.34. The line that is equidistant from the two opposite charges corresponds to zero potential, since at the points on the line, the positive potential from the positive charge cancels the negative potential from the negative charge. Equipotential lines in the cross-sectional plane are closed loops, which are not necessarily circles, since at each point, the net potential is the sum of the potentials from each charge.

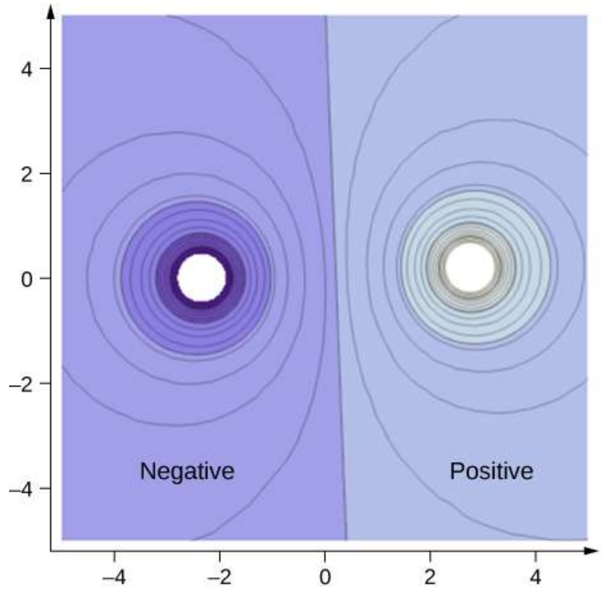


Figure 7.34 A cross-section of the electric potential map of two opposite charges of equal magnitude. The potential is negative near the negative charge and positive near the positive charge.

INTERACTIVE

View this simulation (https://openstax.org/l/21equipsurelec) to observe and modify the equipotential surfaces and electric fields for many standard charge configurations. There's a lot to explore.

One of the most important cases is that of the familiar parallel conducting plates shown in Figure 7.35. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.

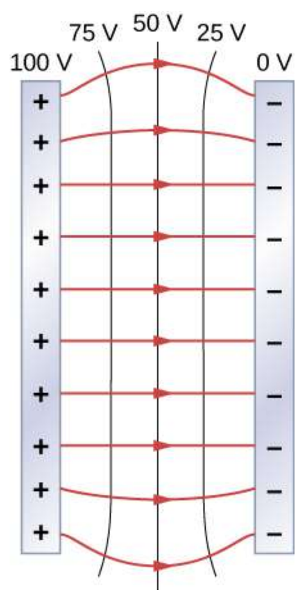


Figure 7.35 The electric field and equipotential lines between two metal plates. Note that the electric field is perpendicular to the equipotentials and hence normal to the plates at their surface as well as in the center of the region between them.

Consider the parallel plates in Figure 7.2. These have equipotential lines that are parallel to the plates in the space between and evenly spaced. An example of this (with sample values) is given in Figure 7.35. We could draw a similar set of equipotential isolines for gravity on the hill shown in Figure 7.2. If the hill has any extent at the same slope, the isolines along that extent would be parallel to each other. Furthermore, in regions of constant slope, the isolines would be evenly spaced. An example of real topographic lines is shown in Figure 7.36.

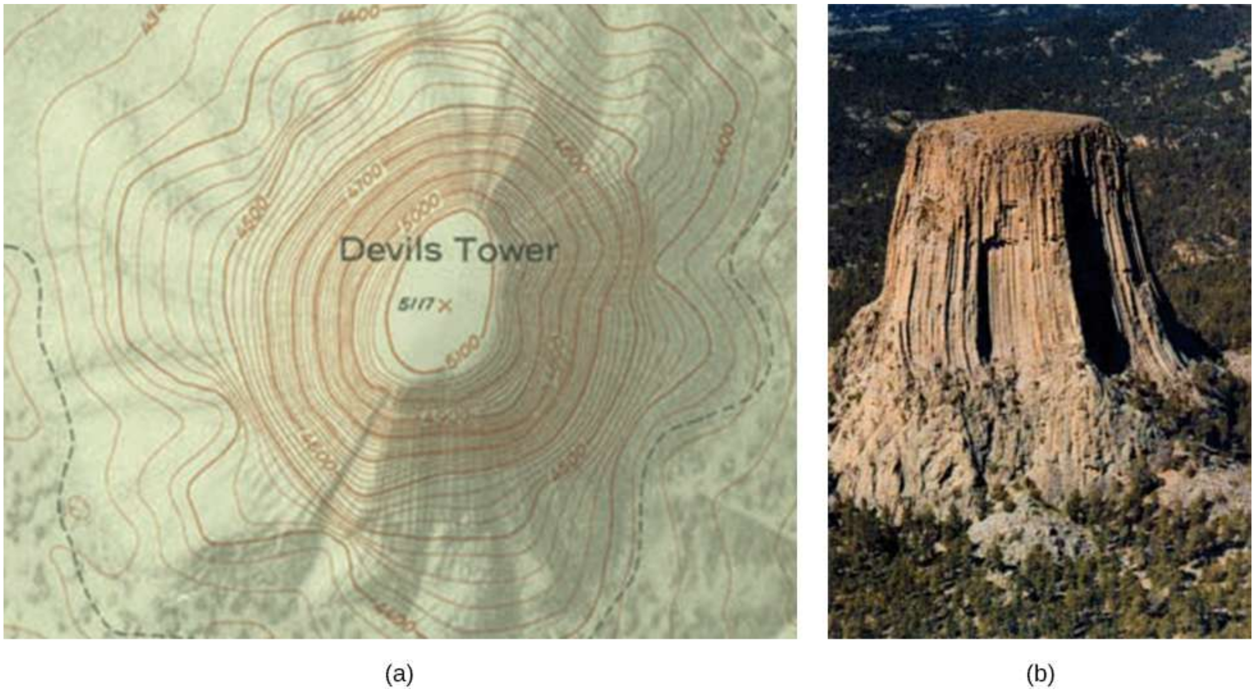


Figure 7.36 A topographical map along a ridge has roughly parallel elevation lines, similar to the equipotential lines in Figure 7.35. (a) A topographical map of Devil's Tower, Wyoming. Lines that are close together indicate very steep terrain. (b) A perspective photo of Devil's Tower shows just how steep its sides are. Notice the top of the tower has the same shape as the center of the topographical map.

EXAMPLE 7.19

Calculating Equipotential Lines

You have seen the equipotential lines of a point charge in Figure 7.30. How do we calculate them? For example, if we have a $+10$-nC charge at the origin, what are the equipotential surfaces at which the potential is (a) 100 V, (b) 50 V, (c) 20 V, and (d) 10 V?

Strategy

Set the equation for the potential of a point charge equal to a constant and solve for the remaining variable(s). Then calculate values as needed.

Solution

In $V = k\frac{q}{r}$, let V be a constant. The only remaining variable is r; hence, $r = k\frac{q}{V} = \text{constant}$. Thus, the equipotential surfaces are spheres about the origin. Their locations are:

a. $r = k\frac{q}{V} = \left(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\right) \frac{\left(10 \times 10^{-9} \text{ C}\right)}{100 \text{ V}} = 0.90 \text{ m};$

b. $r = k\frac{q}{V} = \left(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\right) \frac{\left(10 \times 10^{-9} \text{ C}\right)}{50 \text{ V}} = 1.8 \text{ m};$

c. $r = k\frac{q}{V} = \left(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\right) \frac{\left(10 \times 10^{-9} \text{ C}\right)}{20 \text{ V}} = 4.5 \text{ m};$

d. $r = k\frac{q}{V} = \left(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\right) \frac{\left(10 \times 10^{-9} \text{ C}\right)}{10 \text{ V}} = 9.0 \text{ m}.$

Significance

This means that equipotential surfaces around a point charge are spheres of constant radius, as shown earlier, with well-defined locations.

EXAMPLE 7.20

Potential Difference between Oppositely Charged Parallel Plates

Two large conducting plates carry equal and opposite charges, with a surface charge density σ of magnitude $6.81 \times 10^{-7}\ \text{C/m}^2$, as shown in Figure 7.37. The separation between the plates is $l = 6.50$ mm. (a) What is the electric field between the plates? (b) What is the potential difference between the plates? (c) What is the distance between equipotential planes which differ by 100 V?

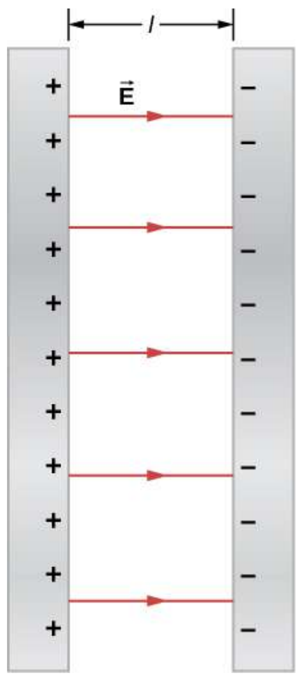


Figure 7.37 The electric field between oppositely charged parallel plates. A portion is released at the positive plate.

Strategy

(a) Since the plates are described as "large" and the distance between them is not, we will approximate each of them as an infinite plane, and apply the result from Gauss's law in the previous chapter.

(b) Use $\Delta V_{AB} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$.

(c) Since the electric field is constant, find the ratio of 100 V to the total potential difference; then calculate this fraction of the distance.

Solution

a. The electric field is directed from the positive to the negative plate as shown in the figure, and its magnitude is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{6.81 \times 10^{-7}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 7.69 \times 10^4\ \text{V/m}.$$

b. To find the potential difference ΔV between the plates, we use a path from the negative to the positive plate that is directed against the field. The displacement vector $d\vec{\mathbf{l}}$ and the electric field $\vec{\mathbf{E}}$ are antiparallel so $\vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -E\,dl$. The potential difference between the positive plate and the negative plate is then

$$\Delta V = -\int E \cdot dl = E\int dl = El = (7.69 \times 10^4\ \text{V/m})(6.50 \times 10^{-3}\ \text{m}) = 500\ \text{V}.$$

c. The total potential difference is 500 V, so 1/5 of the distance between the plates will be the distance between 100-V potential differences. The distance between the plates is 6.5 mm, so there will be 1.3 mm between 100-V potential differences.

Significance

You have now seen a numerical calculation of the locations of equipotentials between two charged parallel plates.

CHECK YOUR UNDERSTANDING 7.12

What are the equipotential surfaces for an infinite line charge?

Distribution of Charges on Conductors

In Example 7.19 with a point charge, we found that the equipotential surfaces were in the form of spheres, with the point charge at the center. Given that a conducting sphere in electrostatic equilibrium is a spherical equipotential surface, we should expect that we could replace one of the surfaces in Example 7.19 with a conducting sphere and have an identical solution outside the sphere. Inside will be rather different, however.

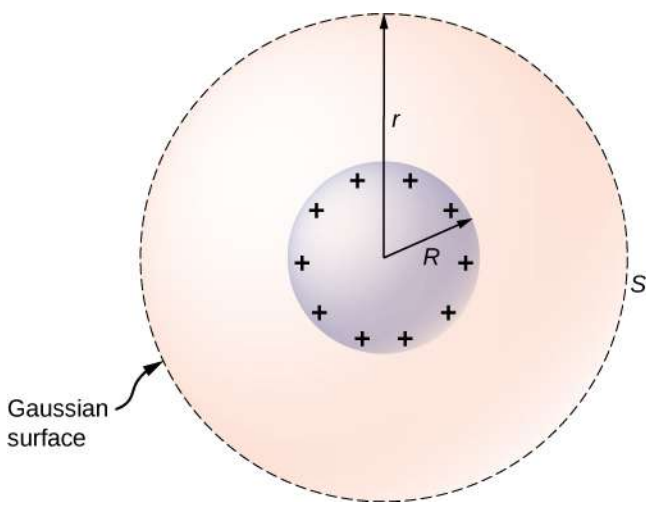


Figure 7.38 An isolated conducting sphere.

To investigate this, consider the isolated conducting sphere of Figure 7.38 that has a radius R and an excess charge q. To find the electric field both inside and outside the sphere, note that the sphere is isolated, so its surface change distribution and the electric field of that distribution are spherically symmetric. We can therefore represent the field as $\vec{\mathbf{E}} = E(r)\hat{\mathbf{r}}$. To calculate $E(r)$, we apply Gauss's law over a closed spherical surface S of radius r that is concentric with the conducting sphere. Since r is constant and $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ on the sphere,

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, da = E(r) \oint da = E(r)\, 4\pi r^2.$$

For $r < R$, S is within the conductor, so recall from our previous study of Gauss's law that $q_{\text{enc}} = 0$ and Gauss's law gives $E(r) = 0$, as expected inside a conductor at equilibrium. If $r > R$, S encloses the conductor so $q_{\text{enc}} = q$. From Gauss's law,

$$E(r)\, 4\pi r^2 = \frac{q}{\varepsilon_0}.$$

The electric field of the sphere may therefore be written as

$$\begin{aligned} E &= 0 && (r < R), \\ E &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} && (r \geq R). \end{aligned}$$

As expected, in the region $r \geq R$, the electric field due to a charge q placed on an isolated conducting sphere of radius R is identical to the electric field of a point charge q located at the center of the sphere.

To find the electric potential inside and outside the sphere, note that for $r \geq R$, the potential must be the same as that of an isolated point charge q located at $r = 0$,

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\ (r \geq R)$$

simply due to the similarity of the electric field.

For $r < R$, $E = 0$, so $V(r)$ is constant in this region. Since $V(R) = q/4\pi\varepsilon_0 R$,

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\ (r < R).$$

We will use this result to show that

$$\sigma_1 R_1 = \sigma_2 R_2,$$

for two conducting spheres of radii R_1 and R_2, with surface charge densities σ_1 and σ_2 respectively, that are connected by a thin wire, as shown in Figure 7.39. The spheres are sufficiently separated so that each can be treated as if it were isolated (aside from the wire). Note that the connection by the wire means that this entire system must be an equipotential.

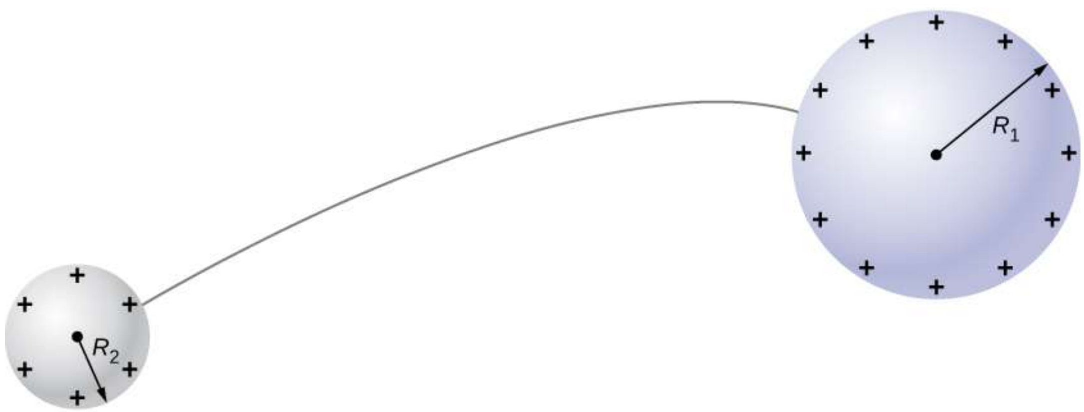


Figure 7.39 Two conducting spheres are connected by a thin conducting wire.

We have just seen that the electrical potential at the surface of an isolated, charged conducting sphere of radius R is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}.$$

Now, the spheres are connected by a conductor and are therefore at the same potential; hence

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2},$$

and

$$\frac{q_1}{R_1} = \frac{q_2}{R_2}.$$

The net charge on a conducting sphere and its surface charge density are related by $q = \sigma(4\pi R^2)$. Substituting this equation into the previous one, we find

$$\sigma_1 R_1 = \sigma_2 R_2.$$

Obviously, two spheres connected by a thin wire do not constitute a typical conductor with a variable radius of curvature. Nevertheless, this result does at least provide a qualitative idea of how charge density varies over the surface of a conductor. The equation indicates that where the radius of curvature is large (points B and D in

Figure 7.40), σ and E are small.

Similarly, the charges tend to be denser where the curvature of the surface is greater, as demonstrated by the charge distribution on oddly shaped metal (Figure 7.40). The surface charge density is higher at locations with a small radius of curvature than at locations with a large radius of curvature.

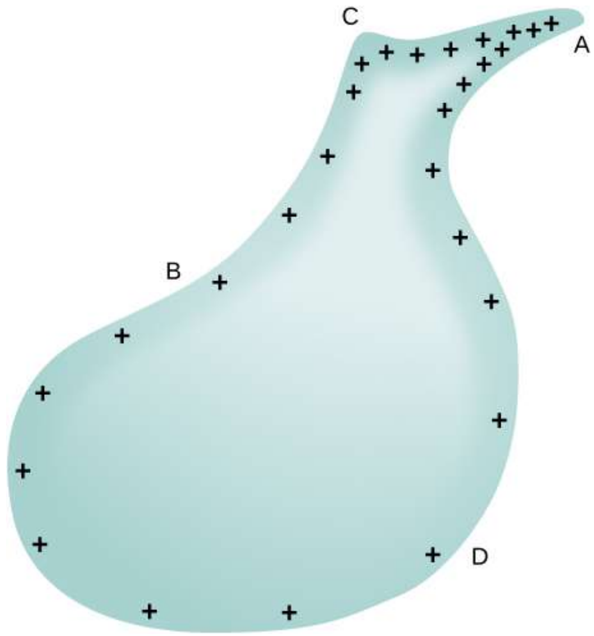


Figure 7.40 The surface charge density and the electric field of a conductor are greater at regions with smaller radii of curvature.

A practical application of this phenomenon is the lightning rod, which is simply a grounded metal rod with a sharp end pointing upward. As positive charge accumulates in the ground due to a negatively charged cloud overhead, the electric field around the sharp point gets very large. When the field reaches a value of approximately 3.0×10^6 N/C (the *dielectric strength* of the air), the free ions in the air are accelerated to such high energies that their collisions with air molecules actually ionize the molecules. The resulting free electrons in the air then flow through the rod to Earth, thereby neutralizing some of the positive charge. This keeps the electric field between the cloud and the ground from getting large enough to produce a lightning bolt in the region around the rod.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart.

INTERACTIVE

Play around with this simulation (https://openstax.org/l/21pointcharsim) to move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more.

7.6 Applications of Electrostatics

Learning Objectives

By the end of this section, you will be able to:

- Describe some of the many practical applications of electrostatics, including several printing technologies
- Relate these applications to Newton's second law and the electric force

The study of electrostatics has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure 7.41 shows a schematic of a large research version. Van de Graaffs use both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

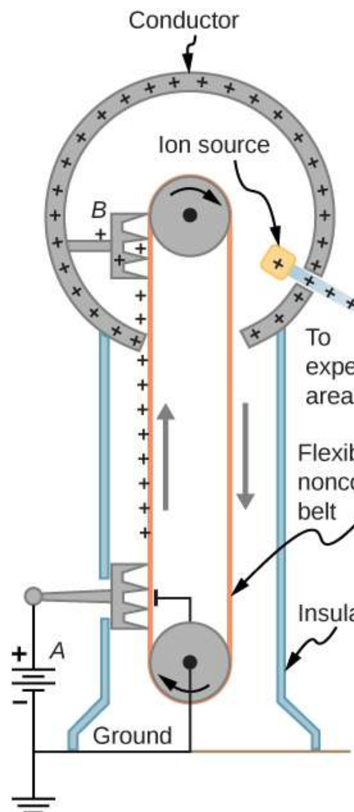


Figure 7.41 Schematic of Van de Graaff generator. A battery (*A*) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (*B*) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in Figure 7.42.

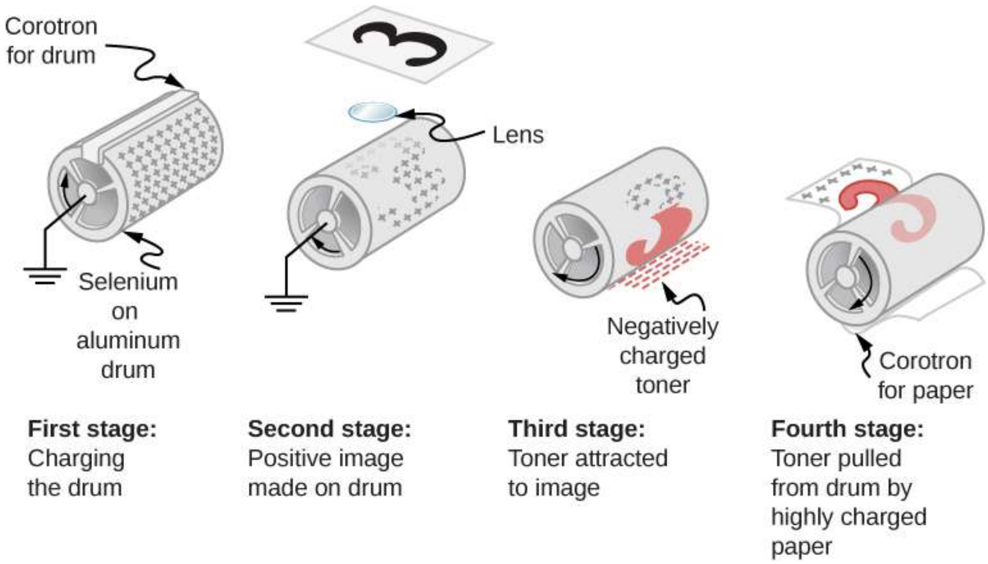


Figure 7.42 Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image, creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is grounded so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. In locations where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it is attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner to the fibers of the paper.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in Figure 7.43. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and in the past may have contained a computer more powerful than the one giving them the raw data to be printed.

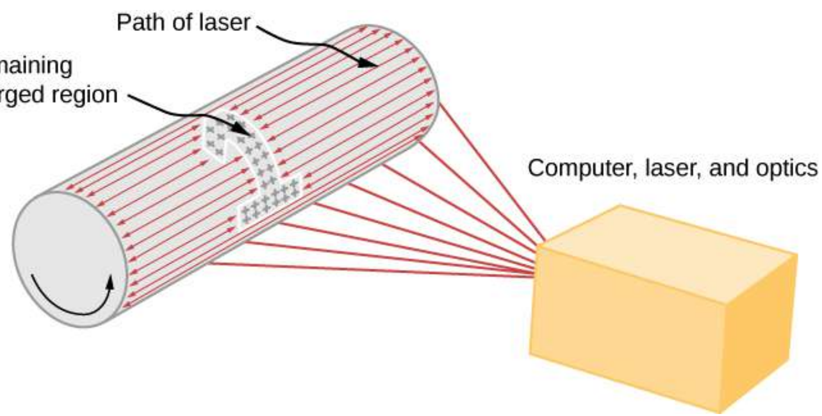


Figure 7.43 In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positively charged image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge (Figure 7.44).

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

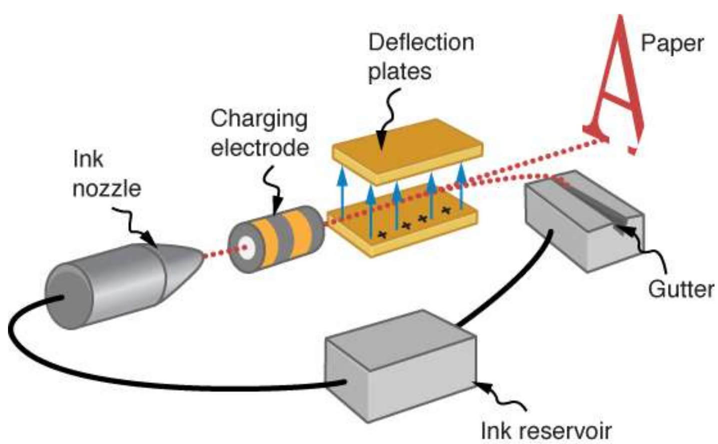


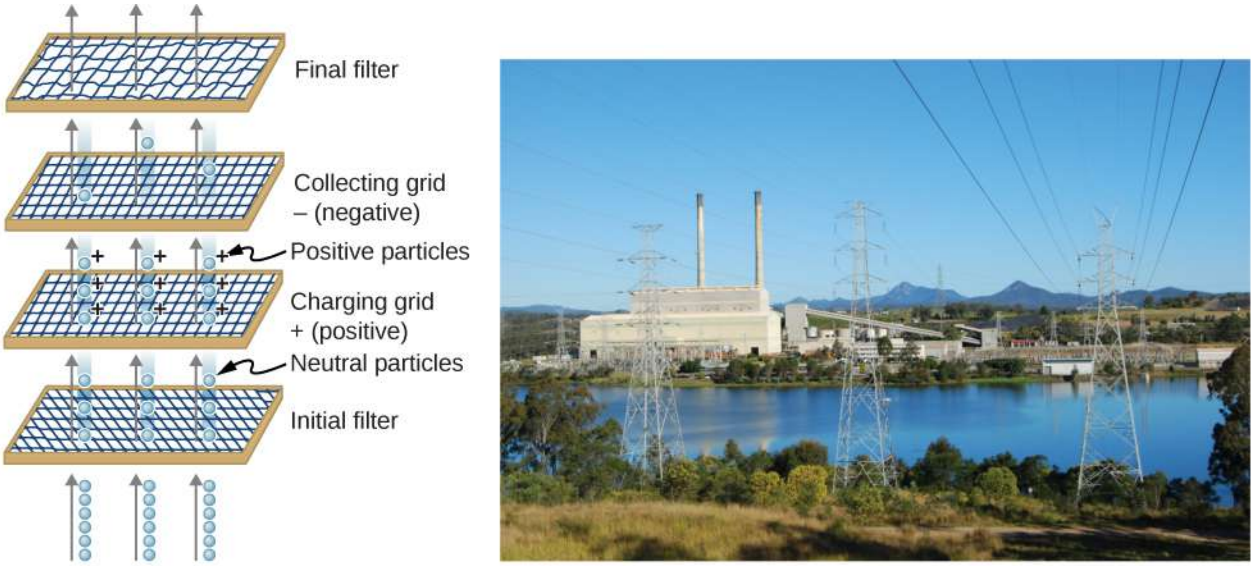
Figure 7.44 The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto oddly shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach hard-to-get-to places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles (Figure 7.45)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.



(a) (b)

Figure 7.45 (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit b: modification of work by “Cmdalgleish”/Wikimedia Commons)

CHAPTER REVIEW

Key Terms

electric dipole system of two equal but opposite charges a fixed distance apart

electric dipole moment quantity defined as $\vec{\mathbf{p}} = q\vec{\mathbf{d}}$ for all dipoles, where the vector points from the negative to positive charge

electric potential potential energy per unit charge

electric potential difference the change in potential energy of a charge q moved between two points, divided by the charge.

electric potential energy potential energy stored in a system of charged objects due to the charges

electron-volt energy given to a fundamental charge accelerated through a potential difference of one volt

electrostatic precipitators filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

equipotential line two-dimensional representation of an equipotential surface

equipotential surface surface (usually in three dimensions) on which all points are at the same potential

grounding process of attaching a conductor to the earth to ensure that there is no potential difference between it and Earth

ink jet printer small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

photoconductor substance that is an insulator until it is exposed to light, when it becomes a conductor

Van de Graaff generator machine that produces a large amount of excess charge, used for experiments with high voltage

voltage change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

xerography dry copying process based on electrostatics

Key Equations

Potential energy of a two-charge system	$U(r) = k\frac{qQ}{r}$
Work done to assemble a system of charges	$W_{12\cdots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}}$ for $i \neq j$
Potential difference	$\Delta V = \frac{\Delta U}{q}$ or $\Delta U = q\Delta V$
Electric potential	$V = \frac{U}{q} = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$
Potential difference between two points	$\Delta V_{BA} = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$
Electric potential of a point charge	$V = \frac{kq}{r}$
Electric potential of a system of point charges	$V_P = k\sum_{1}^{N}\frac{q_i}{r_i}$
Electric dipole moment	$\vec{\mathbf{p}} = q\vec{\mathbf{d}}$
Electric potential due to a dipole	$V_P = k\frac{\vec{\mathbf{p}}\cdot\hat{\mathbf{r}}}{r^2}$

Electric potential of a continuous charge distribution	$V_P = k\int \frac{dq}{r}$
Electric field components	$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$
Del operator in Cartesian coordinates	$\vec{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$
Electric field as gradient of potential	$\vec{\mathbf{E}} = -\vec{\nabla}V$
Del operator in cylindrical coordinates	$\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial\varphi} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$
Del operator in spherical coordinates	$\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}$

Summary

7.1 Electric Potential Energy

- The work done to move a charge from point A to B in an electric field is path independent, and the work around a closed path is zero. Therefore, the electric field and electric force are conservative.
- We can define an electric potential energy, which between point charges is $U(r) = k\frac{qQ}{r}$, with the zero reference taken to be at infinity.
- The superposition principle holds for electric potential energy; the potential energy of a system of multiple charges is the sum of the potential energies of the individual pairs.

7.2 Electric Potential and Potential Difference

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, that is, the change in potential of a charge q moved from A to B, is equal to the change in potential energy divided by the charge.
- Potential difference is commonly called voltage, represented by the symbol ΔV: $\Delta V = \frac{\Delta U}{q}$ or $\Delta U = q\Delta V$.
- An electron-volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form, $1\text{ eV} = \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ J/C}) = 1.60 \times 10^{-19}\text{ J}$.

7.3 Calculations of Electric Potential

- Electric potential is a scalar whereas electric field is a vector.
- Addition of voltages as numbers gives the voltage due to a combination of point charges, allowing us to use the principle of superposition: $V_P = k\sum_1^N \frac{q_i}{r_i}$.
- An electric dipole consists of two equal and opposite charges a fixed distance apart, with a dipole moment $\vec{\mathbf{p}} = q\vec{\mathbf{d}}$.
- Continuous charge distributions may be calculated with $V_P = k\int\frac{dq}{r}$.

7.4 Determining Field from Potential

- Just as we may integrate over the electric field to calculate the potential, we may take the derivative of the potential to calculate the electric field.
- This may be done for individual components of the electric field, or we may calculate the entire electric field vector with the gradient operator.

7.5 Equipotential Surfaces and Conductors

- An equipotential surface is the collection of points in space that are all at the same potential. Equipotential lines are the two-dimensional representation of equipotential surfaces.
- Equipotential surfaces are always perpendicular to electric field lines.
- Conductors in static equilibrium are equipotential surfaces.
- Topographic maps may be thought of as showing gravitational equipotential lines.

7.6 Applications of Electrostatics

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink jet printers, and electrostatic air filters.

Conceptual Questions

7.1 Electric Potential Energy

1. Would electric potential energy be meaningful if the electric field were not conservative?
2. Why do we need to be careful about work done *on* the system versus work done *by* the system in calculations?
3. Does the order in which we assemble a system of point charges affect the total work done?

7.2 Electric Potential and Potential Difference

4. Discuss how potential difference and electric field strength are related. Give an example.
5. What is the strength of the electric field in a region where the electric potential is constant?
6. If a proton is released from rest in an electric field, will it move in the direction of increasing or decreasing potential? Also answer this question for an electron and a neutron. Explain why.
7. Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?
8. If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.
9. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?
10. Voltages are always measured between two points. Why?
11. How are units of volts and electron-volts related? How do they differ?
12. Can a particle move in a direction of increasing electric potential, yet have its electric potential energy decrease? Explain

7.3 Calculations of Electric Potential

13. Compare the electric dipole moments of charges $\pm Q$ separated by a distance *d* and charges $\pm Q/2$ separated by a distance $d/2$.
14. Would Gauss's law be helpful for determining the electric field of a dipole? Why?
15. In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?
16. Can the potential of a nonuniformly charged sphere be the same as that of a point charge? Explain.

7.4 Determining Field from Potential

17. If the electric field is zero throughout a region, must the electric potential also be zero in that region?
18. Explain why knowledge of $\vec{\mathbf{E}}(x, y, z)$ is not sufficient to determine $V(x,y,z)$. What about the other way around?

7.5 Equipotential Surfaces and Conductors

19. If two points are at the same potential, are there any electric field lines connecting them?
20. Suppose you have a map of equipotential surfaces spaced 1.0 V apart. What do the distances between the surfaces in a particular region tell you about the strength of the $\vec{\mathbf{E}}$ in that region?
21. Is the electric potential necessarily constant over the surface of a conductor?
22. Under electrostatic conditions, the excess charge on a conductor resides on its surface. Does this mean that all of the conduction electrons in a conductor are on the surface?
23. Can a positively charged conductor be at a negative potential? Explain.
24. Can equipotential surfaces intersect?

7.6 Applications of Electrostatics

25. Why are the metal support rods for satellite network dishes generally grounded?
26. (a) Why are fish reasonably safe in an electrical storm? (b) Why are swimmers nonetheless ordered to get out of the water in the same circumstance?
27. What are the similarities and differences between the processes in a photocopier and an electrostatic precipitator?

28. About what magnitude of potential is used to charge the drum of a photocopy machine? A web search for “xerography” may be of use.

Problems

7.1 Electric Potential Energy

29. Consider a charge Q_1 ($+5.0\ \mu$C) fixed at a site with another charge Q_2 (charge $+3.0\ \mu$C, mass $6.0\ \mu$g) moving in the neighboring space. (a) Evaluate the potential energy of Q_2 when it is 4.0 cm from Q_1. (b) If Q_2 starts from rest from a point 4.0 cm from Q_1, what will be its speed when it is 8.0 cm from Q_1? (*Note:* Q_1 is held fixed in its place.)

30. Two charges Q_1 ($+2.00\ \mu$C) and Q_2 ($+2.00\ \mu$C) are placed symmetrically along the x-axis at $x = \pm 3.00$ cm. Consider a charge Q_3 of charge $+4.00\ \mu$C and mass 10.0 mg moving along the y-axis. If Q_3 starts from rest at $y = 2.00$ cm, what is its speed when it reaches $y = 4.00$ cm?

31. To form a hydrogen atom, a proton is fixed at a point and an electron is brought from far away to a distance of 0.529×10^{-10} m, the average distance between proton and electron in a hydrogen atom. How much work is done?

32. (a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn’t the defibrillator produce serious burns?

7.2 Electric Potential and Potential Difference

33. Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be 1.67×10^{-27} kg.

34. An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce X-rays. Non-relativistically, what would be the maximum speed of these electrons?

35. Show that units of V/m and N/C for electric field strength are indeed equivalent.

36. What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of 1.50×10^4 V?

37. The electric field strength between two parallel conducting plates separated by 4.00 cm is 7.50×10^4 V/m. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be zero volts. What is the potential 1.00 cm from that plate and 3.00 cm from the other?

38. The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct.) You may assume a uniform electric field.

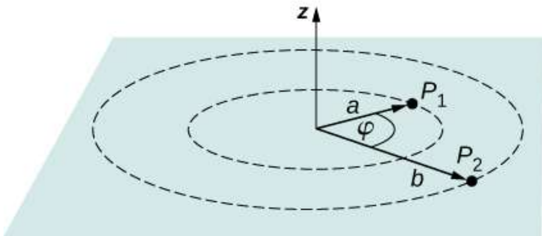
39. Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

40. Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be 3.0×10^6 V/m.

41. An electron is to be accelerated in a uniform electric field having a strength of 2.00×10^6 V/m. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

42. Use the definition of potential difference in terms of electric field to deduce the formula for potential difference between $r = r_a$ and $r = r_b$ for a point charge located at the origin. Here r is the spherical radial coordinate.

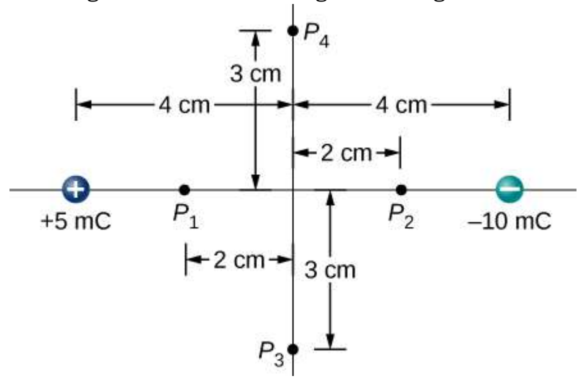
43. The electric field in a region is pointed away from the z-axis and the magnitude depends upon the distance s from the axis. The magnitude of the electric field is given as $E = \frac{\alpha}{s}$ where α is a constant. Find the potential difference between points P_1 and P_2, explicitly stating the path over which you conduct the integration for the line integral.



44. Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

7.3 Calculations of Electric Potential

45. A 0.500-cm-diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0-pC charge on its surface. What is the potential near its surface?
46. How far from a $1.00\text{-}\mu\text{C}$ point charge is the potential 100 V? At what distance is it 2.00×10^2 V?
47. If the potential due to a point charge is 5.00×10^2 V at a distance of 15.0 m, what are the sign and magnitude of the charge?
48. In nuclear fission, a nucleus splits roughly in half. (a) What is the potential 2.00×10^{-14} m from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?
49. A research Van de Graaff generator has a 2.00-m-diameter metal sphere with a charge of 5.00 mC on it. Assume the potential energy is zero at a reference point infinitely far away from the Van de Graaff. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its kinetic energy in MeV when the atom is at the distance found in part b?
50. An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object.
(a) What charge is on the sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?
51. (a) What is the potential between two points situated 10 cm and 20 cm from a $3.0\text{-}\mu\text{C}$ point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?
52. Find the potential at points P_1, P_2, P_3, and P_4 in the diagram due to the two given charges.



53. Two charges $-2.0\,\mu\text{C}$ and $+2.0\,\mu\text{C}$ are separated by 4.0 cm on the z-axis symmetrically about origin, with the positive one uppermost. Two space points of interest P_1 and P_2 are located 3.0 cm and 30 cm from origin at an angle 30° with respect to the z-axis. Evaluate electric potentials at P_1 and P_2 in two ways: (a) Using the exact formula for point charges, and (b) using the approximate dipole potential formula.
54. (a) Plot the potential of a uniformly charged 1-m rod with 1 C/m charge as a function of the perpendicular distance from the center. Draw your graph from $s = 0.1$ m to $s = 1.0$ m. (b) On the same graph, plot the potential of a point charge with a 1-C charge at the origin. (c) Which potential is stronger near the rod? (d) What happens to the difference as the distance increases? Interpret your result.

7.4 Determining Field from Potential

55. Throughout a region, equipotential surfaces are given by $z =$ constant. The surfaces are equally spaced with $V = 100$ V for $z = 0.00$ m, $V = 200$ V for $z = 0.50$ m, $V = 300$ V for $z = 1.00$ m. What is the electric field in this region?
56. In a particular region, the electric potential is given by $V = -xy^2z + 4xy$. What is the electric field in this region?
57. Calculate the electric field of an infinite line charge, throughout space.

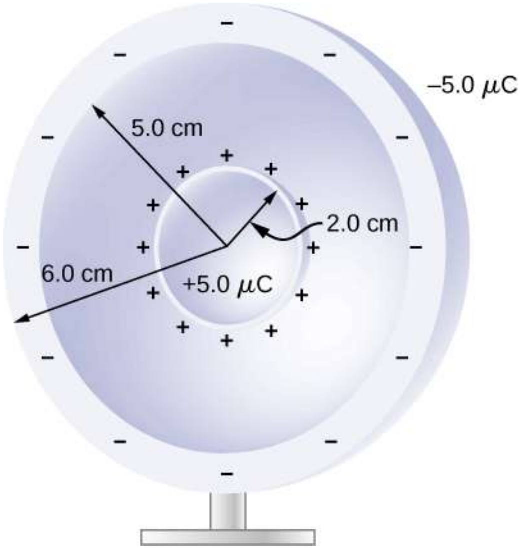
7.5 Equipotential Surfaces and Conductors

58. Two very large metal plates are placed 2.0 cm apart, with a potential difference of 12 V between them. Consider one plate to be at 12 V, and the other at 0 V. (a) Sketch the equipotential surfaces for 0, 4, 8, and 12 V. (b) Next sketch in some electric field lines, and confirm that they

are perpendicular to the equipotential lines.

59. A very large sheet of insulating material has had an excess of electrons placed on it to a surface charge density of -3.00 nC/m^2. (a) As the distance from the sheet increases, does the potential increase or decrease? Can you explain why without any calculations? Does the location of your reference point matter? (b) What is the shape of the equipotential surfaces? (c) What is the spacing between surfaces that differ by 1.00 V?

60. A metallic sphere of radius 2.0 cm is charged with $+5.0\text{-}\mu\text{C}$ charge, which spreads on the surface of the sphere uniformly. The metallic sphere stands on an insulated stand and is surrounded by a larger metallic spherical shell, of inner radius 5.0 cm and outer radius 6.0 cm. Now, a charge of $-5.0\text{-}\mu\text{C}$ is placed on the inside of the spherical shell, which spreads out uniformly on the inside surface of the shell. If potential is zero at infinity, what is the potential of (a) the spherical shell, (b) the sphere, (c) the space between the two, (d) inside the sphere, and (e) outside the shell?

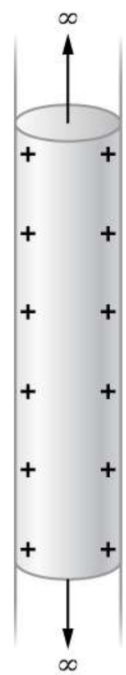


61. Two large charged plates of charge density $\pm 30 \ \mu\text{C/m}^2$ face each other at a separation of 5.0 mm. (a) Find the electric potential everywhere. (b) An electron is released from rest at the negative plate; with what speed will it strike the positive plate?

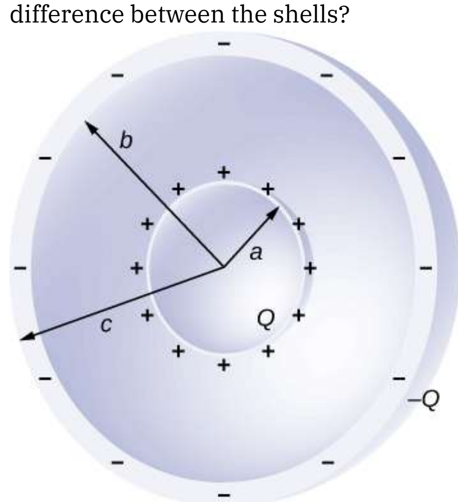
62. A long cylinder of aluminum of radius *R* meters is charged so that it has a uniform charge per unit length on its surface of λ.
(a) Find the electric field inside and outside the cylinder. (b) Find the electric potential inside and outside the cylinder. (c) Plot electric field and electric potential as a function of distance from the center of the rod.

63. Two parallel plates 10 cm on a side are given equal and opposite charges of magnitude 5.0×10^{-9} C. The plates are 1.5 mm apart. What is the potential difference between the plates?

64. The surface charge density on a long straight metallic pipe is σ. What is the electric potential outside and inside the pipe? Assume the pipe has a diameter of $2a$.

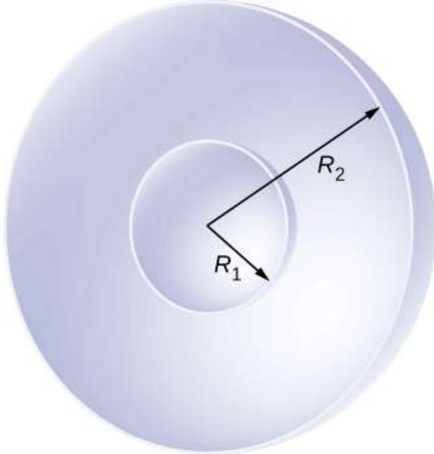


65. Concentric conducting spherical shells carry charges *Q* and –*Q*, respectively. The inner shell has negligible thickness. What is the potential difference between the shells?



66. Shown below are two concentric spherical shells of negligible thicknesses and radii R_1 and R_2. The inner and outer shell carry net charges q_1 and q_2, respectively, where both q_1

and q_2 are positive. What is the electric potential in the regions (a) $r < R_1$, (b) $R_1 < r < R_2$, and (c) $r > R_2$?



67. A solid cylindrical conductor of radius *a* is surrounded by a concentric cylindrical shell of inner radius *b*. The solid cylinder and the shell carry charges *Q* and −*Q*, respectively. Assuming that the length *L* of both conductors is much greater than *a* or *b*, what is the potential difference between the two conductors?

7.6 Applications of Electrostatics

68. (a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00-mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a $2.00\text{-}\mu\text{C}$ charge on the Van de Graaff's belt?
69. (a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.
70. A simple and common technique for accelerating electrons is shown in Figure 7.46, where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is 2.50×10^4 N/C. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.

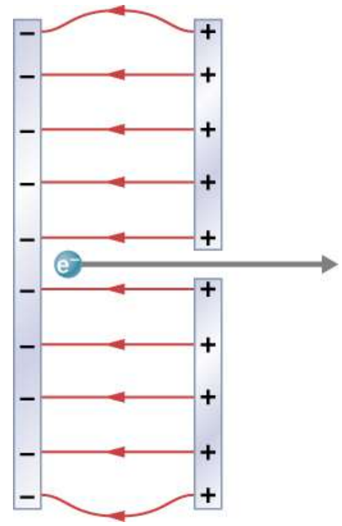
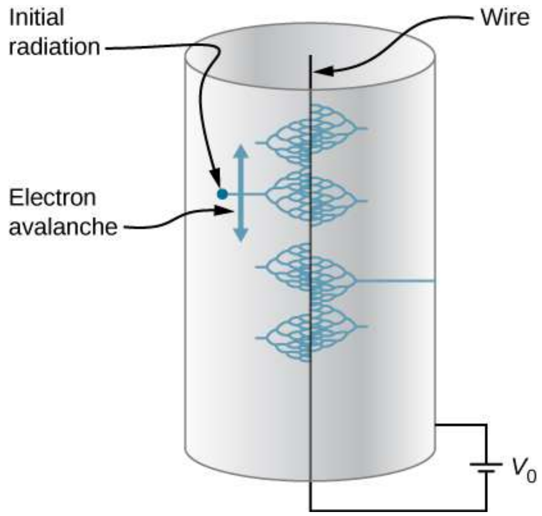


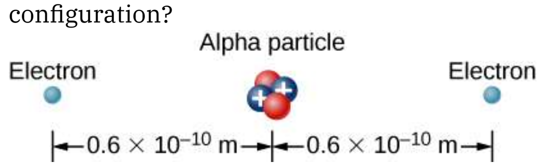
Figure 7.46 Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X- rays.

71. In a Geiger counter, a thin metallic wire at the center of a metallic tube is kept at a high voltage with respect to the metal tube. Ionizing radiation entering the tube knocks electrons off gas molecules or sides of the tube that then accelerate towards the center wire, knocking off even more electrons. This process eventually leads to an avalanche that is detectable as a current. A particular Geiger counter has a tube of radius *R* and the inner wire of radius *a* is at a potential of V_0 volts with respect to the outer metal tube. Consider a point *P* at a distance *s* from the center wire and far away from the ends. (a) Find a formula for the electric field at a point *P* inside using the infinite wire approximation. (b) Find a formula for the electric potential at a point *P* inside. (c) Use $V_0 = 900\text{ V}$, $a = 3.00\text{ mm}$, $R = 2.00\text{ cm}$, and find the value of the electric field at a point 1.00 cm from the center.



72. The practical limit to an electric field in air is about 3.00×10^6 N/C. Above this strength, sparking takes place because air begins to ionize. (a) At this electric field strength, how far would a proton travel before hitting the speed of light (ignore relativistic effects)? (b) Is it practical to leave air in particle accelerators?

73. To form a helium atom, an alpha particle that contains two protons and two neutrons is fixed at one location, and two electrons are brought in from far away, one at a time. The first electron is placed at 0.600×10^{-10} m from the alpha particle and held there while the second electron is brought to 0.600×10^{-10} m from the alpha particle on the other side from the first electron. See the final configuration below. (a) How much work is done in each step? (b) What is the electrostatic energy of the alpha particle and two electrons in the final configuration?

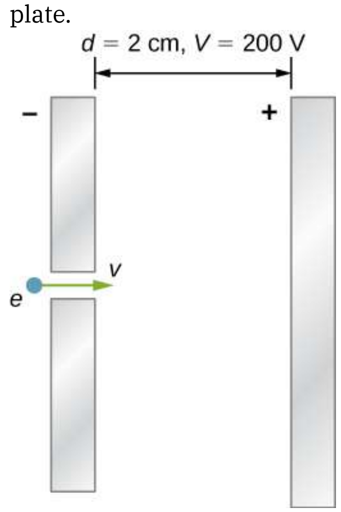


74. Find the electrostatic energy of eight equal charges ($+3\ \mu$C) each fixed at the corners of a cube of side 2 cm.

75. The probability of fusion occurring is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another. (a) Calculate the potential energy of two singly charged nuclei separated by 1.00×10^{-12} m. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

76. A bare helium nucleus has two positive charges and a mass of 6.64×10^{-27} kg. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron-volts? (c) What voltage would be needed to obtain this energy?

77. An electron enters a region between two large parallel plates made of aluminum separated by a distance of 2.0 cm and kept at a potential difference of 200 V. The electron enters through a small hole in the negative plate and moves toward the positive plate. At the time the electron is near the negative plate, its speed is 4.0×10^5 m/s. Assume the electric field between the plates to be uniform, and find the speed of electron at (a) 0.10 cm, (b) 0.50 cm, (c) 1.0 cm, and (d) 1.5 cm from the negative plate, and (e) immediately before it hits the positive plate.



78. How far apart are two conducting plates that have an electric field strength of 4.50×10^3 V/m between them, if their potential difference is 15.0 kV?

79. (a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength of dry air, which is 3.00×10^6 V/m, if the plates are separated by 2.00 mm and a potential difference of 5.0×10^3 V is applied? (b) How close together can the plates be with this applied voltage?

80. Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. What is the voltage across an 8.00-nm-thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

81. A double charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

82. The temperature near the center of the Sun is thought to be 15 million degrees Celsius (1.5×10^7 °C) (or kelvin). Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

83. A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of 1.00×10^2 MV. (a) What energy was dissipated? (b) What mass of water could be raised from 15 °C to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

84. What is the potential 0.530×10^{-10} m from a proton (the average distance between the proton and electron in a hydrogen atom)?

85. (a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

86. What are the sign and magnitude of a point charge that produces a potential of –2.00 V at a distance of 1.00 mm?

87. In one of the classic nuclear physics experiments at the beginning of the twentieth century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

Additional Problems

88. A 12.0-V battery-operated bottle warmer heats 50.0 g of glass, 2.50×10^2 g of baby formula, and 2.00×10^2 g of aluminum from 20.0 °C to 90.0 °C. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (*Hint:* Assume that the specific heat of baby formula is about the same as the specific heat of water.)

89. A battery-operated car uses a 12.0-V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a 2.00×10^2-m high hill, and finally cause it to travel at a constant 25.0 m/s while climbing with 5.00×10^2-N force for an hour.

90. (a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

91. A uniformly charged half-ring of radius 10 cm is placed on a nonconducting table. It is found that 3.0 cm above the center of the half-ring the potential is –3.0 V with respect to zero potential at infinity. How much charge is in the half-ring?

92. A glass ring of radius 5.0 cm is painted with a charged paint such that the charge density around the ring varies continuously given by the following function of the polar angle $\theta, \lambda = \left(3.0 \times 10^{-6}\ \text{C/m}\right) \cos^2\theta$. Find the potential at a point 15 cm above the center.

93. A CD disk of radius ($R = 3.0$ cm) is sprayed with a charged paint so that the charge varies continually with radial distance *r* from the center in the following manner:
$\sigma = -(6.0\ \text{C/m})\, r/R$.
Find the potential at a point 4 cm above the center.

94. (a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graff terminal? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

95. A large metal plate is charged uniformly to a density of $\sigma = 2.0 \times 10^{-9}\ \text{C/m}^2$. How far apart are the equipotential surfaces that represent a potential difference of 25 V?

96. Your friend gets really excited by the idea of making a lightning rod or maybe just a sparking toy by connecting two spheres as shown in Figure 7.39, and making R_2 so small that the electric field is greater than the dielectric strength of air, just from the usual 150 V/m electric field near the surface of the Earth. If R_1 is 10 cm, how small does R_2 need to be, and does this seem practical? (*Hint:* recall the calculation for electric field at the surface of a conductor from Gauss's Law.)

97. (a) Find $x >> L$ limit of the potential of a finite uniformly charged rod and show that it coincides with that of a point charge formula. (b) Why would you expect this result?

98. A small spherical pith ball of radius 0.50 cm is painted with a silver paint and then $-10\ \mu$C of charge is placed on it. The charged pith ball is put at the center of a gold spherical shell of inner radius 2.0 cm and outer radius 2.2 cm. (a) Find the electric potential of the gold shell with respect to zero potential at infinity. (b) How much charge should you put on the gold shell if you want to make its potential 100 V?

99. Two parallel conducting plates, each of cross-sectional area $400\ \text{cm}^2$, are 2.0 cm apart and uncharged. If 1.0×10^{12} electrons are transferred from one plate to the other, (a) what is the potential difference between the plates? (b) What is the potential difference between the positive plate and a point 1.25 cm from it that is between the plates?

100. A point charge of $q = 5.0 \times 10^{-8}$ C is placed at the center of an uncharged spherical conducting shell of inner radius 6.0 cm and outer radius 9.0 cm. Find the electric potential at (a) $r = 4.0$ cm, (b) $r = 8.0$ cm, (c) $r = 12.0$ cm.

101. Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?

102. Point charges of $25.0\ \mu$C and $45.0\ \mu$C are placed 0.500 m apart.
(a) At what point along the line between them is the electric field zero?
(b) What is the electric field halfway between them?

103. What can you say about two charges q_1 and q_2, if the electric field one-fourth of the way from q_1 to q_2 is zero?

104. Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is 0.530×10^{-10} m. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

105. An electron has an initial velocity of 5.00×10^6 m/s in a uniform 2.00×10^5 -N/C electric field. The field accelerates the electron in the direction opposite to its initial velocity.
(a) What is the direction of the electric field?
(b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

Challenge Problems

106. Three Na^+ and three Cl^- ions are placed alternately and equally spaced around a circle of radius 50 nm. Find the electrostatic energy stored.

107. Look up (presumably online, or by dismantling an old device and making measurements) the magnitude of the potential deflection plates (and the space between them) in an ink jet printer. Then look up the speed with which the ink comes out the nozzle. Can you calculate the typical mass of an ink drop?

108. Use the electric field of a finite sphere with constant volume charge density to calculate the electric potential, throughout space. Then check your results by calculating the electric field from the potential.

109. Calculate the electric field of a dipole throughout space from the potential.

CHAPTER 8
Capacitance

Figure 8.1 The tree-like branch patterns in this clear Plexiglas® block are known as a Lichtenberg figure, named for the German physicist Georg Christof Lichtenberg (1742–1799), who was the first to study these patterns. The "branches" are created by the dielectric breakdown produced by a strong electric field. (credit: modification of work by Bert Hickman)

Chapter Outline

INTRODUCTION Capacitors are important components of electrical circuits in many electronic devices, including pacemakers, cell phones, and computers. In this chapter, we study their properties, and, over the next few chapters, we examine their function in combination with other circuit elements. By themselves, capacitors are often used to store electrical energy and release it when needed; with other circuit components, capacitors often act as part of a filter that allows some electrical signals to pass while blocking others. You can see why capacitors are considered one of the fundamental components of electrical circuits.

8.1 Capacitors and Capacitance

Learning Objectives

By the end of this section, you will be able to:

- Explain the concepts of a capacitor and its capacitance
- Describe how to evaluate the capacitance of a system of conductors

A **capacitor** is a device used to store electrical charge and electrical energy. Capacitors are generally with two electrical conductors separated by a distance. (Note that such electrical conductors are sometimes referred to as "electrodes," but more correctly, they are "capacitor plates.") The space between capacitors may simply be a vacuum, and, in that case, a capacitor is then known as a "vacuum capacitor." However, the space is usually filled with an insulating material known as a **dielectric**. (You will learn more about dielectrics in the sections on dielectrics later in this chapter.) The amount of storage in a capacitor is determined by a property called *capacitance*, which you will learn more about a bit later in this section.

Capacitors have applications ranging from filtering static from radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another but not touching, such as those in Figure 8.2. Most of the time, a dielectric is used between the two plates. When battery terminals are connected to an initially uncharged capacitor, the battery potential moves a small amount of charge of magnitude Q from the positive plate to the negative plate. The capacitor remains neutral overall, but with charges $+Q$ and $-Q$ residing on opposite plates.

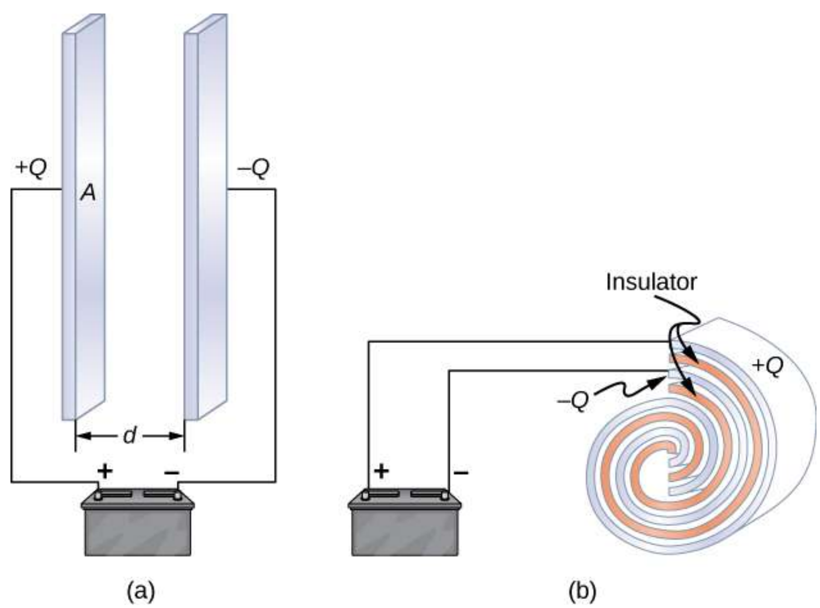


Figure 8.2 Both capacitors shown here were initially uncharged before being connected to a battery. They now have charges of $+Q$ and $-Q$ (respectively) on their plates. (a) A parallel-plate capacitor consists of two plates of opposite charge with area *A* separated by distance *d*. (b) A rolled capacitor has a dielectric material between its two conducting sheets (plates).

A system composed of two identical parallel-conducting plates separated by a distance is called a **parallel-plate capacitor** (Figure 8.3). The magnitude of the electrical field in the space between the parallel plates is $E = \sigma/\varepsilon_0$, where σ denotes the surface charge density on one plate (recall that σ is the charge Q per the surface area A). Thus, the magnitude of the field is directly proportional to Q.

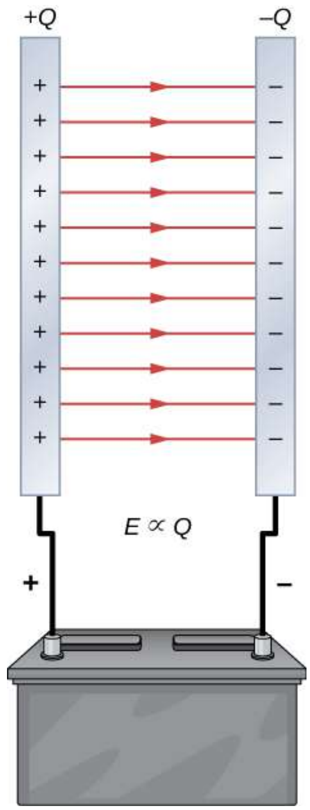


Figure 8.3 The charge separation in a capacitor shows that the charges remain on the surfaces of the capacitor plates. Electrical field lines in a parallel-plate capacitor begin with positive charges and end with negative charges. The magnitude of the electrical field in the space between the plates is in direct proportion to the amount of charge on the capacitor.

Capacitors with different physical characteristics (such as shape and size of their plates) store different amounts of charge for the same applied voltage V across their plates. The **capacitance** C of a capacitor is defined as the ratio of the maximum charge Q that can be stored in a capacitor to the applied voltage V across its plates. In other words, capacitance is the largest amount of charge per volt that can be stored on the device:

$$C = \frac{Q}{V}. \tag{8.1}$$

The SI unit of capacitance is the farad (F), named after Michael Faraday (1791–1867). Since capacitance is the charge per unit voltage, one farad is one coulomb per one volt, or

$$1\,\text{F} = \frac{1\,\text{C}}{1\,\text{V}}.$$

By definition, a 1.0-F capacitor is able to store 1.0 C of charge (a very large amount of charge) when the potential difference between its plates is only 1.0 V. One farad is therefore a very large capacitance. Typical capacitance values range from picofarads ($1\ \text{pF} = 10^{-12}\ \text{F}$) to millifarads ($1\ \text{mF} = 10^{-3}\ \text{F}$), which also includes microfarads ($1\ \mu\text{F} = 10^{-6}\ \text{F}$). Capacitors can be produced in various shapes and sizes (Figure 8.4).

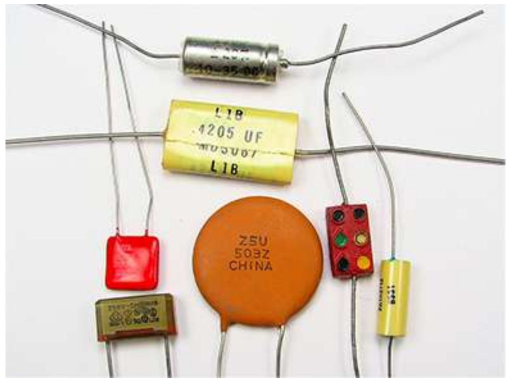


Figure 8.4 These are some typical capacitors used in electronic devices. A capacitor's size is not necessarily related to its capacitance value. (credit: Windell Oskay)

Calculation of Capacitance

We can calculate the capacitance of a pair of conductors with the standard approach that follows.

PROBLEM-SOLVING STRATEGY

Calculating Capacitance

1. Assume that the capacitor has a charge Q.
2. Determine the electrical field $\vec{\mathbf{E}}$ between the conductors. If symmetry is present in the arrangement of conductors, you may be able to use Gauss's law for this calculation.
3. Find the potential difference between the conductors from

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}, \tag{8.2}$$

where the path of integration leads from one conductor to the other. The magnitude of the potential difference is then $V = |V_B - V_A|$.
4. With V known, obtain the capacitance directly from Equation 8.1.

To show how this procedure works, we now calculate the capacitances of parallel-plate, spherical, and cylindrical capacitors. In all cases, we assume vacuum capacitors (empty capacitors) with no dielectric substance in the space between conductors.

Parallel-Plate Capacitor

The parallel-plate capacitor (Figure 8.5) has two identical conducting plates, each having a surface area A, separated by a distance d. When a voltage V is applied to the capacitor, it stores a charge Q, as shown. We can see how its capacitance may depend on A and d by considering characteristics of the Coulomb force. We know that force between the charges increases with charge values and decreases with the distance between them. We should expect that the bigger the plates are, the more charge they can store. Thus, C should be greater for a larger value of A. Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. Therefore, C should be greater for a smaller d.

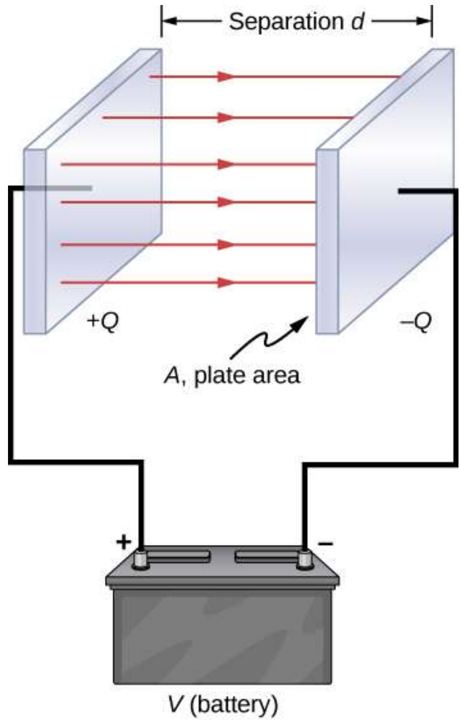


Figure 8.5 In a parallel-plate capacitor with plates separated by a distance d, each plate has the same surface area A.

We define the surface charge density σ on the plates as

$$\sigma = \frac{Q}{A}.$$

We know from previous chapters that when d is small, the electrical field between the plates is fairly uniform (ignoring edge effects) and that its magnitude is given by

$$E = \frac{\sigma}{\varepsilon_0},$$

where the constant ε_0 is the permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m. The SI unit of F/m is equivalent to $\mathrm{C^2/N \cdot m^2}$. Since the electrical field $\vec{\mathbf{E}}$ between the plates is uniform, the potential difference between the plates is

$$V = Ed = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 A}.$$

Therefore Equation 8.1 gives the capacitance of a parallel-plate capacitor as

$$C = \frac{Q}{V} = \frac{Q}{Qd/\varepsilon_0 A} = \varepsilon_0 \frac{A}{d}. \tag{8.3}$$

Notice from this equation that capacitance is a function *only of the geometry* and what material fills the space between the plates (in this case, vacuum) of this capacitor. In fact, this is true not only for a parallel-plate capacitor, but for all capacitors: The capacitance is independent of Q or V. If the charge changes, the potential changes correspondingly so that Q/V remains constant.

EXAMPLE 8.1

Capacitance and Charge Stored in a Parallel-Plate Capacitor

(a) What is the capacitance of an empty parallel-plate capacitor with metal plates that each have an area of $1.00\ \mathrm{m^2}$, separated by 1.00 mm? (b) How much charge is stored in this capacitor if a voltage of 3.00×10^3 V is applied to it?

Strategy

Finding the capacitance C is a straightforward application of Equation 8.3. Once we find C, we can find the charge stored by using Equation 8.1.

Solution

a. Entering the given values into Equation 8.3 yields

$$C = \varepsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}\right) \frac{1.00\,\text{m}^2}{1.00 \times 10^{-3}\,\text{m}} = 8.85 \times 10^{-9}\,\text{F} = 8.85\,\text{nF}.$$

This small capacitance value indicates how difficult it is to make a device with a large capacitance.

b. Inverting Equation 8.1 and entering the known values into this equation gives

$$Q = CV = (8.85 \times 10^{-9}\,\text{F})(3.00 \times 10^3\,\text{V}) = 26.6\,\mu\text{C}.$$

Significance

This charge is only slightly greater than those found in typical static electricity applications. Since air breaks down (becomes conductive) at an electrical field strength of about 3.0 MV/m, no more charge can be stored on this capacitor by increasing the voltage.

EXAMPLE 8.2

A 1-F Parallel-Plate Capacitor

Suppose you wish to construct a parallel-plate capacitor with a capacitance of 1.0 F. What area must you use for each plate if the plates are separated by 1.0 mm?

Solution

Rearranging Equation 8.3, we obtain

$$A = \frac{Cd}{\varepsilon_0} = \frac{(1.0\,\text{F})(1.0 \times 10^{-3}\,\text{m})}{8.85 \times 10^{-12}\,\text{F/m}} = 1.1 \times 10^8\,\text{m}^2.$$

Each square plate would have to be 10 km across. It used to be a common prank to ask a student to go to the laboratory stockroom and request a 1-F parallel-plate capacitor, until stockroom attendants got tired of the joke.

CHECK YOUR UNDERSTANDING 8.1

The capacitance of a parallel-plate capacitor is 2.0 pF. If the area of each plate is $2.4\,\text{cm}^2$, what is the plate separation?

CHECK YOUR UNDERSTANDING 8.2

Verify that σ/V and ε_0/d have the same physical units.

Spherical Capacitor

A spherical capacitor is another set of conductors whose capacitance can be easily determined (Figure 8.6). It consists of two concentric conducting spherical shells of radii R_1 (inner shell) and R_2 (outer shell). The shells are given equal and opposite charges $+Q$ and $-Q$, respectively. From symmetry, the electrical field between the shells is directed radially outward. We can obtain the magnitude of the field by applying Gauss's law over a spherical Gaussian surface of radius r concentric with the shells. The enclosed charge is $+Q$; therefore we have

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = E(4\pi r^2) = \frac{Q}{\varepsilon_0}.$$

Thus, the electrical field between the conductors is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}}.$$

We substitute this $\vec{\mathbf{E}}$ into Equation 8.2 and integrate along a radial path between the shells:

$$V = \int_{R_1}^{R_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \int_{R_1}^{R_2} \left(\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}} \right) \cdot (\hat{\mathbf{r}} \quad dr) = \frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

In this equation, the potential difference between the plates is $V = -(V_2 - V_1) = V_1 - V_2$. We substitute this result into Equation 8.1 to find the capacitance of a spherical capacitor:

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}. \quad (8.4)$$

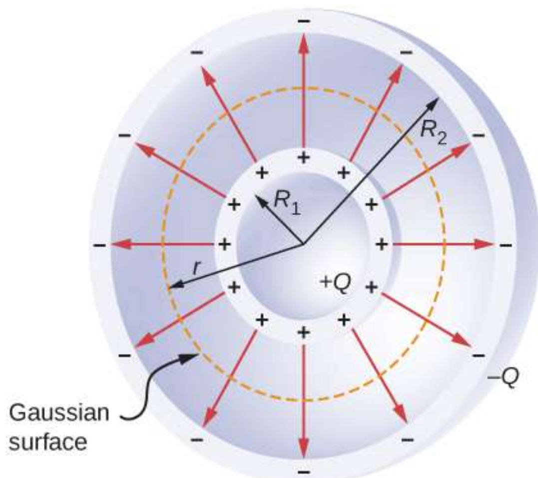


Figure 8.6 A spherical capacitor consists of two concentric conducting spheres. Note that the charges on a conductor reside on its surface.

EXAMPLE 8.3

Capacitance of an Isolated Sphere

Calculate the capacitance of a single isolated conducting sphere of radius R_1 and compare it with Equation 8.4 in the limit as $R_2 \to \infty$.

Strategy

We assume that the charge on the sphere is Q, and so we follow the four steps outlined earlier. We also assume the other conductor to be a concentric hollow sphere of infinite radius.

Solution

On the outside of an isolated conducting sphere, the electrical field is given by Equation 8.2. The magnitude of the potential difference between the surface of an isolated sphere and infinity is

$$V = \int_{R_1}^{+\infty} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{+\infty} \frac{1}{r^2} \hat{\mathbf{r}} \cdot (\hat{\mathbf{r}}\, dr) = \frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{+\infty} \frac{dr}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_1}.$$

The capacitance of an isolated sphere is therefore

$$C = \frac{Q}{V} = Q\frac{4\pi\varepsilon_0 R_1}{Q} = 4\pi\varepsilon_0 R_1.$$

Significance

The same result can be obtained by taking the limit of Equation 8.4 as $R_2 \to \infty$. A single isolated sphere is therefore equivalent to a spherical capacitor whose outer shell has an infinitely large radius.

CHECK YOUR UNDERSTANDING 8.3

The radius of the outer sphere of a spherical capacitor is five times the radius of its inner shell. What are the dimensions of this capacitor if its capacitance is 5.00 pF?

Cylindrical Capacitor

A cylindrical capacitor consists of two concentric, conducting cylinders (Figure 8.7). The inner cylinder, of radius R_1, may either be a shell or be completely solid. The outer cylinder is a shell of inner radius R_2. We assume that the length of each cylinder is l and that the excess charges $+Q$ and $-Q$ reside on the inner and outer cylinders, respectively.

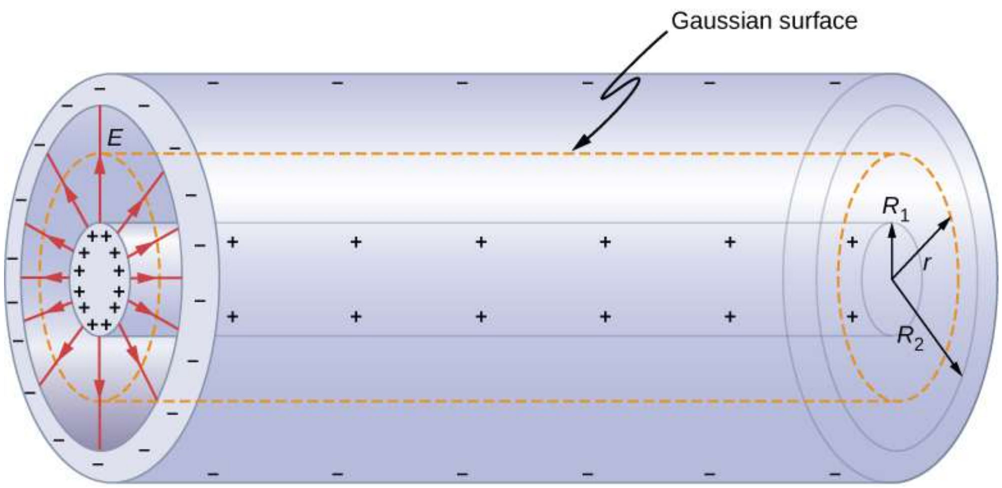


Figure 8.7 A cylindrical capacitor consists of two concentric, conducting cylinders. Here, the charge on the outer surface of the inner cylinder is positive (indicated by +) and the charge on the inner surface of the outer cylinder is negative (indicated by −).

With edge effects ignored, the electrical field between the conductors is directed radially outward from the common axis of the cylinders. Using the Gaussian surface shown in Figure 8.7, we have

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E(2\pi r l) = \frac{Q}{\varepsilon_0}.$$

Therefore, the electrical field between the cylinders is

$$\vec{\mathbf{E}} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{r\,l}\hat{\mathbf{r}}. \qquad 8.5$$

Here $\hat{\mathbf{r}}$ is the unit radial vector along the radius of the cylinder. We can substitute into Equation 8.2 and find the potential difference between the cylinders:

$$V = \int_{R_1}^{R_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}_p = \frac{Q}{2\pi\varepsilon_0\, l}\int_{R_1}^{R_2} \frac{1}{r}\hat{\mathbf{r}} \cdot (\hat{\mathbf{r}}\, dr) = \frac{Q}{2\pi\varepsilon_0\, l}\int_{R_1}^{R_2} \frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0\, l}\ln r\Big|_{R_1}^{R_2} = \frac{Q}{2\pi\varepsilon_0\, l}\ln\frac{R_2}{R_1}.$$

Thus, the capacitance of a cylindrical capacitor is

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \, l}{\ln(R_2/R_1)}. \qquad 8.6$$

As in other cases, this capacitance depends only on the geometry of the conductor arrangement. An important application of Equation 8.6 is the determination of the capacitance per unit length of a *coaxial cable*, which is commonly used to transmit time-varying electrical signals. A coaxial cable consists of two concentric, cylindrical conductors separated by an insulating material. (Here, we assume a vacuum between the conductors, but the physics is qualitatively almost the same when the space between the conductors is filled by a dielectric.) This configuration shields the electrical signal propagating down the inner conductor from stray electrical fields external to the cable. Current flows in opposite directions in the inner and the outer conductors, with the outer conductor usually grounded. Now, from Equation 8.6, the capacitance per unit length of the coaxial cable is given by

$$\frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln(R_2/R_1)}.$$

In practical applications, it is important to select specific values of C/l. This can be accomplished with appropriate choices of radii of the conductors and of the insulating material between them.

CHECK YOUR UNDERSTANDING 8.4

When a cylindrical capacitor is given a charge of 0.500 nC, a potential difference of 20.0 V is measured between the cylinders. (a) What is the capacitance of this system? (b) If the cylinders are 1.0 m long, what is the ratio of their radii?

Several types of practical capacitors are shown in Figure 8.4. Common capacitors are often made of two small pieces of metal foil separated by two small pieces of insulation (see Figure 8.2(b)). The metal foil and insulation are encased in a protective coating, and two metal leads are used for connecting the foils to an external circuit. Some common insulating materials are mica, ceramic, paper, and Teflon™ non-stick coating.

