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University

Phys-ics

Volume 2

CHAPTER 12
Sources of Magnetic Fields

Figure 12.1 An external hard drive attached to a computer works by magnetically encoding information that can be stored or retrieved quickly. A key idea in the development of digital devices is the ability to produce and use magnetic fields in this way. (credit: modification of work by "Miss Karen"/Flickr)

Chapter Outline

INTRODUCTION In the preceding chapter, we saw that a moving charged particle produces a magnetic field. This connection between electricity and magnetism is exploited in electromagnetic devices, such as a computer hard drive. In fact, it is the underlying principle behind most of the technology in modern society, including telephones, television, computers, and the internet.

In this chapter, we examine how magnetic fields are created by arbitrary distributions of electric current, using the Biot-Savart law. Then we look at how current-carrying wires create magnetic fields and deduce the forces that arise between two current-carrying wires due to these magnetic fields. We also study the torques produced by the magnetic fields of current loops. We then generalize these results to an important law of electromagnetism, called Ampère's law.

We examine some devices that produce magnetic fields from currents in geometries based on loops, known as solenoids and toroids. Finally, we look at how materials behave in magnetic fields and categorize materials based on their responses to magnetic fields.

12.1 The Biot-Savart Law

Learning Objectives

By the end of this section, you will be able to:

- Explain how to derive a magnetic field from an arbitrary current in a line segment
- Calculate magnetic field from the Biot-Savart law in specific geometries, such as a current in a line and a current in a circular arc

We have seen that mass produces a gravitational field and also interacts with that field. Charge produces an electric field and also interacts with that field. Since moving charge (that is, current) interacts with a magnetic field, we might expect that it also creates that field—and it does.

The equation used to calculate the magnetic field produced by a current is known as the Biot-Savart law. It is an empirical law named in honor of two scientists who investigated the interaction between a straight, current-carrying wire and a permanent magnet. This law enables us to calculate the magnitude and direction of the magnetic field produced by a current in a wire. The **Biot-Savart law** states that at any point P (Figure 12.2), the magnetic field $d\vec{\mathbf{B}}$ due to an element $d\vec{\mathbf{l}}$ of a current-carrying wire is given by

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}. \qquad 12.1$$

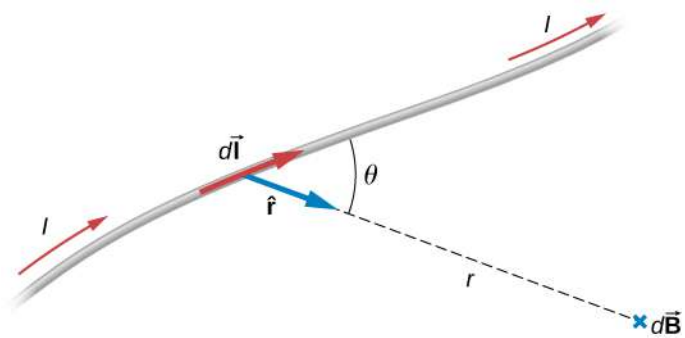


Figure 12.2 A current element $I d\vec{\mathbf{l}}$ produces a magnetic field at point P given by the Biot-Savart law.

The constant μ_0 is known as the **permeability of free space** and is exactly

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\cdot m/A} \qquad 12.2$$

in the SI system. The infinitesimal wire segment $d\vec{\mathbf{l}}$ is in the same direction as the current I (assumed positive), r is the distance from $d\vec{\mathbf{l}}$ to P and $\hat{\mathbf{r}}$ is a unit vector that points from $d\vec{\mathbf{l}}$ to P, as shown in the figure.

The direction of $d\vec{\mathbf{B}}$ is determined by applying the right-hand rule to the vector product $d\vec{\mathbf{l}} \times \hat{\mathbf{r}}$. The magnitude of $d\vec{\mathbf{B}}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2} \qquad 12.3$$

where θ is the angle between $d\vec{\mathbf{l}}$ and $\hat{\mathbf{r}}$. Notice that if $\theta = 0$, then $d\vec{\mathbf{B}} = \vec{\mathbf{0}}$. The field produced by a current element $I d\vec{\mathbf{l}}$ has no component parallel to $d\vec{\mathbf{l}}$.

The magnetic field due to a finite length of current-carrying wire is found by integrating Equation 12.3 along the wire, giving us the usual form of the Biot-Savart law.

Biot-Savart law

The magnetic field $\vec{\mathbf{B}}$ due to an element $d\vec{\mathbf{l}}$ of a current-carrying wire is given by

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I\, d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}. \quad (12.4)$$

Since this is a vector integral, contributions from different current elements may not point in the same direction. Consequently, the integral is often difficult to evaluate, even for fairly simple geometries. The following strategy may be helpful.

PROBLEM-SOLVING STRATEGY

Solving Biot-Savart Problems

To solve Biot-Savart law problems, the following steps are helpful:

1. Identify that the Biot-Savart law is the chosen method to solve the given problem. If there is symmetry in the problem comparing $\vec{\mathbf{B}}$ and $\mathbf{d}\vec{\mathbf{l}}$, Ampère's law may be the preferred method to solve the question.
2. Draw the current element length $d\vec{\mathbf{l}}$ and the unit vector $\hat{\mathbf{r}}$, noting that $d\vec{\mathbf{l}}$ points in the direction of the current and $\hat{\mathbf{r}}$ points from the current element toward the point where the field is desired.
3. Calculate the cross product $d\vec{\mathbf{l}} \times \hat{\mathbf{r}}$. The resultant vector gives the direction of the magnetic field according to the Biot-Savart law.
4. Use Equation 12.4 and substitute all given quantities into the expression to solve for the magnetic field. Note all variables that remain constant over the entire length of the wire may be factored out of the integration.
5. Use the right-hand rule to verify the direction of the magnetic field produced from the current or to write down the direction of the magnetic field if only the magnitude was solved for in the previous part.

EXAMPLE 12.1

Calculating Magnetic Fields of Short Current Segments

A short wire of length 1.0 cm carries a current of 2.0 A in the vertical direction (Figure 12.3). The rest of the wire is shielded so it does not add to the magnetic field produced by the wire. Calculate the magnetic field at point *P*, which is 1 meter from the wire in the *x*-direction.

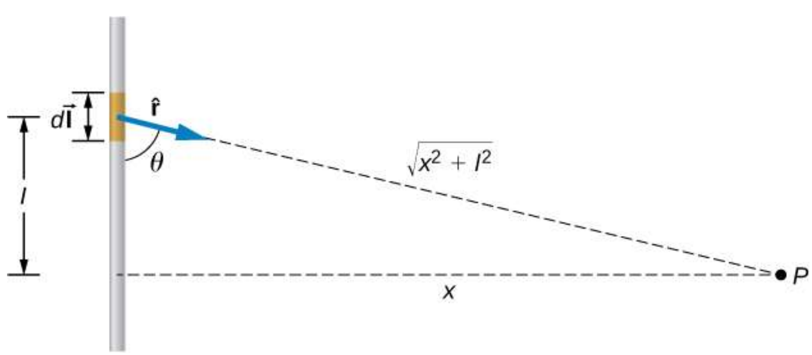


Figure 12.3 A small line segment carries a current *I* in the vertical direction. What is the magnetic field at a distance *x* from the segment?

Strategy

We can determine the magnetic field at point P using the Biot-Savart law. Since the current segment is much smaller than the distance x, we can drop the integral from the expression. The integration is converted back into a summation, but only for small dl, which we now write as Δl. Another way to think about it is that each of the radius values is nearly the same, no matter where the current element is on the line segment, if Δl is small compared to x. The angle θ is calculated using a tangent function. Using the numbers given, we can calculate the magnetic field at P.

Solution

The angle between $\Delta \vec{\mathbf{l}}$ and $\hat{\mathbf{r}}$ is calculated from trigonometry, knowing the distances l and x from the problem:

$$\theta = \tan^{-1}\left(\frac{1\text{ m}}{0.01\text{ m}}\right) = 89.4°.$$

The magnetic field at point P is calculated by the Biot-Savart law:

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta l\sin\theta}{r^2} = (1 \times 10^{-7}\text{T}\cdot\text{m/A})\left(\frac{2\text{ A}(0.01\text{ m})\sin(89.4°)}{(1\text{ m})^2}\right) = 2.0 \times 10^{-9}\text{T}.$$

From the right-hand rule and the Biot-Savart law, the field is directed into the page.

Significance

This approximation is only good if the length of the line segment is very small compared to the distance from the current element to the point. If not, the integral form of the Biot-Savart law must be used over the entire line segment to calculate the magnetic field.

CHECK YOUR UNDERSTANDING 12.1

Using Example 12.1, at what distance would P have to be to measure a magnetic field half of the given answer?

EXAMPLE 12.2

Calculating Magnetic Field of a Circular Arc of Wire

A wire carries a current I in a circular arc with radius R swept through an arbitrary angle θ (Figure 12.4). Calculate the magnetic field at the center of this arc at point P.

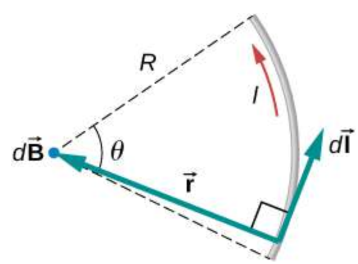


Figure 12.4 A wire segment carrying a current I. The path $d\vec{\mathbf{l}}$ and radial direction $\hat{\mathbf{r}}$ are indicated.

Strategy

We can determine the magnetic field at point P using the Biot-Savart law. The radial and path length directions are always at a right angle, so the cross product turns into multiplication. We also know that the distance along the path dl is related to the radius times the angle θ (in radians). Then we can pull all constants out of the integration and solve for the magnetic field.

Solution

The Biot-Savart law starts with the following equation:

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}.$$

As we integrate along the arc, all the contributions to the magnetic field are in the same direction (out of the page), so we can work with the magnitude of the field. The cross product turns into multiplication because the path dl and the radial direction are perpendicular. We can also substitute the arc length formula, $dl = rd\theta$:

$$B = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{Ir\, d\theta}{r^2}.$$

The current and radius can be pulled out of the integral because they are the same regardless of where we are on the path. This leaves only the integral over the angle,

$$B = \frac{\mu_0 I}{4\pi r} \int_{\text{wire}} d\theta.$$

The angle varies on the wire from 0 to θ; hence, the result is

$$B = \frac{\mu_0 I \theta}{4\pi r}.$$

Significance

The direction of the magnetic field at point P is determined by the right-hand rule, as shown in the previous chapter. If there are other wires in the diagram along with the arc, and you are asked to find the net magnetic field, find each contribution from a wire or arc and add the results by superposition of vectors. Make sure to pay attention to the direction of each contribution. Also note that in a symmetric situation, like a straight or circular wire, contributions from opposite sides of point P cancel each other.

CHECK YOUR UNDERSTANDING 12.2

The wire loop forms a full circle of radius R and current I. What is the magnitude of the magnetic field at the center?

12.2 Magnetic Field Due to a Thin Straight Wire

Learning Objectives

By the end of this section, you will be able to:

- Explain how the Biot-Savart law is used to determine the magnetic field due to a thin, straight wire.
- Determine the dependence of the magnetic field from a thin, straight wire based on the distance from it and the current flowing in the wire.
- Sketch the magnetic field created from a thin, straight wire by using the second right-hand rule.

How much current is needed to produce a significant magnetic field, perhaps as strong as Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted in Chapter 28 that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire? We can use the Biot-Savart law to answer all of these questions, including determining the magnetic field of a long straight wire.

Figure 12.5 shows a section of an infinitely long, straight wire that carries a current I. What is the magnetic field at a point P, located a distance R from the wire?

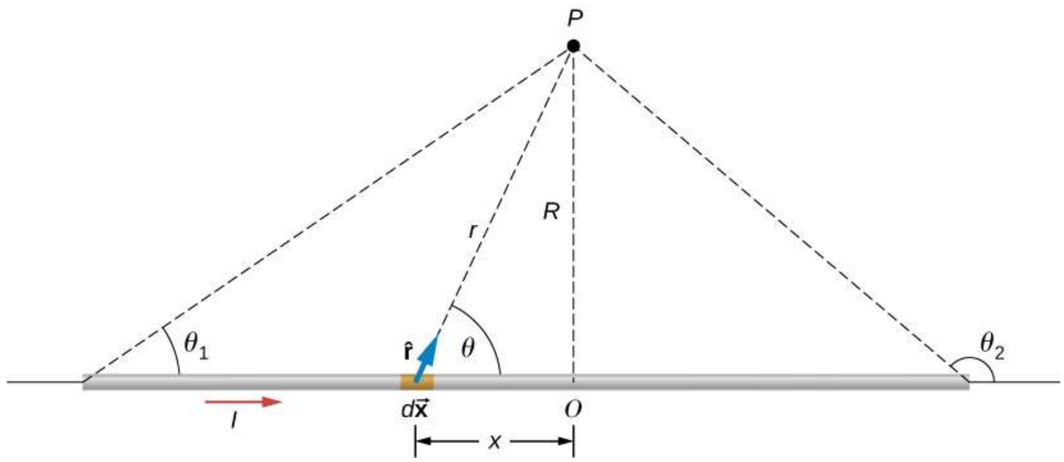


Figure 12.5 A section of a thin, straight current-carrying wire. The independent variable θ has the limits θ_1 and θ_2.

Let's begin by considering the magnetic field due to the current element $I\, d\vec{\mathbf{x}}$ located at the position x. Using the right-hand rule 1 from the previous chapter, $d\vec{\mathbf{x}} \times \hat{\mathbf{r}}$ points out of the page for any element along the wire. At point P, therefore, the magnetic fields due to all current elements have the same direction. This means that we can calculate the net field there by evaluating the scalar sum of the contributions of the elements. With $\left|d\vec{\mathbf{x}} \times \hat{\mathbf{r}}\right| = (dx)(1)\sin\theta$, we have from the Biot-Savart law

$$B = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I \sin\theta\, dx}{r^2}. \quad (12.5)$$

The wire is symmetrical about point O, so we can set the limits of the integration from zero to infinity and double the answer, rather than integrate from negative infinity to positive infinity. Based on the picture and geometry, we can write expressions for r and $\sin\theta$ in terms of x and R, namely:

$$\begin{aligned} r &= \sqrt{x^2 + R^2} \\ \sin\theta &= \frac{R}{\sqrt{x^2+R^2}}. \end{aligned}$$

Substituting these expressions into Equation 12.5, the magnetic field integration becomes

$$B = \frac{\mu_0 I}{2\pi} \int_0^\infty \frac{R\, dx}{(x^2 + R^2)^{3/2}}. \quad (12.6)$$

Evaluating the integral yields

$$B = \frac{\mu_0 I}{2\pi R} \left[\frac{x}{(x^2 + R^2)^{1/2}} \right]_0^\infty. \quad (12.7)$$

Substituting the limits gives us the solution

$$B = \frac{\mu_0 I}{2\pi R}. \quad (12.8)$$

The magnetic field lines of the infinite wire are circular and centered at the wire (Figure 12.6), and they are identical in every plane perpendicular to the wire. Since the field decreases with distance from the wire, the spacing of the field lines must increase correspondingly with distance. The direction of this magnetic field may be found with a second form of the right-hand rule (illustrated in Figure 12.6). If you hold the wire with your right hand so that your thumb points along the current, then your fingers wrap around the wire in the same sense as $\vec{\mathbf{B}}$.

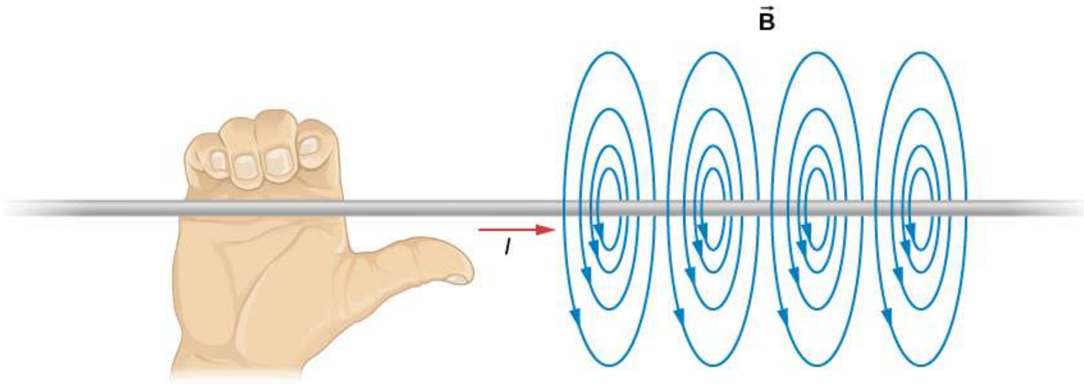


Figure 12.6 Some magnetic field lines of an infinite wire. The direction of $\vec{\mathbf{B}}$ can be found with a form of the right-hand rule.

The direction of the field lines can be observed experimentally by placing several small compass needles on a circle near the wire, as illustrated in Figure 12.7. When there is no current in the wire, the needles align with Earth's magnetic field. However, when a large current is sent through the wire, the compass needles all point tangent to the circle. Iron filings sprinkled on a horizontal surface also delineate the field lines, as shown in Figure 12.7.

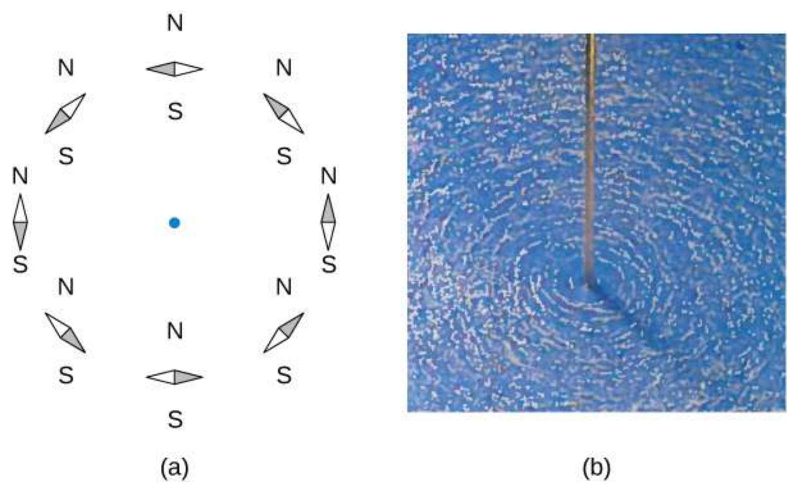


Figure 12.7 The shape of the magnetic field lines of a long wire can be seen using (a) small compass needles and (b) iron filings.

EXAMPLE 12.3

Calculating Magnetic Field Due to Three Wires

Three wires sit at the corners of a square, all carrying currents of 2 amps into the page as shown in Figure 12.8. Calculate the magnitude of the magnetic field at the other corner of the square, point *P*, if the length of each side of the square is 1 cm.

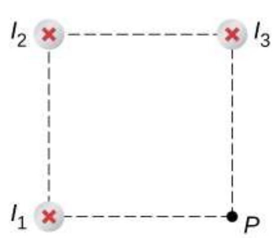


Figure 12.8 Three wires have current flowing into the page. The magnetic field is determined at the fourth corner of the square.

Strategy

The magnetic field due to each wire at the desired point is calculated. The diagonal distance is calculated using the Pythagorean theorem. Next, the direction of each magnetic field's contribution is determined by drawing a circle centered at the point of the wire and out toward the desired point. The direction of the magnetic field contribution from that wire is tangential to the curve. Lastly, working with these vectors, the resultant is calculated.

Solution

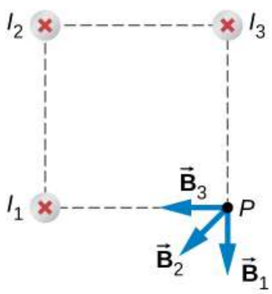
Wires 1 and 3 both have the same magnitude of magnetic field contribution at point *P*:

$$B_1 = B_3 = \frac{\mu_o I}{2\pi R} = \frac{(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(2\text{ A})}{2\pi(0.01\text{ m})} = 4 \times 10^{-5}\,\text{T}.$$

Wire 2 has a longer distance and a magnetic field contribution at point *P* of:

$$B_2 = \frac{\mu_o I}{2\pi R} = \frac{(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(2\text{ A})}{2\pi(0.01414\text{ m})} = 3 \times 10^{-5}\,\text{T}.$$

The vectors for each of these magnetic field contributions are shown.



The magnetic field in the *x*-direction has contributions from wire 3 and the *x*-component of wire 2:

$$B_{\text{net } x} = -4 \times 10^{-5}\,\text{T} - 2.83 \times 10^{-5}\,\text{T}\cos(45°) = -6 \times 10^{-5}\,\text{T}.$$

The *y*-component is similarly the contributions from wire 1 and the *y*-component of wire 2:

$$B_{\text{net } y} = -4 \times 10^{-5}\,\text{T} - 2.83 \times 10^{-5}\,\text{T}\sin(45°) = -6 \times 10^{-5}\,\text{T}.$$

Therefore, the net magnetic field is the resultant of these two components:

$$B_{\text{net}} = \sqrt{B^2_{\text{net } x} + B^2_{\text{net } y}}$$
$$B_{\text{net}} = \sqrt{(-6 \times 10^{-5}\,\text{T})^2 + (-6 \times 10^{-5}\,\text{T})^2}$$
$$B_{\text{net}} = 8 \times 10^{-5}\,\text{T}.$$

Significance

The geometry in this problem results in the magnetic field contributions in the *x*- and *y*-directions having the same magnitude. This is not necessarily the case if the currents were different values or if the wires were located in different positions. Regardless of the numerical results, working on the components of the vectors will yield the resulting magnetic field at the point in need.

CHECK YOUR UNDERSTANDING 12.3

Using Example 12.3, keeping the currents the same in wires 1 and 3, what should the current be in wire 2 to counteract the magnetic fields from wires 1 and 3 so that there is no net magnetic field at point P?

12.3 Magnetic Force between Two Parallel Currents

Learning Objectives

By the end of this section, you will be able to:

- Explain how parallel wires carrying currents can attract or repel each other
- Define the ampere and describe how it is related to current-carrying wires
- Calculate the force of attraction or repulsion between two current-carrying wires

You might expect that two current-carrying wires generate significant forces between them, since ordinary currents produce magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to define the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long, straight, and parallel conductors separated by a distance r can be found by applying what we have developed in the preceding sections. Figure 12.9 shows the wires, their currents, the field created by one wire, and the consequent force the other wire experiences from the created field. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance r is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad \text{12.9}$$

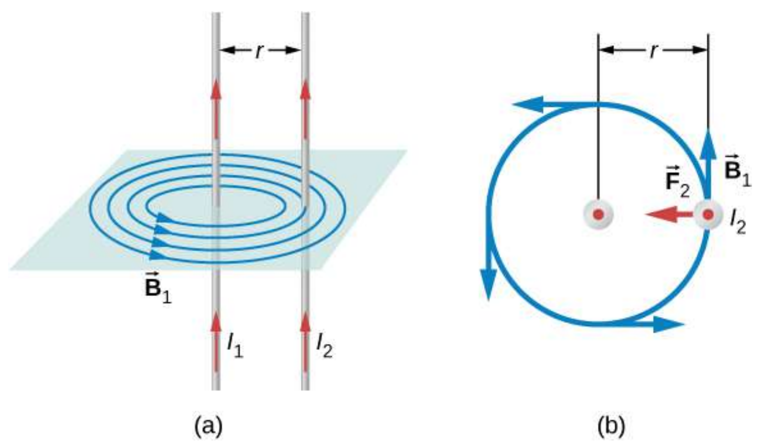


Figure 12.9 (a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by right-hand rule (RHR)-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for wire 1. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

This field is uniform from the wire 1 and perpendicular to it, so the force F_2 it exerts on a length l of wire 2 is given by $F = IlB \sin\theta$ with $\sin\theta = 1$:

$$F_2 = I_2 l B_1. \quad \text{12.10}$$

The forces on the wires are equal in magnitude, so we just write F for the magnitude of F_2. (Note that $\vec{\mathbf{F}}_1 = -\vec{\mathbf{F}}_2$.) Since the wires are very long, it is convenient to think in terms of F/l, the force per unit length. Substituting the expression for B_1 into Equation 12.10 and rearranging terms gives

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}. \quad \text{12.11}$$

The ratio F/l is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and other plasmas. The force exists whether the currents are in wires or not. It is only apparent if the overall charge density is zero; otherwise, the Coulomb repulsion overwhelms the magnetic attraction. In an electric arc, where charges are moving parallel to one another, an attractive force squeezes currents into a smaller tube. In large circuit breakers, such as those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The definition of the ampere is based on the force between current-carrying wires. Note that for long, parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

$$\frac{F}{l} = \frac{\left(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}\right)(1\ \mathrm{A})^2}{(2\pi)(1\ \mathrm{m})} = 2 \times 10^{-7}\ \mathrm{N/m}. \tag{12.12}$$

Since μ_0 is exactly $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ by definition, and because $1\ \mathrm{T} = 1\ \mathrm{N/(A \cdot m)}$, the force per meter is exactly $2 \times 10^{-7}\ \mathrm{N/m}$. This is the basis of the definition of the ampere.

Infinite-length wires are impractical, so in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1\ \mathrm{C} = 1\ \mathrm{A \cdot s}$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

EXAMPLE 12.4

Calculating Forces on Wires

Two wires, both carrying current out of the page, have a current of magnitude 5.0 mA. The first wire is located at (0.0 cm, 3.0 cm) while the other wire is located at (4.0 cm, 0.0 cm) as shown in Figure 12.10. What is the magnetic force per unit length of the first wire on the second and the second wire on the first?

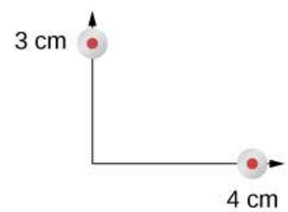


Figure 12.10 Two current-carrying wires at given locations with currents out of the page.

Strategy

Each wire produces a magnetic field felt by the other wire. The distance along the hypotenuse of the triangle between the wires is the radial distance used in the calculation to determine the force per unit length. Since both wires have currents flowing in the same direction, the direction of the force is toward each other.

Solution

The distance between the wires results from finding the hypotenuse of a triangle:

$$r = \sqrt{(3.0\ \mathrm{cm})^2 + (4.0\ \mathrm{cm})^2} = 5.0\ \mathrm{cm}.$$

The force per unit length can then be calculated using the known currents in the wires:

$$\frac{F}{l} = \frac{\left(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}\right)\left(5 \times 10^{-3}\,\mathrm{A}\right)^2}{(2\pi)(5 \times 10^{-2}\,\mathrm{m})} = 1 \times 10^{-10}\ \mathrm{N/m}.$$

The force from the first wire pulls the second wire. The angle between the radius and the x-axis is

$$\theta = \tan^{-1}\left(\frac{3\text{ cm}}{4\text{ cm}}\right) = 36.9°.$$

The unit vector for this is calculated by

$$-\cos(36.9°)\hat{\mathbf{i}} + \sin(36.9°)\hat{\mathbf{j}} = -0.8\hat{\mathbf{i}} + 0.6\hat{\mathbf{j}}.$$

Therefore, the force per unit length from wire one on wire 2 is

$$\frac{\vec{\mathbf{F}}}{l} = (1 \times 10^{-10}\text{ N/m}) \times (-0.8\hat{\mathbf{i}} + 0.6\hat{\mathbf{j}}) = (-8 \times 10^{-11}\hat{\mathbf{i}} + 6 \times 10^{-11}\hat{\mathbf{j}})\text{ N/m}.$$

The force per unit length from wire 2 on wire 1 is the negative of the previous answer:

$$\frac{\vec{\mathbf{F}}}{l} = (8 \times 10^{-11}\hat{\mathbf{i}} - 6 \times 10^{-11}\hat{\mathbf{j}})\text{N/m}.$$

Significance

These wires produced magnetic fields of equal magnitude but opposite directions at each other's locations. Whether the fields are identical or not, the forces that the wires exert on each other are always equal in magnitude and opposite in direction (Newton's third law).

CHECK YOUR UNDERSTANDING 12.4

Two wires, both carrying current out of the page, have a current of magnitude 2.0 mA and 3.0 mA, respectively. The first wire is located at (0.0 cm, 5.0 cm) while the other wire is located at (12.0 cm, 0.0 cm). What is the magnitude of the magnetic force per unit length of the first wire on the second and the second wire on the first?

12.4 Magnetic Field of a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Explain how the Biot-Savart law is used to determine the magnetic field due to a current in a loop of wire at a point along a line perpendicular to the plane of the loop.
- Determine the magnetic field of an arc of current.

The circular loop of Figure 12.11 has a radius R, carries a current I, and lies in the xz-plane. What is the magnetic field due to the current at an arbitrary point P along the axis of the loop?

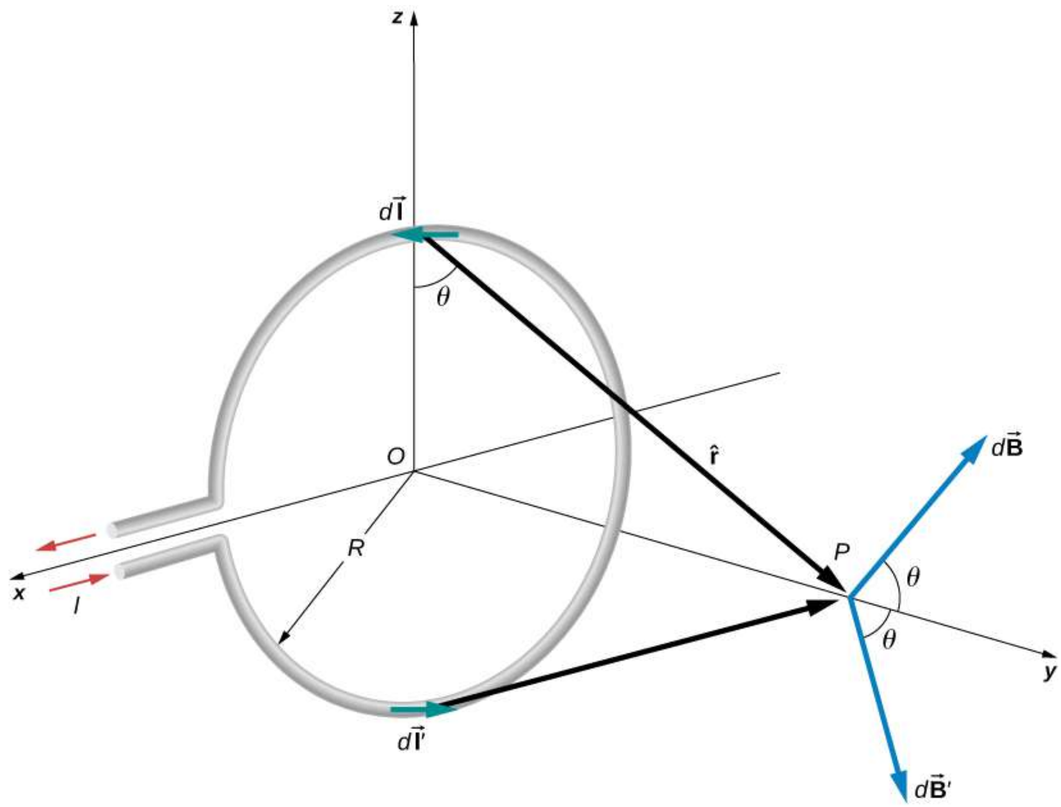


Figure 12.11 Determining the magnetic field at point P along the axis of a current-carrying loop of wire.

We can use the Biot-Savart law to find the magnetic field due to a current. We first consider arbitrary segments on opposite sides of the loop to qualitatively show by the vector results that the net magnetic field direction is along the central axis from the loop. From there, we can use the Biot-Savart law to derive the expression for magnetic field.

Let P be a distance y from the center of the loop. From the right-hand rule, the magnetic field $d\vec{\mathbf{B}}$ at P, produced by the current element $I\,d\vec{\mathbf{l}}$, is directed at an angle θ above the y-axis as shown. Since $d\vec{\mathbf{l}}$ is parallel along the x-axis and $\hat{\mathbf{r}}$ is in the yz-plane, the two vectors are perpendicular, so we have

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\pi/2}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{y^2 + R^2} \quad \textbf{12.13}$$

where we have used $r^2 = y^2 + R^2$.

Now consider the magnetic field $d\vec{\mathbf{B}}'$ due to the current element $I\,d\vec{\mathbf{l}}'$, which is directly opposite $I\,d\vec{\mathbf{l}}$ on the loop. The magnitude of $d\vec{\mathbf{B}}'$ is also given by Equation 12.13, but it is directed at an angle θ below the y-axis. The components of $d\vec{\mathbf{B}}$ and $d\vec{\mathbf{B}}'$ perpendicular to the y-axis therefore cancel, and in calculating the net magnetic field, only the components along the y-axis need to be considered. The components perpendicular to the axis of the loop sum to zero in pairs. Hence at point P:

$$\vec{\mathbf{B}} = \hat{\mathbf{j}}\int_{\text{loop}} dB\,\cos\theta = \hat{\mathbf{j}}\frac{\mu_0 I}{4\pi}\int_{\text{loop}}\frac{\cos\theta\,dl}{y^2 + R^2}. \quad \textbf{12.14}$$

For all elements $d\vec{\mathbf{l}}$ on the wire, y, R, and $\cos\theta$ are constant and are related by

$$\cos\theta = \frac{R}{\sqrt{y^2 + R^2}}.$$

Now from Equation 12.14, the magnetic field at P is

$$\vec{\mathbf{B}} = \hat{\mathbf{j}} \frac{\mu_0 I R}{4\pi (y^2 + R^2)^{3/2}} \int_{\text{loop}} dl = \frac{\mu_0 I R^2}{2 (y^2 + R^2)^{3/2}} \hat{\mathbf{j}} \tag{12.15}$$

where we have used $\int_{\text{loop}} dl = 2\pi R$. As discussed in the previous chapter, the closed current loop is a magnetic dipole of moment $\vec{\mu} = IA\hat{\mathbf{n}}$. For this example, $A = \pi R^2$ and $\hat{\mathbf{n}} = \hat{\mathbf{j}}$, so the magnetic field at P can also be written as

$$\vec{\mathbf{B}} = \frac{\mu_0 \mu \hat{\mathbf{j}}}{2\pi (y^2 + R^2)^{3/2}}. \tag{12.16}$$

By setting $y = 0$ in Equation 12.16, we obtain the magnetic field at the center of the loop:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \hat{\mathbf{j}}. \tag{12.17}$$

This equation becomes $B = \mu_0 n I / (2R)$ for a flat coil of n loops per length. It can also be expressed as

$$\vec{\mathbf{B}} = \frac{\mu_0 \vec{\mu}}{2\pi R^3}. \tag{12.18}$$

If we consider $y \gg R$ in Equation 12.16, the expression reduces to an expression known as the magnetic field from a dipole:

$$\vec{\mathbf{B}} = \frac{\mu_0 \vec{\mu}}{2\pi y^3}. \tag{12.19}$$

The calculation of the magnetic field due to the circular current loop at points off-axis requires rather complex mathematics, so we'll just look at the results. The magnetic field lines are shaped as shown in Figure 12.12. Notice that one field line follows the axis of the loop. This is the field line we just found. Also, very close to the wire, the field lines are almost circular, like the lines of a long straight wire.

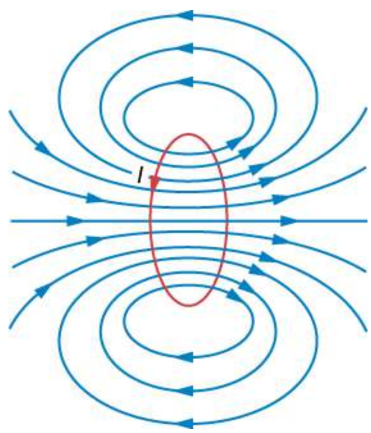


Figure 12.12 Sketch of the magnetic field lines of a circular current loop.

EXAMPLE 12.5

Magnetic Field between Two Loops

Two loops of wire carry the same current of 10 mA, but flow in opposite directions as seen in Figure 12.13. One loop is measured to have a radius of $R = 50$ cm while the other loop has a radius of $2R = 100$ cm. The distance from the first loop to the point where the magnetic field is measured is 0.25 m, and the distance from that point to the second loop is 0.75 m. What is the magnitude of the net magnetic field at point P?

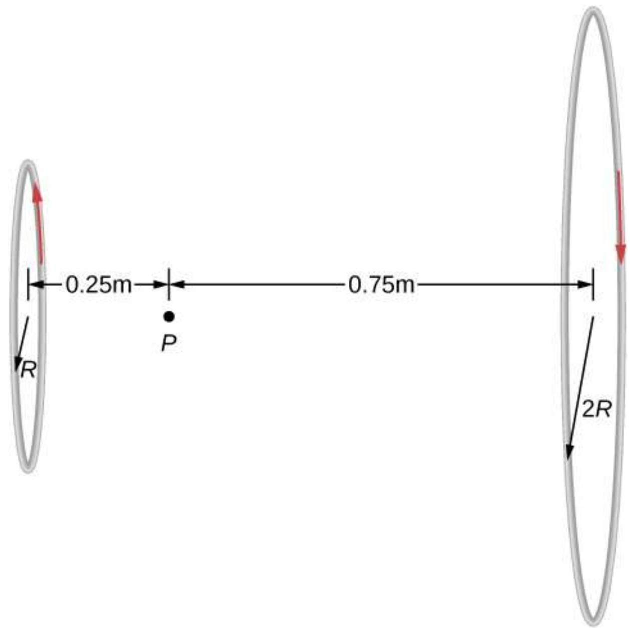


Figure 12.13 Two loops of different radii have the same current but flowing in opposite directions. The magnetic field at point *P* is measured to be zero.

Strategy

The magnetic field at point *P* has been determined in Equation 12.15. Since the currents are flowing in opposite directions, the net magnetic field is the difference between the two fields generated by the coils. Using the given quantities in the problem, the net magnetic field is then calculated.

Solution

Solving for the net magnetic field using Equation 12.15 and the given quantities in the problem yields

$$\begin{aligned} B &= \frac{\mu_0 I R_1{}^2}{2\left(y_1{}^2+R_1{}^2\right)^{3/2}} - \frac{\mu_0 I R_2{}^2}{2\left(y_2{}^2+R_2{}^2\right)^{3/2}} \\ B &= \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.010\,\mathrm{A})(0.5\,\mathrm{m})^2}{2((0.25\,\mathrm{m})^2+(0.5\,\mathrm{m})^2)^{3/2}} - \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.010\,\mathrm{A})(1.0\,\mathrm{m})^2}{2((0.75\,\mathrm{m})^2+(1.0\,\mathrm{m})^2)^{3/2}} \\ B &= 5.77 \times 10^{-9}\,\mathrm{T}\text{ to the right.} \end{aligned}$$

Significance

Helmholtz coils typically have loops with equal radii with current flowing in the same direction to have a strong uniform field at the midpoint between the loops. A similar application of the magnetic field distribution created by Helmholtz coils is found in a magnetic bottle that can temporarily trap charged particles. See Magnetic Forces and Fields for a discussion on this.

CHECK YOUR UNDERSTANDING 12.5

Using Example 12.5, at what distance would you have to move the first coil to have zero measurable magnetic field at point *P*?

12.5 Ampère's Law

Learning Objectives

By the end of this section, you will be able to:

- Explain how Ampère's law relates the magnetic field produced by a current to the value of the current
- Calculate the magnetic field from a long straight wire, either thin or thick, by Ampère's law

A fundamental property of a static magnetic field is that, unlike an electrostatic field, it is not conservative. A conservative vector field is one whose line integral between two end points is the same regardless of the path chosen. Magnetic fields do not have such a property. Instead, there is a relationship between the magnetic field and its source, electric current. It is expressed in terms of the line integral of $\vec{\mathbf{B}}$ and is known as **Ampère's law**. This law can also be derived directly from the Biot-Savart law. We now consider that derivation for the special case of an infinite, straight wire.

Figure 12.14 shows an arbitrary plane perpendicular to an infinite, straight wire whose current I is directed out of the page. The magnetic field lines are circles directed counterclockwise and centered on the wire. To begin, let's consider $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ over the closed paths M and N. Notice that one path (M) encloses the wire, whereas the other (N) does not. Since the field lines are circular, $\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ is the product of B and the projection of dl onto the circle passing through $d\vec{\mathbf{l}}$. If the radius of this particular circle is r, the projection is $r d\theta$, and

$$\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = Br\, d\theta.$$

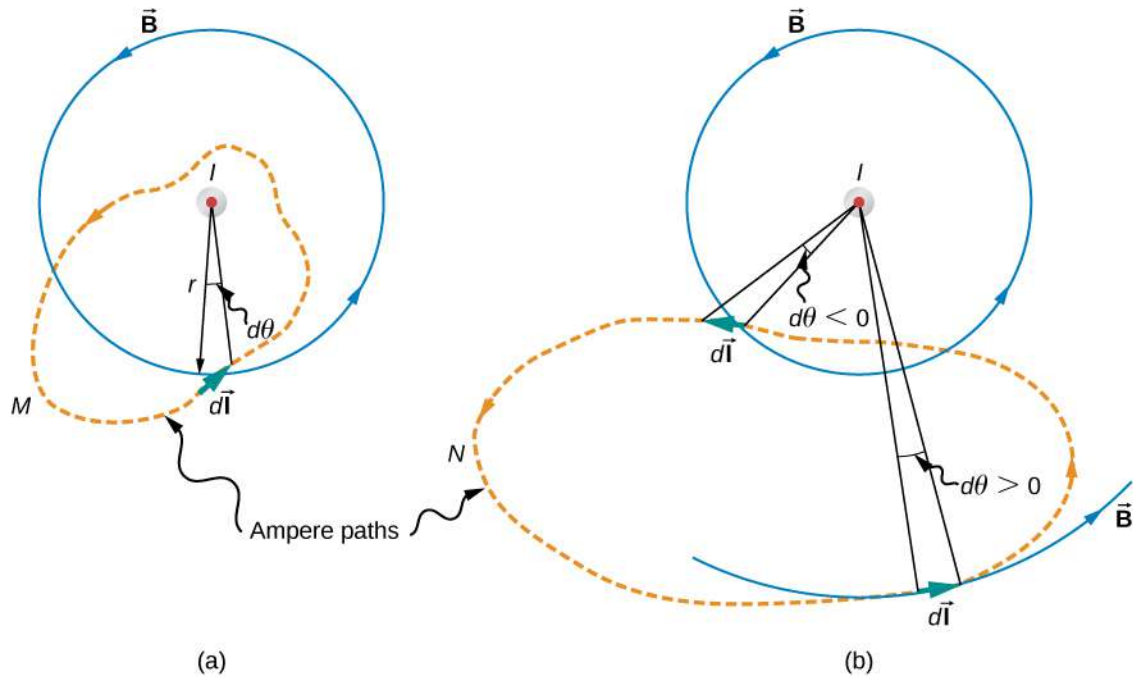


Figure 12.14 The current I of a long, straight wire is directed out of the page. The integral $\oint d\theta$ equals 2π and 0, respectively, for paths M and N.

With $\vec{\mathbf{B}}$ given by Equation 12.9,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \oint \left(\frac{\mu_0 I}{2\pi r}\right) r\, d\theta = \frac{\mu_0 I}{2\pi} \oint d\theta. \quad \textbf{12.20}$$

For path M, which circulates around the wire, $\oint_M d\theta = 2\pi$ and

$$\oint_M \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I. \quad \textbf{12.21}$$

Path N, on the other hand, circulates through both positive (counterclockwise) and negative (clockwise) $d\theta$ (see Figure 12.14), and since it is closed, $\oint_N d\theta = 0$. Thus for path N,

$$\oint_N \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = 0. \quad \textbf{12.22}$$

The extension of this result to the general case is Ampère's law.

Ampère's law

Over an arbitrary closed path,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I \tag{12.23}$$

where I is the total current passing through any open surface S whose perimeter is the path of integration. Only currents inside the path of integration need be considered.

To determine whether a specific current I is positive or negative, curl the fingers of your right hand in the direction of the path of integration, as shown in Figure 12.14. If I passes through S in the same direction as your extended thumb, I is positive; if I passes through S in the direction opposite to your extended thumb, it is negative.

PROBLEM-SOLVING STRATEGY

Ampère's Law

To calculate the magnetic field created from current in wire(s), use the following steps:

1. Identify the symmetry of the current in the wire(s). If there is no symmetry, use the Biot-Savart law to determine the magnetic field.
2. Determine the direction of the magnetic field created by the wire(s) by right-hand rule 2.
3. Chose a path loop where the magnetic field is either constant or zero.
4. Calculate the current inside the loop.
5. Calculate the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ around the closed loop.
6. Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

EXAMPLE 12.6

Using Ampère's Law to Calculate the Magnetic Field Due to a Wire

Use Ampère's law to calculate the magnetic field due to a steady current I in an infinitely long, thin, straight wire as shown in Figure 12.15.

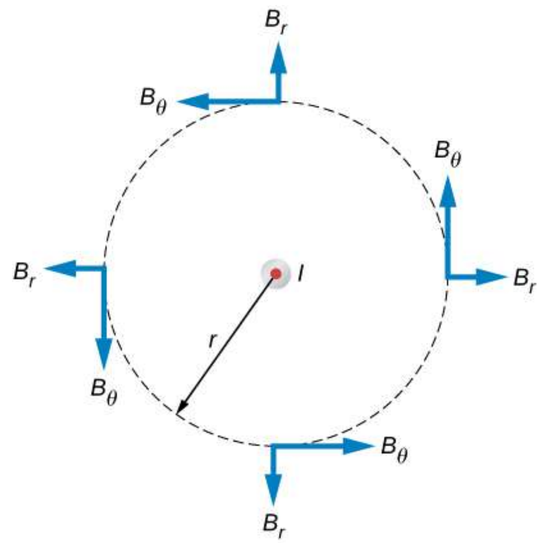


Figure 12.15 The possible components of the magnetic field B due to a current I, which is directed out of the page. The radial component is zero because the angle between the magnetic field and the path is at a right angle.

Strategy

Consider an arbitrary plane perpendicular to the wire, with the current directed out of the page. The possible magnetic field components in this plane, B_r and B_θ, are shown at arbitrary points on a circle of radius r centered on the wire. Since the field is cylindrically symmetric, neither B_r nor B_θ varies with the position on this circle. Also from symmetry, the radial lines, if they exist, must be directed either all inward or all outward from the wire. This means, however, that there must be a net magnetic flux across an arbitrary cylinder concentric with the wire. The radial component of the magnetic field must be zero because $\vec{\mathbf{B}}_r \cdot d\vec{\mathbf{l}} = 0$. Therefore, we can apply Ampère's law to the circular path as shown.

Solution

Over this path $\vec{\mathbf{B}}$ is constant and parallel to $d\vec{\mathbf{l}}$, so

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B_\theta \oint dl = B_\theta(2\pi r).$$

Thus Ampère's law reduces to

$$B_\theta(2\pi r) = \mu_0 I.$$

Finally, since B_θ is the only component of $\vec{\mathbf{B}}$, we can drop the subscript and write

$$B = \frac{\mu_0 I}{2\pi r}.$$

This agrees with the Biot-Savart calculation above.

Significance

Ampère's law works well if you have a path to integrate over which $\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ has results that are easy to simplify. For the infinite wire, this works easily with a path that is circular around the wire so that the magnetic field factors out of the integration. If the path dependence looks complicated, you can always go back to the Biot-Savart law and use that to find the magnetic field.

EXAMPLE 12.7

Calculating the Magnetic Field of a Thick Wire with Ampère's Law

The radius of the long, straight wire of Figure 12.16 is a, and the wire carries a current I_0 that is distributed uniformly over its cross-section. Find the magnetic field both inside and outside the wire.

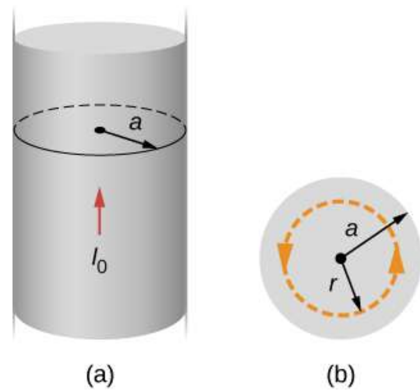


Figure 12.16 (a) A model of a current-carrying wire of radius *a* and current I_0. (b) A cross-section of the same wire showing the radius *a* and the Ampère's loop of radius *r*.

Strategy

This problem has the same geometry as Example 12.6, but the enclosed current changes as we move the integration path from outside the wire to inside the wire, where it doesn't capture the entire current enclosed (see Figure 12.16).

Solution

For any circular path of radius r that is centered on the wire,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \oint Bdl = B\oint dl = B(2\pi r).$$

From Ampère's law, this equals the total current passing through any surface bounded by the path of integration.

Consider first a circular path that is inside the wire $(r \le a)$ such as that shown in part (a) of Figure 12.16. We need the current I passing through the area enclosed by the path. It's equal to the current density J times the area enclosed. Since the current is uniform, the current density inside the path equals the current density in the whole wire, which is $I_0/\pi a^2$. Therefore the current I passing through the area enclosed by the path is

$$I = \frac{\pi r^2}{\pi a^2} I_0 = \frac{r^2}{a^2} I_0.$$

We can consider this ratio because the current density J is constant over the area of the wire. Therefore, the current density of a part of the wire is equal to the current density in the whole area. Using Ampère's law, we obtain

$$B(2\pi r) = \mu_0 \left(\frac{r^2}{a^2}\right) I_0,$$

and the magnetic field inside the wire is

$$B = \frac{\mu_0 I_0}{2\pi} \frac{r}{a^2} \; (r \le a).$$

Outside the wire, the situation is identical to that of the infinite thin wire of the previous example; that is,

$$B = \frac{\mu_0 I_0}{2\pi r} \; (r \geq a).$$

The variation of B with r is shown in Figure 12.17.

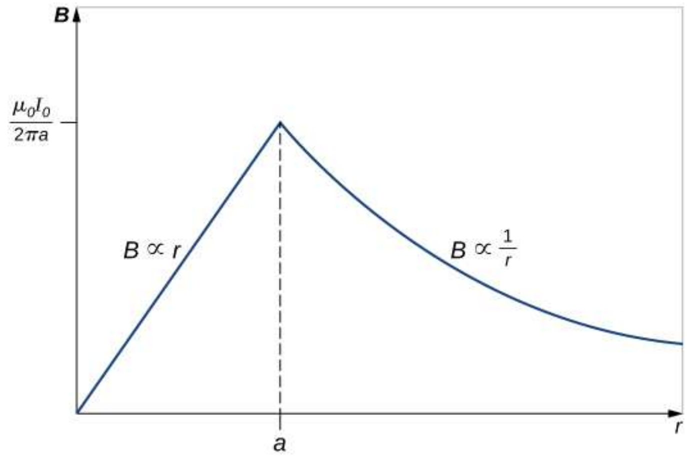


Figure 12.17 Variation of the magnetic field produced by a current I_0 in a long, straight wire of radius *a*.

Significance

The results show that as the radial distance increases inside the thick wire, the magnetic field increases from zero to a familiar value of the magnetic field of a thin wire. Outside the wire, the field drops off regardless of whether it was a thick or thin wire.

This result is similar to how Gauss's law for electrical charges behaves inside a uniform charge distribution, except that Gauss's law for electrical charges has a uniform volume distribution of charge, whereas Ampère's law here has a uniform area of current distribution. Also, the drop-off outside the thick wire is similar to how an electric field drops off outside of a linear charge distribution, since the two cases have the same geometry and neither case depends on the configuration of charges or currents once the loop is outside the distribution.

EXAMPLE 12.8

Using Ampère's Law with Arbitrary Paths

Use Ampère's law to evaluate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ for the current configurations and paths in Figure 12.18.

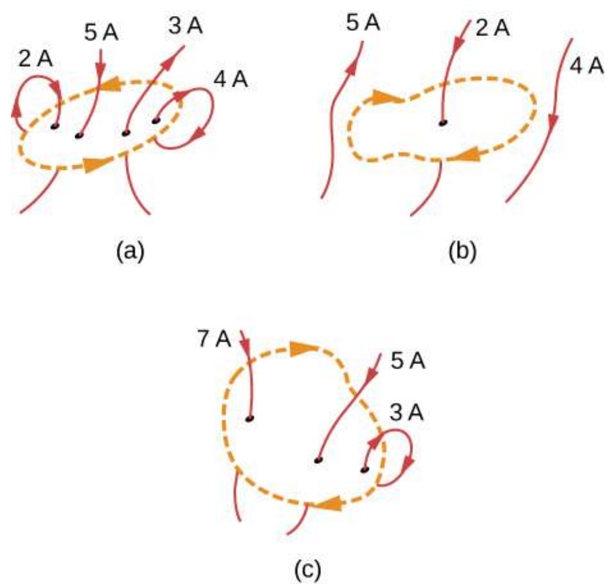


Figure 12.18 Current configurations and paths for Example 12.8.

Strategy

Ampère's law states that $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I$ where I is the total current passing through the enclosed loop. The quickest way to evaluate the integral is to calculate $\mu_0 I$ by finding the net current through the loop. Positive currents flow with your right-hand thumb if your fingers wrap around in the direction of the loop. This will tell us the sign of the answer.

Solution

(a) The current going downward through the loop equals the current going out of the loop, so the net current is zero. Thus, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = 0$.

(b) The only current to consider in this problem is 2A because it is the only current inside the loop. The right-hand rule shows us the current going downward through the loop is in the positive direction. Therefore, the answer is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 (2\text{ A}) = 2.51 \times 10^{-6}\text{T} \cdot \text{m}$.

(c) The right-hand rule shows us the current going downward through the loop is in the positive direction. There are $7\text{A} + 5\text{A} = 12\text{A}$ of current going downward and –3 A going upward. Therefore, the total current is 9 A and $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 (9\text{ A}) = 1.13 \times 10^{-5}\text{T} \cdot \text{m}$.

Significance

If the currents all wrapped around so that the same current went into the loop and out of the loop, the net current would be zero and no magnetic field would be present. This is why wires are very close to each other in an electrical cord. The currents flowing toward a device and away from a device in a wire equal zero total current flow through an Ampère loop around these wires. Therefore, no stray magnetic fields can be present from cords carrying current.

CHECK YOUR UNDERSTANDING 12.6

Consider using Ampère's law to calculate the magnetic fields of a finite straight wire and of a circular loop of wire. Why is it not useful for these calculations?

12.6 Solenoids and Toroids

Learning Objectives

By the end of this section, you will be able to:

- Establish a relationship for how the magnetic field of a solenoid varies with distance and current by using both the Biot-Savart law and Ampère's law
- Establish a relationship for how the magnetic field of a toroid varies with distance and current by using Ampère's law

Two of the most common and useful electromagnetic devices are called solenoids and toroids. In one form or another, they are part of numerous instruments, both large and small. In this section, we examine the magnetic field typical of these devices.

Solenoids

A long wire wound in the form of a helical coil is known as a **solenoid**. Solenoids are commonly used in experimental research requiring magnetic fields. A solenoid is generally easy to wind, and near its center, its magnetic field is quite uniform and directly proportional to the current in the wire.

Figure 12.19 shows a solenoid consisting of N turns of wire tightly wound over a length L. A current I is flowing along the wire of the solenoid. The number of turns per unit length is N/L; therefore, the number of turns in an infinitesimal length dy are $(N/L)dy$ turns. This produces a current

$$dI = \frac{NI}{L}dy. \tag{12.24}$$

We first calculate the magnetic field at the point P of Figure 12.19. This point is on the central axis of the solenoid. We are basically cutting the solenoid into thin slices that are dy thick and treating each as a current loop. Thus, dI is the current through each slice. The magnetic field $d\vec{\mathbf{B}}$ due to the current dI in dy can be found with the help of Equation 12.15 and Equation 12.24:

$$d\vec{\mathbf{B}} = \frac{\mu_0 R^2 dI}{2(y^2 + R^2)^{3/2}}\hat{\mathbf{j}} = \left(\frac{\mu_0 I R^2 N}{2L}\hat{\mathbf{j}}\right)\frac{dy}{(y^2 + R^2)^{3/2}} \tag{12.25}$$

where we used Equation 12.24 to replace dI. The resultant field at P is found by integrating $d\vec{\mathbf{B}}$ along the entire length of the solenoid. It's easiest to evaluate this integral by changing the independent variable from y to θ. From inspection of Figure 12.19, we have:

$$\sin\theta = \frac{y}{\sqrt{y^2 + R^2}}. \tag{12.26}$$

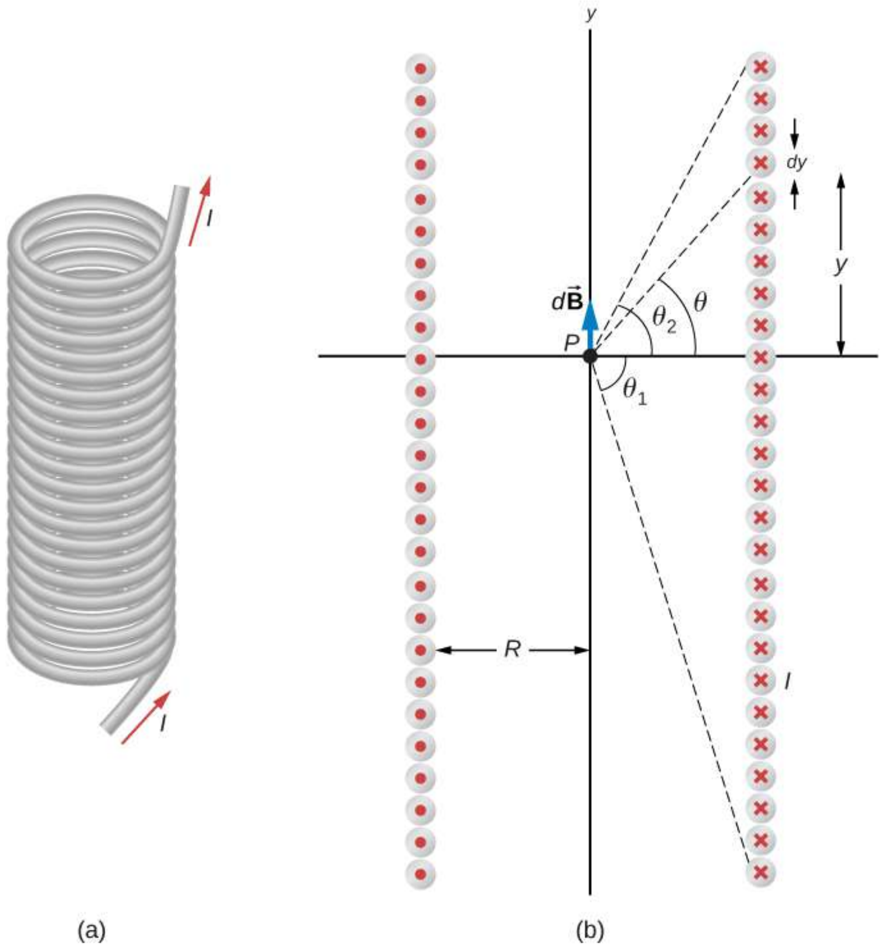


Figure 12.19 (a) A solenoid is a long wire wound in the shape of a helix. (b) The magnetic field at the point *P* on the axis of the solenoid is the net field due to all of the current loops.

Taking the differential of both sides of this equation, we obtain

$$\begin{aligned}\cos\theta\, d\theta &= \left[-\frac{y^2}{(y^2+R^2)^{3/2}} + \frac{1}{\sqrt{y^2+R^2}}\right] dy \\ &= \frac{R^2\, dy}{(y^2+R^2)^{3/2}}.\end{aligned}$$

When this is substituted into the equation for $d\vec{\mathbf{B}}$, we have

$$\vec{\mathbf{B}} = \frac{\mu_0 IN}{2L}\hat{\mathbf{j}}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{\mu_0 IN}{2L}(\sin\theta_2 - \sin\theta_1)\hat{\mathbf{j}}, \tag{12.27}$$

which is the magnetic field along the central axis of a finite solenoid.

Of special interest is the infinitely long solenoid, for which $L \to \infty$. From a practical point of view, the infinite solenoid is one whose length is much larger than its radius ($L \gg R$). In this case, $\theta_1 = \frac{-\pi}{2}$ and $\theta_2 = \frac{\pi}{2}$. Then from Equation 12.27, the magnetic field along the central axis of an infinite solenoid is

$$\vec{\mathbf{B}} = \frac{\mu_0 IN}{2L}\hat{\mathbf{j}}\,[\sin(\pi/2) - \sin(-\pi/2)] = \frac{\mu_0 IN}{L}\hat{\mathbf{j}}$$

or

$$\vec{\mathbf{B}} = \mu_0 n I \hat{\mathbf{j}}, \quad (12.28)$$

where n is the number of turns per unit length. You can find the direction of $\vec{\mathbf{B}}$ with a right-hand rule: Curl your fingers in the direction of the current, and your thumb points along the magnetic field in the interior of the solenoid.

We now use these properties, along with Ampère's law, to calculate the magnitude of the magnetic field at any location inside the infinite solenoid. Consider the closed path of Figure 12.20. Along segment 1, $\vec{\mathbf{B}}$ is uniform and parallel to the path. Along segments 2 and 4, $\vec{\mathbf{B}}$ is perpendicular to part of the path and vanishes over the rest of it. Therefore, segments 2 and 4 do not contribute to the line integral in Ampère's law. Along segment 3, $\vec{\mathbf{B}} = 0$ because the magnetic field is zero outside the solenoid. If you consider an Ampère's law loop outside of the solenoid, the current flows in opposite directions on different segments of wire. Therefore, there is no enclosed current and no magnetic field according to Ampère's law. Thus, there is no contribution to the line integral from segment 3. As a result, we find

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \int_1 \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = Bl. \quad (12.29)$$

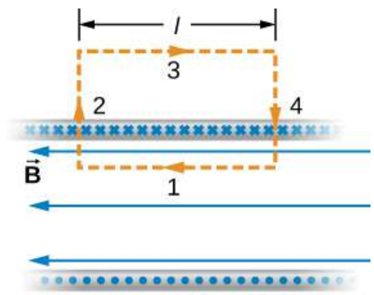


Figure 12.20 The path of integration used in Ampère's law to evaluate the magnetic field of an infinite solenoid.

The solenoid has n turns per unit length, so the current that passes through the surface enclosed by the path is nlI. Therefore, from Ampère's law,

$$Bl = \mu_0 n l I$$

and

$$B = \mu_0 n I \quad (12.30)$$

within the solenoid. This agrees with what we found earlier for B on the central axis of the solenoid. Here, however, the location of segment 1 is arbitrary, so we have found that this equation gives the magnetic field everywhere inside the infinite solenoid.

When a patient undergoes a magnetic resonance imaging (MRI) scan, the person lies down on a table that is moved into the center of a large solenoid that can generate very large magnetic fields. The solenoid is capable of these high fields from high currents flowing through superconducting wires. The large magnetic field is used to change the spin of protons in the patient's body. The time it takes for the spins to align or relax (return to original orientation) is a signature of different tissues that can be analyzed to see if the structures of the tissues is normal (Figure 12.21).

Figure 12.21 In an MRI machine, a large magnetic field is generated by the cylindrical solenoid surrounding the patient. (credit: Liz West)

EXAMPLE 12.9

Magnetic Field Inside a Solenoid

A solenoid has 300 turns wound around a cylinder of diameter 1.20 cm and length 14.0 cm. If the current through the coils is 0.410 A, what is the magnitude of the magnetic field inside and near the middle of the solenoid?

Strategy

We are given the number of turns and the length of the solenoid so we can find the number of turns per unit length. Therefore, the magnetic field inside and near the middle of the solenoid is given by Equation 12.30. Outside the solenoid, the magnetic field is zero.

Solution

The number of turns per unit length is

$$n = \frac{300\text{ turns}}{0.140\text{ m}} = 2.14 \times 10^3\text{ turns/m}.$$

The magnetic field produced inside the solenoid is

$$\begin{aligned} B &= \mu_0 nI = (4\pi \times 10^{-7}\text{ T} \cdot \text{m/A})(2.14 \times 10^3\text{ turns/m})(0.410\text{ A}) \\ B &= 1.10 \times 10^{-3}\text{ T}. \end{aligned}$$

Significance

This solution is valid only if the length of the solenoid is reasonably large compared with its diameter. This example is a case where this is valid.

CHECK YOUR UNDERSTANDING 12.7

What is the ratio of the magnetic field produced from using a finite formula over the infinite approximation for an angle θ of (a) 85°? (b) 89°? The solenoid has 1000 turns in 50 cm with a current of 1.0 A flowing through the coils

Toroids

A toroid is a donut-shaped coil closely wound with one continuous wire, as illustrated in part (a) of Figure 12.22. If the toroid has N windings and the current in the wire is I, what is the magnetic field both inside and outside the toroid?

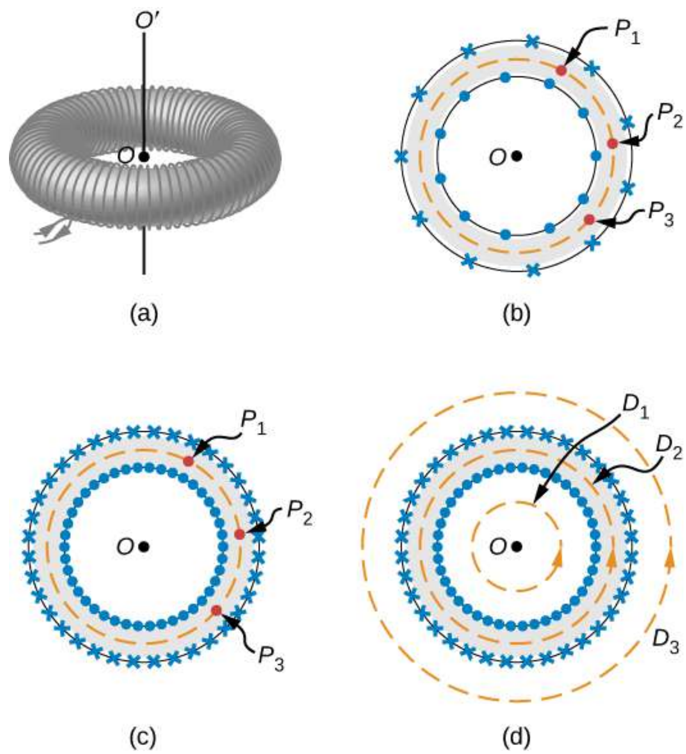


Figure 12.22 (a) A toroid is a coil wound into a donut-shaped object. (b) A loosely wound toroid does not have cylindrical symmetry. (c) In a tightly wound toroid, cylindrical symmetry is a very good approximation. (d) Several paths of integration for Ampère's law.

We begin by assuming cylindrical symmetry around the axis *OO*'. Actually, this assumption is not precisely correct, for as part (b) of Figure 12.22 shows, the view of the toroidal coil varies from point to point (for example, P_1, P_2, and P_3) on a circular path centered around *OO*'. However, if the toroid is tightly wound, all points on the circle become essentially equivalent [part (c) of Figure 12.22], and cylindrical symmetry is an accurate approximation.

With this symmetry, the magnetic field must be tangent to and constant in magnitude along any circular path centered on *OO*'. This allows us to write for each of the paths D_1, D_2, and D_3 shown in part (d) of Figure 12.22,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B(2\pi r). \tag{12.31}$$

Ampère's law relates this integral to the net current passing through any surface bounded by the path of integration. For a path that is external to the toroid, either no current passes through the enclosing surface (path D_1), or the current passing through the surface in one direction is exactly balanced by the current passing through it in the opposite direction (path D_3). In either case, there is no net current passing through the surface, so

$$\oint B(2\pi r) = 0$$

and

$$B = 0 \quad \text{(outside the toroid).} \tag{12.32}$$

The turns of a toroid form a helix, rather than circular loops. As a result, there is a small field external to the coil; however, the derivation above holds if the coils were circular.

For a circular path within the toroid (path D_2), the current in the wire cuts the surface *N* times, resulting in a net current *NI* through the surface. We now find with Ampère's law,

$$B(2\pi r) = \mu_0 NI$$

and

$$B = \frac{\mu_0 N I}{2\pi r} \quad \text{(within the toroid).} \tag{12.33}$$

The magnetic field is directed in the counterclockwise direction for the windings shown. When the current in the coils is reversed, the direction of the magnetic field also reverses.

The magnetic field inside a toroid is not uniform, as it varies inversely with the distance r from the axis OO'. However, if the central radius R (the radius midway between the inner and outer radii of the toroid) is much larger than the cross-sectional diameter of the coils r, the variation is fairly small, and the magnitude of the magnetic field may be calculated by Equation 12.33 where $r = R$.

12.7 Magnetism in Matter

Learning Objectives

By the end of this section, you will be able to:

- Classify magnetic materials as paramagnetic, diamagnetic, or ferromagnetic, based on their response to a magnetic field
- Sketch how magnetic dipoles align with the magnetic field in each type of substance
- Define hysteresis and magnetic susceptibility, which determines the type of magnetic material

Why are certain materials magnetic and others not? And why do certain substances become magnetized by a field, whereas others are unaffected? To answer such questions, we need an understanding of magnetism on a microscopic level.

Within an atom, every electron travels in an orbit and spins on an internal axis. Both types of motion produce current loops and therefore magnetic dipoles. For a particular atom, the net magnetic dipole moment is the vector sum of the magnetic dipole moments. Values of μ for several types of atoms are given in Table 12.1. Notice that some atoms have a zero net dipole moment and that the magnitudes of the nonvanishing moments are typically $10^{-23}\ \text{A} \cdot \text{m}^2$.

Atom	Magnetic Moment $\left(10^{-24}\ \text{A} \cdot \text{m}^2\right)$
H	9.27
He	0
Li	9.27
O	13.9
Na	9.27
S	13.9

Table 12.1 Magnetic Moments of Some Atoms

A handful of matter has approximately 10^{26} atoms and ions, each with its magnetic dipole moment. If no external magnetic field is present, the magnetic dipoles are randomly oriented—as many are pointed up as down, as many are pointed east as west, and so on. Consequently, the net magnetic dipole moment of the sample is zero. However, if the sample is placed in a magnetic field, these dipoles tend to align with the field (see Equation 12.14), and this alignment determines how the sample responds to the field. On the basis of this response, a material is said to be either paramagnetic, ferromagnetic, or diamagnetic.

In a **paramagnetic material**, only a small fraction (roughly one-third) of the magnetic dipoles are aligned with the applied field. Since each dipole produces its own magnetic field, this alignment contributes an extra magnetic field, which enhances the applied field. When a **ferromagnetic material** is placed in a magnetic field, its magnetic dipoles also become aligned; furthermore, they become locked together so that a permanent magnetization results, even when the field is turned off or reversed. This permanent magnetization happens in ferromagnetic materials but not paramagnetic materials. **Diamagnetic materials** are composed of atoms that have no net magnetic dipole moment. However, when a diamagnetic material is placed in a magnetic field, a magnetic dipole moment is directed opposite to the applied field and therefore produces a magnetic field that opposes the applied field. We now consider each type of material in greater detail.

Paramagnetic Materials

For simplicity, we assume our sample is a long, cylindrical piece that completely fills the interior of a long, tightly wound solenoid. When there is no current in the solenoid, the magnetic dipoles in the sample are randomly oriented and produce no net magnetic field. With a solenoid current, the magnetic field due to the solenoid exerts a torque on the dipoles that tends to align them with the field. In competition with the aligning torque are thermal collisions that tend to randomize the orientations of the dipoles. The relative importance of these two competing processes can be estimated by comparing the energies involved. From Equation 12.14, the energy difference between a magnetic dipole aligned with and against a magnetic field is $U_B = 2\mu B$. If $\mu = 9.3 \times 10^{-24}\,\text{A} \cdot \text{m}^2$ (the value of atomic hydrogen) and B = 1.0 T, then

$$U_B = 1.9 \times 10^{-23}\,\text{J}.$$

At a room temperature of 27 °C, the thermal energy per atom is

$$U_T \approx kT = (1.38 \times 10^{-23}\,\text{J/K})(300\,\text{K}) = 4.1 \times 10^{-21}\,\text{J},$$

which is about 220 times greater than U_B. Clearly, energy exchanges in thermal collisions can seriously interfere with the alignment of the magnetic dipoles. As a result, only a small fraction of the dipoles is aligned at any instant.

The four sketches of Figure 12.23 furnish a simple model of this alignment process. In part (a), before the field of the solenoid (not shown) containing the paramagnetic sample is applied, the magnetic dipoles are randomly oriented and there is no net magnetic dipole moment associated with the material. With the introduction of the field, a partial alignment of the dipoles takes place, as depicted in part (b). The component of the net magnetic dipole moment that is perpendicular to the field vanishes. We may then represent the sample by part (c), which shows a collection of magnetic dipoles completely aligned with the field. By treating these dipoles as current loops, we can picture the dipole alignment as equivalent to a current around the surface of the material, as in part (d). This fictitious surface current produces its own magnetic field, which enhances the field of the solenoid.

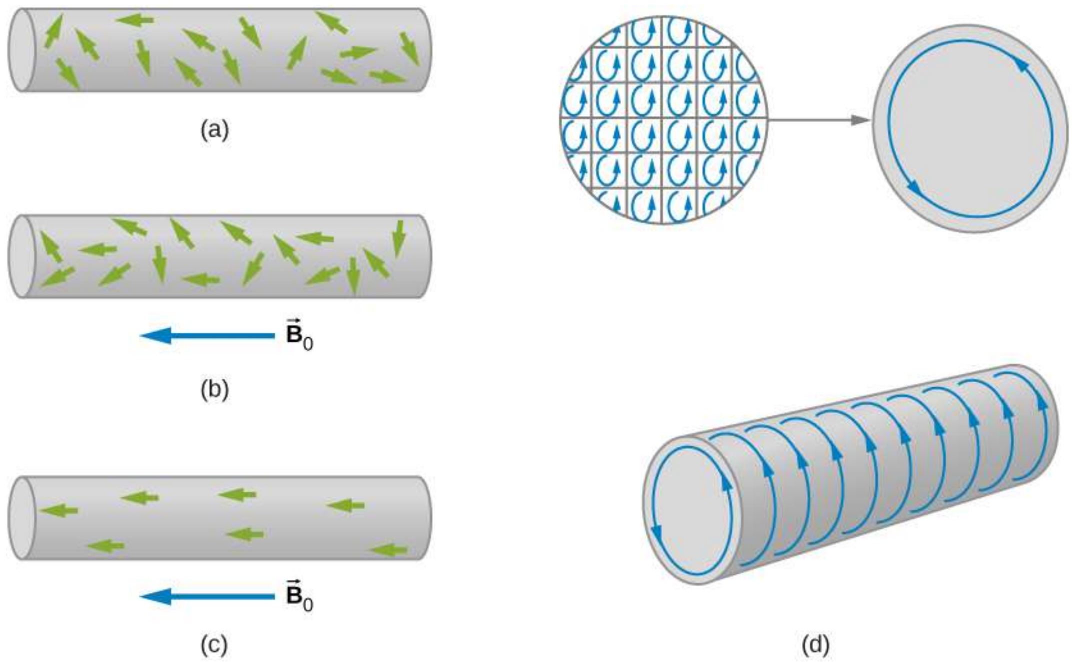


Figure 12.23 The alignment process in a paramagnetic material filling a solenoid (not shown). (a) Without an applied field, the magnetic dipoles are randomly oriented. (b) With a field, partial alignment occurs. (c) An equivalent representation of part (b). (d) The internal currents cancel, leaving an effective surface current that produces a magnetic field similar to that of a finite solenoid.

We can express the total magnetic field $\vec{\mathbf{B}}$ in the material as

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \vec{\mathbf{B}}_m, \tag{12.34}$$

where $\vec{\mathbf{B}}_0$ is the field due to the current I_0 in the solenoid and $\vec{\mathbf{B}}_m$ is the field due to the surface current I_m around the sample. Now $\vec{\mathbf{B}}_m$ is usually proportional to $\vec{\mathbf{B}}_0$, a fact we express by

$$\vec{\mathbf{B}}_m = \chi \vec{\mathbf{B}}_0, \tag{12.35}$$

where χ is a dimensionless quantity called the **magnetic susceptibility**. Values of χ for some paramagnetic materials are given in Table 12.2. Since the alignment of magnetic dipoles is so weak, χ is very small for paramagnetic materials. By combining Equation 12.34 and Equation 12.35, we obtain:

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \chi \vec{\mathbf{B}}_0 = (1 + \chi)\vec{\mathbf{B}}_0. \tag{12.36}$$

For a sample within an infinite solenoid, this becomes

$$B = (1 + \chi)\mu_0 n I. \tag{12.37}$$

This expression tells us that the insertion of a paramagnetic material into a solenoid increases the field by a factor of $(1 + \chi)$. However, since χ is so small, the field isn't enhanced very much.

The quantity

$$\mu = (1 + \chi)\mu_0. \tag{12.38}$$

is called the magnetic permeability of a material. In terms of μ, Equation 12.37 can be written as

$$B = \mu n I \tag{12.39}$$

for the filled solenoid.

Paramagnetic Materials	χ	Diamagnetic Materials	χ
Aluminum	2.2×10^{-5}	Bismuth	-1.7×10^{-5}
Calcium	1.4×10^{-5}	Carbon (diamond)	-2.2×10^{-5}
Chromium	3.1×10^{-4}	Copper	-9.7×10^{-6}
Magnesium	1.2×10^{-5}	Lead	-1.8×10^{-5}
Oxygen gas (1 atm)	1.8×10^{-6}	Mercury	-2.8×10^{-5}
Oxygen liquid (90 K)	3.5×10^{-3}	Hydrogen gas (1 atm)	-2.2×10^{-9}
Tungsten	6.8×10^{-5}	Nitrogen gas (1 atm)	-6.7×10^{-9}
Air (1 atm)	3.6×10^{-7}	Water	-9.1×10^{-6}

Table 12.2 Magnetic Susceptibilities *Note: Unless otherwise specified, values given are for room temperature.

Diamagnetic Materials

A magnetic field always induces a magnetic dipole in an atom. This induced dipole points opposite to the applied field, so its magnetic field is also directed opposite to the applied field. In paramagnetic and ferromagnetic materials, the induced magnetic dipole is masked by much stronger permanent magnetic dipoles of the atoms. However, in diamagnetic materials, whose atoms have no permanent magnetic dipole moments, the effect of the induced dipole is observable.

We can now describe the magnetic effects of diamagnetic materials with the same model developed for paramagnetic materials. In this case, however, the fictitious surface current flows opposite to the solenoid current, and the magnetic susceptibility χ is negative. Values of χ for some diamagnetic materials are also given in Table 12.2.

INTERACTIVE

Water is a common diamagnetic material. Animals are mostly composed of water. Experiments have been performed on frogs (https://openstax.org/l/21frogs) and mice (https://openstax.org/l/21mice) in diverging magnetic fields. The water molecules are repelled from the applied magnetic field against gravity until the animal reaches an equilibrium. The result is that the animal is levitated by the magnetic field.

Ferromagnetic Materials

Common magnets are made of a ferromagnetic material such as iron or one of its alloys. Experiments reveal that a ferromagnetic material consists of tiny regions known as **magnetic domains**. Their volumes typically range from 10^{-12} to $10^{-8} m^3$, and they contain about 10^{17} to 10^{21} atoms. Within a domain, the magnetic dipoles are rigidly aligned in the same direction by coupling among the atoms. This coupling, which is due to quantum mechanical effects, is so strong that even thermal agitation at room temperature cannot break it. The result is that each domain has a net dipole moment. Some materials have weaker coupling and are ferromagnetic only at lower temperatures.

If the domains in a ferromagnetic sample are randomly oriented, as shown in Figure 12.24, the sample has no net magnetic dipole moment and is said to be unmagnetized. Suppose that we fill the volume of a solenoid with an unmagnetized ferromagnetic sample. When the magnetic field $\vec{\mathbf{B}}_0$ of the solenoid is turned on, the dipole moments of the domains rotate so that they align somewhat with the field, as depicted in Figure 12.24. In addition, the aligned domains tend to increase in size at the expense of unaligned ones. The net effect of these two processes is the creation of a net magnetic dipole moment for the ferromagnet that is directed along the applied magnetic field. This net magnetic dipole moment is much larger than that of a paramagnetic sample, and the domains, with their large numbers of atoms, do not become misaligned by thermal agitation. Consequently, the field due to the alignment of the domains is quite large.

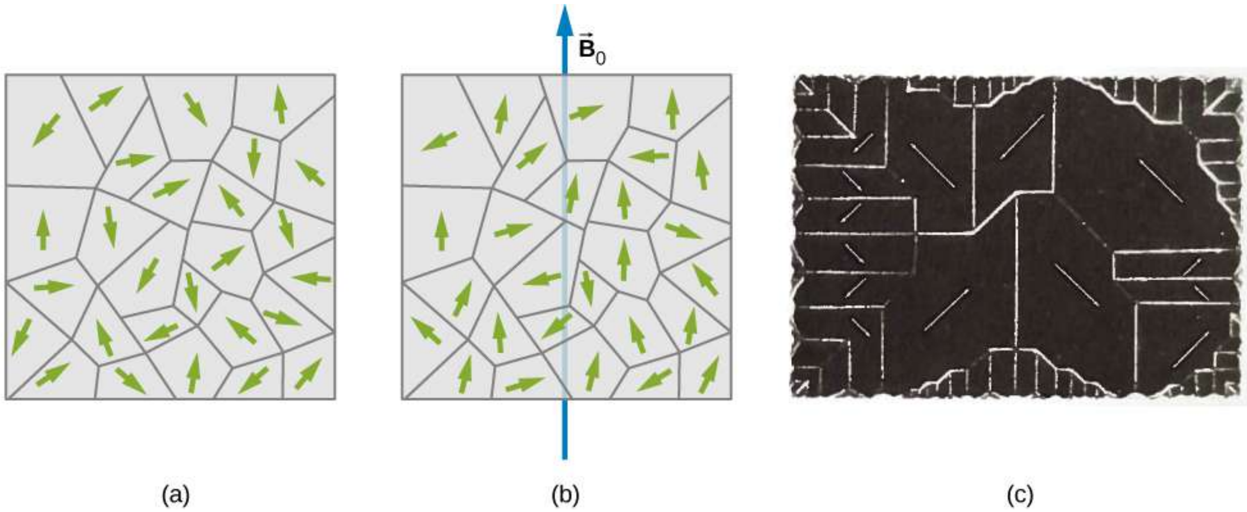


Figure 12.24 (a) Domains are randomly oriented in an unmagnetized ferromagnetic sample such as iron. The arrows represent the orientations of the magnetic dipoles within the domains. (b) In an applied magnetic field, the domains align somewhat with the field. (c) The domains of a single crystal of nickel. The white lines show the boundaries of the domains. These lines are produced by iron oxide powder sprinkled on the crystal.

Besides iron, only four elements contain the magnetic domains needed to exhibit ferromagnetic behavior: cobalt, nickel, gadolinium, and dysprosium. Many alloys of these elements are also ferromagnetic. Ferromagnetic materials can be described using Equation 12.34 through Equation 12.39, the paramagnetic equations. However, the value of χ for ferromagnetic material is usually on the order of 10^3 to 10^4, and it also depends on the history of the magnetic field to which the material has been subject. A typical plot of B (the total field in the material) versus B_0 (the applied field) for an initially unmagnetized piece of iron is shown in Figure 12.25. Some sample numbers are (1) for $B_0 = 1.0 \times 10^{-4}$ T, $B = 0.60$ T, and $\chi = \left({0.60}/{1.0 \times 10^{-4}} \right) - 1 \approx 6.0 \times 10^3$; (2) for $B_0 = 6.0 \times 10^{-4}$ T, $B = 1.5$ T, and $\chi = \left({1.5}/{6.0 \times 10^{-4}} \right) - 1 \approx 2.5 \times 10^3$.

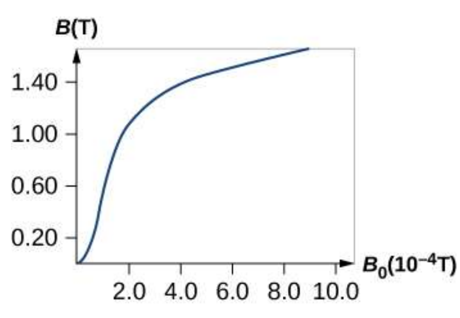


Figure 12.25 (a) The magnetic field B in annealed iron as a function of the applied field B_0.

When B_0 is varied over a range of positive and negative values, B is found to behave as shown in Figure 12.26. Note that the same B_0 (corresponding to the same current in the solenoid) can produce different values of B in the material. The magnetic field B produced in a ferromagnetic material by an applied field B_0 depends on the magnetic history of the material. This effect is called **hysteresis**, and the curve of Figure 12.26 is called a hysteresis loop. Notice that B does not disappear when $B_0 = 0$ (i.e., when the current in the solenoid is turned off). The iron stays magnetized, which means that it has become a permanent magnet.

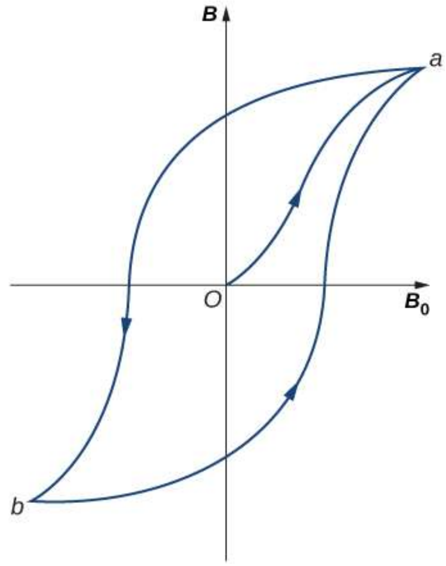


Figure 12.26 A typical hysteresis loop for a ferromagnet. When the material is first magnetized, it follows a curve from 0 to *a*. When B_0 is reversed, it takes the path shown from *a* to *b*. If B_0 is reversed again, the material follows the curve from *b* to *a*.

Like the paramagnetic sample of Figure 12.23, the partial alignment of the domains in a ferromagnet is equivalent to a current flowing around the surface. A bar magnet can therefore be pictured as a tightly wound solenoid with a large current circulating through its coils (the surface current). You can see in Figure 12.27 that this model fits quite well. The fields of the bar magnet and the finite solenoid are strikingly similar. The figure also shows how the poles of the bar magnet are identified. To form closed loops, the field lines outside the magnet leave the north (N) pole and enter the south (S) pole, whereas inside the magnet, they leave S and enter N.

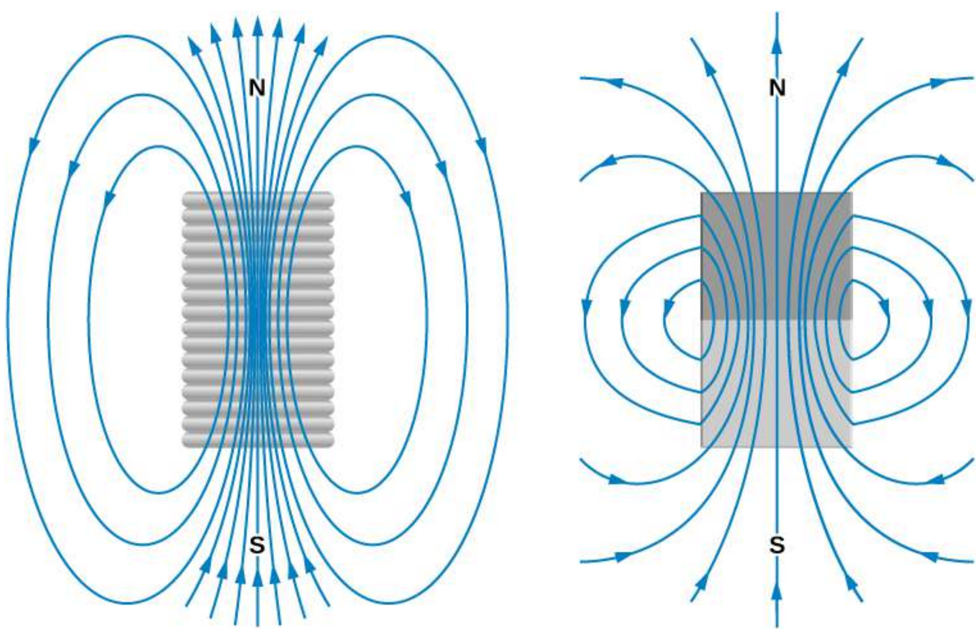


Figure 12.27 Comparison of the magnetic fields of a finite solenoid and a bar magnet.

Ferromagnetic materials are found in computer hard disk drives and permanent data storage devices (Figure 12.28). A material used in your hard disk drives is called a spin valve, which has alternating layers of ferromagnetic (aligning with the external magnetic field) and antiferromagnetic (each atom is aligned opposite to the next) metals. It was observed that a significant change in resistance was discovered based on whether an applied magnetic field was on the spin valve or not. This large change in resistance creates a quick and consistent way for recording or reading information by an applied current.

Figure 12.28 The inside of a hard disk drive. The silver disk contains the information, whereas the thin stylus on top of the disk reads and writes information to the disk.

EXAMPLE 12.10

Iron Core in a Coil

A long coil is tightly wound around an iron cylinder whose magnetization curve is shown in Figure 12.25. (a) If $n = 20$ turns per centimeter, what is the applied field B_0 when $I_0 = 0.20\text{ A}$? (b) What is the net magnetic field for this same current? (c) What is the magnetic susceptibility in this case?

Strategy

(a) The magnetic field of a solenoid is calculated using Equation 12.28. (b) The graph is read to determine the net magnetic field for this same current. (c) The magnetic susceptibility is calculated using Equation 12.37.

Solution

a. The applied field B_0 of the coil is

$$\begin{aligned} B_0 &= \mu_0 n I_0 = (4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(2000/\text{m})(0.20\text{ A}) \\ B_0 &= 5.0 \times 10^{-4}\,\text{T}. \end{aligned}$$

b. From inspection of the magnetization curve of Figure 12.25, we see that, for this value of B_0, $B = 1.4\text{ T}$. Notice that the internal field of the aligned atoms is much larger than the externally applied field.

c. The magnetic susceptibility is calculated to be

$$\chi = \frac{B}{B_0} - 1 = \frac{1.4\text{ T}}{5.0 \times 10^{-4}\,\text{T}} - 1 = 2.8 \times 10^3.$$

Significance

Ferromagnetic materials have susceptibilities in the range of 10^3 which compares well to our results here. Paramagnetic materials have fractional susceptibilities, so their applied field of the coil is much greater than the magnetic field generated by the material.

CHECK YOUR UNDERSTANDING 12.8

Repeat the calculations from the previous example for $I_0 = 0.040\text{ A}$.

CHAPTER REVIEW

Key Terms

Ampère's law physical law that states that the line integral of the magnetic field around an electric current is proportional to the current

Biot-Savart law an equation giving the magnetic field at a point produced by a current-carrying wire

diamagnetic materials their magnetic dipoles align oppositely to an applied magnetic field; when the field is removed, the material is unmagnetized

ferromagnetic materials contain groups of dipoles, called domains, that align with the applied magnetic field; when this field is removed, the material is still magnetized

hysteresis property of ferromagnets that is seen when a material's magnetic field is examined versus the applied magnetic field; a loop is created resulting from sweeping the applied field forward and reverse

magnetic domains groups of magnetic dipoles that are all aligned in the same direction and are coupled together quantum mechanically

magnetic susceptibility ratio of the magnetic field in the material over the applied field at that time; positive susceptibilities are either paramagnetic or ferromagnetic (aligned with the field) and negative susceptibilities are diamagnetic (aligned oppositely with the field)

paramagnetic materials their magnetic dipoles align partially in the same direction as the applied magnetic field; when this field is removed, the material is unmagnetized

permeability of free space μ_0, measure of the ability of a material, in this case free space, to support a magnetic field

solenoid thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

toroid donut-shaped coil closely wound around that is one continuous wire

Key Equations

Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$
Contribution to magnetic field from a current element	$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$
Biot–Savart law	$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}$
Magnetic field due to a long straight wire	$B = \frac{\mu_0 I}{2\pi R}$
Force between two parallel currents	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$
Magnetic field of a current loop	$B = \frac{\mu_0 I}{2R}$ (at center of loop)
Ampère's law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I$
Magnetic field strength inside a solenoid	$B = \mu_0 n I$
Magnetic field strength inside a toroid	$B = \frac{\mu_o N I}{2\pi r}$

Magnetic permeability	$\mu = (1+\chi)\mu_0$
Magnetic field of a solenoid filled with paramagnetic material	$B = \mu n I$

Summary

12.1 The Biot-Savart Law

- The magnetic field created by a current-carrying wire is found by the Biot-Savart law.
- The current element $Id\vec{\mathbf{l}}$ produces a magnetic field a distance r away.

12.2 Magnetic Field Due to a Thin Straight Wire

- The strength of the magnetic field created by current in a long straight wire is given by $B = \frac{\mu_0 I}{2\pi R}$ (long straight wire) where I is the current, R is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/s}$ is the permeability of free space.
- The direction of the magnetic field created by a long straight wire is given by right-hand rule 2 (RHR-2): Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops created by it.

12.3 Magnetic Force between Two Parallel Currents

- The force between two parallel currents I_1 and I_2, separated by a distance r, has a magnitude per unit length given by $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$.
- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

12.4 Magnetic Field of a Current Loop

- The magnetic field strength at the center of a circular loop is given by $B = \frac{\mu_0 I}{2R}$ (at center of loop), where R is the radius of the loop. RHR-2 gives the direction of the field about the loop.

12.5 Ampère's Law

- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampère's law.
- Ampère's law can be used to determine the magnetic field from a thin wire or thick wire by a geometrically convenient path of integration. The results are consistent with the Biot-Savart law.

12.6 Solenoids and Toroids

- The magnetic field strength inside a solenoid is
$$B = \mu_0 n I \quad \text{(inside a solenoid)}$$
where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.
- The magnetic field strength inside a toroid is
$$B = \frac{\mu_o N I}{2\pi r} \quad \text{(within the toroid)}$$
where N is the number of windings. The field inside a toroid is not uniform and varies with the distance as $1/r$.

12.7 Magnetism in Matter

- Materials are classified as paramagnetic, diamagnetic, or ferromagnetic, depending on how they behave in an applied magnetic field.
- Paramagnetic materials have partial alignment of their magnetic dipoles with an applied magnetic field. This is a positive magnetic susceptibility. Only a surface current remains, creating a solenoid-like magnetic field.
- Diamagnetic materials exhibit induced dipoles opposite to an applied magnetic field. This is a negative magnetic susceptibility.
- Ferromagnetic materials have groups of dipoles, called domains, which align with the applied magnetic field. However, when the field is removed, the ferromagnetic material remains magnetized, unlike paramagnetic materials. This magnetization of the material versus the applied field effect is called hysteresis.

Conceptual Questions

12.1 The Biot-Savart Law

1. For calculating magnetic fields, what are the advantages and disadvantages of the Biot-Savart law?
2. Describe the magnetic field due to the current in two wires connected to the two terminals of a source of emf and twisted tightly around each other.
3. How can you decide if a wire is infinite?
4. Identical currents are carried in two circular loops; however, one loop has twice the diameter as the other loop. Compare the magnetic fields created by the loops at the center of each loop.

12.2 Magnetic Field Due to a Thin Straight Wire

5. How would you orient two long, straight, current-carrying wires so that there is no net magnetic force between them? (*Hint*: What orientation would lead to one wire not experiencing a magnetic field from the other?)

12.3 Magnetic Force between Two Parallel Currents

6. Compare and contrast the electric field of an infinite line of charge and the magnetic field of an infinite line of current.
7. Is $\vec{\mathbf{B}}$ constant in magnitude for points that lie on a magnetic field line?

12.4 Magnetic Field of a Current Loop

8. Is the magnetic field of a current loop uniform?
9. What happens to the length of a suspended spring when a current passes through it?
10. Two concentric circular wires with different diameters carry currents in the same direction. Describe the force on the inner wire.

12.5 Ampère's Law

11. Is Ampère's law valid for all closed paths? Why isn't it normally useful for calculating a magnetic field?

12.6 Solenoids and Toroids

12. Is the magnetic field inside a toroid completely uniform? Almost uniform?
13. Explain why $\vec{\mathbf{B}} = 0$ inside a long, hollow copper pipe that is carrying an electric current parallel to the axis. Is $\vec{\mathbf{B}} = 0$ outside the pipe?

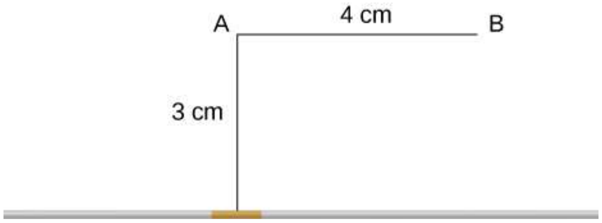
12.7 Magnetism in Matter

14. A diamagnetic material is brought close to a permanent magnet. What happens to the material?
15. If you cut a bar magnet into two pieces, will you end up with one magnet with an isolated north pole and another magnet with an isolated south pole? Explain your answer.

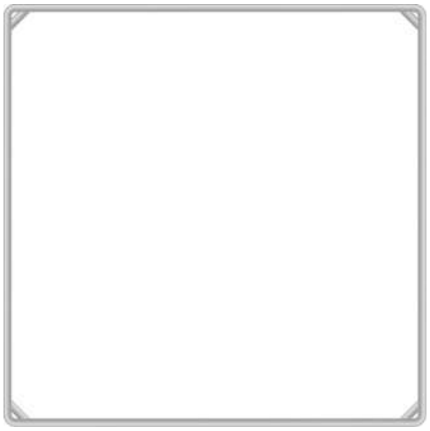
Problems

12.1 The Biot-Savart Law

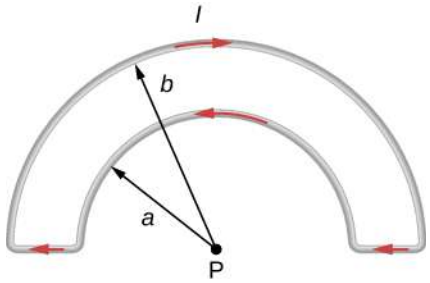
16. A 10-A current flows through the wire shown. What is the magnitude of the magnetic field due to a 0.5-mm segment of wire as measured at (a) point A and (b) point B?



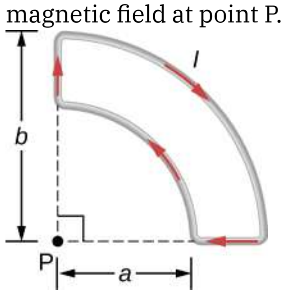
17. Ten amps flow through a square loop where each side is 20 cm in length. At each corner of the loop is a 0.01-cm segment that connects the longer wires as shown. Calculate the magnitude of the magnetic field at the center of the loop.



