Basic Review

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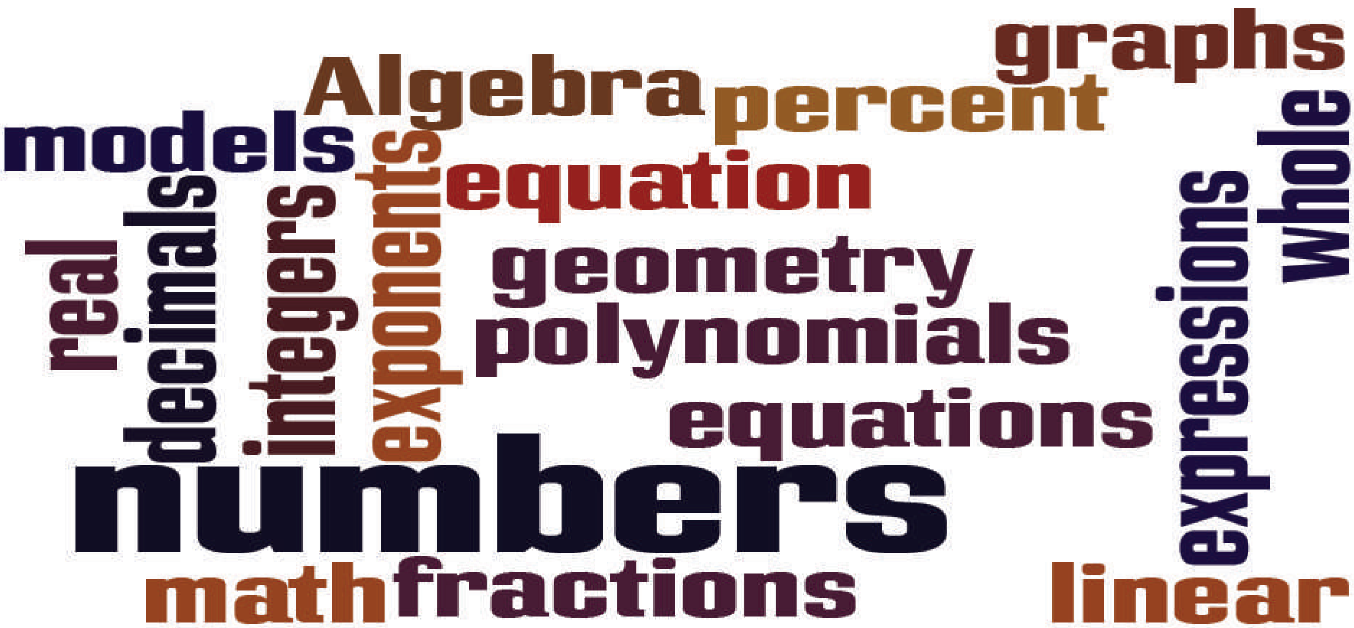
I

CHAPTER 1 Whole Numbers, Integers, and Introduction to Algebra

Algebra has a language of its own. The picture shows just some of the words you may see and use in your study of algebra.



You may not realize it, but you already use algebra every day. Perhaps you figure out how much to tip a server in a restaurant. Maybe you calculate the amount of change you should get when you pay for something. It could even be when you compare batting averages of your favorite players. You can describe the algebra you use in specific words, and follow an orderly process. In this chapter, you will explore the words used to describe algebra and start on your path to solving algebraic problems easily, both in class and in your everyday life.

1.1 Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Use place value with whole numbers
- Identify multiples and apply divisibility tests
- Find prime factorization and least common multiples

As we begin our study of intermediate algebra, we need to refresh some of our skills and vocabulary. This chapter and the next will focus on whole numbers, integers, fractions, decimals, and real numbers. We will also begin our use of algebraic notation and vocabulary.

Use Place Value with Whole Numbers

The most basic numbers used in algebra are the numbers we use to count objects in our world: 1, 2, 3, 4, and so on. These are called the counting number**s**. Counting numbers are also called *natural numbers*. If we add zero to the counting numbers, we get the set of whole number**s**.

Counting Numbers: 1, 2, 3, …

Whole Numbers: 0, 1, 2, 3, …

The notation "…" is called ellipsis and means "and so on," or that the pattern continues endlessly.

We can visualize counting numbers and whole numbers on a number line .See Figure 1.
The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left. While this number line shows only the whole numbers 0 through 6, the numbers keep going without end.

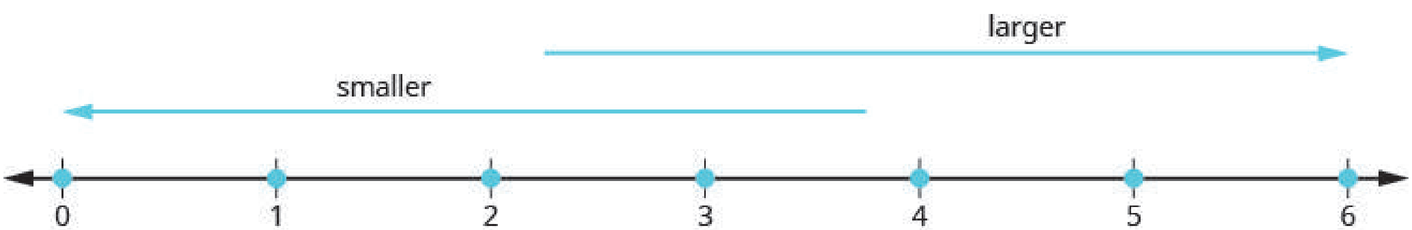


Figure 1

Our number system is called a place value system, because the value of a digit depends on its position in a number. Figure 2 shows the place values. The place values are separated into groups of three, which are called periods. The periods are *ones, thousands, millions, billions, trillions*, and so on. In a written number, commas separate the periods.

The number 5,278,194 is shown in the chart. The digit 5 is in the millions place. The digit 2 is in the hundred-thousands place. The digit 7 is in the ten-thousands place. The digit 8 is in the thousands place. The digit 1 is in the hundreds place. The digit 9 is in the tens place. The digit 4 is in the ones place.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

Figure 2

EXAMPLE 1

In the number 63,407,218, find the place value of each digit:

a. 7

b. 0

c. 1

d. 6

e. 3

Solution
Place the number in the place value chart:

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

a) The 7 is in the thousands place.
b) The 0 is in the ten thousands place.
c) The 1 is in the tens place.
d) The 6 is in the ten-millions place.
e) The 3 is in the millions place.

TRY IT 1.1

For the number 27,493,615, find the place value of each digit:

a) 2 b) 1 c) 4 d) 7 e) 5

Answer
a) ten millions b) tens c) hundred thousands d) millions e) ones

TRY IT 1.2

For the number 519,711,641,328, find the place value of each digit:

a) 9 b) 4 c) 2 d) 6 e) 7

Answer
a) billions b) ten thousands c) tens d) hundred thousands e) hundred millions

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period, followed by the name of the period, without the *s* at the end. Start at the left, where the periods have the largest value. The ones period is not named. The commas separate the periods, so wherever there is a comma in the number, put a comma between the words (see Figure 3). The number 74,218,369 is written as seventy-four million, two hundred eighteen thousand, three hundred sixty-nine.