Another popular type of capacitor is an electrolytic capacitor. It consists of an oxidized metal in a conducting paste. The main advantage of an electrolytic capacitor is its high capacitance relative to other common types of capacitors. For example, capacitance of one type of aluminum electrolytic capacitor can be as high as 1.0 F. However, you must be careful when using an electrolytic capacitor in a circuit, because it only functions correctly when the metal foil is at a higher potential than the conducting paste. When reverse polarization occurs, electrolytic action destroys the oxide film. This type of capacitor cannot be connected across an alternating current source, because half of the time, ac voltage would have the wrong polarity, as an alternating current reverses its polarity (see Alternating-Current Circuits on alternating-current circuits).

A variable air capacitor (Figure 8.8) has two sets of parallel plates. One set of plates is fixed (indicated as "stator"), and the other set of plates is attached to a shaft that can be rotated (indicated as "rotor"). By turning the shaft, the cross-sectional area in the overlap of the plates can be changed; therefore, the capacitance of this system can be tuned to a desired value. Capacitor tuning has applications in any type of radio transmission and in receiving radio signals from electronic devices. Any time you tune your car radio to your favorite station, think of capacitance.

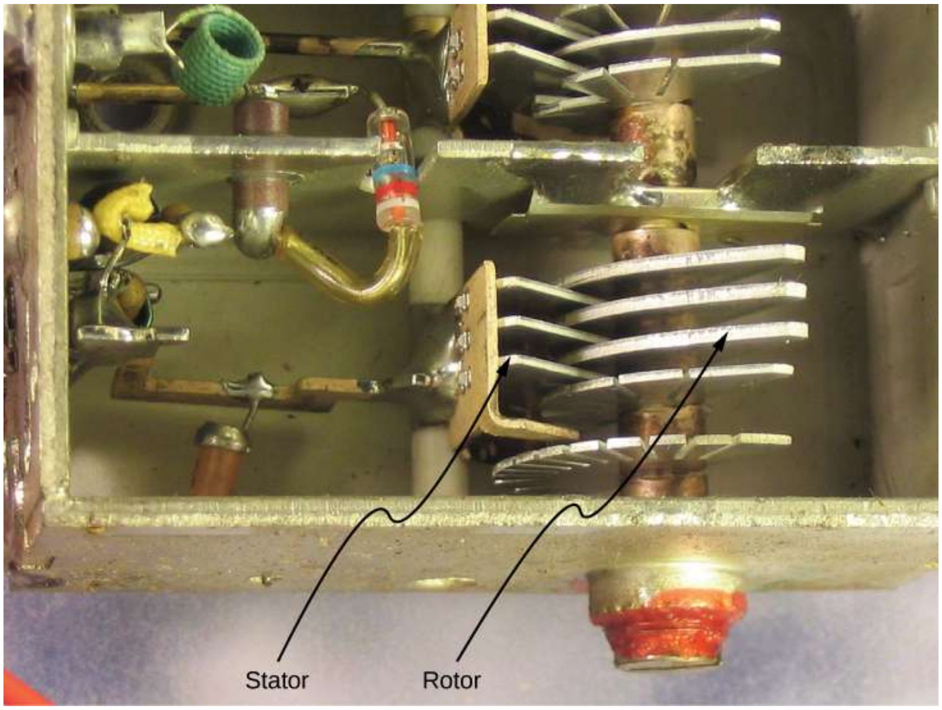


Figure 8.8 In a variable air capacitor, capacitance can be tuned by changing the effective area of the plates. (credit: modification of work by Robbie Sproule)

The symbols shown in Figure 8.9 are circuit representations of various types of capacitors. We generally use the symbol shown in Figure 8.9(a). The symbol in Figure 8.9(c) represents a variable-capacitance capacitor. Notice the similarity of these symbols to the symmetry of a parallel-plate capacitor. An electrolytic capacitor is represented by the symbol in part Figure 8.9(b), where the curved plate indicates the negative terminal.

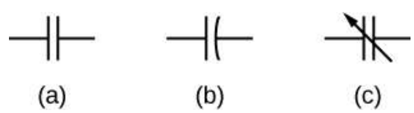


Figure 8.9 This shows three different circuit representations of capacitors. The symbol in (a) is the most commonly used one. The symbol in (b) represents an electrolytic capacitor. The symbol in (c) represents a variable-capacitance capacitor.

An interesting applied example of a capacitor model comes from cell biology and deals with the electrical potential in the plasma membrane of a living cell (Figure 8.10). Cell membranes separate cells from their surroundings but allow some selected ions to pass in or out of the cell. The potential difference across a membrane is about 70 mV. The cell membrane may be 7 to 10 nm thick. Treating the cell membrane as a nano-sized capacitor, the estimate of the smallest electrical field strength across its ‘plates’ yields the value $E = \frac{V}{d} = \frac{70 \times 10^{-3}\,\text{V}}{10 \times 10^{-9}\,\text{m}} = 7 \times 10^{6}\ \text{V/m} > 3\,\text{MV/m}.$

This magnitude of electrical field is great enough to create an electrical spark in the air.

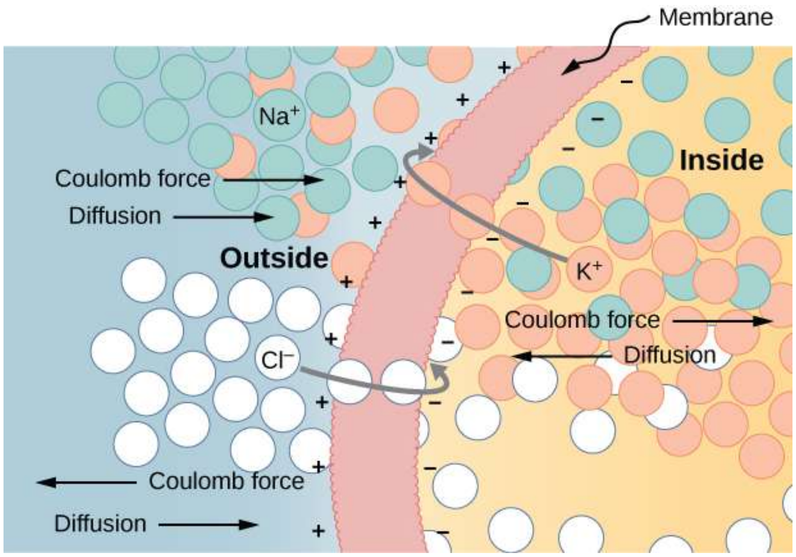


Figure 8.10 The semipermeable membrane of a biological cell has different concentrations of ions on its interior surface than on its exterior. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. In this way, the exterior of the membrane acquires a positive charge and its interior surface acquires a negative charge, creating a potential difference across the membrane. The membrane is normally impermeable to Na+ (sodium ions).

INTERACTIVE

Visit the PhET Explorations: Capacitor Lab (https://openstax.org/l/21phetcapacitor) to explore how a capacitor works. Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electrical field in the capacitor. Measure the voltage and the electrical field.

8.2 Capacitors in Series and in Parallel

Learning Objectives

By the end of this section, you will be able to:

- Explain how to determine the equivalent capacitance of capacitors in series and in parallel combinations
- Compute the potential difference across the plates and the charge on the plates for a capacitor in a network and determine the net capacitance of a network of capacitors

Several capacitors can be connected together to be used in a variety of applications. Multiple connections of capacitors behave as a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. Capacitors can be arranged in two simple and common types of connections, known as *series* and *parallel*, for which we can easily calculate the total capacitance. These two basic combinations, series and parallel, can also be used as part of more complex connections.

The Series Combination of Capacitors

Figure 8.11 illustrates a series combination of three capacitors, arranged in a row within the circuit. As for any capacitor, the capacitance of the combination is related to the charge and voltage by using Equation 8.1. When this series combination is connected to a battery with voltage V, each of the capacitors acquires an identical charge Q. To explain, first note that the charge on the plate connected to the positive terminal of the battery is $+Q$ and the charge on the plate connected to the negative terminal is $-Q$. Charges are then induced on the other plates so that the sum of the charges on all plates, and the sum of charges on any pair of capacitor plates, is zero. However, the potential drop $V_1 = Q/C_1$ on one capacitor may be different from the potential drop $V_2 = Q/C_2$ on another capacitor, because, generally, the capacitors may have different capacitances. The series combination of two or three capacitors resembles a single capacitor with a smaller capacitance.

Generally, any number of capacitors connected in series is equivalent to one capacitor whose capacitance (called the *equivalent capacitance*) is smaller than the smallest of the capacitances in the series combination. Charge on this equivalent capacitor is the same as the charge on any capacitor in a series combination: That is, *all capacitors of a series combination have the same charge*. This occurs due to the conservation of charge in the circuit. When a charge Q in a series circuit is removed from a plate of the first capacitor (which we denote as $-Q$), it must be placed on a plate of the second capacitor (which we denote as $+Q$), and so on.

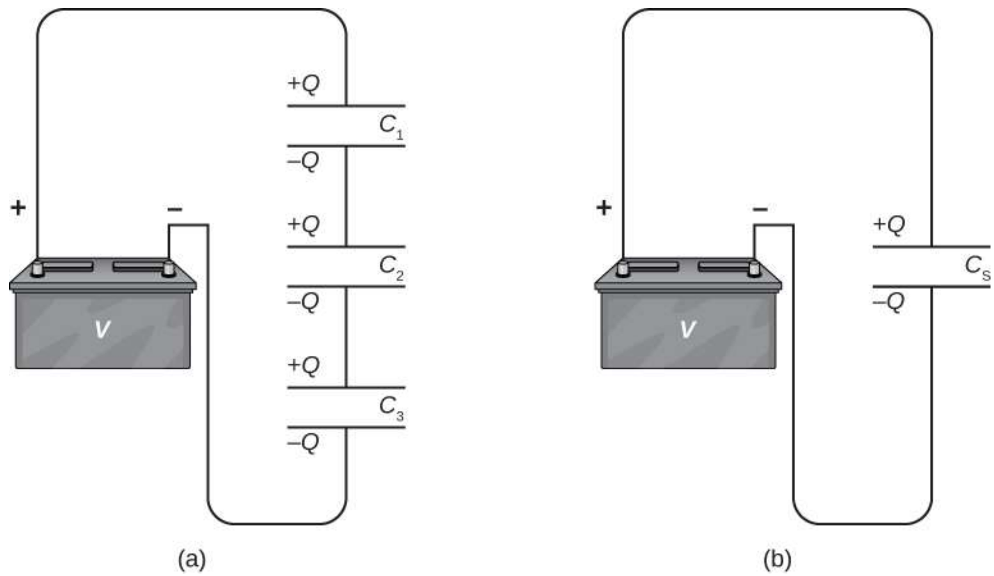


Figure 8.11 (a) Three capacitors are connected in series. The magnitude of the charge on each plate is Q. (b) The network of capacitors in (a) is equivalent to one capacitor that has a smaller capacitance than any of the individual capacitances in (a), and the charge on its plates is Q.

We can find an expression for the total (equivalent) capacitance by considering the voltages across the individual capacitors. The potentials across capacitors 1, 2, and 3 are, respectively, $V_1 = Q/C_1$, $V_2 = Q/C_2$, and $V_3 = Q/C_3$. These potentials must sum up to the voltage of the battery, giving the following potential balance:

$$V = V_1 + V_2 + V_3.$$

Potential V is measured across an equivalent capacitor that holds charge Q and has an equivalent capacitance C_S. Entering the expressions for V_1, V_2, and V_3, we get

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Canceling the charge Q, we obtain an expression containing the equivalent capacitance, C_S, of three capacitors connected in series:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

This expression can be generalized to any number of capacitors in a series network.

Series Combination

For capacitors connected in a **series combination**, the reciprocal of the equivalent capacitance is the sum of reciprocals of individual capacitances:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \quad (8.7)$$

EXAMPLE 8.4

Equivalent Capacitance of a Series Network

Find the total capacitance for three capacitors connected in series, given their individual capacitances are $1.000\ \mu\text{F}$, $5.000\ \mu\text{F}$, and $8.000\ \mu\text{F}$.

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using Equation 8.7 with three terms.

Solution

We enter the given capacitances into Equation 8.7:

$$\begin{aligned}\frac{1}{C_S} &= \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\\ &= \frac{1}{1.000\ \mu\text{F}}+\frac{1}{5.000\ \mu\text{F}}+\frac{1}{8.000\ \mu\text{F}}\\ \frac{1}{C_S} &= \frac{1.325}{\mu\text{F}}.\end{aligned}$$

Now we invert this result and obtain $C_S = \frac{\mu\text{F}}{1.325} = 0.755\ \mu\text{F}$.

Significance

Note that in a series network of capacitors, the equivalent capacitance is always less than the smallest individual capacitance in the network.

The Parallel Combination of Capacitors

A parallel combination of three capacitors, with one plate of each capacitor connected to one side of the circuit and the other plate connected to the other side, is illustrated in Figure 8.12(a). Since the capacitors are connected in parallel, *they all have the same voltage V across their plates*. However, each capacitor in the parallel network may store a different charge. To find the equivalent capacitance C_P of the parallel network, we note that the total charge Q stored by the network is the sum of all the individual charges:

$$Q = Q_1 + Q_2 + Q_3.$$

On the left-hand side of this equation, we use the relation $Q = C_P V$, which holds for the entire network. On the right-hand side of the equation, we use the relations $Q_1 = C_1 V, Q_2 = C_2 V$, and $Q_3 = C_3 V$ for the three capacitors in the network. In this way we obtain

$$C_P V = C_1 V + C_2 V + C_3 V.$$

This equation, when simplified, is the expression for the equivalent capacitance of the parallel network of three capacitors:

$$C_P = C_1 + C_2 + C_3.$$

This expression is easily generalized to any number of capacitors connected in parallel in the network.

Parallel Combination

For capacitors connected in a **parallel combination**, the equivalent (net) capacitance is the sum of all individual capacitances in the network,

$$C_P = C_1 + C_2 + C_3 + \cdots. \tag{8.8}$$

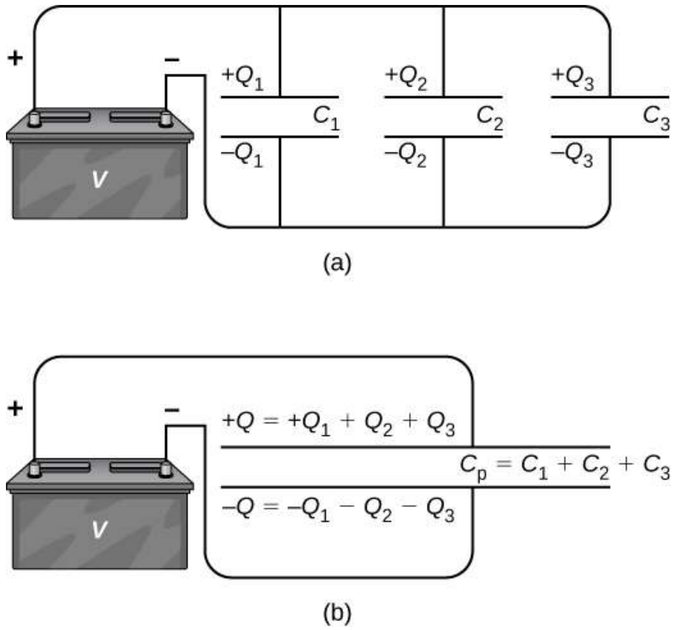


Figure 8.12 (a) Three capacitors are connected in parallel. Each capacitor is connected directly to the battery. (b) The charge on the equivalent capacitor is the sum of the charges on the individual capacitors.

EXAMPLE 8.5

Equivalent Capacitance of a Parallel Network

Find the net capacitance for three capacitors connected in parallel, given their individual capacitances are $1.0\ \mu\text{F}, 5.0\ \mu\text{F},$ and $8.0\ \mu\text{F}.$

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using Equation 8.8 with three terms.

Solution

Entering the given capacitances into Equation 8.8 yields

$$\begin{aligned} C_\text{P} &= C_1 + C_2 + C_3 = 1.0\ \mu\text{F} + 5.0\ \mu\text{F} + 8.0\ \mu\text{F} \\ C_\text{P} &= 14.0\ \mu\text{F}. \end{aligned}$$

Significance

Note that in a parallel network of capacitors, the equivalent capacitance is always larger than any of the individual capacitances in the network.

Capacitor networks are usually some combination of series and parallel connections, as shown in Figure 8.13. To find the net capacitance of such combinations, we identify parts that contain only series or only parallel connections, and find their equivalent capacitances. We repeat this process until we can determine the equivalent capacitance of the entire network. The following example illustrates this process.

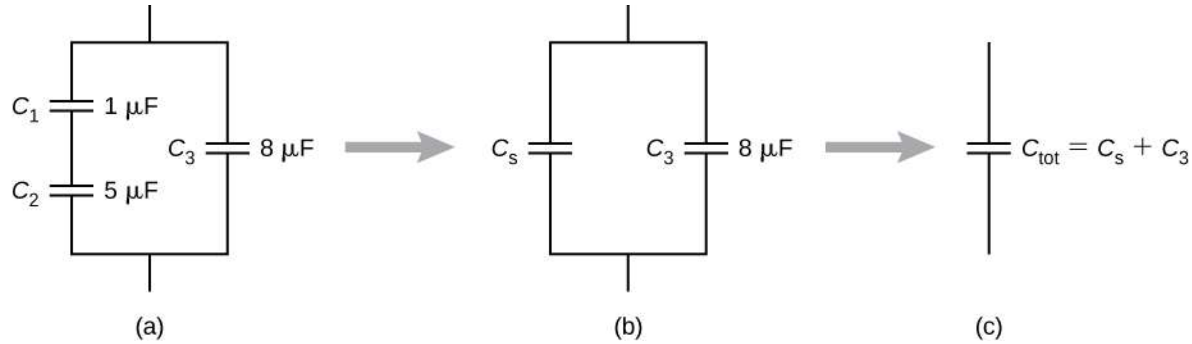


Figure 8.13 (a) This circuit contains both series and parallel connections of capacitors. (b) C_1 and C_2 are in series; their equivalent capacitance is C_S. (c) The equivalent capacitance C_S is connected in parallel with C_3. Thus, the equivalent capacitance of the entire network is the sum of C_S and C_3.

EXAMPLE 8.6

Equivalent Capacitance of a Network

Find the total capacitance of the combination of capacitors shown in Figure 8.13. Assume the capacitances are known to three decimal places ($C_1 = 1.000\ \mu\text{F}, C_2 = 5.000\ \mu\text{F}, C_3 = 8.000\ \mu\text{F}$). Round your answer to three decimal places.

Strategy

We first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S, is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their equivalent capacitance C_S is obtained with Equation 8.7:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000\ \mu\text{F}} + \frac{1}{5.000\ \mu\text{F}} = \frac{1.200}{\mu\text{F}} \Rightarrow C_S = 0.833\ \mu\text{F}.$$

Capacitance C_S is connected in parallel with the third capacitance C_3, so we use Equation 8.8 to find the equivalent capacitance C of the entire network:

$$C = C_S + C_3 = 0.833\ \mu\text{F} + 8.000\ \mu\text{F} = 8.833\ \mu\text{F}.$$

EXAMPLE 8.7

Network of Capacitors

Determine the net capacitance C of the capacitor combination shown in Figure 8.14 when the capacitances are $C_1 = 12.0\ \mu\text{F}, C_2 = 2.0\ \mu\text{F}$, and $C_3 = 4.0\ \mu\text{F}$. When a 12.0-V potential difference is maintained across the combination, find the charge and the voltage across each capacitor.

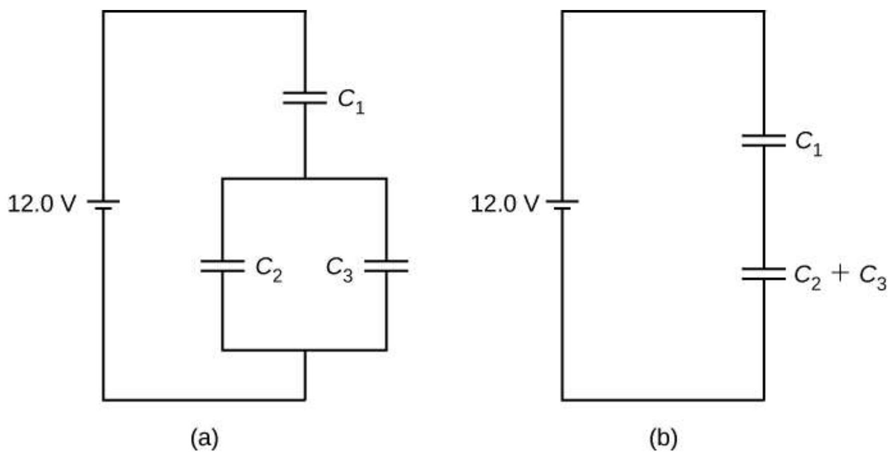


Figure 8.14 (a) A capacitor combination. (b) An equivalent two-capacitor combination.

Strategy

We first compute the net capacitance C_{23} of the parallel connection C_2 and C_3. Then C is the net capacitance of the series connection C_1 and C_{23}. We use the relation $C = Q/V$ to find the charges Q_1, Q_2, and Q_3, and the voltages V_1, V_2, and V_3, across capacitors 1, 2, and 3, respectively.

Solution

The equivalent capacitance for C_2 and C_3 is

$$C_{23} = C_2 + C_3 = 2.0\ \mu\text{F} + 4.0\ \mu\text{F} = 6.0\ \mu\text{F}.$$

The entire three-capacitor combination is equivalent to two capacitors in series,

$$\frac{1}{C} = \frac{1}{12.0\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}} = \frac{1}{4.0\ \mu\text{F}} \Rightarrow C = 4.0\ \mu\text{F}.$$

Consider the equivalent two-capacitor combination in Figure 8.14(b). Since the capacitors are in series, they have the same charge, $Q_1 = Q_{23}$. Also, the capacitors share the 12.0-V potential difference, so

$$12.0\ \text{V} = V_1 + V_{23} = \frac{Q_1}{C_1} + \frac{Q_{23}}{C_{23}} = \frac{Q_1}{12.0\ \mu\text{F}} + \frac{Q_1}{6.0\ \mu\text{F}} \Rightarrow Q_1 = 48.0\ \mu\text{C}.$$

Now the potential difference across capacitor 1 is

$$V_1 = \frac{Q_1}{C_1} = \frac{48.0\ \mu\text{C}}{12.0\ \mu\text{F}} = 4.0\ \text{V}.$$

Because capacitors 2 and 3 are connected in parallel, they are at the same potential difference:

$$V_2 = V_3 = 12.0\ \text{V} - 4.0\ \text{V} = 8.0\ \text{V}.$$

Hence, the charges on these two capacitors are, respectively,

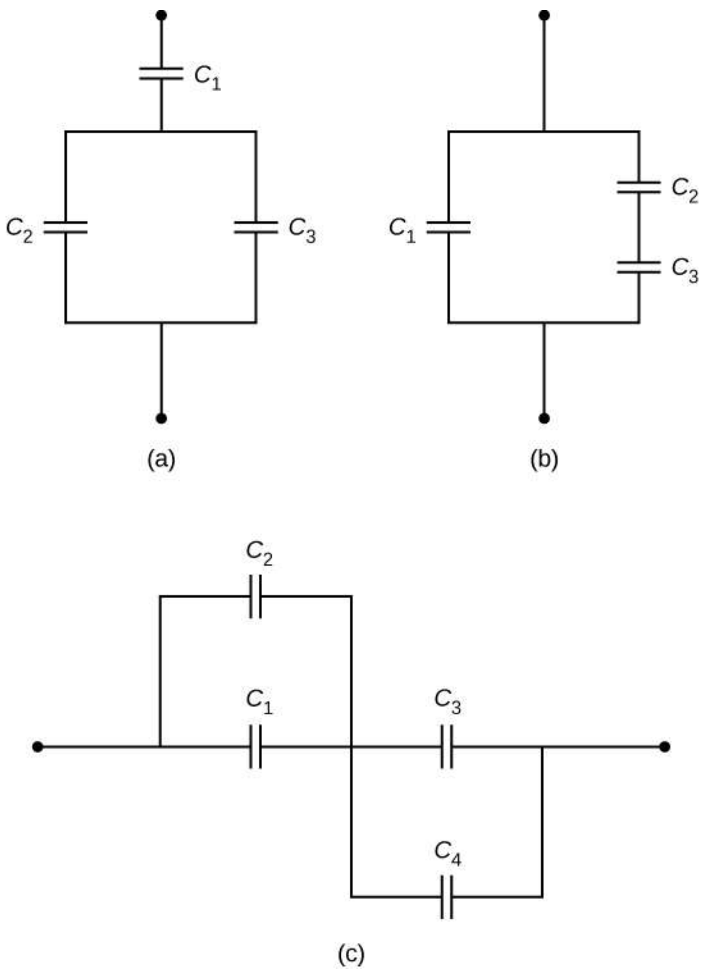
$$Q_2 = C_2 V_2 = (2.0\ \mu\text{F})(8.0\ \text{V}) = 16.0\ \mu\text{C},$$
$$Q_3 = C_3 V_3 = (4.0\ \mu\text{F})(8.0\ \text{V}) = 32.0\ \mu\text{C}.$$

Significance

As expected, the net charge on the parallel combination of C_2 and C_3 is $Q_{23} = Q_2 + Q_3 = 48.0\ \mu\text{C}$.

CHECK YOUR UNDERSTANDING 8.5

Determine the net capacitance C of each network of capacitors shown below. Assume that $C_1 = 1.0$ pF, $C_2 = 2.0$ pF, $C_3 = 4.0$ pF, and $C_4 = 5.0$ pF. Find the charge on each capacitor, assuming there is a potential difference of 12.0 V across each network.



8.3 Energy Stored in a Capacitor

Learning Objectives

By the end of this section, you will be able to:

- Explain how energy is stored in a capacitor
- Use energy relations to determine the energy stored in a capacitor network

Most of us have seen dramatizations of medical personnel using a defibrillator to pass an electrical current through a patient's heart to get it to beat normally. Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics to supply energy when batteries are charged (Figure 8.15). Capacitors are also used to supply energy for flash lamps on cameras.

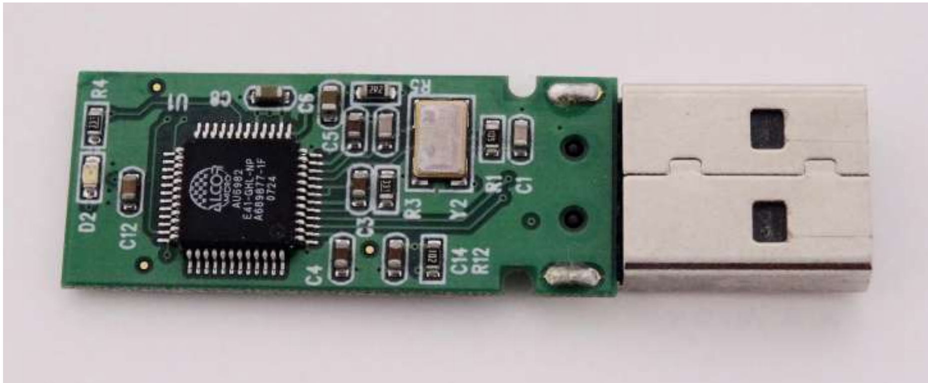


Figure 8.15 The capacitors on the circuit board for an electronic device follow a labeling convention that identifies each one with a code that begins with the letter "C." (credit: Windell Oskay)

The energy U_C stored in a capacitor is electrostatic potential energy and is thus related to the charge Q and voltage V between the capacitor plates. A charged capacitor stores energy in the electrical field between its plates. As the capacitor is being charged, the electrical field builds up. When a charged capacitor is disconnected from a battery, its energy remains in the field in the space between its plates.

To gain insight into how this energy may be expressed (in terms of Q and V), consider a charged, empty, parallel-plate capacitor; that is, a capacitor without a dielectric but with a vacuum between its plates. The space between its plates has a volume Ad, and it is filled with a uniform electrostatic field E. The total energy U_C of the capacitor is contained within this space. The **energy density** u_E in this space is simply U_C divided by the volume Ad. If we know the energy density, the energy can be found as $U_C = u_E(Ad)$. We will learn in Electromagnetic Waves (after completing the study of Maxwell's equations) that the energy density u_E in a region of free space occupied by an electrical field E depends only on the magnitude of the field and is

$$u_E = \frac{1}{2}\varepsilon_0 E^2. \tag{8.9}$$

If we multiply the energy density by the volume between the plates, we obtain the amount of energy stored between the plates of a parallel-plate capacitor:$U_C = u_E(Ad) = \frac{1}{2}\varepsilon_0 E^2 Ad = \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2} Ad = \frac{1}{2}V^2 \varepsilon_0 \frac{A}{d} = \frac{1}{2}V^2 C.$

In this derivation, we used the fact that the electrical field between the plates is uniform so that $E = V/d$ and $C = \varepsilon_0 A/d$. Because $C = Q/V$, we can express this result in other equivalent forms:

$$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV. \tag{8.10}$$

The expression in Equation 8.10 for the energy stored in a parallel-plate capacitor is generally valid for all types of capacitors. To see this, consider any uncharged capacitor (not necessarily a parallel-plate type). At some instant, we connect it across a battery, giving it a potential difference $V = q/C$ between its plates. Initially, the charge on the plates is $Q = 0$. As the capacitor is being charged, the charge gradually builds up on its plates, and after some time, it reaches the value Q. To move an infinitesimal charge dq from the negative plate to the positive plate (from a lower to a higher potential), the amount of work dW that must be done on dq is $dW = V dq = \frac{q}{C}dq$.

This work becomes the energy stored in the electrical field of the capacitor. In order to charge the capacitor to a charge Q, the total work required is

$$W = \int_0^{W(Q)} dW = \int_0^{Q} \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C}.$$

Since the geometry of the capacitor has not been specified, this equation holds for any type of capacitor. The total work W needed to charge a capacitor is the electrical potential energy U_C stored in it, or $U_C = W$. When the charge is expressed in coulombs, potential is expressed in volts, and the capacitance is expressed in farads, this relation gives the energy in joules.

Knowing that the energy stored in a capacitor is $U_C = Q^2/(2C)$, we can now find the energy density u_E stored in a vacuum between the plates of a charged parallel-plate capacitor. We just have to divide U_C by the volume Ad of space between its plates and take into account that for a parallel-plate capacitor, we have $E = \sigma/\varepsilon_0$ and $C = \varepsilon_0 A/d$. Therefore, we obtain

$$u_E = \frac{U_C}{Ad} = \frac{1}{2}\frac{Q^2}{C}\frac{1}{Ad} = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A/d}\frac{1}{Ad} = \frac{1}{2}\frac{1}{\varepsilon_0}\left(\frac{Q}{A}\right)^2 = \frac{\sigma^2}{2\varepsilon_0} = \frac{(E\varepsilon_0)^2}{2\varepsilon_0} = \frac{\varepsilon_0}{2}E^2.$$

We see that this expression for the density of energy stored in a parallel-plate capacitor is in accordance with the general relation expressed in Equation 8.9. We could repeat this calculation for either a spherical capacitor or a cylindrical capacitor—or other capacitors—and in all cases, we would end up with the general relation given by Equation 8.9.

EXAMPLE 8.8

Energy Stored in a Capacitor

Calculate the energy stored in the capacitor network in Figure 8.14(a) when the capacitors are fully charged and when the capacitances are $C_1 = 12.0\ \mu\text{F}$, $C_2 = 2.0\ \mu\text{F}$, and $C_3 = 4.0\ \mu\text{F}$, respectively.

Strategy

We use Equation 8.10 to find the energy U_1, U_2, and U_3 stored in capacitors 1, 2, and 3, respectively. The total energy is the sum of all these energies.

Solution

We identify $C_1 = 12.0\ \mu\text{F}$ and $V_1 = 4.0\ \text{V}$, $C_2 = 2.0\ \mu\text{F}$ and $V_2 = 8.0\ \text{V}$, $C_3 = 4.0\ \mu\text{F}$ and $V_3 = 8.0\ \text{V}$. The energies stored in these capacitors are

$$\begin{aligned} U_1 &= \tfrac{1}{2}C_1V_1^2 = \tfrac{1}{2}(12.0\ \mu\text{F})(4.0\ \text{V})^2 = 96\ \mu\text{J}, \\ U_2 &= \tfrac{1}{2}C_2V_2^2 = \tfrac{1}{2}(2.0\ \mu\text{F})(8.0\ \text{V})^2 = 64\ \mu\text{J}, \\ U_3 &= \tfrac{1}{2}C_3V_3^2 = \tfrac{1}{2}(4.0\ \mu\text{F})(8.0\ \text{V})^2 = 130\ \mu\text{J}. \end{aligned}$$

The total energy stored in this network is

$$U_C = U_1 + U_2 + U_3 = 96\ \mu\text{J} + 64\ \mu\text{J} + 130\ \mu\text{J} = 0.29\ \text{mJ}.$$

Significance

We can verify this result by calculating the energy stored in the single 4.0-μF capacitor, which is found to be equivalent to the entire network. The voltage across the network is 12.0 V. The total energy obtained in this way agrees with our previously obtained result, $U_C = \frac{1}{2}CV^2 = \frac{1}{2}(4.0\ \mu\text{F})(12.0\ \text{V})^2 = 0.29\ \text{mJ}$.

CHECK YOUR UNDERSTANDING 8.6

The potential difference across a 5.0-pF capacitor is 0.40 V. (a) What is the energy stored in this capacitor? (b) The potential difference is now increased to 1.20 V. By what factor is the stored energy increased?

In a cardiac emergency, a portable electronic device known as an automated external defibrillator (AED) can be a lifesaver. A **defibrillator** (Figure 8.16) delivers a large charge in a short burst, or a shock, to a person's heart to correct abnormal heart rhythm (an arrhythmia). A heart attack can arise from the onset of fast, irregular beating of the heart—called cardiac or ventricular fibrillation. Applying a large shock of electrical energy can terminate the arrhythmia and allow the body's natural pacemaker to resume its normal rhythm. Today, it is common for ambulances to carry AEDs. AEDs are also found in many public places. These are designed to be used by lay persons. The device automatically diagnoses the patient's heart rhythm and then applies the shock with appropriate energy and waveform. CPR (cardiopulmonary resuscitation) is recommended in many cases before using a defibrillator.

Figure 8.16 Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack. (credit: Owain Davies)

EXAMPLE 8.9

Capacitance of a Heart Defibrillator

A heart defibrillator delivers 4.00×10^2 J of energy by discharging a capacitor initially at 1.00×10^4 V. What is its capacitance?

Strategy

We are given U_C and V, and we are asked to find the capacitance C. We solve Equation 8.10 for C and substitute.

Solution

Solving this expression for C and entering the given values yields $C = 2\frac{U_C}{V^2} = 2\frac{4.00 \times 10^2 \text{ J}}{(1.00 \times 10^4 \text{ V})^2} = 8.00\ \mu\text{F}$.

8.4 Capacitor with a Dielectric

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects a dielectric in a capacitor has on capacitance and other properties
- Calculate the capacitance of a capacitor containing a dielectric

As we discussed earlier, an insulating material placed between the plates of a capacitor is called a dielectric. Inserting a dielectric between the plates of a capacitor affects its capacitance. To see why, let's consider an experiment described in Figure 8.17. Initially, a capacitor with capacitance C_0 when there is air between its plates is charged by a battery to voltage V_0. When the capacitor is fully charged, the battery is disconnected. A charge Q_0 then resides on the plates, and the potential difference between the plates is measured to be V_0. Now, suppose we insert a dielectric that *totally* fills the gap between the plates. If we monitor the voltage, we find that the voltmeter reading has dropped to a *smaller* value V. We write this new voltage value as a fraction of the original voltage V_0, with a positive number κ, $\kappa > 1$:

$$V = \frac{1}{\kappa} V_0.$$

The constant κ in this equation is called the **dielectric constant** of the material between the plates, and its value is characteristic for the material. A detailed explanation for why the dielectric reduces the voltage is given in the next section. Different materials have different dielectric constants (a table of values for typical materials is provided in the next section). Once the battery becomes disconnected, there is no path for a charge to flow to the battery from the capacitor plates. Hence, the insertion of the dielectric has no effect on the charge on the plate, which remains at a value of Q_0. Therefore, we find that the capacitance of the capacitor with a dielectric is

$$C = \frac{Q_0}{V} = \frac{Q_0}{V_0/\kappa} = \kappa \frac{Q_0}{V_0} = \kappa C_0. \tag{8.11}$$

This equation tells us that the *capacitance* C_0 *of an empty (vacuum) capacitor can be increased by a factor of* κ *when we insert a dielectric material to completely fill the space between its plates*. Note that Equation 8.11 can also be used for an empty capacitor by setting $\kappa = 1$. In other words, we can say that the dielectric constant of the vacuum is 1, which is a reference value.

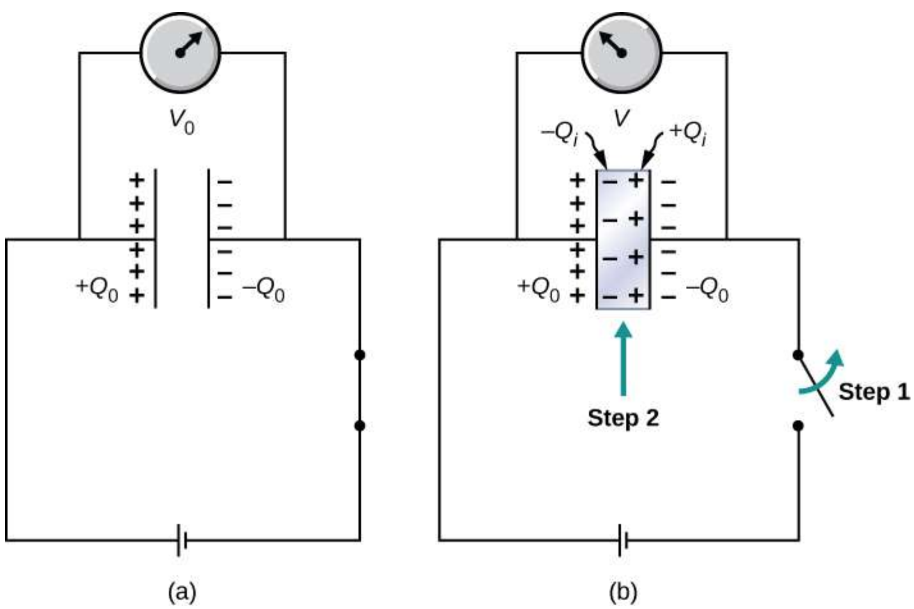


Figure 8.17 (a) When fully charged, a vacuum capacitor has a voltage V_0 and charge Q_0 (the charges remain on plate's inner surfaces; the schematic indicates the sign of charge on each plate). (b) In step 1, the battery is disconnected. Then, in step 2, a dielectric (that is electrically neutral) is inserted into the charged capacitor. When the voltage across the capacitor is now measured, it is found that the voltage value has decreased to $V = V_0/\kappa$. The schematic indicates the sign of the induced charge that is now present on the surfaces of the dielectric material between the plates.

The principle expressed by Equation 8.11 is widely used in the construction industry (Figure 8.18). Metal plates in an electronic stud finder act effectively as a capacitor. You place a stud finder with its flat side on the wall and move it continually in the horizontal direction. When the finder moves over a wooden stud, the capacitance of its plates changes, because wood has a different dielectric constant than a gypsum wall. This change triggers a signal in a circuit, and thus the stud is detected.

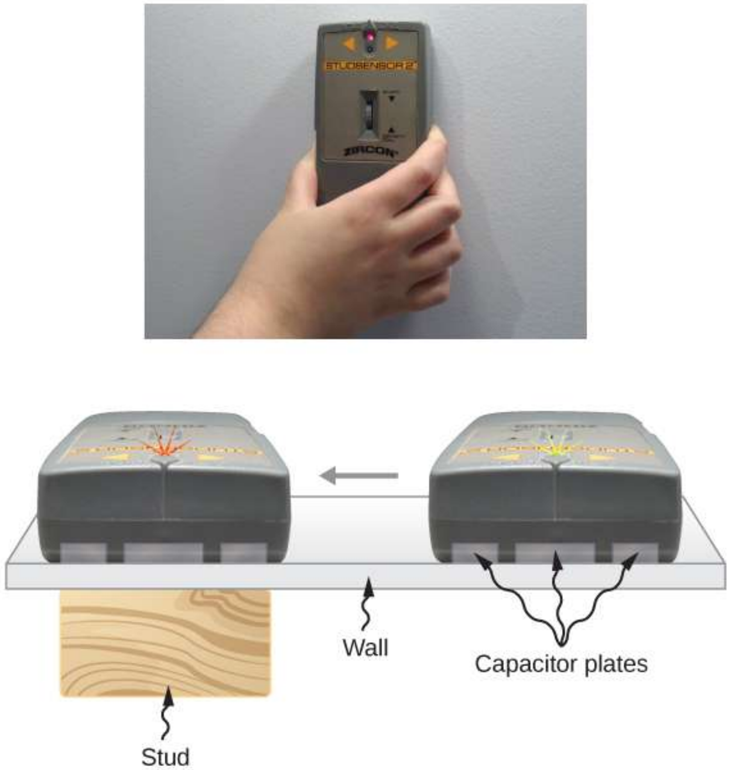


Figure 8.18 An electronic stud finder is used to detect wooden studs behind drywall. (credit top: modification of work by Jane Whitney)

The electrical energy stored by a capacitor is also affected by the presence of a dielectric. When the energy stored in an empty capacitor is U_0, the energy U stored in a capacitor with a dielectric is smaller by a factor of κ,

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{\kappa C_0} = \frac{1}{\kappa}U_0. \tag{8.12}$$

As a dielectric material sample is brought near an empty charged capacitor, the sample reacts to the electrical field of the charges on the capacitor plates. Just as we learned in Electric Charges and Fields on electrostatics, there will be the induced charges on the surface of the sample; however, they are not free charges like in a conductor, because a perfect insulator does not have freely moving charges. These induced charges on the dielectric surface are of an opposite sign to the free charges on the plates of the capacitor, and so they are attracted by the free charges on the plates. Consequently, the dielectric is "pulled" into the gap, and the work to polarize the dielectric material between the plates is done at the expense of the stored electrical energy, which is reduced, in accordance with Equation 8.12.

EXAMPLE 8.10

Inserting a Dielectric into an Isolated Capacitor

An empty 20.0-pF capacitor is charged to a potential difference of 40.0 V. The charging battery is then disconnected, and a piece of Teflon™ with a dielectric constant of 2.1 is inserted to completely fill the space between the capacitor plates (see Figure 8.17). What are the values of (a) the capacitance, (b) the charge of the plate, (c) the potential difference between the plates, and (d) the energy stored in the capacitor with and without dielectric?

Strategy

We identify the original capacitance $C_0 = 20.0\ \text{pF}$ and the original potential difference $V_0 = 40.0\ \text{V}$ between the plates. We combine Equation 8.11 with other relations involving capacitance and substitute.

Solution

a. The capacitance increases to
$$C = \kappa C_0 = 2.1(20.0\ \text{pF}) = 42.0\ \text{pF}.$$

b. Without dielectric, the charge on the plates is
$$Q_0 = C_0 V_0 = (20.0\ \text{pF})(40.0\ \text{V}) = 0.8\ \text{nC}.$$
Since the battery is disconnected before the dielectric is inserted, the plate charge is unaffected by the dielectric and remains at 0.8 nC.

c. With the dielectric, the potential difference becomes
$$V = \frac{1}{\kappa} V_0 = \frac{1}{2.1} 40.0\ \text{V} = 19.0\ \text{V}.$$

d. The stored energy without the dielectric is
$$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2}(20.0\ \text{pF})(40.0\ \text{V})^2 = 16.0\ \text{nJ}.$$
With the dielectric inserted, we use Equation 8.12 to find that the stored energy decreases to
$$U = \frac{1}{\kappa} U_0 = \frac{1}{2.1} 16.0\ \text{nJ} = 7.6\ \text{nJ}.$$

Significance

Notice that the effect of a dielectric on the capacitance of a capacitor is a drastic increase of its capacitance. This effect is far more profound than a mere change in the geometry of a capacitor.

CHECK YOUR UNDERSTANDING 8.7

When a dielectric is inserted into an isolated and charged capacitor, the stored energy decreases to 33% of its original value. (a) What is the dielectric constant? (b) How does the capacitance change?

8.5 Molecular Model of a Dielectric

Learning Objectives

By the end of this section, you will be able to:

- Explain the polarization of a dielectric in a uniform electrical field
- Describe the effect of a polarized dielectric on the electrical field between capacitor plates
- Explain dielectric breakdown

We can understand the effect of a dielectric on capacitance by looking at its behavior at the molecular level. As we have seen in earlier chapters, in general, all molecules can be classified as either *polar* or *nonpolar*. There is a net separation of positive and negative charges in an isolated polar molecule, whereas there is no charge separation in an isolated nonpolar molecule (Figure 8.19). In other words, polar molecules have permanent *electric-dipole moments* and nonpolar molecules do not. For example, a molecule of water is polar, and a molecule of oxygen is nonpolar. Nonpolar molecules can become polar in the presence of an external electrical field, which is called *induced polarization*.

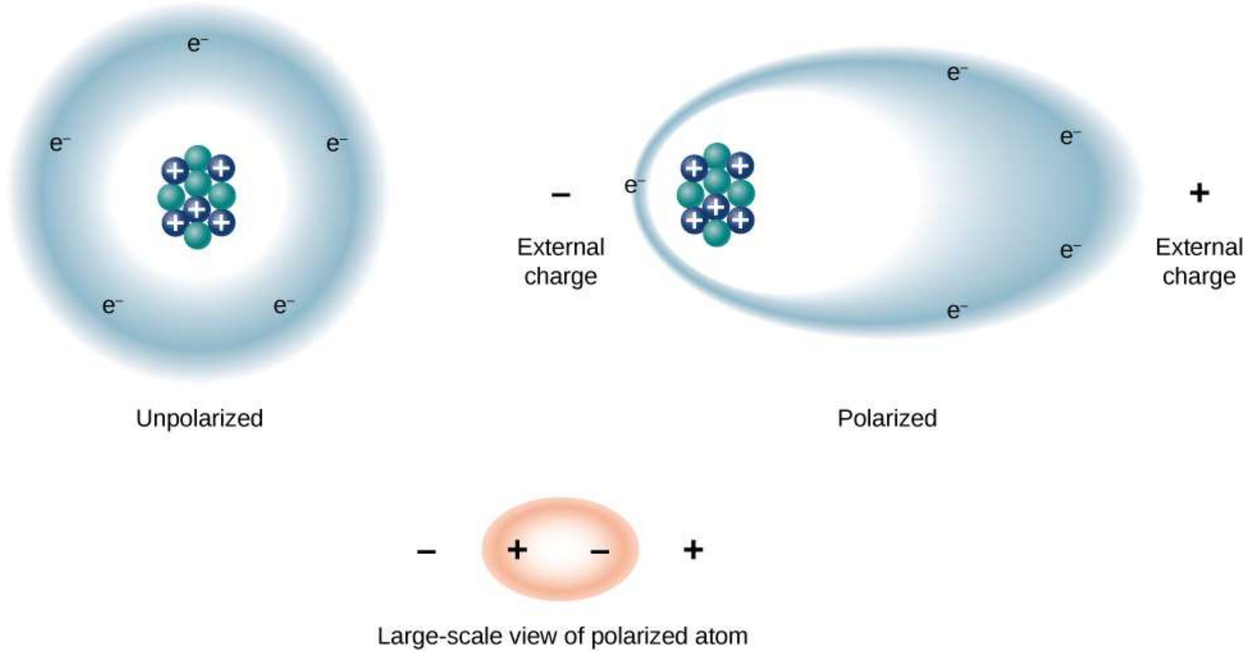


Figure 8.19 The concept of polarization: In an unpolarized atom or molecule, a negatively charged electron cloud is evenly distributed around positively charged centers, whereas a polarized atom or molecule has an excess of negative charge at one side so that the other side has an excess of positive charge. However, the entire system remains electrically neutral. The charge polarization may be caused by an external electrical field. Some molecules and atoms are permanently polarized (electric dipoles) even in the absence of an external electrical field (polar molecules and atoms).

Let's first consider a dielectric composed of polar molecules. In the absence of any external electrical field, the electric dipoles are oriented randomly, as illustrated in Figure 8.20(a). However, if the dielectric is placed in an external electrical field $\vec{\mathbf{E}}_0$, the polar molecules align with the external field, as shown in part (b) of the figure. Opposite charges on adjacent dipoles within the volume of dielectric neutralize each other, so there is no net charge within the dielectric (see the dashed circles in part (b)). However, this is not the case very close to the upper and lower surfaces that border the dielectric (the region enclosed by the dashed rectangles in part (b)), where the alignment does produce a net charge. Since the external electrical field merely aligns the dipoles, the dielectric as a whole is neutral, and the surface charges induced on its opposite faces are equal and opposite. These **induced surface charges** $+Q_i$ and $-Q_i$ produce an additional electrical field $\vec{\mathbf{E}}_{\mathrm{i}}$ (an **induced electrical field**), which *opposes* the external field $\vec{\mathbf{E}}_0$, as illustrated in part (c).

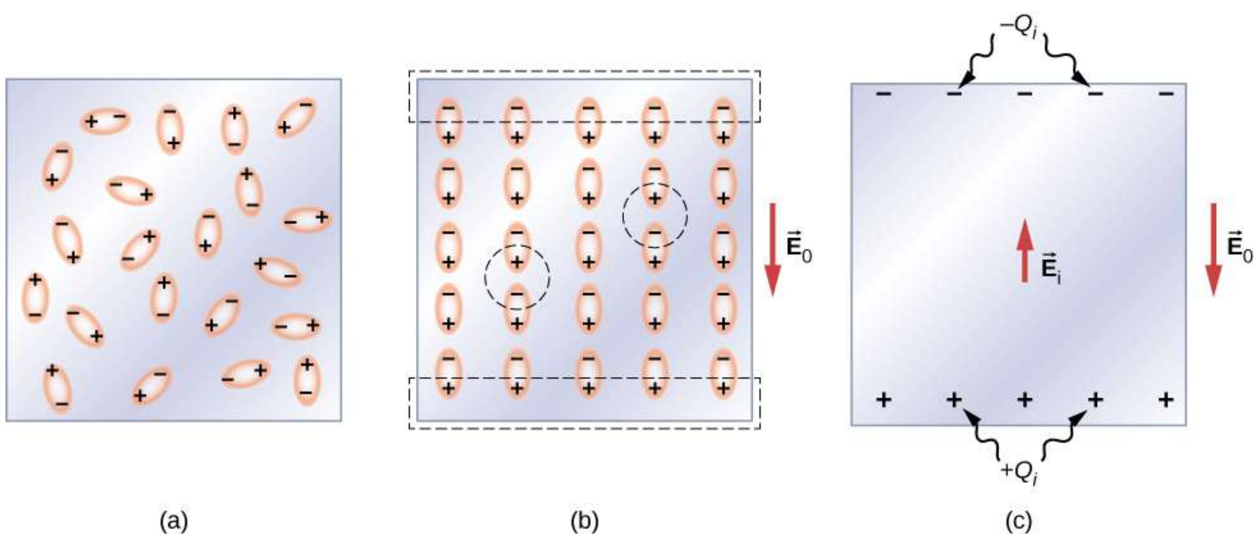


Figure 8.20 A dielectric with polar molecules: (a) In the absence of an external electrical field; (b) in the presence of an external electrical

field $\vec{\mathbf{E}}_0$. The dashed lines indicate the regions immediately adjacent to the capacitor plates. (c) The induced electrical field $\vec{\mathbf{E}}_\text{i}$ inside the dielectric produced by the induced surface charge Q_i of the dielectric. Note that, in reality, the individual molecules are not perfectly aligned with an external field because of thermal fluctuations; however, the *average* alignment is along the field lines as shown.

The same effect is produced when the molecules of a dielectric are nonpolar. In this case, a nonpolar molecule acquires an **induced electric-dipole moment** because the external field $\vec{\mathbf{E}}_0$ causes a separation between its positive and negative charges. The induced dipoles of the nonpolar molecules align with $\vec{\mathbf{E}}_0$ in the same way as the permanent dipoles of the polar molecules are aligned (shown in part (b)). Hence, the electrical field within the dielectric is weakened regardless of whether its molecules are polar or nonpolar.

Therefore, when the region between the parallel plates of a charged capacitor, such as that shown in Figure 8.21(a), is filled with a dielectric, within the dielectric there is an electrical field $\vec{\mathbf{E}}_0$ due to the *free charge* Q_0 on the capacitor plates and an electrical field $\vec{\mathbf{E}}_\text{i}$ due to the induced charge Q_i on the surfaces of the dielectric. Their vector sum gives the net electrical field $\vec{\mathbf{E}}$ within the dielectric between the capacitor plates (shown in part (b) of the figure):

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 + \vec{\mathbf{E}}_\text{i}. \quad (8.13)$$

This net field can be considered to be the field produced by an *effective charge* $Q_0 - Q_i$ on the capacitor.

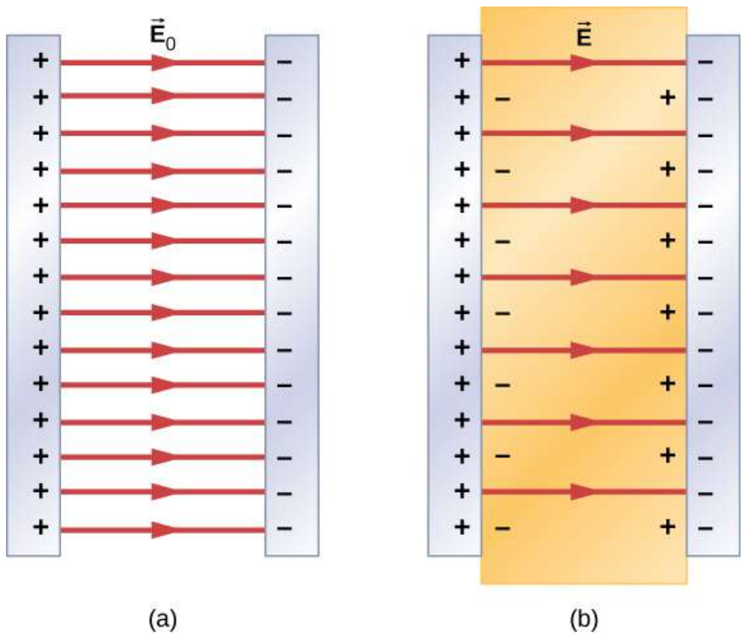


Figure 8.21 Electrical field: (a) In an empty capacitor, electrical field $\vec{\mathbf{E}}_0$. (b) In a dielectric-filled capacitor, electrical field $\vec{\mathbf{E}}$.

In most dielectrics, the net electrical field $\vec{\mathbf{E}}$ is proportional to the field $\vec{\mathbf{E}}_0$ produced by the free charge. In terms of these two electrical fields, the dielectric constant κ of the material is defined as

$$\kappa = \frac{E_0}{E}. \quad (8.14)$$

Since $\vec{\mathbf{E}}_0$ and $\vec{\mathbf{E}}_\text{i}$ point in opposite directions, the magnitude E is smaller than the magnitude E_0 and therefore $\kappa > 1$. Combining Equation 8.14 with Equation 8.13, and rearranging the terms, yields the following expression for the induced electrical field in a dielectric:

$$\vec{\mathbf{E}}_\text{i} = \left(\frac{1}{\kappa} - 1\right)\vec{\mathbf{E}}_0. \quad (8.15)$$

When the magnitude of an external electrical field becomes too large, the molecules of dielectric material start to become ionized. A molecule or an atom is ionized when one or more electrons are removed from it and become free electrons, no longer bound to the molecular or atomic structure. When this happens, the material can conduct, thereby allowing charge to move through the dielectric from one capacitor plate to the other. This phenomenon is called **dielectric breakdown**. (Figure 8.1 shows typical random-path patterns of electrical discharge during dielectric breakdown.) The critical value, E_c, of the electrical field at which the molecules of an insulator become ionized is called the **dielectric strength** of the material. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation in a capacitor. For example, the dielectric strength of air is $E_c = 3.0\,\text{MV/m}$, so for an air-filled capacitor with a plate separation of $d = 1.00\,\text{mm}$, the limit on the potential difference that can be safely applied across its plates without causing dielectric breakdown is $V = E_c \quad d = (3.0 \times 10^6\ \text{V/m})(1.00 \times 10^{-3}\ \text{m}) = 3.0\,\text{kV}$.

However, this limit becomes 60.0 kV when the same capacitor is filled with Teflon™, whose dielectric strength is about $60.0\,\text{MV/m}$. Because of this limit imposed by the dielectric strength, the amount of charge that an air-filled capacitor can store is only $Q_0 = \kappa_{\text{air}} C_0 (3.0\,\text{kV})$ and the charge stored on the same Teflon™-filled capacitor can be as much as

$$Q = \kappa_{\text{teflon}} C_0 (60.0\,\text{kV}) = \kappa_{\text{teflon}} \frac{Q_0}{\kappa_{\text{air}}(3.0\,\text{kV})}(60.0\,\text{kV}) = 20\frac{\kappa_{\text{teflon}}}{\kappa_{\text{air}}} Q_0 = 20\frac{2.1}{1.00059} Q_0 \cong 42 Q_0,$$

which is about 42 times greater than a charge stored on an air-filled capacitor. Typical values of dielectric constants and dielectric strengths for various materials are given in Table 8.1. Notice that the dielectric constant κ is exactly 1.0 for a vacuum (the empty space serves as a reference condition) and very close to 1.0 for air under normal conditions (normal pressure at room temperature). These two values are so close that, in fact, the properties of an air-filled capacitor are essentially the same as those of an empty capacitor.

Material	Dielectric constant κ	Dielectric strength $E_c\,[\times 10^6\,\text{V/m}]$
Vacuum	1	∞
Dry air (1 atm)	1.00059	3.0
Teflon™	2.1	60 to 173
Paraffin	2.3	11
Silicon oil	2.5	10 to 15
Polystyrene	2.56	19.7
Nylon	3.4	14
Paper	3.7	16
Fused quartz	3.78	8
Glass	4 to 6	9.8 to 13.8
Concrete	4.5	–
Bakelite	4.9	24
Diamond	5.5	2,000

Material	Dielectric constant κ	Dielectric strength $E_c[\times 10^6 \text{V/m}]$
Pyrex glass	5.6	14
Mica	6.0	118
Neoprene rubber	6.7	15.7 to 26.7
Water	80	–
Sulfuric acid	84 to 100	–
Titanium dioxide	86 to 173	–
Strontium titanate	310	8
Barium titanate	1,200 to 10,000	–
Calcium copper titanate	> 250,000	–

Table 8.1 Representative Values of Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Not all substances listed in the table are good insulators, despite their high dielectric constants. Water, for example, consists of polar molecules and has a large dielectric constant of about 80. In a water molecule, electrons are more likely found around the oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogens end slightly positive, which makes the molecule easy to align along an external electrical field, and thus water has a large dielectric constant. However, the polar nature of water molecules also makes water a good solvent for many substances, which produces undesirable effects, because any concentration of free ions in water conducts electricity.

EXAMPLE 8.11

Electrical Field and Induced Surface Charge

Suppose that the distance between the plates of the capacitor in Example 8.10 is 2.0 mm and the area of each plate is $4.5 \times 10^{-3}\ \text{m}^2$. Determine: (a) the electrical field between the plates before and after the Teflon™ is inserted, and (b) the surface charge induced on the Teflon™ surfaces.

Strategy

In part (a), we know that the voltage across the empty capacitor is $V_0 = 40\ \text{V}$, so to find the electrical fields we use the relation $V = Ed$ and Equation 8.14. In part (b), knowing the magnitude of the electrical field, we use the expression for the magnitude of electrical field near a charged plate $E = \sigma/\varepsilon_0$, where σ is a uniform surface charge density caused by the surface charge. We use the value of free charge $Q_0 = 8.0 \times 10^{-10}\ \text{C}$ obtained in Example 8.10.

Solution

a. The electrical field E_0 between the plates of an empty capacitor is

$$E_0 = \frac{V_0}{d} = \frac{40\ \text{V}}{2.0 \times 10^{-3}\ \text{m}} = 2.0 \times 10^4\ \text{V/m}.$$

The electrical field E with the Teflon™ in place is

$$E = \frac{1}{\kappa}E_0 = \frac{1}{2.1}2.0 \times 10^4 \text{ V/m} = 9.5 \times 10^3 \text{ V/m}.$$

b. The effective charge on the capacitor is the difference between the free charge Q_0 and the induced charge Q_i. The electrical field in the Teflon™ is caused by this effective charge. Thus

$$E = \frac{1}{\varepsilon_0}\sigma = \frac{1}{\varepsilon_0}\frac{Q_0 - Q_i}{A}.$$

We invert this equation to obtain Q_i, which yields

$$\begin{aligned} Q_i &= Q_0 - \varepsilon_0 AE \\ &= 8.0 \times 10^{-10}\text{C} - \left(8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(4.5 \times 10^{-3}\ \text{m}^2\right)\left(9.5 \times 10^3\ \frac{\text{V}}{\text{m}}\right) \\ &= 4.2 \times 10^{-10}\text{C} = 0.42\ \text{nC}. \end{aligned}$$

EXAMPLE 8.12

Inserting a Dielectric into a Capacitor Connected to a Battery

When a battery of voltage V_0 is connected across an empty capacitor of capacitance C_0, the charge on its plates is Q_0, and the electrical field between its plates is E_0. A dielectric of dielectric constant κ is inserted between the plates *while the battery remains in place*, as shown in Figure 8.22. (a) Find the capacitance C, the voltage V across the capacitor, and the electrical field E between the plates after the dielectric is inserted. (b) Obtain an expression for the free charge Q on the plates of the filled capacitor and the induced charge Q_i on the dielectric surface in terms of the original plate charge Q_0.

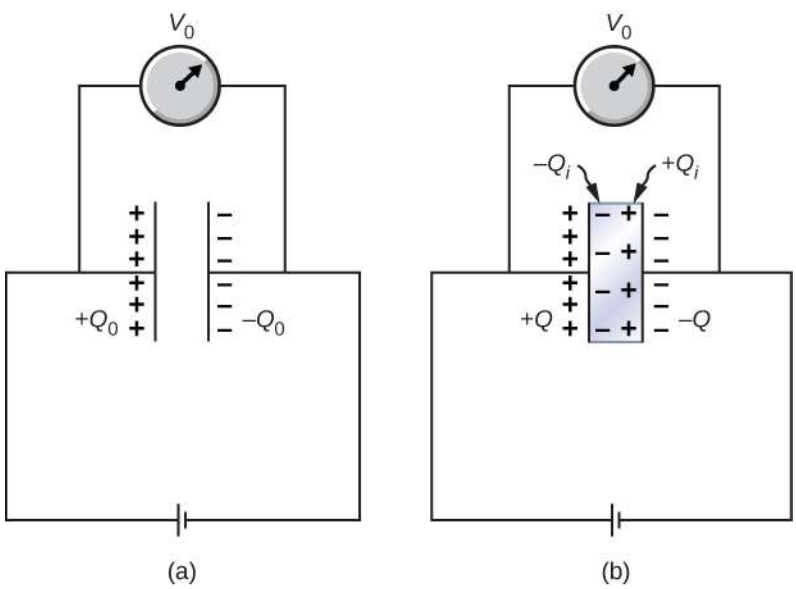


Figure 8.22 A dielectric is inserted into the charged capacitor while the capacitor remains connected to the battery.

Strategy

We identify the known values: V_0, C_0, E_0, κ, and Q_0. Our task is to express the unknown values in terms of these known values.

Solution

(a) The capacitance of the filled capacitor is $C = \kappa C_0$. Since the battery is always connected to the capacitor plates, the potential difference between them does not change; hence, $V = V_0$. Because of that, the electrical field in the filled capacitor is the same as the field in the empty capacitor, so we can obtain directly that

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0.$$

(b) For the filled capacitor, the free charge on the plates is

$$Q = CV = (\kappa C_0)V_0 = \kappa(C_0 V_0) = \kappa Q_0.$$

The electrical field E in the filled capacitor is due to the effective charge $Q - Q_i$ (Figure 8.22(b)). Since $E = E_0$, we have

$$\frac{Q - Q_i}{\varepsilon_0 A} = \frac{Q_0}{\varepsilon_0 A}.$$

Solving this equation for Q_i, we obtain for the induced charge

$$Q_i = Q - Q_0 = \kappa Q_0 - Q_0 = (\kappa - 1)Q_0.$$

Significance

Notice that for materials with dielectric constants larger than 2 (see Table 8.1), the induced charge on the surface of dielectric is larger than the charge on the plates of a vacuum capacitor. The opposite is true for gasses like air whose dielectric constant is smaller than 2.

CHECK YOUR UNDERSTANDING 8.8

Continuing with Example 8.12, show that when the battery is connected across the plates the energy stored in dielectric-filled capacitor is $U = \kappa U_0$ (larger than the energy U_0 of an empty capacitor kept at the same voltage). Compare this result with the result $U = U_0/\kappa$ found previously for an isolated, charged capacitor.

CHECK YOUR UNDERSTANDING 8.9

Repeat the calculations of Example 8.10 for the case in which the battery remains connected while the dielectric is placed in the capacitor.

CHAPTER REVIEW

Key Terms

capacitance amount of charge stored per unit volt

capacitor device that stores electrical charge and electrical energy

dielectric insulating material used to fill the space between two plates

dielectric breakdown phenomenon that occurs when an insulator becomes a conductor in a strong electrical field

dielectric constant factor by which capacitance increases when a dielectric is inserted between the plates of a capacitor

dielectric strength critical electrical field strength above which molecules in insulator begin to break down and the insulator starts to conduct

energy density energy stored in a capacitor divided by the volume between the plates

induced electric-dipole moment dipole moment that a nonpolar molecule may acquire when it is placed in an electrical field

induced electrical field electrical field in the dielectric due to the presence of induced charges

induced surface charges charges that occur on a dielectric surface due to its polarization

parallel combination components in a circuit arranged with one side of each component connected to one side of the circuit and the other sides of the components connected to the other side of the circuit

parallel-plate capacitor system of two identical parallel conducting plates separated by a distance

series combination components in a circuit arranged in a row one after the other in a circuit

Key Equations

Capacitance	$C = \frac{Q}{V}$
Capacitance of a parallel-plate capacitor	$C = \varepsilon_0 \frac{A}{d}$
Capacitance of a vacuum spherical capacitor	$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$
Capacitance of a vacuum cylindrical capacitor	$C = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}$
Capacitance of a series combination	$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$
Capacitance of a parallel combination	$C_P = C_1 + C_2 + C_3 + \cdots$
Energy density	$u_E = \frac{1}{2}\varepsilon_0 E^2$
Energy stored in a capacitor	$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$
Capacitance of a capacitor with dielectric	$C = \kappa C_0$
Energy stored in an isolated capacitor with dielectric	$U = \frac{1}{\kappa}U_0$
Dielectric constant	$\kappa = \frac{E_0}{E}$
Induced electrical field in a dielectric	$\vec{\mathbf{E}}_i = \left(\frac{1}{\kappa} - 1\right)\vec{\mathbf{E}}_0$

Summary

8.1 Capacitors and Capacitance

- A capacitor is a device that stores an electrical charge and electrical energy. The amount of charge a vacuum capacitor can store depends on two major factors: the voltage applied and the capacitor's physical characteristics, such as its size and geometry.
- The capacitance of a capacitor is a parameter that tells us how much charge can be stored in the capacitor per unit potential difference between its plates. Capacitance of a system of conductors depends only on the geometry of their arrangement and physical properties of the insulating material that fills the space between the conductors. The unit of capacitance is the farad, where $1 \text{ F} = 1 \text{ C}/1 \text{ V}$.

8.2 Capacitors in Series and in Parallel

- When several capacitors are connected in a series combination, the reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitances.
- When several capacitors are connected in a parallel combination, the equivalent capacitance is the sum of the individual capacitances.
- When a network of capacitors contains a combination of series and parallel connections, we identify the series and parallel networks, and compute their equivalent capacitances step by step until the entire network becomes reduced to one equivalent capacitance.

8.3 Energy Stored in a Capacitor

- Capacitors are used to supply energy to a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps.
- The energy stored in a capacitor is the work required to charge the capacitor, beginning with no charge on its plates. The energy is stored in the electrical field in the space between the capacitor plates. It depends on the amount of electrical charge on the plates and on the potential difference between the plates.
- The energy stored in a capacitor network is the sum of the energies stored on individual capacitors in the network. It can be computed as the energy stored in the equivalent capacitor of the network.

8.4 Capacitor with a Dielectric

- The capacitance of an empty capacitor is increased by a factor of κ when the space between its plates is completely filled by a dielectric with dielectric constant κ.
- Each dielectric material has its specific dielectric constant.
- The energy stored in an empty isolated capacitor is decreased by a factor of κ when the space between its plates is completely filled with a dielectric with dielectric constant κ while disconnecting the battery and keeping the charge on the capacitor constant.

8.5 Molecular Model of a Dielectric

- When a dielectric is inserted between the plates of a capacitor, equal and opposite surface charge is induced on the two faces of the dielectric. The induced surface charge produces an induced electrical field that opposes the field of the free charge on the capacitor plates.
- The dielectric constant of a material is the ratio of the electrical field in vacuum to the net electrical field in the material. A capacitor filled with dielectric has a larger capacitance than an empty capacitor.
- The dielectric strength of an insulator represents a critical value of electrical field at which the molecules in an insulating material start to become ionized. When this happens, the material can conduct and dielectric breakdown is observed.

Conceptual Questions

8.1 Capacitors and Capacitance

1. Does the capacitance of a device depend on the applied voltage? Does the capacitance of a device depend on the charge residing on it?

2. Would you place the plates of a parallel-plate capacitor closer together or farther apart to increase their capacitance?

3. The value of the capacitance is zero if the plates are not charged. True or false?

4. If the plates of a capacitor have different areas, will they acquire the same charge when the capacitor is connected across a battery?

5. Does the capacitance of a spherical capacitor depend on which sphere is charged positively or negatively?

8.2 Capacitors in Series and in Parallel

6. If you wish to store a large amount of charge in a capacitor bank, would you connect capacitors in series or in parallel? Explain.
7. What is the maximum capacitance you can get by connecting three $1.0\text{-}\mu\text{F}$ capacitors? What is the minimum capacitance?

8.3 Energy Stored in a Capacitor

8. If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

8.4 Capacitor with a Dielectric

9. Discuss what would happen if a conducting slab rather than a dielectric were inserted into the gap between the capacitor plates.
10. Discuss how the energy stored in an empty but charged capacitor changes when a dielectric is inserted if (a) the capacitor is isolated so that its charge does not change; (b) the capacitor remains connected to a battery so that the potential difference between its plates does not change.

8.5 Molecular Model of a Dielectric

11. Distinguish between dielectric strength and dielectric constant.
12. Water is a good solvent because it has a high dielectric constant. Explain.
13. Water has a high dielectric constant. Explain why it is then not used as a dielectric material in capacitors.
14. Elaborate on why molecules in a dielectric material experience net forces on them in a non-uniform electrical field but not in a uniform field.
15. Explain why the dielectric constant of a substance containing permanent molecular electric dipoles decreases with increasing temperature.
16. Give a reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. How does a dielectric material also allow a greater voltage to be applied to a capacitor? (The dielectric thus increases C and permits a greater V.)
17. Elaborate on the way in which the polar character of water molecules helps to explain water's relatively large dielectric constant.
18. Sparks will occur between the plates of an air-filled capacitor at a lower voltage when the air is humid than when it is dry. Discuss why, considering the polar character of water molecules.

Problems

8.1 Capacitors and Capacitance

19. What charge is stored in a $180.0\text{-}\mu\text{F}$ capacitor when 120.0 V is applied to it?
20. Find the charge stored when 5.50 V is applied to an 8.00-pF capacitor.
21. Calculate the voltage applied to a $2.00\text{-}\mu\text{F}$ capacitor when it holds $3.10\ \mu\text{C}$ of charge.
22. What voltage must be applied to an 8.00-nF capacitor to store 0.160 mC of charge?
23. What capacitance is needed to store $3.00\ \mu\text{C}$ of charge at a voltage of 120 V?
24. What is the capacitance of a large Van de Graaff generator's terminal, given that it stores 8.00 mC of charge at a voltage of 12.0 MV?
25. The plates of an empty parallel-plate capacitor of capacitance 5.0 pF are 2.0 mm apart. What is the area of each plate?
26. A 60.0-pF vacuum capacitor has a plate area of $0.010\ \text{m}^2$. What is the separation between its plates?
27. A set of parallel plates has a capacitance of $5.0\mu\text{F}$. How much charge must be added to the plates to increase the potential difference between them by 100 V?
28. Consider Earth to be a spherical conductor of radius 6400 km and calculate its capacitance.
29. If the capacitance per unit length of a cylindrical capacitor is 20 pF/m, what is the ratio of the radii of the two cylinders?
30. An empty parallel-plate capacitor has a capacitance of $20\ \mu\text{F}$. How much charge must leak off its plates before the voltage across them is reduced by 100 V?

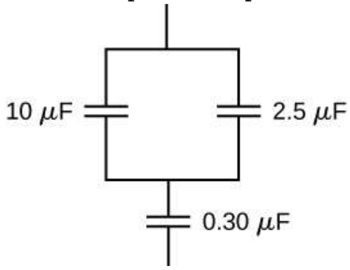
8.2 Capacitors in Series and in Parallel

31. A 4.00-pF is connected in series with an

8.00-pF capacitor and a 400-V potential difference is applied across the pair. (a) What is the charge on each capacitor? (b) What is the voltage across each capacitor?

32. Three capacitors, with capacitances of $C_1 = 2.0\ \mu\text{F}, C_2 = 3.0\ \mu\text{F}$, and $C_3 = 6.0\ \mu\text{F}$, respectively, are connected in parallel. A 500-V potential difference is applied across the combination. Determine the voltage across each capacitor and the charge on each capacitor.

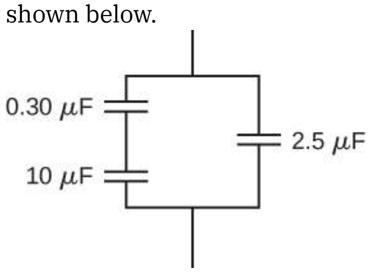
33. Find the total capacitance of this combination of series and parallel capacitors shown below.



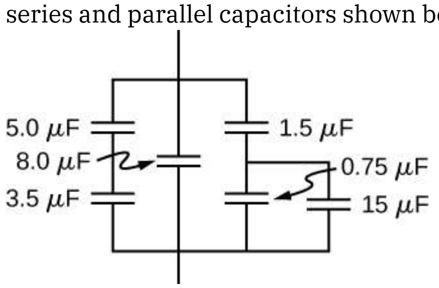
34. Suppose you need a capacitor bank with a total capacitance of 0.750 F but you have only 1.50-mF capacitors at your disposal. What is the smallest number of capacitors you could connect together to achieve your goal, and how would you connect them?

35. What total capacitances can you make by connecting a $5.00\text{-}\mu\text{F}$ and a $8.00\text{-}\mu\text{F}$ capacitor?

36. Find the equivalent capacitance of the combination of series and parallel capacitors shown below.



37. Find the net capacitance of the combination of series and parallel capacitors shown below.



38. A 40-pF capacitor is charged to a potential difference of 500 V. Its terminals are then connected to those of an uncharged 10-pF capacitor. Calculate: (a) the original charge on the 40-pF capacitor; (b) the charge on each capacitor after the connection is made; and (c) the potential difference across the plates of each capacitor after the connection.

39. A $2.0\text{-}\mu\text{F}$ capacitor and a $4.0\text{-}\mu\text{F}$ capacitor are connected in series across a 1.0-kV potential. The charged capacitors are then disconnected from the source and connected to each other with terminals of like sign together. Find the charge on each capacitor and the voltage across each capacitor.