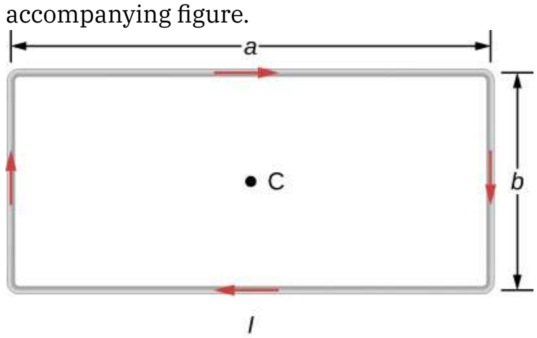
18. What is the magnetic field at P due to the current I in the wire shown?



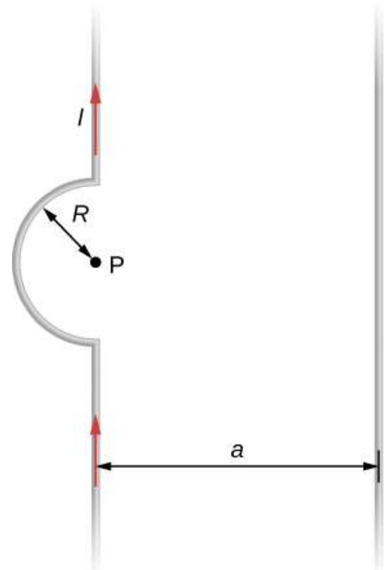
19. The accompanying figure shows a current loop consisting of two concentric circular arcs and two perpendicular radial lines. Determine the magnetic field at point P.



20. Find the magnetic field at the center C of the rectangular loop of wire shown in the accompanying figure.



21. Two long wires, one of which has a semicircular bend of radius R, are positioned as shown in the accompanying figure. If both wires carry a current I, how far apart must their parallel sections be so that the net magnetic field at P is zero? Does the current in the straight wire flow up or down?



12.2 Magnetic Field Due to a Thin Straight Wire

22. A typical current in a lightning bolt is 10^4 A. Estimate the magnetic field 1 m from the bolt.

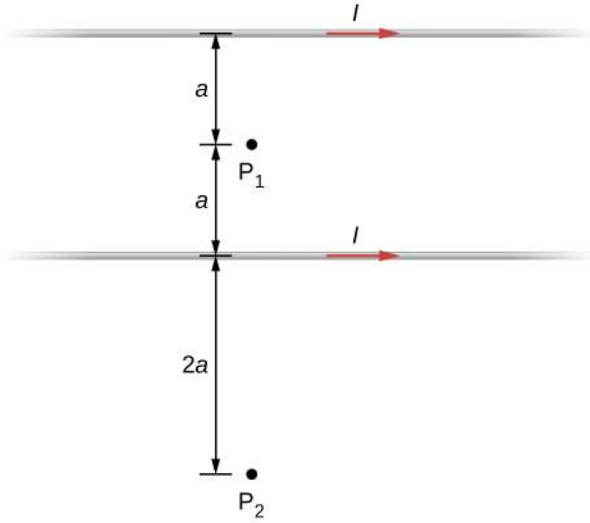
23. The magnitude of the magnetic field 50 cm from a long, thin, straight wire is $8.0\ \mu\text{T}$. What is the current through the long wire?

24. A transmission line strung 7.0 m above the ground carries a current of 500 A. What is the magnetic field on the ground directly below the wire? Compare your answer with the magnetic field of Earth.

25. A long, straight, horizontal wire carries a left-to-right current of 20 A. If the wire is placed in a uniform magnetic field of magnitude $4.0 \times 10^{-5}\,\text{T}$ that is directed vertically downward, what is the resultant magnitude of the magnetic field 20 cm above the wire? 20 cm below the wire?

26. The two long, parallel wires shown in the accompanying figure carry currents in the same direction. If $I_1 = 10$ A and $I_2 = 20$ A, what is the magnetic field at point P?

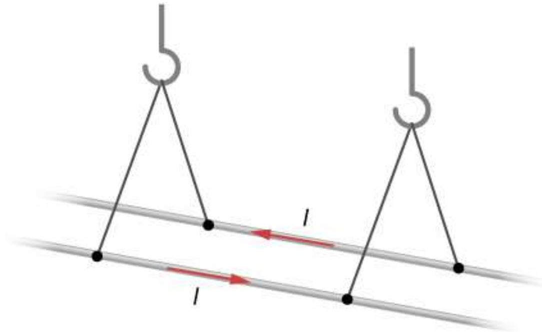
27. The accompanying figure shows two long, straight, horizontal wires that are parallel and a distance $2a$ apart. If both wires carry current I in the same direction, (a) what is the magnetic field at P_1? (b) P_2?



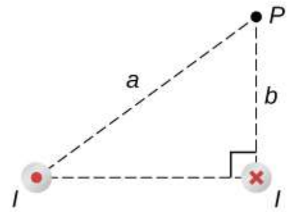
28. Repeat the calculations of the preceding problem with the direction of the current in the lower wire reversed.
29. Consider the area between the wires of the preceding problem. At what distance from the top wire is the net magnetic field a minimum? Assume that the currents are equal and flow in opposite directions.

12.3 Magnetic Force between Two Parallel Currents

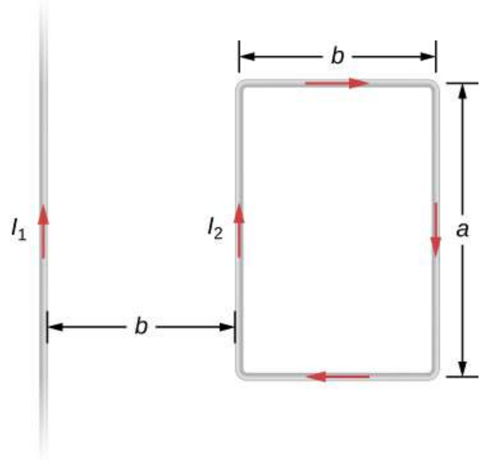
30. Two long, straight wires are parallel and 25 cm apart. (a) If each wire carries a current of 50 A in the same direction, what is the magnetic force per meter exerted on each wire? (b) Does the force pull the wires together or push them apart? (c) What happens if the currents flow in opposite directions?
31. Two long, straight wires are parallel and 10 cm apart. One carries a current of 2.0 A, the other a current of 5.0 A. (a) If the two currents flow in opposite directions, what is the magnitude and direction of the force per unit length of one wire on the other? (b) What is the magnitude and direction of the force per unit length if the currents flow in the same direction?
32. Two long, parallel wires are hung by cords of length 5.0 cm, as shown in the accompanying figure. Each wire has a mass per unit length of 30 g/m, and they carry the same current in opposite directions. What is the current if the cords hang at 6.0° with respect to the vertical?



33. A circuit with current I has two long parallel wire sections that carry current in opposite directions. Find magnetic field at a point P near these wires that is a distance a from one wire and b from the other wire as shown in the figure.



34. The infinite, straight wire shown in the accompanying figure carries a current I_1. The rectangular loop, whose long sides are parallel to the wire, carries a current I_2. What are the magnitude and direction of the force on the rectangular loop due to the magnetic field of the wire?



12.4 Magnetic Field of a Current Loop

35. When the current through a circular loop is 6.0 A, the magnetic field at its center is $2.0 \times 10^{-4}\,\text{T}$. What is the radius of the loop?
36. How many turns must be wound on a flat, circular coil of radius 20 cm in order to produce a magnetic field of magnitude $4.0 \times 10^{-5}\,\text{T}$ at

the center of the coil when the current through it is 0.85 A?

37. A flat, circular loop has 20 turns. The radius of the loop is 10.0 cm and the current through the wire is 0.50 A. Determine the magnitude of the magnetic field at the center of the loop.

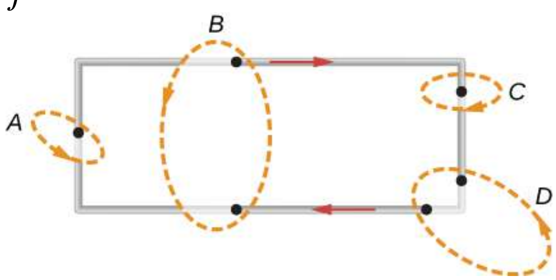
38. A circular loop of radius R carries a current I. At what distance along the axis of the loop is the magnetic field one-half its value at the center of the loop?

39. Two flat, circular coils, each with a radius R and wound with N turns, are mounted along the same axis so that they are parallel a distance d apart. What is the magnetic field at the midpoint of the common axis if a current I flows in the same direction through each coil?

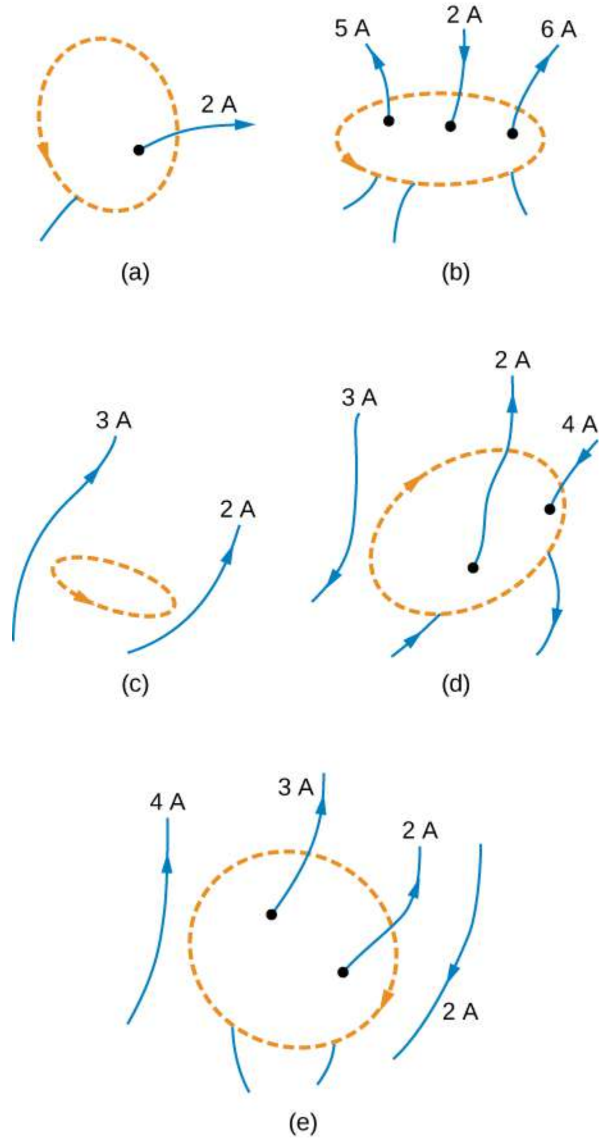
40. For the coils in the preceding problem, what is the magnetic field at the center of either coil?

12.5 Ampère's Law

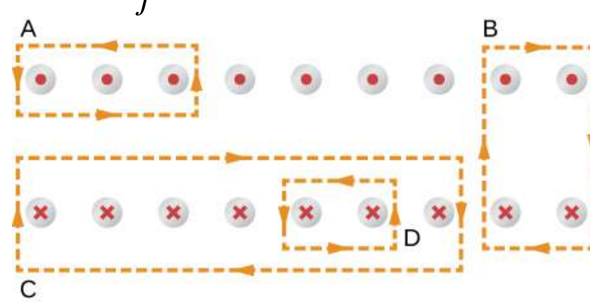
41. A current I flows around the rectangular loop shown in the accompanying figure. Evaluate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ for the paths A, B, C, and D.



42. Evaluate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ for each of the cases shown in the accompanying figure.



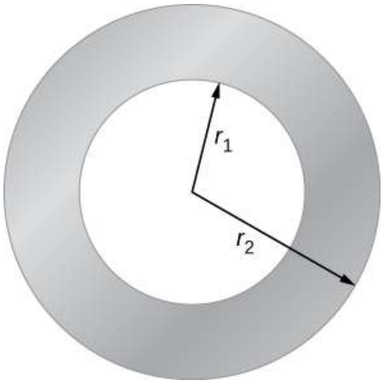
43. The coil whose lengthwise cross section is shown in the accompanying figure carries a current I and has N evenly spaced turns distributed along the length l. Evaluate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ for the paths indicated.



44. A superconducting wire of diameter 0.25 cm carries a current of 1000 A. What is the magnetic field just outside the wire?

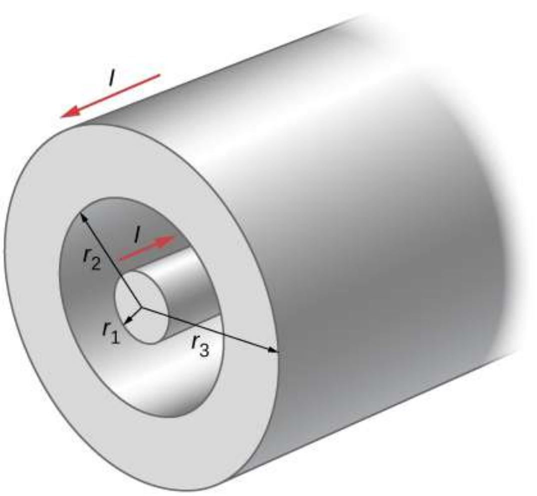
45. A long, straight wire of radius R carries a current I that is distributed uniformly over the cross-section of the wire. At what distance from the axis of the wire is the magnitude of the magnetic field a maximum?

46. The accompanying figure shows a cross-section of a long, hollow, cylindrical conductor of inner radius $r_1 = 3.0$ cm and outer radius $r_2 = 5.0$ cm. A 50-A current distributed uniformly over the cross-section flows into the page. Calculate the magnetic field at $r = 2.0$ cm, $r = 4.0$ cm, and $r = 6.0$ cm.



47. A long, solid, cylindrical conductor of radius 3.0 cm carries a current of 50 A distributed uniformly over its cross-section. Plot the magnetic field as a function of the radial distance r from the center of the conductor.

48. A portion of a long, cylindrical coaxial cable is shown in the accompanying figure. A current I flows down the center conductor, and this current is returned in the outer conductor. Determine the magnetic field in the regions (a) $r \le r_1$, (b) $r_2 \ge r \ge r_1$, (c) $r_3 \ge r \ge r_2$, and (d) $r \ge r_3$. Assume that the current is distributed uniformly over the cross sections of the two parts of the cable.



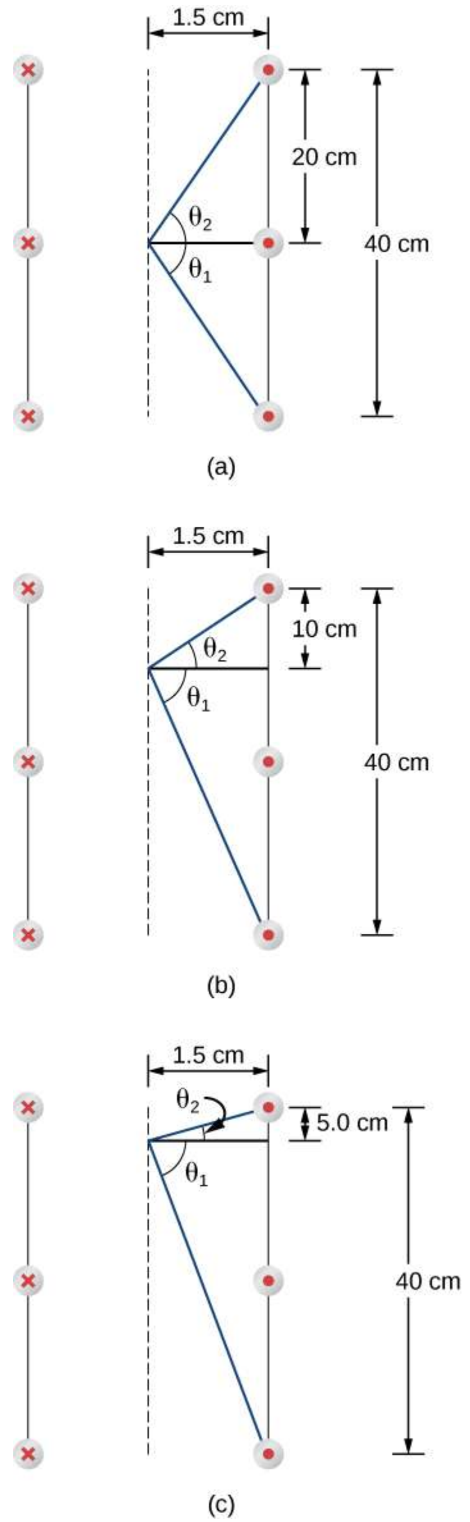
12.6 Solenoids and Toroids

49. A solenoid is wound with 2000 turns per meter. When the current is 5.2 A, what is the magnetic field within the solenoid?

50. A solenoid has 12 turns per centimeter. What current will produce a magnetic field of $2.0 \times 10^{-2}\,\text{T}$ within the solenoid?

51. If a current is 2.0 A, how many turns per centimeter must be wound on a solenoid in order to produce a magnetic field of $2.0 \times 10^{-3}\,\text{T}$ within it?

52. A solenoid is 40 cm long, has a diameter of 3.0 cm, and is wound with 500 turns. If the current through the windings is 4.0 A, what is the magnetic field at a point on the axis of the solenoid that is (a) at the center of the solenoid, (b) 10.0 cm from one end of the solenoid, and (c) 5.0 cm from one end of the solenoid? (d) Compare these answers with the infinite-solenoid case.



53. Determine the magnetic field on the central axis at the opening of a semi-infinite solenoid. (That is, take the opening to be at $x = 0$ and the other end to be at

$x = \infty$.)

54. By how much is the approximation $B = \mu_0 nI$ in error at the center of a solenoid that is 15.0 cm long, has a diameter of 4.0 cm, is wrapped with n turns per meter, and carries a current I?

55. A solenoid with 25 turns per centimeter carries a current I. An electron moves within the solenoid in a circle that has a radius of 2.0 cm and is perpendicular to the axis of the solenoid. If the speed of the electron is 2.0×10^5 m/s, what is I?

56. A toroid has 250 turns of wire and carries a current of 20 A. Its inner and outer radii are 8.0 and 9.0 cm. What are the values of its magnetic field at $r = 8.1, 8.5,$ and 8.9 cm?

57. A toroid with a square cross section 3.0 cm $\times$ 3.0 cm has an inner radius of 25.0 cm. It is wound with 500 turns of wire, and it carries a current of 2.0 A. What is the strength of the magnetic field at the center of the square cross section?

12.7 Magnetism in Matter

58. The magnetic field in the core of an air-filled solenoid is 1.50 T. By how much will this magnetic field decrease if the air is pumped out of the core while the current is held constant?

59. A solenoid has a ferromagnetic core, $n = 1000$ turns per meter, and $I = 5.0$ A. If B inside the solenoid is 2.0 T, what is χ for the core material?

60. A 20-A current flows through a solenoid with 2000 turns per meter. What is the magnetic field inside the solenoid if its core is (a) a vacuum and (b) filled with liquid oxygen at 90 K?

61. The magnetic dipole moment of the iron atom is about $2.1 \times 10^{-23} \text{A} \cdot \text{m}^2$. (a) Calculate the maximum magnetic dipole moment of a domain consisting of 10^{19} iron atoms. (b) What current would have to flow through a single circular loop of wire of diameter 1.0 cm to produce this magnetic dipole moment?

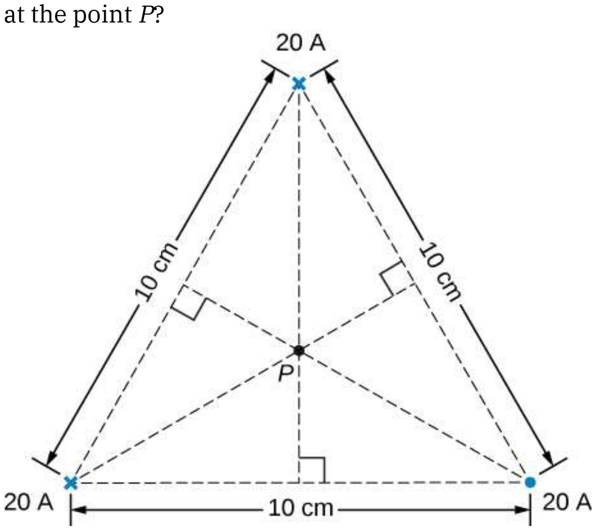
62. Suppose you wish to produce a 1.2-T magnetic field in a toroid with an iron core for which $\chi = 4.0 \times 10^3$. The toroid has a mean radius of 15 cm and is wound with 500 turns. What current is required?

63. A current of 1.5 A flows through the windings of a large, thin toroid with 200 turns per meter and a radius of 1 meter. If the toroid is filled with iron for which $\chi = 3.0 \times 10^3$, what is the magnetic field within it?

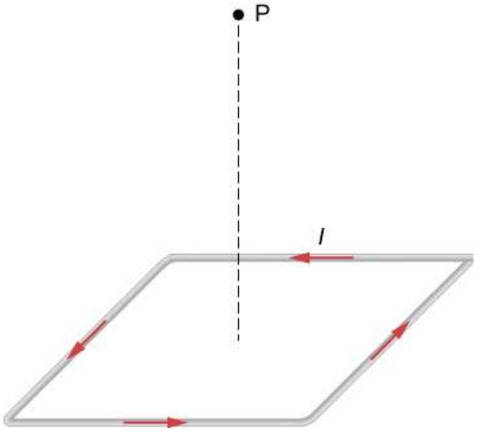
64. A solenoid with an iron core is 25 cm long and is wrapped with 100 turns of wire. When the current through the solenoid is 10 A, the magnetic field inside it is 2.0 T. For this current, what is the permeability of the iron? If the current is turned off and then restored to 10 A, will the magnetic field necessarily return to 2.0 T?

Additional Problems

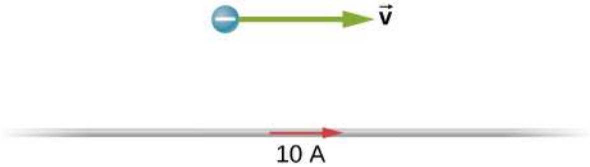
65. Three long, straight, parallel wires, all carrying 20 A, are positioned as shown in the accompanying figure. What is the magnitude of the magnetic field at the point *P*?



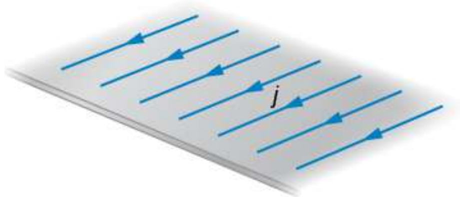
66. A current *I* flows around a wire bent into the shape of a square of side *a*. What is the magnetic field at the point P that is a distance *z* above the center of the square (see the accompanying figure)?



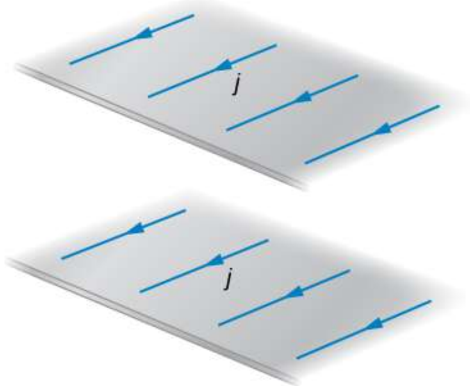
67. The accompanying figure shows a long, straight wire carrying a current of 10 A. What is the magnetic force on an electron at the instant it is 20 cm from the wire, traveling parallel to the wire with a speed of 2.0×10^5 m/s? Describe qualitatively the subsequent motion of the electron.



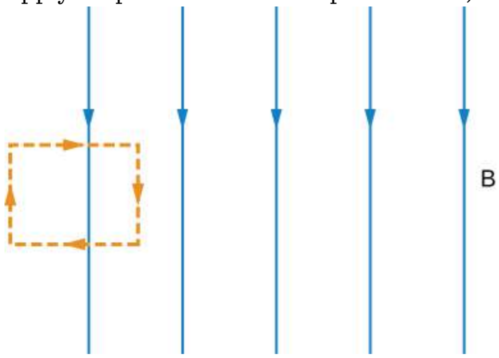
68. Current flows along a thin, infinite sheet as shown in the accompanying figure. The current per unit length along the sheet is *J* in amperes per meter. (a) Use the Biot-Savart law to show that $B = \mu_0 J/2$ on either side of the sheet. What is the direction of $\vec{\mathbf{B}}$ on each side? (b) Now use Ampère's law to calculate the field.



69. (a) Use the result of the previous problem to calculate the magnetic field between, above, and below the pair of infinite sheets shown in the accompanying figure. (b) Repeat your calculations if the direction of the current in the lower sheet is reversed.



70. We often assume that the magnetic field is uniform in a region and zero everywhere else. Show that in reality it is impossible for a magnetic field to drop abruptly to zero, as illustrated in the accompanying figure. (*Hint*: Apply Ampère's law over the path shown.)



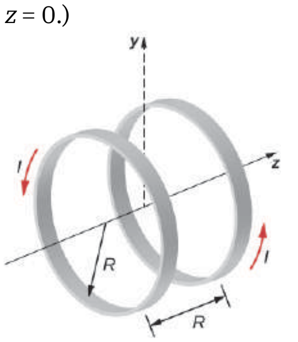
71. How is the fractional change in the strength of the magnetic field across the face of the toroid related to the fractional change in the radial distance from the axis of the toroid?

72. Show that the expression for the magnetic field of a toroid reduces to that for the field of an infinite solenoid in the limit that the central radius goes to infinity.

73. A toroid with an inner radius of 20 cm and an outer radius of 22 cm is tightly wound with one layer of wire that has a diameter of 0.25 mm. (a) How many turns are there on the toroid? (b) If the current through the toroid windings is 2.0 A, what is the strength of the magnetic field at the center of the toroid?

74. A wire element has $d\vec{\mathbf{l}}$, $Id\vec{\mathbf{l}} = \mathbf{J}Adl = \mathbf{J}dv$, where A and dv are the cross-sectional area and volume of the element, respectively. Use this, the Biot-Savart law, and $\mathbf{J} = ne\mathbf{v}$ to show that the magnetic field of a moving point charge q is given by:
$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$

75. A reasonably uniform magnetic field over a limited region of space can be produced with the Helmholtz coil, which consists of two parallel coils centered on the same axis. The coils are connected so that they carry the same current *I*. Each coil has *N* turns and radius *R*, which is also the distance between the coils. (a) Find the magnetic field at any point on the *z*-axis shown in the accompanying figure. (b) Show that dB/dz and d^2B/dz^2 are both zero at $z = 0$. (These vanishing derivatives demonstrate that the magnetic field varies only slightly near $z = 0$.)

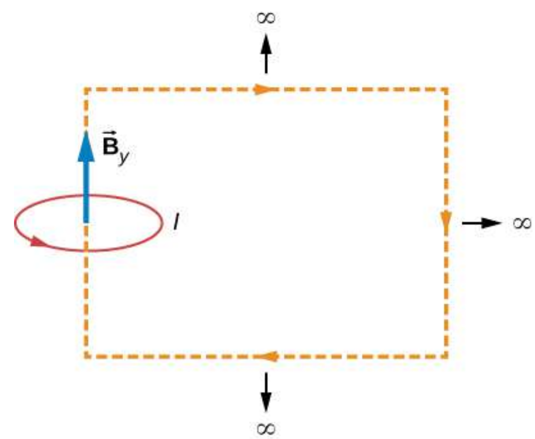


76. A charge of 4.0 μC is distributed uniformly around a thin ring of insulating material. The ring has a radius of 0.20 m and rotates at 2.0×10^4 rev/min around the axis that passes through its center and is perpendicular to the plane of the ring. What is the magnetic field at the center of the ring?

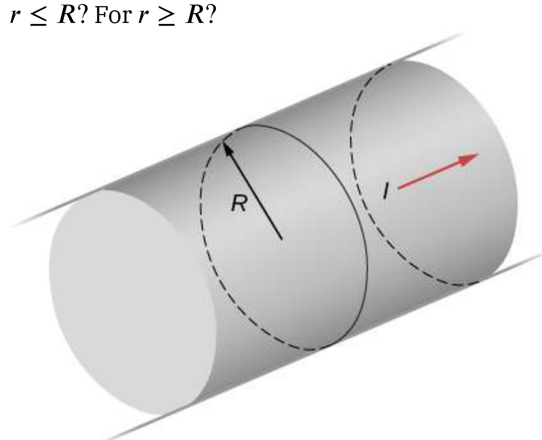
77. A thin, nonconducting disk of radius *R* is free to rotate around the axis that passes through its center and is perpendicular to the face of the disk. The disk is charged uniformly with a total charge *q*. If the disk rotates at a constant angular velocity ω, what is the magnetic field at its center?

78. Consider the disk in the previous problem. Calculate the magnetic field at a point on its central axis that is a distance *y* above the disk.

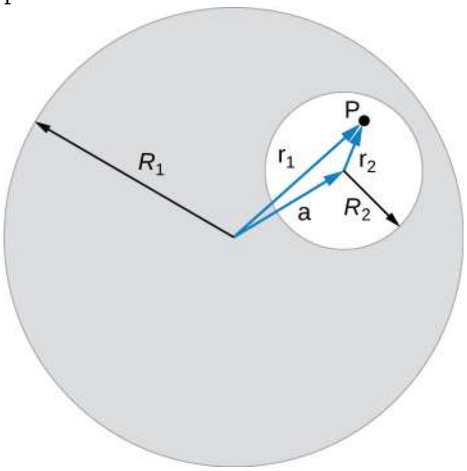
79. Consider the axial magnetic field $B_y = \mu_0 IR^2/2(y^2 + R^2)^{3/2}$ of the circular current loop shown below. (a) Evaluate $\int_{-a}^{a} B_y dy$. Also show that $\lim_{a\to\infty}\int_{-a}^{a} B_y dy = \mu_0 I$. (b) Can you deduce this limit without evaluating the integral? (*Hint:* See the accompanying figure.)



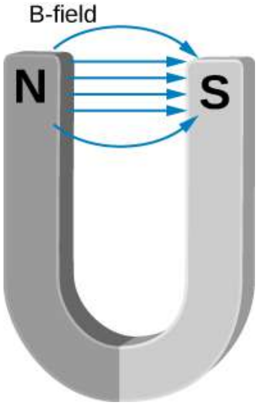
80. The current density in the long, cylindrical wire shown in the accompanying figure varies with distance *r* from the center of the wire according to $J = cr$, where *c* is a constant. (a) What is the current through the wire? (b) What is the magnetic field produced by this current for $r \le R$? For $r \ge R$?



81. A long, straight, cylindrical conductor contains a cylindrical cavity whose axis is displaced by a from the axis of the conductor, as shown in the accompanying figure. The current density in the conductor is given by $\vec{\mathbf{J}} = J_0\hat{\mathbf{k}}$, where J_0 is a constant and $\hat{\mathbf{k}}$ is along the axis of the conductor. Calculate the magnetic field at an arbitrary point P in the cavity by superimposing the field of a solid cylindrical conductor with radius R_1 and current density $\vec{\mathbf{J}}$ onto the field of a solid cylindrical conductor with radius R_2 and current density $-\vec{\mathbf{J}}$. Then use the fact that the appropriate azimuthal unit vectors can be expressed as $\hat{\theta}_1 = \hat{k} \times \hat{r}_1$ and $\hat{\theta}_2 = \hat{k} \times \hat{r}_2$ to show that everywhere inside the cavity the magnetic field is given by the constant $\vec{\mathbf{B}} = \frac{1}{2}\mu_0 J_0 \mathbf{k} \times \mathbf{a}$, where $\mathbf{a} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{r}_1 = r_1\hat{r}_1$ is the position of P relative to the center of the conductor and $\mathbf{r}_2 = r_2\hat{r}_2$ is the position of P relative to the center of the cavity.

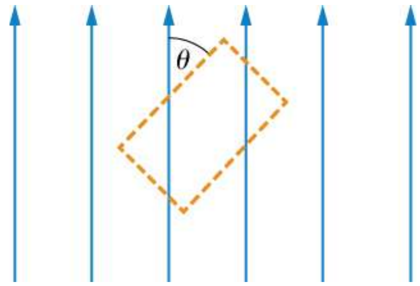


82. Between the two ends of a horseshoe magnet the field is uniform as shown in the diagram. As you move out to outside edges, the field bends. Show by Ampère's law that the field must bend and thereby the field weakens due to these bends.



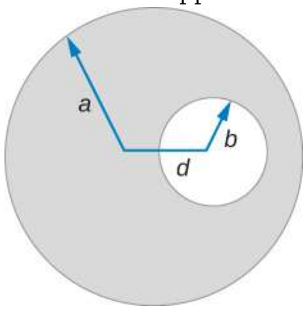
83. Show that the magnetic field of a thin wire and that of a current loop are zero if you are infinitely far away.

84. An Ampère loop is chosen as shown by dashed lines for a parallel constant magnetic field as shown by solid arrows. Calculate $\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ for each side of the loop then find the entire $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$. Can you think of an Ampère loop that would make the problem easier? Do those results match these?

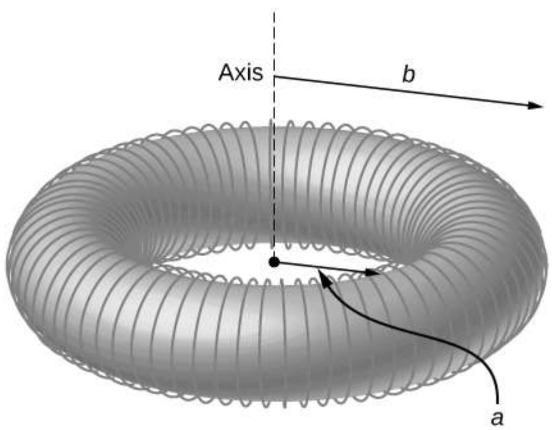


85. A very long, thick cylindrical wire of radius R carries a current density J that varies across its cross-section. The magnitude of the current density at a point a distance r from the center of the wire is given by $J = J_0 \frac{r}{R}$, where J_0 is a constant. Find the magnetic field (a) at a point outside the wire and (b) at a point inside the wire. Write your answer in terms of the net current I through the wire.

86. A very long, cylindrical wire of radius a has a circular hole of radius b in it at a distance d from the center. The wire carries a uniform current of magnitude I through it. The direction of the current in the figure is out of the paper. Find the magnetic field (a) at a point at the edge of the hole closest to the center of the thick wire, (b) at an arbitrary point inside the hole, and (c) at an arbitrary point outside the wire. (*Hint:* Think of the hole as a sum of two wires carrying current in the opposite directions.)



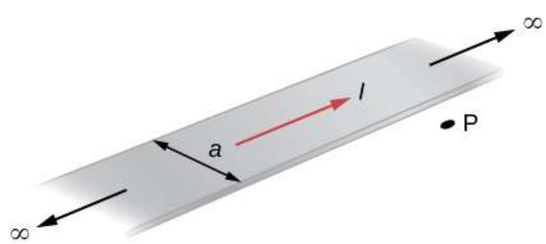
87. Magnetic field inside a torus. Consider a torus of rectangular cross-section with inner radius *a* and outer radius *b*. *N* turns of an insulated thin wire are wound evenly on the torus tightly all around the torus and connected to a battery producing a steady current *I* in the wire. Assume that the current on the top and bottom surfaces in the figure is radial, and the current on the inner and outer radii surfaces is vertical. Find the magnetic field inside the torus as a function of radial distance *r* from the axis.



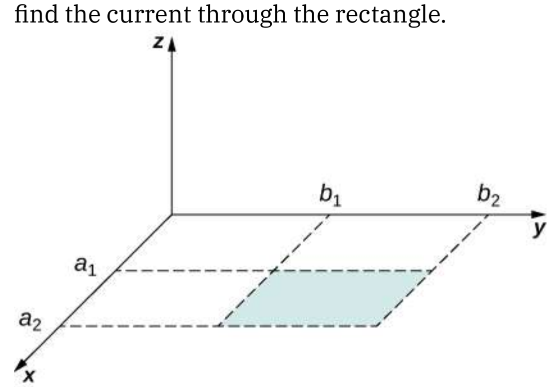
88. Two long coaxial copper tubes, each of length L, are connected to a battery of voltage *V*. The inner tube has inner radius *a* and outer radius *b*, and the outer tube has inner radius *c* and outer radius *d*. The tubes are then disconnected from the battery and rotated in the same direction at angular speed of ω radians per second about their common axis. Find the magnetic field (a) at a point inside the space enclosed by the inner tube $r < a$, and (b) at a point between the tubes $b < r < c$, and (c) at a point outside the tubes $r > d$. (*Hint:* Think of copper tubes as a capacitor and find the charge density based on the voltage applied, $Q = VC$, $C = \frac{2\pi\varepsilon_0 L}{\ln(c/b)}$.)

Challenge Problems

89. The accompanying figure shows a flat, infinitely long sheet of width *a* that carries a current *I* uniformly distributed across it. Find the magnetic field at the point P, which is in the plane of the sheet and at a distance *x* from one edge. Test your result for the limit $a \to 0$.



90. A hypothetical current flowing in the *z*-direction creates the field $\vec{\mathbf{B}} = C\left[\left(x/y^2\right)\hat{\mathbf{i}} + (1/y)\hat{\mathbf{j}}\right]$ in the rectangular region of the *xy*-plane shown in the accompanying figure. Use Ampère's law to find the current through the rectangle.



91. A nonconducting hard rubber circular disk of radius R is painted with a uniform surface charge density σ. It is rotated about its axis with angular speed ω. (a) Find the magnetic field produced at a point on the axis a distance h meters from the center of the disk. (b) Find the numerical value of magnitude of the magnetic field when $\sigma = 1\text{C/m}^2$, $R = 20$ cm, $h = 2$ cm, and $\omega = 400$ rad/sec, and compare it with the magnitude of magnetic field of Earth, which is about 1/2 Gauss.

CHAPTER 13
Electromagnetic Induction

Figure 13.1 The black strip found on the back of credit cards and driver's licenses is a very thin layer of magnetic material with information stored on it. Reading and writing the information on the credit card is done with a swiping motion. The physical reason why this is necessary is called electromagnetic induction and is discussed in this chapter. (credit: modification of work by Jane Whitney)

Chapter Outline

INTRODUCTION We have been considering electric fields created by fixed charge distributions and magnetic fields produced by constant currents, but electromagnetic phenomena are not restricted to these stationary situations. Most of the interesting applications of electromagnetism are, in fact, time-dependent. To investigate some of these applications, we now remove the time-independent assumption that we have been making and allow the fields to vary with time. In this and the next several chapters, you will see a wonderful symmetry in the behavior exhibited by time-varying electric and magnetic fields. Mathematically, this symmetry is expressed by an additional term in Ampère's law and by another key equation of electromagnetism called Faraday's law. We also discuss how moving a wire through a magnetic field produces an emf or voltage. Lastly, we describe applications of these principles, such as the card reader shown above.

13.1 Faraday's Law

Learning Objectives

By the end of this section, you will be able to:

- Determine the magnetic flux through a surface, knowing the strength of the magnetic field, the surface area, and the angle between the normal to the surface and the magnetic field
- Use Faraday's law to determine the magnitude of induced emf in a closed loop due to changing magnetic flux through the loop

The first productive experiments concerning the effects of time-varying magnetic fields were performed by Michael Faraday in 1831. One of his early experiments is represented in Figure 13.2. An emf is induced when the magnetic field in the coil is changed by pushing a bar magnet into or out of the coil. Emfs of opposite signs are produced by motion in opposite directions, and the directions of emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.

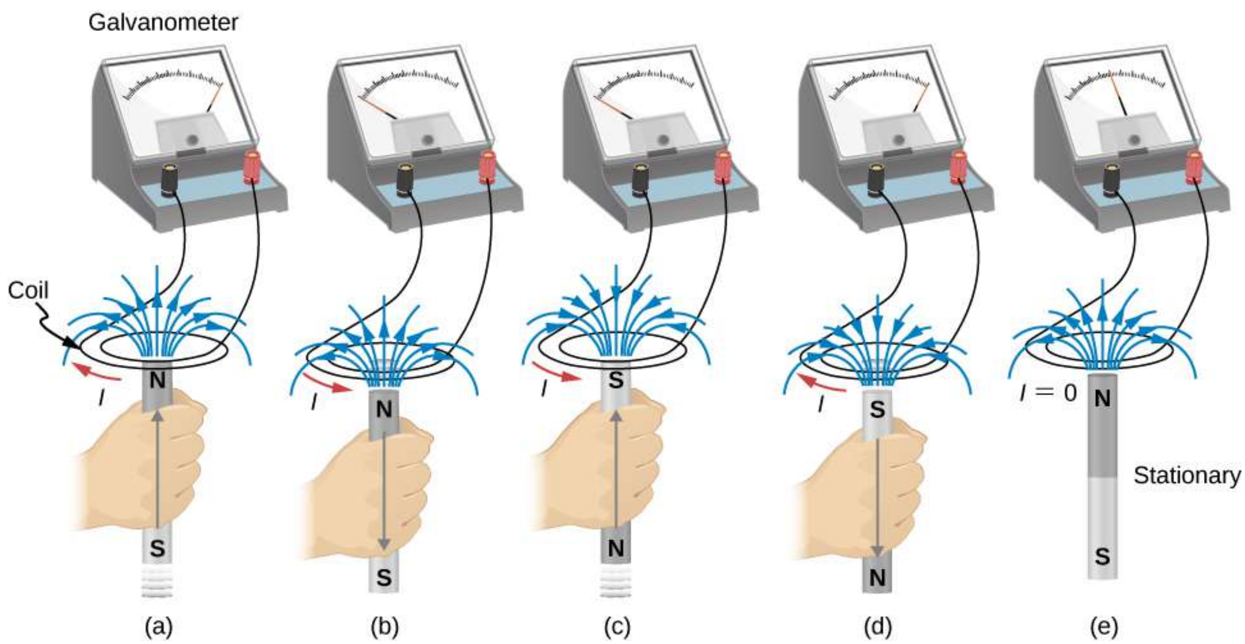


Figure 13.2 Movement of a magnet relative to a coil produces emfs as shown (a–d). The same emfs are produced if the coil is moved relative to the magnet. This short-lived emf is only present during the motion. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion, as shown in (e).

Faraday also discovered that a similar effect can be produced using two circuits—a changing current in one circuit induces a current in a second, nearby circuit. For example, when the switch is closed in circuit 1 of Figure 13.3(a), the ammeter needle of circuit 2 momentarily deflects, indicating that a short-lived current surge has been induced in that circuit. The ammeter needle quickly returns to its original position, where it remains. However, if the switch of circuit 1 is now suddenly opened, another short-lived current surge in the direction opposite from before is observed in circuit 2.

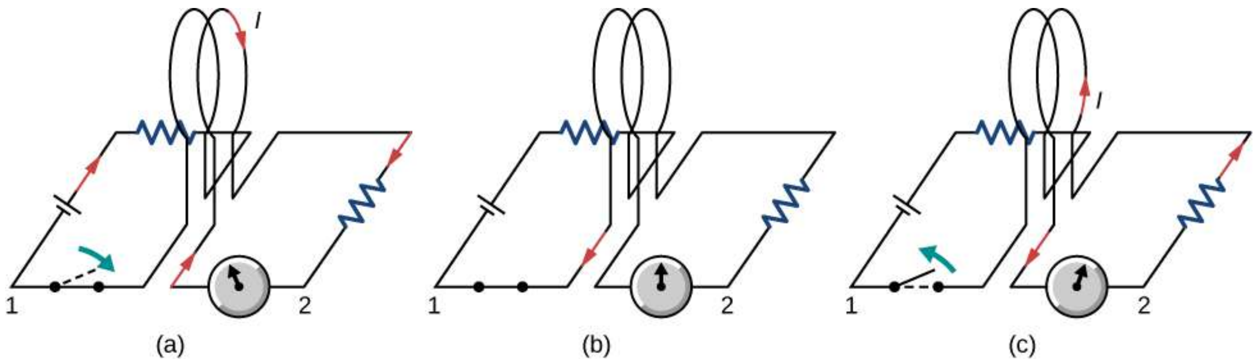


Figure 13.3 (a) Closing the switch of circuit 1 produces a short-lived current surge in circuit 2. (b) If the switch remains closed, no current is observed in circuit 2. (c) Opening the switch again produces a short-lived current in circuit 2 but in the opposite direction from before.

Faraday realized that in both experiments, a current flowed in the circuit containing the ammeter only when the magnetic field in the region occupied by that circuit was *changing*. As the magnet of the figure was moved, the strength of its magnetic field at the loop changed; and when the current in circuit 1 was turned on or off, the strength of its magnetic field at circuit 2 changed. Faraday was eventually able to interpret these and all other experiments involving magnetic fields that vary with time in terms of the following law:

Faraday's Law

The emf ε induced is the negative change in the magnetic flux Φ_{m} per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

The **magnetic flux** is a measurement of the amount of magnetic field lines through a given surface area, as seen in Figure 13.4. This definition is similar to the electric flux studied earlier. This means that if we have

$$\Phi_{\mathrm{m}} = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA, \tag{13.1}$$

then the **induced emf** or the voltage generated by a conductor or coil moving in a magnetic field is

$$\varepsilon = -\frac{d}{dt} \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = -\frac{d\Phi_{\mathrm{m}}}{dt}. \tag{13.2}$$

The negative sign describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with a rule known as Lenz's law, which we will discuss shortly.

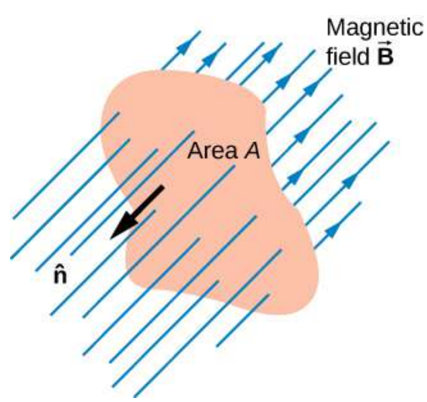


Figure 13.4 The magnetic flux is the amount of magnetic field lines cutting through a surface area A defined by the unit area vector $\hat{\mathbf{n}}$. If

the angle between the unit area $\mathbf{\hat{n}}$ and magnetic field vector $\vec{\mathbf{B}}$ are parallel or antiparallel, as shown in the diagram, the magnetic flux is the highest possible value given the values of area and magnetic field.

Part (a) of Figure 13.5 depicts a circuit and an arbitrary surface S that it bounds. Notice that S is an *open surface*. It can be shown that *any* open surface bounded by the circuit in question can be used to evaluate Φ_m. For example, Φ_m is the same for the various surfaces $S_1, S_2, \ldots$ of part (b) of the figure.

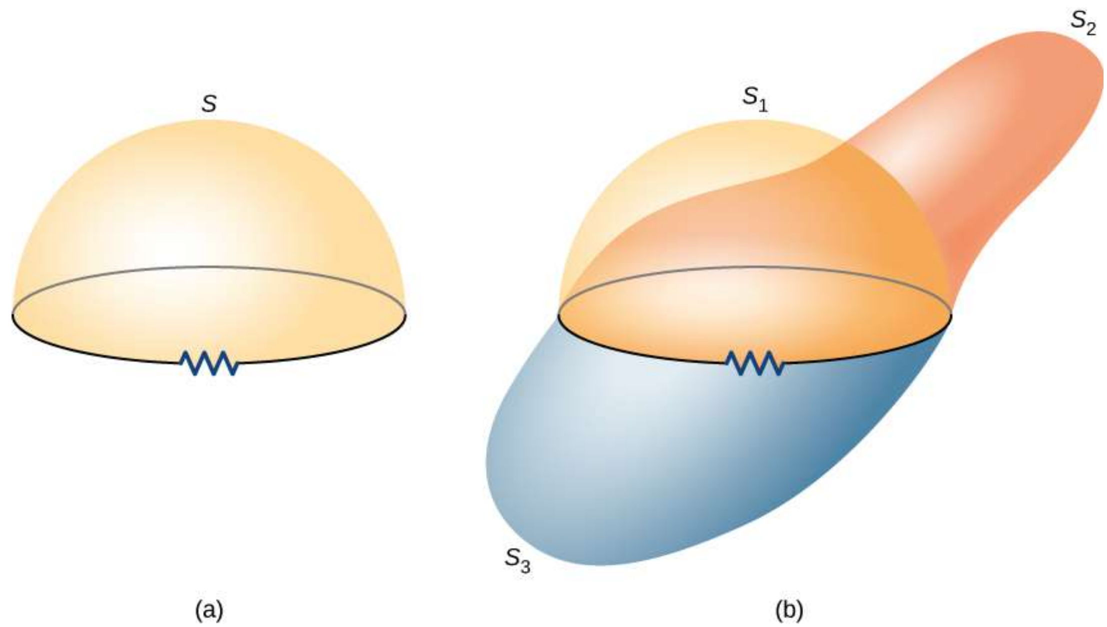


Figure 13.5 (a) A circuit bounding an arbitrary open surface *S*. The planar area bounded by the circuit is not part of *S*. (b) Three arbitrary open surfaces bounded by the same circuit. The value of Φ_m is the same for all these surfaces.

The SI unit for magnetic flux is the weber (Wb),

$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

Occasionally, the magnetic field unit is expressed as webers per square meter (Wb/m^2) instead of teslas, based on this definition. In many practical applications, the circuit of interest consists of a number N of tightly wound turns (see Figure 13.6). Each turn experiences the same magnetic flux. Therefore, the net magnetic flux through the circuits is N times the flux through one turn, and Faraday's law is written as

$$\varepsilon = -\frac{d}{dt}(N\Phi_m) = -N\frac{d\Phi_m}{dt}. \quad 13.3$$

EXAMPLE 13.1

A Square Coil in a Changing Magnetic Field

The square coil of Figure 13.6 has sides $l = 0.25$ m long and is tightly wound with $N = 200$ turns of wire. The resistance of the coil is $R = 5.0\ \Omega$. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt = -0.040$ T/s. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

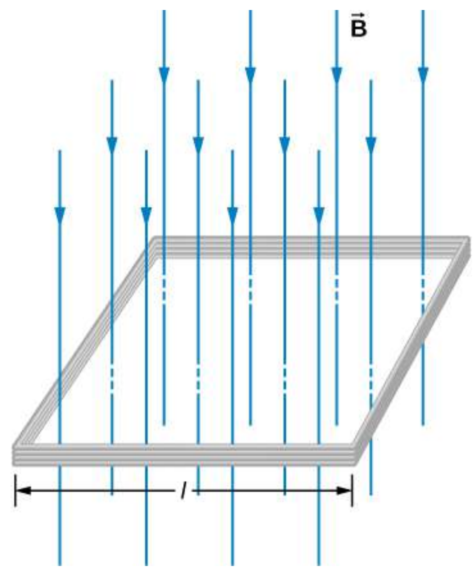


Figure 13.6 A square coil with N turns of wire with uniform magnetic field $\vec{\mathbf{B}}$ directed in the downward direction, perpendicular to the coil.

Strategy

The area vector, or $\hat{\mathbf{n}}$ direction, is perpendicular to area covering the loop. We will choose this to be pointing downward so that $\vec{\mathbf{B}}$ is parallel to $\hat{\mathbf{n}}$ and that the flux turns into multiplication of magnetic field times area. The area of the loop is not changing in time, so it can be factored out of the time derivative, leaving the magnetic field as the only quantity varying in time. Lastly, we can apply Ohm's law once we know the induced emf to find the current in the loop.

Solution

a. The flux through one turn is

$$\Phi_m = BA = Bl^2,$$

so we can calculate the magnitude of the emf from Faraday's law. The sign of the emf will be discussed in the next section, on Lenz's law:

$$\begin{aligned} |\varepsilon| &= \left|-N\frac{d\Phi_m}{dt}\right| = Nl^2\frac{dB}{dt} \\ &= (200)(0.25\text{ m})^2(0.040\text{ T/s}) = 0.50\text{ V}. \end{aligned}$$

b. The magnitude of the current induced in the coil is

$$I = \frac{\varepsilon}{R} = \frac{0.50\text{ V}}{5.0\,\Omega} = 0.10\text{ A}.$$

Significance

If the area of the loop were changing in time, we would not be able to pull it out of the time derivative. Since the loop is a closed path, the result of this current would be a small amount of heating of the wires until the magnetic field stops changing. This may increase the area of the loop slightly as the wires are heated.

CHECK YOUR UNDERSTANDING 13.1

A closely wound coil has a radius of 4.0 cm, 50 turns, and a total resistance of $40\ \Omega$. At what rate must a magnetic field perpendicular to the face of the coil change in order to produce Joule heating in the coil at a rate of 2.0 mW?

13.2 Lenz's Law

Learning Objectives

By the end of this section, you will be able to:

- Use Lenz's law to determine the direction of induced emf whenever a magnetic flux changes
- Use Faraday's law with Lenz's law to determine the induced emf in a coil and in a solenoid

The direction in which the induced emf drives current around a wire loop can be found through the negative sign. However, it is usually easier to determine this direction with **Lenz's law**, named in honor of its discoverer, Heinrich Lenz (1804–1865). (Faraday also discovered this law, independently of Lenz.) We state Lenz's law as follows:

Lenz's Law

The direction of the induced emf drives current around a wire loop to always *oppose* the change in magnetic flux that causes the emf.

Lenz's law can also be considered in terms of conservation of energy. If pushing a magnet into a coil causes current, the energy in that current must have come from somewhere. If the induced current causes a magnetic field opposing the increase in field of the magnet we pushed in, then the situation is clear. We pushed a magnet against a field and did work on the system, and that showed up as current. If it were not the case that the induced field opposes the change in the flux, the magnet would be pulled in produce a current without anything having done work. Electric potential energy would have been created, violating the conservation of energy.

To determine an induced emf ε, you first calculate the magnetic flux Φ_m and then obtain $d\Phi_m/dt$. The magnitude of ε is given by $\varepsilon = |d\Phi_m/dt|$. Finally, you can apply Lenz's law to determine the sense of ε. This will be developed through examples that illustrate the following problem-solving strategy.

PROBLEM-SOLVING STRATEGY

Lenz's Law

To use Lenz's law to determine the directions of induced magnetic fields, currents, and emfs:

1. Make a sketch of the situation for use in visualizing and recording directions.
2. Determine the direction of the applied magnetic field $\vec{\mathbf{B}}$.
3. Determine whether its magnetic flux is increasing or decreasing.
4. Now determine the direction of the induced magnetic field $\vec{\mathbf{B}}$. The induced magnetic field tries to reinforce a magnetic flux that is decreasing or opposes a magnetic flux that is increasing. Therefore, the induced magnetic field adds or subtracts to the applied magnetic field, depending on the change in magnetic flux.
5. Use right-hand rule 2 (RHR-2; see Magnetic Forces and Fields) to determine the direction of the induced current I that is responsible for the induced magnetic field $\vec{\mathbf{B}}$.
6. The direction (or polarity) of the induced emf can now drive a conventional current in this direction.

Let's apply Lenz's law to the system of Figure 13.7(a). We designate the "front" of the closed conducting loop as the region containing the approaching bar magnet, and the "back" of the loop as the other region. As the north pole of the magnet moves toward the loop, the flux through the loop due to the field of the magnet increases because the strength of field lines directed from the front to the back of the loop is increasing. A current is therefore induced in the loop. By Lenz's law, the direction of the induced current must be such that its own magnetic field is directed in a way to *oppose* the changing flux caused by the field of the approaching magnet. Hence, the induced current circulates so that its magnetic field lines through the loop are directed from the back to the front of the loop. By RHR-2, place your thumb pointing against the magnetic field lines, which is toward the bar magnet. Your fingers wrap in a counterclockwise direction as viewed from the bar magnet.

Alternatively, we can determine the direction of the induced current by treating the current loop as an electromagnet that *opposes* the approach of the north pole of the bar magnet. This occurs when the induced current flows as shown, for then the face of the loop nearer the approaching magnet is also a north pole.

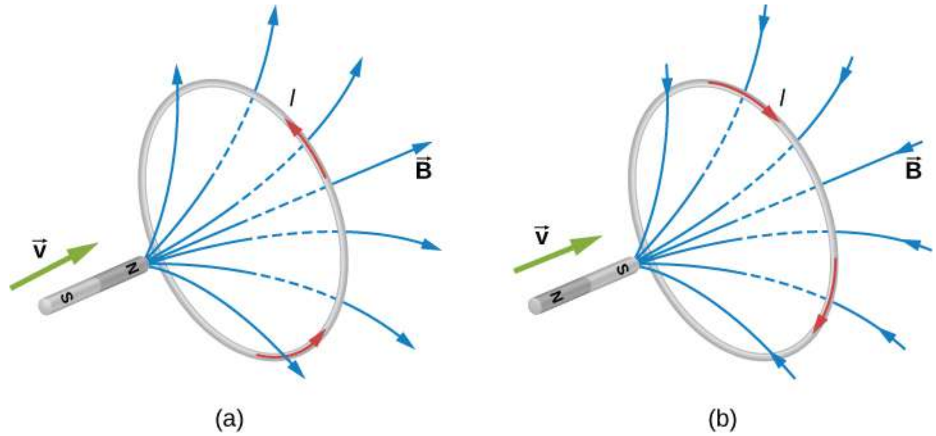


Figure 13.7 The change in magnetic flux caused by the approaching magnet induces a current in the loop. (a) An approaching north pole induces a counterclockwise current with respect to the bar magnet. (b) An approaching south pole induces a clockwise current with respect to the bar magnet.

Part (b) of the figure shows the south pole of a magnet moving toward a conducting loop. In this case, the flux through the loop due to the field of the magnet increases because the number of field lines directed from the back to the front of the loop is increasing. To oppose this change, a current is induced in the loop whose field lines through the loop are directed from the front to the back. Equivalently, we can say that the current flows in a direction so that the face of the loop nearer the approaching magnet is a south pole, which then repels the approaching south pole of the magnet. By RHR-2, your thumb points away from the bar magnet. Your fingers wrap in a clockwise fashion, which is the direction of the induced current.

Another example illustrating the use of Lenz's law is shown in Figure 13.8. When the switch is opened, the decrease in current through the solenoid causes a decrease in magnetic flux through its coils, which induces an emf in the solenoid. This emf must oppose the change (the termination of the current) causing it. Consequently, the induced emf has the polarity shown and drives in the direction of the original current. This may generate an arc across the terminals of the switch as it is opened.

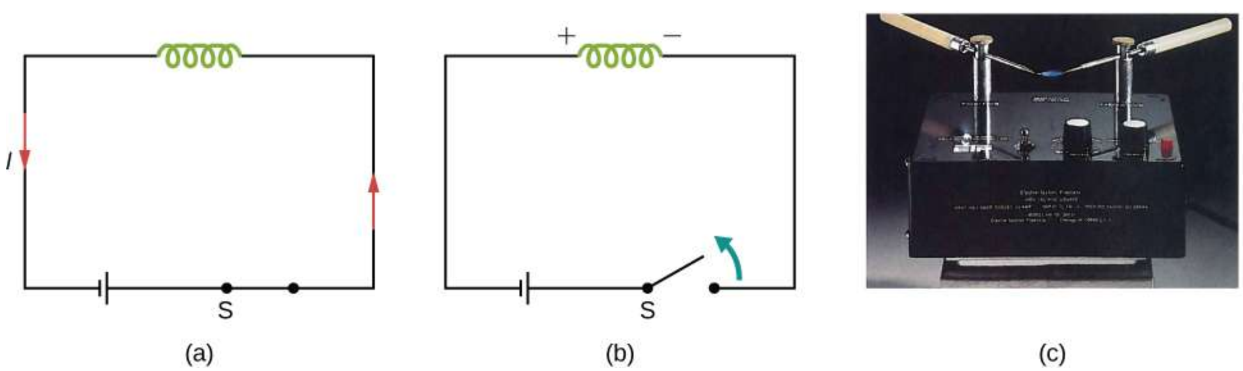


Figure 13.8 (a) A solenoid connected to a source of emf. (b) Opening switch S terminates the current, which in turn induces an emf in the solenoid. (c) A potential difference between the ends of the sharply pointed rods is produced by inducing an emf in a coil. This potential difference is large enough to produce an arc between the sharp points.

CHECK YOUR UNDERSTANDING 13.2

Find the direction of the induced current in the wire loop shown below as the magnet enters, passes through, and leaves the loop.

CHECK YOUR UNDERSTANDING 13.3

Verify the directions of the induced currents in Figure 13.3.

EXAMPLE 13.2

A Circular Coil in a Changing Magnetic Field

A magnetic field $\vec{\mathbf{B}}$ is directed outward perpendicular to the plane of a circular coil of radius $r = 0.50\text{ m}$ (Figure 13.9). The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to $B = (1.5T)e^{-(5.0\text{s}^{-1})t}$, where B is in teslas and t is in seconds. (a) Calculate the emf induced in the coil at the times $t_1 = 0$, $t_2 = 5.0 \times 10^{-2}\text{s}$, and $t_3 = 1.0\text{ s}$. (b) Determine the current in the coil at these three times if its resistance is $10\,\Omega$.

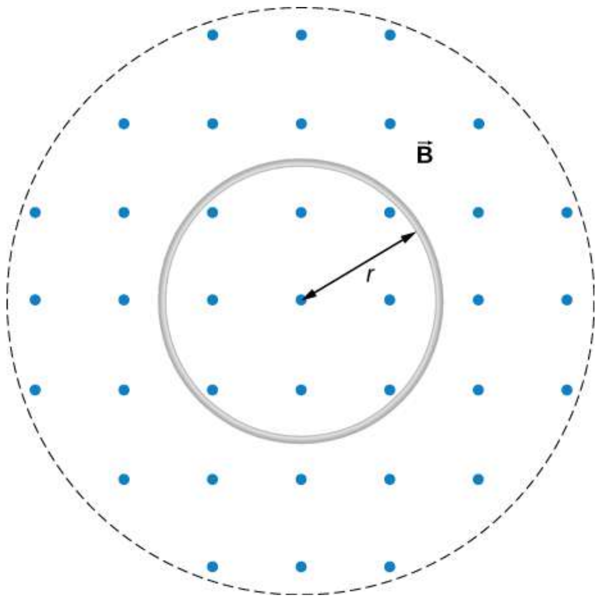


Figure 13.9 A circular coil in a decreasing magnetic field.

Strategy

Since the magnetic field is perpendicular to the plane of the coil and constant over each spot in the coil, the dot product of the magnetic field $\vec{\mathbf{B}}$ and normal to the area unit vector $\hat{\mathbf{n}}$ turns into a multiplication. The magnetic field can be pulled out of the integration, leaving the flux as the product of the magnetic field times area. We need to take the time derivative of the exponential function to calculate the emf using Faraday's law. Then we use Ohm's law to calculate the current.

Solution

a. Since $\vec{\mathbf{B}}$ is perpendicular to the plane of the coil, the magnetic flux is given by

$$\begin{aligned}\Phi_\text{m} &= B\pi r^2 = (1.5e^{-5.0t}\text{ T})\pi(0.50\text{ m})^2 \\ &= 1.2e^{-(5.0\text{s}^{-1})t}\text{ Wb}.\end{aligned}$$

From Faraday's law, the magnitude of the induced emf is

$$\varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\frac{d}{dt}(1.2e^{-(5.0\text{s}^{-1})t}\text{ Wb})\right| = 6.0\,e^{-(5.0\text{s}^{-1})t}\text{ V}.$$

Since $\vec{\mathbf{B}}$ is directed out of the page and is decreasing, the induced current must flow counterclockwise when viewed from above so that the magnetic field it produces through the coil also points out of the page. For all three times, the sense of ε is counterclockwise; its magnitudes are

$$\varepsilon\left(t_1\right) = 6.0\text{ V};\quad \varepsilon\left(t_2\right) = 4.7\text{ V};\quad \varepsilon\left(t_3\right) = 0.040\text{ V}.$$

b. From Ohm's law, the respective currents are

$$\begin{aligned} I(t_1) &= \frac{\varepsilon(t_1)}{R} = \frac{6.0\text{ V}}{10\,\Omega} = 0.60\text{ A}; \\ I(t_2) &= \frac{4.7\text{ V}}{10\,\Omega} = 0.47\text{ A}; \end{aligned}$$

and

$$I(t_3) = \frac{0.040\text{ V}}{10\,\Omega} = 4.0 \times 10^{-3}\text{ A}.$$

Significance

An emf voltage is created by a changing magnetic flux over time. If we know how the magnetic field varies with time over a constant area, we can take its time derivative to calculate the induced emf.

EXAMPLE 13.3

Changing Magnetic Field Inside a Solenoid

The current through the windings of a solenoid with $n = 2000$ turns per meter is changing at a rate $dI/dt = 3.0$ A/s. (See Sources of Magnetic Fields for a discussion of solenoids.) The solenoid is 50-cm long and has a cross-sectional diameter of 3.0 cm. A small coil consisting of $N = 20$ closely wound turns wrapped in a circle of diameter 1.0 cm is placed in the middle of the solenoid such that the plane of the coil is perpendicular to the central axis of the solenoid. Assuming that the infinite-solenoid approximation is valid at the location of the small coil, determine the magnitude of the emf induced in the coil.

Strategy

The magnetic field in the middle of the solenoid is a uniform value of $\mu_0 nI$. This field is producing a maximum magnetic flux through the coil as it is directed along the length of the solenoid. Therefore, the magnetic flux through the coil is the product of the solenoid's magnetic field times the area of the coil. Faraday's law involves a time derivative of the magnetic flux. The only quantity varying in time is the current, the rest can be pulled out of the time derivative. Lastly, we include the number of turns in the coil to determine the induced emf in the coil.

Solution

Since the field of the solenoid is given by $B = \mu_0 nI$, the flux through each turn of the small coil is

$$\Phi_\text{m} = \mu_0 nI\left(\frac{\pi d^2}{4}\right),$$

where d is the diameter of the coil. Now from Faraday's law, the magnitude of the emf induced in the coil is

$$\begin{aligned} \varepsilon &= \left|N\frac{d\Phi_\text{m}}{dt}\right| = \left|N\mu_0 n\frac{\pi d^2}{4}\ \frac{dI}{dt}\right| \\ &= 20\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/s}\right)\left(2000\text{ m}^{-1}\right)\frac{\pi(0.010\text{ m})^2}{4}(3.0\text{ A/s}) \\ &= 1.2 \times 10^{-5}\text{ V}. \end{aligned}$$

Significance

When the current is turned on in a vertical solenoid, as shown in Figure 13.10, the ring has an induced emf from the solenoid's changing magnetic flux that opposes the change. The result is that the ring is fired vertically into the air.

Figure 13.10 The jumping ring. When a current is turned on in the vertical solenoid, a current is induced in the metal ring. The stray field produced by the solenoid causes the ring to jump off the solenoid.

INTERACTIVE

Visit this website (https://openstax.org/l/21mitjumpring) for a demonstration of the jumping ring from MIT.

13.3 Motional Emf

Learning Objectives

By the end of this section, you will be able to:

- Determine the magnitude of an induced emf in a wire moving at a constant speed through a magnetic field
- Discuss examples that use motional emf, such as a rail gun and a tethered satellite

Magnetic flux depends on three factors: the strength of the magnetic field, the area through which the field lines pass, and the orientation of the field with the surface area. If any of these quantities varies, a corresponding variation in magnetic flux occurs. So far, we've only considered flux changes due to a changing field. Now we look at another possibility: a changing area through which the field lines pass including a change in the orientation of the area.

Two examples of this type of flux change are represented in Figure 13.11. In part (a), the flux through the rectangular loop increases as it moves into the magnetic field, and in part (b), the flux through the rotating coil varies with the angle θ.

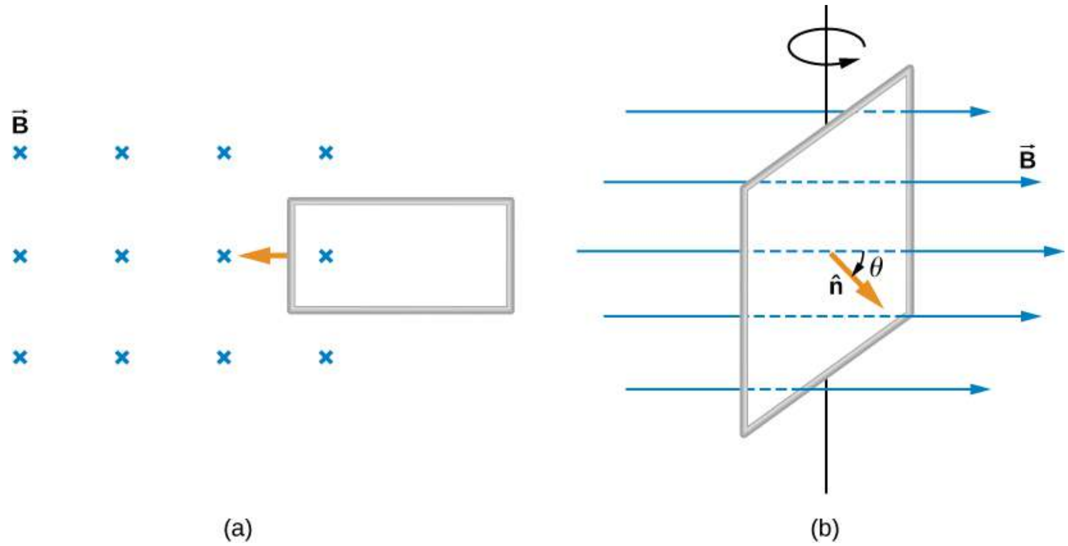


Figure 13.11 (a) Magnetic flux changes as a loop moves into a magnetic field; (b) magnetic flux changes as a loop rotates in a magnetic field.

It's interesting to note that what we perceive as the cause of a particular flux change actually depends on the frame of reference we choose. For example, if you are at rest relative to the moving coils of Figure 13.11, you would see the flux vary because of a changing magnetic field—in part (a), the field moves from left to right in your reference frame, and in part (b), the field is rotating. It is often possible to describe a flux change through a coil that is moving in one particular reference frame in terms of a changing magnetic field in a second frame, where the coil is stationary. However, reference-frame questions related to magnetic flux are beyond the level of this textbook. We'll avoid such complexities by always working in a frame at rest relative to the laboratory and explain flux variations as due to either a changing field or a changing area.

Now let's look at a conducting rod pulled in a circuit, changing magnetic flux. The area enclosed by the circuit 'MNOP' of Figure 13.12 is lx and is perpendicular to the magnetic field, so we can simplify the integration of Equation 13.1 into a multiplication of magnetic field and area. The magnetic flux through the open surface is therefore

$$\Phi_{\mathrm{m}} = Blx. \tag{13.4}$$

Since B and l are constant and the velocity of the rod is $v = dx/dt$, we can now restate Faraday's law, Equation 13.2, for the magnitude of the emf in terms of the moving conducting rod as

$$\varepsilon = \frac{d\Phi_{\mathrm{m}}}{dt} = Bl\frac{dx}{dt} = Blv. \tag{13.5}$$

The current induced in the circuit is the emf divided by the resistance or

$$I = \frac{Blv}{R}.$$

Furthermore, the direction of the induced emf satisfies Lenz's law, as you can verify by inspection of the figure.

This calculation of motionally induced emf is not restricted to a rod moving on conducting rails. With $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ as the starting point, it can be shown that $\varepsilon = -d\Phi_{\mathrm{m}}/dt$ holds for any change in flux caused by the motion of a conductor. We saw in Faraday's Law that the emf induced by a time-varying magnetic field obeys this same relationship, which is Faraday's law. Thus Faraday's law *holds for all flux changes*, whether they are produced by a changing magnetic field, by motion, or by a combination of the two.

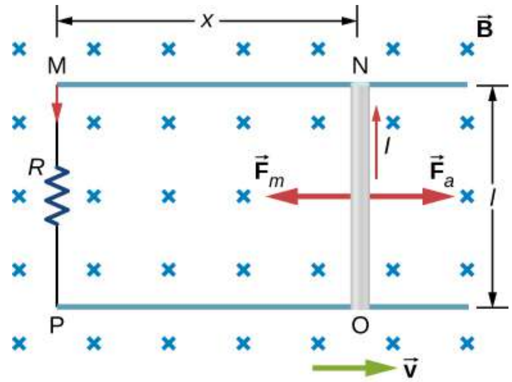


Figure 13.12 A conducting rod is pushed to the right at constant velocity. The resulting change in the magnetic flux induces a current in the circuit.

From an energy perspective, $\vec{\mathbf{F}}_a$ produces power $F_a v$, and the resistor dissipates power $I^2 R$. Since the rod is moving at constant velocity, the applied force F_a must balance the magnetic force $F_\mathrm{m} = IlB$ on the rod when it is carrying the induced current I. Thus the power produced is

$$F_a v = IlBv = \frac{Blv}{R} \cdot lBv = \frac{l^2 B^2 v^2}{R}. \tag{13.6}$$

The power dissipated is

$$P = I^2 R = \left(\frac{Blv}{R}\right)^2 R = \frac{l^2 B^2 v^2}{R}. \tag{13.7}$$

In satisfying the principle of energy conservation, the produced and dissipated powers are equal.

This principle can be seen in the operation of a rail gun. A rail gun is an electromagnetic projectile launcher that uses an apparatus similar to Figure 13.12 and is shown in schematic form in Figure 13.13. The conducting rod is replaced with a projectile or weapon to be fired. So far, we've only heard about how motion causes an emf. In a rail gun, the optimal shutting off/ramping down of a magnetic field decreases the flux in between the rails, causing a current to flow in the rod (armature) that holds the projectile. This current through the armature experiences a magnetic force and is propelled forward. Rail guns, however, are not used widely in the military due to the high cost of production and high currents: Nearly one million amps is required to produce enough energy for a rail gun to be an effective weapon.

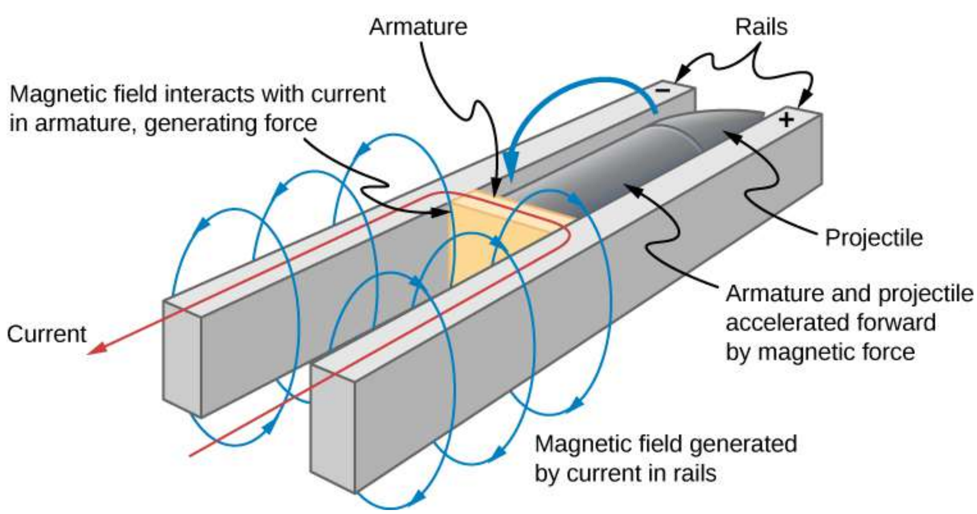


Figure 13.13 Current through two rails drives a conductive projectile forward by the magnetic force created.

We can calculate a **motionally induced emf** with Faraday's law *even when an actual closed circuit is not present*. We simply imagine an enclosed area whose boundary includes the moving conductor, calculate Φ_m, and then find the emf from Faraday's law. For example, we can let the moving rod of Figure 13.14 be one side of the imaginary rectangular area represented by the dashed lines. The area of the rectangle is lx, so the magnetic flux through it is $\Phi_m = Blx$. Differentiating this equation, we obtain

$$\frac{d\Phi_m}{dt} = Bl\frac{dx}{dt} = Blv, \tag{13.8}$$

which is identical to the potential difference between the ends of the rod that we determined earlier.

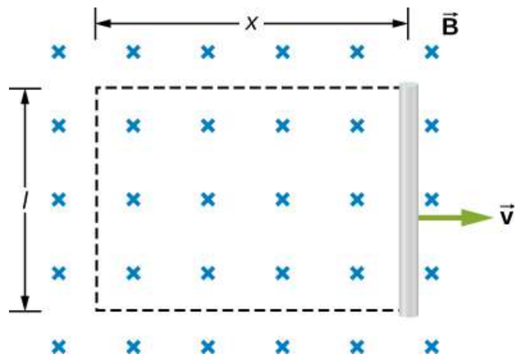


Figure 13.14 With the imaginary rectangle shown, we can use Faraday's law to calculate the induced emf in the moving rod.

Motional emfs in Earth's weak magnetic field are not ordinarily very large, or we would notice voltage along metal rods, such as a screwdriver, during ordinary motions. For example, a simple calculation of the motional emf of a 1.0-m rod moving at 3.0 m/s perpendicular to the Earth's field gives

$$\text{emf} = B\ell v = (5.0 \times 10^{-5}\ \text{T})(1.0\ \text{m})(3.0\ \text{m/s}) = 150\mu\text{V}.$$

This small value is consistent with experience. There is a spectacular exception, however. In 1992 and 1996, attempts were made with the space shuttle to create large motional emfs. The tethered satellite was to be let out on a 20-km length of wire, as shown in Figure 13.15, to create a 5-kV emf by moving at orbital speed through Earth's field. This emf could be used to convert some of the shuttle's kinetic and potential energy into electrical energy if a complete circuit could be made. To complete the circuit, the stationary ionosphere was to supply a return path through which current could flow. (The ionosphere is the rarefied and partially ionized atmosphere at orbital altitudes. It conducts because of the ionization. The ionosphere serves the same function as the stationary rails and connecting resistor in Figure 13.13, without which there would not be a complete circuit.) Drag on the current in the cable due to the magnetic force $F = I\ell B\sin\theta$ does the work that reduces the shuttle's kinetic and potential energy, and allows it to be converted into electrical energy. Both tests were unsuccessful. In the first, the cable hung up and could only be extended a couple of hundred meters; in the second, the cable broke when almost fully extended. Example 13.4 indicates feasibility in principle.

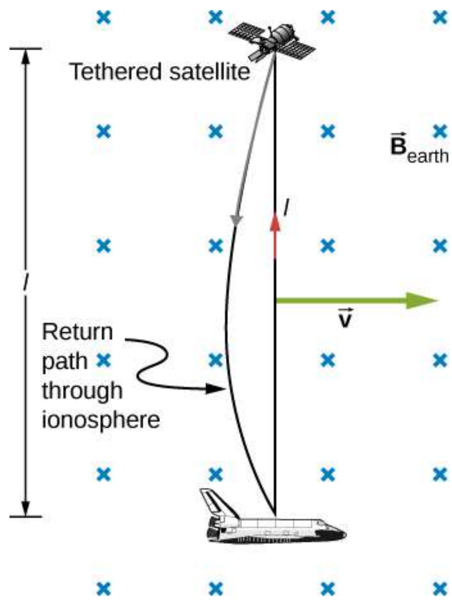


Figure 13.15 Motional emf as electrical power conversion for the space shuttle was the motivation for the tethered satellite experiment. A 5-kV emf was predicted to be induced in the 20-km tether while moving at orbital speed in Earth's magnetic field. The circuit is completed by a return path through the stationary ionosphere.

EXAMPLE 13.4

Calculating the Large Motional Emf of an Object in Orbit

Calculate the motional emf induced along a 20.0-km conductor moving at an orbital speed of 7.80 km/s perpendicular to Earth's 5.00×10^{-5} T magnetic field.

Strategy

This is a great example of using the equation motional $\varepsilon = B\ell v$.

Solution

Entering the given values into $\varepsilon = B\ell v$ gives

$$\begin{aligned} \varepsilon &= B\ell v \\ &= (5.00 \times 10^{-5}\ \text{T})(2.00 \times 10^{4}\ \text{m})(7.80 \times 10^{3}\ \text{m/s}) \\ &= 7.80 \times 10^{3}\ \text{V}. \end{aligned}$$

Significance

The value obtained is greater than the 5-kV measured voltage for the shuttle experiment, since the actual orbital motion of the tether is not perpendicular to Earth's field. The 7.80-kV value is the maximum emf obtained when $\theta = 90°$ and so $\sin\theta = 1$.

EXAMPLE 13.5

A Metal Rod Rotating in a Magnetic Field

Part (a) of Figure 13.16 shows a metal rod *OS* that is rotating in a horizontal plane around point *O*. The rod slides along a wire that forms a circular arc *PST* of radius *r*. The system is in a constant magnetic field $\vec{\mathbf{B}}$ that is directed out of the page. (a) If you rotate the rod at a constant angular velocity ω, what is the current *I* in the closed loop *OPSO*? Assume that the resistor *R* furnishes all of the resistance in the closed loop. (b) Calculate the work per unit time that you do while rotating the rod and show that it is equal to the power dissipated in the resistor.

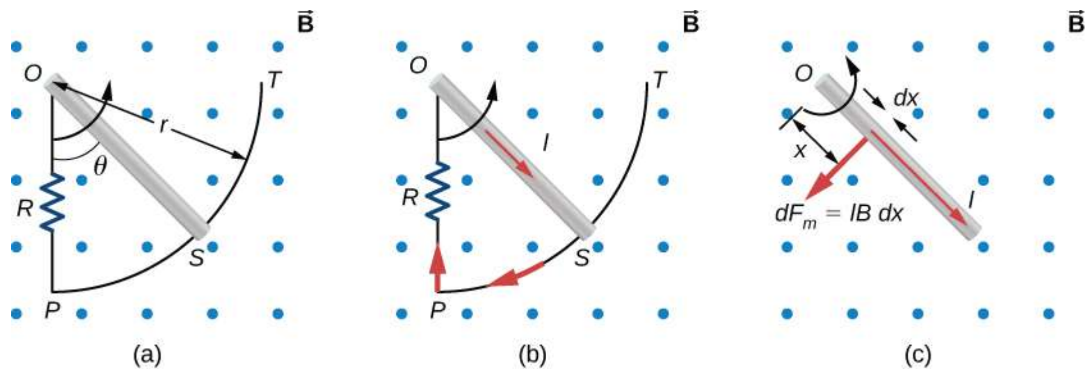


Figure 13.16 (a) The end of a rotating metal rod slides along a circular wire in a horizontal plane. (b) The induced current in the rod. (c) The magnetic force on an infinitesimal current segment.

Strategy

The magnetic flux is the magnetic field times the area of the quarter circle or $A = r^2\theta/2$. When finding the emf through Faraday's law, all variables are constant in time but θ, with$\omega = d\theta/dt$. To calculate the work per unit time, we know this is related to the torque times the angular velocity. The torque is calculated by knowing the force on a rod and integrating it over the length of the rod.

Solution

a. From geometry, the area of the loop *OPSO* is $A = \frac{r^2\theta}{2}$. Hence, the magnetic flux through the loop is

$$\Phi_\text{m} = BA = B\frac{r^2\theta}{2}.$$

Differentiating with respect to time and using $\omega = d\theta/dt$, we have

$$\varepsilon = \left|\frac{d\Phi_\text{m}}{dt}\right| = \frac{Br^2\omega}{2}.$$

When divided by the resistance R of the loop, this yields for the magnitude of the induced current

$$I = \frac{\varepsilon}{R} = \frac{Br^2\omega}{2R}.$$

As θ increases, so does the flux through the loop due to $\vec{\mathbf{B}}$. To counteract this increase, the magnetic field due to the induced current must be directed into the page in the region enclosed by the loop. Therefore, as part (b) of Figure 13.16 illustrates, the current circulates clockwise.

b. You rotate the rod by exerting a torque on it. Since the rod rotates at constant angular velocity, this torque is equal and opposite to the torque exerted on the current in the rod by the original magnetic field. The magnetic force on the infinitesimal segment of length dx shown in part (c) of Figure 13.16 is $dF_\text{m} = IBdx$, so the magnetic torque on this segment is

$$d\tau_\text{m} = x \cdot dF_\text{m} = IBxdx.$$

The net magnetic torque on the rod is then

$$\tau_\text{m} = \int_0^r d\tau_\text{m} = IB\int_0^r x\,dx = \frac{1}{2}IBr^2.$$

The torque τ that you exert on the rod is equal and opposite to τ_m, and the work that you do when the rod rotates through an angle $d\theta$ is $dW = \tau d\theta$. Hence, the work per unit time that you do on the rod is

$$\frac{dW}{dt} = \tau\frac{d\theta}{dt} = \frac{1}{2}IBr^2\frac{d\theta}{dt} = \frac{1}{2}\left(\frac{Br^2\omega}{2R}\right)Br^2\omega = \frac{B^2r^4\omega^2}{4R},$$

where we have substituted for I. The power dissipated in the resister is $P = I^2R$, which can be written as

$$P = \left(\frac{Br^2\omega}{2R}\right)^2 R = \frac{B^2r^4\omega^2}{4R}.$$

Therefore, we see that

$$P = \frac{dW}{dt}.$$

Hence, the power dissipated in the resistor is equal to the work per unit time done in rotating the rod.

Significance

An alternative way of looking at the induced emf from Faraday's law is to integrate in space instead of time. The solution, however, would be the same. The motional emf is

$$|\varepsilon| = \int Bv dl.$$

The velocity can be written as the angular velocity times the radius and the differential length written as *dr*. Therefore,

$$|\varepsilon| = B\int v dr = B\omega \int_0^l r dr = \frac{1}{2}B\omega l^2,$$

which is the same solution as before.

EXAMPLE 13.6

A Rectangular Coil Rotating in a Magnetic Field

A rectangular coil of area A and N turns is placed in a uniform magnetic field $\vec{\mathbf{B}} = B\hat{\mathbf{j}}$, as shown in Figure 13.17. The coil is rotated about the z-axis through its center at a constant angular velocity ω. Obtain an expression for the induced emf in the coil.

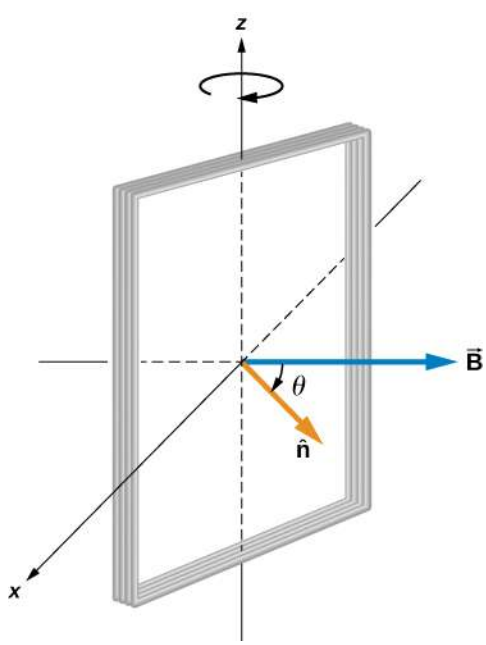


Figure 13.17 A rectangular coil rotating in a uniform magnetic field.

Strategy

According to the diagram, the angle between the perpendicular to the surface ($\hat{n}$) and the magnetic field ($\vec{\mathbf{B}}$) is θ. The dot product of $\vec{\mathbf{B}} \cdot \hat{\mathbf{n}}$ simplifies to only the cos θ component of the magnetic field, namely where the magnetic field projects onto the unit area vector $\hat{n}$. The magnitude of the magnetic field and the area of the loop are fixed over time, which makes the integration simplify quickly. The induced emf is written out using Faraday's law.

Solution

When the coil is in a position such that its normal vector $\hat{\mathbf{n}}$ makes an angle θ with the magnetic field $\vec{\mathbf{B}}$, the magnetic flux through a single turn of the coil is

$$\Phi_{\mathrm{m}} = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = BA\cos\theta.$$

From Faraday's law, the emf induced in the coil is

$$\varepsilon = -N\frac{d\Phi_{\mathrm{m}}}{dt} = NBA\sin\theta\frac{d\theta}{dt}.$$

The constant angular velocity is $\omega = d\theta/dt$. The angle θ represents the time evolution of the angular velocity or ωt. This is changes the function to time space rather than θ. The induced emf therefore varies sinusoidally with time according to

$$\varepsilon = \varepsilon_0 \sin\omega t,$$

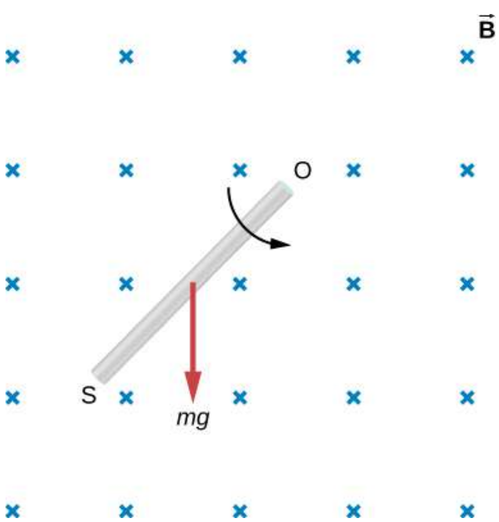
where $\varepsilon_0 = NBA\omega$.

Significance

If the magnetic field strength or area of the loop were also changing over time, these variables wouldn't be able to be pulled out of the time derivative to simply the solution as shown. This example is the basis for an electric generator, as we will give a full discussion in Applications of Newton's Law.

CHECK YOUR UNDERSTANDING 13.4

Shown below is a rod of length l that is rotated counterclockwise around the axis through O by the torque due to $m\vec{\mathbf{g}}$. Assuming that the rod is in a uniform magnetic field $\vec{\mathbf{B}}$, what is the emf induced between the ends of the rod when its angular velocity is ω? Which end of the rod is at a higher potential?



CHECK YOUR UNDERSTANDING 13.5

A rod of length 10 cm moves at a speed of 10 m/s perpendicularly through a 1.5-T magnetic field. What is the potential difference between the ends of the rod?

13.4 Induced Electric Fields

Learning Objectives

By the end of this section, you will be able to:

- Connect the relationship between an induced emf from Faraday's law to an electric field, thereby showing that a changing magnetic flux creates an electric field
- Solve for the electric field based on a changing magnetic flux in time

The fact that emfs are induced in circuits implies that work is being done on the conduction electrons in the wires. What can possibly be the source of this work? We know that it's neither a battery nor a magnetic field, for a battery does not have to be present in a circuit where current is induced, and magnetic fields never do work on moving charges. The answer is that the source of the work is an electric field $\vec{\mathbf{E}}$ that is induced in the wires. The work done by $\vec{\mathbf{E}}$ in moving a unit charge completely around a circuit is the induced emf ε; that is,

$$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}, \tag{13.9}$$

where $\oint$ represents the line integral around the circuit. Faraday's law can be written in terms of the **induced electric field** as

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_{\text{m}}}{dt}. \tag{13.10}$$

There is an important distinction between the electric field induced by a changing magnetic field and the electrostatic field produced by a fixed charge distribution. Specifically, the induced electric field is nonconservative because it does net work in moving a charge over a closed path, whereas the electrostatic field is conservative and does no net work over a closed path. Hence, electric potential can be associated with the electrostatic field, but not with the induced field. The following equations represent the distinction between the two types of electric field:

$$\begin{aligned} \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} &\neq 0 \ \text{(induced)}; \\ \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} &= 0 \ \text{(electrostatic)}. \end{aligned} \tag{13.11}$$

Our results can be summarized by combining these equations:

$$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_{\text{m}}}{dt}. \tag{13.12}$$

EXAMPLE 13.7

Induced Electric Field in a Circular Coil

What is the induced electric field in the circular coil of Example 13.2 (and Figure 13.9) at the three times indicated?

Strategy

Using cylindrical symmetry, the electric field integral simplifies into the electric field times the circumference of a circle. Since we already know the induced emf, we can connect these two expressions by Faraday's law to solve for the induced electric field.

Solution

The induced electric field in the coil is constant in magnitude over the cylindrical surface, similar to how Ampere's law problems with cylinders are solved. Since $\vec{\mathbf{E}}$ is tangent to the coil,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \oint E dl = 2\pi r E.$$

When combined with Equation 13.12, this gives

$$E = \frac{\varepsilon}{2\pi r}.$$

The direction of ε is counterclockwise, and $\vec{\mathbf{E}}$ circulates in the same direction around the coil. The values of E are

$$\begin{aligned} E(t_1) &= \frac{6.0\text{ V}}{2\pi\,(0.50\text{ m})} = 1.9\text{ V/m}; \\ E(t_2) &= \frac{4.7\text{ V}}{2\pi\,(0.50\text{ m})} = 1.5\text{ V/m}; \\ E(t_3) &= \frac{0.040\text{ V}}{2\pi\,(0.50\text{ m})} = 0.013\text{ V/m}. \end{aligned}$$

Significance

When the magnetic flux through a circuit changes, a nonconservative electric field is induced, which drives current through the circuit. But what happens if $dB/dt \neq 0$ in free space where there isn't a conducting path? The answer is that this case can be treated *as if a conducting path were present*; that is, nonconservative electric fields are induced wherever $dB/dt \neq 0$, whether or not there is a conducting path present.

These nonconservative electric fields always satisfy Equation 13.12. For example, if the circular coil of Figure 13.9 were removed, an electric field *in free space* at $r = 0.50$ m would still be directed counterclockwise, and its magnitude would still be 1.9 V/m at $t = 0$, 1.5 V/m at $t = 5.0 \times 10^{-2}$ s, etc. The existence of induced electric fields is certainly *not* restricted to wires in circuits.

EXAMPLE 13.8

Electric Field Induced by the Changing Magnetic Field of a Solenoid

Part (a) of Figure 13.18 shows a long solenoid with radius R and n turns per unit length; its current decreases with time according to $I = I_0 e^{-\alpha t}$. What is the magnitude of the induced electric field at a point a distance r from the central axis of the solenoid (a) when $r > R$ and (b) when $r < R$ [see part (b) of Figure 13.18]. (c) What is the direction of the induced field at both locations? Assume that the infinite-solenoid approximation is valid throughout the regions of interest.

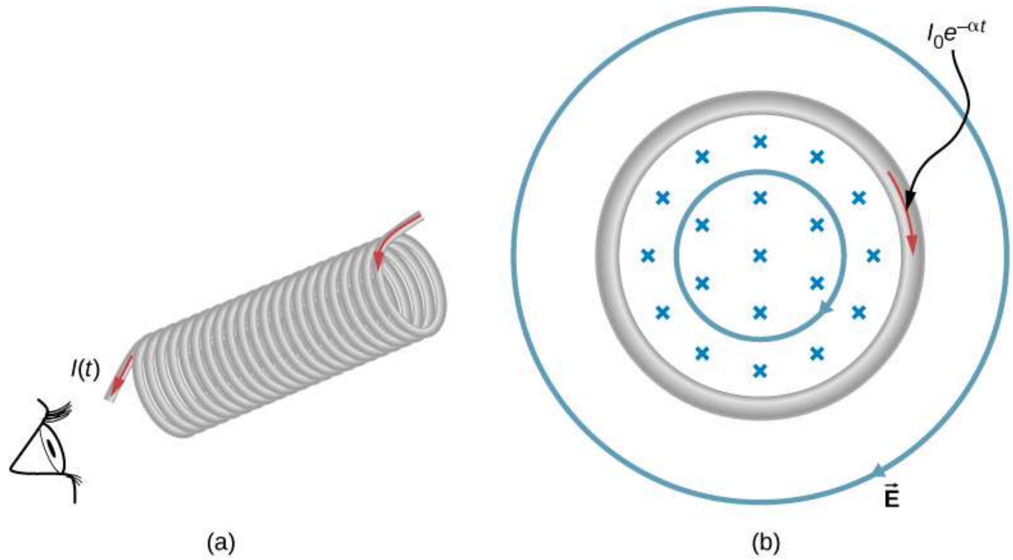


Figure 13.18 (a) The current in a long solenoid is decreasing exponentially. (b) A cross-sectional view of the solenoid from its left end. The cross-section shown is near the middle of the solenoid. An electric field is induced both inside and outside the solenoid.

Strategy

Using the formula for the magnetic field inside an infinite solenoid and Faraday's law, we calculate the induced emf. Since we have cylindrical symmetry, the electric field integral reduces to the electric field times the circumference of the integration path. Then we solve for the electric field.

Solution

a. The magnetic field is confined to the interior of the solenoid where

$$B = \mu_0 nI = \mu_0 nI_0 e^{-\alpha t}.$$

Thus, the magnetic flux through a circular path whose radius r is greater than R, the solenoid radius, is

$$\Phi_m = BA = \mu_0 nI_0 \pi R^2 e^{-\alpha t}.$$

The induced field $\vec{\mathbf{E}}$ is tangent to this path, and because of the cylindrical symmetry of the system, its magnitude is constant on the path. Hence, we have

$$\left|\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}\right| = \left|\frac{d\Phi_m}{dt}\right|,$$

$$E(2\pi r) = \left|\frac{d}{dt}(\mu_0 nI_0 \pi R^2 e^{-\alpha t})\right| = \alpha\mu_0 nI_0 \pi R^2 e^{-\alpha t},$$

$$E = \frac{\alpha\mu_0 nI_0 R^2}{2r} e^{-\alpha t} \quad (r > R).$$

b. For a path of radius r inside the solenoid, $\Phi_m = B\pi r^2$, so

$$E(2\pi r) = \left|\frac{d}{dt}(\mu_0 nI_0 \pi r^2 e^{-\alpha t})\right| = \alpha\mu_0 nI_0 \pi r^2 e^{-\alpha t},$$

and the induced field is

$$E = \frac{\alpha\mu_0 nI_0 r}{2} e^{-\alpha t} \ (r < R).$$

c. The magnetic field points into the page as shown in part (b) and is decreasing. If either of the circular paths were occupied by conducting rings, the currents induced in them would circulate as shown, in conformity with Lenz's law. The induced electric field must be so directed as well.

Significance

In part (b), note that $\left|\vec{\mathbf{E}}\right|$ increases with r inside and decreases as $1/r$ outside the solenoid, as shown in Figure 13.19.

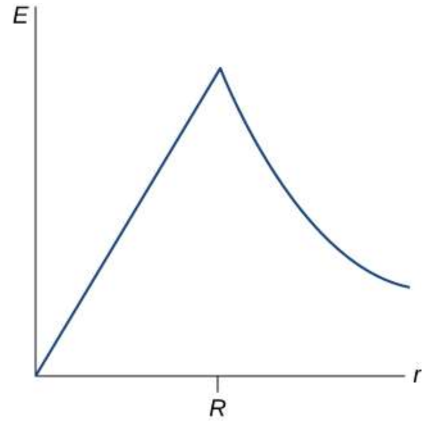


Figure 13.19 The electric field vs. distance r. When $r < R$, the electric field rises linearly, whereas when $r > R$, the electric field falls of proportional to $1/r$.