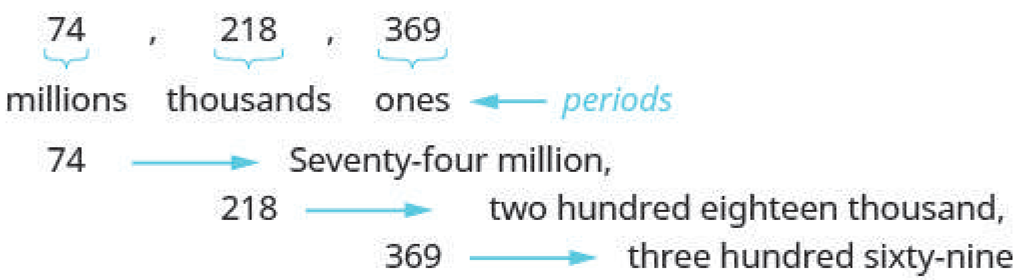


Figure 3

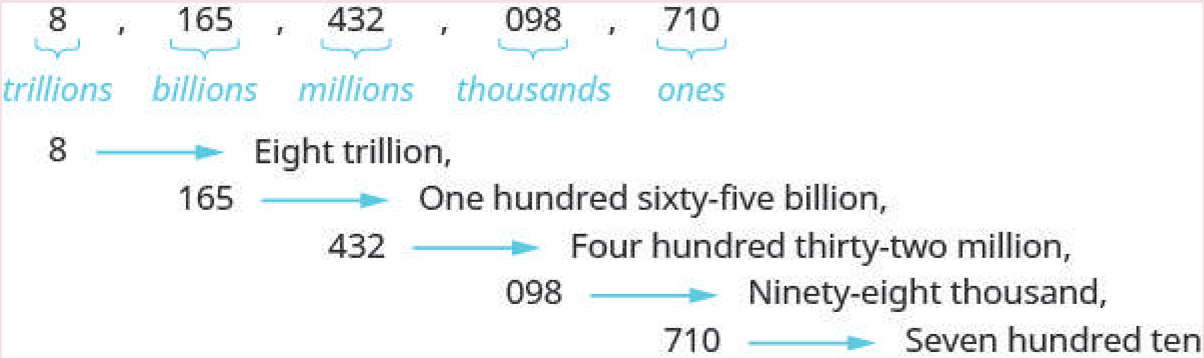
HOW TO: Name a Whole Number in Words.

1. Start at the left and name the number in each period, followed by the period name.
2. Put commas in the number to separate the periods.
3. Do not name the ones period.

EXAMPLE 2

Name the number 8,165,432,098,710 using words.

Solution
Name the number in each period, followed by the period name.



Put the commas in to separate the periods.

So, $8, 165, 432, 098, 710$ is named as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

TRY IT 2.1

Name the number $9, 258, 137, 904, 061$ using words.

Answer
nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

TRY IT 2.2

Name the number $17, 864, 325, 619, 004$ using words.

Answer
seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand four

We are now going to reverse the process by writing the digits from the name of the number. To write the number in digits, we first look for the clue words that indicate the periods. It is helpful to draw three blanks for the needed periods and then fill in the blanks with the numbers, separating the periods with commas.

HOW TO: Write a Whole Number Using Digits.

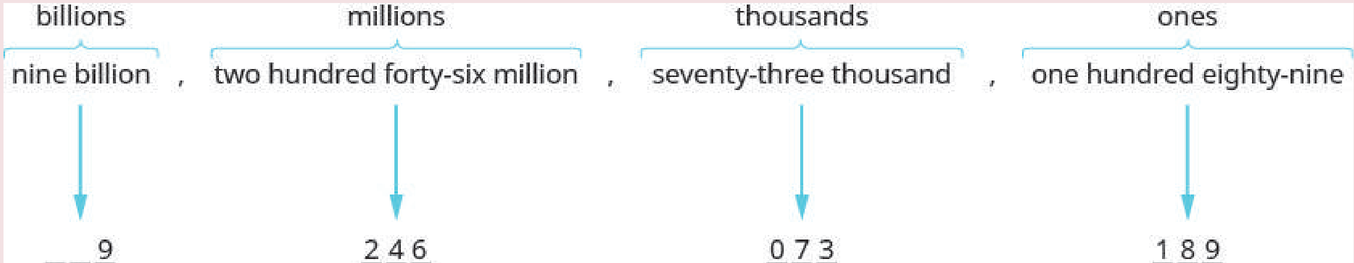
1. Identify the words that indicate periods. (Remember, the ones period is never named.)
2. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
3. Name the number in each period and place the digits in the correct place value position.

EXAMPLE 3

Write *nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine* as a whole number using digits.

Solution

Identify the words that indicate periods.
Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Then write the digits in each period.



The number is 9,246,073,189.

TRY IT 3.1

Write the number two billion, four hundred sixty-six million, seven hundred fourteen thousand, fifty-one as a whole number using digits.

Answer
$2,466,714,051$

TRY IT 3.2

Write the number eleven billion, nine hundred twenty-one million, eight hundred thirty thousand, one hundred six as a whole number using digits.

Answer
$11,921,830,106$

In 2016, Statistics Canada estimated the population of Toronto as 13,448,494. We could say the population of Toronto was approximately 13.4 million. In many cases, you don't need the exact value; an approximate number is good enough.

The process of approximating a number is called rounding. Numbers are rounded to a specific place value, depending on how much accuracy is needed. Saying that the population of Toronto is approximately 13.4 million means that we rounded to the hundred thousands place.

EXAMPLE 4

Round 23,658 to the nearest hundred.

Solution

Step 1. Locate the given place value with an arrow. All digits to the left do not change.	Locate the hundreds place in 23,658.	hundredths place ↓ 23,658

Step 2. Underline the digit to the right of the given place value.	Underline the 5, which is to the right of the hundreds place.	hundredths place ↓ 23,6<u>5</u>8
Step 3. Is this digit greater than or equal to 5? Yes–add 1 to the digit in the given place value. No–do <u>not</u> change the digit in the given place value.	Add 1 to the 6 in the hundreds place, since 5 is greater than or equal to 5.	23,658 add 1
Step 4. Replace all digits to the right of the given place value with zeros.	Replace all digits to the right of the hundreds place with zeros.	23,700 add 1 replace with 0s So, 23,700 is rounded to the nearest hundred.

TRY IT 4.1

Round to the nearest hundred: $17,852$.

Answer
$17,900$

TRY IT 4.2

Round to the nearest hundred: $468,751$.

Answer
$468,800$

HOW TO: Round Whole Numbers.

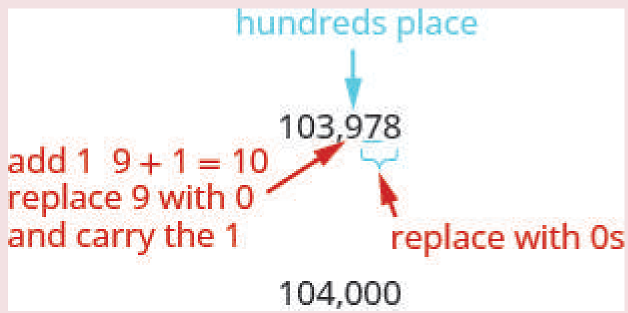
1. Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
2. Underline the digit to the right of the given place value.
3. Is this digit greater than or equal to 5?
    - Yes–add 1 to the digit in the given place value.
    - No–do <u>not</u> change the digit in the given place value.
4. Replace all digits to the right of the given place value with zeros.