8.3 Energy Stored in a Capacitor

40. How much energy is stored in an $8.00\text{-}\mu\text{F}$ capacitor whose plates are at a potential difference of 6.00 V?

41. A capacitor has a charge of $2.5\ \mu\text{C}$ when connected to a 6.0-V battery. How much energy is stored in this capacitor?

42. How much energy is stored in the electrical field of a metal sphere of radius 2.0 m that is kept at a 10.0-V potential?

43. (a) What is the energy stored in the $10.0\text{-}\mu\text{F}$ capacitor of a heart defibrillator charged to $9.00 \times 10^3\ \text{V}$? (b) Find the amount of the stored charge.

44. In open-heart surgery, a much smaller amount of energy will defibrillate the heart. (a) What voltage is applied to the $8.00\text{-}\mu\text{F}$ capacitor of a heart defibrillator that stores 40.0 J of energy? (b) Find the amount of the stored charge.

45. A $165\text{-}\mu\text{F}$ capacitor is used in conjunction with a dc motor. How much energy is stored in it when 119 V is applied?

46. Suppose you have a 9.00-V battery, a $2.00\text{-}\mu\text{F}$ capacitor, and a $7.40\text{-}\mu\text{F}$ capacitor. (a) Find the charge and energy stored if the capacitors are connected to the battery in series. (b) Do the same for a parallel connection.

47. An anxious physicist worries that the two metal shelves of a wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. (a) What is the capacitance of the empty shelves if they have area $1.00 \times 10^2\ \text{m}^2$ and are 0.200 m apart? (b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them? (c) To show that this voltage poses a small hazard, calculate the energy stored. (d) The actual shelves have an area 100 times smaller than these hypothetical shelves with a connection to the same voltage. Are his fears justified?

48. A parallel-plate capacitor is made of two square plates 25 cm on a side and 1.0 mm apart. The capacitor is connected to a 50.0-V battery. With the battery still connected, the plates are pulled apart to a separation of 2.00 mm. What are the energies stored in the capacitor before and after the plates are pulled farther apart? Why does the energy decrease even though work is done in separating the plates?

49. Suppose that the capacitance of a variable capacitor can be manually changed from 100 pF to 800 pF by turning a dial, connected to one set of plates by a shaft, from $0°$ to $180°$. With the dial set at $180°$ (corresponding to $C = 800\,\text{pF}$), the capacitor is connected to a 500-V source. After charging, the capacitor is disconnected from the source, and the dial is turned to $0°$. If friction is negligible, how much work is required to turn the dial from $180°$ to $0°$?

8.4 Capacitor with a Dielectric

50. Show that for a given dielectric material, the maximum energy a parallel-plate capacitor can store is directly proportional to the volume of dielectric.

51. An air-filled capacitor is made from two flat parallel plates 1.0 mm apart. The inside area of each plate is $8.0\,\text{cm}^2$. (a) What is the capacitance of this set of plates? (b) If the region between the plates is filled with a material whose dielectric constant is 6.0, what is the new capacitance?

52. A capacitor is made from two concentric spheres, one with radius 5.00 cm, the other with radius 8.00 cm. (a) What is the capacitance of this set of conductors? (b) If the region between the conductors is filled with a material whose dielectric constant is 6.00, what is the capacitance of the system?

53. A parallel-plate capacitor has charge of magnitude $9.00\,\mu\text{C}$ on each plate and capacitance $3.00\,\mu\text{F}$ when there is air between the plates. The plates are separated by 2.00 mm. With the charge on the plates kept constant, a dielectric with $\kappa = 5$ is inserted between the plates, completely filling the volume between the plates. (a) What is the potential difference between the plates of the capacitor, before and after the dielectric has been inserted? (b) What is the electrical field at the point midway between the plates before and after the dielectric is inserted?

54. Some cell walls in the human body have a layer of negative charge on the inside surface. Suppose that the surface charge densities are $\pm 0.50 \times 10^{-3}\,\text{C/m}^2$, the cell wall is $5.0 \times 10^{-9}\,\text{m}$ thick, and the cell wall material has a dielectric constant of $\kappa = 5.4$. (a) Find the magnitude of the electric field in the wall between two charge layers. (b) Find the potential difference between the inside and the outside of the cell. Which is at higher potential? (c) A typical cell in the human body has volume $10^{-16}\,\text{m}^3$. Estimate the total electrical field energy stored in the wall of a cell of this size when assuming that the cell is spherical. (*Hint*: Calculate the volume of the cell wall.)

55. A parallel-plate capacitor with only air between its plates is charged by connecting the capacitor to a battery. The capacitor is then disconnected from the battery, without any of the charge leaving the plates. (a) A voltmeter reads 45.0 V when placed across the capacitor. When a dielectric is inserted between the plates, completely filling the space, the voltmeter reads 11.5 V. What is the dielectric constant of the material? (b) What will the voltmeter read if the dielectric is now pulled away out so it fills only one-third of the space between the plates?

8.5 Molecular Model of a Dielectric

56. Two flat plates containing equal and opposite charges are separated by material 4.0 mm thick with a dielectric constant of 5.0. If the electrical field in the dielectric is 1.5 MV/m, what are (a) the charge density on the capacitor plates, and (b) the induced charge density on the surfaces of the dielectric?

57. For a Teflon™-filled, parallel-plate capacitor, the area of the plate is $50.0\,\text{cm}^2$ and the spacing between the plates is 0.50 mm. If the capacitor is connected to a 200-V battery, find (a) the free charge on the capacitor plates, (b) the electrical field in the dielectric, and (c) the induced charge on the dielectric surfaces.

58. Find the capacitance of a parallel-plate capacitor having plates with a surface area of $5.00\,m^2$ and separated by 0.100 mm of Teflon™.

59. (a) What is the capacitance of a parallel-plate capacitor with plates of area $1.50\,\text{m}^2$ that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?

60. Two parallel plates have equal and opposite charges. When the space between the plates is evacuated, the electrical field is

$E = 3.20 \times 10^5$ V/m. When the space is filled with dielectric, the electrical field is $E = 2.50 \times 10^5$ V/m. (a) What is the surface charge density on each surface of the dielectric? (b) What is the dielectric constant?

61. The dielectric to be used in a parallel-plate capacitor has a dielectric constant of 3.60 and a dielectric strength of 1.60×10^7 V/m. The capacitor has to have a capacitance of 1.25 nF and must be able to withstand a maximum potential difference 5.5 kV. What is the minimum area the plates of the capacitor may have?

62. When a 360-nF air capacitor is connected to a power supply, the energy stored in the capacitor is $18.5\ \mu$J. While the capacitor is connected to the power supply, a slab of dielectric is inserted that completely fills the space between the plates. This increases the stored energy by $23.2\ \mu$J. (a) What is the potential difference between the capacitor plates? (b) What is the dielectric constant of the slab?

63. A parallel-plate capacitor has square plates that are 8.00 cm on each side and 3.80 mm apart. The space between the plates is completely filled with two square slabs of dielectric, each 8.00 cm on a side and 1.90 mm thick. One slab is Pyrex glass and the other slab is polystyrene. If the potential difference between the plates is 86.0 V, find how much electrical energy can be stored in this capacitor.

Additional Problems

64. A capacitor is made from two flat parallel plates placed 0.40 mm apart. When a charge of $0.020\ \mu$C is placed on the plates the potential difference between them is 250 V. (a) What is the capacitance of the plates? (b) What is the area of each plate? (c) What is the charge on the plates when the potential difference between them is 500 V? (d) What maximum potential difference can be applied between the plates so that the magnitude of electrical fields between the plates does not exceed 3.0 MV/m?

65. An air-filled (empty) parallel-plate capacitor is made from two square plates that are 25 cm on each side and 1.0 mm apart. The capacitor is connected to a 50-V battery and fully charged. It is then disconnected from the battery and its plates are pulled apart to a separation of 2.00 mm. (a) What is the capacitance of this new capacitor? (b) What is the charge on each plate? (c) What is the electrical field between the plates?

66. Suppose that the capacitance of a variable capacitor can be manually changed from 100 to 800 pF by turning a dial connected to one set of plates by a shaft, from $0°$ to $180°$. With the dial set at $180°$ (corresponding to $C = 800$ pF), the capacitor is connected to a 500-V source. After charging, the capacitor is disconnected from the source, and the dial is turned to $0°$. (a) What is the charge on the capacitor? (b) What is the voltage across the capacitor when the dial is set to $0°$?

67. Earth can be considered as a spherical capacitor with two plates, where the negative plate is the surface of Earth and the positive plate is the bottom of the ionosphere, which is located at an altitude of approximately 70 km. The potential difference between Earth's surface and the ionosphere is about 350,000 V. (a) Calculate the capacitance of this system. (b) Find the total charge on this capacitor. (c) Find the energy stored in this system.

68. A $4.00\text{-}\mu$F capacitor and a $6.00\text{-}\mu$F capacitor are connected in parallel across a 600-V supply line. (a) Find the charge on each capacitor and voltage across each. (b) The charged capacitors are disconnected from the line and from each other. They are then reconnected to each other with terminals of unlike sign together. Find the final charge on each capacitor and the voltage across each.

69. Three capacitors having capacitances of 8.40, 8.40, and 4.20 μF, respectively, are connected in series across a 36.0-V potential difference. (a) What is the charge on the $4.20\text{-}\mu$F capacitor? (b) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other with the positively charged plates connected together. What is the voltage across each capacitor in the parallel combination?

70. A parallel-plate capacitor with capacitance $5.0\ \mu$F is charged with a 12.0-V battery, after which the battery is disconnected. Determine the minimum work required to increase the separation between the plates by a factor of 3.

71. (a) How much energy is stored in the electrical fields in the capacitors (in total) shown below? (b) Is this energy equal to the work done by the 400-V source in charging the capacitors?

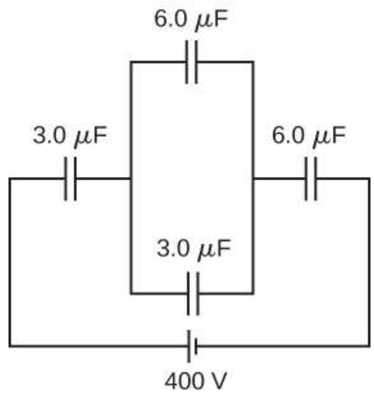


Figure 8.23

72. Three capacitors having capacitances 8.4, 8.4, and 4.2 μF are connected in series across a 36.0-V potential difference. (a) What is the total energy stored in all three capacitors? (b) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other with the positively charged plates connected together. What is the total energy now stored in the capacitors?

73. (a) An $8.00\text{-}\mu\text{F}$ capacitor is connected in parallel to another capacitor, producing a total capacitance of $5.00\ \mu\text{F}$. What is the capacitance of the second capacitor? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

74. (a) On a particular day, it takes 9.60×10^3 J of electrical energy to start a truck's engine. Calculate the capacitance of a capacitor that could store that amount of energy at 12.0 V. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

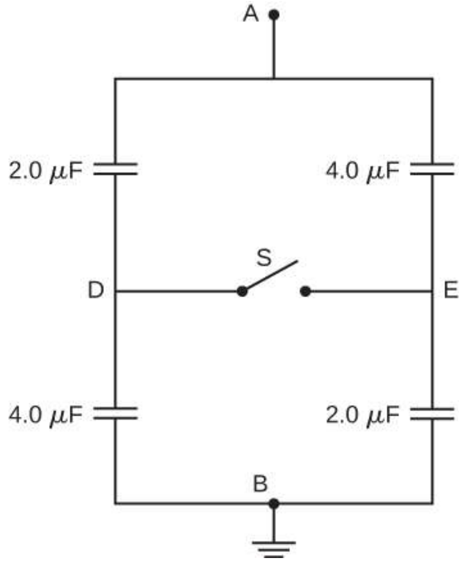
75. (a) A certain parallel-plate capacitor has plates of area $4.00\ \text{m}^2$, separated by 0.0100 mm of nylon, and stores 0.170 C of charge. What is the applied voltage? (b) What is unreasonable about this result? (c) Which assumptions are responsible or inconsistent?

76. A prankster applies 450 V to an $80.0\text{-}\mu\text{F}$ capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200 g of flesh. Estimate, what is the temperature increase of the flesh? Is it reasonable to assume that no thermodynamic phase change happened?

Challenge Problems

77. A spherical capacitor is formed from two concentric spherical conducting spheres separated by vacuum. The inner sphere has radius 12.5 cm and the outer sphere has radius 14.8 cm. A potential difference of 120 V is applied to the capacitor. (a) What is the capacitance of the capacitor? (b) What is the magnitude of the electrical field at $r = 12.6$ cm, just outside the inner sphere? (c) What is the magnitude of the electrical field at $r = 14.7$ cm, just inside the outer sphere? (d) For a parallel-plate capacitor the electrical field is uniform in the region between the plates, except near the edges of the plates. Is this also true for a spherical capacitor?

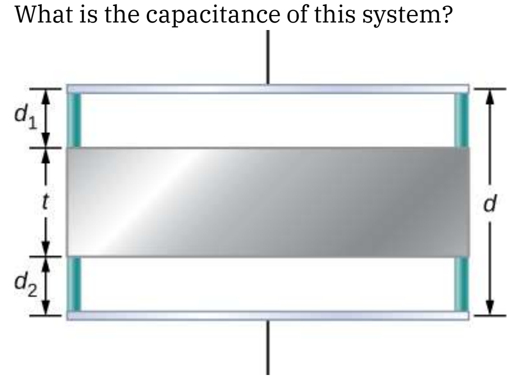
78. The network of capacitors shown below are all uncharged when a 300-V potential is applied between points A and B with the switch S open. (a) What is the potential difference $V_E - V_D$? (b) What is the potential at point E after the switch is closed? (c) How much charge flows through the switch after it is closed?



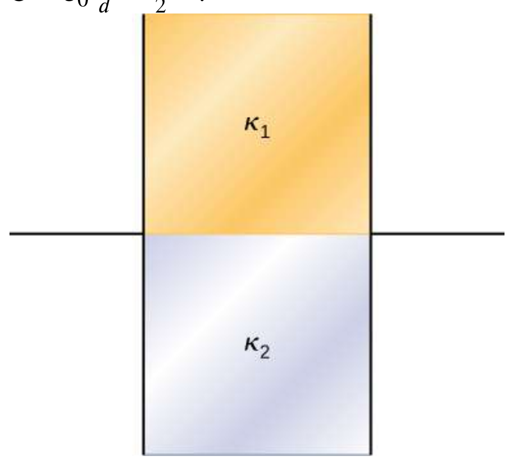
79. Electronic flash units for cameras contain a capacitor for storing the energy used to produce the flash. In one such unit the flash lasts for 1/675 fraction of a second with an average light power output of 270 kW. (a) If the conversion of electrical energy to light is 95% efficient (because the rest of the energy goes to thermal energy), how much energy must be stored in the capacitor for one flash? (b) The capacitor has a potential difference between its plates of 125 V when the stored energy equals the value stored in part (a). What is the capacitance?

80. A spherical capacitor is formed from two concentric spherical conducting shells separated by a vacuum. The inner sphere has radius 12.5 cm and the outer sphere has radius 14.8 cm. A potential difference of 120 V is applied to the capacitor. (a) What is the energy density at $r = 12.6$ cm, just outside the inner sphere? (b) What is the energy density at $r = 14.7$ cm, just inside the outer sphere? (c) For the parallel-plate capacitor the energy density is uniform in the region between the plates, except near the edges of the plates. Is this also true for the spherical capacitor?

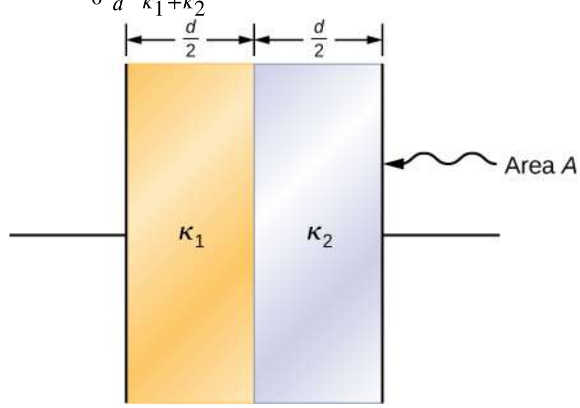
81. A metal plate of thickness *t* is held in place between two capacitor plates by plastic pegs, as shown below. The effect of the pegs on the capacitance is negligible. The area of each capacitor plate and the area of the top and bottom surfaces of the inserted plate are all *A*. What is the capacitance of this system?



82. A parallel-plate capacitor is filled with two dielectrics, as shown below. When the plate area is *A* and separation between plates is *d*, show that the capacitance is given by $C = \varepsilon_0 \frac{A}{d} \frac{\kappa_1 + \kappa_2}{2}$.



83. A parallel-plate capacitor is filled with two dielectrics, as shown below. Show that the capacitance is given by $C = 2\varepsilon_0 \frac{A}{d} \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}$.



84. A capacitor has parallel plates of area 12 cm^2 separated by 2.0 mm. The space between the plates is filled with polystyrene. (a) Find the maximum permissible voltage across the capacitor to avoid dielectric breakdown. (b) When the voltage equals the value found in part (a), find the surface charge density on the surface of the dielectric.

CHAPTER 9
Current and Resistance

Figure 9.1 Magnetic resonance imaging (MRI) uses superconducting magnets and produces high-resolution images without the danger of radiation. The image on the left shows the spacing of vertebrae along a human spinal column, with the circle indicating where the vertebrae are too close due to a ruptured disc. On the right is a picture of the MRI instrument, which surrounds the patient on all sides. A large amount of electrical current is required to operate the electromagnets (credit right: modification of work by “digital cat”/Flickr).

Chapter Outline

INTRODUCTION In this chapter, we study the electrical current through a material, where the electrical current is the rate of flow of charge. We also examine a characteristic of materials known as the resistance. Resistance is a measure of how much a material impedes the flow of charge, and it will be shown that the resistance depends on temperature. In general, a good conductor, such as copper, gold, or silver, has very low resistance. Some materials, called superconductors, have zero resistance at very low temperatures.

High currents are required for the operation of electromagnets. Superconductors can be used to make electromagnets that are 10 times stronger than the strongest conventional electromagnets. These superconducting magnets are used in the construction of magnetic resonance imaging (MRI) devices that can be used to make high-resolution images of the human body. The chapter-opening picture shows an MRI image of the vertebrae of a human subject and the MRI device itself. Superconducting magnets have many other uses. For example, superconducting magnets are used in the Large Hadron Collider (LHC) to curve the path of protons in the ring.

9.1 Electrical Current

Learning Objectives

By the end of this section, you will be able to:

- Describe an electrical current
- Define the unit of electrical current
- Explain the direction of current flow

Up to now, we have considered primarily static charges. When charges did move, they were accelerated in response to an electrical field created by a voltage difference. The charges lost potential energy and gained kinetic energy as they traveled through a potential difference where the electrical field did work on the charge.

Although charges do not require a material to flow through, the majority of this chapter deals with understanding the movement of charges through a material. The rate at which the charges flow past a location—that is, the amount of charge per unit time—is known as the *electrical current*. When charges flow through a medium, the current depends on the voltage applied, the material through which the charges flow, and the state of the material. Of particular interest is the motion of charges in a conducting wire. In previous chapters, charges were accelerated due to the force provided by an electrical field, losing potential energy and gaining kinetic energy. In this chapter, we discuss the situation of the force provided by an electrical field in a conductor, where charges lose kinetic energy to the material reaching a constant velocity, known as the "*drift velocity*." This is analogous to an object falling through the atmosphere and losing kinetic energy to the air, reaching a constant terminal velocity.

If you have ever taken a course in first aid or safety, you may have heard that in the event of electric shock, it is the current, not the voltage, which is the important factor on the severity of the shock and the amount of damage to the human body. Current is measured in units called amperes; you may have noticed that circuit breakers in your home and fuses in your car are rated in amps (or amperes). But what is the ampere and what does it measure?

Defining Current and the Ampere

Electrical current is defined to be the rate at which charge flows. When there is a large current present, such as that used to run a refrigerator, a large amount of charge moves through the wire in a small amount of time. If the current is small, such as that used to operate a handheld calculator, a small amount of charge moves through the circuit over a long period of time.

Electrical Current

The average electrical current I is the rate at which charge flows,

$$I_{\text{ave}} = \frac{\Delta Q}{\Delta t}, \tag{9.1}$$

where ΔQ is the amount of net charge passing through a given cross-sectional area in time Δt (Figure 9.2). The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \frac{\Delta Q}{\Delta t}$, we see that an ampere is defined as one coulomb of charge passing through a given area per second:

$$1\text{A} \equiv 1\frac{\text{C}}{\text{s}}. \tag{9.2}$$

The instantaneous electrical current, or simply the **electrical current**, is the time derivative of the charge that flows and is found by taking the limit of the average electrical current as $\Delta t \to 0$:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}. \tag{9.3}$$

Most electrical appliances are rated in amperes (or amps) required for proper operation, as are fuses and circuit breakers.

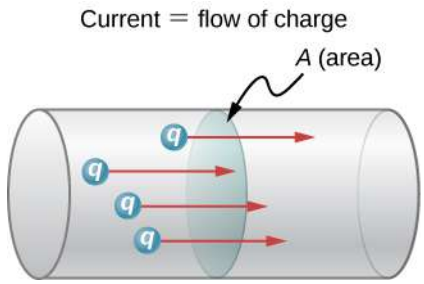


Figure 9.2 The rate of flow of charge is current. An ampere is the flow of one coulomb of charge through an area in one second. A current of one amp would result from 6.25×10^{18} electrons flowing through the area *A* each second.

EXAMPLE 9.1

Calculating the Average Current

The main purpose of a battery in a car or truck is to run the electric starter motor, which starts the engine. The operation of starting the vehicle requires a large current to be supplied by the battery. Once the engine starts, a device called an alternator takes over supplying the electric power required for running the vehicle and for charging the battery.

(a) What is the average current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow from the battery?

Strategy

We can use the definition of the average current in the equation $I = \frac{\Delta Q}{\Delta t}$ to find the average current in part (a), since charge and time are given. For part (b), once we know the average current, we can its definition $I = \frac{\Delta Q}{\Delta t}$ to find the time required for 1.00 C of charge to flow from the battery.

Solution

a. Entering the given values for charge and time into the definition of current gives

$$I = \frac{\Delta Q}{\Delta t} = \frac{720\,\text{C}}{4.00\,\text{s}} = 180\,\text{C/s} = 180\,\text{A}.$$

b. Solving the relationship $I = \frac{\Delta Q}{\Delta t}$ for time Δt and entering the known values for charge and current gives

$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00\,\text{C}}{180\,\text{C/s}} = 5.56 \times 10^{-3}\,\text{s} = 5.56\,\text{ms}.$$

Significance

a. This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large to overcome the inertia of the engine. b. A high current requires a short time to supply a large amount of charge. This large current is needed to supply the large amount of energy needed to start the engine.

EXAMPLE 9.2

Calculating Instantaneous Currents

Consider a charge moving through a cross-section of a wire where the charge is modeled as $Q(t) = Q_M\left(1 - e^{-t/\tau}\right)$. Here, Q_M is the charge after a long period of time, as time approaches infinity, with units of coulombs, and τ is a time constant with units of seconds (see Figure 9.3). What is the current through the wire?

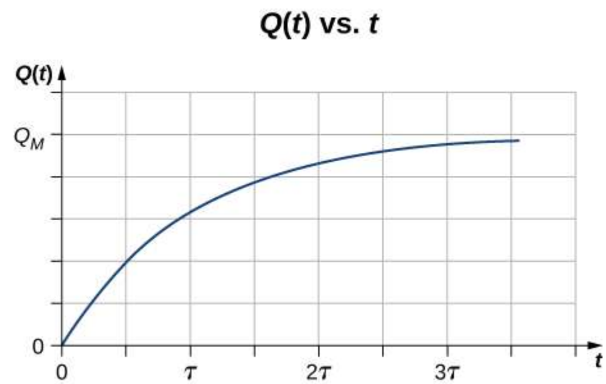


Figure 9.3 A graph of the charge moving through a cross-section of a wire over time.

Strategy

The current through the cross-section can be found from $I = \frac{dQ}{dt}$. Notice from the figure that the charge increases to Q_M and the derivative decreases, approaching zero, as time increases (Figure 9.4).

Solution

The derivative can be found using $\frac{d}{dx}e^u = e^u\frac{du}{dx}$.

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[Q_M\left(1 - e^{-t/\tau}\right)\right] = \frac{Q_M}{\tau}e^{-t/\tau}.$$

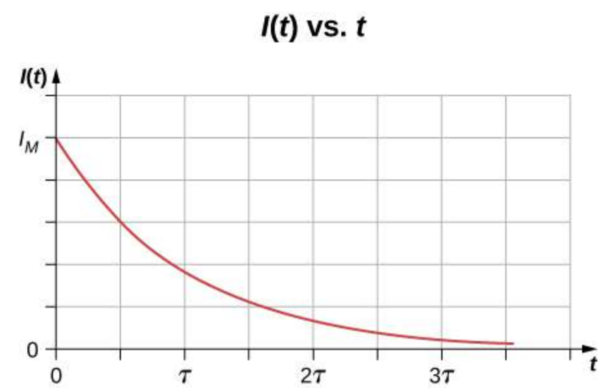


Figure 9.4 A graph of the current flowing through the wire over time.

Significance

The current through the wire in question decreases exponentially, as shown in Figure 9.4. In later chapters, it will be shown that a time-dependent current appears when a capacitor charges or discharges through a resistor. Recall that a capacitor is a device that stores charge. You will learn about the resistor in Model of Conduction in Metals.

CHECK YOUR UNDERSTANDING 9.1

Handheld calculators often use small solar cells to supply the energy required to complete the calculations needed to complete your next physics exam. The current needed to run your calculator can be as small as 0.30 mA. How long would it take for 1.00 C of charge to flow from the solar cells? Can solar cells be used, instead of batteries, to start traditional internal combustion engines presently used in most cars and trucks?

CHECK YOUR UNDERSTANDING 9.2

Circuit breakers in a home are rated in amperes, normally in a range from 10 amps to 30 amps, and are used to protect the residents from harm and their appliances from damage due to large currents. A single 15-amp circuit breaker may be used to protect several outlets in the living room, whereas a single 20-amp circuit breaker may be used to protect the refrigerator in the kitchen. What can you deduce from this about current used by the various appliances?

Current in a Circuit

In the previous paragraphs, we defined the current as the charge that flows through a cross-sectional area per unit time. In order for charge to flow through an appliance, such as the headlight shown in Figure 9.5, there must be a complete path (or **circuit**) from the positive terminal to the negative terminal. Consider a simple circuit of a car battery, a switch, a headlight lamp, and wires that provide a current path between the components. In order for the lamp to light, there must be a complete path for current flow. In other words, a charge must be able to leave the positive terminal of the battery, travel through the component, and back to the negative terminal of the battery. The switch is there to control the circuit. Part (a) of the figure shows the simple circuit of a car battery, a switch, a conducting path, and a headlight lamp. Also shown is the **schematic** of the circuit [part (b)]. A schematic is a graphical representation of a circuit and is very useful in visualizing the main features of a circuit. Schematics use standardized symbols to represent the components in a circuits and solid lines to represent the wires connecting the components. The battery is shown as a series of long and short lines, representing the historic voltaic pile. The lamp is shown as a circle with a loop inside, representing the filament of an incandescent bulb. The switch is shown as two points with a conducting bar to connect the two points and the wires connecting the components are shown as solid lines. The schematic in part (c) shows the direction of current flow when the switch is closed.

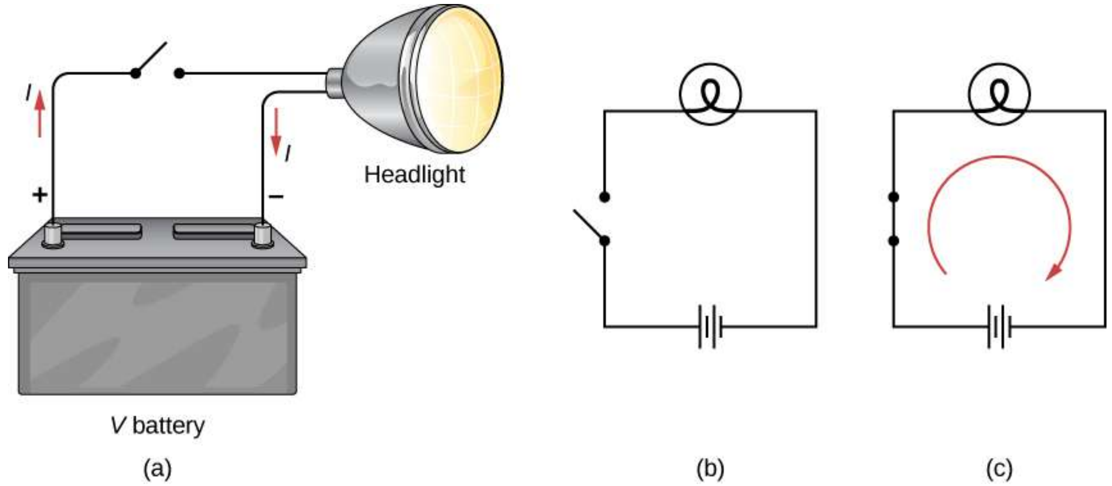


Figure 9.5 (a) A simple electric circuit of a headlight (lamp), a battery, and a switch. When the switch is closed, an uninterrupted path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by parallel lines, which resemble plates in the original design of a battery. The longer lines indicate the positive terminal. The conducting wires are shown as solid lines. The switch is shown, in the open position, as two terminals with a line representing a conducting bar that can make contact between the two terminals. The lamp is represented by a circle encompassing a filament, as would be seen in an incandescent light bulb. (c) When the switch is closed, the circuit is complete and current flows from the positive terminal to the negative terminal of the battery.

When the switch is closed in Figure 9.5(c), there is a complete path for charges to flow, from the positive terminal of the battery, through the switch, then through the headlight and back to the negative terminal of the battery. Note that the direction of current flow is from positive to negative. The direction of **conventional current** is always represented in the direction that positive charge would flow, from the positive terminal to the negative terminal.

The conventional current flows from the positive terminal to the negative terminal, but depending on the actual situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator, used for nuclear research, can produce a current of pure positive charges, such as protons. In the Tevatron Accelerator at Fermilab, before it was shut down in 2011, beams of protons and antiprotons traveling in opposite directions were collided. The protons are positive and therefore their current is in the same direction as they travel. The antiprotons are negativity charged and thus their current is in the opposite direction that the actual particles travel.

A closer look at the current flowing through a wire is shown in Figure 9.6. The figure illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American scientist and statesman Benjamin Franklin in the 1700s. Having no knowledge of the particles that make up the atom (namely the proton, electron, and neutron), Franklin believed that electrical current flowed from a material that had more of an "electrical fluid" and to a material that had less of this "electrical fluid." He coined the term *positive* for the material that had more of this electrical fluid and *negative* for the material that lacked the electrical fluid. He surmised that current would flow from the material with more electrical fluid—the positive material—to the negative material, which has less electrical fluid. Franklin called this direction of current a positive current flow. This was pretty advanced thinking for a man who knew nothing about the atom.

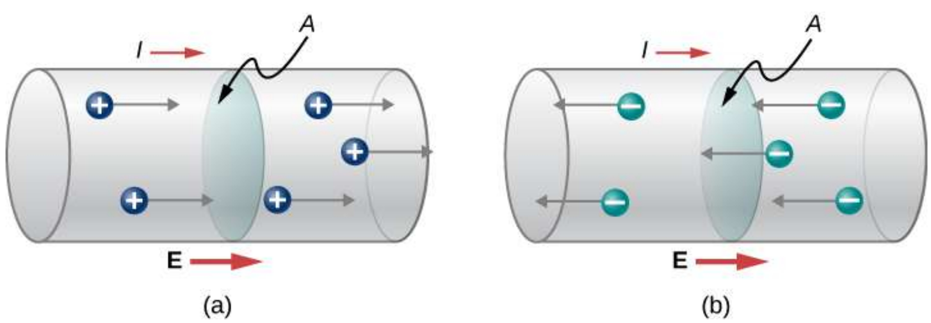


Figure 9.6 Current *I* is the rate at which charge moves through an area *A*, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electrical field. (a) Positive charges move in the direction of the electrical field, which is the same direction as conventional current. (b) Negative charges move in the direction opposite to the electrical field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

We now know that a material is positive if it has a greater number of protons than electrons, and it is negative if it has a greater number of electrons than protons. In a conducting metal, the current flow is due primarily to electrons flowing from the negative material to the positive material, but for historical reasons, we consider the positive current flow and the current is shown to flow from the positive terminal of the battery to the negative terminal.

It is important to realize that an electrical field is present in conductors and is responsible for producing the current (Figure 9.6). In previous chapters, we considered the static electrical case, where charges in a conductor quickly redistribute themselves on the surface of the conductor in order to cancel out the external electrical field and restore equilibrium. In the case of an electrical circuit, the charges are prevented from ever reaching equilibrium by an external source of electric potential, such as a battery. The energy needed to move the charge is supplied by the electric potential from the battery.

Although the electrical field is responsible for the motion of the charges in the conductor, the work done on the charges by the electrical field does not increase the kinetic energy of the charges. We will show that the electrical field is responsible for keeping the electric charges moving at a "drift velocity."

9.2 Model of Conduction in Metals

Learning Objectives

By the end of this section, you will be able to:

- Define the drift velocity of charges moving through a metal
- Define the vector current density
- Describe the operation of an incandescent lamp

When electrons move through a conducting wire, they do not move at a constant velocity, that is, the electrons do not move in a straight line at a constant speed. Rather, they interact with and collide with atoms and other free electrons in the conductor. Thus, the electrons move in a zig-zag fashion and drift through the wire. We should also note that even though it is convenient to discuss the direction of current, current is a scalar quantity. When discussing the velocity of charges in a current, it is more appropriate to discuss the current density. We will come back to this idea at the end of this section.

Drift Velocity

Electrical signals move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a light switch is moved to the 'on' position. Most electrical signals carried by currents travel at speeds on the order of 10^8 m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move much slower on average, typically drifting at speeds on the order of 10^{-4} m/s. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in Figure 9.7, the incoming charge pushes other charges ahead of it due to the repulsive force between like charges. These moving charges push on charges farther down the line. The density of charge in a system cannot easily be increased, so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this fast-moving signal, or shock wave, is a rapidly propagating change in the electrical field.

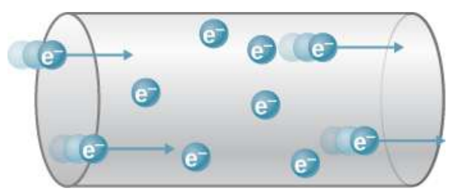


Figure 9.7 When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges. In metals, the free charges are free electrons. (In fact, good electrical conductors are often good heat conductors too, because large numbers of free electrons can transport thermal energy as well as carry electrical current.) Figure 9.8 shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electrical field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** $\vec{v}_d$ is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

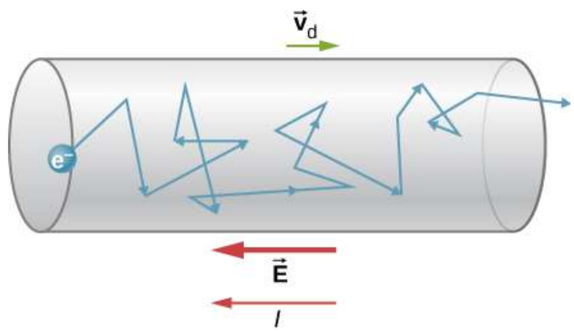


Figure 9.8 Free electrons moving in a conductor make many collisions with other electrons and other particles. A typical path of one electron is shown. The average velocity of the free charges is called the drift velocity $\vec{v}_d$ and for electrons, it is in the direction opposite to the electrical field. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Free-electron collisions transfer energy to the atoms of the conductor. The electrical field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed) of the electrons. The work is transferred to the conductor's atoms, often increasing temperature. Thus, a continuous power input is required to keep a current flowing. (An exception is superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings.) For a conductor that is not a superconductor, the supply of energy can be useful, as in an incandescent light bulb filament (Figure 9.9). The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

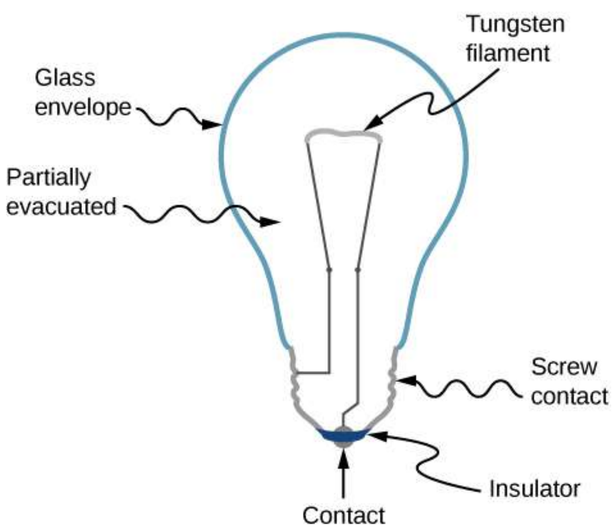


Figure 9.9 The incandescent lamp is a simple design. A tungsten filament is placed in a partially evacuated glass envelope. One end of the filament is attached to the screw base, which is made out of a conducting material. The second end of the filament is attached to a second contact in the base of the bulb. The two contacts are separated by an insulating material. Current flows through the filament, and the temperature of the filament becomes large enough to cause the filament to glow and produce light. However, these bulbs are not very energy efficient, as evident from the heat coming from the bulb. In the year 2012, the United States, along with many other countries, began to phase out incandescent lamps in favor of more energy-efficient lamps, such as light-emitting diode (LED) lamps and compact fluorescent lamps (CFL) (credit right: modification of work by Serge Saint).

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in Figure 9.10. The number of free charges per unit volume, or the number density of free charges, is given the symbol n where $n = \frac{\text{number of charges}}{\text{volume}}$. The value of n depends on the material. The shaded segment has a volume $Av_d\,dt$, so that the number of free charges in the volume is $nAv_d\,dt$. The charge dQ in this segment is thus $qnAv_d\,dt$, where q is the amount of charge on each carrier. (The magnitude of the charge of electrons is $q = 1.60 \times 10^{-19}$ C.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time dt, the current is

$$I = \frac{dQ}{dt} = qnAv_{\mathrm{d}}.$$

Rearranging terms gives

$$v_{\mathrm{d}} = \frac{I}{nqA} \tag{9.4}$$

where v_{d} is the drift velocity, n is the free charge density, A is the cross-sectional area of the wire, and I is the current through the wire. The carriers of the current each have charge q and move with a drift velocity of magnitude v_{d}.

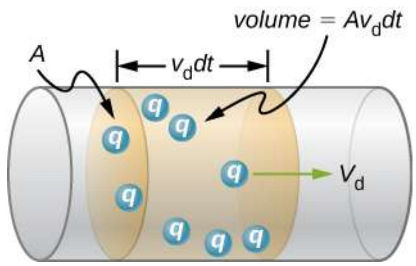


Figure 9.10 All the charges in the shaded volume of this wire move out in a time dt, having a drift velocity of magnitude v_{d}.

Note that simple drift velocity is not the entire story. The speed of an electron is sometimes much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do move might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons?

Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as strongly as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a “sea” of electrons. When an electrical field is applied, these free electrons respond by accelerating. As they move, they collide with the atoms in the lattice and with other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

As you know, electric power is usually supplied to equipment and appliances through round wires made of a conducting material (copper, aluminum, silver, or gold) that are stranded or solid. The diameter of the wire determines the current-carrying capacity—the larger the diameter, the greater the current-carrying capacity. Even though the current-carrying capacity is determined by the diameter, wire is not normally characterized by the diameter directly. Instead, wire is commonly sold in a unit known as “gauge.” Wires are manufactured by passing the material through circular forms called “drawing dies.” In order to make thinner wires, manufacturers draw the wires through multiple dies of successively thinner diameter. Historically, the gauge of the wire was related to the number of drawing processes required to manufacture the wire. For this reason, the larger the gauge, the smaller the diameter. In the United States, the American Wire Gauge (AWG) was developed to standardize the system. Household wiring commonly consists of 10-gauge (2.588-mm diameter) to 14-gauge (1.628-mm diameter) wire. A device used to measure the gauge of wire is shown in Figure 9.11.

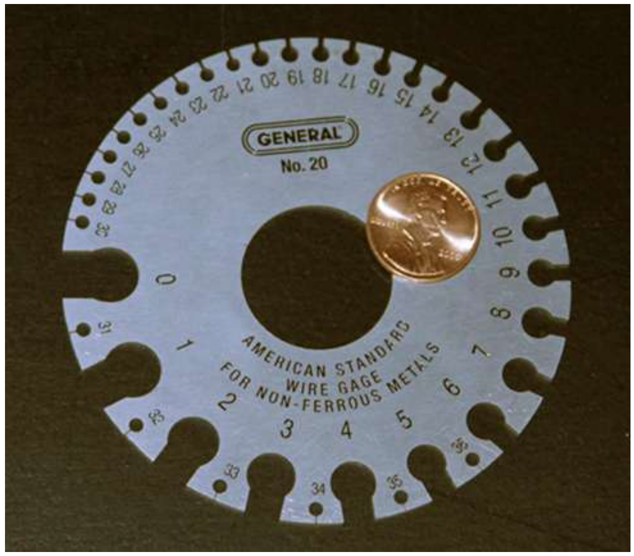


Figure 9.11 A device for measuring the gauge of electrical wire. As you can see, higher gauge numbers indicate thinner wires. (credit: Joseph J. Trout)

EXAMPLE 9.3

Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a copper wire with a diameter of 2.053 mm (12-gauge) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20.0 A.) The density of copper is $8.80 \times 10^3\ \text{kg/m}^3$ and the atomic mass of copper is 63.54 g/mol.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_\text{d}$. The current is $I = 20.00\ \text{A}$ and $q = 1.60 \times 10^{-19}\,\text{C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the diameter. The given diameter is 2.053 mm, so r is 1.0265 mm. We are given the density of copper, $8.80 \times 10^3\ \text{kg/m}^3$, and the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, 6.02×10^{23} atoms/mol, to determine n, the number of free electrons per cubic meter.

Solution

First, we calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, the number of free electrons is the same as the number of copper atoms per m^3. We can now find n as follows:

$$\begin{aligned} n &= \frac{1\,e^-}{\text{atom}} \times \frac{6.02 \times 10^{23}\,\text{atoms}}{\text{mol}} \times \frac{1\ \text{mol}}{63.54\ \text{g}} \times \frac{1000\ \text{g}}{\text{kg}} \times \frac{8.80 \times 10^3\,\text{kg}}{1\ \text{m}^3} \\ &= 8.34 \times 10^{28}\,e^-/\text{m}^3. \end{aligned}$$

The cross-sectional area of the wire is

$$A = \pi r^2 = \pi \left(\frac{2.05 \times 10^{-3}\,\text{m}}{2} \right)^2 = 3.30 \times 10^{-6}\,\text{m}^2.$$

Rearranging $I = nqAv_\text{d}$ to isolate drift velocity gives

$$v_\text{d} = \frac{I}{nqA} = \frac{20.00\ \text{A}}{(8.34 \times 10^{28}/\text{m}^3)(-1.60 \times 10^{-19}\,\text{C})(3.30 \times 10^{-6}\,\text{m}^2)} = -4.54 \times 10^{-4}\ \text{m/s}.$$

Significance

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4} m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^{8} m/s) than the charges that carry it.

CHECK YOUR UNDERSTANDING 9.3

In Example 9.4, the drift velocity was calculated for a 2.053-mm diameter (12-gauge) copper wire carrying a 20-amp current. Would the drift velocity change for a 1.628-mm diameter (14-gauge) wire carrying the same 20-amp current?

Current Density

Although it is often convenient to attach a negative or positive sign to indicate the overall direction of motion of the charges, current is a scalar quantity, $I = \frac{dQ}{dt}$. It is often necessary to discuss the details of the motion of the charge, instead of discussing the overall motion of the charges. In such cases, it is necessary to discuss the current density, $\vec{\mathbf{J}}$, a vector quantity. The **current density** is the flow of charge through an infinitesimal area, divided by the area. The current density must take into account the local magnitude and direction of the charge flow, which varies from point to point. The unit of current density is ampere per meter squared, and the direction is defined as the direction of net flow of positive charges through the area.

The relationship between the current and the current density can be seen in Figure 9.12. The differential current flow through the area $d\vec{\mathbf{A}}$ is found as

$$dI = \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = J\,dA\cos\theta,$$

where θ is the angle between the area and the current density. The total current passing through area $d\vec{\mathbf{A}}$ can be found by integrating over the area,

$$I = \iint_{\text{area}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}. \tag{9.5}$$

Consider the magnitude of the current density, which is the current divided by the area:

$$J = \frac{I}{A} = \frac{n\,|q|\,Av_{\mathrm{d}}}{A} = n\,|q|\,v_{\mathrm{d}}.$$

Thus, the current density is $\vec{\mathbf{J}} = nq\vec{\mathbf{v}}_{\mathrm{d}}$. If q is positive, $\vec{\mathbf{v}}_{\mathrm{d}}$ is in the same direction as the electrical field $\vec{\mathbf{E}}$. If q is negative, $\vec{\mathbf{v}}_{\mathrm{d}}$ is in the opposite direction of $\vec{\mathbf{E}}$. Either way, the direction of the current density $\vec{\mathbf{J}}$ is in the direction of the electrical field $\vec{\mathbf{E}}$.

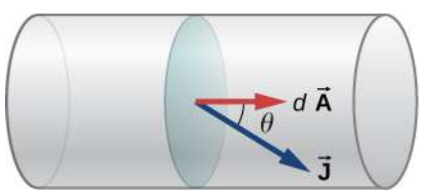


Figure 9.12 The current density $\vec{\mathbf{J}}$ is defined as the current passing through an infinitesimal cross-sectional area divided by the area. The direction of the current density is the direction of the net flow of positive charges and the magnitude is equal to the current divided by the infinitesimal area.

EXAMPLE 9.4

Calculating the Current Density in a Wire

The current supplied to a lamp with a 100-W light bulb is 0.87 amps. The lamp is wired using a copper wire with diameter 2.588 mm (10-gauge). Find the magnitude of the current density.

Strategy

The current density is the current moving through an infinitesimal cross-sectional area divided by the area. We can calculate the magnitude of the current density using $J = \frac{I}{A}$. The current is given as 0.87 A. The cross-sectional area can be calculated to be $A = 5.26\ \text{mm}^2$.

Solution

Calculate the current density using the given current $I = 0.87\ \text{A}$ and the area, found to be $A = 5.26\ \text{mm}^2$.

$$J = \frac{I}{A} = \frac{0.87\ \text{A}}{5.26 \times 10^{-6}\text{m}^2} = 1.65 \times 10^5\ \frac{\text{A}}{\text{m}^2}.$$

Significance

The current density in a conducting wire depends on the current through the conducting wire and the cross-sectional area of the wire. For a given current, as the diameter of the wire increases, the charge density decreases.

CHECK YOUR UNDERSTANDING 9.4

The current density is proportional to the current and inversely proportional to the area. If the current density in a conducting wire increases, what would happen to the drift velocity of the charges in the wire?

What is the significance of the current density? The current density is proportional to the current, and the current is the number of charges that pass through a cross-sectional area per second. The charges move through the conductor, accelerated by the electric force provided by the electrical field. The electrical field is created when a voltage is applied across the conductor. In Ohm's Law, we will use this relationship between the current density and the electrical field to examine the relationship between the current through a conductor and the voltage applied.

9.3 Resistivity and Resistance

Learning Objectives

By the end of this section, you will be able to:

- Differentiate between resistance and resistivity
- Define the term conductivity
- Describe the electrical component known as a resistor
- State the relationship between resistance of a resistor and its length, cross-sectional area, and resistivity
- State the relationship between resistivity and temperature

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—that are necessary to maintain a current. All such devices create a potential difference and are referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electrical field. The electrical field, in turn, exerts force on free charges, causing current. The amount of current depends not only on the magnitude of the voltage, but also on the characteristics of the material that the current is flowing through. The material can resist the flow of the charges, and the measure of how much a material resists the flow of charges is known as the *resistivity*. This resistivity is crudely analogous to the friction between two materials that resists motion.

Resistivity

When a voltage is applied to a conductor, an electrical field $\vec{\mathbf{E}}$ is created, and charges in the conductor feel a force due to the electrical field. The current density $\vec{\mathbf{J}}$ that results depends on the electrical field and the properties of the material. This dependence can be very complex. In some materials, including metals at a given temperature, the current density is approximately proportional to the electrical field. In these cases, the current density can be modeled as

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}},$$

where σ is the **electrical conductivity**. The electrical conductivity is analogous to thermal conductivity and is a measure of a material's ability to conduct or transmit electricity. Conductors have a higher electrical conductivity than insulators. Since the electrical conductivity is $\sigma = J/E$, the units are

$$\sigma = \frac{[J]}{[E]} = \frac{\text{A/m}^2}{\text{V/m}} = \frac{\text{A}}{\text{V} \cdot \text{m}}.$$

Here, we define a unit named the **ohm** with the Greek symbol uppercase omega, Ω. The unit is named after Georg Simon Ohm, whom we will discuss later in this chapter. The Ω is used to avoid confusion with the number 0. One ohm equals one volt per amp: $1\ \Omega = 1\ \text{V/A}$. The units of electrical conductivity are therefore $(\Omega \cdot \text{m})^{-1}$.

Conductivity is an intrinsic property of a material. Another intrinsic property of a material is the **resistivity**, or electrical resistivity. The resistivity of a material is a measure of how strongly a material opposes the flow of electrical current. The symbol for resistivity is the lowercase Greek letter rho, ρ, and resistivity is the reciprocal of electrical conductivity:

$$\rho = \frac{1}{\sigma}.$$

The unit of resistivity in SI units is the ohm-meter $(\Omega \cdot \text{m})$. We can define the resistivity in terms of the electrical field and the current density,

$$\rho = \frac{E}{J}. \tag{9.6}$$

The greater the resistivity, the larger the field needed to produce a given current density. The lower the resistivity, the larger the current density produced by a given electrical field. Good conductors have a high conductivity and low resistivity. Good insulators have a low conductivity and a high resistivity. Table 9.1 lists resistivity and conductivity values for various materials.

Material	Conductivity, σ $(\Omega \cdot \text{m})^{-1}$	Resistivity, ρ $(\Omega \cdot \text{m})$	Temperature Coefficient, α $(°\text{C})^{-1}$
Conductors			
Silver	6.29×10^7	1.59×10^{-8}	0.0038
Copper	5.95×10^7	1.68×10^{-8}	0.0039
Gold	4.10×10^7	2.44×10^{-8}	0.0034
Aluminum	3.77×10^7	2.65×10^{-8}	0.0039
Tungsten	1.79×10^7	5.60×10^{-8}	0.0045

Material	**Conductivity, σ** $(\Omega \cdot m)^{-1}$	**Resistivity, ρ** $(\Omega \cdot m)$	**Temperature Coefficient, α** $(°C)^{-1}$
Iron	1.03×10^{7}	9.71×10^{-8}	0.0065
Platinum	0.94×10^{7}	10.60×10^{-8}	0.0039
Steel	0.50×10^{7}	20.00×10^{-8}	
Lead	0.45×10^{7}	22.00×10^{-8}	
Manganin (Cu, Mn, Ni alloy)	0.21×10^{7}	48.20×10^{-8}	0.000002
Constantan (Cu, Ni alloy)	0.20×10^{7}	49.00×10^{-8}	0.00003
Mercury	0.10×10^{7}	98.00×10^{-8}	0.0009
Nichrome (Ni, Fe, Cr alloy)	0.10×10^{7}	100.00×10^{-8}	0.0004
Semiconductors[1]			
Carbon (pure)	2.86×10^{4}	3.50×10^{-5}	−0.0005
Carbon	$(2.86 - 1.67) \times 10^{-6}$	$(3.5 - 60) \times 10^{-5}$	−0.0005
Germanium (pure)		600×10^{-3}	−0.048
Germanium		$(1 - 600) \times 10^{-3}$	−0.050
Silicon (pure)		2300	−0.075
Silicon		0.1 – 2300	−0.07
Insulators			
Amber	2.00×10^{-15}	5×10^{14}	
Glass	$10^{-9} - 10^{-14}$	$10^{9} - 10^{14}$	
Lucite	$<10^{-13}$	$>10^{13}$	
Mica	$10^{-11} - 10^{-15}$	$10^{11} - 10^{15}$	
Quartz (fused)	1.33×10^{-18}	75×10^{16}	
Rubber (hard)	$10^{-13} - 10^{-16}$	$10^{13} - 10^{16}$	
Sulfur	10^{-15}	10^{15}	

Material	Conductivity, σ $(\Omega \cdot \text{m})^{-1}$	Resistivity, ρ $(\Omega \cdot \text{m})$	Temperature Coefficient, α $(°\text{C})^{-1}$
Teflon™	$<10^{-13}$	$>10^{13}$	
Wood	$10^{-8} - 10^{-11}$	$10^{8} - 10^{11}$	

Table 9.1 Resistivities and Conductivities of Various Materials at 20 °C [1] Values depend strongly on amounts and types of impurities.

The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivity. Conductors have the smallest resistivity, and insulators have the largest; semiconductors have intermediate resistivity. Conductors have varying but large, free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as we will explore in later chapters.

CHECK YOUR UNDERSTANDING 9.5

Copper wires use routinely used for extension cords and house wiring for several reasons. Copper has the highest electrical conductivity rating, and therefore the lowest resistivity rating, of all nonprecious metals. Also important is the tensile strength, where the tensile strength is a measure of the force required to pull an object to the point where it breaks. The tensile strength of a material is the maximum amount of tensile stress it can take before breaking. Copper has a high tensile strength, $2 \times 10^8 \, \frac{\text{N}}{\text{m}^2}$. A third important characteristic is ductility. Ductility is a measure of a material's ability to be drawn into wires and a measure of the flexibility of the material, and copper has a high ductility. Summarizing, for a conductor to be a suitable candidate for making wire, there are at least three important characteristics: low resistivity, high tensile strength, and high ductility. What other materials are used for wiring and what are the advantages and disadvantages?

INTERACTIVE

View this interactive simulation (https://openstax.org/l/21resistwire) to see what the effects of the cross-sectional area, the length, and the resistivity of a wire are on the resistance of a conductor. Adjust the variables using slide bars and see if the resistance becomes smaller or larger.

Temperature Dependence of Resistivity

Looking back at Table 9.1, you will see a column labeled "Temperature Coefficient." The resistivity of some materials has a strong temperature dependence. In some materials, such as copper, the resistivity increases with increasing temperature. In fact, in most conducting metals, the resistivity increases with increasing temperature. The increasing temperature causes increased vibrations of the atoms in the lattice structure of the metals, which impede the motion of the electrons. In other materials, such as carbon, the resistivity decreases with increasing temperature. In many materials, the dependence is approximately linear and can be modeled using a linear equation:

$$\rho \approx \rho_0 \left[1 + \alpha (T - T_0)\right], \tag{9.7}$$

where ρ is the resistivity of the material at temperature T, α is the temperature coefficient of the material, and ρ_0 is the resistivity at T_0, usually taken as $T_0 = 20.00\ °\text{C}$.

Note also that the temperature coefficient α is negative for the semiconductors listed in Table 9.1, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

Resistance

We now consider the resistance of a wire or component. The resistance is a measure of how difficult it is to pass current through a wire or component. Resistance depends on the resistivity. The resistivity is a characteristic of the material used to fabricate a wire or other electrical component, whereas the resistance is a characteristic of the wire or component.

To calculate the resistance, consider a section of conducting wire with cross-sectional area A, length L, and resistivity ρ. A battery is connected across the conductor, providing a potential difference ΔV across it (Figure 9.13). The potential difference produces an electrical field that is proportional to the current density, according to $\vec{\mathbf{E}} = \rho\vec{\mathbf{J}}$.

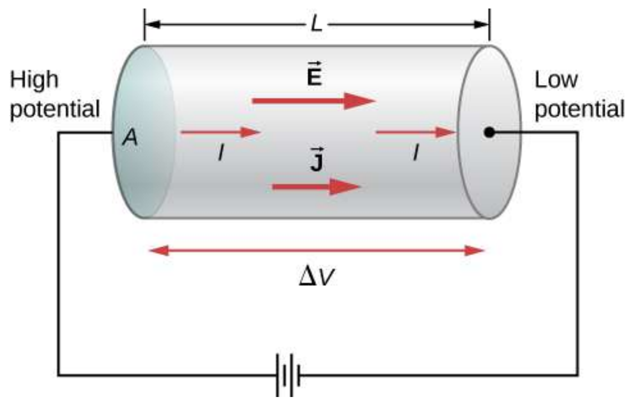


Figure 9.13 A potential provided by a battery is applied to a segment of a conductor with a cross-sectional area A and a length L.

The magnitude of the electrical field across the segment of the conductor is equal to the voltage divided by the length, $E = V/L$, and the magnitude of the current density is equal to the current divided by the cross-sectional area, $J = I/A$. Using this information and recalling that the electrical field is proportional to the resistivity and the current density, we can see that the voltage is proportional to the current:

$$\begin{aligned} E &= \rho J \\ \frac{V}{L} &= \rho\frac{I}{A} \\ V &= \left(\rho\frac{L}{A}\right) I. \end{aligned}$$

Resistance

The ratio of the voltage to the current is defined as the **resistance** R:

$$R \equiv \frac{V}{I}. \quad (9.8)$$

The resistance of a cylindrical segment of a conductor is equal to the resistivity of the material times the length divided by the area:

$$R \equiv \frac{V}{I} = \rho\frac{L}{A}. \quad (9.9)$$

The unit of resistance is the ohm, Ω. For a given voltage, the higher the resistance, the lower the current.

Resistors

A common component in electronic circuits is the resistor. The resistor can be used to reduce current flow or provide a voltage drop. Figure 9.14 shows the symbols used for a resistor in schematic diagrams of a circuit. Two commonly used standards for circuit diagrams are provided by the American National Standard Institute (ANSI, pronounced "AN-see") and the International Electrotechnical Commission (IEC). Both systems are commonly used. We use the ANSI standard in this text for its visual recognition, but we note that for larger, more complex circuits, the IEC standard may have a cleaner presentation, making it easier to read.

Figure 9.14 Symbols for a resistor used in circuit diagrams. (a) The ANSI symbol; (b) the IEC symbol.

Material and shape dependence of resistance

A resistor can be modeled as a cylinder with a cross-sectional area A and a length L, made of a material with a resistivity ρ (Figure 9.15). The resistance of the resistor is $R = \rho \frac{L}{A}$.

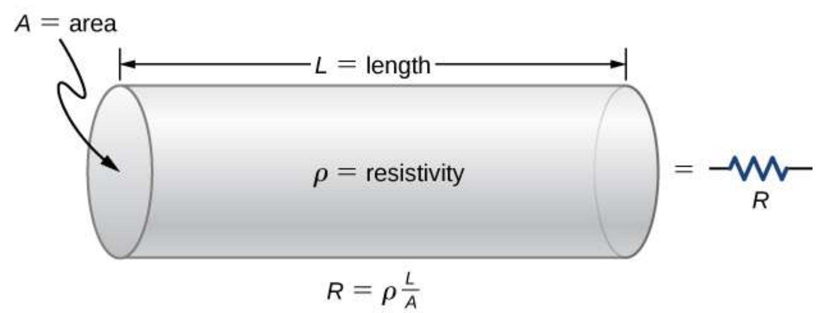
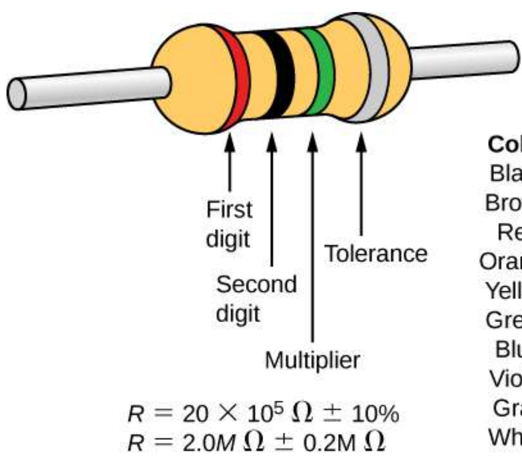


Figure 9.15 A model of a resistor as a uniform cylinder of length L and cross-sectional area A. Its resistance to the flow of current is analogous to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance.

The most common material used to make a resistor is carbon. A carbon track is wrapped around a ceramic core, and two copper leads are attached. A second type of resistor is the metal film resistor, which also has a ceramic core. The track is made from a metal oxide material, which has semiconductive properties similar to carbon. Again, copper leads are inserted into the ends of the resistor. The resistor is then painted and marked for identification. A resistor has four colored bands, as shown in Figure 9.16.



Color	Digit	Multiplier
Black	0	10^0
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Tolerance	
Gold	5%
Silver	10%

Figure 9.16 Many resistors resemble the figure shown above. The four bands are used to identify the resistor. The first two colored bands represent the first two digits of the resistance of the resistor. The third color is the multiplier. The fourth color represents the tolerance of

the resistor. The resistor shown has a resistance of $20 \times 10^5\ \Omega \pm 10\%$.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^5\ \Omega$, whereas the resistance of the human heart is about $10^3\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all at low temperatures. As we have seen, resistance is related to the shape of an object and the material of which it is composed.

EXAMPLE 9.5

Current Density, Resistance, and Electrical field for a Current-Carrying Wire

Calculate the current density, resistance, and electrical field of a 5-m length of copper wire with a diameter of 2.053 mm (12-gauge) carrying a current of $I = 10\ \text{mA}$.

Strategy

We can calculate the current density by first finding the cross-sectional area of the wire, which is $A = 3.31\ \text{mm}^2$, and the definition of current density $J = \frac{I}{A}$. The resistance can be found using the length of the wire $L = 5.00\ \text{m}$, the area, and the resistivity of copper $\rho = 1.68 \times 10^{-8}\ \Omega \cdot \text{m}$, where $R = \rho \frac{L}{A}$. The resistivity and current density can be used to find the electrical field.

Solution

First, we calculate the current density:

$$J = \frac{I}{A} = \frac{10 \times 10^{-3}\,\text{A}}{3.31 \times 10^{-6}\,\text{m}^2} = 3.02 \times 10^3\,\frac{\text{A}}{\text{m}^2}.$$

The resistance of the wire is

$$R = \rho\frac{L}{A} = \left(1.68 \times 10^{-8}\ \Omega \cdot \text{m}\right)\frac{5.00\ \text{m}}{3.31 \times 10^{-6}\,\text{m}^2} = 0.025\,\Omega\,.$$

Finally, we can find the electrical field:

$$E = \rho J = 1.68 \times 10^{-8}\ \Omega \cdot \text{m}\left(3.02 \times 10^3\,\frac{\text{A}}{\text{m}^2}\right) = 5.07 \times 10^{-5}\,\frac{\text{V}}{\text{m}}.$$

Significance

From these results, it is not surprising that copper is used for wires for carrying current because the resistance is quite small. Note that the current density and electrical field are independent of the length of the wire, but the voltage depends on the length.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ. For a cylinder, we know $R = \rho\frac{L}{A}$, so if L and A do not change greatly with temperature, R has the same temperature dependence as ρ. (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, so the effect of temperature on L and A is about two orders of magnitude less than on ρ.) Thus,

$$R = R_0(1 + \alpha\Delta T) \qquad \textbf{9.10}$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance (usually taken to be $20.00\ °\text{C}$) and R is the resistance after a temperature change ΔT. The color code gives the resistance of the resistor at a temperature of $T = 20.00\ °\text{C}$.

Numerous thermometers are based on the effect of temperature on resistance (Figure 9.17). One of the most common thermometers is based on the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.

Figure 9.17 These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance.

EXAMPLE 9.6

Calculating Resistance

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than $100\ °\text{C}$, for tungsten, the equations work reasonably well for very large temperature changes. A tungsten filament at $20\ °\text{C}$ has a resistance of $0.350\ \Omega$. What would the resistance be if the temperature is increased to $2850\ °\text{C}$?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament is given as $R_0 = 0.350\ \Omega$ and the temperature change is $\Delta T = 2830\ °\text{C}$.

Solution

The resistance of the hotter filament R is obtained by entering known values into the above equation:

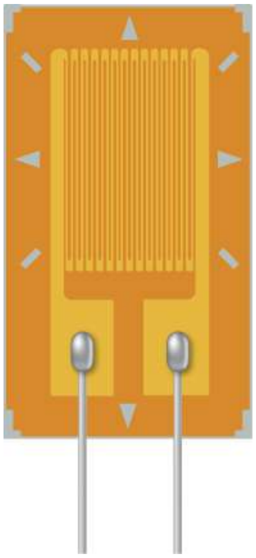
$$R = R_0\,(1 + \alpha\Delta T) = (0.350\ \Omega)\left[1 + \left(\frac{4.5\ \times\ 10^{-3}}{°\text{C}}\right)(2830\ °\text{C})\right] = 4.8\ \Omega\,.$$

Significance

Notice that the resistance changes by more than a factor of 10 as the filament warms to the high temperature and the current through the filament depends on the resistance of the filament and the voltage applied. If the filament is used in an incandescent light bulb, the initial current through the filament when the bulb is first energized will be higher than the current after the filament reaches the operating temperature.

CHECK YOUR UNDERSTANDING 9.6

A strain gauge is an electrical device to measure strain, as shown below. It consists of a flexible, insulating backing that supports a conduction foil pattern. The resistance of the foil changes as the backing is stretched. How does the strain gauge resistance change? Is the strain gauge affected by temperature changes?



EXAMPLE 9.7

The Resistance of Coaxial Cable

Long cables can sometimes act like antennas, picking up electronic noise, which are signals from other equipment and appliances. Coaxial cables are used for many applications that require this noise to be eliminated. For example, they can be found in the home in cable TV connections or other audiovisual connections. Coaxial cables consist of an inner conductor of radius r_{i} surrounded by a second, outer concentric conductor with radius r_{o} (Figure 9.18). The space between the two is normally filled with an insulator such as polyethylene plastic. A small amount of radial leakage current occurs between the two conductors. Determine the resistance of a coaxial cable of length L.

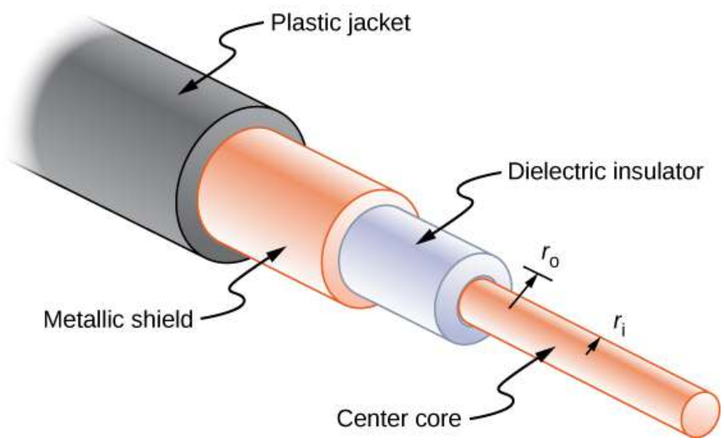


Figure 9.18 Coaxial cables consist of two concentric conductors separated by insulation. They are often used in cable TV or other audiovisual connections.

Strategy

We cannot use the equation $R = \rho\frac{L}{A}$ directly. Instead, we look at concentric cylindrical shells, with thickness dr, and integrate.

Solution

We first find an expression for dR and then integrate from r_{i} to r_{o},

$$\begin{aligned} dR &= \frac{\rho}{A} dr = \frac{\rho}{2\pi r L} dr, \\ R &= \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} dR = \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{\rho}{2\pi r L} dr = \frac{\rho}{2\pi L} \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{1}{r} dr = \frac{\rho}{2\pi L} \ln \frac{r_{\mathrm{o}}}{r_{\mathrm{i}}}. \end{aligned}$$

Significance

The resistance of a coaxial cable depends on its length, the inner and outer radii, and the resistivity of the material separating the two conductors. Since this resistance is not infinite, a small leakage current occurs between the two conductors. This leakage current leads to the attenuation (or weakening) of the signal being sent through the cable.

CHECK YOUR UNDERSTANDING 9.7

The resistance between the two conductors of a coaxial cable depends on the resistivity of the material separating the two conductors, the length of the cable and the inner and outer radius of the two conductor. If you are designing a coaxial cable, how does the resistance between the two conductors depend on these variables?

INTERACTIVE

View this simulation (https://openstax.org/l/21batteryresist) to see how the voltage applied and the resistance of the material the current flows through affects the current through the material. You can visualize the collisions of the electrons and the atoms of the material effect the temperature of the material.

9.4 Ohm's Law

Learning Objectives

By the end of this section, you will be able to:

- Describe Ohm's law
- Recognize when Ohm's law applies and when it does not

We have been discussing three electrical properties so far in this chapter: current, voltage, and resistance. It turns out that many materials exhibit a simple relationship among the values for these properties, known as Ohm's law. Many other materials do not show this relationship, so despite being called Ohm's law, it is not considered a law of nature, like Newton's laws or the laws of thermodynamics. But it is very useful for calculations involving materials that do obey Ohm's law.

Description of Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$

This important relationship is the basis for **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law, which is to say that it is an experimentally observed phenomenon, like friction. Such a linear relationship doesn't always occur. Any material, component, or device that obeys Ohm's law, where the current through the device is proportional to the voltage applied, is known as an **ohmic** material or ohmic component. Any material or component that does not obey Ohm's law is known as a **nonohmic** material or nonohmic component.

Ohm’s Experiment

In a paper published in 1827, Georg Ohm described an experiment in which he measured voltage across and current through various simple electrical circuits containing various lengths of wire. A similar experiment is shown in Figure 9.19. This experiment is used to observe the current through a resistor that results from an applied voltage. In this simple circuit, a resistor is connected in series with a battery. The voltage is measured with a voltmeter, which must be placed across the resistor (in parallel with the resistor). The current is measured with an ammeter, which must be in line with the resistor (in series with the resistor).

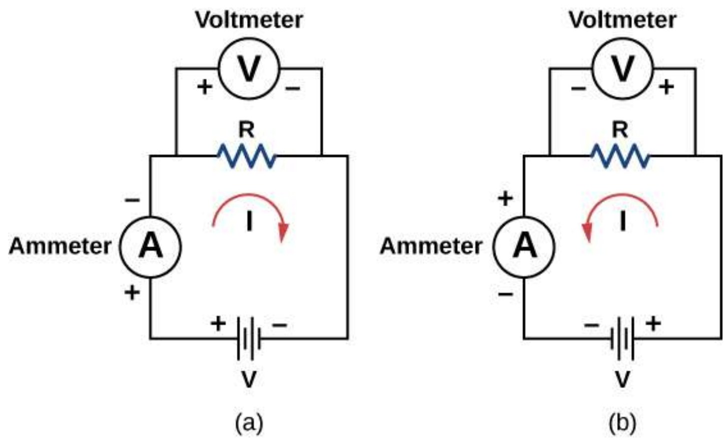


Figure 9.19 The experimental set-up used to determine if a resistor is an ohmic or nonohmic device. (a) When the battery is attached, the current flows in the clockwise direction and the voltmeter and ammeter have positive readings. (b) When the leads of the battery are switched, the current flows in the counterclockwise direction and the voltmeter and ammeter have negative readings.

In this updated version of Ohm’s original experiment, several measurements of the current were made for several different voltages. When the battery was hooked up as in Figure 9.19(a), the current flowed in the clockwise direction and the readings of the voltmeter and ammeter were positive. Does the behavior of the current change if the current flowed in the opposite direction? To get the current to flow in the opposite direction, the leads of the battery can be switched. When the leads of the battery were switched, the readings of the voltmeter and ammeter readings were negative because the current flowed in the opposite direction, in this case, counterclockwise. Results of a similar experiment are shown in Figure 9.20.

I(A)	V(V)
–2.74	–10.00
–2.25	–9.00
–2.00	–8.00
–1.75	–7.00
–1.50	–6.00
–1.49	–5.00
–1.00	–4.00
–0.51	–3.00
–0.74	–2.00
–0.49	–1.00
+0.00	+0.00
+0.49	+1.00
+0.50	+2.00
+0.99	+3.00
+0.76	+4.00
+1.01	+5.00
+1.74	+6.00
+1.75	+7.00
+2.00	+8.00
+2.49	+9.00
+2.50	+10.00

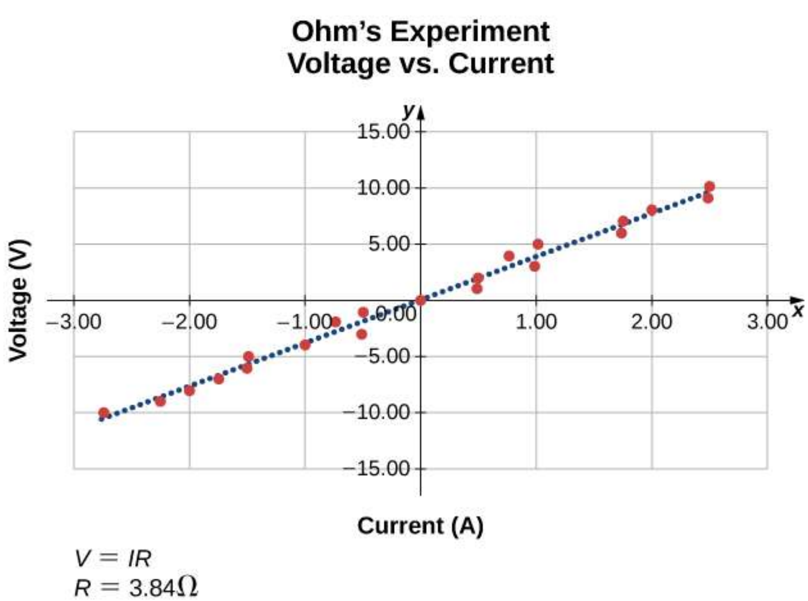


Figure 9.20 A resistor is placed in a circuit with a battery. The voltage applied varies from −10.00 V to +10.00 V, increased by 1.00-V

increments. A plot shows values of the voltage versus the current typical of what a casual experimenter might find.

In this experiment, the voltage applied across the resistor varies from −10.00 to +10.00 V, by increments of 1.00 V. The current through the resistor and the voltage across the resistor are measured. A plot is made of the voltage versus the current, and the result is approximately linear. The slope of the line is the resistance, or the voltage divided by the current. This result is known as Ohm's law:

$$V = IR, \tag{9.11}$$

where V is the voltage measured in volts across the object in question, I is the current measured through the object in amps, and R is the resistance in units of ohms. As stated previously, any device that shows a linear relationship between the voltage and the current is known as an ohmic device. A resistor is therefore an ohmic device.

EXAMPLE 9.8

Measuring Resistance

A carbon resistor at room temperature (20 °C) is attached to a 9.00-V battery and the current measured through the resistor is 3.00 mA. (a) What is the resistance of the resistor measured in ohms? (b) If the temperature of the resistor is increased to 60 °C by heating the resistor, what is the current through the resistor?

Strategy

(a) The resistance can be found using Ohm's law. Ohm's law states that $V = IR$, so the resistance can be found using $R = V/I$.

(b) First, the resistance is temperature dependent so the new resistance after the resistor has been heated can be found using $R = R_0\,(1 + \alpha\Delta T)$. The current can be found using Ohm's law in the form $I = V/R$.