CHECK YOUR UNDERSTANDING 13.6

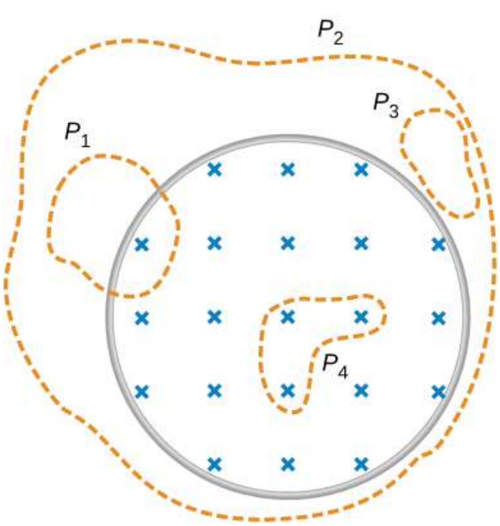
Suppose that the coil of Example 13.2 is a square rather than circular. Can Equation 13.12 be used to calculate (a) the induced emf and (b) the induced electric field?

CHECK YOUR UNDERSTANDING 13.7

What is the magnitude of the induced electric field in Example 13.8 at $t = 0$ if $r = 6.0\,\text{cm}$, $R = 2.0\,\text{cm}$, $n = 2000$ turns per meter, $I_0 = 2.0\,\text{A}$, and $\alpha = 200\,\text{s}^{-1}$?

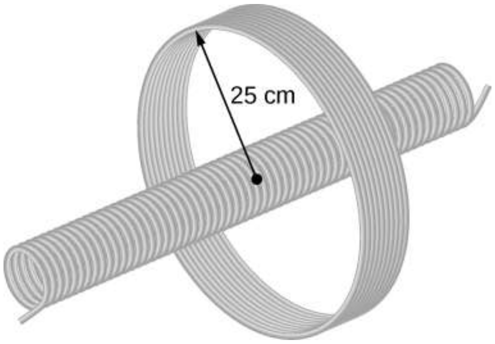
CHECK YOUR UNDERSTANDING 13.8

The magnetic field shown below is confined to the cylindrical region shown and is changing with time. Identify those paths for which $\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} \neq 0$.



CHECK YOUR UNDERSTANDING 13.9

A long solenoid of cross-sectional area 5.0 cm^2 is wound with 25 turns of wire per centimeter. It is placed in the middle of a closely wrapped coil of 10 turns and radius 25 cm, as shown below. (a) What is the emf induced in the coil when the current through the solenoid is decreasing at a rate $dI/dt = -0.20 \text{ A/s}$? (b) What is the electric field induced in the coil?



13.5 Eddy Currents

Learning Objectives

By the end of this section, you will be able to:

- Explain how eddy currents are created in metals
- Describe situations where eddy currents are beneficial and where they are not helpful

As discussed two sections earlier, a motional emf is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional emf can cause a current in the conductor, we refer to that current as an **eddy current**.

Magnetic Damping

Eddy currents can produce significant drag, called **magnetic damping**, on the motion involved. Consider the apparatus shown in Figure 13.20, which swings a pendulum bob between the poles of a strong magnet. (This is another favorite physics demonstration.) If the bob is metal, significant drag acts on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in part (b) of the figure, the magnet produces a much smaller effect. There is no discernible effect on a bob made of an insulator. Why does drag occur in both directions, and are there any uses for magnetic drag?

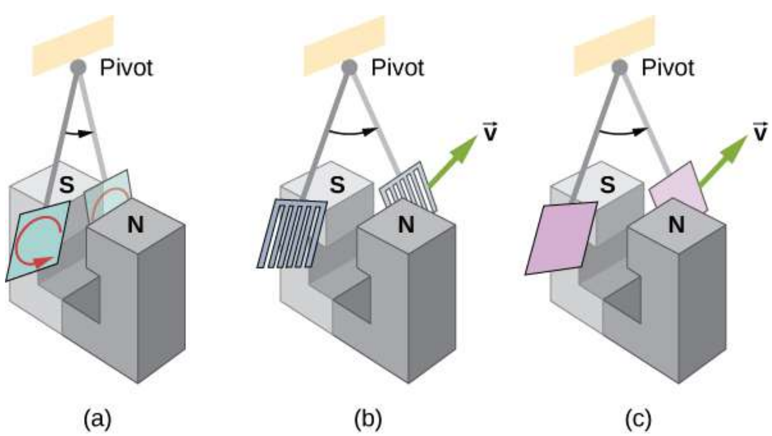


Figure 13.20 A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents. (b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.

Figure 13.21 shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, setting up an eddy current (Faraday's law) in the counterclockwise direction (Lenz's law), as shown. Only the right-hand side of the current loop is in the field, so an unopposed force acts on it to the left (RHR-1). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.

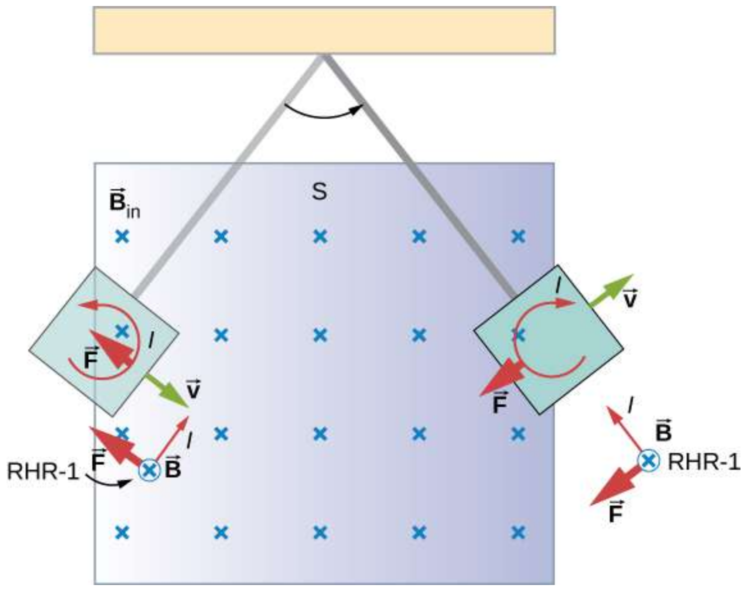


Figure 13.21 A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.

When a slotted metal plate enters the field (Figure 13.22), an emf is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they must be slotted or constructed of thin layers of conducting material separated by insulating sheets.

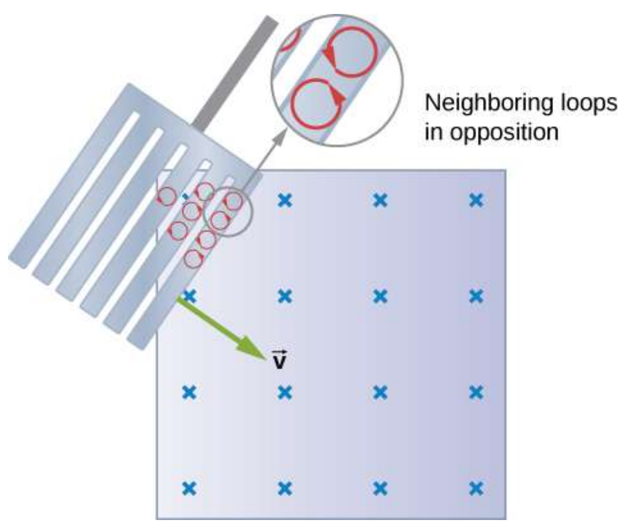


Figure 13.22 Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to

cancel, thereby making magnetic drag almost zero.

Applications of Magnetic Damping

One use of magnetic damping is found in sensitive laboratory balances. To have maximum sensitivity and accuracy, the balance must be as friction-free as possible. But if it is friction-free, then it will oscillate for a very long time. Magnetic damping is a simple and ideal solution. With magnetic damping, drag is proportional to speed and becomes zero at zero velocity. Thus, the oscillations are quickly damped, after which the damping force disappears, allowing the balance to be very sensitive (Figure 13.23). In most balances, magnetic damping is accomplished with a conducting disc that rotates in a fixed field.

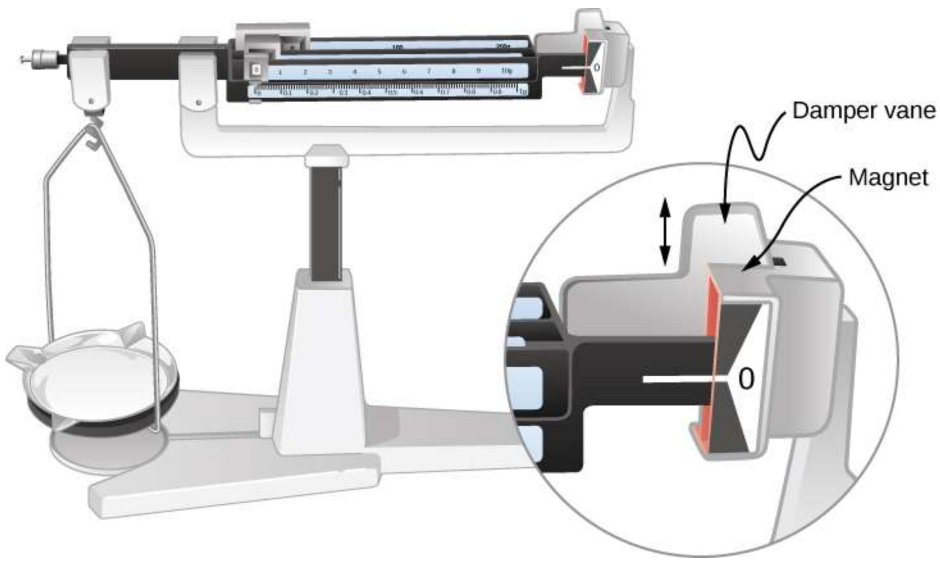


Figure 13.23 Magnetic damping of this sensitive balance slows its oscillations. Since Faraday's law of induction gives the greatest effect for the most rapid change, damping is greatest for large oscillations and goes to zero as the motion stops.

Since eddy currents and magnetic damping occur only in conductors, recycling centers can use magnets to separate metals from other materials. Trash is dumped in batches down a ramp, beneath which lies a powerful magnet. Conductors in the trash are slowed by magnetic damping while nonmetals in the trash move on, separating from the metals (Figure 13.24). This works for all metals, not just ferromagnetic ones. A magnet can separate out the ferromagnetic materials alone by acting on stationary trash.

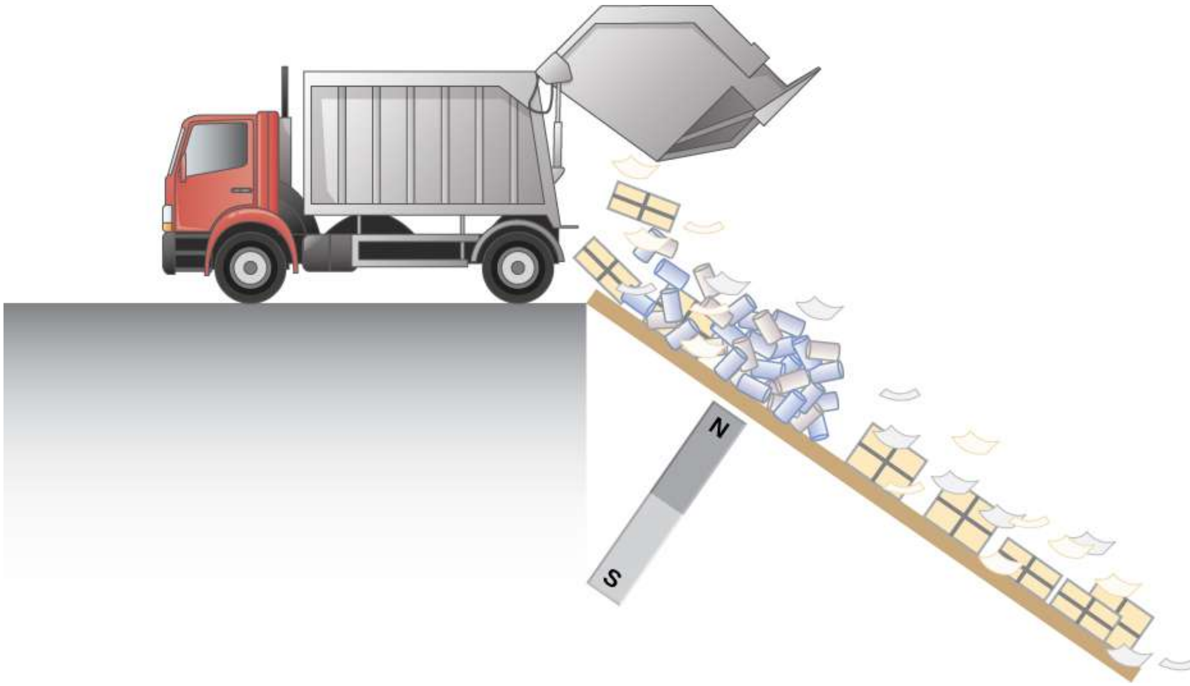


Figure 13.24 Metals can be separated from other trash by magnetic drag. Eddy currents and magnetic drag are created in the metals sent down this ramp by the powerful magnet beneath it. Nonmetals move on.

Other major applications of eddy currents appear in metal detectors and braking systems in trains and roller coasters. Portable metal detectors (Figure 13.25) consist of a primary coil carrying an alternating current and a secondary coil in which a current is induced. An eddy current is induced in a piece of metal close to the detector, causing a change in the induced current within the secondary coil. This can trigger some sort of signal, such as a shrill noise.

Figure 13.25 A soldier in Iraq uses a metal detector to search for explosives and weapons. (credit: U.S. Army)

Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother. However, eddy currents cannot bring the motion to a complete stop, since the braking force produced decreases as speed is reduced. Thus, speed can be reduced from say 20 m/s to 5 m/s, but another form of braking is needed to completely stop the vehicle. Generally, powerful rare-earth magnets such as neodymium magnets are used in roller coasters. Figure 13.26 shows rows of magnets in such an application. The vehicle has metal fins (normally containing copper) that pass through the magnetic field, slowing the vehicle down in much the same way as with the pendulum bob shown in Figure 13.20.

Figure 13.26 The rows of rare-earth magnets (protruding horizontally) are used for magnetic braking in roller coasters. (credit: Stefan Scheer)

Induction cooktops have electromagnets under their surface. The magnetic field is varied rapidly, producing eddy currents in the base of the pot, causing the pot and its contents to increase in temperature. Induction cooktops have high efficiencies and good response times when the base of the pot is a conductor, such as iron or steel.

13.6 Electric Generators and Back Emf

Learning Objectives

By the end of this section, you will be able to:

- Explain how an electric generator works
- Determine the induced emf in a loop at any time interval, rotating at a constant rate in a magnetic field
- Show that rotating coils have an induced emf; in motors this is called back emf because it opposes the emf input to the motor

A variety of important phenomena and devices can be understood with Faraday's law. In this section, we examine two of these.

Electric Generators

Electric generators induce an emf by rotating a coil in a magnetic field, as briefly discussed in Motional Emf. We now explore generators in more detail. Consider the following example.

EXAMPLE 13.9

Calculating the Emf Induced in a Generator Coil

The generator coil shown in Figure 13.27 is rotated through one-fourth of a revolution (from $\theta = 0°$ to$\theta = 90°$) in 15.0 ms. The 200-turn circular coil has a 5.00-cm radius and is in a uniform 0.80-T magnetic field. What is the emf induced?

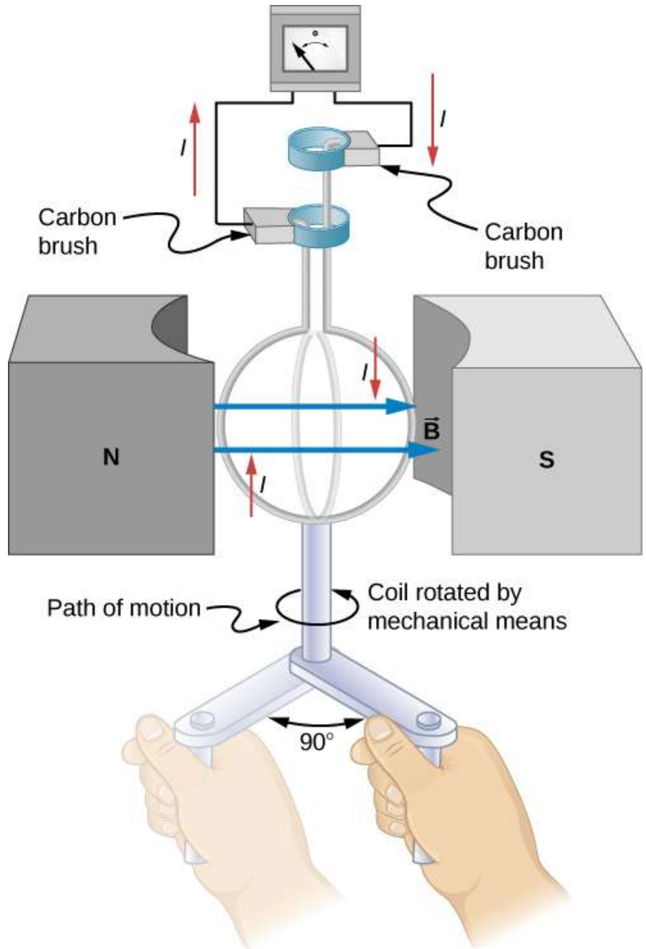


Figure 13.27 When this generator coil is rotated through one-fourth of a revolution, the magnetic flux Φ_m changes from its maximum to zero, inducing an emf.

Strategy

Faraday's law of induction is used to find the emf induced:

$$\varepsilon = -N\frac{d\Phi_m}{dt}.$$

We recognize this situation as the same one in Example 13.6. According to the diagram, the projection of the surface normal vector $\hat{\mathbf{n}}$ to the magnetic field is initiallycos θ, and this is inserted by the definition of the dot product. The magnitude of the magnetic field and area of the loop are fixed over time, which makes the integration simplify quickly. The induced emf is written out using Faraday's law:

$$\varepsilon = NBA\sin\theta\frac{d\theta}{dt}.$$

Solution

We are given that $N = 200$, $B = 0.80\,\text{T}$, $\theta = 90°$, $d\theta = 90° = \pi/2$, and $dt = 15.0\,\text{ms}$. The area of the loop is

$$A = \pi r^2 = (3.14)\,(0.0500\,\text{m})^2 = 7.85\,\times\,10^{-3}\,\text{m}^2.$$

Entering this value gives

$$\varepsilon = (200)(0.80\,\text{T})(7.85\,\times\,10^{-3}\,\text{m}^2)\sin\,(90°)\,\frac{\pi/2}{15.0\,\times\,10^{-3}\,\text{s}} = 131\,\text{V}.$$

Significance

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in Example 13.9 is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf as a function of time by considering the motional emf on a rotating rectangular coil of width w and height l in a uniform magnetic field, as illustrated in Figure 13.28.

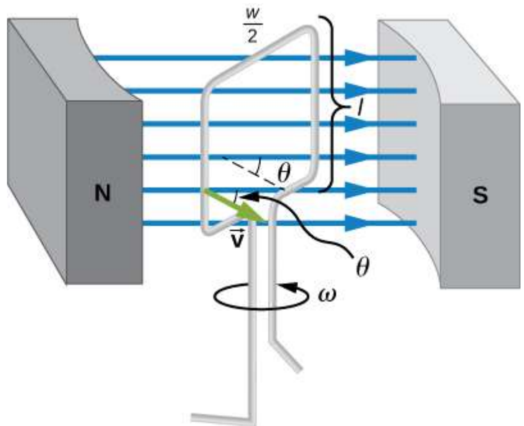


Figure 13.28 A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be $\varepsilon = Blv$, where the velocity v is perpendicular to the magnetic field B. Here the velocity is at an angle θ with B, so that its component perpendicular to B is $v \sin\theta$ (see Figure 13.28). Thus, in this case, the emf induced on each side is $\varepsilon = Blv \sin\theta$, and they are in the same direction. The total emf around the loop is then

$$\varepsilon = 2Blv \sin\theta. \quad (13.13)$$

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil rotates at a constant angular velocity ω. The angle θ is related to angular velocity by $\theta = \omega t$, so that

$$\varepsilon = 2Blv \sin(\omega t). \quad (13.14)$$

Now, linear velocity v is related to angular velocity ω by $v = r\omega$. Here, $r = w/2$, so that $v = (w/2)\,\omega$, and

$$\varepsilon = 2Bl\frac{w}{2}\omega \sin\omega t = (lw)B\omega \sin\omega t. \quad (13.15)$$

Noting that the area of the loop is $A = lw$, and allowing for N loops, we find that

$$\varepsilon = NBA\omega \sin(\omega t). \quad (13.16)$$

This is the emf induced in a generator coil of N turns and area A rotating at a constant angular velocity ω in a uniform magnetic field B. This can also be expressed as

$$\varepsilon = \varepsilon_0 \sin\omega t, \quad (13.17)$$

where

$$\varepsilon_0 = NAB\omega \quad (13.18)$$

is the peak emf, since the maximum value of $\sin(wt) = 1$. Note that the frequency of the oscillation is $f = \omega/2\pi$

and the period is $T = 1/f = 2\pi/\omega$. Figure 13.29 shows a graph of emf as a function of time, and it now seems reasonable that ac voltage is sinusoidal.

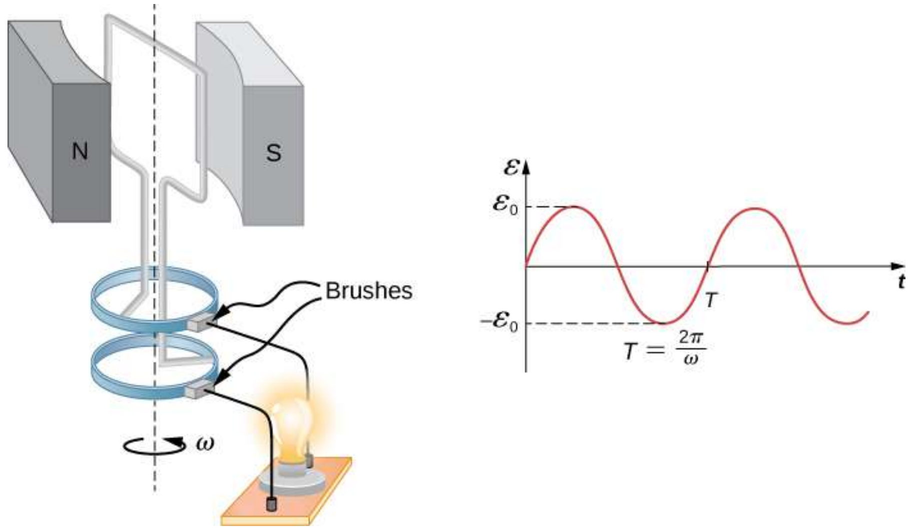


Figure 13.29 The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time, where ε_0 is the peak emf. The period is $T = 1/f = 2\pi/\omega$, where *f* is the frequency.

The fact that the peak emf is $\varepsilon_0 = NBA\omega$ makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater ω), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties.

Figure 13.30 shows a scheme by which a generator can be made to produce pulsed dc. More elaborate arrangements of multiple coils and split rings can produce smoother dc, although electronic rather than mechanical means are usually used to make ripple-free dc.

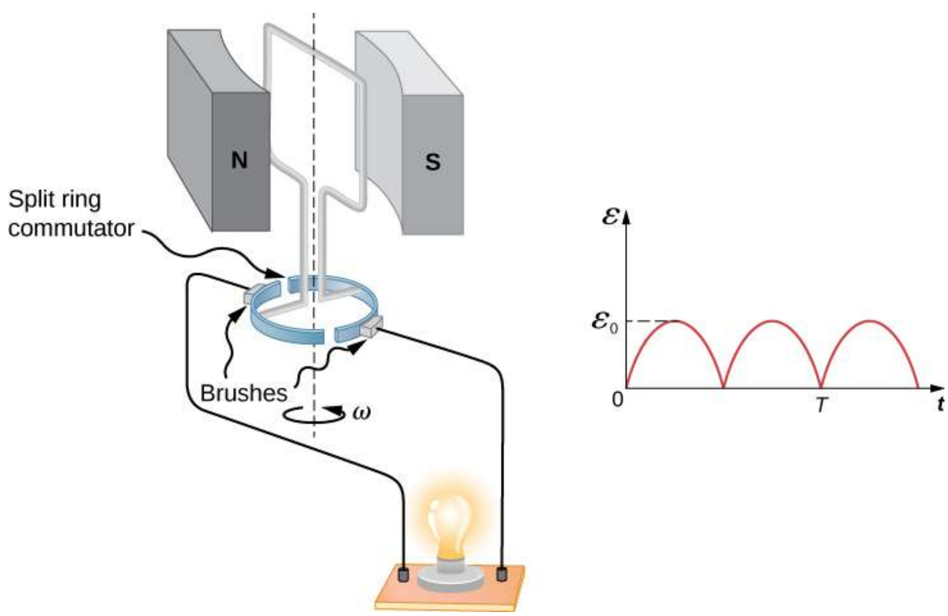


Figure 13.30 Split rings, called commutators, produce a pulsed dc emf output in this configuration.

In real life, electric generators look a lot different from the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the burning of fossil fuels, or the kinetic energy of wind. Figure 13.31 shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator. The generation of electrical energy from mechanical energy is the basic principle of all power that is sent through our electrical grids to our homes.

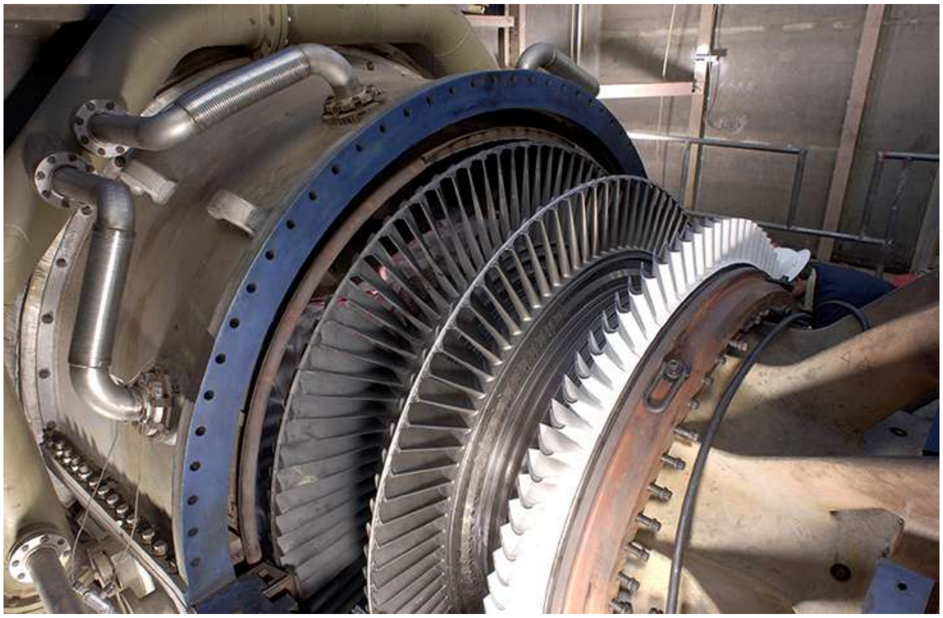


Figure 13.31 Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft, which is connected to the generator.

Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In the next section, we further explore the action of a motor as a generator.

Back Emf

Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Thus, it is not surprising that motors and generators have the same general construction. A motor works by sending a current through a loop of wire located in a magnetic field. As a result, the magnetic field exerts torque on the loop. This rotates a shaft, thereby extracting mechanical work out of the electrical current sent in initially. (Refer to Force and Torque on a Current Loop for a discussion on motors that will help you understand more about them before proceeding.)

When the coil of a motor is turned, magnetic flux changes through the coil, and an emf (consistent with Faraday's law) is induced. The motor thus acts as a generator whenever its coil rotates. This happens whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an emf is generated. Lenz's law tells us the emf opposes any change, so that the input emf that powers the motor is opposed by the motor's self-generated emf, called the **back emf** of the motor (Figure 13.32).

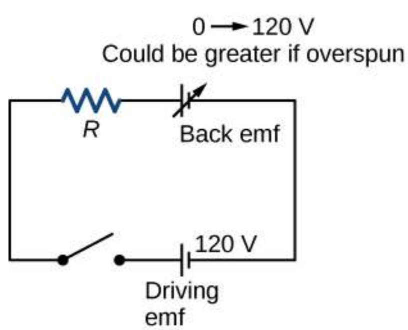


Figure 13.32 The coil of a dc motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the emf driving the motor. Back emf is zero when the motor is not turning and increases proportionally to the motor's angular velocity.

The generator output of a motor is the difference between the supply voltage and the back emf. The back emf is zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster, the back emf grows, always opposing the driving emf, and reduces both the voltage across the coil and the amount of current it draws. This effect is noticeable in many common situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the *IR* drop produced in feeder lines by the large current drawn by the motor.

When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil, $P = I^2 R$), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it increases its angular velocity ω until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Eddy currents in iron cores of motors can cause troublesome energy losses. These are usually minimized by constructing the cores out of thin, electrically insulated sheets of iron. The magnetic properties of the core are hardly affected by the lamination of the insulating sheet, while the resistive heating is reduced considerably. Consider, for example, the motor coils represented in Figure 13.32. The coils have an equivalent resistance of $0.400\ \Omega$ and are driven by an emf of 48.0 V. Shortly after being turned on, they draw a current

$$I = V/R = (48.0\ \text{V})/(0.400\ \Omega) = 120\ \text{A}$$

and thus dissipate $P = I^2 R = 5.76\ \text{kW}$ of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is 40.0 V. Then at operating speed, the total voltage across the coils is 8.0 V (48.0 V minus the 40.0 V back emf), and the current drawn is

$$I = V/R = (8.0\ \text{V})/(0.400\ \Omega) = 20\ \text{A}\ .$$

Under normal load, then, the power dissipated is $P = IV = (20\ \text{A})(8.0\ \text{V}) = 160\ \text{W}$. This does not cause a problem for this motor, whereas the former 5.76 kW would burn out the coils if sustained.

EXAMPLE 13.10

A Series-Wound Motor in Operation

The total resistance $\left(R_f + R_a\right)$ of a series-wound dc motor is $2.0\ \Omega$ (Figure 13.33). When connected to a 120-V source (ε_S), the motor draws 10 A while running at constant angular velocity. (a) What is the back emf induced in the rotating coil, ε_i? (b) What is the mechanical power output of the motor? (c) How much power is dissipated in the resistance of the coils? (d) What is the power output of the 120-V source? (e) Suppose the load on the motor increases, causing it to slow down to the point where it draws 20 A. Answer parts (a) through (d) for this situation.

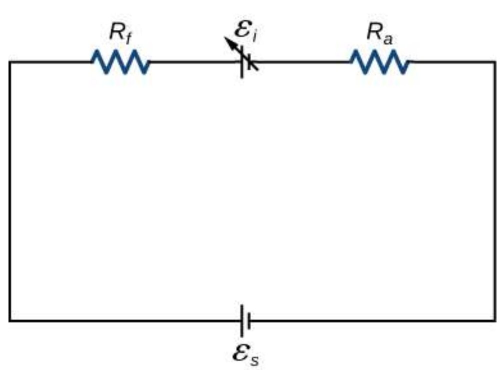


Figure 13.33 Circuit representation of a series-wound direct current motor.

Strategy

The back emf is calculated based on the difference between the supplied voltage and the loss from the current through the resistance. The power from each device is calculated from one of the power formulas based on the given information.

Solution

a. The back emf is

$$\varepsilon_i = \varepsilon_s - I(R_f + R_a) = 120\text{ V} - (10\text{ A})(2.0\,\Omega) = 100\text{ V}.$$

b. Since the potential across the armature is 100 V when the current through it is 10 A, the power output of the motor is

$$P_\text{m} = \varepsilon_i I = (100\text{ V})(10\text{ A}) = 1.0 \times 10^3\text{ W}.$$

c. A 10-A current flows through coils whose combined resistance is 2.0 Ω, so the power dissipated in the coils is

$$P_R = I^2 R = (10\text{ A})^2(2.0\,\Omega) = 2.0 \times 10^2\text{ W}.$$

d. Since 10 A is drawn from the 120-V source, its power output is

$$P_s = \varepsilon_s\, I = (120\text{ V})(10\text{ A}) = 1.2 \times 10^3\text{ W}.$$

e. Repeating the same calculations with $I = 20\text{ A}$, we find

$$\varepsilon_\text{i} = 80\text{ V}, P_\text{m} = 1.6 \times 10^3\text{ W}, P_R = 8.0 \times 10^2\text{ W}, \text{ and } P_s = 2.4 \times 10^3\text{ W}.$$

The motor is turning more slowly in this case, so its power output and the power of the source are larger.

Significance

Notice that we have an energy balance in part (d): $1.2 \times 10^3\text{ W} = 1.0 \times 10^3\text{ W} + 2.0 \times 10^2\text{ W}.$

13.7 Applications of Electromagnetic Induction

Learning Objectives

By the end of this section, you will be able to:

- Explain how computer hard drives and graphic tablets operate using magnetic induction
- Explain how hybrid/electric vehicles and transcranial magnetic stimulation use magnetic induction to their advantage

Modern society has numerous applications of Faraday's law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that involve recording information using magnetic fields.

Some computer hard drives apply the principle of magnetic induction. Recorded data are made on a coated, spinning disk. Historically, reading these data was made to work on the principle of induction. However, most input information today is carried in digital rather than analog form—a series of 0s or 1s are written upon the spinning hard drive. Therefore, most hard drive readout devices do not work on the principle of induction, but use a technique known as giant magnetoresistance. Giant magnetoresistance is the effect of a large change of electrical resistance induced by an applied magnetic field to thin films of alternating ferromagnetic and nonmagnetic layers. This is one of the first large successes of nanotechnology.

Graphics tablets, or tablet computers where a specially designed pen is used to draw digital images, also applies induction principles. The tablets discussed here are labeled as passive tablets, since there are other designs that use either a battery-operated pen or optical signals to write with. The passive tablets are different than the touch tablets and phones many of us use regularly, but may still be found when signing your signature at a cash register. Underneath the screen, shown in Figure 13.34, are tiny wires running across the length and width of the screen. The pen has a tiny magnetic field coming from the tip. As the tip brushes across the screen, a changing magnetic field is felt in the wires which translates into an induced emf that is converted into the line you just drew.

Figure 13.34 A tablet with a specially designed pen to write with is another application of magnetic induction. (credit: Jane Whitney)

Another application of induction is the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape, in which a playback head reads personal information from your card.

INTERACTIVE

Check out this video (https://openstax.org/l/21flashmagind) to see how flashlights can use magnetic induction. A magnet moves by your mechanical work through a wire. The induced current charges a capacitor that stores the charge that will light the lightbulb even while you are not doing this mechanical work.

Electric and hybrid vehicles also take advantage of electromagnetic induction. One limiting factor that inhibits widespread acceptance of 100% electric vehicles is that the lifetime of the battery is not as long as the time you get to drive on a full tank of gas. To increase the amount of charge in the battery during driving, the motor can act as a generator whenever the car is braking, taking advantage of the back emf produced. This extra emf can be newly acquired stored energy in the car's battery, prolonging the life of the battery.

Another contemporary area of research in which electromagnetic induction is being successfully implemented is transcranial magnetic stimulation (TMS). A host of disorders, including depression and hallucinations, can be traced to irregular localized electrical activity in the brain. In transcranial magnetic stimulation, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. The usage of TMS as a diagnostic technique is well established.

INTERACTIVE

Check out this Youtube video (https://openstax.org/l/21randrelectro) to see how rock-and-roll instruments like electric guitars use electromagnetic induction to get those strong beats.

CHAPTER REVIEW

Key Terms

back emf emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor

eddy current current loop in a conductor caused by motional emf

electric generator device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field

Faraday's law induced emf is created in a closed loop due to a change in magnetic flux through the loop

induced electric field created based on the changing magnetic flux with time

induced emf short-lived voltage generated by a conductor or coil moving in a magnetic field

Lenz's law direction of an induced emf opposes the change in magnetic flux that produced it; this is the negative sign in Faraday's law

magnetic damping drag produced by eddy currents

magnetic flux measurement of the amount of magnetic field lines through a given area

motionally induced emf voltage produced by the movement of a conducting wire in a magnetic field

peak emf maximum emf produced by a generator

Key Equations

Magnetic flux	$\Phi_{\mathrm{m}} = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA$
Faraday's law	$\varepsilon = -N \frac{d\Phi_{\mathrm{m}}}{dt}$
Motionally induced emf	$\varepsilon = Blv$
Motional emf around a circuit	$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_{\mathrm{m}}}{dt}$
Emf produced by an electric generator	$\varepsilon = NBA\omega \sin(\omega t)$

Summary

13.1 Faraday's Law

- The magnetic flux through an enclosed area is defined as the amount of field lines cutting through a surface area A defined by the unit area vector.
- The units for magnetic flux are webers, where $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.
- The induced emf in a closed loop due to a change in magnetic flux through the loop is known as Faraday's law. If there is no change in magnetic flux, no induced emf is created.

13.2 Lenz's Law

- We can use Lenz's law to determine the directions of induced magnetic fields, currents, and emfs.
- The direction of an induced emf always opposes the change in magnetic flux that causes the emf, a result known as Lenz's law.

13.3 Motional Emf

- The relationship between an induced emf ε in a wire moving at a constant speed v through a magnetic field B is given by $\varepsilon = Blv$.
- An induced emf from Faraday's law is created from a motional emf that opposes the change in flux.

13.4 Induced Electric Fields

- A changing magnetic flux induces an electric field.
- Both the changing magnetic flux and the induced electric field are related to the induced emf from Faraday's law.

13.5 Eddy Currents

- Current loops induced in moving conductors are called eddy currents. They can create significant drag, called magnetic damping.
- Manipulation of eddy currents has resulted in applications such as metal detectors, braking in trains or roller coasters, and induction cooktops.

13.6 Electric Generators and Back Emf

- An electric generator rotates a coil in a magnetic field, inducing an emf given as a function of time by $\varepsilon = NBA\omega \sin(\omega t)$ where A is the area of an N-turn coil rotated at a constant angular velocity ω in a uniform magnetic field $\vec{\mathbf{B}}$.
- The peak emf of a generator is $\varepsilon_0 = NBA\omega$.
- Any rotating coil produces an induced emf. In motors, this is called back emf because it opposes the emf input to the motor.

13.7 Applications of Electromagnetic Induction

- Hard drives utilize magnetic induction to read/write information.
- Other applications of magnetic induction can be found in graphics tablets, electric and hybrid vehicles, and in transcranial magnetic stimulation.

Conceptual Questions

13.1 Faraday's Law

1. A stationary coil is in a magnetic field that is changing with time. Does the emf induced in the coil depend on the actual values of the magnetic field?

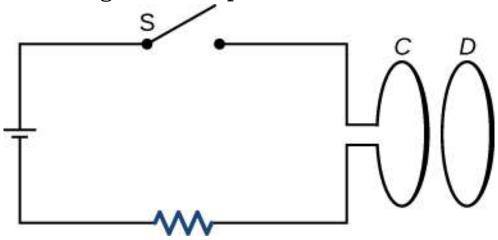
2. In Faraday's experiments, what would be the advantage of using coils with many turns?

3. A copper ring and a wooden ring of the same dimensions are placed in magnetic fields so that there is the same change in magnetic flux through them. Compare the induced electric fields and currents in the rings.

4. Discuss the factors determining the induced emf in a closed loop of wire.

5. (a) Does the induced emf in a circuit depend on the resistance of the circuit? (b) Does the induced current depend on the resistance of the circuit?

6. How would changing the radius of loop D shown below affect its emf, assuming C and D are much closer together compared to their radii?



7. Can there be an induced emf in a circuit at an instant when the magnetic flux through the circuit is zero?

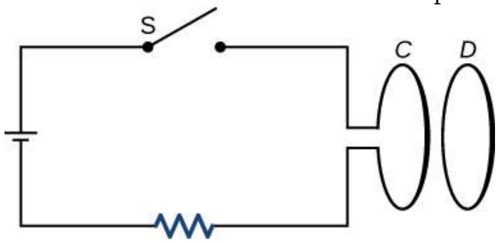
8. Does the induced emf always act to decrease the magnetic flux through a circuit?

9. How would you position a flat loop of wire in a changing magnetic field so that there is no induced emf in the loop?

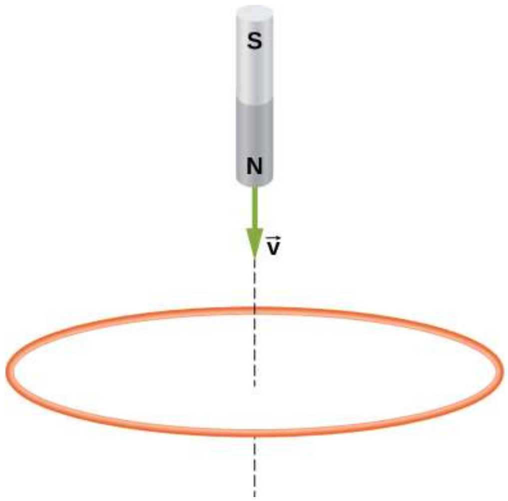
10. The normal to the plane of a single-turn conducting loop is directed at an angle θ to a spatially uniform magnetic field $\vec{\mathbf{B}}$. It has a fixed area and orientation relative to the magnetic field. Show that the emf induced in the loop is given by $\varepsilon = (dB/dt)(A \cos\theta)$, where A is the area of the loop.

13.2 Lenz's Law

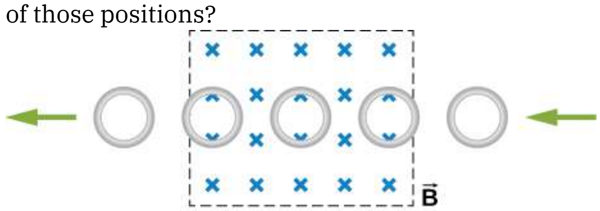
11. The circular conducting loops shown in the accompanying figure are parallel, perpendicular to the plane of the page, and coaxial. (a) When the switch S is closed, what is the direction of the current induced in D? (b) When the switch is opened, what is the direction of the current induced in loop D?



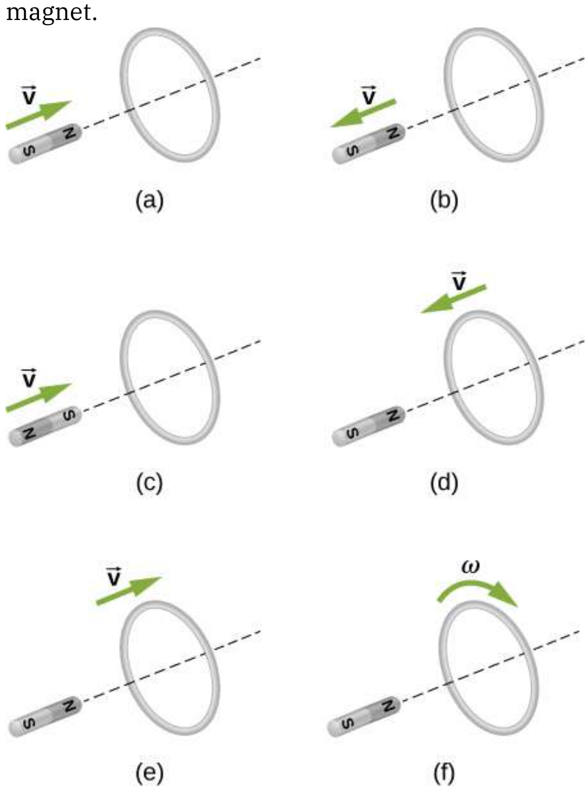
12. The north pole of a magnet is moved toward a copper loop, as shown below. If you are looking at the loop from above the magnet, will you say the induced current is circulating clockwise or counterclockwise?



13. The accompanying figure shows a conducting ring at various positions as it moves through a magnetic field. What is the sense of the induced emf for each of those positions?



14. Show that ε and $d\Phi_m/dt$ have the same units.
15. State the direction of the induced current for each case shown below, observing from the side of the magnet.

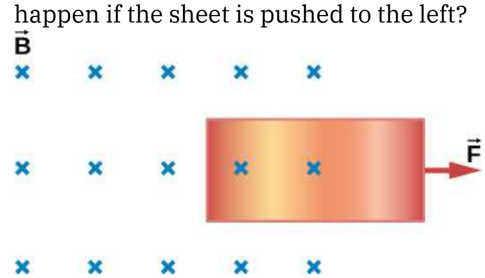


13.3 Motional Emf

16. A bar magnet falls under the influence of gravity along the axis of a long copper tube. If air resistance is negligible, will there be a force to oppose the descent of the magnet? If so, will the magnet reach a terminal velocity?
17. Around the geographic North Pole (or magnetic South Pole), Earth's magnetic field is almost vertical. If an airplane is flying northward in this region, which side of the wing is positively charged and which is negatively charged?
18. A wire loop moves translationally (no rotation) in a uniform magnetic field. Is there an emf induced in the loop?

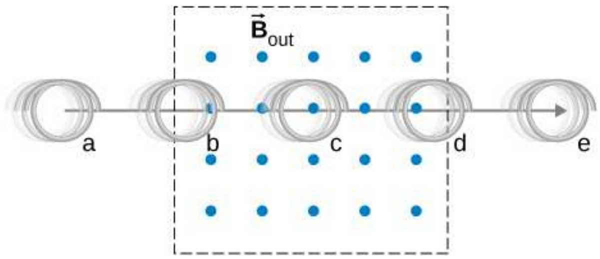
13.4 Induced Electric Fields

19. Is the work required to accelerate a rod from rest to a speed v in a magnetic field greater than the final kinetic energy of the rod? Why?
20. The copper sheet shown below is partially in a magnetic field. When it is pulled to the right, a resisting force pulls it to the left. Explain. What happen if the sheet is pushed to the left?



13.5 Eddy Currents

21. A conducting sheet lies in a plane perpendicular to a magnetic field $\vec{\mathbf{B}}$ that is below the sheet. If $\vec{\mathbf{B}}$ oscillates at a high frequency and the conductor is made of a material of low resistivity, the region above the sheet is effectively shielded from $\vec{\mathbf{B}}$. Explain why. Will the conductor shield this region from static magnetic fields?
22. Electromagnetic braking can be achieved by applying a strong magnetic field to a spinning metal disk attached to a shaft. (a) How can a magnetic field slow the spinning of a disk? (b) Would the brakes work if the disk was made of plastic instead of metal?
23. A coil is moved through a magnetic field as shown below. The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?



Problems

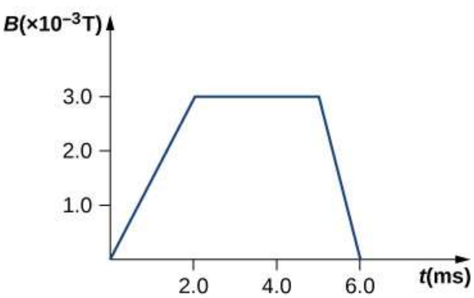
13.1 Faraday's Law

24. A 50-turn coil has a diameter of 15 cm. The coil is placed in a spatially uniform magnetic field of magnitude 0.50 T so that the face of the coil and the magnetic field are perpendicular. Find the magnitude of the emf induced in the coil if the magnetic field is reduced to zero uniformly in (a) 0.10 s, (b) 1.0 s, and (c) 60 s.

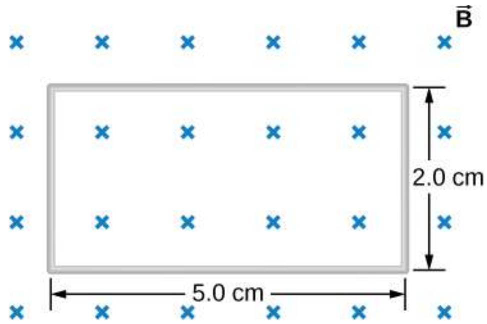
25. Repeat your calculations of the preceding problem's time of 0.1 s with the plane of the coil making an angle of (a) 30°, (b) 60°, and (c) 90° with the magnetic field.

26. A square loop whose sides are 6.0-cm long is made with copper wire of radius 1.0 mm. If a magnetic field perpendicular to the loop is changing at a rate of 5.0 mT/s, what is the current in the loop?

27. The magnetic field through a circular loop of radius 10.0 cm varies with time as shown below. The field is perpendicular to the loop. Plot the magnitude of the induced emf in the loop as a function of time.



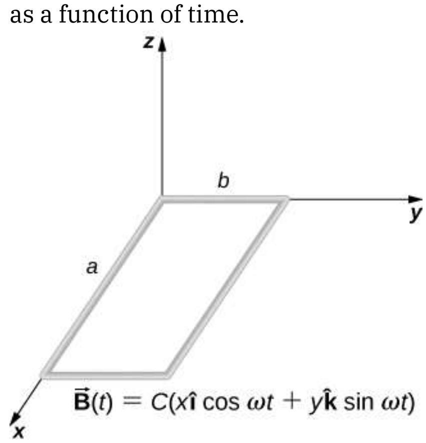
28. The accompanying figure shows a single-turn rectangular coil that has a resistance of 2.0 Ω. The magnetic field at all points inside the coil varies according to $B = B_0 e^{-\alpha t}$, where $B_0 = 0.25$ T and $\alpha = 200$ Hz. What is the current induced in the coil at (a) $t = 0.001$ s, (b) 0.002 s, (c) 2.0 s?



29. How would the answers to the preceding problem change if the coil consisted of 20 closely spaced turns?

30. A long solenoid with $n = 10$ turns per centimeter has a cross-sectional area of 5.0 cm^2 and carries a current of 0.25 A. A coil with five turns encircles the solenoid. When the current through the solenoid is turned off, it decreases to zero in 0.050 s. What is the average emf induced in the coil?

31. A rectangular wire loop with length a and width b lies in the xy-plane, as shown below. Within the loop there is a time-dependent magnetic field given by $\vec{\mathbf{B}}(t) = C\left((x \cos \omega t)\hat{\mathbf{i}} + (y \sin \omega t)\hat{\mathbf{k}}\right)$, with $\vec{\mathbf{B}}(t)$ in tesla. Determine the emf induced in the loop as a function of time.



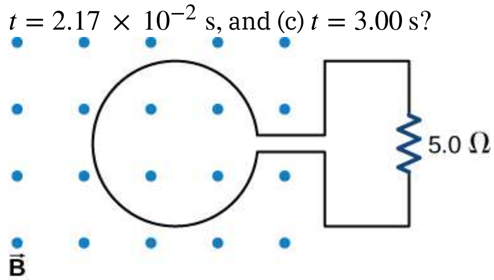
32. The magnetic field perpendicular to a single wire loop of diameter 10.0 cm decreases from 0.50 T to zero. The wire is made of copper and has a diameter of 2.0 mm and length 1.0 cm. How much charge moves through the wire while the field is changing?

13.2 Lenz's Law

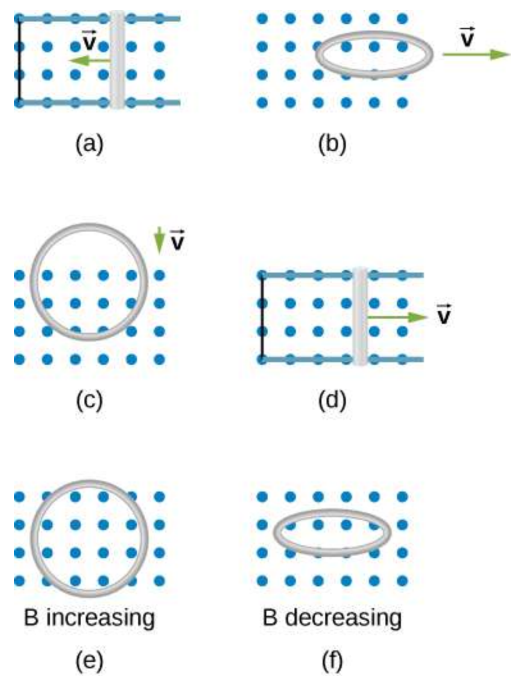
33. A single-turn circular loop of wire of radius 50 mm lies in a plane perpendicular to a spatially uniform magnetic field. During a 0.10-s time interval, the magnitude of the field increases uniformly from 200 to 300 mT. (a) Determine the emf induced in the loop. (b) If the magnetic field is directed out of the page, what is the direction of the current induced in the loop?

34. When a magnetic field is first turned on, the flux through a 20-turn loop varies with time according to $\Phi_m = 5.0t^2 - 2.0t$, where Φ_m is in milliwebers, t is in seconds, and the loop is in the plane of the page with the unit normal pointing outward. (a) What is the emf induced in the loop as a function of time? What is the direction of the induced current at (b) $t = 0$, (c) 0.10, (d) 1.0, and (e) 2.0 s?

35. The magnetic flux through the loop shown in the accompanying figure varies with time according to $\Phi_m = 2.00e^{-3t}\sin(120\pi t)$, where Φ_m is in milliwebers. What are the direction and magnitude of the current through the 5.00-Ω resistor at (a) $t = 0$; (b) $t = 2.17 \times 10^{-2}$ s, and (c) $t = 3.00$ s?



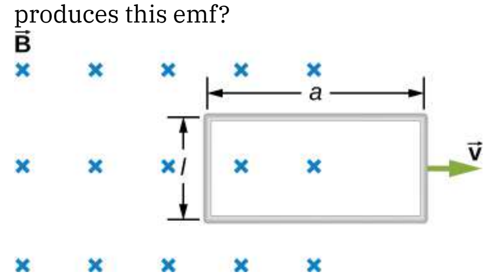
36. Use Lenz's law to determine the direction of induced current in each case.



13.3 Motional Emf

37. An automobile with a radio antenna 1.0 m long travels at 100.0 km/h in a location where the Earth's horizontal magnetic field is 5.5×10^{-5} T. What is the maximum possible emf induced in the antenna due to this motion?

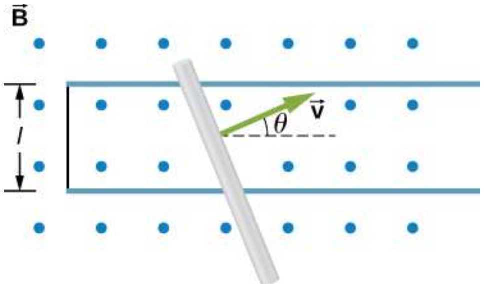
38. The rectangular loop of N turns shown below moves to the right with a constant velocity $\vec{v}$ while leaving the poles of a large electromagnet. (a) Assuming that the magnetic field is uniform between the pole faces and negligible elsewhere, determine the induced emf in the loop. (b) What is the source of work that produces this emf?



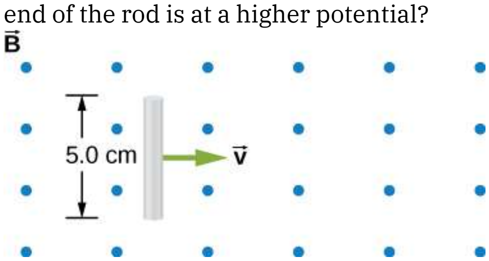
39. Suppose the magnetic field of the preceding problem oscillates with time according to $B = B_0 \sin \omega t$. What then is the emf induced in the loop when its trailing side is a distance d from the right edge of the magnetic field region?

40. A coil of 1000 turns encloses an area of 25 cm^2. It is rotated in 0.010 s from a position where its plane is perpendicular to Earth's magnetic field to one where its plane is parallel to the field. If the strength of the field is 6.0×10^{-5} T, what is the average emf induced in the coil?

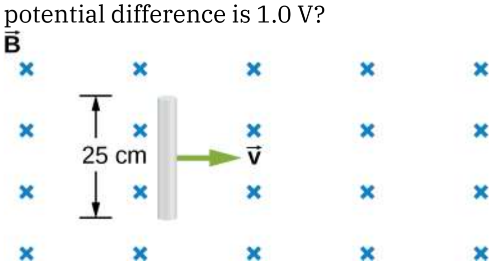
41. In the circuit shown in the accompanying figure, the rod slides along the conducting rails at a constant velocity $\vec{\mathbf{v}}$. The velocity is in the same plane as the rails and directed at an angle θ to them. A uniform magnetic field $\vec{\mathbf{B}}$ is directed out of the page. What is the emf induced in the rod?



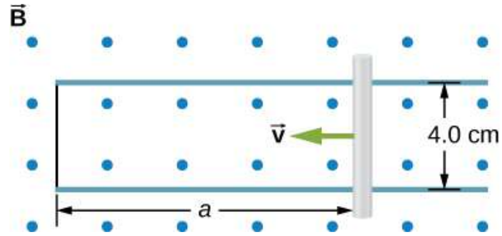
42. The rod shown in the accompanying figure is moving through a uniform magnetic field of strength $B = 0.50$ T with a constant velocity of magnitude $v = 8.0$ m/s. What is the potential difference between the ends of the rod? Which end of the rod is at a higher potential?



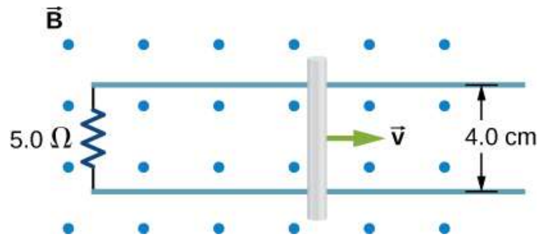
43. A 25-cm rod moves at 5.0 m/s in a plane perpendicular to a magnetic field of strength 0.25 T. The rod, velocity vector, and magnetic field vector are mutually perpendicular, as indicated in the accompanying figure. Calculate (a) the magnetic force on an electron in the rod, (b) the electric field in the rod, and (c) the potential difference between the ends of the rod. (d) What is the speed of the rod if the potential difference is 1.0 V?



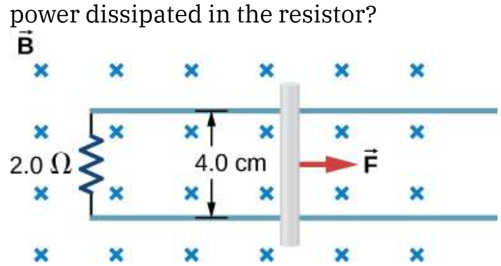
44. In the accompanying figure, the rails, connecting end piece, and rod all have a resistance per unit length of 2.0 Ω/cm. The rod moves to the left at $v = 3.0$ m/s. If $B = 0.75$ T everywhere in the region, what is the current in the circuit (a) when $a = 8.0$ cm? (b) when $a = 5.0$ cm? Specify also the sense of the current flow.



45. The rod shown below moves to the right on essentially zero-resistance rails at a speed of $v = 3.0$ m/s. If $B = 0.75$ T everywhere in the region, what is the current through the 5.0-Ω resistor? Does the current circulate clockwise or counterclockwise?



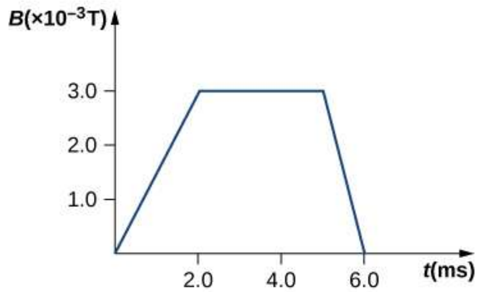
46. Shown below is a conducting rod that slides along metal rails. The apparatus is in a uniform magnetic field of strength 0.25 T, which is directly into the page. The rod is pulled to the right at a constant speed of 5.0 m/s by a force $\vec{\mathbf{F}}$. The only significant resistance in the circuit comes from the 2.0-Ω resistor shown. (a) What is the emf induced in the circuit? (b) What is the induced current? Does it circulate clockwise or counter clockwise? (c) What is the magnitude of $\vec{\mathbf{F}}$? (d) What are the power output of $\vec{\mathbf{F}}$ and the power dissipated in the resistor?



13.4 Induced Electric Fields

47. Calculate the induced electric field in a 50-turn coil with a diameter of 15 cm that is placed in a spatially uniform magnetic field of magnitude 0.50 T so that the face of the coil and the magnetic field are perpendicular. This magnetic field is reduced to zero in 0.10 seconds. Assume that the magnetic field is cylindrically symmetric with respect to the central axis of the coil.

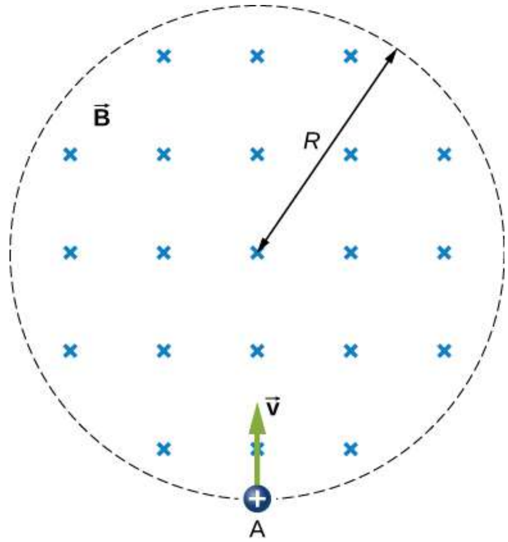
48. The magnetic field through a circular loop of radius 10.0 cm varies with time as shown in the accompanying figure. The field is perpendicular to the loop. Assuming cylindrical symmetry with respect to the central axis of the loop, plot the induced electric field in the loop as a function of time.



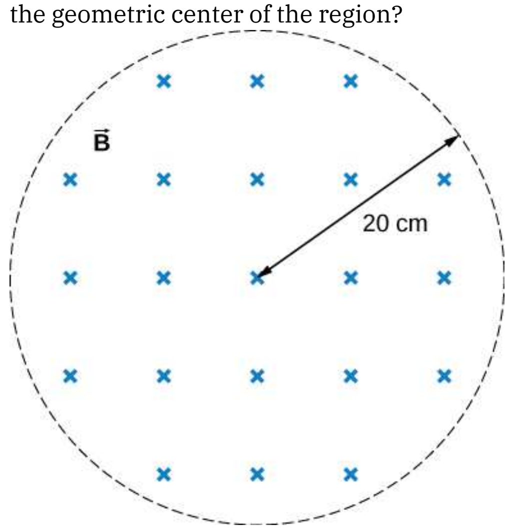
49. The current I through a long solenoid with n turns per meter and radius R is changing with time as given by dI/dt. Calculate the induced electric field as a function of distance r from the central axis of the solenoid.

50. Calculate the electric field induced both inside and outside the solenoid of the preceding problem if $I = I_0 \sin \omega t$.

51. Over a region of radius R, there is a spatially uniform magnetic field $\vec{\mathbf{B}}$. (See below.) At $t = 0$, $B = 1.0\,\text{T}$, after which it decreases at a constant rate to zero in 30 s. (a) What is the electric field in the regions where $r \le R$ and $r \ge R$ during that 30-s interval? (b) Assume that $R = 10.0\,\text{cm}$. How much work is done by the electric field on a proton that is carried once clock wise around a circular path of radius 5.0 cm? (c) How much work is done by the electric field on a proton that is carried once counterclockwise around a circular path of any radius $r \ge R$? (d) At the instant when $B = 0.50\,\text{T}$, a proton enters the magnetic field at A, moving a velocity $\vec{\mathbf{v}}$ $\left(v = 5.0 \times 10^6 \text{ m/s}\right)$ as shown. What are the electric and magnetic forces on the proton at that instant?



52. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the accompanying figure starts at 1.0 T and decreases uniformly to zero in 20 s. What is the electric field (both magnitude and direction) as a function of r, the distance from the geometric center of the region?



53. The current in a long solenoid with 20 turns per centimeter of radius 3 cm is varied with time at a rate of 2 A/s. A circular loop of wire of radius 5 cm and resistance 2 Ω surrounds the solenoid. Find the electrical current induced in the loop.

54. The current in a long solenoid of radius 3 cm and 20 turns/cm is varied with time at a rate of 2 A/s. Find the electric field at a distance of 4 cm from the center of the solenoid.

13.6 Electric Generators and Back Emf

55. Design a current loop that, when rotated in a uniform magnetic field of strength 0.10 T, will produce an emf $\varepsilon = \varepsilon_0 \sin \omega t$, where

$\varepsilon_0 = 110\text{ V}$ and $\omega = 120\pi\text{ rad/s}$.

56. A flat, square coil of 20 turns that has sides of length 15.0 cm is rotating in a magnetic field of strength 0.050 T. If the maximum emf produced in the coil is 30.0 mV, what is the angular velocity of the coil?

57. A 50-turn rectangular coil with dimensions $0.15\text{ m} \times 0.40\text{ m}$ rotates in a uniform magnetic field of magnitude 0.75 T at 3600 rev/min. (a) Determine the emf induced in the coil as a function of time. (b) If the coil is connected to a $1000\text{-}\Omega$ resistor, what is the power as a function of time required to keep the coil turning at 3600 rpm? (c) Answer part (b) if the coil is connected to a $2000\text{-}\Omega$ resistor.

58. The square armature coil of an alternating current generator has 200 turns and is 20.0 cm on side. When it rotates at 3600 rpm, its peak output voltage is 120 V. (a) What is the frequency of the output voltage? (b) What is the strength of the magnetic field in which the coil is turning?

59. A flip coil is a relatively simple device used to measure a magnetic field. It consists of a circular coil of *N* turns wound with fine conducting wire. The coil is attached to a ballistic galvanometer, a device that measures the total charge that passes through it. The coil is placed in a magnetic field $\vec{\mathbf{B}}$ such that its face is perpendicular to the field. It is then flipped through $180°$, and the total charge *Q* that flows through the galvanometer is measured. (a) If the total resistance of the coil and galvanometer is *R*, what is the relationship between *B* and *Q*? Because the coil is very small, you can assume that $\vec{\mathbf{B}}$ is uniform over it. (b) How can you determine whether or not the magnetic field is perpendicular to the face of the coil?

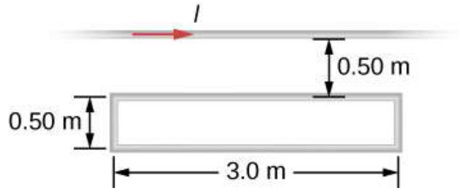
60. The flip coil of the preceding problem has a radius of 3.0 cm and is wound with 40 turns of copper wire. The total resistance of the coil and ballistic galvanometer is $0.20\ \Omega$. When the coil is flipped through $180°$ in a magnetic field $\vec{\mathbf{B}}$, a change of 0.090 C flows through the ballistic galvanometer. (a) Assuming that $\vec{\mathbf{B}}$ and the face of the coil are initially perpendicular, what is the magnetic field? (b) If the coil is flipped through $90°$, what is the reading of the galvanometer?

61. A 120-V, series-wound motor has a field resistance of 80 Ω and an armature resistance of 10 Ω. When it is operating at full speed, a back emf of 75 V is generated. (a) What is the initial current drawn by the motor? When the motor is operating at full speed, where are (b) the current drawn by the motor, (c) the power output of the source, (d) the power output of the motor, and (e) the power dissipated in the two resistances?

62. A small series-wound dc motor is operated from a 12-V car battery. Under a normal load, the motor draws 4.0 A, and when the armature is clamped so that it cannot turn, the motor draws 24 A. What is the back emf when the motor is operating normally?

Additional Problems

63. Shown in the following figure is a long, straight wire and a single-turn rectangular loop, both of which lie in the plane of the page. The wire is parallel to the long sides of the loop and is 0.50 m away from the closer side. At an instant when the emf induced in the loop is 2.0 V, what is the time rate of change of the current in the wire?



64. A metal bar of mass 500 g slides outward at a constant speed of 1.5 cm/s over two parallel rails separated by a distance of 30 cm which are part of a U-shaped conductor. There is a uniform magnetic field of magnitude 2 T pointing out of the page over the entire area. The railings and metal bar have an equivalent resistance of $150\ \Omega$. (a) Determine the induced current, both magnitude and direction. (b) Find the direction of the induced current if the magnetic field is pointing into the page. (c) Find the direction of the induced current if the magnetic field is pointed into the page and the bar moves inwards.

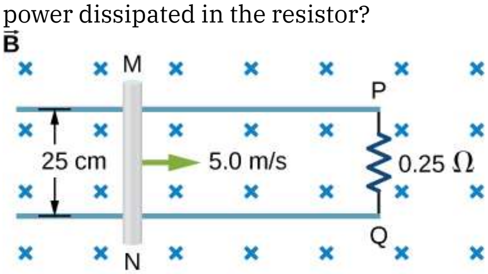
65. A current is induced in a circular loop of radius 1.5 cm between two poles of a horseshoe electromagnet when the current in the electromagnet is varied. The magnetic field in the area of the loop is perpendicular to the area and has a uniform magnitude. If the rate of change of magnetic field is 10 T/s, find the magnitude and direction of the induced current if resistance of the loop is $25\ \Omega$.

66. A metal bar of length 25 cm is placed perpendicular to a uniform magnetic field of strength 3 T. (a) Determine the induced emf between the ends of the rod when it is not moving. (b) Determine the emf when the rod is moving perpendicular to its length and magnetic field with a speed of 50 cm/s.

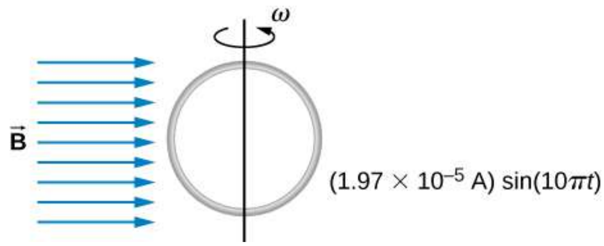
67. A coil with 50 turns and area 10 cm^2 is oriented with its plane perpendicular to a 0.75-T magnetic field. If the coil is flipped over (rotated through 180°) in 0.20 s, what is the average emf induced in it?

68. A 2-turn planer loop of flexible wire is placed inside a long solenoid of n turns per meter that carries a constant current I_0. The area A of the loop is changed by pulling on its sides while ensuring that the plane of the loop always remains perpendicular to the axis of the solenoid. If $n = 500$ turns per meter, $I_0 = 20$ A, and $A = 20\ \text{cm}^2$, what is the emf induced in the loop when $dA/dt = 100$?

69. The conducting rod shown in the accompanying figure moves along parallel metal rails that are 25-cm apart. The system is in a uniform magnetic field of strength 0.75 T, which is directed into the page. The resistances of the rod and the rails are negligible, but the section PQ has a resistance of $0.25\ \Omega$. (a) What is the emf (including its sense) induced in the rod when it is moving to the right with a speed of 5.0 m/s? (b) What force is required to keep the rod moving at this speed? (c) What is the rate at which work is done by this force? (d) What is the power dissipated in the resistor?

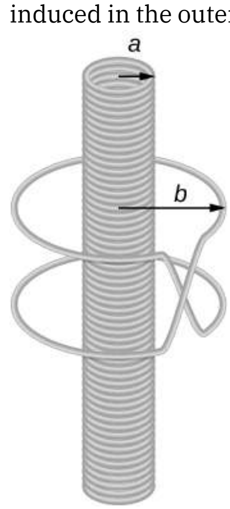


70. A circular loop of wire of radius 10 cm is mounted on a vertical shaft and rotated at a frequency of 5 cycles per second in a region of uniform magnetic field of 2 Gauss perpendicular to the axis of rotation. (a) Find an expression for the time-dependent flux through the ring. (b) Determine the time-dependent current through the ring if it has a resistance of 10 Ω.



71. The magnetic field between the poles of a horseshoe electromagnet is uniform and has a cylindrical symmetry about an axis from the middle of the South Pole to the middle of the North Pole. The magnitude of the magnetic field changes as a rate of dB/dt due to the changing current through the electromagnet. Determine the electric field at a distance r from the center.

72. A long solenoid of radius a with n turns per unit length is carrying a time-dependent current $I(t) = I_0 \sin(\omega t)$, where I_0 and ω are constants. The solenoid is surrounded by a wire of resistance R that has two circular loops of radius b with $b > a$ (see the following figure). Find the magnitude and direction of current induced in the outer loops at time $t = 0$.



73. A 120-V, series-wound dc motor draws 0.50 A from its power source when operating at full speed, and it draws 2.0 A when it starts. The resistance of the armature coils is $10\ \Omega$. (a) What is the resistance of the field coils? (b) What is the back emf of the motor when it is running at full speed? (c) The motor operates at a different speed and draws 1.0 A from the source. What is the back emf in this case?

74. The armature and field coils of a series-wound motor have a total resistance of $3.0\ \Omega$. When connected to a 120-V source and running at normal speed, the motor draws 4.0 A. (a) How large is the back emf? (b) What current will the motor draw just after it is turned on? Can you suggest a way to avoid this large initial current?

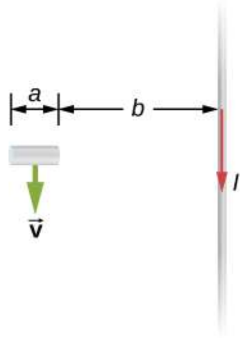
Challenge Problems

75. A copper wire of length L is fashioned into a circular coil with N turns. When the magnetic field through the coil changes with time, for what value of N is the induced emf a maximum?

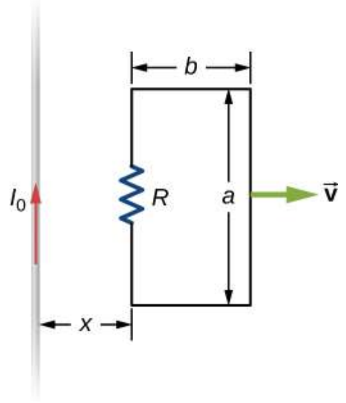
76. A 0.50-kg copper sheet drops through a uniform horizontal magnetic field of 1.5 T, and it reaches a terminal velocity of 2.0 m/s. (a) What is the net magnetic force on the sheet after it reaches terminal velocity? (b) Describe the mechanism responsible for this force. (c) How much power is dissipated as Joule heating while the sheet moves at terminal velocity?

77. A circular copper disk of radius 7.5 cm rotates at 2400 rpm around the axis through its center and perpendicular to its face. The disk is in a uniform magnetic field $\vec{\mathbf{B}}$ of strength 1.2 T that is directed along the axis. What is the potential difference between the rim and the axis of the disk?

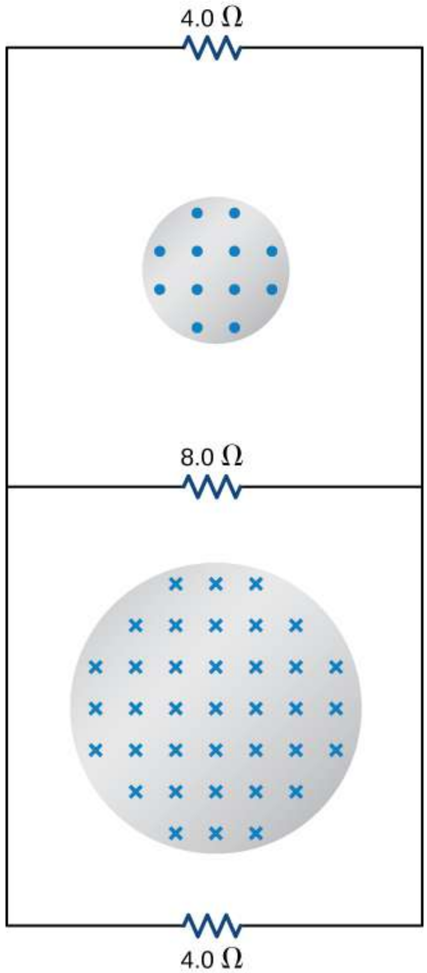
78. A short rod of length a moves with its velocity $\vec{\mathbf{v}}$ parallel to an infinite wire carrying a current I (see below). If the end of the rod nearer the wire is a distance b from the wire, what is the emf induced in the rod?



79. A rectangular circuit containing a resistance R is pulled at a constant velocity $\vec{\mathbf{v}}$ away from a long, straight wire carrying a current I_0 (see below). Derive an equation that gives the current induced in the circuit as a function of the distance x between the near side of the circuit and the wire.

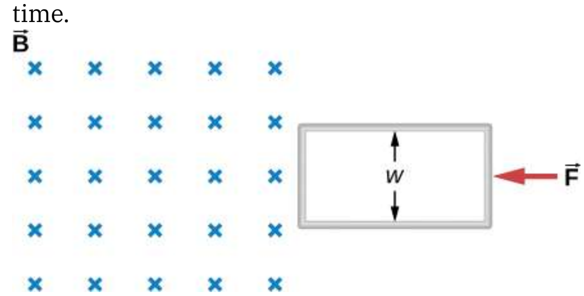


80. Two infinite solenoids cross the plane of the circuit as shown below. The radii of the solenoids are 0.10 and 0.20 m, respectively, and the current in each solenoid is changing such that $dB/dt = 50.0\ \text{T/s}$. What are the currents in the resistors of the circuit?

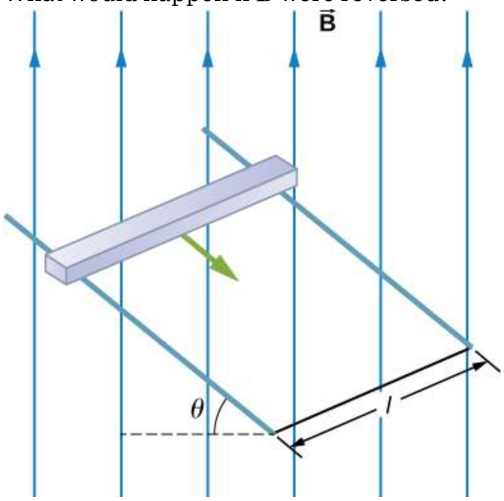


81. An eight-turn coil is *tightly wrapped* around the outside of the long solenoid as shown below. The radius of the solenoid is 2.0 cm and it has 10 turns per centimeter. The current through the solenoid increases according to $I = I_0(1 - e^{-\alpha t})$, where $I_0 = 4.0\ \text{A}$ and $\alpha = 2.0 \times 10^{-2}\ \text{s}^{-1}$. What is the emf induced in the coil when (a) $t = 0$, (b) $t = 1.0 \times 10^2$ s, and (c) $t \to \infty$?

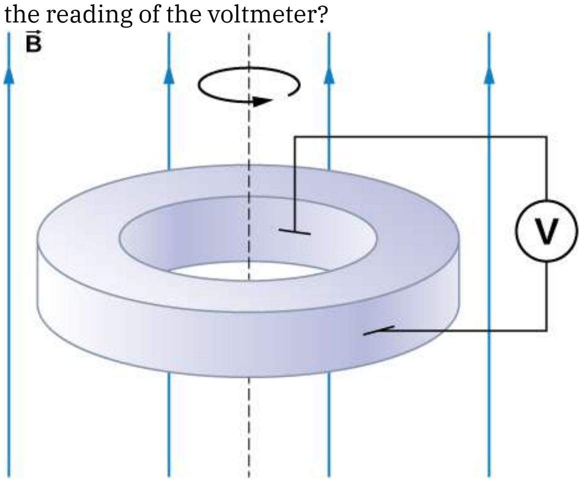
82. Shown below is a long rectangular loop of width w, length l, mass m, and resistance R. The loop starts from rest at the edge of a uniform magnetic field $\vec{\mathbf{B}}$ and is pushed into the field by a constant force $\vec{\mathbf{F}}$. Calculate the speed of the loop as a function of time.



83. A square bar of mass m and resistance R is sliding without friction down very long, parallel conducting rails of negligible resistance (see below). The two rails are a distance l apart and are connected to each other at the bottom of the incline by a zero-resistance wire. The rails are inclined at an angle θ, and there is a uniform vertical magnetic field $\vec{\mathbf{B}}$ throughout the region. (a) Show that the bar acquires a terminal velocity given by $v = \frac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$. (b) Calculate the work per unit time done by the force of gravity. (c) Compare this with the power dissipated in the Joule heating of the bar. (d) What would happen if $\vec{\mathbf{B}}$ were reversed?

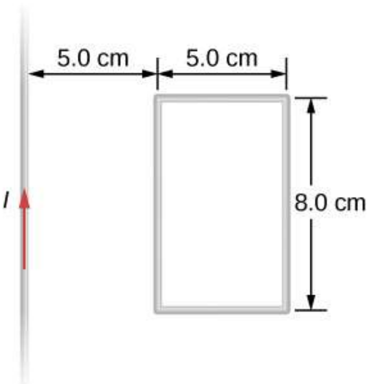


84. The accompanying figure shows a metal disk of inner radius r_1 and other radius r_2 rotating at an angular velocity $\vec{\omega}$ while in a uniform magnetic field directed parallel to the rotational axis. The brush leads of a voltmeter are connected to the dark's inner and outer surfaces as shown. What is the reading of the voltmeter?



85. A long solenoid with 10 turns per centimeter is placed inside a copper ring such that both objects have the same central axis. The radius of the ring is 10.0 cm, and the radius of the solenoid is 5.0 cm. (a) What is the emf induced in the ring when the current I through the solenoid is 5.0 A and changing at a rate of 100 A/s? (b) What is the emf induced in the ring when $I = 2.0$ A and $dI/dt = 100$ A/s? (c) What is the electric field inside the ring for these two cases? (d) Suppose the ring is moved so that its central axis and the central axis of the solenoid are still parallel but no longer coincide. (You should assume that the solenoid is still inside the ring.) Now what is the emf induced in the ring? (e) Can you calculate the electric field in the ring as you did in part (c)?

86. The current in the long, straight wire shown in the accompanying figure is given by $I = I_0 \sin \omega t$, where $I_0 = 15$ A and $\omega = 120\pi$ rad/s. What is the current induced in the rectangular loop at (a) $t = 0$ and (b) $t = 2.1 \times 10^{-3}$ s? The resistance of the loop is 2.0 Ω.

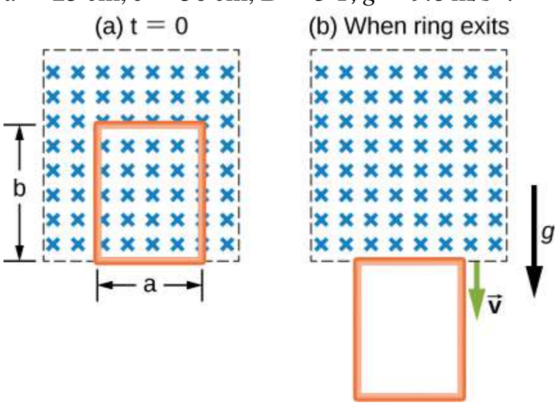


87. A 500-turn coil with a 0.250-m^2 area is spun in Earth's 5.00×10^{-5} T magnetic field, producing a 12.0-kV maximum emf. (a) At what angular velocity must the coil be spun? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

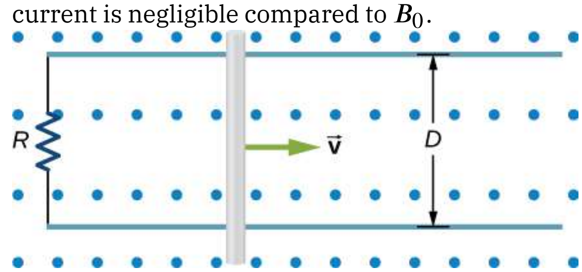
88. A circular loop of wire of radius 10 cm is mounted on a vertical shaft and rotated at a frequency of 5 cycles per second in a region of uniform magnetic field of $2 \times 10^{-4} T$ perpendicular to the axis of rotation. (a) Find an expression for the time-dependent flux through the ring (b) Determine the time-dependent current through the ring if it has a resistance of 10 Ω.

89. A long solenoid of radius a with n turns per unit length is carrying a time-dependent current $I(t) = I_0 \sin \omega t$ where I_0 and ω are constants. The solenoid is surrounded by a wire of resistance R that has two circular loops of radius b with $b > a$. Find the magnitude and direction of current induced in the outer loops at time $t = 0$.

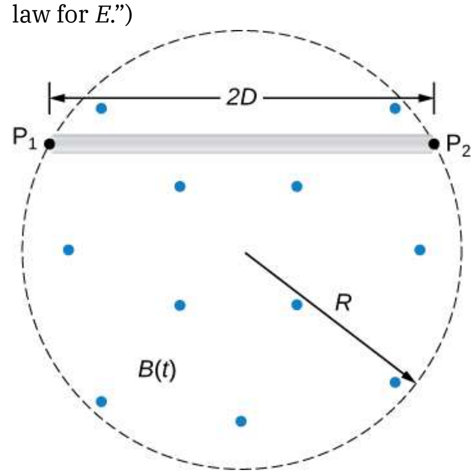
90. A rectangular copper loop of mass 100 g and resistance $0.2\ \Omega$ is in a region of uniform magnetic field that is perpendicular to the area enclosed by the ring and horizontal to Earth's surface (see below). The loop is let go from rest when it is at the edge of the nonzero magnetic field region. (a) Find an expression for the speed when the loop just exits the region of uniform magnetic field. (b) If it was let go at $t = 0$, what is the time when it exits the region of magnetic field for the following values: $a = 25\ \text{cm}$, $b = 50\ \text{cm}$, $B = 3\ \text{T}$, $g = 9.8\ \text{m/s}^2$?



91. A metal bar of mass m slides without friction over two rails a distance D apart in the region that has a uniform magnetic field of magnitude B_0 and direction perpendicular to the rails (see below). The two rails are connected at one end to a resistor whose resistance is much larger than the resistance of the rails and the bar. The bar is given an initial speed of v_0. It is found to slow down. How far does the bar go before coming to rest? Assume that the magnetic field of the induced current is negligible compared to B_0.



92. A time-dependent uniform magnetic field of magnitude $B(t)$ is confined in a cylindrical region of radius R. A conducting rod of length $2D$ is placed in the region, as shown below. Show that the emf between the ends of the rod is given by $\frac{dB}{dt} D\sqrt{R^2 - D^2}$. (*Hint:* To find the emf between the ends, we need to integrate the electric field from one end to the other. To find the electric field, use Faraday's law as "Ampère's law for E.")



CHAPTER 14
Inductance

Figure 14.1 A smartphone charging mat contains a coil that receives alternating current, or current that is constantly increasing and decreasing. The varying current induces an emf in the smartphone, which charges its battery. Note that the black box containing the electrical plug also contains a transformer (discussed in Alternating-Current Circuits) that modifies the current from the outlet to suit the needs of the smartphone. (credit: modification of work by "LG"/Flickr)

Chapter Outline

INTRODUCTION In Electromagnetic Induction, we discussed how a time-varying magnetic flux induces an emf in a circuit. In many of our calculations, this flux was due to an applied time-dependent magnetic field. The reverse of this phenomenon also occurs: The current flowing in a circuit produces its own magnetic field.

In Electric Charges and Fields, we saw that induction is the process by which an emf is induced by changing electric flux and separation of a dipole. So far, we have discussed some examples of induction, although some of these applications are more effective than others. The smartphone charging mat in the chapter opener photo also works by induction. Is there a useful physical quantity related to how "effective" a given device is? The answer is yes, and that physical quantity is *inductance*. In this chapter, we look at the applications of inductance in electronic devices and how inductors are used in circuits.

14.1 Mutual Inductance

Learning Objectives

By the end of this section, you will be able to:

- Correlate two nearby circuits that carry time-varying currents with the emf induced in each circuit
- Describe examples in which mutual inductance may or may not be desirable

Inductance is the property of a device that tells us how effectively it induces an emf in another device. In other words, it is a physical quantity that expresses the effectiveness of a given device.

When two circuits carrying time-varying currents are close to one another, the magnetic flux through each circuit varies because of the changing current I in the other circuit. Consequently, an emf is induced in each circuit by the changing current in the other. This type of emf is therefore called a *mutually induced emf*, and the phenomenon that occurs is known as **mutual inductance (*M*)**. As an example, let's consider two tightly wound coils (Figure 14.2). Coils 1 and 2 have N_1 and N_2 turns and carry currents I_1 and I_2, respectively. The flux through a single turn of coil 2 produced by the magnetic field of the current in coil 1 is Φ_{21}, whereas the flux through a single turn of coil 1 due to the magnetic field of I_2 is Φ_{12}.

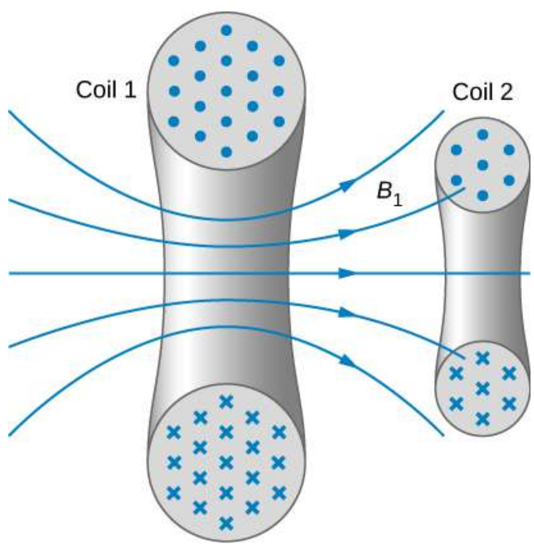


Figure 14.2 Some of the magnetic field lines produced by the current in coil 1 pass through coil 2.

The mutual inductance M_{21} of coil 2 with respect to coil 1 is the ratio of the flux through the N_2 turns of coil 2 produced by the magnetic field of the current in coil 1, divided by that current, that is,

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}. \tag{14.1}$$

Similarly, the mutual inductance of coil 1 with respect to coil 2 is

$$M_{12} = \frac{N_1 \Phi_{12}}{I_2}. \tag{14.2}$$

Like capacitance, mutual inductance is a geometric quantity. It depends on the shapes and relative positions of the two coils, and it is independent of the currents in the coils. The SI unit for mutual inductance M is called the **henry (H)** in honor of Joseph Henry (1799–1878), an American scientist who discovered induced emf independently of Faraday. Thus, we have $1 \text{ H} = 1 \text{ V} \cdot \text{s/A}$. From Equation 14.1 and Equation 14.2, we can show that $M_{21} = M_{12}$, so we usually drop the subscripts associated with mutual inductance and write

$$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_1 \Phi_{12}}{I_2}. \tag{14.3}$$

The emf developed in either coil is found by combining Faraday's law and the definition of mutual inductance. Since $N_2 \Phi_{21}$ is the total flux through coil 2 due to I_1, we obtain

$$\varepsilon_2 = -\frac{d}{dt}(N_2 \Phi_{21}) = -\frac{d}{dt}(M I_1) = -M \frac{dI_1}{dt} \tag{14.4}$$

where we have used the fact that M is a time-independent constant because the geometry is time-independent. Similarly, we have

$$\varepsilon_1 = -M \frac{dI_2}{dt}. \tag{14.5}$$

In Equation 14.5, we can see the significance of the earlier description of mutual inductance (M) as a geometric quantity. The value of M neatly encapsulates the physical properties of circuit elements and allows us to separate the physical layout of the circuit from the dynamic quantities, such as the emf and the current. Equation 14.5 defines the mutual inductance in terms of properties in the circuit, whereas the previous definition of mutual inductance in Equation 14.1 is defined in terms of the magnetic flux experienced, regardless of circuit elements. You should be careful when using Equation 14.4 and Equation 14.5 because ε_1 and ε_2 do not necessarily represent the total emfs in the respective coils. Each coil can also have an emf induced in it because of its *self-inductance* (self-inductance will be discussed in more detail in a later section).

A large mutual inductance M may or may not be desirable. We want a transformer to have a large mutual inductance. But an appliance, such as an electric clothes dryer, can induce a dangerous emf on its metal case if the mutual inductance between its coils and the case is large. One way to reduce mutual inductance is to counter-wind coils to cancel the magnetic field produced (Figure 14.3).

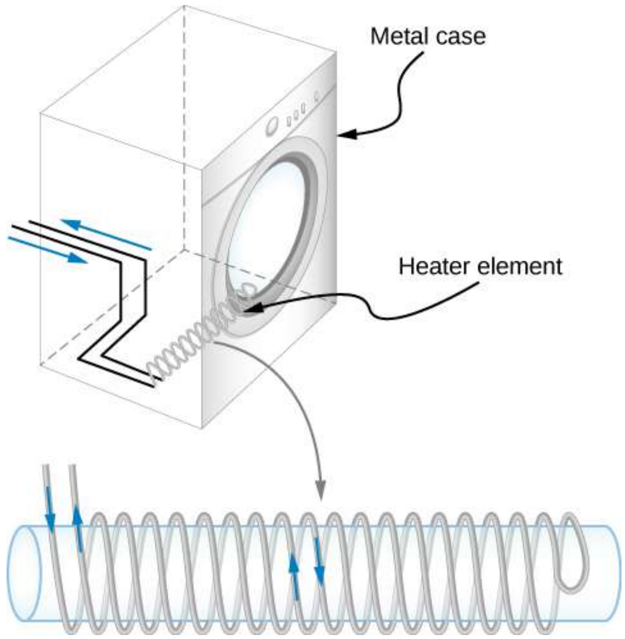


Figure 14.3 The heating coils of an electric clothes dryer can be counter-wound so that their magnetic fields cancel one another, greatly reducing the mutual inductance with the case of the dryer.

Digital signal processing is another example in which mutual inductance is reduced by counter-winding coils. The rapid on/off emf representing 1s and 0s in a digital circuit creates a complex time-dependent magnetic field. An emf can be generated in neighboring conductors. If that conductor is also carrying a digital signal, the induced emf may be large enough to switch 1s and 0s, with consequences ranging from inconvenient to disastrous.

EXAMPLE 14.1

Mutual Inductance

Figure 14.4 shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1, and N_1 turns. (a) What is the mutual inductance of the two coils? (b) If $N_1 = 500$ turns, $N_2 = 10$ turns, $R_1 = 3.10$ cm, $l_1 = 75.0$ cm, and the current in the solenoid is changing at a rate of 200 A/s, what is the emf induced in the surrounding coil?

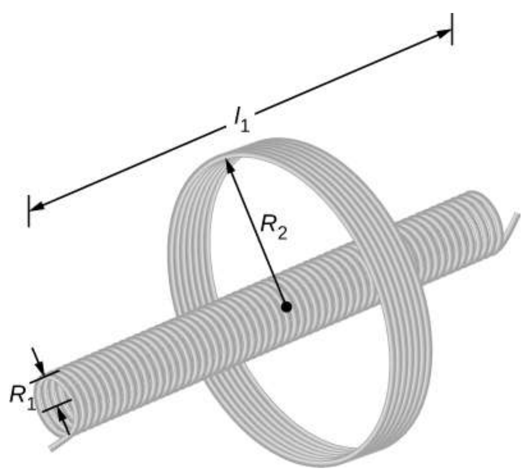


Figure 14.4 A solenoid surrounded by a coil.

Strategy

There is no magnetic field outside the solenoid, and the field inside has magnitude $B_1 = \mu_0(N_1/l_1)I_1$ and is directed parallel to the solenoid's axis. We can use this magnetic field to find the magnetic flux through the surrounding coil and then use this flux to calculate the mutual inductance for part (a), using Equation 14.3. We solve part (b) by calculating the mutual inductance from the given quantities and using Equation 14.4 to calculate the induced emf.

Solution

a. The magnetic flux Φ_{21} through the surrounding coil is

$$\Phi_{21} = B_1 \pi R_1^2 = \frac{\mu_0 N_1 I_1}{l_1} \pi R_1^2.$$

Now from Equation 14.3, the mutual inductance is

$$M = \frac{N_2 \Phi_{21}}{I_1} = \left(\frac{N_2}{I_1}\right)\left(\frac{\mu_0 N_1 I_1}{l_1}\right)\pi R_1^2 = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}.$$

b. Using the previous expression and the given values, the mutual inductance is

$$\begin{aligned} M &= \frac{(4\pi \times 10^{-7}\ \text{T·m/A})(500)(10)\pi(0.0310\ \text{m})^2}{0.750\ \text{m}} \\ &= 2.53 \times 10^{-5}\ \text{H}. \end{aligned}$$

Thus, from Equation 14.4, the emf induced in the surrounding coil is

$$\begin{aligned} \varepsilon_2 &= -M\frac{dI_1}{dt} = -(2.53 \times 10^{-5}\ \text{H})(200\ \text{A/s}) \\ &= -5.06 \times 10^{-3}\ \text{V}. \end{aligned}$$

Significance

Notice that M in part (a) is independent of the radius R_2 of the surrounding coil because the solenoid's magnetic field is confined to its interior. In principle, we can also calculate M by finding the magnetic flux through the solenoid produced by the current in the surrounding coil. This approach is much more difficult because Φ_{12} is so complicated. However, since $M_{12} = M_{21}$, we do know the result of this calculation.

CHECK YOUR UNDERSTANDING 14.1

A current $I(t) = (5.0\ \text{A})\sin((120\pi\ \text{rad/s})t)$ flows through the solenoid of part (b) of Example 14.1. What is the maximum emf induced in the surrounding coil?

14.2 Self-Inductance and Inductors

Learning Objectives

By the end of this section, you will be able to:

- Correlate the rate of change of current to the induced emf created by that current in the same circuit
- Derive the self-inductance for a cylindrical solenoid
- Derive the self-inductance for a rectangular toroid

Mutual inductance arises when a current in one circuit produces a changing magnetic field that induces an emf in another circuit. But can the magnetic field affect the current in the original circuit that produced the field? The answer is yes, and this is the phenomenon called *self-inductance*.

Inductors

Figure 14.5 shows some of the magnetic field lines due to the current in a circular loop of wire. If the current is constant, the magnetic flux through the loop is also constant. However, if the current I were to vary with time—say, immediately after switch S is closed—then the magnetic flux Φ_m would correspondingly change. Then Faraday's law tells us that an emf ε would be induced in the circuit, where

$$\varepsilon = -\frac{d\Phi_m}{dt}. \qquad 14.6$$

Since the magnetic field due to a current-carrying wire is directly proportional to the current, the flux due to this field is also proportional to the current; that is,

$$\Phi_m \propto I. \tag{14.7}$$

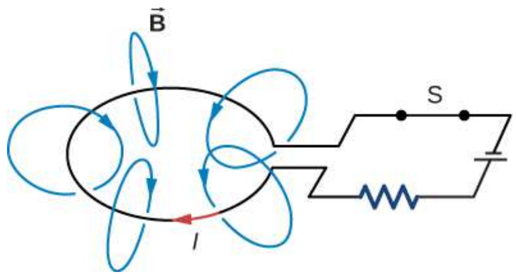


Figure 14.5 A magnetic field is produced by the current I in the loop. If I were to vary with time, the magnetic flux through the loop would also vary and an emf would be induced in the loop.