EXAMPLE 5

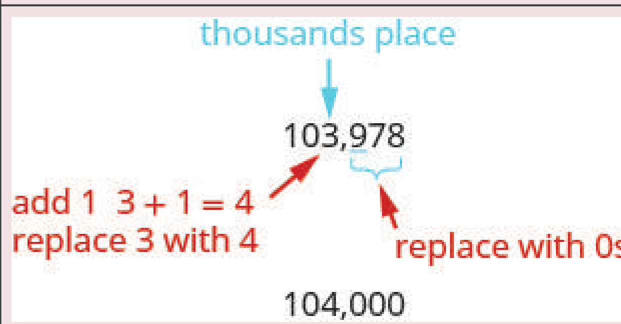
Round $103,978$ to the nearest:

a. hundred
b. thousand
c. ten thousand

Solution
a)

Locate the hundreds place in 103,978.	
Underline the digit to the right of the hundreds place.	
Since 7 is greater than or equal to 5, add 1 to the 9. Replace all digits to the right of the hundreds place with zeros.	
	So, 104,000 is 103,978 rounded to the nearest hundred.

b)

Locate the thousands place and underline the digit to the right of the thousands place.	
Since 9 is greater than or equal to 5, add 1 to the 3. Replace all digits to the right of the hundreds place with zeros.	
	So, 104,000 is 103,978 rounded to the nearest thousand.

c)

Locate the ten thousands place and underline the digit to the right of the ten thousands place.	ten thousands place ↓ 103,978
Since 3 is less than 5, we leave the 0 as is, and then replace the digits to the right with zeros.	100,000
	So, 100,000 is 103,978 rounded to the nearest ten thousand.

TRY IT 5.1

Round 206,981 to the nearest: a) hundred b) thousand c) ten thousand.

Answer
a) 207,000 b) 207,000 c) 210,000

TRY IT 5.2

Round 784,951 to the nearest: a) hundred b) thousand c) ten thousand.

Answer
a) 785,000 b) 785,000 c) 780,000

Identify Multiples and Apply Divisibility Tests

The numbers 2, 4, 6, 8, 10, and 12 are called multiples of 2. A multiple of 2 can be written as the product of a counting number and 2

$$\begin{array}{cccccc} 2, & 4, & 6, & 8, & 10, & 12, \ldots \\ 2\cdot 1, & 2\cdot 2, & 2\cdot 3, & 2\cdot 4, & 2\cdot 5, & 2\cdot 6 \end{array}$$

Similarly, a multiple of 3 would be the product of a counting number and 3

$$\begin{array}{cccccc} 3, & 6, & 9, & 12, & 15, & 18, \ldots \\ 3\cdot 1, & 3\cdot 2, & 3\cdot 3, & 3\cdot 4, & 3\cdot 5, & 3\cdot 6 \end{array}$$

We could find the multiples of any number by continuing this process.

The Table 1 below shows the multiples of 2 through 9 for the first 12 counting numbers.

Table 1

Counting Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Multiples of 2	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108
Multiples of 10	10	20	30	40	50	60	70	80	90	100	110	120

Multiple of a Number

A number is a **multiple** of *n* if it is the product of a counting number and *n*.

Another way to say that 15 is a multiple of 3 is to say that 15 is divisible by 3. That means that when we divide 3 into 15, we get a counting number. In fact, $15 \div 3$ is 5, so 15 is $5 \cdot 3$.

Divisible by a Number

If a number *m* is a multiple of *n*, then *m* is **divisible** by *n*.

Look at the multiples of 5 in Table 1. They all end in 5 or 0. Numbers with last digit of 5 or 0 are divisible by 5. Looking for other patterns in Table 1 that shows multiples of the numbers 2 through 9, we can discover the following divisibility tests:

Divisibility Tests

A number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8.
- 3 if the sum of the digits is divisible by 3.
- 5 if the last digit is 5 or 0.
- 6 if it is divisible by both 2 and 3.
- 10 if it ends with 0.

EXAMPLE 6

Is 5,625 divisible by 2? By 3? By 5? By 6? By 10?

Solution

Is 5,625 divisible by 2?	
Does it end in 0,2,4,6, or 8?	No. 5,625 is not divisible by 2.
Is 5,625 divisible by 3?	
What is the sum of the digits?	$5+6+2+5=18$
Is the sum divisible by 3?	Yes. 5,625 is divisble by 3.
Is 5,625 divisible by 5 or 10?	
What is the last digit? It is 5.	5,625 is divisble by 5 but not by 10.
Is 5,625 divisible by 6?	
Is it divisible by both 2 or 3?	No, 5,625 is not divisible by 2, so 5,625 is not divisible by 6.

TRY IT 6.1

Determine whether 4,962 is divisible by 2, by 3, by 5, by 6, and by 10

Answer
by 2, 3, and 6

TRY IT 6.2

Determine whether 3,765 is divisible by 2, by 3, by 5, by 6, and by 10

Answer
by 3 and 5

Find Prime Factorization and Least Common Multiples

In mathematics, there are often several ways to talk about the same ideas. So far, we've seen that if *m* is a multiple of *n*, we can say that *m* is divisible by *n*. For example, since 72 is a multiple of 8, we say 72 is divisible by 8. Since 72 is a multiple of 9, we say 72 is divisible by 9. We can express this still another way.

Since $8 \cdot 9 = 72$, we say that 8 and 9 are factors of 72. When we write $72 = 8 \cdot 9$, we say we have factored 72

Other ways to factor 72 are $1 \cdot 72, 2 \cdot 36, 3 \cdot 24, 4 \cdot 18,$ and $6 \cdot 12$. Seventy-two has many factors: 1, 2, 3, 4, 6, 8, 9, 12, 18, 36, and 72

Factors

If $a \cdot b = m$, then *a* and *b* are factors of *m*.

Some numbers, like 72, have many factors. Other numbers have only two factors.

Prime Number and Composite Number

A **prime number** is a counting number greater than 1, whose only factors are 1 and itself.

A composite number is a counting number that is not prime. A composite number has factors other than 1 and itself.

The counting numbers from 2 to 19 are listed in Figure 4, with their factors. Make sure to agree with the “prime” or “composite” label for each!

Number	Factors	Prime or Composite?
2	1,2	Prime
3	1,3	Prime
4	1,2,4	Composite
5	1,5	Prime
6	1,2,3,6	Composite
7	1,7	Prime
8	1,2,4,8	Composite
9	1,3,9	Composite
10	1,2,5,10	Composite

Number	Factors	Prime or Composite?
11	1,11	Prime
12	1,2,3,4,6,12	Composite
13	1,13	Prime
14	1,2,7,14	Composite
15	1,3,5,15	Composite
16	1,2,4,8,16	Composite
17	1,17	Prime
18	1,2,3,6,9,18	Composite
19	1,19	Prime

Figure 4

The prime number**s** less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. Notice that the only even prime number is 2

A composite number can be written as a unique product of primes. This is called the prime factorization of the number. Finding the prime factorization of a composite number will be useful later in this course.

Prime Factorization

The prime factorization of a number is the product of prime numbers that equals the number.

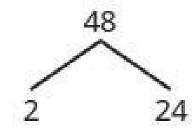
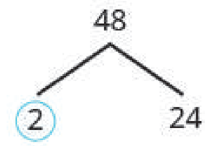
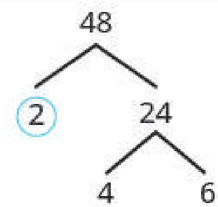
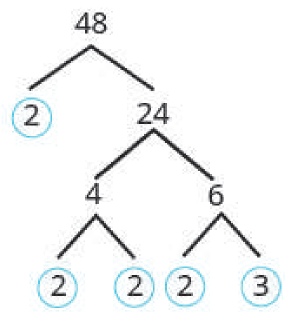
To find the prime factorization of a composite number, find any two factors of the number and use them to create two branches. If a factor is prime, that branch is complete. Circle that prime!

If the factor is not prime, find two factors of the number and continue the process. Once all the branches have circled primes at the end, the factorization is complete. The composite number can now be written as a product of prime numbers.