Solution

a. Using Ohm's law and solving for the resistance yields the resistance at room temperature:

$$R = \frac{V}{I} = \frac{9.00\text{ V}}{3.00 \times 10^{-3}\text{ A}} = 3.00 \times 10^{3}\ \Omega = 3.00\text{ k}\,\Omega.$$

b. The resistance at 60 °C can be found using $R = R_0\,(1 + \alpha\Delta T)$ where the temperature coefficient for carbon is $\alpha = -0.0005$. $R = R_0\,(1 + \alpha\Delta T) = 3.00 \times 10^3\,(1 - 0.0005\,(60\,°\text{C} - 20\,°\text{C})) = 2.94\text{ k}\,\Omega.$ The current through the heated resistor is

$$I = \frac{V}{R} = \frac{9.00\text{ V}}{2.94 \times 10^{3}\ \Omega} = 3.06 \times 10^{-3}\text{ A} = 3.06\text{ mA}.$$

Significance

A change in temperature of 40 °C resulted in a 2.00% change in current. This may not seem like a very great change, but changing electrical characteristics can have a strong effect on the circuits. For this reason, many electronic appliances, such as computers, contain fans to remove the heat dissipated by components in the electric circuits.

CHECK YOUR UNDERSTANDING 9.8

The voltage supplied to your house varies as $V(t) = V_{\max}\sin(2\pi f t)$. If a resistor is connected across this voltage, will Ohm's law $V = IR$ still be valid?

INTERACTIVE

See how the [equation form of Ohm's law (https://openstax.org/l/21ohmslaw)](https://openstax.org/l/21ohmslaw) relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

Nonohmic devices do not exhibit a linear relationship between the voltage and the current. One such device is the semiconducting circuit element known as a diode. A **diode** is a circuit device that allows current flow in only one direction. A diagram of a simple circuit consisting of a battery, a diode, and a resistor is shown in Figure 9.21. Although we do not cover the theory of the diode in this section, the diode can be tested to see if it is an ohmic or a nonohmic device.

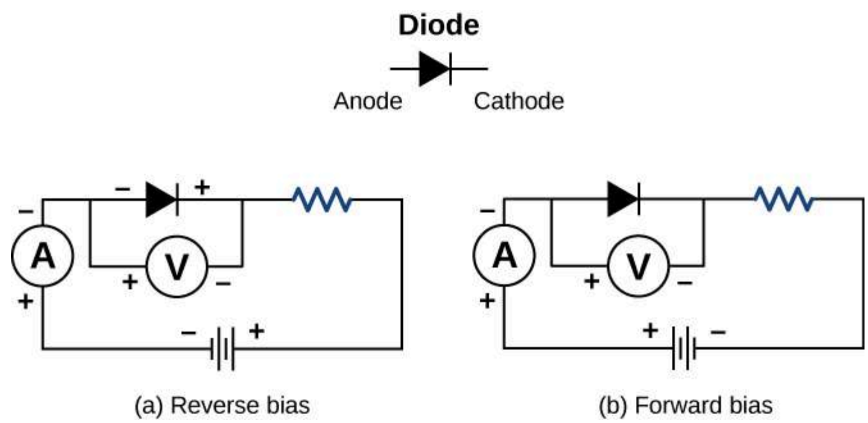


Figure 9.21 A diode is a semiconducting device that allows current flow only if the diode is forward biased, which means that the anode is positive and the cathode is negative.

A plot of current versus voltage is shown in Figure 9.22. Note that the behavior of the diode is shown as current versus voltage, whereas the resistor operation was shown as voltage versus current. A diode consists of an anode and a cathode. When the anode is at a negative potential and the cathode is at a positive potential, as shown in part (a), the diode is said to have reverse bias. With reverse bias, the diode has an extremely large resistance and there is very little current flow—essentially zero current—through the diode and the resistor. As the voltage applied to the circuit increases, the current remains essentially zero, until the voltage reaches the breakdown voltage and the diode conducts current, as shown in Figure 9.22. When the battery and the potential across the diode are reversed, making the anode positive and the cathode negative, the diode conducts and current flows through the diode if the voltage is greater than 0.7 V. The resistance of the diode is close to zero. (This is the reason for the resistor in the circuit; if it were not there, the current would become very large.) You can see from the graph in Figure 9.22 that the voltage and the current do not have a linear relationship. Thus, the diode is an example of a nonohmic device.

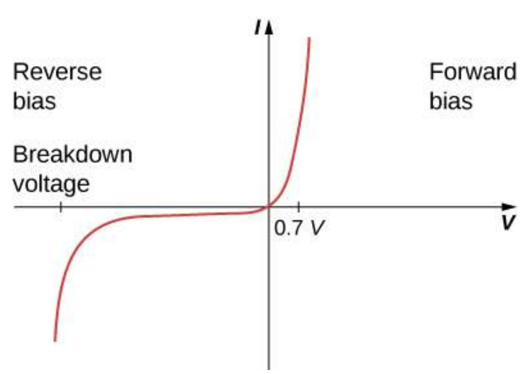


Figure 9.22 When the voltage across the diode is negative and small, there is very little current flow through the diode. As the voltage reaches the breakdown voltage, the diode conducts. When the voltage across the diode is positive and greater than 0.7 V (the actual voltage value depends on the diode), the diode conducts. As the voltage applied increases, the current through the diode increases, but the voltage across the diode remains approximately 0.7 V.

Ohm's law is commonly stated as $V = IR$, but originally it was stated as a microscopic view, in terms of the current density, the conductivity, and the electrical field. This microscopic view suggests the proportionality $V \propto I$ comes from the drift velocity of the free electrons in the metal that results from an applied electrical field. As stated earlier, the current density is proportional to the applied electrical field. The reformulation of Ohm's law is credited to Gustav Kirchhoff, whose name we will see again in the next chapter.

9.5 Electrical Energy and Power

Learning Objectives

By the end of this section, you will be able to:

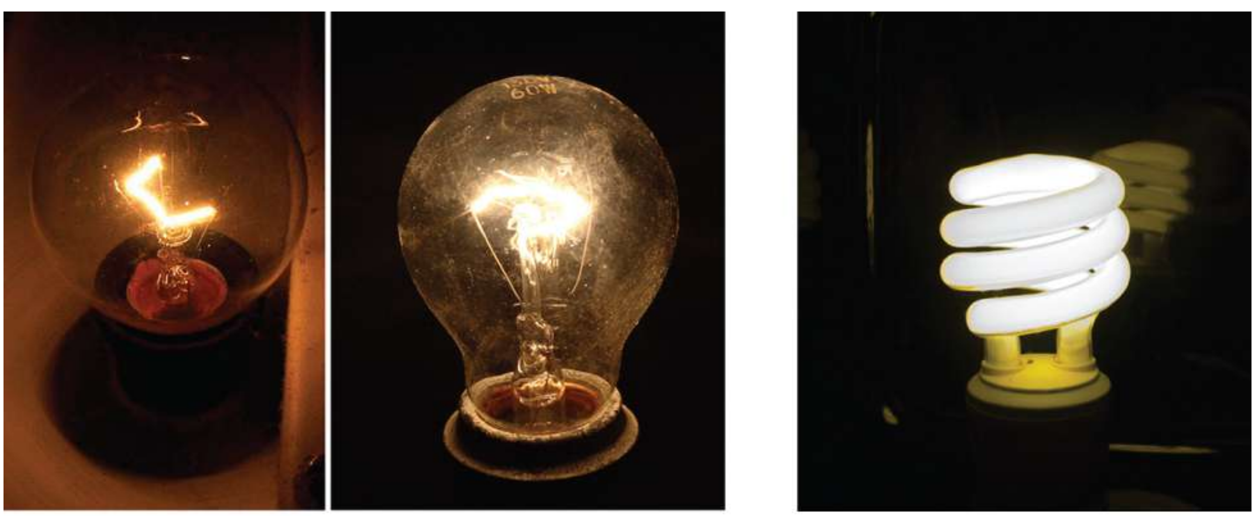
- Express electrical power in terms of the voltage and the current
- Describe the power dissipated by a resistor in an electric circuit
- Calculate the energy efficiency and cost effectiveness of appliances and equipment

In an electric circuit, electrical energy is continuously converted into other forms of energy. For example, when a current flows in a conductor, electrical energy is converted into thermal energy within the conductor. The electrical field, supplied by the voltage source, accelerates the free electrons, increasing their kinetic energy for a short time. This increased kinetic energy is converted into thermal energy through collisions with the ions of the lattice structure of the conductor. In Work and Kinetic Energy, we defined power as the rate at which work is done by a force measured in watts. Power can also be defined as the rate at which energy is transferred. In this section, we discuss the time rate of energy transfer, or power, in an electric circuit.

Power in Electric Circuits

Power is associated by many people with electricity. Power transmission lines might come to mind. We also think of light bulbs in terms of their power ratings in watts. What is the expression for **electric power**?

Let us compare a 25-W bulb with a 60-W bulb (Figure 9.23(a)). The 60-W bulb glows brighter than the 25-W bulb. Although it is not shown, a 60-W light bulb is also warmer than the 25-W bulb. The heat and light is produced by from the conversion of electrical energy. The kinetic energy lost by the electrons in collisions is converted into the internal energy of the conductor and radiation. How are voltage, current, and resistance related to electric power?



(a) (b)

Figure 9.23 (a) Pictured above are two incandescent bulbs: a 25-W bulb (left) and a 60-W bulb (right). The 60-W bulb provides a higher intensity light than the 25-W bulb. The electrical energy supplied to the light bulbs is converted into heat and light. (b) This compact fluorescent light (CFL) bulb puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit a: modification of works by "Dickbauch"/Wikimedia Commons and Greg Westfall; credit b: modification of work by "dbgg1979"/Flickr)

To calculate electric power, consider a voltage difference existing across a material (Figure 9.24). The electric potential V_1 is higher than the electric potential at V_2, and the voltage difference is negative $V = V_2 - V_1$. As discussed in Electric Potential, an electrical field exists between the two potentials, which points from the higher potential to the lower potential. Recall that the electrical potential is defined as the potential energy per charge, $V = \Delta U/q$, and the charge ΔQ loses potential energy moving through the potential difference.

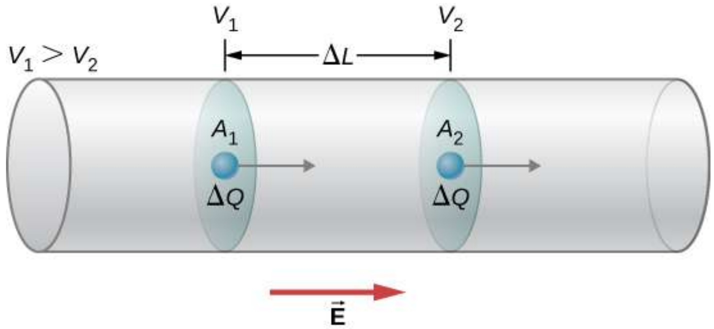


Figure 9.24 When there is a potential difference across a conductor, an electrical field is present that points in the direction from the higher potential to the lower potential.

If the charge is positive, the charge experiences a force due to the electrical field $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = \Delta Q\vec{\mathbf{E}}$. This force is necessary to keep the charge moving. This force does not act to accelerate the charge through the entire distance ΔL because of the interactions of the charge with atoms and free electrons in the material. The speed, and therefore the kinetic energy, of the charge do not increase during the entire trip across ΔL, and charge passing through area A_2 has the same drift velocity v_{d} as the charge that passes through area A_1. However, work is done on the charge, by the electrical field, which changes the potential energy. Since the change in the electrical potential difference is negative, the electrical field is found to be

$$E = -\frac{(V_2 - V_1)}{\Delta L} = \frac{V}{\Delta L}.$$

The work done on the charge is equal to the electric force times the length at which the force is applied,

$$W = F\Delta L = (\Delta Q E)\,\Delta L = \left(\Delta Q\frac{V}{\Delta L}\right)\Delta L = \Delta QV = \Delta U.$$

The charge moves at a drift velocity v_{d} so the work done on the charge results in a loss of potential energy, but the average kinetic energy remains constant. The lost electrical potential energy appears as thermal energy in the material. On a microscopic scale, the energy transfer is due to collisions between the charge and the molecules of the material, which leads to an increase in temperature in the material. The loss of potential energy results in an increase in the temperature of the material, which is dissipated as radiation. In a resistor, it is dissipated as heat, and in a light bulb, it is dissipated as heat and light.

The power dissipated by the material as heat and light is equal to the time rate of change of the work:

$$P = \frac{\Delta U}{\Delta t} = -\frac{\Delta QV}{\Delta t} = IV.$$

With a resistor, the voltage drop across the resistor is dissipated as heat. Ohm's law states that the voltage across the resistor is equal to the current times the resistance, $V = IR$. The power dissipated by the resistor is therefore

$$P = IV = I\,(IR) = I^2 R \text{ or } P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}.$$

If a resistor is connected to a battery, the power dissipated as radiant energy by the wires and the resistor is equal to $P = IV = I^2 R = \frac{V^2}{R}$. The power supplied from the battery is equal to current times the voltage, $P = IV$.

Electric Power

The electric power gained or lost by any device has the form

$$P = IV. \tag{9.12}$$

The power dissipated by a resistor has the form

$$P = I^2 R = \frac{V^2}{R}. \tag{9.13}$$

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature, its resistance is higher, too.

EXAMPLE 9.9

Calculating Power in Electric Devices

A DC winch motor is rated at 20.00 A with a voltage of 115 V. When the motor is running at its maximum power, it can lift an object with a weight of 4900.00 N a distance of 10.00 m, in 30.00 s, at a constant speed. (a) What is the power consumed by the motor? (b) What is the power used in lifting the object? Ignore air resistance. (c) Assuming that the difference in the power consumed by the motor and the power used lifting the object are dissipated as heat by the resistance of the motor, estimate the resistance of the motor?

Strategy

(a) The power consumed by the motor can be found using $P = IV$. (b) The power used in lifting the object at a constant speed can be found using $P = Fv$, where the speed is the distance divided by the time. The upward force supplied by the motor is equal to the weight of the object because the acceleration is zero. (c) The resistance of the motor can be found using $P = I^2 R$.

Solution

a. The power consumed by the motor is equal to $P = IV$ and the current is given as 20.00 A and the voltage is 115.00 V:

$$P = IV = (20.00\text{ A})\,115.00\text{ V} = 2300.00\text{ W}.$$

b. The power used lifting the object is equal to $P = Fv$ where the force is equal to the weight of the object (1960 N) and the magnitude of the velocity is $v = \frac{10.00\text{ m}}{30.00\text{ s}} = 0.33\,\frac{\text{m}}{\text{s}}$,

$$P = Fv = (4900\text{ N})\,0.33\text{ m/s} = 1633.33\text{ W}.$$

c. The difference in the power equals 2300.00 W − 1633.33 W = 666.67 W and the resistance can be found using $P = I^2 R$:

$$R = \frac{P}{I^2} = \frac{666.67\text{ W}}{(20.00\text{ A})^2} = 1.67\,\Omega.$$

Significance

The resistance of the motor is quite small. The resistance of the motor is due to many windings of copper wire. The power dissipated by the motor can be significant since the thermal power dissipated by the motor is proportional to the square of the current $\left(P = I^2 R\right)$.

CHECK YOUR UNDERSTANDING 9.9

Electric motors have a reasonably high efficiency. A 100-hp motor can have an efficiency of 90% and a 1-hp motor can have an efficiency of 80%. Why is it important to use high-performance motors?

A fuse (Figure 9.25) is a device that protects a circuit from currents that are too high. A fuse is basically a short piece of wire between two contacts. As we have seen, when a current is running through a conductor, the kinetic energy of the charge carriers is converted into thermal energy in the conductor. The piece of wire in the fuse is under tension and has a low melting point. The wire is designed to heat up and break at the rated current. The fuse is destroyed and must be replaced, but it protects the rest of the circuit. Fuses act quickly, but there is a small time delay while the wire heats up and breaks.

Figure 9.25 A fuse consists of a piece of wire between two contacts. When a current passes through the wire that is greater than the rated current, the wire melts, breaking the connection. Pictured is a "blown" fuse where the wire broke protecting a circuit (credit: modification of work by "Shardayyy"/Flickr).

Circuit breakers are also rated for a maximum current, and open to protect the circuit, but can be reset. Circuit breakers react much faster. The operation of circuit breakers is not within the scope of this chapter and will be discussed in later chapters. Another method of protecting equipment and people is the ground fault circuit interrupter (GFCI), which is common in bathrooms and kitchens. The GFCI outlets respond very quickly to changes in current. These outlets open when there is a change in magnetic field produced by current-carrying conductors, which is also beyond the scope of this chapter and is covered in a later chapter.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = \frac{dE}{dt}$, we see that

$$E = \int P dt$$

is the energy used by a device using power P for a time interval t. If power is delivered at a constant rate, then then the energy can be found by $E = Pt$. For example, the more light bulbs burning, the greater P used; the longer they are on, the greater t is.

The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electrical appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted into joules. You can prove to yourself that $1 \text{ kW} \cdot \text{h} = 3.6 \times 10^6 \text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This not only reduces the cost but also results in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, and the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFLs). (See Figure 9.23(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.)

The heat transfer from these CFLs is less, and they last up to 10 times longer than incandescent bulbs. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last five times longer than CFLs.

EXAMPLE 9.10

Calculating the Cost Effectiveness of LED Bulb

The typical replacement for a 100-W incandescent bulb is a 20-W LED bulb. The 20-W LED bulb can provide the same amount of light output as the 100-W incandescent light bulb. What is the cost savings for using the LED bulb in place of the incandescent bulb for one year, assuming $0.10 per kilowatt-hour is the average energy rate charged by the power company? Assume that the bulb is turned on for three hours a day.

Strategy

(a) Calculate the energy used during the year for each bulb, using $E = Pt$.

(b) Multiply the energy by the cost.

Solution

a. Calculate the power for each bulb.

$$\begin{aligned} E_{\text{Incandescent}} &= Pt = 100\text{ W}\left(\frac{1\text{ kW}}{1000\text{ W}}\right)\left(\frac{3\text{ h}}{\text{day}}\right)(365\text{ days}) = 109.5\text{ kW}\cdot\text{h} \\ E_{\text{LED}} &= Pt = 20\text{ W}\left(\frac{1\text{ kW}}{1000\text{ W}}\right)\left(\frac{3\text{ h}}{\text{day}}\right)(365\text{ days}) = 21.90\text{ kW}\cdot\text{h} \end{aligned}.$$

b. Calculate the cost for each.

$$\begin{aligned} \text{cost}_{\text{Incandescent}} &= 109.5\text{ kW-h}\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right) = \$10.95 \\ \text{cost}_{\text{LED}} &= 21.90\text{ kW-h}\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right) = \$2.19 \end{aligned}.$$

Significance

A LED bulb uses 80% less energy than the incandescent bulb, saving $8.76 over the incandescent bulb for one year. The LED bulb can cost $20.00 and the 100-W incandescent bulb can cost $0.75, which should be calculated into the computation. A typical lifespan of an incandescent bulb is 1200 hours and is 50,000 hours for the LED bulb. The incandescent bulb would last 1.08 years at 3 hours a day and the LED bulb would last 45.66 years. The initial cost of the LED bulb is high, but the cost to the home owner will be $0.69 for the incandescent bulbs versus $0.44 for the LED bulbs per year. (Note that the LED bulbs are coming down in price.) The cost savings per year is approximately $8.50, and that is just for one bulb.

CHECK YOUR UNDERSTANDING 9.10

Is the efficiency of the various light bulbs the only consideration when comparing the various light bulbs?

Changing light bulbs from incandescent bulbs to CFL or LED bulbs is a simple way to reduce energy consumption in homes and commercial sites. CFL bulbs operate with a much different mechanism than do incandescent lights. The mechanism is complex and beyond the scope of this chapter, but here is a very general description of the mechanism. CFL bulbs contain argon and mercury vapor housed within a spiral-shaped tube. The CFL bulbs use a “ballast” that increases the voltage used by the CFL bulb. The ballast produce an electrical current, which passes through the gas mixture and excites the gas molecules. The excited gas molecules produce ultraviolet (UV) light, which in turn stimulates the fluorescent coating on the inside of the tube. This coating fluoresces in the visible spectrum, emitting visible light. Traditional fluorescent tubes and CFL bulbs had a short time delay of up to a few seconds while the mixture was being “warmed up” and the molecules reached an excited state. It should be noted that these bulbs do contain mercury, which is poisonous, but if the bulb is broken, the mercury is never released. Even if the bulb is broken, the mercury tends to remain in the fluorescent coating. The amount is also quite small and the advantage of the energy saving may outweigh the disadvantage of using mercury.

The CFL light bulbs are being replaced with LED light bulbs, where LED stands for “light-emitting diode.” The diode was briefly discussed as a nonohmic device, made of semiconducting material, which essentially permits current flow in one direction. LEDs are a special type of diode made of semiconducting materials infused with impurities in combinations and concentrations that enable the extra energy from the movement of the electrons during electrical excitation to be converted into visible light. Semiconducting devices will be explained in greater detail in Condensed Matter Physics.

Commercial LEDs are quickly becoming the standard for commercial and residential lighting, replacing incandescent and CFL bulbs. They are designed for the visible spectrum and are constructed from gallium doped with arsenic and phosphorous atoms. The color emitted from an LED depends on the materials used in the semiconductor and the current. In the early years of LED development, small LEDs found on circuit boards were red, green, and yellow, but LED light bulbs can now be programmed to produce millions of colors of light as well as many different hues of white light.

Comparison of Incandescent, CFL, and LED Light Bulbs

The energy savings can be significant when replacing an incandescent light bulb or a CFL light bulb with an LED light. Light bulbs are rated by the amount of power that the bulb consumes, and the amount of light output is measured in lumens. The lumen (lm) is the SI -derived unit of luminous flux and is a measure of the total quantity of visible light emitted by a source. A 60-W incandescent light bulb can be replaced with a 13- to 15-W CFL bulb or a 6- to 8-W LED bulb, all three of which have a light output of approximately 800 lm. A table of light output for some commonly used light bulbs appears in Table 9.2.

The life spans of the three types of bulbs are significantly different. An LED bulb has a life span of 50,000 hours, whereas the CFL has a lifespan of 8000 hours and the incandescent lasts a mere 1200 hours. The LED bulb is the most durable, easily withstanding rough treatment such as jarring and bumping. The incandescent light bulb has little tolerance to the same treatment since the filament and glass can easily break. The CFL bulb is also less durable than the LED bulb because of its glass construction. The amount of heat emitted is 3.4 btu/h for the 8-W LED bulb, 85 btu/h for the 60-W incandescent bulb, and 30 btu/h for the CFL bulb. As mentioned earlier, a major drawback of the CFL bulb is that it contains mercury, a neurotoxin, and must be disposed of as hazardous waste. From these data, it is easy to understand why the LED light bulb is quickly becoming the standard in lighting.

Light Output (lumens)	LED Light Bulbs (watts)	Incandescent Light Bulbs (watts)	CFL Light Bulbs (watts)
450	4–5	40	9–13
800	6–8	60	13–15
1100	9–13	75	18–25
1600	16–20	100	23–30
2600	25–28	150	30–55

Table 9.2 Light Output of LED, Incandescent, and CFL Light Bulbs

Summary of Relationships

In this chapter, we have discussed relationships between voltages, current, resistance, and power. Figure 9.26 shows a summary of the relationships between these measurable quantities for ohmic devices. (Recall that ohmic devices follow Ohm's law $V = IR$.) For example, if you need to calculate the power, use the pink section, which shows that $P = VI$, $P = \frac{V^2}{R}$, and $P = I^2 R$.

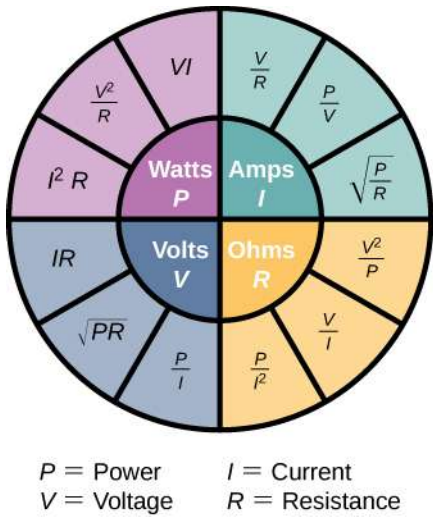


Figure 9.26 This circle shows a summary of the equations for the relationships between power, current, voltage, and resistance.

Which equation you use depends on what values you are given, or you measure. For example if you are given the current and the resistance, use $P = I^2 R$. Although all the possible combinations may seem overwhelming, don't forget that they all are combinations of just two equations, Ohm's law ($V = IR$) and power ($P = IV$).

9.6 Superconductors

Learning Objectives

By the end of this section, you will be able to:

- Describe the phenomenon of superconductivity
- List applications of superconductivity

Touch the power supply of your laptop computer or some other device. It probably feels slightly warm. That heat is an unwanted byproduct of the process of converting household electric power into a current that can be used by your device. Although electric power is reasonably efficient, other losses are associated with it. As discussed in the section on power and energy, transmission of electric power produces $I^2 R$ line losses. These line losses exist whether the power is generated from conventional power plants (using coal, oil, or gas), nuclear plants, solar plants, hydroelectric plants, or wind farms. These losses can be reduced, but not eliminated, by transmitting using a higher voltage. It would be wonderful if these line losses could be eliminated, but that would require transmission lines that have zero resistance. In a world that has a global interest in not wasting energy, the reduction or elimination of this unwanted thermal energy would be a significant achievement. Is this possible?

The Resistance of Mercury

In 1911, Heike Kamerlingh Onnes of Leiden University, a Dutch physicist, was looking at the temperature dependence of the resistance of the element mercury. He cooled the sample of mercury and noticed the familiar behavior of a linear dependence of resistance on temperature; as the temperature decreased, the resistance decreased. Kamerlingh Onnes continued to cool the sample of mercury, using liquid helium. As the temperature approached 4.2 K (−269.2 °C), the resistance abruptly went to zero (Figure 9.27). This temperature is known as the **critical temperature** T_c for mercury. The sample of mercury entered into a phase where the resistance was absolutely zero. This phenomenon is known as **superconductivity**. (*Note:* If you connect the leads of a three-digit ohmmeter across a conductor, the reading commonly shows up as

$0.00\ \Omega$. The resistance of the conductor is not actually zero, it is less than $0.01\ \Omega$.) There are various methods to measure very small resistances, such as the four-point method, but an ohmmeter is not an acceptable method to use for testing resistance in superconductivity.

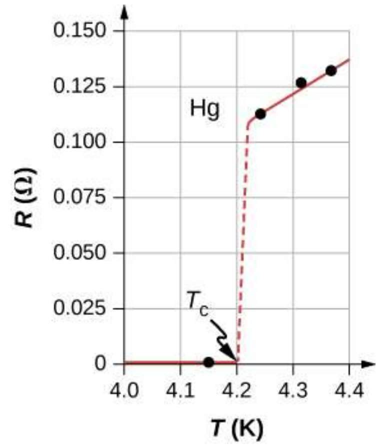


Figure 9.27 The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to the temperature of about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Other Superconducting Materials

As research continued, several other materials were found to enter a superconducting phase, when the temperature reached near absolute zero. In 1941, an alloy of niobium-nitride was found that could become superconducting at $T_c = 16\ \text{K}\ (-257\ °\text{C})$ and in 1953, vanadium-silicon was found to become superconductive at $T_c = 17.5\ \text{K}(-255.7\ °\text{C})$. The temperatures for the transition into superconductivity were slowly creeping higher. Strangely, many materials that make good conductors, such as copper, silver, and gold, do not exhibit superconductivity. Imagine the energy savings if transmission lines for electric power-generating stations could be made to be superconducting at temperatures near room temperature! A resistance of zero ohms means no $I^2 R$ losses and a great boost to reducing energy consumption. The problem is that $T_c = 17.5\ \text{K}$ is still very cold and in the range of liquid helium temperatures. At this temperature, it is not cost effective to transmit electrical energy because of the cooling requirements.

A large jump was seen in 1986, when a team of researchers, headed by Dr. Ching Wu Chu of Houston University, fabricated a brittle, ceramic compound with a transition temperature of $T_c = 92\ \text{K}\ (-181\ °\text{C})$. The ceramic material, composed of yttrium barium copper oxide (YBCO), was an insulator at room temperature. Although this temperature still seems quite cold, it is near the boiling point of liquid nitrogen, a liquid commonly used in refrigeration. You may have noticed refrigerated trucks traveling down the highway labeled as “Liquid Nitrogen Cooled.”

YBCO ceramic is a material that could be useful for transmitting electrical energy because the cost saving of reducing the $I^2 R$ losses are larger than the cost of cooling the superconducting cable, making it financially feasible. There were and are many engineering problems to overcome. For example, unlike traditional electrical cables, which are flexible and have a decent tensile strength, ceramics are brittle and would break rather than stretch under pressure. Processes that are rather simple with traditional cables, such as making connections, become difficult when working with ceramics. The problems are difficult and complex, and material scientists and engineers are coming up with innovative solutions.

An interesting consequence of the resistance going to zero is that once a current is established in a superconductor, it persists without an applied voltage source. Current loops in a superconductor have been set up and the current loops have been observed to persist for years without decaying.

Zero resistance is not the only interesting phenomenon that occurs as the materials reach their transition temperatures. A second effect is the exclusion of magnetic fields. This is known as the **Meissner effect** (Figure 9.28). A light, permanent magnet placed over a superconducting sample will levitate in a stable position above the superconductor. High-speed trains have been developed that levitate on strong superconducting magnets, eliminating the friction normally experienced between the train and the tracks. In Japan, the Yamanashi Maglev test line opened on April 3, 1997. In April 2015, the MLX01 test vehicle attained a speed of 374 mph (603 km/h).

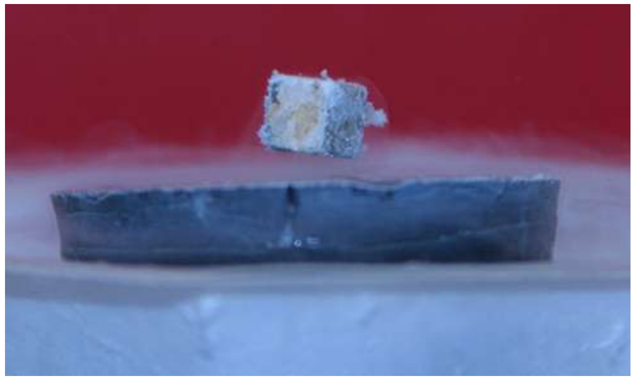


Figure 9.28 A small, strong magnet levitates over a superconductor cooled to liquid nitrogen temperature. The magnet levitates because the superconductor excludes magnetic fields. (credit: Joseph J. Trout)

Table 9.3 shows a select list of elements, compounds, and high-temperature superconductors, along with the critical temperatures for which they become superconducting. Each section is sorted from the highest critical temperature to the lowest. Also listed is the critical magnetic field for some of the materials. This is the strength of the magnetic field that destroys superconductivity. Finally, the type of the superconductor is listed.

There are two types of superconductors. There are 30 pure metals that exhibit zero resistivity below their critical temperature and exhibit the Meissner effect, the property of excluding magnetic fields from the interior of the superconductor while the superconductor is at a temperature below the critical temperature. These metals are called Type I superconductors. The superconductivity exists only below their critical temperatures and below a critical magnetic field strength. Type I superconductors are well described by the BCS theory (described next). Type I superconductors have limited practical applications because the strength of the critical magnetic field needed to destroy the superconductivity is quite low.

Type II superconductors are found to have much higher critical magnetic fields and therefore can carry much higher current densities while remaining in the superconducting state. A collection of various ceramics containing barium-copper-oxide have much higher critical temperatures for the transition into a superconducting state. Superconducting materials that belong to this subcategory of the Type II superconductors are often categorized as high-temperature superconductors.

Introduction to BCS Theory

Type I superconductors, along with some Type II superconductors can be modeled using the BCS theory, proposed by John Bardeen, Leon Cooper, and Robert Schrieffer. Although the theory is beyond the scope of this chapter, a short summary of the theory is provided here. (More detail is provided in Condensed Matter Physics.) The theory considers pairs of electrons and how they are coupled together through lattice-vibration interactions. Through the interactions with the crystalline lattice, electrons near the Fermi energy level feel a small attractive force and form pairs (Cooper pairs), and the coupling is known as a phonon interaction. Single electrons are fermions, which are particles that obey the Pauli exclusion principle. The Pauli exclusion principle in quantum mechanics states that two identical fermions (particles with half-integer spin) cannot occupy the same quantum state simultaneously. Each electron has four quantum numbers (n, l, m_l, m_s). The principal quantum number (n) describes the energy of the electron, the orbital angular momentum quantum number (l) indicates the most probable distance from the nucleus, the magnetic quantum number (m_l) describes the energy levels in the subshell, and the electron spin quantum number (m_s) describes the orientation of the spin of the electron, either up or down. As the material enters a superconducting state, pairs of electrons act more like bosons, which can condense into the same energy level and need not obey the Pauli exclusion principle. The electron pairs have a slightly lower energy and leave an energy gap above them on the order of 0.001 eV. This energy gap inhibits collision interactions that lead to ordinary resistivity. When the material is below the critical temperature, the thermal energy is less than the band gap and the material exhibits zero resistivity.

Material	Symbol or Formula	Critical Temperature T_c (K)	Critical Magnetic Field H_c (T)	Type
Elements				
Lead	Pb	7.19	0.08	I
Lanthanum	La	(α) 4.90 – (β) 6.30		I
Tantalum	Ta	4.48	0.09	I
Mercury	Hg	(α) 4.15 – (β) 3.95	0.04	I
Tin	Sn	3.72	0.03	I
Indium	In	3.40	0.03	I
Thallium	Tl	2.39	0.03	I
Rhenium	Re	2.40	0.03	I
Thorium	Th	1.37	0.013	I
Protactinium	Pa	1.40		I
Aluminum	Al	1.20	0.01	I
Gallium	Ga	1.10	0.005	I
Zinc	Zn	0.86	0.014	I
Titanium	Ti	0.39	0.01	I
Uranium	U	(α) 0.68 – (β) 1.80		I
Cadmium	Cd	11.4	4.00	I
Compounds				
Niobium-germanium	Nb_3Ge	23.20	37.00	II
Niobium-tin	Nb_3Sn	18.30	30.00	II
Niobium-nitrite	NbN	16.00		II
Niobium-titanium	NbTi	10.00	15.00	II
High-Temperature Oxides				

Material	Symbol or Formula	Critical Temperature T_c (K)	Critical Magnetic Field H_c (T)	Type
	$HgBa_2CaCu_2O_8$	134.00		II
	$Tl_2Ba_2Ca_2Cu_3O_{10}$	125.00		II
	$YBa_2Cu_3O_7$	92.00	120.00	II

Table 9.3 Superconductor Critical Temperatures

Applications of Superconductors

Superconductors can be used to make superconducting magnets. These magnets are 10 times stronger than the strongest electromagnets. These magnets are currently in use in magnetic resonance imaging (MRI), which produces high-quality images of the body interior without dangerous radiation.

Another interesting application of superconductivity is the **SQUID** (superconducting quantum interference device). A SQUID is a very sensitive magnetometer used to measure extremely subtle magnetic fields. The operation of the SQUID is based on superconducting loops containing Josephson junctions. A **Josephson junction** is the result of a theoretical prediction made by B. D. Josephson in an article published in 1962. In the article, Josephson described how a supercurrent can flow between two pieces of superconductor separated by a thin layer of insulator. This phenomenon is now called the Josephson effect. The SQUID consists of a superconducting current loop containing two Josephson junctions, as shown in Figure 9.29. When the loop is placed in even a very weak magnetic field, there is an interference effect that depends on the strength of the magnetic field.

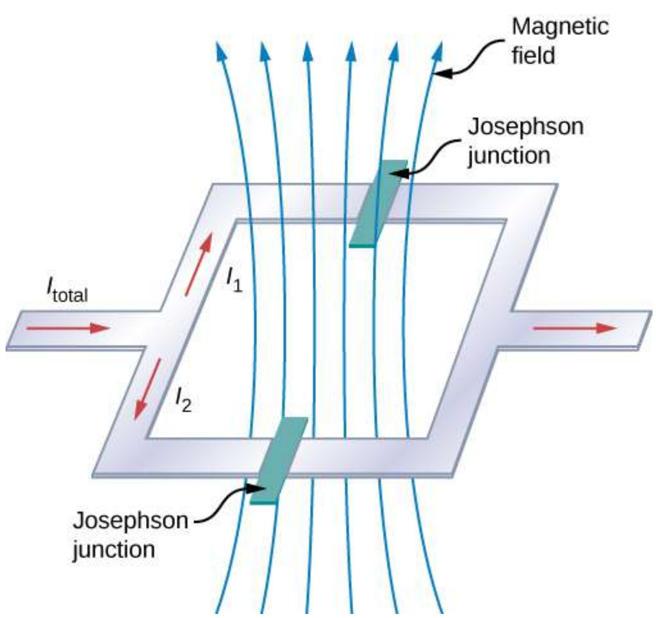


Figure 9.29 The SQUID (superconducting quantum interference device) uses a superconducting current loop and two Josephson junctions to detect magnetic fields as low as 10^{-14} T (Earth's magnet field is on the order of 0.3×10^{-5} T).

Superconductivity is a fascinating and useful phenomenon. At critical temperatures near the boiling point of liquid nitrogen, superconductivity has special applications in MRIs, particle accelerators, and high-speed trains. Will we reach a state where we can have materials enter the superconducting phase at near room temperatures? It seems a long way off, but if scientists in 1911 were asked if we would reach liquid-nitrogen temperatures with a ceramic, they might have thought it implausible.

CHAPTER REVIEW

Key Terms

ampere (amp) SI unit for current; $1\text{ A} = 1\text{ C/s}$

circuit complete path that an electrical current travels along

conventional current current that flows through a circuit from the positive terminal of a battery through the circuit to the negative terminal of the battery

critical temperature temperature at which a material reaches superconductivity

current density flow of charge through a cross-sectional area divided by the area

diode nonohmic circuit device that allows current flow in only one direction

drift velocity velocity of a charge as it moves nearly randomly through a conductor, experiencing multiple collisions, averaged over a length of a conductor, whose magnitude is the length of conductor traveled divided by the time it takes for the charges to travel the length

electrical conductivity measure of a material's ability to conduct or transmit electricity

electrical current rate at which charge flows, $I = \frac{dQ}{dt}$

electrical power time rate of change of energy in an electric circuit

Josephson junction junction of two pieces of superconducting material separated by a thin layer of insulating material, which can carry a supercurrent

Meissner effect phenomenon that occurs in a superconducting material where all magnetic fields are expelled

nonohmic type of a material for which Ohm's law is not valid

ohm (Ω) unit of electrical resistance, $1\,\Omega = 1\text{ V/A}$

Ohm's law empirical relation stating that the current I is proportional to the potential difference V; it is often written as $V = IR$, where R is the resistance

ohmic type of a material for which Ohm's law is valid, that is, the voltage drop across the device is equal to the current times the resistance

resistance electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

resistivity intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ

schematic graphical representation of a circuit using standardized symbols for components and solid lines for the wire connecting the components

SQUID (Superconducting Quantum Interference Device) device that is a very sensitive magnetometer, used to measure extremely subtle magnetic fields

superconductivity phenomenon that occurs in some materials where the resistance goes to exactly zero and all magnetic fields are expelled, which occurs dramatically at some low critical temperature (T_C)

Key Equations

Average electrical current	$I_{\text{ave}} = \frac{\Delta Q}{\Delta t}$
Definition of an ampere	$1\text{ A} = 1\text{ C/s}$
Electrical current	$I = \frac{dQ}{dt}$
Drift velocity	$v_d = \frac{I}{nqA}$
Current density	$I = \iint_{\text{area}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$
Resistivity	$\rho = \frac{E}{J}$

Common expression of Ohm's law	$V = IR$
Resistivity as a function of temperature	$\rho = \rho_0 \left[1 + \alpha (T - T_0)\right]$
Definition of resistance	$R \equiv \frac{V}{I}$
Resistance of a cylinder of material	$R = \rho \frac{L}{A}$
Temperature dependence of resistance	$R = R_0 (1 + \alpha \Delta T)$
Electric power	$P = IV$
Power dissipated by a resistor	$P = I^2 R = \frac{V^2}{R}$

Summary

9.1 Electrical Current

- The average electrical current I_{ave} is the rate at which charge flows, given by $I_{\text{ave}} = \frac{\Delta Q}{\Delta t}$, where ΔQ is the amount of charge passing through an area in time Δt.
- The instantaneous electrical current, or simply the current I, is the rate at which charge flows. Taking the limit as the change in time approaches zero, we have $I = \frac{dQ}{dt}$, where $\frac{dQ}{dt}$ is the time derivative of the charge.
- The direction of conventional current is taken as the direction in which positive charge moves. In a simple direct-current (DC) circuit, this will be from the positive terminal of the battery to the negative terminal.
- The SI unit for current is the ampere, or simply the amp (A), where $1 \text{ A} = 1 \text{ C/s}$.
- Current consists of the flow of free charges, such as electrons, protons, and ions.

9.2 Model of Conduction in Metals

- The current through a conductor depends mainly on the motion of free electrons.
- When an electrical field is applied to a conductor, the free electrons in a conductor do not move through a conductor at a constant speed and direction; instead, the motion is almost random due to collisions with atoms and other free electrons.
- Even though the electrons move in a nearly random fashion, when an electrical field is applied to the conductor, the overall velocity of the electrons can be defined in terms of a drift velocity.
- The current density is a vector quantity defined as the current through an infinitesimal area divided by the area.
- The current can be found from the current density, $I = \iint_{\text{area}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$.
- An incandescent light bulb is a filament of wire enclosed in a glass bulb that is partially evacuated. Current runs through the filament, where the electrical energy is converted to light and heat.

9.3 Resistivity and Resistance

- Resistance has units of ohms (Ω), related to volts and amperes by $1 \ \Omega = 1 \text{ V/A}$.
- The resistance R of a cylinder of length L and cross-sectional area A is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in Table 9.1 show that materials fall into three groups—conductors, semiconductors, and insulators.
- Temperature affects resistivity; for relatively small temperature changes ΔT, resistivity is $\rho = \rho_0 (1 + \alpha \Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0 (1 + \alpha \Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

9.4 Ohm's Law

- Ohm's law is an empirical relationship for current, voltage, and resistance for some

common types of circuit elements, including resistors. It does not apply to other devices, such as diodes.
- One statement of Ohm's law gives the relationship among current I, voltage V, and resistance R in a simple circuit as $V = IR$.
- Another statement of Ohm's law, on a microscopic level, is $J = \sigma E$.

9.5 Electrical Energy and Power

- Electric power is the rate at which electric energy is supplied to a circuit or consumed by a load.
- Power dissipated by a resistor depends on the square of the current through the resistor and is equal to $P = I^2 R = \frac{V^2}{R}$.
- The SI unit for electric power is the watt and the SI unit for electric energy is the joule. Another common unit for electric energy, used by power companies, is the kilowatt-hour $(\text{kW} \cdot \text{h})$.
- The total energy used over a time interval can be found by $E = \int P dt$.

9.6 Superconductors

- Superconductivity is a phenomenon that occurs in some materials when cooled to very low critical temperatures, resulting in a resistance of exactly zero and the expulsion of all magnetic fields.
- Materials that are normally good conductors (such as copper, gold, and silver) do not experience superconductivity.
- Superconductivity was first observed in mercury by Heike Kamerlingh Onnes in 1911. In 1986, Dr. Ching Wu Chu of Houston University fabricated a brittle, ceramic compound with a critical temperature close to the temperature of liquid nitrogen.
- Superconductivity can be used in the manufacture of superconducting magnets for use in MRIs and high-speed, levitated trains.

Conceptual Questions

9.1 Electrical Current

1. Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

2. Car batteries are rated in ampere-hours $(\text{A} \cdot \text{h})$. To what physical quantity do ampere-hours correspond (voltage, current, charge, energy, power,...)?

3. When working with high-power electric circuits, it is advised that whenever possible, you work "one-handed" or "keep one hand in your pocket." Why is this a sensible suggestion?

9.2 Model of Conduction in Metals

4. Incandescent light bulbs are being replaced with more efficient LED and CFL light bulbs. Is there any obvious evidence that incandescent light bulbs might not be that energy efficient? Is energy converted into anything but visible light?

5. It was stated that the motion of an electron appears nearly random when an electrical field is applied to the conductor. What makes the motion nearly random and differentiates it from the random motion of molecules in a gas?

6. Electric circuits are sometimes explained using a conceptual model of water flowing through a pipe. In this conceptual model, the voltage source is represented as a pump that pumps water through pipes and the pipes connect components in the circuit. Is a conceptual model of water flowing through a pipe an adequate representation of the circuit? How are electrons and wires similar to water molecules and pipes? How are they different?

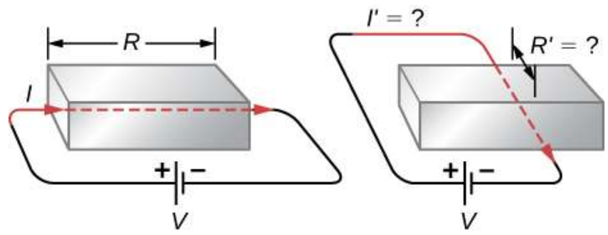
7. An incandescent light bulb is partially evacuated. Why do you suppose that is?

9.3 Resistivity and Resistance

8. The *IR* drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

9. Do impurities in semiconducting materials listed in Table 9.1 supply free charges? (*Hint*: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

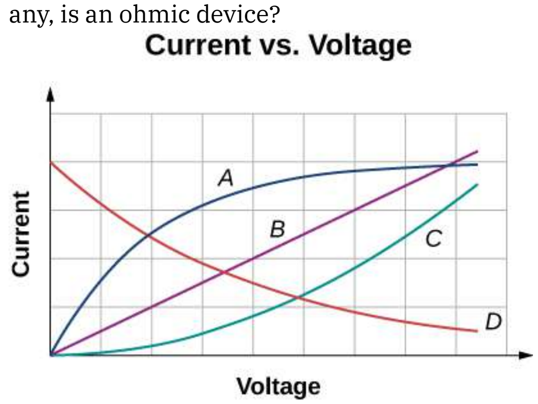
10. Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width?



11. If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

9.4 Ohm's Law

12. In Determining Field from Potential, resistance was defined as $R \equiv \frac{V}{I}$. In this section, we presented Ohm's law, which is commonly expressed as $V = IR$. The equations look exactly alike. What is the difference between Ohm's law and the definition of resistance?
13. Shown below are the results of an experiment where four devices were connected across a variable voltage source. The voltage is increased and the current is measured. Which device, if any, is an ohmic device?



14. The current I is measured through a sample of an ohmic material as a voltage V is applied. (a) What is the current when the voltage is doubled to $2V$ (assume the change in temperature of the material is negligible)? (b) What is the voltage applied is the current measured is $0.2I$ (assume the change in temperature of the material is negligible)? What will happen to the current if the material if the voltage remains constant, but the temperature of the material increases significantly?

9.5 Electrical Energy and Power

15. Common household appliances are rated at 110 V, but power companies deliver voltage in the kilovolt range and then step the voltage down using transformers to 110 V to be used in homes. You will learn in later chapters that transformers consist of many turns of wire, which warm up as current flows through them, wasting some of the energy that is given off as heat. This sounds inefficient. Why do the power companies transport electric power using this method?
16. Your electric bill gives your consumption in units of kilowatt-hour ($\text{kW} \cdot \text{h}$). Does this unit represent the amount of charge, current, voltage, power, or energy you buy?
17. Resistors are commonly rated at $\frac{1}{8}\,\text{W}$, $\frac{1}{4}\,\text{W}$, $\frac{1}{2}\,\text{W}$, 1 W and 2 W for use in electrical circuits. If a current of $I = 2.00\ \text{A}$ is accidentally passed through a $R = 1.00\ \Omega$ resistor rated at 1 W, what would be the most probable outcome? Is there anything that can be done to prevent such an accident?
18. An immersion heater is a small appliance used to heat a cup of water for tea by passing current through a resistor. If the voltage applied to the appliance is doubled, will the time required to heat the water change? By how much? Is this a good idea?

9.6 Superconductors

19. What requirement for superconductivity makes current superconducting devices expensive to operate?
20. Name two applications for superconductivity listed in this section and explain how superconductivity is used in the application. Can you think of a use for superconductivity that is not listed?

Problems

9.1 Electrical Current

21. A Van de Graaff generator is one of the original particle accelerators and can be used to accelerate charged particles like protons or electrons. You may have seen it used to make human hair stand on end or produce large sparks. One application of the Van de Graaff generator is to create X-rays by bombarding a hard metal target with the beam. Consider a beam of protons at 1.00 keV and a current of 5.00 mA produced by the generator. (a) What is the speed of the protons? (b) How many protons

are produced each second?

22. A cathode ray tube (CRT) is a device that produces a focused beam of electrons in a vacuum. The electrons strike a phosphor-coated glass screen at the end of the tube, which produces a bright spot of light. The position of the bright spot of light on the screen can be adjusted by deflecting the electrons with electrical fields, magnetic fields, or both. Although the CRT tube was once commonly found in televisions, computer displays, and oscilloscopes, newer appliances use a liquid crystal display (LCD) or plasma screen. You still may come across a CRT in your study of science. Consider a CRT with an electron beam average current of 25.00μ A. How many electrons strike the screen every minute?

23. How many electrons flow through a point in a wire in 3.00 s if there is a constant current of $I = 4.00$ A?

24. A conductor carries a current that is decreasing exponentially with time. The current is modeled as $I = I_0 e^{-t/\tau}$, where $I_0 = 3.00$ A is the current at time $t = 0.00$ s and $\tau = 0.50$ s is the time constant. How much charge flows through the conductor between $t = 0.00$ s and $t = 3\tau$?

25. The quantity of charge through a conductor is modeled as $Q = 4.00 \frac{\text{C}}{\text{s}^4} t^4 - 1.00 \frac{\text{C}}{\text{s}} t + 6.00$ mC. What is the current at time $t = 3.00$ s?

26. The current through a conductor is modeled as $I(t) = I_m \sin(2\pi [60 \text{ Hz}] t)$. Write an equation for the charge as a function of time.

27. The charge on a capacitor in a circuit is modeled as $Q(t) = Q_{\max} \cos(\omega t + \phi)$. What is the current through the circuit as a function of time?

9.2 Model of Conduction in Metals

28. An aluminum wire 1.628 mm in diameter (14-gauge) carries a current of 3.00 amps. (a) What is the absolute value of the charge density in the wire? (b) What is the drift velocity of the electrons? (c) What would be the drift velocity if the same gauge copper were used instead of aluminum? The density of copper is 8.96 g/cm^3 and the density of aluminum is 2.70 g/cm^3. The molar mass of aluminum is 26.98 g/mol and the molar mass of copper is 63.5 g/mol. Assume each atom of metal contributes one free electron.

29. The current of an electron beam has a measured current of $I = 50.00\ \mu$A with a radius of 1.00 mm. What is the magnitude of the current density of the beam?

30. A high-energy proton accelerator produces a proton beam with a radius of $r = 0.90$ mm. The beam current is $I = 9.00\ \mu$A and is constant. The charge density of the beam is $n = 6.00 \times 10^{11}$ protons per cubic meter. (a) What is the current density of the beam? (b) What is the drift velocity of the beam? (c) How much time does it take for 1.00×10^{10} protons to be emitted by the accelerator?

31. Consider a wire of a circular cross-section with a radius of $R = 3.00$ mm. The magnitude of the current density is modeled as $J = cr^2 = 5.00 \times 10^6 \frac{\text{A}}{\text{m}^4} r^2$. What is the current through the inner section of the wire from the center to $r = 0.5R$?

32. A cylindrical wire has a current density from the center of the wire's cross section as $J(r) = Cr^2$ where r is in meters, J is in amps per square meter, and $C = 10^3$ A/m^4. This current density continues to the end of the wire at a radius of 1.0 mm. Calculate the current just outside of this wire.

33. The current supplied to an air conditioner unit is 4.00 amps. The air conditioner is wired using a 10-gauge (diameter 2.588 mm) wire. The charge density is $n = 8.48 \times 10^{28} \frac{\text{electrons}}{\text{m}^3}$. Find the magnitude of (a) current density and (b) the drift velocity.

9.3 Resistivity and Resistance

34. What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

35. Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

36. How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

37. What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

38. The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

39. If the 0.100-mm-diameter tungsten filament in a light bulb is to have a resistance of $0.200\ \Omega$ at $20.0\ °\text{C}$, how long should it be?

40. A lead rod has a length of 30.00 cm and a resistance of $5.00\ \mu\Omega$. What is the radius of the rod?

41. Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

42. What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when 1.00×10^3 V is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

43. (a) To what temperature must you raise a copper wire, originally at $20.0\ °C$, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

44. A resistor made of nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at $20.0\ °C$. Over what temperature range can it be used?

45. Of what material is a resistor made if its resistance is 40.0% greater at $100.0\ °C$ than at $20.0\ °C$?

46. An electronic device designed to operate at any temperature in the range from $-10.0\ °C$ to $55.0\ °C$ contains pure carbon resistors. By what factor does their resistance increase over this range?

47. (a) Of what material is a wire made, if it is 25.0 m long with a diameter of 0.100 mm and has a resistance of $77.7\ \Omega$ at $20.0\ °C$? (b) What is its resistance at $150.0\ °C$?

48. Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at $20.0\ °C$?

49. A copper wire has a resistance of $0.500\ \Omega$ at $20.0\ °C$, and an iron wire has a resistance of $0.525\ \Omega$ at the same temperature. At what temperature are their resistances equal?

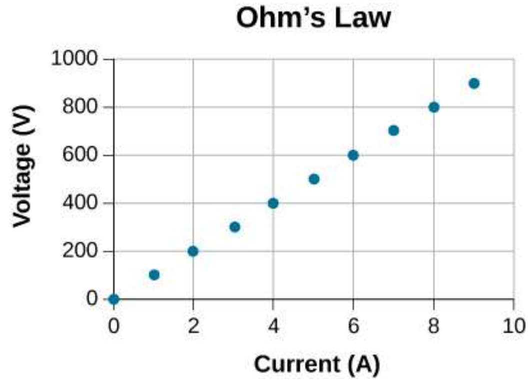
9.4 Ohm's Law

50. A $2.2\text{-k}\ \Omega$ resistor is connected across a D cell battery (1.5 V). What is the current through the resistor?

51. A resistor rated at $250\ \text{k}\ \Omega$ is connected across two D cell batteries (each 1.50 V) in series, with a total voltage of 3.00 V. The manufacturer advertises that their resistors are within 5% of the rated value. What are the possible minimum current and maximum current through the resistor?

52. A resistor is connected in series with a power supply of 20.00 V. The current measure is 0.50 A. What is the resistance of the resistor?

53. A resistor is placed in a circuit with an adjustable voltage source. The voltage across and the current through the resistor and the measurements are shown below. Estimate the resistance of the resistor.



54. The following table show the measurements of a current through and the voltage across a sample of material. Plot the data, and assuming the object is an ohmic device, estimate the resistance.

I(A)	V(V)
0	3
2	23
4	39
6	58
8	77
10	100
12	119
14	142
16	162

9.5 Electrical Energy and Power

55. A 20.00-V battery is used to supply current to a $10\text{-k}\ \Omega$ resistor. Assume the voltage drop across any wires used for connections is negligible. (a) What is the current through the resistor? (b) What is the power dissipated by the resistor? (c) What is the power input from the battery, assuming all the electrical power is dissipated by the resistor? (d) What happens to the energy dissipated by the resistor?

56. What is the maximum voltage that can be applied to a $20\text{-k}\,\Omega$ resistor rated at $\frac{1}{4}\text{W}$?

57. A heater is being designed that uses a coil of 14-gauge nichrome wire to generate 300 W using a voltage of $V = 110\text{ V}$. How long should the engineer make the wire?

58. An alternative to CFL bulbs and incandescent bulbs are light-emitting diode (LED) bulbs. A 100-W incandescent bulb can be replaced by a 16-W LED bulb. Both produce 1600 lumens of light. Assuming the cost of electricity is $0.10 per kilowatt-hour, how much does it cost to run the bulb for one year if it runs for four hours a day?

59. The power dissipated by a resistor with a resistance of $R = 100\,\Omega$ is $P = 2.0\text{ W}$. What are the current through and the voltage drop across the resistor?

60. Running late to catch a plane, a driver accidentally leaves the headlights on after parking the car in the airport parking lot. During takeoff, the driver realizes the mistake. Having just replaced the battery, the driver knows that the battery is a 12-V automobile battery, rated at $100\text{ A}\cdot\text{h}$. The driver, knowing there is nothing that can be done, estimates how long the lights will shine, assuming there are two 12-V headlights, each rated at 40 W. What did the driver conclude?

61. A physics student has a single-occupancy dorm room. The student has a small refrigerator that runs with a current of 3.00 A and a voltage of 110 V, a lamp that contains a 100-W bulb, an overhead light with a 60-W bulb, and various other small devices adding up to 3.00 W. (a) Assuming the power plant that supplies 110 V electricity to the dorm is 10 km away and the two aluminum transmission cables use 0-gauge wire with a diameter of 8.252 mm, estimate the percentage of the total power supplied by the power company that is lost in the transmission. (b) What would be the result is the power company delivered the electric power at 110 kV?

62. A 0.50-W, $220\text{-}\,\Omega$ resistor carries the maximum current possible without damaging the resistor. If the current were reduced to half the value, what would be the power consumed?

9.6 Superconductors

63. Consider a power plant is located 60 km away from a residential area uses 0-gauge $\left(A = 42.40\text{ mm}^2\right)$ wire of copper to transmit power at a current of $I = 100.00\text{ A}$. How much more power is dissipated in the copper wires than it would be in superconducting wires?

64. A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

65. Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.06/°\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37 °C (normal body temperature)?

66. Electrical power generators are sometimes "load tested" by passing current through a large vat of water. A similar method can be used to test the heat output of a resistor. A $R = 30\,\Omega$ resistor is connected to a 9.0-V battery and the resistor leads are waterproofed and the resistor is placed in 1.0 kg of room temperature water ($T = 20\,°\text{C}$). Current runs through the resistor for 20 minutes. Assuming all the electrical energy dissipated by the resistor is converted to heat, what is the final temperature of the water?

67. A 12-gauge gold wire has a length of 1 meter. (a) What would be the length of a silver 12-gauge wire with the same resistance? (b) What are their respective resistances at the temperature of boiling water?

68. What is the change in temperature required to decrease the resistance for a carbon resistor by 10%?

Additional Problems

69. A coaxial cable consists of an inner conductor with radius $r_i = 0.25$ cm and an outer radius of $r_o = 0.5$ cm and has a length of 10 meters. Plastic, with a resistivity of $\rho = 2.00 \times 10^{13}\ \Omega \cdot \text{m}$, separates the two conductors. What is the resistance of the cable?

70. A 10.00-meter long wire cable that is made of copper has a resistance of 0.051 ohms. (a) What is the weight if the wire was made of copper? (b) What is the weight of a 10.00-meter-long wire of the same gauge made of aluminum? (c)What is the resistance of the aluminum wire? The density of copper is $8960\ \text{kg/m}^3$ and the density of aluminum is $2760\ \text{kg/m}^3$.

71. A nichrome rod that is 3.00 mm long with a cross-sectional area of $1.00\ \text{mm}^2$ is used for a digital thermometer. (a) What is the resistance at room temperature? (b) What is the resistance at body temperature?

72. The temperature in Philadelphia, PA can vary between $68.00\ °\text{F}$ and $100.00\ °\text{F}$ in one summer day. By what percentage will an aluminum wire's resistance change during the day?

73. When 100.0 V is applied across a 5-gauge (diameter 4.621 mm) wire that is 10 m long, the magnitude of the current density is $2.0 \times 10^8\ \text{A/m}^2$. What is the resistivity of the wire?

74. A wire with a resistance of $5.0\ \Omega$ is drawn out through a die so that its new length is twice times its original length. Find the resistance of the longer wire. You may assume that the resistivity and density of the material are unchanged.

75. What is the resistivity of a wire of 5-gauge wire $(A = 16.8 \times 10^{-6}\text{m}^2)$, 5.00 m length, and $5.10\ \text{m}\,\Omega$ resistance?

76. Coils are often used in electrical and electronic circuits. Consider a coil which is formed by winding 1000 turns of insulated 20-gauge copper wire (area $0.52\ \text{mm}^2$) in a single layer on a cylindrical non-conducting core of radius 2.0 mm. What is the resistance of the coil? Neglect the thickness of the insulation.

77. Currents of approximately 0.06 A can be potentially fatal. Currents in that range can make the heart fibrillate (beat in an uncontrolled manner). The resistance of a dry human body can be approximately $100\ \text{k}\,\Omega$. (a) What voltage can cause 0.06 A through a dry human body? (b) When a human body is wet, the resistance can fall to $100\ \Omega$. What voltage can cause harm to a wet body?

78. A 20.00-ohm, 5.00-watt resistor is placed in series with a power supply. (a) What is the maximum voltage that can be applied to the resistor without harming the resistor? (b) What would be the current through the resistor?

79. A battery with an emf of 24.00 V delivers a constant current of 2.00 mA to an appliance. How much work does the battery do in three minutes?

80. A 12.00-V battery has an internal resistance of a tenth of an ohm. (a) What is the current if the battery terminals are momentarily shorted together? (b) What is the terminal voltage if the battery delivers 0.25 amps to a circuit?

Challenge Problems

81. A 10-gauge copper wire has a cross-sectional area $A = 5.26\ \text{mm}^2$ and carries a current of $I = 5.00$ A. The density of copper is $\rho = 8.95\ \text{g/cm}^3$. One mole of copper atoms $\left(6.02 \times 10^{23}\,\text{atoms}\right)$ has a mass of approximately 63.50 g. What is the magnitude of the drift velocity of the electrons, assuming that each copper atom contributes one free electron to the current?

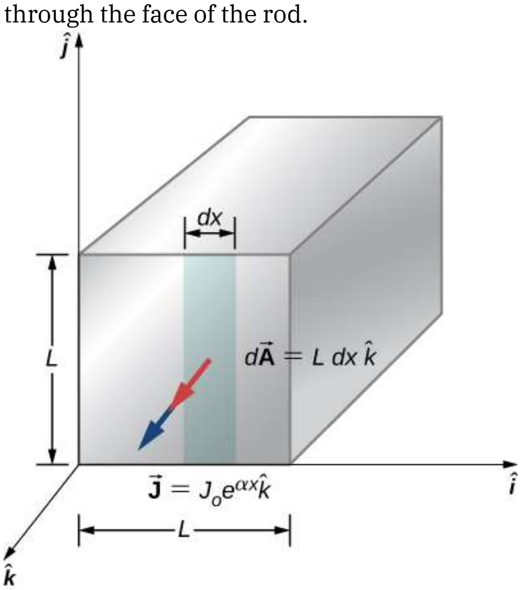
82. The current through a 12-gauge wire is given as $I(t) = (5.00\ \text{A}) \sin\ (2\pi 60\ \text{Hz}\ t)$. What is the current density at time 15.00 ms?

83. A particle accelerator produces a beam with a radius of 1.25 mm with a current of 2.00 mA. Each proton has a kinetic energy of 10.00 MeV. (a) What is the velocity of the protons? (b) What is the number (n) of protons per unit volume? (b) How many electrons pass a cross sectional area each second?

84. In this chapter, most examples and problems involved direct current (DC). DC circuits have the current flowing in one direction, from positive to negative. When the current was changing, it was changed linearly from $I = -I_{\max}$ to $I = +I_{\max}$ and the voltage changed linearly from $V = -V_{\max}$ to $V = +V_{\max}$, where $V_{\max} = I_{\max} R$. Suppose a voltage source is placed in series with a resistor of $R = 10\,\Omega$ that supplied a current that alternated as a sine wave, for example, $I(t) = (3.00\text{ A})\sin\left(\frac{2\pi}{4.00\text{ s}}t\right)$. (a) What would a graph of the voltage drop across the resistor *V*(*t*) versus time look like? (b) What would a plot of *V*(*t*) versus *I*(*t*) for one period look like? (*Hint*: If you are not sure, try plotting *V*(*t*) versus *I*(*t*) using a spreadsheet.)

85. A current of $I = 25A$ is drawn from a 100-V battery for 30 seconds. By how much is the chemical energy reduced?

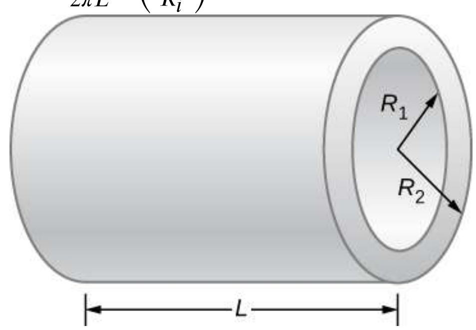
86. Consider a square rod of material with sides of length $L = 3.00$ cm with a current density of $\vec{\mathbf{J}} = J_0 e^{\alpha x}\hat{k} = \left(0.35\frac{\text{A}}{\text{m}^2}\right)e^{\left(2.1\times10^{-3}\text{m}^{-1}\right)x}\hat{\mathbf{k}}$ as shown below. Find the current that passes through the face of the rod.



87. A resistor of an unknown resistance is placed in an insulated container filled with 0.75 kg of water. A voltage source is connected in series with the resistor and a current of 1.2 amps flows through the resistor for 10 minutes. During this time, the temperature of the water is measured and the temperature change during this time is $\Delta T = 10.00\,°\text{C}$. (a) What is the resistance of the resistor? (b) What is the voltage supplied by the power supply?

88. The charge that flows through a point in a wire as a function of time is modeled as $q(t) = q_0 e^{-t/T} = 10.0\,\text{C}e^{-t/5\text{ s}}$. (a) What is the initial current through the wire at time $t = 0.00$ s? (b) Find the current at time $t = \frac{1}{2}T$. (c) At what time *t* will the current be reduced by one-half $I = \frac{1}{2}I_0$?

89. Consider a resistor made from a hollow cylinder of carbon as shown below. The inner radius of the cylinder is $R_i = 0.20$ mm and the outer radius is $R_0 = 0.30$ mm. The length of the resistor is $L = 0.90$ mm. The resistivity of the carbon is $\rho = 3.5 \times 10^{-5}\ \Omega\cdot\text{m}$. (a) Prove that the resistance perpendicular from the axis is $R = \frac{\rho}{2\pi L}\ln\left(\frac{R_0}{R_i}\right)$. (b) What is the resistance?



90. What is the current through a cylindrical wire of radius $R = 0.1$ mm if the current density is $J = \frac{J_0}{R}r$, where $J_0 = 32000\frac{\text{A}}{\text{m}^2}$?

91. A student uses a 100.00-W, 115.00-V radiant heater to heat the student's dorm room, during the hours between sunset and sunrise, 6:00 p.m. to 7:00 a.m. (a) What current does the heater operate at? (b) How many electrons move through the heater? (c) What is the resistance of the heater? (d) How much heat was added to the dorm room?

92. A 12-V car battery is used to power a 20.00-W, 12.00-V lamp during the physics club camping trip/star party. The cable to the lamp is 2.00 meters long, 14-gauge copper wire with a charge density of $n = 9.50 \times 10^{28}\,\text{m}^{-3}$. (a) What is the current draw by the lamp? (b) How long would it take an electron to get from the battery to the lamp?

93. A physics student uses a 115.00-V immersion heater to heat 400.00 grams (almost two cups) of water for herbal tea. During the two minutes it takes the water to heat, the physics student becomes bored and decides to figure out the resistance of the heater. The student starts with the assumption that the water is initially at the temperature of the room $T_i = 25.00\ °\text{C}$ and reaches $T_f = 100.00\ °\text{C}$. The specific heat of the water is $c = 4180 \frac{\text{J}}{\text{kg} \cdot \text{K}}$. What is the resistance of the heater?

CHAPTER 10
Direct-Current Circuits

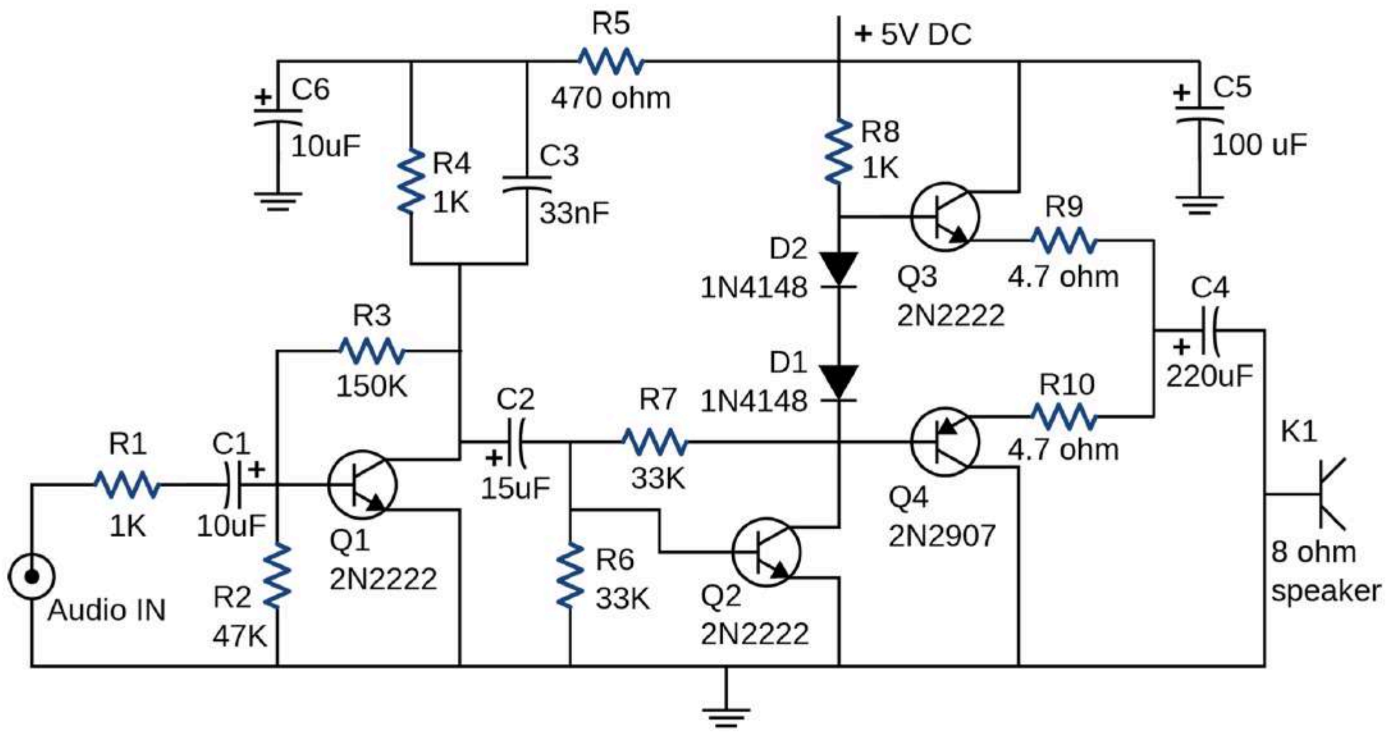


Figure 10.1 This circuit shown is used to amplify small signals and power the earbud speakers attached to a cellular phone. This circuit's components include resistors, capacitors, and diodes, all of which have been covered in previous chapters, as well as transistors, which are semi-conducting devices covered in Condensed Matter Physics. Circuits using similar components are found in all types of equipment and appliances you encounter in everyday life, such as alarm clocks, televisions, computers, and refrigerators.

Chapter Outline

INTRODUCTION In the preceding few chapters, we discussed electric components, including capacitors, resistors, and diodes. In this chapter, we use these electric components in circuits. A circuit is a collection of electrical components connected to accomplish a specific task. Figure 10.1 shows an amplifier circuit, which takes a small-amplitude signal and amplifies it to power the speakers in earbuds. Although the circuit looks complex, it actually consists of a set of series, parallel, and series-parallel circuits. The second section of this chapter covers the analysis of series and parallel circuits that consist of resistors. Later in this chapter, we introduce the basic equations and techniques to analyze any circuit, including those that are not reducible through simplifying parallel and series elements. But first, we need to understand how to power a circuit.

10.1 Electromotive Force

Learning Objectives

By the end of the section, you will be able to:

- Describe the electromotive force (emf) and the internal resistance of a battery
- Explain the basic operation of a battery

If you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they suddenly blink off when the battery's energy is gone? Their gradual dimming implies that the battery output voltage decreases as the battery is depleted. The reason for the decrease in output voltage for depleted batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an internal resistance. In this section, we examine the energy source and the internal resistance.

Introduction to Electromotive Force

Voltage has many sources, a few of which are shown in Figure 10.2. All such devices create a **potential difference** and can supply current if connected to a circuit. A special type of potential difference is known as **electromotive force (emf)**. The emf is not a force at all, but the term 'electromotive force' is used for historical reasons. It was coined by Alessandro Volta in the 1800s, when he invented the first battery, also known as the voltaic pile. Because the electromotive force is not a force, it is common to refer to these sources simply as sources of emf (pronounced as the letters "ee-em-eff"), instead of sources of electromotive force.

Figure 10.2 A variety of voltage sources. (a) The Brazos Wind Farm in Fluvanna, Texas; (b) the Krasnoyarsk Dam in Russia; (c) a solar

farm; (d) a group of nickel metal hydride batteries. The voltage output of each device depends on its construction and load. The voltage output equals emf only if there is no load. (credit a: modification of work by Stig Nygaard; credit b: modification of work by "vadimpl"/Wikimedia Commons; credit c: modification of work by "The tdog"/Wikimedia Commons; credit d: modification of work by "Itrados"/Wikimedia Commons)

If the electromotive force is not a force at all, then what is the emf and what is a source of emf? To answer these questions, consider a simple circuit of a 12-V lamp attached to a 12-V battery, as shown in Figure 10.3. The battery can be modeled as a two-terminal device that keeps one terminal at a higher electric potential than the second terminal. The higher electric potential is sometimes called the positive terminal and is labeled with a plus sign. The lower-potential terminal is sometimes called the negative terminal and labeled with a minus sign. This is the source of the emf.

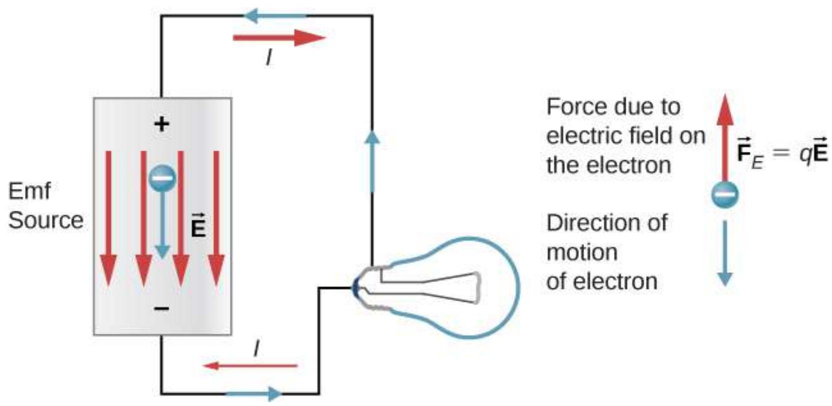


Figure 10.3 A source of emf maintains one terminal at a higher electric potential than the other terminal, acting as a source of current in a circuit.

When the emf source is not connected to the lamp, there is no net flow of charge within the emf source. Once the battery is connected to the lamp, charges flow from one terminal of the battery, through the lamp (causing the lamp to light), and back to the other terminal of the battery. If we consider positive (conventional) current flow, positive charges leave the positive terminal, travel through the lamp, and enter the negative terminal.

Positive current flow is useful for most of the circuit analysis in this chapter, but in metallic wires and resistors, electrons contribute the most to current, flowing in the opposite direction of positive current flow. Therefore, it is more realistic to consider the movement of electrons for the analysis of the circuit in Figure 10.3. The electrons leave the negative terminal, travel through the lamp, and return to the positive terminal. In order for the emf source to maintain the potential difference between the two terminals, negative charges (electrons) must be moved from the positive terminal to the negative terminal. The emf source acts as a charge pump, moving negative charges from the positive terminal to the negative terminal to maintain the potential difference. This increases the potential energy of the charges and, therefore, the electric potential of the charges.