This can also be written as

$$\Phi_m = LI \tag{14.8}$$

where the constant of proportionality L is known as the **self-inductance** of the wire loop. If the loop has N turns, this equation becomes

$$N\Phi_m = LI. \tag{14.9}$$

By convention, the positive sense of the normal to the loop is related to the current by the right-hand rule, so in Figure 14.5, the normal points downward. With this convention, Φ_m is positive in Equation 14.9, so L *always has a positive value.*

For a loop with N turns, $\varepsilon = -N d\Phi_m/dt$, so the induced emf may be written in terms of the self-inductance as

$$\varepsilon = -L\frac{dI}{dt}. \tag{14.10}$$

When using this equation to determine L, it is easiest to ignore the signs of ε and dI/dt, and calculate L as

$$L = \frac{|\varepsilon|}{|dI/dt|}.$$

Since self-inductance is associated with the magnetic field produced by a current, any configuration of conductors possesses self-inductance. For example, besides the wire loop, a long, straight wire has self-inductance, as does a coaxial cable. A coaxial cable is most commonly used by the cable television industry and may also be found connecting to your cable modem. Coaxial cables are used due to their ability to transmit electrical signals with minimal distortions. Coaxial cables have two long cylindrical conductors that possess current and a self-inductance that may have undesirable effects.

A circuit element used to provide self-inductance is known as an **inductor**. It is represented by the symbol shown in Figure 14.6, which resembles a coil of wire, the basic form of the inductor. Figure 14.7 shows several types of inductors commonly used in circuits.

Figure 14.6 Symbol used to represent an inductor in a circuit.

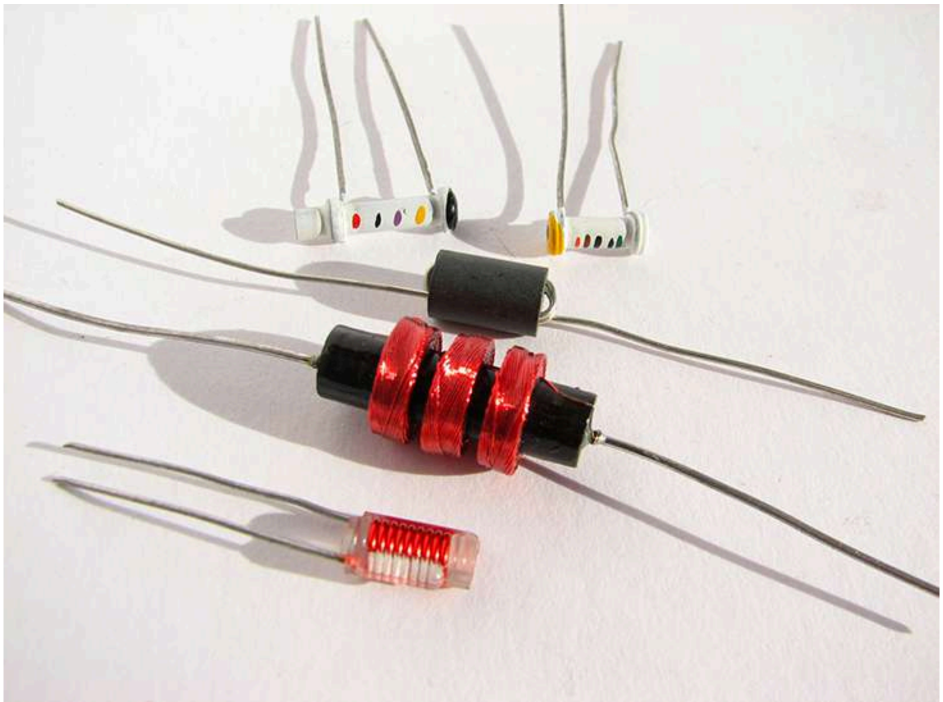


Figure 14.7 A variety of inductors. Whether they are encapsulated like the top three shown or wound around in a coil like the bottom-most one, each is simply a relatively long coil of wire. (credit: Windell Oskay)

In accordance with Lenz's law, the negative sign in Equation 14.10 indicates that the induced emf across an inductor always has a polarity that *opposes* the change in the current. For example, if the current flowing from A to B in Figure 14.8(a) were increasing, the induced emf (represented by the imaginary battery) would have the polarity shown in order to oppose the increase. If the current from A to B were decreasing, then the induced emf would have the opposite polarity, again to oppose the change in current (Figure 14.8(b)). Finally, if the current through the inductor were constant, no emf would be induced in the coil.

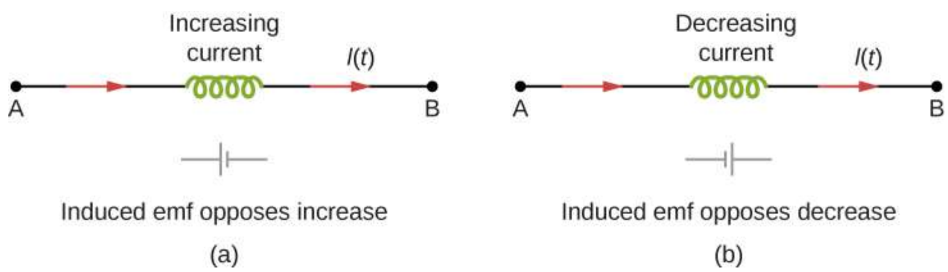


Figure 14.8 The induced emf across an inductor always acts to oppose the change in the current. This can be visualized as an imaginary battery causing current to flow to oppose the change in (a) and reinforce the change in (b).

One common application of inductance is to allow traffic signals to sense when vehicles are waiting at a street intersection. An electrical circuit with an inductor is placed in the road underneath the location where a waiting car will stop. The body of the car increases the inductance and the circuit changes, sending a signal to the traffic lights to change colors. Similarly, metal detectors used for airport security employ the same technique. A coil or inductor in the metal detector frame acts as both a transmitter and a receiver. The pulsed signal from the transmitter coil induces a signal in the receiver. The self-inductance of the circuit is affected by any metal object in the path (Figure 14.9). Metal detectors can be adjusted for sensitivity and can also sense the presence of metal on a person.

Figure 14.9 The familiar security gate at an airport not only detects metals, but can also indicate their approximate height above the floor. (credit: "Alexbuirds"/Wikimedia Commons)

Large induced voltages are found in camera flashes. Camera flashes use a battery, two inductors that function as a transformer, and a switching system or *oscillator* to induce large voltages. Recall from Oscillations on oscillations that "oscillation" is defined as the fluctuation of a quantity, or repeated regular fluctuations of a quantity, between two extreme values around an average value. Also recall (from Electromagnetic Induction on electromagnetic induction) that we need a changing magnetic field, brought about by a changing current, to induce a voltage in another coil. The oscillator system does this many times as the battery voltage is boosted to over 1000 volts. (You may hear the high-pitched whine from the transformer as the capacitor is being charged.) A capacitor stores the high voltage for later use in powering the flash.

EXAMPLE 14.2

Self-Inductance of a Coil

An induced emf of 20 mV is measured across a coil of 50 closely wound turns while the current through it increases uniformly from 0.0 to 5.0 A in 0.10 s. (a) What is the self-inductance of the coil? (b) With the current at 5.0 A, what is the flux through each turn of the coil?

Strategy

Both parts of this problem give all the information needed to solve for the self-inductance in part (a) or the flux through each turn of the coil in part (b). The equations needed are Equation 14.10 for part (a) and Equation 14.9 for part (b).

Solution

a. Ignoring the negative sign and using magnitudes, we have, from Equation 14.10,

$$L = \frac{\varepsilon}{dI/dt} = \frac{20\,\text{mV}}{5.0\,\text{A}/0.10\,\text{s}} = 4.0 \times 10^{-2}\,\text{H}.$$

b. From Equation 14.9, the flux is given in terms of the current by $\Phi_\text{m} = LI/N$, so

$$\Phi_\text{m} = \frac{(4.0 \times 10^{-2}\,\text{H})(5.0\,\text{A})}{50\,\text{turns}} = 4.0 \times 10^{-3}\,\text{Wb}.$$

Significance

The self-inductance and flux calculated in parts (a) and (b) are typical values for coils found in contemporary devices. If the current is not changing over time, the flux is not changing in time, so no emf is induced.

CHECK YOUR UNDERSTANDING 14.2

Current flows through the inductor in Figure 14.8 from B to A instead of from A to B as shown. Is the current increasing or decreasing in order to produce the emf given in diagram (a)? In diagram (b)?

CHECK YOUR UNDERSTANDING 14.3

A changing current induces an emf of 10 V across a 0.25-H inductor. What is the rate at which the current is changing?

A good approach for calculating the self-inductance of an inductor consists of the following steps:

PROBLEM-SOLVING STRATEGY

Self-Inductance

1. Assume a current I is flowing through the inductor.
2. Determine the magnetic field $\vec{\mathbf{B}}$ produced by the current. If there is appropriate symmetry, you may be able to do this with Ampère's law.
3. Obtain the magnetic flux, Φ_m.
4. With the flux known, the self-inductance can be found from Equation 14.9, $L = N\Phi_m/I$.

To demonstrate this procedure, we now calculate the self-inductances of two inductors.

Cylindrical Solenoid

Consider a long, cylindrical solenoid with length l, cross-sectional area A, and N turns of wire. We assume that the length of the solenoid is so much larger than its diameter that we can take the magnetic field to be $B = \mu_0 nI$ throughout the interior of the solenoid, that is, we ignore end effects in the solenoid. With a current I flowing through the coils, the magnetic field produced within the solenoid is

$$B = \mu_0 \left(\frac{N}{l}\right) I, \quad 14.11$$

so the magnetic flux through one turn is

$$\Phi_m = BA = \frac{\mu_0 NA}{l} I. \quad 14.12$$

Using Equation 14.9, we find for the self-inductance of the solenoid,

$$L_{\text{solenoid}} = \frac{N\Phi_m}{I} = \frac{\mu_0 N^2 A}{l}. \quad 14.13$$

If $n = N/l$ is the number of turns per unit length of the solenoid, we may write Equation 14.13 as

$$L = \mu_0 \left(\frac{N}{l}\right)^2 Al = \mu_0 n^2 Al = \mu_0 n^2 (V), \quad 14.14$$

where $V = Al$ is the volume of the solenoid. Notice that *the self-inductance of a long solenoid depends only on its physical properties* (such as the number of turns of wire per unit length and the volume), and not on the magnetic field or the current. This is true for inductors in general.

Rectangular Toroid

A toroid with a rectangular cross-section is shown in Figure 14.10. The inner and outer radii of the toroid are R_1 and R_2, and h is the height of the toroid. Applying Ampère's law in the same manner as we did in Example

13.8 for a toroid with a circular cross-section, we find the magnetic field inside a rectangular toroid is also given by

$$B = \frac{\mu_0 NI}{2\pi r}, \tag{14.15}$$

where r is the distance from the central axis of the toroid. Because the field changes within the toroid, we must calculate the flux by integrating over the toroid's cross-section. Using the infinitesimal cross-sectional area element $da = h\,dr$ shown in Figure 14.10, we obtain

$$\Phi_\text{m} = \int B\,da = \int_{R_1}^{R_2} \left(\frac{\mu_0 NI}{2\pi r}\right)(hdr) = \frac{\mu_0 NhI}{2\pi} \ln \frac{R_2}{R_1}. \tag{14.16}$$

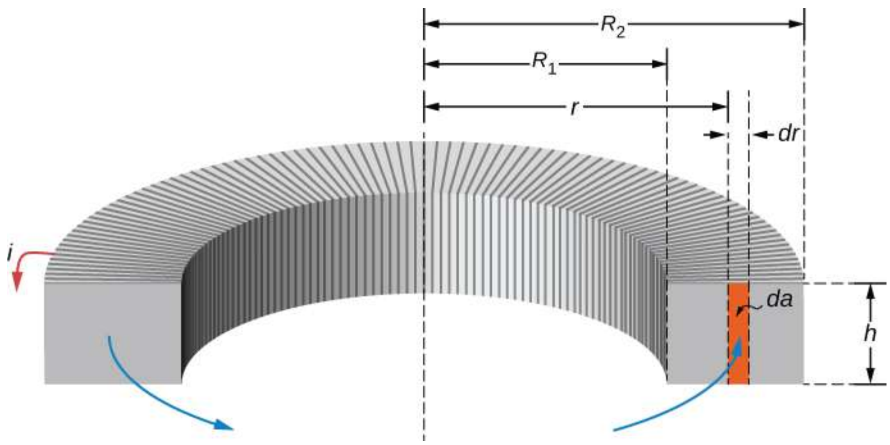


Figure 14.10 Calculating the self-inductance of a rectangular toroid.

Now from Equation 14.16, we obtain for the self-inductance of a rectangular toroid

$$L = \frac{N\Phi_\text{m}}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{R_2}{R_1}. \tag{14.17}$$

As expected, the self-inductance is a constant determined by only the physical properties of the toroid.

CHECK YOUR UNDERSTANDING 14.4

(a) Calculate the self-inductance of a solenoid that is tightly wound with wire of diameter 0.10 cm, has a cross-sectional area of $0.90\ \text{cm}^2$, and is 40 cm long. (b) If the current through the solenoid decreases uniformly from 10 to 0 A in 0.10 s, what is the emf induced between the ends of the solenoid?

CHECK YOUR UNDERSTANDING 14.5

(a) What is the magnetic flux through one turn of a solenoid of self-inductance 8.0×10^{-5} H when a current of 3.0 A flows through it? Assume that the solenoid has 1000 turns and is wound from wire of diameter 1.0 mm. (b) What is the cross-sectional area of the solenoid?

14.3 Energy in a Magnetic Field

Learning Objectives

By the end of this section, you will be able to:

- Explain how energy can be stored in a magnetic field
- Derive the equation for energy stored in a coaxial cable given the magnetic energy density

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability to store energy, but in its magnetic field. This energy can be found by integrating the **magnetic**

energy density,

$$u_{\mathrm{m}} = \frac{B^2}{2\mu_0} \quad \textbf{14.18}$$

over the appropriate volume. To understand where this formula comes from, let's consider the long, cylindrical solenoid of the previous section. Again using the infinite solenoid approximation, we can assume that the magnetic field is essentially constant and given by $B = \mu_0 nI$ everywhere inside the solenoid. Thus, the energy stored in a solenoid or the magnetic energy density times volume is equivalent to

$$U = u_{\mathrm{m}}(V) = \frac{(\mu_0 nI)^2}{2\mu_0}(Al) = \frac{1}{2}\left(\mu_0 n^2 Al\right) I^2. \quad \textbf{14.19}$$

With the substitution of Equation 14.14, this becomes

$$U = \frac{1}{2}LI^2. \quad \textbf{14.20}$$

Although derived for a special case, this equation gives the energy stored in the magnetic field of *any* inductor. We can see this by considering an arbitrary inductor through which a changing current is passing. At any instant, the magnitude of the induced emf is $\varepsilon = Ldi/dt$, where i is the induced current at that instance. Therefore, the power absorbed by the inductor is

$$P = \varepsilon i = L\frac{di}{dt}i. \quad \textbf{14.21}$$

The total energy stored in the magnetic field when the current increases from 0 to I in a time interval from 0 to t can be determined by integrating this expression:

$$U = \int_0^t Pdt' = \int_0^t L\frac{di}{dt'}idt' = L\int_0^I idi = \frac{1}{2}LI^2. \quad \textbf{14.22}$$

EXAMPLE 14.3

Self-Inductance of a Coaxial Cable

Figure 14.11 shows two long, concentric cylindrical shells of radii R_1 and R_2. As discussed in Capacitance on capacitance, this configuration is a simplified representation of a coaxial cable. The capacitance per unit length of the cable has already been calculated. Now (a) determine the magnetic energy stored per unit length of the coaxial cable and (b) use this result to find the self-inductance per unit length of the cable.

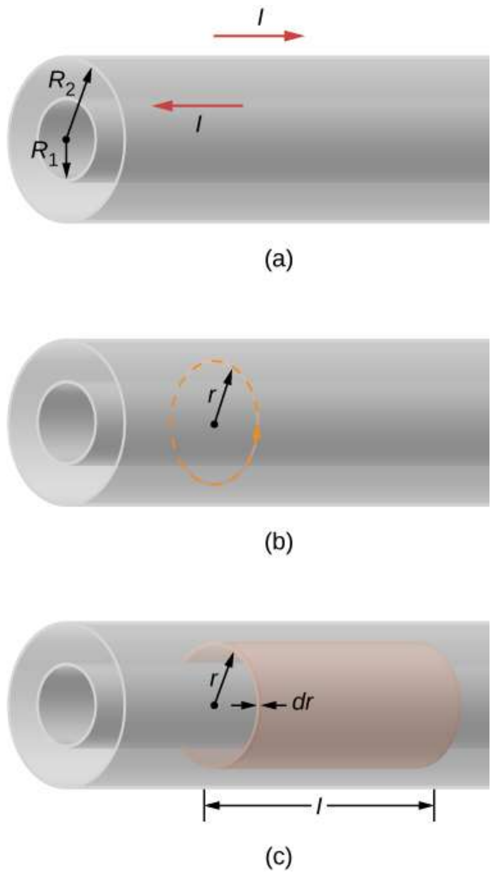


Figure 14.11 (a) A coaxial cable is represented here by two hollow, concentric cylindrical conductors along which electric current flows in opposite directions. (b) The magnetic field between the conductors can be found by applying Ampère's law to the dashed path. (c) The cylindrical shell is used to find the magnetic energy stored in a length *l* of the cable.

Strategy

The magnetic field both inside and outside the coaxial cable is determined by Ampère's law. Based on this magnetic field, we can use Equation 14.22 to calculate the energy density of the magnetic field. The magnetic energy is calculated by an integral of the magnetic energy density times the differential volume over the cylindrical shell. After the integration is carried out, we have a closed-form solution for part (a). The self-inductance per unit length is determined based on this result and Equation 14.22.

Solution

a. We determine the magnetic field between the conductors by applying Ampère's law to the dashed circular path shown in Figure 14.11(b). Because of the cylindrical symmetry, $\vec{\mathbf{B}}$ is constant along the path, and

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B(2\pi r) = \mu_0 I.$$

This gives us

$$B = \frac{\mu_0 I}{2\pi r}.$$

In the region outside the cable, a similar application of Ampère's law shows that $B = 0$, since no net current crosses the area bounded by a circular path where $r > R_2$. This argument also holds when $r < R_1$; that is, in the region within the inner cylinder. All the magnetic energy of the cable is therefore stored between the two conductors. Since the energy density of the magnetic field is

$$u_{\mathrm{m}} = \frac{B^2}{2\mu_0}$$

the energy stored in a cylindrical shell of inner radius r, outer radius $r + dr$, and length l (see part (c) of the figure) is

$$u_{\text{m}} = \frac{\mu_0 I^2}{8\pi^2 r^2}.$$

Thus, the total energy of the magnetic field in a length l of the cable is

$$U = \int_{R_1}^{R_2} dU = \int_{R_1}^{R_2} \frac{\mu_0 I^2}{8\pi^2 r^2}(2\pi r l)dr = \frac{\mu_0 I^2 l}{4\pi} \ln \frac{R_2}{R_1},$$

and the energy per unit length is $(\mu_0 I^2/4\pi)\ln(R_2/R_1)$.

b. From Equation 14.22,

$$U = \frac{1}{2}LI^2,$$

where L is the self-inductance of a length l of the coaxial cable. Equating the previous two equations, we find that the self-inductance per unit length of the cable is

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{R_2}{R_1}.$$

Significance

The inductance per unit length depends only on the inner and outer radii as seen in the result. To increase the inductance, we could either increase the outer radius (R_2) or decrease the inner radius (R_1). In the limit as the two radii become equal, the inductance goes to zero. In this limit, there is no coaxial cable. Also, the magnetic energy per unit length from part (a) is proportional to the square of the current.

CHECK YOUR UNDERSTANDING 14.6

How much energy is stored in the inductor of Example 14.2 after the current reaches its maximum value?

14.4 RL Circuits

Learning Objectives

By the end of this section, you will be able to:

- Analyze circuits that have an inductor and resistor in series
- Describe how current and voltage exponentially grow or decay based on the initial conditions

A circuit with resistance and self-inductance is known as an *RL* circuit. Figure 14.12(a) shows an *RL* circuit consisting of a resistor, an inductor, a constant source of emf, and switches S_1 and S_2. When S_1 is closed, the circuit is equivalent to a single-loop circuit consisting of a resistor and an inductor connected across a source of emf (Figure 14.12(b)). When S_1 is opened and S_2 is closed, the circuit becomes a single-loop circuit with only a resistor and an inductor (Figure 14.12(c)).

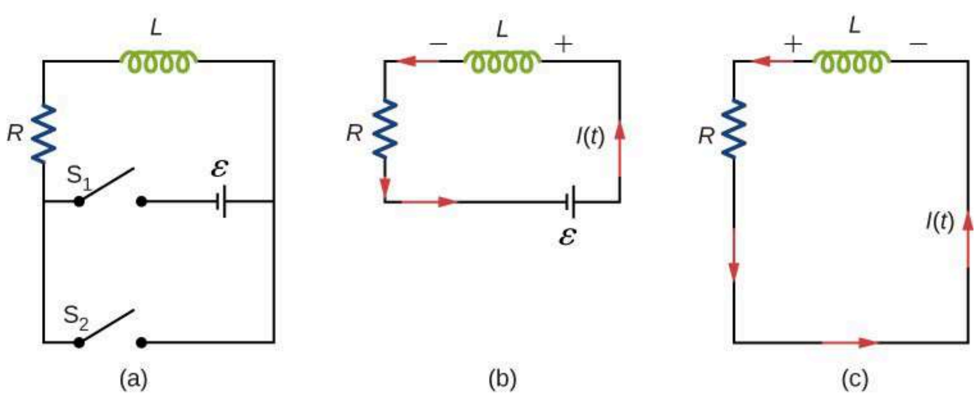


Figure 14.12 (a) An *RL* circuit with switches S_1 and S_2. (b) The equivalent circuit with S_1 closed and S_2 open. (c) The equivalent circuit

after S_1 is opened and S_2 is closed.

We first consider the *RL* circuit of Figure 14.12(b). Once S_1 is closed and S_2 is open, the source of emf produces a current in the circuit. If there were no self-inductance in the circuit, the current would rise immediately to a steady value of ε/R. However, from Faraday's law, the increasing current produces an emf $V_L = -L(dI/dt)$ across the inductor. In accordance with Lenz's law, the induced emf counteracts the increase in the current and is directed as shown in the figure. As a result, *I(t)* starts at zero and increases asymptotically to its final value.

Applying Kirchhoff's loop rule to this circuit, we obtain

$$\varepsilon - L\frac{dI}{dt} - IR = 0, \quad (14.23)$$

which is a first-order differential equation for *I(t)*. Notice its similarity to the equation for a capacitor and resistor in series (See *RC* Circuits). Similarly, the solution to Equation 14.23 can be found by making substitutions in the equations relating the capacitor to the inductor. This gives

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right), \quad (14.24)$$

where

$$\tau_L = L/R \quad (14.25)$$

is the **inductive time constant** of the circuit.

The current *I(t)* is plotted in Figure 14.13(a). It starts at zero, and as $t \to \infty$, *I(t)* approaches ε/R asymptotically. The induced emf $V_L(t)$ is directly proportional to dI/dt, or the slope of the curve. Hence, while at its greatest immediately after the switches are thrown, the induced emf decreases to zero with time as the current approaches its final value of ε/R. The circuit then becomes equivalent to a resistor connected across a source of emf.

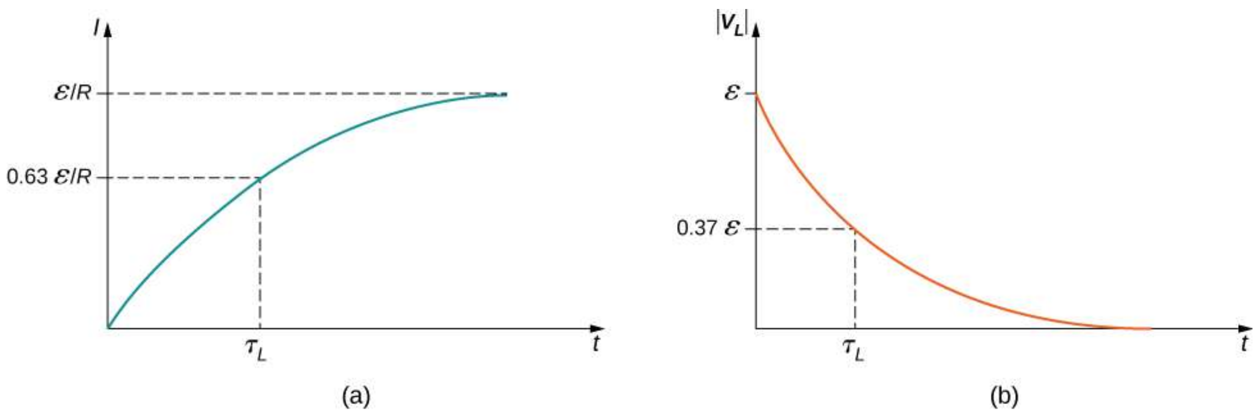


Figure 14.13 Time variation of (a) the electric current and (b) the magnitude of the induced voltage across the coil in the circuit of Figure 14.12(b).

The energy stored in the magnetic field of an inductor is

$$U_L = \frac{1}{2}LI^2. \quad (14.26)$$

Thus, as the current approaches the maximum current ε/R, the stored energy in the inductor increases from zero and asymptotically approaches a maximum of $L(\varepsilon/R)^2/2$.

The time constant τ_L tells us how rapidly the current increases to its final value. At $t = \tau_L$, the current in the circuit is, from Equation 14.24,

$$I(\tau_L) = \frac{\varepsilon}{R}(1 - e^{-1}) = 0.63\frac{\varepsilon}{R}, \quad \textbf{14.27}$$

which is 63% of the final value ε/R. The smaller the inductive time constant $\tau_L = L/R$, the more rapidly the current approaches ε/R.

We can find the time dependence of the induced voltage across the inductor in this circuit by using $V_L(t) = -L(dI/dt)$ and Equation 14.24:

$$V_L(t) = -L\frac{dI}{dt} = -\varepsilon e^{-t/\tau_L}. \quad \textbf{14.28}$$

The magnitude of this function is plotted in Figure 14.13(b). The greatest value of $L(dI/dt)$ is ε; it occurs when dI/dt is greatest, which is immediately after S_1 is closed and S_2 is opened. In the approach to steady state, dI/dt decreases to zero. As a result, the voltage across the inductor also vanishes as $t \to \infty$.

The time constant τ_L also tells us how quickly the induced voltage decays. At $t = \tau_L$, the magnitude of the induced voltage is

$$|V_L(\tau_L)| = \varepsilon e^{-1} = 0.37\varepsilon = 0.37V(0). \quad \textbf{14.29}$$

The voltage across the inductor therefore drops to about 37% of its initial value after one time constant. The shorter the time constant τ_L, the more rapidly the voltage decreases.

After enough time has elapsed so that the current has essentially reached its final value, the positions of the switches in Figure 14.12(a) are reversed, giving us the circuit in part (c). At $t = 0$, the current in the circuit is $I(0) = \varepsilon/R$. With Kirchhoff's loop rule, we obtain

$$IR + L\frac{dI}{dt} = 0. \quad \textbf{14.30}$$

The solution to this equation is similar to the solution of the equation for a discharging capacitor, with similar substitutions. The current at time t is then

$$I(t) = \frac{\varepsilon}{R}e^{-t/\tau_L}. \quad \textbf{14.31}$$

The current starts at $I(0) = \varepsilon/R$ and decreases with time as the energy stored in the inductor is depleted (Figure 14.14).

The time dependence of the voltage across the inductor can be determined from $V_L = -L(dI/dt)$:

$$V_L(t) = \varepsilon e^{-t/\tau L}. \quad \textbf{14.32}$$

This voltage is initially $V_L(0) = \varepsilon$, and it decays to zero like the current. The energy stored in the magnetic field of the inductor, $LI^2/2$, also decreases exponentially with time, as it is dissipated by Joule heating in the resistance of the circuit.

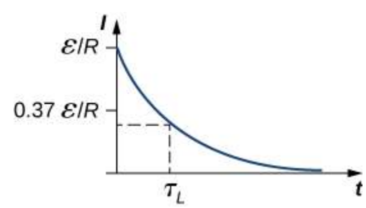


Figure 14.14 Time variation of electric current in the *RL* circuit of Figure 14.12(c). The induced voltage across the coil also decays exponentially.

EXAMPLE 14.4

An *RL* Circuit with a Source of emf

In the circuit of Figure 14.12(a), let $\varepsilon = 2.0V$, $R = 4.0\,\Omega$, and $L = 4.0\,\text{H}$. With S_1 closed and S_2 open (Figure

14.12(b)), (a) what is the time constant of the circuit? (b) What are the current in the circuit and the magnitude of the induced emf across the inductor at $t = 0,$ at $t = 2.0\tau_L$, and as $t \to \infty$?

Strategy

The time constant for an inductor and resistor in a series circuit is calculated using Equation 14.25. The current through and voltage across the inductor are calculated by the scenarios detailed from Equation 14.24 and Equation 14.32.

Solution

a. The inductive time constant is

$$\tau_L = \frac{L}{R} = \frac{4.0\,\mathrm{H}}{4.0\,\Omega} = 1.0\,\mathrm{s}.$$

b. The current in the circuit of Figure 14.12(b) increases according to Equation 14.24:

$$I(t) = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L}).$$

At $t = 0,$

$$(1 - e^{-t/\tau_L}) = (1 - 1) = 0;\ \text{so } I(0) = 0.$$

At $t = 2.0\tau_L$ and $t \to \infty$, we have, respectively,

$$I(2.0\tau_L) = \frac{\varepsilon}{R}(1 - e^{-2.0}) = (0.50\,\mathrm{A})(0.86) = 0.43\,\mathrm{A},$$

and

$$I(\infty) = \frac{\varepsilon}{R} = 0.50\,\mathrm{A}.$$

From Equation 14.32, the magnitude of the induced emf decays as

$$|V_L(t)| = \varepsilon e^{-t/\tau_L}.$$

At $t = 0, t = 2.0\tau_L$, and as $t \to \infty$, we obtain

$$\begin{aligned} |V_L\,(0)| &= \varepsilon = 2.0\,\mathrm{V}, \\ |V_L\,(2.0\tau_L)| &= (2.0\,\mathrm{V})\ e^{-2.0} = 0.27\,\mathrm{V} \\ &\text{and} \\ \left|V_L(\infty)\right| &= 0. \end{aligned}$$

Significance

If the time of the measurement were much larger than the time constant, we would not see the decay or growth of the voltage across the inductor or resistor. The circuit would quickly reach the asymptotic values for both of these. See Figure 14.15.

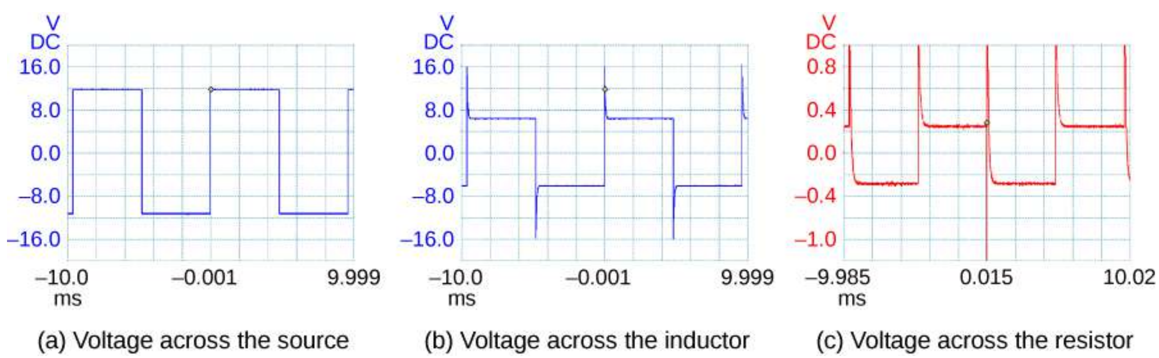


Figure 14.15 A generator in an *RL* circuit produces a square-pulse output in which the voltage oscillates between zero and some set value. These oscilloscope traces show (a) the voltage across the source; (b) the voltage across the inductor; (c) the voltage across the resistor.

EXAMPLE 14.5

An *RL* Circuit without a Source of emf

After the current in the *RL* circuit of Example 14.4 has reached its final value, the positions of the switches are reversed so that the circuit becomes the one shown in Figure 14.12(c). (a) How long does it take the current to drop to half its initial value? (b) How long does it take before the energy stored in the inductor is reduced to 1.0% of its maximum value?

Strategy

The current in the inductor will now decrease as the resistor dissipates this energy. Therefore, the current falls as an exponential decay. We can also use that same relationship as a substitution for the energy in an inductor formula to find how the energy decreases at different time intervals.

Solution

a. With the switches reversed, the current decreases according to

$$I(t) = \frac{\varepsilon}{R} e^{-t/\tau_L} = I(0) e^{-t/\tau_L}.$$

At a time t when the current is one-half its initial value, we have

$$I(t) = 0.50 I(0) \text{ so } e^{-t/\tau_L} = 0.50,$$

and

$$t = -[\ln(0.50)]\tau_L = 0.69(1.0 \text{ s}) = 0.69 \text{ s},$$

where we have used the inductive time constant found in Example 14.4.

b. The energy stored in the inductor is given by

$$U_L(t) = \frac{1}{2} L [I(t)]^2 = \frac{1}{2} L \left(\frac{\varepsilon}{R} e^{-t/\tau_L} \right)^2 = \frac{L\varepsilon^2}{2R^2} e^{-2t/\tau_L}.$$

If the energy drops to 1.0% of its initial value at a time t, we have

$$U_L(t) = (0.010) U_L(0) \text{ or } \frac{L\varepsilon^2}{2R^2} e^{-2t/\tau_L} = (0.010) \frac{L\varepsilon^2}{2R^2}.$$

Upon canceling terms and taking the natural logarithm of both sides, we obtain

$$-\frac{2t}{\tau_L} = \ln(0.010),$$

so

$$t = -\frac{1}{2} \tau_L \ln(0.010).$$

Since $\tau_L = 1.0$ s, the time it takes for the energy stored in the inductor to decrease to 1.0% of its initial value is

$$t = -\frac{1}{2}(1.0 \text{ s}) \ln(0.010) = 2.3 \text{ s}.$$

Significance

This calculation only works if the circuit is at maximum current in situation (b) prior to this new situation. Otherwise, we start with a lower initial current, which will decay by the same relationship.

CHECK YOUR UNDERSTANDING 14.7

Verify that *RC* and *L*/*R* have the dimensions of time.

CHECK YOUR UNDERSTANDING 14.8

(a) If the current in the circuit of in Figure 14.12(b) increases to 90% of its final value after 5.0 s, what is the inductive time constant? (b) If $R = 20\ \Omega$, what is the value of the self-inductance? (c) If the $20\text{-}\Omega$ resistor is replaced with a $100\text{-}\Omega$ resister, what is the time taken for the current to reach 90% of its final value?

CHECK YOUR UNDERSTANDING 14.9

For the circuit of in Figure 14.12(b), show that when steady state is reached, the difference in the total energies produced by the battery and dissipated in the resistor is equal to the energy stored in the magnetic field of the coil.

14.5 Oscillations in an LC Circuit

Learning Objectives

By the end of this section, you will be able to:

- Explain why charge or current oscillates between a capacitor and inductor, respectively, when wired in series
- Describe the relationship between the charge and current oscillating between a capacitor and inductor wired in series

It is worth noting that both capacitors and inductors store energy, in their electric and magnetic fields, respectively. A circuit containing both an inductor (L) and a capacitor (C) can oscillate without a source of emf by shifting the energy stored in the circuit between the electric and magnetic fields. Thus, the concepts we develop in this section are directly applicable to the exchange of energy between the electric and magnetic fields in electromagnetic waves, or light. We start with an idealized circuit of zero resistance that contains an inductor and a capacitor, an ***LC* circuit**.

An LC circuit is shown in Figure 14.16. If the capacitor contains a charge q_0 before the switch is closed, then all the energy of the circuit is initially stored in the electric field of the capacitor (Figure 14.16(a)). This energy is

$$U_C = \frac{1}{2}\frac{q_0^2}{C}. \tag{14.33}$$

When the switch is closed, the capacitor begins to discharge, producing a current in the circuit. The current, in turn, creates a magnetic field in the inductor. The net effect of this process is a transfer of energy from the capacitor, with its diminishing electric field, to the inductor, with its increasing magnetic field.

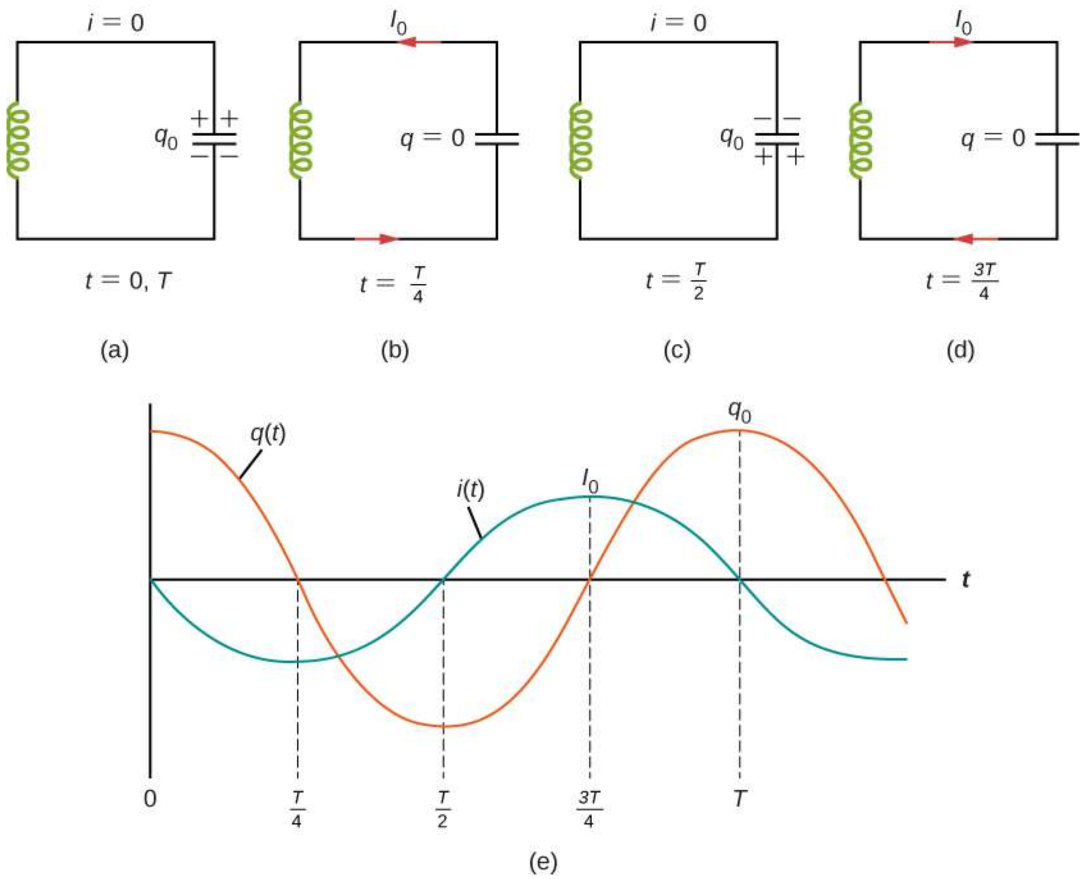


Figure 14.16 (a–d) The oscillation of charge storage with changing directions of current in an *LC* circuit. (e) The graphs show the distribution of charge and current between the capacitor and inductor.

In Figure 14.16(b), the capacitor is completely discharged and all the energy is stored in the magnetic field of the inductor. At this instant, the current is at its maximum value I_0 and the energy in the inductor is

$$U_L = \frac{1}{2}LI_0^2. \qquad \textbf{14.34}$$

Since there is no resistance in the circuit, no energy is lost through Joule heating; thus, the maximum energy stored in the capacitor is equal to the maximum energy stored at a later time in the inductor:

$$\frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}LI_0^2. \qquad \textbf{14.35}$$

At an arbitrary time when the capacitor charge is *q(t)* and the current is *i(t)*, the total energy *U* in the circuit is given by

$$\frac{q^2(t)}{2C} + \frac{Li^2(t)}{2}.$$

Because there is no energy dissipation,

$$U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}LI_0^2. \qquad \textbf{14.36}$$

After reaching its maximum I_0, the current *i(t)* continues to transport charge between the capacitor plates, thereby recharging the capacitor. Since the inductor resists a change in current, current continues to flow, even though the capacitor is discharged. This continued current causes the capacitor to charge with opposite polarity. The electric field of the capacitor increases while the magnetic field of the inductor diminishes, and the overall effect is a transfer of energy from the inductor *back* to the capacitor. From the law of energy conservation, the maximum charge that the capacitor re-acquires is q_0. However, as Figure 14.16(c) shows, the capacitor plates are charged *opposite* to what they were initially.

When fully charged, the capacitor once again transfers its energy to the inductor until it is again completely discharged, as shown in Figure 14.16(d). Then, in the last part of this cyclic process, energy flows back to the capacitor, and the initial state of the circuit is restored.

We have followed the circuit through one complete cycle. Its electromagnetic oscillations are analogous to the mechanical oscillations of a mass at the end of a spring. In this latter case, energy is transferred back and forth between the mass, which has kinetic energy $mv^2/2$, and the spring, which has potential energy $kx^2/2$. With the absence of friction in the mass-spring system, the oscillations would continue indefinitely. Similarly, the oscillations of an LC circuit with no resistance would continue forever if undisturbed; however, this ideal zero-resistance LC circuit is not practical, and any LC circuit will have at least a small resistance, which will radiate and lose energy over time.

The frequency of the oscillations in a resistance-free LC circuit may be found by analogy with the mass-spring system. For the circuit, $i(t) = dq(t)/dt$, the total electromagnetic energy U is

$$U = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}. \tag{14.37}$$

For the mass-spring system, $v(t) = dx(t)/dt$, the total mechanical energy E is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2. \tag{14.38}$$

The equivalence of the two systems is clear. To go from the mechanical to the electromagnetic system, we simply replace m by L, v by i, k by $1/C$, and x by q. Now $x(t)$ is given by

$$x(t) = A\cos(\omega t + \phi) \tag{14.39}$$

where $\omega = \sqrt{k/m}$. Hence, the charge on the capacitor in an LC circuit is given by

$$q(t) = q_0 \cos(\omega t + \phi) \tag{14.40}$$

where the angular frequency of the oscillations in the circuit is

$$\omega = \sqrt{\frac{1}{LC}}. \tag{14.41}$$

Finally, the current in the LC circuit is found by taking the time derivative of $q(t)$:

$$i(t) = \frac{dq(t)}{dt} = -\omega q_0 \sin(\omega t + \phi). \tag{14.42}$$

The time variations of q and I are shown in Figure 14.16(e) for $\phi = 0$.

EXAMPLE 14.6

An *LC* Circuit

In an LC circuit, the self-inductance is 2.0×10^{-2} H and the capacitance is 8.0×10^{-6} F. At $t = 0$, all of the energy is stored in the capacitor, which has charge 1.2×10^{-5} C. (a) What is the angular frequency of the oscillations in the circuit? (b) What is the maximum current flowing through circuit? (c) How long does it take the capacitor to become completely discharged? (d) Find an equation that represents $q(t)$.

Strategy

The angular frequency of the LC circuit is given by Equation 14.41. To find the maximum current, the maximum energy in the capacitor is set equal to the maximum energy in the inductor. The time for the capacitor to become discharged if it is initially charged is a quarter of the period of the cycle, so if we calculate the period of the oscillation, we can find out what a quarter of that is to find this time. Lastly, knowing the initial charge and angular frequency, we can set up a cosine equation to find $q(t)$.

Solution

a. From Equation 14.41, the angular frequency of the oscillations is

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(2.0 \times 10^{-2}\text{ H})(8.0 \times 10^{-6}\text{ F})}} = 2.5 \times 10^3 \text{ rad/s}.$$

b. The current is at its maximum I_0 when all the energy is stored in the inductor. From the law of energy conservation,

$$\frac{1}{2}LI_0^2 = \frac{1}{2}\frac{q_0^2}{C},$$

so

$$I_0 = \sqrt{\frac{1}{LC}}q_0 = (2.5 \times 10^3 \text{ rad/s})(1.2 \times 10^{-5}\text{ C}) = 3.0 \times 10^{-2}\text{ A}.$$

This result can also be found by an analogy to simple harmonic motion, where current and charge are the velocity and position of an oscillator.

c. The capacitor becomes completely discharged in one-fourth of a cycle, or during a time $T/4$, where T is the period of the oscillations. Since

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.5 \times 10^3 \text{ rad/s}} = 2.5 \times 10^{-3}\text{ s},$$

the time taken for the capacitor to become fully discharged is $(2.5 \times 10^{-3}\text{ s})/4 = 6.3 \times 10^{-4}\text{ s}$.

d. The capacitor is completely charged at $t = 0$, so $q(0) = q_0$. Using Equation 14.20, we obtain

$$q(0) = q_0 = q_0 \cos\phi.$$

Thus, $\phi = 0$, and

$$q(t) = (1.2 \times 10^{-5}\text{ C})\cos(2.5 \times 10^3 t).$$

Significance

The energy relationship set up in part (b) is not the only way we can equate energies. At most times, some energy is stored in the capacitor and some energy is stored in the inductor. We can put both terms on each side of the equation. By examining the circuit only when there is no charge on the capacitor or no current in the inductor, we simplify the energy equation.

CHECK YOUR UNDERSTANDING 14.10

The angular frequency of the oscillations in an LC circuit is 2.0×10^3 rad/s. (a) If $L = 0.10$ H, what is C? (b) Suppose that at $t = 0$, all the energy is stored in the inductor. What is the value of ϕ? (c) A second identical capacitor is connected in parallel with the original capacitor. What is the angular frequency of this circuit?

14.6 RLC Series Circuits

Learning Objectives

By the end of this section, you will be able to:

- Determine the angular frequency of oscillation for a resistor, inductor, capacitor (RLC) series circuit
- Relate the RLC circuit to a damped spring oscillation

When the switch is closed in the ***RLC* circuit** of Figure 14.17(a), the capacitor begins to discharge and electromagnetic energy is dissipated by the resistor at a rate i^2R. With U given by Equation 14.19, we have

$$\frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = -i^2 R \qquad 14.43$$

where i and q are time-dependent functions. This reduces to

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0. \qquad 14.44$$

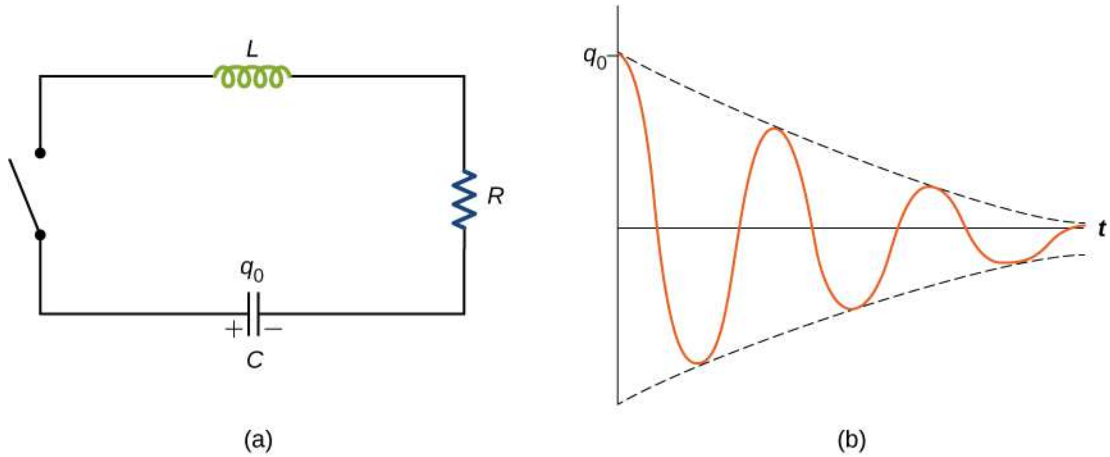


Figure 14.17 (a) An *RLC* circuit. Electromagnetic oscillations begin when the switch is closed. The capacitor is fully charged initially. (b) Damped oscillations of the capacitor charge are shown in this curve of charge versus time, or q versus t. The capacitor contains a charge q_0 before the switch is closed.

This equation is analogous to

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0,$$

which is the equation of motion for a *damped mass-spring system* (you first encountered this equation in Oscillations). As we saw in that chapter, it can be shown that the solution to this differential equation takes three forms, depending on whether the angular frequency of the undamped spring is greater than, equal to, or less than $b/2m$. Therefore, the result can be underdamped ($\sqrt{k/m} > b/2m$), critically damped ($\sqrt{k/m} = b/2m$), or overdamped ($\sqrt{k/m} < b/2m$). By analogy, the solution $q(t)$ to the *RLC* differential equation has the same feature. Here we look only at the case of under-damping. By replacing m by L, b by R, k by $1/C$, and x by q in Equation 14.44, and assuming $\sqrt{1/LC} > R/2L$, we obtain

$$q(t) = q_0 e^{-Rt/2L}\cos(\omega' t + \phi) \qquad 14.45$$

where the angular frequency of the oscillations is given by

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \qquad 14.46$$

This underdamped solution is shown in Figure 14.17(b). Notice that the amplitude of the oscillations decreases as energy is dissipated in the resistor. Equation 14.45 can be confirmed experimentally by measuring the voltage across the capacitor as a function of time. This voltage, multiplied by the capacitance of the capacitor, then gives $q(t)$.

INTERACTIVE

Try an interactive circuit construction kit (https://openstax.org/l/21phetcirconstr) that allows you to graph current and voltage as a function of time. You can add inductors and capacitors to work with any combination of R, L, and C circuits with both dc and ac sources.

INTERACTIVE

Try out a circuit-based java applet website (https://openstax.org/l/21cirphysbascur) that has many problems with both dc and ac sources that will help you practice circuit problems.

CHECK YOUR UNDERSTANDING 14.11

In an RLC circuit, $L = 5.0\,\text{mH}, C = 6.0\mu\text{F},$ and $R = 200\,\Omega$. (a) Is the circuit underdamped, critically damped, or overdamped? (b) If the circuit starts oscillating with a charge of 3.0×10^{-3} C on the capacitor, how much energy has been dissipated in the resistor by the time the oscillations cease?

CHAPTER REVIEW

Key Terms

henry (H) unit of inductance, $1 \text{ H} = 1 \,\Omega \cdot \text{s}$; it is also expressed as a volt second per ampere

inductance property of a device that tells how effectively it induces an emf in another device

inductive time constant denoted by τ, the characteristic time given by quantity L/R of a particular series RL circuit

inductor part of an electrical circuit to provide self-inductance, which is symbolized by a coil of wire

***LC* circuit** circuit composed of an ac source, inductor, and capacitor

magnetic energy density energy stored per volume in a magnetic field

mutual inductance geometric quantity that expresses how effective two devices are at inducing emfs in one another

***RLC* circuit** circuit with an ac source, resistor, inductor, and capacitor all in series.

self-inductance effect of the device inducing emf in itself

Key Equations

Mutual inductance by flux	$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_1 \Phi_{12}}{I_2}$
Mutual inductance in circuits	$\varepsilon_1 = -M \frac{dI_2}{dt}$
Self-inductance in terms of magnetic flux	$N\Phi_{\text{m}} = LI$
Self-inductance in terms of emf	$\varepsilon = -L \frac{dI}{dt}$
Self-inductance of a solenoid	$L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l}$
Self-inductance of a toroid	$L_{\text{toroid}} = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{R_2}{R_1}.$
Energy stored in an inductor	$U = \frac{1}{2} LI^2$
Current as a function of time for a *RL* circuit	$I(t) = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$
Time constant for a *RL* circuit	$\tau_L = L/R$
Charge oscillation in *LC* circuits	$q(t) = q_0 \cos(\omega t + \phi)$
Angular frequency in *LC* circuits	$\omega = \sqrt{\frac{1}{LC}}$
Current oscillations in *LC* circuits	$i(t) = -\omega q_0 \sin(\omega t + \phi)$
Charge as a function of time in *RLC* circuit	$q(t) = q_0 e^{-Rt/2L} \cos(\omega' t + \phi)$
Angular frequency in *RLC* circuit	$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

Summary

14.1 Mutual Inductance

- Inductance is the property of a device that expresses how effectively it induces an emf in another device.
- Mutual inductance is the effect of two devices inducing emfs in each other.
- A change in current dI_1/dt in one circuit induces an emf (ε_2) in the second:
$$\varepsilon_2 = -M\frac{dI1}{dt},$$
where M is defined to be the mutual inductance between the two circuits and the minus sign is due to Lenz's law.
- Symmetrically, a change in current dI_2/dt through the second circuit induces an emf (ε_1) in the first:
$$\varepsilon_1 = -M\frac{dI_2}{dt},$$
where M is the same mutual inductance as in the reverse process.

14.2 Self-Inductance and Inductors

- Current changes in a device induce an emf in the device itself, called self-inductance,
$$\varepsilon = -L\frac{dI}{dt},$$
where L is the self-inductance of the inductor and dI/dt is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz's law. The unit of self-inductance and inductance is the henry (H), where $1\ \text{H} = 1\ \Omega \cdot \text{s}$.
- The self-inductance of a solenoid is
$$L = \frac{\mu_0 N^2 A}{l},$$
where N is its number of turns in the solenoid, A is its cross-sectional area, l is its length, and $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$ is the permeability of free space.
- The self-inductance of a toroid is
$$L = \frac{\mu_0 N^2 h}{2\pi}\ln\frac{R_2}{R_1},$$
where N is its number of turns in the toroid, R_1 and R_2 are the inner and outer radii of the toroid, h is the height of the toroid, and $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$ is the permeability of free space.

14.3 Energy in a Magnetic Field

- The energy stored in an inductor U is
$$U = \frac{1}{2}LI^2.$$
- The self-inductance per unit length of coaxial cable is
$$\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{R_2}{R_1}.$$

14.4 RL Circuits

- When a series connection of a resistor and an inductor—an RL circuit—is connected to a voltage source, the time variation of the current is
$I(t) = \frac{\varepsilon}{R}(1 - e^{-Rt/L}) = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$ (turning on),
where the initial current is $I_0 = \varepsilon/R$.
- The characteristic time constant τ is $\tau_L = L/R$, where L is the inductance and R is the resistance.
- In the first time constant τ, the current rises from zero to $0.632I_0$, and to 0.632 of the remainder in every subsequent time interval τ.
- When the inductor is shorted through a resistor, current decreases as
$I(t) = \frac{\varepsilon}{R}e^{-t/\tau_L}$ (turning off).
Current falls to $0.368I_0$ in the first time interval τ, and to 0.368 of the remainder toward zero in each subsequent time τ.

14.5 Oscillations in an LC Circuit

- The energy transferred in an oscillatory manner between the capacitor and inductor in an LC circuit occurs at an angular frequency
$\omega = \sqrt{\frac{1}{LC}}$.
- The charge and current in the circuit are given by
$$\begin{aligned} q(t) &= q_0\cos(\omega t + \phi), \\ i(t) &= -\omega q_0 \sin(\omega t + \phi). \end{aligned}$$

14.6 RLC Series Circuits

- The underdamped solution for the capacitor charge in an RLC circuit is
$$q(t) = q_0 e^{-Rt/2L}\cos(\omega' t + \phi).$$
- The angular frequency given in the underdamped solution for the RLC circuit is
$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

Conceptual Questions

14.1 Mutual Inductance

1. Show that $N\Phi_m/I$ and $\varepsilon/(dI/dt)$, which are both expressions for self-inductance, have the same units.
2. A 10-H inductor carries a current of 20 A. Describe how a 50-V emf can be induced across it.
3. The ignition circuit of an automobile is powered by a 12-V battery. How are we able to generate large voltages with this power source?
4. When the current through a large inductor is interrupted with a switch, an arc appears across the open terminals of the switch. Explain.

14.2 Self-Inductance and Inductors

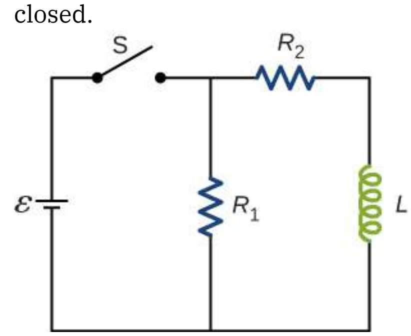
5. Does self-inductance depend on the value of the magnetic flux? Does it depend on the current through the wire? Correlate your answers with the equation $N\Phi_m = LI$.
6. Would the self-inductance of a 1.0 m long, tightly wound solenoid differ from the self-inductance per meter of an infinite, but otherwise identical, solenoid?
7. Discuss how you might determine the self-inductance per unit length of a long, straight wire.
8. The self-inductance of a coil is zero if there is no current passing through the windings. True or false?
9. How does the self-inductance per unit length near the center of a solenoid (away from the ends) compare with its value near the end of the solenoid?

14.3 Energy in a Magnetic Field

10. Show that $LI^2/2$ has units of energy.

14.4 RL Circuits

11. Use Lenz's law to explain why the initial current in the *RL* circuit of Figure 14.12(b) is zero.
12. When the current in the *RL* circuit of Figure 14.12(b) reaches its final value ε/R, what is the voltage across the inductor? Across the resistor?
13. Does the time required for the current in an *RL* circuit to reach any fraction of its steady-state value depend on the emf of the battery?
14. An inductor is connected across the terminals of a battery. Does the current that eventually flows through the inductor depend on the internal resistance of the battery? Does the time required for the current to reach its final value depend on this resistance?
15. At what time is the voltage across the inductor of the *RL* circuit of Figure 14.12(b) a maximum?
16. In the simple *RL* circuit of Figure 14.12(b), can the emf induced across the inductor ever be greater than the emf of the battery used to produce the current?
17. If the emf of the battery of Figure 14.12(b) is reduced by a factor of 2, by how much does the steady-state energy stored in the magnetic field of the inductor change?
18. A steady current flows through a circuit with a large inductive time constant. When a switch in the circuit is opened, a large spark occurs across the terminals of the switch. Explain.
19. Describe how the currents through R_1 and R_2 shown below vary with time after switch S is closed.



20. Discuss possible practical applications of *RL* circuits.

14.5 Oscillations in an LC Circuit

21. Do Kirchhoff's rules apply to circuits that contain inductors and capacitors?
22. Can a circuit element have both capacitance and inductance?
23. In an *LC* circuit, what determines the frequency and the amplitude of the energy oscillations in either the inductor or capacitor?

14.6 RLC Series Circuits

24. When a wire is connected between the two ends of a solenoid, the resulting circuit can oscillate like an *RLC* circuit. Describe what causes the capacitance in this circuit.
25. Describe what effect the resistance of the connecting wires has on an oscillating *LC* circuit.
26. Suppose you wanted to design an *LC* circuit with

a frequency of 0.01 Hz. What problems might you encounter?

27. A radio receiver uses an *RLC* circuit to pick out particular frequencies to listen to in your house or car without hearing other unwanted frequencies. How would someone design such a circuit?

Problems

14.1 Mutual Inductance

28. When the current in one coil changes at a rate of 5.6 A/s, an emf of 6.3×10^{-3} V is induced in a second, nearby coil. What is the mutual inductance of the two coils?

29. An emf of 9.7×10^{-3} V is induced in a coil while the current in a nearby coil is decreasing at a rate of 2.7 A/s. What is the mutual inductance of the two coils?

30. Two coils close to each other have a mutual inductance of 32 mH. If the current in one coil decays according to $I = I_0 e^{-\alpha t}$, where $I_0 = 5.0$ A and $\alpha = 2.0 \times 10^3\ \text{s}^{-1}$, what is the emf induced in the second coil immediately after the current starts to decay? At $t = 1.0 \times 10^{-3}$ s?

31. A coil of 40 turns is wrapped around a long solenoid of cross-sectional area $7.5 \times 10^{-3}\ \text{m}^2$. The solenoid is 0.50 m long and has 500 turns. (a) What is the mutual inductance of this system? (b) The outer coil is replaced by a coil of 40 turns whose radius is three times that of the solenoid. What is the mutual inductance of this configuration?

32. A 600-turn solenoid is 0.55 m long and 4.2 cm in diameter. Inside the solenoid, a small $(1.1\ \text{cm} \times 1.4\ \text{cm})$, single-turn rectangular coil is fixed in place with its face perpendicular to the long axis of the solenoid. What is the mutual inductance of this system?

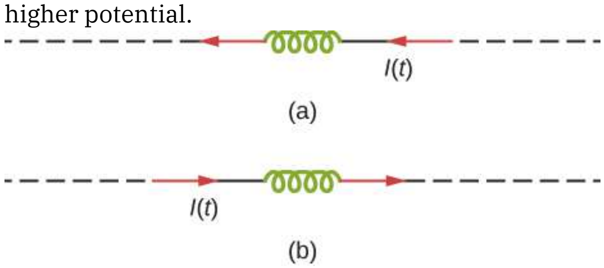
33. A toroidal coil has a mean radius of 16 cm and a cross-sectional area of $0.25\ \text{cm}^2$; it is wound uniformly with 1000 turns. A second toroidal coil of 750 turns is wound uniformly over the first coil. Ignoring the variation of the magnetic field within a toroid, determine the mutual inductance of the two coils.

34. A solenoid of N_1 turns has length l_1 and radius R_1, and a second smaller solenoid of N_2 turns has length l_2 and radius R_2. The smaller solenoid is placed completely inside the larger solenoid so that their long axes coincide. What is the mutual inductance of the two solenoids?

14.2 Self-Inductance and Inductors

35. An emf of 0.40 V is induced across a coil when the current through it changes uniformly from 0.10 to 0.60 A in 0.30 s. What is the self-inductance of the coil?

36. The current shown in part (a) below is increasing, whereas that shown in part (b) is decreasing. In each case, determine which end of the inductor is at the higher potential.



37. What is the rate at which the current though a 0.30-H coil is changing if an emf of 0.12 V is induced across the coil?

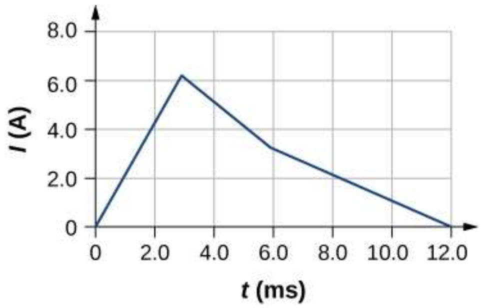
38. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

39. A coil with a self-inductance of 2.0 H carries a current that varies with time according to $I(t) = (2.0\ \text{A})\sin 120\pi t$. Find an expression for the emf induced in the coil.

40. A solenoid 50 cm long is wound with 500 turns of wire. The cross-sectional area of the coil is $2.0\ \text{cm}^2$ What is the self-inductance of the solenoid?

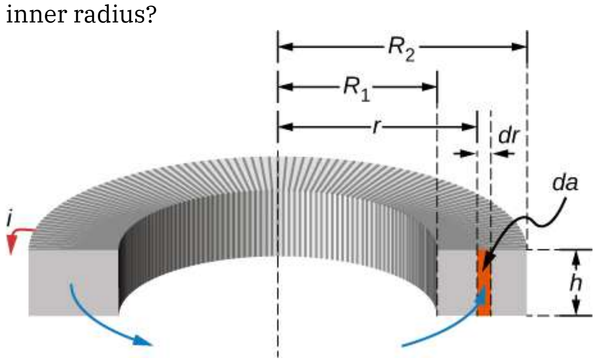
41. A coil with a self-inductance of 3.0 H carries a current that decreases at a uniform rate $dI/dt = -0.050$ A/s. What is the emf induced in the coil? Describe the polarity of the induced emf.

42. The current *I(t)* through a 5.0-mH inductor varies with time, as shown below. The resistance of the inductor is 5.0 Ω. Calculate the voltage across the inductor at $t = 2.0$ ms, $t = 4.0$ ms, and $t = 8.0$ ms.



43. A long, cylindrical solenoid with 100 turns per centimeter has a radius of 1.5 cm. (a) Neglecting end effects, what is the self-inductance per unit length of the solenoid? (b) If the current through the solenoid changes at the rate 5.0 A/s, what is the emf induced per unit length?

44. Suppose that a rectangular toroid has 2000 windings and a self-inductance of 0.040 H. If $h = 0.10$ m, what is the ratio of its outer radius to its inner radius?



45. What is the self-inductance per meter of a coaxial cable whose inner radius is 0.50 mm and whose outer radius is 4.00 mm?

14.3 Energy in a Magnetic Field

46. At the instant a current of 0.20 A is flowing through a coil of wire, the energy stored in its magnetic field is 6.0×10^{-3} J. What is the self-inductance of the coil?

47. Suppose that a rectangular toroid has 2000 windings and a self-inductance of 0.040 H. If $h = 0.10$ m, what is the current flowing through a rectangular toroid when the energy in its magnetic field is 2.0×10^{-6} J?

48. Solenoid *A* is tightly wound while solenoid *B* has windings that are evenly spaced with a gap equal to the diameter of the wire. The solenoids are otherwise identical. Determine the ratio of the energies stored per unit length of these solenoids when the same current flows through each.

49. A 10-H inductor carries a current of 20 A. How much ice at 0° C could be melted by the energy stored in the magnetic field of the inductor? (*Hint*: Use the value $L_f = 334$ J/g for ice.)

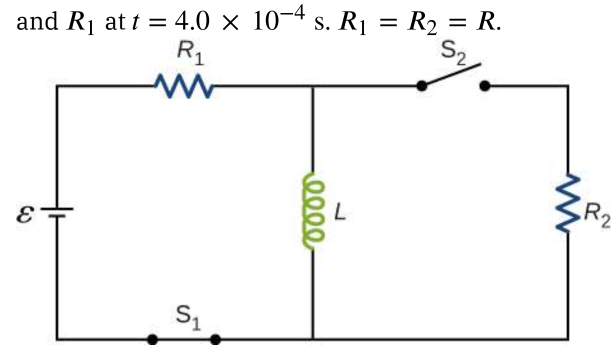
50. A coil with a self-inductance of 3.0 H and a resistance of 100 Ω carries a steady current of 2.0 A. (a) What is the energy stored in the magnetic field of the coil? (b) What is the energy per second dissipated in the resistance of the coil?

51. A current of 1.2 A is flowing in a coaxial cable whose outer radius is five times its inner radius. What is the magnetic field energy stored in a 3.0-m length of the cable?

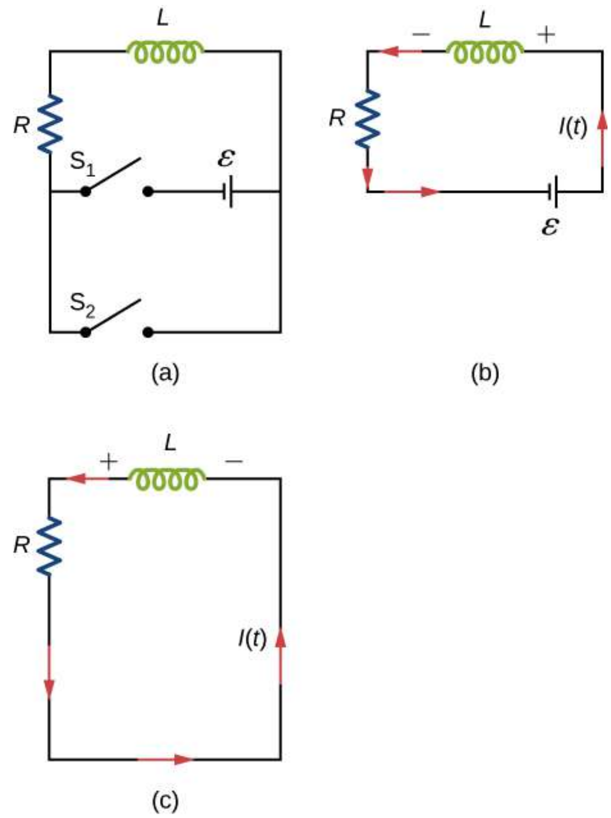
14.4 RL Circuits

52. In Figure 14.12, $\varepsilon = 12$ V, $L = 20$ mH, and $R = 5.0\,\Omega$. Determine (a) the time constant of the circuit, (b) the initial current through the resistor, (c) the final current through the resistor, (d) the current through the resistor when $t = 2\tau_L$, and (e) the voltages across the inductor and the resistor when $t = 2\tau_L$.

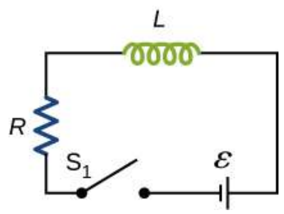
53. For the circuit shown below, $\varepsilon = 20$ V, $L = 4.0$ mH, and $R = 5.0\,\Omega$. After steady state is reached with S_1 closed and S_2 open, S_2 is closed and immediately thereafter (at $t = 0$) S_1 is opened. Determine (a) the current through L at $t = 0$, (b) the current through L at $t = 4.0 \times 10^{-4}$ s, and (c) the voltages across L and R_1 at $t = 4.0 \times 10^{-4}$ s. $R_1 = R_2 = R$.



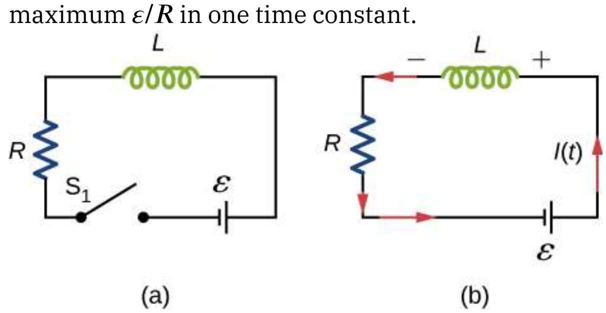
54. The current in the *RL* circuit shown here increases to 40% of its steady-state value in 2.0 s. What is the time constant of the circuit?



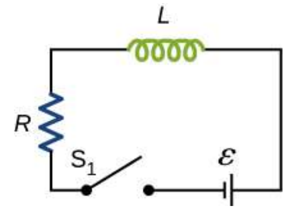
55. How long after switch S_1 is thrown does it take the current in the circuit shown to reach half its maximum value? Express your answer in terms of the time constant of the circuit.



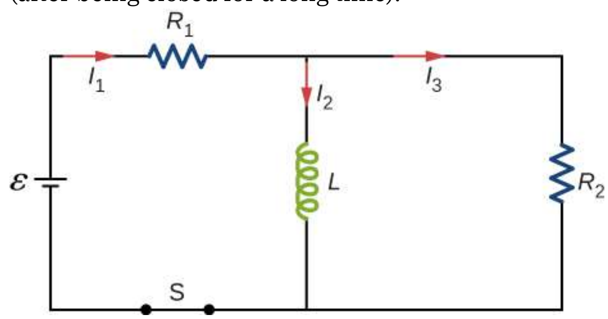
56. Examine the circuit shown below in part (a). Determine *dI/dt* at the instant after the switch is thrown in the circuit of (a), thereby producing the circuit of (b). Show that if *I* were to continue to increase at this initial rate, it would reach its maximum ε/R in one time constant.



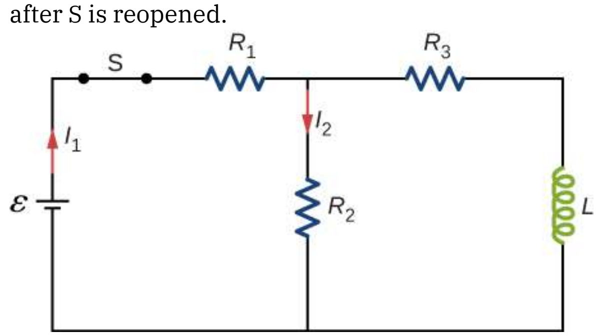
57. The current in the *RL* circuit shown below reaches half its maximum value in 1.75 ms after the switch S_1 is thrown. Determine (a) the time constant of the circuit and (b) the resistance of the circuit if $L = 250\,\text{mH}$.



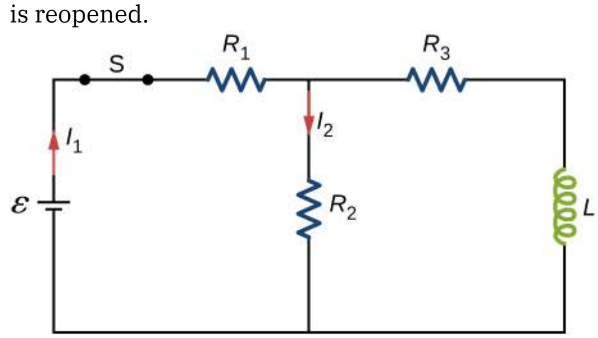
58. Consider the circuit shown below. Find $I_1, I_2,$ and I_3 when (a) the switch S is first closed, (b) after the currents have reached steady-state values, and (c) at the instant the switch is reopened (after being closed for a long time).



59. For the circuit shown below, $\varepsilon = 50\text{ V}$, $R_1 = 10\,\Omega$,, $R_2 = R_3 = 19.4\,\Omega$,, and $L = 2.0\,\text{mH}$. Find the values of I_1 and I_2 (a) immediately after switch S is closed, (b) a long time after S is closed, (c) immediately after S is reopened, and (d) a long time after S is reopened.

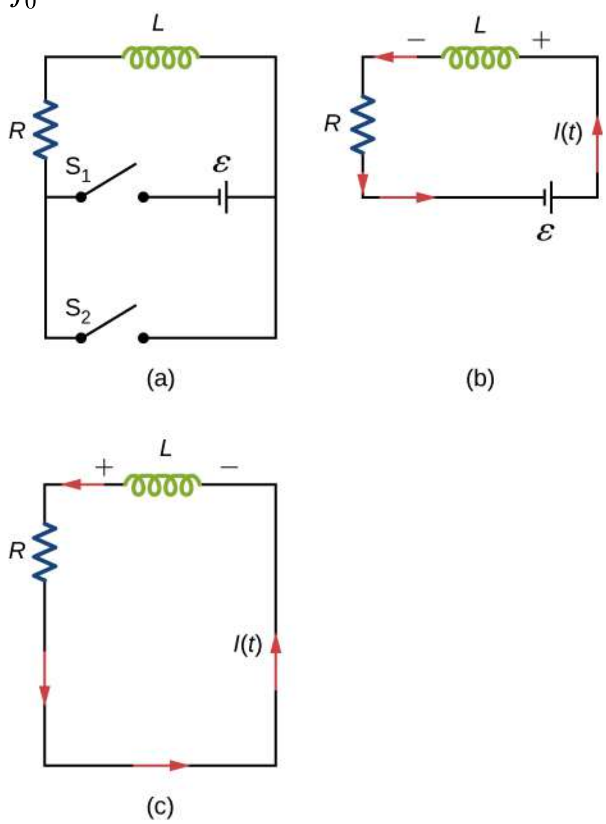


60. For the circuit shown below, find the current through the inductor 2.0×10^{-5} s after the switch is reopened.



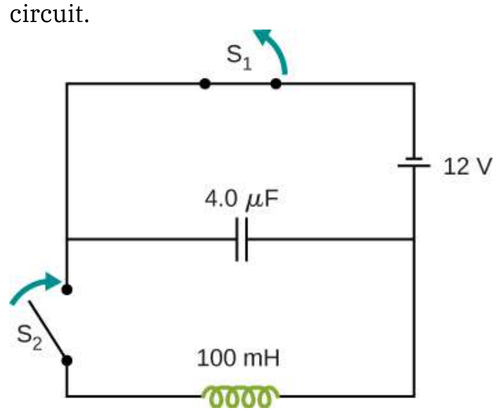
61. Show that for the circuit shown below, the initial energy stored in the inductor, $LI^2(0)/2$, is equal to the total energy eventually dissipated in the resistor,

$$\int_0^\infty I^2(t)Rdt.$$



14.5 Oscillations in an LC Circuit

62. A 5000-pF capacitor is charged to 100 V and then quickly connected to an 80-mH inductor. Determine (a) the maximum energy stored in the magnetic field of the inductor, (b) the peak value of the current, and (c) the frequency of oscillation of the circuit.
63. The self-inductance and capacitance of an *LC* circuit are 0.20 mH and 5.0 pF. What is the angular frequency at which the circuit oscillates?
64. What is the self-inductance of an *LC* circuit that oscillates at 60 Hz when the capacitance is $10\ \mu\text{F}$?
65. In an oscillating *LC* circuit, the maximum charge on the capacitor is 2.0×10^{-6} C and the maximum current through the inductor is 8.0 mA. (a) What is the period of the oscillations? (b) How much time elapses between an instant when the capacitor is uncharged and the next instant when it is fully charged?
66. The self-inductance and capacitance of an oscillating *LC* circuit are $L = 20$ mH and $C = 1.0\ \mu\text{F}$, respectively. (a) What is the frequency of the oscillations? (b) If the maximum potential difference between the plates of the capacitor is 50 V, what is the maximum current in the circuit?
67. In an oscillating *LC* circuit, the maximum charge on the capacitor is q_m. Determine the charge on the capacitor and the current through the inductor when energy is shared equally between the electric and magnetic fields. Express your answer in terms of q_m, L, and C.
68. In the circuit shown below, S_1 is opened and S_2 is closed simultaneously. Determine (a) the frequency of the resulting oscillations, (b) the maximum charge on the capacitor, (c) the maximum current through the inductor, and (d) the electromagnetic energy of the oscillating circuit.



69. An *LC* circuit in an AM tuner (in a car stereo) uses a coil with an inductance of 2.5 mH and a variable capacitor. If the natural frequency of the circuit is to be adjustable over the range 540 to 1600 kHz (the AM broadcast band), what range of capacitance is required?

14.6 RLC Series Circuits

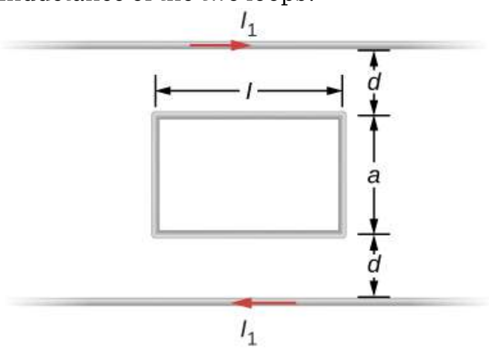
70. In an oscillating *RLC* circuit, $R = 5.0\ \Omega$, $L = 5.0$ mH, and $C = 500\ \mu\text{F}$. What is the angular frequency of the oscillations?
71. In an oscillating *RLC* circuit with $L = 10$ mH, $C = 1.5\ \mu\text{F}$, and $R = 2.0\ \Omega$, how much time elapses before the amplitude of the oscillations drops to half its initial value?
72. What resistance *R* must be connected in series with a 200-mH inductor and a $10\mu\text{F}$ capacitor of the resulting *RLC* oscillating circuit is to decay to 50% of its initial value of charge in 50 cycles? To 0.10% of its initial value in 50 cycles?

Additional Problems

73. Show that the self-inductance per unit length of an infinite, straight, thin wire is infinite.

74. Two long, parallel wires carry equal currents in opposite directions. The radius of each wire is *a*, and the distance between the centers of the wires is *d*. Show that if the magnetic flux within the wires themselves can be ignored, the self-inductance of a length *l* of such a pair of wires is
$L = \frac{\mu_0 l}{\pi} \ln \frac{d-a}{a}$.
(*Hint*: Calculate the magnetic flux through a rectangle of length *l* between the wires and then use $L = N\Phi/I$.)

75. A small, rectangular single loop of wire with dimensions *l*, and *a* is placed, as shown below, in the plane of a much larger, rectangular single loop of wire. The two short sides of the larger loop are so far from the smaller loop that their magnetic fields over the smaller fields over the smaller loop can be ignored. What is the mutual inductance of the two loops?

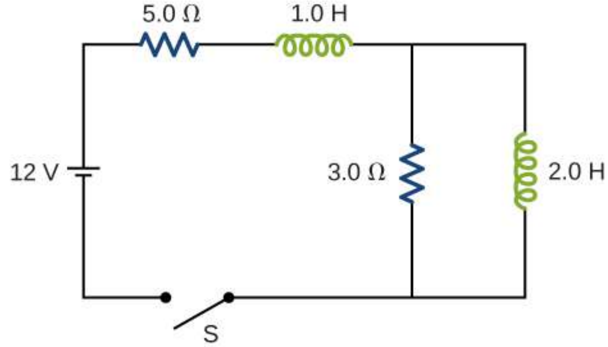


76. Suppose that a cylindrical solenoid is wrapped around a core of iron whose magnetic susceptibility is ***x***. Using Equation 14.9, show that the self-inductance of the solenoid is given by
$L = \frac{(1+x)\mu_0 N^2 A}{l}$,
where *l* is its length, *A* its cross-sectional area, and *N* its total number of turns.

77. A solenoid with $4 \text{ x } 10^7$ turns/m has an iron core placed in it whose magnetic susceptibility is 4.0×10^3. (a) If a current of 2.0 A flows through the solenoid, what is the magnetic field in the iron core? (b) What is the effective surface current formed by the aligned atomic current loops in the iron core? (c) What is the self-inductance of the filled solenoid?

78. A rectangular toroid with inner radius $R_1 = 7.0 \text{ cm}$, outer radius $R_2 = 9.0 \text{ cm}$, height $h = 3.0$, and $N = 3000$ turns is filled with an iron core of magnetic susceptibility 5.2×10^3. (a) What is the self-inductance of the toroid? (b) If the current through the toroid is 2.0 A, what is the magnetic field at the center of the core? (c) For this same 2.0-A current, what is the effective surface current formed by the aligned atomic current loops in the iron core?

79. The switch S of the circuit shown below is closed at $t = 0$. Determine (a) the initial current through the battery and (b) the steady-state current through the battery.



80. In an oscillating *RLC* circuit, $R = 7.0 \, \Omega, L = 10 \text{ mH}, \text{ and } C = 3.0 \, \mu\text{F}$. Initially, the capacitor has a charge of $8.0 \, \mu\text{C}$ and the current is zero. Calculate the charge on the capacitor (a) five cycles later and (b) 50 cycles later.

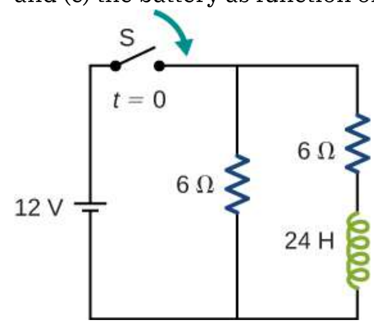
81. A 25.0-H inductor has 100 A of current turned off in 1.00 ms. (a) What voltage is induced to oppose this? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Challenge Problems

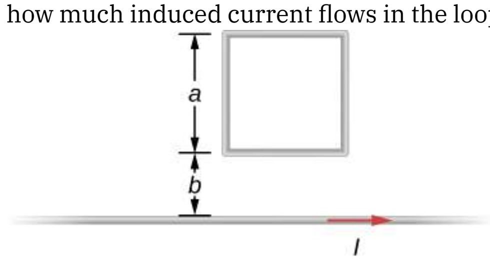
82. A coaxial cable has an inner conductor of radius a, and outer thin cylindrical shell of radius b. A current I flows in the inner conductor and returns in the outer conductor. The self-inductance of the structure will depend on how the current in the inner cylinder tends to be distributed. Investigate the following two extreme cases. (a) Let current in the inner conductor be distributed only on the surface and find the self-inductance. (b) Let current in the inner cylinder be distributed uniformly over its cross-section and find the self-inductance. Compare with your results in (a).

83. In a damped oscillating circuit the energy is dissipated in the resistor. The Q-factor is a measure of the persistence of the oscillator against the dissipative loss. (a) Prove that for a lightly damped circuit the energy, U, in the circuit decreases according to the following equation.
$\frac{dU}{dt} = -2\beta U$, where $\beta = \frac{R}{2L}$.
(b) Using the definition of the Q-factor as energy divided by the loss over the next cycle, prove that Q-factor of a lightly damped oscillator as defined in this problem is
$Q \equiv \frac{U_{\text{begin}}}{\Delta U_{\text{one cycle}}} = \frac{1}{2\pi R}\sqrt{\frac{L}{C}}$.
(*Hint:* For (b), to obtain Q, divide E at the beginning of one cycle by the change ΔE over the next cycle.)

84. The switch in the circuit shown below is closed at $t = 0$ s. Find currents through (a) R_1, (b) R_2, and (c) the battery as function of time.



85. A square loop of side 2 cm is placed 1 cm from a long wire carrying a current that varies with time at a constant rate of 3 A/s as shown below. (a) Use Ampère's law and find the magnetic field. (b) Determine the magnetic flux through the loop. (c) If the loop has a resistance of 3 Ω, how much induced current flows in the loop?



CHAPTER 15
Alternating-Current Circuits

Figure 15.1 The current we draw into our houses is an alternating current (ac). Power lines transmit ac to our neighborhoods, where local power stations and transformers distribute it to our homes. In this chapter, we discuss how a transformer works and how it allows us to transmit power at very high voltages and minimal heating losses across the lines.

Chapter Outline

INTRODUCTION Electric power is delivered to our homes by alternating current (ac) through high-voltage transmission lines. As explained in Transformers, transformers can then change the amplitude of the alternating potential difference to a more useful form. This lets us transmit power at very high voltages, minimizing resistive heating losses in the lines, and then furnish that power to homes at lower, safer voltages. Because constant potential differences are unaffected by transformers, this capability is more difficult to achieve with direct-current transmission.

In this chapter, we use Kirchhoff's laws to analyze four simple circuits in which ac flows. We have discussed the use of the resistor, capacitor, and inductor in circuits with batteries. These components are also part of ac circuits. However, because ac is required, the constant source of emf supplied by a battery is replaced by an ac voltage source, which produces an oscillating emf.

15.1 AC Sources

Learning Objectives

By the end of this section, you will be able to:

- Explain the differences between direct current (dc) and alternating current (ac)
- Define characteristic features of alternating current and voltage, such as the amplitude or peak and the frequency

Most examples dealt with so far in this book, particularly those using batteries, have constant-voltage sources. Thus, once the current is established, it is constant. **Direct current (dc)** is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit.

Most well-known applications, however, use a time-varying voltage source. **Alternating current (ac)** is the flow of electric charge that periodically reverses direction. An ac is produced by an alternating emf, which is generated in a power plant, as described in Induced Electric Fields. If the ac source varies periodically, particularly sinusoidally, the circuit is known as an ac circuit. Examples include the commercial and residential power that serves so many of our needs.

The ac voltages and frequencies commonly used in businesses and homes vary around the world. In a typical house, the potential difference between the two sides of an electrical outlet alternates sinusoidally with a frequency of 60 or 50 Hz and an amplitude of 170 or 311 V, depending on whether you live in the United States or Europe, respectively. Most people know the potential difference for electrical outlets is 120 V or 220 V in the US or Europe, but as explained later in the chapter, these voltages are not the peak values given here but rather are related to the common voltages we see in our electrical outlets. Figure 15.2 shows graphs of voltage and current versus time for typical dc and ac power in the United States.

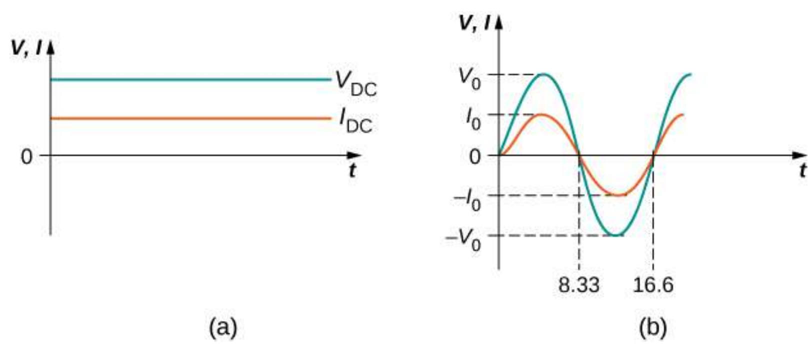


Figure 15.2 (a) The dc voltage and current are constant in time, once the current is established. (b) The voltage and current versus time are quite different for ac power. In this example, which shows 60-Hz ac power and time *t* in milliseconds, voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of ac sources differ greatly.

Suppose we hook up a resistor to an ac voltage source and determine how the voltage and current vary in time across the resistor. Figure 15.3 shows a schematic of a simple circuit with an ac voltage source. The voltage fluctuates sinusoidally with time at a fixed frequency, as shown, on either the battery terminals or the resistor. Therefore, the **ac voltage**, or the "voltage at a plug," can be given by

$$v = V_0 \sin \omega t, \tag{15.1}$$

where v is the voltage at time t, V_0 is the peak voltage, and ω is the angular frequency in radians per second. For a typical house in the United States, $V_0 = 170$ V and $\omega = 120\pi$ rad/s, whereas in Europe, $V_0 = 311$ V and $\omega = 100\pi$ rad/s.

For this simple resistance circuit, $I = V/R$, so the **ac current**, meaning the current that fluctuates sinusoidally with time at a fixed frequency, is

$$i = I_0 \sin \omega t, \quad 15.2$$

where i is the current at time t and I_0 is the peak current and is equal to V_0/R. For this example, the voltage and current are said to be in phase, meaning that their sinusoidal functional forms have peaks, troughs, and nodes in the same place. They oscillate in sync with each other, as shown in Figure 15.2(b). In these equations, and throughout this chapter, we use lowercase letters (such as i) to indicate instantaneous values and capital letters (such as I) to indicate maximum, or peak, values.

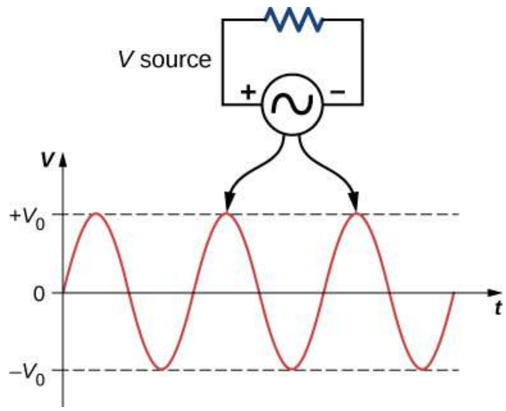


Figure 15.3 The potential difference V between the terminals of an ac voltage source fluctuates, so the source and the resistor have ac sine waves on top of each other. The mathematical expression for v is given by $v = V_0 \sin \omega t$.

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see the stroboscopic effect of ac.

CHECK YOUR UNDERSTANDING 15.1

If a European ac voltage source is considered, what is the time difference between the zero crossings on an ac voltage-versus-time graph?

15.2 Simple AC Circuits

Learning Objectives

By the end of this section, you will be able to:

- Interpret phasor diagrams and apply them to ac circuits with resistors, capacitors, and inductors
- Define the reactance for a resistor, capacitor, and inductor to help understand how current in the circuit behaves compared to each of these devices

In this section, we study simple models of ac voltage sources connected to three circuit components: (1) a resistor, (2) a capacitor, and (3) an inductor. The power furnished by an ac voltage source has an emf given by

$$v(t) = V_0 \sin \omega t,$$

as shown in Figure 15.4. This sine function assumes we start recording the voltage when it is $v = 0$ V at a time of $t = 0$ s. A phase constant may be involved that shifts the function when we start measuring voltages, similar to the phase constant in the waves we studied in Waves. However, because we are free to choose when we start examining the voltage, we can ignore this phase constant for now. We can measure this voltage across the circuit components using one of two methods: (1) a quantitative approach based on our knowledge of circuits, or (2) a graphical approach that is explained in the coming sections.

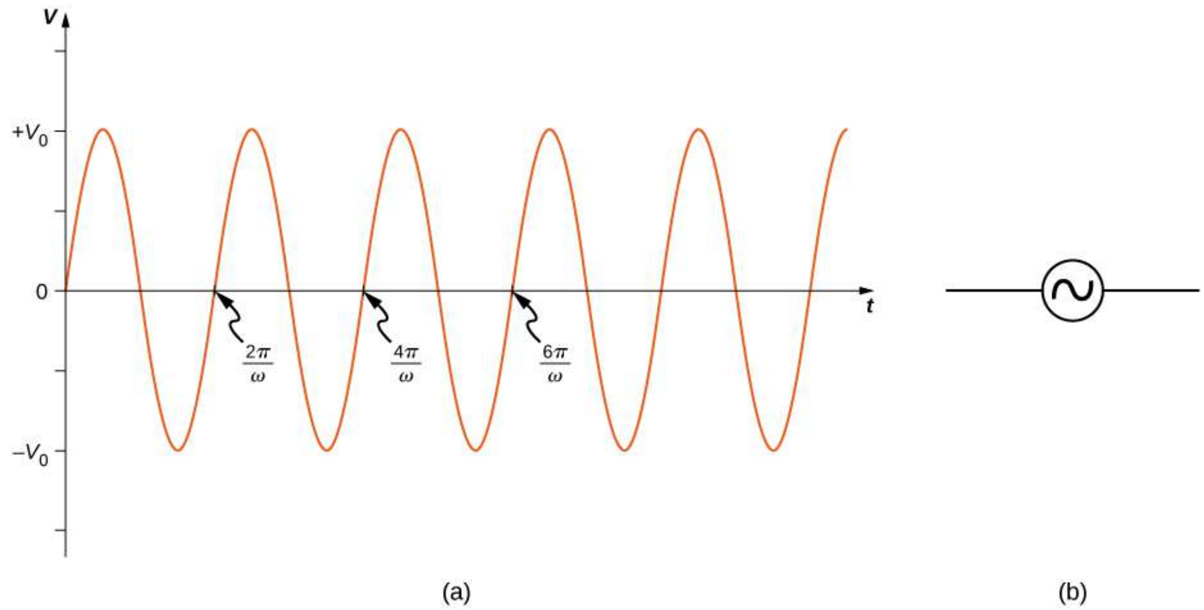


Figure 15.4 (a) The output $v(t) = V_0 \sin \omega t$ of an ac generator. (b) Symbol used to represent an ac voltage source in a circuit diagram.

Resistor

First, consider a resistor connected across an ac voltage source. From Kirchhoff's loop rule, the instantaneous voltage across the resistor of Figure 15.5(a) is

$$v_R(t) = V_0 \sin \omega t$$

and the instantaneous current through the resistor is

$$i_R(t) = \frac{v_R(t)}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t.$$

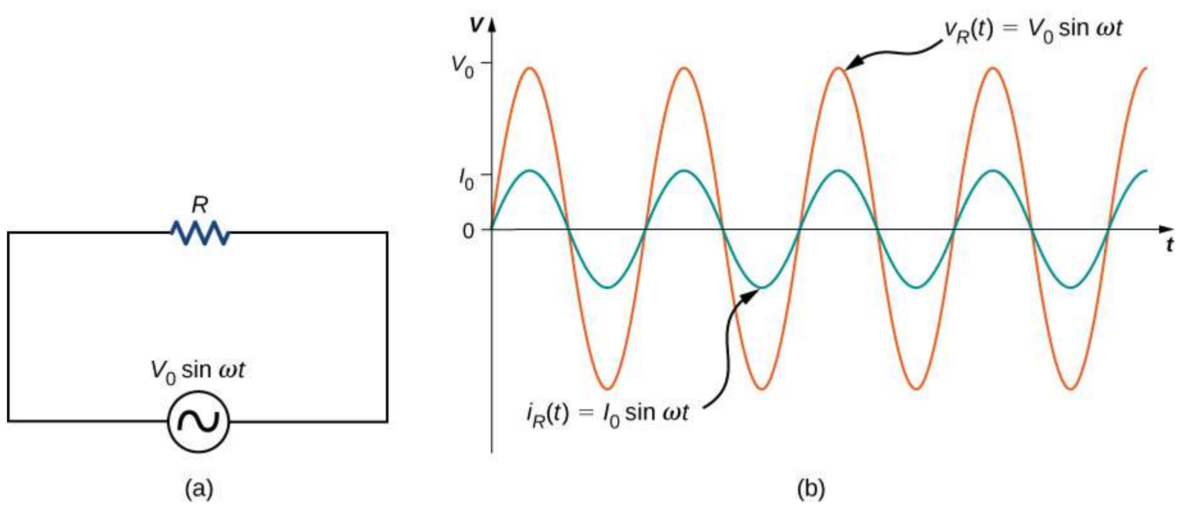


Figure 15.5 (a) A resistor connected across an ac voltage source. (b) The current $i_R\,(t)$ through the resistor and the voltage $v_R\,(t)$ across the resistor. The two quantities are in phase.

Here, $I_0 = V_0/R$ is the amplitude of the time-varying current. Plots of $i_R\,(t)$ and $v_R\,(t)$ are shown in Figure 15.5(b). Both curves reach their maxima and minima at the same times, that is, the current through and the voltage across the resistor are in phase.

Graphical representations of the phase relationships between current and voltage are often useful in the analysis of ac circuits. Such representations are called *phasor diagrams*. The phasor diagram for $i_R\,(t)$ is shown in Figure 15.6(a), with the current on the vertical axis. The arrow (or phasor) is rotating counterclockwise at a constant angular frequency ω, so we are viewing it at one instant in time. If the length of the arrow corresponds to the current amplitude I_0, the projection of the rotating arrow onto the vertical axis is $i_R(t) = I_0 \sin \omega t$, which is the instantaneous current.

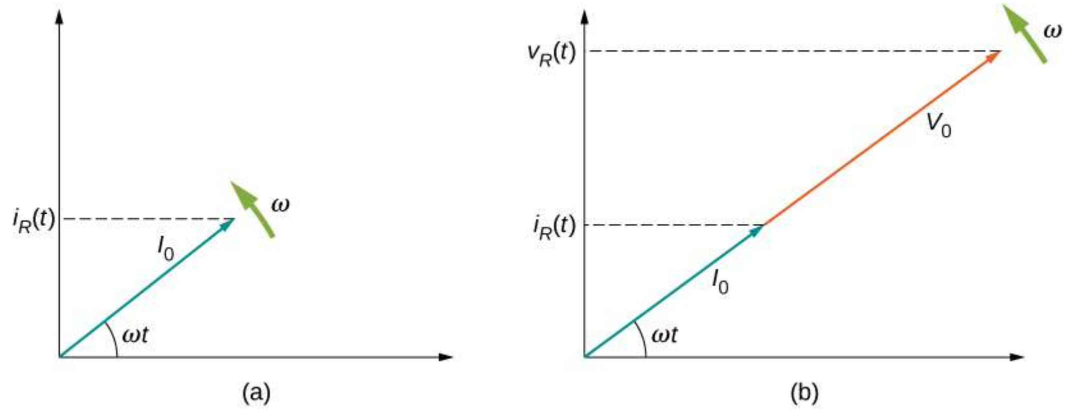


Figure 15.6 (a) The phasor diagram representing the current through the resistor of Figure 15.5. (b) The phasor diagram representing both $i_R(t)$ and $v_R(t)$.