EXAMPLE 7

Factor 48.

Solution

Step 1. Find two factors whose product is the given number. Use these numbers to create two branches.	$48 = 2 \cdot 24$	
Step 2. If a factor is prime, that branch is complete. Circle the prime.	2 is prime. Circle the prime.	
Step 3. If a factor is not prime, write it as the product of two factors and continue the process.	24 is not prime. Break it into 2 more factors. 4 and 6 are not prime. Break them each into two factors. 2 and 3 are prime, so circle them.	 
Step 4. Write the composite number as the product of all the circled primes.		$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

We say $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ is the prime factorization of 48. We generally write the primes in ascending order. Be sure to multiply the factors to verify your answer!

If we first factored 48 in a different way, for example as $6 \cdot 8$, the result would still be the same. Finish the prime factorization and verify this for yourself.

TRY IT 7.1

Find the prime factorization of 80.

Answer

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

TRY IT 7.2

Find the prime factorization of 60.

Answer
$2 \cdot 2 \cdot 3 \cdot 5$

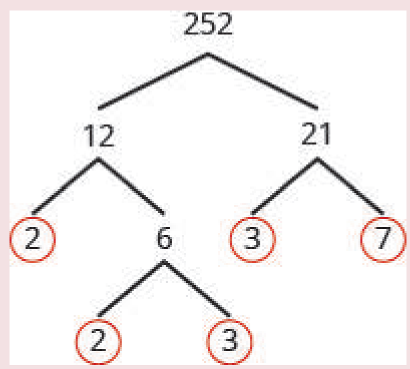
HOW TO: Find the Prime Factorization of a Composite Number.

1. Find two factors whose product is the given number, and use these numbers to create two branches.
2. If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
3. If a factor is not prime, write it as the product of two factors and continue the process.
4. Write the composite number as the product of all the circled primes.

EXAMPLE 8

Find the prime factorization of 252

Solution

Step 1. Find two factors whose product is 252. 12 and 21 are not prime. Break 12 and 21 into two more factors. Continue until all primes are factored.	
Step 2. Write 252 as the product of all the circled primes.	$252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$

TRY IT 8.1

Find the prime factorization of 126

Answer
$2 \cdot 3 \cdot 3 \cdot 7$

TRY IT 8.2

Find the prime factorization of 294

Answer

$$2 \cdot 3 \cdot 7 \cdot 7$$

One of the reasons we look at multiples and primes is to use these techniques to find the least common multiple of two numbers. This will be useful when we add and subtract fractions with different denominators. Two methods are used most often to find the least common multiple and we will look at both of them.

The first method is the Listing Multiples Method. To find the least common multiple of 12 and 18, we list the first few multiples of 12 and 18:

12: 12, 24, **36**, 48, 60, **72**, 84, 96, **108**...

18: 18, **36**, 54, **72**, 90, **108**...

Common Multiples: 36, 72, 108...

Least Common Multiple: 36

Notice that some numbers appear in both lists. They are the **common multiples** of 12 and 18

We see that the first few common multiples of 12 and 18 are 36, 72, and 108. Since 36 is the smallest of the common multiples, we call it the *least common multiple.* We often use the abbreviation LCM.

Least Common Multiple

The least common multiple (LCM) of two numbers is the smallest number that is a multiple of both numbers.

The procedure box lists the steps to take to find the LCM using the prime factors method we used above for 12 and 18

HOW TO: Find the Least Common Multiple by Listing Multiples.

1. List several multiples of each number.
2. Look for the smallest number that appears on both lists.
3. This number is the LCM.

EXAMPLE 9

Find the least common multiple of 15 and 20 by listing multiples.

Solution

Make lists of the first few multiples of 15 and of 20, and use them to find the least common multiple.	**15:** 15, 30, 45, **60**, 75, 90, 105, 120 **20:** 20, 40, **60**, 80, 100, 120, 140, 160
Look for the smallest number that appears in both lists.	The first number to appear on both lists is 60, so 60 is the least common multiple of 15 and 20.

Notice that 120 is in both lists, too. It is a common multiple, but it is not the *least* common multiple.

TRY IT 9.1

Find the least common multiple by listing multiples: 9 and 12

Answer
36

TRY IT 9.2

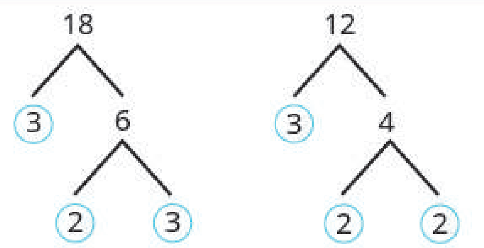
Find the least common multiple by listing multiples: 18 and 24

Answer
72

Our second method to find the least common multiple of two numbers is to use The Prime Factors Method. Let's find the LCM of 12 and 18 again, this time using their prime factors.

EXAMPLE 10

Find the Least Common Multiple (LCM) of 12 and 18 using the prime factors method.
Solution

Step 1. Write each number as a product of primes.		
Step 2. List the primes of each number. Match primes vertically when possible.	List the primes of 12. List the primes of 18. Line up with the primes of 12 when possible. If not create a new column.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$
Step 3. Bring down the number from each column.		$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3$
Step 4. Multiply the factors.		$\text{LCM} = 36$

Notice that the prime factors of 12 $(2 \cdot 2 \cdot 3)$ and the prime factors of 18 $(2 \cdot 3 \cdot 3)$ are included in the LCM $(2 \cdot 2 \cdot 3 \cdot 3)$. So 36 is the least common multiple of 12 and 18

By matching up the common primes, each common prime factor is used only once. This way you are sure that 36 is the *least* common multiple.

TRY IT 10.1

Find the LCM using the prime factors method: 9 and 12

Answer
36

TRY IT 10.2

Find the LCM using the prime factors method: 18 and 24

Answer
72

HOW TO: Find the Least Common Multiple Using the Prime Factors Method.

1. Write each number as a prime factors.
2. List the primes of each number. Match primes vertically when possible.
3. Bring down the columns.
4. Multiply the factors.

EXAMPLE 11

Find the Least Common Multiple (LCM) of 24 and 36 using the prime factors method.

Solution

Find the prime factors of 24 and 36. Match primes vertically when possible. Bring down all columns.	$24 = 2 \cdot 2 \cdot 2 \cdot 3$ $36 = 2 \cdot 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
Multiply the factors.	$\text{LCM} = 72$
	The LCM of 24 and 36 is 72.

TRY IT 11.1

Find the LCM using the prime factors method: 21 and 28

Answer
84

TRY IT 11.2

Find the LCM using the prime factors method: 24 and 32

Answer
96

Key Concepts

- **Place Value** as in Figure 2.
- **Name a Whole Number in Words**
    1. Start at the left and name the number in each period, followed by the period name.
    2. Put commas in the number to separate the periods.
    3. Do not name the ones period.
- **Write a Whole Number Using Digits**
    1. Identify the words that indicate periods. (Remember the ones period is never named.)
    2. Draw 3 blanks to indicate the number of places needed in each period. Separate the periods by commas.
    3. Name the number in each period and place the digits in the correct place value position.
- **Round Whole Numbers**
    1. Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
    2. Underline the digit to the right of the given place value.
    3. Is this digit greater than or equal to 5?
        - Yes—add 1 to the digit in the given place value.
        - No—do not change the digit in the given place value.
    4. Replace all digits to the right of the given place value with zeros.
- **Divisibility Tests:** A number is divisible by:
    - 2 if the last digit is 0, 2, 4, 6, or 8.
    - 3 if the sum of the digits is divisible by 3.
    - 5 if the last digit is 5 or 0.
    - 6 if it is divisible by both 2 and 3.
    - 10 if it ends with 0.
- **Find the Prime Factorization of a Composite Number**
    1. Find two factors whose product is the given number, and use these numbers to create two branches.
    2. If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
    3. If a factor is not prime, write it as the product of two factors and continue the process.