The force on the negative charge from the electric field is in the opposite direction of the electric field, as shown in Figure 10.3. In order for the negative charges to be moved to the negative terminal, work must be done on the negative charges. This requires energy, which comes from chemical reactions in the battery. The potential is kept high on the positive terminal and low on the negative terminal to maintain the potential difference between the two terminals. The emf is equal to the work done on the charge per unit charge $\left(\varepsilon = \frac{dW}{dq}\right)$ when there is no current flowing. Since the unit for work is the joule and the unit for charge is the coulomb, the unit for emf is the volt $(1\ \text{V} = 1\ \text{J/C})$.

The **terminal voltage** V_{terminal} of a battery is voltage measured across the terminals of the battery. An ideal battery is an emf source that maintains a constant terminal voltage, independent of the current between the two terminals. An ideal battery has no internal resistance, and the terminal voltage is equal to the emf of the battery. In the next section, we will show that a real battery does have internal resistance and the terminal voltage is always less than the emf of the battery.

The Origin of Battery Potential

The combination of chemicals and the makeup of the terminals in a battery determine its emf. The lead acid battery used in cars and other vehicles is one of the most common combinations of chemicals. Figure 10.4 shows a single cell (one of six) of this battery. The cathode (positive) terminal of the cell is connected to a lead oxide plate, whereas the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

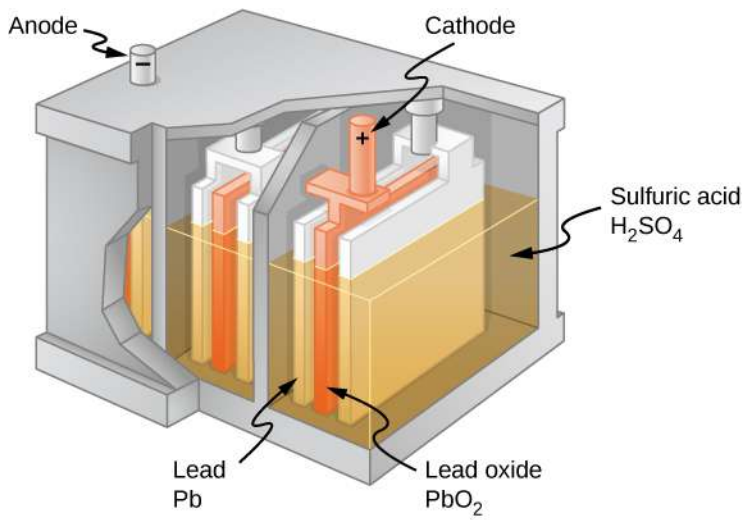


Figure 10.4 Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge, as well as participates in the chemical reaction.

Knowing a little about how the chemicals in a lead-acid battery interact helps in understanding the potential created by the battery. Figure 10.5 shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplies two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction does not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.

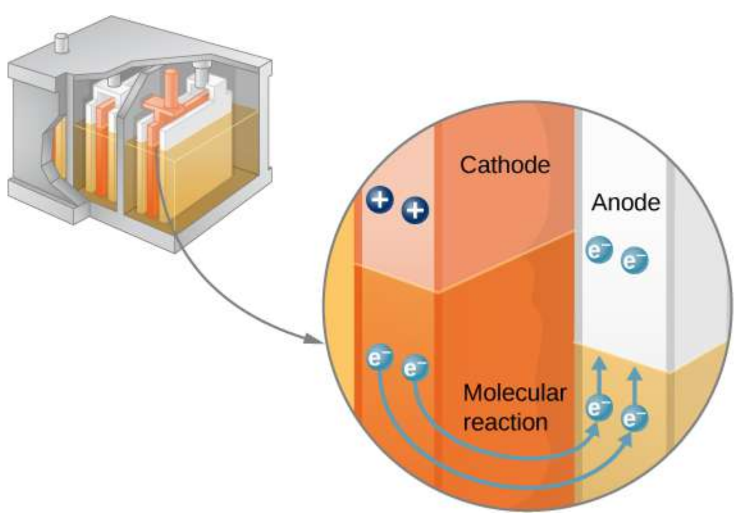


Figure 10.5 In a lead-acid battery, two electrons are forced onto the anode of a cell, and two electrons are removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a

closed circuit to proceed, since the two electrons must be supplied to the cathode.

Internal Resistance and Terminal Voltage

The amount of resistance to the flow of current within the voltage source is called the **internal resistance**. The internal resistance r of a battery can behave in complex ways. It generally increases as a battery is depleted, due to the oxidation of the plates or the reduction of the acidity of the electrolyte. However, internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted. A simple model for a battery consists of an idealized emf source ε and an internal resistance r (Figure 10.6).

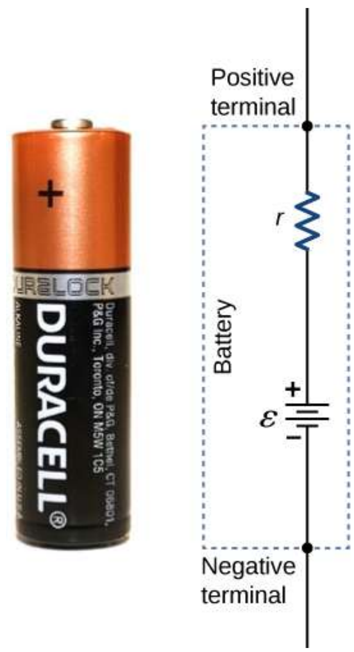


Figure 10.6 A battery can be modeled as an idealized emf (ε) with an internal resistance (r). The terminal voltage of the battery is $V_{\text{terminal}} = \varepsilon - Ir$.

Suppose an external resistor, known as the load resistance R, is connected to a voltage source such as a battery, as in Figure 10.7. The figure shows a model of a battery with an emf ε, an internal resistance r, and a load resistor R connected across its terminals. Using conventional current flow, positive charges leave the positive terminal of the battery, travel through the resistor, and return to the negative terminal of the battery. The terminal voltage of the battery depends on the emf, the internal resistance, and the current, and is equal to

$$V_{\text{terminal}} = \varepsilon - Ir. \quad (10.1)$$

For a given emf and internal resistance, the terminal voltage decreases as the current increases due to the potential drop Ir of the internal resistance.

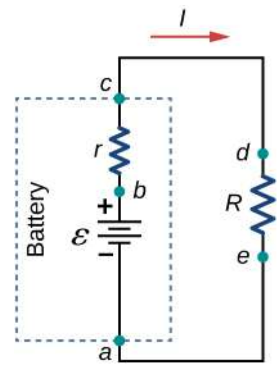


Figure 10.7 Schematic of a voltage source and its load resistor *R*. Since the internal resistance *r* is in series with the load, it can significantly affect the terminal voltage and the current delivered to the load.

A graph of the potential difference across each element the circuit is shown in Figure 10.8. A current I runs through the circuit, and the potential drop across the internal resistor is equal to *Ir*. The terminal voltage is equal to $\varepsilon - Ir$, which is equal to the **potential drop** across the load resistor $IR = \varepsilon - Ir$. As with potential energy, it is the change in voltage that is important. When the term "voltage" is used, we assume that it is actually the change in the potential, or ΔV. However, Δ is often omitted for convenience.

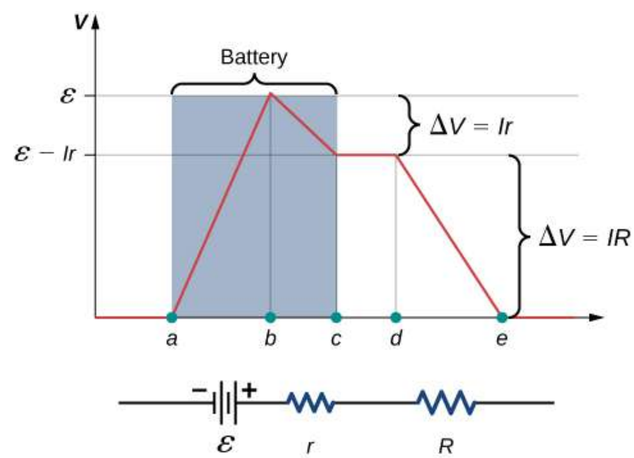


Figure 10.8 A graph of the voltage through the circuit of a battery and a load resistance. The electric potential increases the emf of the battery due to the chemical reactions doing work on the charges. There is a decrease in the electric potential in the battery due to the internal resistance. The potential decreases due to the internal resistance $(-Ir)$, making the terminal voltage of the battery equal to $(\varepsilon - Ir)$. The voltage then decreases by (*IR*). The current is equal to $I = \frac{\varepsilon}{r+R}$.

The current through the load resistor is $I = \frac{\varepsilon}{r+R}$. We see from this expression that the smaller the internal resistance *r*, the greater the current the voltage source supplies to its load *R*. As batteries are depleted, *r* increases. If *r* becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

EXAMPLE 10.1

Analyzing a Circuit with a Battery and a Load

A given battery has a 12.00-V emf and an internal resistance of $0.100\ \Omega$. (a) Calculate its terminal voltage when connected to a $10.00\text{-}\Omega$ load. (b) What is the terminal voltage when connected to a $0.500\text{-}\Omega$ load? (c) What power does the $0.500\text{-}\Omega$ load dissipate? (d) If the internal resistance grows to $0.500\ \Omega$, find the current, terminal voltage, and power dissipated by a $0.500\text{-}\Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated by using the equation $V_{\text{terminal}} = \varepsilon - Ir$. Once current is found, we can also find the power dissipated by the resistor.

Solution

a. Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

$$I = \frac{\varepsilon}{R + r} = \frac{12.00\text{ V}}{10.10\ \Omega} = 1.188\text{ A}.$$

Enter the known values into the equation $V_{\text{terminal}} = \varepsilon - Ir$ to get the terminal voltage:

$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\text{ V} - (1.188\text{ A})(0.100\ \Omega) = 11.90\text{ V}.$$

The terminal voltage here is only slightly lower than the emf, implying that the current drawn by this light load is not significant.

b. Similarly, with $R_{\text{load}} = 0.500\ \Omega$, the current is

$$I = \frac{\varepsilon}{R + r} = \frac{12.00\text{ V}}{0.600\ \Omega} = 20.00\text{ A}.$$

The terminal voltage is no

$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\text{ V} - (20.00\text{ A})(0.100\ \Omega) = 10.00\text{ V}.$$

The terminal voltage exhibits a more significant reduction compared with emf, implying $0.500\ \Omega$ is a heavy load for this battery. A "heavy load" signifies a larger draw of current from the source but not a larger resistance.

c. The power dissipated by the $0.500\text{-}\Omega$ load can be found using the formula $P = I^2R$. Entering the known values gives

$$P = I^2R = (20.0\text{ A})^2(0.500\ \Omega) = 2.00 \times 10^2\text{ W}.$$

Note that this power can also be obtained using the expression $\frac{V^2}{R}$ or IV, where V is the terminal voltage (10.0 V in this case).

d. Here, the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

$$I = \frac{\varepsilon}{R + r} = \frac{12.00\text{ V}}{1.00\ \Omega} = 12.00\text{ A}.$$

Now the terminal voltage is

$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\text{ V} - (12.00\text{ A})(0.500\ \Omega) = 6.00\text{ V},$$

and the power dissipated by the load is

$$P = I^2R = (12.00\text{ A})^2(0.500\ \Omega) = 72.00\text{ W}.$$

We see that the increased internal resistance has significantly decreased the terminal voltage, current, and power delivered to a load.

Significance

The internal resistance of a battery can increase for many reasons. For example, the internal resistance of a rechargeable battery increases as the number of times the battery is recharged increases. The increased internal resistance may have two effects on the battery. First, the terminal voltage will decrease. Second, the battery may overheat due to the increased power dissipated by the internal resistance.

CHECK YOUR UNDERSTANDING 10.1

If you place a wire directly across the two terminal of a battery, effectively shorting out the terminals, the battery will begin to get hot. Why do you suppose this happens?

Battery Testers

Battery testers, such as those in Figure 10.9, use small load resistors to intentionally draw current to determine whether the terminal potential drops below an acceptable level. Although it is difficult to measure the internal resistance of a battery, battery testers can provide a measurement of the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.

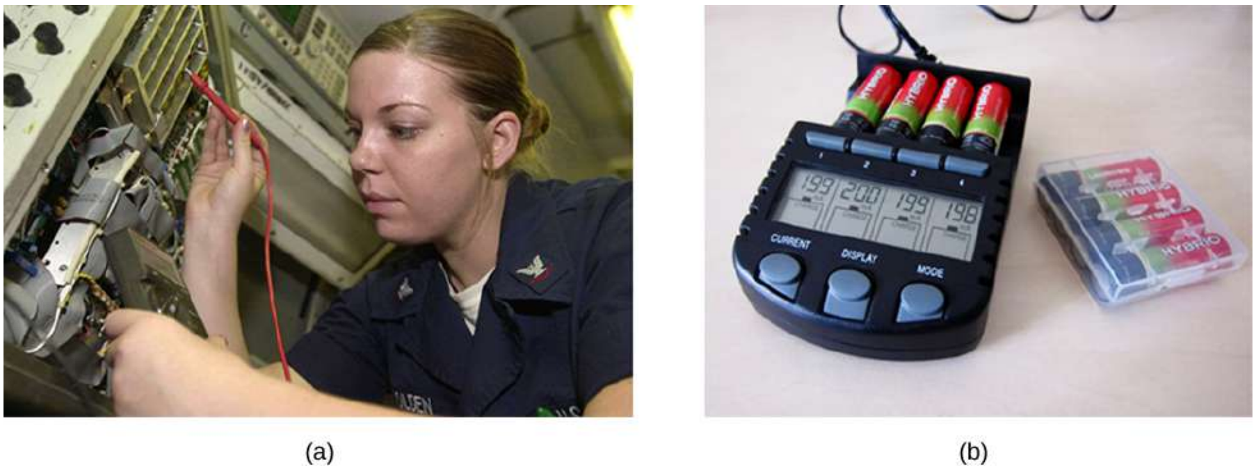


Figure 10.9 Battery testers measure terminal voltage under a load to determine the condition of a battery. (a) A US Navy electronics technician uses a battery tester to test large batteries aboard the aircraft carrier USS *Nimitz*. The battery tester she uses has a small resistance that can dissipate large amounts of power. (b) The small device shown is used on small batteries and has a digital display to indicate the acceptability of the terminal voltage. (credit a: modification of work by Jason A. Johnston; credit b: modification of work by Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to an appliance. This is done routinely in cars and in batteries for small electrical appliances and electronic devices (Figure 10.10). The voltage output of the battery charger must be greater than the emf of the battery to reverse the current through it. This causes the terminal voltage of the battery to be greater than the emf, since $V = \varepsilon - Ir$ and I is now negative.

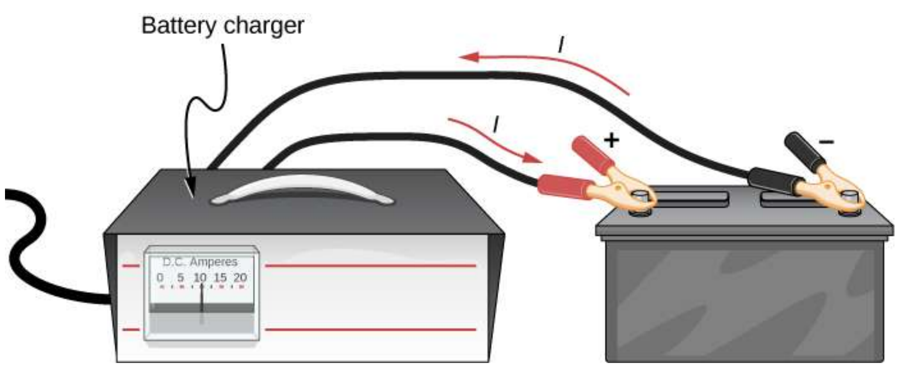


Figure 10.10 A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

It is important to understand the consequences of the internal resistance of emf sources, such as batteries and solar cells, but often, the analysis of circuits is done with the terminal voltage of the battery, as we have done in the previous sections. The terminal voltage is referred to as simply as V, dropping the subscript "terminal." This is because the internal resistance of the battery is difficult to measure directly and can change over time.

10.2 Resistors in Series and Parallel

Learning Objectives

By the end of this section, you will be able to:

- Define the term equivalent resistance
- Calculate the equivalent resistance of resistors connected in series
- Calculate the equivalent resistance of resistors connected in parallel

In Current and Resistance, we described the term 'resistance' and explained the basic design of a resistor. Basically, a resistor limits the flow of charge in a circuit and is an ohmic device where $V = IR$. Most circuits have more than one resistor. If several resistors are connected together and connected to a battery, the current supplied by the battery depends on the **equivalent resistance** of the circuit.

The equivalent resistance of a combination of resistors depends on both their individual values and how they are connected. The simplest combinations of resistors are series and parallel connections (Figure 10.11). In a series circuit, the output current of the first resistor flows into the input of the second resistor; therefore, the current is the same in each resistor. In a parallel circuit, all of the resistor leads on one side of the resistors are connected together and all the leads on the other side are connected together. In the case of a parallel configuration, each resistor has the same potential drop across it, and the currents through each resistor may be different, depending on the resistor. The sum of the individual currents equals the current that flows into the parallel connections.

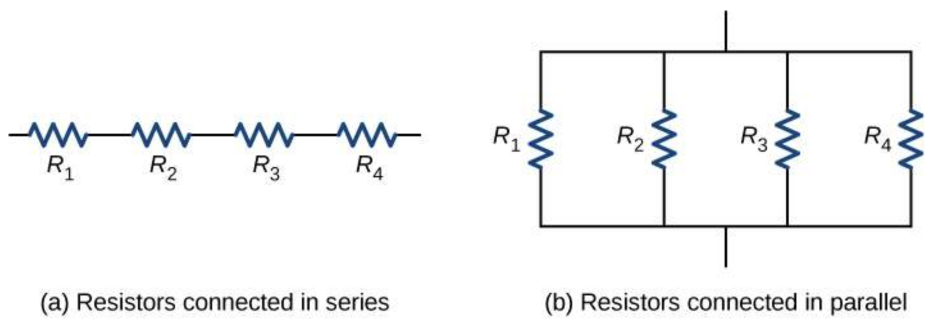


Figure 10.11 (a) For a series connection of resistors, the current is the same in each resistor. (b) For a parallel connection of resistors, the voltage is the same across each resistor.

Resistors in Series

Resistors are said to be in series whenever the current flows through the resistors sequentially. Consider Figure 10.12, which shows three resistors in series with an applied voltage equal to V_{ab}. Since there is only one path for the charges to flow through, the current is the same through each resistor. The equivalent resistance of a set of resistors in a series connection is equal to the algebraic sum of the individual resistances.

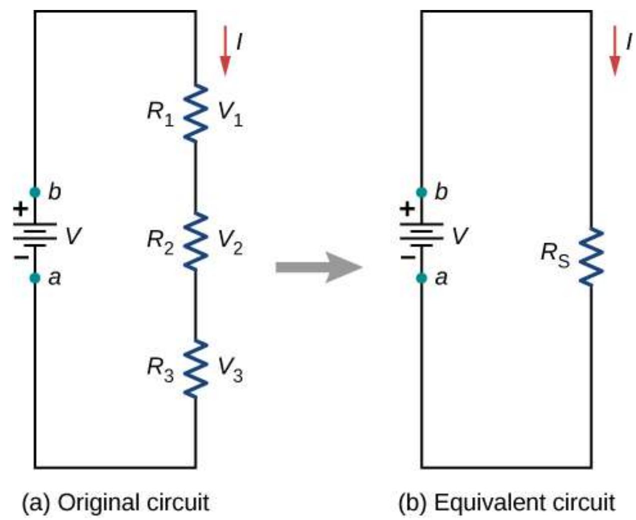


Figure 10.12 (a) Three resistors connected in series to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

In Figure 10.12, the current coming from the voltage source flows through each resistor, so the current through each resistor is the same. The current through the circuit depends on the voltage supplied by the voltage source and the resistance of the resistors. For each resistor, a potential drop occurs that is equal to the loss of electric potential energy as a current travels through each resistor. According to Ohm's law, the potential drop V across a resistor when a current flows through it is calculated using the equation $V = IR$, where I is the current in amps (A) and R is the resistance in ohms (Ω). Since energy is conserved, and the voltage is equal to the potential energy per charge, the sum of the voltage applied to the circuit by the source and the potential drops across the individual resistors around a loop should be equal to zero:

$$\sum_{i=1}^{N} V_i = 0.$$

This equation is often referred to as Kirchhoff's loop law, which we will look at in more detail later in this chapter. For Figure 10.12, the sum of the potential drop of each resistor and the voltage supplied by the voltage source should equal zero:

$$\begin{aligned} V - V_1 - V_2 - V_3 &= 0, \\ V &= V_1 + V_2 + V_3, \\ &= IR_1 + IR_2 + IR_3, \\ I &= \frac{V}{R_1 + R_2 + R_3} = \frac{V}{R_S}. \end{aligned}$$

Since the current through each component is the same, the equality can be simplified to an equivalent resistance, which is just the sum of the resistances of the individual resistors.

Any number of resistors can be connected in series. If N resistors are connected in series, the equivalent resistance is

$$R_S = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i. \quad (10.2)$$

One result of components connected in a series circuit is that if something happens to one component, it affects all the other components. For example, if several lamps are connected in series and one bulb burns out, all the other lamps go dark.

EXAMPLE 10.2

Equivalent Resistance, Current, and Power in a Series Circuit

A battery with a terminal voltage of 9 V is connected to a circuit consisting of four $20\text{-}\Omega$ and one $10\text{-}\Omega$ resistors all in series (Figure 10.13). Assume the battery has negligible internal resistance. (a) Calculate the equivalent resistance of the circuit. (b) Calculate the current through each resistor. (c) Calculate the potential drop across each resistor. (d) Determine the total power dissipated by the resistors and the power supplied by the battery.

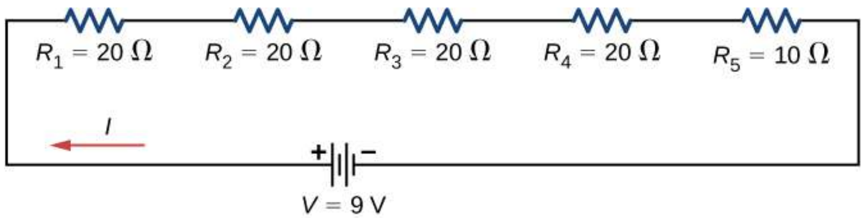


Figure 10.13 A simple series circuit with five resistors.

Strategy

In a series circuit, the equivalent resistance is the algebraic sum of the resistances. The current through the circuit can be found from Ohm's law and is equal to the voltage divided by the equivalent resistance. The potential drop across each resistor can be found using Ohm's law. The power dissipated by each resistor can be found using $P = I^2 R$, and the total power dissipated by the resistors is equal to the sum of the power dissipated by each resistor. The power supplied by the battery can be found using $P = I\varepsilon$.

Solution

a. The equivalent resistance is the algebraic sum of the resistances:
$$R_S = R_1 + R_2 + R_3 + R_4 + R_5 = 20\ \Omega + 20\ \Omega + 20\ \Omega + 20\ \Omega + 10\ \Omega = 90\ \Omega.$$

b. The current through the circuit is the same for each resistor in a series circuit and is equal to the applied voltage divided by the equivalent resistance:
$$I = \frac{V}{R_S} = \frac{9\text{ V}}{90\ \Omega} = 0.1\text{ A}.$$

c. The potential drop across each resistor can be found using Ohm's law:
$$\begin{aligned} V_1 &= V_2 = V_3 = V_4 = (0.1\text{ A})\,20\ \Omega = 2\text{ V}, \\ V_5 &= (0.1\text{ A})\,10\ \Omega = 1\text{ V}, \\ V_1 &+ V_2 + V_3 + V_4 + V_5 = 9\text{ V}. \end{aligned}$$

Note that the sum of the potential drops across each resistor is equal to the voltage supplied by the battery.

d. The power dissipated by a resistor is equal to $P = I^2 R$, and the power supplied by the battery is equal to $P = I\varepsilon$:
$$\begin{aligned} P_1 &= P_2 = P_3 = P_4 = (0.1\text{ A})^2\,(20\ \Omega) = 0.2\text{ W}, \\ P_5 &= (0.1\text{ A})^2\,(10\ \Omega) = 0.1\text{ W}, \\ P_{\text{dissipated}} &= 0.2\text{ W} + 0.2\text{ W} + 0.2\text{ W} + 0.2\text{ W} + 0.1\text{ W} = 0.9\text{ W}, \\ P_{\text{source}} &= I\varepsilon = (0.1\text{ A})\,(9\text{ V}) = 0.9\text{ W}. \end{aligned}$$

Significance

There are several reasons why we would use multiple resistors instead of just one resistor with a resistance equal to the equivalent resistance of the circuit. Perhaps a resistor of the required size is not available, or we need to dissipate the heat generated, or we want to minimize the cost of resistors. Each resistor may cost a few cents to a few dollars, but when multiplied by thousands of units, the cost saving may be appreciable.

CHECK YOUR UNDERSTANDING 10.2

Some strings of miniature holiday lights are made to short out when a bulb burns out. The device that causes the short is called a shunt, which allows current to flow around the open circuit. A "short" is like putting a piece of wire across the component. The bulbs are usually grouped in series of nine bulbs. If too many bulbs burn out, the shunts eventually open. What causes this?

Let's briefly summarize the major features of resistors in series:

1. Series resistances add together to get the equivalent resistance:

$$R_S = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i.$$

2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it. The total potential drop across a series configuration of resistors is equal to the sum of the potential drops across each resistor.

Resistors in Parallel

Figure 10.14 shows resistors in parallel, wired to a voltage source. Resistors are in parallel when one end of all the resistors are connected by a continuous wire of negligible resistance and the other end of all the resistors are also connected to one another through a continuous wire of negligible resistance. The potential drop across each resistor is the same. Current through each resistor can be found using Ohm's law $I = V/R$, where the voltage is constant across each resistor. For example, an automobile's headlights, radio, and other systems are wired in parallel, so that each subsystem utilizes the full voltage of the source and can operate completely independently. The same is true of the wiring in your house or any building.

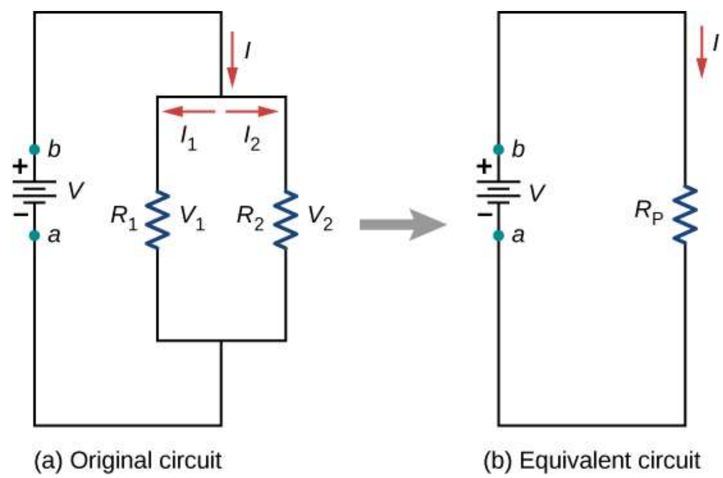


Figure 10.14 (a) Two resistors connected in parallel to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

The current flowing from the voltage source in Figure 10.14 depends on the voltage supplied by the voltage source and the equivalent resistance of the circuit. In this case, the current flows from the voltage source and enters a junction, or node, where the circuit splits flowing through resistors R_1 and R_2. As the charges flow from the battery, some go through resistor R_1 and some flow through resistor R_2. The sum of the currents flowing into a junction must be equal to the sum of the currents flowing out of the junction:

$$\sum I_{in} = \sum I_{out}.$$

This equation is referred to as Kirchhoff's junction rule and will be discussed in detail in the next section. In Figure 10.14, the junction rule gives $I = I_1 + I_2$. There are two loops in this circuit, which leads to the equations $V = I_1 R_1$ and $I_1 R_1 = I_2 R_2$. Note the voltage across the resistors in parallel are the same $(V = V_1 = V_2)$ and the current is additive:

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{V_1}{R_1} + \frac{V_2}{R_2} \\ &= \frac{V}{R_1} + \frac{V}{R_2} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_P} \\ R_P &= \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}. \end{aligned}$$

Generalizing to any number of N resistors, the equivalent resistance R_P of a parallel connection is related to the individual resistances by

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_{N-1}} + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}. \qquad 10.3$$

This relationship results in an equivalent resistance R_P that is less than the smallest of the individual resistances. When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, so the total resistance is lower.

EXAMPLE 10.3

Analysis of a Parallel Circuit

Three resistors $R_1 = 1.00\,\Omega$, $R_2 = 2.00\,\Omega$, and $R_3 = 2.00\,\Omega$, are connected in parallel. The parallel connection is attached to a $V = 3.00\text{ V}$ voltage source. (a) What is the equivalent resistance? (b) Find the current supplied by the source to the parallel circuit. (c) Calculate the currents in each resistor and show that these add together to equal the current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source and show that it equals the total power dissipated by the resistors.

Strategy

(a) The total resistance for a parallel combination of resistors is found using $R_P = \left(\sum_i \frac{1}{R_i}\right)^{-1}$.

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

(b) The current supplied by the source can be found from Ohm's law, substituting R_P for the total resistance $I = \frac{V}{R_P}$.

(c) The individual currents are easily calculated from Ohm's law $\left(I_i = \frac{V_i}{R_i}\right)$, since each resistor gets the full voltage. The total current is the sum of the individual currents: $I = \sum_i I_i$.

(d) The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P_i = V^2/R_i$, since each resistor gets full voltage.

(e) The total power can also be calculated in several ways, use $P = IV$.

Solution

a. The total resistance for a parallel combination of resistors is found using Equation 10.3. Entering known values gives

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{1.00\,\Omega} + \frac{1}{2.00\,\Omega} + \frac{1}{2.00\,\Omega}\right)^{-1} = 0.50\,\Omega.$$

The total resistance with the correct number of significant digits is $R_P = 0.50\ \Omega$. As predicted, R_P is less than the smallest individual resistance.

b. The total current can be found from Ohm's law, substituting R_P for the total resistance. This gives

$$I = \frac{V}{R_P} = \frac{3.00\ \text{V}}{0.50\ \Omega} = 6.00\ \text{A}.$$

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

c. The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{3.00\ \text{V}}{1.00\ \Omega} = 3.00\ \text{A}.$$

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{3.00\ \text{V}}{2.00\ \Omega} = 1.50\ \text{A}$$

and

$$I_3 = \frac{V}{R_3} = \frac{3.00\ \text{V}}{2.00\ \Omega} = 1.50\ \text{A}.$$

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 6.00\ \text{A}.$$

d. The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = V^2/R$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R_1} = \frac{(3.00\ \text{V})^2}{1.00\ \Omega} = 9.00\ \text{W}.$$

Similarly,

$$P_2 = \frac{V^2}{R_2} = \frac{(3.00\ \text{V})^2}{2.00\ \Omega} = 4.50\ \text{W}$$

and

$$P_3 = \frac{V^2}{R_3} = \frac{(3.00\ \text{V})^2}{2.00\ \Omega} = 4.50\ \text{W}.$$

e. The total power can also be calculated in several ways. Choosing $P = IV$ and entering the total current yields

$$P = IV = (6.00\ \text{A})(3.00\ \text{V}) = 18.00\ \text{W}.$$

Significance

Total power dissipated by the resistors is also 18.00 W:

$$P_1 + P_2 + P_3 = 9.00\ \text{W} + 4.50\ \text{W} + 4.50\ \text{W} = 18.00\ \text{W}.$$

Notice that the total power dissipated by the resistors equals the power supplied by the source.

CHECK YOUR UNDERSTANDING 10.3

Consider the same potential difference ($V = 3.00$ V) applied to the same three resistors connected in series. Would the equivalent resistance of the series circuit be higher, lower, or equal to the three resistor in parallel? Would the current through the series circuit be higher, lower, or equal to the current provided by the same voltage applied to the parallel circuit? How would the power dissipated by the resistor in series compare to the power dissipated by the resistors in parallel?

CHECK YOUR UNDERSTANDING 10.4

How would you use a river and two waterfalls to model a parallel configuration of two resistors? How does this analogy break down?

Let us summarize the major features of resistors in parallel:

1. Equivalent resistance is found from
$$R_\mathrm{P} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_{N-1}} + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1},$$
and is smaller than any individual resistance in the combination.
2. The potential drop across each resistor in parallel is the same.
3. Parallel resistors do not each get the total current; they divide it. The current entering a parallel combination of resistors is equal to the sum of the current through each resistor in parallel.

In this chapter, we introduced the equivalent resistance of resistors connect in series and resistors connected in parallel. You may recall that in Capacitance, we introduced the equivalent capacitance of capacitors connected in series and parallel. Circuits often contain both capacitors and resistors. Table 10.1 summarizes the equations used for the equivalent resistance and equivalent capacitance for series and parallel connections.

	Series combination	Parallel combination
Equivalent capacitance	$\frac{1}{C_\mathrm{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$C_\mathrm{P} = C_1 + C_2 + C_3 + \cdots$
Equivalent resistance	$R_\mathrm{S} = R_1 + R_2 + R_3 + \cdots = \sum_{i=1}^{N} R_i$	$\frac{1}{R_\mathrm{P}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Table 10.1 Summary for Equivalent Resistance and Capacitance in Series and Parallel Combinations

Combinations of Series and Parallel

More complex connections of resistors are often just combinations of series and parallel connections. Such combinations are common, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in Figure 10.15. Various parts can be identified as either series or parallel connections, reduced to their equivalent resistances, and then further reduced until a single equivalent resistance is left. The process is more time consuming than difficult. Here, we note the equivalent resistance as R_eq.

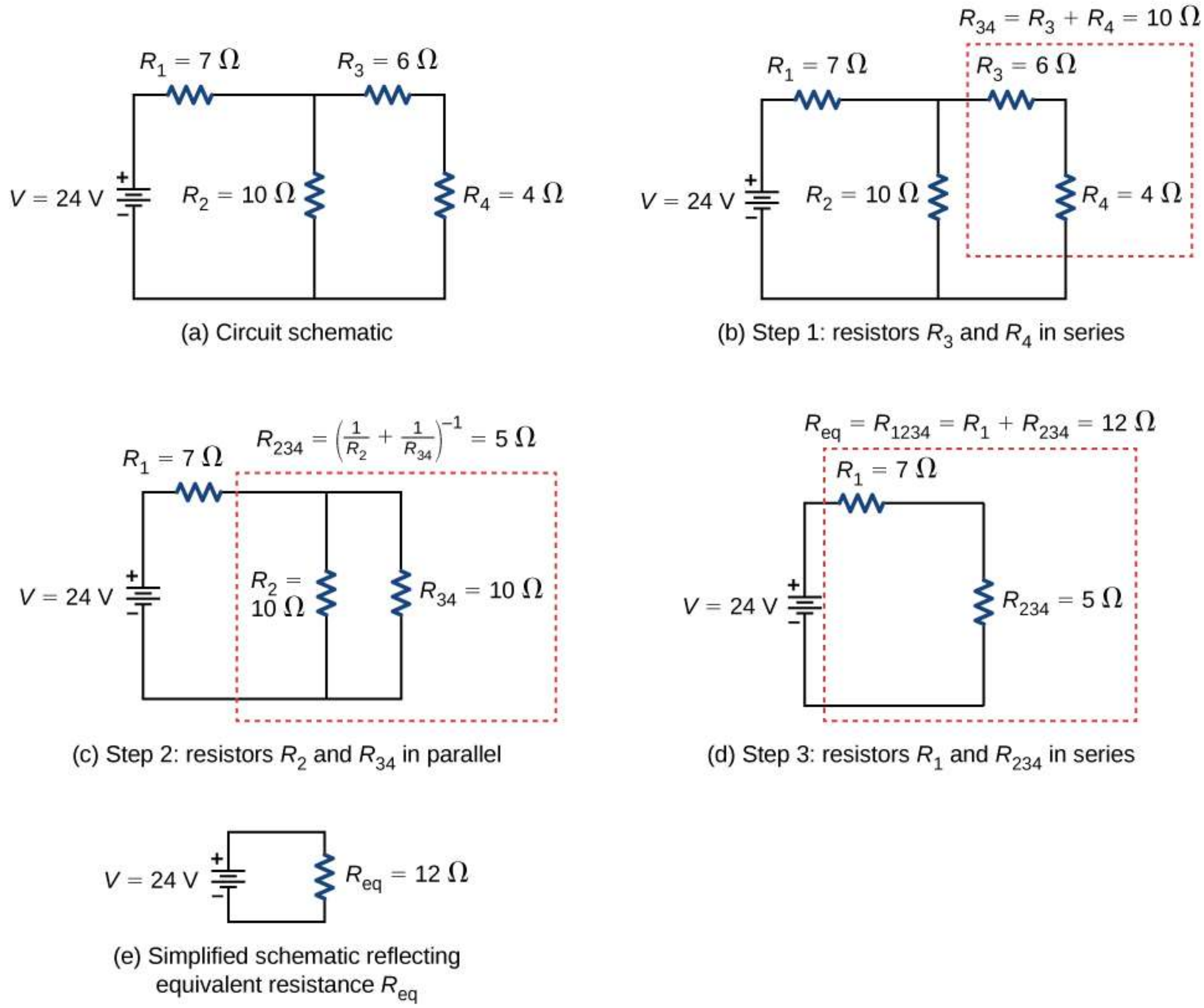


Figure 10.15 (a) The original circuit of four resistors. (b) Step 1: The resistors R_3 and R_4 are in series and the equivalent resistance is $R_{34} = 10\,\Omega$. (c) Step 2: The reduced circuit shows resistors R_2 and R_{34} are in parallel, with an equivalent resistance of $R_{234} = 5\,\Omega$. (d) Step 3: The reduced circuit shows that R_1 and R_{234} are in series with an equivalent resistance of $R_{1234} = 12\,\Omega$, which is the equivalent resistance R_{eq}. (e) The reduced circuit with a voltage source of $V = 24\text{ V}$ with an equivalent resistance of $R_{\text{eq}} = 12\,\Omega$. This results in a current of $I = 2\text{ A}$ from the voltage source.

Notice that resistors R_3 and R_4 are in series. They can be combined into a single equivalent resistance. One method of keeping track of the process is to include the resistors as subscripts. Here the equivalent resistance of R_3 and R_4 is

$$R_{34} = R_3 + R_4 = 6\,\Omega + 4\,\Omega = 10\,\Omega.$$

The circuit now reduces to three resistors, shown in Figure 10.15(c). Redrawing, we now see that resistors R_2 and R_{34} constitute a parallel circuit. Those two resistors can be reduced to an equivalent resistance:

$$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_{34}}\right)^{-1} = \left(\frac{1}{10\,\Omega} + \frac{1}{10\,\Omega}\right)^{-1} = 5\,\Omega.$$

This step of the process reduces the circuit to two resistors, shown in in Figure 10.15(d). Here, the circuit reduces to two resistors, which in this case are in series. These two resistors can be reduced to an equivalent resistance, which is the equivalent resistance of the circuit:

$$R_{\text{eq}} = R_{1234} = R_1 + R_{234} = 7\,\Omega + 5\,\Omega = 12\,\Omega.$$

The main goal of this circuit analysis is reached, and the circuit is now reduced to a single resistor and single voltage source.

Now we can analyze the circuit. The current provided by the voltage source is $I = \frac{V}{R_{eq}} = \frac{24\text{ V}}{12\,\Omega} = 2\text{ A}$. This current runs through resistor R_1 and is designated as I_1. The potential drop across R_1 can be found using Ohm's law:

$$V_1 = I_1 R_1 = (2\text{ A})(7\,\Omega) = 14\text{ V}.$$

Looking at Figure 10.15(c), this leaves $24\text{ V} - 14\text{ V} = 10\text{ V}$ to be dropped across the parallel combination of R_2 and R_{34}. The current through R_2 can be found using Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{10\text{ V}}{10\,\Omega} = 1\text{ A}.$$

The resistors R_3 and R_4 are in series so the currents I_3 and I_4 are equal to

$$I_3 = I_4 = I - I_2 = 2\text{ A} - 1\text{ A} = 1\text{ A}.$$

Using Ohm's law, we can find the potential drop across the last two resistors. The potential drops are $V_3 = I_3 R_3 = 6\text{ V}$ and $V_4 = I_4 R_4 = 4\text{ V}$. The final analysis is to look at the power supplied by the voltage source and the power dissipated by the resistors. The power dissipated by the resistors is

$$\begin{aligned}
P_1 &= I_1^2 R_1 = (2\text{ A})^2 (7\,\Omega) = 28\text{ W}, \\
P_2 &= I_2^2 R_2 = (1\text{ A})^2 (10\,\Omega) = 10\text{ W}, \\
P_3 &= I_3^2 R_3 = (1\text{ A})^2 (6\,\Omega) = 6\text{ W}, \\
P_4 &= I_4^2 R_4 = (1\text{ A})^2 (4\,\Omega) = 4\text{ W}, \\
P_{\text{dissipated}} &= P_1 + P_2 + P_3 + P_4 = 48\text{ W}.
\end{aligned}$$

The total energy is constant in any process. Therefore, the power supplied by the voltage source is $P_s = IV = (2\text{ A})(24\text{ V}) = 48\text{ W}$. Analyzing the power supplied to the circuit and the power dissipated by the resistors is a good check for the validity of the analysis; they should be equal.

EXAMPLE 10.4

Combining Series and Parallel Circuits

Figure 10.16 shows resistors wired in a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the equivalent resistance of the circuit. (b) What is the potential drop V_1 across resistor R_1? (c) Find the current I_2 through resistor R_2. (d) What power is dissipated by R_2?

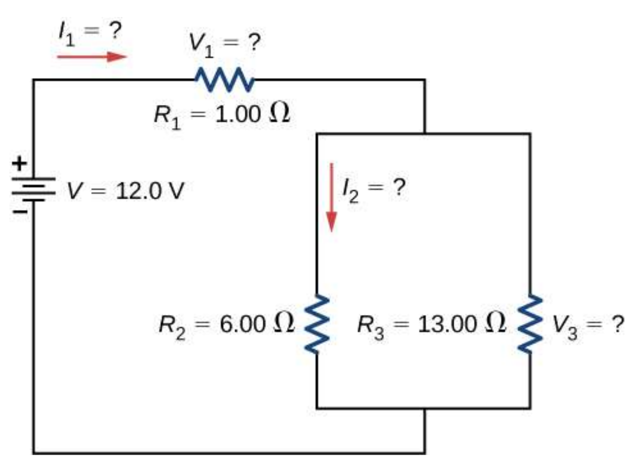


Figure 10.16 These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy

(a) To find the equivalent resistance, first find the equivalent resistance of the parallel connection of R_2 and R_3. Then use this result to find the equivalent resistance of the series connection with R_1.

(b) The current through R_1 can be found using Ohm's law and the voltage applied. The current through R_1 is equal to the current from the battery. The potential drop V_1 across the resistor R_1 (which represents the resistance in the connecting wires) can be found using Ohm's law.

(c) The current through R_2 can be found using Ohm's law $I_2 = \frac{V_2}{R_2}$. The voltage across R_2 can be found using $V_2 = V - V_1$.

(d) Using Ohm's law ($V_2 = I_2 R_2$), the power dissipated by the resistor can also be found using $P_2 = I_2^2 R_2 = \frac{V_2^2}{R_2}$.

Solution

a. To find the equivalent resistance of the circuit, notice that the parallel connection of R_2 and R_3 is in series with R_1, so the equivalent resistance is

$$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 1.00\,\Omega + \left(\frac{1}{6.00\,\Omega} + \frac{1}{13.00\,\Omega}\right)^{-1} = 5.10\,\Omega.$$

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\,\Omega$ and $0.804\,\Omega$, respectively).

b. The current through R_1 is equal to the current supplied by the battery:

$$I_1 = I = \frac{V}{R_{eq}} = \frac{12.0\text{ V}}{5.10\,\Omega} = 2.35\text{ A}.$$

The voltage across R_1 is

$$V_1 = I_1 R_1 = (2.35\text{ A})(1\,\Omega) = 2.35\text{ V}.$$

The voltage applied to R_2 and R_3 is less than the voltage supplied by the battery by an amount V_1. When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3.

c. To find the current through R_2, we must first find the voltage applied to it. The voltage across the two resistors in parallel is the same:

$$V_2 = V_3 = V - V_1 = 12.0\text{ V} - 2.35\text{ V} = 9.65\text{ V}.$$

Now we can find the current I_2 through resistance R_2 using Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{9.65\text{ V}}{6.00\,\Omega} = 1.61\text{ A}.$$

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

d. The power dissipated by R_2 is given by

$$P_2 = I_2^2 R_2 = (1.61\text{ A})^2(6.00\,\Omega) = 15.5\text{ W}.$$

Significance

The analysis of complex circuits can often be simplified by reducing the circuit to a voltage source and an equivalent resistance. Even if the entire circuit cannot be reduced to a single voltage source and a single equivalent resistance, portions of the circuit may be reduced, greatly simplifying the analysis.

CHECK YOUR UNDERSTANDING 10.5

Consider the electrical circuits in your home. Give at least two examples of circuits that must use a combination of series and parallel circuits to operate efficiently.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the *IR* drop in the wires can also be significant and may become apparent from the heat generated in the cord.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in Figure 10.17. The device represented by R_3 has a very low resistance, so when it is switched on, a large current flows. This increased current causes a larger *IR* drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

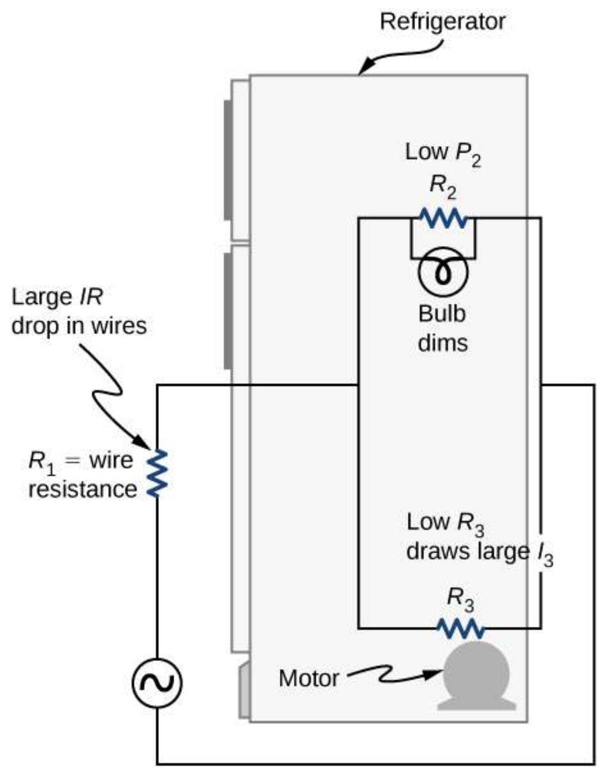


Figure 10.17 Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant *IR* drop in the wires and reduces the voltage across the light.

PROBLEM-SOLVING STRATEGY

Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the known values for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel.

5. Check to see whether the answers are reasonable and consistent.

EXAMPLE 10.5

Combining Series and Parallel Circuits

Two resistors connected in series (R_1, R_2) are connected to two resistors that are connected in parallel (R_3, R_4). The series-parallel combination is connected to a battery. Each resistor has a resistance of 10.00 Ohms. The wires connecting the resistors and battery have negligible resistance. A current of 2.00 Amps runs through resistor R_1. What is the voltage supplied by the voltage source?

Strategy

Use the steps in the preceding problem-solving strategy to find the solution for this example.

Solution

1. Draw a clear circuit diagram (Figure 10.18).

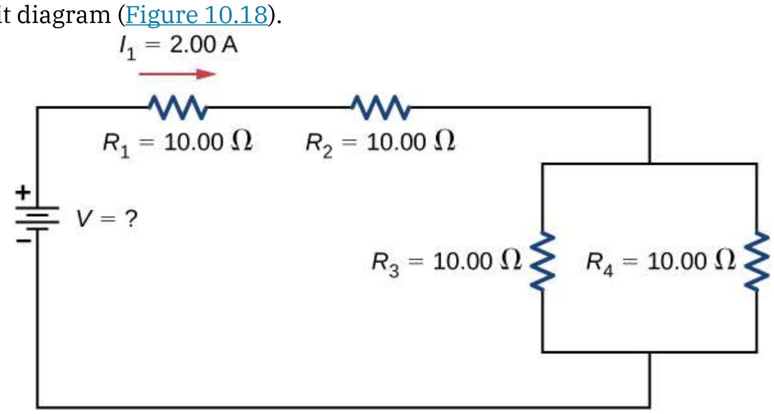


Figure 10.18 To find the unknown voltage, we must first find the equivalent resistance of the circuit.

2. The unknown is the voltage of the battery. In order to find the voltage supplied by the battery, the equivalent resistance must be found.
3. In this circuit, we already know that the resistors R_1 and R_2 are in series and the resistors R_3 and R_4 are in parallel. The equivalent resistance of the parallel configuration of the resistors R_3 and R_4 is in series with the series configuration of resistors R_1 and R_2.
4. The voltage supplied by the battery can be found by multiplying the current from the battery and the equivalent resistance of the circuit. The current from the battery is equal to the current through R_1 and is equal to 2.00 A. We need to find the equivalent resistance by reducing the circuit. To reduce the circuit, first consider the two resistors in parallel. The equivalent resistance is $R_{34} = \left(\frac{1}{10.00\ \Omega} + \frac{1}{10.00\ \Omega}\right)^{-1} = 5.00\ \Omega$. This parallel combination is in series with the other two resistors, so the equivalent resistance of the circuit is $R_{\text{eq}} = R_1 + R_2 + R_{34} = 25.00\ \Omega$. The voltage supplied by the battery is therefore $V = IR_{\text{eq}} = 2.00\ \text{A}\ (25.00\ \Omega) = 50.00\ \text{V}$.
5. One way to check the consistency of your results is to calculate the power supplied by the battery and the power dissipated by the resistors. The power supplied by the battery is $P_{\text{batt}} = IV = 100.00\ \text{W}$. Since they are in series, the current through R_2 equals the current through R_1. Since $R_3 = R_4$, the current through each will be 1.00 Amps. The power dissipated by the resistors is equal to the sum of the power dissipated by each resistor:
$$P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 = 40.00\ \text{W} + 40.00\ \text{W} + 10.00\ \text{W} + 10.00\ \text{W} = 100.00\ \text{W}.$$
Since the power dissipated by the resistors equals the power supplied by the battery, our solution seems consistent.

Significance

If a problem has a combination of series and parallel, as in this example, it can be reduced in steps by using the preceding problem-solving strategy and by considering individual groups of series or parallel connections. When finding R_{eq} for a parallel connection, the reciprocal must be taken with care. In addition, units and numerical results must be reasonable. Equivalent series resistance should be greater, whereas equivalent parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

10.3 Kirchhoff's Rules

Learning Objectives

By the end of this section, you will be able to:

- State Kirchhoff's junction rule
- State Kirchhoff's loop rule
- Analyze complex circuits using Kirchhoff's rules

We have just seen that some circuits may be analyzed by reducing a circuit to a single voltage source and an equivalent resistance. Many complex circuits cannot be analyzed with the series-parallel techniques developed in the preceding sections. In this section, we elaborate on the use of Kirchhoff's rules to analyze more complex circuits. For example, the circuit in Figure 10.19 is known as a multi-loop circuit, which consists of junctions. A junction, also known as a node, is a connection of three or more wires. In this circuit, the previous methods cannot be used, because not all the resistors are in clear series or parallel configurations that can be reduced. Give it a try. The resistors R_1 and R_2 are in series and can be reduced to an equivalent resistance. The same is true of resistors R_4 and R_5. But what do you do then?

Even though this circuit cannot be analyzed using the methods already learned, two circuit analysis rules can be used to analyze any circuit, simple or complex. The rules are known as **Kirchhoff's rules**, after their inventor Gustav Kirchhoff (1824–1887).

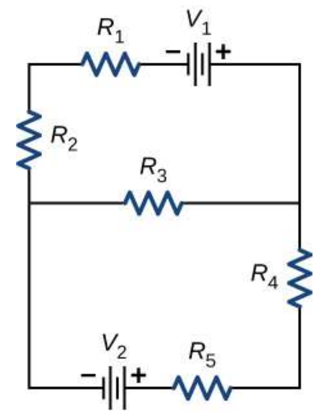


Figure 10.19 This circuit cannot be reduced to a combination of series and parallel connections. However, we can use Kirchhoff's rules to analyze it.

Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}}. \quad (10.4)$$

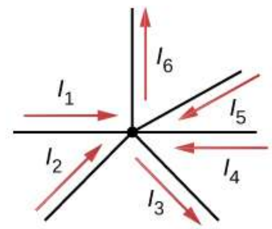
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero:

$$\sum V = 0. \quad (10.5)$$

We now provide explanations of these two rules, followed by problem-solving hints for applying them and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) applies to the charge entering and leaving a junction (Figure 10.20). As stated earlier, a junction, or node, is a connection of three or more wires. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out.



$$\sum I_{in} = \sum I_{out}$$
$$I_1 + I_2 + I_4 + I_5 = I_3 + I_6$$

Figure 10.20 Charge must be conserved, so the sum of currents into a junction must be equal to the sum of currents out of the junction.

Although it is an over-simplification, an analogy can be made with water pipes connected in a plumbing junction. If the wires in Figure 10.20 were replaced by water pipes, and the water was assumed to be incompressible, the volume of water flowing into the junction must equal the volume of water flowing out of the junction.

Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) applies to potential differences. The loop rule is stated in terms of potential V rather than potential energy, but the two are related since $U = qV$. In a closed loop, whatever energy is supplied by a voltage source, the energy must be transferred into other forms by the devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Kirchhoff's loop rule states that the algebraic sum of potential differences, including voltage supplied by the voltage sources and resistive elements, in any loop must be equal to zero. For example, consider a simple loop with no junctions, as in Figure 10.21.

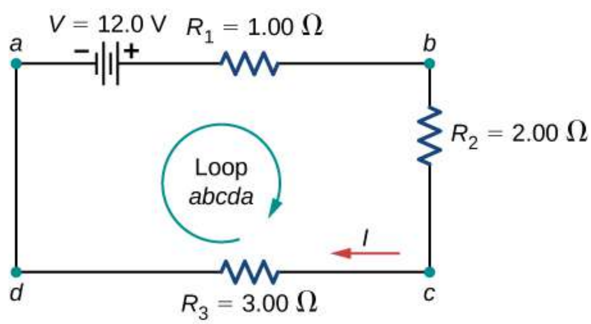


Figure 10.21 A simple loop with no junctions. Kirchhoff's loop rule states that the algebraic sum of the voltage differences is equal to zero.

The circuit consists of a voltage source and three external load resistors. The labels *a*, *b*, *c*, and *d* serve as references, and have no other significance. The usefulness of these labels will become apparent soon. The loop is designated as Loop *abcda*, and the labels help keep track of the voltage differences as we travel around the circuit. Start at point *a* and travel to point *b*. The voltage of the voltage source is added to the equation and the potential drop of the resistor R_1 is subtracted. From point *b* to *c*, the potential drop across R_2 is subtracted. From *c* to *d*, the potential drop across R_3 is subtracted. From points *d* to *a*, nothing is done because there are no components.

Figure 10.22 shows a graph of the voltage as we travel around the loop. Voltage increases as we cross the battery, whereas voltage decreases as we travel across a resistor. The potential drop, or change in the electric potential, is equal to the current through the resistor times the resistance of the resistor. Since the wires have negligible resistance, the voltage remains constant as we cross the wires connecting the components.

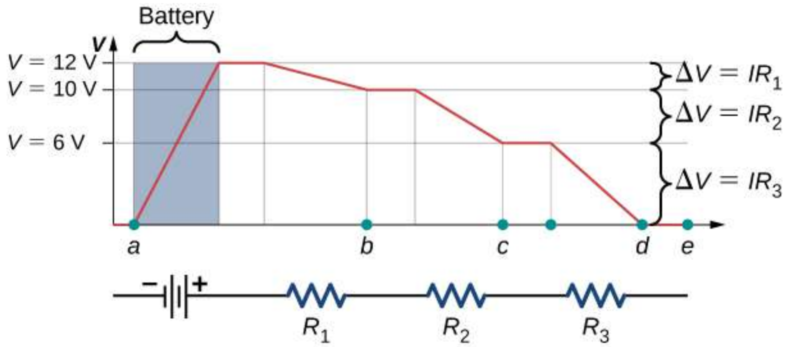


Figure 10.22 A voltage graph as we travel around the circuit. The voltage increases as we cross the battery and decreases as we cross each resistor. Since the resistance of the wire is quite small, we assume that the voltage remains constant as we cross the wires connecting the components.

Then Kirchhoff's loop rule states

$$V - IR_1 - IR_2 - IR_3 = 0.$$

The loop equation can be used to find the current through the loop:

$$I = \frac{V}{R_1 + R_2 + R_2} = \frac{12.00\text{ V}}{1.00\,\Omega + 2.00\,\Omega + 3.00\,\Omega} = 2.00\text{ A}.$$

This loop could have been analyzed using the previous methods, but we will demonstrate the power of Kirchhoff's method in the next section.

Applying Kirchhoff's Rules

By applying Kirchhoff's rules, we generate a set of linear equations that allow us to find the unknown values in circuits. These may be currents, voltages, or resistances. Each time a rule is applied, it produces an equation. If there are as many independent equations as unknowns, then the problem can be solved.

Using Kirchhoff's method of analysis requires several steps, as listed in the following procedure.

PROBLEM-SOLVING STRATEGY

Kirchhoff's Rules

1. Label points in the circuit diagram using lowercase letters *a*, *b*, *c*, ... These labels simply help with orientation.
2. Locate the junctions in the circuit. The junctions are points where three or more wires connect. Label each junction with the currents and directions into and out of it. Make sure at least one current points into the junction and at least one current points out of the junction.
3. Choose the loops in the circuit. Every component must be contained in at least one loop, but a component may be contained in more than one loop.
4. Apply the junction rule. Again, some junctions should not be included in the analysis. You need only use enough nodes to include every current.
5. Apply the loop rule. Use the map in Figure 10.23.

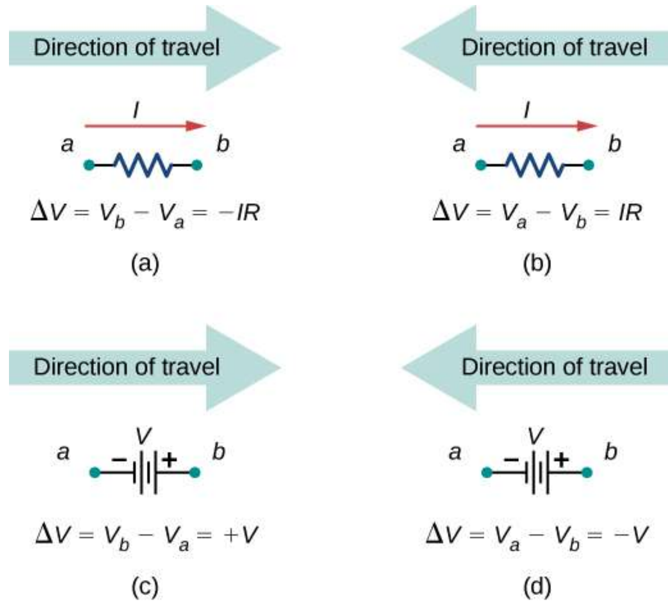


Figure 10.23 Each of these resistors and voltage sources is traversed from *a* to *b*. (a) When moving across a resistor in the same direction as the current flow, subtract the potential drop. (b) When moving across a resistor in the opposite direction as the current flow, add the potential drop. (c) When moving across a voltage source from the negative terminal to the positive terminal, add the potential drop. (d) When moving across a voltage source from the positive terminal to the negative terminal, subtract the potential drop.

Let's examine some steps in this procedure more closely. When locating the junctions in the circuit, do not be concerned about the direction of the currents. If the direction of current flow is not obvious, choosing any direction is sufficient as long as at least one current points into the junction and at least one current points out of the junction. If the arrow is in the opposite direction of the conventional current flow, the result for the current in question will be negative but the answer will still be correct.

The number of nodes depends on the circuit. Each current should be included in a node and thus included in at least one junction equation. Do not include nodes that are not linearly independent, meaning nodes that contain the same information.

Consider Figure 10.24. There are two junctions in this circuit: Junction *b* and Junction *e*. Points *a*, *c*, *d*, and *f* are not junctions, because a junction must have three or more connections. The equation for Junction *b* is $I_1 = I_2 + I_3$, and the equation for Junction *e* is $I_2 + I_3 = I_1$. These are equivalent equations, so it is necessary to keep only one of them.

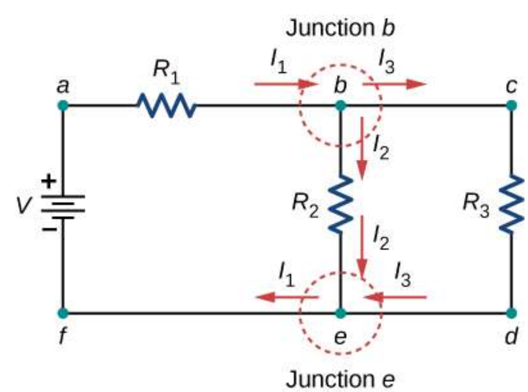


Figure 10.24 At first glance, this circuit contains two junctions, Junction *b* and Junction *e*, but only one should be considered because their junction equations are equivalent.

When choosing the loops in the circuit, you need enough loops so that each component is covered once, without repeating loops. Figure 10.25 shows four choices for loops to solve a sample circuit; choices (a), (b), and (c) have a sufficient amount of loops to solve the circuit completely. Option (d) reflects more loops than necessary to solve the circuit.

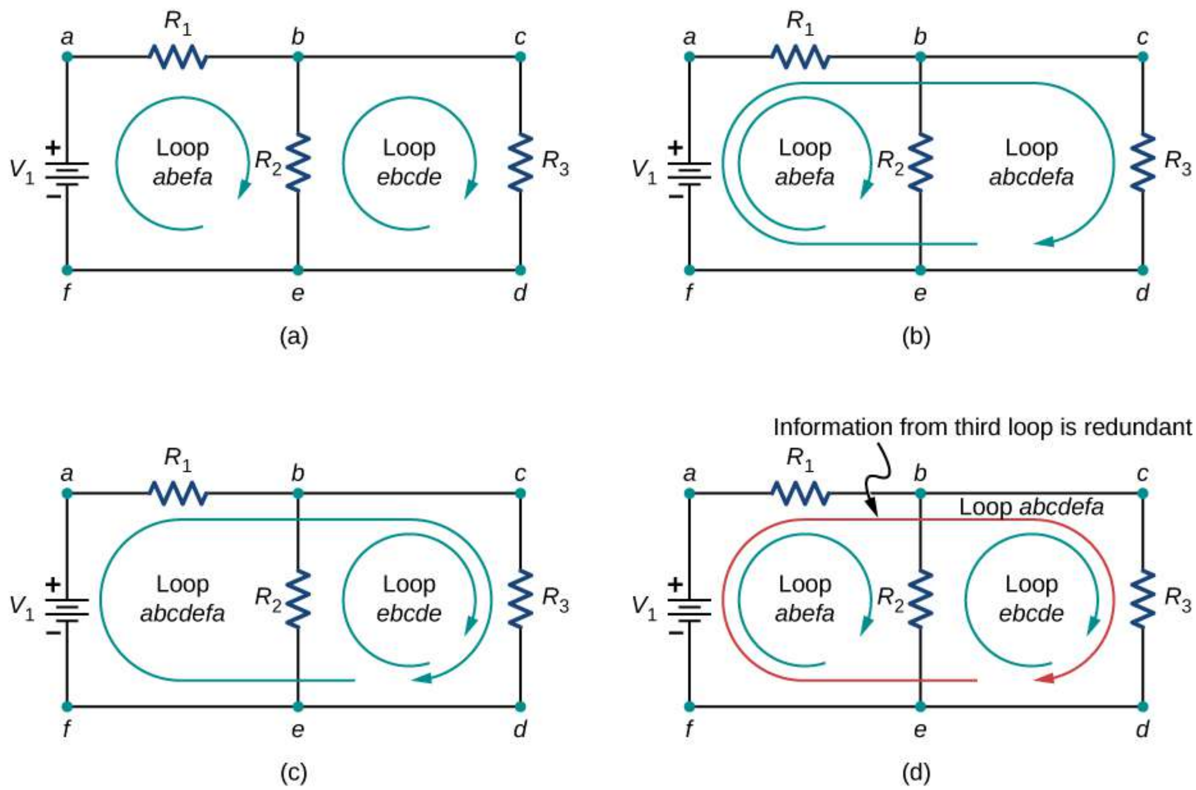


Figure 10.25 Panels (a)–(c) are sufficient for the analysis of the circuit. In each case, the two loops shown contain all the circuit elements necessary to solve the circuit completely. Panel (d) shows three loops used, which is more than necessary. Any two loops in the system will contain all information needed to solve the circuit. Adding the third loop provides redundant information.

Consider the circuit in Figure 10.26(a). Let us analyze this circuit to find the current through each resistor. First, label the circuit as shown in part (b).

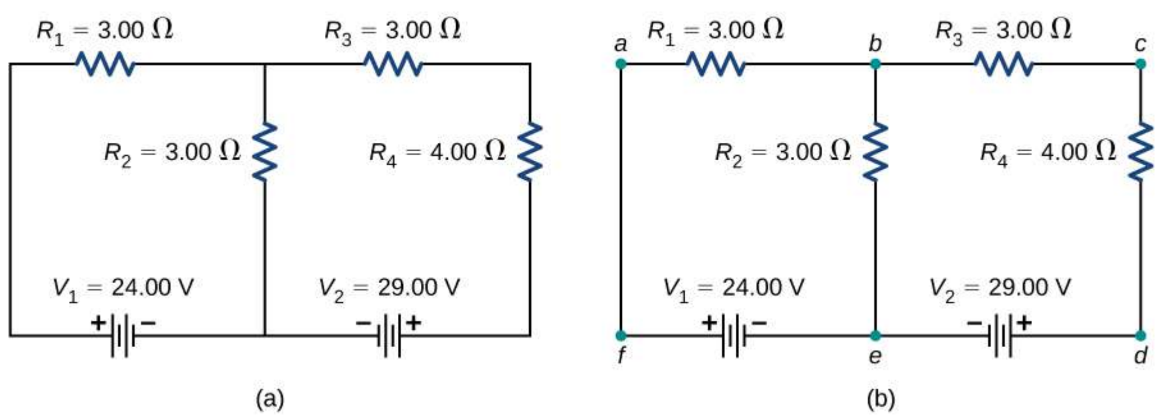


Figure 10.26 (a) A multi-loop circuit. (b) Label the circuit to help with orientation.

Next, determine the junctions. In this circuit, points *b* and *e* each have three wires connected, making them junctions. Start to apply Kirchhoff's junction rule $\left(\sum I_{\text{in}} = \sum I_{\text{out}}\right)$ by drawing arrows representing the currents and labeling each arrow, as shown in Figure 10.27(b). Junction *b* shows that $I_1 = I_2 + I_3$ and Junction *e* shows that $I_2 + I_3 = I_1$. Since Junction *e* gives the same information of Junction *b*, it can be disregarded. This circuit has three unknowns, so we need three linearly independent equations to analyze it.

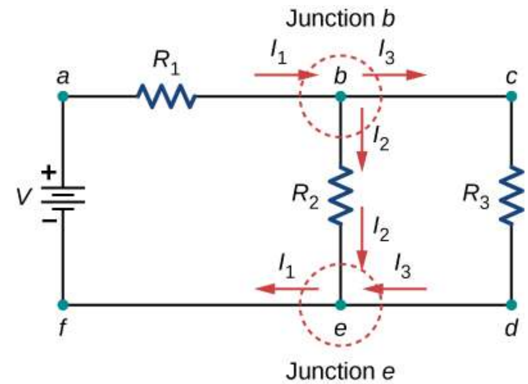


Figure 10.27 (a) This circuit has two junctions, labeled *b* and *e*, but only node *b* is used in the analysis. (b) Labeled arrows represent the currents into and out of the junctions.

Next we need to choose the loops. In Figure 10.28, Loop *abefa* includes the voltage source V_1 and resistors R_1 and R_2. The loop starts at point *a*, then travels through points *b*, *e*, and *f*, and then back to point *a*. The second loop, Loop *ebcde*, starts at point *e* and includes resistors R_2 and R_3, and the voltage source V_2.

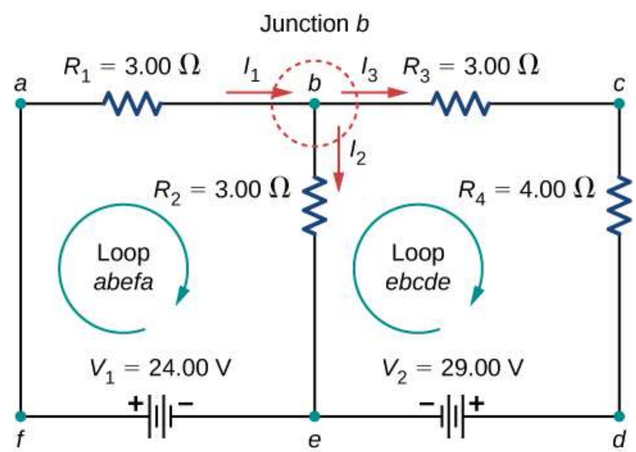


Figure 10.28 Choose the loops in the circuit.

Now we can apply Kirchhoff's loop rule, using the map in Figure 10.23. Starting at point *a* and moving to point *b*, the resistor R_1 is crossed in the same direction as the current flow I_1, so the potential drop $I_1 R_1$ is subtracted. Moving from point *b* to point *e*, the resistor R_2 is crossed in the same direction as the current flow I_2 so the potential drop $I_2 R_2$ is subtracted. Moving from point *e* to point *f*, the voltage source V_1 is crossed from the negative terminal to the positive terminal, so V_1 is added. There are no components between points *f* and *a*. The sum of the voltage differences must equal zero:

$$\text{Loop } abefa: \quad -I_1 R_1 - I_2 R_2 + V_1 = 0 \text{ or } V_1 = I_1 R_1 + I_2 R_2.$$

Finally, we check loop *ebcde*. We start at point *e* and move to point *b*, crossing R_2 in the opposite direction as the current flow I_2. The potential drop $I_2 R_2$ is added. Next, we cross R_3 and R_4 in the same direction as the current flow I_3 and subtract the potential drops $I_3 R_3$ and $I_3 R_4$. Note that the current is the same through resistors R_3 and R_4, because they are connected in series. Finally, the voltage source is crossed from the positive terminal to the negative terminal, and the voltage source V_2 is subtracted. The sum of these voltage differences equals zero and yields the loop equation

$$\text{Loop } ebcde: \ I_2 R_2 - I_3 (R_3 + R_4) - V_2 = 0.$$

We now have three equations, which we can solve for the three unknowns.

$$\begin{aligned}&(1)\ \text{Junction } b: I_1 - I_2 - I_3 = 0.\\&(2)\ \text{Loop } abefa: \quad I_1 R_1 + I_2 R_2 = V_1.\\&(3)\ \text{Loop } ebcde: I_2 R_2 - I_3 (R_3 + R_4) = V_2.\end{aligned}$$

To solve the three equations for the three unknown currents, start by eliminating current I_2. First add Eq. (1) times R_2 to Eq. (2). The result is labeled as Eq. (4):

$$(R_1 + R_2) I_1 - R_2 I_3 = V_1.$$

$$(4)\ 6\,\Omega I_1 - 3\,\Omega I_3 = 24\text{ V}.$$

Next, subtract Eq. (3) from Eq. (2). The result is labeled as Eq. (5):

$$I_1 R_1 + I_3 (R_3 + R_4) = V_1 - V_2.$$

$$(5)\ 3\,\Omega I_1 + 7\,\Omega I_3 = -5\text{ V}.$$

We can solve Eqs. (4) and (5) for current I_1. Adding seven times Eq. (4) and three times Eq. (5) results in $51\,\Omega I_1 = 153\text{ V}$, or $I_1 = 3.00\text{ A}$. Using Eq. (4) results in $I_3 = -2.00\text{ A}$. Finally, Eq. (1) yields $I_2 = I_1 - I_3 = 5.00\text{ A}$. One way to check that the solutions are consistent is to check the power supplied by the voltage sources and the power dissipated by the resistors:

$$\begin{aligned}P_{\text{in}} &= I_1 V_1 + I_3 V_2 = 130\text{ W},\\P_{\text{out}} &= I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_3^2 R_4 = 130\text{ W}.\end{aligned}$$

Note that the solution for the current I_3 is negative. This is the correct answer, but suggests that the arrow originally drawn in the junction analysis is the direction opposite of conventional current flow. The power supplied by the second voltage source is 58 W and not −58 W.

EXAMPLE 10.6

Calculating Current by Using Kirchhoff's Rules

Find the currents flowing in the circuit in Figure 10.29.

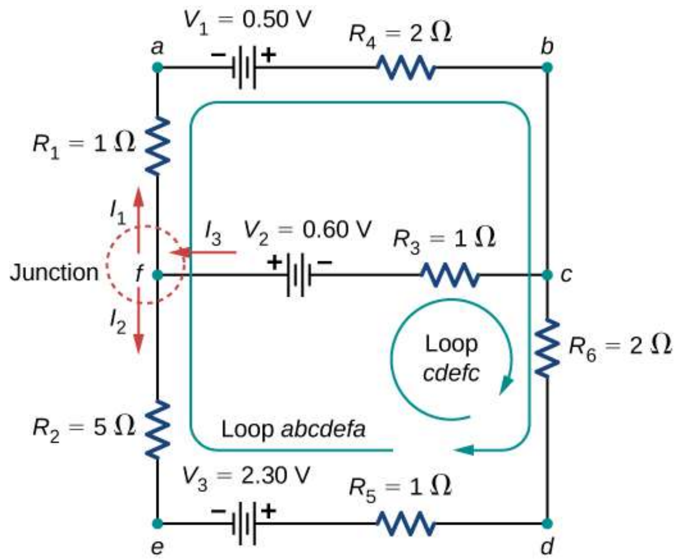


Figure 10.29 This circuit is combination of series and parallel configurations of resistors and voltage sources. This circuit cannot be analyzed using the techniques discussed in Electromotive Force but can be analyzed using Kirchhoff's rules.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1, I_2, and I_3 in the figure, and assumptions have been made about their directions. Locations on the diagram have been labeled with letters *a* through *h*. In the solution, we apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

Applying the junction and loop rules yields the following three equations. We have three unknowns, so three equations are required.

$$\begin{aligned}
&\text{Junction } c: \; I_1 + I_2 = I_3. \\
&\text{Loop } abcdefa: \; I_1 (R_1 + R_4) - I_2 (R_2 + R_5 + R_6) = V_1 - V_3. \\
&\text{Loop } cdefc: \; I_2 (R_2 + R_5 + R_6) + I_3 R_3 = V_2 + V_3.
\end{aligned}$$

Simplify the equations by placing the unknowns on one side of the equations.

$$\begin{aligned}
&\text{Junction } c: \; I_1 + I_2 - I_3 = 0. \\
&\text{Loop } abcdefa: \; I_1 (3\ \Omega) - I_2 (8\ \Omega) = 0.5\ \text{V} - 2.30\ \text{V}. \\
&\text{Loop } cdefc: \; I_2 (8\ \Omega) + I_3 (1\ \Omega) = 0.6\ \text{V} + 2.30\ \text{V}.
\end{aligned}$$

Simplify the equations. The first loop equation can be simplified by dividing both sides by 3.00. The second loop equation can be simplified by dividing both sides by 6.00.

$$\begin{aligned}
&\text{Junction } c: \; I_1 + I_2 - I_3 = 0. \\
&\text{Loop } abcdefa: \; I_1 (3\ \Omega) - I_2 (8\ \Omega) = -1.8\ \text{V}. \\
&\text{Loop } cdefc: \; I_2 (8\ \Omega) + I_3 (1\ \Omega) = 2.9\ \text{V}.
\end{aligned}$$

The results are

$$I_1 = 0.20\ \text{A}, \quad I_2 = 0.30\ \text{A}, \quad I_3 = 0.50\ \text{A}.$$

Significance

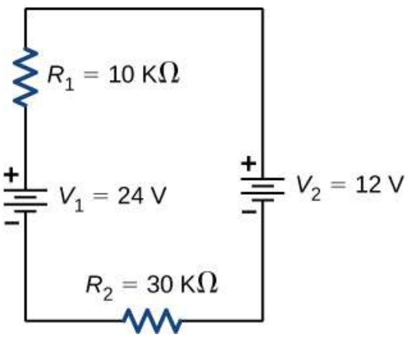
A method to check the calculations is to compute the power dissipated by the resistors and the power supplied by the voltage sources:

$$
\begin{aligned}
P_{R_1} &= I_1^2 R_1 = 0.04\ \text{W}. \\
P_{R_2} &= I_2^2 R_2 = 0.45\ \text{W}. \\
P_{R_3} &= I_3^2 R_3 = 0.25\ \text{W}. \\
P_{R_4} &= I_1^2 R_4 = 0.08\ \text{W}. \\
P_{R_5} &= I_2^2 R_5 = 0.09\ \text{W}. \\
P_{R_6} &= I_2^2 R_6 = 0.18\ \text{W}. \\
P_{\text{dissipated}} &= 1.09\ \text{W}. \\
P_{\text{source}} &= I_1 V_1 + I_2 V_3 + I_3 V_2 = 0.10\ \text{W} + 0.69\ \text{W} + 0.30\ \text{W} = 1.09\ \text{W}.
\end{aligned}
$$

The power supplied equals the power dissipated by the resistors.