The vertical axis on a phasor diagram could be either the voltage or the current, depending on the phasor that is being examined. In addition, several quantities can be depicted on the same phasor diagram. For example, both the current $i_R(t)$ and the voltage $v_R(t)$ are shown in the diagram of Figure 15.6(b). Since they have the same frequency and are in phase, their phasors point in the same direction and rotate together. The relative lengths of the two phasors are arbitrary because they represent different quantities; however, the ratio of the lengths of the two phasors can be represented by the resistance, since one is a voltage phasor and the other is a current phasor.

Capacitor

Now let's consider a capacitor connected across an ac voltage source. From Kirchhoff's loop rule, the instantaneous voltage across the capacitor of Figure 15.7(a) is

$$v_C(t) = V_0 \sin \omega t.$$

Recall that the charge in a capacitor is given by $Q = CV$. This is true at any time measured in the ac cycle of voltage. Consequently, the instantaneous charge on the capacitor is

$$q(t) = Cv_C(t) = CV_0 \sin \omega t.$$

Since the current in the circuit is the rate at which charge enters (or leaves) the capacitor,

$$i_C(t) = \frac{dq(t)}{dt} = \omega CV_0 \cos \omega t = I_0 \cos \omega t,$$

where $I_0 = \omega CV_0$ is the current amplitude. Using the trigonometric relationship $\cos \omega t = \sin(\omega t + \pi/2)$, we may express the instantaneous current as

$$i_C(t) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right).$$

Dividing V_0 by I_0, we obtain an equation that looks similar to Ohm's law:

$$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C. \tag{15.3}$$

The quantity X_C is analogous to resistance in a dc circuit in the sense that both quantities are a ratio of a voltage to a current. As a result, they have the same unit, the ohm. Keep in mind, however, that a capacitor stores and discharges electric energy, whereas a resistor dissipates it. The quantity X_C is known as the **capacitive reactance** of the capacitor, or the opposition of a capacitor to a change in current. It depends inversely on the frequency of the ac source—high frequency leads to low capacitive reactance.

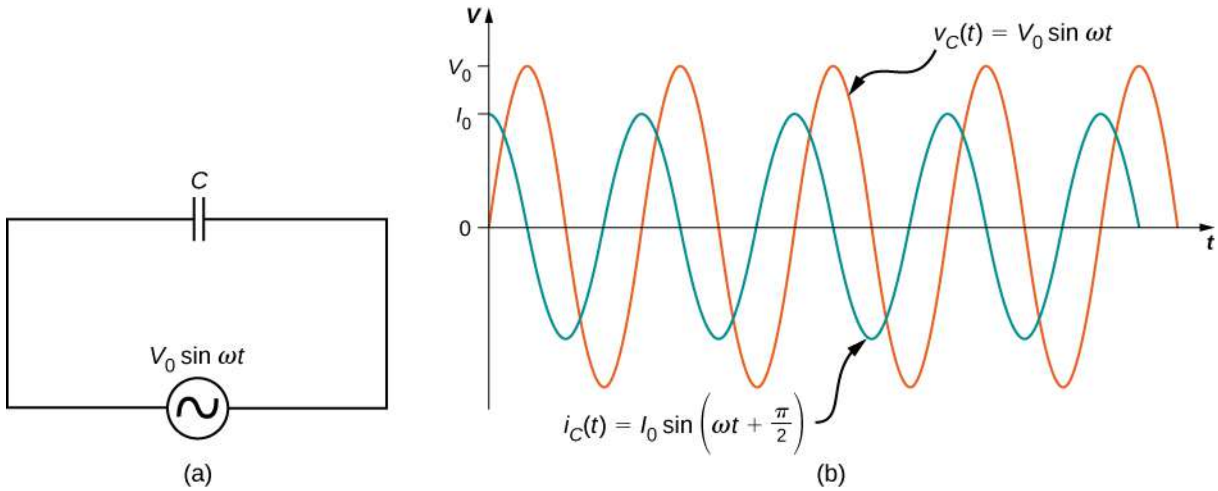


Figure 15.7 (a) A capacitor connected across an ac generator. (b) The current $i_C\ (t)$ through the capacitor and the voltage $v_C\ (t)$ across the capacitor. Notice that $i_C\ (t)$ leads $v_C\ (t)$ by $\pi/2$ rad.

A comparison of the expressions for $v_C\ (t)$ and $i_C\ (t)$ shows that there is a phase difference of $\pi/2$ rad between them. When these two quantities are plotted together, the current peaks a quarter cycle (or $\pi/2$ rad) ahead of the voltage, as illustrated in Figure 15.7(b). The current through a capacitor leads the voltage across a capacitor by $\pi/2$ rad, or a quarter of a cycle.

The corresponding phasor diagram is shown in Figure 15.8. Here, the relationship between $i_C\ (t)$ and $v_C\ (t)$ is represented by having their phasors rotate at the same angular frequency, with the current phasor leading by $\pi/2$ rad.

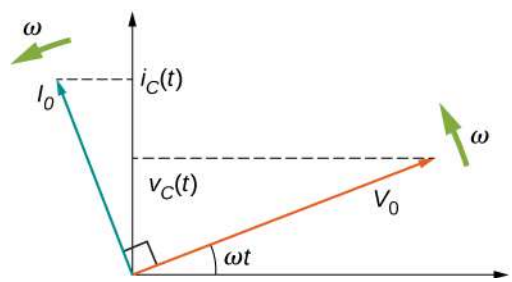


Figure 15.8 The phasor diagram for the capacitor of Figure 15.7. The current phasor leads the voltage phasor by $\pi/2$ rad as they both rotate with the same angular frequency.

To this point, we have exclusively been using peak values of the current or voltage in our discussion, namely, I_0 and V_0. However, if we average out the values of current or voltage, these values are zero. Therefore, we often use a second convention called the root mean square value, or rms value, in discussions of current and voltage. The rms operates in reverse of the terminology. First, you square the function, next, you take the mean, and then, you find the square root. As a result, the rms values of current and voltage are not zero. Appliances and devices are commonly quoted with rms values for their operations, rather than peak values. We indicate rms values with a subscript attached to a capital letter (such as I_{rms}).

Although a capacitor is basically an open circuit, an **rms current**, or the root mean square of the current, appears in a circuit with an ac voltage applied to a capacitor. Consider that

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}, \qquad 15.4$$

where I_0 is the peak current in an ac system. The **rms voltage**, or the root mean square of the voltage, is

$$V_{\mathrm{rms}} = \frac{V_0}{\sqrt{2}}, \tag{15.5}$$

where V_0 is the peak voltage in an ac system. The rms current appears because the voltage is continually reversing, charging, and discharging the capacitor. If the frequency goes to zero, which would be a dc voltage, X_C tends to infinity, and the current is zero once the capacitor is charged. At very high frequencies, the capacitor's reactance tends to zero—it has a negligible reactance and does not impede the current (it acts like a simple wire).

Inductor

Lastly, let's consider an inductor connected to an ac voltage source. From Kirchhoff's loop rule, the voltage across the inductor L of Figure 15.9(a) is

$$v_L(t) = V_0 \sin \omega t. \tag{15.6}$$

The emf across an inductor is equal to $\varepsilon = -L\,(di_L/dt)$; however, the potential difference across the inductor is $v_L(t) = L di_L(t)/dt$, because if we consider that the voltage around the loop must equal zero, the voltage gained from the ac source must dissipate through the inductor. Therefore, connecting this with the ac voltage source, we have

$$\frac{di_L(t)}{dt} = \frac{V_0}{L} \sin \omega t.$$

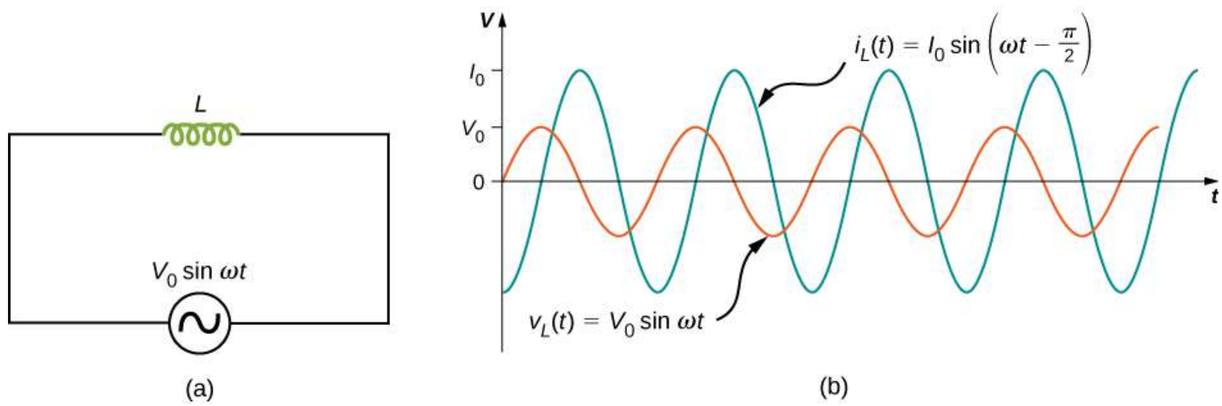


Figure 15.9 (a) An inductor connected across an ac generator. (b) The current $i_L(t)$ through the inductor and the voltage $v_L(t)$ across the inductor. Here $i_L(t)$ lags $v_L(t)$ by $\pi/2$ rad.

The current $i_L(t)$ is found by integrating this equation. Since the circuit does not contain a source of constant emf, there is no steady current in the circuit. Hence, we can set the constant of integration, which represents the steady current in the circuit, equal to zero, and we have

$$i_L(t) = -\frac{V_0}{\omega L}\cos \omega t = \frac{V_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) = I_0 \sin\left(\omega t - \frac{\pi}{2}\right), \tag{15.7}$$

where $I_0 = V_0/\omega L$. The relationship between V_0 and I_0 may also be written in a form analogous to Ohm's law:

$$\frac{V_0}{I_0} = \omega L = X_L. \tag{15.8}$$

The quantity X_L is known as the **inductive reactance** of the inductor, or the opposition of an inductor to a change in current; its unit is also the ohm. Note that X_L varies directly as the frequency of the ac source—high frequency causes high inductive reactance.

A phase difference of $\pi/2$ rad occurs between the current through and the voltage across the inductor. From Equation 15.6 and Equation 15.7, the current through an inductor lags the potential difference across an inductor by $\pi/2$ rad, or a quarter of a cycle. The phasor diagram for this case is shown in Figure 15.10.

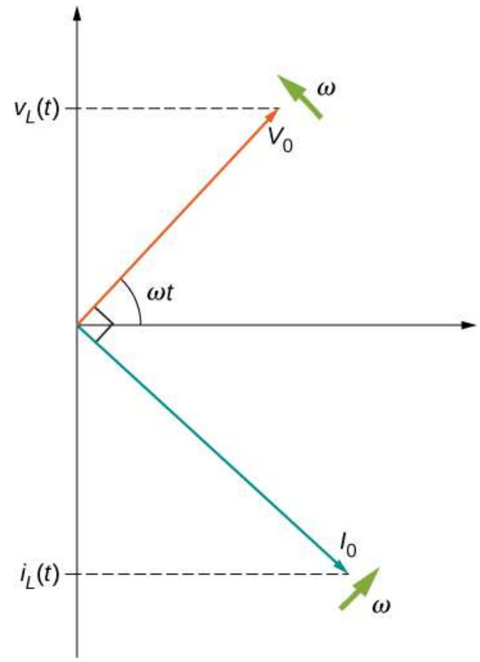


Figure 15.10 The phasor diagram for the inductor of Figure 15.9. The current phasor lags the voltage phasor by $\pi/2$ rad as they both rotate with the same angular frequency.

INTERACTIVE

An animation from the University of New South Wales AC Circuits (https://openstax.org/l/21accircuits) illustrates some of the concepts we discuss in this chapter. They also include wave and phasor diagrams that evolve over time so that you can get a better picture of how each changes over time.

EXAMPLE 15.1

Simple AC Circuits

An ac generator produces an emf of amplitude 10 V at a frequency $f = 60\,\text{Hz}$. Determine the voltages across and the currents through the circuit elements when the generator is connected to (a) a $100 - \Omega$ resistor, (b) a $10 - \mu\text{F}$ capacitor, and (c) a 15-mH inductor.

Strategy

The entire AC voltage across each device is the same as the source voltage. We can find the currents by finding the reactance X of each device and solving for the peak current using $I_0 = V_0/X$.

Solution

The voltage across the terminals of the source is

$$v(t) = V_0 \sin \omega t = (10\,\text{V}) \sin 120\pi t,$$

where $\omega = 2\pi f = 120\pi$ rad/s is the angular frequency. Since $v(t)$ is also the voltage across each of the elements, we have

$$v(t) = v_R(t) = v_C(t) = v_L(t) = (10\,\text{V}) \sin 120\pi t.$$

a. When $R = 100\,\Omega$, the amplitude of the current through the resistor is

$$I_0 = V_0/R = 10\,\text{V}/100\,\Omega = 0.10\,\text{A},$$

so

$$i_R(t) = (0.10\,\text{A}) \sin 120\pi t.$$

b. From Equation 15.3, the capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{(120\pi\ \text{rad/s})(10 \times 10^{-6}\text{F})} = 265\ \Omega,$$

so the maximum value of the current is

$$I_0 = \frac{V_0}{X_C} = \frac{10\ \text{V}}{265\ \Omega} = 3.8 \times 10^{-2}\ \text{A}$$

and the instantaneous current is given by

$$i_C(t) = (3.8 \times 10^{-2}\ \text{A}) \sin\left(120\pi t + \frac{\pi}{2}\right).$$

c. From Equation 15.8, the inductive reactance is

$$X_L = \omega L = (120\pi\ \text{rad/s})(15 \times 10^{-3}\ \text{H}) = 5.7\ \Omega.$$

The maximum current is therefore

$$I_0 = \frac{10\ \text{V}}{5.7\ \Omega} = 1.8\ \text{A}$$

and the instantaneous current is

$$i_L(t) = (1.8\ \text{A})\ \sin\left(120\pi t - \frac{\pi}{2}\right).$$

Significance

Although the voltage across each device is the same, the peak current has different values, depending on the reactance. The reactance for each device depends on the values of resistance, capacitance, or inductance.

CHECK YOUR UNDERSTANDING 15.2

Repeat Example 15.1 for an ac source of amplitude 20 V and frequency 100 Hz.

15.3 RLC Series Circuits with AC

Learning Objectives

By the end of this section, you will be able to:

- Describe how the current varies in a resistor, a capacitor, and an inductor while in series with an ac power source
- Use phasors to understand the phase angle of a resistor, capacitor, and inductor ac circuit and to understand what that phase angle means
- Calculate the impedance of a circuit

The ac circuit shown in Figure 15.11, called an *RLC* series circuit, is a series combination of a resistor, capacitor, and inductor connected across an ac source. It produces an emf of

$$v(t) = V_0 \sin \omega t.$$

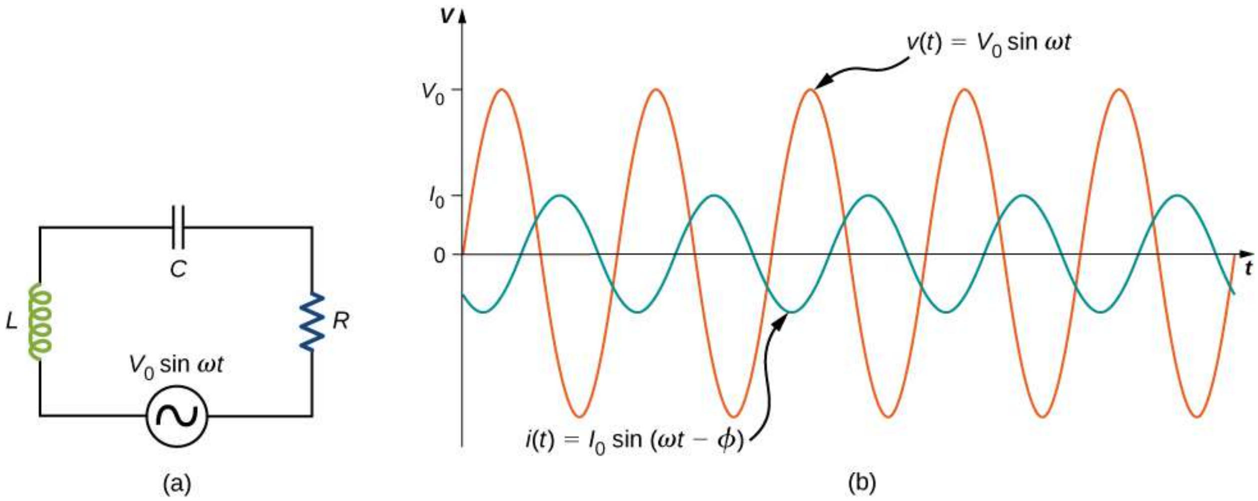


Figure 15.11 (a) An *RLC* series circuit. (b) A comparison of the generator output voltage and the current. The value of the phase difference ϕ depends on the values of *R*, *C*, and *L*.

Since the elements are in series, the same current flows through each element at all points in time. The relative phase between the current and the emf is not obvious when all three elements are present. Consequently, we represent the current by the general expression

$$i(t) = I_0 \sin(\omega t - \phi),$$

where I_0 is the current amplitude and ϕ is the **phase angle** between the current and the applied voltage. The phase angle is thus the amount by which the voltage and current are out of phase with each other in a circuit. Our task is to find I_0 and ϕ.

A phasor diagram involving $i(t), v_R(t), v_C(t),$ and $v_L(t)$ is helpful for analyzing the circuit. As shown in Figure 15.12, the phasor representing $v_R(t)$ points in the same direction as the phasor for $i(t)$; its amplitude is $V_R = I_0 R$. The $v_C(t)$ phasor lags the $i(t)$ phasor by $\pi/2$ rad and has the amplitude $V_C = I_0 X_C$. The phasor for $v_L(t)$ leads the $i(t)$ phasor by $\pi/2$ rad and has the amplitude $V_L = I_0 X_L$.

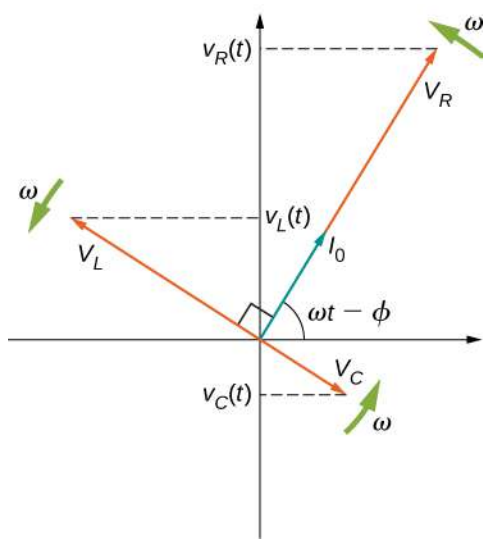


Figure 15.12 The phasor diagram for the *RLC* series circuit of Figure 15.11.

At any instant, the voltage across the *RLC* combination is $v_R(t) + v_L(t) + v_C(t) = v(t)$, the emf of the source. Since a component of a sum of vectors is the sum of the components of the individual vectors—for example, $(A + B)_y = A_y + B_y$ —the projection of the vector sum of phasors onto the vertical axis is the sum of the vertical projections of the individual phasors. Hence, if we add vectorially the phasors representing $v_R(t), v_L(t),$ and $v_C(t)$ and then find the projection of the resultant onto the vertical axis, we obtain

$$v_R(t) + v_L(t) + v_C(t) = v(t) = V_0 \sin \omega t.$$

The vector sum of the phasors is shown in Figure 15.13. The resultant phasor has an amplitude V_0 and is directed at an angle ϕ with respect to the $v_R\ (t)$, or $i(t)$, phasor. The projection of this resultant phasor onto the vertical axis is $v(t) = V_0 \sin \omega t$. We can easily determine the unknown quantities I_0 and ϕ from the geometry of the phasor diagram. For the phase angle,

$$\phi = \tan^{-1} \frac{V_L - V_C}{V_R} = \tan^{-1} \frac{I_0 X_L - I_0 X_C}{I_0 R},$$

and after cancellation of I_0, this becomes

$$\phi = \tan^{-1} \frac{X_L - X_C}{R}. \tag{15.9}$$

Furthermore, from the Pythagorean theorem,

$$V_0 = \sqrt{V_R{}^2 + (V_L - V_C)^2} = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2}.$$

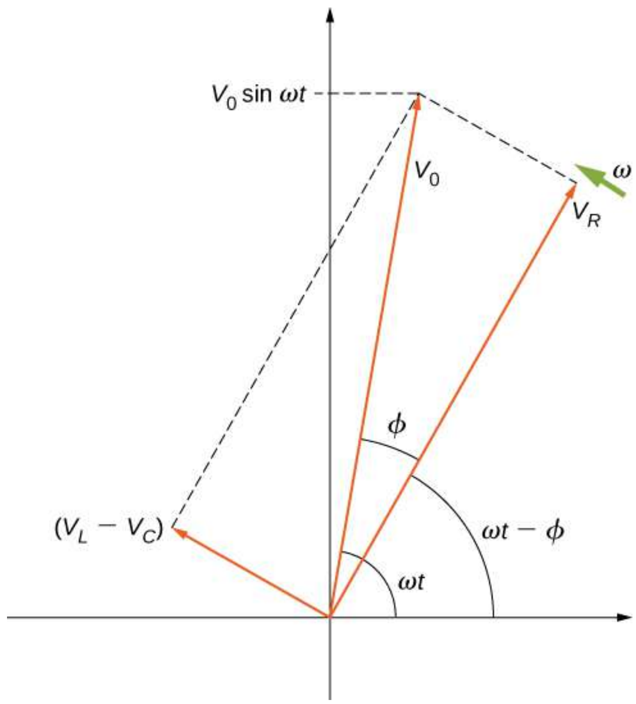


Figure 15.13 The resultant of the phasors for $v_L\ (t)$, $v_C\ (t)$, and $v_R\ (t)$ is equal to the phasor for $v(t) = V_0 \sin \omega t$. The $i(t)$ phasor (not shown) is aligned with the $v_R\ (t)$ phasor.

The current amplitude is therefore the ac version of Ohm's law:

$$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_0}{Z}, \tag{15.10}$$

where

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{15.11}$$

is known as the **impedance** of the circuit. Its unit is the ohm, and it is the ac analog to resistance in a dc circuit, which measures the combined effect of resistance, capacitive reactance, and inductive reactance (Figure 15.14).

Figure 15.14 Power capacitors are used to balance the impedance of the effective inductance in transmission lines.

The *RLC* circuit is analogous to the wheel of a car driven over a corrugated road (Figure 15.15). The regularly spaced bumps in the road drive the wheel up and down; in the same way, a voltage source increases and decreases. The shock absorber acts like the resistance of the *RLC* circuit, damping and limiting the amplitude of the oscillation. Energy within the wheel system goes back and forth between kinetic and potential energy stored in the car spring, analogous to the shift between a maximum current, with energy stored in an inductor, and no current, with energy stored in the electric field of a capacitor. The amplitude of the wheel's motion is at a maximum if the bumps in the road are hit at the resonant frequency, which we describe in more detail in Resonance in an AC Circuit.

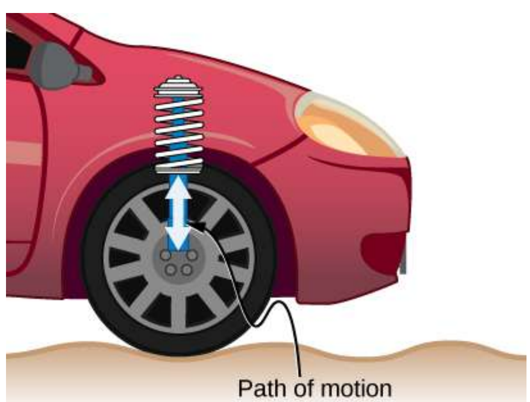


Figure 15.15 On a car, the shock absorber damps motion and dissipates energy. This is much like the resistance in an *RLC* circuit. The mass and spring determine the resonant frequency.

PROBLEM-SOLVING STRATEGY

AC Circuits

To analyze an ac circuit containing resistors, capacitors, and inductors, it is helpful to think of each device's reactance and find the equivalent reactance using the rules we used for equivalent resistance in the past. Phasors are a great method to determine whether the emf of the circuit has positive or negative phase (namely, leads or lags other values). A mnemonic device of "ELI the ICE man" is sometimes used to remember that the emf (E) leads the current (I) in an inductor (L) and the current (I) leads the emf (E) in a capacitor (C).

Use the following steps to determine the emf of the circuit by phasors:

1. Draw the phasors for voltage across each device: resistor, capacitor, and inductor, including the phase angle in the circuit.
2. If there is both a capacitor and an inductor, find the net voltage from these two phasors, since they are antiparallel.
3. Find the equivalent phasor from the phasor in step 2 and the resistor's phasor using trigonometry or components of the phasors. The equivalent phasor found is the emf of the circuit.

EXAMPLE 15.2

An *RLC* Series Circuit

The output of an ac generator connected to an *RLC* series combination has a frequency of 200 Hz and an amplitude of 0.100 V. If $R = 4.00\ \Omega$, $L = 3.00 \times 10^{-3}$ H, and $C = 8.00 \times 10^{-4}$ F, what are (a) the capacitive reactance, (b) the inductive reactance, (c) the impedance, (d) the current amplitude, and (e) the phase difference between the current and the emf of the generator?

Strategy

The reactances and impedance in (a)–(c) are found by substitutions into Equation 15.3, Equation 15.8, and Equation 15.11, respectively. The current amplitude is calculated from the peak voltage and the impedance. The phase difference between the current and the emf is calculated by the inverse tangent of the difference between the reactances divided by the resistance.

Solution

a. From Equation 15.3, the capacitive reactance is
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\,(200\ \text{Hz})\left(8.00 \times 10^{-4}\ \text{F}\right)} = 0.995\ \Omega.$$

b. From Equation 15.8, the inductive reactance is
$$X_L = \omega L = 2\pi\,(200\ \text{Hz})\left(3.00 \times 10^{-3}\ \text{H}\right) = 3.77\ \Omega.$$

c. Substituting the values of *R*, X_C, and X_L into Equation 15.11, we obtain for the impedance
$$Z = \sqrt{(4.00\ \Omega)^2 + (3.77\ \Omega - 0.995\ \Omega)^2} = 4.87\ \Omega.$$

d. The current amplitude is
$$I_0 = \frac{V_0}{Z} = \frac{0.100\ \text{V}}{4.87\ \Omega} = 2.05 \times 10^{-2}\,\text{A}.$$

e. From Equation 15.9, the phase difference between the current and the emf is
$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2.77\ \Omega}{4.00\ \Omega} = 0.607\ \text{rad}.$$

Significance

The phase angle is positive because the reactance of the inductor is larger than the reactance of the capacitor.

CHECK YOUR UNDERSTANDING 15.3

Find the voltages across the resistor, the capacitor, and the inductor in the circuit of Figure 15.11 using $v(t) = V_0 \sin \omega t$ as the output of the ac generator.

15.4 Power in an AC Circuit

Learning Objectives

By the end of this section, you will be able to:

- Describe how average power from an ac circuit can be written in terms of peak current and voltage and of rms current and voltage
- Determine the relationship between the phase angle of the current and voltage and the average power, known as the power factor

A circuit element dissipates or produces power according to $P = IV$, where I is the current through the element and V is the voltage across it. Since the current and the voltage both depend on time in an ac circuit, the instantaneous power $p(t) = i(t)v(t)$ is also time dependent. A plot of $p(t)$ for various circuit elements is shown in Figure 15.16. For a resistor, $i(t)$ and $v(t)$ are in phase and therefore always have the same sign (see Figure 15.5). For a capacitor or inductor, the relative signs of $i(t)$ and $v(t)$ vary over a cycle due to their phase differences (see Figure 15.7 and Figure 15.9). Consequently, $p(t)$ is positive at some times and negative at others, indicating that capacitive and inductive elements produce power at some instants and absorb it at others.

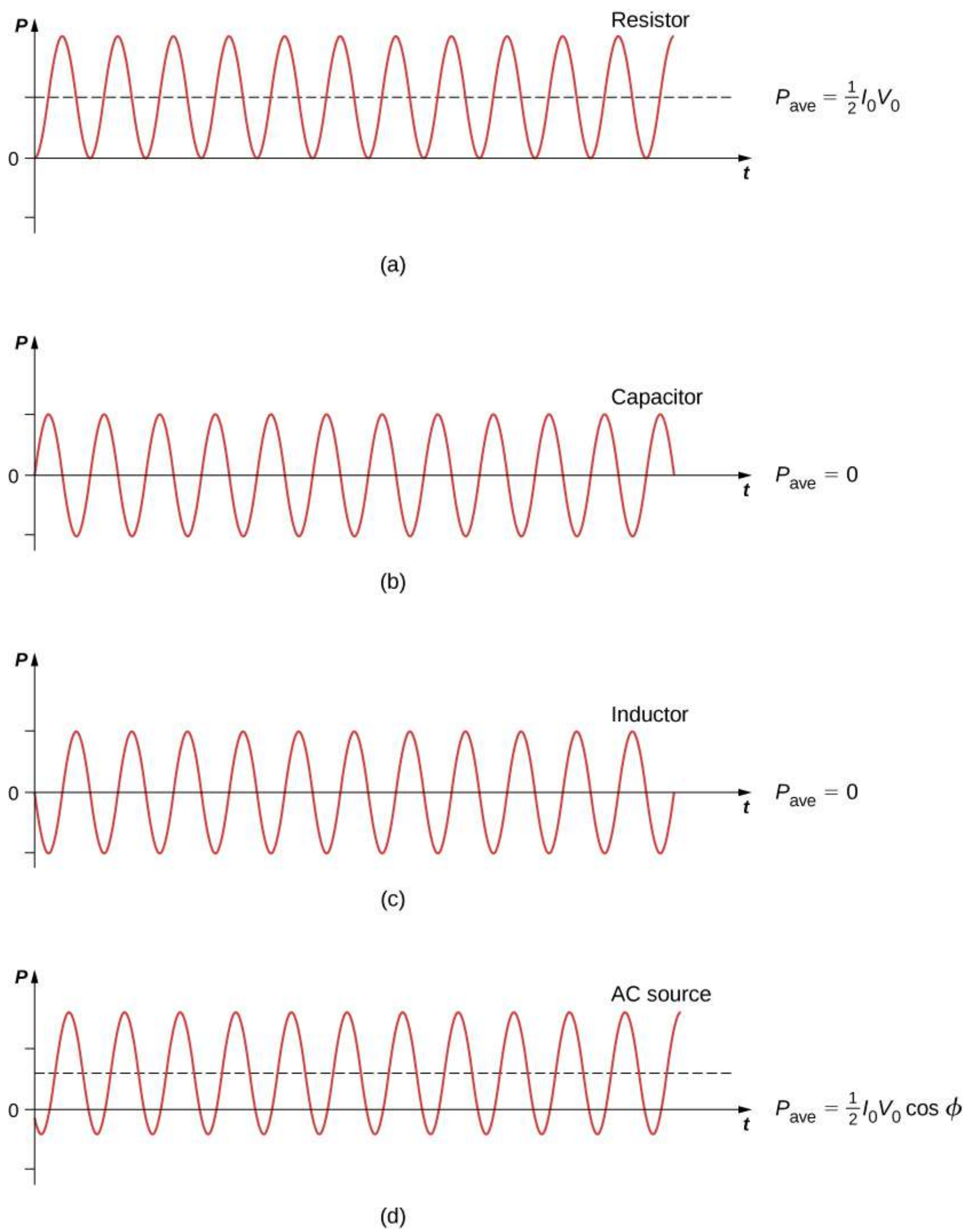


Figure 15.16 Graph of instantaneous power for various circuit elements. (a) For the resistor, $P_{ave} = I_0V_0/2$, whereas for (b) the capacitor and (c) the inductor, $P_{ave} = 0$. (d) For the source, $P_{ave} = I_0V_0(\cos \phi)/2$, which may be positive, negative, or zero, depending on ϕ.

Because instantaneous power varies in both magnitude and sign over a cycle, it seldom has any practical importance. What we're almost always concerned with is the power averaged over time, which we refer to as the **average power**. It is defined by the time average of the instantaneous power over one cycle:

$$P_{ave} = \frac{1}{T}\int_0^T p(t)dt,$$

where $T = 2\pi/\omega$ is the period of the oscillations. With the substitutions $v(t) = V_0 \sin \omega t$ and $i(t) = I_0 \sin(\omega t - \phi)$, this integral becomes

$$P_{\text{ave}} = \frac{I_0 V_0}{T} \int_0^T \sin(\omega t - \phi) \sin \omega t \, dt.$$

Using the trigonometric relation $\sin(A - B) = \sin A \cos B - \sin B \cos A$, we obtain

$$P_{\text{ave}} = \frac{I_0 V_0 \cos\phi}{T} \int_0^T \sin^2 \omega t dt - \frac{I_0 V_0 \sin\phi}{T} \int_0^T \sin \omega t \cos \omega t dt.$$

Evaluation of these two integrals yields

$$\frac{1}{T} \int_0^T \sin^2 \omega t dt = \frac{1}{2}$$

and

$$\frac{1}{T} \int_0^T \sin \omega t \cos \omega t dt = 0.$$

Hence, the average power associated with a circuit element is given by

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos\phi. \qquad \textbf{15.12}$$

In engineering applications, $\cos\phi$ is known as the **power factor**, which is the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase. For a resistor, $\phi = 0$, so the average power dissipated is

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0.$$

A comparison of $p(t)$ and P_{ave} is shown in Figure 15.16(d). To make $P_{\text{ave}} = (1/2) I_0 V_0$ look like its dc counterpart, we use the rms values I_{rms} and V_{rms} of the current and the voltage. By definition, these are

$$I_{\text{rms}} = \sqrt{i^2_{\text{ave}}} \text{ and } V_{\text{rms}} = \sqrt{v^2_{\text{ave}}},$$

where

$$i^2_{\text{ave}} = \frac{1}{T} \int_0^T i^2(t) dt \text{ and } v^2_{\text{ave}} = \frac{1}{T} \int_0^T v^2(t) dt.$$

With $i(t) = I_0 \sin(\omega t - \phi)$ and $v(t) = V_0 \sin \omega t$, we obtain

$$I_{\text{rms}} = \frac{1}{\sqrt{2}} I_0 \text{ and } V_{\text{rms}} = \frac{1}{\sqrt{2}} V_0.$$

We may then write for the average power dissipated by a resistor,

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 = I_{\text{rms}} V_{\text{rms}} = I^2_{\text{rms}} R. \qquad \textbf{15.13}$$

This equation further emphasizes why the rms value is chosen in discussion rather than peak values. Both equations for average power are correct for Equation 15.13, but the rms values in the formula give a cleaner representation, so the extra factor of 1/2 is not necessary.

Alternating voltages and currents are usually described in terms of their rms values. For example, the 110 V from a household outlet is an rms value. The amplitude of this source is $110\sqrt{2}\text{ V} = 156\text{ V}$. Because most ac meters are calibrated in terms of rms values, a typical ac voltmeter placed across a household outlet will read 110 V.

For a capacitor and an inductor, $\phi = \pi/2$ and $-\pi/2$ rad, respectively. Since $\cos\pi/2 = \cos(-\pi/2) = 0$, we find from Equation 15.12 that the average power dissipated by either of these elements is $P_{\text{ave}} = 0$. Capacitors and inductors absorb energy from the circuit during one half-cycle and then discharge it back to the circuit during the other half-cycle. This behavior is illustrated in the plots of Figure 15.16, (b) and (c), which show *p*(*t*) oscillating sinusoidally about zero.

The phase angle for an ac generator may have any value. If $\cos\phi > 0$, the generator produces power; if $\cos\phi < 0$, it absorbs power. In terms of rms values, the average power of an ac generator is written as

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos\phi.$$

For the generator in an *RLC* circuit,

$$\tan\phi = \frac{X_L - X_C}{R}$$

and

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{Z}.$$

Hence the average power of the generator is

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos\phi = \frac{V_{\text{rms}}}{Z} V_{\text{rms}} \frac{R}{Z} = \frac{V_{\text{rms}}^2 R}{Z^2}. \qquad \textbf{15.14}$$

This can also be written as

$$P_{\text{ave}} = I_{\text{rms}}^2 R,$$

which designates that the power produced by the generator is dissipated in the resistor. As we can see, Ohm's law for the rms ac is found by dividing the rms voltage by the impedance.

EXAMPLE 15.3

Power Output of a Generator

An ac generator whose emf is given by

$$v(t) = (4.00\text{ V}) \sin\left[\left(1.00 \times 10^4 \text{rad/s}\right) t\right]$$

is connected to an *RLC* circuit for which $L = 2.00 \times 10^{-3}$ H, $C = 4.00 \times 10^{-6}$ F, and $R = 5.00\,\Omega$. (a) What is the rms voltage across the generator? (b) What is the impedance of the circuit? (c) What is the average power output of the generator?

Strategy

The rms voltage is the amplitude of the voltage times $1/\sqrt{2}$. The impedance of the circuit involves the resistance and the reactances of the capacitor and the inductor. The average power is calculated by Equation 15.14, or more specifically, the last part of the equation, because we have the impedance of the circuit *Z*, the rms voltage V_{rms}, and the resistance *R*.

Solution

a. Since $V_0 = 4.00$ V, the rms voltage across the generator is

$$V_{\text{rms}} = \frac{1}{\sqrt{2}}(4.00\text{ V}) = 2.83\text{ V}.$$

b. The impedance of the circuit is

$$
\begin{aligned}
Z &= \sqrt{R^2 + (X_L - X_C)^2} \\
&= \left\{ (5.00\,\Omega)^2 + \left[\left(1.00 \times 10^4 \text{ rad/s}\right) \left(2.00 \times 10^{-3}\text{ H}\right) - \frac{1}{\left(1.00 \times 10^4 \text{ rad/s}\right)\left(4.00 \times 10^{-6}\text{ F}\right)} \right]^2 \right\}^{1/2} \\
&= 7.07\,\Omega.
\end{aligned}
$$

c. From Equation 15.14, the average power transferred to the circuit is

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2 R}{Z^2} = \frac{(2.83\text{ V})^2\,(5.00\,\Omega)}{(7.07\,\Omega)^2} = 0.801\text{ W}.$$

Significance

If the resistance is much larger than the reactance of the capacitor or inductor, the average power is a dc circuit equation of $P = V^2/R$, where V replaces the rms voltage.

CHECK YOUR UNDERSTANDING 15.4

An ac voltmeter attached across the terminals of a 45-Hz ac generator reads 7.07 V. Write an expression for the emf of the generator.

CHECK YOUR UNDERSTANDING 15.5

Show that the rms voltages across a resistor, a capacitor, and an inductor in an ac circuit where the rms current is I_{rms} are given by $I_{\text{rms}}R$, $I_{\text{rms}}X_C$, and $I_{\text{rms}}X_L$, respectively. Determine these values for the components of the *RLC* circuit of Equation 15.12.

15.5 Resonance in an AC Circuit

Learning Objectives

By the end of this section, you will be able to:

- Determine the peak ac resonant angular frequency for a RLC circuit
- Explain the width of the average power versus angular frequency curve and its significance using terms like bandwidth and quality factor

In the *RLC* series circuit of Figure 15.11, the current amplitude is, from Equation 15.10,

$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \qquad (15.15)$$

If we can vary the frequency of the ac generator while keeping the amplitude of its output voltage constant, then the current changes accordingly. A plot of I_0 versus ω is shown in Figure 15.17.

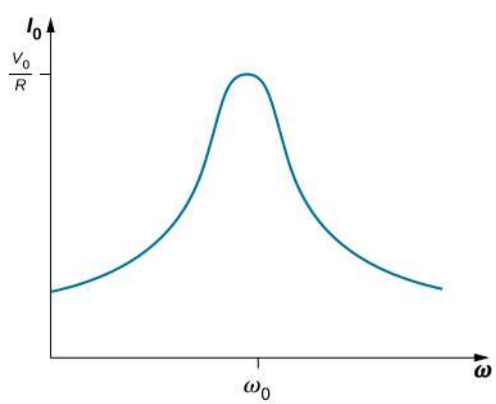


Figure 15.17 At an *RLC* circuit's resonant frequency, $\omega_0 = \sqrt{1/LC}$, the current amplitude is at its maximum value.

In Oscillations, we encountered a similar graph where the amplitude of a damped harmonic oscillator was plotted against the angular frequency of a sinusoidal driving force (see Forced Oscillations). This similarity is more than just a coincidence, as shown earlier by the application of Kirchhoff's loop rule to the circuit of Figure 15.11. This yields

$$L\frac{di}{dt} + iR + \frac{q}{C} = V_0 \sin \omega t, \tag{15.16}$$

or

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V_0 \sin \omega t,$$

where we substituted $dq(\mathrm{t})/dt$ for $i(\mathrm{t})$. A comparison of Equation 15.16 and, from Oscillations, Damped Oscillations for damped harmonic motion clearly demonstrates that the driven *RLC* series circuit is the electrical analog of the driven damped harmonic oscillator.

The **resonant frequency** f_0 of the *RLC* circuit is the frequency at which the amplitude of the current is a maximum and the circuit would oscillate if not driven by a voltage source. By inspection, this corresponds to the angular frequency $\omega_0 = 2\pi f_0$ at which the impedance Z in Equation 15.15 is a minimum, or when

$$\omega_0 L = \frac{1}{\omega_0 C}$$

and

$$\omega_0 = \sqrt{\frac{1}{LC}}. \tag{15.17}$$

This is the resonant angular frequency of the circuit. Substituting ω_0 into Equation 15.9, Equation 15.10, and Equation 15.11, we find that at resonance,

$$\phi = \tan^{-1}(0) = 0, \quad I_0 = V_0/R, \quad \text{and} \quad Z = R.$$

Therefore, at resonance, an *RLC* circuit is purely resistive, with the applied emf and current in phase.

What happens to the power at resonance? Equation 15.14 tells us how the average power transferred from an ac generator to the *RLC* combination varies with frequency. In addition, P_{ave} reaches a maximum when Z, which depends on the frequency, is a minimum, that is, when $X_L = X_C$ and $Z = R$. Thus, at resonance, the average power output of the source in an *RLC* series circuit is a maximum. From Equation 15.14, this maximum is V_{rms}^2/R.

Figure 15.18 is a typical plot of P_{ave} versus ω in the region of maximum power output. The **bandwidth** $\Delta\omega$ of the resonance peak is defined as the range of angular frequencies ω over which the average power P_{ave} is greater than one-half the maximum value of P_{ave}. The sharpness of the peak is described by a dimensionless quantity known as the **quality factor** Q of the circuit. By definition,

$$Q = \frac{\omega_0}{\Delta\omega}, \tag{15.18}$$

where ω_0 is the resonant angular frequency. A high Q indicates a sharp resonance peak. We can give Q in terms of the circuit parameters as

$$Q = \frac{\omega_0 L}{R}. \tag{15.19}$$

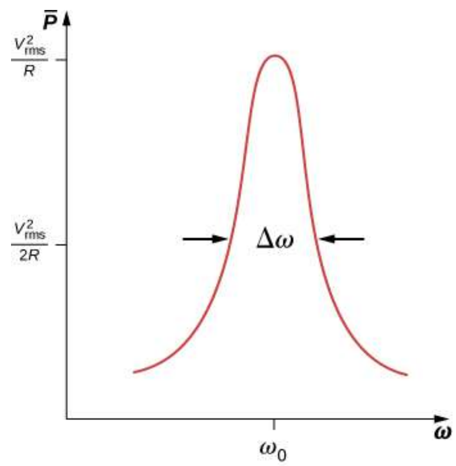


Figure 15.18 Like the current, the average power transferred from an ac generator to an *RLC* circuit peaks at the resonant frequency.

Resonant circuits are commonly used to pass or reject selected frequency ranges. This is done by adjusting the value of one of the elements and hence "tuning" the circuit to a particular resonant frequency. For example, in radios, the receiver is tuned to the desired station by adjusting the resonant frequency of its circuitry to match the frequency of the station. If the tuning circuit has a high Q, it will have a small bandwidth, so signals from other stations at frequencies even slightly different from the resonant frequency encounter a high impedance and are not passed by the circuit. Cell phones work in a similar fashion, communicating with signals of around 1 GHz that are tuned by an inductor-capacitor circuit. One of the most common applications of capacitors is their use in ac-timing circuits, based on attaining a resonant frequency. A metal detector also uses a shift in resonance frequency in detecting metals (Figure 15.19).

Figure 15.19 When a metal detector comes near a piece of metal, the self-inductance of one of its coils changes. This causes a shift in the resonant frequency of a circuit containing the coil. That shift is detected by the circuitry and transmitted to the diver by means of the headphones. (credit: modification of work by Eric Lippmann, U.S. Navy)

EXAMPLE 15.4

Resonance in an *RLC* Series Circuit

(a) What is the resonant frequency of a circuit using the voltage and LRC values all wired in series from

Example 15.1? (b) If the ac generator is set to this frequency without changing the amplitude of the output voltage, what is the amplitude of the current?

Strategy

The resonant frequency for a *RLC* circuit is calculated from Equation 15.17, which comes from a balance between the reactances of the capacitor and the inductor. Since the circuit is at resonance, the impedance is equal to the resistor. Then, the peak current is calculated by the voltage divided by the resistance.

Solution

a. The resonant frequency is found from Equation 15.17:

$$\begin{aligned} f_0 &= \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{\left(3.00 \times 10^{-3}\text{ H}\right)\left(8.00 \times 10^{-4}\text{ F}\right)}} \\ &= 1.03 \times 10^2\text{ Hz.} \end{aligned}$$

b. At resonance, the impedance of the circuit is purely resistive, and the current amplitude is

$$I_0 = \frac{0.100\text{ V}}{4.00\ \Omega} = 2.50 \times 10^{-2}\text{A.}$$

Significance

If the circuit were not set to the resonant frequency, we would need the impedance of the entire circuit to calculate the current.

EXAMPLE 15.5

Power Transfer in an *RLC* Series Circuit at Resonance

(a) What is the resonant angular frequency of an *RLC* circuit with $R = 0.200\ \Omega$, $L = 4.00 \times 10^{-3}$ H, and $C = 2.00 \times 10^{-6}$ F? (b) If an ac source of constant amplitude 4.00 V is set to this frequency, what is the average power transferred to the circuit? (c) Determine Q and the bandwidth of this circuit.

Strategy

The resonant angular frequency is calculated from Equation 15.17. The average power is calculated from the rms voltage and the resistance in the circuit. The quality factor is calculated from Equation 15.19 and by knowing the resonant frequency. The bandwidth is calculated from Equation 15.18 and by knowing the quality factor.

Solution

a. The resonant angular frequency is

$$\begin{aligned} \omega_0 &= \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{\left(4.00 \times 10^{-3}\text{ H}\right)\left(2.00 \times 10^{-6}\text{ F}\right)}} \\ &= 1.12 \times 10^4\text{ rad/s.} \end{aligned}$$

b. At this frequency, the average power transferred to the circuit is a maximum. It is

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R} = \frac{\left[\left(1/\sqrt{2}\right)(4.00\text{ V})\right]^2}{0.200\ \Omega} = 40.0\text{ W.}$$

c. The quality factor of the circuit is

$$Q = \frac{\omega_0 L}{R} = \frac{\left(1.12 \times 10^4 \text{ rad/s}\right)\left(4.00 \times 10^{-3} \text{ H}\right)}{0.200 \,\Omega} = 224.$$

We then find for the bandwidth

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{1.12 \times 10^4 \text{ rad/s}}{224} = 50.0 \text{ rad/s}.$$

Significance

If a narrower bandwidth is desired, a lower resistance or higher inductance would help. However, a lower resistance increases the power transferred to the circuit, which may not be desirable, depending on the maximum power that could possibly be transferred.

CHECK YOUR UNDERSTANDING 15.6

In the circuit of Figure 15.11, $L = 2.0 \times 10^{-3}$ H, $C = 5.0 \times 10^{-4}$ F, and $R = 40\,\Omega$. (a) What is the resonant frequency? (b) What is the impedance of the circuit at resonance? (c) If the voltage amplitude is 10 V, what is $i(t)$ at resonance? (d) The frequency of the AC generator is now changed to 200 Hz. Calculate the phase difference between the current and the emf of the generator.

CHECK YOUR UNDERSTANDING 15.7

What happens to the resonant frequency of an *RLC* series circuit when the following quantities are increased by a factor of 4: (a) the capacitance, (b) the self-inductance, and (c) the resistance?

CHECK YOUR UNDERSTANDING 15.8

The resonant angular frequency of an *RLC* series circuit is 4.0×10^2 rad/s. An ac source operating at this frequency transfers an average power of 2.0×10^{-2} W to the circuit. The resistance of the circuit is $0.50\,\Omega$. Write an expression for the emf of the source.

15.6 Transformers

Learning Objectives

By the end of this section, you will be able to:

- Explain why power plants transmit electricity at high voltages and low currents and how they do this
- Develop relationships among current, voltage, and the number of windings in step-up and step-down transformers

Although ac electric power is produced at relatively low voltages, it is sent through transmission lines at very high voltages (as high as 500 kV). The same power can be transmitted at different voltages because power is the product $I_{\text{rms}} V_{\text{rms}}$. (For simplicity, we ignore the phase factor $\cos \phi$.) A particular power requirement can therefore be met with a low voltage and a high current or with a high voltage and a low current. The advantage of the high-voltage/low-current choice is that it results in lower $I_{\text{rms}}^2 R$ ohmic losses in the transmission lines, which can be significant in lines that are many kilometers long (Figure 15.20).

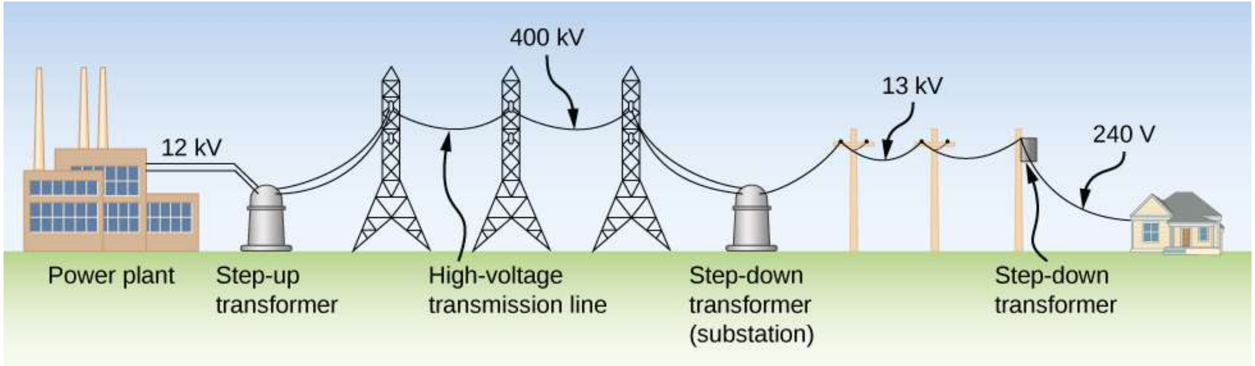


Figure 15.20 The rms voltage from a power plant eventually needs to be stepped down from 12 kV to 240 V so that it can be safely introduced into a home. A high-voltage transmission line allows a low current to be transmitted via a substation over long distances.

Typically, the alternating emfs produced at power plants are “stepped up” to very high voltages before being transmitted through power lines; then, they must be “stepped down” to relatively safe values (110 or 220 V rms) before they are introduced into homes. The device that transforms voltages from one value to another using induction is the **transformer** (Figure 15.21).

Figure 15.21 Transformers are used to step down the high voltages in transmission lines to the 110 to 220 V used in homes. (credit: modification of work by “Fortyseven”/Flickr)

As Figure 15.22 illustrates, a transformer basically consists of two separated coils, or windings, wrapped around a soft iron core. The primary winding has N_P loops, or turns, and is connected to an alternating voltage $v_P(t)$. The secondary winding has N_S turns and is connected to a load resistor R_S. We assume the ideal case for which all magnetic field lines are confined to the core so that the same magnetic flux permeates each turn of both the primary and the secondary windings. We also neglect energy losses to magnetic hysteresis, to ohmic heating in the windings, and to ohmic heating of the induced eddy currents in the core. A good transformer can have losses as low as 1% of the transmitted power, so this is not a bad assumption.

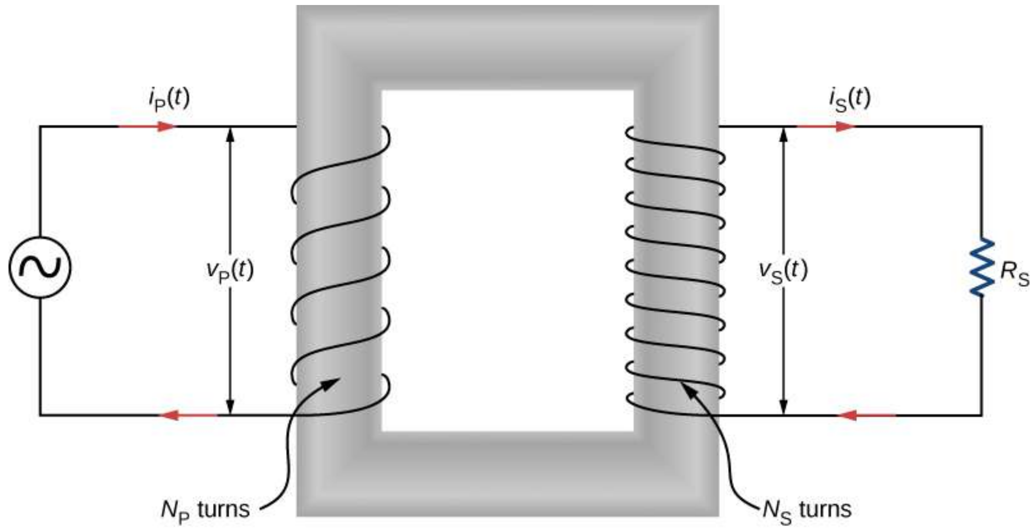


Figure 15.22 A step-up transformer (more turns in the secondary winding than in the primary winding). The two windings are wrapped around a soft iron core.

To analyze the transformer circuit, we first consider the primary winding. The input voltage $v_P(t)$ is equal to the potential difference induced across the primary winding. From Faraday's law, the induced potential difference is $-N_P\,(d\Phi/dt)$, where Φ is the flux through one turn of the primary winding. Thus,

$$v_P\,(t) = -N_P \frac{d\Phi}{dt}.$$

Similarly, the output voltage $v_S(t)$ delivered to the load resistor must equal the potential difference induced across the secondary winding. Since the transformer is ideal, the flux through every turn of the secondary winding is also $\boldsymbol{\Phi}$, and

$$v_S(t) = -N_S \frac{d\Phi}{dt}.$$

Combining the last two equations, we have

$$v_S\,(t) = \frac{N_S}{N_P} v_P\,(t)\,. \qquad 15.20$$

Hence, with appropriate values for N_S and N_P, the input voltage $v_P(t)$ may be "stepped up" ($N_S > N_P$) or "stepped down" ($N_S < N_P$) to $v_S(t)$, the output voltage. This is often abbreviated as the **transformer equation**,

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}, \qquad 15.21$$

which shows that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of turns in their windings. For a **step-up transformer**, which increases voltage and decreases current, this ratio is greater than one; for a **step-down transformer**, which decreases voltage and increases current, this ratio is less than one.

From the law of energy conservation, the power introduced at any instant by $v_P(t)$ to the primary winding must be equal to the power dissipated in the resistor of the secondary circuit; thus,

$$i_P(t)v_P(t) = i_S(t)v_S(t).$$

When combined with Equation 15.20, this gives

$$i_S\,(t) = \frac{N_P}{N_S} i_P\,(t)\,. \qquad 15.22$$

If the voltage is stepped up, the current is stepped down, and vice versa.

Finally, we can use $i_S(t) = v_S(t)/R_S$, along with Equation 15.20 and Equation 15.22, to obtain

$$v_P(t) = i_P\left[\left(\frac{N_P}{N_S}\right)^2 R_S\right],$$

which tells us that the input voltage $v_P(t)$ “sees” not a resistance R_S but rather a resistance

$$R_P = \left(\frac{N_P}{N_S}\right)^2 R_S.$$

Our analysis has been based on instantaneous values of voltage and current. However, the resulting equations are not limited to instantaneous values; they hold also for maximum and rms values.

EXAMPLE 15.6

A Step-Down Transformer

A transformer on a utility pole steps the rms voltage down from 12 kV to 240 V. (a) What is the ratio of the number of secondary turns to the number of primary turns? (b) If the input current to the transformer is 2.0 A, what is the output current? (c) Determine the power loss in the transmission line.

Strategy

The number of turns related to the voltages is found from Equation 15.20. The output current is calculated using Equation 15.22.

Solution

a. Using Equation 15.20 with rms values V_P and V_S, we have

$$\frac{N_S}{N_P} = \frac{240\text{ V}}{12 \times 10^3\text{ V}} = \frac{1}{50},$$

so the primary winding has 50 times the number of turns in the secondary winding.

b. From Equation 15.22, the output rms current I_S is found using the transformer equation with current

$$I_S = \frac{N_P}{N_S} I_P \qquad \text{15.23}$$

such that

$$I_S = \frac{N_P}{N_S} I_P = (50)(2.0\text{ A}) = 100\text{ A}.$$

c. The power loss in the transmission line is calculated to be

$$P_{\text{loss}} = I_P^2 R = (2.0\text{ A})^2 (6000\ \Omega) = 24.000\text{ W}.$$

d. If there were no transformer, the power would have to be sent at 240 V to work for these houses, and the power loss would be

$$P_{\text{loss}} = I_S^2 R = (100\text{ A})^2 (200\ \Omega) = 2 \times 10^6\text{ W}.$$

Therefore, when power needs to be transmitted, we want to avoid power loss. Thus, lines are sent with high voltages and low currents and adjusted with a transformer before power is sent into homes.

Significance

This application of a step-down transformer allows a home that uses 240-V outlets to have 100 A available to draw upon. This can power many devices in the home.

CHECK YOUR UNDERSTANDING 15.9

A transformer steps the line voltage down from 110 to 9.0 V so that a current of 0.50 A can be delivered to a doorbell. (a) What is the ratio of the number of turns in the primary and secondary windings? (b) What is the current in the primary winding? (c) What is the resistance seen by the 110-V source?

CHAPTER REVIEW

Key Terms

ac current current that fluctuates sinusoidally with time at a fixed frequency

ac voltage voltage that fluctuates sinusoidally with time at a fixed frequency

alternating current (ac) flow of electric charge that periodically reverses direction

average power time average of the instantaneous power over one cycle

bandwidth range of angular frequencies over which the average power is greater than one-half the maximum value of the average power

capacitive reactance opposition of a capacitor to a change in current

direct current (dc) flow of electric charge in only one direction

impedance ac analog to resistance in a dc circuit, which measures the combined effect of resistance, capacitive reactance, and inductive reactance

inductive reactance opposition of an inductor to a change in current

phase angle amount by which the voltage and current are out of phase with each other in a circuit

power factor amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase

quality factor dimensionless quantity that describes the sharpness of the peak of the bandwidth; a high quality factor is a sharp or narrow resonance peak

resonant frequency frequency at which the amplitude of the current is a maximum and the circuit would oscillate if not driven by a voltage source

rms current root mean square of the current

rms voltage root mean square of the voltage

step-down transformer transformer that decreases voltage and increases current

step-up transformer transformer that increases voltage and decreases current

transformer device that transforms voltages from one value to another using induction

transformer equation equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of turns in their windings

Key Equations

AC voltage	$v = V_0 \sin \omega t$
AC current	$i = I_0 \sin \omega t$
capacitive reactance	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
rms voltage	$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$
rms current	$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$
inductive reactance	$\frac{V_0}{I_0} = \omega L = X_L$
Phase angle of an RLC series circuit	$\phi = \tan^{-1} \frac{X_L - X_C}{R}$
AC version of Ohm’s law	$I_0 = \frac{V_0}{Z}$
Impedance of an RLC series circuit	$Z = \sqrt{R^2 + (X_L - X_C)^2}$

Average power associated with a circuit element	$P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos \phi$
Average power dissipated by a resistor	$P_{\text{ave}} = \frac{1}{2} I_0 V_0 = I_{\text{rms}} V_{\text{rms}} = I_{\text{rms}}^2 R$
Resonant angular frequency of a circuit	$\omega_0 = \sqrt{\frac{1}{LC}}$
Quality factor of a circuit	$Q = \frac{\omega_0}{\Delta\omega}$
Quality factor of a circuit in terms of the circuit parameters	$Q = \frac{\omega_0 L}{R}$
Transformer equation with voltage	$\frac{V_{\text{S}}}{V_{\text{P}}} = \frac{N_{\text{S}}}{N_{\text{P}}}$
Transformer equation with current	$I_{\text{S}} = \frac{N_{\text{P}}}{N_{\text{S}}} I_{\text{P}}$

Summary

15.1 AC Sources

- Direct current (dc) refers to systems in which the source voltage is constant.
- Alternating current (ac) refers to systems in which the source voltage varies periodically, particularly sinusoidally.
- The voltage source of an ac system puts out a voltage that is calculated from the time, the peak voltage, and the angular frequency.
- In a simple circuit, the current is found by dividing the voltage by the resistance. An ac current is calculated using the peak current (determined by dividing the peak voltage by the resistance), the angular frequency, and the time.

15.2 Simple AC Circuits

- For resistors, the current through and the voltage across are in phase.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle. Since a capacitor can stop current when fully charged, it limits current and offers another form of ac resistance, called capacitive reactance, which has units of ohms.
- For inductors in ac circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle.
- The opposition of an inductor to a change in current is expressed as a type of ac reactance. This inductive reactance, which has units of ohms, varies with the frequency of the ac source.

15.3 RLC Series Circuits with AC

- An *RLC* series circuit is a resistor, capacitor, and inductor series combination across an ac source.
- The same current flows through each element of an *RLC* series circuit at all points in time.
- The counterpart of resistance in a dc circuit is impedance, which measures the combined effect of resistors, capacitors, and inductors. The maximum current is defined by the ac version of Ohm's law.
- Impedance has units of ohms and is found using the resistance, the capacitive reactance, and the inductive reactance.

15.4 Power in an AC Circuit

- The average ac power is found by multiplying the rms values of current and voltage.
- Ohm's law for the rms ac is found by dividing the rms voltage by the impedance.
- In an ac circuit, there is a phase angle between the source voltage and the current, which can be found by dividing the resistance by the impedance.
- The average power delivered to an *RLC* circuit is affected by the phase angle.
- The power factor ranges from –1 to 1.

15.5 Resonance in an AC Circuit

- At the resonant frequency, inductive reactance equals capacitive reactance.
- The average power versus angular frequency plot for a *RLC* circuit has a peak located at the resonant frequency; the sharpness or width of the peak is known as the bandwidth.
- The bandwidth is related to a dimensionless quantity called the quality factor. A high quality factor value is a sharp or narrow peak.

15.6 Transformers

- Power plants transmit high voltages at low currents to achieve lower ohmic losses in their many kilometers of transmission lines.
- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils, or windings, are related by the transformer equation.
- The currents in the primary and secondary windings are related by the number of primary and secondary loops, or turns, in the windings of the transformer.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

Conceptual Questions

15.1 AC Sources

1. What is the relationship between frequency and angular frequency?

15.2 Simple AC Circuits

2. Explain why at high frequencies a capacitor acts as an ac short, whereas an inductor acts as an open circuit.

15.3 RLC Series Circuits with AC

3. In an *RLC* series circuit, can the voltage measured across the capacitor be greater than the voltage of the source? Answer the same question for the voltage across the inductor.

15.4 Power in an AC Circuit

4. For what value of the phase angle ϕ between the voltage output of an ac source and the current is the average power output of the source a maximum?

5. Discuss the differences between average power and instantaneous power.

6. The average ac current delivered to a circuit is zero. Despite this, power is dissipated in the circuit. Explain.

7. Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?

8. The power rating of a resistor used in ac circuits refers to the maximum average power dissipated in the resistor. How does this compare with the maximum instantaneous power dissipated in the resistor?

15.6 Transformers

9. Why do transmission lines operate at very high voltages while household circuits operate at fairly small voltages?

10. How can you distinguish the primary winding from the secondary winding in a step-up transformer?

11. Battery packs in some electronic devices are charged using an adapter connected to a wall socket. Speculate as to the purpose of the adapter.

12. Will a transformer work if the input is a dc voltage?

13. Why are the primary and secondary coils of a transformer wrapped around the same closed loop of iron?

Problems

15.1 AC Sources

14. Write an expression for the output voltage of an ac source that has an amplitude of 12 V and a frequency of 200 Hz.

15.2 Simple AC Circuits

15. Calculate the reactance of a $5.0\text{-}\mu\text{F}$ capacitor at (a) 60 Hz, (b) 600 Hz, and (c) 6000 Hz.

16. What is the capacitance of a capacitor whose reactance is $10\,\Omega$ at 60 Hz?

17. Calculate the reactance of a 5.0-mH inductor at

(a) 60 Hz, (b) 600 Hz, and (c) 6000 Hz.

18. What is the self-inductance of a coil whose reactance is $10\ \Omega$ at 60 Hz?

19. At what frequency is the reactance of a $20\text{-}\mu\text{F}$ capacitor equal to that of a 10-mH inductor?

20. At 1000 Hz, the reactance of a 5.0-mH inductor is equal to the reactance of a particular capacitor. What is the capacitance of the capacitor?

21. A $50\text{-}\Omega$ resistor is connected across the emf $v(t) = (160\ \text{V})\ \sin\ (120\pi t)$. Write an expression for the current through the resistor.

22. A $25\text{-}\mu\text{F}$ capacitor is connected to an emf given by $v(t) = (160\ \text{V})\ \sin\ (120\pi t)$. (a) What is the reactance of the capacitor? (b) Write an expression for the current output of the source.

23. A 100-mH inductor is connected across the emf of the preceding problem. (a) What is the reactance of the inductor? (b) Write an expression for the current through the inductor.

15.3 RLC Series Circuits with AC

24. What is the impedance of a series combination of a $50\text{-}\Omega$ resistor, a $5.0\text{-}\mu\text{F}$ capacitor, and a $10\text{-}\mu\text{F}$ capacitor at a frequency of 2.0 kHz?

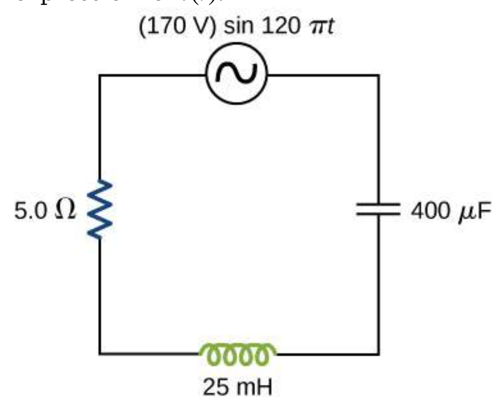
25. A resistor and capacitor are connected in series across an ac generator. The emf of the generator is given by $v(t) = V_0 \cos \omega t$, where $V_0 = 120\ \text{V}$, $\omega = 120\pi\ \text{rad/s}$, $R = 400\ \Omega$, and $C = 4.0\mu\text{F}$. (a) What is the impedance of the circuit? (b) What is the amplitude of the current through the resistor? (c) Write an expression for the current through the resistor. (d) Write expressions representing the voltages across the resistor and across the capacitor.

26. A resistor and inductor are connected in series across an ac generator. The emf of the generator is given by $v(t) = V_0 \cos \omega t$, where $V_0 = 120\ \text{V}$ and $\omega = 120\pi\ \text{rad/s}$; also, $R = 400\ \Omega$ and $L = 1.5\ \text{H}$. (a) What is the impedance of the circuit? (b) What is the amplitude of the current through the resistor? (c) Write an expression for the current through the resistor. (d) Write expressions representing the voltages across the resistor and across the inductor.

27. In an *RLC* series circuit, the voltage amplitude and frequency of the source are 100 V and 500 Hz, respectively, an $R = 500\ \Omega$, $L = 0.20\ \text{H}$, and $C = 2.0\mu\text{F}$. (a) What is the impedance of the circuit? (b) What is the amplitude of the current from the source? (c) If the emf of the source is given by $v(t) = (100\ \text{V})\ \sin 1000\pi t$, how does the current vary with time? (d) Repeat the calculations with *C* changed to $0.20\mu\text{F}$.

28. An *RLC* series circuit with $R = 600\ \Omega$, $L = 30\ \text{mH}$, and $C = 0.050\mu\text{F}$ is driven by an ac source whose frequency and voltage amplitude are 500 Hz and 50 V, respectively. (a) What is the impedance of the circuit? (b) What is the amplitude of the current in the circuit? (c) What is the phase angle between the emf of the source and the current?

29. For the circuit shown below, what are (a) the total impedance and (b) the phase angle between the current and the emf? (c) Write an expression for $i(t)$.



15.4 Power in an AC Circuit

30. The emf of an ac source is given by $v(t) = V_0 \sin \omega t$, where $V_0 = 100\ \text{V}$ and $\omega = 200\pi\ \text{rad/s}$. Calculate the average power output of the source if it is connected across (a) a $20\text{-}\mu\text{F}$ capacitor, (b) a 20-mH inductor, and (c) a $50\text{-}\Omega$ resistor.

31. Calculate the rms currents for an ac source is given by $v(t) = V_0 \sin \omega t$, where $V_0 = 100\ \text{V}$ and $\omega = 200\pi\ \text{rad/s}$ when connected across (a) a $20\text{-}\mu\text{F}$ capacitor, (b) a 20-mH inductor, and (c) a $50\text{-}\Omega$ resistor.

32. A 40-mH inductor is connected to a 60-Hz AC source whose voltage amplitude is 50 V. If an AC voltmeter is placed across the inductor, what does it read?

33. For an *RLC* series circuit, the voltage amplitude and frequency of the source are 100 V and 500 Hz, respectively; $R = 500\ \Omega$; and $L = 0.20\ \text{H}$. Find the average power dissipated in the resistor for the following values for the capacitance: (a) $C = 2.0\mu\text{F}$ and (b) $C = 0.20\ \mu\text{F}$.

34. An ac source of voltage amplitude 10 V delivers electric energy at a rate of 0.80 W when its current output is 2.5 A. What is the phase angle ϕ between the emf and the current?

35. An *RLC* series circuit has an impedance of $60\ \Omega$

and a power factor of 0.50, with the voltage lagging the current. (a) Should a capacitor or an inductor be placed in series with the elements to raise the power factor of the circuit? (b) What is the value of the reactance across the inductor that will raise the power factor to unity?

15.5 Resonance in an AC Circuit

36. (a) Calculate the resonant angular frequency of an *RLC* series circuit for which $R = 20\,\Omega$, $L = 75$ mH, and $C = 4.0\mu\text{F}$. (b) If R is changed to $300\,\Omega$, what happens to the resonant angular frequency?

37. The resonant frequency of an *RLC* series circuit is 2.0×10^3 Hz. If the self-inductance in the circuit is 5.0 mH, what is the capacitance in the circuit?

38. (a) What is the resonant frequency of an *RLC* series circuit with $R = 20\,\Omega$, $L = 2.0$ mH, and $C = 4.0\mu\text{F}$? (b) What is the impedance of the circuit at resonance?

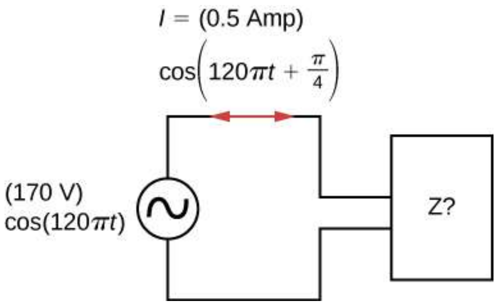
39. For an *RLC* series circuit, $R = 100\,\Omega$, $L = 150$ mH, and $C = 0.25\mu\text{F}$. (a) If an ac source of variable frequency is connected to the circuit, at what frequency is maximum power dissipated in the resistor? (b) What is the quality factor of the circuit?

40. An ac source of voltage amplitude 100 V and variable frequency *f* drives an *RLC* series circuit with $R = 10\,\Omega$, $L = 2.0$ mH, and $C = 25\mu\text{F}$. (a) Plot the current through the resistor as a function of the frequency *f*. (b) Use the plot to determine the resonant frequency of the circuit.

41. (a) What is the resonant frequency of a resistor, capacitor, and inductor connected in series if $R = 100\,\Omega$, $L = 2.0$ H, and $C = 5.0\mu\text{F}$? (b) If this combination is connected to a 100-V source operating at the resonant frequency, what is the power output of the source? (c) What is the *Q* of the circuit? (d) What is the bandwidth of the circuit?

42. Suppose a coil has a self-inductance of 20.0 H and a resistance of $200\,\Omega$. What (a) capacitance and (b) resistance must be connected in series with the coil to produce a circuit that has a resonant frequency of 100 Hz and a *Q* of 10?

43. An ac generator is connected to a device whose internal circuits are not known. We only know current and voltage outside the device, as shown below. Based on the information given, what can you infer about the electrical nature of the device and its power usage?



15.6 Transformers

44. A step-up transformer is designed so that the output of its secondary winding is 2000 V (rms) when the primary winding is connected to a 110-V (rms) line voltage. (a) If there are 100 turns in the primary winding, how many turns are there in the secondary winding? (b) If a resistor connected across the secondary winding draws an rms current of 0.75 A, what is the current in the primary winding?

45. A step-up transformer connected to a 110-V line is used to supply a hydrogen-gas discharge tube with 5.0 kV (rms). The tube dissipates 75 W of power. (a) What is the ratio of the number of turns in the secondary winding to the number of turns in the primary winding? (b) What are the rms currents in the primary and secondary windings? (c) What is the effective resistance seen by the 110-V source?

46. An ac source of emf delivers 5.0 mW of power at an rms current of 2.0 mA when it is connected to the primary coil of a transformer. The rms voltage across the secondary coil is 20 V. (a) What are the voltage across the primary coil and the current through the secondary coil? (b) What is the ratio of secondary to primary turns for the transformer?

47. A transformer is used to step down 110 V from a wall socket to 9.0 V for a radio. (a) If the primary winding has 500 turns, how many turns does the secondary winding have? (b) If the radio operates at a current of 500 mA, what is the current through the primary winding?

48. A transformer is used to supply a 12-V model train with power from a 110-V wall plug. The train operates at 50 W of power. (a) What is the rms current in the secondary coil of the transformer? (b) What is the rms current in the primary coil? (c) What is the ratio of the number of primary to secondary turns? (d) What is the resistance of the train? (e) What is the resistance seen by the 110-V source?

Additional Problems

49. The emf of an ac source is given by $v(t) = V_0 \sin \omega t$, where $V_0 = 100 \text{ V}$ and $\omega = 200\pi$ rad/s. Find an expression that represents the output current of the source if it is connected across (a) a 20-μF capacitor, (b) a 20-mH inductor, and (c) a 50-Ω resistor.

50. A 700-pF capacitor is connected across an ac source with a voltage amplitude of 160 V and a frequency of 20 kHz. (a) Determine the capacitive reactance of the capacitor and the amplitude of the output current of the source. (b) If the frequency is changed to 60 Hz while keeping the voltage amplitude at 160 V, what are the capacitive reactance and the current amplitude?

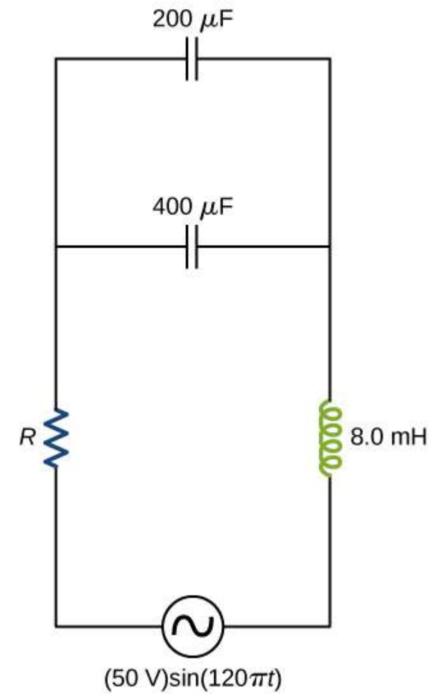
51. A 20-mH inductor is connected across an AC source with a variable frequency and a constant-voltage amplitude of 9.0 V. (a) Determine the reactance of the circuit and the maximum current through the inductor when the frequency is set at 20 kHz. (b) Do the same calculations for a frequency of 60 Hz.

52. A 30-μF capacitor is connected across a 60-Hz ac source whose voltage amplitude is 50 V. (a) What is the maximum charge on the capacitor? (b) What is the maximum current into the capacitor? (c) What is the phase relationship between the capacitor charge and the current in the circuit?

53. A 7.0-mH inductor is connected across a 60-Hz ac source whose voltage amplitude is 50 V. (a) What is the maximum current through the inductor? (b) What is the phase relationship between the current through and the potential difference across the inductor?

54. What is the impedance of an *RLC* series circuit at the resonant frequency?

55. What is the resistance *R* in the circuit shown below if the amplitude of the ac through the inductor is 4.24 A?



56. An ac source of voltage amplitude 100 V and frequency 1.0 kHz drives an *RLC* series circuit with $R = 20 \,\Omega$, $L = 4.0 \text{ mH}$, and $C = 50 \mu\text{F}$. (a) Determine the rms current through the circuit. (b) What are the rms voltages across the three elements? (c) What is the phase angle between the emf and the current? (d) What is the power output of the source? (e) What is the power dissipated in the resistor?

57. In an *RLC* series circuit, $R = 200 \,\Omega$, $L = 1.0 \text{ H}$, $C = 50 \mu\text{F}$, $V_0 = 120 \text{ V}$, and $f = 50 \text{ Hz}$. What is the power output of the source?

58. A power plant generator produces 100 A at 15 kV (rms). A transformer is used to step up the transmission line voltage to 150 kV (rms). (a) What is rms current in the transmission line? (b) If the resistance per unit length of the line is $8.6 \times 10^{-8} \,\Omega/\text{m}$, what is the power loss per meter in the line? (c) What would the power loss per meter be if the line voltage were 15 kV (rms)?

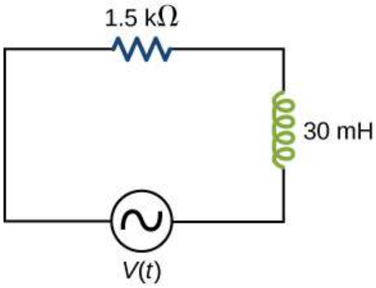
59. Consider a power plant located 25 km outside a town delivering 50 MW of power to the town. The transmission lines are made of aluminum cables with a 7 cm^2 cross-sectional area. Find the loss of power in the transmission lines if it is transmitted at (a) 200 kV (rms) and (b) 120 V (rms).

60. Neon signs require 12-kV for their operation. A transformer is to be used to change the voltage from 220-V (rms) ac to 12-kV (rms) ac. What must the ratio be of turns in the secondary winding to the turns in the primary winding? (b) What is the maximum rms current the neon lamps can draw if the fuse in the primary winding goes off at 0.5 A? (c) How much power is used by the neon sign when it is drawing the maximum current allowed by the fuse in the primary winding?

Challenge Problems

61. The 335-kV ac electricity from a power transmission line is fed into the primary winding of a transformer. The ratio of the number of turns in the secondary winding to the number in the primary winding is $N_s/N_p = 1000$. (a) What voltage is induced in the secondary winding? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

62. A $1.5\text{-k}\Omega$ resistor and 30-mH inductor are connected in series, as shown below, across a 120-V (rms) ac power source oscillating at 60-Hz frequency. (a) Find the current in the circuit. (b) Find the voltage drops across the resistor and inductor. (c) Find the impedance of the circuit. (d) Find the power dissipated in the resistor. (e) Find the power dissipated in the inductor. (f) Find the power produced by the source.

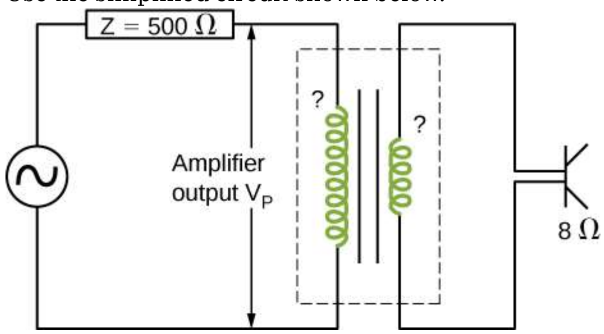


63. A $20\text{-}\Omega$ resistor, $50\text{-}\mu\text{F}$ capacitor, and 30-mH inductor are connected in series with an ac source of amplitude 10 V and frequency 125 Hz. (a) What is the impedance of the circuit? (b) What is the amplitude of the current in the circuit? (c) What is the phase constant of the current? Is it leading or lagging the source voltage? (d) Write voltage drops across the resistor, capacitor, and inductor and the source voltage as a function of time. (e) What is the power factor of the circuit? (f) How much energy is used by the resistor in 2.5 s?

64. A $200\text{-}\Omega$ resistor, $150\text{-}\mu\text{F}$ capacitor, and 2.5-H inductor are connected in series with an ac source of amplitude 10 V and variable angular frequency ω. (a) What is the value of the resonance frequency ω_R? (b) What is the amplitude of the current if $\omega = \omega_R$? (c) What is the phase constant of the current when $\omega = \omega_R$? Is it leading or lagging the source voltage, or is it in phase? (d) Write an equation for the voltage drop across the resistor as a function of time when $\omega = \omega_R$. (e) What is the power factor of the circuit when $\omega = \omega_R$? (f) How much energy is used up by the resistor in 2.5 s when $\omega = \omega_R$?

65. Find the reactances of the following capacitors and inductors in ac circuits with the given frequencies in each case: (a) 2-mH inductor with a frequency 60-Hz of the ac circuit; (b) 2-mH inductor with a frequency 600-Hz of the ac circuit; (c) 20-mH inductor with a frequency 6-Hz of the ac circuit; (d) 20-mH inductor with a frequency 60-Hz of the ac circuit; (e) 2-mF capacitor with a frequency 60-Hz of the ac circuit; and (f) 2-mF capacitor with a frequency 600-Hz of the AC circuit.

66. An output impedance of an audio amplifier has an impedance of $500\ \Omega$ and has a mismatch with a low-impedance $8\text{-}\Omega$ loudspeaker. You are asked to insert an appropriate transformer to match the impedances. What turns ratio will you use, and why? Use the simplified circuit shown below.



67. Show that the SI unit for capacitive reactance is the ohm. Show that the SI unit for inductive reactance is also the ohm.

68. A coil with a self-inductance of 16 mH and a resistance of $6.0\ \Omega$ is connected to an ac source whose frequency can be varied. At what frequency will the voltage across the coil lead the current through the coil by $45°$?

69. An *RLC* series circuit consists of a $50\text{-}\Omega$ resistor, a $200\text{-}\mu\text{F}$ capacitor, and a 120-mH inductor whose coil has a resistance of $20\ \Omega$. The source for the circuit has an rms emf of 240 V at a frequency of 60 Hz. Calculate the rms voltages across the (a) resistor, (b) capacitor, and (c) inductor.

70. An *RLC* series circuit consists of a $10\text{-}\Omega$ resistor, an $8.0\text{-}\mu\text{F}$ capacitor, and a 50-mH inductor. A 110-V (rms) source of variable frequency is connected across the combination. What is the power output of the source when its frequency is set to one-half the resonant frequency of the circuit?

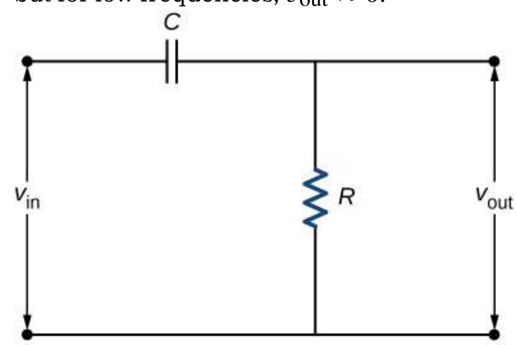
71. Shown below are two circuits that act as crude high-pass filters. The input voltage to the circuits is v_{in}, and the output voltage is v_{out}. (a) Show that for the capacitor circuit,

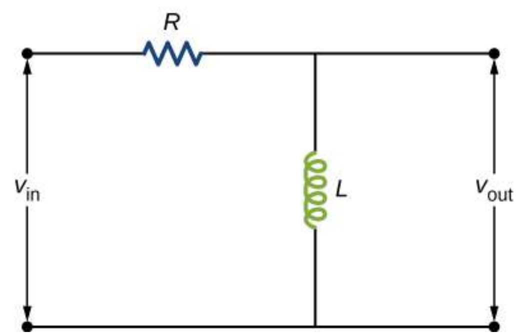
$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{1}{\sqrt{1+1/\omega^2 R^2 C^2}},$$

and for the inductor circuit,

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}}.$$

(b) Show that for high frequencies, $v_{\text{out}} \approx v_{\text{in}}$, but for low frequencies, $v_{\text{out}} \approx 0$.





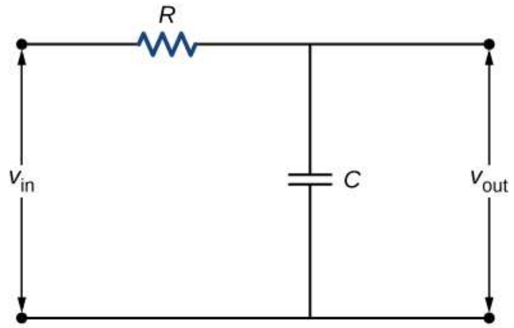
72. The two circuits shown below act as crude low-pass filters. The input voltage to the circuits is v_{in}, and the output voltage is v_{out}. (a) Show that for the capacitor circuit,

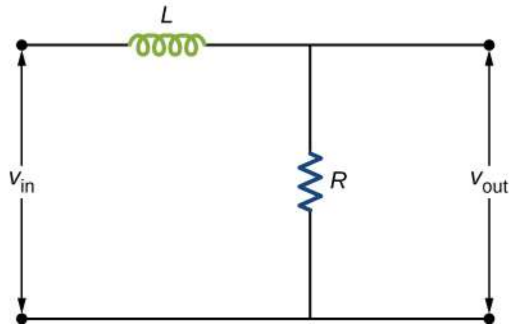
$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}},$$

and for the inductor circuit,

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{R}{\sqrt{R^2+\omega^2 L^2}}.$$

(b) Show that for low frequencies, $v_{\text{out}} \approx v_{\text{in}}$, but for high frequencies, $v_{\text{out}} \approx 0$.





CHAPTER 16
Electromagnetic Waves

Figure 16.1 The pressure from sunlight predicted by Maxwell's equations helped produce the tail of Comet McNaught. (credit: modification of work by Sebastian Deiries—ESO)

Chapter Outline

INTRODUCTION Our view of objects in the sky at night, the warm radiance of sunshine, the sting of sunburn, our cell phone conversations, and the X-rays revealing a broken bone—all are brought to us by electromagnetic waves. It would be hard to overstate the practical importance of electromagnetic waves, through their role in vision, through countless technological applications, and through their ability to transport the energy from the Sun through space to sustain life and almost all of its activities on Earth.

Theory predicted the general phenomenon of electromagnetic waves before anyone realized that light is a form of an electromagnetic wave. In the mid-nineteenth century, James Clerk Maxwell formulated a single theory combining all the electric and magnetic effects known at that time. Maxwell's equations, summarizing this theory, predicted the existence of electromagnetic waves that travel at the speed of light. His theory also predicted how these waves behave, and how they carry both energy and momentum. The tails of comets, such as Comet McNaught in Figure 16.1, provide a spectacular example. Energy carried by light from the Sun warms the comet to release dust and gas. The momentum carried by the light exerts a weak force that shapes the dust into a tail of the kind seen here. The flux of particles emitted by the Sun, called the solar wind, typically produces an additional, second tail, as described in detail in this chapter.

In this chapter, we explain Maxwell's theory and show how it leads to his prediction of electromagnetic waves. We use his theory to examine what electromagnetic waves are, how they are produced, and how they transport energy and momentum. We conclude by summarizing some of the many practical applications of electromagnetic waves.

16.1 Maxwell's Equations and Electromagnetic Waves

Learning Objectives

By the end of this section, you will be able to:

- Explain Maxwell's correction of Ampère's law by including the displacement current
- State and apply Maxwell's equations in integral form
- Describe how the symmetry between changing electric and changing magnetic fields explains Maxwell's prediction of electromagnetic waves
- Describe how Hertz confirmed Maxwell's prediction of electromagnetic waves

James Clerk Maxwell (1831–1879) was one of the major contributors to physics in the nineteenth century (Figure 16.2). Although he died young, he made major contributions to the development of the kinetic theory of gases, to the understanding of color vision, and to the nature of Saturn's rings. He is probably best known for having combined existing knowledge of the laws of electricity and of magnetism with insights of his own into a complete overarching electromagnetic theory, represented by **Maxwell's equations**.

Figure 16.2 James Clerk Maxwell, a nineteenth-century physicist, developed a theory that explained the relationship between electricity and magnetism, and correctly predicted that visible light consists of electromagnetic waves.

Maxwell's Correction to the Laws of Electricity and Magnetism

The four basic laws of electricity and magnetism had been discovered experimentally through the work of physicists such as Oersted, Coulomb, Gauss, and Faraday. Maxwell discovered logical inconsistencies in these earlier results and identified the incompleteness of Ampère's law as their cause.

Recall that according to Ampère's law, the integral of the magnetic field around a closed loop C is proportional to the current I passing through any surface whose boundary is loop C itself:

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I. \tag{16.1}$$

There are infinitely many surfaces that can be attached to any loop, and Ampère's law stated in Equation 16.1 is independent of the choice of surface.

Consider the set-up in Figure 16.3. A source of emf is abruptly connected across a parallel-plate capacitor so that a time-dependent current I develops in the wire. Suppose we apply Ampère's law to loop C shown at a time before the capacitor is fully charged, so that $I \neq 0$. Surface S_1 gives a nonzero value for the enclosed current I, whereas surface S_2 gives zero for the enclosed current because no current passes through it:

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \begin{cases} \mu_0 I & \text{if surface } S_1 \text{ is used} \\ 0 & \text{if surface } S_2 \text{ is used} \end{cases}.$$

Clearly, Ampère's law in its usual form does not work here. This is an internal contradiction in the theory which requires a modification to the theory, Ampère's law, itself.

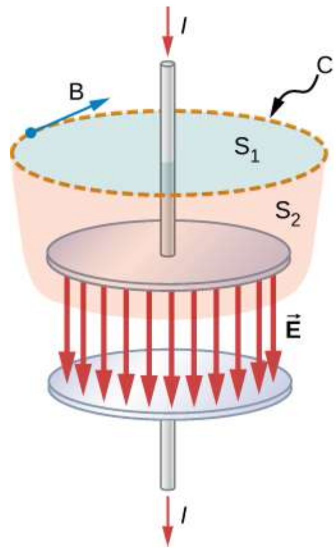


Figure 16.3 The currents through surface S_1 and surface S_2 are unequal, despite having the same boundary loop C.

How can Ampère's law be modified so that it works in all situations? Maxwell suggested including an additional contribution, called the displacement current I_d, to the real current I,

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I + I_d) \tag{16.2}$$

where the displacement current is defined to be

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}. \tag{16.3}$$

Here ε_0 is the permittivity of free space and Φ_E is the electric flux, defined as

$$\Phi_E = \iint_{\text{Surface } S} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

The **displacement current** is analogous to a real current in Ampère's law, entering into Ampère's law in the same way. It is produced, however, by a changing electric field. It accounts for a changing electric field producing a magnetic field, just as a real current does, but the displacement current can produce a magnetic field even where no real current is present. When this extra term is included, the modified Ampère's law equation becomes

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \tag{16.4}$$

and is independent of the surface S through which the current I is measured.

We can now examine this modified version of Ampère's law to confirm that it holds independent of whether the surface S_1 or the surface S_2 in Figure 16.3 is chosen. The electric field $\vec{\mathbf{E}}$ corresponding to the flux Φ_E in Equation 16.3 is between the capacitor plates. Therefore, the $\vec{\mathbf{E}}$ field and the displacement current through the surface S_1 are both zero, and Equation 16.2 takes the form

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I. \tag{16.5}$$

We must now show that for surface S_2, through which no actual current flows, the displacement current leads to the same value $\mu_0 I$ for the right side of the Ampère's law equation. For surface S_2, the equation becomes

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \frac{d}{dt}\left[\varepsilon_0 \iint_{\text{Surface } S_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}\right]. \tag{16.6}$$

Gauss's law for electric charge requires a closed surface and cannot ordinarily be applied to a surface like S_1 alone or S_2 alone. But the two surfaces S_1 and S_2 form a closed surface in Figure 16.3 and can be used in Gauss's law. Because the electric field is zero on S_1, the flux contribution through S_1 is zero. This gives us

$$\begin{aligned}\oiint_{\text{Surface } S_1+S_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} &= \iint_{\text{Surface } S_1} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \iint_{\text{Surface } S_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} \\ &= 0 + \iint_{\text{Surface } S_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} \\ &= \iint_{\text{Surface } S_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.\end{aligned}$$

Therefore, we can replace the integral over S_2 in Equation 16.6 with the closed Gaussian surface $S_1 + S_2$ and apply Gauss's law to obtain

$$\oint_{S_1} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \frac{dQ_{\text{in}}}{dt} = \mu_0 I. \tag{16.7}$$

Thus, the modified Ampère's law equation is the same using surface S_2, where the right-hand side results from the displacement current, as it is for the surface S_1, where the contribution comes from the actual flow of electric charge.

EXAMPLE 16.1

Displacement current in a charging capacitor

A parallel-plate capacitor with capacitance C whose plates have area A and separation distance d is connected to a resistor R and a battery of voltage V. The current starts to flow at $t = 0$. (a) Find the displacement current between the capacitor plates at time t. (b) From the properties of the capacitor, find the corresponding real current $I = \frac{dQ}{dt}$, and compare the answer to the expected current in the wires of the corresponding RC circuit.

Strategy

We can use the equations from the analysis of an RC circuit (Alternating-Current Circuits) plus Maxwell's version of Ampère's law.

Solution

a. The voltage between the plates at time t is given by

$$V_C = \frac{1}{C}Q(t) = V_0\left(1 - e^{-t/RC}\right).$$

Let the z-axis point from the positive plate to the negative plate. Then the z-component of the electric field between the plates as a function of time t is

$$E_z(t) = \frac{V_0}{d}\left(1 - e^{-t/RC}\right).$$

Therefore, the z-component of the displacement current I_d between the plates is

$$I_d(t) = \varepsilon_0 A \frac{\partial E_z(t)}{\partial t} = \varepsilon_0 A \frac{V_0}{d} \times \frac{1}{RC} e^{-t/RC} = \frac{V_0}{R} e^{-t/RC},$$

where we have used $C = \varepsilon_0 \frac{A}{d}$ for the capacitance.

b. From the expression for V_C, the charge on the capacitor is

$$Q(t) = CV_C = CV_0\left(1 - e^{-t/RC}\right).$$

The current into the capacitor after the circuit is closed, is therefore

$$I = \frac{dQ}{dt} = \frac{V_0}{R} e^{-t/RC}.$$

This current is the same as I_d found in (a).

Maxwell's Equations

With the correction for the displacement current, Maxwell's equations take the form

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0} \qquad \left(\text{Gauss's law}\right) \tag{16.8}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \qquad \left(\text{Gauss's law for magnetism}\right) \tag{16.9}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_m}{dt} \qquad \left(\text{Faraday's law}\right) \tag{16.10}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt} \qquad \left(\text{Ampère-Maxwell law}\right). \tag{16.11}$$

Once the fields have been calculated using these four equations, the Lorentz force equation

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \tag{16.12}$$

gives the force that the fields exert on a particle with charge q moving with velocity $\vec{\mathbf{v}}$. The Lorentz force equation combines the force of the electric field and of the magnetic field on the moving charge. The magnetic and electric forces have been examined in earlier modules. These four Maxwell's equations are, respectively,

Maxwell's Equations

1. **Gauss's law**

The electric flux through any closed surface is equal to the electric charge Q_{in} enclosed by the surface. Gauss's law [Equation 16.7] describes the relation between an electric charge and the electric field it produces. This is often pictured in terms of electric field lines originating from positive charges and terminating on negative charges, and indicating the direction of the electric field at each point in space.

2. **Gauss's law for magnetism**

The magnetic field flux through any closed surface is zero [Equation 16.8]. This is equivalent to the statement that magnetic field lines are continuous, having no beginning or end. Any magnetic field line entering the region enclosed by the surface must also leave it. No magnetic monopoles, where magnetic field lines would terminate, are known to exist (see Magnetic Fields and Lines).

3. **Faraday's law**

A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change. This third of Maxwell's equations, Equation 16.9, is Faraday's law of induction and includes Lenz's law. The electric field from a changing magnetic field has field lines that form closed loops, without any beginning or end.

4. **Ampère-Maxwell law**

Magnetic fields are generated by moving charges or by changing electric fields. This fourth of Maxwell's equations, Equation 16.10, encompasses Ampère's law and adds another source of magnetic fields, namely changing electric fields.

Maxwell's equations and the Lorentz force law together encompass all the laws of electricity and magnetism. The symmetry that Maxwell introduced into his mathematical framework may not be immediately apparent. Faraday's law describes how changing magnetic fields produce electric fields. The displacement current introduced by Maxwell results instead from a changing electric field and accounts for a changing electric field producing a magnetic field. The equations for the effects of both changing electric fields and changing magnetic fields differ in form only where the absence of magnetic monopoles leads to missing terms. This symmetry between the effects of changing magnetic and electric fields is essential in explaining the nature of electromagnetic waves.

Later application of Einstein's theory of relativity to Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate but are different manifestations of the same thing—the electromagnetic force. The electromagnetic force and weak nuclear force are similarly unified as the electroweak force. This unification of forces has been one motivation for attempts to unify all of the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces (see Particle Physics and Cosmology).