4. Write the composite number as the product of all the circled primes.

- **Find the Least Common Multiple by Listing Multiples**

    1. List several multiples of each number.
    2. Look for the smallest number that appears on both lists.
    3. This number is the LCM.

- **Find the Least Common Multiple Using the Prime Factors Method**

    1. Write each number as a product of primes.
    2. List the primes of each number. Match primes vertically when possible.
    3. Bring down the columns.
    4. Multiply the factors.

Glossary

composite number
A composite number is a counting number that is not prime. A composite number has factors other than 1 and itself.

counting numbers
The counting numbers are the numbers 1, 2, 3, …

divisible by a number
If a number m is a multiple of n, then m is divisible by n. (If 6 is a multiple of 3, then 6 is divisible by 3.)

factors
If $ab = m$, then $a \text{ and } b$ are factors of m. Since 3 · 4 = 12, then 3 and 4 are factors of 12.

least common multiple
The least common multiple of two numbers is the smallest number that is a multiple of both numbers.

multiple of a number
A number is a multiple of *n* if it is the product of a counting number and *n*.

number line
A number line is used to visualize numbers. The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left.

origin
The origin is the point labeled 0 on a number line.

prime factorization
The prime factorization of a number is the product of prime numbers that equals the number.

prime number
A prime number is a counting number greater than 1, whose only factors are 1 and itself.

whole numbers
The whole numbers are the numbers 0, 1, 2, 3, ….

Practice Makes Perfect

Use Place Value with Whole Numbers

In the following exercises, find the place value of each digit in the given numbers.

1. 51,493 a) 1 b) 4 c) 9 d) 5 e) 3	2. 87,210 a) 2 b) 8 c) 0 d) 7 e) 1
3. 164,285 a) 5 b) 6 c) 1 d) 8 e) 2	4. 395,076 a) 5 b) 3 c) 7 d) 0 e) 9
5. 93,285,170 a) 9 b) 8 c) 7 d) 5 e) 3	6. 36,084,215 a) 8 b) 6 c) 5 d) 4 e) 3
7. 7,284,915,860,132 a) 7 b) 4 c) 5 d) 3 e) 0	8. 2,850,361,159,433 a) 9 b) 8 c) 6 d) 4 e) 2

In the following exercises, name each number using words.

9. 1,078	10. 5,902
11. 364,510	12. 146,023
13. 5,846,103	14. 1,458,398
15. 37,889,005	16. 62,008,465

In the following exercises, write each number as a whole number using digits.

17. four hundred twelve	18. two hundred fifty-three
19. thirty-five thousand, nine hundred seventy-five	20. sixty-one thousand, four hundred fifteen
21. eleven million, forty-four thousand, one hundred sixty-seven	22. eighteen million, one hundred two thousand, seven hundred eighty-three
23. three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen	24. eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

In the following, round to the indicated place value.

25. Round to the nearest ten. a) 386 b) 2,931	26. Round to the nearest ten. a) 792 b) 5,647
27. Round to the nearest hundred. a) 13,748 b) 391,794	28. Round to the nearest hundred. a) 28,166 b) 481,628
29. Round to the nearest ten. a) 1,492 b) 1,497	30. Round to the nearest ten. a) 2,791 b) 2,795
31. Round to the nearest hundred. a) 63,994 b) 63,940	32. Round to the nearest hundred. a) 49,584 b) 49,548

In the following exercises, round each number to the nearest a) hundred, b) thousand, c) ten thousand.

33. 392,546	34. 619,348
35. 2,586,991	36. 4,287,965

Identify Multiples and Factors

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, 3, 5, 6, and 10

37. 84	38. 9,696
39. 75	40. 78
41. 900	42. 800
43. 986	44. 942
45. 350	46. 550
47. 22,335	48. 39,075

Find Prime Factorizations and Least Common Multiples

In the following exercises, find the prime factorization.

49. 86	50. 78
51. 132	52. 455
53. 693	54. 400
55. 432	56. 627
57. 2,160	58. 2,520

In the following exercises, find the least common multiple of the each pair of numbers using the multiples method.

59. 8, 12	60. 4, 3
61. 12, 16	62. 30, 40
63. 20, 30	64. 44, 55

In the following exercises, find the least common multiple of each pair of numbers using the prime factors method.

65. 8, 12	66. 12, 16
67. 28, 40	68. 84, 90
69. 55, 88	70. 60, 72

Everyday Math

71. **Writing a Check** Jorge bought a car for $24,493. He paid for the car with a check. Write the purchase price in words.	72. **Writing a Check** Marissa's kitchen remodeling cost $18,549. She wrote a check to the contractor. Write the amount paid in words.
73. **Buying a Car** Jorge bought a car for $24,493. Round the price to the nearest a) ten b) hundred c) thousand; and d) ten-thousand.	74. **Remodeling a Kitchen** Marissa's kitchen remodeling cost $18,549, Round the cost to the nearest a) ten b) hundred c) thousand and d) ten-thousand.
75. **Population** The population of China was 1,339,724,852 on November 1, 2010. Round the population to the nearest a) billion b) hundred-million; and c) million.	76. **Astronomy** The average distance between Earth and the sun is 149,597,888 kilometres. Round the distance to the nearest a) hundred-million b) ten-million; and c) million.
77. **Grocery Shopping** Hot dogs are sold in packages of 10, but hot dog buns come in packs of eight. What is the smallest number that makes the hot dogs and buns come out even?	78. **Grocery Shopping** Paper plates are sold in packages of 12 and party cups come in packs of eight. What is the smallest number that makes the plates and cups come out even?

Writing Exercises

79. What is the difference between prime numbers and composite numbers?	80. Give an everyday example where it helps to round numbers.
81. Explain in your own words how to find the prime factorization of a composite number, using any method you prefer.	

Answers

1. a) thousands b) hundreds c) tens d) ten thousands e) ones	3. a) ones b) ten thousands c) hundred thousands d) tens e) hundreds	5. a) ten millions b) ten thousands c) tens d) thousands e) millions
7. a) trillions b) billions c) millions d) tens e) thousands	9. one thousand, seventy-eight	11. three hundred sixty-four thousand, five hundred ten
13. five million, eight hundred forty-six thousand, one hundred three	15. thirty-seven million, eight hundred eighty-nine thousand, five	17. 412
19. 35,975	21. 11,044,167	23. 3,226,512,017
25. a) 390 b) 2,930	27. a) 13,700 b) 391,800	29. a) 1,490 b) 1,500
31. a) 64,000 b) 63,900	33. a) 392,500 b) 393,000 c) 390,000	35. a) 2,587,000 b) 2,587,000 c) 2,590,000
37. divisible by 2, 3, and 6	39. divisible by 3 and 5	41. divisible by 2, 3, 5, 6, and 10
43. divisible by 2	45. divisible by 2, 5, and 10	47. divisible by 3 and 5
49. $2 \cdot 43$	51. $2 \cdot 2 \cdot 3 \cdot 11$	53. $3 \cdot 3 \cdot 7 \cdot 11$
55. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$	57. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$	59. 24
61. 48	63. 60	65. 24
67. 280	69. 440	71. twenty-four thousand, four hundred ninety-three dollars
73. a) $24,490 b) $24,500 c) $24,000 d) $20,000	75. a) 1,000,000,000 b) 1,300,000,000 c) 1,340,000,000	77. 40
79. Answers may vary.	81. Answers may vary.	