CHECK YOUR UNDERSTANDING 10.6

In considering the following schematic and the power supplied and consumed by a circuit, will a voltage source always provide power to the circuit, or can a voltage source consume power?



EXAMPLE 10.7

Calculating Current by Using Kirchhoff's Rules

Find the current flowing in the circuit in Figure 10.30.

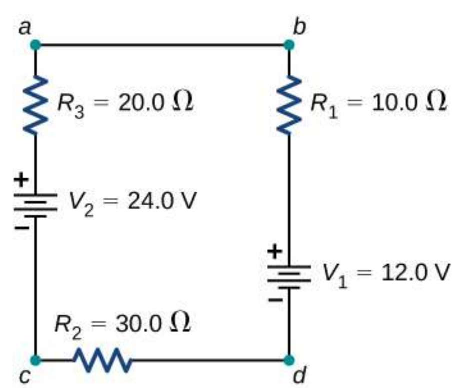


Figure 10.30 This circuit consists of three resistors and two batteries connected in series. Note that the batteries are connected with opposite polarities.

Strategy

This circuit can be analyzed using Kirchhoff's rules. There is only one loop and no nodes. Choose the direction of current flow. For this example, we will use the clockwise direction from point *a* to point *b*. Consider Loop *abcda* and use Figure 10.23 to write the loop equation. Note that according to Figure 10.23, battery V_1 will be added and battery V_2 will be subtracted.

Solution

Applying the junction rule yields the following three equations. We have one unknown, so one equation is required:

$$\text{Loop } abcda: \ -IR_1 - V_1 - IR_2 + V_2 - IR_3 = 0.$$

Simplify the equations by placing the unknowns on one side of the equations. Use the values given in the figure.

$$I\left(R_1 + R_2 + R_3\right) = V_2 - V_1.$$

$$I = \frac{V_2 - V_1}{R_1 + R_2 + R_3} = \frac{24\text{ V} - 12\text{ V}}{10.0\ \Omega + 30.0\ \Omega + 10.0\ \Omega} = 0.20\text{ A}.$$

Significance

The power dissipated or consumed by the circuit equals the power supplied to the circuit, but notice that the current in the battery V_1 is flowing through the battery from the positive terminal to the negative terminal and consumes power.

$$\begin{aligned}
P_{R_1} &= I^2 R_1 = 0.40\text{ W}\\
P_{R_2} &= I^2 R_2 = 1.20\text{ W}\\
P_{R_3} &= I^2 R_3 = 0.80\text{ W}\\
P_{V_1} &= IV_1 = 2.40\text{ W}\\
P_{\text{dissipated}} &= 4.80\text{ W}\\
P_{\text{source}} &= IV_2 = 4.80\text{ W}
\end{aligned}$$

The power supplied equals the power dissipated by the resistors and consumed by the battery V_1.

CHECK YOUR UNDERSTANDING 10.7

When using Kirchhoff's laws, you need to decide which loops to use and the direction of current flow through each loop. In analyzing the circuit in Example 10.7, the direction of current flow was chosen to be clockwise, from point *a* to point *b*. How would the results change if the direction of the current was chosen to be counterclockwise, from point *b* to point *a*?

Multiple Voltage Sources

Many devices require more than one battery. Multiple voltage sources, such as batteries, can be connected in series configurations, parallel configurations, or a combination of the two.

In series, the positive terminal of one battery is connected to the negative terminal of another battery. Any number of voltage sources, including batteries, can be connected in series. Two batteries connected in series are shown in Figure 10.31. Using Kirchhoff's loop rule for the circuit in part (b) gives the result

$$\begin{aligned}
&\varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2 - IR = 0,\\
&\left[\left(\varepsilon_1 + \varepsilon_2\right) - I\left(r_1 + r_2\right)\right] - IR = 0.
\end{aligned}$$

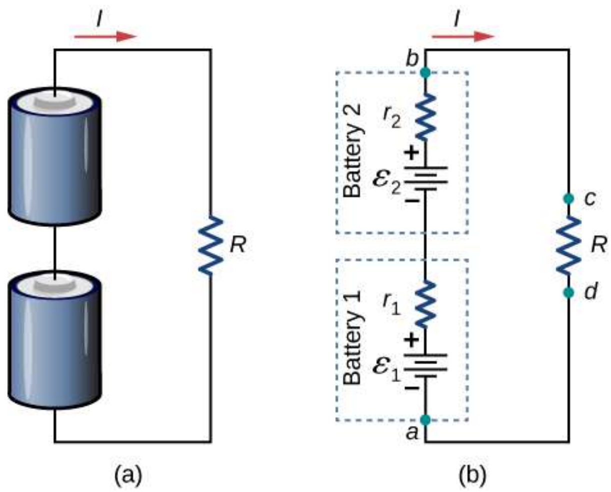


Figure 10.31 (a) Two batteries connected in series with a load resistor. (b) The circuit diagram of the two batteries and the load resistor, with each battery modeled as an idealized emf source and an internal resistance.

When voltage sources are in series, their internal resistances can be added together and their emfs can be added together to get the total values. Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf. In Figure 10.31, the terminal voltage is

$$V_{\text{terminal}} = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2) = [(\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)] = (\varepsilon_1 + \varepsilon_2) + Ir_{\text{eq}}.$$

Note that the same current I is found in each battery because they are connected in series. The disadvantage of series connections of cells is that their internal resistances are additive.

Batteries are connected in series to increase the voltage supplied to the circuit. For instance, an LED flashlight may have two AAA cell batteries, each with a terminal voltage of 1.5 V, to provide 3.0 V to the flashlight.

Any number of batteries can be connected in series. For N batteries in series, the terminal voltage is equal to

$$V_{\text{terminal}} = \left(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_{N-1} + \varepsilon_N\right) - I\left(r_1 + r_2 + \cdots + r_{N-1} + r_N\right) = \sum_{i=1}^{N} \varepsilon_i - Ir_{\text{eq}} \qquad \text{10.6}$$

where the equivalent resistance is $r_{\text{eq}} = \sum_{i=1}^{N} r_i$.

When a load is placed across voltage sources in series, as in Figure 10.32, we can find the current:

$$(\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2) = IR,$$
$$Ir_1 + Ir_2 + IR = \varepsilon_1 + \varepsilon_2,$$
$$I = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R}.$$

As expected, the internal resistances increase the equivalent resistance.

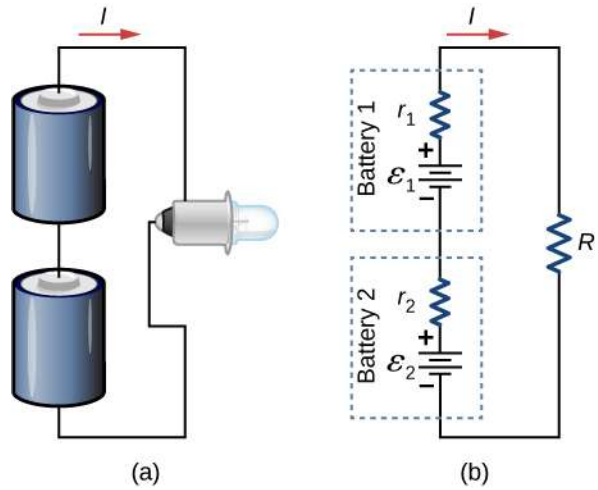


Figure 10.32 Two batteries connect in series to an LED bulb, as found in a flashlight.

Voltage sources, such as batteries, can also be connected in parallel. Figure 10.33 shows two batteries with identical emfs in parallel and connected to a load resistance. When the batteries are connect in parallel, the positive terminals are connected together and the negative terminals are connected together, and the load resistance is connected to the positive and negative terminals. Normally, voltage sources in parallel have identical emfs. In this simple case, since the voltage sources are in parallel, the total emf is the same as the individual emfs of each battery.

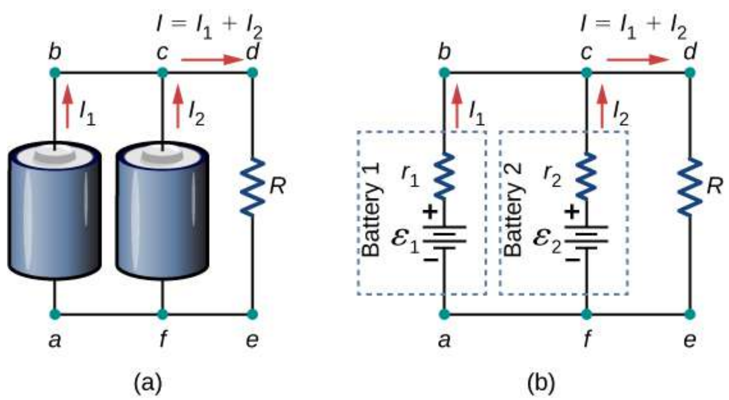


Figure 10.33 (a) Two batteries connect in parallel to a load resistor. (b) The circuit diagram shows the shows battery as an emf source and an internal resistor. The two emf sources have identical emfs (each labeled by ε) connected in parallel that produce the same emf.

Consider the Kirchhoff analysis of the circuit in Figure 10.33(b). There are two loops and a node at point *b* and $\varepsilon = \varepsilon_1 = \varepsilon_2$.

Node *b*: $I_1 + I_2 - I = 0$.

Loop *abcfa*: $$\begin{aligned} \varepsilon - I_1 r_1 + I_2 r_2 - \varepsilon &= 0, \\ I_1 r_1 &= I_2 r_2. \end{aligned}$$

Loop *fcdef*: $$\begin{aligned} \varepsilon_2 - I_2 r_2 - IR &= 0, \\ \varepsilon - I_2 r_2 - IR &= 0. \end{aligned}$$

Solving for the current through the load resistor results in $I = \frac{\varepsilon}{r_{\text{eq}} + R}$, where $r_{\text{eq}} = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1}$. The terminal voltage is equal to the potential drop across the load resistor $IR = \left(\frac{\varepsilon}{r_{\text{eq}} + R}\right)$. The parallel connection reduces the internal resistance and thus can produce a larger current.

Any number of batteries can be connected in parallel. For *N* batteries in parallel, the terminal voltage is equal to

$$V_{\text{terminal}} = \varepsilon - I\left(\frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_{N-1}} + \frac{1}{r_N}\right)^{-1} = \varepsilon - Ir_{\text{eq}} \tag{10.7}$$

where the equivalent resistance is $r_{\text{eq}} = \left(\sum_{i=1}^{N} \frac{1}{r_i}\right)^{-1}$.

As an example, some diesel trucks use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.

In summary, the terminal voltage of batteries in series is equal to the sum of the individual emfs minus the sum of the internal resistances times the current. When batteries are connected in parallel, they usually have equal emfs and the terminal voltage is equal to the emf minus the equivalent internal resistance times the current, where the equivalent internal resistance is smaller than the individual internal resistances. Batteries are connected in series to increase the terminal voltage to the load. Batteries are connected in parallel to increase the current to the load.

Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation, which is the conversion of sunlight directly into electricity, is based upon the photoelectric effect. The photoelectric effect is beyond the scope of this chapter and is covered in Photons and Matter Waves, but in general, photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight falling on the cell (the incident solar radiation known as the insolation). Under bright noon sunlight, a current per unit area of about $100\ \text{mA/cm}^2$ of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical energy needs. They can be wired together in series or in parallel—connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

Solar cells, like batteries, provide a direct current (dc) voltage. Current from a dc voltage source is unidirectional. Most household appliances need an alternating current (ac) voltage.

10.4 Electrical Measuring Instruments

Learning Objectives

By the end of this section, you will be able to:

- Describe how to connect a voltmeter in a circuit to measure voltage
- Describe how to connect an ammeter in a circuit to measure current
- Describe the use of an ohmmeter

Ohm's law and Kirchhoff's method are useful to analyze and design electrical circuits, providing you with the voltages across, the current through, and the resistance of the components that compose the circuit. To measure these parameters require instruments, and these instruments are described in this section.

DC Voltmeters and Ammeters

Whereas **voltmeters** measure voltage, **ammeters** measure current. Some of the meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are actually voltmeters or ammeters (Figure 10.34). The internal construction of the simplest of these meters and how they are connected to the system they monitor give further insight into applications of series and parallel connections.

Figure 10.34 The fuel and temperature gauges (far right and far left, respectively) in this 1996 Volkswagen are voltmeters that register the voltage output of “sender” units. These units are proportional to the amount of gasoline in the tank and to the engine temperature. (credit: Christian Giersing)

Measuring Current with an Ammeter

To measure the current through a device or component, the ammeter is placed in series with the device or component. A series connection is used because objects in series have the same current passing through them. (See Figure 10.35, where the ammeter is represented by the symbol A.)

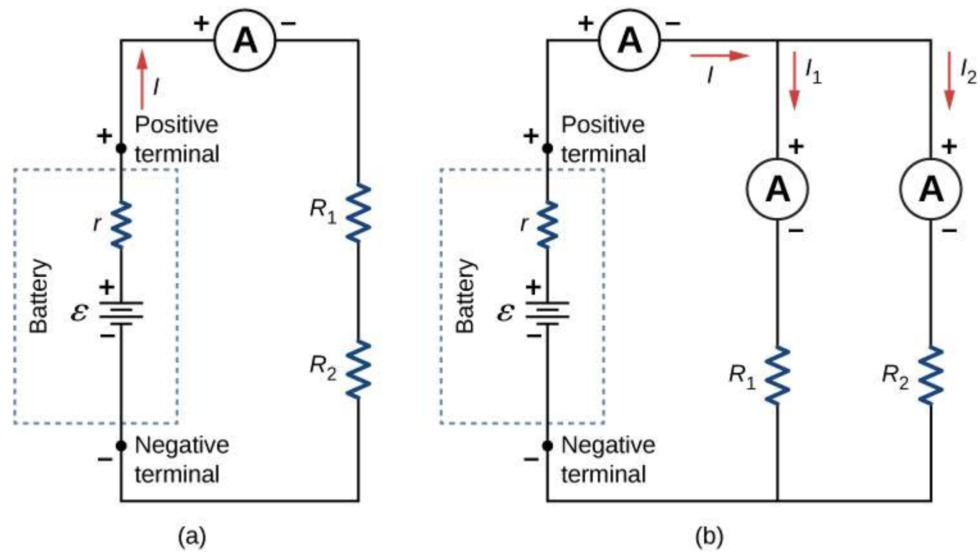


Figure 10.35 (a) When an ammeter is used to measure the current through two resistors connected in series to a battery, a single ammeter is placed in series with the two resistors because the current is the same through the two resistors in series. (b) When two resistors are connected in parallel with a battery, three meters, or three separate ammeter readings, are necessary to measure the current from the battery and through each resistor. The ammeter is connected in series with the component in question.

Ammeters need to have a very low resistance, a fraction of a milliohm. If the resistance is not negligible, placing the ammeter in the circuit would change the equivalent resistance of the circuit and modify the current that is being measured. Since the current in the circuit travels through the meter, ammeters normally contain a fuse to protect the meter from damage from currents which are too high.

Measuring Voltage with a Voltmeter

A voltmeter is connected in parallel with whatever device it is measuring. A parallel connection is used because objects in parallel experience the same potential difference. (See Figure 10.36, where the voltmeter is represented by the symbol V.)

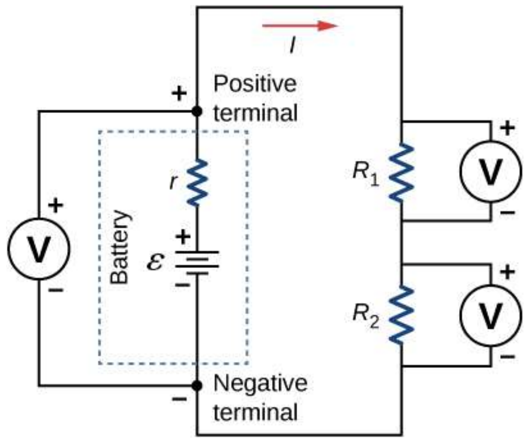


Figure 10.36 To measure potential differences in this series circuit, the voltmeter (V) is placed in parallel with the voltage source or either of the resistors. Note that terminal voltage is measured between the positive terminal and the negative terminal of the battery or voltage source. It is not possible to connect a voltmeter directly across the emf without including the internal resistance *r* of the battery.

Since voltmeters are connected in parallel, the voltmeter must have a very large resistance. Digital voltmeters convert the analog voltage into a digital value to display on a digital readout (Figure 10.37). Inexpensive voltmeters have resistances on the order of $R_M = 10\ \text{M}\,\Omega$, whereas high-precision voltmeters have resistances on the order of $R_M = 10\ \text{G}\,\Omega$. The value of the resistance may vary, depending on which scale is used on the meter.

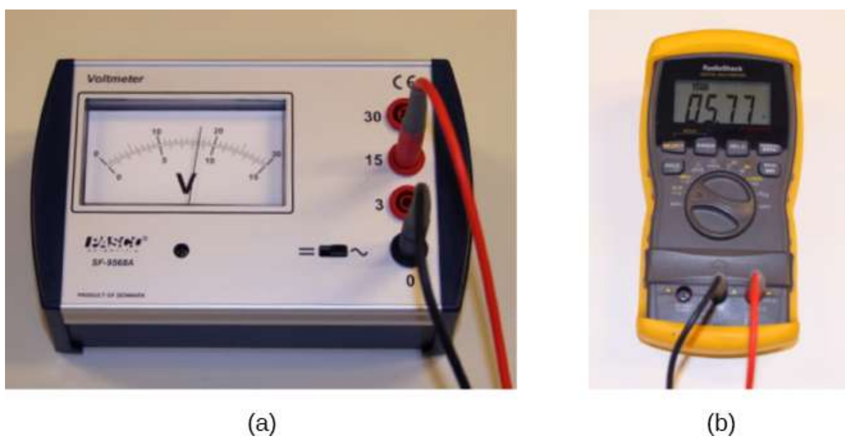


Figure 10.37 (a) An analog voltmeter uses a galvanometer to measure the voltage. (b) Digital meters use an analog-to-digital converter to measure the voltage. (credit: modification of works by Joseph J. Trout)

Analog and Digital Meters

You may encounter two types of meters in the physics lab: analog and digital. The term ‘analog’ refers to signals or information represented by a continuously variable physical quantity, such as voltage or current. An analog meter uses a galvanometer, which is essentially a coil of wire with a small resistance, in a magnetic field, with a pointer attached that points to a scale. Current flows through the coil, causing the coil to rotate. To use the galvanometer as an ammeter, a small resistance is placed in parallel with the coil. For a voltmeter, a large resistance is placed in series with the coil. A digital meter uses a component called an analog-to-digital (A to D) converter and expresses the current or voltage as a series of the digits 0 and 1, which are used to run a digital display. Most analog meters have been replaced by digital meters.

CHECK YOUR UNDERSTANDING 10.8

Digital meters are able to detect smaller currents than analog meters employing galvanometers. How does this explain their ability to measure voltage and current more accurately than analog meters?

INTERACTIVE

In this virtual lab (https://openstax.org/l/21cirreslabsim) simulation, you may construct circuits with resistors, voltage sources, ammeters and voltmeters to test your knowledge of circuit design.

Ohmmeters

An ohmmeter is an instrument used to measure the resistance of a component or device. The operation of the ohmmeter is based on Ohm's law. Traditional ohmmeters contained an internal voltage source (such as a battery) that would be connected across the component to be tested, producing a current through the component. A galvanometer was then used to measure the current and the resistance was deduced using Ohm's law. Modern digital meters use a constant current source to pass current through the component, and the voltage difference across the component is measured. In either case, the resistance is measured using Ohm's law $(R = V/I)$, where the voltage is known and the current is measured, or the current is known and the voltage is measured.

The component of interest should be isolated from the circuit; otherwise, you will be measuring the equivalent resistance of the circuit. An ohmmeter should never be connected to a "live" circuit, one with a voltage source connected to it and current running through it. Doing so can damage the meter.

10.5 RC Circuits

Learning Objectives

By the end of this section, you will be able to:

- Describe the charging process of a capacitor
- Describe the discharging process of a capacitor
- List some applications of RC circuits

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and several other phenomena that involve charging and discharging capacitors are discussed in this module.

Circuits with Resistance and Capacitance

An ***RC* circuit** is a circuit containing resistance and capacitance. As presented in Capacitance, the capacitor is an electrical component that stores electric charge, storing energy in an electric field.

Figure 10.38(a) shows a simple *RC* circuit that employs a dc (direct current) voltage source ε, a resistor R, a capacitor C, and a two-position switch. The circuit allows the capacitor to be charged or discharged, depending on the position of the switch. When the switch is moved to position A, the capacitor charges, resulting in the circuit in part (b). When the switch is moved to position B, the capacitor discharges through the resistor.

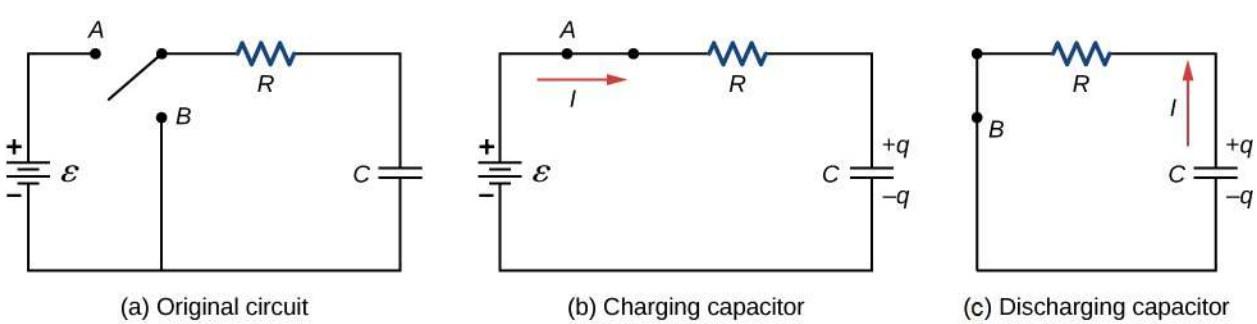


Figure 10.38 (a) An *RC* circuit with a two-pole switch that can be used to charge and discharge a capacitor. (b) When the switch is moved to position *A*, the circuit reduces to a simple series connection of the voltage source, the resistor, the capacitor, and the switch. (c) When the switch is moved to position *B*, the circuit reduces to a simple series connection of the resistor, the capacitor, and the switch. The voltage source is removed from the circuit.

Charging a Capacitor

We can use Kirchhoff's loop rule to understand the charging of the capacitor. This results in the equation $\varepsilon - V_R - V_c = 0$. This equation can be used to model the charge as a function of time as the capacitor charges. Capacitance is defined as $C = q/V$, so the voltage across the capacitor is $V_C = \frac{q}{C}$. Using Ohm's law, the potential drop across the resistor is $V_R = IR$, and the current is defined as $I = dq/dt$.

$$\begin{aligned} &\varepsilon - V_R - V_c = 0, \\ &\varepsilon - IR - \frac{q}{C} = 0, \\ &\varepsilon - R\frac{dq}{dt} - \frac{q}{C} = 0. \end{aligned}$$

This differential equation can be integrated to find an equation for the charge on the capacitor as a function of time.

$$\begin{aligned} &\varepsilon - R\frac{dq}{dt} - \frac{q}{C} = 0, \\ &\frac{dq}{dt} = \frac{\varepsilon C - q}{RC}, \\ &\int_0^q \frac{dq}{\varepsilon C - q} = \frac{1}{RC}\int_0^t dt. \end{aligned}$$

Let $u = \varepsilon C - q$, then $du = -dq$. The result is

$$\begin{aligned} &-\int_0^q \frac{du}{u} = \frac{1}{RC}\int_0^t dt, \\ &\ln\left(\frac{\varepsilon C - q}{\varepsilon C}\right) = -\frac{1}{RC}t, \\ &\frac{\varepsilon C - q}{\varepsilon C} = e^{-t/RC}. \end{aligned}$$

Simplifying results in an equation for the charge on the charging capacitor as a function of time:

$$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right). \quad \mathbf{10.8}$$

A graph of the charge on the capacitor versus time is shown in Figure 10.39(a). First note that as time approaches infinity, the exponential goes to zero, so the charge approaches the maximum charge $Q = C\varepsilon$ and has units of coulombs. The units of RC are seconds, units of time. This quantity is known as the time constant:

$$\tau = RC. \quad \mathbf{10.9}$$

At time $t = \tau = RC$, the charge is equal to $1 - e^{-1} = 1 - 0.368 = 0.632$ of the maximum charge $Q = C\varepsilon$. Notice that the time rate change of the charge is the slope at a point of the charge versus time plot. The slope of the graph is large at time $t = 0.0$ s and approaches zero as time increases.

As the charge on the capacitor increases, the current through the resistor decreases, as shown in Figure

10.39(b). The current through the resistor can be found by taking the time derivative of the charge.

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left[C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)\right],$$

$$I(t) = C\varepsilon\left(\frac{1}{RC}\right)e^{-\frac{t}{RC}} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}},$$

$$I(t) = I_0 e^{-t/\tau}. \quad \textbf{10.10}$$

At time $t = 0.00$ s, the current through the resistor is $I_0 = \frac{\varepsilon}{R}$. As time approaches infinity, the current approaches zero. At time $t = \tau$, the current through the resistor is $I(t = \tau) = I_0 e^{-1} = 0.368 I_0$.

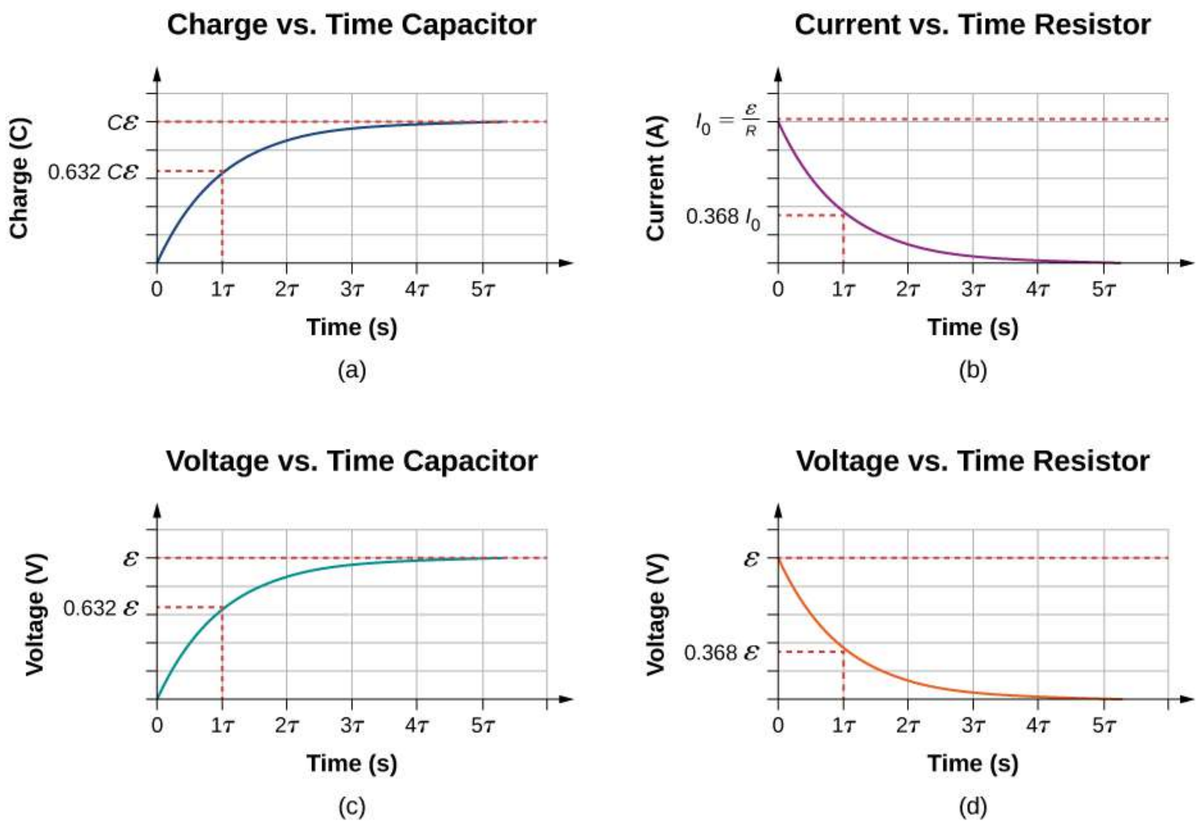


Figure 10.39 (a) Charge on the capacitor versus time as the capacitor charges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

Figure 10.39(c) and Figure 10.39(d) show the voltage differences across the capacitor and the resistor, respectively. As the charge on the capacitor increases, the current decreases, as does the voltage difference across the resistor $V_R(t) = (I_0 R)e^{-t/\tau} = \varepsilon e^{-t/\tau}$. The voltage difference across the capacitor increases as $V_C(t) = \varepsilon\left(1 - e^{-t/\tau}\right)$.

Discharging a Capacitor

When the switch in Figure 10.38(a) is moved to position *B*, the circuit reduces to the circuit in part (c), and the charged capacitor is allowed to discharge through the resistor. A graph of the charge on the capacitor as a function of time is shown in Figure 10.40(a). Using Kirchhoff's loop rule to analyze the circuit as the capacitor discharges results in the equation $-V_R - V_c = 0$, which simplifies to $IR + \frac{q}{C} = 0$. Using the definition of current $\frac{dq}{dt}R = -\frac{q}{C}$ and integrating the loop equation yields an equation for the charge on the capacitor as a function of time:

$$q(t) = Qe^{-t/\tau}. \qquad (10.11)$$

Here, Q is the initial charge on the capacitor and $\tau = RC$ is the time constant of the circuit. As shown in the graph, the charge decreases exponentially from the initial charge, approaching zero as time approaches infinity.

The current as a function of time can be found by taking the time derivative of the charge:

$$I(t) = -\frac{Q}{RC}e^{-t/\tau}. \qquad (10.12)$$

The negative sign shows that the current flows in the opposite direction of the current found when the capacitor is charging. Figure 10.40(b) shows an example of a plot of charge versus time and current versus time. A plot of the voltage difference across the capacitor and the voltage difference across the resistor as a function of time are shown in parts (c) and (d) of the figure. Note that the magnitudes of the charge, current, and voltage all decrease exponentially, approaching zero as time increases.

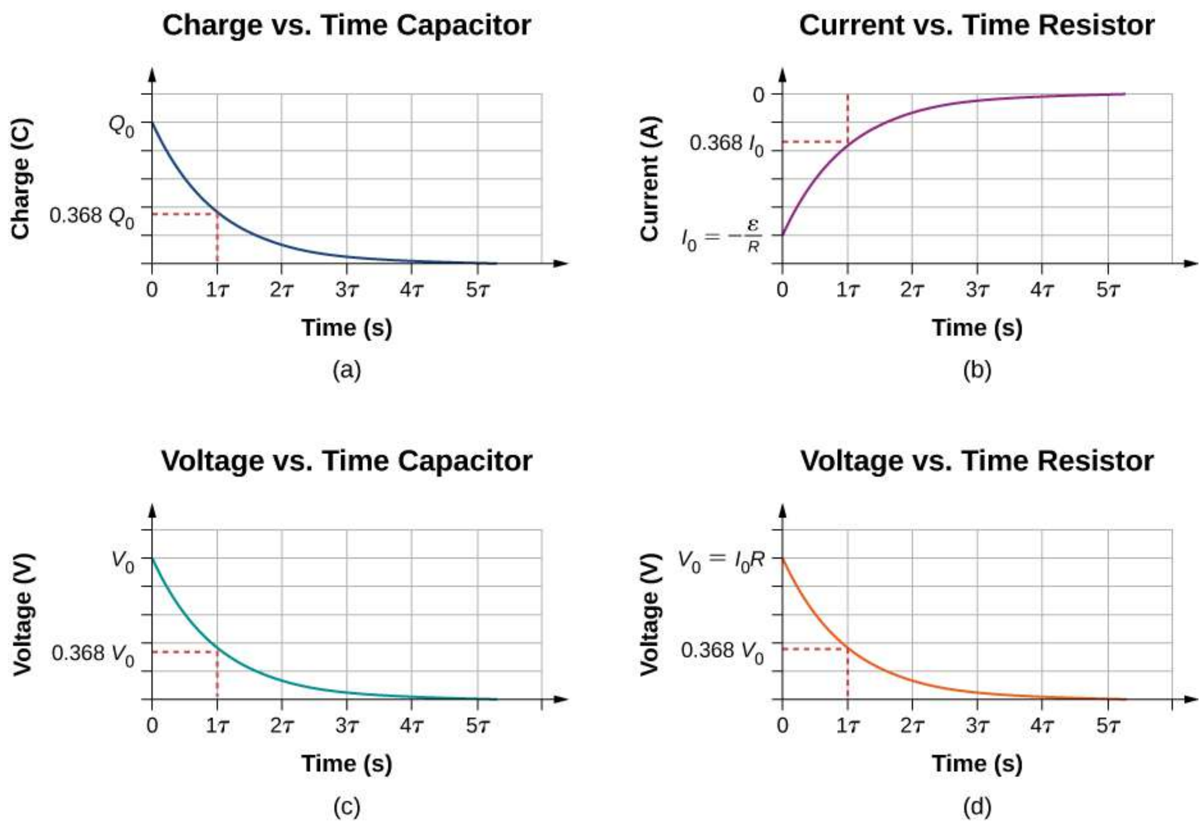


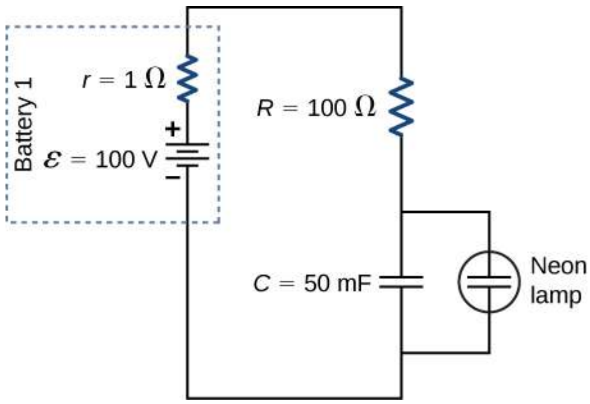
Figure 10.40 (a) Charge on the capacitor versus time as the capacitor discharges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

Now we can explain why the flash camera mentioned at the beginning of this section takes so much longer to charge than discharge: The resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower.

EXAMPLE 10.8

The Relaxation Oscillator

One application of an RC circuit is the relaxation oscillator, as shown below. The relaxation oscillator consists of a voltage source, a resistor, a capacitor, and a neon lamp. The neon lamp acts like an open circuit (infinite resistance) until the potential difference across the neon lamp reaches a specific voltage. At that voltage, the lamp acts like a short circuit (zero resistance), and the capacitor discharges through the neon lamp and produces light. In the relaxation oscillator shown, the voltage source charges the capacitor until the voltage across the capacitor is 80 V. When this happens, the neon in the lamp breaks down and allows the capacitor to discharge through the lamp, producing a bright flash. After the capacitor fully discharges through the neon lamp, it begins to charge again, and the process repeats. Assuming that the time it takes the capacitor to discharge is negligible, what is the time interval between flashes?



Strategy

The time period can be found from considering the equation $V_C(t) = \varepsilon\left(1 - e^{-t/\tau}\right)$, where $\tau = (R + r)C$.

Solution

The neon lamp flashes when the voltage across the capacitor reaches 80 V. The RC time constant is equal to $\tau = (R + r)C = (101\ \Omega)\left(50 \times 10^{-3}\text{F}\right) = 5.05\ \text{s}$. We can solve the voltage equation for the time it takes the capacitor to reach 80 V:

$$\begin{aligned}
V_C(t) &= \varepsilon\left(1 - e^{-t/\tau}\right), \\
e^{-t/\tau} &= 1 - \frac{V_C(t)}{\varepsilon}, \\
\ln\left(e^{-t/\tau}\right) &= \ln\left(1 - \frac{V_C(t)}{\varepsilon}\right), \\
t &= -\tau \ln\left(1 - \frac{V_C(t)}{\varepsilon}\right) = -5.05\ \text{s} \cdot \ln\left(1 - \frac{80\ \text{V}}{100\ \text{V}}\right) = 8.13\ \text{s}.
\end{aligned}$$

Significance

One application of the relaxation oscillator is for controlling indicator lights that flash at a frequency determined by the values for R and C. In this example, the neon lamp will flash every 8.13 seconds, a frequency of $f = \frac{1}{T} = \frac{1}{8.13\ \text{s}} = 0.123\ \text{Hz}$. The relaxation oscillator has many other practical uses. It is often used in electronic circuits, where the neon lamp is replaced by a transistor or a device known as a tunnel diode. The description of the transistor and tunnel diode is beyond the scope of this chapter, but you can think of them as voltage controlled switches. They are normally open switches, but when the right voltage is applied, the switch closes and conducts. The “switch” can be used to turn on another circuit, turn on a light, or run a small motor. A relaxation oscillator can be used to make the turn signals of your car blink or your cell phone to vibrate.

RC circuits have many applications. They can be used effectively as timers for applications such as intermittent windshield wipers, pace makers, and strobe lights. Some models of intermittent windshield wipers use a variable resistor to adjust the interval between sweeps of the wiper. Increasing the resistance increases the *RC* time constant, which increases the time between the operation of the wipers.

Another application is the pacemaker. The heart rate is normally controlled by electrical signals, which cause the muscles of the heart to contract and pump blood. When the heart rhythm is abnormal (the heartbeat is too high or too low), pace makers can be used to correct this abnormality. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during physical activities, thus meeting the increased need for blood and oxygen, and an *RC* timing circuit can be used to control the time between voltage signals to the heart.

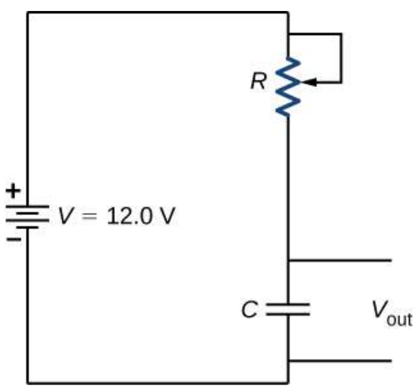
Looking ahead to the study of ac circuits (Alternating-Current Circuits), ac voltages vary as sine functions with specific frequencies. Periodic variations in voltage, or electric signals, are often recorded by scientists. These voltage signals could come from music recorded by a microphone or atmospheric data collected by radar. Occasionally, these signals can contain unwanted frequencies known as “noise.” *RC* filters can be used to filter out the unwanted frequencies.

In the study of electronics, a popular device known as a 555 timer provides timed voltage pulses. The time between pulses is controlled by an *RC* circuit. These are just a few of the countless applications of *RC* circuits.

EXAMPLE 10.9

Intermittent Windshield Wipers

A relaxation oscillator is used to control a pair of windshield wipers. The relaxation oscillator consists of a 10.00-mF capacitor and a $10.00\text{-k}\Omega$ variable resistor known as a rheostat. A knob connected to the variable resistor allows the resistance to be adjusted from $0.00\ \Omega$ to $10.00\ \text{k}\Omega$. The output of the capacitor is used to control a voltage-controlled switch. The switch is normally open, but when the output voltage reaches 10.00 V, the switch closes, energizing an electric motor and discharging the capacitor. The motor causes the windshield wipers to sweep once across the windshield and the capacitor begins to charge again. To what resistance should the rheostat be adjusted for the period of the wiper blades be 10.00 seconds?



Strategy

The resistance considers the equation $V_{\text{out}}\,(t) = V\left(1 - e^{-t/\tau}\right)$, where $\tau = RC$. The capacitance, output voltage, and voltage of the battery are given. We need to solve this equation for the resistance.

Solution

The output voltage will be 10.00 V and the voltage of the battery is 12.00 V. The capacitance is given as 10.00 mF. Solving for the resistance yields

$$\begin{aligned}
V_{\text{out}}(t) &= V\left(1 - e^{-t/\tau}\right), \\
e^{-t/RC} &= 1 - \frac{V_{\text{out}}(t)}{V}, \\
\ln\left(e^{-t/RC}\right) &= \ln\left(1 - \frac{V_{\text{out}}(t)}{V}\right), \\
-\frac{t}{RC} &= \ln\left(1 - \frac{V_{\text{out}}(t)}{V}\right), \\
R &= \frac{-t}{C\ln\left(1 - \frac{V_C(t)}{V}\right)} = \frac{-10.00\text{ s}}{10 \times 10^{-3}\text{ F}\ln\left(1 - \frac{10\text{ V}}{12\text{ V}}\right)} = 558.11\ \Omega.
\end{aligned}$$

Significance

Increasing the resistance increases the time delay between operations of the windshield wipers. When the resistance is zero, the windshield wipers run continuously. At the maximum resistance, the period of the operation of the wipers is:

$$t = -RC\ln\left(1 - \frac{V_{\text{out}}(t)}{V}\right) = -\left(10 \times 10^{-3}\text{ F}\right)\left(10 \times 10^{3}\ \Omega\right)\ln\left(1 - \frac{10\text{ V}}{12\text{ V}}\right) = 179.18\text{ s} = 2.98\text{ min}.$$

The *RC* circuit has thousands of uses and is a very important circuit to study. Not only can it be used to time circuits, it can also be used to filter out unwanted frequencies in a circuit and used in power supplies, like the one for your computer, to help turn ac voltage to dc voltage.

10.6 Household Wiring and Electrical Safety

Learning Objectives

By the end of this section, you will be able to:

- List the basic concepts involved in house wiring
- Define the terms thermal hazard and shock hazard
- Describe the effects of electrical shock on human physiology and their relationship to the amount of current through the body
- Explain the function of fuses and circuit breakers

Electricity presents two known hazards: thermal and shock. A **thermal hazard** is one in which an excessive electric current causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when an electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. In this section, we consider these hazards and the various factors affecting them in a quantitative manner. We also examine systems and devices for preventing electrical hazards.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted into thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the short circuit, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in Figure 10.41. A toaster is plugged into a common household electrical outlet. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact, or "short." As a result, thermal energy can quickly raise the temperature of surrounding materials, melting the insulation and perhaps causing a fire.

The circuit diagram shows a symbol that consists of a sine wave enclosed in a circle. This symbol represents an alternating current (ac) voltage source. In an ac voltage source, the voltage oscillates between a positive and negative maximum amplitude. Up to now, we have been considering direct current (dc) voltage sources, but many of the same concepts are applicable to ac circuits.

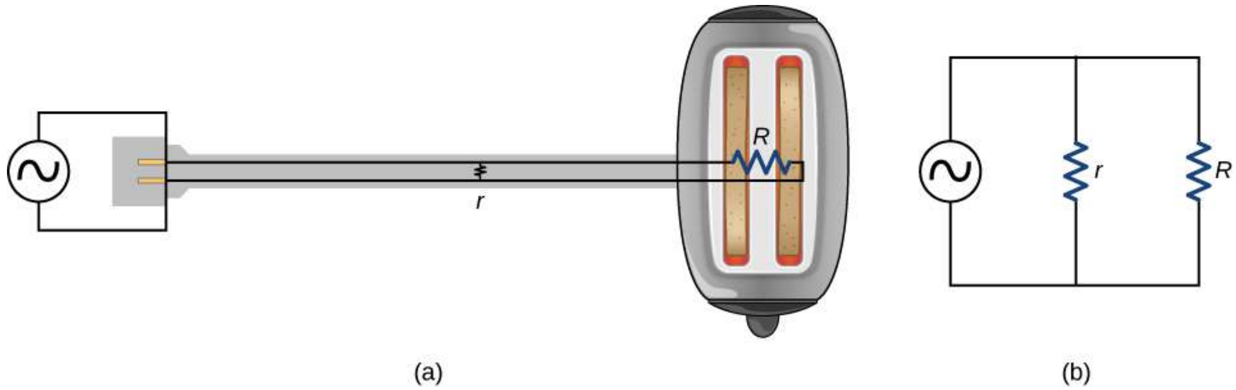


Figure 10.41 A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance *r*. Since $P = V^2/r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

Another serious thermal hazard occurs when wires supplying power to an appliance are overloaded. Electrical wires and appliances are often rated for the maximum current they can safely handle. The term "overloaded" refers to a condition where the current exceeds the rated maximum current. As current flows through a wire, the power dissipated in the supply wires is $P = I^2 R_W$, where R_W is the resistance of the wires and I is the current flowing through the wires. If either I or R_W is too large, the wires overheat. Fuses and circuit breakers are used to limit excessive currents.

Shock Hazards

Electric shock is the physiological reaction or injury caused by an external electric current passing through the body. The effect of an electric shock can be negative or positive. When a current with a magnitude above 300 mA passes through the heart, death may occur. Most electrical shock fatalities occur because a current causes ventricular fibrillation, a massively irregular and often fatal, beating of the heart. On the other hand, a heart attack victim, whose heart is in fibrillation, can be saved by an electric shock from a defibrillator.

The effects of an undesirable electric shock can vary in severity: a slight sensation at the point of contact, pain, loss of voluntary muscle control, difficulty breathing, heart fibrillation, and possibly death. The loss of voluntary muscle control can cause the victim to not be able to let go of the source of the current.

The major factors upon which the severity of the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for dc)

Our bodies are relatively good electric conductors due to the body's water content. A dangerous condition occurs when the body is in contact with a voltage source and "ground." The term "ground" refers to a large sink or source of electrons, for example, the earth (thus, the name). When there is a direct path to ground, large currents will pass through the parts of the body with the lowest resistance and a direct path to ground. A safety precaution used by many professions is the wearing of insulated shoes. Insulated shoes prohibit a pathway to ground for electrons through the feet by providing a large resistance. Whenever working with high-power tools, or any electric circuit, ensure that you do not provide a pathway for current flow (especially across the heart). A common safety precaution is to work with one hand, reducing the possibility of providing a current path through the heart.

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 5–30 mA and above, the current can stimulate sustained muscular contractions, much as regular nerve impulses do (Figure 10.42). Very large currents (above 300 mA) cause the heart and diaphragm of the lung to contract for the duration of the shock. Both the heart and respiration stop. Both often return to normal following the shock.

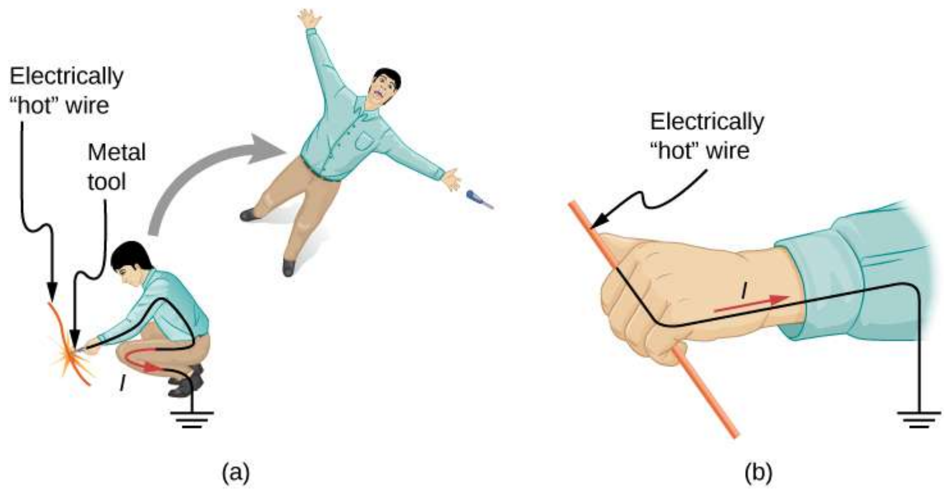


Figure 10.42 An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Current is the major factor determining shock severity. A larger voltage is more hazardous, but since $I = V/R$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\ \text{k}\Omega$. If he comes into contact with 120-V ac, a current

$$I = (120\ \text{V})/(200\ \text{k}\Omega) = 0.6\ \text{mA}$$

passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\ \text{k}\Omega$ and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

Electrical Safety: Systems and Devices

Figure 10.43(a) shows the schematic for a simple ac circuit with no safety features. This is not how power is distributed in practice. Modern household and industrial wiring requires the **three-wire system**, shown schematically in part (b), which has several safety features, with live, neutral, and ground wires. First is the familiar circuit breaker (or fuse) to prevent thermal overload. Second is a protective case around the appliance, such as a toaster or refrigerator. The case's safety feature is that it prevents a person from touching exposed wires and coming into electrical contact with the circuit, helping prevent shocks.

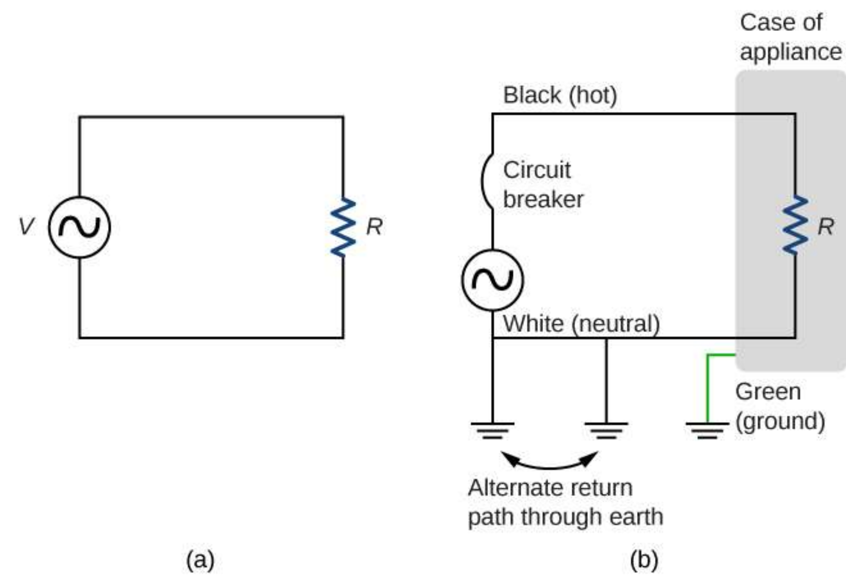


Figure 10.43 (a) Schematic of a simple ac circuit with a voltage source and a single appliance represented by the resistance *R*. There are no safety features in this circuit. (b) The three-wire system connects the neutral wire to ground at the voltage source and user location, forcing it to be at zero volts and supplying an alternative return path for the current through ground. Also grounded to zero volts is the case of the appliance. A circuit breaker or fuse protects against thermal overload and is in series on the active (live/hot) wire.

There are three connections to ground shown in Figure 10.43(b). Recall that a ground connection is a low-resistance path directly to ground. The two ground connections on the neutral wire force it to be at zero volts relative to ground, giving the wire its name. This wire is therefore safe to touch even if its insulation, usually white, is missing. The neutral wire is the return path for the current to follow to complete the circuit. Furthermore, the two ground connections supply an alternative path through ground (a good conductor) to complete the circuit. The ground connection closest to the power source could be at the generating plant, whereas the other is at the user's location. The third ground is to the case of the appliance, through the green ground wire, forcing the case, too, to be at zero volts. The live or hot wire (hereafter referred to as "live/hot") supplies voltage and current to operate the appliance. Figure 10.44 shows a more pictorial version of how the three-wire system is connected through a three-prong plug to an appliance.

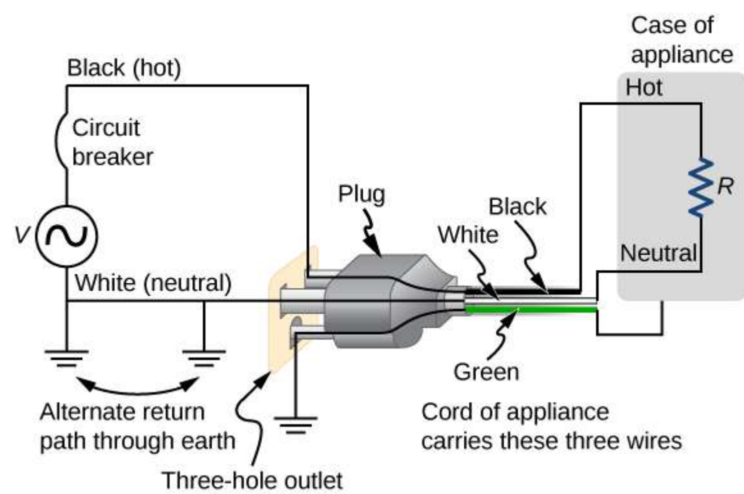


Figure 10.44 The standard three-prong plug can only be inserted in one way, to ensure proper function of the three-wire system.

Insulating plastic is color-coded to identify live/hot, neutral, and ground wires, but these codes vary around the world. It is essential to determine the color code in your region. Striped coatings are sometimes used for the benefit of those who are colorblind.

Grounding the case solves more than one problem. The simplest problem is worn insulation on the live/hot wire that allows it to contact the case, as shown in Figure 10.45. Lacking a ground connection, a severe shock is possible. This is particularly dangerous in the kitchen, where a good connection to ground is available through water on the floor or a water faucet. With the ground connection intact, the circuit breaker will trip, forcing repair of the appliance.

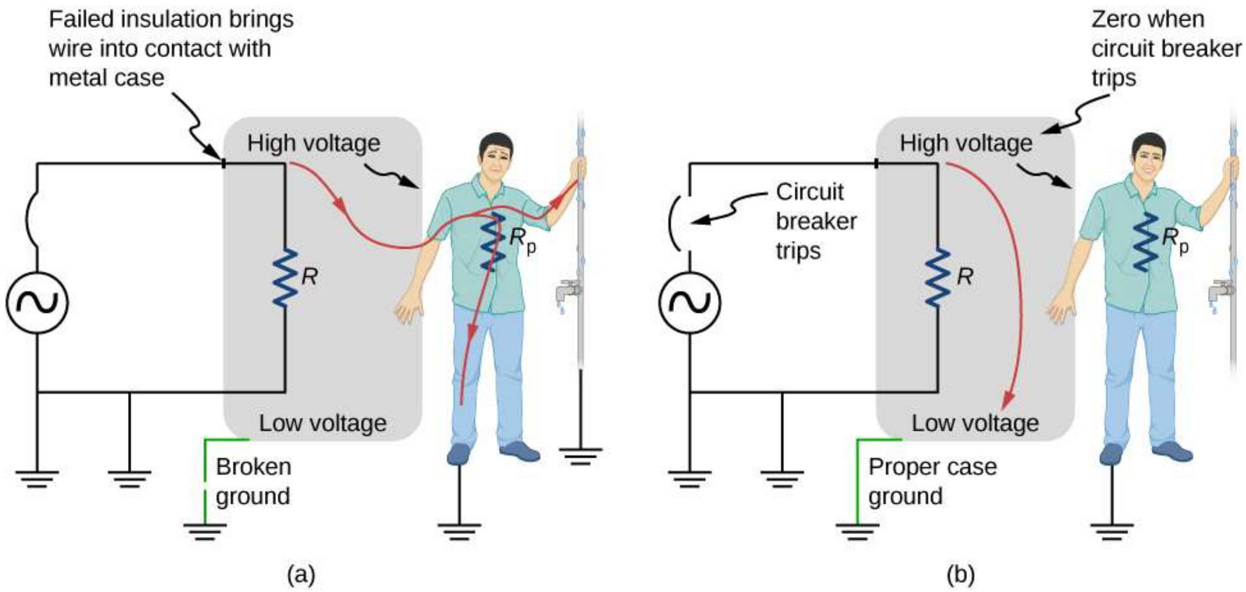


Figure 10.45 Worn insulation allows the live/hot wire to come into direct contact with the metal case of this appliance. (a) The ground connection being broken, the person is severely shocked. The appliance may operate normally in this situation. (b) With a proper ground, the circuit breaker trips, forcing repair of the appliance.

A ground fault circuit interrupter (GFCI) is a safety device found in updated kitchen and bathroom wiring that works based on electromagnetic induction. GFCIs compare the currents in the live/hot and neutral wires. When live/hot and neutral currents are not equal, it is almost always because current in the neutral is less than in the live/hot wire. Then some of the current, called a leakage current, is returning to the voltage source by a path other than through the neutral wire. It is assumed that this path presents a hazard. GFCIs are usually set to interrupt the circuit if the leakage current is greater than 5 mA, the accepted maximum harmless shock. Even if the leakage current goes safely to ground through an intact ground wire, the GFCI will trip, forcing repair of the leakage.

CHAPTER REVIEW

Key Terms

ammeter instrument that measures current

electromotive force (emf) energy produced per unit charge, drawn from a source that produces an electrical current

equivalent resistance resistance of a combination of resistors; it can be thought of as the resistance of a single resistor that can replace a combination of resistors in a series and/or parallel circuit

internal resistance amount of resistance to the flow of current within the voltage source

junction rule sum of all currents entering a junction must equal the sum of all currents leaving the junction

Kirchhoff's rules set of two rules governing current and changes in potential in an electric circuit

loop rule algebraic sum of changes in potential around any closed circuit path (loop) must be zero

potential difference difference in electric potential between two points in an electric circuit, measured in volts

potential drop loss of electric potential energy as a current travels across a resistor, wire, or other component

***RC* circuit** circuit that contains both a resistor and a capacitor

shock hazard hazard in which an electric current passes through a person

terminal voltage potential difference measured across the terminals of a source when there is no load attached

thermal hazard hazard in which an excessive electric current causes undesired thermal effects

three-wire system wiring system used at present for safety reasons, with live, neutral, and ground wires

voltmeter instrument that measures voltage

Key Equations

Terminal voltage of a single voltage source	$V_{\text{terminal}} = \varepsilon - Ir_{\text{eq}}$
Equivalent resistance of a series circuit	$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i$
Equivalent resistance of a parallel circuit	$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R2} + \cdots + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}$
Junction rule	$\sum I_{\text{in}} = \sum I_{\text{out}}$
Loop rule	$\sum V = 0$
Terminal voltage of N voltage sources in series	$V_{\text{terminal}} = \sum_{i=1}^{N} \varepsilon_i - I\sum_{i=1}^{N} r_i = \sum_{i=1}^{N} \varepsilon_i - Ir_{\text{eq}}$
Terminal voltage of N voltage sources in parallel	$V_{\text{terminal}} = \varepsilon - I\sum_{i=1}^{N}\left(\frac{1}{r_i}\right)^{-1} = \varepsilon - Ir_{\text{eq}}$
Charge on a charging capacitor	$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right)$
Time constant	$\tau = RC$

Current during charging of a capacitor	$I = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$
Charge on a discharging capacitor	$q(t) = Qe^{-\frac{t}{\tau}}$
Current during discharging of a capacitor	$I(t) = -\frac{Q}{RC} e^{-\frac{t}{\tau}}$

Summary

10.1 Electromotive Force

- All voltage sources have two fundamental parts: a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r. The emf is the work done per charge to keep the potential difference of a source constant. The emf is equal to the potential difference across the terminals when no current is flowing. The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V_{terminal} and is given by $V_{\text{terminal}} = \varepsilon - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source and r is the internal resistance.

10.2 Resistors in Series and Parallel

- The equivalent resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances:
$$R_s = R_1 + R_2 + R_3 + \cdots = \sum_{i=1}^{N} R_i .$$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The potential drop, or power dissipation, across each individual resistor in a series is different, and their combined total is the power source input.
- The equivalent resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula
$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1} .$$
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

10.3 Kirchhoff's Rules

- Kirchhoff’s rules can be used to analyze any circuit, simple or complex. The simpler series and parallel connection rules are special cases of Kirchhoff’s rules.
- Kirchhoff’s first rule, also known as the junction rule, applies to the charge to a junction. Current is the flow of charge; thus, whatever charge flows into the junction must flow out.
- Kirchhoff’s second rule, also known as the loop rule, states that the voltage drop around a loop is zero.
- When calculating potential and current using Kirchhoff’s rules, a set of conventions must be followed for determining the correct signs of various terms.
- When multiple voltage sources are in series, their internal resistances add together and their emfs add together to get the total values.
- When multiple voltage sources are in parallel, their internal resistances combine to an equivalent resistance that is less than the individual resistance and provides a higher current than a single cell.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

10.4 Electrical Measuring Instruments

- Voltmeters measure voltage, and ammeters measure current. Analog meters are based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current or voltage. Digital meters are based on analog-to-digital converters and provide a discrete or digital measurement of the current or voltage.

- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.
- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.
- Standard voltmeters and ammeters alter the circuit they are connected to and are thus limited in accuracy.
- Ohmmeters are used to measure resistance. The component in which the resistance is to be measured should be isolated (removed) from the circuit.

10.5 RC Circuits

- An RC circuit is one that has both a resistor and a capacitor.
- The time constant τ for an RC circuit is $\tau = RC$.
- When an initially uncharged ($q = 0$ at $t = 0$) capacitor in series with a resistor is charged by a dc voltage source, the capacitor asymptotically approaches the maximum charge.
- As the charge on the capacitor increases, the current exponentially decreases from the initial current: $I_0 = \varepsilon/R$.
- If a capacitor with an initial charge Q is discharged through a resistor starting at $t = 0$, then its charge decreases exponentially. The current flows in the opposite direction, compared to when it charges, and the magnitude of the charge decreases with time.

10.6 Household Wiring and Electrical Safety

- The two types of electric hazards are thermal (excessive power) and shock (current through a person). Electrical safety systems and devices are employed to prevent thermal and shock hazards.
- Shock severity is determined by current, path, duration, and ac frequency.
- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault circuit interrupter (GFCI) prevents shock by detecting the loss of current to unintentional paths.

Conceptual Questions

10.1 Electromotive Force

1. What effect will the internal resistance of a rechargeable battery have on the energy being used to recharge the battery?

2. A battery with an internal resistance of r and an emf of 10.00 V is connected to a load resistor $R = r$. As the battery ages, the internal resistance triples. How much is the current through the load resistor reduced?

3. Show that the power dissipated by the load resistor is maximum when the resistance of the load resistor is equal to the internal resistance of the battery.

10.2 Resistors in Series and Parallel

4. A voltage occurs across an open switch. What is the power dissipated by the open switch?

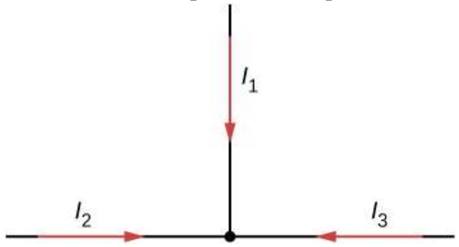
5. The severity of a shock depends on the magnitude of the current through your body. Would you prefer to be in series or in parallel with a resistance, such as the heating element of a toaster, if you were shocked by it? Explain.

6. Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

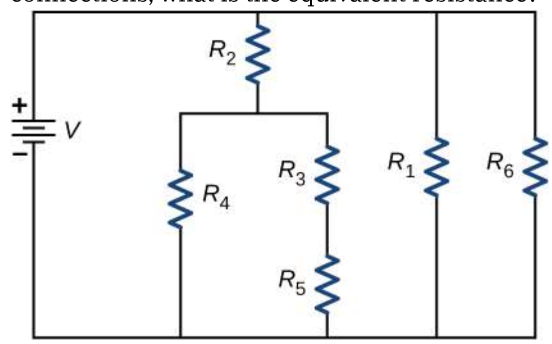
7. Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

10.3 Kirchhoff's Rules

8. Can all of the currents going into the junction shown below be positive? Explain.



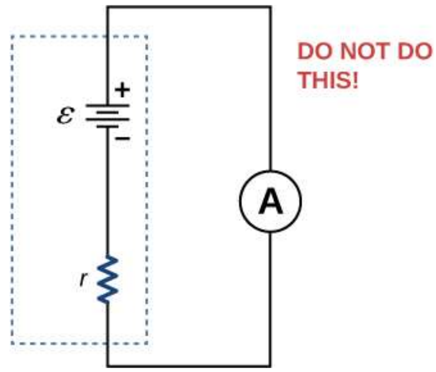
9. Consider the circuit shown below. Does the analysis of the circuit require Kirchhoff's method, or can it be redrawn to simplify the circuit? If it is a circuit of series and parallel connections, what is the equivalent resistance?



10. Do batteries in a circuit always supply power to a circuit, or can they absorb power in a circuit? Give an example.
11. What are the advantages and disadvantages of connecting batteries in series? In parallel?
12. Semi-tractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck's other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck's engine (a very heavy load)?

10.4 Electrical Measuring Instruments

13. What would happen if you placed a voltmeter in series with a component to be tested?
14. What is the basic operation of an ohmmeter as it measures a resistor?
15. Why should you not connect an ammeter directly across a voltage source as shown below?



10.5 RC Circuits

16. A battery, switch, capacitor, and lamp are connected in series. Describe what happens to the lamp when the switch is closed.
17. When making an ECG measurement, it is important to measure voltage variations over small time intervals. The time is limited by the *RC* constant of the circuit—it is not possible to measure time variations shorter than *RC*. How would you manipulate *R* and *C* in the circuit to allow the necessary measurements?

10.6 Household Wiring and Electrical Safety

18. Why isn't a short circuit necessarily a shock hazard?
19. We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why?

Problems

10.1 Electromotive Force

20. A car battery with a 12-V emf and an internal resistance of $0.050\ \Omega$ is being charged with a current of 60 A. Note that in this process, the battery is being charged. (a) What is the potential difference across its terminals? (b) At what rate is thermal energy being dissipated in the battery? (c) At what rate is electric energy being converted into chemical energy?
21. The label on a battery-powered radio recommends the use of a rechargeable nickel-cadmium cell (nicads), although it has a 1.25-V emf, whereas an alkaline cell has a 1.58-V emf. The radio has a $3.20\ \Omega$ resistance. (a) Draw a circuit diagram of the radio and its battery. Now, calculate the power delivered to the radio (b) when using a nicad cells, each having an internal resistance of $0.0400\ \Omega$, and (c) when using an alkaline cell, having an internal resistance of $0.200\ \Omega$. (d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?
22. An automobile starter motor has an equivalent resistance of $0.0500\ \Omega$ and is supplied by a 12.0-V battery with a $0.0100\text{-}\Omega$ internal resistance. (a) What is the current to the motor? (b) What voltage is applied to it? (c) What power is supplied to the motor? (d) Repeat these

calculations for when the battery connections are corroded and add $0.0900\ \Omega$ to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

23. (a) What is the internal resistance of a voltage source if its terminal potential drops by 2.00 V when the current supplied increases by 5.00 A? (b) Can the emf of the voltage source be found with the information supplied?

24. A person with body resistance between his hands of $10.0\ \text{k}\Omega$ accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!) (a) Draw a circuit diagram to represent the situation. (b) If the internal resistance of the power supply is $2000\ \Omega$, what is the current through his body? (c) What is the power dissipated in his body? (d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be 1.00 mA or less? (e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

25. A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A. (a) What is the battery's internal resistance? (b) What power is dissipated inside the battery? (c) At what rate (in $^{\circ}\text{C/min}$) will its temperature increase if its mass is 20.0 kg and it has a specific heat of $0.300\ \text{kcal/kg} \cdot {}^{\circ}\text{C}$, assuming no heat escapes?

10.2 Resistors in Series and Parallel

26. (a) What is the resistance of a $1.00 \times 10^{2}\text{-}\Omega$, a $2.50\text{-k}\Omega$, and a $4.00\text{-k}\Omega$ resistor connected in series? (b) In parallel?

27. What are the largest and smallest resistances you can obtain by connecting a $36.0\text{-}\Omega$, a $50.0\text{-}\Omega$, and a $700\text{-}\Omega$ resistor together?

28. An 1800-W toaster, a 1400-W speaker, and a 75-W lamp are plugged into the same outlet in a 15-A fuse and 120-V circuit. (The three devices are in parallel when plugged into the same socket.) (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

29. Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

30. (a) Given a 48.0-V battery and $24.0\text{-}\Omega$ and $96.0\text{-}\Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

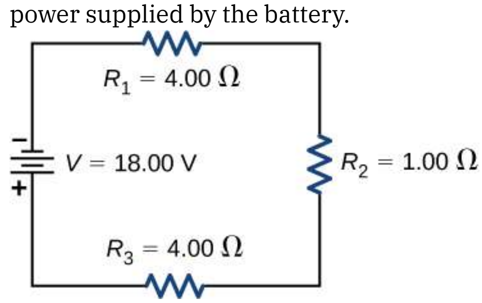
31. Referring to the example combining series and parallel circuits and Figure 10.16, calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts, explicitly show how you follow the steps in the Figure 10.17.

32. Referring to Figure 10.16, (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

33. Refer to Figure 10.17 and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.800\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

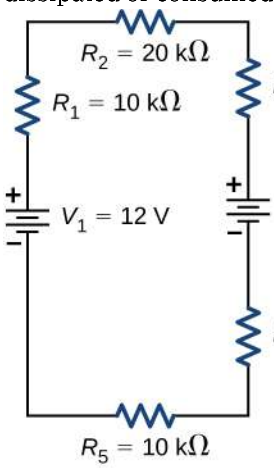
34. Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 \gg R_2$), (a) their series resistance is very nearly equal to the greater resistance R_1 and (b) their parallel resistance is very nearly equal to the smaller resistance R_2.

35. Consider the circuit shown below. The terminal voltage of the battery is $V = 18.00\ \text{V}$. (a) Find the equivalent resistance of the circuit. (b) Find the current through each resistor. (c) Find the potential drop across each resistor. (d) Find the power dissipated by each resistor. (e) Find the power supplied by the battery.

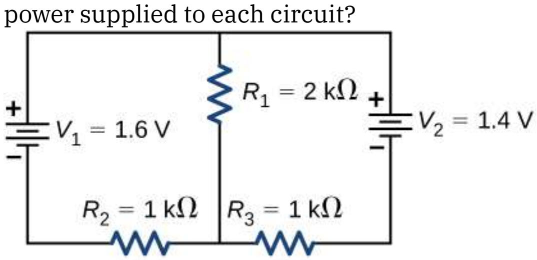


10.3 Kirchhoff's Rules

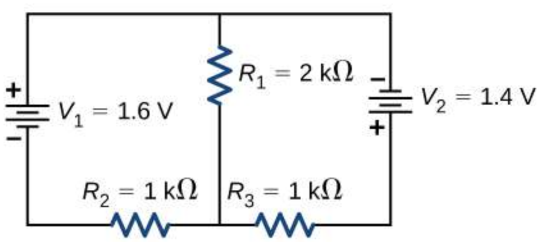
36. Consider the circuit shown below. (a) Find the voltage across each resistor. (b)What is the power supplied to the circuit and the power dissipated or consumed by the circuit?



37. Consider the circuits shown below. (a) What is the current through each resistor in part (a)? (b) What is the current through each resistor in part (b)? (c) What is the power dissipated or consumed by each circuit? (d) What is the power supplied to each circuit?

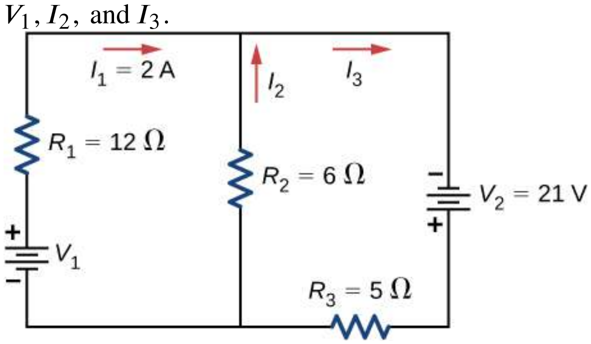


(a)

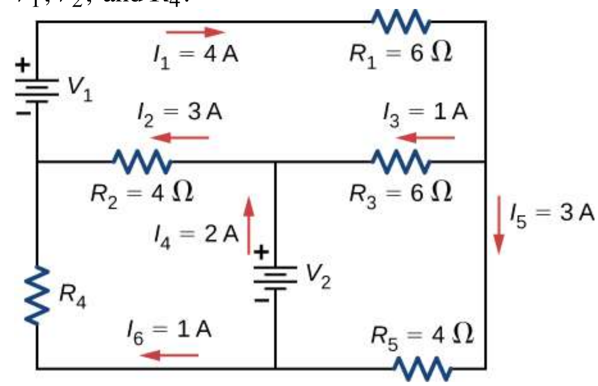


(b)

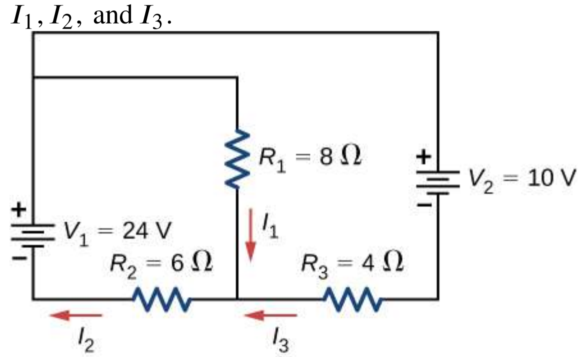
38. Consider the circuit shown below. Find $V_1, I_2,$ and I_3.



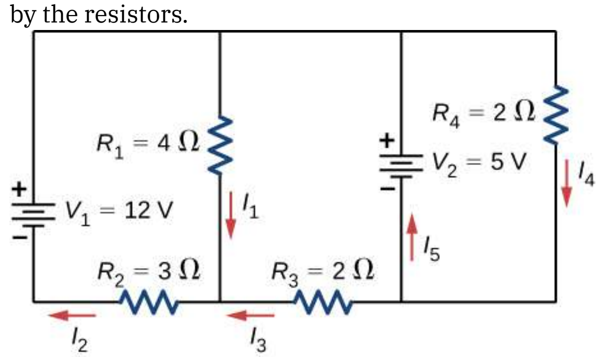
39. Consider the circuit shown below. Find $V_1, V_2,$ and R_4.