The Mechanism of Electromagnetic Wave Propagation

To see how the symmetry introduced by Maxwell accounts for the existence of combined electric and magnetic waves that propagate through space, imagine a time-varying magnetic field $\vec{\mathbf{B}}_0(t)$ produced by the high-frequency alternating current seen in Figure 16.4. We represent $\vec{\mathbf{B}}_0(t)$ in the diagram by one of its field lines. From Faraday's law, the changing magnetic field through a surface induces a time-varying electric field $\vec{\mathbf{E}}_0(t)$ at the boundary of that surface. The displacement current source for the electric field, like the Faraday's law source for the magnetic field, produces only closed loops of field lines, because of the mathematical symmetry involved in the equations for the induced electric and induced magnetic fields. A field line representation of $\vec{\mathbf{E}}_0(t)$ is shown. In turn, the changing electric field $\vec{\mathbf{E}}_0(t)$ creates a magnetic field $\vec{\mathbf{B}}_1(t)$ according to the modified Ampère's law. This changing field induces $\vec{\mathbf{E}}_1(t)$, which induces $\vec{\mathbf{B}}_2(t)$, and so on. We then have a self-continuing process that leads to the creation of time-varying electric and magnetic fields in regions farther and farther away from *O*. This process may be visualized as the propagation of an electromagnetic wave through space.

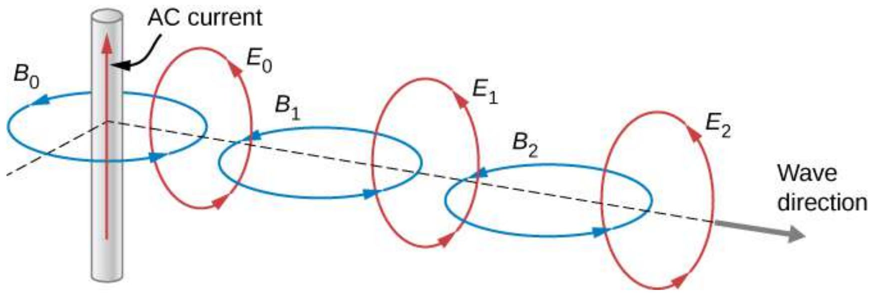


Figure 16.4 How changing $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields propagate through space.

In the next section, we show in more precise mathematical terms how Maxwell's equations lead to the prediction of electromagnetic waves that can travel through space without a material medium, implying a speed of electromagnetic waves equal to the speed of light.

Prior to Maxwell's work, experiments had already indicated that light was a wave phenomenon, although the nature of the waves was yet unknown. In 1801, Thomas Young (1773–1829) showed that when a light beam was separated by two narrow slits and then recombined, a pattern made up of bright and dark fringes was formed on a screen. Young explained this behavior by assuming that light was composed of waves that added constructively at some points and destructively at others (see Interference). Subsequently, Jean Foucault (1819–1868), with measurements of the speed of light in various media, and Augustin Fresnel (1788–1827), with detailed experiments involving interference and diffraction of light, provided further conclusive evidence that light was a wave. So, light was known to be a wave, and Maxwell had predicted the existence of electromagnetic waves that traveled at the speed of light. The conclusion seemed inescapable: Light must be a form of electromagnetic radiation. But Maxwell's theory showed that other wavelengths and frequencies than those of light were possible for electromagnetic waves. He showed that electromagnetic radiation with the same fundamental properties as visible light should exist at any frequency. It remained for others to test, and confirm, this prediction.

CHECK YOUR UNDERSTANDING 16.1

When the emf across a capacitor is turned on and the capacitor is allowed to charge, when does the magnetic field induced by the displacement current have the greatest magnitude?

Hertz's Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves but also verified that they travel at the speed of light.

Hertz used an alternating-current *RLC* (resistor-inductor-capacitor) circuit that resonates at a known frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$ and connected it to a loop of wire, as shown in Figure 16.5. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and helped generate electromagnetic waves.

Across the laboratory, Hertz placed another loop attached to another *RLC* circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could thus be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.

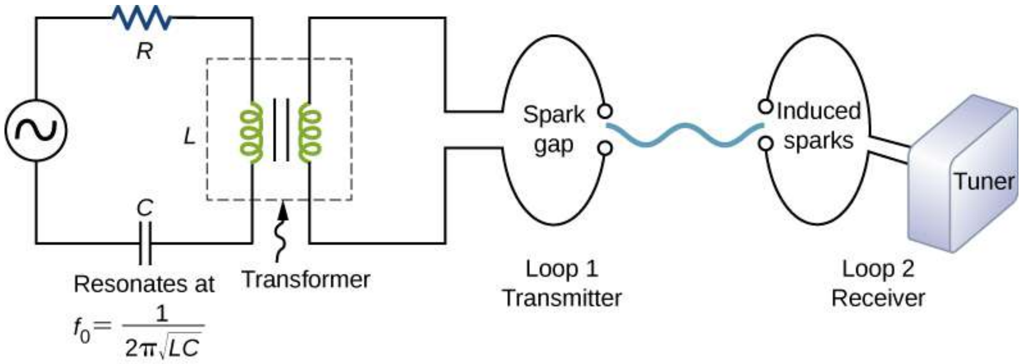


Figure 16.5 The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, confirming their wave character. He was able to determine the wavelengths from the interference patterns, and knowing their frequencies, he could calculate the propagation speed using the equation $v = f\lambda$, where v is the speed of a wave, f is its frequency, and λ is its wavelength. Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz ($1\ \text{Hz} = 1\ \text{cycle/s}$), is named in his honor.

CHECK YOUR UNDERSTANDING 16.2

Could a purely electric field propagate as a wave through a vacuum without a magnetic field? Justify your answer.

16.2 Plane Electromagnetic Waves

Learning Objectives

By the end of this section, you will be able to:

- Describe how Maxwell's equations predict the relative directions of the electric fields and magnetic fields, and the direction of propagation of plane electromagnetic waves
- Explain how Maxwell's equations predict that the speed of propagation of electromagnetic waves in free space is exactly the speed of light
- Calculate the relative magnitude of the electric and magnetic fields in an electromagnetic plane wave
- Describe how electromagnetic waves are produced and detected

Mechanical waves travel through a medium such as a string, water, or air. Perhaps the most significant prediction of Maxwell's equations is the existence of combined electric and magnetic (or electromagnetic) fields that propagate through space as electromagnetic waves. Because Maxwell's equations hold in free space, the predicted electromagnetic waves, unlike mechanical waves, do not require a medium for their propagation.

A general treatment of the physics of electromagnetic waves is beyond the scope of this textbook. We can, however, investigate the special case of an electromagnetic wave that propagates through free space along the x-axis of a given coordinate system.

Electromagnetic Waves in One Direction

An electromagnetic wave consists of an electric field, defined as usual in terms of the force per charge on a stationary charge, and a magnetic field, defined in terms of the force per charge on a moving charge. The electromagnetic field is assumed to be a function of only the x-coordinate and time. The y-component of the electric field is then written as $E_y(x, t)$, the z-component of the magnetic field as $B_z(x, t)$, etc. Because we are assuming free space, there are no free charges or currents, so we can set $Q_{\text{in}} = 0$ and $I = 0$ in Maxwell's equations.

The transverse nature of electromagnetic waves

We examine first what Gauss's law for electric fields implies about the relative directions of the electric field and the propagation direction in an electromagnetic wave. Assume the Gaussian surface to be the surface of a rectangular box whose cross-section is a square of side l and whose third side has length Δx, as shown in Figure 16.6. Because the electric field is a function only of x and t, the y-component of the electric field is the same on both the top (labeled Side 2) and bottom (labeled Side 1) of the box, so that these two contributions to the flux cancel. The corresponding argument also holds for the net flux from the z-component of the electric field through Sides 3 and 4. Any net flux through the surface therefore comes entirely from the x-component of the electric field. Because the electric field has no y- or z-dependence, $E_x\,(x, t)$ is constant over the face of the box with area A and has a possibly different value $E_x\,(x + \Delta x, t)$ that is constant over the opposite face of the box. Applying Gauss's law gives

$$\text{Net flux} = -E_x\,(x, t)\,A + E_x\,(x + \Delta x, t)\,A = \frac{Q_{\text{in}}}{\varepsilon_0} \tag{16.13}$$

where $A = l \times l$ is the area of the front and back faces of the rectangular surface. But the charge enclosed is $Q_{\text{in}} = 0$, so this component's net flux is also zero, and Equation 16.13 implies $E_x\,(x, t) = E_x\,(x + \Delta x, t)$ for any Δx. Therefore, if there is an x-component of the electric field, it cannot vary with x. A uniform field of that kind would merely be superposed artificially on the traveling wave, for example, by having a pair of parallel-charged plates. Such a component $E_x(x, t)$ would not be part of an electromagnetic wave propagating along the x-axis; so $E_x(x, t) = 0$ for this wave. Therefore, the only nonzero components of the electric field are $E_y(x, t)$ and $E_z(x, t)$, perpendicular to the direction of propagation of the wave.

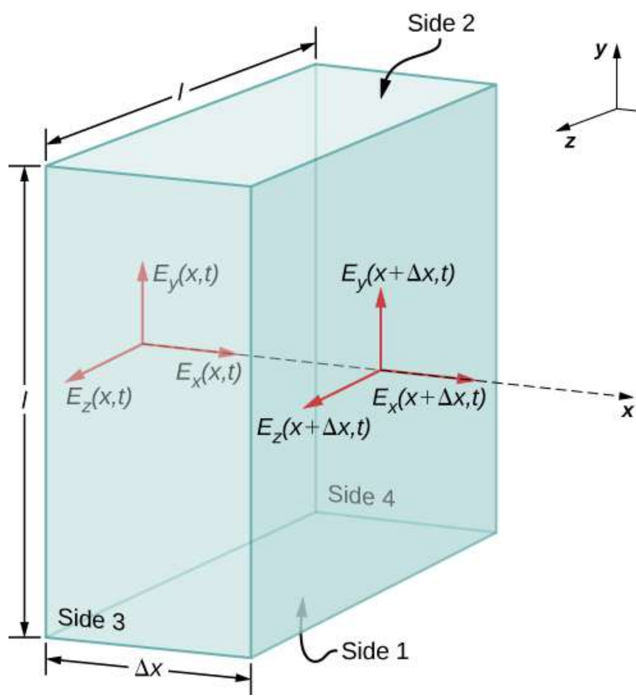


Figure 16.6 The surface of a rectangular box of dimensions $l \times l \times \Delta x$ is our Gaussian surface. The electric field shown is from an electromagnetic wave propagating along the x-axis.

A similar argument holds by substituting E for B and using Gauss's law for magnetism instead of Gauss's law for electric fields. This shows that the B field is also perpendicular to the direction of propagation of the wave. The electromagnetic wave is therefore a transverse wave, with its oscillating electric and magnetic fields perpendicular to its direction of propagation.

The speed of propagation of electromagnetic waves

We can next apply Maxwell's equations to the description given in connection with Figure 16.4 in the previous section to obtain an equation for the E field from the changing B field, and for the B field from a changing E field. We then combine the two equations to show how the changing E and B fields propagate through space at a speed precisely equal to the speed of light.

First, we apply Faraday's law over Side 3 of the Gaussian surface, using the path shown in Figure 16.7. Because $E_x(x,t) = 0$, we have

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E_y\,(x,t)\,l + E_y\,(x+\Delta x,t)\,l.$$

Assuming Δx is small and approximating $E_y\,(x+\Delta x,t)$ by

$$E_y\,(x+\Delta x,t) = E_y\,(x,t) + \frac{\partial E_y\,(x,t)}{\partial x}\Delta x,$$

we obtain

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = \frac{\partial E_y\,(x,t)}{\partial x}(l\Delta x)\,.$$

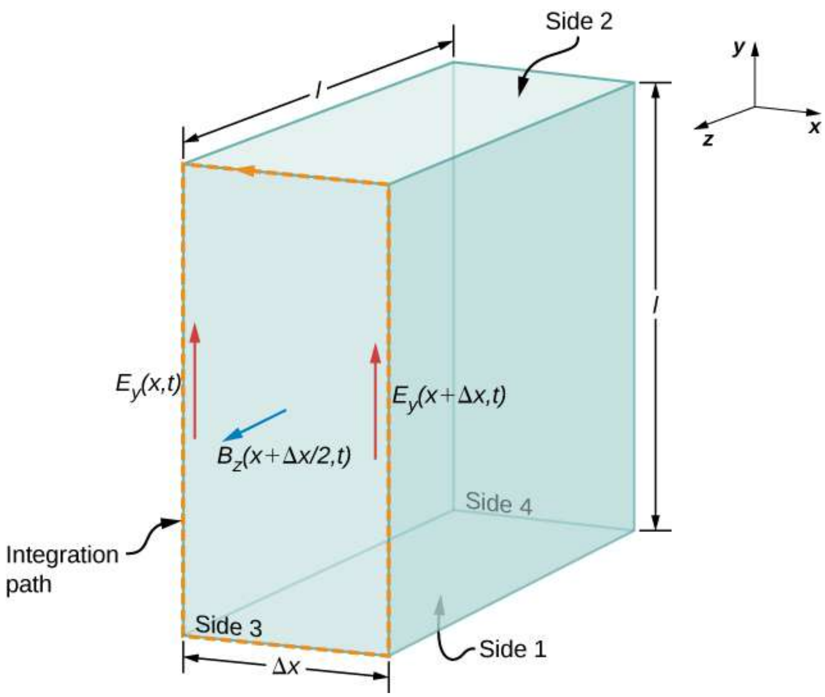


Figure 16.7 We apply Faraday's law to the front of the rectangle by evaluating $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ along the rectangular edge of Side 3 in the direction indicated, taking the *B* field crossing the face to be approximately its value in the middle of the area traversed.

Because Δx is small, the magnetic flux through the face can be approximated by its value in the center of the area traversed, namely $B_z\,\left(x+\frac{\Delta x}{2},t\right)$. The flux of the *B* field through Face 3 is then the *B* field times the area,

$$\oint_S \vec{\mathbf{B}} \cdot \vec{\mathbf{n}} dA = B_z\left(x+\frac{\Delta x}{2},t\right)(l\Delta x)\,. \quad \textbf{16.14}$$

From Faraday's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int_S \vec{\mathbf{B}} \cdot \vec{\mathbf{n}} dA. \quad \textbf{16.15}$$

Therefore, from Equation 16.13 and Equation 16.14,

$$\frac{\partial E_y\,(x,t)}{\partial x}(l\Delta x) = -\frac{\partial}{\partial t}\left[B_z\left(x+\frac{\Delta x}{2},t\right)\right](l\Delta x)\,.$$

Canceling $l\Delta x$ and taking the limit as $\Delta x = 0$, we are left with

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}. \qquad \textbf{16.16}$$

We could have applied Faraday's law instead to the top surface (numbered 2) in Figure 16.7, to obtain the resulting equation

$$\frac{\partial E_z(x,t)}{\partial x} = -\frac{\partial B_y(x,t)}{\partial t}. \qquad \textbf{16.17}$$

This is the equation describing the spatially dependent E field produced by the time-dependent B field.

Next we apply the Ampère-Maxwell law (with $I = 0$) over the same two faces (Surface 3 and then Surface 2) of the rectangular box of Figure 16.7. Applying Equation 16.10,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 (d/dt) \int_S \vec{\mathbf{E}} \cdot \mathbf{n}\, da$$

to Surface 3, and then to Surface 2, yields the two equations

$$\frac{\partial B_y(x,t)}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E_z(x,t)}{\partial t}, \text{ and} \qquad \textbf{16.18}$$

$$\frac{\partial B_z(x,t)}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t}. \qquad \textbf{16.19}$$

These equations describe the spatially dependent B field produced by the time-dependent E field.

We next combine the equations showing the changing B field producing an E field with the equation showing the changing E field producing a B field. Taking the derivative of Equation 16.16 with respect to x and using Equation 16.26 gives

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \frac{\partial}{\partial t}\left(\varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}\right)$$

or

$$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2}. \qquad \textbf{16.20}$$

This is the form taken by the general wave equation for our plane wave. Because the equations describe a wave traveling at some as-yet-unspecified speed c, we can assume the field components are each functions of $x - ct$ for the wave traveling in the $+x$-direction, that is,

$$E_y(x,t) = f(\xi) \qquad \text{where } \xi = x - ct. \qquad \textbf{16.21}$$

It is left as a mathematical exercise to show, using the chain rule for differentiation, that Equation 16.17 and Equation 16.18 imply

$$1 = \varepsilon_0 \mu_0 c^2.$$

The speed of the electromagnetic wave in free space is therefore given in terms of the permeability and the permittivity of free space by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}. \qquad \textbf{16.22}$$

We could just as easily have assumed an electromagnetic wave with field components $E_z(x,t)$ and $B_y(x,t)$. The same type of analysis with Equation 16.25 and Equation 16.24 would also show that the speed of an electromagnetic wave is $c = 1/\sqrt{\varepsilon_0 \mu_0}$.

The physics of traveling electromagnetic fields was worked out by Maxwell in 1873. He showed in a more general way than our derivation that electromagnetic waves always travel in free space with a speed given by

Equation 16.18. If we evaluate the speed $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, we find that

$$c = \frac{1}{\sqrt{\left(8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}\right)\left(4\pi \times 10^{-7} \frac{\mathrm{T \cdot m}}{\mathrm{A}}\right)}} = 3.00 \times 10^8 \text{ m/s},$$

which is the speed of light. Imagine the excitement that Maxwell must have felt when he discovered this equation! He had found a fundamental connection between two seemingly unrelated phenomena: electromagnetic fields and light.

CHECK YOUR UNDERSTANDING 16.3

The wave equation was obtained by (1) finding the E field produced by the changing B field, (2) finding the B field produced by the changing E field, and combining the two results. Which of Maxwell's equations was the basis of step (1) and which of step (2)?

How the *E* and *B* Fields Are Related

So far, we have seen that the rates of change of different components of the E and B fields are related, that the electromagnetic wave is transverse, and that the wave propagates at speed c. We next show what Maxwell's equations imply about the ratio of the E and B field magnitudes and the relative directions of the E and B fields.

We now consider solutions to Equation 16.16 in the form of plane waves for the electric field:

$$E_y(x, t) = E_0 \cos(kx - \omega t). \tag{16.23}$$

We have arbitrarily taken the wave to be traveling in the $+x$-direction and chosen its phase so that the maximum field strength occurs at the origin at time $t = 0$. We are justified in considering only sines and cosines in this way, and generalizing the results, because Fourier's theorem implies we can express any wave, including even square step functions, as a superposition of sines and cosines.

At any one specific point in space, the E field oscillates sinusoidally at angular frequency ω between $+E_0$ and $-E_0$, and similarly, the B field oscillates between $+B_0$ and $-B_0$. The amplitude of the wave is the maximum value of $E_y(x, t)$. The period of oscillation T is the time required for a complete oscillation. The frequency f is the number of complete oscillations per unit of time, and is related to the angular frequency ω by $\omega = 2\pi f$. The wavelength λ is the distance covered by one complete cycle of the wave, and the wavenumber k is the number of wavelengths that fit into a distance of 2π in the units being used. These quantities are related in the same way as for a mechanical wave:

$$\omega = 2\pi f, \quad f = \frac{1}{T}, \quad k = \frac{2\pi}{\lambda}, \quad \text{and} \quad c = f\lambda = \omega/k.$$

Given that the solution of E_y has the form shown in Equation 16.20, we need to determine the B field that accompanies it. From Equation 16.24, the magnetic field component B_z must obey

$$\begin{aligned} \frac{\partial B_Z}{\partial t} &= -\frac{\partial E_y}{\partial x} \\ \frac{\partial B_Z}{\partial t} &= -\frac{\partial}{\partial x} E_0 \cos(kx - \omega t) = kE_0 \sin(kx - \omega t). \end{aligned} \tag{16.24}$$

Because the solution for the B-field pattern of the wave propagates in the $+x$-direction at the same speed c as the E-field pattern, it must be a function of $k(x - ct) = kx - \omega t$. Thus, we conclude from Equation 16.21 that B_z is

$$B_z(x, t) = \frac{k}{\omega} E_0 \cos(kx - \omega t) = \frac{1}{c} E_0 \cos(kx - \omega t).$$

These results may be written as

$$\begin{aligned} E_y(x, t) &= E_0 \cos(kx - \omega t) \\ B_z(x, t) &= B_0 \cos(kx - \omega t) \end{aligned} \tag{16.25}$$

$$\frac{E_y}{B_z} = \frac{E_0}{B_0} = c. \qquad (16.26)$$

Therefore, the peaks of the *E* and *B* fields coincide, as do the troughs of the wave, and at each point, the *E* and *B* fields are in the same ratio equal to the speed of light *c*. The plane wave has the form shown in Figure 16.8.

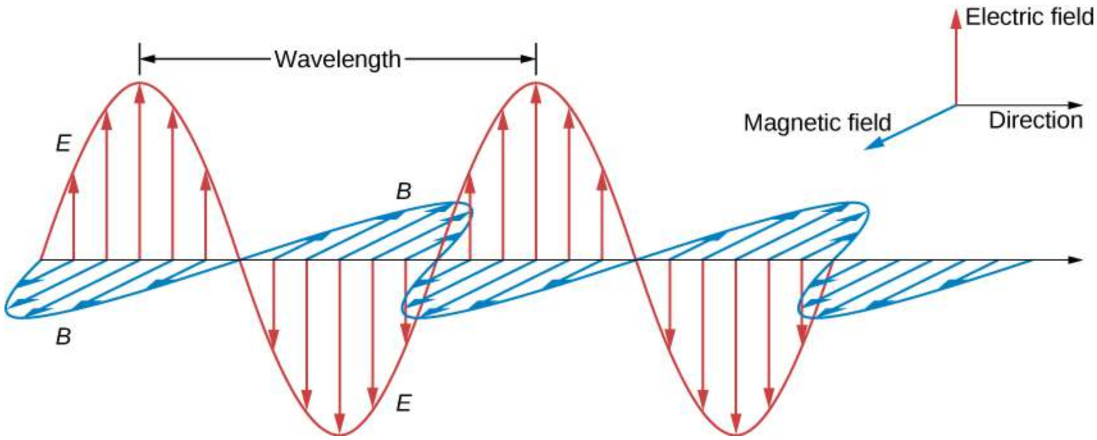


Figure 16.8 The plane wave solution of Maxwell's equations has the *B* field directly proportional to the *E* field at each point, with the relative directions shown.

EXAMPLE 16.2

Calculating *B*-Field Strength in an Electromagnetic Wave

What is the maximum strength of the *B* field in an electromagnetic wave that has a maximum *E*-field strength of 1000 V/m?

Strategy

To find the *B*-field strength, we rearrange Equation 16.23 to solve for *B*, yielding

$$B = \frac{E}{c}.$$

Solution

We are given *E*, and *c* is the speed of light. Entering these into the expression for *B* yields

$$B = \frac{1000\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = 3.33 \times 10^{-6}\ \text{T}.$$

Significance

The *B*-field strength is less than a tenth of Earth's admittedly weak magnetic field. This means that a relatively strong electric field of 1000 V/m is accompanied by a relatively weak magnetic field.

Changing electric fields create relatively weak magnetic fields. The combined electric and magnetic fields can be detected in electromagnetic waves, however, by taking advantage of the phenomenon of resonance, as Hertz did. A system with the same natural frequency as the electromagnetic wave can be made to oscillate. All radio and TV receivers use this principle to pick up and then amplify weak electromagnetic waves, while rejecting all others not at their resonant frequency.

CHECK YOUR UNDERSTANDING 16.4

What conclusions did our analysis of Maxwell's equations lead to about these properties of a plane electromagnetic wave:

(a) the relative directions of wave propagation, of the E field, and of B field,
(b) the speed of travel of the wave and how the speed depends on frequency, and
(c) the relative magnitudes of the E and B fields.

Production and Detection of Electromagnetic Waves

A steady electric current produces a magnetic field that is constant in time and which does not propagate as a wave. Accelerating charges, however, produce electromagnetic waves. An electric charge oscillating up and down, or an alternating current or flow of charge in a conductor, emit radiation at the frequencies of their oscillations. The electromagnetic field of a *dipole antenna* is shown in Figure 16.9. The positive and negative charges on the two conductors are made to reverse at the desired frequency by the output of a transmitter as the power source. The continually changing current accelerates charge in the antenna, and this results in an oscillating electric field a distance away from the antenna. The changing electric fields produce changing magnetic fields that in turn produce changing electric fields, which thereby propagate as electromagnetic waves. The frequency of this radiation is the same as the frequency of the ac source that is accelerating the electrons in the antenna. The two conducting elements of the dipole antenna are commonly straight wires. The total length of the two wires is typically about one-half of the desired wavelength (hence, the alternative name *half-wave antenna*), because this allows standing waves to be set up and enhances the effectiveness of the radiation.

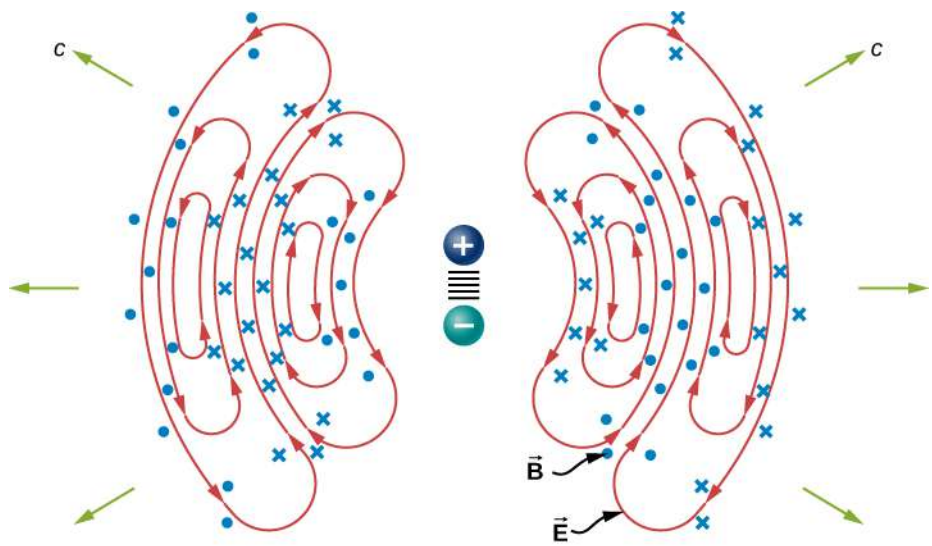


Figure 16.9 The oscillatory motion of the charges in a dipole antenna produces electromagnetic radiation.

The electric field lines in one plane are shown. The magnetic field is perpendicular to this plane. This radiation field has cylindrical symmetry around the axis of the dipole. Field lines near the dipole are not shown. The pattern is not at all uniform in all directions. The strongest signal is in directions perpendicular to the axis of the antenna, which would be horizontal if the antenna is mounted vertically. There is zero intensity along the axis of the antenna. The fields detected far from the antenna are from the changing electric and magnetic fields inducing each other and traveling as electromagnetic waves. Far from the antenna, the wave fronts, or surfaces of equal phase for the electromagnetic wave, are almost spherical. Even farther from the antenna, the radiation propagates like electromagnetic plane waves.

The electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving electromagnetic signals works in reverse. Incoming electromagnetic waves induce oscillating currents in the antenna, each at its own frequency. The radio receiver includes a tuner circuit, whose resonant frequency can be adjusted. The tuner responds strongly to the desired frequency but not others, allowing the user to tune to the desired broadcast. Electrical components amplify the signal formed by the moving electrons. The signal is then converted into an audio and/or video format.

INTERACTIVE

Use this simulation (https://openstax.org/l/21radwavsim) to broadcast radio waves. Wiggle the transmitter electron manually or have it oscillate automatically. Display the field as a curve or vectors. The strip chart shows the electron positions at the transmitter and at the receiver.

16.3 Energy Carried by Electromagnetic Waves

Learning Objectives

By the end of this section, you will be able to:

- Express the time-averaged energy density of electromagnetic waves in terms of their electric and magnetic field amplitudes
- Calculate the Poynting vector and the energy intensity of electromagnetic waves
- Explain how the energy of an electromagnetic wave depends on its amplitude, whereas the energy of a photon is proportional to its frequency

Anyone who has used a microwave oven knows there is energy in electromagnetic waves. Sometimes this energy is obvious, such as in the warmth of the summer Sun. Other times, it is subtle, such as the unfelt energy of gamma rays, which can destroy living cells.

Electromagnetic waves bring energy into a system by virtue of their electric and magnetic fields. These fields can exert forces and move charges in the system and, thus, do work on them. However, there is energy in an electromagnetic wave itself, whether it is absorbed or not. Once created, the fields carry energy away from a source. If some energy is later absorbed, the field strengths are diminished and anything left travels on.

Clearly, the larger the strength of the electric and magnetic fields, the more work they can do and the greater the energy the electromagnetic wave carries. In electromagnetic waves, the amplitude is the maximum field strength of the electric and magnetic fields (Figure 16.10). The wave energy is determined by the wave amplitude.

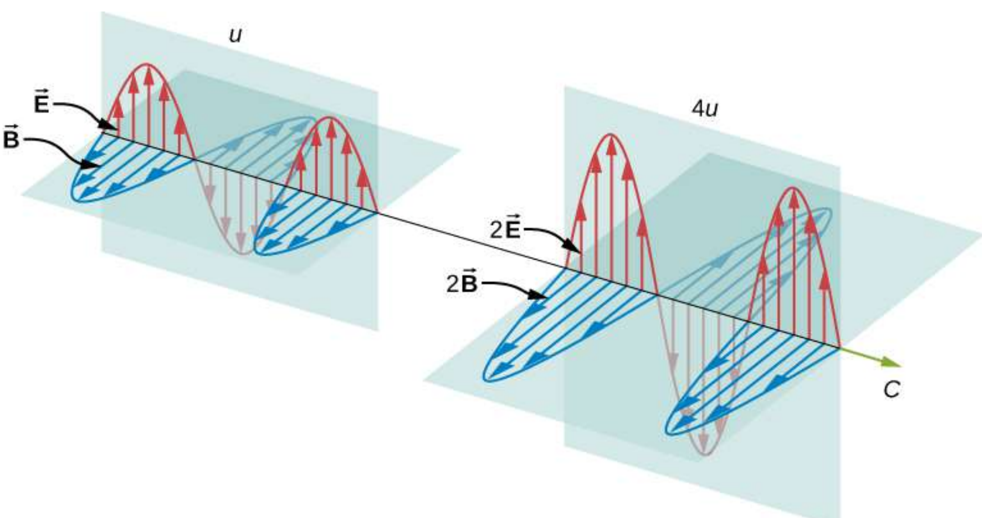


Figure 16.10 Energy carried by a wave depends on its amplitude. With electromagnetic waves, doubling the *E* fields and *B* fields quadruples the energy density *u* and the energy flux *uc*.

For a plane wave traveling in the direction of the positive x-axis with the phase of the wave chosen so that the wave maximum is at the origin at $t = 0$, the electric and magnetic fields obey the equations

$$E_y(x, t) = E_0 \cos\ (kx - \omega t)$$
$$B_z(x, t) = B_0 \cos\ (kx - \omega t)\,.$$

The energy in any part of the electromagnetic wave is the sum of the energies of the electric and magnetic fields. This energy per unit volume, or energy density u, is the sum of the energy density from the electric field and the energy density from the magnetic field. Expressions for both field energy densities were discussed earlier (u_E in Capacitance and u_B in Inductance). Combining these the contributions, we obtain

$$u(x,t) = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2.$$

The expression $E = cB = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}B$ then shows that the magnetic energy density u_B and electric energy density u_E are equal, despite the fact that changing electric fields generally produce only small magnetic fields. The equality of the electric and magnetic energy densities leads to

$$u(x,t) = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}. \tag{16.27}$$

The energy density moves with the electric and magnetic fields in a similar manner to the waves themselves.

We can find the rate of transport of energy by considering a small time interval Δt. As shown in Figure 16.11, the energy contained in a cylinder of length $c\Delta t$ and cross-sectional area A passes through the cross-sectional plane in the interval Δt.

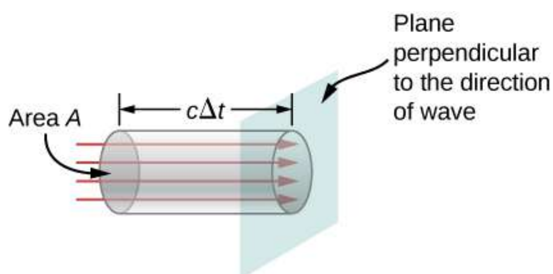


Figure 16.11 The energy $uAc\Delta t$ contained in the electric and magnetic fields of the electromagnetic wave in the volume $Ac\Delta t$ passes through the area A in time Δt.

The energy passing through area A in time Δt is

$$u \times \text{volume} = uAc\Delta t.$$

The energy per unit area per unit time passing through a plane perpendicular to the wave, called the energy flux and denoted by S, can be calculated by dividing the energy by the area A and the time interval Δt.

$$S = \frac{\text{Energy passing area } A \text{ in time } \Delta t}{A\Delta t} = uc = \varepsilon_0 cE^2 = \frac{1}{\mu_0}EB.$$

More generally, the flux of energy through any surface also depends on the orientation of the surface. To take the direction into account, we introduce a vector $\vec{\mathbf{S}}$, called the **Poynting vector**, with the following definition:

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}. \tag{16.28}$$

The cross-product of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ points in the direction perpendicular to both vectors. To confirm that the direction of $\vec{\mathbf{S}}$ is that of wave propagation, and not its negative, return to Figure 16.7. Note that Lenz's and Faraday's laws imply that when the magnetic field shown is increasing in time, the electric field is greater at x than at $x + \Delta x$. The electric field is decreasing with increasing x at the given time and location. The proportionality between electric and magnetic fields requires the electric field to increase in time along with the magnetic field. This is possible only if the wave is propagating to the right in the diagram, in which case, the relative orientations show that $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ is specifically in the direction of propagation of the electromagnetic wave.

The energy flux at any place also varies in time, as can be seen by substituting u from Equation 16.23 into Equation 16.27.

$$S(x,t) = c\varepsilon_0 E_0^2 \cos^2(kx - \omega t) \tag{16.29}$$

Because the frequency of visible light is very high, of the order of 10^{14} Hz, the energy flux for visible light through any area is an extremely rapidly varying quantity. Most measuring devices, including our eyes, detect only an average over many cycles. The time average of the energy flux is the intensity I of the electromagnetic wave and is the power per unit area. It can be expressed by averaging the cosine function in Equation 16.29 over one complete cycle, which is the same as time-averaging over many cycles (here, T is one period):

$$I = S_{\text{avg}} = c\varepsilon_0 E_0^2 \frac{1}{T}\int_0^T \cos^2\left(2\pi\frac{t}{T}\right) dt. \tag{16.30}$$

We can either evaluate the integral, or else note that because the sine and cosine differ merely in phase, the average over a complete cycle for $\cos^2(\xi)$ is the same as for $\sin^2(\xi)$, to obtain

$$\left\langle \cos^2\xi \right\rangle = \frac{1}{2}\left[\left\langle \cos^2\xi \right\rangle + \left\langle \sin^2\xi \right\rangle\right] = \frac{1}{2}\langle 1 \rangle = \frac{1}{2}.$$

where the angle brackets $\langle\cdots\rangle$ stand for the time-averaging operation. The intensity of light moving at speed c in vacuum is then found to be

$$I = S_{\text{avg}} = \frac{1}{2}c\varepsilon_0 E_0^2 \tag{16.31}$$

in terms of the maximum electric field strength E_0, which is also the electric field amplitude. Algebraic manipulation produces the relationship

$$I = \frac{cB_0^2}{2\mu_0} \tag{16.32}$$

where B_0 is the magnetic field amplitude, which is the same as the maximum magnetic field strength. One more expression for I_{avg} in terms of both electric and magnetic field strengths is useful. Substituting the fact that $cB_0 = E_0$, the previous expression becomes

$$I = \frac{E_0 B_0}{2\mu_0}. \tag{16.33}$$

We can use whichever of the three preceding equations is most convenient, because the three equations are really just different versions of the same result: The energy in a wave is related to amplitude squared. Furthermore, because these equations are based on the assumption that the electromagnetic waves are sinusoidal, the peak intensity is twice the average intensity; that is, $I_0 = 2I$.

EXAMPLE 16.3

A Laser Beam

The beam from a small laboratory laser typically has an intensity of about $1.0 \times 10^{-3}\ \text{W/m}^2$. Assuming that the beam is composed of plane waves, calculate the amplitudes of the electric and magnetic fields in the beam.

Strategy

Use the equation expressing intensity in terms of electric field to calculate the electric field from the intensity.

Solution

From Equation 16.31, the intensity of the laser beam is

$$I = \frac{1}{2}c\varepsilon_0 E_0^2.$$

The amplitude of the electric field is therefore

$$E_0 = \sqrt{\frac{2}{c\varepsilon_0} I} = \sqrt{\frac{2}{\left(3.00 \times 10^8 \text{ m/s}\right)\left(8.85 \times 10^{-12} \text{ F/m}\right)}\left(1.0 \times 10^{-3} \text{ W/m}^2\right)} = 0.87 \text{ V/m}.$$

The amplitude of the magnetic field can be obtained from Equation 16.20:

$$B_0 = \frac{E_0}{c} = 2.9 \times 10^{-9} \text{ T}.$$

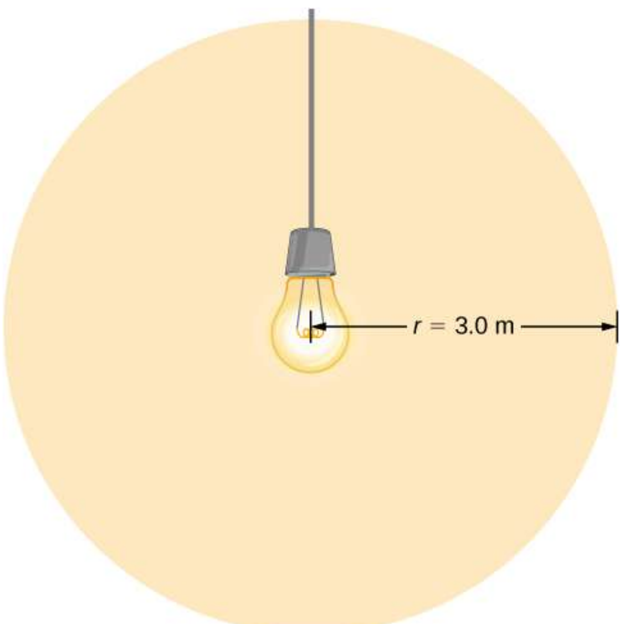
EXAMPLE 16.4

Light Bulb Fields

A light bulb emits 5.00 W of power as visible light. What are the average electric and magnetic fields from the light at a distance of 3.0 m?

Strategy

Assume the bulb's power output P is distributed uniformly over a sphere of radius 3.0 m to calculate the intensity, and from it, the electric field.



Solution

The power radiated as visible light is then

$$\begin{aligned} I &= \frac{P}{4\pi r^2} = \frac{c\varepsilon_0 E_0^2}{2}, \\ E_0 &= \sqrt{2\frac{P}{4\pi r^2 c\varepsilon_0}} = \sqrt{2\frac{5.00 \text{ W}}{4\pi(3.0 \text{ m})^2\left(3.00 \times 10^8 \text{ m/s}\right)\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)}} = 5.77 \text{ N/C}, \\ B_0 &= E_0/c = 1.92 \times 10^{-8} \text{ T}. \end{aligned}$$

Significance

The intensity I falls off as the distance squared if the radiation is dispersed uniformly in all directions.

EXAMPLE 16.5

Radio Range

A 60-kW radio transmitter on Earth sends its signal to a satellite 100 km away (Figure 16.12). At what distance in the same direction would the signal have the same maximum field strength if the transmitter's output power were increased to 90 kW?

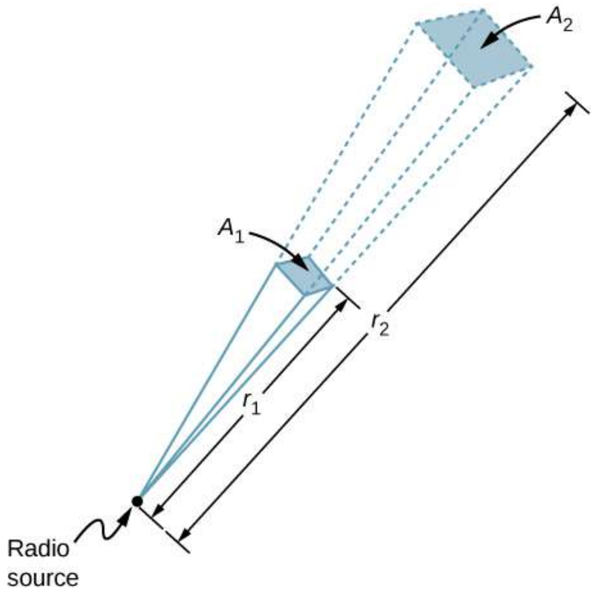


Figure 16.12 In three dimensions, a signal spreads over a solid angle as it travels outward from its source.

Strategy

The area over which the power in a particular direction is dispersed increases as distance squared, as illustrated in the figure. Change the power output P by a factor of (90 kW/60 kW) and change the area by the same factor to keep $I = \frac{P}{A} = \frac{c\varepsilon_0 E_0^2}{2}$ the same. Then use the proportion of area A in the diagram to distance squared to find the distance that produces the calculated change in area.

Solution

Using the proportionality of the areas to the squares of the distances, and solving, we obtain from the diagram

$$\begin{aligned} \frac{r_2^2}{r_1^2} &= \frac{A_2}{A_1} = \frac{90\ \text{W}}{60\ \text{W}}, \\ r_2 &= \sqrt{\frac{90}{60}}(100\ \text{km}) = 122\ \text{km}. \end{aligned}$$

Significance

The range of a radio signal is the maximum distance between the transmitter and receiver that allows for normal operation. In the absence of complications such as reflections from obstacles, the intensity follows an inverse square law, and doubling the range would require multiplying the power by four.

16.4 Momentum and Radiation Pressure

Learning Objectives

By the end of this section, you will be able to:

- Describe the relationship of the radiation pressure and the energy density of an electromagnetic wave
- Explain how the radiation pressure of light, while small, can produce observable astronomical effects

Material objects consist of charged particles. An electromagnetic wave incident on the object exerts forces on the charged particles, in accordance with the Lorentz force, Equation 16.11. These forces do work on the particles of the object, increasing its energy, as discussed in the previous section. The energy that sunlight carries is a familiar part of every warm sunny day. A much less familiar feature of electromagnetic radiation is the extremely weak pressure that electromagnetic radiation produces by exerting a force in the direction of the wave. This force occurs because electromagnetic waves contain and transport momentum.

To understand the direction of the force for a very specific case, consider a plane electromagnetic wave incident on a metal in which electron motion, as part of a current, is damped by the resistance of the metal, so that the average electron motion is in phase with the force causing it. This is comparable to an object moving against friction and stopping as soon as the force pushing it stops (Figure 16.13). When the electric field is in the direction of the positive y-axis, electrons move in the negative y-direction, with the magnetic field in the direction of the positive z-axis. By applying the right-hand rule, and accounting for the negative charge of the electron, we can see that the force on the electron from the magnetic field is in the direction of the positive x-axis, which is the direction of wave propagation. When the E field reverses, the B field does too, and the force is again in the same direction. Maxwell's equations together with the Lorentz force equation imply the existence of radiation pressure much more generally than this specific example, however.

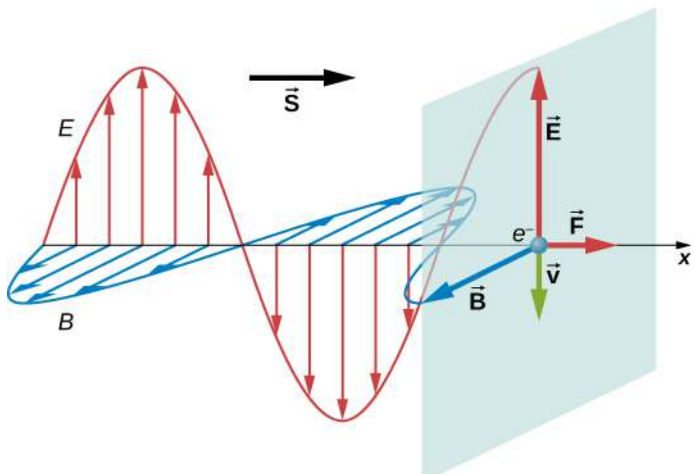


Figure 16.13 Electric and magnetic fields of an electromagnetic wave can combine to produce a force in the direction of propagation, as illustrated for the special case of electrons whose motion is highly damped by the resistance of a metal.

Maxwell predicted that an electromagnetic wave carries momentum. An object absorbing an electromagnetic wave would experience a force in the direction of propagation of the wave. The force corresponds to radiation pressure exerted on the object by the wave. The force would be twice as great if the radiation were reflected rather than absorbed.

Maxwell's prediction was confirmed in 1903 by Nichols and Hull by precisely measuring radiation pressures with a torsion balance. The schematic arrangement is shown in Figure 16.14. The mirrors suspended from a fiber were housed inside a glass container. Nichols and Hull were able to obtain a small measurable deflection of the mirrors from shining light on one of them. From the measured deflection, they could calculate the unbalanced force on the mirror, and obtained agreement with the predicted value of the force.

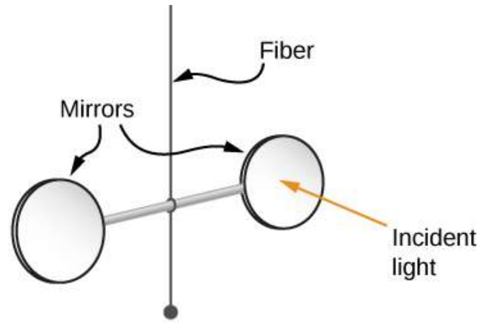


Figure 16.14 Simplified diagram of the central part of the apparatus Nichols and Hull used to precisely measure radiation pressure and confirm Maxwell's prediction.

The **radiation pressure** p_{rad} applied by an electromagnetic wave on a perfectly absorbing surface turns out to be equal to the energy density of the wave:

$$p_{\text{rad}} = u \text{ (Perfect absorber).} \tag{16.34}$$

If the material is perfectly reflecting, such as a metal surface, and if the incidence is along the normal to the surface, then the pressure exerted is twice as much because the momentum direction reverses upon reflection:

$$p_{\text{rad}} = 2u \text{ (Perfect reflector).} \tag{16.35}$$

We can confirm that the units are right:

$$[u] = \frac{\text{J}}{\text{m}^3} = \frac{\text{N} \cdot \text{m}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2} = \text{units of pressure.}$$

Equation 16.34 and Equation 16.35 give the instantaneous pressure, but because the energy density oscillates rapidly, we are usually interested in the time-averaged radiation pressure, which can be written in terms of intensity:

$$p = \langle p_{\text{rad}} \rangle = \begin{cases} I/c & \text{Perfect absorber} \\ 2I/c & \text{Perfect reflector.} \end{cases} \tag{16.36}$$

Radiation pressure plays a role in explaining many observed astronomical phenomena, including the appearance of comets. Comets are basically chunks of icy material in which frozen gases and particles of rock and dust are embedded. When a comet approaches the Sun, it warms up and its surface begins to evaporate. The *coma* of the comet is the hazy area around it from the gases and dust. Some of the gases and dust form tails when they leave the comet. Notice in Figure 16.15 that a comet has *two* tails. The *ion tail* (or *gas tail* in Figure 16.15) is composed mainly of ionized gases. These ions interact electromagnetically with the solar wind, which is a continuous stream of charged particles emitted by the Sun. The force of the solar wind on the ionized gases is strong enough that the ion tail almost always points directly away from the Sun. The second tail is composed of dust particles. Because the *dust tail* is electrically neutral, it does not interact with the solar wind. However, this tail is affected by the radiation pressure produced by the light from the Sun. Although quite small, this pressure is strong enough to cause the dust tail to be displaced from the path of the comet.

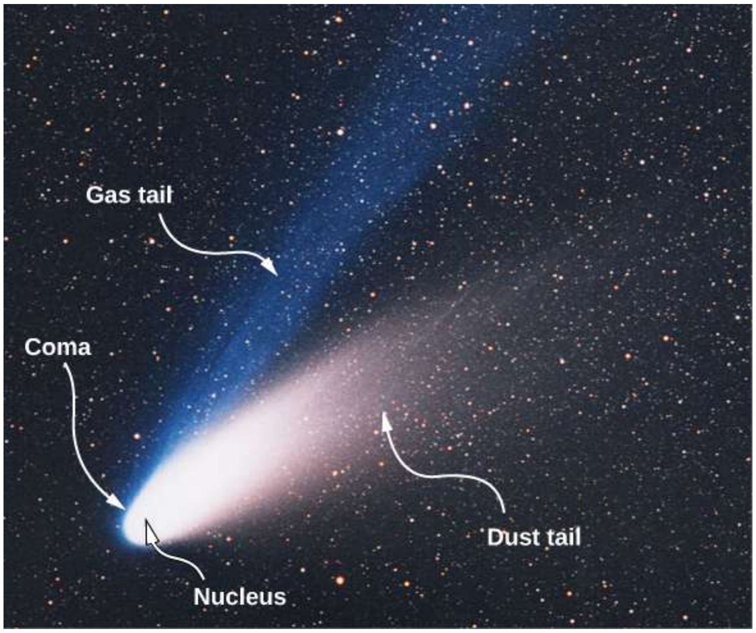


Figure 16.15 Evaporation of material being warmed by the Sun forms two tails, as shown in this photo of Comet Ison. (credit: modification of work by E. Slawik—ESO)

EXAMPLE 16.6

Halley's Comet

On February 9, 1986, Comet Halley was at its closest point to the Sun, about 9.0×10^{10} m from the center of the Sun. The average power output of the Sun is 3.8×10^{26} W.

(a) Calculate the radiation pressure on the comet at this point in its orbit. Assume that the comet reflects all the incident light.

(b) Suppose that a 10-kg chunk of material of cross-sectional area $4.0 \times 10^{-2}\ \text{m}^2$ breaks loose from the comet. Calculate the force on this chunk due to the solar radiation. Compare this force with the gravitational force of the Sun.

Strategy

Calculate the intensity of solar radiation at the given distance from the Sun and use that to calculate the radiation pressure. From the pressure and area, calculate the force.

Solution

a. The intensity of the solar radiation is the average solar power per unit area. Hence, at 9.0×10^{10} m from the center of the Sun, we have

$$I = S_{\text{avg}} = \frac{3.8 \times 10^{26}\ \text{W}}{4\pi\left(9.0 \times 10^{10}\ \text{m}\right)^2} = 3.7 \times 10^3\ \text{W/m}^2.$$

Assuming the comet reflects all the incident radiation, we obtain from Equation 16.36

$$p = \frac{2I}{c} = \frac{2\left(3.7 \times 10^3\ \text{W/m}^2\right)}{3.00 \times 10^8\ \text{m/s}} = 2.5 \times 10^{-5}\ \text{N/m}^2.$$

b. The force on the chunk due to the radiation is

$$\begin{aligned} F &= pA = \left(2.5 \times 10^{-5}\ \text{N/m}^2\right)\left(4.0 \times 10^{-2}\ \text{m}^2\right) \\ &= 1.0 \times 10^{-6}\ \text{N}, \end{aligned}$$

whereas the gravitational force of the Sun is

$$F_g = \frac{GMm}{r^2} = \frac{\left(6.67 \times 10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2\right)\left(2.0 \times 10^{30}\,\mathrm{kg}\right)(10\,\mathrm{kg})}{\left(9.0 \times 10^{10}\,\mathrm{m}\right)^2} = 0.16\,\mathrm{N}.$$

Significance

The gravitational force of the Sun on the chunk is therefore much greater than the force of the radiation.

After Maxwell showed that light carried momentum as well as energy, a novel idea eventually emerged, initially only as science fiction. Perhaps a spacecraft with a large reflecting light sail could use radiation pressure for propulsion. Such a vehicle would not have to carry fuel. It would experience a constant but small force from solar radiation, instead of the short bursts from rocket propulsion. It would accelerate slowly, but by being accelerated continuously, it would eventually reach great speeds. A spacecraft with small total mass and a sail with a large area would be necessary to obtain a usable acceleration.

When the space program began in the 1960s, the idea started to receive serious attention from NASA. The most recent development in light propelled spacecraft has come from a citizen-funded group, the Planetary Society. It is currently testing the use of light sails to propel a small vehicle built from *CubeSats*, tiny satellites that NASA places in orbit for various research projects during space launches intended mainly for other purposes.

The *LightSail* spacecraft shown below (Figure 16.16) consists of three *CubeSats* bundled together. It has a total mass of only about 5 kg and is about the size as a loaf of bread. Its sails are made of very thin Mylar and open after launch to have a surface area of 32 m^2.

Figure 16.16 Two small *CubeSat* satellites deployed from the International Space Station in May, 2016. The solar sails open out when the CubeSats are far enough away from the Station. (credit: modification of work by NASA)

INTERACTIVE

The first *LightSail* spacecraft was launched in 2015 to test the sail deployment system. It was placed in low-earth orbit in 2015 by hitching a ride on an Atlas 5 rocket launched for an unrelated mission. The test was successful, but the low-earth orbit allowed too much drag on the spacecraft to accelerate it by sunlight. Eventually, it burned in the atmosphere, as expected. The next Planetary Society's *LightSail* solar sailing spacecraft is scheduled for 2016. An illustration (https://openstax.org/l/21lightsail) of the spacecraft, as it is expected to appear in flight, can be seen on the Planetary Society's website.

EXAMPLE 16.7

LightSail Acceleration

The intensity of energy from sunlight at a distance of 1 AU from the Sun is $1370\ \text{W/m}^2$. The *LightSail* spacecraft has sails with total area of $32\ \text{m}^2$ and a total mass of 5.0 kg. Calculate the maximum acceleration LightSail spacecraft could achieve from radiation pressure when it is about 1 AU from the Sun.

Strategy

The maximum acceleration can be expected when the sail is opened directly facing the Sun. Use the light intensity to calculate the radiation pressure and from it, the force on the sails. Then use Newton's second law to calculate the acceleration.

Solution

The radiation pressure is

$$F = pA = 2uA = \frac{2I}{c}A = \frac{2\left(1370\ \text{W/m}^2\right)\left(32\ \text{m}^2\right)}{\left(3.00 \times 10^8\ \text{m/s}\right)} = 2.92 \times 10^{-4}\ \text{N}.$$

The resulting acceleration is

$$a = \frac{F}{m} = \frac{2.92 \times 10^{-4}\ \text{N}}{5.0\ \text{kg}} = 5.8 \times 10^{-5}\ \text{m/s}^2.$$

Significance

If this small acceleration continued for a year, the craft would attain a speed of 1829 m/s, or 6600 km/h.

CHECK YOUR UNDERSTANDING 16.5

How would the speed and acceleration of a radiation-propelled spacecraft be affected as it moved farther from the Sun on an interplanetary space flight?

16.5 The Electromagnetic Spectrum

Learning Objectives

By the end of this section, you will be able to:

- Explain how electromagnetic waves are divided into different ranges, depending on wavelength and corresponding frequency
- Describe how electromagnetic waves in different categories are produced
- Describe some of the many practical everyday applications of electromagnetic waves

Electromagnetic waves have a vast range of practical everyday applications that includes such diverse uses as communication by cell phone and radio broadcasting, WiFi, cooking, vision, medical imaging, and treating cancer. In this module, we discuss how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on. We also summarize some of the main applications for each range.

The different categories of electromagnetic waves differ in their wavelength range, or equivalently, in their corresponding frequency ranges. Their properties change smoothly from one frequency range to the next, with different applications in each range. A brief overview of the production and utilization of electromagnetic waves is found in Table 16.1.

Type of wave	Production	Applications	Issues
Radio	Accelerating charges	Communications Remote controls MRI	Requires control for band use
Microwaves	Accelerating charges and thermal agitation	Communications Ovens Radar Cell phone use	
Infrared	Thermal agitation and electronic transitions	Thermal imaging Heating	Absorbed by atmosphere Greenhouse effect
Visible light	Thermal agitation and electronic transitions	Photosynthesis Human vision	
Ultraviolet	Thermal agitation and electronic transitions	Sterilization Vitamin D production	Ozone depletion Cancer causing
X-rays	Inner electronic transitions and fast collisions	Security Medical diagnosis Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicine Security Medical diagnosis Cancer therapy	Cancer causing Radiation damage

Table 16.1 Electromagnetic Waves

The relationship $c = f\lambda$ between frequency f and wavelength λ applies to all waves and ensures that greater frequency means smaller wavelength. Figure 16.17 shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies—that is, it shows the electromagnetic spectrum.

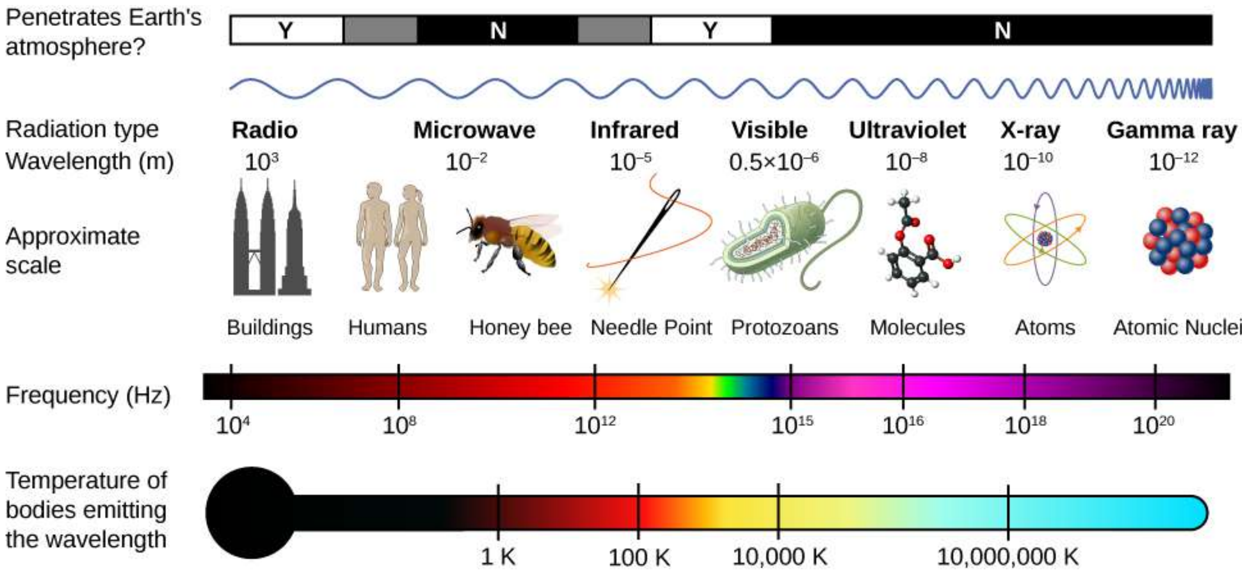


Figure 16.17 The electromagnetic spectrum, showing the major categories of electromagnetic waves.

Radio Waves

The term **radio waves** refers to electromagnetic radiation with wavelengths greater than about 0.1 m. Radio waves are commonly used for audio communications (i.e., for radios), but the term is used for electromagnetic waves in this range regardless of their application. Radio waves typically result from an alternating current in the wires of a broadcast antenna. They cover a very broad wavelength range and are divided into many subranges, including microwaves, electromagnetic waves used for AM and FM radio, cellular telephones, and TV signals.

There is no lowest frequency of radio waves, but ELF waves, or "extremely low frequency" are among the lowest frequencies commonly encountered, from 3 Hz to 3 kHz. The accelerating charge in the ac currents of electrical power lines produce electromagnetic waves in this range. ELF waves are able to penetrate sea water, which strongly absorbs electromagnetic waves of higher frequency, and therefore are useful for submarine communications.

In order to use an electromagnetic wave to transmit information, the amplitude, frequency, or phase of the wave is *modulated*, or varied in a controlled way that encodes the intended information into the wave. In AM radio transmission, the amplitude of the wave is modulated to mimic the vibrations of the sound being conveyed. Fourier's theorem implies that the modulated AM wave amounts to a superposition of waves covering some narrow frequency range. Each AM station is assigned a specific carrier frequency that, by international agreement, is allowed to vary by ± 5 kHz. In FM radio transmission, the frequency of the wave is modulated to carry this information, as illustrated in Figure 16.18, and the frequency of each station is allowed to use 100 kHz on each side of its carrier frequency. The electromagnetic wave produces a current in a receiving antenna, and the radio or television processes the signal to produce the sound and any image. The higher the frequency of the radio wave used to carry the data, the greater the detailed variation of the wave that can be carried by modulating it over each time unit, and the more data that can be transmitted per unit of time. The assigned frequencies for AM broadcasting are 540 to 1600 kHz, and for FM are 88 MHz to108 MHz.

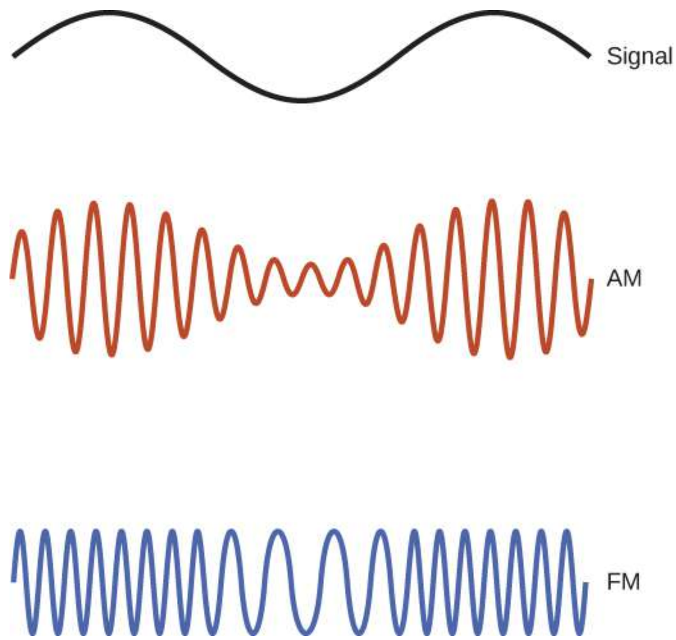


Figure 16.18 Electromagnetic waves are used to carry communications signals by varying the wave's amplitude (AM), its frequency (FM), or its phase.

Cell phone conversations, and television voice and video images are commonly transmitted as digital data, by converting the signal into a sequence of binary ones and zeros. This allows clearer data transmission when the signal is weak, and allows using computer algorithms to compress the digital data to transmit more data in each frequency range. Computer data as well is transmitted as a sequence of binary ones and zeros, each one or zero constituting one bit of data.

Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about $10^9\,\text{Hz}$ to nearly $10^{12}\,\text{Hz}$. Their high frequencies correspond to short wavelengths compared with other radio waves—hence the name "microwave." Microwaves also occur naturally as the cosmic background radiation left over from the origin of the universe. Along with other ranges of electromagnetic waves, they are part of the radiation that any object above absolute zero emits and absorbs because of **thermal agitation**, that is, from the thermal motion of its atoms and molecules.

Most satellite-transmitted information is carried on microwaves. **Radar** is a common application of microwaves. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds, aircraft, or even the surface of Venus.

Microwaves of 2.45 GHz are commonly used in microwave ovens. The electrons in a water molecule tend to remain closer to the oxygen nucleus than the hydrogen nuclei (Figure 16.19). This creates two separated centers of equal and opposite charges, giving the molecule a dipole moment (see Electric Field). The oscillating electric field of the microwaves inside the oven exerts a torque that tends to align each molecule first in one direction and then in the other, with the motion of each molecule coupled to others around it. This pumps energy into the continual thermal motion of the water to heat the food. The plate under the food contains no water, and remains relatively unheated.

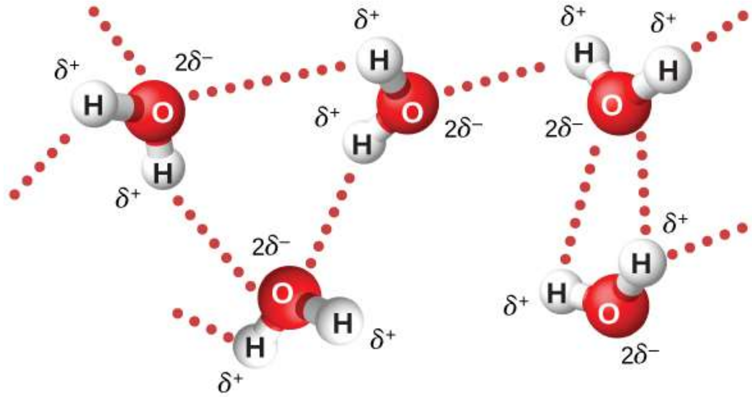


Figure 16.19 The oscillating electric field in a microwave oven exerts a torque on water molecules because of their dipole moment, and the torque reverses direction 4.90×10^9 times per second. Interactions between the molecules distributes the energy being pumped into them. The δ^+ and δ^- denote the charge distribution on the molecules.

The microwaves in a microwave oven reflect off the walls of the oven, so that the superposition of waves produces standing waves, similar to the standing waves of a vibrating guitar or violin string (see Normal Modes of a Standing Sound Wave). A rotating fan acts as a stirrer by reflecting the microwaves in different directions, and food turntables, help spread out the hot spots.

EXAMPLE 16.8

Why Microwave Ovens Heat Unevenly

How far apart are the hotspots in a 2.45-GHz microwave oven?

Strategy

Consider the waves along one direction in the oven, being reflected at the opposite wall from where they are generated.

Solution

The antinodes, where maximum intensity occurs, are half the wavelength apart, with separation

$$d = \frac{1}{2}\lambda = \frac{1}{2}\frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{2\left(2.45 \times 10^9 \text{ Hz}\right)} = 6.02 \text{ cm}.$$

Significance

The distance between the hot spots in a microwave oven are determined by the wavelength of the microwaves.

A cell phone has a radio receiver and a weak radio transmitter, both of which can quickly tune to hundreds of specifically assigned microwave frequencies. The low intensity of the transmitted signal gives it an intentionally limited range. A ground-based system links the phone to only to the broadcast tower assigned to the specific small area, or cell, and smoothly transitions its connection to the next cell when the signal reception there is the stronger one. This enables a cell phone to be used while changing location.

Microwaves also provide the WiFi that enables owners of cell phones, laptop computers, and similar devices to connect wirelessly to the Internet at home and at coffee shops and airports. A wireless WiFi router is a device that exchanges data over the Internet through the cable or another connection, and uses microwaves to exchange the data wirelessly with devices such as cell phones and computers. The term WiFi itself refers to the standards followed in modulating and analyzing the microwaves so that wireless routers and devices from different manufacturers work compatibly with one another. The computer data in each direction consist of sequences of binary zeros and ones, each corresponding to a binary bit. The microwaves are in the range of 2.4 GHz to 5.0 GHz range.

Other wireless technologies also use microwaves to provide everyday communications between devices. Bluetooth developed alongside WiFi as a standard for radio communication in the 2.4-GHz range between nearby devices, for example, to link to headphones and audio earpieces to devices such as radios, or a driver's cell phone to a hands-free device to allow answering phone calls without fumbling directly with the cell phone.

Microwaves find use also in radio tagging, using RFID (radio frequency identification) technology. Examples are RFID tags attached to store merchandize, transponder for toll booths use attached to the windshield of a car, or even a chip embedded into a pet's skin. The device responds to a microwave signal by emitting a signal of its own with encoded information, allowing stores to quickly ring up items at their cash registers, drivers to charge tolls to their account without stopping, and lost pets to be reunited with their owners. NFC (near field communication) works similarly, except it is much shorter range. Its mechanism of interaction is the induced magnetic field at microwave frequencies between two coils. Cell phones that have NFC capability and the right software can supply information for purchases using the cell phone instead of a physical credit card. The very short range of the data transfer is a desired security feature in this case.

Infrared Radiation

The boundary between the microwave and infrared regions of the electromagnetic spectrum is not well defined (see Figure 16.17). **Infrared radiation** is generally produced by thermal motion, and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce infrared radiation. About half of the solar energy arriving at Earth is in the infrared region, with most of the rest in the visible part of the spectrum. About 23% of the solar energy is absorbed in the atmosphere, about 48% is absorbed at Earth's surface, and about 29% is reflected back into space.[1]

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means "below red." Water molecules rotate and vibrate particularly well at infrared frequencies. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, including those called *quartz heaters*, to preferentially warm us because we absorb infrared better than our surroundings.

The familiar handheld "remotes" for changing channels and settings on television sets often transmit their signal by modulating an infrared beam. If you try to use a TV remote without the infrared emitter being in direct line of sight with the infrared detector, you may find the television not responding. Some remotes use Bluetooth instead and reduce this annoyance.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum between about 400 nm and about 750 nm to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions.

Red light has the lowest frequencies and longest wavelengths, whereas violet has the highest frequencies and shortest wavelengths (Figure 16.20). Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the sun yellowish in appearance.

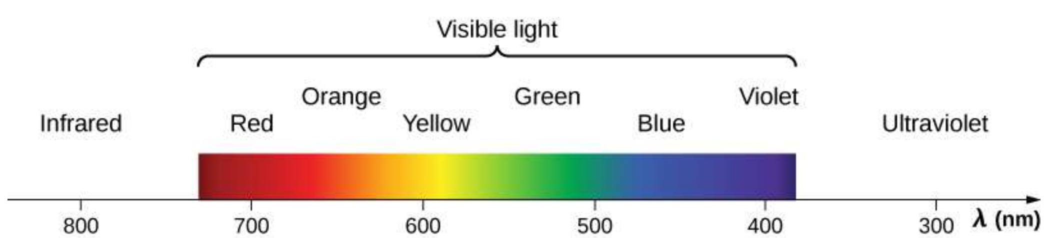


Figure 16.20 A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

1 http://earthobservatory.nasa.gov/Features/EnergyBalance/page4.php

Living things—plants and animals—have evolved to utilize and respond to parts of the electromagnetic spectrum in which they are embedded. We enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis uses parts of the visible spectrum to make sugars.

Ultraviolet Radiation

Ultraviolet means "above violet." The electromagnetic frequencies of **ultraviolet radiation (UV)** extend upward from violet, the highest-frequency visible light. The highest-frequency ultraviolet overlaps with the lowest-frequency X-rays. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies. Ultraviolet is produced by atomic and molecular motions and electronic transitions.

UV radiation from the Sun is broadly subdivided into three wavelength ranges: UV-A (320–400 nm) is the lowest frequency, then UV-B (290–320 nm) and UV-C (220–290 nm). Most UV-B and all UV-C are absorbed by ozone (O_3) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching Earth's surface is UV-A.

Sunburn is caused by large exposures to UV-B and UV-C, and repeated exposure can increase the likelihood of skin cancer. The tanning response is a defense mechanism in which the body produces pigments in inert skin layers to reduce exposure of the living cells below.

As examined in a later chapter, the shorter the wavelength of light, the greater the energy change of an atom or molecule that absorbs the light in an electronic transition. This makes short-wavelength ultraviolet light damaging to living cells. It also explains why ultraviolet radiation is better able than visible light to cause some materials to glow, or *fluoresce*.

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin results from exposure to UV-B radiation, generally from sunlight. Several studies suggest vitamin D deficiency is associated with the development of a range of cancers (prostate, breast, colon), as well as osteoporosis. Low-intensity ultraviolet has applications such as providing the energy to cause certain dyes to fluoresce and emit visible light, for example, in printed money to display hidden watermarks as counterfeit protection.

X-Rays

X-rays have wavelengths from about $10^{-8}\,\text{m}$ to $10^{-12}\,\text{m}$. They have shorter wavelengths, and higher frequencies, than ultraviolet, so that the energy they transfer at an atomic level is greater. As a result, X-rays have adverse effects on living cells similar to those of ultraviolet radiation, but they are more penetrating. Cancer and genetic defects can be induced by X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted, and these were designated as alpha, beta, and gamma rays. The most penetrating nuclear radiation, the **gamma ray (γ ray)**, was later found to be an extremely high-frequency electromagnetic wave.

The lower end of the γ- ray frequency range overlaps the upper end of the X-ray range. Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. The name "gamma rays" is generally used for electromagnetic radiation emitted by a nucleus, while X-rays are generally produced by bombarding a target with energetic electrons in an X-ray tube. At higher frequencies, γ rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

INTERACTIVE

Use this simulation (https://openstax.org/l/21simlightmol) to explore how light interacts with molecules in our atmosphere.

Explore how light interacts with molecules in our atmosphere.

Identify that absorption of light depends on the molecule and the type of light.

Relate the energy of the light to the resulting motion.

Identify that energy increases from microwave to ultraviolet.

Predict the motion of a molecule based on the type of light it absorbs.

CHECK YOUR UNDERSTANDING 16.6

How do the electromagnetic waves for the different kinds of electromagnetic radiation differ?

CHAPTER REVIEW

Key Terms

displacement current extra term in Maxwell's equations that is analogous to a real current but accounts for a changing electric field producing a magnetic field, even when the real current is present

gamma ray (γ ray) extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons; the lower end of the γ -ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

infrared radiation region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from $0.74\ \mu\text{m}$ to $300\ \mu\text{m}$

Maxwell's equations set of four equations that comprise a complete, overarching theory of electromagnetism

microwaves electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

Poynting vector vector equal to the cross product of the electric-and magnetic fields, that describes the flow of electromagnetic energy through a surface

radar common application of microwaves; radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm

radiation pressure force divided by area applied by an electromagnetic wave on a surface

radio waves electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

thermal agitation thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation

ultraviolet radiation electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

visible light narrow segment of the electromagnetic spectrum to which the normal human eye responds, from about 400 to 750 nm

X-ray invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ-ray range

Key Equations

Displacement current	$I_\text{d} = \varepsilon_0 \frac{d\Phi_\text{E}}{dt}$
Gauss's law	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_\text{in}}{\varepsilon_0}$
Gauss's law for magnetism	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$
Faraday's law	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_\text{m}}{dt}$
Ampère-Maxwell law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_\text{E}}{dt}$
Wave equation for plane EM wave	$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2}$
Speed of EM waves	$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

Ratio of E field to B field in electromagnetic wave	$c = \frac{E}{B}$
Energy flux (Poynting) vector	$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$
Average intensity of an electromagnetic wave	$I = S_{\text{avg}} = \frac{c\varepsilon_0 E_0^2}{2} = \frac{cB_0^2}{2\mu_0} = \frac{E_0 B_0}{2\mu_0}$
Radiation pressure	$p = \begin{cases} I/c & \text{Perfect absorber} \\ 2I/c & \text{Perfect reflector} \end{cases}$

Summary

16.1 Maxwell's Equations and Electromagnetic Waves

- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- The four Maxwell's equations together with the Lorentz force law encompass the major laws of electricity and magnetism. The first of these is Gauss's law for electricity; the second is Gauss's law for magnetism; the third is Faraday's law of induction (including Lenz's law); and the fourth is Ampère's law in a symmetric formulation that adds another source of magnetism, namely changing electric fields.
- The symmetry introduced between electric and magnetic fields through Maxwell's displacement current explains the mechanism of electromagnetic wave propagation, in which changing magnetic fields produce changing electric fields and vice versa.
- Although light was already known to be a wave, the nature of the wave was not understood before Maxwell. Maxwell's equations also predicted electromagnetic waves with wavelengths and frequencies outside the range of light. These theoretical predictions were first confirmed experimentally by Heinrich Hertz.

16.2 Plane Electromagnetic Waves

- Maxwell's equations predict that the directions of the electric and magnetic fields of the wave, and the wave's direction of propagation, are all mutually perpendicular. The electromagnetic wave is a transverse wave.
- The strengths of the electric and magnetic parts of the wave are related by $c = E/B$, which implies that the magnetic field B is very weak relative to the electric field E.
- Accelerating charges create electromagnetic waves (for example, an oscillating current in a wire produces electromagnetic waves with the same frequency as the oscillation).

16.3 Energy Carried by Electromagnetic Waves

- The energy carried by any wave is proportional to its amplitude squared. For electromagnetic waves, this means intensity can be expressed as

$$I = \frac{c\varepsilon_0 E_0^2}{2}$$

where I is the average intensity in W/m^2 and E_0 is the maximum electric field strength of a continuous sinusoidal wave. This can also be expressed in terms of the maximum magnetic field strength B_0 as

$$I = \frac{cB_0^2}{2\mu_0}$$

and in terms of both electric and magnetic fields as

$$I = \frac{E_0 B_0}{2\mu_0}.$$

The three expressions for I_{avg} are all equivalent.

16.4 Momentum and Radiation Pressure

- Electromagnetic waves carry momentum and exert radiation pressure.
- The radiation pressure of an electromagnetic wave is directly proportional to its energy density.
- The pressure is equal to twice the electromagnetic energy intensity if the wave is reflected and equal to the incident energy intensity if the wave is absorbed.

16.5 The Electromagnetic Spectrum

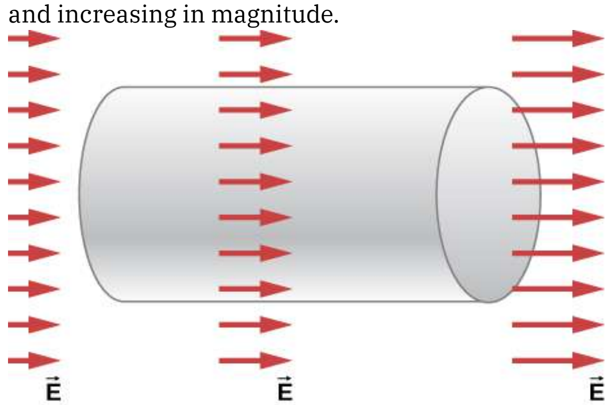
- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by $v = f\lambda$, so that for electromagnetic waves, $c = f\lambda$, where f is the frequency, λ is the wavelength, and c is the speed of light.
- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.

Conceptual Questions

16.1 Maxwell's Equations and Electromagnetic Waves

1. Explain how the displacement current maintains the continuity of current in a circuit containing a capacitor.

2. Describe the field lines of the induced magnetic field along the edge of the imaginary horizontal cylinder shown below if the cylinder is in a spatially uniform electric field that is horizontal, pointing to the right, and increasing in magnitude.

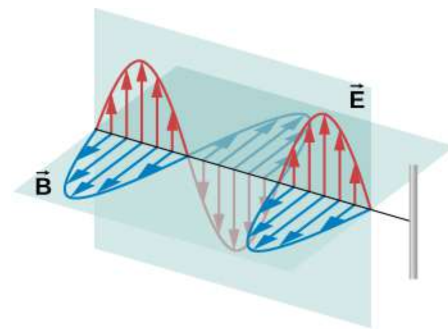


3. Why is it much easier to demonstrate in a student lab that a changing magnetic field induces an electric field than it is to demonstrate that a changing electric field produces a magnetic field?

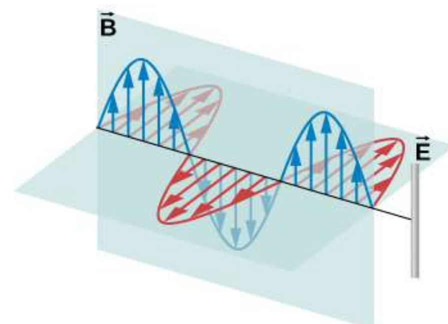
16.2 Plane Electromagnetic Waves

4. If the electric field of an electromagnetic wave is oscillating along the z-axis and the magnetic field is oscillating along the x-axis, in what possible direction is the wave traveling?

5. In which situation shown below will the electromagnetic wave be more successful in inducing a current in the wire? Explain.

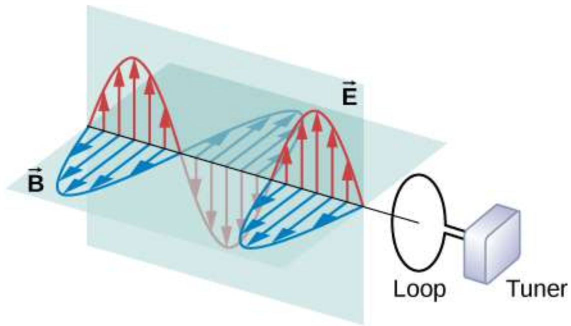


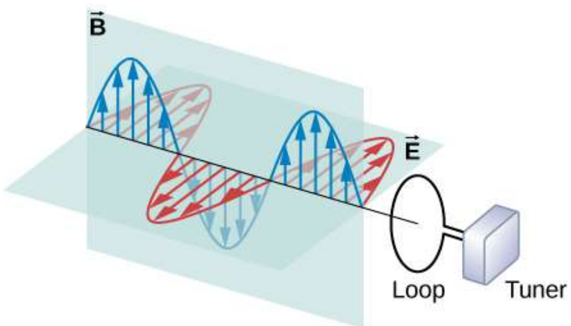
(a)



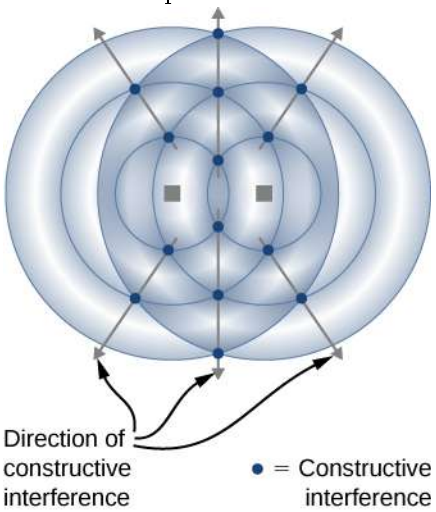
(b)

6. In which situation shown below will the electromagnetic wave be more successful in inducing a current in the loop? Explain.





7. Under what conditions might wires in a circuit where the current flows in only one direction emit electromagnetic waves?
8. Shown below is the interference pattern of two radio antennas broadcasting the same signal. Explain how this is analogous to the interference pattern for sound produced by two speakers. Could this be used to make a directional antenna system that broadcasts preferentially in certain directions? Explain.



16.3 Energy Carried by Electromagnetic Waves

9. When you stand outdoors in the sunlight, why can you feel the energy that the sunlight carries, but not the momentum it carries?
10. How does the intensity of an electromagnetic wave depend on its electric field? How does it depend on its magnetic field?
11. What is the physical significance of the Poynting vector?
12. A 2.0-mW helium-neon laser transmits a continuous beam of red light of cross-sectional area $0.25\ \text{cm}^2$. If the beam does not diverge appreciably, how would its rms electric field vary with distance from the laser? Explain.

16.4 Momentum and Radiation Pressure

13. Why is the radiation pressure of an electromagnetic wave on a perfectly reflecting surface twice as large as the pressure on a perfectly absorbing surface?
14. Why did the early Hubble Telescope photos of Comet Ison approaching Earth show it to have merely a fuzzy coma around it, and not the pronounced double tail that developed later (see below)?

Figure 16.21 (credit: modification of work by NASA, ESA, J.-Y. Li (Planetary Science Institute), and the Hubble Comet ISON Imaging Science Team)

15. (a) If the electric field and magnetic field in a sinusoidal plane wave were interchanged, in which direction relative to before would the energy propagate?
(b) What if the electric and the magnetic fields were both changed to their negatives?

16.5 The Electromagnetic Spectrum

16. Compare the speed, wavelength, and frequency of radio waves and X-rays traveling in a vacuum.

17. Accelerating electric charge emits electromagnetic radiation. How does this apply in each case: (a) radio waves, (b) infrared radiation.

18. Compare and contrast the meaning of the prefix “micro” in the names of SI units in the term *microwaves*.

19. Part of the light passing through the air is scattered in all directions by the molecules comprising the atmosphere. The wavelengths of visible light are larger than molecular sizes, and the scattering is strongest for wavelengths of light closest to sizes of molecules.
(a) Which of the main colors of light is scattered the most? (b) Explain why this would give the sky its familiar background color at midday.

20. When a bowl of soup is removed from a microwave oven, the soup is found to be steaming hot, whereas the bowl is only warm to the touch. Discuss the temperature changes that have occurred in terms of energy transfer.

21. Certain orientations of a broadcast television antenna give better reception than others for a particular station. Explain.

22. What property of light corresponds to loudness in sound?

23. Is the visible region a major portion of the electromagnetic spectrum?

24. Can the human body detect electromagnetic radiation that is outside the visible region of the spectrum?

25. Radio waves normally have their E and B fields in specific directions, whereas visible light usually has its E and B fields in random and rapidly changing directions that are perpendicular to each other and to the propagation direction. Can you explain why?

26. Give an example of resonance in the reception of electromagnetic waves.

27. Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light).

28. In which part of the electromagnetic spectrum are each of these waves:
(a) $f = 10.0$ kHz, (b) $f = \lambda = 750$ nm,
(c) $f = 1.25 \times 10^8$ Hz, (d) 0.30 nm

29. In what range of electromagnetic radiation are the electromagnetic waves emitted by power lines in a country that uses 50-Hz ac current?

30. If a microwave oven could be modified to merely tune the waves generated to be in the infrared range instead of using microwaves, how would this affect the uneven heating of the oven?

31. A leaky microwave oven in a home can sometimes cause interference with the homeowner’s WiFi system. Why?

32. When a television news anchor in a studio speaks to a reporter in a distant country, there is sometimes a noticeable lag between when the anchor speaks in the studio and when the remote reporter hears it and replies. Explain what causes this delay.

Problems

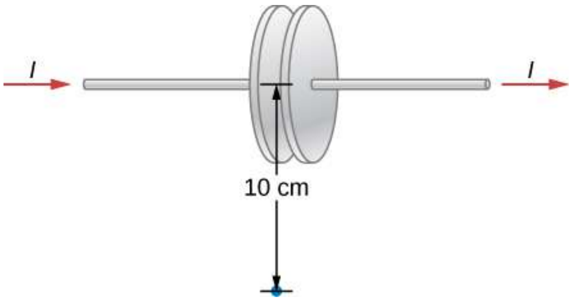
16.1 Maxwell’s Equations and Electromagnetic Waves

33. Show that the magnetic field at a distance r from the axis of two circular parallel plates, produced by placing charge $Q(t)$ on the plates is $B_{\text{ind}} = \frac{\mu_0}{2\pi r}\frac{dQ(t)}{dt}$.

34. Express the displacement current in a capacitor in terms of the capacitance and the rate of change of the voltage across the capacitor.

35. A potential difference $V(t) = V_0 \sin \omega t$ is maintained across a parallel-plate capacitor with capacitance C consisting of two circular parallel plates. A thin wire with resistance R connects the centers of the two plates, allowing charge to leak between plates while they are charging.
(a) Obtain expressions for the leakage current $I_{\text{res}}(t)$ in the thin wire. Use these results to obtain an expression for the current $I_{\text{real}}(t)$ in the wires connected to the capacitor.
(b) Find the displacement current in the space between the plates from the changing electric field between the plates.
(c) Compare $I_{\text{real}}(t)$ with the sum of the displacement current $I_{\text{d}}(t)$ and resistor current $I_{\text{res}}(t)$ between the plates, and explain why the relationship you observe would be expected.

36. Suppose the parallel-plate capacitor shown below is accumulating charge at a rate of 0.010 C/s. What is the induced magnetic field at a distance of 10 cm from the capacitator?



37. The potential difference $V(t)$ between parallel plates shown above is instantaneously increasing at a rate of 10^7 V/s. What is the displacement current between the plates if the separation of the plates is 1.00 cm and they have an area of $0.200\ \text{m}^2$?

38. A parallel-plate capacitor has a plate area of $A = 0.250\ \text{m}^2$ and a separation of 0.0100 m. What must be must be the angular frequency ω for a voltage $V(t) = V_0 \sin \omega t$ with $V_0 = 100$ V to produce a maximum displacement induced current of 1.00 A between the plates?

39. The voltage across a parallel-plate capacitor with area $A = 800\ \text{cm}^2$ and separation $d = 2$ mm varies sinusoidally as $V = (15\ \text{mV}) \cos(150t)$, where t is in seconds. Find the displacement current between the plates.

40. The voltage across a parallel-plate capacitor with area A and separation d varies with time t as $V = at^2$, where a is a constant. Find the displacement current between the plates.

16.2 Plane Electromagnetic Waves

41. If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is 1.496×10^{11} m away?

42. What is the maximum electric field strength in an electromagnetic wave that has a maximum magnetic field strength of 5.00×10^{-4} T (about 10 times Earth's magnetic field)?

43. An electromagnetic wave has a frequency of 12 MHz. What is its wavelength in vacuum?

44. If electric and magnetic field strengths vary sinusoidally in time at frequency 1.00 GHz, being zero at $t = 0$, then $E = E_0 \sin 2\pi f t$ and $B = B_0 \sin 2\pi f t$. (a) When are the field strengths next equal to zero? (b) When do they reach their most negative value? (c) How much time is needed for them to complete one cycle?

45. The electric field of an electromagnetic wave traveling in vacuum is described by the following wave function:

$$\vec{\mathbf{E}} = (5.00\ \text{V/m}) \cos\left[kx - \left(6.00 \times 10^9\ \text{s}^{-1}\right)t + 0.40\right]\hat{\mathbf{j}}$$

where k is the wavenumber in rad/m, x is in m, t is in s. Find the following quantities:
(a) amplitude
(b) frequency
(c) wavelength
(d) the direction of the travel of the wave
(e) the associated magnetic field wave

46. A plane electromagnetic wave of frequency 20 GHz moves in the positive y-axis direction such that its electric field is pointed along the z-axis. The amplitude of the electric field is 10 V/m. The start of time is chosen so that at $t = 0$, the electric field has a value 10 V/m at the origin. (a) Write the wave function that will describe the electric field wave. (b) Find the wave function that will describe the associated magnetic field wave.

47. The following represents an electromagnetic wave traveling in the direction of the positive y-axis:

$$E_x = 0;\ E_y = E_0 \cos(kx - \omega t);\ E_z = 0$$
$$B_x = 0;\ B_y = 0;\ B_z = B_0 \cos(kx - \omega t).$$

The wave is passing through a wide tube of circular cross-section of radius R whose axis is along the y-axis. Find the expression for the displacement current through the tube.

16.3 Energy Carried by Electromagnetic Waves

48. While outdoors on a sunny day, a student holds a large convex lens of radius 4.0 cm above a sheet of paper to produce a bright spot on the paper that is 1.0 cm in radius, rather than a sharp focus. By what factor is the electric field in the bright spot of light related to the electric field of sunlight leaving the side of the lens facing the paper?

49. A plane electromagnetic wave travels northward. At one instant, its electric field has a magnitude of 6.0 V/m and points eastward. What are the magnitude and direction of the magnetic field at this instant?

50. The electric field of an electromagnetic wave is given by $E =$

$$\left(6.0 \times 10^{-3}\ \text{V/m}\right) \sin\left[2\pi\left(\frac{x}{18\ \text{m}} - \frac{t}{6.0 \times 10^{-8}\ \text{s}}\right)\right]\hat{\mathbf{j}}.$$

Write the equations for the associated magnetic field and Poynting vector.

51. A radio station broadcasts at a frequency of 760 kHz. At a receiver some distance from the antenna, the maximum magnetic field of the

electromagnetic wave detected is 2.15×10^{-11} T.
(a) What is the maximum electric field? (b) What is the wavelength of the electromagnetic wave?

52. The filament in a clear incandescent light bulb radiates visible light at a power of 5.00 W. Model the glass part of the bulb as a sphere of radius $r_0 = 3.00$ cm and calculate the amount of electromagnetic energy from visible light inside the bulb.

53. At what distance does a 100-W lightbulb produce the same intensity of light as a 75-W lightbulb produces 10 m away? (Assume both have the same efficiency for converting electrical energy in the circuit into emitted electromagnetic energy.)

54. An incandescent light bulb emits only 2.6 W of its power as visible light. What is the rms electric field of the emitted light at a distance of 3.0 m from the bulb?

55. A 150-W lightbulb emits 5% of its energy as electromagnetic radiation. What is the magnitude of the average Poynting vector 10 m from the bulb?

56. A small helium-neon laser has a power output of 2.5 mW. What is the electromagnetic energy in a 1.0-m length of the beam?

57. At the top of Earth's atmosphere, the time-averaged Poynting vector associated with sunlight has a magnitude of about $1.4\ \text{kW/m}^2$.
(a) What are the maximum values of the electric and magnetic fields for a wave of this intensity? (b) What is the total power radiated by the sun? Assume that the Earth is 1.5×10^{11} m from the Sun and that sunlight is composed of electromagnetic plane waves.

58. The magnetic field of a plane electromagnetic wave moving along the z axis is given by $\vec{\mathbf{B}} = B_0 (\cos kz + \omega t)\,\hat{\mathbf{j}}$, where $B_0 = 5.00 \times 10^{-10}$ T and $k = 3.14 \times 10^{-2}\ \text{m}^{-1}$.
(a) Write an expression for the electric field associated with the wave. (b) What are the frequency and the wavelength of the wave? (c) What is its average Poynting vector?

59. What is the intensity of an electromagnetic wave with a peak electric field strength of 125 V/m?

60. Assume the helium-neon lasers commonly used in student physics laboratories have power outputs of 0.500 mW. (a) If such a laser beam is projected onto a circular spot 1.00 mm in diameter, what is its intensity? (b) Find the peak magnetic field strength. (c) Find the peak electric field strength.

61. An AM radio transmitter broadcasts 50.0 kW of power uniformly in all directions. (a) Assuming all of the radio waves that strike the ground are completely absorbed, and that there is no absorption by the atmosphere or other objects, what is the intensity 30.0 km away? (*Hint:* Half the power will be spread over the area of a hemisphere.) (b) What is the maximum electric field strength at this distance?

62. Suppose the maximum safe intensity of microwaves for human exposure is taken to be $1.00\ \text{W/m}^2$. (a) If a radar unit leaks 10.0 W of microwaves (other than those sent by its antenna) uniformly in all directions, how far away must you be to be exposed to an intensity considered to be safe? Assume that the power spreads uniformly over the area of a sphere with no complications from absorption or reflection. (b) What is the maximum electric field strength at the safe intensity? (Note that early radar units leaked more than modern ones do. This caused identifiable health problems, such as cataracts, for people who worked near them.)

63. A 2.50-m-diameter university communications satellite dish receives TV signals that have a maximum electric field strength (for one channel) of $7.50\ \mu\text{V/m}$ (see below). (a) What is the intensity of this wave? (b) What is the power received by the antenna? (c) If the orbiting satellite broadcasts uniformly over an area of $1.50 \times 10^{13}\ \text{m}^2$ (a large fraction of North America), how much power does it radiate?

64. Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to initiate nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of 1.00×10^{11} V/m for a time of 1.00 ns. (a) What is the maximum magnetic field strength in the wave? (b) What is the intensity of the beam? (c) What energy does it deliver on an 1.00-mm^2 area?

16.4 Momentum and Radiation Pressure

65. A 150-W lightbulb emits 5% of its energy as electromagnetic radiation. What is the radiation pressure on an absorbing sphere of radius 10 m that surrounds the bulb?
66. What pressure does light emitted uniformly in all directions from a 100-W incandescent light bulb exert on a mirror at a distance of 3.0 m, if 2.6 W of the power is emitted as visible light?
67. A microscopic spherical dust particle of radius $2\ \mu\text{m}$ and mass $10\ \mu\text{g}$ is moving in outer space at a constant speed of 30 cm/sec. A wave of light strikes it from the opposite direction of its motion and gets absorbed. Assuming the particle accelerates opposite to the motion uniformly to zero speed in one second, what is the average electric field amplitude in the light?
68. A Styrofoam spherical ball of radius 2 mm and mass $20\ \mu\text{g}$ is to be suspended by the radiation pressure in a vacuum tube in a lab. How much intensity will be required if the light is completely absorbed the ball?
69. Suppose that $\vec{\mathbf{S}}_{\text{avg}}$ for sunlight at a point on the surface of Earth is $900\ \text{W/m}^2$. (a) If sunlight falls perpendicularly on a kite with a reflecting surface of area $0.75\ \text{m}^2$, what is the average force on the kite due to radiation pressure? (b) How is your answer affected if the kite material is black and absorbs all sunlight?
70. Sunlight reaches the ground with an intensity of about $1.0\ \text{kW/m}^2$. A sunbather has a body surface area of $0.8\ \text{m}^2$ facing the sun while reclining on a beach chair on a clear day. (a) how much energy from direct sunlight reaches the sunbather's skin per second? (b) What pressure does the sunlight exert if it is absorbed?
71. Suppose a spherical particle of mass m and radius R in space absorbs light of intensity I for time t. (a) How much work does the radiation pressure do to accelerate the particle from rest in the given time it absorbs the light? (b) How much energy carried by the electromagnetic waves is absorbed by the particle over this time based on the radiant energy incident on the particle?

16.5 The Electromagnetic Spectrum

72. How many helium atoms, each with a radius of about 31 pm, must be placed end to end to have a length equal to one wavelength of 470 nm blue light?
73. If you wish to detect details of the size of atoms (about 0.2 nm) with electromagnetic radiation, it must have a wavelength of about this size. (a) What is its frequency? (b) What type of electromagnetic radiation might this be?
74. Find the frequency range of visible light, given that it encompasses wavelengths from 380 to 760 nm.
75. (a) Calculate the wavelength range for AM radio given its frequency range is 540 to 1600 kHz. (b) Do the same for the FM frequency range of 88.0 to 108 MHz.
76. Radio station WWVB, operated by the National Institute of Standards and Technology (NIST) from Fort Collins, Colorado, at a low frequency of 60 kHz, broadcasts a time synchronization signal whose range covers the entire continental US. The timing of the synchronization signal is controlled by a set of atomic clocks to an accuracy of 1×10^{-12} s, and repeats every 1 minute. The signal is used

for devices, such as radio-controlled watches, that automatically synchronize with it at preset local times. WWVB's long wavelength signal tends to propagate close to the ground.
(a) Calculate the wavelength of the radio waves from WWVB.
(b) Estimate the error that the travel time of the signal causes in synchronizing a radio controlled watch in Norfolk, Virginia, which is 1570 mi (2527 km) from Fort Collins, Colorado.

77. An outdoor WiFi unit for a picnic area has a 100-mW output and a range of about 30 m. What output power would reduce its range to 12 m for use with the same devices as before? Assume there are no obstacles in the way and that microwaves into the ground are simply absorbed.

78. **7**. The prefix "mega" (M) and "kilo" (k), when referring to amounts of computer data, refer to factors of 1024 or 2^{10} rather than 1000 for the prefix *kilo*, and $1024^2 = 2^{20}$ rather than 1,000,000 for the prefix *Mega* (M). If a wireless (WiFi) router transfers 150 Mbps of data, how many bits per second is that in decimal arithmetic?

79. A computer user finds that his wireless router transmits data at a rate of 75 Mbps (megabits per second). Compare the average time to transmit one bit of data with the time difference between the wifi signal reaching an observer's cell phone directly and by bouncing back to the observer from a wall 8.00 m past the observer.

80. (a) The ideal size (most efficient) for a broadcast antenna with one end on the ground is one-fourth the wavelength (λ/4) of the electromagnetic radiation being sent out. If a new radio station has such an antenna that is 50.0 m high, what frequency does it broadcast most efficiently? Is this in the AM or FM band? (b) Discuss the analogy of the fundamental resonant mode of an air column closed at one end to the resonance of currents on an antenna that is one-fourth their wavelength.