Attributions

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1.2 Use the Language of Algebra

Learning Objectives

By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Identify expressions and equations
- Simplify expressions with exponents
- Simplify expressions using the order of operations

Use Variables and Algebraic Symbols

Greg and Alex have the same birthday, but they were born in different years. This year Greg is 20 years old and Alex is 23, so Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right?

In the language of algebra, we say that Greg's age and Alex's age are variable and the three is a constant. The ages change, or vary, so age is a variable. The 3 years between them always stays the same, so the age difference is the constant.

In algebra, letters of the alphabet are used to represent variables. Suppose we call Greg's age g. Then we could use $g+3$ to represent Alex's age. See the table below.

Greg's age	**Alex's age**
12	15
20	23
35	38
g	$g+3$

Letters are used to represent variables. Letters often used for variables are $x, y, a, b, \text{and } c$.

Variables and Constants

A variable is a letter that represents a number or quantity whose value may change.

A constant is a number whose value always stays the same.

To write algebraically, we need some symbols as well as numbers and variables. There are several types of symbols we will be using. In 1.1 Whole Numbers, we introduced the symbols for the four basic arithmetic operations: addition, subtraction, multiplication, and division. We will summarize them here, along with words we use for the operations and the result.

Operation	**Notation**	**Say:**	**The result is…**
Addition	$a+b$	a plus b	the sum of a and b
Subtraction	$a-b$	a minus b	the difference of a and b
Multiplication	$a\cdot b, (a)(b), (a)b, a(b)$	a times b	The product of a and b
Division	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$	a divided by b	The quotient of a and b

In algebra, the cross symbol, ×, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ (three times y) or $3\cdot x\cdot y$ (three times x times y)? To make it clear, use • or parentheses for multiplication.

We perform these operations on two numbers. When translating from symbolic form to words, or from words to symbolic form, pay attention to the words *of* or *and* to help you find the numbers.

- The *sum **of*** 5 ***and*** 3 means add 5 plus 3, which we write as $5+3$.
- The *difference **of*** 9 ***and*** 2 means subtract 9 minus 2, which we write as $9-2$.
- The *product **of*** 4 ***and*** 8 means multiply 4 times 8, which we can write as $4\cdot 8$.
- The *quotient **of*** 20 ***and*** 5 means divide 20 by 5, which we can write as $20\div 5$.

EXAMPLE 1

Translate from algebra to words:

a. $12+14$
b. $(30)(5)$
c. $64\div 8$
d. $x-y$

Solution

a.
$12+14$
12 plus 14
the sum of twelve and fourteen

b.
$(30)(5)$
30 times 5
the product of thirty and five

c.
$64 \div 8$
64 divided by 8
the quotient of sixty-four and eight

d.
$x - y$
x minus y
the difference of x and y

TRY IT 1.1

Translate from algebra to words.

a. $18 + 11$
b. $(27)(9)$
c. $84 \div 7$
d. $p - q$

Answer

a. 18 plus 11; the sum of eighteen and eleven
b. 27 times 9; the product of twenty-seven and nine
c. 84 divided by 7; the quotient of eighty-four and seven
d. *p* minus *q*; the difference of *p* and *q*

TRY IT 1.2

Translate from algebra to words.

a. $47 - 19$
b. $72 \div 9$
c. $m + n$
d. $(13)(7)$

Answer

a. 47 minus 19; the difference of forty-seven and nineteen
b. 72 divided by 9; the quotient of seventy-two and nine
c. *m* plus *n*; the sum of *m* and *n*
d. 13 times 7; the product of thirteen and seven

When two quantities have the same value, we say they are equal and connect them with an *equal sign*.

Equality Symbol

$a = b$ is read a is equal to b

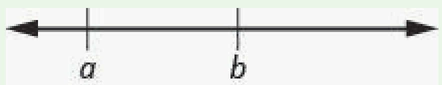
The symbol $=$ is called the equal sign.

An inequality is used in algebra to compare two quantities that may have different values. The number line can help you understand inequalities. Remember that on the number line the numbers get larger as they go from left to right. So if we know that b is greater than a, it means that b is to the right of a on the number line. We use the symbols < and > for inequalities.

Inequality

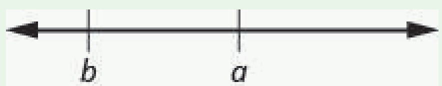
$a < b$ is read a is less than b

a is to the left of b on the number line



$a > b$ is read a is greater than b

a is to the right of b on the number line



The expressions $a < b$ and $a > b$ can be read from left-to-right or right-to-left, though in English we usually read from left-to-right. In general,

$$a < b \text{ is equivalent to } b > a. \qquad \text{For example, } 7 < 11 \text{ is equivalent to } 11 > 7.$$
$$a > b \text{ is equivalent to } b < a. \qquad \text{For example, } 17 > 4 \text{ is equivalent to } 4 < 17.$$

When we write an inequality symbol with a line under it, such as $a \leq b$, it means $a < b$ or $a = b$. We read this a is less than or equal to b. Also, if we put a slash through an equal sign, $\neq$ it means not equal.

We summarize the symbols of equality and inequality in the table below.

Algebraic Notation	**Say**
$a = b$	a is equal to b
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b

Symbols < and >

The symbols < and > each have a smaller side and a larger side.

smaller side < larger side
larger side > smaller side

The smaller side of the symbol faces the smaller number and the larger faces the larger number.

EXAMPLE 2

Translate from algebra to words:

a. $20 \leq 35$
b. $11 \neq 15 - 3$
c. $9 > 10 \div 2$
d. $x + 2 < 10$

Solution

a.
$20 \leq 35$
20 is less than or equal to 35

b.
$11 \neq 15 - 3$
11 is not equal to 15 minus 3

c.
$9 > 10 \div 2$
9 is greater than 10 divided by 2

d.
$x + 2 < 10$
x plus 2 is less than 10

TRY IT 2.1

Translate from algebra to words.

a. $14 \leq 27$
b. $19 - 2 \neq 8$

c. $12 > 4 \div 2$
d. $x - 7 < 1$

Answer

a. fourteen is less than or equal to twenty-seven
b. nineteen minus two is not equal to eight
c. twelve is greater than four divided by two
d. *x* minus seven is less than one

TRY IT 2.2

Translate from algebra to words.

a. $19 \geq 15$
b. $7 = 12 - 5$
c. $15 \div 3 < 8$
d. $y - 3 > 6$

Answer

a. nineteen is greater than or equal to fifteen
b. seven is equal to twelve minus five
c. fifteen divided by three is less than eight
d. *y* minus three is greater than six

EXAMPLE 3

The information in (Figure 1) compares the fuel economy in miles-per-gallon (mpg) of several cars. Write the appropriate symbol symbol, $=$, $<$, or $>$. in each expression to compare the fuel economy of the cars.