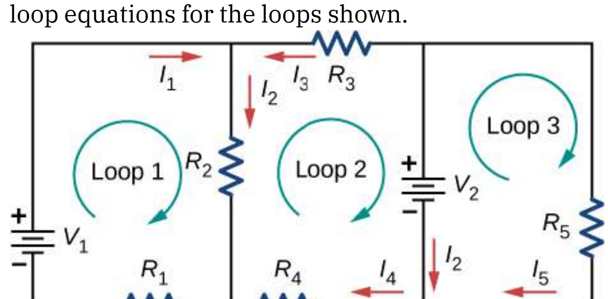
40. Consider the circuit shown below. Find $I_1, I_2,$ and I_3.



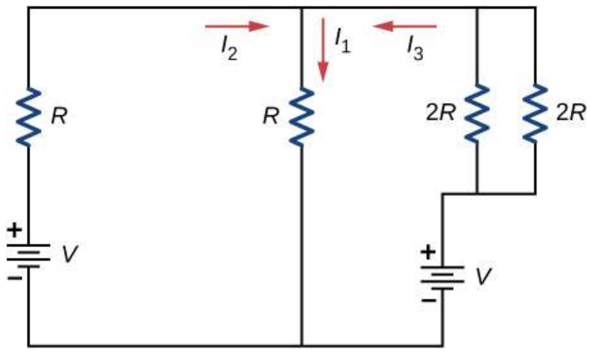
41. Consider the circuit shown below. (a) Find $I_1, I_2, I_3, I_4,$ and I_5. (b) Find the power supplied by the voltage sources. (c) Find the power dissipated by the resistors.



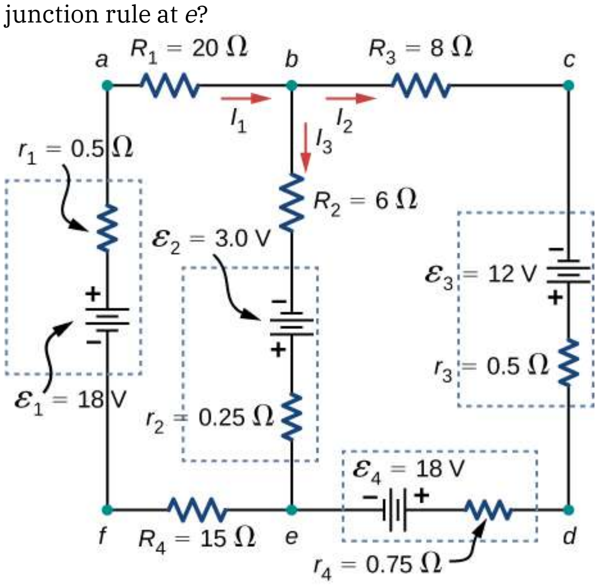
42. Consider the circuit shown below. Write the three loop equations for the loops shown.



43. Consider the circuit shown below. Write equations for the three currents in terms of R and V.



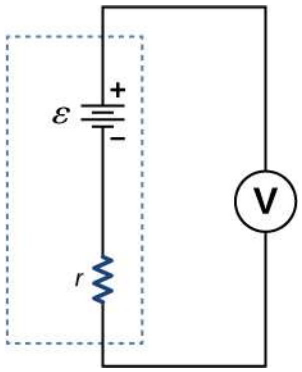
44. Consider the circuit shown in the preceding problem. Write equations for the power supplied by the voltage sources and the power dissipated by the resistors in terms of *R* and *V*.
45. A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of $0.0200\ \Omega$ in series with a 1.53-V carbon-zinc dry cell having a $0.100\text{-}\Omega$ internal resistance. The load resistance is $10.0\ \Omega$. (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?
46. Apply the junction rule to Junction *b* shown below. Is any new information gained by applying the junction rule at *e*?



47. Apply the loop rule to Loop *afedcba* in the preceding problem.

10.4 Electrical Measuring Instruments

48. Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of $0.100\ \Omega$ by placing a $1.00\text{-k}\Omega$ voltmeter across its terminals (see below). (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.



10.5 RC Circuits

49. The timing device in an automobile's intermittent wiper system is based on an *RC* time constant and utilizes a $0.500\text{-}\mu\text{F}$ capacitor and a variable resistor. Over what range must *R* be made to vary to achieve time constants from 2.00 to 15.0 s?
50. A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?
51. The duration of a photographic flash is related to an *RC* time constant, which is $0.100\mu\text{s}$ for a certain camera. (a) If the resistance of the flash lamp is $0.0400\ \Omega$ during discharge, what is the size of the capacitor supplying its energy? (b) What is the time constant for charging the capacitor, if the charging resistance is $800\ \text{k}\Omega$?
52. A 2.00- and a $7.50\text{-}\mu\text{F}$ capacitor can be connected in series or parallel, as can a 25.0- and a $100\text{-k}\Omega$ resistor. Calculate the four *RC* time constants possible from connecting the resulting capacitance and resistance in series.
53. A $500\text{-}\Omega$ resistor, an uncharged $1.50\text{-}\mu\text{F}$ capacitor, and a 6.16-V emf are connected in series. (a) What is the initial current? (b) What is the *RC* time constant? (c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?
54. A heart defibrillator being used on a patient has an *RC* time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has a capacitance of $8.00\mu\text{F}$, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial

voltage is 12.0 kV, how long does it take to decline to 6.00×10^2 V?

55. An ECG monitor must have an *RC* time constant less than 1.00×10^2 μs to be able to measure variations in voltage over small time intervals. (a) If the resistance of the circuit (due mostly to that of the patient's chest) is 1.00 kΩ, what is the maximum capacitance of the circuit? (b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)?

56. Using the exact exponential treatment, determine how much time is required to charge an initially uncharged 100-pF capacitor through a 75.0-MΩ resistor to 90.0% of its final voltage.

57. If you wish to take a picture of a bullet traveling at 500 m/s, then a very brief flash of light produced by an *RC* discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one *RC* constant is acceptable, and given that the flash is driven by a 600-μF capacitor, what is the resistance in the flash tube?

10.6 Household Wiring and Electrical Safety

58. (a) How much power is dissipated in a short circuit of 240-V ac through a resistance of $0.250\ \Omega$? (b) What current flows?

59. What voltage is involved in a 1.44-kW short circuit through a 0.100-Ω resistance?

60. Find the current through a person and identify the likely effect on her if she touches a 120-V ac source: (a) if she is standing on a rubber mat and offers a total resistance of 300 kΩ; (b) if she is standing barefoot on wet grass and has a resistance of only 4000 kΩ.

61. While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $4000\ \Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

62. A man foolishly tries to fish a burning piece of bread from a toaster with a metal butter knife and comes into contact with 120-V ac. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

63. (a) During surgery, a current as small as $20.0\ \mu$A applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

64. (a) What is the resistance of a 220-V ac short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage were 120 V ac?

65. A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

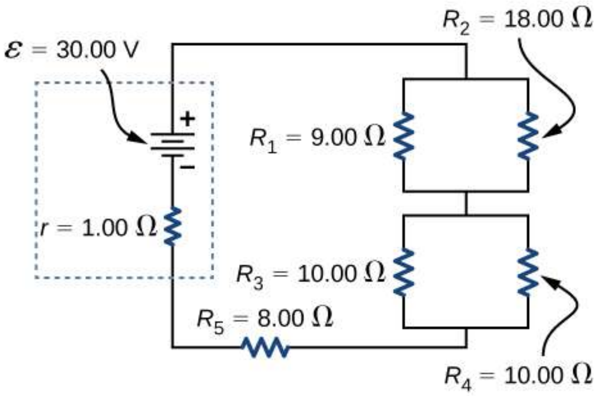
66. A short circuit in a 120-V appliance cord has a 0.500-Ω resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200\ \text{cal/g} \cdot {}^\circ\text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

Additional Problems

67. A circuit contains a D cell battery, a switch, a 20-Ω resistor, and four 20-mF capacitors connected in series. (a) What is the equivalent capacitance of the circuit? (b) What is the *RC* time constant? (c) How long before the current decreases to 50% of the initial value once the switch is closed?

68. A circuit contains a D-cell battery, a switch, a 20-Ω resistor, and three 20-mF capacitors. The capacitors are connected in parallel, and the parallel connection of capacitors are connected in series with the switch, the resistor and the battery. (a) What is the equivalent capacitance of the circuit? (b) What is the *RC* time constant? (c) How long before the current decreases to 50% of the initial value once the switch is closed?

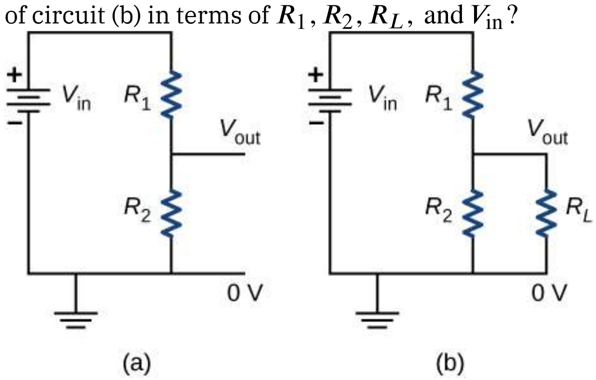
69. Consider the circuit below. The battery has an emf of $\varepsilon = 30.00$ V and an internal resistance of $r = 1.00\ \Omega$. (a) Find the equivalent resistance of the circuit and the current out of the battery. (b) Find the current through each resistor. (c) Find the potential drop across each resistor. (d) Find the power dissipated by each resistor. (e) Find the total power supplied by the batteries.



70. A homemade capacitor is constructed of 2 sheets of aluminum foil with an area of 2.00 square meters, separated by paper, 0.05 mm thick, of the same area and a dielectric constant of 3.7. The homemade capacitor is connected in series with a $100.00\text{-}\Omega$ resistor, a switch, and a 6.00-V voltage source. (a) What is the *RC* time constant of the circuit? (b) What is the initial current through the circuit, when the switch is closed? (c) How long does it take the current to reach one third of its initial value?

71. A student makes a homemade resistor from a graphite pencil 5.00 cm long, where the graphite is 0.05 mm in diameter. The resistivity of the graphite is $\rho = 1.38 \times 10^{-5}$ Ω/m. The homemade resistor is place in series with a switch, a 10.00-mF uncharged capacitor and a 0.50-V power source. (a) What is the *RC* time constant of the circuit? (b) What is the potential drop across the pencil 1.00 s after the switch is closed?

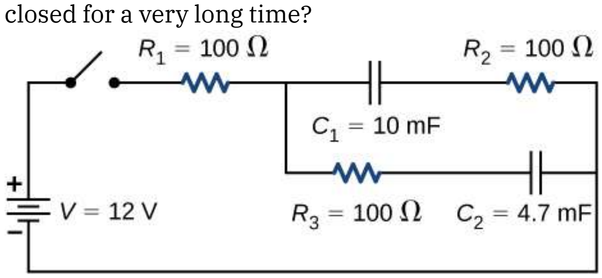
72. The rather simple circuit shown below is known as a voltage divider. The symbol consisting of three horizontal lines is represents "ground" and can be defined as the point where the potential is zero. The voltage divider is widely used in circuits and a single voltage source can be used to provide reduced voltage to a load resistor as shown in the second part of the figure. (a) What is the output voltage V_{out} of circuit (a) in terms of R_1, R_2, and V_{in}? (b) What is the output voltage V_{out} of circuit (b) in terms of R_1, R_2, R_L, and V_{in}?



73. Three $300\text{-}\Omega$ resistors are connect in series with an AAA battery with a rating of 3 AmpHours. (a) How long can the battery supply the resistors with power? (b) If the resistors are connected in parallel, how long can the battery last?

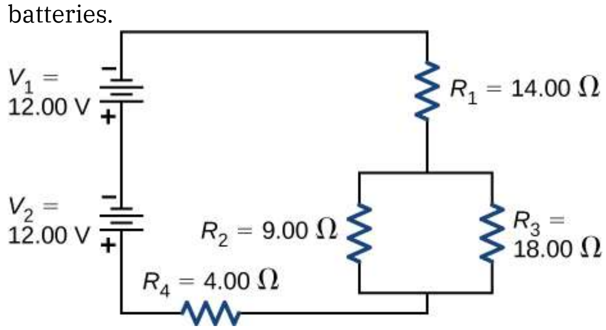
74. Consider a circuit that consists of a real battery with an emf ε and an internal resistance of r connected to a variable resistor R. (a) In order for the terminal voltage of the battery to be equal to the emf of the battery, what should the resistance of the variable resistor be adjusted to? (b) In order to get the maximum current from the battery, what should the resistance of the variable resistor be adjusted to? (c) In order for the maximum power output of the battery to be reached, what should the resistance of the variable resistor be set to?

75. Consider the circuit shown below. What is the energy stored in each capacitor after the switch has been closed for a very long time?

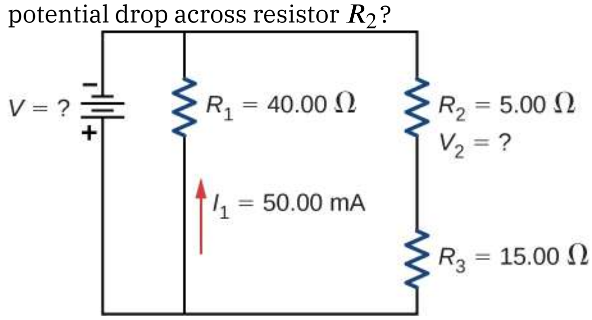


76. Consider a circuit consisting of a battery with an emf ε and an internal resistance of r connected in series with a resistor R and a capacitor C. Show that the total energy supplied by the battery while charging the battery is equal to $\varepsilon^2 C$.

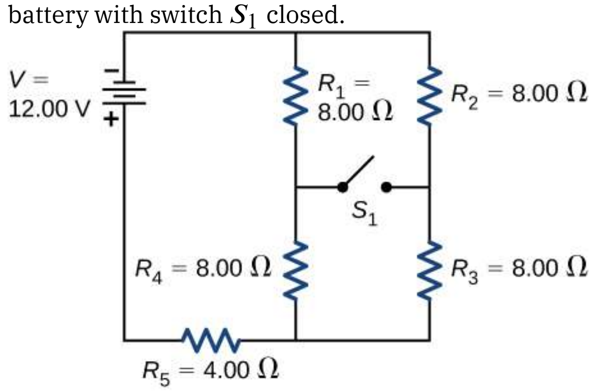
77. Consider the circuit shown below. The terminal voltages of the batteries are shown. (a) Find the equivalent resistance of the circuit and the current out of the battery. (b) Find the current through each resistor. (c) Find the potential drop across each resistor. (d) Find the power dissipated by each resistor. (e) Find the total power supplied by the batteries.



78. Consider the circuit shown below. (a) What is the terminal voltage of the battery? (b) What is the potential drop across resistor R_2?



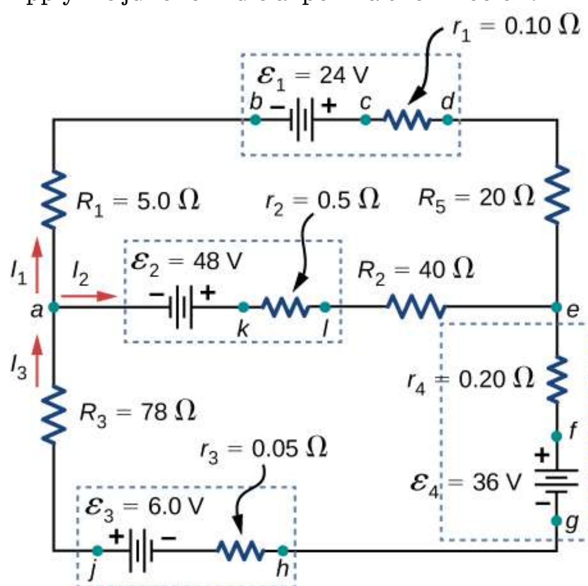
79. Consider the circuit shown below. (a)Determine the equivalent resistance and the current from the battery with switch S_1 open. (b) Determine the equivalent resistance and the current from the battery with switch S_1 closed.



80. Two resistors, one having a resistance of 145 Ω, are connected in parallel to produce a total resistance of 150 Ω. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

81. Two resistors, one having a resistance of 900 kΩ, are connected in series to produce a total resistance of 0.500 MΩ. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

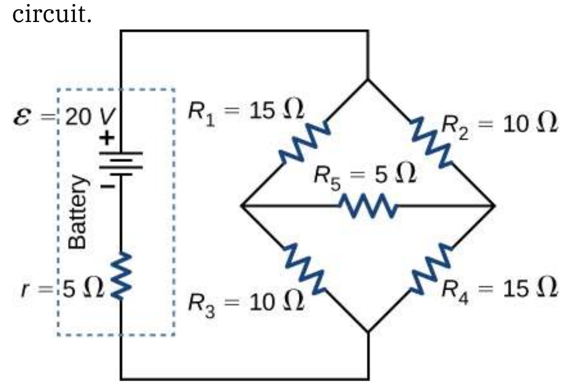
82. Apply the junction rule at point *a* shown below.



83. Apply the loop rule to Loop *akledcba* in the preceding problem.

84. Find the currents flowing in the circuit in the preceding problem. Explicitly show how you follow the steps in the [Problem-Solving Strategy: Series and Parallel Resistors](#).

85. Consider the circuit shown below. (a) Find the current through each resistor. (b) Check the calculations by analyzing the power in the circuit.



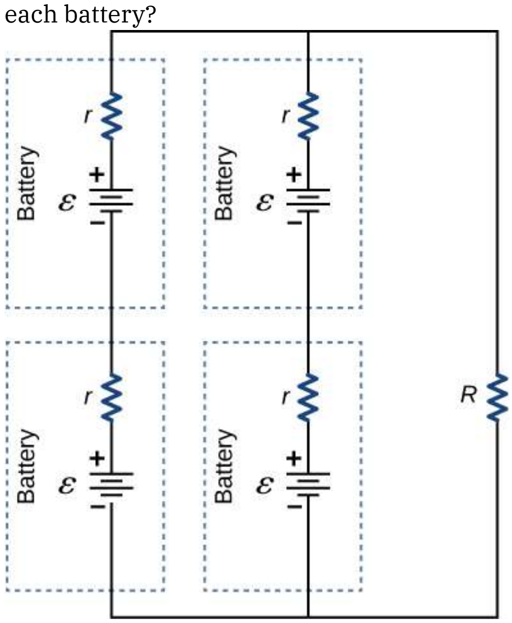
86. A flashing lamp in a Christmas earring is based on an *RC* discharge of a capacitor through its resistance. The effective duration of the flash is 0.250 s, during which it produces an average 0.500 W from an average 3.00 V. (a) What energy does it dissipate? (b) How much charge moves through the lamp? (c) Find the capacitance. (d) What is the resistance of the lamp? (Since average values are given for some quantities, the shape of the pulse profile is not needed.)

87. A $160\text{-}\mu\text{F}$ capacitor charged to 450 V is discharged through a $31.2\text{-k}\Omega$ resistor. (a) Find the time constant. (b) Calculate the temperature increase of the resistor, given that its mass is 2.50 g and its specific heat is $1.67\ \text{kJ/kg} \cdot {}^\circ\text{C}$, noting that most of the thermal energy is retained in the short time of the discharge. (c) Calculate the new resistance, assuming it is pure carbon. (d) Does this change in resistance seem significant?

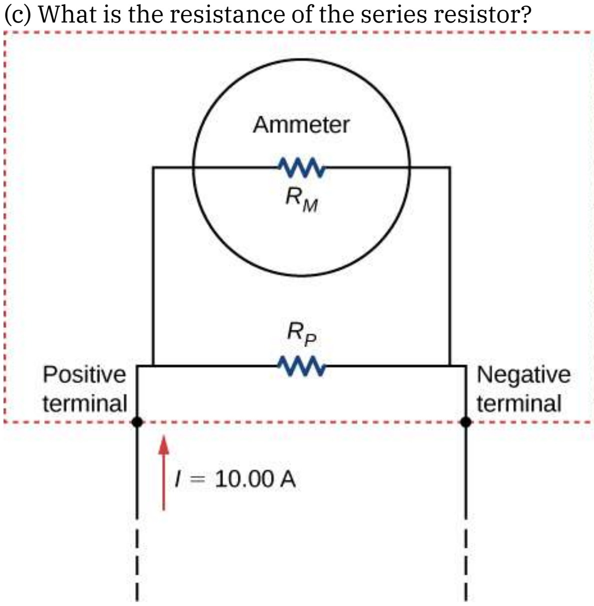
Challenge Problems

88. Some camera flashes use flash tubes that require a high voltage. They obtain a high voltage by charging capacitors in parallel and then internally changing the connections of the capacitors to place them in series. Consider a circuit that uses four AAA batteries connected in series to charge six 10-mF capacitors through an equivalent resistance of $100\ \Omega$. The connections are then switched internally to place the capacitors in series. The capacitors discharge through a lamp with a resistance of $100\ \Omega$. (a) What is the *RC* time constant and the initial current out of the batteries while they are connected in parallel? (b) How long does it take for the capacitors to charge to 90% of the terminal voltages of the batteries? (c) What is the *RC* time constant and the initial current of the capacitors connected in series assuming it discharges at 90% of full charge? (d) How long does it take the current to decrease to 10% of the initial value?

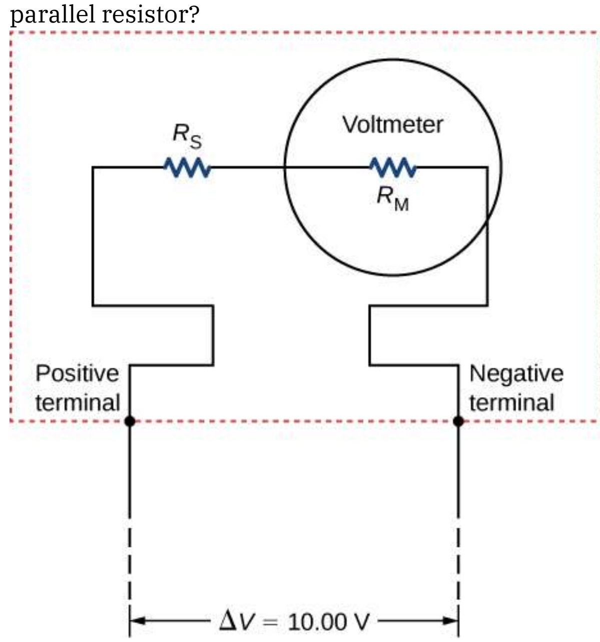
89. Consider the circuit shown below. Each battery has an emf of 1.50 V and an internal resistance of $1.00\ \Omega$. (a) What is the current through the external resistor, which has a resistance of 10.00 ohms? (b) What is the terminal voltage of each battery?



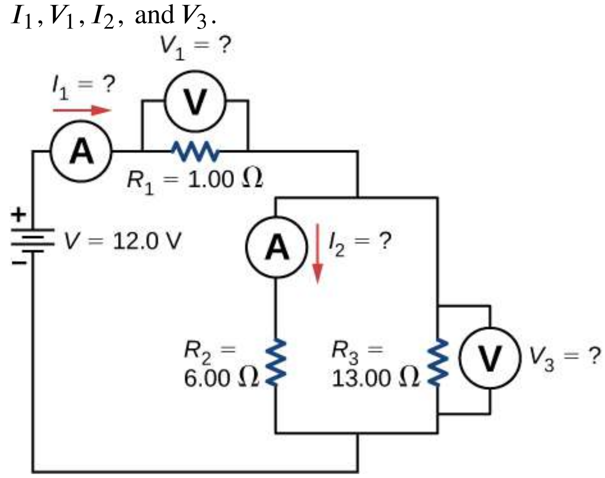
90. Analog meters use a galvanometer, which essentially consists of a coil of wire with a small resistance and a pointer with a scale attached. When current runs through the coil, the pointer turns; the amount the pointer turns is proportional to the amount of current running through the coil. Galvanometers can be used to make an ammeter if a resistor is placed in parallel with the galvanometer. Consider a galvanometer that has a resistance of $25.00\ \Omega$ and gives a full scale reading when a $50\text{-}\mu\text{A}$ current runs through it. The galvanometer is to be used to make an ammeter that has a full scale reading of 10.00 A, as shown below. Recall that an ammeter is connected in series with the circuit of interest, so all 10 A must run through the meter. (a) What is the current through the parallel resistor in the meter? (b) What is the voltage across the parallel resistor? (c) What is the resistance of the series resistor?



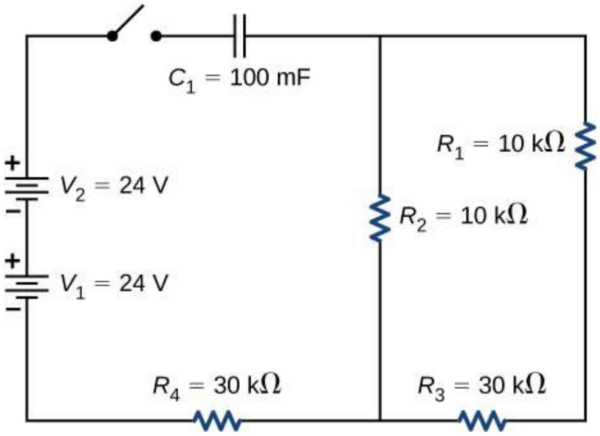
91. Analog meters use a galvanometer, which essentially consists of a coil of wire with a small resistance and a pointer with a scale attached. When current runs through the coil, the point turns; the amount the pointer turns is proportional to the amount of current running through the coil. Galvanometers can be used to make a voltmeter if a resistor is placed in series with the galvanometer. Consider a galvanometer that has a resistance of $25.00\ \Omega$ and gives a full scale reading when a $50\text{-}\mu\text{A}$ current runs through it. The galvanometer is to be used to make an voltmeter that has a full scale reading of 10.00 V, as shown below. Recall that a voltmeter is connected in parallel with the component of interest, so the meter must have a high resistance or it will change the current running through the component. (a) What is the potential drop across the series resistor in the meter? (b) What is the resistance of the parallel resistor?



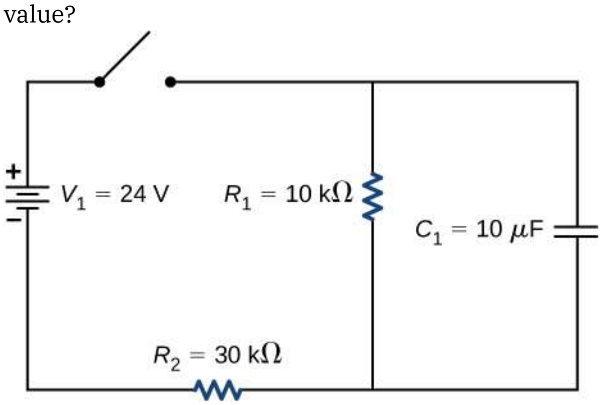
92. Consider the circuit shown below. Find $I_1, V_1, I_2,$ and V_3.



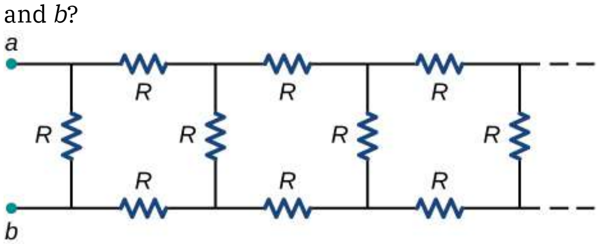
93. Consider the circuit below. (a) What is the *RC* time constant of the circuit? (b) What is the initial current in the circuit once the switch is closed? (c) How much time passes between the instant the switch is closed and the time the current has reached half of the initial current?



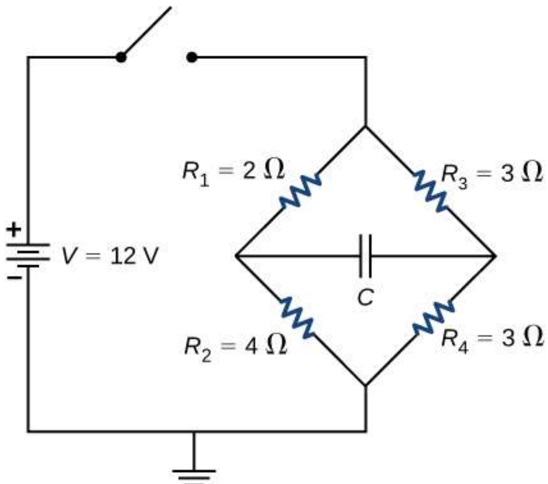
94. Consider the circuit below. (a) What is the initial current through resistor R_2 when the switch is closed? (b) What is the current through resistor R_2 when the capacitor is fully charged, long after the switch is closed? (c) What happens if the switch is opened after it has been closed for some time? (d) If the switch has been closed for a time period long enough for the capacitor to become fully charged, and then the switch is opened, how long before the current through resistor R_1 reaches half of its initial value?



95. Consider the infinitely long chain of resistors shown below. What is the resistance between terminals *a* and *b*?



96. Consider the circuit below. The capacitor has a capacitance of 10 mF. The switch is closed and after a long time the capacitor is fully charged. (a) What is the current through each resistor a long time after the switch is closed? (b) What is the voltage across each resistor a long time after the switch is closed? (c) What is the voltage across the capacitor a long time after the switch is closed? (d) What is the charge on the capacitor a long time after the switch is closed? (e) The switch is then opened. The capacitor discharges through the resistors. How long from the time before the current drops to one fifth of the initial value?



97. A 120-V immersion heater consists of a coil of wire that is placed in a cup to boil the water. The heater can boil one cup of $20.00\ °C$ water in 180.00 seconds. You buy one to use in your dorm room, but you are worried that you will overload the circuit and trip the 15.00-A, 120-V circuit breaker, which supplies your dorm room. In your dorm room, you have four 100.00-W incandescent lamps and a 1500.00-W space heater. (a) What is the power rating of the immersion heater? (b) Will it trip the breaker when everything is turned on? (c) If it you replace the incandescent bulbs with 18.00-W LED, will the breaker trip when everything is turned on?

98. Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 3000-V full-scale reading. Include a circuit diagram with your solution.

99. Find the resistance that must be placed in parallel with a $60.0\text{-}\Omega$ galvanometer having a 1.00-mA sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 25.0-A full-scale reading. Include a circuit diagram with your solution.

CHAPTER 11
Magnetic Forces and Fields

Figure 11.1 An industrial electromagnet is capable of lifting thousands of pounds of metallic waste. (credit: modification of work by “BedfordAl”/Flickr)

Chapter Outline

INTRODUCTION For the past few chapters, we have been studying electrostatic forces and fields, which are caused by electric charges at rest. These electric fields can move other free charges, such as producing a current in a circuit; however, the electrostatic forces and fields themselves come from other static charges. In this chapter, we see that when an electric charge moves, it generates other forces and fields. These additional forces and fields are what we commonly call magnetism.

Before we examine the origins of magnetism, we first describe what it is and how magnetic fields behave. Once we are more familiar with magnetic effects, we can explain how they arise from the behavior of atoms and molecules, and how magnetism is related to electricity. The connection between electricity and magnetism is fascinating from a theoretical point of view, but it is also immensely practical, as shown by an industrial electromagnet that can lift thousands of pounds of metal.

11.1 Magnetism and Its Historical Discoveries

Learning Objectives

By the end of this section, you will be able to:

- Explain attraction and repulsion by magnets
- Describe the historical and contemporary applications of magnetism

Magnetism has been known since the time of the ancient Greeks, but it has always been a bit mysterious. You can see electricity in the flash of a lightning bolt, but when a compass needle points to magnetic north, you can't see any force causing it to rotate. People learned about magnetic properties gradually, over many years, before several physicists of the nineteenth century connected magnetism with electricity. In this section, we review the basic ideas of magnetism and describe how they fit into the picture of a magnetic field.

Brief History of Magnetism

Magnets are commonly found in everyday objects, such as toys, hangers, elevators, doorbells, and computer devices. Experimentation on these magnets shows that all magnets have two poles: One is labeled north (N) and the other is labeled south (S). Magnetic poles repel if they are alike (both N or both S), they attract if they are opposite (one N and the other S), and both poles of a magnet attract unmagnetized pieces of iron. An important point to note here is that you cannot isolate an individual magnetic pole. Every piece of a magnet, no matter how small, which contains a north pole must also contain a south pole.

INTERACTIVE

Visit this website (https://openstax.org/l/21magnetcompass) for an interactive demonstration of magnetic north and south poles.

An example of a magnet is a compass needle. It is simply a thin bar magnet suspended at its center, so it is free to rotate in a horizontal plane. Earth itself also acts like a very large bar magnet, with its south-seeking pole near the geographic North Pole (Figure 11.2). The north pole of a compass is attracted toward Earth's geographic North Pole because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole. Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, "**north magnetic pole**" is actually a misnomer—it should be called the **south magnetic pole**. [Note that the orientation of Earth's magnetic field is not permanent but changes ("flips") after long time intervals. Eventually, Earth's north magnetic pole may be located near its geographic North Pole.]

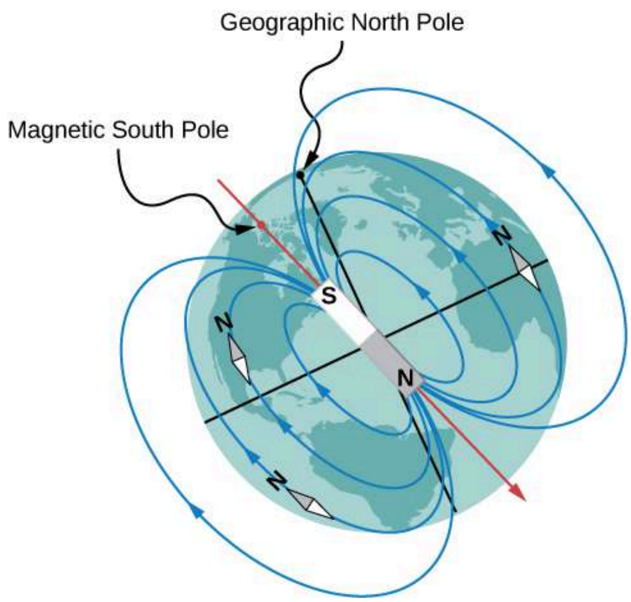


Figure 11.2 The north pole of a compass needle points toward the south pole of a magnet, which is how today's magnetic field is oriented from inside Earth. It also points toward Earth's geographic North Pole because the geographic North Pole is near the magnetic south pole.

Back in 1819, the Danish physicist Hans Oersted was performing a lecture demonstration for some students and noticed that a compass needle moved whenever current flowed in a nearby wire. Further investigation of this phenomenon convinced Oersted that an electric current could somehow cause a magnetic force. He reported this finding to an 1820 meeting of the French Academy of Science.

Soon after this report, Oersted's investigations were repeated and expanded upon by other scientists. Among those whose work was especially important were Jean-Baptiste Biot and Felix Savart, who investigated the forces exerted on magnets by currents; André Marie Ampère, who studied the forces exerted by one current on another; François Arago, who found that iron could be magnetized by a current; and Humphry Davy, who discovered that a magnet exerts a force on a wire carrying an electric current. Within 10 years of Oersted's discovery, Michael Faraday found that the relative motion of a magnet and a metallic wire induced current in the wire. This finding showed not only that a current has a magnetic effect, but that a magnet can generate electric current. You will see later that the names of Biot, Savart, Ampère, and Faraday are linked to some of the fundamental laws of electromagnetism.

The evidence from these various experiments led Ampère to propose that electric current is the source of all magnetic phenomena. To explain permanent magnets, he suggested that matter contains microscopic current loops that are somehow aligned when a material is magnetized. Today, we know that permanent magnets are actually created by the alignment of spinning electrons, a situation quite similar to that proposed by Ampère. This model of permanent magnets was developed by Ampère almost a century before the atomic nature of matter was understood. (For a full quantum mechanical treatment of magnetic spins, see Quantum Mechanics and Atomic Structure.)

Contemporary Applications of Magnetism

Today, magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect both individuals and society. The electronic tablet in your purse or backpack, for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale (Figure 11.3). Weak changes in a magnetic field in a thin film of iron and chromium were discovered to bring about much larger changes in resistance, called giant magnetoresistance. Information can then be recorded magnetically based on the direction in which the iron layer is magnetized. As a result of the discovery of giant magnetoresistance and its applications to digital storage, the 2007 Nobel Prize in Physics was awarded to Albert Fert from France and Peter Grunberg from Germany.

Figure 11.3 Engineering technology like computer storage would not be possible without a deep understanding of magnetism. (credit: Klaus Eifert)

All electric motors—with uses as diverse as powering refrigerators, starting cars, and moving elevators—contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Research into using magnetic containment of fusion as a future energy source has been continuing for several years. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is involved in the structure of atomic energy levels, as well as the motion of cosmic rays and charged particles trapped in the Van Allen belts around Earth. Once again, we see that all these disparate phenomena are linked by a small number of underlying physical principles.

11.2 Magnetic Fields and Lines

Learning Objectives

By the end of this section, you will be able to:

- Define the magnetic field based on a moving charge experiencing a force
- Apply the right-hand rule to determine the direction of a magnetic force based on the motion of a charge in a magnetic field
- Sketch magnetic field lines to understand which way the magnetic field points and how strong it is in a region of space

We have outlined the properties of magnets, described how they behave, and listed some of the applications of magnetic properties. Even though there are no such things as isolated magnetic charges, we can still define the attraction and repulsion of magnets as based on a field. In this section, we define the magnetic field, determine its direction based on the right-hand rule, and discuss how to draw magnetic field lines.

Defining the Magnetic Field

A magnetic field is defined by the force that a charged particle experiences moving in this field, after we account for the gravitational and any additional electric forces possible on the charge. The magnitude of this force is proportional to the amount of charge q, the speed of the charged particle v, and the magnitude of the applied magnetic field. The direction of this force is perpendicular to both the direction of the moving charged particle and the direction of the applied magnetic field. Based on these observations, we define the magnetic field strength B based on the **magnetic force** $\vec{\mathbf{F}}$ on a charge q moving at velocity $\vec{\mathbf{v}}$ as the cross product of the velocity and magnetic field, that is,

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}. \tag{11.1}$$

In fact, this is how we define the magnetic field $\vec{\mathbf{B}}$—in terms of the force on a charged particle moving in a magnetic field. The magnitude of the force is determined from the definition of the cross product as it relates to the magnitudes of each of the vectors. In other words, the magnitude of the force satisfies

$$F = qvB\sin\theta \quad (11.2)$$

where θ is the angle between the velocity and the magnetic field.

The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943), where

$$1\ \text{T} = \frac{1\ \text{N}}{\text{A} \cdot \text{m}}. \quad (11.3)$$

A smaller unit, called the **gauss** (G), where $1\ \text{G} = 10^{-4}\ \text{T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. Earth's magnetic field on its surface is only about $5 \times 10^{-5}\ \text{T}$, or 0.5 G.

PROBLEM-SOLVING STRATEGY

Direction of the Magnetic Field by the Right-Hand Rule

The direction of the magnetic force $\vec{\mathbf{F}}$ is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, as determined by the **right-hand rule-1** (or RHR-1), which is illustrated in Figure 11.4.

1. Orient your right hand so that your fingers curl in the plane defined by the velocity and magnetic field vectors.
2. Using your right hand, sweep from the velocity toward the magnetic field with your fingers through the smallest angle possible.
3. The magnetic force is directed where your thumb is pointing.
4. If the charge was negative, reverse the direction found by these steps.

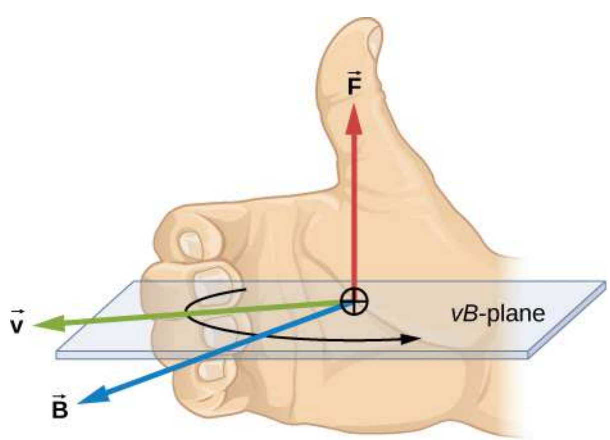


Figure 11.4 Magnetic fields exert forces on moving charges. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ and follows the right-hand rule-1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and the sine of the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.

INTERACTIVE

Visit this website (https://openstax.org/l/21magfields) for additional practice with the direction of magnetic fields.

There is no magnetic force on static charges. However, there is a magnetic force on charges moving at an angle to a magnetic field. When charges are stationary, their electric fields do not affect magnets. However, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic forces emerges—each affects the other.

EXAMPLE 11.1

An Alpha-Particle Moving in a Magnetic Field

An alpha-particle $\left(q = 3.2 \times 10^{-19}\,\text{C}\right)$ moves through a uniform magnetic field whose magnitude is 1.5 T. The field is directly parallel to the positive z-axis of the rectangular coordinate system of Figure 11.5. What is the magnetic force on the alpha-particle when it is moving (a) in the positive x-direction with a speed of $5.0 \times 10^4\,\text{m/s}$? (b) in the negative y-direction with a speed of $5.0 \times 10^4\,\text{m/s}$? (c) in the positive z-direction with a speed of $5.0 \times 10^4\,\text{m/s}$? (d) with a velocity $\vec{\mathbf{v}} = \left(2.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}}\right) \times 10^4\,\text{m/s}$?

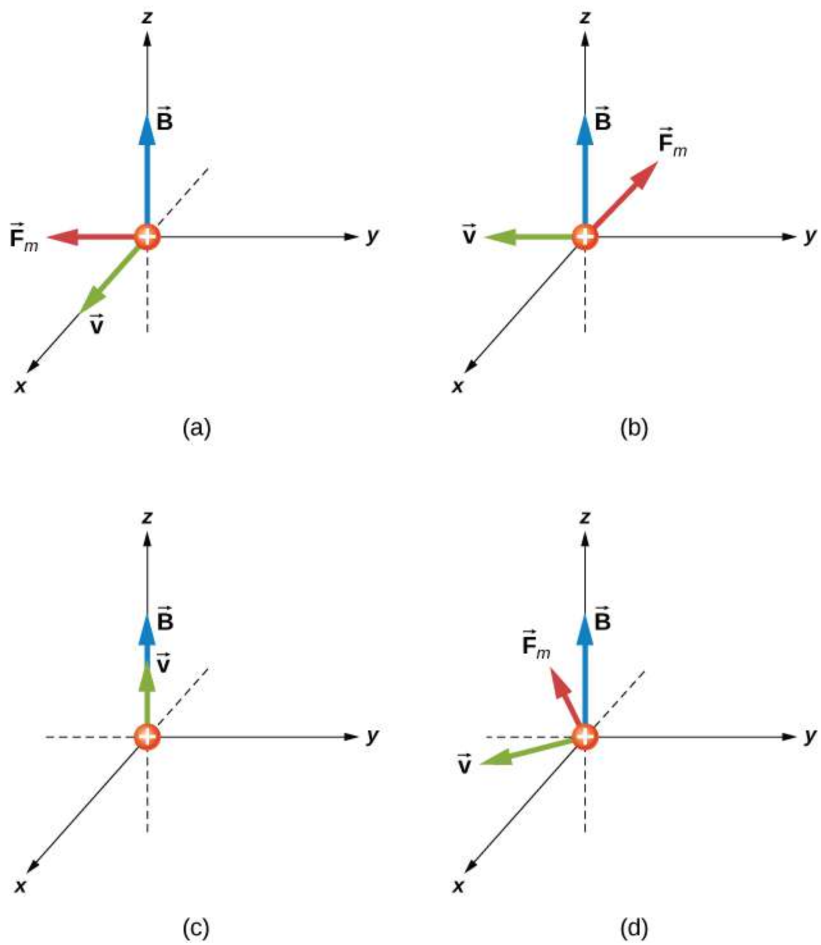


Figure 11.5 The magnetic forces on an alpha-particle moving in a uniform magnetic field. The field is the same in each drawing, but the velocity is different.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ or $F = qvB\sin\theta$ to calculate the force. The direction of the force is determined by RHR-1.

Solution

a. First, to determine the direction, start with your fingers pointing in the positive x-direction. Sweep your fingers upward in the direction of magnetic field. Your thumb should point in the negative y-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \left(3.2 \times 10^{-19}\,\text{C}\right)\left(5.0 \times 10^{4}\,\text{m/s}\,\hat{\mathbf{i}}\right) \times \left(1.5\,\text{T}\,\hat{\mathbf{k}}\right) = -2.4 \times 10^{-14}\,\text{N}\,\hat{\mathbf{j}}.$$

b. First, to determine the directionality, start with your fingers pointing in the negative y-direction. Sweep your fingers upward in the direction of magnetic field as in the previous problem. Your thumb should be open in the negative x-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \left(3.2 \times 10^{-19}\,\text{C}\right)\left(-5.0 \times 10^{4}\,\text{m/s}\,\hat{\mathbf{j}}\right) \times \left(1.5\,\text{T}\,\hat{\mathbf{k}}\right) = -2.4 \times 10^{-14}\,\text{N}\,\hat{\mathbf{i}}.$$

An alternative approach is to use Equation 11.2 to find the magnitude of the force. This applies for both parts (a) and (b). Since the velocity is perpendicular to the magnetic field, the angle between them is 90 degrees. Therefore, the magnitude of the force is:

$$F = qvB\sin\theta = \left(3.2 \times 10^{-19}\,\text{C}\right)\left(5.0 \times 10^{4}\,\text{m/s}\right)(1.5\,\text{T})\sin(90°) = 2.4 \times 10^{-14}\,\text{N}.$$

c. Since the velocity and magnetic field are parallel to each other, there is no orientation of your hand that will result in a force direction. Therefore, the force on this moving charge is zero. This is confirmed by the cross product. When you cross two vectors pointing in the same direction, the result is equal to zero.

d. First, to determine the direction, your fingers could point in any orientation; however, you must sweep your fingers upward in the direction of the magnetic field. As you rotate your hand, notice that the thumb can point in any x- or y-direction possible, but not in the z-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\begin{aligned}\vec{\mathbf{F}} &= q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \left(3.2 \times 10^{-19}\,\text{C}\right)\left(\left(2.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}}\right) \times 10^{4}\,\text{m/s}\right) \times \left(1.5\,\text{T}\,\hat{\mathbf{k}}\right)\\ &= \left(-14.4\hat{\mathbf{i}} - 9.6\hat{\mathbf{j}}\right) \times 10^{-15}\,\text{N}.\end{aligned}$$

This solution can be rewritten in terms of a magnitude and angle in the xy-plane:

$$\begin{aligned}\left|\vec{\mathbf{F}}\right| &= \sqrt{F_x^2 + F_y^2} = \sqrt{(-14.4)^2 + (-9.6)^2} \times 10^{-15}\,\text{N} = 1.7 \times 10^{-14}\,\text{N}\\ \theta &= \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-9.6 \times 10^{-15}\,\text{N}}{-14.4 \times 10^{-15}\,\text{N}}\right) = 34°.\end{aligned}$$

The magnitude of the force can also be calculated using Equation 11.2. The velocity in this question, however, has three components. The z-component of the velocity can be neglected, because it is parallel to the magnetic field and therefore generates no force. The magnitude of the velocity is calculated from the x- and y-components. The angle between the velocity in the xy-plane and the magnetic field in the z-plane is 90 degrees. Therefore, the force is calculated to be:

$$\begin{aligned}\left|\vec{\mathbf{v}}\right| &= \sqrt{(2)^2 + (-3)^2} \times 10^{4}\,\tfrac{\text{m}}{\text{s}} = 3.6 \times 10^{4}\,\tfrac{\text{m}}{\text{s}}\\ F &= qvB\sin\theta = (3.2 \times 10^{-19}\,\text{C})(3.6 \times 10^{4}\,\text{m/s})(1.5\,\text{T})\sin(90°) = 1.7 \times 10^{-14}\,\text{N}.\end{aligned}$$

This is the same magnitude of force calculated by unit vectors.

Significance

The cross product in this formula results in a third vector that must be perpendicular to the other two. Other physical quantities, such as angular momentum, also have three vectors that are related by the cross product. Note that typical force values in magnetic force problems are much larger than the gravitational force. Therefore, for an isolated charge, the magnetic force is the dominant force governing the charge's motion.

CHECK YOUR UNDERSTANDING 11.1

Repeat the previous problem with the magnetic field in the x-direction rather than in the z-direction. Check your answers with RHR-1.

Representing Magnetic Fields

The representation of magnetic fields by **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in Figure 11.6, each of these lines forms a closed loop, even if not shown by the constraints of the space available for the figure. The field lines emerge from the north pole (N), loop around to the south pole (S), and continue through the bar magnet back to the north pole.

Magnetic field lines have several hard-and-fast rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without a beginning or end. They are directed from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which generally begin on positive charges and end on negative charges or at infinity. If isolated magnetic charges (referred to as magnetic monopoles) existed, then magnetic field lines would begin and end on them.

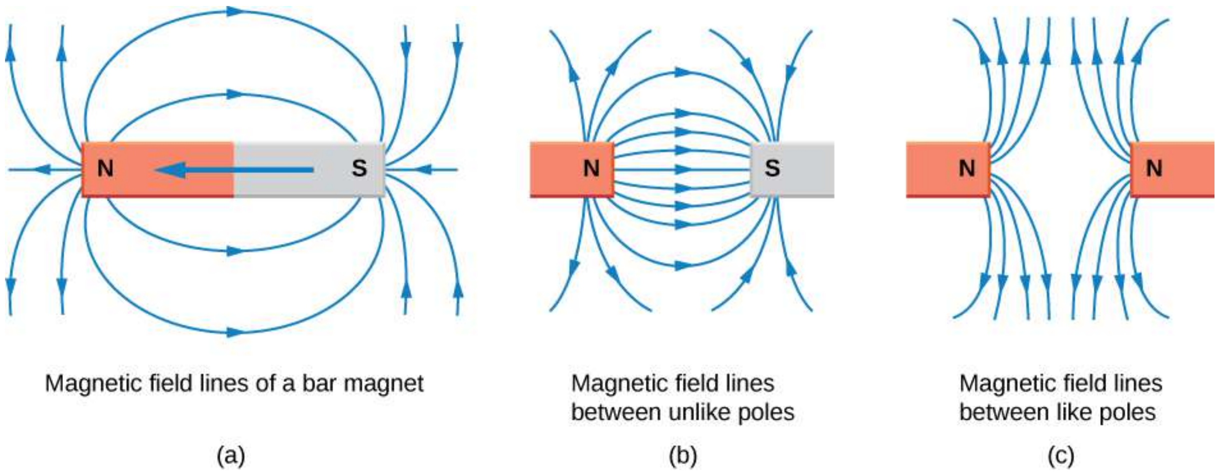


Figure 11.6 Magnetic field lines are defined to have the direction in which a small compass points when placed at a location in the field. The strength of the field is proportional to the closeness (or density) of the lines. If the interior of the magnet could be probed, the field lines would be found to form continuous, closed loops. To fit in a reasonable space, some of these drawings may not show the closing of the loops; however, if enough space were provided, the loops would be closed.

11.3 Motion of a Charged Particle in a Magnetic Field

Learning Objectives

By the end of this section, you will be able to:

- Explain how a charged particle in an external magnetic field undergoes circular motion
- Describe how to determine the radius of the circular motion of a charged particle in a magnetic field

A charged particle experiences a force when moving through a magnetic field. What happens if this field is uniform over the motion of the charged particle? What path does the particle follow? In this section, we discuss the circular motion of the charged particle as well as other motion that results from a charged particle entering a magnetic field.

The simplest case occurs when a charged particle moves perpendicular to a uniform *B*-field (Figure 11.7). If the field is in a vacuum, the magnetic field is the dominant factor determining the motion. Since the magnetic force is perpendicular to the direction of travel, a charged particle follows a curved path in a magnetic field. The particle continues to follow this curved path until it forms a complete circle. Another way to look at this is that the magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected but not the speed.

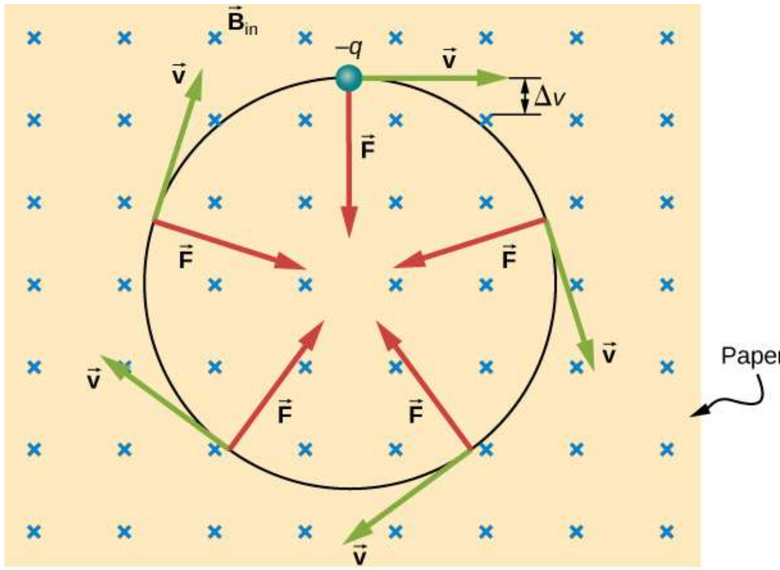


Figure 11.7 A negatively charged particle moves in the plane of the paper in a region where the magnetic field is perpendicular to the paper (represented by the small × 's—like the tails of arrows). The magnetic force is perpendicular to the velocity, so velocity changes in direction but not magnitude. The result is uniform circular motion. (Note that because the charge is negative, the force is opposite in direction to the prediction of the right-hand rule.)

In this situation, the magnetic force supplies the centripetal force $F_c = \frac{mv^2}{r}$. Noting that the velocity is perpendicular to the magnetic field, the magnitude of the magnetic force is reduced to $F = qvB$. Because the magnetic force F supplies the centripetal force F_c, we have

$$qvB = \frac{mv^2}{r}. \tag{11.4}$$

Solving for r yields

$$r = \frac{mv}{qB}. \tag{11.5}$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q, moving at a speed v that is perpendicular to a magnetic field of strength B. The time for the charged particle to go around the circular path is defined as the period, which is the same as the distance traveled (the circumference) divided by the speed. Based on this and Equation 11.4, we can derive the period of motion as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}. \tag{11.6}$$

If the velocity is not perpendicular to the magnetic field, then we can compare each component of the velocity separately with the magnetic field. The component of the velocity perpendicular to the magnetic field produces a magnetic force perpendicular to both this velocity and the field:

$$v_{\text{perp}} = v\sin\theta, \quad v_{\text{para}} = v\cos\theta. \tag{11.7}$$

where θ is the angle between v and B. The component parallel to the magnetic field creates constant motion along the same direction as the magnetic field, also shown in Equation 11.7. The parallel motion determines the *pitch* p of the helix, which is the distance between adjacent turns. This distance equals the parallel component of the velocity times the period:

$$p = v_{para} T. \quad (11.8)$$

The result is a **helical motion**, as shown in the following figure.

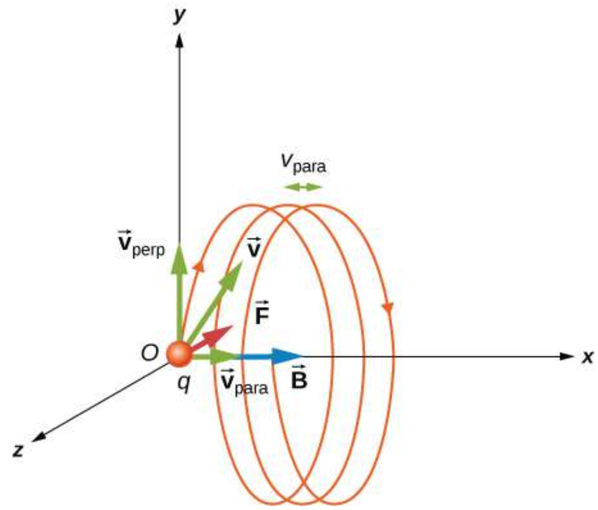


Figure 11.8 A charged particle moving with a velocity not in the same direction as the magnetic field. The velocity component perpendicular to the magnetic field creates circular motion, whereas the component of the velocity parallel to the field moves the particle along a straight line. The pitch is the horizontal distance between two consecutive circles. The resulting motion is helical.

While the charged particle travels in a helical path, it may enter a region where the magnetic field is not uniform. In particular, suppose a particle travels from a region of strong magnetic field to a region of weaker field, then back to a region of stronger field. The particle may reflect back before entering the stronger magnetic field region. This is similar to a wave on a string traveling from a very light, thin string to a hard wall and reflecting backward. If the reflection happens at both ends, the particle is trapped in a so-called magnetic bottle.

Trapped particles in magnetic fields are found in the Van Allen radiation belts around Earth, which are part of Earth's magnetic field. These belts were discovered by James Van Allen while trying to measure the flux of **cosmic rays** on Earth (high-energy particles that come from outside the solar system) to see whether this was similar to the flux measured on Earth. Van Allen found that due to the contribution of particles trapped in Earth's magnetic field, the flux was much higher on Earth than in outer space. Aurorae, like the famous aurora borealis (northern lights) in the Northern Hemisphere (Figure 11.9), are beautiful displays of light emitted as ions recombine with electrons entering the atmosphere as they spiral along magnetic field lines. (The ions are primarily oxygen and nitrogen atoms that are initially ionized by collisions with energetic particles in Earth's atmosphere.) Aurorae have also been observed on other planets, such as Jupiter and Saturn.

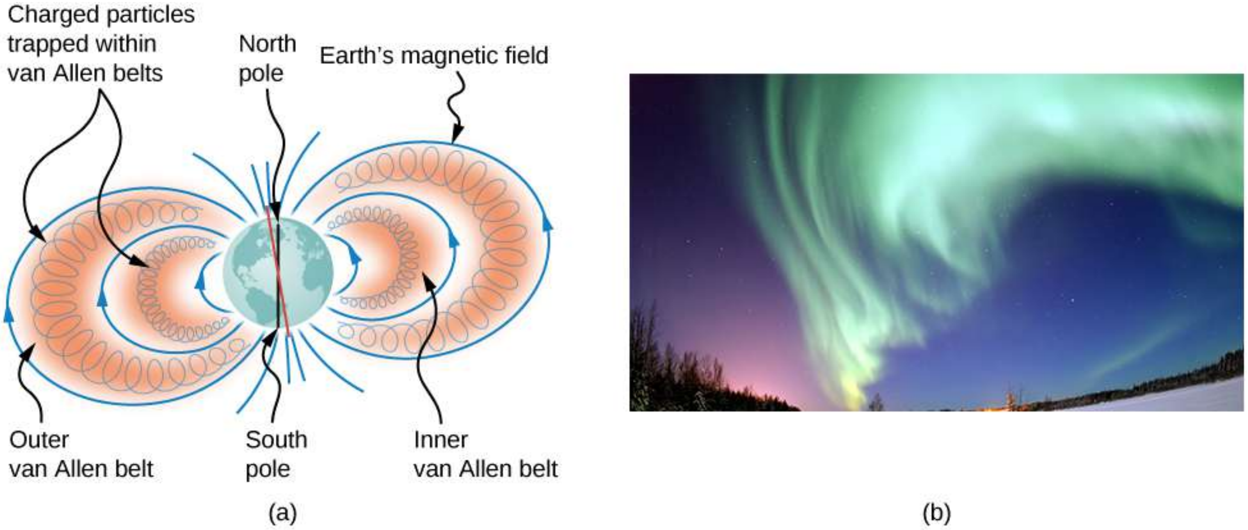


Figure 11.9 (a) The Van Allen radiation belts around Earth trap ions produced by cosmic rays striking Earth's atmosphere. (b) The magnificent spectacle of the aurora borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by Earth's magnetic field, this light is produced by glowing molecules and ions of oxygen and nitrogen. (credit b: modification of work by USAF Senior Airman Joshua Strang)

EXAMPLE 11.2

Beam Deflector

A research group is investigating short-lived radioactive isotopes. They need to design a way to transport alpha-particles (helium nuclei) from where they are made to a place where they will collide with another material to form an isotope. The beam of alpha-particles $\left(m = 6.64 \times 10^{-27}\,\text{kg}, q = 3.2 \times 10^{-19}\,\text{C}\right)$ bends through a 90-degree region with a uniform magnetic field of 0.050 T (Figure 11.10). (a) In what direction should the magnetic field be applied? (b) How much time does it take the alpha-particles to traverse the uniform magnetic field region?

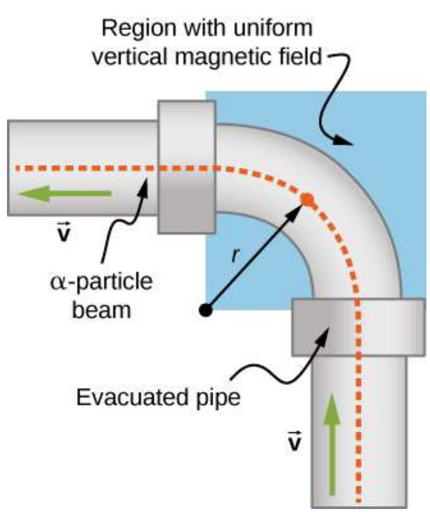


Figure 11.10 Top view of the beam deflector setup.

Strategy

a. The direction of the magnetic field is shown by the RHR-1. Your fingers point in the direction of v, and your thumb needs to point in the direction of the force, to the left. Therefore, since the alpha-particles are positively charged, the magnetic field must point down.

b. The period of the alpha-particle going around the circle is

$$T = \frac{2\pi m}{qB}. \qquad 11.9$$

Because the particle is only going around a quarter of a circle, we can take 0.25 times the period to find the time it takes to go around this path.

Solution

a. Let's start by focusing on the alpha-particle entering the field near the bottom of the picture. First, point your thumb up the page. In order for your palm to open to the left where the centripetal force (and hence the magnetic force) points, your fingers need to change orientation until they point into the page. This is the direction of the applied magnetic field.
b. The period of the charged particle going around a circle is calculated by using the given mass, charge, and magnetic field in the problem. This works out to be

$$T = \frac{2\pi m}{qB} = \frac{2\pi\left(6.64 \times 10^{-27}\,\text{kg}\right)}{\left(3.2 \times 10^{-19}\,\text{C}\right)(0.050\,\text{T})} = 2.6 \times 10^{-6}\,\text{s}.$$

However, for the given problem, the alpha-particle goes around a quarter of the circle, so the time it takes would be

$$t = 0.25 \times 2.61 \times 10^{-6}\,\text{s} = 6.5 \times 10^{-7}\,\text{s}.$$

Significance

This time may be quick enough to get to the material we would like to bombard, depending on how short-lived the radioactive isotope is and continues to emit alpha-particles. If we could increase the magnetic field applied in the region, this would shorten the time even more. The path the particles need to take could be shortened, but this may not be economical given the experimental setup.

CHECK YOUR UNDERSTANDING 11.2

A uniform magnetic field of magnitude 1.5 T is directed horizontally from west to east. (a) What is the magnetic force on a proton at the instant when it is moving vertically downward in the field with a speed of 4×10^7 m/s? (b) Compare this force with the weight w of a proton.

EXAMPLE 11.3

Helical Motion in a Magnetic Field

A proton enters a uniform magnetic field of $1.0 \times 10^{-4}\,\text{T}$ with a speed of 5×10^5 m/s. At what angle must the magnetic field be from the velocity so that the pitch of the resulting helical motion is equal to the radius of the helix?

Strategy

The pitch of the motion relates to the parallel velocity times the period of the circular motion, whereas the radius relates to the perpendicular velocity component. After setting the radius and the pitch equal to each other, solve for the angle between the magnetic field and velocity or θ.

Solution

The pitch is given by Equation 11.8, the period is given by Equation 11.6, and the radius of circular motion is given by Equation 11.5. Note that the velocity in the radius equation is related to only the perpendicular velocity, which is where the circular motion occurs. Therefore, we substitute the sine component of the overall velocity into the radius equation to equate the pitch and radius:

$$\begin{aligned} p &= r \\ v_{\parallel} T &= \frac{m v_{\perp}}{qB} \\ v\cos\theta \frac{2\pi m}{qB} &= \frac{mv\sin\theta}{qB} \\ 2\pi &= \tan\theta \\ \theta &= 81.0°. \end{aligned}$$

Significance

If this angle were $0°$, only parallel velocity would occur and the helix would not form, because there would be no circular motion in the perpendicular plane. If this angle were $90°$, only circular motion would occur and there would be no movement of the circles perpendicular to the motion. That is what creates the helical motion.

11.4 Magnetic Force on a Current-Carrying Conductor

Learning Objectives

By the end of this section, you will be able to:

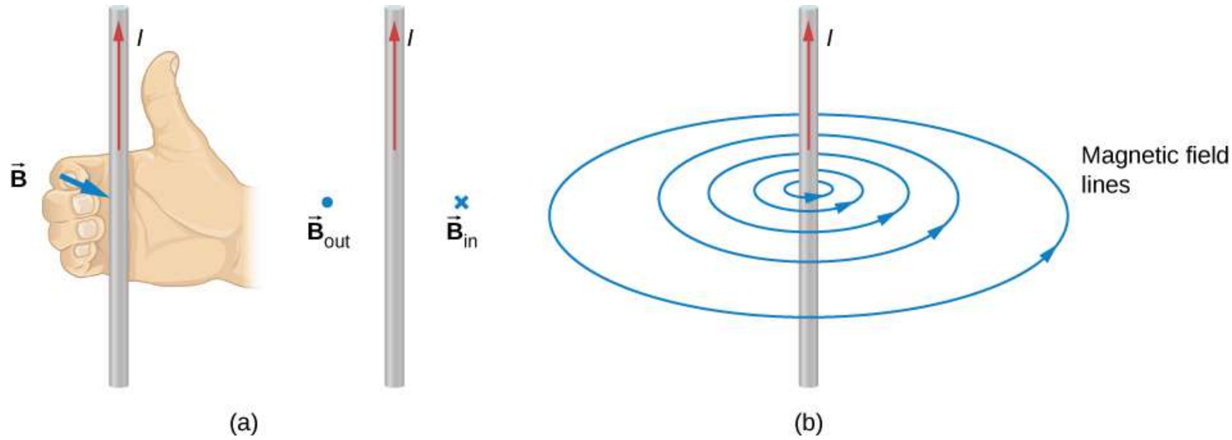
- Determine the direction in which a current-carrying wire experiences a force in an external magnetic field
- Calculate the force on a current-carrying wire in an external magnetic field

Moving charges experience a force in a magnetic field. If these moving charges are in a wire—that is, if the wire is carrying a current—the wire should also experience a force. However, before we discuss the force exerted on a current by a magnetic field, we first examine the magnetic field generated by an electric current. We are studying two separate effects here that interact closely: A current-carrying wire generates a magnetic field and the magnetic field exerts a force on the current-carrying wire.

Magnetic Fields Produced by Electrical Currents

When discussing historical discoveries in magnetism, we mentioned Oersted's finding that a wire carrying an electrical current caused a nearby compass to deflect. A connection was established that electrical currents produce magnetic fields. (This connection between electricity and magnetism is discussed in more detail in Sources of Magnetic Fields.)

The compass needle near the wire experiences a force that aligns the needle tangent to a circle around the wire. Therefore, a current-carrying wire produces circular loops of magnetic field. To determine the direction of the magnetic field generated from a wire, we use a second right-hand rule. In RHR-2, your thumb points in the direction of the current while your fingers wrap around the wire, pointing in the direction of the magnetic field produced (Figure 11.11). If the magnetic field were coming at you or out of the page, we represent this with a dot. If the magnetic field were going into the page, we represent this with an $\times$. These symbols come from considering a vector arrow: An arrow pointed toward you, from your perspective, would look like a dot or the tip of an arrow. An arrow pointed away from you, from your perspective, would look like a cross or an $\times$. A composite sketch of the magnetic circles is shown in Figure 11.11, where the field strength is shown to decrease as you get farther from the wire by loops that are farther separated.



(a) (b)

Figure 11.11 (a) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow). (b) A long and straight wire creates a field with magnetic field lines forming circular loops.

Calculating the Magnetic Force

Electric current is an ordered movement of charge. A current-carrying wire in a magnetic field must therefore experience a force due to the field. To investigate this force, let's consider the infinitesimal section of wire as shown in Figure 11.12. The length and cross-sectional area of the section are dl and A, respectively, so its volume is $V = A \cdot dl$. The wire is formed from material that contains n charge carriers per unit volume, so the number of charge carriers in the section is $nA \cdot dl$. If the charge carriers move with drift velocity $\vec{\mathbf{v}}_{\mathrm{d}}$, the current I in the wire is (from Current and Resistance)

$$I = neAv_d.$$

The magnetic force on any single charge carrier is $e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}$, so the total magnetic force $d\vec{\mathbf{F}}$ on the $nA \cdot dl$ charge carriers in the section of wire is

$$d\vec{\mathbf{F}} = (nA \cdot dl)e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}. \tag{11.10}$$

We can define dl to be a vector of length dl pointing along $\vec{\mathbf{v}}_{\mathrm{d}}$, which allows us to rewrite this equation as

$$d\vec{\mathbf{F}} = neAv_{\mathrm{d}}\vec{\mathbf{dl}} \times \vec{\mathbf{B}}, \tag{11.11}$$

or

$$d\vec{\mathbf{F}} = I\vec{\mathbf{dl}} \times \vec{\mathbf{B}}. \tag{11.12}$$

This is the magnetic force on the section of wire. Note that it is actually the net force exerted by the field on the charge carriers themselves. The direction of this force is given by RHR-1, where you point your fingers in the direction of the current and curl them toward the field. Your thumb then points in the direction of the force.

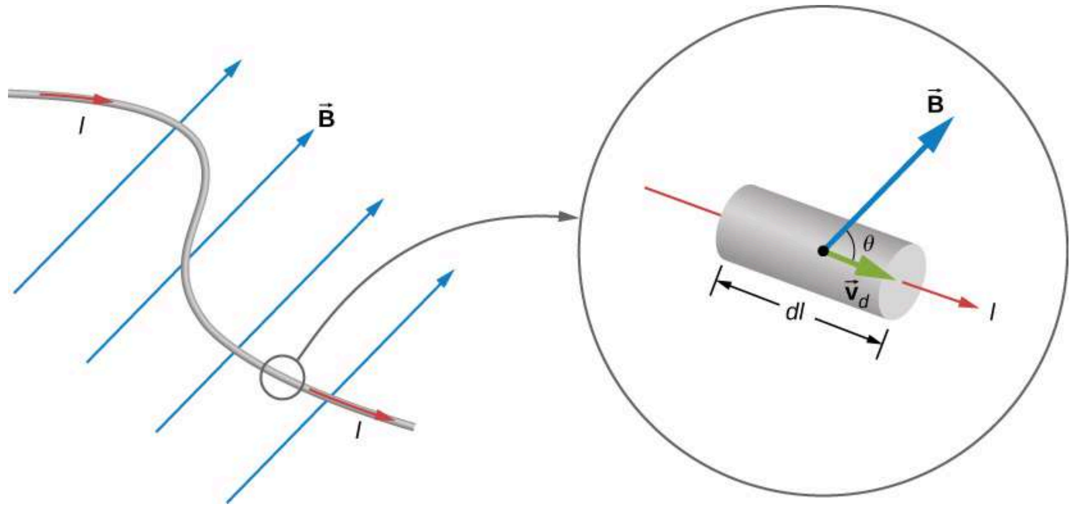


Figure 11.12 An infinitesimal section of current-carrying wire in a magnetic field.

To determine the magnetic force $\vec{F}$ on a wire of arbitrary length and shape, we must integrate Equation 11.12 over the entire wire. If the wire section happens to be straight and B is uniform, the equation differentials become absolute quantities, giving us

$$\vec{F} = I\vec{l} \times \vec{B}. \tag{11.13}$$

This is the force on a straight, current-carrying wire in a uniform magnetic field.

EXAMPLE 11.4

Balancing the Gravitational and Magnetic Forces on a Current-Carrying Wire

A wire of length 50 cm and mass 10 g is suspended in a horizontal plane by a pair of flexible leads (Figure 11.13). The wire is then subjected to a constant magnetic field of magnitude 0.50 T, which is directed as shown. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?

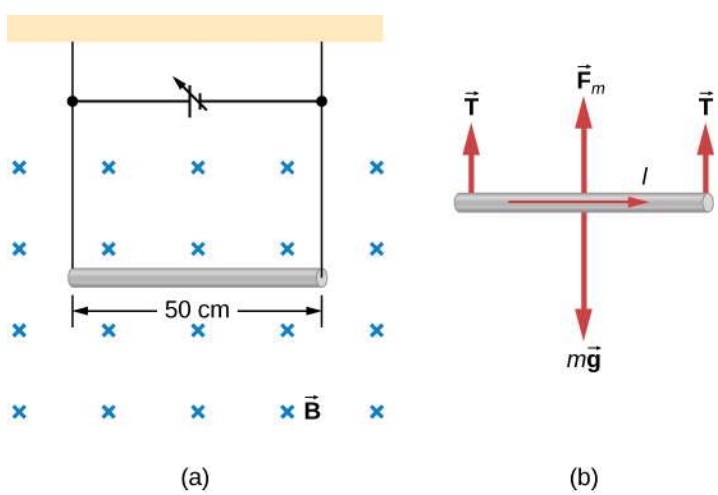


Figure 11.13 (a) A wire suspended in a magnetic field. (b) The free-body diagram for the wire.

Strategy

From the free-body diagram in the figure, the tensions in the supporting leads go to zero when the gravitational and magnetic forces balance each other. Using the RHR-1, we find that the magnetic force points up. We can then determine the current I by equating the two forces.

Solution

Equate the two forces of weight and magnetic force on the wire:

$$mg = IlB.$$

Thus,

$$I = \frac{mg}{lB} = \frac{(0.010\ \text{kg})(9.8\ \text{m/s}^2)}{(0.50\ \text{m})(0.50\ \text{T})} = 0.39\ \text{A}.$$

Significance

This large magnetic field creates a significant force on a length of wire to counteract the weight of the wire.

EXAMPLE 11.5

Calculating Magnetic Force on a Current-Carrying Wire

A long, rigid wire lying along the y-axis carries a 5.0-A current flowing in the positive y-direction. (a) If a constant magnetic field of magnitude 0.30 T is directed along the positive x-axis, what is the magnetic force per unit length on the wire? (b) If a constant magnetic field of 0.30 T is directed 30 degrees from the $+x$-axis towards the $+y$-axis, what is the magnetic force per unit length on the wire?

Strategy

The magnetic force on a current-carrying wire in a magnetic field is given by $\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. For part a, since the current and magnetic field are perpendicular in this problem, we can simplify the formula to give us the magnitude and find the direction through the RHR-1. The angle θ is 90 degrees, which means $\sin\theta = 1$. Also, the length can be divided over to the left-hand side to find the force per unit length. For part b, the current times length is written in unit vector notation, as well as the magnetic field. After the cross product is taken, the directionality is evident by the resulting unit vector.