81. What are the wavelengths of (a) X-rays of frequency 2.0×10^{17} Hz? (b) Yellow light of frequency 5.1×10^{14} Hz? (c) Gamma rays of frequency 1.0×10^{23} Hz?

82. For red light of $\lambda = 660$ nm, what are f, ω, and k?

83. A radio transmitter broadcasts plane electromagnetic waves whose maximum electric field at a particular location is 1.55×10^{-3} V/m. What is the maximum magnitude of the oscillating magnetic field at that location? How does it compare with Earth's magnetic field?

84. (a) Two microwave frequencies authorized for use in microwave ovens are: 915 and 2450 MHz. Calculate the wavelength of each. (b) Which frequency would produce smaller hot spots in foods due to interference effects?

85. During normal beating, the heart creates a maximum 4.00-mV potential across 0.300 m of a person's chest, creating a 1.00-Hz electromagnetic wave. (a) What is the maximum electric field strength created? (b) What is the corresponding maximum magnetic field strength in the electromagnetic wave? (c) What is the wavelength of the electromagnetic wave?

86. Distances in space are often quoted in units of light-years, the distance light travels in 1 year. (a) How many meters is a light-year? (b) How many meters is it to Andromeda, the nearest large galaxy, given that it is 2.54×10^6 ly away? (c) The most distant galaxy yet discovered is 13.4×10^9 ly away. How far is this in meters?

87. A certain 60.0-Hz ac power line radiates an electromagnetic wave having a maximum electric field strength of 13.0 kV/m. (a) What is the wavelength of this very-low-frequency electromagnetic wave? (b) What type of electromagnetic radiation is this wave (b) What is its maximum magnetic field strength?

88. (a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this electromagnetic radiation can ablate (reshape) the cornea is directly proportional to wavelength, how much more accurate can this UV radiation be than the shortest visible wavelength of light?

Additional Problems

89. In a region of space, the electric field is pointed along the x-axis, but its magnitude changes as described by

$$E_x = (10\,\text{N/C})\ \sin\ (20x - 500t)$$
$$E_y = E_z = 0$$

where t is in nanoseconds and x is in cm. Find the displacement current through a circle of radius 3 cm in the $x = 0$ plane at $t = 0$.

90. A microwave oven uses electromagnetic waves of frequency $f = 2.45 \times 10^9$ Hz to heat foods. The waves reflect from the inside walls of the oven to produce an interference pattern of standing waves whose antinodes are hot spots that can leave observable pit marks in some foods. The pit marks are measured to be 6.0 cm apart. Use the method employed by Heinrich Hertz to calculate the speed of electromagnetic waves this implies.

Use the Appendix D for the next two exercises

91. Galileo proposed measuring the speed of light by uncovering a lantern and having an assistant a known distance away uncover his lantern when he saw the light from Galileo's lantern, and timing the delay. How far away must the assistant be for the delay to equal the human reaction time of about 0.25 s?

92. Show that the wave equation in one dimension

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

is satisfied by any doubly differentiable function of either the form $f\,(x - vt)$ or $f\,(x + vt)$.

93. On its highest power setting, a microwave oven increases the temperature of 0.400 kg of spaghetti by 45.0 °C in 120 s. (a) What was the rate of energy absorption by the spaghetti, given that its specific heat is 3.76×10^3 J/kg · °C ? Assume the spaghetti is perfectly absorbing. (b) Find the average intensity of the microwaves, given that they are absorbed over a circular area 20.0 cm in diameter. (c) What is the peak electric field strength of the microwave? (d) What is its peak magnetic field strength?

94. A certain microwave oven projects 1.00 kW of microwaves onto a 30-cm-by-40-cm area. (a) What is its intensity in W/m^2 ? (b) Calculate the maximum electric field strength E_0 in these waves. (c) What is the maximum magnetic field strength B_0 ?

95. Electromagnetic radiation from a 5.00-mW laser is concentrated on a 1.00-mm^2 area. (a) What is the intensity in W/m^2 ? (b) Suppose a 2.00-nC electric charge is in the beam. What is the maximum electric force it experiences? (c) If the electric charge moves at 400 m/s, what maximum magnetic force can it feel?

96. A 200-turn flat coil of wire 30.0 cm in diameter acts as an antenna for FM radio at a frequency of 100 MHz. The magnetic field of the incoming electromagnetic wave is perpendicular to the coil and has a maximum strength of 1.00×10^{-12} T. (a) What power is incident on the coil? (b) What average emf is induced in the coil over one-fourth of a cycle? (c) If the radio receiver has an inductance of 2.50 μH, what capacitance must it have to resonate at 100 MHz?

97. Suppose a source of electromagnetic waves radiates uniformly in all directions in empty space where there are no absorption or interference effects. (a) Show that the intensity is inversely proportional to r^2, the distance from the source squared. (b) Show that the magnitudes of the electric and magnetic fields are inversely proportional to r.

98. A radio station broadcasts its radio waves with a power of 50,000 W. What would be the intensity of this signal if it is received on a planet orbiting Proxima Centuri, the closest star to our Sun, at 4.243 ly away?

99. The Poynting vector describes a flow of energy whenever electric and magnetic fields are present. Consider a long cylindrical wire of radius r with a current I in the wire, with resistance R and voltage V. From the expressions for the electric field along the wire and the magnetic field around the wire, obtain the magnitude and direction of the Poynting vector at the surface. Show that it accounts for an energy flow into the wire from the fields around it that accounts for the Ohmic heating of the wire.

100. The Sun's energy strikes Earth at an intensity of $1.37\ \text{kW/m}^2$. Assume as a model approximation that all of the light is absorbed. (Actually, about 30% of the light intensity is reflected out into space.)
(a) Calculate the total force that the Sun's radiation exerts on Earth.
(b) Compare this to the force of gravity between the Sun and Earth.
Earth's mass is $5.972\ \times\ 10^{24}\ \text{kg}$.

101. If a *Lightsail* spacecraft were sent on a Mars mission, by what ratio of the final force to the initial force would its propulsion be reduced when it reached Mars?

102. Lunar astronauts placed a reflector on the Moon's surface, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. (a) To what accuracy in meters can the distance to the Moon be determined, if this time can be measured to 0.100 ns? (b) What percent accuracy is this, given the average distance to the Moon is 384,400 km?

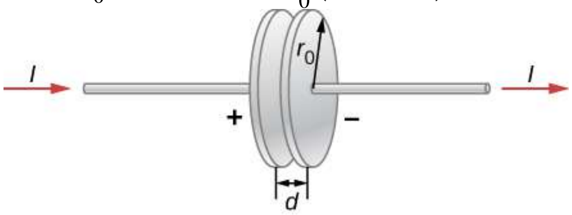
103. Radar is used to determine distances to various objects by measuring the round-trip time for an echo from the object. (a) How far away is the planet Venus if the echo time is 1000 s? (b) What is the echo time for a car 75.0 m from a highway police radar unit? (c) How accurately (in nanoseconds) must you be able to measure the echo time to an airplane 12.0 km away to determine its distance within 10.0 m?

104. Calculate the ratio of the highest to lowest frequencies of electromagnetic waves the eye can see, given the wavelength range of visible light is from 380 to 760 nm. (Note that the ratio of highest to lowest frequencies the ear can hear is 1000.)

105. How does the wavelength of radio waves for an AM radio station broadcasting at 1030 KHz compare with the wavelength of the lowest audible sound waves (of 20 Hz). The speed of sound in air at $20\ °\text{C}$ is about 343 m/s.

Challenge Problems

106. A parallel-plate capacitor with plate separation d is connected to a source of emf that places a time-dependent voltage $V(t)$ across its circular plates of radius r_0 and area $A = \pi r_0^2$ (see below).



(a) Write an expression for the time rate of change of energy inside the capacitor in terms of $V(t)$ and $dV(t)/dt$.
(b) Assuming that $V(t)$ is increasing with time, identify the directions of the electric field lines inside the capacitor and of the magnetic field lines at the edge of the region between the plates, and then the direction of the Poynting vector $\vec{\mathbf{S}}$ at this location.
(c) Obtain expressions for the time dependence of $E(t)$, for $B(t)$ from the displacement current, and for the magnitude of the Poynting vector at the edge of the region between the plates.
(d) From $\vec{\mathbf{S}}$, obtain an expression in terms of $V(t)$ and $dV(t)/dt$ for the rate at which electromagnetic field energy enters the region between the plates.
(e) Compare the results of parts (a) and (d) and explain the relationship between them.

107. A particle of cosmic dust has a density $\rho = 2.0 \text{ g/cm}^3$. (a) Assuming the dust particles are spherical and light absorbing, and are at the same distance as Earth from the Sun, determine the particle size for which radiation pressure from sunlight is equal to the Sun's force of gravity on the dust particle. (b) Explain how the forces compare if the particle radius is smaller. (c) Explain what this implies about the sizes of dust particle likely to be present in the inner solar system compared with outside the Oort cloud.

APPENDIX A

Units

Quantity	Common Symbol	Unit	Unit in Terms of Base SI Units
Acceleration	$\vec{\mathbf{a}}$	m/s^2	m/s^2
Amount of substance	n	**mole**	mol
Angle	θ, ϕ	radian (rad)	
Angular acceleration	$\vec{\alpha}$	rad/s^2	s^{-2}
Angular frequency	ω	rad/s	s^{-1}
Angular momentum	$\vec{\mathbf{L}}$	kg · m^2/s	kg · m^2/s
Angular velocity	$\vec{\omega}$	rad/s	s^{-1}
Area	A	m^2	m^2
Atomic number	Z		
Capacitance	C	farad (F)	A^2 · s^4/kg · m^2
Charge	q, Q, e	coulomb (C)	A · s
Charge density:			
Line	λ	C/m	A · s/m
Surface	σ	C/m^2	A · s/m^2
Volume	ρ	C/m^3	A · s/m^3
Conductivity	σ	1/Ω · m	A^2 · s^3/kg · m^3
Current	I	**ampere**	A
Current density	$\vec{\mathbf{J}}$	A/m^2	A/m^2
Density	ρ	kg/m^3	kg/m^3
Dielectric constant	κ		
Electric dipole moment	$\vec{\mathbf{p}}$	C · m	A · s · m

Quantity	Common Symbol	Unit	Unit in Terms of Base SI Units
Electric field	$\vec{\mathbf{E}}$	N/C	$\text{kg} \cdot \text{m/A} \cdot \text{s}^3$
Electric flux	Φ	$\text{N} \cdot \text{m}^2\text{/C}$	$\text{kg} \cdot \text{m}^3\text{/A} \cdot \text{s}^3$
Electromotive force	ε	volt (V)	$\text{kg} \cdot \text{m}^2\text{/A} \cdot \text{s}^3$
Energy	E,U,K	joule (J)	$\text{kg} \cdot \text{m}^2\text{/s}^2$
Entropy	S	J/K	$\text{kg} \cdot \text{m}^2\text{/s}^2 \cdot \text{K}$
Force	$\vec{\mathbf{F}}$	newton (N)	$\text{kg} \cdot \text{m/s}^2$
Frequency	f	hertz (Hz)	s^{-1}
Heat	Q	joule (J)	$\text{kg} \cdot \text{m}^2\text{/s}^2$
Inductance	L	henry (H)	$\text{kg} \cdot \text{m}^2\text{/A}^2 \cdot \text{s}^2$
Length:	ℓ, L	**meter**	m
Displacement	$\Delta x, \Delta\vec{\mathbf{r}}$		
Distance	d, h		
Position	$x, y, z, \vec{\mathbf{r}}$		
Magnetic dipole moment	$\vec{\mu}$	$\text{N} \cdot \text{J/T}$	$\text{A} \cdot \text{m}^2$
Magnetic field	$\vec{\mathbf{B}}$	tesla (T) $= \left(\text{Wb/m}^2\right)$	$\text{kg/A} \cdot \text{s}^2$
Magnetic flux	Φ_m	weber (Wb)	$\text{kg} \cdot \text{m}^2\text{/A} \cdot \text{s}^2$
Mass	m, M	**kilogram**	kg
Molar specific heat	C	$\text{J/mol} \cdot \text{K}$	$\text{kg} \cdot \text{m}^2\text{/s}^2 \cdot \text{mol} \cdot \text{K}$
Moment of inertia	I	$\text{kg} \cdot \text{m}^2$	$\text{kg} \cdot \text{m}^2$
Momentum	$\vec{\mathbf{p}}$	$\text{kg} \cdot \text{m/s}$	$\text{kg} \cdot \text{m/s}$
Period	T	s	s
Permeability of free space	μ_0	$\text{N/A}^2 = \text{(H/m)}$	$\text{kg} \cdot \text{m/A}^2 \cdot \text{s}^2$
Permittivity of free space	ε_0	$\text{C}^2\text{/N} \cdot \text{m}^2 = \text{(F/m)}$	$\text{A}^2 \cdot \text{s}^4\text{/kg} \cdot \text{m}^3$
Potential	V	volt (V) = (J/C)	$\text{kg} \cdot \text{m}^2\text{/A} \cdot \text{s}^3$

Quantity	Common Symbol	Unit	Unit in Terms of Base SI Units
Power	P	watt (W) = (J/s)	$\text{kg} \cdot \text{m}^2/\text{s}^3$
Pressure	p	pastcal (P) = $\left(\text{N/m}^2\right)$	$\text{kg/m} \cdot \text{s}^2$
Resistance	R	ohm (Ω) = (V/A)	$\text{kg} \cdot \text{m}^2/\text{A}^2 \cdot \text{s}^3$
Specific heat	c	$\text{J/kg} \cdot \text{K}$	$\text{m}^2/\text{s}^2 \cdot \text{K}$
Speed	v	m/s	m/s
Temperature	T	**kelvin**	K
Time	t	**second**	s
Torque	$\vec{\tau}$	$\text{N} \cdot \text{m}$	$\text{kg} \cdot \text{m}^2/\text{s}^2$
Velocity	$\vec{\mathbf{v}}$	m/s	m/s
Volume	V	m^3	m^3
Wavelength	λ	m	m
Work	W	joule (J) = $(\text{N} \cdot \text{m})$	$\text{kg} \cdot \text{m}^2/\text{s}^2$

Table A1 Units Used in Physics (Fundamental units in bold)

APPENDIX B

Conversion Factors

	m	cm	km
1 meter	1	10^2	10^{-3}
1 centimeter	10^{-2}	1	10^{-5}
1 kilometer	10^3	10^5	1
1 inch	2.540×10^{-2}	2.540	2.540×10^{-5}
1 foot	0.3048	30.48	3.048×10^{-4}
1 mile	1609	1.609×10^4	1.609
1 angstrom	10^{-10}		
1 fermi	10^{-15}		
1 light-year			9.460×10^{12}
	in.	**ft**	**mi**
1 meter	39.37	3.281	6.214×10^{-4}
1 centimeter	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 kilometer	3.937×10^4	3.281×10^3	0.6214
1 inch	1	8.333×10^{-2}	1.578×10^{-5}
1 foot	12	1	1.894×10^{-4}
1 mile	6.336×10^4	5280	1

Table B1 Length

Area

$1\ \text{cm}^2 = 0.155\ \text{in.}^2$

$1\ \text{m}^2 = 10^4\ \text{cm}^2 = 10.76\ \text{ft}^2$

$1\ \text{in.}^2 = 6.452\ \text{cm}^2$

$1\ \text{ft}^2 = 144\ \text{in.}^2 = 0.0929\ \text{m}^2$

Volume

1 liter $= 1000\ \text{cm}^3 = 10^{-3}\ \text{m}^3 = 0.03531\ \text{ft}^3 = 61.02\ \text{in.}^3$

1 $\text{ft}^3 = 0.02832\ \text{m}^3 = 28.32$ liters $= 7.477$ gallons

1 gallon $= 3.788$ liters

	s	min	h	day	yr
1 second	1	1.667×10^{-2}	2.778×10^{-4}	1.157×10^{-5}	3.169×10^{-8}
1 minute	60	1	1.667×10^{-2}	6.944×10^{-4}	1.901×10^{-6}
1 hour	3600	60	1	4.167×10^{-2}	1.141×10^{-4}
1 day	8.640×10^{4}	1440	24	1	2.738×10^{-3}
1 year	3.156×10^{7}	5.259×10^{5}	8.766×10^{3}	365.25	1

Table B2 Time

	m/s	cm/s	ft/s	mi/h
1 meter/second	1	10^{2}	3.281	2.237
1 centimeter/second	10^{-2}	1	3.281×10^{-2}	2.237×10^{-2}
1 foot/second	0.3048	30.48	1	0.6818
1 mile/hour	0.4470	44.70	1.467	1

Table B3 Speed

Acceleration

$1\ \text{m/s}^2 = 100\ \text{cm/s}^2 = 3.281\ \text{ft/s}^2$

$1\ \text{cm/s}^2 = 0.01\ \text{m/s}^2 = 0.03281\ \text{ft/s}^2$

$1\ \text{ft/s}^2 = 0.3048\ \text{m/s}^2 = 30.48\ \text{cm/s}^2$

$1\ \text{mi/h} \cdot \text{s} = 1.467\ \text{ft/s}^2$

	kg	g	slug	u
1 kilogram	1	10^{3}	6.852×10^{-2}	6.024×10^{26}
1 gram	10^{-3}	1	6.852×10^{-5}	6.024×10^{23}
1 slug	14.59	1.459×10^{4}	1	8.789×10^{27}
1 atomic mass unit	1.661×10^{-27}	1.661×10^{-24}	1.138×10^{-28}	1
1 metric ton	1000			

Table B4 Mass

	N	dyne	lb
1 newton	1	10^5	0.2248
1 dyne	10^{-5}	1	2.248×10^{-6}
1 pound	4.448	4.448×10^5	1

Table B5 Force

	Pa	dyne/cm^2	atm	cmHg	lb/in.^{2}
1 pascal	1	10	9.869×10^{-6}	7.501×10^{-4}	1.450×10^{-4}
1 dyne/centimeter^2	10^{-1}	1	9.869×10^{-7}	7.501×10^{-5}	1.450×10^{-5}
1 atmosphere	1.013×10^5	1.013×10^6	1	76	14.70
1 centimeter mercury*	1.333×10^3	1.333×10^4	1.316×10^{-2}	1	0.1934
1 pound/inch^2	6.895×10^3	6.895×10^4	6.805×10^{-2}	5.171	1
1 bar	10^5				
1 torr				1 (mmHg)	

*Where the acceleration due to gravity is 9.80665 m/s^2 and the temperature is 0°C

Table B6 Pressure

	J	erg	ft.lb
1 joule	1	10^7	0.7376
1 erg	10^{-7}	1	7.376×10^{-8}
1 foot-pound	1.356	1.356×10^7	1
1 electron-volt	1.602×10^{-19}	1.602×10^{-12}	1.182×10^{-19}
1 calorie	4.186	4.186×10^7	3.088
1 British thermal unit	1.055×10^3	1.055×10^{10}	7.779×10^2
1 kilowatt-hour	3.600×10^6		
	eV	**cal**	**Btu**
1 joule	6.242×10^{18}	0.2389	9.481×10^{-4}
1 erg	6.242×10^{11}	2.389×10^{-8}	9.481×10^{-11}

	J	erg	ft.lb
1 foot-pound	8.464×10^{18}	0.3239	1.285×10^{-3}
1 electron-volt	1	3.827×10^{-20}	1.519×10^{-22}
1 calorie	2.613×10^{19}	1	3.968×10^{-3}
1 British thermal unit	6.585×10^{21}	2.520×10^{2}	1

Table B7 Work, Energy, Heat

Power

$1\ \text{W} = 1\ \text{J/s}$

$1\ \text{hp} = 746\ \text{W} = 550\ \text{ft} \cdot \text{lb/s}$

$1\ \text{Btu/h} = 0.293\ \text{W}$

Angle

$1\ \text{rad} = 57.30° = 180°/\pi$

$1° = 0.01745\ \text{rad} = \pi/180\ \text{rad}$

$1\ \text{revolution} = 360° = 2\pi\ \text{rad}$

$1\ \text{rev/min (rpm)} = 0.1047\ \text{rad/s}$

APPENDIX C

Fundamental Constants

Quantity	Symbol	Value
Atomic mass unit	u	$1.660\ 538\ 782\ (83) \times 10^{-27}$ kg $931.494\ 028\ (23)$ MeV/c^2
Avogadro's number	N_A	$6.02214076 \times 10^{23}$ reciprocal mole$\left(\text{mol}^{-1}\right)$
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	$9.274\ 009\ 15\ (23) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \frac{\hbar^2}{m_e e^2 k_e}$	$5.291\ 772\ 085\ 9\ (36) \times 10^{-11}$ m
Boltzmann's constant	$k_B = \frac{R}{N_A}$	1.380649×10^{-23} joule per kelvin$\left(\text{J} \cdot \text{K}^{-1}\right)$
Compton wavelength	$\lambda_C = \frac{h}{m_e c}$	$2.426\ 310\ 217\ 5\ (33) \times 10^{-12}$ m
Coulomb constant	$k_e = \frac{1}{4\pi\varepsilon_0}$	$8.987\ 551\ 788... \times 10^9$ N · m^2/C^2 (exact)
Deuteron mass	m_d	$3.343\ 583\ 20\ (17) \times 10^{-27}$ kg $2.013\ 553\ 212\ 724\ (78)$ u $1875.612\ 859$ MeV/c^2
Electron mass	m_e	$9.109\ 382\ 15\ (45) \times 10^{-31}$ kg $5.485\ 799\ 094\ 3\ (23) \times 10^{-4}$ u $0.510\ 998\ 910\ (13)$ MeV/c^2
Electron volt	eV	$1.602\ 176\ 487\ (40) \times 10^{-19}$ J
Elementary charge	e	$1.602176634 \times 10^{-19}$ C
Gas constant	R	$8.314\ 472\ (15)$ J/mol · K
Gravitational constant	G	$6.674\ 28\ (67) \times 10^{-11}$ N · m^2/kg^2
Neutron mass	m_n	$1.674\ 927\ 211\ (84) \times 10^{-27}$ kg $1.008\ 664\ 915\ 97\ (43)$ u $939.565\ 346\ (23)$ MeV/c^2

Quantity	Symbol	Value
Nuclear magneton	$\mu_n = \frac{e\hbar}{2m_p}$	$5.050\,783\,24\ (13) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T · m/A (exact)
Permittivity of free space	$\varepsilon_0 = \frac{1}{\mu_0 c^2}$	$8.854\,187\,817... \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$ (exact)
Planck’s constant	h $\hbar = \frac{h}{2\pi}$	$6.62607015 \times 10^{-34}\ \mathrm{kg \cdot m^2 \cdot s^{-1}}$ $1.05457182 \times 10^{-34}\ \mathrm{kg \cdot m^2 \cdot s^{-1}}$
Proton mass	m_p	$1.672\,621\,637\ (83) \times 10^{-27}$ kg $1.007\,276\,466\,77\ (10)$ u $938.272\,013\ (23)\ \mathrm{MeV}/c^2$
Rydberg constant	R_H	$1.097\,373\,156\,852\,7\ (73) \times 10^7\ \mathrm{m^{-1}}$
Speed of light in vacuum	c	$2.997\,924\,58 \times 10^8$ m/s (exact)

Table C1 Fundamental Constants *Note:* These constants are the values recommended in 2006 by CODATA, based on a least-squares adjustment of data from different measurements. The numbers in parentheses for the values represent the uncertainties of the last two digits.

Useful combinations of constants for calculations:

$hc = 12{,}400\ \mathrm{eV \cdot \AA} = 1240\ \mathrm{eV \cdot nm} = 1240\ \mathrm{MeV \cdot fm}$

$\hbar c = 1973\ \mathrm{eV \cdot \AA} = 197.3\ \mathrm{eV \cdot nm} = 197.3\ \mathrm{MeV \cdot fm}$

$k_e e^2 = 14.40\ \mathrm{eV \cdot \AA} = 1.440\ \mathrm{eV \cdot nm} = 1.440\ \mathrm{MeV \cdot fm}$

$k_\mathrm{B} T = 0.02585$ eV at $T = 300$ K

APPENDIX D

Astronomical Data

Celestial Object	Mean Distance from Sun (million km)	Period of Revolution (d = days) (y = years)	Period of Rotation at Equator	Eccentricity of Orbit
Sun	–	–	27 d	–
Mercury	57.9	88 d	59 d	0.206
Venus	108.2	224.7 d	243 d	0.007
Earth	149.6	365.25 d	23 h 56 min 4 s	0.017
Mars	227.9	687 d	24 h 37 min 23 s	0.093
Jupiter	778.4	11.9 y	9 h 50 min 30 s	0.048
Saturn	1426.7	29.5 y	10 h 14 min	0.054
Uranus	2871.0	84.0 y	17 h 14 min	0.047
Neptune	4498.3	164.8 y	16 h	0.009
Earth's Moon	149.6 (0.386 from Earth)	27.3 d	27.3 d	0.055
Celestial Object	**Equatorial Diameter (km)**	**Mass (Earth = 1)**	**Density (g/cm^3)**	
Sun	1,392,000	333,000.00	1.4	
Mercury	4879	0.06	5.4	
Venus	12,104	0.82	5.2	
Earth	12,756	1.00	5.5	
Mars	6794	0.11	3.9	
Jupiter	142,984	317.83	1.3	
Saturn	120,536	95.16	0.7	
Uranus	51,118	14.54	1.3	
Neptune	49,528	17.15	1.6	

Celestial Object	Mean Distance from Sun (million km)	Period of Revolution (d = days) (y = years)	Period of Rotation at Equator	Eccentricity of Orbit
Earth's Moon	3476	0.01	3.3	

Table D1 Astronomical Data

Other Data:

Mass of Earth: 5.97×10^{24} kg

Mass of the Moon: 7.36×10^{22} kg

Mass of the Sun: 1.99×10^{30} kg

APPENDIX E

Mathematical Formulas

Quadratic formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Triangle of base b and height h	Area $= \frac{1}{2}bh$	
Circle of radius r	Circumference $= 2\pi r$	Area $= \pi r^2$
Sphere of radius r	Surface area $= 4\pi r^2$	Volume $= \frac{4}{3}\pi r^3$
Cylinder of radius r and height h	Area of curved surface $= 2\pi rh$	Volume $= \pi r^2 h$

Table E1 Geometry

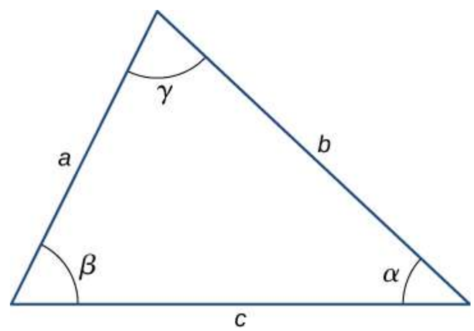
Trigonometry

Trigonometric Identities

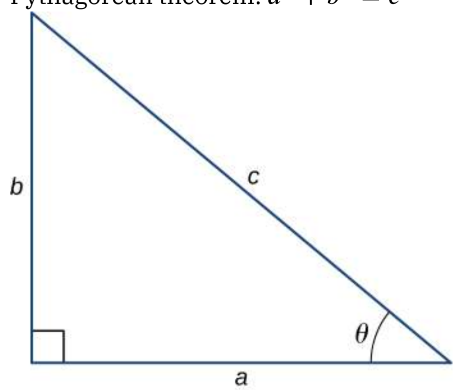
1. $\sin\theta = 1/\csc\theta$
2. $\cos\theta = 1/\sec\theta$
3. $\tan\theta = 1/\cot\theta$
4. $\sin\left(90^0 - \theta\right) = \cos\theta$
5. $\cos\left(90^0 - \theta\right) = \sin\theta$
6. $\tan\left(90^0 - \theta\right) = \cot\theta$
7. $\sin^2\theta + \cos^2\theta = 1$
8. $\sec^2\theta - \tan^2\theta = 1$
9. $\tan\theta = \sin\theta/\cos\theta$
10. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
11. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
12. $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$
13. $\sin 2\theta = 2\sin\theta\cos\theta$
14. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
15. $\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$
16. $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$

Triangles

1. Law of sines: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
2. Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$



3. Pythagorean theorem: $a^2 + b^2 = c^2$



Series expansions

1. Binomial theorem: $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \cdots$
2. $(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \cdots \left(x^2 < 1\right)$
3. $(1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \cdots \left(x^2 < 1\right)$
4. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
5. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$
6. $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$
7. $e^x = 1 + x + \frac{x^2}{2!} + \cdots$
8. $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots (|x| < 1)$

Derivatives

1. $\frac{d}{dx}[af(x)] = a\frac{d}{dx}f(x)$
2. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
3. $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$
4. $\frac{d}{dx}f(u) = \left[\frac{d}{du}f(u)\right]\frac{du}{dx}$
5. $\frac{d}{dx}x^m = mx^{m-1}$
6. $\frac{d}{dx}\sin x = \cos x$
7. $\frac{d}{dx}\cos x = -\sin x$
8. $\frac{d}{dx}\tan x = \sec^2 x$
9. $\frac{d}{dx}\cot x = -\csc^2 x$
10. $\frac{d}{dx}\sec x = \tan x \sec x$
11. $\frac{d}{dx}\csc x = -\cot x \csc x$
12. $\frac{d}{dx}e^x = e^x$

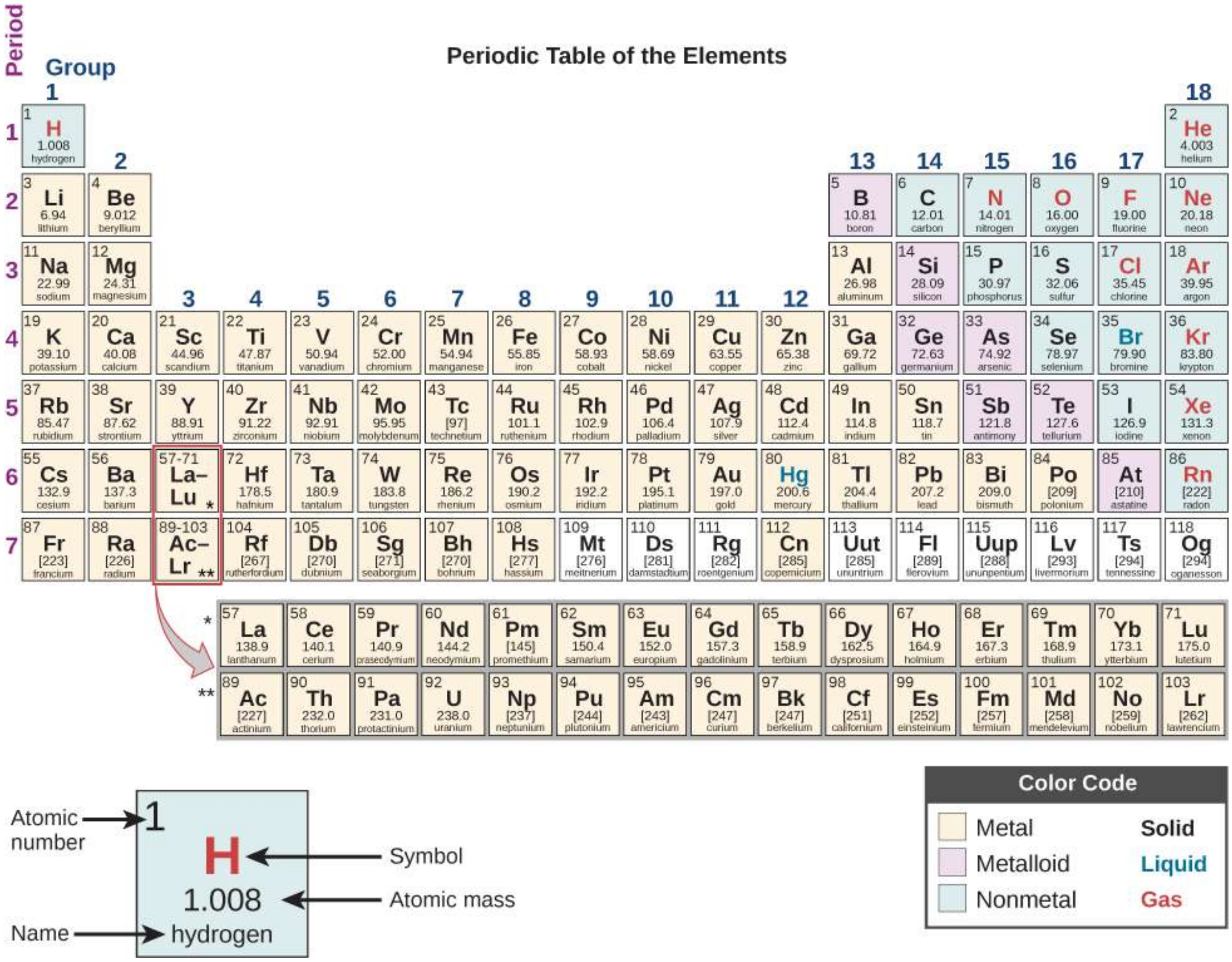
13. $\frac{d}{dx} \ln x = \frac{1}{x}$
14. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$
16. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

Integrals

1. $\int af(x)\,dx = a\int f(x)\,dx$
2. $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$
3. $\int x^m\,dx = \frac{x^{m+1}}{m+1}\ (m \neq -1)$
   $= \ln x (m = -1)$
4. $\int \sin x\,dx = -\cos x$
5. $\int \cos x\,dx = \sin x$
6. $\int \tan x\,dx = \ln|\sec x|$
7. $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$
8. $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$
9. $\int \sin ax \cos ax\,dx = -\frac{\cos 2ax}{4a}$
10. $\int e^{ax}\,dx = \frac{1}{a}e^{ax}$
11. $\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1)$
12. $\int \ln ax\,dx = x \ln ax - x$
13. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$
14. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right|$
15. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a}$
16. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$
17. $\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$
18. $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$

APPENDIX F

Chemistry



APPENDIX G

The Greek Alphabet

Name	Capital	Lowercase	Name	Capital	Lowercase
Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ε	Rho	P	ρ
Zeta	Z	ζ	Sigma	Σ	σ
Eta	H	η	Tau	T	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Table G1 The Greek Alphabet

ANSWER KEY

Chapter 1

Check Your Understanding

1.1 The actual amount (mass) of gasoline left in the tank when the gauge hits “empty” is less in the summer than in the winter. The gasoline has the same volume as it does in the winter when the “add fuel” light goes on, but because the gasoline has expanded, there is less mass.

1.2 Not necessarily, as the thermal stress is also proportional to Young’s modulus.

1.3 To a good approximation, the heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case. (As we will see in the next section, the answer would have been different if the object had been made of some substance that changes phase anywhere between $30\ ^\circ\text{C}$ and $50\ ^\circ\text{C}$.)

1.4 The ice and liquid water are in thermal equilibrium, so that the temperature stays at the freezing temperature as long as ice remains in the liquid. (Once all of the ice melts, the water temperature will start to rise.)

1.5 Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be transferred from the air, even if the air is above $0\ ^\circ\text{C}$.

1.6 Conduction: Heat transfers into your hands as you hold a hot cup of coffee. Convection: Heat transfers as the barista “steams” cold milk to make hot cocoa. Radiation: Heat transfers from the Sun to a jar of water with tea leaves in it to make “Sun tea.” A great many other answers are possible.

1.7 Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled $\left(A_{\text{final}} = (2d)^2 = 4d^2 = 4A_{\text{initial}}\right)$. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two: $P_{\text{final}} = \frac{kA_{\text{final}}(T_{\text{h}}-T_{\text{c}})}{d_{\text{final}}} = \frac{k(4A_{\text{final}}(T_{\text{h}}-T_{\text{c}}))}{2d_{\text{initial}}} = 2\frac{kA_{\text{final}}(T_{\text{h}}-T_{\text{c}})}{d_{\text{initial}}} = 2P_{\text{initial}}.$

1.8 Using a fan increases the flow of air: Warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air “feels” cooler than still air.

1.9 The radiated heat is proportional to the fourth power of the *absolute temperature*. Because $T_1 = 293\ \text{K}$ and $T_2 = 313\ \text{K}$, the rate of heat transfer increases by about 30% of the original rate.

Conceptual Questions

1. They are at the same temperature, and if they are placed in contact, no net heat flows between them.

3. The reading will change.

5. The cold water cools part of the inner surface, making it contract, while the rest remains expanded. The strain is too great for the strength of the material. Pyrex contracts less, so it experiences less strain.

7. In principle, the lid expands more than the jar because metals have higher coefficients of expansion than glass. That should make unscrewing the lid easier. (In practice, getting the lid and jar wet may make gripping them more difficult.)

9. After being heated, the length is $(1 + 300\alpha)\ (1\ \text{m})$. After being cooled, the length is $(1 - 300\ \alpha)\ (1 + 300\ \alpha)\ (1\ \text{m})$. That answer is not 1 m, but it should be. The explanation is that even if α is exactly constant, the relation $\Delta L = \alpha L \Delta T$ is strictly true only in the limit of small ΔT. Since α values are small, the discrepancy is unimportant in practice.

11. Temperature differences cause heat transfer.

13. No, it is stored as thermal energy. A thermodynamic system does not have a well-defined quantity of heat.

15. It raises the boiling point, so the water, which the food gains heat from, is at a higher temperature.

17. Yes, by raising the pressure above 56 atm.

19. work

21. $0\ ^\circ\text{C}$ (at or near atmospheric pressure)

23. Condensation releases heat, so it speeds up the melting.
25. Because of water's high specific heat, it changes temperature less than land. Also, evaporation reduces temperature rises. The air tends to stay close to equilibrium with the water, so its temperature does not change much where there's a lot of water around, as in San Francisco but not Sacramento.
27. The liquid is oxygen, whose boiling point is above that of nitrogen but whose melting point is below the boiling point of liquid nitrogen. The crystals that sublime are carbon dioxide, which has no liquid phase at atmospheric pressure. The crystals that melt are water, whose melting point is above carbon dioxide's sublimation point. The water came from the instructor's breath.
29. Increasing circulation to the surface will warm the person, as the temperature of the water is warmer than human body temperature. Sweating will cause no evaporative cooling under water or in the humid air immediately above the tub.
31. It spread the heat over the area above the heating elements, evening the temperature there, but does not spread the heat much beyond the heating elements.
33. Heat is conducted from the fire through the fire box to the circulating air and then convected by the air into the room (forced convection).
35. The tent is heated by the Sun and transfers heat to you by all three processes, especially radiation.
37. If shielded, it measures the air temperature. If not, it measures the combined effect of air temperature and net radiative heat gain from the Sun.
39. Turn the thermostat down. To have the house at the normal temperature, the heating system must replace all the heat that was lost. For all three mechanisms of heat transfer, the greater the temperature difference between inside and outside, the more heat is lost and must be replaced. So the house should be at the lowest temperature that does not allow freezing damage.
41. Air is a good insulator, so there is little conduction, and the heated air rises, so there is little convection downward.

Problems

43. That must be Celsius. Your Fahrenheit temperature is $102\ °\text{F}$. Yes, it is time to get treatment.
45. a. $\Delta T_\text{C} = 22.2\ °\text{C}$; b. We know that $\Delta T_\text{F} = T_\text{F2} - T_\text{F1}$. We also know that $T_\text{F2} = \frac{9}{5}T_\text{C2} + 32$ and $T_\text{F1} = \frac{9}{5}T_\text{C1} + 32$. So, substituting, we have $\Delta T_\text{F} = \left(\frac{9}{5}T_\text{C2} + 32\right) - \left(\frac{9}{5}T_\text{C1} + 32\right)$. Partially solving and rearranging the equation, we have $\Delta T_\text{F} = \frac{9}{5}(T_\text{C2} - T_\text{C1})$. Therefore, $\Delta T_\text{F} = \frac{9}{5}\Delta T_\text{C}$.
47. a. $-40°$; b. 575 K
49. Using Table 1.2 to find the coefficient of thermal expansion of marble:
    $L = L_0 + \Delta L = L_0\,(1 + \alpha\Delta T) = 170\ \text{m}\left[1 + \left(2.5 \times 10^{-6}/°\text{C}\right)(-45.0\ °\text{C})\right] = 169.98\ \text{m}.$
    (Answer rounded to five significant figures to show the slight difference in height.)
51. We use β instead of α since this is a volume expansion with constant surface area. Therefore:
    $\Delta L = \alpha L\Delta T = \left(6.0 \times 10^{-5}/°\text{C}\right)(0.0300\ \text{m})(3.00\ °\text{C}) = 5.4 \times 10^{-6}\ \text{m}.$
53. On the warmer day, our tape measure will expand linearly. Therefore, each measured dimension will be smaller than the actual dimension of the land. Calling these measured dimensions l' and w', we will find a new area, A. Let's calculate these measured dimensions:
    $l' = l_0 - \Delta l = (20\ \text{m}) - (20\ °\text{C})(20\ \text{m})\left(\frac{1.2 \times 10^{-5}}{°\text{C}}\right) = 19.9952\ \text{m};$
    $A' = l \times w' = (29.9928\ \text{m})(19.9952\ \text{m}) = 599.71\ \text{m}^2;$
    $\text{Cost change} = (A - A')\left(\frac{\$60{,}000}{\text{m}^2}\right) = \left((600 - 599.71)\ \text{m}^2\right)\left(\frac{\$60{,}000}{\text{m}^2}\right) = \$17{,}000.$
    Because the area gets smaller, the price of the land *decreases* by about \$17,000.
55. a. Use Table 1.2 to find the coefficients of thermal expansion of steel and aluminum. Then
    $\Delta L_\text{Al} - \Delta L_\text{steel} = (\alpha_\text{Al} - \alpha_\text{steel})L_0\Delta T = \left(\frac{2.5 \times 10^{-5}}{°\text{C}} - \frac{1.2 \times 10^{-5}}{°\text{C}}\right)(1.00\ \text{m})(22\ °\text{C}) = 2.9 \times 10^{-4}\ \text{m}.$
    b. By the same method with $L_0 = 30.0\ \text{m}$, we have $\Delta L = 8.6 \times 10^{-3}\ \text{m}$.
57. $\Delta V = 0.475\ \text{L}$
59. If we start with the freezing of water, then it would expand to

$\left(1 \text{ m}^3\right)\left(\frac{1000 \text{ kg/m}^3}{917 \text{ kg/m}^3}\right) = 1.09 \text{ m}^3 = 1.98 \times 10^8 \text{ N/m}^2$ of ice.

61. $m = 5.20 \times 10^8$ J

63. $Q = mc\Delta T \Rightarrow \Delta T = \frac{Q}{mc}$; a. 21.0 °C; b. 25.0 °C; c. 29.3 °C; d. 50.0 °C

65. $Q = mc\Delta T \Rightarrow c = \frac{Q}{m\Delta T} = \frac{1.04 \text{ kcal}}{(0.250 \text{ kg})(45.0 \text{ °C})} = 0.0924 \text{ kcal/kg} \cdot \text{°C}$. It is copper.

67. a. $Q = m_w c_w \Delta T + m_{A1} c_{A1} \Delta T = (m_w c_w + m_{A1} c_{A1}) \Delta T$;

$$Q = \begin{bmatrix} (0.500 \text{ kg})(1.00 \text{ kcal/kg} \cdot \text{°C}) + \\ (0.100 \text{ kg})(0.215 \text{ kcal/kg} \cdot \text{°C}) \end{bmatrix} (54.9 \text{ °C}) = 28.63 \text{ kcal};$$

$\frac{Q}{m_p} = \frac{28.63 \text{ kcal}}{5.00 \text{ g}} = 5.73 \text{ kcal/g}$; b. $\frac{Q}{m_p} = \frac{200 \text{ kcal}}{33 \text{ g}} = 6 \text{ kcal/g}$, which is consistent with our results to part (a), to one significant figure.

69. 0.139 °C

71. It should be lower. The beaker will not make much difference: 16.3 °C

73. a. 1.00×10^5 J; b. 3.68×10^5 J; c. The ice is much more effective in absorbing heat because it first must be melted, which requires a lot of energy, and then it gains the same amount of heat as the bag that started with water. The first 2.67×10^5 J of heat is used to melt the ice, then it absorbs the 1.00×10^5 J of heat as water.

75. 58.1 g

77. Let M be the mass of pool water and m be the mass of pool water that evaporates.

$$Mc\Delta T = mL_{V(37\text{ °C})} \Rightarrow \frac{m}{M} = \frac{c\Delta T}{L_{V(37\text{ °C})}} = \frac{(1.00 \text{ kcal/kg}\cdot\text{°C})(1.50 \text{ °C})}{580 \text{ kcal/kg}} = 2.59 \times 10^{-3};$$

(Note that L_V for water at 37 °C is used here as a better approximation than L_V for 100 °C water.)

79. a. 1.47×10^{15} kg; b. 4.90×10^{20} J; c. 48.5 y

81. a. 9.35 L; b. Crude oil is less dense than water, so it floats on top of the water, thereby exposing it to the oxygen in the air, which it uses to burn. Also, if the water is under the oil, it is less able to absorb the heat generated by the oil.

83. a. 319 kcal; b. 2.00 °C

85. First bring the ice up to 0 °C and melt it with heat Q_1 : 4.74 kcal. This lowers the temperature of water by ΔT_2 : 23.15 °C. Now, the heat lost by the hot water equals that gained by the cold water (T_f is the final temperature): 20.6 °C

87. Let the subscripts r, e, v, and w represent rock, equilibrium, vapor, and water, respectively.

$m_r c_r (T_1 - T_e) = m_V L_V + m_W c_W (T_e - T_2)$;

$$\begin{aligned} m_r &= \frac{m_V L_V + m_W c_W (T_e - T_2)}{c_r (T_1 - T_e)} \\ &= \frac{(0.0250 \text{ kg})\left(2256 \times 10^3 \text{ J/kg}\right) + (3.975 \text{ kg})\left(4186 \times 10^3 \text{ J/kg}\cdot\text{°C}\right)(100 \text{ °C} - 15 \text{ °C})}{(840 \text{ J/kg}\cdot\text{°C})(500 \text{ °C} - 100 \text{ °C})} \\ &= 4.38 \text{ kg} \end{aligned}$$

89. a. 1.01×10^3 W; b. One 1-kilowatt room heater is needed.

91. 84.0 W

93. 2.59 kg

95. a. 39.7 W; b. 820 kcal

97. $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$, so that

$$\frac{(Q/t)_{\text{wall}}}{(Q/t)_{\text{window}}} = \frac{k_{\text{wall}} A_{\text{wall}} d_{\text{window}}}{k_{\text{window}} A_{\text{window}} d_{\text{wall}}} = \frac{(2 \times 0.042 \text{ J/s}\cdot\text{m}\cdot\text{°C})\left(10.0 \text{ m}^2\right)\left(0.750 \times 10^{-2} \text{m}\right)}{(0.84 \text{ J/s}\cdot\text{m}\cdot\text{°C})\left(2.00 \text{ m}^2\right)\left(13.0 \times 10^{-2} \text{m}\right)}$$

This gives 0.0288 wall: window, or 35:1 window: wall

99. $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d} = \frac{kA\Delta T}{d} \Rightarrow$

$$\Delta T = \frac{d(Q/t)}{kA} = \frac{\left(6.00 \times 10^{-3} \text{ m}\right)(2256 \text{ W})}{(0.84 \text{ J/s}\cdot\text{m}\cdot\text{°C})\left(1.54 \times 10^{-2} \text{ m}^2\right)} = 1046 \text{ °C} = 1.05 \times 10^3 \text{K}$$

101. We found in the preceding problem that $P = 126\Delta T\ \text{W} \cdot °\text{C}$ as baseline energy use. So the total heat loss during this period is $Q = (126\ \text{J/s} \cdot °\text{C})\,(15.0\ °\text{C})\,(120\ \text{days})\,\left(86.4 \times 10^3\ \text{s/day}\right) = 1960 \times 10^6\ \text{J}$. At the cost of \$1/MJ, the cost is \$1960. From an earlier problem, the savings is 12% or \$235/y. We need $150\ \text{m}^2$ of insulation in the attic. At $\$4/\text{m}^2$, this is a \$500 cost. So the payback period is $\$600/\left(\$235/\text{y}\right) = 2.6$ years (excluding labor costs).

Additional Problems

103. 7.39%

105. $\frac{F}{A} = \left(210 \times 10^9\ \text{Pa}\right)\left(12 \times 10^{-6}/°\text{C}\right)(40\ °\text{C} - (-15\ °\text{C})) = 1.4 \times 10^8\ \text{N/m}^2$.

107. a. 1.06 cm; b. 1.11 cm

109. 1.7 kJ/(kg · ºC)

111. a. 1.57×10^4 kcal; b. 18.3 kW · h; c. 1.29×10^4 kcal

113. 6.3 °C. All of the ice melted.

115. 63.9 °C, all the ice melted

117. a. 83 W; b. 1.97×10^3 W; The single-pane window has a rate of heat conduction equal to 1969/83, or 24 times that of a double-pane window.

119. The rate of heat transfer by conduction is 20.0 W. On a daily basis, this is 1,728 kJ/day. Daily food intake is 2400 kcal/d $\times$ 4186 J/kcal $=$ 10,050 kJ/day. So only 17.2% of energy intake goes as heat transfer by conduction to the environment at this ΔT.

121. 620 K

Challenge Problems

123. Denoting the period by *P*, we know $P = 2\pi\sqrt{L/g}$. When the temperature increases by *dT*, the length increases by $\alpha L dT$. Then the new length is a.

$$P = 2\pi\sqrt{\frac{L+\alpha L dT}{g}} = 2\pi\sqrt{\frac{L}{g}(1+\alpha dT)} = 2\pi\sqrt{\frac{L}{g}}\left(1+\frac{1}{2}\alpha dT\right) = P\left(1+\frac{1}{2}\alpha dT\right)$$

by the binomial expansion. b. The clock runs slower, as its new period is 1.00019 s. It loses 16.4 s per day.

125. The amount of heat to melt the ice and raise it to 100 °C is not enough to condense the steam, but it is more than enough to lower the steam's temperature by 50 °C, so the final state will consist of steam and liquid water in equilibrium, and the final temperature is 100 °C; 9.5 g of steam condenses, so the final state contains 49.5 g of steam and 40.5 g of liquid water.

127. a. $dL/dT = kT/\rho L$; b. $L = \sqrt{2kTt/\rho L_\text{f}}$; c. yes

129. a. $4\left(\pi R^2\right) T_\text{s}{}^4$; b. $4e\sigma\pi R^2 T_\text{s}{}^4$; c. $8e\sigma\pi R^2 T_\text{e}{}^4$; d. $T_s^4 = 2T_e^4$; e. $e\sigma T_\text{s}^4 + \frac{1}{4}(1-A)S = \sigma T_\text{s}^4$; f. $288K$

Chapter 2

Check Your Understanding

2.1 We first need to calculate the molar mass (the mass of one mole) of niacin. To do this, we must multiply the number of atoms of each element in the molecule by the element's molar mass.

$$(6\ \text{mol of carbon})\,(12.0\ \text{g/mol}) + (5\ \text{mol hydrogen})\,(1.0\ \text{g/mol})$$
$$+ (1\ \text{mol nitrogen})\,(14\ \text{g/mol}) + (2\ \text{mol oxygen})\,(16.0\ \text{g/mol}) = 123\ \text{g/mol}$$

Then we need to calculate the number of moles in 14 mg.

$$\left(\frac{14\ \text{mg}}{123\ \text{g/mol}}\right)\left(\frac{1\ \text{g}}{1000\ \text{mg}}\right) = 1.14 \times 10^{-4}\ \text{mol}.$$

Then, we use Avogadro's number to calculate the number of molecules:

$$N = nN_A = \left(1.14 \times 10^{-4}\ \text{mol}\right)\left(6.02 \times 10^{23}\ \text{molecules/mol}\right) = 6.85 \times 10^{19}\ \text{molecules}.$$

2.2 The density of a gas is equal to a constant, the average molecular mass, times the number density *N*/*V*. From the ideal gas law, $pV = Nk_\text{B}T$, we see that $N/V = p/k_\text{B}T$. Therefore, at constant temperature, if the density and, consequently, the number density are reduced by half, the pressure must also be reduced by half, and $p_\text{f} = 0.500$ atm.

2.3 Density is mass per unit volume, and volume is proportional to the size of a body (such as the radius of a sphere) cubed. So if the distance between molecules increases by a factor of 10, then the volume occupied increases by a factor of 1000, and the density decreases by a factor of 1000. Since we assume molecules are in contact in liquids and solids, the distance between their centers is on the order of their typical size, so the distance in gases is on the order of 10 times as great.

2.4 Yes. Such fluctuations actually occur for a body of any size in a gas, but since the numbers of molecules are immense for macroscopic bodies, the fluctuations are a tiny percentage of the number of collisions, and the averages spoken of in this section vary imperceptibly. Roughly speaking, the fluctuations are inversely proportional to the square root of the number of collisions, so for small bodies, they can become significant. This was actually observed in the nineteenth century for pollen grains in water and is known as Brownian motion.

2.5 In a liquid, the molecules are very close together, constantly colliding with one another. For a gas to be nearly ideal, as air is under ordinary conditions, the molecules must be very far apart. Therefore the mean free path is much longer in the air.

2.6 As the number of moles is equal and we know the molar heat capacities of the two gases are equal, the temperature is halfway between the initial temperatures, 300 K.

Conceptual Questions

1. 2 moles, as that will contain twice as many molecules as the 1 mole of oxygen

3. pressure

5. The flame contains hot gas (heated by combustion). The pressure is still atmospheric pressure, in mechanical equilibrium with the air around it (or roughly so). The density of the hot gas is proportional to its number density N/V (neglecting the difference in composition between the gas in the flame and the surrounding air). At higher temperature than the surrounding air, the ideal gas law says that $N/V = p/k_B T$ is less than that of the surrounding air. Therefore the hot air has lower density than the surrounding air and is lifted by the buoyant force.

7. The mean free path is inversely proportional to the square of the radius, so it decreases by a factor of 4. The mean free time is proportional to the mean free path and inversely proportional to the rms speed, which in turn is inversely proportional to the square root of the mass. That gives a factor of $\sqrt{8}$ in the numerator, so the mean free time decreases by a factor of $\sqrt{2}$.

9. Since they're more massive, their gravity is stronger, so the escape velocity from them is higher. Since they're farther from the Sun, they're colder, so the speeds of atmospheric molecules including hydrogen and helium are lower. The combination of those facts means that relatively few hydrogen and helium molecules have escaped from the outer planets.

11. One where nitrogen is stored, as excess CO_2 will cause a feeling of suffocating, but excess nitrogen and insufficient oxygen will not.

13. Less, because at lower temperatures their heat capacity was only $3RT/2$.

15. a. false; b. true; c. true; d. true

17. 1200 K

Problems

19. a. 0.137 atm; b. $p_g = (1\text{ atm})\frac{T_2 V_1}{T_1 V_2} - 1\text{ atm}$. Because of the expansion of the glass, $V_2 = 0.99973$. Multiplying by that factor does not make any significant difference.

21. a. 1.79×10^{-3} mol; b. 0.227 mol; c. 1.08×10^{21} molecules for the nitrogen, 1.37×10^{23} molecules for the carbon dioxide

23. 7.84×10^{-2} mol

25. 1.87×10^{3}

27. 2.47×10^{7} molecules

29. 6.95×10^{5} Pa; 6.86 atm

31. a. 9.14×10^{6} Pa; b. 8.22×10^{6} Pa; c. 2.15 K; d. no

33. 40.7 km

35. a. 0.61 N; b. 0.20 Pa
37. a. 5.88 m/s; b. 5.89 m/s
39. 177 m/s
41. 4.54×10^3
43. a. 0.0352 mol; b. 5.65×10^{-21} J; c. 139 J
45. 21.1 kPa
47. 458 K
49. 3.22×10^3 K
51. a. 1.004; b. 764 K; c. This temperature is equivalent to 915 °F, which is high but not impossible to achieve. Thus, this process is feasible. At this temperature, however, there may be other considerations that make the process difficult. (In general, uranium enrichment by gaseous diffusion is indeed difficult and requires many passes.)
53. 65 mol
55. a. 0.76 atm; b. 0.29 atm; c. The pressure there is barely above the quickly fatal level.
57. 4.92×10^5 K; Yes, that's an impractically high temperature.
59. polyatomic
61. 3.08×10^3 J
63. 29.2 °C
65. −1.6 °C
67. 0.00157
69. About 0.072. Answers may vary slightly. A more accurate answer is 0.074.
71. a. 419 m/s; b. 472 m/s; c. 513 m/s
73. 541 K
75. 2400 K for all three parts

Additional Problems

77. a. 1.20 kg/m^3; b. 65.9 kg/m^3
79. 7.9 m
81. a. supercritical fluid; b. 3.00×10^7 Pa
83. 40.18%
85. a. 2.21×10^{27} molecules/m^3; b. 3.67×10^3 mol/m^3
87. 8.2 mm
89. a. 1080 J/kg °C; b. 12%
91. $2\sqrt{e}/3$ or about 1.10
93. a. 411 m/s; b. According to Table 2.3, the C_V of H_2S is significantly different from the theoretical value, so the ideal gas model does not describe it very well at room temperature and pressure, and the Maxwell-Boltzmann speed distribution for ideal gases may not hold very well, even less well at a lower temperature.

Challenge Problems

95. 29.5 N/m
97. Substituting $v = \sqrt{\frac{2k_B T}{m}}u$ and $dv = \sqrt{\frac{2k_B T}{m}}du$ gives

$$\int_0^\infty \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T} dv = \int_0^\infty \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_B T}\right)^{3/2}\left(\frac{2k_B T}{m}\right) u^2 e^{-u^2}\sqrt{\frac{2k_B T}{m}}du$$
$$= \int_0^\infty \frac{4}{\sqrt{\pi}} u^2 e^{-u^2} du = \frac{4}{\sqrt{\pi}}\,\frac{\sqrt{\pi}}{4} = 1$$

99. Making the scaling transformation as in the previous problems, we find that

$$\overline{v^2} = \int_0^\infty \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_B T}\right)^{3/2} v^2 v^2 e^{-mv^2/2k_B T} dv = \int_0^\infty \frac{4}{\sqrt{\pi}}\,\frac{2k_B T}{m} u^4 e^{-u^2} du.$$

As in the previous problem, we integrate by parts:

$$\int_0^\infty u^4 e^{-u^2}\,du = \left[-\tfrac{1}{2}u^3 e^{-u^2}\right]_0^\infty + \tfrac{3}{2}\int_0^\infty u^2 e^{-u^2}\,du.$$

Again, the first term is 0, and we were given in an earlier problem that the integral in the second term equals $\frac{\sqrt{\pi}}{4}$. We now have

$$\overline{v^2} = \frac{4}{\sqrt{\pi}}\frac{2k_BT}{m}\frac{3}{2}\frac{\sqrt{\pi}}{4} = \frac{3k_BT}{m}.$$

Taking the square root of both sides gives the desired result: $v_{rms} = \sqrt{\frac{3k_BT}{m}}$.

Chapter 3

Check Your Understanding

3.1 $p_2(V_2 - V_1)$

3.2 Line 1, $\Delta E_{int} = 40\,J$; line 2, $W = 50\,J$ and $\Delta E_{int} = 40\,J$; line 3, $Q = 80\,J$ and $\Delta E_{int} = 40\,J$; and line 4, $Q = 0$ and $\Delta E_{int} = 40\,J$

3.3 So that the process is represented by the curve $p = nRT/V$ on the pV plot for the evaluation of work.

3.4 1.26×10^3 J.

Conceptual Questions

1. a. SE; b. ES; c. ES

3. Some of the energy goes into changing the phase of the liquid to gas.

5. Yes, as long as the work done equals the heat added there will be no change in internal energy and thereby no change in temperature. When water freezes or when ice melts while removing or adding heat, respectively, the temperature remains constant.

7. If more work is done on the system than heat added, the internal energy of the system will actually decrease.

9. The system must be in contact with a heat source that allows heat to flow into the system.

11. Isothermal processes must be slow to make sure that as heat is transferred, the temperature does not change. Even for isobaric and isochoric processes, the system must be in thermal equilibrium with slow changes of thermodynamic variables.

13. Typically C_p is greater than C_V because when expansion occurs under constant pressure, it does work on the surroundings. Therefore, heat can go into internal energy and work. Under constant volume, all heat goes into internal energy. In this example, water contracts upon heating, so if we add heat at constant pressure, work is done on the water by surroundings and therefore, C_p is less than C_V.

15. No, it is always greater than 1.

17. An adiabatic process has a change in temperature but no heat flow. The isothermal process has no change in temperature but has heat flow.

Problems

19. $p(V - b) = -c_T$ is the temperature scale desired and mirrors the ideal gas if under constant volume.

21. $V - bpT + cT^2 = 0$

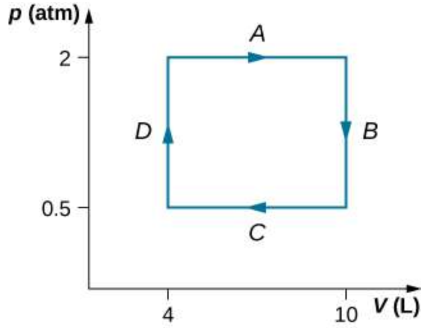
23. 74 K

25. 0.31

27. pVln(4)

29. a. 160 J; b. −160 J

31.



$W = 900\,\text{J}$

33. 3.53×10^4 J

35. a. 1:1; b. 10:1

37. a. 600 J; b. 0; c. 500 J; d. 200 J; e. 800 J; f. 500 J

39. 580 J

41. a. 600 J; b. 600 J; c. 800 J

43. a. 0; b. 160 J; c. −160 J

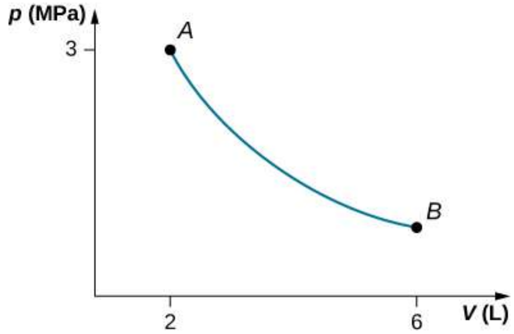
45. a. 20 J; b. 90 J

47. No work is done and they reach the same common temperature.

49. 54,500 J

51. a. $(p_1 + 3V_1^2)(V_2 - V_1) - 3V_1(V_2^2 - V_1^2) + (V_2^3 - V_1^3)$; b. $\frac{3}{2}(p_2V_2 - p_1V_1)$; c. the sum of parts (a) and (b); d. $T_1 = \frac{p_1V_1}{nR}$ and $T_2 = \frac{p_2V_2}{nR}$

53. a.



;

b. $W = 4.39\,\text{kJ}$, $\Delta E_{\text{int}} = -4.39\,\text{kJ}$

55. a. 1660 J; b. −2730 J; c. It does not depend on the process.

57. a. 700 J; b. 500 J

59. a. −3 400 J; b. 3400 J enters the gas

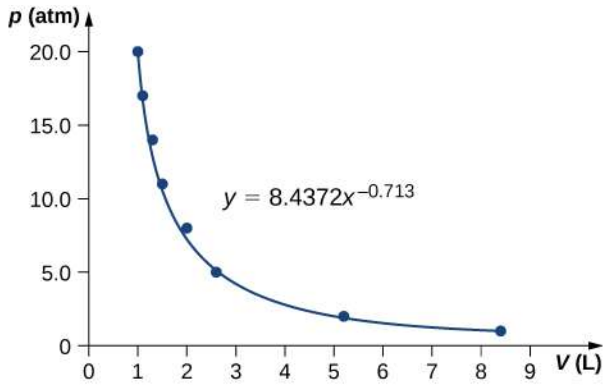
61. 100 J

63. a. 370 J; b. 100 J; c. 500 J

65. 850 J

67. pressure decreased by 0.31 times the original pressure

69.



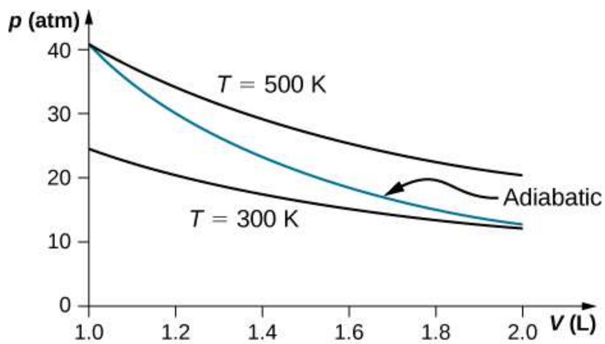
;
$\gamma = 0.713$

71. 84 K

73. An adiabatic expansion has less work done and no heat flow, thereby a lower internal energy comparing to an isothermal expansion which has both heat flow and work done. Temperature decreases during adiabatic expansion.

75. Isothermal has a greater final pressure and does not depend on the type of gas.

77.



Additional Problems

79. a. $W_{AB} = 0, W_{BC} = 2026\,\text{J}, W_{AD} = 810.4\,\text{J}, W_{DC} = 0$; b. $\Delta E_{AB} = 3600\,\text{J}, \Delta E_{BC} = 374\,\text{J}$; c. $\Delta E_{AC} = 3974\,\text{J}$; d. $Q_{ADC} = 4784\,\text{J}$; e. No, because heat was added for both parts *AD* and *DC*. There is not enough information to figure out how much is from each segment of the path.

81. 300 J

83. a. 59.5 J; b. 170 N

85. 2.4×10^3 J

87. a. 15,000 J; b. 10,000 J; c. 25,000 J

89. 78 J

91. A cylinder containing three moles of nitrogen gas is heated at a constant pressure of 2 atm. a. –1220 J; b. +1220 J

93. a. 7.6 L, 61.6 K; b. 81.3 K; c. $3.63\,\text{L}\cdot\text{atm} = 367\,\text{J}$; d. –367 J

Challenge Problems

95. a. 1700 J; b. 1200 J; c. 2400 J

97. a. 2.2 mol; b. $V_A = 2.6 \times 10^{-2}\text{m}^3$, $V_B = 7.4 \times 10^{-2}\text{m}^3$; c. $T_A = 1220\,\text{K}, T_B = 430\,\text{K}$; d. 30,500 J

Chapter 4

Check Your Understanding

4.1 A perfect heat engine would have $Q_c = 0$, which would lead to $e = 1 - Q_c/Q_h = 1$. A perfect refrigerator would need zero work, that is, $W = 0$, which leads to $K_R = Q_c/W \to \infty$.

4.2 From the engine on the right, we have $W = Q'_h - Q'_c$. From the refrigerator on the right, we have $Q_h = Q_c + W$. Thus, $W = Q'_h - Q'_c = Q_h - Q_c$.

4.3 a. $e = 1 - T_c/T_h = 0.55$; b. $Q_h = eW = 9.1$ J; c. $Q_c = Q_h - W = 4.1$ J; d. −273 °C and 400 °C

4.4 a. $K_R = T_c/(T_h - T_c) = 10.9$; b. $Q_c = K_R W = 2.18$ kJ; c. $Q_h = Q_c + W = 2.38$ kJ

4.5 When heat flows from the reservoir to the ice, the internal (mainly kinetic) energy of the ice goes up, resulting in a higher average speed and thus an average greater position variance of the molecules in the ice. The reservoir does become more ordered, but due to its much larger amount of molecules, it does not offset the change in entropy in the system.

4.6 $-Q/T_h$; Q/T_c; and $Q(T_h - T_c)/(T_h T_c)$

4.7 a. 4.71 J/K; b. −4.18 J/K; c. 0.53 J/K

Conceptual Questions

1. Some possible solutions are frictionless movement; restrained compression or expansion; energy transfer as heat due to infinitesimal temperature nonuniformity; electric current flow through a zero resistance; restrained chemical reaction; and mixing of two samples of the same substance at the same state.

3. The temperature increases since the heat output behind the refrigerator is greater than the cooling from the inside of the refrigerator.

5. If we combine a perfect engine and a real refrigerator with the engine converting heat Q from the hot reservoir into work $W = Q$ to drive the refrigerator, then the heat dumped to the hot reservoir by the refrigerator will be $W + \Delta Q$, resulting in a perfect refrigerator transferring heat ΔQ from the cold reservoir to hot reservoir without any other effect.

7. Heat pumps can efficiently extract heat from the ground to heat on cooler days or pull heat out of the house on warmer days. The disadvantage of heat pumps are that they are more costly than alternatives, require maintenance, and will not work efficiently when temperature differences between the inside and outside are very large. Electric heating is much cheaper to purchase than a heat pump; however, it may be more costly to run depending on the electric rates and amount of usage.

9. A nuclear reactor needs to have a lower temperature to operate, so its efficiency will not be as great as a fossil-fuel plant. This argument does not take into consideration the amount of energy per reaction: Nuclear power has a far greater energy output than fossil fuels.

11. In order to increase the efficiency, the temperature of the hot reservoir should be raised, and the cold reservoir should be lowered as much as possible. This can be seen in Equation 4.3.

13. adiabatic and isothermal processes

15. Entropy will not change if it is a reversible transition but will change if the process is irreversible.

17. Entropy is a function of disorder, so all the answers apply here as well.

Problems

19. 11.0×10^3 J

21. $4.5\,pV_0$

23. 0.667

25. a. 0.200; b. 25 J

27. a. 0.67; b. 75 J; c. 25 J

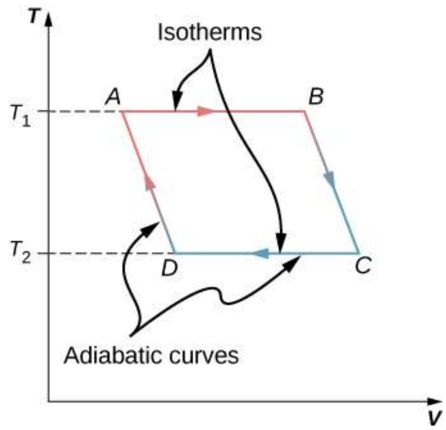
29. a. 600 J; b. 800 J

31. a. 69 J; b. 11 J

33. 2.0

35. 50 J

37.



39. a. 381 J; b. 619 J
41. a. 546 K; b. 137 K
43. −1 J/K
45. −13 J(K mole)
47. $-\frac{Q}{T_h}, \frac{Q}{T_c}, Q\left(\frac{1}{T_c} - \frac{1}{T_h}\right)$
49. a. −709 J/K; b. 1300 J/K; c. 591 J/K
51. a. $Q = nR\Delta T$; b. $S = nR\ln(T_2/T_1)$
53. 3.78×10^{-3} W/K
55. 430 J/K
57. 80 °C, 80 °C, 6.70×10^4 J, 215 J/K, −190 J/K, 25 J/K
59. $\Delta S_{H_2O} = 215$ J/K, $\Delta S_R = -208$ J/K, $\Delta S_U = 7$ J/K
61. a. 1200 J; b. 600 J; c. 600 J; d. 0.50
63. $\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right) + nC_p \ln\left(\frac{T_3}{T_2}\right)$
65. a. 0.33, 0.39; b. 0.91

Additional Problems

67. 1.45×10^7 J
69. a. $V_B = 0.042$ m^3, $V_D = 0.018$ m^3; b. 13,000 J; c. 13,000 J; d. −8,000 J; e. −8,000 J; f. 6200 J; g. −6200 J; h. 39%; with temperatures efficiency is 40%, which is off likely by rounding errors.
71. −670 J/K
73. a. −570 J/K; b. 570 J/K
75. 82 J/K
77. a. 2000 J; b. 40%
79. 60%
81. 64.4%

Challenge Problems

83. derive
85. derive
87. 18 J/K
89. proof
91. $K_R = \frac{3(p_1 - p_2)V_1}{5p_2V_3 - 3p_1V_1 - p_2V_1}$
93. $W = 110{,}000$ J

Chapter 5

Check Your Understanding

5.1 The force would point outward.
5.2 The net force would point 58° below the $-x$-axis.
5.3 $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$
5.4 We will no longer be able to take advantage of symmetry. Instead, we will need to calculate each of the two components of the electric field with their own integral.
5.5 The point charge would be $Q = \sigma ab$ where a and b are the sides of the rectangle but otherwise identical.
5.6 The electric field would be zero in between, and have magnitude $\frac{\sigma}{\varepsilon_0}$ everywhere else.

Conceptual Questions

1. There are mostly equal numbers of positive and negative charges present, making the object electrically neutral.
3. a. no; b. yes
5. Take an object with a known charge, either positive or negative, and bring it close to the rod. If the known charged object is positive and it is repelled from the rod, the rod is charged positive. If the positively charged object is attracted to the rod, the rod is negatively charged.
7. No, the dust is attracted to both because the dust particle molecules become polarized in the direction of the silk.
9. Yes, polarization charge is induced on the conductor so that the positive charge is nearest the charged rod, causing an attractive force.
11. Charging by conduction is charging by contact where charge is transferred to the object. Charging by induction first involves producing a polarization charge in the object and then connecting a wire to ground to allow some of the charge to leave the object, leaving the object charged.
13. This is so that any excess charge is transferred to the ground, keeping the gasoline receptacles neutral. If there is excess charge on the gasoline receptacle, a spark could ignite it.
15. The dryer charges the clothes. If they are damp, the presence of water molecules suppresses the charge.
17. There are only two types of charge, attractive and repulsive. If you bring a charged object near the quartz, only one of these two effects will happen, proving there is not a third kind of charge.
19. a. No, since a polarization charge is induced. b. Yes, since the polarization charge would produce only an attractive force.
21. The force holding the nucleus together must be greater than the electrostatic repulsive force on the protons.
23. Either sign of the test charge could be used, but the convention is to use a positive test charge.
25. The charges are of the same sign.
27. At infinity, we would expect the field to go to zero, but because the sheet is infinite in extent, this is not the case. Everywhere you are, you see an infinite plane in all directions.
29. The infinite charged plate would have $E = \frac{\sigma}{2\varepsilon_0}$ everywhere. The field would point toward the plate if it were negatively charged and point away from the plate if it were positively charged. The electric field of the parallel plates would be zero between them if they had the same charge, and E would be $E = \frac{\sigma}{\varepsilon_0}$ everywhere else. If the charges were opposite, the situation is reversed, zero outside the plates and $E = \frac{\sigma}{\varepsilon_0}$ between them.
31. yes; no
33. At the surface of Earth, the gravitational field is always directed in toward Earth's center. An electric field could move a charged particle in a different direction than toward the center of Earth. This would indicate an electric field is present.
35. 10

Problems

37. a. $2.00 \times 10^{-9}\ \text{C}\left(\frac{1}{1.602 \times 10^{-19}}\ \text{e/C}\right) = 1.248 \times 10^{10}$ electrons;

b. $0.500 \times 10^{-6}\ \text{C}\left(\frac{1}{1.602 \times 10^{-19}}\ \text{e/C}\right) = 3.121 \times 10^{12}$ electrons

39. $\frac{3.750 \times 10^{21}\ \text{e}}{6.242 \times 10^{18}\ \text{e/C}} = -600.8\ \text{C}$

41. a. $2.0 \times 10^{-9}\ \text{C}\ (6.242 \times 10^{18}\ \text{e/C}) = 1.248 \times 10^{10}$ e;

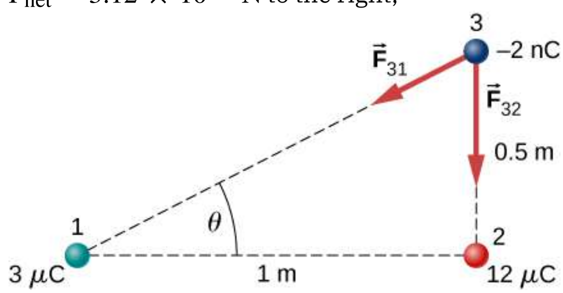
b. $9.109 \times 10^{-31}\ \text{kg}\ (1.248 \times 10^{10}\ \text{e}) = 1.137 \times 10^{-20}$ kg,
$\frac{1.137 \times 10^{-20}\ \text{kg}}{2.5 \times 10^{-3}\ \text{kg}} = 4.548 \times 10^{-18}$ or $4.545 \times 10^{-16}\%$

43. $5.00 \times 10^{-9}\ \text{C}\ (6.242 \times 10^{18}\ \text{e/C}) = 3.121 \times 10^{10}$ e;
$3.121 \times 10^{10}\ \text{e} + 1.0000 \times 10^{12}\ \text{e} = 1.0312 \times 10^{12}$ e

45. atomic mass of copper atom times 1 u $= 1.055 \times 10^{-25}$ kg;
number of copper atoms $= 4.739 \times 10^{23}$ atoms;
number of electrons equals 29 times number of atoms or 1.374×10^{25} electrons;
$\frac{2.00 \times 10^{-6}\ \text{C}(6.242 \times 10^{18}\ \text{e/C})}{1.374 \times 10^{25}\ \text{e}} = 9.083 \times 10^{-13}$ or $9.083 \times 10^{-11}\%$

47. $244.00\ \text{u}(1.66 \times 10^{-27}\ \text{kg/u}) = 4.050 \times 10^{-25}$ kg;
$\frac{4.00\ \text{kg}}{4.050 \times 10^{-25}\ \text{kg}} = 9.877 \times 10^{24}$ atoms $\quad 9.877 \times 10^{24}(94) = 9.284 \times 10^{26}$ protons;
$9.284 \times 10^{26}(1.602 \times 10^{-19}\ \text{C/p}) = 1.487 \times 10^{8}$ C

49. a. charge 1 is 3 μC; charge 2 is 12 μC, $F_{31} = 2.16 \times 10^{-4}$ N to the left,
$F_{32} = 8.63 \times 10^{-4}$ N to the right,
$F_{net} = 6.47 \times 10^{-4}$ N to the right;
b. $F_{31} = 2.16 \times 10^{-4}$ N to the right,
$F_{32} = 9.59 \times 10^{-5}$ N to the right,
$F_{net} = 3.12 \times 10^{-4}$ N to the right,



;

c. $\vec{\mathbf{F}}_{31x} = -2.76 \times 10^{-5}\ \text{N}\ \hat{\mathbf{i}}$,
$\vec{\mathbf{F}}_{31y} = -1.38 \times 10^{-5}\ \text{N}\ \hat{\mathbf{j}}$,
$\vec{\mathbf{F}}_{32y} = -8.63 \times 10^{-4}\ \text{N}\ \hat{\mathbf{j}}$
$\vec{\mathbf{F}}_{net} = -3.86 \times 10^{-5}\ \text{N}\ \hat{\mathbf{i}} - 8.83 \times 10^{-4}\ \text{N}\ \hat{\mathbf{j}}$

51. $F = 230.7$ N

53. $F = 53.94$ N

55. The tension is $T = 0.049$ N. The horizontal component of the tension is 0.0043 N
$d = 0.088$ m, $\quad q = 6.1 \times 10^{-8}$ C.
The charges can be positive or negative, but both have to be the same sign.

57. Let the charge on one of the spheres be nQ, where n is a fraction between 0 and 1. In the numerator of Coulomb's law, the term involving the charges is $nQ(1-n)Q$. This is equal to $(n-n^2)Q^2$. Finding the maximum of this term gives $1 - 2n = 0 \Rightarrow n = \frac{1}{2}$

59. Define right to be the positive direction and hence left is the negative direction, then $F = -0.05$ N

61. The particles form triangle of sides 13, 13, and 24 cm. The x-components cancel, whereas there is a contribution to the y-component from both charges 24 cm apart. The y-axis passing through the third charge bisects the 24-cm line, creating two right triangles of sides 5, 12, and 13 cm.

$F_y = 2.56$ N in the negative y-direction since the force is attractive. The net force from both charges is $\vec{\mathbf{F}}_{\text{net}} = -5.12\ \text{N}\hat{\mathbf{j}}$.

63. The diagonal is $\sqrt{2}a$ and the components of the force due to the diagonal charge has a factor $\cos\theta = \frac{1}{\sqrt{2}}$;

$\vec{\mathbf{F}}_{\text{net}} = \left[k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}\right]\hat{\mathbf{i}} - \left[k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}\right]\hat{\mathbf{j}}$

65. a. $E = 2.0 \times 10^2\ \frac{\text{N}}{\text{C}}$ up;
b. $F = 2.0 \times 10^{-6}$ N down

67. a. $E = 2.88 \times 10^{11}$ N/C;
b. $E = 1.44 \times 10^{11}$ N/C;
c. $F = 4.61 \times 10^{-8}$ N on alpha particle;
$F = 4.61 \times 10^{-8}$ N on electron

69. $E = \left(-2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}\right)$ N

71. $F = 3.204 \times 10^{-14}$ N,
$a = 3.517 \times 10^{16}\ \text{m/s}^2$

73. $q = 2.78 \times 10^{-9}$ C

75. a. $E = 1.15 \times 10^{12}$ N/C;
b. $F = 1.47 \times 10^{-6}$ N

77. If the q_2 is to the right of q_1, the electric field vector from both charges point to the right. a.
$E = 2.70 \times 10^6$ N/C;
b. $F = 54.0$ N

79. There is 45° right triangle geometry. The x-components of the electric field at $y = 3$ m cancel. The y-components give $E(y = 3\ \text{m}) = 2.83 \times 10^3$ N/C.
At the origin we have a a negative charge of magnitide
$q = -2.83 \times 10^{-6}$ C.

81. $\vec{\mathbf{E}}(z) = 3.6 \times 10^4\ \text{N/C}\hat{\mathbf{k}}$

83. $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x+a)^2},\quad E = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{l+a} - \frac{1}{a}\right]$

85. $\sigma = 0.02\ \text{C/m}^2\quad E = 2.26 \times 10^9$ N/C

87. At P_1: $\vec{\mathbf{E}}(y) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{y\sqrt{y^2+\frac{L^2}{4}}}\hat{\mathbf{j}} \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q}{\frac{a}{2}\sqrt{(\frac{a}{2})^2+\frac{L^2}{4}}}\hat{\mathbf{j}} = \frac{1}{\pi\varepsilon_0}\frac{q}{a\sqrt{a^2+L^2}}\hat{\mathbf{j}}$

At P_2: Put the origin at the end of L.
$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x+a)^2},\quad \vec{\mathbf{E}} = -\frac{q}{4\pi\varepsilon_0 l}\left[\frac{1}{l+a} - \frac{1}{a}\right]\hat{\mathbf{i}}$

89. a. $\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_x}{b}\hat{\mathbf{i}} + \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_y}{a}\hat{\mathbf{j}}$; b. $\frac{1}{4\pi\varepsilon_0}\frac{2(\lambda_x+\lambda_y)}{c}\hat{\mathbf{k}}$

91. a. $\vec{\mathbf{F}} = 3.2 \times 10^{-17}\ \text{N}\hat{\mathbf{i}}$,
$\vec{\mathbf{a}} = 1.92 \times 10^{10}\ \text{m/s}^2\hat{\mathbf{i}}$;
b. $\vec{\mathbf{F}} = -3.2 \times 10^{-17}\ \text{N}\hat{\mathbf{i}}$,
$\vec{\mathbf{a}} = -3.51 \times 10^{13}\ \text{m/s}^2\hat{\mathbf{i}}$

93. $m = 6.5 \times 10^{-11}$ kg,
$E = 1.6 \times 10^7$ N/C

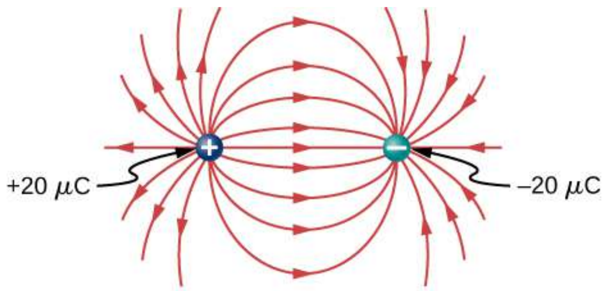
95. $E = 1.70 \times 10^6$ N/C,
$F = 1.53 \times 10^{-3}\ \text{N}\quad T\cos\theta = mg\quad T\sin\theta = qE$,
$\tan\theta = 0.62 \Rightarrow \theta = 32.0°$,
This is independent of the length of the string.

97. circular arc $dE_x(-\hat{\mathbf{i}}) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}\cos\theta(-\hat{\mathbf{i}})$,
$\vec{\mathbf{E}}_x = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{\mathbf{i}})$,
$dE_y(-\hat{\mathbf{i}}) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}\sin\theta(-\hat{\mathbf{j}})$,

$\vec{\mathbf{E}}_y = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{\mathbf{j}});$
y-axis: $\vec{\mathbf{E}}_x = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{\mathbf{i}});$
x-axis: $\vec{\mathbf{E}}_y = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{\mathbf{j}}),$
$\vec{\mathbf{E}} = \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{\mathbf{i}}) + \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{\mathbf{j}})$

99. a. $W = \frac{1}{2}m(v^2 - v_0^2)$, $\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_0}\right) = \frac{1}{2}m(v^2 - v_0^2) \Rightarrow r_0 - r = \frac{4\pi\varepsilon_0}{Qq}\,\frac{1}{2}rr_0 m(v^2 - v_0^2)$; b. $r_0 - r$ is negative; therefore, $v_0 > v, r \to \infty,$ and $v \to 0$: $\frac{Qq}{4\pi\varepsilon_0}\left(-\frac{1}{r_0}\right) = -\frac{1}{2}mv_0^2 \Rightarrow v_0 = \sqrt{\frac{Qq}{2\pi\varepsilon_0 m r_0}}$

101.



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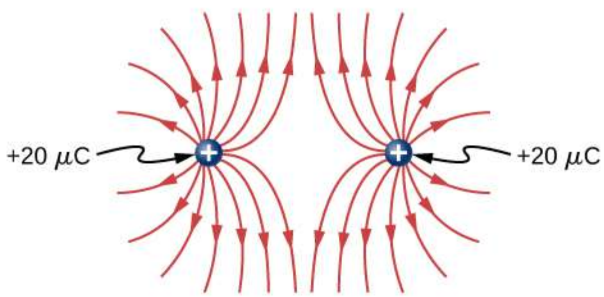

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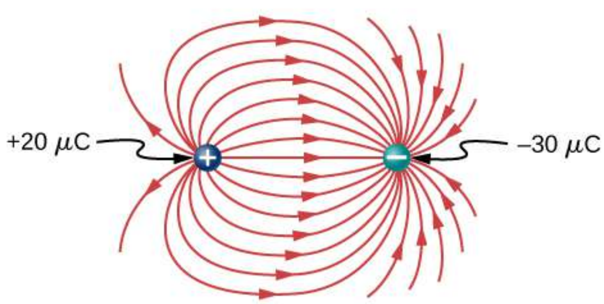

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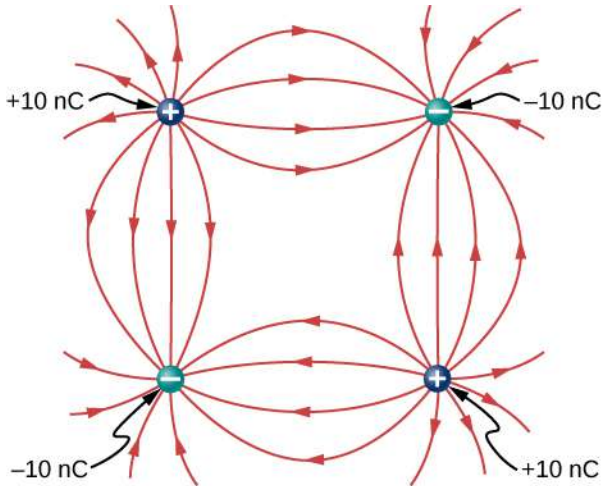
103.



105. $E_x = 0,$
$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{(x^2+a^2)} \frac{a}{\sqrt{(x^2+a^2)}}\right]$
$\Rightarrow x \gg a \Rightarrow \frac{1}{2\pi\varepsilon_0} \frac{qa}{x^3},$
$E_y = \frac{q}{4\pi\varepsilon_0}\left[\frac{2ya+2ya}{(y-a)^2(y+a)^2}\right]$
$\Rightarrow y \gg a \Rightarrow \frac{1}{\pi\varepsilon_0} \frac{qa}{y^3}$

107. The net dipole moment of the molecule is the vector sum of the individual dipole moments between the two O-H. The separation O-H is 0.9578 angstroms:
$\vec{\mathbf{p}} = 1.889 \times 10^{-29}\ \mathrm{Cm}\,\hat{\mathbf{i}}$

Additional Problems

109. $\vec{\mathbf{F}}_{\text{net}} = [-8.99 \times 10^9 \frac{3.0 \times 10^{-6}(5.0 \times 10^{-6})}{(3.0\ \mathrm{m})^2} - 8.99 \times 10^9 \frac{9.0 \times 10^{-6}(5.0 \times 10^{-6})}{(3.0\ \mathrm{m})^2}]\hat{\mathbf{i}},$
$-8.99 \times 10^9 \frac{6.0 \times 10^{-6}(5.0 \times 10^{-6})}{(3.0\ \mathrm{m})^2}\hat{\mathbf{j}} = -0.06\ \mathrm{N}\hat{\mathbf{i}} - 0.03\ \mathrm{N}\hat{\mathbf{j}}$

111. Charges Q and q form a right triangle of sides 1 m and $3 + \sqrt{3}$ m. Charges $2Q$ and q form a right triangle of sides 1 m and $\sqrt{3}$ m.
$F_x = 0.049\ \mathrm{N},$
$F_y = 0.093\ \mathrm{N},$
$\vec{\mathbf{F}}_{\text{net}} = 0.036\ \mathrm{N}\,\hat{\mathbf{i}} + 0.09\ \mathrm{N}\,\hat{\mathbf{j}}$

113. $W = 0.054\ \mathrm{J}$

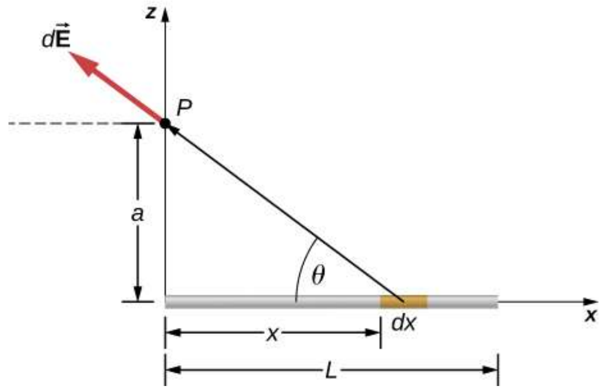
115. a. $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}(\frac{q}{(2a)^2} - \frac{q}{a^2})\hat{\mathbf{i}}$; b. $\vec{\mathbf{E}} = \frac{\sqrt{3}}{4\pi\varepsilon_0} \frac{q}{a^2}(-\hat{\mathbf{j}})$; c. $\vec{\mathbf{E}} = \frac{2}{\pi\varepsilon_0} \frac{q}{a^2} \frac{1}{\sqrt{2}}(-\hat{\mathbf{j}})$

117. $\vec{\mathbf{E}}_{\text{net}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 + \vec{\mathbf{E}}_4 = \left(4.65\hat{\mathbf{i}} + 1.44\hat{\mathbf{j}}\right) \times 10^7\ \mathrm{N/C}$

119. $F = qE_0\,(1 + x/a) \quad W = \frac{1}{2}m(v^2 - v_0^2),$
$\frac{1}{2}mv^2 = qE_0(\frac{15a}{2})\ \mathrm{J}$

121. Electric field of wire at x: $\vec{\mathbf{E}}(x) = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda_y}{x}\hat{\mathbf{i}},$
$dF = \frac{\lambda_y \lambda_x}{2\pi\varepsilon_0}(\ln b - \ln a)$

123.



$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+a^2)}\frac{x}{\sqrt{x^2+a^2}},$$

$$\vec{\mathbf{E}}_x = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{L^2+a^2}} - \frac{1}{a}\right]\hat{\mathbf{i}},$$

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+a^2)}\frac{a}{\sqrt{x^2+a^2}},$$

$$\vec{\mathbf{E}}_z = \frac{\lambda}{4\pi\varepsilon_0 a}\frac{L}{\sqrt{L^2+a^2}}\hat{\mathbf{k}},$$

Substituting z for a, we have:

$$\vec{\mathbf{E}}(z) = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{z}\right]\hat{\mathbf{i}} + \frac{\lambda}{4\pi\varepsilon_0 z}\frac{L}{\sqrt{L^2+z^2}}\hat{\mathbf{k}}$$

125. There is a net force only in the y-direction. Let θ be the angle the vector from dx to q makes with the x-axis. The components along the x-axis cancel due to symmetry, leaving the y-component of the force.

$$dF_y = \frac{1}{4\pi\varepsilon_0}\frac{aq\lambda dx}{(x^2+a^2)^{3/2}},$$

$$F_y = \frac{1}{2\pi\varepsilon_0}\frac{q\lambda}{a}\left[\frac{l/2}{((l/2)^2+a^2)^{1/2}}\right]$$

Chapter 6

Check Your Understanding

6.1 Place it so that its unit normal is perpendicular to $\vec{\mathbf{E}}$.

6.2 $mab^2/2$

6.3 a. $3.4 \times 10^5\ \mathrm{N \cdot m^2/C}$; b. $-3.4 \times 10^5\ \mathrm{N \cdot m^2/C}$; c. $3.4 \times 10^5\ \mathrm{N \cdot m^2/C}$; d. 0

6.4 In this case, there is only $\vec{\mathbf{E}}_{\text{out}}$. So, yes.

6.5 $\vec{\mathbf{E}} = \frac{\lambda_0}{2\pi\varepsilon_0}\frac{1}{d}\hat{\mathbf{r}}$; This agrees with the calculation of Example 5.5 where we found the electric field by integrating over the charged wire. Notice how much simpler the calculation of this electric field is with Gauss's law.

6.6 If there are other charged objects around, then the charges on the surface of the sphere will not necessarily be spherically symmetrical; there will be more in certain direction than in other directions.

Conceptual Questions

1. a. If the planar surface is perpendicular to the electric field vector, the maximum flux would be obtained. b. If the planar surface were parallel to the electric field vector, the minimum flux would be obtained.

3. False. The net electric flux crossing a closed surface is always zero if and only if the net charge enclosed is zero.

5. Since the electric field vector has a $\frac{1}{r^2}$ dependence, the fluxes are the same since $A = 4\pi r^2$.

7. a. no; b. zero

9. Both fields vary as $\frac{1}{r^2}$. Because the gravitational constant is so much smaller than $\frac{1}{4\pi\varepsilon_0}$, the gravitational field is orders of magnitude weaker than the electric field. Also, the gravitational flux through a closed surface is zero or positive; however, the electric flux is positive, negative, or zero, depending on the definition of flux for the given situation.
11. No, it is produced by all charges both inside and outside the Gaussian surface.
13. No, since the situation does not have symmetry, making Gauss's law challenging to simplify.
15. Any shape of the Gaussian surface can be used. The only restriction is that the Gaussian integral must be calculable; therefore, a box or a cylinder are the most convenient geometrical shapes for the Gaussian surface.
17. No. If a metal was in a region of zero electric field, all the conduction electrons would be distributed uniformly throughout the metal.
19. Since the electric field is zero inside a conductor, a charge of $-2.0\ \mu$C is induced on the inside surface of the cavity. This will put a charge of $+2.0\ \mu C$ on the outside surface leaving a net charge of $-3.0\ \mu$C on the surface.

Problems

21. $\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \to EA\cos\theta = 2.2 \times 10^4 \mathrm{N \cdot m^2/C}$ electric field in direction of unit normal;
$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \to EA\cos\theta = -2.2 \times 10^4 \mathrm{N \cdot m^2/C}$ electric field opposite to unit normal
23. $\frac{3 \times 10^{-5}\ \mathrm{N \cdot m^2/C}}{(0.05\ \mathrm{m})^2} = E \Rightarrow \sigma = 2.12 \times 10^{-13}\ \mathrm{C/m^2}$
25. a. $\Phi = 0.17\ \mathrm{N \cdot m^2/C}$;
b. $\Phi = 0$; c. $\Phi = EA\cos 0° = 1.0 \times 10^3\ \mathrm{N/C}(2.0 \times 10^{-4}\ \mathrm{m})^2 \cos 0° = 0.20\ \mathrm{N \cdot m^2/C}$
27. $\Phi = 3.8 \times 10^4\ \mathrm{N \cdot m^2/C}$
29. $\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{z}\hat{\mathbf{k}}, \ \int \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{\lambda}{\varepsilon_0} l$
31. a. $\Phi = 3.39 \times 10^3\ \mathrm{N \cdot m^2/C}$; b. $\Phi = 0$;
c. $\Phi = -2.25 \times 10^5\ \mathrm{N \cdot m^2/C}$;
d. $\Phi = 90.4\ \mathrm{N \cdot m^2/C}$
33. $\Phi = 1.13 \times 10^6\ \mathrm{N \cdot m^2/C}$
35. Make a cube with q at the center, using the cube of side a. This would take four cubes of side a to make one side of the large cube. The shaded side of the small cube would be 1/24th of the total area of the large cube; therefore, the flux through the shaded area would be
$\Phi = \frac{1}{24}\frac{q}{\varepsilon_0}$.
37. $q = 3.54 \times 10^{-7}\ \mathrm{C}$
39. zero, also because flux in equals flux out
41. $r > R,\ E = \frac{Q}{4\pi\varepsilon_0 r^2};\ r < R,\ E = \frac{qr}{4\pi\varepsilon_0 R^3}$
43. $EA = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = 4.50 \times 10^7\ \mathrm{N/C}$
45. a. 0; b. 0; c. $\vec{\mathbf{E}} = 6.74 \times 10^6\ \mathrm{N/C}(-\hat{\mathbf{r}})$
47. a. 0; b. $E = 2.70 \times 10^6\ \mathrm{N/C}$
49. a. Yes, the length of the rod is much greater than the distance to the point in question. b. No, The length of the rod is of the same order of magnitude as the distance to the point in question. c. Yes, the length of the rod is much greater than the distance to the point in question. d. No. The length of the rod is of the same order of magnitude as the distance to the point in question.
51. a. $\vec{\mathbf{E}} = \frac{R\sigma_0}{\varepsilon_0}\frac{1}{r}\hat{\mathbf{r}} \Rightarrow \sigma_0 = 5.31 \times 10^{-11}\ \mathrm{C/m^2}$,
$\lambda = 3.33 \times 10^{-12}\ \mathrm{C/m}$;
b. $\Phi = \frac{q_{\mathrm{enc}}}{\varepsilon_0} = \frac{3.33 \times 10^{-12}\ \mathrm{C/m}(0.05\ \mathrm{m})}{\varepsilon_0} = 0.019\ \mathrm{N \cdot m^2/C}$
53. $E2\pi rl = \frac{\rho\pi r^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{2\varepsilon_0}\ (r \le R)$;

$E2\pi rl = \frac{\rho\pi R^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}\ (r \geq R)$

55. $\Phi = \frac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow q_{\text{enc}} = -1.0 \times 10^{-9}\ \text{C}$

57. $q_{\text{enc}} = \frac{4}{5}\pi\alpha r^5,$

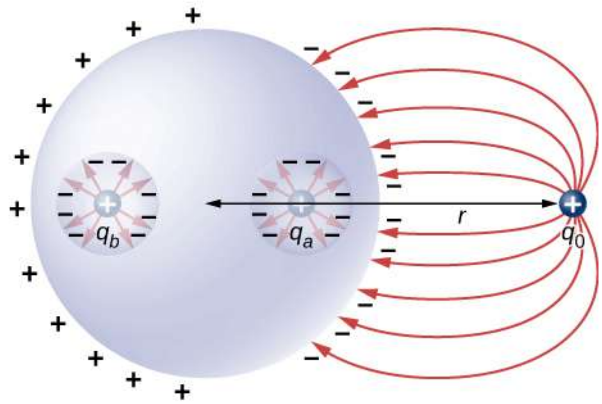
$E4\pi r^2 = \frac{4\pi\alpha r^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha r^3}{5\varepsilon_0}\ (r \leq R),$

$q_{\text{enc}} = \frac{4}{5}\pi\alpha R^5,\ E4\pi r^2 = \frac{4\pi\alpha R^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha R^5}{5\varepsilon_0 r^2}\ (r \geq R)$

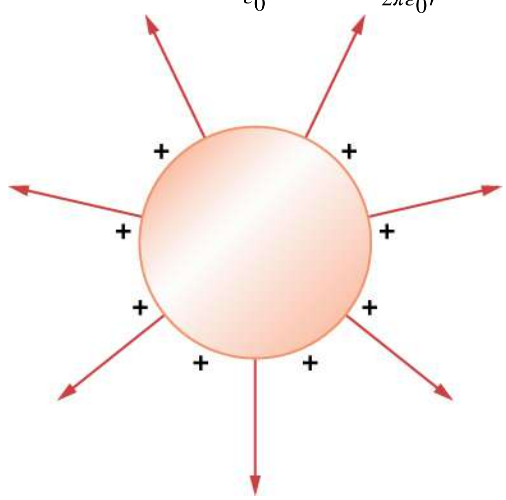
59. integrate by parts:

$q_{\text{enc}} = 4\pi\rho_0\left[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}\right] \Rightarrow E = \frac{\rho_0}{r^2\varepsilon_0}\left[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}\right]$

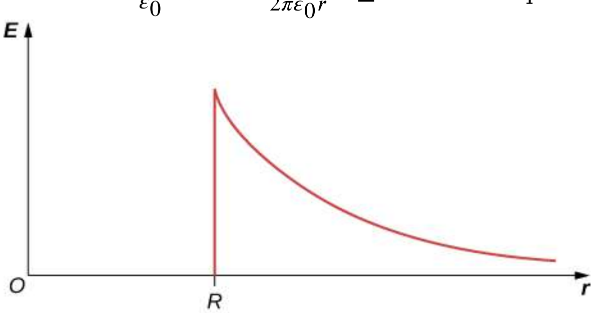
61.