(credit: modification of work by Bernard Goldbach, Wikimedia Commons)

Car	Prius 	Mini Cooper 	Toyota Corolla 	Versa 	Honda Fit 
Fuel economy (mpg)	48	27	28	26	27

Figure 1

a. MPG of Prius_____ MPG of Mini Cooper
b. MPG of Versa_____ MPG of Fit
c. MPG of Mini Cooper_____ MPG of Fit
d. MPG of Corolla_____ MPG of Versa

e. MPG of Corolla_____ MPG of Prius

Solution

a.	
	MPG of Prius____MPG of Mini Cooper
Find the values in the chart.	48____27
Compare.	48 > 27
	MPG of Prius > MPG of Mini Cooper

b.	
	MPG of Versa____MPG of Fit
Find the values in the chart.	26____27
Compare.	26 < 27
	MPG of Versa < MPG of Fit

c.	
	MPG of Mini Cooper____MPG of Fit
Find the values in the chart.	27____27
Compare.	27 = 27
	MPG of Mini Cooper = MPG of Fit

d.	
	MPG of Corolla____MPG of Versa
Find the values in the chart.	28____26
Compare.	28 > 26
	MPG of Corolla > MPG of Versa

e.	
	MPG of Corolla____MPG of Prius
Find the values in the chart.	28____48
Compare.	28 < 48
	MPG of Corolla < MPG of Prius

TRY IT 3.1

Use Figure 1 to fill in the appropriate $\text{symbol}, =,$ < , or >.

a. MPG of Prius_____MPG of Versa
b. MPG of Mini Cooper_____ MPG of Corolla

Answer

a. $>$
b. $<$

TRY IT 3.2

Use Figure 1 to fill in the appropriate $\text{symbol}, =,$ $<$, or $>$.

a. MPG of Fit_____ MPG of Prius
b. MPG of Corolla _____ MPG of Fit

Answer

a. $<$
b. $<$

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in written language. They indicate which expressions are to be kept together and separate from other expressions. The table below lists three of the most commonly used grouping symbols in algebra.

Common Grouping Symbols

parentheses	$(\ )$
brackets	$[\ ]$
braces	$\{\ \}$

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.

$8\left(14-8\right)$ $\quad$ $21-3\left[2+4\left(9-8\right)\right]$ $\quad$ $24\div\left\{13-2\left[1\left(6-5\right)+4\right]\right\}$

Identify Expressions and Equations

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb.

In algebra, we have *expressions* and *equations*. An expression is like a phrase. Here are some examples of expressions and how they relate to word phrases:

Expression	Words	Phrase
$3+5$	3 plus 5	the sum of three and five
$n-1$	n minus one	the difference of n and one
$6 \cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	x divided by y	the quotient of x and y

Notice that the phrases do not form a complete sentence because the phrase does not have a verb. An equation is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb. Here are some examples of equations:

Equation	Sentence
$3+5=8$	The sum of three and five is equal to eight.
$n-1=14$	n minus one equals fourteen.
$6 \cdot 7=42$	The product of six and seven is equal to forty-two.
$x=53$	x is equal to fifty-three.
$y+9=2y-3$	y plus nine is equal to two y minus three.

Expressions and Equations

An expression is a number, a variable, or a combination of numbers and variables and operation symbols.

An equation is made up of two expressions connected by an equal sign.

EXAMPLE 4

Determine if each is an expression or an equation:

a. $16-6=10$
b. $4 \cdot 2+1$
c. $x \div 25$
d. $y+8=40$

Solution

a. $16-6=10$	This is an equation—two expressions are connected with an equal sign.
b. $4 \cdot 2+1$	This is an expression—no equal sign.
c. $x \div 25$	This is an expression—no equal sign.
d. $y+8=40$	This is an equation—two expressions are connected with an equal sign.

TRY IT 4.1

Determine if each is an expression or an equation:

a. $23 + 6 = 29$
b. $7 \cdot 3 - 7$

Answer

a. equation
b. expression

TRY IT 4.2

Determine if each is an expression or an equation:

a. $y \div 14$
b. $x - 6 = 21$

Answer

a. expression
b. equation

Simplify Expressions with Exponents

To simplify a numerical expression means to do all the math possible. For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:

$4 \cdot 2 + 1$
$8 + 1$
9

Suppose we have the expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. We could write this more compactly using exponential notation. Exponential notation is used in algebra to represent a quantity multiplied by itself several times. We write $2 \cdot 2 \cdot 2$ as 2^3 and $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^9. In expressions such as 2^3, the 2 is called the base and the 3 is called the exponent. The exponent tells us how many factors of the base we have to multiply.

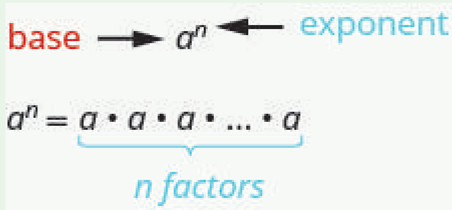
$\text{means multiply three factors of 2}$

We say 2^3 is in exponential notation and $2 \cdot 2 \cdot 2$ is in expanded notation.

Exponential Notation

For any expression a^n, a is a factor multiplied by itself n times if n is a positive integer.

$a^n \text{means multiply } n \text{ factors of } a$



The expression a^n is read a to the n^{th} power.

For powers of $n = 2$ and $n = 3$, we have special names.
a^2 is read as "a squared"
a^3 is read as "a cubed"
The table below lists some examples of expressions written in exponential notation.

Exponential Notation	**In Words**
7^2	7 to the second power, or 7 squared
5^3	5 to the third power, or 5 cubed
9^4	9 to the fourth power
12^5	12 to the fifth power

EXAMPLE 5

Write each expression in exponential form:

a. $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$
b. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
c. $x \cdot x \cdot x \cdot x$
d. $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

Solution

a. The base 16 is a factor 7 times.	16^7
b. The base 9 is a factor 5 times.	9^5
c. The base x is a factor 4 times.	x^4
d. The base a is a factor 8 times.	a^8

TRY IT 5.1

Write each expression in exponential form:
$41 \cdot 41 \cdot 41 \cdot 41 \cdot 41$

Answer
41^5

TRY IT 5.2

Write each expression in exponential form:

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

Answer
7^9

EXAMPLE 6

Write each exponential expression in expanded form:

a. 8^6
b. x^5

Solution

a. The base is 8 and the exponent is 6, so 8^6 means $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

b. The base is x and the exponent is 5, so x^5 means $x \cdot x \cdot x \cdot x \cdot x$

TRY IT 6.1

Write each exponential expression in expanded form:

a. 4^8
b. a^7

Answer

a. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
b. $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

TRY IT 6.2

Write each exponential expression in expanded form:

a. 8^8
b. b^6

Answer

a. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$
b. $b \cdot b \cdot b \cdot b \cdot b \cdot b$

To simplify an exponential expression without using a calculator, we write it in expanded form and then multiply the factors.

EXAMPLE 7

Simplify: 3^4.

Solution

	3^4
Expand the expression.	$3 \cdot 3 \cdot 3 \cdot 3$
Multiply left to right.	$9 \cdot 3 \cdot 3$
	$27 \cdot 3$
Multiply.	81

TRY IT 7.1

Simplify:

a. 5^3
b. 1^7

Answer

a. 125
b. 1

TRY IT 7.2

Simplify:

a. 7^2
b. 0^5

Answer

a. 49
b. 0

Simplify Expressions Using the Order of Operations

We've introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values.

For example, consider the expression:

$4 + 3 \cdot 7$

Some students say it simplifies to 49.		Some students say it simplifies to 25.	
	$4 + 3 \cdot 7$		$4 + 3 \cdot 7$
Since $4 + 3$ gives 7.	$7 \cdot 7$	Since $3 \cdot 7$ is 21.	$4 + 21$
And $7 \cdot 7$ is 49.	49	And $21 + 4$ makes 25.	25

Imagine the confusion that could result if every problem had several different correct answers. The same expression should give the same result. So mathematicians established some guidelines called the order of operations, which outlines the order in which parts of an expression must be simplified.