Solution

a. We start with the general formula for the magnetic force on a wire. We are looking for the force per unit length, so we divide by the length to bring it to the left-hand side. We also set $\sin\theta = 1$. The solution therefore is

$$\begin{aligned} F &= IlB\sin\theta \\ \tfrac{F}{l} &= (5.0\ \text{A})(0.30\ \text{T}) \\ \tfrac{F}{l} &= 1.5\ \text{N/m}. \end{aligned}$$

Directionality: Point your fingers in the positive y-direction and curl your fingers in the positive x-direction. Your thumb will point in the $-\vec{\mathbf{k}}$ direction. Therefore, with directionality, the solution is

$$\frac{\vec{\mathbf{F}}}{l} = -1.5\vec{\mathbf{k}}\ \text{N/m}.$$

b. The current times length and the magnetic field are written in unit vector notation. Then, we take the cross product to find the force:

$$\begin{aligned} \vec{\mathbf{F}} &= I\vec{\mathbf{l}} \times \vec{\mathbf{B}} = (5.0A)\,l\,\hat{\mathbf{j}} \times \left(0.30T\cos(30°)\,\hat{\mathbf{i}} + 0.30T\sin(30°)\,\hat{\mathbf{j}}\right) \\ \vec{\mathbf{F}}/l &= -1.30\hat{\mathbf{k}}\ \text{N/m}. \end{aligned}$$

Significance

This large magnetic field creates a significant force on a small length of wire. As the angle of the magnetic field becomes more closely aligned to the current in the wire, there is less of a force on it, as seen from comparing parts a and b.

CHECK YOUR UNDERSTANDING 11.3

A straight, flexible length of copper wire is immersed in a magnetic field that is directed into the page. (a) If the wire's current runs in the $+x$-direction, which way will the wire bend? (b) Which way will the wire bend if the current runs in the $-x$-direction?

EXAMPLE 11.6

Force on a Circular Wire

A circular current loop of radius R carrying a current I is placed in the xy-plane. A constant uniform magnetic field cuts through the loop parallel to the y-axis (Figure 11.14). Find the magnetic force on the upper half of the loop, the lower half of the loop, and the total force on the loop.

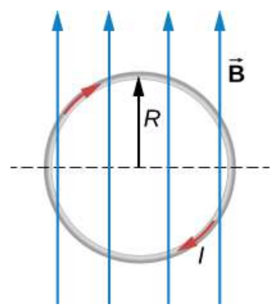


Figure 11.14 A loop of wire carrying a current in a magnetic field.

Strategy

The magnetic force on the upper loop should be written in terms of the differential force acting on each segment of the loop. If we integrate over each differential piece, we solve for the overall force on that section of the loop. The force on the lower loop is found in a similar manner, and the total force is the addition of these two forces.

Solution

A differential force on an arbitrary piece of wire located on the upper ring is:

$$dF = IB\sin\theta \, dl.$$

where θ is the angle between the magnetic field direction ($+y$) and the segment of wire. A differential segment is located at the same radius, so using an arc-length formula, we have:

$$\begin{aligned} dl &= R\,d\theta \\ dF &= IBR\sin\theta \, d\theta. \end{aligned}$$

In order to find the force on a segment, we integrate over the upper half of the circle, from 0 to π. This results in:

$$F = IBR\int_0^{\pi} \sin\theta \, d\theta = IBR(-\cos\pi + \cos 0) = 2IBR.$$

The lower half of the loop is integrated from π to zero, giving us:

$$F = IBR\int_{\pi}^{0} \sin\theta \, d\theta = IBR(-\cos 0 + \cos\pi) = -2IBR.$$

The net force is the sum of these forces, which is zero.

Significance

The total force on any closed loop in a uniform magnetic field is zero. Even though each piece of the loop has a force acting on it, the net force on the system is zero. (Note that there is a net torque on the loop, which we consider in the next section.)

11.5 Force and Torque on a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Evaluate the net force on a current loop in an external magnetic field
- Evaluate the net torque on a current loop in an external magnetic field
- Define the magnetic dipole moment of a current loop

Motors are the most common application of magnetic force on current-carrying wires. Motors contain loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted into mechanical work in the process. Once the loop's surface area is aligned with the magnetic field, the direction of current is reversed, so there is a continual torque on the loop (Figure 11.15). This reversal of the current is done with commutators and brushes. The commutator is set to reverse the current flow at set points to keep continual motion in the motor. A basic commutator has three contact areas to avoid and dead spots where the loop would have zero instantaneous torque at that point. The brushes press against the commutator, creating electrical contact between parts of the commutator during the spinning motion.

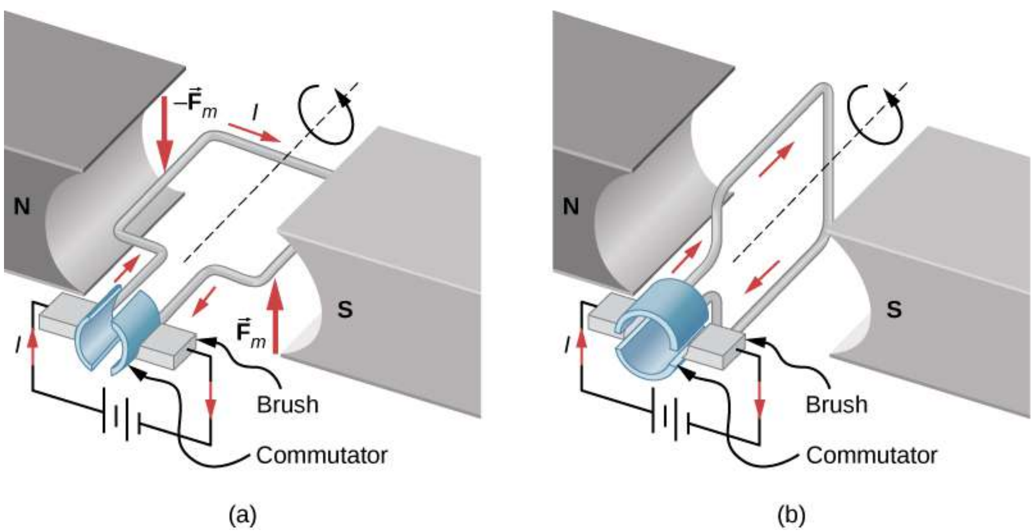


Figure 11.15 A simplified version of a dc electric motor. (a) The rectangular wire loop is placed in a magnetic field. The forces on the wires closest to the magnetic poles (N and S) are opposite in direction as determined by the right-hand rule-1. Therefore, the loop has a net torque and rotates to the position shown in (b). (b) The brushes now touch the commutator segments so that no current flows through the loop. No torque acts on the loop, but the loop continues to spin from the initial velocity given to it in part (a). By the time the loop flips over, current flows through the wires again but now in the opposite direction, and the process repeats as in part (a). This causes continual rotation of the loop.

In a uniform magnetic field, a current-carrying loop of wire, such as a loop in a motor, experiences both forces and torques on the loop. Figure 11.16 shows a rectangular loop of wire that carries a current I and has sides of lengths a and b. The loop is in a uniform magnetic field: $\vec{\mathbf{B}} = B\hat{\mathbf{j}}$. The magnetic force on a straight current-carrying wire of length l is given by $I\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. To find the net force on the loop, we have to apply this equation to each of the four sides. The force on side 1 is

$$\vec{\mathbf{F}}_1 = IaB\sin(90° - \theta)\hat{\mathbf{i}} = IaB\cos\theta\hat{\mathbf{i}} \tag{11.14}$$

where the direction has been determined with the RHR-1. The current in side 3 flows in the opposite direction to that of side 1, so

$$\vec{\mathbf{F}}_3 = -IaB\sin(90° + \theta)\hat{\mathbf{i}} = -IaB\cos\theta\hat{\mathbf{i}}. \quad (11.15)$$

The currents in sides 2 and 4 are perpendicular to $\vec{\mathbf{B}}$ and the forces on these sides are

$$\vec{\mathbf{F}}_2 = IbB\hat{\mathbf{k}}, \quad \vec{\mathbf{F}}_4 = -IbB\hat{\mathbf{k}}. \quad (11.16)$$

We can now find the net force on the loop:

$$\sum \vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = 0. \quad (11.17)$$

Although this result $(\Sigma F = 0)$ has been obtained for a rectangular loop, it is far more general and holds for current-carrying loops of arbitrary shapes; that is, there is no net force on a current loop in a uniform magnetic field.

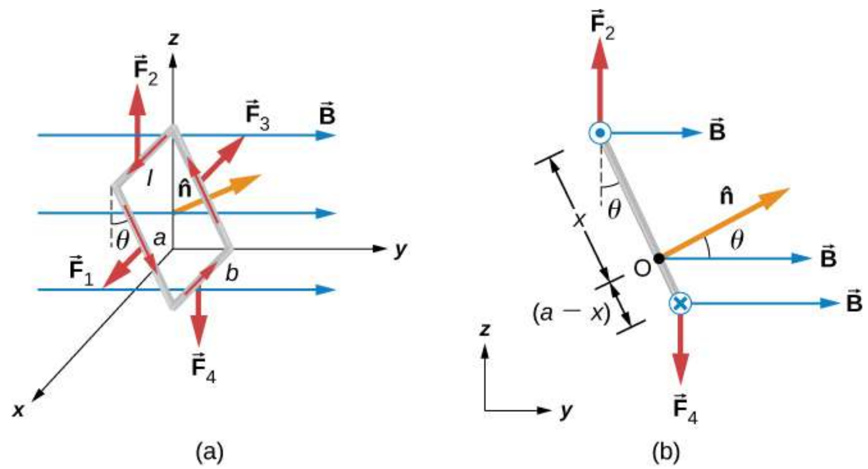


Figure 11.16 (a) A rectangular current loop in a uniform magnetic field is subjected to a net torque but not a net force. (b) A side view of the coil.

To find the net torque on the current loop shown in Figure 11.16, we first consider F_1 and F_3. Since they have the same line of action and are equal and opposite, the sum of their torques about any axis is zero (see Fixed-Axis Rotation). Thus, if there is any torque on the loop, it must be furnished by F_2 and F_4. Let's calculate the torques around the axis that passes through point O of Figure 11.16 (a side view of the coil) and is perpendicular to the plane of the page. The point O is a distance x from side 2 and a distance $(a - x)$ from side 4 of the loop. The moment arms of F_2 and F_4 are $x\sin\theta$ and $(a - x)\sin\theta$, respectively, so the net torque on the loop is

$$\begin{aligned}\sum \vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\tau}}_1 + \vec{\boldsymbol{\tau}}_2 + \vec{\boldsymbol{\tau}}_3 + \vec{\boldsymbol{\tau}}_4 &= F_2 x\sin\theta\hat{\mathbf{i}} - F_4(a-x)\sin(\theta)\hat{\mathbf{i}} \\ &= -IbBx\sin\theta\hat{\mathbf{i}} - IbB(a-x)\sin\theta\hat{\mathbf{i}}.\end{aligned} \quad (11.18)$$

This simplifies to

$$\vec{\boldsymbol{\tau}} = -IAB\sin\theta\hat{\mathbf{i}} \quad (11.19)$$

where $A = ab$ is the area of the loop.

Notice that this torque is independent of x; it is therefore independent of where point O is located in the plane of the current loop. Consequently, the loop experiences the same torque from the magnetic field about any axis in the plane of the loop and parallel to the x-axis.

A closed-current loop is commonly referred to as a **magnetic dipole** and the term IA is known as its **magnetic dipole moment** μ. Actually, the magnetic dipole moment is a vector that is defined as

$$\vec{\boldsymbol{\mu}} = IA\hat{\mathbf{n}} \quad (11.20)$$

where $\hat{\mathbf{n}}$ is a unit vector directed perpendicular to the plane of the loop (see Figure 11.16). The direction of $\hat{\mathbf{n}}$ is obtained with the RHR-2—if you curl the fingers of your right hand in the direction of current flow in the loop, then your thumb points along $\hat{\mathbf{n}}$. If the loop contains N turns of wire, then its magnetic dipole moment is given by

$$\vec{\boldsymbol{\mu}} = NIA\hat{\mathbf{n}}. \qquad (11.21)$$

In terms of the magnetic dipole moment, the torque on a current loop due to a uniform magnetic field can be written simply as

$$\vec{\tau} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}. \qquad (11.22)$$

This equation holds for a current loop in a two-dimensional plane of arbitrary shape.

Using a calculation analogous to that found in Capacitance for an electric dipole, the potential energy of a magnetic dipole is

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}. \qquad (11.23)$$

EXAMPLE 11.7

Forces and Torques on Current-Carrying Loops

A circular current loop of radius 2.0 cm carries a current of 2.0 mA. (a) What is the magnitude of its magnetic dipole moment? (b) If the dipole is oriented at 30 degrees to a uniform magnetic field of magnitude 0.50 T, what is the magnitude of the torque it experiences and what is its potential energy?

Strategy

The dipole moment is defined by the current times the area of the loop. The area of the loop can be calculated from the area of the circle. The torque on the loop and potential energy are calculated from identifying the magnetic moment, magnetic field, and angle oriented in the field.

Solution

a. The magnetic moment μ is calculated by the current times the area of the loop or πr^2.

$$\mu = IA = (2.0 \times 10^{-3}\ \text{A})(\pi(0.02\ \text{m})^2) = 2.5 \times 10^{-6}\ \text{A} \cdot \text{m}^2$$

b. The torque and potential energy are calculated by identifying the magnetic moment, magnetic field, and the angle between these two vectors. The calculations of these quantities are:

$$\begin{aligned} \tau &= \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} = \mu B \sin\theta = \left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\sin(30°) = 6.3 \times 10^{-7}\text{N} \cdot \text{m} \\ U &= -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}} = -\mu B \cos\theta = -\left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\cos(30°) = -1.1 \times 10^{-6}\text{J}. \end{aligned}$$

Significance

The concept of magnetic moment at the atomic level is discussed in the next chapter. The concept of aligning the magnetic moment with the magnetic field is the functionality of devices like magnetic motors, whereby switching the external magnetic field results in a constant spinning of the loop as it tries to align with the field to minimize its potential energy.

CHECK YOUR UNDERSTANDING 11.4

In what orientation would a magnetic dipole have to be to produce (a) a maximum torque in a magnetic field? (b) A maximum energy of the dipole?

11.6 The Hall Effect

Learning Objectives

By the end of this section, you will be able to:

- Explain a scenario where the magnetic and electric fields are crossed and their forces balance each other as a charged particle moves through a velocity selector
- Compare how charge carriers move in a conductive material and explain how this relates to the Hall effect

In 1879, E.H. Hall devised an experiment that can be used to identify the sign of the predominant charge carriers in a conducting material. From a historical perspective, this experiment was the first to demonstrate that the charge carriers in most metals are negative.

INTERACTIVE

Visit this website (https://openstax.org/l/21halleffect) to find more information about the Hall effect.

We investigate the **Hall effect** by studying the motion of the free electrons along a metallic strip of width l in a constant magnetic field (Figure 11.17). The electrons are moving from left to right, so the magnetic force they experience pushes them to the bottom edge of the strip. This leaves an excess of positive charge at the top edge of the strip, resulting in an electric field E directed from top to bottom. The charge concentration at both edges builds up until the electric force on the electrons in one direction is balanced by the magnetic force on them in the opposite direction. Equilibrium is reached when:

$$eE = ev_d B \tag{11.24}$$

where e is the magnitude of the electron charge, v_d is the drift speed of the electrons, and E is the magnitude of the electric field created by the separated charge. Solving this for the drift speed results in

$$v_d = \frac{E}{B}. \tag{11.25}$$

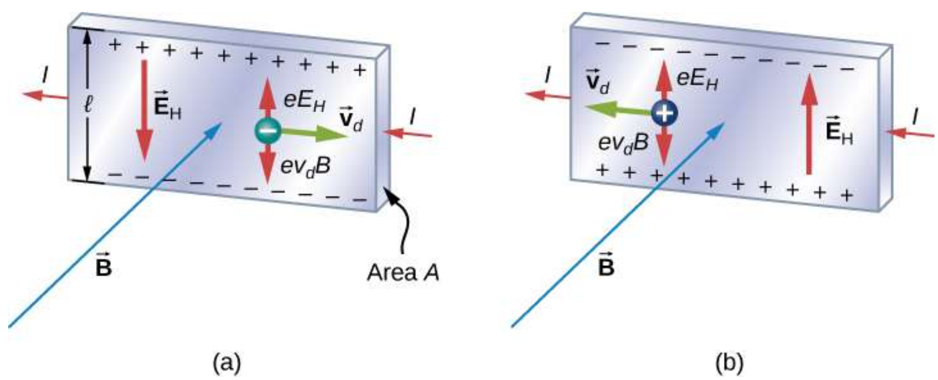


Figure 11.17 In the Hall effect, a potential difference between the top and bottom edges of the metal strip is produced when moving charge carriers are deflected by the magnetic field. (a) Hall effect for negative charge carriers; (b) Hall effect for positive charge carriers.

A scenario where the electric and magnetic fields are perpendicular to one another is called a crossed-field situation. If these fields produce equal and opposite forces on a charged particle with the velocity that equates the forces, these particles are able to pass through an apparatus, called a **velocity selector**, undeflected. This velocity is represented in Equation 11.26. Any other velocity of a charged particle sent into the same fields would be deflected by the magnetic force or electric force.

Going back to the Hall effect, if the current in the strip is I, then from Current and Resistance, we know that

$$I = nev_d A \tag{11.26}$$

where n is the number of charge carriers per volume and A is the cross-sectional area of the strip. Combining the equations for v_d and I results in

$$I = ne\left(\frac{E}{B}\right)A. \tag{11.27}$$

The field E is related to the potential difference V between the edges of the strip by

$$E = \frac{V}{l}. \tag{11.28}$$

The quantity V is called the Hall potential and can be measured with a voltmeter. Finally, combining the equations for I and E gives us

$$V = \frac{IBl}{neA} \tag{11.29}$$

where the upper edge of the strip in Figure 11.17 is positive with respect to the lower edge.

We can also combine Equation 11.24 and Equation 11.28 to get an expression for the Hall voltage in terms of the magnetic field:

$$V = Blv_d. \tag{11.30}$$

What if the charge carriers are positive, as in Figure 11.17? For the same current I, the magnitude of V is still given by Equation 11.29. However, the upper edge is now negative with respect to the lower edge. Therefore, by simply measuring the sign of V, we can determine the sign of the majority charge carriers in a metal.

Hall potential measurements show that electrons are the dominant charge carriers in most metals. However, Hall potentials indicate that for a few metals, such as tungsten, beryllium, and many semiconductors, the majority of charge carriers are positive. It turns out that conduction by positive charge is caused by the migration of missing electron sites (called holes) on ions. Conduction by holes is studied later in Condensed Matter Physics.

The Hall effect can be used to measure magnetic fields. If a material with a known density of charge carriers n is placed in a magnetic field and V is measured, then the field can be determined from Equation 11.29. In research laboratories where the fields of electromagnets used for precise measurements have to be extremely steady, a "Hall probe" is commonly used as part of an electronic circuit that regulates the field.

EXAMPLE 11.8

Velocity Selector

An electron beam enters a crossed-field velocity selector with magnetic and electric fields of 2.0 mT and 6.0×10^3 N/C, respectively. (a) What must the velocity of the electron beam be to traverse the crossed fields undeflected? If the electric field is turned off, (b) what is the acceleration of the electron beam and (c) what is the radius of the circular motion that results?

Strategy

The electron beam is not deflected by either of the magnetic or electric fields if these forces are balanced. Based on these balanced forces, we calculate the velocity of the beam. Without the electric field, only the magnetic force is used in Newton's second law to find the acceleration. Lastly, the radius of the path is based on the resulting circular motion from the magnetic force.

Solution

a. The velocity of the unperturbed beam of electrons with crossed fields is calculated by Equation 11.25:

$$v_d = \frac{E}{B} = \frac{6 \times 10^3 \text{ N/C}}{2 \times 10^{-3} \text{ T}} = 3 \times 10^6 \text{m/s}.$$

b. The acceleration is calculated from the net force from the magnetic field, equal to mass times acceleration.

The magnitude of the acceleration is:

$$\begin{aligned} ma &= qvB \\ a &= \frac{qvB}{m} = \frac{(1.6 \times 10^{-19}\,\text{C})(3 \times 10^{6}\ \text{m/s})(2 \times 10^{-3}\,\text{T})}{9.1 \times 10^{-31}\,\text{kg}} = 1.1 \times 10^{15}\ \text{m/s}^2. \end{aligned}$$

c. The radius of the path comes from a balance of the circular and magnetic forces, or Equation 11.25:

$$r = \frac{mv}{qB} = \frac{(9.1 \times 10^{-31}\,\text{kg})(3 \times 10^{6}\ \text{m/s})}{(1.6 \times 10^{-19}\,\text{C})(2 \times 10^{-3}\,\text{T})} = 8.5 \times 10^{-3}\,\text{m}.$$

Significance

If electrons in the beam had velocities above or below the answer in part (a), those electrons would have a stronger net force exerted by either the magnetic or electric field. Therefore, only those electrons at this specific velocity would make it through.

EXAMPLE 11.9

The Hall Potential in a Silver Ribbon

Figure 11.18 shows a silver ribbon whose cross section is 1.0 cm by 0.20 cm. The ribbon carries a current of 100 A from left to right, and it lies in a uniform magnetic field of magnitude 1.5 T. Using a density value of $n = 5.9 \times 10^{28}$ electrons per cubic meter for silver, find the Hall potential between the edges of the ribbon.

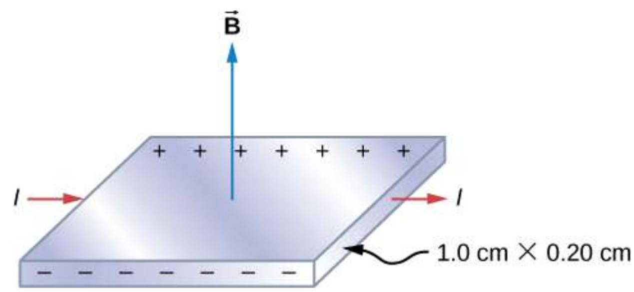


Figure 11.18 Finding the Hall potential in a silver ribbon in a magnetic field is shown.

Strategy

Since the majority of charge carriers are electrons, the polarity of the Hall voltage is that indicated in the figure. The value of the Hall voltage is calculated using Equation 11.29:

$$V = \frac{IBl}{neA}.$$

Solution

When calculating the Hall voltage, we need to know the current through the material, the magnetic field, the length, the number of charge carriers, and the area. Since all of these are given, the Hall voltage is calculated as:

$$V = \frac{IBl}{neA} = \frac{(100\ \text{A})(1.5\ \text{T})\left(1.0 \times 10^{-2}\text{m}\right)}{\left(5.9 \times 10^{28}/\text{m}^3\right)\left(1.6 \times 10^{-19}\text{C}\right)\left(2.0 \times 10^{-5}\text{m}^2\right)} = 7.9 \times 10^{-6}\text{V}.$$

Significance

As in this example, the Hall potential is generally very small, and careful experimentation with sensitive equipment is required for its measurement.

CHECK YOUR UNDERSTANDING 11.5

A Hall probe consists of a copper strip, $n = 8.5 \times 10^{28}$ electrons per cubic meter, which is 2.0 cm wide and

0.10 cm thick. What is the magnetic field when $I = 50$ A and the Hall potential is (a) 4.0μV and (b) 6.0μV?

11.7 Applications of Magnetic Forces and Fields

Learning Objectives

By the end of this section, you will be able to:

- Explain how a mass spectrometer works to separate charges
- Explain how a cyclotron works

Being able to manipulate and sort charged particles allows deeper experimentation to understand what matter is made of. We first look at a mass spectrometer to see how we can separate ions by their charge-to-mass ratio. Then we discuss cyclotrons as a method to accelerate charges to very high energies.

Mass Spectrometer

The **mass spectrometer** is a device that separates ions according to their charge-to-mass ratios. One particular version, the Bainbridge mass spectrometer, is illustrated in Figure 11.19. Ions produced at a source are first sent through a velocity selector, where the magnetic force is equally balanced with the electric force. These ions all emerge with the same speed $v = E/B$ since any ion with a different velocity is deflected preferentially by either the electric or magnetic force, and ultimately blocked from the next stage. They then enter a uniform magnetic field B_0 where they travel in a circular path whose radius R is given by Equation 11.3. The radius is measured by a particle detector located as shown in the figure.

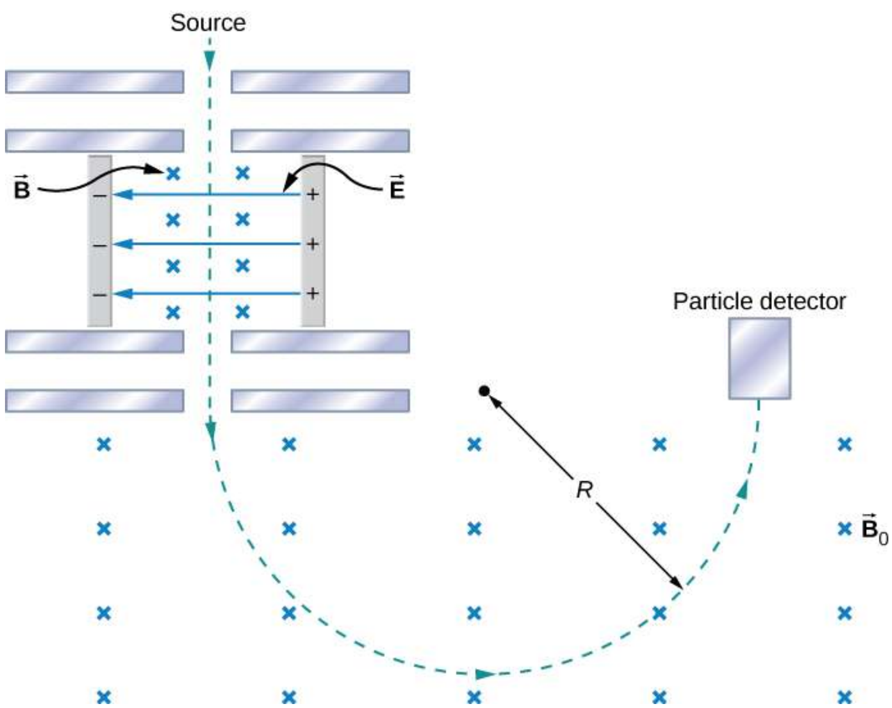


Figure 11.19 A schematic of the Bainbridge mass spectrometer, showing charged particles leaving a source, followed by a velocity selector where the electric and magnetic forces are balanced, followed by a region of uniform magnetic field where the particle is ultimately detected.

The relationship between the charge-to-mass ratio q/m and the radius R is determined by combining Equation 11.3 and Equation 11.25:

$$\frac{q}{m} = \frac{E}{BB_0 R}. \tag{11.31}$$

Since most ions are singly charged $\left(q = 1.6 \times 10^{-19}\ \text{C}\right)$, measured values of R can be used with this equation to determine the mass of ions. With modern instruments, masses can be determined to one part in 10^8.

An interesting use of a spectrometer is as part of a system for detecting very small leaks in a research apparatus. In low-temperature physics laboratories, a device known as a dilution refrigerator uses a mixture of He-3, He-4, and other cryogens to reach temperatures well below 1 K. The performance of the refrigerator is severely hampered if even a minute leak between its various components occurs. Consequently, before it is cooled down to the desired temperature, the refrigerator is subjected to a leak test. A small quantity of gaseous helium is injected into one of its compartments, while an adjacent, but supposedly isolated, compartment is connected to a high-vacuum pump to which a mass spectrometer is attached. A heated filament ionizes any helium atoms evacuated by the pump. The detection of these ions by the spectrometer then indicates a leak between the two compartments of the dilution refrigerator.

In conjunction with gas chromatography, mass spectrometers are used widely to identify unknown substances. While the gas chromatography portion breaks down the substance, the mass spectrometer separates the resulting ionized molecules. This technique is used with fire debris to ascertain the cause, in law enforcement to identify illegal drugs, in security to identify explosives, and in many medicinal applications.

Cyclotron

The **cyclotron** was developed by E.O. Lawrence to accelerate charged particles (usually protons, deuterons, or alpha-particles) to large kinetic energies. These particles are then used for nuclear-collision experiments to produce radioactive isotopes. A cyclotron is illustrated in Figure 11.20. The particles move between two flat, semi-cylindrical metallic containers D1 and D2, called **dees**. The dees are enclosed in a larger metal container, and the apparatus is placed between the poles of an electromagnet that provides a uniform magnetic field. Air is removed from the large container so that the particles neither lose energy nor are deflected because of collisions with air molecules. The dees are connected to a high-frequency voltage source that provides an alternating electric field in the small region between them. Because the dees are made of metal, their interiors are shielded from the electric field.

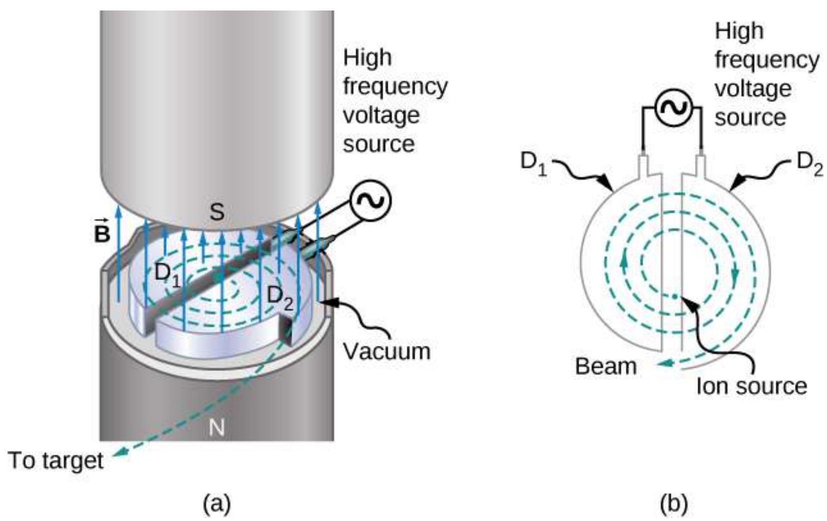


Figure 11.20 The inside of a cyclotron. A uniform magnetic field is applied as circulating protons travel through the dees, gaining energy as they traverse through the gap between the dees.

Suppose a positively charged particle is injected into the gap between the dees when D2 is at a positive potential relative to D1. The particle is then accelerated across the gap and enters D1 after gaining kinetic energy qV, where V is the average potential difference the particle experiences between the dees. When the particle is inside D1, only the uniform magnetic field $\vec{\mathbf{B}}$ of the electromagnet acts on it, so the particle moves in a circle of radius

$$r = \frac{mv}{qB} \quad (11.32)$$

with a period of

$$T = \frac{2\pi m}{qB}. \qquad 11.33$$

The period of the alternating voltage course is set at T, so while the particle is inside D1, moving along its semicircular orbit in a time $T/2$, the polarity of the dees is reversed. When the particle reenters the gap, D1 is positive with respect to D2, and the particle is again accelerated across the gap, thereby gaining a kinetic energy qV. The particle then enters D2, circulates in a slightly larger circle, and emerges from D2 after spending a time $T/2$ in this dee. This process repeats until the orbit of the particle reaches the boundary of the dees. At that point, the particle (actually, a beam of particles) is extracted from the cyclotron and used for some experimental purpose.

The operation of the cyclotron depends on the fact that, in a uniform magnetic field, a particle's orbital period is independent of its radius and its kinetic energy. Consequently, the period of the alternating voltage source need only be set at the one value given by Equation 11.33. With that setting, the electric field accelerates particles every time they are between the dees.

If the maximum orbital radius in the cyclotron is R, then from Equation 11.32, the maximum speed of a circulating particle of mass m and charge q is

$$v_{\max} = \frac{qBR}{m}. \qquad 11.34$$

Thus, its kinetic energy when ejected from the cyclotron is

$$\frac{1}{2}mv_{\max}{}^2 = \frac{q^2B^2R^2}{2m}. \qquad 11.35$$

The maximum kinetic energy attainable with this type of cyclotron is approximately 30 MeV. Above this energy, relativistic effects become important, which causes the orbital period to increase with the radius. Up to energies of several hundred MeV, the relativistic effects can be compensated for by making the magnetic field gradually increase with the radius of the orbit. However, for higher energies, much more elaborate methods must be used to accelerate particles.

Particles are accelerated to very high energies with either linear accelerators or synchrotrons. The linear accelerator accelerates particles continuously with the electric field of an electromagnetic wave that travels down a long evacuated tube. The Stanford Linear Accelerator (SLAC) is about 3.3 km long and accelerates electrons and positrons (positively charged electrons) to energies of 50 GeV. The synchrotron is constructed so that its bending magnetic field increases with particle speed in such a way that the particles stay in an orbit of fixed radius. The world's highest-energy synchrotron is located at CERN, which is on the Swiss-French border near Geneva. CERN has been of recent interest with the verified discovery of the Higgs Boson (see Particle Physics and Cosmology). This synchrotron can accelerate beams of approximately 10^{13} protons to energies of about 10^3 GeV.

EXAMPLE 11.10

Accelerating Alpha-Particles in a Cyclotron

A cyclotron used to accelerate alpha-particles ($m = 6.64 \times 10^{-27}\,\text{kg}, q = 3.2 \times 10^{-19}\,\text{C}$) has a radius of 0.50 m and a magnetic field of 1.8 T. (a) What is the period of revolution of the alpha-particles? (b) What is their maximum kinetic energy?

Strategy

a. The period of revolution is approximately the distance traveled in a circle divided by the speed. Identifying that the magnetic force applied is the centripetal force, we can derive the period formula.
b. The kinetic energy can be found from the maximum speed of the beam, corresponding to the maximum radius within the cyclotron.

Solution

a. By identifying the mass, charge, and magnetic field in the problem, we can calculate the period:

$$T = \frac{2\pi m}{qB} = \frac{2\pi\left(6.64 \times 10^{-27}\,\text{kg}\right)}{\left(3.2 \times 10^{-19}\,\text{C}\right)(1.8\text{T})} = 7.3 \times 10^{-8}\,\text{s}.$$

b. By identifying the charge, magnetic field, radius of path, and the mass, we can calculate the maximum kinetic energy:

$$\frac{1}{2}m{v_{\text{max}}}^2 = \frac{q^2B^2R^2}{2m} = \frac{\left(3.2 \times 10^{-19}\,\text{C}\right)^2(1.8\text{T})^2(0.50\text{m})^2}{2(6.65 \times 10^{-27}\,\text{kg})} = 6.2 \times 10^{-12}\,\text{J} = 39\text{MeV}.$$

CHECK YOUR UNDERSTANDING 11.6

A cyclotron is to be designed to accelerate protons to kinetic energies of 20 MeV using a magnetic field of 2.0 T. What is the required radius of the cyclotron?

CHAPTER REVIEW

Key Terms

cosmic rays comprised of particles that originate mainly from outside the solar system and reach Earth

cyclotron device used to accelerate charged particles to large kinetic energies

dees large metal containers used in cyclotrons that serve contain a stream of charged particles as their speed is increased

gauss G, unit of the magnetic field strength; $1\text{ G} = 10^{-4}\text{T}$

Hall effect creation of voltage across a current-carrying conductor by a magnetic field

helical motion superposition of circular motion with a straight-line motion that is followed by a charged particle moving in a region of magnetic field at an angle to the field

magnetic dipole closed-current loop

magnetic dipole moment term *IA* of the magnetic dipole, also called μ

magnetic field lines continuous curves that show the direction of a magnetic field; these lines point in the same direction as a compass points, toward the magnetic south pole of a bar magnet

magnetic force force applied to a charged particle moving through a magnetic field

mass spectrometer device that separates ions according to their charge-to-mass ratios

motor (dc) loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted into mechanical work in the process

north magnetic pole currently where a compass points to north, near the geographic North Pole; this is the effective south pole of a bar magnet but has flipped between the effective north and south poles of a bar magnet multiple times over the age of Earth

right-hand rule-1 using your right hand to determine the direction of either the magnetic force, velocity of a charged particle, or magnetic field

south magnetic pole currently where a compass points to the south, near the geographic South Pole; this is the effective north pole of a bar magnet but has flipped just like the north magnetic pole

tesla SI unit for magnetic field: 1 T = 1 N/A-m

velocity selector apparatus where the crossed electric and magnetic fields produce equal and opposite forces on a charged particle moving with a specific velocity; this particle moves through the velocity selector not affected by either field while particles moving with different velocities are deflected by the apparatus

Key Equations

Force on a charge in a magnetic field	$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
Magnitude of magnetic force	$F = qvB\sin\theta$
Radius of a particle's path in a magnetic field	$r = \frac{mv}{qB}$
Period of a particle's motion in a magnetic field	$T = \frac{2\pi m}{qB}$
Force on a current-carrying wire in a uniform magnetic field	$\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$
Magnetic dipole moment	$\vec{\boldsymbol{\mu}} = NIA\hat{\mathbf{n}}$
Torque on a current loop	$\vec{\tau} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$
Energy of a magnetic dipole	$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$

Drift velocity in crossed electric and magnetic fields	$v_d = \frac{E}{B}$
Hall potential	$V = \frac{IBl}{neA}$
Hall potential in terms of drift velocity	$V = Blv_d$
Charge-to-mass ratio in a mass spectrometer	$\frac{q}{m} = \frac{E}{BB_0 R}$
Maximum speed of a particle in a cyclotron	$v_{\max} = \frac{qBR}{m}$

Summary

11.1 Magnetism and Its Historical Discoveries

- Magnets have two types of magnetic poles, called the north magnetic pole and the south magnetic pole. North magnetic poles are those that are attracted toward Earth's geographic North Pole.
- Like poles repel and unlike poles attract.
- Discoveries of how magnets respond to currents by Oersted and others created a framework that led to the invention of modern electronic devices, electric motors, and magnetic imaging technology.

11.2 Magnetic Fields and Lines

- Charges moving across a magnetic field experience a force determined by $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. The force is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.
- The direction of the force on a moving charge is given by the right hand rule 1 (RHR-1): Sweep your fingers in a velocity, magnetic field plane. Start by pointing them in the direction of velocity and sweep towards the magnetic field. Your thumb points in the direction of the magnetic force for positive charges.
- Magnetic fields can be pictorially represented by magnetic field lines, which have the following properties:
  1. The field is tangent to the magnetic field line.
  2. Field strength is proportional to the line density.
  3. Field lines cannot cross.
  4. Field lines form continuous, closed loops.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

11.3 Motion of a Charged Particle in a Magnetic Field

- A magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius $r = \frac{mv}{qB}$.
- The period of circular motion for a charged particle moving in a magnetic field perpendicular to the plane of motion is $T = \frac{2\pi m}{qB}$.
- Helical motion results if the velocity of the charged particle has a component parallel to the magnetic field as well as a component perpendicular to the magnetic field.

11.4 Magnetic Force on a Current-Carrying Conductor

- An electrical current produces a magnetic field around the wire.
- The directionality of the magnetic field produced is determined by the right hand rule-2, where your thumb points in the direction of the current and your fingers wrap around the wire in the direction of the magnetic field.
- The magnetic force on current-carrying conductors is given by $\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$ where I is the current and l is the length of a wire in a uniform magnetic field B.

11.5 Force and Torque on a Current Loop

- The net force on a current-carrying loop of any plane shape in a uniform magnetic field is zero.
- The net torque τ on a current-carrying loop of any shape in a uniform magnetic field is calculated using $\tau = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ where $\vec{\mu}$ is the magnetic dipole moment and $\vec{\mathbf{B}}$ is the magnetic field strength.
- The magnetic dipole moment μ is the product of

the number of turns of wire N, the current in the loop I, and the area of the loop A or $\vec{\boldsymbol{\mu}} = NIA\hat{\mathbf{n}}$.

11.6 The Hall Effect

- Perpendicular electric and magnetic fields exert equal and opposite forces for a specific velocity of entering particles, thereby acting as a velocity selector. The velocity that passes through undeflected is calculated by $v = \frac{E}{B}$.
- The Hall effect can be used to measure the sign of the majority of charge carriers for metals. It can also be used to measure a magnetic field.

11.7 Applications of Magnetic Forces and Fields

- A mass spectrometer is a device that separates ions according to their charge-to-mass ratios by first sending them through a velocity selector, then a uniform magnetic field.
- Cyclotrons are used to accelerate charged particles to large kinetic energies through applied electric and magnetic fields.

Conceptual Questions

11.2 Magnetic Fields and Lines

1. Discuss the similarities and differences between the electrical force on a charge and the magnetic force on a charge.

2. (a) Is it possible for the magnetic force on a charge moving in a magnetic field to be zero? (b) Is it possible for the electric force on a charge moving in an electric field to be zero? (c) Is it possible for the resultant of the electric and magnetic forces on a charge moving simultaneously through both fields to be zero?

11.3 Motion of a Charged Particle in a Magnetic Field

3. At a given instant, an electron and a proton are moving with the same velocity in a constant magnetic field. Compare the magnetic forces on these particles. Compare their accelerations.

4. Does increasing the magnitude of a uniform magnetic field through which a charge is traveling necessarily mean increasing the magnetic force on the charge? Does changing the direction of the field necessarily mean a change in the force on the charge?

5. An electron passes through a magnetic field without being deflected. What do you conclude about the magnetic field?

6. If a charged particle moves in a straight line, can you conclude that there is no magnetic field present?

7. How could you determine which pole of an electromagnet is north and which pole is south?

11.4 Magnetic Force on a Current-Carrying Conductor

8. Describe the error that results from accidently using your left rather than your right hand when determining the direction of a magnetic force.

9. Considering the magnetic force law, are the velocity and magnetic field always perpendicular? Are the force and velocity always perpendicular? What about the force and magnetic field?

10. Why can a nearby magnet distort a cathode ray tube television picture?

11. A magnetic field exerts a force on the moving electrons in a current carrying wire. What exerts the force on a wire?

12. There are regions where the magnetic field of earth is almost perpendicular to the surface of Earth. What difficulty does this cause in the use of a compass?

11.6 The Hall Effect

13. Hall potentials are much larger for poor conductors than for good conductors. Why?

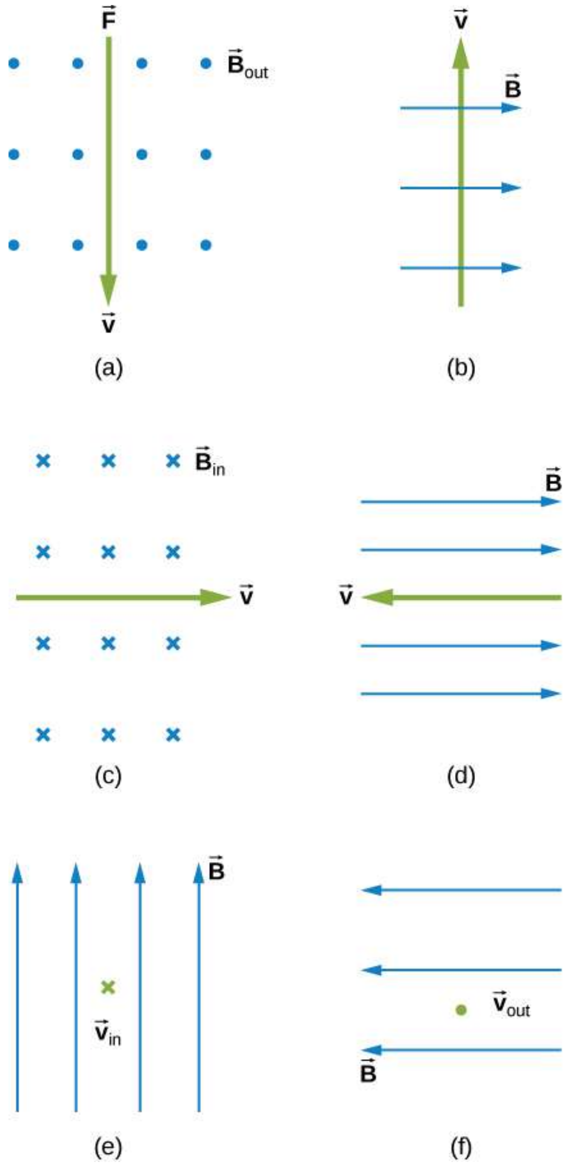
11.7 Applications of Magnetic Forces and Fields

14. Describe the primary function of the electric field and the magnetic field in a cyclotron.

Problems

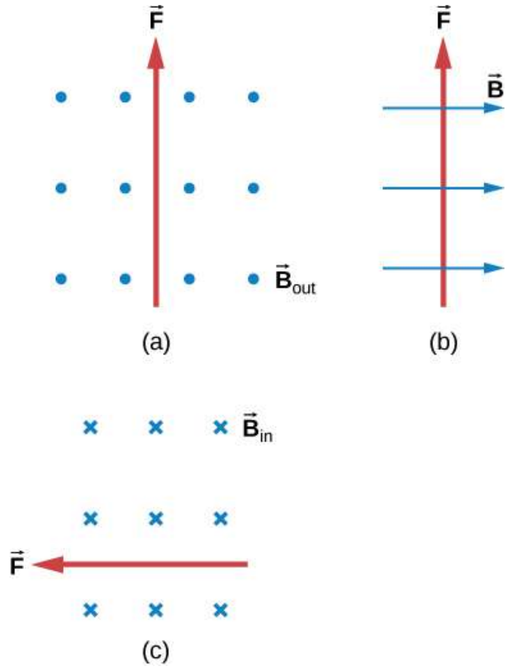
11.2 Magnetic Fields and Lines

15. What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases?



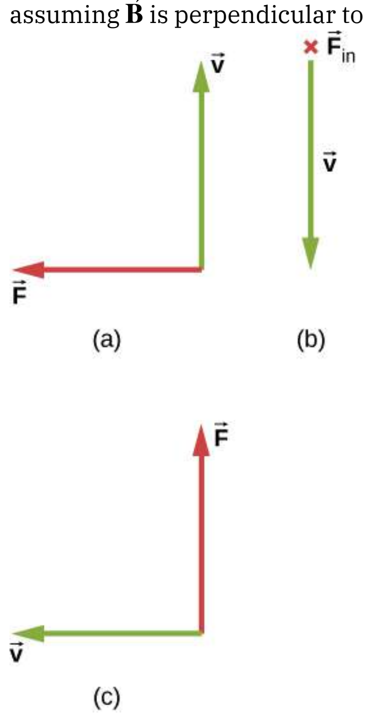
16. Repeat previous exercise for a negative charge.

17. What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases, assuming it moves perpendicular to B?



18. Repeat previous exercise for a positive charge.

19. What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases, assuming $\vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{v}}$?



20. Repeat previous exercise for a negative charge.

21. (a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a 0.500-μC charge and flies due west at a speed of 660. m/s over Earth's south magnetic pole, where the $8.00 \times 10^{-5} - \mathrm{T}$ magnetic field points straight down into the ground. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

22. (a) A cosmic ray proton moving toward Earth at 5.00×10^7 m/s experiences a magnetic force of 1.70×10^{-16} N. What is the strength of the magnetic field if there is a 45° angle between it and the proton's velocity? (b) Is the value obtained in part a. consistent with the known strength of Earth's magnetic field on its surface? Discuss.

23. An electron moving at 4.00×10^3 m/s in a 1.25-T magnetic field experiences a magnetic force of 1.40×10^{-16} N. What angle does the velocity of the electron make with the magnetic field? There are two answers.

24. (a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than 1.00×10^{-12} N. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in Earth's field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

11.3 Motion of a Charged Particle in a Magnetic Field

25. A cosmic-ray electron moves at 7.5×10^6 m/s perpendicular to Earth's magnetic field at an altitude where the field strength is 1.0×10^{-5} T. What is the radius of the circular path the electron follows?

26. (a) Viewers of Star Trek have heard of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at 5.0×10^7 m/s in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

27. (a) An oxygen-16 ion with a mass of 2.66×10^{-26} kg travels at 5.0×10^6 m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

28. An electron in a TV CRT moves with a speed of 6.0×10^6 m/s, in a direction perpendicular to Earth's field, which has a strength of 5.0×10^{-5} T. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

29. (a) At what speed will a proton move in a circular path of the same radius as the electron in the previous exercise? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

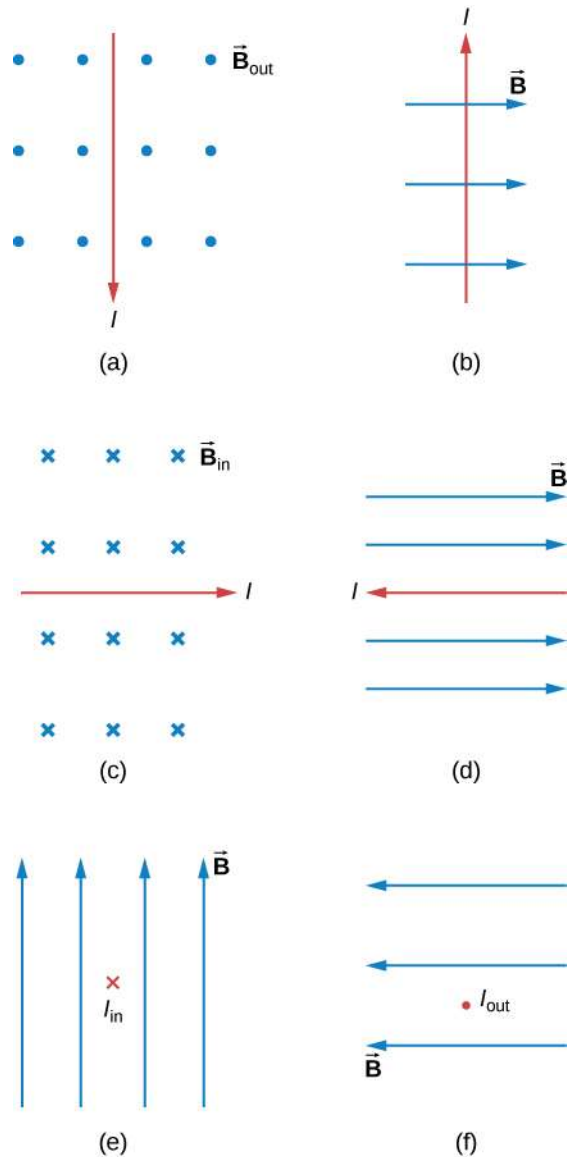
30. (a) What voltage will accelerate electrons to a speed of 6.00×10^{-7} m/s? (b) Find the radius of curvature of the path of a proton accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

31. An alpha-particle $\left(m = 6.64 \times 10^{-27}\text{ kg}, q = 3.2 \times 10^{-19}\text{ C}\right)$ travels in a circular path of radius 25 cm in a uniform magnetic field of magnitude 1.5 T. (a) What is the speed of the particle? (b) What is the kinetic energy in electron-volts? (c) Through what potential difference must the particle be accelerated in order to give it this kinetic energy?

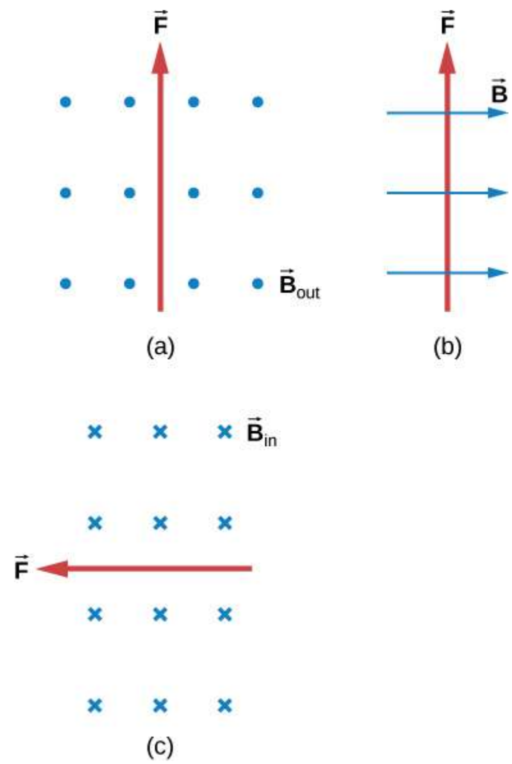
32. A particle of charge q and mass m is accelerated from rest through a potential difference V, after which it encounters a uniform magnetic field B. If the particle moves in a plane perpendicular to B, what is the radius of its circular orbit?

11.4 Magnetic Force on a Current-Carrying Conductor

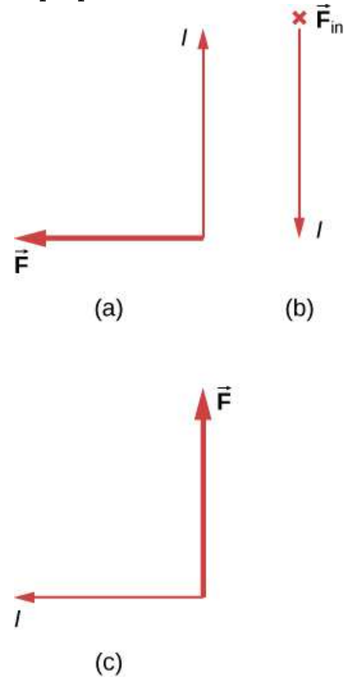
33. What is the direction of the magnetic force on the current in each of the six cases?



34. What is the direction of a current that experiences the magnetic force shown in each of the three cases, assuming the current runs perpendicular to $\vec{\mathbf{B}}$?



35. What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases, assuming $\vec{\mathbf{B}}$ is perpendicular to I?



36. (a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to Earth's $3.0 \times 10^{-5}\,\mathrm{T}$ field? (b) What is the direction of the force if the current is straight up and Earth's field direction is due north, parallel to the ground?

37. (a) A dc power line for a light-rail system carries 1000 A at an angle of 30.0° to Earth's 5.0×10^{-5} T field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

38. A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

11.5 Force and Torque on a Current Loop

39. (a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?

40. (a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is 10.9°?

41. Find the current through a loop needed to create a maximum torque of $9.0 \text{ N} \cdot \text{m}$. The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

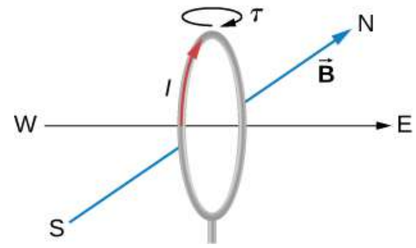
42. Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of 300 N · m if the loop is carrying 25.0 A.

43. Since the equation for torque on a current-carrying loop is $\tau = NIAB \sin \theta$, the units of N · m must equal units of A · m^2 T. Verify this.

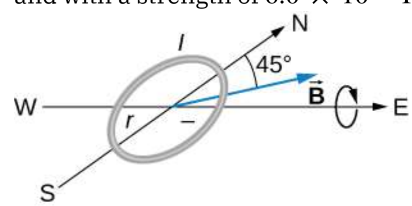
44. (a) At what angle θ is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?

45. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^{4} A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

46. (a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. Earth's field here is due north, parallel to the ground, with a strength of 3.0×10^{-5} T. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?



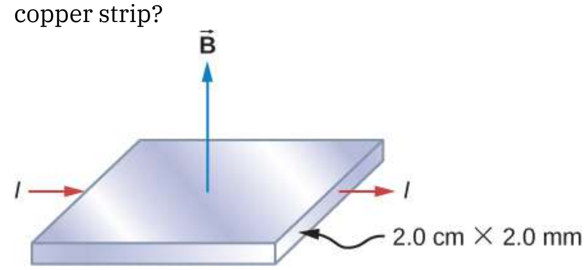
47. Repeat the previous problem, but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where Earth's field is north, but at an angle 45.0° below the horizontal and with a strength of 6.0×10^{-5} T.



11.6 The Hall Effect

48. A strip of copper is placed in a uniform magnetic field of magnitude 2.5 T. The Hall electric field is measured to be 1.5×10^{-3} V/m. (a) What is the drift speed of the conduction electrons? (b) Assuming that n = 8.0×10^{28} electrons per cubic meter and that the cross-sectional area of the strip is $5.0 \times 10^{-6} \text{m}^2$, calculate the current in the strip. (c) What is the Hall coefficient 1/nq?

49. The cross-sectional dimensions of the copper strip shown are 2.0 cm by 2.0 mm. The strip carries a current of 100 A, and it is placed in a magnetic field of magnitude B = 1.5 T. What are the value and polarity of the Hall potential in the copper strip?



50. The magnitudes of the electric and magnetic fields in a velocity selector are 1.8×10^{5} V/m and 0.080 T, respectively. (a) What speed must a proton have to pass through the selector? (b) Also calculate the speeds required for an alpha-particle and a singly ionized $_{s}O^{16}$ atom to pass through the selector.

51. A charged particle moves through a velocity selector at constant velocity. In the selector, $E = 1.0 \times 10^4$ N/C and $B = 0.250$ T. When the electric field is turned off, the charged particle travels in a circular path of radius 3.33 mm. Determine the charge-to-mass ratio of the particle.

52. A Hall probe gives a reading of $1.5\mu V$ for a current of 2 A when it is placed in a magnetic field of 1 T. What is the magnetic field in a region where the reading is $2\mu V$ for 1.7 A of current?

11.7 Applications of Magnetic Forces and Fields

53. A physicist is designing a cyclotron to accelerate protons to one-tenth the speed of light. The magnetic field will have a strength of 1.5 T. Determine (a) the rotational period of the circulating protons and (b) the maximum radius of the protons' orbit.

54. The strengths of the fields in the velocity selector of a Bainbridge mass spectrometer are $B = 0.500$ T and $E = 1.2 \times 10^5$ V/m, and the strength of the magnetic field that separates the ions is $B_o = 0.750$ T. A stream of singly charged Li ions is found to bend in a circular arc of radius 2.32 cm. What is the mass of the Li ions?

55. The magnetic field in a cyclotron is 1.25 T, and the maximum orbital radius of the circulating protons is 0.40 m. (a) What is the kinetic energy of the protons when they are ejected from the cyclotron? (b) What is this energy in MeV? (c) Through what potential difference would a proton have to be accelerated to acquire this kinetic energy? (d) What is the period of the voltage source used to accelerate the protons? (e) Repeat the calculations for alpha-particles.

56. A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is 2.66×10^{-26} kg, and they are singly charged and travel at 5.00×10^6 m/s in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

57. (a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are 3.90×10^{-25} kg and 3.95×10^{-25} kg, respectively, and they travel at 3.0×10^5 m/s in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

Additional Problems

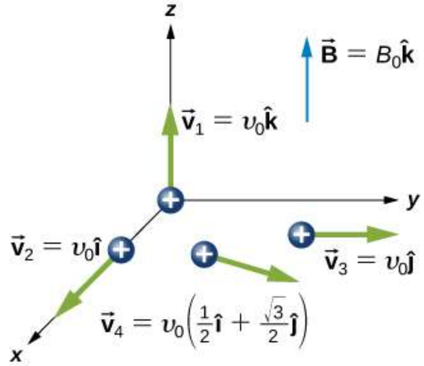
58. Calculate the magnetic force on a hypothetical particle of charge 1.0×10^{-19} C moving with a velocity of $6.0 \times 10^4 \hat{\mathbf{i}}$ m/s in a magnetic field of $1.2\hat{\mathbf{k}}$T.

59. Repeat the previous problem with a new magnetic field of $(0.4\hat{\mathbf{i}} + 1.2\hat{\mathbf{k}})$T.

60. An electron is projected into a uniform magnetic field $(0.5\hat{\mathbf{i}} + 0.8\hat{\mathbf{k}})$T with a velocity of $(3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}) \times 10^6$ m/s. What is the magnetic force on the electron?

61. The mass and charge of a water droplet are 1.0×10^{-4} g and 2.0×10^{-8} C, respectively. If the droplet is given an initial horizontal velocity of $5.0 \times 10^5 \hat{\mathbf{i}}$ m/s, what magnetic field will keep it moving in this direction? Why must gravity be considered here?

62. Four different proton velocities are given. For each case, determine the magnetic force on the proton in terms of e, v_0, and B_0.



63. An electron of kinetic energy 2000 eV passes between parallel plates that are 1.0 cm apart and kept at a potential difference of 300 V. What is the strength of the uniform magnetic field B that will allow the electron to travel undeflected through the plates? Assume E and B are perpendicular.

64. An alpha-particle $\left(m = 6.64 \times 10^{-27}\,\text{kg},\ q = 3.2 \times 10^{-19}\,\text{C}\right)$ moving with a velocity $\vec{\mathbf{v}} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{k}}) \times 10^6\,\text{m/s}$ enters a region where $\vec{\mathbf{E}} = (5.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}}) \times 10^4$ V/m and $\vec{\mathbf{B}} = (1.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{k}}) \times 10^{-2}\,\text{T}$. What is the initial force on it?

65. An electron moving with a velocity $\vec{\mathbf{v}} = \left(4.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{k}}\right) \times 10^6\,\text{m/s}$ enters a region where there is a uniform electric field and a uniform magnetic field. The magnetic field is given by $\vec{B} = \left(1.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}} + 4.0\hat{\mathbf{k}}\right) \times 10^{-2}\,\text{T}$. If the electron travels through a region without being deflected, what is the electric field?

66. At a particular instant, an electron is traveling west to east with a kinetic energy of 10 keV. Earth's magnetic field has a horizontal component of $1.8 \times 10^{-5}\,\text{T}$ north and a vertical component of $5.0 \times 10^{-5}\,\text{T}$ down. (a) What is the path of the electron? (b) What is the radius of curvature of the path?

67. What is the (a) path of a proton and (b) the magnetic force on the proton that is traveling west to east with a kinetic energy of 10 keV in Earth's magnetic field that has a horizontal component of 1.8 x 10^{-5} T north and a vertical component of 5.0 x 10^{-5} T down?

68. What magnetic field is required in order to confine a proton moving with a speed of $4.0 \times 10^6\,\text{m/s}$ to a circular orbit of radius 10 cm?

69. An electron and a proton move with the same speed in a plane perpendicular to a uniform magnetic field. Compare the radii and periods of their orbits.

70. A proton and an alpha-particle have the same kinetic energy and both move in a plane perpendicular to a uniform magnetic field. Compare the periods of their orbits.

71. A singly charged ion takes $2.0 \times 10^{-3}\,\text{s}$ to complete eight revolutions in a uniform magnetic field of magnitude $2.0 \times 10^{-2}\,\text{T}$. What is the mass of the ion?

72. A particle moving downward at a speed of $6.0 \times 10^6\,\text{m/s}$ enters a uniform magnetic field that is horizontal and directed from east to west. (a) If the particle is deflected initially to the north in a circular arc, is its charge positive or negative? (b) If B = 0.25 T and the charge-to-mass ratio (q/m) of the particle is $4.0 \times 10^7\,\text{C/kg}$, what is the radius of the path? (c) What is the speed of the particle after it has moved in the field for $1.0 \times 10^{-5}\,\text{s}$? for 2.0 s?

73. A proton, deuteron, and an alpha-particle are all accelerated from rest through the same potential difference. They then enter the same magnetic field, moving perpendicular to it. Compute the ratios of the radii of their circular paths. Assume that $m_d = 2m_p$ and $m_\alpha = 4m_p$.

74. A singly charged ion is moving in a uniform magnetic field of $7.5 \times 10^{-2}\,\text{T}$ completes 10 revolutions in $3.47 \times 10^{-4}\,\text{s}$. Identify the ion.

75. Two particles have the same linear momentum, but particle A has four times the charge of particle B. If both particles move in a plane perpendicular to a uniform magnetic field, what is the ratio R_A/R_B of the radii of their circular orbits?

76. A uniform magnetic field of magnitude $\boldsymbol{B}$ is directed parallel to the z-axis. A proton enters the field with a velocity $\vec{\mathbf{v}} = (4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \times 10^6\,\text{m/s}$ and travels in a helical path with a radius of 5.0 cm. (a) What is the value of $\boldsymbol{B}$? (b) What is the time required for one trip around the helix? (c) Where is the proton $5.0 \times 10^{-7}\,\text{s}$ after entering the field?

77. An electron moving along the $+x$ -axis at $5.0 \times 10^6\,\text{m/s}$ enters a magnetic field that makes a 75° angle with the x-axis of magnitude 0.20 T. Calculate the (a) pitch and (b) radius of the trajectory.

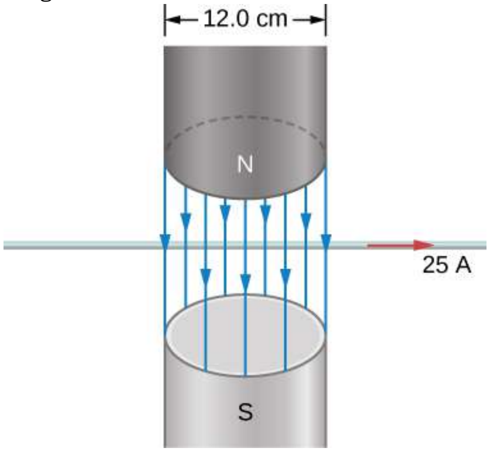
78. (a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of 60° with Earth's $5.5 \times 10^{-5}\,\text{T}$ field. What is the current when the wire experiences a force of $7.0 \times 10^{-3}\,\text{N}$? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

79. (a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N? (b) What is the force on the wire if it is rotated to make an angle of 90° with the field?

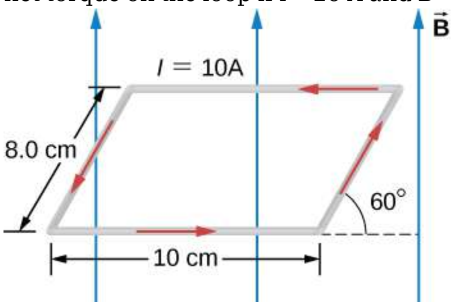
80. A 1.0-m-long segment of wire lies along the x-axis and carries a current of 2.0 A in the positive x-direction. Around the wire is the magnetic field of $\left(3.0\hat{\mathbf{i}} \times 4.0\hat{\mathbf{k}}\right) \times 10^{-3}\text{T}$. Find the magnetic force on this segment.

81. A 5.0-m section of a long, straight wire carries a current of 10 A while in a uniform magnetic field of magnitude $8.0 \times 10^{-3}\text{T}$. Calculate the magnitude of the force on the section if the angle between the field and the direction of the current is (a) 45°; (b) 90°; (c) 0°; or (d) 180°.

82. An electromagnet produces a magnetic field of magnitude 1.5 T throughout a cylindrical region of radius 6.0 cm. A straight wire carrying a current of 25 A passes through the field as shown in the accompanying figure. What is the magnetic force on the wire?



83. The current loop shown in the accompanying figure lies in the plane of the page, as does the magnetic field. Determine the net force and the net torque on the loop if I = 10 A and B = 1.5 T.



84. A circular coil of radius 5.0 cm is wound with five turns and carries a current of 5.0 A. If the coil is placed in a uniform magnetic field of strength 5.0 T, what is the maximum torque on it?

85. A circular coil of wire of radius 5.0 cm has 20 turns and carries a current of 2.0 A. The coil lies in a magnetic field of magnitude 0.50 T that is directed parallel to the plane of the coil. (a) What is the magnetic dipole moment of the coil? (b) What is the torque on the coil?

86. A current-carrying coil in a magnetic field experiences a torque that is 75% of the maximum possible torque. What is the angle between the magnetic field and the normal to the plane of the coil?

87. A 4.0-cm by 6.0-cm rectangular current loop carries a current of 10 A. What is the magnetic dipole moment of the loop?

88. A circular coil with 200 turns has a radius of 2.0 cm. (a) What current through the coil results in a magnetic dipole moment of 3.0 Am^2? (b) What is the maximum torque that the coil will experience in a uniform field of strength $5.0 \times 10^{-2}\text{T}$? (c) If the angle between μ and B is 45°, what is the magnitude of the torque on the coil? (d) What is the magnetic potential energy of coil for this orientation?

89. The current through a circular wire loop of radius 10 cm is 5.0 A. (a) Calculate the magnetic dipole moment of the loop. (b) What is the torque on the loop if it is in a uniform 0.20-T magnetic field such that μ and B are directed at 30° to each other? (c) For this position, what is the potential energy of the dipole?

90. A wire of length 1.0 m is wound into a single-turn planar loop. The loop carries a current of 5.0 A, and it is placed in a uniform magnetic field of strength 0.25 T. (a) What is the maximum torque that the loop will experience if it is square? (b) If it is circular? (c) At what angle relative to B would the normal to the circular coil have to be oriented so that the torque on it would be the same as the maximum torque on the square coil?

91. Consider an electron rotating in a circular orbit of radius r. Show that the magnitudes of the magnetic dipole moment μ and the angular momentum L of the electron are related by:

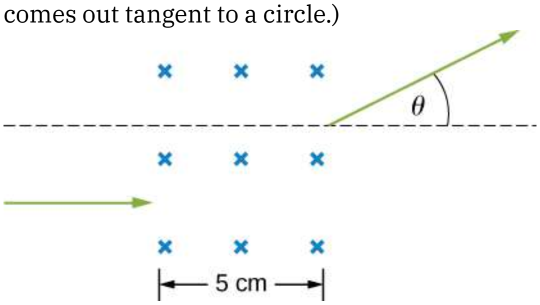
$$\frac{\mu}{L} = \frac{e}{2m}.$$

92. The Hall effect is to be used to find the sign of charge carriers in a semiconductor sample. The probe is placed between the poles of a magnet so that magnetic field is pointed up. A current is passed through a rectangular sample placed horizontally. As current is passed through the sample in the east direction, the north side of the sample is found to be at a higher potential than the south side. Decide if the number density of charge carriers is positively or negatively charged.
93. The density of charge carriers for copper is 8.47×10^{28} electrons per cubic meter. What will be the Hall voltage reading from a probe made up of $3\text{ cm} \times 2\text{ cm} \times 1\text{ cm}$ $(\text{L} \times \text{W} \times \text{T})$ copper plate when a current of 1.5 A is passed through it in a magnetic field of 2.5 T perpendicular to the $3\text{ cm} \times 2\text{ cm}$.
94. The Hall effect is to be used to find the density of charge carriers in an unknown material. A Hall voltage 40 μV for 3-A current is observed in a 3-T magnetic field for a rectangular sample with length 2 cm, width 1.5 cm, and height 0.4 cm. Determine the density of the charge carriers.
95. Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)
96. A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of 4.0×10^6 m/s? (b) What is the voltage between the plates if they are separated by 1.00 cm?
97. Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.
98. **Unreasonable results** To construct a non-mechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?
99. **Unreasonable results** A charged particle having mass 6.64×10^{-27} kg (that of a helium atom) moving at 8.70×10^5 m/s perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?
100. **Unreasonable results** An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to Earth's 5.00×10^{-5} T field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?
101. **Unreasonable results** Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

Challenge Problems

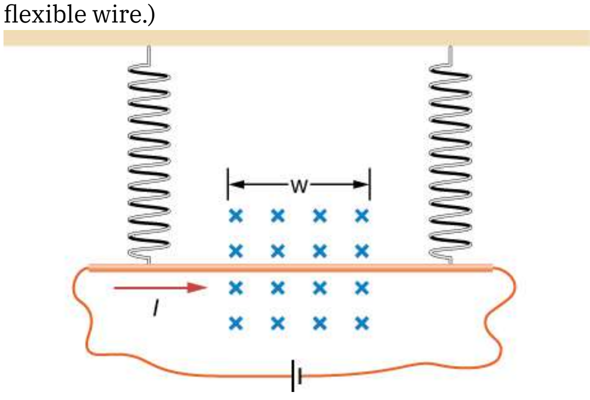
102. A particle of charge $+q$ and mass m moves with velocity $\vec{\mathbf{v}}_0$ pointed in the $+y$-direction as it crosses the x-axis at $x = R$ at a particular time. There is a negative charge $-Q$ fixed at the origin, and there exists a uniform magnetic field $\vec{\mathbf{B}}_0$ pointed in the $+z$-direction. It is found that the particle describes a circle of radius R about $-Q$. Find $\vec{\mathbf{B}}_0$ in terms of the given quantities.
103. A proton of speed $v = 6 \times 10^5$ m/s enters a region of uniform magnetic field of $B = 0.5$ T at an angle of $q = 30°$ to the magnetic field. In the region of magnetic field proton describes a helical path with radius R and pitch p (distance between loops). Find R and p.

104. A particle's path is bent when it passes through a region of non-zero magnetic field although its speed remains unchanged. This is very useful for "beam steering" in particle accelerators. Consider a proton of speed 4×10^6 m/s entering a region of uniform magnetic field 0.2 T over a 5-cm-wide region. Magnetic field is perpendicular to the velocity of the particle. By how much angle will the path of the proton be bent? (Hint: The particle comes out tangent to a circle.)

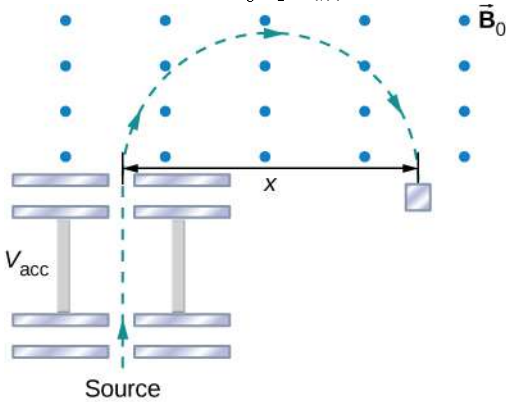


105. In a region a non-uniform magnetic field exists such that $B_x = 0, B_y = 0, \textit{ and } B_z = ax$, where *a* is a constant. At some time *t*, a wire of length *L* is carrying a current *I* is located along the *x*-axis from origin to *x* = *L*. Find the magnetic force on the wire at this instant in time.

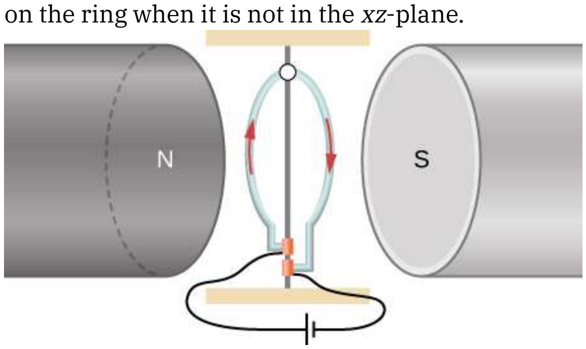
106. A copper rod of mass *m* and length *L* is hung from the ceiling using two springs of spring constant *k*. A uniform magnetic field of magnitude B_0 pointing perpendicular to the rod and spring (coming into the page in the figure) exists in a region of space covering a length *w* of the copper rod. The ends of the rod are then connected by flexible copper wire across the terminals of a battery of voltage *V*. Determine the change in the length of the springs when *a* current *I* runs through the copper rod in the direction shown in figure. (Ignore any force by the flexible wire.)



107. The accompanied figure shows an arrangement for measuring mass of ions by an instrument called the mass spectrometer. An ion of mass *m* and charge $+q$ is produced essentially at rest in source *S*, a chamber in which a gas discharge is taking place. The ion is accelerated by a potential difference V_{acc} and allowed to enter a region of constant magnetic field $\vec{\mathbf{B}}_0$. In the uniform magnetic field region, the ion moves in a semicircular path striking a photographic plate at a distance *x* from the entry point. Derive a formula for mass m in terms of B_0, *q*, V_{acc}, and *x*.



108. A wire is made into a circular shape of radius *R* and pivoted along a central support. The two ends of the wire are touching a brush that is connected to a dc power source. The structure is between the poles of a magnet such that we can assume there is a uniform magnetic field on the wire. In terms of a coordinate system with origin at the center of the ring, magnetic field is $B_x = B_0, B_y = B_z = 0$, and the ring rotates about the *z*-axis. Find the torque on the ring when it is not in the *xz*-plane.



109. A long-rigid wire lies along the *x*-axis and carries a current of 2.5 A in the positive *x*-direction. Around the wire is the magnetic field $\vec{\mathbf{B}} = 2.0\hat{\mathbf{i}} + 5.0x^2\hat{\mathbf{j}}$, with *x* in meters and *B* in millitesla. Calculate the magnetic force on the segment of wire between *x* = 2.0 m and *x* = 4.0 m.

110. A circular loop of wire of area $10\ \text{cm}^2$ carries a current of 25 A. At a particular instant, the loop lies in the *xy*-plane and is subjected to a magnetic field $\vec{\mathbf{B}} = \left(2.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}} + 8.0\hat{\mathbf{k}}\right) \times 10^{-3}\ \text{T}$. As viewed from above the *xy*-plane, the current is circulating clockwise. (a) What is the magnetic dipole moment of the current loop? (b) At this instant, what is the magnetic torque on the loop?