63. a. Outside: $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{3.0\ \text{C/m}}{2\pi\varepsilon_0 r}$; Inside $E_{\text{in}} = 0$; b.



65. a. $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}\ r \geq R$ E inside equals 0; b.



67. $E = 5.65 \times 10^4\ \text{N/C}$

69. $\lambda = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{a\sigma}{\varepsilon_0 r}\ r \geq a,\ E = 0$ inside since q enclosed $= 0$

71. a. $E = 0$; b. $E2\pi rL = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 rL}$; c. $E = 0$ since r would be either inside the second shell or if outside then q enclosed equals 0.

Additional Problems

73. $\int \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = a^4$

75. a. $\int \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E_0 r^2 \pi$; b. zero, since the flux through the upper half cancels the flux through the lower half of the sphere

77. $\Phi = \frac{q_{\text{enc}}}{\varepsilon_0}$; There are two contributions to the surface integral: one at the side of the rectangle at $x = 0$ and the other at the side at $x = 2.0$ m;
$-E(0)[1.5\text{ m}^2] + E(2.0\text{ m})[1.5\text{ m}^2] = \frac{q_{\text{enc}}}{\varepsilon_0} = -100\text{ Nm}^2/\text{C}$
where the minus sign indicates that at $x = 0$, the electric field is along positive x and the unit normal is along negative x. At $x = 2$, the unit normal and the electric field vector are in the same direction:
$q_{\text{enc}} = \varepsilon_0\, \Phi = -8.85 \times 10^{-10}$ C.

79. didn't keep consistent directions for the area vectors, or the electric fields

81. a. $\sigma = 3.0 \times 10^{-3}\text{ C/m}^2$, $+3 \times 10^{-3}\text{ C/m}^2$ on one and $-3 \times 10^{-3}\text{ C/m}^2$ on the other; b. $E = 3.39 \times 10^8$ N/C

83. Construct a Gaussian cylinder along the z-axis with cross-sectional area A.
$|z| \geq \frac{a}{2}\; q_{\text{enc}} = \rho A a,\; \Phi = \frac{\rho A a}{\varepsilon_0} \Rightarrow E = \frac{\rho a}{2\varepsilon_0}$,
$|z| \leq \frac{a}{2}\; q_{\text{enc}} = \rho A 2z,\; E(2A) = \frac{\rho A 2z}{\varepsilon_0} \Rightarrow E = \frac{\rho z}{\varepsilon_0}$

85. a. $r > b_2$ $E4\pi r^2 = \frac{\frac{4}{3}\pi[\rho_1(b_1^3 - a_1^3) + \rho_2(b_2^3 - a_2^3)}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(b_1^3 - a_1^3) + \rho_2(b_2^3 - a_2^3)}{3\varepsilon_0 r^2}$;

b. $a_2 < r < b_2$ $\quad E4\pi r^2 = \frac{\frac{4}{3}\pi[\rho_1(b_1^3 - a_1^3) + \rho_2(r^3 - a_2^3)]}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(b_1^3 - a_1^3) + \rho_2(r^3 - a_2^3)}{3\varepsilon_0 r^2}$;

c. $b_1 < r < a_2$ $\quad E4\pi r^2 = \frac{\frac{4}{3}\pi\rho_1(b_1^3 - a_1^3)}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(b_1^3 - a_1^3)}{3\varepsilon_0 r^2}$;

d. $a_1 < r < b_1$ $\quad E4\pi r^2 = \frac{\frac{4}{3}\pi\rho_1(r^3 - a_1^3)}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(r^3 - a_1^3)}{3\varepsilon_0 r^2}$; e. 0

87. Electric field due to plate without hole: $E = \frac{\sigma}{2\varepsilon_0}$.

Electric field of just hole filled with $-\sigma$ $E = \frac{-\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2 + z^2}}\right)$.

Thus, $E_{\textbf{net}} = \frac{\sigma}{2\varepsilon_0}\;\frac{h}{\sqrt{R^2 + h^2}}$.

89. a. $E = 0$; b. $E = \frac{q_1}{4\pi\varepsilon_0 r^2}$; c. $E = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2}$; d. 0, $q_1, -q_1,\; q_1 + q_2$

Challenge Problems

91. Given the referenced link, using a distance to Vega of 237×10^{15} m[4] and a diameter of 2.4 m for the primary mirror,[5] we find that at a wavelength of 555.6 nm, Vega is emitting 2.44×10^{24} J/s at that wavelength. Note that the flux through the mirror is essentially constant.

93. The symmetry of the system forces $\vec{\mathbf{E}}$ to be perpendicular to the sheet and constant over any plane parallel to the sheet. To calculate the electric field, we choose the cylindrical Gaussian surface shown. The cross-section area and the height of the cylinder are A and $2x$, respectively, and the cylinder is positioned so that it is bisected by the plane sheet. Since E is perpendicular to each end and parallel to the side of the

4 http://webviz.u-strasbg.fr/viz-bin/VizieR-5?-source=I/311&HIP=91262

5 http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19910003124.pdf

cylinder, we have EA as the flux through each end and there is no flux through the side. The charge enclosed by the cylinder is σA, so from Gauss's law, $2EA = \frac{\sigma A}{\varepsilon_0}$, and the electric field of an infinite sheet of charge is
$E = \frac{\sigma}{2\varepsilon_0}$, in agreement with the calculation of in the text.

95. There is $Q/2$ on each side of the plate since the net charge is Q: $\sigma = \frac{Q}{2A}$,
$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{2\sigma\Delta A}{\varepsilon_0} \Rightarrow E_P = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 2A}$

Chapter 7

Check Your Understanding

7.1 $K = \frac{1}{2}mv^2,\ v = \sqrt{2\frac{K}{m}} = \sqrt{2\frac{4.5 \times 10^{-7}\text{ J}}{4.00 \times 10^{-9}\text{ kg}}} = 15\text{ m/s}$

7.2 It has kinetic energy of 4.5×10^{-7} J at point r_2 and potential energy of 9.0×10^{-7} J, which means that as Q approaches infinity, its kinetic energy totals three times the kinetic energy at r_2, since all of the potential energy gets converted to kinetic.

7.3 positive, negative, and these quantities are the same as the work you would need to do to bring the charges in from infinity

7.4 $\Delta U = q\Delta V = (100\text{ C})(1.5\text{ V}) = 150\text{ J}$

7.5 –2.00 C, $n_e = 1.25 \times 10^{19}$ electrons

7.6 It would be going in the opposite direction, with no effect on the calculations as presented.

7.7 Given a fixed maximum electric field strength, the potential at which a strike occurs increases with increasing height above the ground. Hence, each electron will carry more energy. Determining if there is an effect on the total number of electrons lies in the future.

7.8 $V = k\frac{q}{r} = \left(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-3.00 \times 10^{-9}\text{ C}}{5.00 \times 10^{-3}\text{ m}}\right) = -5390\text{ V}$; recall that the electric field inside a conductor is zero. Hence, any path from a point on the surface to any point in the interior will have an integrand of zero when calculating the change in potential, and thus the potential in the interior of the sphere is identical to that on the surface.

7.9 The x-axis the potential is zero, due to the equal and opposite charges the same distance from it. On the z-axis, we may superimpose the two potentials; we will find that for $z >> d$, again the potential goes to zero due to cancellation.

7.10 It will be zero, as at all points on the axis, there are equal and opposite charges equidistant from the point of interest. Note that this distribution will, in fact, have a dipole moment.

7.11 Any, but cylindrical is closest to the symmetry of a dipole.

7.12 infinite cylinders of constant radius, with the line charge as the axis

Conceptual Questions

1. No. We can only define potential energies for conservative fields.

3. No, though certain orderings may be simpler to compute.

5. The electric field strength is zero because electric potential differences are directly related to the field strength. If the potential difference is zero, then the field strength must also be zero.

7. Potential difference is more descriptive because it indicates that it is the difference between the electric potential of two points.

9. They are very similar, but potential difference is a feature of the system; when a charge is introduced to the system, it will have a potential energy which may be calculated by multiplying the magnitude of the charge by the potential difference.

11. An electron-volt is a volt multiplied by the charge of an electron. Volts measure potential difference, electron-volts are a unit of energy.

13. The second has 1/4 the dipole moment of the first.

15. The region outside of the sphere will have a potential indistinguishable from a point charge; the interior of the sphere will have a different potential.

17. No. It will be constant, but not necessarily zero.

19. no

21. No; it might not be at electrostatic equilibrium.

23. Yes. It depends on where the zero reference for potential is. (Though this might be unusual.)

25. So that lightning striking them goes into the ground instead of the television equipment.

27. They both make use of static electricity to stick small particles to another surface. However, the precipitator has to charge a wide variety of particles, and is not designed to make sure they land in a particular place.

Problems

29. a. $U = 3.4$ J;
b. $\frac{1}{2}mv^2 = Q_1 Q_2 \left(\frac{1}{r_i} - \frac{1}{r_f}\right) \rightarrow v = 2.4 \text{ x } 10^4 \text{ m/s}$

31. $U = 4.36 \times 10^{-18}$ J

33. $$\begin{aligned} \frac{1}{2}m_e v_e^2 &= qV, \ \frac{1}{2}m_{\text{H}} v_{\text{H}}^2 = qV, \text{ so that} \\ \frac{m_e v_e^2}{m_{\text{H}} v_{\text{H}}^2} &= 1 \text{ or } \frac{v_e}{v_{\text{H}}} = 42.8 \end{aligned}$$

35. 1 V = 1 J/C; 1 J = 1 N · m → 1 V/m = 1 N/C

37. a. $V_{AB} = 3.00$ kV; b. $V_{AB} = 750$ V

39. a. $V_{AB} = Ed \rightarrow E = 5.63$ kV/m;
b. $V_{AB} = 563$ V

41. a. $$\begin{aligned} \Delta K &= q\Delta V \text{ and } V_{AB} = Ed, \text{ so that} \\ \Delta K &= 800 \text{ keV}; \end{aligned}$$
b. $d = 25.0$ km

43. One possibility is to stay at constant radius and go along the arc from P_1 to P_2, which will have zero potential due to the path being perpendicular to the electric field. Then integrate from *a* to *b*:
$V_{ab} = \alpha \ln\left(\frac{b}{a}\right)$

45. $V = 144$ V

47. $V = \frac{kQ}{r} \rightarrow Q = 8.33 \times 10^{-7}$ C;
The charge is positive because the potential is positive.

49. a. $V = 45.0$ MV;
b. $V = \frac{kQ}{r} \rightarrow r = 45.0$ m;
c. $\Delta U = 132$ MeV

51. $V = kQ/r$; a. Relative to origin, find the potential at each point and then calculate the difference.
$\Delta V = 135 \times 10^3$ V;
b. To double the potential difference, move the point from 20 cm to infinity; the potential at 20 cm is halfway between zero and that at 10 cm.

53. a. $V_{P1} = 7.4 \times 10^5$ V
and $V_{P2} = 6.9 \times 10^3$ V;
b. $V_{P1} = 6.9 \times 10^5$ V and $V_{P2} = 6.9 \times 10^3$ V

55. The problem is describing a uniform field, so $E = 200$ V/m in the $-z$-direction.

57. Apply $\vec{\mathbf{E}} = -\vec{\nabla} V$ with $\vec{\nabla} = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \widehat{\varphi} \frac{1}{r} \frac{\partial}{\partial \varphi} + \hat{\mathbf{z}} \frac{\partial}{\partial z}$ to the potential calculated earlier,
$V = -2k\lambda \ln s$: $\vec{\mathbf{E}} = 2k\lambda \frac{1}{r} \hat{\mathbf{r}}$ as expected.

59. a. increases; the constant (negative) electric field has this effect, the reference point only matters for magnitude; b. they are planes parallel to the sheet; c. 0.006 m/V

61. a. from the previous chapter, the electric field has magnitude $\frac{\sigma}{\varepsilon_0}$ in the region between the plates and zero outside; defining the negatively charged plate to be at the origin and zero potential, with the positively charged plate located at +5 mm in the z-direction, $V = 1.7 \times 10^4$ V so the potential is 0 for $z < 0$, $1.7 \times 10^4 \text{ V}\left(\frac{z}{5 \text{ mm}}\right)$ for $0 \leq z \leq 5$ mm, 1.7×10^4 V for $z > 5$ mm;

b. $qV = \frac{1}{2}mv^2 \rightarrow v = 7.7 \times 10^7$ m/s

63. $V = 85$ V

65. In the region $a \leq r \leq b$, $\vec{\mathbf{E}} = \frac{kQ}{r^2}\hat{\mathbf{r}}$, and E is zero elsewhere; hence, the potential difference is $V = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$.

67. From previous results $V_P - V_R = -2k\lambda \ln\frac{s_P}{s_R}$., note that b is a very convenient location to define the zero level of potential: $\Delta V = -2k\frac{Q}{L}\ln\frac{a}{b}$.

69. a. $F = 5.58 \times 10^{-11}$ N/C;
The electric field is towards the surface of Earth. b. The coulomb force is much stronger than gravity.

71. We know from the Gauss's law chapter that the electric field for an infinite line charge is $\vec{\mathbf{E}}_P = 2k\lambda\frac{1}{s}\hat{\mathbf{s}}$, and from earlier in this chapter that the potential of a wire-cylinder system of this sort is $V_P = -2k\lambda \ln\frac{s_P}{R}$ by integration. We are not given λ, but we are given a fixed V_0; thus, we know that $V_0 = -2k\lambda \ln\frac{a}{R}$ and hence $\lambda = -\frac{V_0}{2k\,\ln\left(\frac{a}{R}\right)}$. We may substitute this back in to find a. $\vec{\mathbf{E}}_P = -\frac{V_0}{\ln\left(\frac{a}{R}\right)}\,\frac{1}{s}\hat{\mathbf{s}}$; b. $V_P = V_0\frac{\ln\left(\frac{s_P}{R}\right)}{\ln\left(\frac{a}{R}\right)}$;
c. 4.74×10^4 N/C

73. a. $\begin{aligned} U_1 &= 7.68 \times 10^{-18}\ \text{J} \\ U_2 &= 5.76 \times 10^{-18}\ \text{J} \end{aligned}$;
b. $U_1 + U_2 = -1.34 \times 10^{-17}$ J

75. a. $U = 2.30 \times 10^{-16}$ J;
b. $\overline{K} = \frac{3}{2}kT \rightarrow T = 1.11 \times 10^7$ K

77. a. 1.9×10^6 m/s; b. 4.2×10^6 m/s; c. 5.9×10^6 m/s; d. 7.3×10^6 m/s; e. 8.4×10^6 m/s

79. a. $E = 2.5 \times 10^6\ \text{V/m} < 3 \times 10^6\ \text{V/m}$;
No, the field strength is smaller than the breakdown strength for air.
b. $d = 1.7$ mm

81. $\begin{aligned} K_{\text{f}} &= qV_{\text{AB}} = qEd \rightarrow \\ E &= 8.00 \times 10^5\ \text{V/m} \end{aligned}$

83. a. Energy $= 2.00 \times 10^9$ J;
b. $\begin{aligned} Q &= m\left(c\Delta T + L_{\text{V}}\right) \\ m &= 766\ \text{kg} \end{aligned}$;
c. The expansion of the steam upon boiling can literally blow the tree apart.

85. a. $V = \frac{kQ}{r} \rightarrow r = 1.80$ km; b. A 1-C charge is a very large amount of charge; a sphere of 1.80 km is impractical.

87. The alpha particle approaches the gold nucleus until its original energy is converted to potential energy.
5.00 MeV $= 8.00 \times 10^{-13}$ J, so
$\begin{aligned} E_0 &= \frac{qkQ}{r} \rightarrow \\ r &= 4.54 \times 10^{-14}\ \text{m} \end{aligned}$
(Size of gold nucleus is about 7×10^{-15} m).

Additional Problems

89. $\begin{aligned} E_{\text{tot}} &= 4.67 \times 10^7\ \text{J} \\ E_{\text{tot}} &= qV \rightarrow q = \frac{E_{\text{tot}}}{V} = 3.89 \times 10^6\ \text{C} \end{aligned}$

91. $V_P = k\frac{q_{\text{tot}}}{\sqrt{z^2+R^2}} \rightarrow q_{\text{tot}} = -3.5 \times 10^{-11}$ C

93. $V_P = -2.2$ GV

95. Recall from the previous chapter that the electric field $E_P = \frac{\sigma_0}{2\varepsilon_0}$ is uniform throughout space, and that for uniform fields we have $E = -\frac{\Delta V}{\Delta z}$ for the relation. Thus, we get $\frac{\sigma}{2\varepsilon_0} = \frac{\Delta V}{\Delta z} \rightarrow \Delta z = 0.22$ m for the distance between 25-V equipotentials.

97. a. Take the result from Example 7.13, divide both the numerator and the denominator by x, take the limit of that, and then apply a Taylor expansion to the resulting log to get: $V_P \approx k\lambda \frac{L}{x}$; b. which is the result we expect, because at great distances, this should look like a point charge of $q = \lambda L$

99. a. $V = 9.0 \times 10^3$ V; b. $-9.0 \times 10^3 \text{ V} \left(\frac{1.25 \text{ cm}}{2.0 \text{ cm}}\right) = -5.7 \times 10^3$ V

101. a. $E = \frac{KQ}{r^2} \rightarrow Q = -6.76 \times 10^5$ C;

b. $\begin{aligned} F &= ma = qE \rightarrow \\ a &= \frac{qE}{m} = 2.63 \times 10^{13} \text{ m/s}^2 \text{ (upwards)} \end{aligned}$;

c. $F = -mg = qE \rightarrow m = \frac{-qE}{g} = 2.45 \times 10^{-18}$ kg

103. If the electric field is zero ¼ from the way of q_1 and q_2, then we know from $E = k\frac{Q}{r^2}$ that $|E_1| = |E_2| \rightarrow \frac{Kq_1}{x^2} = \frac{Kq_2}{(3x)^2}$ so that $\frac{q_2}{q_1} = \frac{(3x)^2}{x^2} = 9$; the charge q_2 is 9 times larger than q_1.

105. a. The field is in the direction of the electron's initial velocity.

b. $\begin{aligned} v^2 &= v_0^2 + 2ax \rightarrow x = -\frac{v_0^2}{2a}(v = 0). \text{ Also, } F = ma = qE \rightarrow a = \frac{qE}{m}, \\ x &= 3.56 \times 10^{-4} \text{ m;} \end{aligned}$

c. $\begin{aligned} v_2 &= v_0 + at \rightarrow t = -\frac{v_0 m}{qE}(v = 0), \\ \therefore t &= 1.42 \times 10^{-10} \text{ s;} \end{aligned}$

d. $v = -\left(\frac{2qEx}{m}\right)^{1/2} -5.00 \times 10^6$ m/s (opposite its initial velocity)

Challenge Problems

107. Answers will vary. This appears to be proprietary information, and ridiculously difficult to find. Speeds will be 20 m/s or less, and there are claims of ~10^{-7} grams for the mass of a drop.

109. Apply $\vec{\mathbf{E}} = -\vec{\nabla}V$ with $\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r \sin\theta}\frac{\partial}{\partial \varphi}$ to the potential calculated earlier, $V_P = k\frac{\vec{\mathbf{p}}\cdot\hat{\mathbf{r}}}{r^2}$ with $\vec{\mathbf{p}} = q\vec{\mathbf{d}}$, and assume that the axis of the dipole is aligned with the z-axis of the coordinate system. Thus, the potential is $V_P = k\frac{q\vec{\mathbf{d}}\cdot\hat{\mathbf{r}}}{r^2} = k\frac{qd\cos\theta}{r^2}$.

$\vec{\mathbf{E}} = 2kqd\left(\frac{\cos\theta}{r^3}\right)\hat{\mathbf{r}} + kqd\left(\frac{\sin\theta}{r^3}\right)\hat{\theta}$

Chapter 8

Check Your Understanding

8.1 1.1×10^{-3} m

8.3 3.59 cm, 17.98 cm

8.4 a. 25.0 pF; b. 9.2

8.5 a. $C = 0.86$ pF, $Q_1 = 10$ pC, $Q_2 = 3.4$ pC, $Q_3 = 6.8$ pC;
b. $C = 2.3$ pF, $Q_1 = 12$ pC, $Q_2 = Q_3 = 16$ pC;
c. $C = 2.3$ pF, $Q_1 = 9.0$ pC, $Q_2 = 18$ pC, $Q_3 = 12$ pC, $Q_4 = 15$ pC

8.6 a.4.0×10^{-13}J; b. 9 times

8.7 a. 3.0; b. $C = 3.0\, C_0$

8.9 a. $C_0 = 20$ pF, $C = 42$ pF; b. $Q_0 = 0.8$ nC, $Q = 1.7$ nC; c. $V_0 = V = 40$ V; d. $U_0 = 16$ nJ, $U = 34$ nJ

Conceptual Questions

1. no; yes

3. false

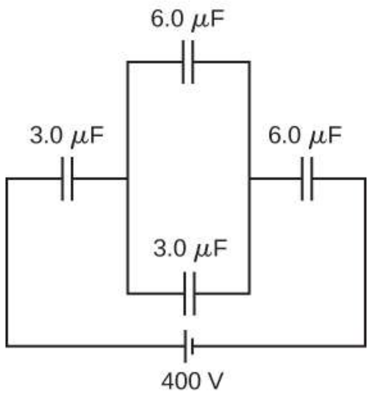
5. no
7. 3.0 μF, 0.33 μF
9. answers may vary
11. Dielectric strength is a critical value of an electrical field above which an insulator starts to conduct; a dielectric constant is the ratio of the electrical field in vacuum to the net electrical field in a material.
13. Water is a good solvent.
15. When energy of thermal motion is large (high temperature), an electrical field must be large too in order to keep electric dipoles aligned with it.
17. answers may vary

Problems

19. 21.6 mC
21. 1.55 V
23. 25.0 nF
25. $1.1 \times 10^{-3} m^2$
27. 500 μC
29. 1:16
31. a. 1.07 nC; b. 267 V, 133 V
33. 0.29 μF
34. 500 capacitors; connected in parallel
35. 3.08 μF (series) and 13.0 μF (parallel)
37. 11.4 μF
39. 0.89 mC; 1.78 mC; 444 V
41. 7.5 μJ
43. a. 405 J; b. 90.0 mC
45. 1.17 J
47. a. 4.43×10^{-9} F; b. 0.453 V; c. 4.53×10^{-10} J; d. no
49. 0.7 mJ
51. a. 7.1 pF; b. 42 pF
53. a. before 3.00 V; after 0.600 V; b. before 1500 V/m; after 300 V/m
55. a. 3.91; b. 22.8 V
57. a. 37 nC; b. 0.4 MV/m; c. 19 nC
59. a. 4.4 μF; b. 4.0×10^{-5} C
61. 0.0135 m^2
63. 0.185 μJ

Additional Problems

65. a. 0.277 nF; b. 27.7 nC; c. 50 kV/m
67. a. 0.065 F; b. 23,000 C; c. 4.0 GJ
69. a. 75.6 μC; b. 10.8 V
71. a. 0.13 J; b. no, because of resistive heating in connecting wires that is always present, but the circuit schematic does not indicate resistors



73. a. $-3.00\ \mu F$; b. You cannot have a negative C_2 capacitance. c. The assumption that they were hooked up in parallel, rather than in series, is incorrect. A parallel connection always produces a greater capacitance, while here a smaller capacitance was assumed. This could only happen if the capacitors are connected in series.

75. a. 14.2 kV; b. The voltage is unreasonably large, more than 100 times the breakdown voltage of nylon. c. The assumed charge is unreasonably large and cannot be stored in a capacitor of these dimensions.

Challenge Problems

77. a. 89.6 pF; b. 6.09 kV/m; c. 4.47 kV/m; d. no

79. a. 421 J; b. 53.9 mF

81. $C = \varepsilon_0 A/(d_1 + d_2)$

83. proof

Chapter 9

Check Your Understanding

9.1 The time for 1.00 C of charge to flow would be $\Delta t = \frac{\Delta Q}{I} = \frac{1.00\ \text{C}}{0.300 \times 10^{-3}\ \text{C/s}} = 3.33 \times 10^3\,\text{s}$, slightly less than an hour. This is quite different from the 5.55 ms for the truck battery. The calculator takes a very small amount of energy to operate, unlike the truck's starter motor. There are several reasons that vehicles use batteries and not solar cells. Aside from the obvious fact that a light source to run the solar cells for a car or truck is not always available, the large amount of current needed to start the engine cannot easily be supplied by present-day solar cells. Solar cells can possibly be used to charge the batteries. Charging the battery requires a small amount of energy when compared to the energy required to run the engine and the other accessories such as the heater and air conditioner. Present day solar-powered cars are powered by solar panels, which may power an electric motor, instead of an internal combustion engine.

9.2 The total current needed by all the appliances in the living room (a few lamps, a television, and your laptop) draw less current and require less power than the refrigerator.

9.3 The diameter of the 14-gauge wire is smaller than the diameter of the 12-gauge wire. Since the drift velocity is inversely proportional to the cross-sectional area, the drift velocity in the 14-gauge wire is larger than the drift velocity in the 12-gauge wire carrying the same current. The number of electrons per cubic meter will remain constant.

9.4 The current density in a conducting wire increases due to an increase in current. The drift velocity is inversely proportional to the current $\left(v_d = \frac{I}{nqA}\right)$, so the drift velocity would decrease.

9.5 Silver, gold, and aluminum are all used for making wires. All four materials have a high conductivity, silver having the highest. All four can easily be drawn into wires and have a high tensile strength, though not as high as copper. The obvious disadvantage of gold and silver is the cost, but silver and gold wires are used for special applications, such as speaker wires. Gold does not oxidize, making better connections between components. Aluminum wires do have their drawbacks. Aluminum has a higher resistivity than copper, so a larger diameter is needed to match the resistance per length of copper wires, but aluminum is cheaper than copper, so this is not a major drawback. Aluminum wires do not have as high of a ductility and tensile strength as copper, but the ductility and tensile strength is within acceptable levels. There are a few concerns that must be addressed in using aluminum and care must be used when making connections. Aluminum has a higher rate of thermal expansion than copper, which can lead to loose connections and a possible fire hazard. The oxidation of aluminum does not conduct and can cause problems. Special techniques must be used when using aluminum wires and components, such as electrical outlets, must be designed to accept aluminum wires.

9.6 The foil pattern stretches as the backing stretches, and the foil tracks become longer and thinner. Since the resistance is calculated as $R = \rho \frac{L}{A}$, the resistance increases as the foil tracks are stretched. When the temperature changes, so does the resistivity of the foil tracks, changing the resistance. One way to combat this is to use two strain gauges, one used as a reference and the other used to measure the strain. The two strain gauges are kept at a constant temperature

9.7 The longer the length, the smaller the resistance. The greater the resistivity, the higher the resistance. The larger the difference between the outer radius and the inner radius, that is, the greater the ratio between the two, the greater the resistance. If you are attempting to maximize the resistance, the choice of the values for these variables will depend on the application. For example, if the cable must be flexible, the choice of materials may be limited.

9.8 Yes, Ohm's law is still valid. At every point in time the current is equal to $I(t) = V(t)/R$, so the current is also a function of time, $I\,(t) = \frac{V_{\text{max}}}{R}\,\sin\,(2\pi f t)$.

9.9 Even though electric motors are highly efficient 10–20% of the power consumed is wasted, not being used for doing useful work. Most of the 10–20% of the power lost is transferred into heat dissipated by the copper wires used to make the coils of the motor. This heat adds to the heat of the environment and adds to the demand on power plants providing the power. The demand on the power plant can lead to increased greenhouse gases, particularly if the power plant uses coal or gas as fuel.

9.10 No, the efficiency is a very important consideration of the light bulbs, but there are many other considerations. As mentioned above, the cost of the bulbs and the life span of the bulbs are important considerations. For example, CFL bulbs contain mercury, a neurotoxin, and must be disposed of as hazardous waste. When replacing incandescent bulbs that are being controlled by a dimmer switch with LED, the dimmer switch may need to be replaced. The dimmer switches for LED lights are comparably priced to the incandescent light switches, but this is an initial cost which should be considered. The spectrum of light should also be considered, but there is a broad range of color temperatures available, so you should be able to find one that fits your needs. None of these considerations mentioned are meant to discourage the use of LED or CFL light bulbs, but they are considerations.

Conceptual Questions

1. If a wire is carrying a current, charges enter the wire from the voltage source's positive terminal and leave at the negative terminal, so the total charge remains zero while the current flows through it.

3. Using one hand will reduce the possibility of "completing the circuit" and having current run through your body, especially current running through your heart.

5. Even though the electrons collide with atoms and other electrons in the wire, they travel from the negative terminal to the positive terminal, so they drift in one direction. Gas molecules travel in completely random directions.

7. In the early years of light bulbs, the bulbs are partially evacuated to reduce the amount of heat conducted through the air to the glass envelope. Dissipating the heat would cool the filament, increasing the amount of energy needed to produce light from the filament. It also protects the glass from the heat produced from the hot filament. If the glass heats, it expands, and as it cools, it contacts. This expansion and contraction could cause the glass to become brittle and crack, reducing the life of the bulbs. Many bulbs are now partially filled with an inert gas. It is also useful to remove the oxygen to reduce the possibility of the filament actually burning. When the original filaments were replaced with more efficient tungsten filaments, atoms from the tungsten would evaporate off the filament at such high temperatures. The atoms collide with the atoms of the inert gas and land back on the filament.

9. In carbon, resistivity increases with the amount of impurities, meaning fewer free charges. In silicon and germanium, impurities decrease resistivity, meaning more free electrons.
11. Copper has a lower resistivity than aluminum, so if length is the same, copper must have the smaller diameter.
13. Device *B* shows a linear relationship and the device is ohmic.
15. Although the conductors have a low resistance, the lines from the power company can be kilometers long. Using a high voltage reduces the current that is required to supply the power demand and that reduces line losses.
17. The resistor would overheat, possibly to the point of causing the resistor to burn. Fuses are commonly added to circuits to prevent such accidents.
19. Very low temperatures necessitate refrigeration. Some materials require liquid nitrogen to cool them below their critical temperatures. Other materials may need liquid helium, which is even more costly.

Problems

21. a. $v = 4.38 \times 10^5 \frac{\text{m}}{\text{s}}$;
b. $\Delta q = 5.00 \times 10^{-3}\,\text{C}, \quad \text{no. of protons} = 3.13 \times 10^{16}$
23. $I = \frac{\Delta Q}{\Delta t}, \quad \Delta Q = 12.00\,\text{C}$
no. of electrons = 7.5×10^{19}
25. $$I(t) = 0.016\frac{\text{C}}{\text{s}^4}t^3 - 0.001\frac{\text{C}}{\text{s}}$$
$$I(3.00\,\text{s}) = 0.431\,\text{A}$$
27. $I(t) = -I_{\text{max}} \sin(\omega t + \phi)$
29. $|J| = 15.92\,\text{A/m}^2$
31. $I = 3.98 \times 10^{-5}\,\text{A}$
33. a. $|J| = 7.60 \times 10^5 \frac{\text{A}}{\text{m}^2}$; b. $v_{\text{d}} = 5.60 \times 10^{-5} \frac{\text{m}}{\text{s}}$
35. $R = 6.750\,\text{k}\,\Omega$
37. $R = 0.10\,\Omega$
39. $$R = \rho\frac{L}{A}$$
$$L = 3\,\text{cm}$$
41. $$\frac{R_{\text{Al}}/L_{\text{Al}}}{R_{\text{Cu}}/L_{\text{Cu}}} = \frac{\rho_{\text{Al}}\frac{1}{\pi\left(\frac{D_{\text{Al}}}{2}\right)^2}}{\rho_{\text{Cu}}\frac{1}{\pi\left(\frac{D_{\text{Cu}}}{2}\right)^2}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}\left(\frac{D_{\text{Cu}}}{D_{\text{Al}}}\right)^2 = 1, \quad \frac{D_{\text{Al}}}{D_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{Cu}}}$$
43. a. $R = R_0(1 + \alpha\Delta T), \quad 2 = 1 + \alpha\Delta T, \quad \Delta T = 256.4\,°\text{C}, \quad T = 276.4\,°\text{C}$;
b. Under normal conditions, no it should not occur.
45. $$R = R_0(1 + \alpha\Delta T)$$, iron
$$\alpha = 0.006\,°\text{C}^{-1}$$
47. a. $R = \rho\frac{L}{A}, \quad \rho = 2.44 \times 10^{-8}\,\Omega \cdot \text{m}$, gold;

$$R = \rho \frac{L}{A}(1 + \alpha \Delta T)$$

b.

$$R = 2.44 \times 10^{-8}\ \Omega \cdot \text{m} \left(\frac{25\text{ m}}{\pi \left(\frac{0.100 \times 10^{-3}\text{m}}{2} \right)^2} \right) \left(1 + 0.0034\ °\text{C}^{-1}\ (150\ °\text{C} - 20\ °\text{C})\right)$$

$$R = 112\ \Omega$$

49.

$$R_{\text{Fe}} = 0.525\ \Omega\,, \quad R_{\text{Cu}} = 0.500\ \Omega\,, \quad \alpha_{\text{Fe}} = 0.0065\ °\text{C}^{-1} \quad \alpha_{\text{Cu}} = 0.0039\ °\text{C}^{-1}$$
$$R_{\text{Fe}} = R_{\text{Cu}}$$
$$R_{0\ \text{Fe}}\,(1 + \alpha_{\text{Fe}}\,(T - T_0)) = R_{0\ \text{Cu}}\,(1 + \alpha_{\text{Cu}}\,(T - T_0))$$
$$\frac{R_{0\ \text{Fe}}}{R_{0\ \text{Cu}}}(1 + \alpha_{\text{Fe}}\,(T - T_0)) = 1 + \alpha_{\text{Cu}}\,(T - T_0)$$
$$T = 2.91\ °\text{C}$$

51.

$$R_{\text{min}} = 2.375 \times 10^5\ \Omega\,, \quad I_{\text{min}} = 12.63\ \mu\,\text{A}$$
$$R_{\text{max}} = 2.625 \times 10^5\ \Omega\,, \quad I_{\text{max}} = 11.43\ \mu\,\text{A}$$

53. $R = 100\ \Omega$

55. a. $I = 2$ mA; b. $P = 0.04$ W; c. $P = 0.04$ W; d. It is converted into heat.

57.

$$A = 2.08\text{mm}^2$$
$$\rho = 100 \times 10^{-8}\ \Omega \cdot \text{m}$$
$$P = \frac{V^2}{R}\ ,\quad R = \rho \frac{L}{A}$$
$$R = 40\ \Omega \quad L = 83\text{ m}$$

59. $I = 0.14\text{ A}, \quad V = 14\text{ V}$

61.

$$I \approx 3.00\text{ A} + \frac{100\text{ W}}{110\text{ V}} + \frac{60\text{ W}}{110\text{ V}} + \frac{3.00\text{ W}}{110\text{ V}} = 4.48\text{ A}$$

a.

$$P = 493\text{ W}$$
$$R = 9.91\ \Omega\,,$$
$$P_{\text{loss}} = 200.\text{ W}$$
$$\%\text{loss} = 40\%$$
$$P = 493\text{ W}$$
$$I = 0.0045\text{ A}$$

b.

$$R = 9.91\ \Omega$$
$$P_{\text{loss}} = 201\,\mu\,\text{W}$$
$$\%\text{loss} = 0.00004\%$$

63.

$$R_{\text{copper}} = 23.77\ \Omega$$
$$P = 2.377 \times 10^5\text{W}$$

65.

$$R = R_0\,(1 + \alpha\,(T - T_0))$$
$$0.82R_0 = R_0\,(1 + \alpha\,(T - T_0))\,, \quad 0.82 = 1 - 0.06\,(T - 37\ °\text{C})\,, \quad T = 40\ °\text{C}$$

67. a. $R_{\text{Au}} = R_{\text{Ag}}, \quad \rho_{\text{Au}} \frac{L_{\text{Au}}}{A_{\text{Au}}} = \rho_{\text{Ag}} \frac{L_{\text{Ag}}}{A_{\text{Ag}}}, \quad L_{\text{Ag}} = 1.53 \text{ m};$

b. $R_{\text{Au},20\ °\text{C}} = 0.0074\ \Omega, \quad R_{\text{Au},100\ °\text{C}} = 0.0094\ \Omega, \quad R_{\text{Ag},100\ °\text{C}} = 0.0096\ \Omega$

Additional Problems

69.
$$dR = \frac{\rho}{2\pi r L} dr$$
$$R = \frac{\rho}{2\pi L} \ln \frac{r_\text{o}}{r_\text{i}}$$
$$R = 2.21 \times 10^{11}\ \Omega$$

71. a.
$R_0 = 0.003\ \Omega$; b.
$T_\text{c} = 37.0\ °\text{C}$
$R = 0.00302\ \Omega$

73. $\rho = 5.00 \times 10^{-8}\ \Omega \cdot \text{m}$

75. $\rho = 1.71 \times 10^{-8}\ \Omega \cdot \text{m}$

77. a. $V = 6000$ V; b. $V = 6$ V

79. $P = \frac{W}{t}, \quad W = 8.64$ J

Challenge Problems

81.
$V = 7.09\ \text{cm}^3$
$n = 8.49 \times 10^{28} \frac{\text{electrons}}{\text{m}^3}$
$v_\text{d} = 7.00 \times 10^{-5} \frac{\text{m}}{\text{s}}$

83. a. 4.38 x 10^7 m/s b. $v = 5.81 \times 10^{13} \frac{\text{protons}}{\text{m}^3}$ c. 1.25 $\frac{\text{electrons}}{\text{m}^3}$

85. $E = 75$ kJ

87. a. ; b. $V = 43.54$ V
$P = 52$ W
$R = 36\ \Omega$

89. a. $R = \frac{\rho}{2\pi L} \ln\left(\frac{R_0}{R_i}\right)$; b. $R = 2.5\ \text{m}\,\Omega$

91. (a) 0.870 A
(b) #electrons = 2.54×10^{23} electrons
(c) 132 ohms
(d) $q = 4.68 \times 10^6$ J

93. $P = 1045\ \text{W}, \quad P = \frac{V^2}{R}, \quad R = 12.27\ \Omega$

Chapter 10

Check Your Understanding

10.1 If a wire is connected across the terminals, the load resistance is close to zero, or at least considerably less than the internal resistance of the battery. Since the internal resistance is small, the current through the circuit will be large, $I = \frac{\varepsilon}{R+r} = \frac{\varepsilon}{0+r} = \frac{\varepsilon}{r}$. The large current causes a high power to be dissipated by the internal resistance $\left(P = I^2 r\right)$. The power is dissipated as heat.

10.2 The equivalent resistance of nine bulbs connected in series is $9R$. The current is $I = V/9\ R$. If one bulb burns out, the equivalent resistance is $8R$, and the voltage does not change, but the current increases $(I = V/8\ R)$. As more bulbs burn out, the current becomes even higher. Eventually, the current becomes too high, burning out the shunt.

10.3 The equivalent of the series circuit would be $R_{eq} = 1.00\,\Omega + 2.00\,\Omega + 2.00\,\Omega = 5.00\,\Omega$, which is higher than the equivalent resistance of the parallel circuit $R_{eq} = 0.50\,\Omega$. The equivalent resistor of any number of resistors is always higher than the equivalent resistance of the same resistors connected in parallel. The current through for the series circuit would be $I = \frac{3.00\text{ V}}{5.00\,\Omega} = 0.60\text{ A}$, which is lower than the sum of the currents through each resistor in the parallel circuit, $I = 6.00\text{ A}$. This is not surprising since the equivalent resistance of the series circuit is higher. The current through a series connection of any number of resistors will always be lower than the current into a parallel connection of the same resistors, since the equivalent resistance of the series circuit will be higher than the parallel circuit. The power dissipated by the resistors in series would be $P = 1.80\text{ W}$, which is lower than the power dissipated in the parallel circuit $P = 18.00\text{ W}$.

10.4 A river, flowing horizontally at a constant rate, splits in two and flows over two waterfalls. The water molecules are analogous to the electrons in the parallel circuits. The number of water molecules that flow in the river and falls must be equal to the number of molecules that flow over each waterfall, just like sum of the current through each resistor must be equal to the current flowing into the parallel circuit. The water molecules in the river have energy due to their motion and height. The potential energy of the water molecules in the river is constant due to their equal heights. This is analogous to the constant change in voltage across a parallel circuit. Voltage is the potential energy across each resistor.
The analogy quickly breaks down when considering the energy. In the waterfall, the potential energy is converted into kinetic energy of the water molecules. In the case of electrons flowing through a resistor, the potential drop is converted into heat and light, not into the kinetic energy of the electrons.

10.5 1. All the overhead lighting circuits are in parallel and connected to the main supply line, so when one bulb burns out, all the overhead lighting does not go dark. Each overhead light will have at least one switch in series with the light, so you can turn it on and off. 2. A refrigerator has a compressor and a light that goes on when the door opens. There is usually only one cord for the refrigerator to plug into the wall. The circuit containing the compressor and the circuit containing the lighting circuit are in parallel, but there is a switch in series with the light. A thermostat controls a switch that is in series with the compressor to control the temperature of the refrigerator.

10.6 The circuit can be analyzed using Kirchhoff's loop rule. The first voltage source supplies power: $P_{in} = IV_1 = 7.20\text{ mW}$. The second voltage source consumes power: $P_{out} = IV_2 + I^2R_1 + I^2R_2 = 7.2\text{ mW}$.

10.7 The current calculated would be equal to $I = -0.20\text{ A}$ instead of $I = 0.20\text{ A}$. The sum of the power dissipated and the power consumed would still equal the power supplied.

10.8 Since digital meters require less current than analog meters, they alter the circuit less than analog meters. Their resistance as a voltmeter can be far greater than an analog meter, and their resistance as an ammeter can be far less than an analog meter. Consult Figure 10.36 and Figure 10.35 and their discussion in the text.

Conceptual Questions

1. Some of the energy being used to recharge the battery will be dissipated as heat by the internal resistance.

3. $P = I^2R = \left(\frac{\varepsilon}{r+R}\right)^2 R = \varepsilon^2 R(r+R)^{-2}, \quad \frac{dP}{dR} = \varepsilon^2\left[(r+R)^{-2} - 2R(r+R)^{-3}\right] = 0,$
$\left[\frac{(r+R)-2R}{(r+R)^3}\right] = 0, \quad r = R$

5. It would probably be better to be in series because the current will be less than if it were in parallel.

7. two filaments, a low resistance and a high resistance, connected in parallel

9. It can be redrawn.

$$R_{\text{eq}} = \left[\frac{1}{R_6} + \frac{1}{R_1} + \frac{1}{R_2 + \left(\frac{1}{R_4} + \frac{1}{R_3 + R_5}\right)^{-1}}\right]^{-1}$$

11. In series the voltages add, but so do the internal resistances, because the internal resistances are in series. In parallel, the terminal voltage is the same, but the equivalent internal resistance is smaller than the smallest individual internal resistance and a higher current can be provided.

13. The voltmeter would put a large resistance in series with the circuit, significantly changing the circuit. It would probably give a reading, but it would be meaningless.

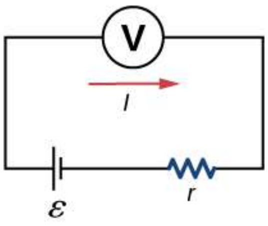
15. The ammeter has a small resistance; therefore, a large current will be produced and could damage the meter and/or overheat the battery.

17. The time constant can be shortened by using a smaller resistor and/or a smaller capacitor. Care should be taken when reducing the resistance because the initial current will increase as the resistance decreases.

19. Not only might water drip into the switch and cause a shock, but also the resistance of your body is lower when you are wet.

Problems

21. a.



b. 0.476W; c. 0.691 W; d. As R_L is lowered, the power difference decreases; therefore, at higher volumes, there is no significant difference.

23. a. 0.400 Ω; b. No, there is only one independent equation, so only r can be found.

25. a. 0.400 Ω; b. 40.0 W; c. 0.0956 °C/min

27. largest, 786 Ω, smallest, 20.32 Ω

29. 29.6 W

31. a. 0.74 A; b. 0.742 A

33. a. 60.8 W; b. 1.56 kW

35. a. $R_s = 9.00\ \Omega$; b. $I_1 = I_2 = I_3 = 2.00\ \text{A}$;
c. $V_1 = 8.00\ \text{V}, V_2 = 2.00\ \text{V}, V_3 = 8.00\ \text{V}$; d. $P_1 = 16.00\ \text{W}, P_2 = 4.00\ \text{W}, P_3 = 16.00\ \text{W}$; e. $P = 36.00\ \text{W}$

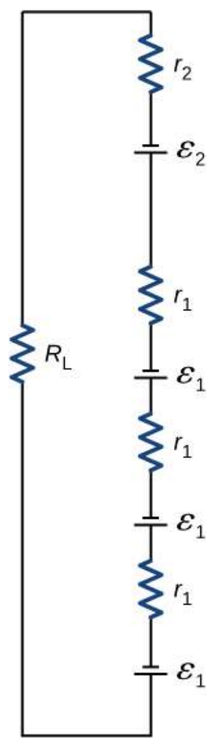
37. a. $I_1 = 0.6\ \text{mA}, \quad I_2 = 0.4\ \text{mA}, \quad I_3 = 0.2\ \text{mA}$;
b. $I_1 = 0.04\ \text{mA}, \quad I_2 = 1.52\ \text{mA}, \quad I_3 = -1.48\ \text{mA}$; c. $P_{\text{out}} = 0.92\ \text{mW}, \quad P_{\text{out}} = 4.50\ \text{mW}$;
d. $P_{\text{in}} = 0.92\ \text{mW}, \quad P_{\text{in}} = 4.50\ \text{mW}$

39. $V_1 = 42\ \text{V}, V_2 = 6\ \text{V}, R_4 = 18\ \Omega$

41. a. $I_1 = 1.5\ \text{A}, I_2 = 2\ \text{A}, I_3 = 0.5\ \text{A}, \ I_4 = 2.5\ \text{A}, I_5 = 2\ \text{A}$; b. $P_{\text{in}} = I_2 V_1 + I_5 V_5 = 34\ \text{W}$;
c. $P_{\text{out}} = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 = 34\ \text{W}$

43. $I_1 = \frac{2}{3}\frac{V}{R}, I_2 = \frac{V}{3R}, I_3 = \frac{V}{3R}$

45. a.



;
b. 0.617 A; c. 3.81 W; d. 18.0 Ω

47. $I_1 r_1 - \varepsilon_1 + I_1 R_4 + \varepsilon_4 + I_2 r_4 + I_4 r_3 - \varepsilon_3 + I_2 R_3 + I_1 R_1 = 0$

49. 4.00 to 30.0 MΩ

51. a. 2.50 μF; b. 2.00 s

53. a. 12.3 mA; b. 7.50×10^{-4} s; c. 4.53 mA; d. 3.89 V

55. a. 1.00×10^{-7} F; b. No, in practice it would not be difficult to limit the capacitance to less than 100 nF, since typical capacitors range from fractions of a picofarad (pF) to milifarad (mF).

57. $3.33 \times 10^{-3}\ \Omega$

59. 12.0 V

61. 400 V

63. a. 6.00 mV; b. It would not be necessary to take extra precautions regarding the power coming from the wall. However, it is possible to generate voltages of approximately this value from static charge built up on gloves, for instance, so some precautions are necessary.

65. a. 5.00×10^{-2} C; b. 10.0 kV; c. 1.00 kΩ ; d. 1.79×10^{-2} °C

Additional Problems

67. a. $C_{eq} = 5.00$ mF; b. $\tau = 0.1$ s; c. 0.069 s

69. a. $R_{eq} = 20.00\ \Omega$;
b. $I_r = 1.50$ A, $I_1 = 1.00$ A, $I_2 = 0.50$ A, $I_3 = 0.75$ A, $I_4 = 0.75$ A, $I_5 = 1.50$ A;
c. $V_r = 1.50$ V, $V_1 = 9.00$ V, $V_2 = 9.00$ V, $V_3 = 7.50$ V, $V_4 = 7.50$ V, $V_5 = 12.00$ V;
d. $P_r = 2.25$ W, $P_1 = 9.00$ W, $P_2 = 4.50$ W, $P_3 = 5.625$ W, $P_4 = 5.625$ W, $P_5 = 18.00$ W;
e. $P = 45.00$ W

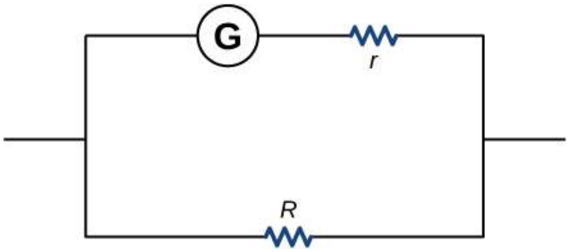
71. a. $\tau = \left(1.38 \times 10^{-5}\,\Omega\text{m}\left(\frac{5.00 \times 10^{-2}\text{ m}}{3.14\left(\frac{0.05 \times 10^{-3}}{2}\right)^2}\right)\right)10 \times 10^{-3}\text{ F} = 3.52\text{ s}$; b.

$V = 0.014\text{ A}\left(e^{-\frac{1.00\text{ s}}{3.52\text{ s}}}\right)351.59\ \Omega = 0.376\text{ V}$

73. a. $t = \frac{3\ \text{A}\cdot\text{h}}{\frac{1.5\ \text{V}}{900\ \Omega}} = 1800\ \text{h}$; b. $t = \frac{3\ \text{A}\cdot\text{h}}{\frac{1.5\ \text{V}}{100\ \Omega}} = 200\ \text{h}$
75. $U_1 = C_1 V_1^2 = 0.72\text{J}, \quad U_2 = C_2 V_2^2 = 0.338\ \text{J}$
77. a. $R_{\text{eq}} = 24.00\ \Omega$; b. $I_1 = 1.00\ \text{A}, I_2 = 0.67\ \text{A}, I_3 = 0.33\ \text{A}, I_4 = 1.00\ \text{A}$;
c. $V_1 = 14.00\ \text{V}, V_2 = 6.00\ \text{V}, V_3 = 6.00\ \text{V}, V_4 = 4.00\ \text{V}$;
d. $P_1 = 14.00\ \text{W}, P_2 = 4.04\ \text{W}, P_3 = 1.96\ \text{W}, P_4 = 4.00\ \text{W}$; e. $P = 24.00\ \text{W}$
79. a. $R_{\text{eq}} = 12.00\ \Omega, I = 1.00\ \text{A}$; b. $R_{\text{eq}} = 12.00\ \Omega, I = 1.00\ \text{A}$
81. a. $-400\ \text{k}\Omega$; b. You cannot have negative resistance. c. The assumption that $R_{\text{eq}} < R_1$ is unreasonable. Series resistance is always greater than any of the individual resistances.
83. $E_2 - I_2 r_2 - I_2 R_2 + I_1 R_5 + I_1 r_1 - E_1 + I_1 R_1 = 0$
85. a. $I = 1.17\ \text{A}, I_1 = 0.50\ \text{A}, I_2 = 0.67\ \text{A}, I_3 = 0.67\ \text{A}, I_4 = 0.50\ \text{A}, I_5 = 0.17\ \text{A}$;
b. $P_{\text{output}} = 23.4\ \text{W}, P_{\text{input}} = 23.4\ \text{W}$
87. a. 4.99 s; b. 3.87 °C; c. $3.11 \times 10^4\ \Omega$; d. No, this change does not seem significant. It probably would not be noticed.

Challenge Problems

89. a. 0.273 A; b. $V_T = 1.36\ \text{V}$
91. a. $V_s = V - I_M R_M = 9.99875\ \text{V}$; b. $R_S = \frac{V_P}{I_M} = 199.975\ \text{k}\Omega$
93. a. $\tau = 3800\ \text{s}$; b. 1.26 mA; c. $t = 2633.96\ \text{s}$
95. $R_{\text{eq}} = \left(\sqrt{3} - 1\right) R$
97. a. $P_{\text{imheater}} = \dfrac{1\text{cup}\left(\frac{0.000237\ \text{m}^3}{\text{cup}}\right)\left(\frac{1000\ \text{kg}}{\text{m}^3}\right)\left(4186\frac{\text{J}}{\text{kg}\ °\text{C}}\right)(100\ °\text{C} - 20\ °\text{C})}{180.00\ \text{s}} \approx 441\ \text{W}$;
b. $I = \frac{441\ \text{W}}{120\ \text{V}} + 4\left(\frac{100\ \text{W}}{120\ \text{V}}\right) + \frac{1500\ \text{W}}{120\ \text{V}} = 19.51\ \text{A}$; Yes, the breaker will trip.
c. $I = \frac{441\ \text{W}}{120\ \text{V}} + 4\left(\frac{18\ \text{W}}{120\ \text{V}}\right) + \frac{1500\ \text{W}}{120\ \text{V}} = 16.78\ \text{A}$; Yes, the breaker will trip.
99.



,
$2.40 \times 10^{-3}\ \Omega$

Chapter 11

Check Your Understanding

11.1 a. 0 N; b. $2.4 \times 10^{-14}\hat{\mathbf{k}}\text{N}$; c. $2.4 \times 10^{-14}\hat{\mathbf{j}}\ \text{N}$; d. $(7.2\hat{\mathbf{j}} + 2.2\hat{\mathbf{k}}) \times 10^{-15}\text{N}$
11.2 a. $9.6 \times 10^{-12}\text{N}$ toward the south; b. $\frac{w}{F_{\text{m}}} = 1.7 \times 10^{-15}$
11.3 a. bends upward; b. bends downward
11.4 a. aligned or anti-aligned; b. perpendicular
11.5 a. 1.1 T; b. 1.6 T
11.6 0.32 m

Conceptual Questions

1. Both are field dependent. Electrical force is dependent on charge, whereas magnetic force is dependent on current or rate of charge flow.

3. The magnitude of the proton and electron magnetic forces are the same since they have the same amount of charge. The direction of these forces however are opposite of each other. The accelerations are opposite in direction and the electron has a larger acceleration than the proton due to its smaller mass.
5. The magnetic field must point parallel or anti-parallel to the velocity.
7. A compass points toward the north pole of an electromagnet.
9. Velocity and magnetic field can be set together in any direction. If there is a force, the velocity is perpendicular to it. The magnetic field is also perpendicular to the force if it exists.
11. A force on a wire is exerted by an external magnetic field created by a wire or another magnet.
13. Poor conductors have a lower charge carrier density, *n*, which, based on the Hall effect formula, relates to a higher Hall potential. Good conductors have a higher charge carrier density, thereby a lower Hall potential.

Problems

15. a. left; b. into the page; c. up the page; d. no force; e. right; f. down
17. a. right; b. into the page; c. down
19. a. into the page; b. left; c. out of the page
21. a. 2.64×10^{-8} N; north b. The force is very small, so this implies that the effect of static charges on airplanes is negligible.
23. $10.1°$; $169.9°$
25. 4.27 m
27. a. 4.80×10^{-19} C; b. 3; c. This ratio must be an integer because charges must be integer numbers of the basic charge of an electron. There are no free charges with values less than this basic charge, and all charges are integer multiples of this basic charge.
29. (a) $3.27 \text{ x } 10^4$ m/s (b) 12,525 m (c) 292 m (d) 6.83 m.
31. a. 1.8×10^7 m/s; b. 6.8×10^6 eV; c. 3.4×10^6 V
33. a. left; b. into the page; c. up; d. no force; e. right; f. down
35. a. into the page; b. left; c. out of the page
37. a. 2.50 N; b. This means that the light-rail power lines must be attached in order not to be moved by the force caused by Earth's magnetic field.
39. a. $\tau = NIAB$, so τ decreases by 5.00% if B decreases by 5.00%; b. 5.26% increase
41. 10.0 A
43. $\text{A} \cdot \text{m}^2 \cdot \text{T} = \text{A} \cdot \text{m}^2 \cdot \frac{\text{N}}{\text{A} \cdot \text{m}} = \text{N} \cdot \text{m}$
45. 3.48×10^{-26} N · m
47. 0.666 N · m
49. 5.8×10^{-6} V
51. 4.8×10^7 C/kg
53. a. 4.4×10^{-8} s; b. 0.21 m
55. a. 1.92×10^{-12} J; b. 12 MeV; c. 12 MV; d. 5.2×10^{-8} s; e. 1.92×10^{-12} J, 12 MeV, 12 V, 10.4×10^{-8} s
57. a. 2.50×10^{-2} m; b. Yes, this distance between their paths is clearly big enough to separate the U-235 from the U-238, since it is a distance of 2.5 cm.

Additional Problems

59. $-7.2 \times 10^{-15} \text{N}\hat{\mathbf{j}}$
61. $9.8 \times 10^{-5} \hat{\mathbf{j}}\text{T}$; the magnetic and gravitational forces must balance to maintain dynamic equilibrium
63. 1.13×10^{-3} T
65. $(1.6\hat{\mathbf{i}} - 1.4\hat{\mathbf{j}} - 1.1\hat{\mathbf{k}}) \times 10^5$ V/m
67. a. circular motion in a north, down plane; b. $(1.61\hat{\mathbf{j}} - 0.58\hat{\mathbf{k}}) \times 10^{-14}$ N
69. The proton has more mass than the electron; therefore, its radius and period will be larger.
71. 1.3×10^{-25} kg
73. 1:0.707:1
75. 1/4

77. a. 2.3×10^{-4} m; b. 1.37×10^{-4} m
79. a. 30.0°; b. 4.80 N
81. a. 0.283 N; b. 0.4 N; c. 0 N; d. 0 N
83. 0 N and 0.012 Nm
85. a. 0.31 Am^2; b. 0.16 Nm
87. 0.024 Am^2
89. a. 0.16 Am^2; b. 0.016 Nm; c. 0.028 J
91. (Proof)
93. 4.65×10^{-7} V
95. Since $E = Blv$, where the width is twice the radius, $I = 2r$, $I = nqAv_d$,
$v_\text{d} = \frac{I}{nqA} = \frac{I}{nq\pi r^2}$ so $E = B \times 2r \times \frac{\text{I}}{nq\pi r^2} = \frac{2IB}{nq\pi r} \propto \frac{1}{r} \propto \frac{1}{d}$.
The Hall voltage is inversely proportional to the diameter of the wire.
97. 6.92×10^7 m/s; 0.602 m
99. a. 2.4×10^{-19} C; b. not an integer multiple of e; c. need to assume all charges have multiples of e, could be other forces not accounted for
101. a. B = 5 T; b. very large magnet; c. applying such a large voltage

Challenge Problems

103. $R = (mv\sin\theta)/qB; p = \left(\frac{2\pi m}{eB}\right) v\cos\theta$
105. $IaL^2/2$
107. $m = \frac{qB_0{}^2}{8V_\text{acc}} x^2$
109. 0.23 N

Chapter 12

Check Your Understanding

12.1 1.41 meters
12.2 $\frac{\mu_0 I}{2R}$
12.3 4 amps flowing out of the page
12.4 Both have a force per unit length of 9.23×10^{-12} N/m
12.5 0.608 meters
12.6 In these cases the integrals around the Ampèrian loop are very difficult because there is no symmetry, so this method would not be useful.
12.7 a. 1.00382; b. 1.00015
12.8 a. 1.0×10^{-4} T; b. 0.60 T; c. 6.0×10^3

Conceptual Questions

1. Biot-Savart law's advantage is that it works with any magnetic field produced by a current loop. The disadvantage is that it can take a long time.
3. If you were to go to the start of a line segment and calculate the angle θ to be approximately 0°, the wire can be considered infinite. This judgment is based also on the precision you need in the result.
5. You would make sure the currents flow perpendicular to one another.
7. A magnetic field line gives the direction of the magnetic field at any point in space. The density of magnetic field lines indicates the strength of the magnetic field.
9. The spring reduces in length since each coil with have a north pole-produced magnetic field next to a south pole of the next coil.
11. Ampère's law is valid for all closed paths, but it is not useful for calculating fields when the magnetic field produced lacks symmetry that can be exploited by a suitable choice of path.
13. If there is no current inside the loop, there is no magnetic field (see Ampère's law). Outside the pipe, there may be an enclosed current through the copper pipe, so the magnetic field may not be zero outside the

pipe.

15. The bar magnet will then become two magnets, each with their own north and south poles. There are no magnetic monopoles or single pole magnets.

Problems

17. $5.66 \times 10^{-5}\text{T}$

19. $B = \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right)$ out of the page

21. $a = \frac{2R}{\pi}$; the current in the wire to the right must flow up the page.

23. 20 A

25. Both answers have the magnitude of magnetic field of $4.5 \times 10^{-5}\text{T}$.

27. At P1, the net magnetic field is zero. At P2, $B = \frac{3\mu_0 I}{8\pi a}$ into the page.

29. The magnetic field is at a minimum at distance *a* from the top wire, or half-way between the wires.

31. a. $F/l = 8 \times 10^{-6}\text{N/m}$ away from the other wire; b. $F/l = 8 \times 10^{-6}\text{N/m}$ toward the other wire

33. $B = \frac{\mu_0 I}{2\pi a^2 b}\left(\left(a_2 + b_2\right)\hat{\mathbf{i}} + b\sqrt{\left(a^2 - b^2\right)}\hat{\mathbf{j}}\right)$

35. 0.019 m

37. $\text{N} \times 6.28 \times 10^{-5}\text{T}$

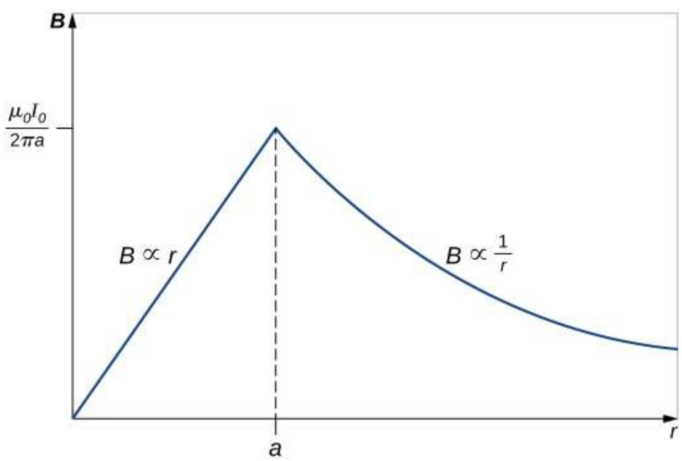
39. $B = \frac{\mu_0 I R^2}{\left(\left(\frac{d}{2}\right)^2 + R^2\right)^{3/2}}$

41. a. $\mu_0 I$; b. 0; c. $\mu_0 I$; d. 0

43. a. $3\mu_0 I$; b. 0; c. $7\mu_0 I$; d. $-2\mu_0 I$

45. at the radius R

47.



49. $B = 1.3 \times 10^{-2}\text{T}$

51. roughly eight turns per cm

53. $B = \frac{1}{2}\mu_0 n I$

55. 0.0181 A

57. 0.0008 T

59. 317.31

61. $2.1 \times 10^{-4}\text{A} \cdot \text{m}^2$
2.7 A

63. 0.18 T

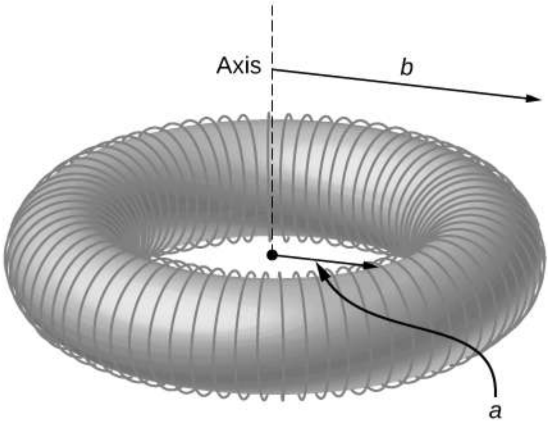
Additional Problems

65. $B = 1.4 \times 10^{-4}\text{T}$

67. $3.2 \times 10^{-19}N$ in an arc away from the wire

69. a. above and below $B = \mu_0 j$, in the middle $B = 0$; b. above and below $B = 0$, in the middle $B = \mu_0 j$

71. $\frac{dB}{B} = -\frac{dr}{r}$
73. a. 5026 turns; b. 0.00957 T
75. $B_1(x) = \frac{\mu_0 I R^2}{2\left(R^2+z^2\right)^{3/2}}$
77. $B = \frac{\mu_0 \sigma\omega}{2} R$
79. derivation
81. derivation
83. As the radial distance goes to infinity, the magnetic fields of each of these formulae go to zero.
85. a. $B = \frac{\mu_0 I}{2\pi r}$; b. $B = \frac{\mu_0 J_0 r^2}{3R}$
87. $B(r) = \mu_0 NI/2\pi r$



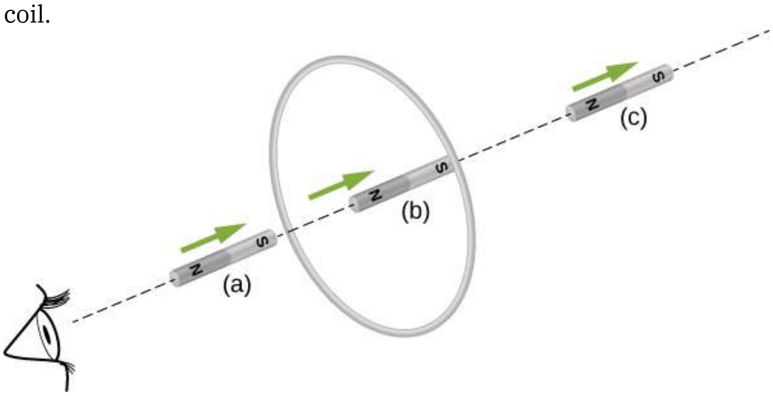
Challenge Problems

89. $B = \frac{\mu_0 I}{2\pi a} \ln \frac{x+a}{x}$.
91. a. $B = \frac{\mu_0 \sigma\omega}{2}\left[\frac{2h^2+R^2}{\sqrt{R^2+h^2}} - 2h\right]$; b. $B = 4.09 \times 10^{-5}$ T, 82% of Earth's magnetic field

Chapter 13

Check Your Understanding

13.1 1.1 T/s
13.2 To the observer shown, the current flows clockwise as the magnet approaches, decreases to zero when the magnet is centered in the plane of the coil, and then flows counterclockwise as the magnet leaves the coil.



13.4 $\varepsilon = Bl^2\omega/2$, with O at a higher potential than S
13.5 1.5 V

13.6 a. yes; b. Yes; however there is a lack of symmetry between the electric field and coil, making $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ a more complicated relationship that can't be simplified as shown in the example.
13.7 3.4×10^{-3} V/m
13.8 P_1, P_2, P_4
13.9 a. 3.1×10^{-6} V; b. 2.0×10^{-7} V/m

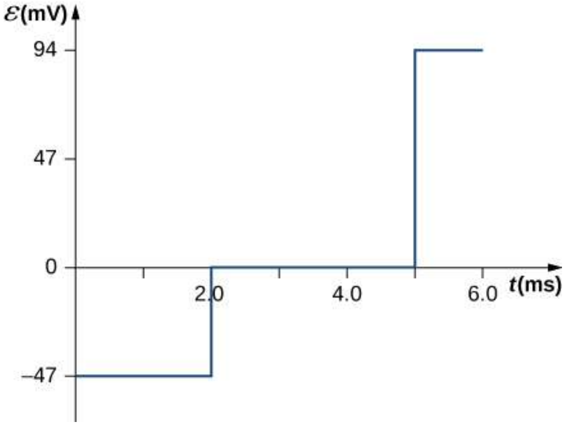
Conceptual Questions

1. The emf depends on the rate of change of the magnetic field.
3. Both have the same induced electric fields; however, the copper ring has a much higher induced emf because it conducts electricity better than the wooden ring.
5. a. no; b. yes
7. As long as the magnetic flux is changing from positive to negative or negative to positive, there could be an induced emf.
9. Position the loop so that the field lines run perpendicular to the area vector or parallel to the surface.
11. a. CW as viewed from the circuit; b. CCW as viewed from the circuit
13. As the loop enters, the induced emf creates a CCW current while as the loop leaves the induced emf creates a CW current. While the loop is fully inside the magnetic field, there is no flux change and therefore no induced current.
15. a. CCW viewed from the magnet; b. CW viewed from the magnet; c. CW viewed from the magnet; d. CCW viewed from the magnet; e. CW viewed from the magnet; f. no current
17. Positive charges on the wings would be to the west, or to the left of the pilot while negative charges would be pulled east or to the right of the pilot. Thus, the left hand tips of the wings would be positive and the right hand tips would be negative.
19. The work is greater than the kinetic energy because it takes energy to counteract the induced emf.
21. The conducting sheet is shielded from the changing magnetic fields by creating an induced emf. This induced emf creates an induced magnetic field that opposes any changes in magnetic fields from the field underneath. Therefore, there is no net magnetic field in the region above this sheet. If the field were due to a static magnetic field, no induced emf will be created since you need a changing magnetic flux to induce an emf. Therefore, this static magnetic field will not be shielded.
23. a. zero induced current, zero force; b. clockwise induced current, force is to the left; c. zero induced current, zero force; d. counterclockwise induced current, force is to the left; e. zero induced current, zero force.

Problems

25. a. 3.8 V; b. 2.2 V; c. 0 V

27.
$$\begin{aligned}
B &= 1.5t, 0 \le t < 2.0\text{ ms}, B = 3.0\text{ mT}, 2.0\text{ ms} \le t \le 5.0\text{ ms}, \\
B &= -3.0t + 18\text{ mT}, 5.0\text{ ms} < t \le 6.0\text{ ms}, \\
\varepsilon &= -\frac{d\Phi_\text{m}}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt}, \\
\varepsilon &= -\pi(0.100\text{ m})^2\,(1.5\text{ T/s}) \\
&= -47\text{ mV}\,(0 \le t < 2.0\text{ ms}), \\
\varepsilon &= \pi(0.100\text{ m})^2\,(0) = 0\,(2.0\text{ ms} \le t \le 5.0\text{ ms}), \\
\varepsilon &= -\pi(0.100\text{ m})^2\,(-3.0\text{ T/s}) = 94\text{ mV}\ \ (5.0\text{ ms} < t < 6.0\text{ ms}).
\end{aligned}$$



29. Each answer is 20 times the previously given answers.

31.
$$\begin{aligned}\hat{\mathbf{n}} &= \hat{k},\ d\Phi_{\mathrm{m}} = \mathrm{C}y\sin(\omega t)\,dxdy, \\ \Phi_{\mathrm{m}} &= \frac{Cab^2\sin(\omega t)}{2}, \\ \varepsilon &= -\frac{Cab^2\omega\cos(\omega t)}{2}.\end{aligned}$$

33. a. 7.8×10^{-3} V; b. CCW from the same view as the magnetic field

35. a. 150 A downward through the resistor; b. 46 A upward through the resistor; c. 0.019 A downward through the resistor

37. 0.0015 V

39. $\varepsilon = -B_0 l d\omega \cos(\Omega t)\mathrm{ld} + B_0 \sin(\Omega t)\mathrm{lv}$

41. $\varepsilon = Blv\cos\theta$

43. a. $2 \times 10^{-19} T$; b. 1.25 V/m; c. 0.3125 V; d. 16 m/s

45. 0.018 A, CW as seen in the diagram

47. 9.375 V/m

49. Inside, $B = \mu_0 nI$, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \left(\pi r^2\right)\mu_0 n\frac{dI}{dt}$, so, $E = \frac{\mu_0 nr}{2}\cdot\frac{dI}{dt}$ (inside). Outside, $E(2\pi r) = \pi R^2 \mu_0 n\frac{dI}{dt}$, so, $E = \frac{\mu_0 nR^2}{2r}\cdot\frac{dI}{dt}$ (outside)

51. a. $E_{\text{inside}} = -\frac{r}{2}\frac{dB}{dt}$, $E_{\text{outside}} = -\frac{dB}{dt}\frac{R^2}{2r}$; b. $W = 4.19 \times 10^{-23}$ J; c. 0 J; d. $F_{\text{mag}} = 4 \times 10^{-13}$ N, $F_{\text{elec}} = 2.7 \times 10^{-22}$ N

53. 7.1 μA

55. three turns with an area of 1 m^2

57. a.
$$\begin{aligned}\omega &= 120\pi \text{ rad/s}, \\ \varepsilon &= 850\sin 120\,\pi\mathrm{t}\ \mathrm{V};\end{aligned}$$
b. $P = 720\sin^2 120\,\pi t$ W;
c. $P = 360\sin^2 120\,\pi t$ W

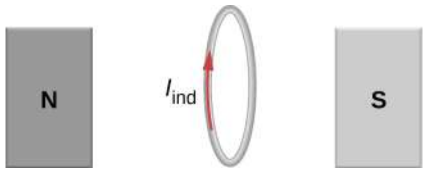
59. a. B is proportional to Q; b. If the coin turns easily, the magnetic field is perpendicular. If the coin is at an equilibrium position, it is parallel.

61. a. 1.33 A; b. 0.50 A; c. 60 W; d. 37.5 W; e. 22.5W

Additional Problems

63. 4.8×10^6 A/s

65. 2.83×10^{-4} A, the direction as follows for increasing magnetic field:



67. 0.375 V
69. a. 0.94 V; b. 0.70 N; c. 3.52 J/s; d. 3.52 W
71. $\left(\frac{dB}{dt}\right)\frac{A}{2\pi r}$
73. a. $R_f + R_a = \frac{120\text{ V}}{2.0\text{ A}} = 60\,\Omega,\text{ so }R_f = 50\,\Omega;$
b. $I = \frac{\varepsilon_S - \varepsilon_i}{R_f + R_a}, \Rightarrow \varepsilon_i = 90\text{ V};$
c. $\varepsilon_i = 60\text{ V}$

Challenge Problems

75. N is a maximum number of turns allowed.
77. 0.848 V

79. $\Phi = \frac{\mu_0 I_0 a}{2\pi}\ln\left(1 + \frac{b}{x}\right),\ \varepsilon = \frac{\mu_0 I_0 abv}{2\pi x(x+b)},$
so $I = \frac{\mu_0 I_0 abv}{2\pi Rx(x+b)}$
81. a. 1.01×10^{-6} V; b. 1.37×10^{-7} V; c. 0 V
83. a. $v = \frac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$; b. $mgv\ \sin\ \theta$; c. $mc\Delta T$; d. current would reverse direction but bar would still slide at the same speed
85. a.

$B = \mu_0 nI,\ \Phi_m = BA = \mu_0 nIA,$
$\varepsilon = 9.9 \times 10^{-4}\text{ V};$
b. 9.9×10^{-4} V;
c. $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \varepsilon,\ \Rightarrow\ E = 1.6 \times 10^{-3}\text{ V/m}$; d. 9.9×10^{-4} V;
e. no, because there is no cylindrical symmetry
87. a. 1.92×10^6 rad/s $= 1.83 \times 10^7$ rpm; b. This angular velocity is unreasonably high, higher than can be obtained for any mechanical system. c. The assumption that a voltage as great as 12.0 kV could be obtained is unreasonable.
89. $\frac{2\mu_0 \pi a^2 I_0 n\omega}{R}$
91. $\frac{mRv_0}{B^2 D^2}$

Chapter 14

Check Your Understanding

14.1 4.77×10^{-2} V
14.2 a. decreasing; b. increasing; Since the current flows in the opposite direction of the diagram, in order to get a positive emf on the left-hand side of diagram (a), we need to decrease the current to the left, which creates a reinforced emf where the positive end is on the left-hand side. To get a positive emf on the right-hand side of diagram (b), we need to increase the current to the left, which creates a reinforced emf where the positive end is on the right-hand side.
14.3 40 A/s

14.4 a. 4.5×10^{-5} H; b. 4.5×10^{-3} V
14.5 a. 2.4×10^{-7} Wb; b. $6.4 \times 10^{-5}\ m^2$
14.6 0.50 J
14.8 a. 2.2 s; b. 43 H; c. 1.0 s
14.10 a. $2.5\mu F$; b. $\pi/2$ rad or $3\pi/2$ rad; c. 1.4×10^3 rad/s
14.11 a. overdamped; b. 0.75 J

Conceptual Questions

1. $\frac{\text{Wb}}{\text{A}} = \frac{\text{T}\cdot\text{m}^2}{\text{A}} = \frac{\text{V}\cdot\text{s}}{\text{A}} = \frac{\text{V}}{\text{A/s}}$
3. The induced current from the 12-V battery goes through an inductor, generating a large voltage.
5. Self-inductance is proportional to the magnetic flux and inversely proportional to the current. However, since the magnetic flux depends on the current I, these effects cancel out. This means that the self-inductance does not depend on the current. If the emf is induced across an element, it does depend on how the current changes with time.
7. Consider the ends of a wire a part of an *RL* circuit and determine the self-inductance from this circuit.
9. The magnetic field will flare out at the end of the solenoid so there is less flux through the last turn than through the middle of the solenoid.
11. As current flows through the inductor, there is a back current by Lenz's law that is created to keep the net current at zero amps, the initial current.
13. no
15. At $t = 0$, or when the switch is first thrown.
17. 1/4
19. Initially, $I_{R1} = \frac{\varepsilon}{R_1}$ and $I_{R2} = 0$, and after a long time has passed, $I_{R1} = \frac{\varepsilon}{R_1}$ and $I_{R2} = \frac{\varepsilon}{R_2}$.
21. yes
23. The amplitude of energy oscillations depend on the initial energy of the system. The frequency in a *LC* circuit depends on the values of inductance and capacitance.
25. This creates an *RLC* circuit that dissipates energy, causing oscillations to decrease in amplitude slowly or quickly depending on the value of resistance.
27. You would have to pick out a resistance that is small enough so that only one station at a time is picked up, but big enough so that the tuner doesn't have to be set at exactly the correct frequency. The inductance or capacitance would have to be varied to tune into the station however practically speaking, variable capacitors are a lot easier to build in a circuit.

Problems

29. $M = 3.6 \times 10^{-3}$ H
31. a. 3.8×10^{-4} H; b. 3.8×10^{-4} H
33. $M_{21} = 2.3 \times 10^{-5}$ H
35. 0.24 H
37. 0.4 A/s
39. $\varepsilon = 480\pi \sin(120\pi t - \pi/2)$ V
41. 0.15 V. This is the same polarity as the emf driving the current.
43. a. 0.089 H/m; b. 0.44 V/m
45. $\frac{L}{l} = 4.16 \times 10^{-7}$ H/m
47. 0.01 A
49. 6.0 g
51. $U_m = 7.0 \times 10^{-7}$ J
53. a. 4.0 A; b. 2.4 A; c. on R: $V = 12$ V; on L: $V = 7.9$ V
55. 0.69τ
57. a. 2.52 ms; b. 99.2 Ω
59. a. $I_1 = I_2 = 1.7\ A$; b. $I_1 = 2.54\ A$, $I_2 = 1.27\ A$; c. $I_1 = 0$, $I_2 = 1.27\ A$; d. $I_1 = I_2 = 0$
61. proof

63. $\omega = 3.2 \times 10^7$ rad/s
65. a. 1.57×10^{-6} s; b. 3.93×10^{-7} s
67. $q = \frac{q_m}{\sqrt{2}}, I = \frac{q_m}{\sqrt{2LC}}$

$$C = \frac{1}{4\pi^2 f^2 L}$$

69.

$$f_1 = 540\,\text{Hz}; \quad C_1 = 3.5 \times 10^{-11}\,\text{F}$$
$$f_2 = 1600\,\text{Hz}; \quad C_2 = 4.0 \times 10^{-12}\,\text{F}$$

71. 6.9 ms

Additional Problems

73. Let a equal the radius of the long, thin wire, r the location where the magnetic field is measured, and R the upper limit of the problem where we will take R as it approaches infinity.

$$\text{Outside,} \quad B = \frac{\mu_0 I}{2\pi r} \quad \text{Inside,} \quad B = \frac{\mu_0 I r}{2\pi a^2}$$

proof

$$U = \frac{\mu_0 I^2 l}{4\pi}\left(\frac{1}{4} + \ln \frac{R}{a}\right)$$

$$\text{So,} \quad \frac{2U}{I^2} = \frac{\mu_0 l}{2\pi}\left(\frac{1}{4} + \ln \frac{R}{a}\right) \quad \text{and} \quad L = \infty$$

75. $M = \frac{\mu_0 l}{\pi} \ln \frac{d+a}{d}$
77. a. 100 T; b. 2 A; c. 0.50 H
79. a. 0 A; b. 2.4 A
81. a. 2.50×10^6 V; (b) The voltage is so extremely high that arcing would occur and the current would not be reduced so rapidly. (c) It is not reasonable to shut off such a large current in such a large inductor in such an extremely short time.

Challenge Problems

83. proof
85. a. $\frac{dB}{dt} = 6 \times 10^{-6}$ T/s; b. $\Phi = \frac{\mu_0 a I}{2\pi} \ln\left(\frac{a+b}{b}\right)$; c. 4.4 nA

Chapter 15

Check Your Understanding

15.1 10 ms
15.2 a. $(20\text{ V})\ \sin 200\pi t,\ (0.20\text{ A})\ \sin 200\pi t$; b. $(20\text{ V})\ \sin 200\pi t,\ (0.13\text{ A})\ \sin\ (200\pi t + \pi/2)$; c. $(20\text{ V})\ \sin 200\pi t,\ (2.1\text{ A})\ \sin\ (200\pi t - \pi/2)$
15.3 $v_R = (V_0 R/Z)\ \sin\ (\omega t - \phi)$; $v_C = (V_0 X_C/Z)\ \sin\ (\omega t - \phi + \pi/2) = -(V_0 X_C/Z) \cos(\omega t - \phi)$; $v_L = (V_0 X_L/Z)\ \sin\ (\omega t - \phi - \pi/2) = (V_0 X_L/Z) \cos(\omega t - \phi)$
15.4 $v(t) = (10.0\text{ V}) \sin 90\pi t$
15.5 2.00 V; 10.01 V; 8.01 V
15.6 a. 160 Hz; b. 40 Ω; c. $(0.25\text{ A})\ \sin 10^3 t$; d. 0.023 rad
15.7 a. halved; b. halved; c. same
15.8 $v(t) = (0.14\text{ V})\ \sin(4.0 \times 10^2 t)$
15.9 a. 12:1; b. 0.042 A; c. $2.6 \times 10^3\ \Omega$

Conceptual Questions

1. Angular frequency is 2π times frequency.
3. yes for both
5. The instantaneous power is the power at a given instant. The average power is the power averaged over a cycle or number of cycles.
7. The instantaneous power can be negative, but the power output can't be negative.

9. There is less thermal loss if the transmission lines operate at low currents and high voltages.
11. The adapter has a step-down transformer to have a lower voltage and possibly higher current at which the device can operate.
13. so each loop can experience the same changing magnetic flux

Problems

15. a. 530 Ω; b. 53 Ω; c. 5.3 Ω
17. a. 1.9 Ω; b. 19 Ω; c. 190 Ω
19. 360 Hz
21. $i\,(t) = (3.2\text{ A})\ \sin\ (120\pi t)$
23. a. 38 Ω; b. $i\,(t) = (4.24\text{A})\ \sin\ (120\pi t - \pi/2)$
25. a. 770 Ω; b. 0.16 A; c. $I = (0.16\text{ A})\cos(120\pi t{-}0.33\pi)$; d. $v_R = 62\cos(120\pi t)$; $v_C = 103\cos(120\pi t - \pi/2)$
27. a. 690 Ω; b. 0.15 A; c. $I = (0.15\text{A})\ \sin\ (1000\pi t - 0.753)$; d. 1100 Ω, 0.092 A, $I = (0.092\text{A})\ \sin\ (1000\pi t + 1.09)$
29. a. 5.7 Ω; b. 29°; c. $I \quad = (30.\text{ A})\cos(120\pi t + 0.51)$
31. a. 0.89 A; b. 5.6A; c. 1.4 A
33. a. 5.3 W; b. 2.1 W
35. a. inductor; b. $X_L = 52\,\Omega$
37. 1.3×10^{-6} F
39. a. 820 Hz; b. 7.8
41. a. 50 Hz; b. 50 W; c. 6.32; d. 50 rad/s
43. The reactance of the capacitor is larger than the reactance of the inductor because the current leads the voltage. The power usage is 30 W.
45. a. 45:1; b. 0.68 A, 0.015 A; c. 160 Ω
47. a. 41 turns; b. 40.9 mA

Additional Problems

49. a. $i\,(t) = (1.26\text{A})\ \sin\ (200\pi t + \pi/2)$; b. $i\,(t) = (7.96\text{A})\ \sin\ (200\pi t - \pi/2)$; c. $i\,(t) = (2\text{A})\ \sin\ (200\pi t)$
51. a. $2.5 \times 10^3\,\Omega$, 3.6×10^{-3} A; b. 7.5 Ω, 1.2A
53. a. 19 A; b. inductor leads by 90°
55. 11.7 Ω
57. 14 W
59. a. 5.9×10^4 W; b. 1.64×10^{11} W

Challenge Problems

61. a. 335 MV; b. the result is way too high, well beyond the breakdown voltage of air over reasonable distances; c. the input voltage is too high
63. a. 20 Ω; b. 0.5 A; c. 5.4°, lagging;
d. $V_R = (9.96\text{ V})\cos(250\pi t + 5.4°)$, $V_C = (12.7\text{ V})\cos(250\pi t + 5.4° - 90°)$, $V_L = (11.8\text{ V})\cos(250\pi t + 5.4° + 90°)$, $V_{\text{source}} = \left(10.0 \quad \text{V}\right)\cos(250\pi t)$; e. 0.995; f. 6.25 J
65. a. 0.75 Ω; b. 7.5 Ω; c. 0.75 Ω; d. 7.5 Ω; e. 1.3 Ω; f. 0.13 Ω
67. The units as written for inductive reactance Equation 15.8 are $\frac{\text{rad}}{\text{s}}$H. Radians can be ignored in unit analysis. The Henry can be defined as $\text{H} = \frac{\text{V}\cdot\text{s}}{\text{A}} = \Omega\cdot\text{s}$. Combining these together results in a unit of Ω for reactance.
69. a. 156 V; b. 42 V; c. 154 V
71. a. $\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{1}{\sqrt{1+1/\omega^2R^2C^2}}$ and $\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}$; b. $v_{\text{out}} \approx v_{\text{in}}$ and $v_{\text{out}} \approx 0$

Chapter 16

Check Your Understanding

16.1 It is greatest immediately after the current is switched on. The displacement current and the magnetic field from it are proportional to the rate of change of electric field between the plates, which is greatest when the plates first begin to charge.

16.2 No. The changing electric field according to the modified version of Ampère's law would necessarily induce a changing magnetic field.

16.3 (1) Faraday's law, (2) the Ampère-Maxwell law

16.4 a. The directions of wave propagation, of the E field, and of B field are all mutually perpendicular. b. The speed of the electromagnetic wave is the speed of light $c = 1/\sqrt{\varepsilon_0 \mu_0}$ independent of frequency. c. The ratio of electric and magnetic field amplitudes is $E/B = c$.

16.5 Its acceleration would decrease because the radiation force is proportional to the intensity of light from the Sun, which decreases with distance. Its speed, however, would not change except for the effects of gravity from the Sun and planets.

16.6 They fall into different ranges of wavelength, and therefore also different corresponding ranges of frequency.

Conceptual Questions

1. The current into the capacitor to change the electric field between the plates is equal to the displacement current between the plates.

3. The first demonstration requires simply observing the current produced in a wire that experiences a changing magnetic field. The second demonstration requires moving electric charge from one location to another, and therefore involves electric currents that generate a changing electric field. The magnetic fields from these currents are not easily separated from the magnetic field that the displacement current produces.

5. in (a), because the electric field is parallel to the wire, accelerating the electrons

7. A steady current in a dc circuit will not produce electromagnetic waves. If the magnitude of the current varies while remaining in the same direction, the wires will emit electromagnetic waves, for example, if the current is turned on or off.

9. The amount of energy (about $100\ \text{W/m}^2$) is can quickly produce a considerable change in temperature, but the light pressure (about $3.00 \times 10^{-7}\text{N/m}^2$) is much too small to notice.

11. It has the magnitude of the energy flux and points in the direction of wave propagation. It gives the direction of energy flow and the amount of energy per area transported per second.

13. The force on a surface acting over time Δt is the momentum that the force would impart to the object. The momentum change of the light is doubled if the light is reflected back compared with when it is absorbed, so the force acting on the object is twice as great.

15. a. According to the right hand rule, the direction of energy propagation would reverse. b. This would leave the vector $\vec{\mathbf{S}}$, and therefore the propagation direction, the same.

17. a. Radio waves are generally produced by alternating current in a wire or an oscillating electric field between two plates; b. Infrared radiation is commonly produced by heated bodies whose atoms and the charges in them vibrate at about the right frequency.

19. a. blue; b. Light of longer wavelengths than blue passes through the air with less scattering, whereas more of the blue light is scattered in different directions in the sky to give it is blue color.

21. A typical antenna has a stronger response when the wires forming it are orientated parallel to the electric field of the radio wave.

23. No, it is very narrow and just a small portion of the overall electromagnetic spectrum.

25. Visible light is typically produced by changes of energies of electrons in randomly oriented atoms and molecules. Radio waves are typically emitted by an ac current flowing along a wire, that has fixed orientation and produces electric fields pointed in particular directions.

27. Radar can observe objects the size of an airplane and uses radio waves of about 0.5 cm in wavelength. Visible light can be used to view single biological cells and has wavelengths of about 10^{-7} m.

29. ELF radio waves

31. The frequency of 2.45 GHz of a microwave oven is close to the specific frequencies in the 2.4 GHz band used for WiFi.

Problems

33.
$$\begin{aligned} B_{\text{ind}} &= \frac{\mu_0}{2\pi r} I_{\text{ind}} = \frac{\mu_0}{2\pi r}\varepsilon_0 \frac{\partial \Phi_E}{\partial t} = \frac{\mu_0}{2\pi r}\varepsilon_0 \left(A\frac{\partial E}{\partial t}\right) = \frac{\mu_0}{2\pi r}\varepsilon_0 A\left(\frac{1}{d}\frac{dV(t)}{dt}\right) \\ &= \frac{\mu_0}{2\pi r}\left[\frac{\varepsilon_0 A}{d}\right]\left[\frac{1}{C}\frac{dQ(t)}{dt}\right] = \frac{\mu_0}{2\pi r}\frac{dQ(t)}{dt} \qquad \text{because } C = \frac{\varepsilon_0 A}{d} \end{aligned}$$

35. a. $I_{\text{res}} = \frac{V_0 \sin \omega t}{R}$; b. $I_{\text{d}} = CV_0\omega \cos \omega t$;
c. $I_{\text{real}} = I_{\text{res}} + \frac{dQ}{dt} = \frac{V_0 \sin \omega t}{R} + CV_0 \frac{d}{dt}\sin \omega t = \frac{V_0 \sin \omega t}{R} + CV_0\omega\cos\omega t$; which is the sum of I_{res} and I_{real}, consistent with how the displacement current maintaining the continuity of current.

37. 1.77×10^{-3} A

39. $I_{\text{d}} = \left(7.97 \times 10^{-10}\text{ A}\right)\sin\left(150\, t\right)$

41. 499 s

43. 25 m

45. a. 5.00 V/m; b. 9.55×10^8 Hz; c. 31.4 cm; d. toward the $+x$-axis;
e. $\mathbf{B} = \left(1.67 \times 10^{-8}\text{ T}\right)\cos\left[kx - \left(6 \times 10^9\text{ s}^{-1}\right)t + 0.40\right]\hat{\mathbf{k}}$

47. $I_{\text{d}} = \pi\varepsilon_0\omega R^2 E_0 \sin\left(kx - \omega t\right)$

49. The magnetic field is downward, and it has magnitude 2.00×10^{-8} T.

51. a. 6.45×10^{-3} V/m; b. 394 m

53. 11.5 m

55. $5.97 \times 10^{-3}\text{ W/m}^2$

57. a. $E_0 = 1027$ V/m, $B_0 = 3.42 \times 10^{-6}$ T; b. 3.96×10^{26} W

59. 20.8 W/m^2

61. a. $4.42 \times 10^{-6}\text{ W/m}^2$; b. 5.77×10^{-2} V/m

63. a. $7.47 \times 10^{-14}\text{ W/m}^2$; b. 3.66×10^{-13} W; c. 1.12 W

65. $1.99 \times 10^{-11}\text{ N/m}^2$

67.
$$\begin{aligned} F &= ma = (p)\left(\pi r^2\right),\ p = \frac{ma}{\pi r^2} = \frac{\varepsilon_0}{2}E_0^2 \\ E_0 &= \sqrt{\frac{2ma}{\varepsilon_0 \pi r^2}} = \sqrt{\frac{2\left(10^{-8}\text{ kg}\right)\left(0.30\text{ m/s}^2\right)}{\left(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(\pi)\left(2\times10^{-6}\text{ m}\right)^2}} \\ E_0 &= 7.34 \times 10^6\text{ V/m} \end{aligned}$$

69. a. 4.50×10^{-6} N; b. it is reduced to half the pressure, 2.25×10^{-6} N

71. a. $W = \frac{1}{2}\frac{\pi^2 r^4}{mc^2} I^2 t^2$; b. $E = \pi r^2 I t$

73. a. 1.5×10^{18} Hz; b. X-rays

75. a. The wavelength range is 187 m to 556 m. b. The wavelength range is 2.78 m to 3.41 m.

77. $P' = \left(\frac{12\text{ m}}{30\text{ m}}\right)^2 (100\text{ mW}) = 16\text{ mW}$

79. time for 1 bit = 1.27×10^{-8} s, difference in travel time is 5.34×10^{-8} s

81. a. 1.5×10^{-9} m; b. 5.9×10^{-7} m; c. 3.0×10^{-15} m

83. 5.17×10^{-12} T, the non-oscillating geomagnetic field of 25–65 μT is much larger

85. a. 1.33×10^{-2} V/m; b. 4.44×10^{-11} T; c. 3.00×10^8 m

87. a. 5.00×10^6 m; b. radio wave; c. 4.33×10^{-5} T

Additional Problems

89. $I_{\text{d}} = (10\text{ N/C})\left(8.845 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\pi(0.03\text{ m})^2\left(5000\frac{1}{\text{s}}\right) = 1.25 \times 10^{-9}$ A

91. 3.75×10^7 km, which is much greater than Earth's circumference

93. a. 564 W; b. $1.80 \times 10^4\text{ W/m}^2$; c. 3.68×10^3 V/m; d. 1.23×10^{-5} T

95. a. $5.00 \times 10^3\text{ W/m}^2$; b. 3.88×10^{-6} N; c. 5.18×10^{-12} N

97. a. $I = \frac{P}{A} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$; b. $I \propto E_0^2\,,\ B_0^2 \Rightarrow E_0^2\,,\ B_0^2 \propto \frac{1}{r^2} \Rightarrow E_0, B_0 \propto \frac{1}{r}$

99.
$$\begin{aligned}\text{Power into the wire} &= \int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = \left(\frac{1}{\mu_0} EB\right)(2\pi rL) \\ &= \frac{1}{\mu_0}\left(\frac{V}{L}\right)\left(\frac{\mu_0 i}{2\pi r}\right)(2\pi rL) = iV = i^2 R\end{aligned}$$

101. 0.431

103. a. 1.5×10^{11} m; b. 5.0×10^{-7} s; c. 33 ns

105.
$$\begin{aligned}\text{sound: } \lambda_{\text{sound}} &= \frac{v_s}{f} = \frac{343 \text{ m/s}}{20.0 \text{ Hz}} = 17.2 \text{ m} \\ \text{radio: } \lambda_{\text{radio}} &= \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1030 \times 10^3 \text{ Hz}} = 291 \text{ m; or } 17.1\ \lambda_{\text{sound}}\end{aligned}$$

Challenge Problems

107. a. 0.29 μm; b. The radiation pressure is greater than the Sun’s gravity if the particle size is smaller, because the gravitational force varies as the radius cubed while the radiation pressure varies as the radius squared. c. The radiation force outward implies that particles smaller than this are less likely to be near the Sun than outside the range of the Sun’s radiation pressure.

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