Order of Operations

When simplifying mathematical expressions perform the operations in the following order:

1. **P**arentheses and other Grouping Symbols
   - Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
2. **E**xponents
   - Simplify all expressions with exponents.
3. **M**ultiplication and **D**ivision
   - Perform all multiplication and division in order from left to right. These operations have equal priority.
4. **A**ddition and **S**ubtraction
   - Perform all addition and subtraction in order from left to right. These operations have equal priority.

Students often ask, “How will I remember the order?” Here is a way to help you remember: Take the first letter of each key word and substitute the silly phrase.

Please **E**xcuse **M**y **D**ear **A**unt **S**ally.

Please	**P**arentheses
Excuse	**E**xponents
My **D**ear	**M**ultiplication and **D**ivision
Aunt **S**ally	**A**ddition and **S**ubtraction

It’s good that ‘**M**y **D**ear’ goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, ‘**A**unt **S**ally’ goes together and so reminds us that **a**ddition and **s**ubtraction also have equal priority and we do them in order from left to right.

EXAMPLE 8

Simplify the expressions:

a. $4 + 3 \cdot 7$
b. $(4 + 3) \cdot 7$

Solution

a.	
	$4+3\cdot 7$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply first.	$4+3\cdot 7$
Add.	$4+21$
	25

b.	
	$(4+3)\cdot 7$
Are there any **p**arentheses? Yes.	$(4+3)\cdot 7$
Simplify inside the parentheses.	$(7)7$
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply.	49

TRY IT 8.1

Simplify the expressions:

a. $12-5\cdot 2$
b. $(12-5)\cdot 2$

Answer

a. 2
b. 14

TRY IT 8.2

Simplify the expressions:

a. $8+3\cdot 9$
b. $(8+3)\cdot 9$

Answer

a. 35
b. 99

EXAMPLE 9

Simplify:

a. $18 \div 9 \cdot 2$
b. $18 \cdot 9 \div 2$

Solution

a.	
	$18 \div 9 \cdot 2$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply and divide from left to right. Divide.	$2 \cdot 2$
Multiply.	4

b.	
	$18 \cdot 9 \div 2$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply and divide from left to right.	
Multiply.	$162 \div 2$
Divide.	81

TRY IT 9.1

Simplify:

$42 \div 7 \cdot 3$

Answer
18

TRY IT 9.2

Simplify:

$12 \cdot 3 \div 4$

Answer
9

EXAMPLE 10

Simplify: $18 \div 6 + 4\left(5 - 2\right)$.

Solution

	$18 \div 6 + 4(5 - 2)$
Parentheses? Yes, subtract first.	$18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

TRY IT 10.1

Simplify:

$30 \div 5 + 10\left(3 - 2\right)$

Answer

16

TRY IT 10.2

Simplify:

$70 \div 10 + 4\left(6 - 2\right)$

Answer

23

When there are multiple grouping symbols, we simplify the innermost parentheses first and work outward.

EXAMPLE 11

Simplify: $5 + 2^3 + 3\left[6 - 3\left(4 - 2\right)\right]$.

Solution

	$5 + 2^3 + 3[6 - 3(4 - 2)]$
Are there any parentheses (or other grouping symbol)? Yes.	
Focus on the parentheses that are inside the brackets.	$5 + 2^3 + 3[6 - 3(4 - 2)]$
Subtract.	$5 + 2^3 + 3[6 - 3(2)]$
Continue inside the brackets and multiply.	$5 + 2^3 + 3[6 - 6]$
Continue inside the brackets and subtract.	$5 + 2^3 + 3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	
Simplify exponents.	$5 + 2^3 + 3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5 + 8 + 3[0]$
Is there any addition or subtraction? Yes.	
Add.	$5 + 8 + 0$
Add.	$13 + 0$
	13

TRY IT 11.1

Simplify:

$$9 + 5^3 - [4(9 + 3)]$$

Answer

86

TRY IT 11.2

Simplify:

$$7^2 - 2[4(5 + 1)]$$

Answer

1

EXAMPLE 12

Simplify: $2^3 + 3^4 \div 3 - 5^2$.

Solution

	$2^3 + 3^4 \div 3 - 5^2$
If an expression has several exponents, they may be simplified in the same step.	
Simplify exponents.	$2^3 + 3^4 \div 3 - 5^2$
Divide.	$8 + 81 \div 3 - 25$
Add.	$8 + 27 - 25$
Subtract.	$35 - 25$
	10

TRY IT 12.1

Simplify:

$3^2 + 2^4 \div 2 + 4^3$

Answer

81

TRY IT 12.2

Simplify:

$6^2 - 5^3 \div 5 + 8^2$

Answer

75

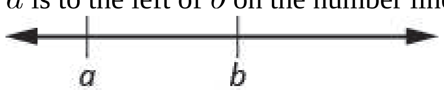
ACCESS ADDITIONAL ONLINE RESOURCES

- Order of Operations
- Order of Operations – The Basics
- Ex: Evaluate an Expression Using the Order of Operations
- Example 3: Evaluate an Expression Using The Order of Operations

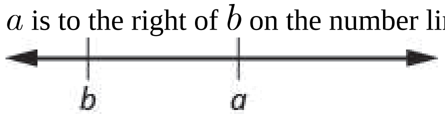
Key Concepts

Operation	**Notation**	**Say:**	**The result is…**
Addition	$a+b$	a plus b	the sum of a and b
Multiplication	$a\cdot b, (a)(b), (a)b, a(b)$	a times b	The product of a and b
Subtraction	$a-b$	a minus b	the difference of a and b
Division	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$	a divided by b	The quotient of a and b

- **Equality Symbol**
  - $a=b$ is read as a is equal to b
  - The symbol $=$ is called the equal sign.
- **Inequality**
  - $a<b$ is read a is less than b
  - a is to the left of b on the number line

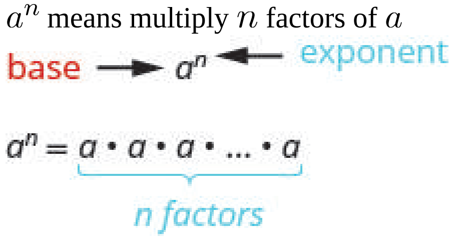


  - $a>b$ is read a is greater than b
  - a is to the right of b on the number line



Algebraic Notation	**Say**
$a=b$	a is equal to b
$a\neq b$	a is not equal to b
$a<b$	a is less than b
$a>b$	a is greater than b
$a\leq b$	a is less than or equal to b
$a\geq b$	a is greater than or equal to b

- **Exponential Notation**
  - For any expression a^n is a factor multiplied by itself n times, if n is a positive integer.
  - a^n means multiply n factors of a



  - The expression of a^n is read a to the $n\text{th}$ power.

Order of Operations When simplifying mathematical expressions perform the operations in the following order:

- Parentheses and other Grouping Symbols: Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
- Exponents: Simplify all expressions with exponents.
- Multiplication and Division: Perform all multiplication and division in order from left to right. These operations have equal priority.
- Addition and Subtraction: Perform all addition and subtraction in order from left to right. These operations have equal priority.

Glossary

expressions
An expression is a number, a variable, or a combination of numbers and variables and operation symbols.
equation
An equation is made up of two expressions connected by an equal sign.

Practice Makes Perfect

Use Variables and Algebraic Symbols

In the following exercises, translate from algebraic notation to words.

1. $16 - 9$	2. $25 - 7$
3. $5 \cdot 6$	4. $3 \cdot 9$
5. $28 \div 4$	6. $45 \div 5$
7. $x + 8$	8. $x + 11$
9. $(2)\,(7)$	10. $(4)\,(8)$
11. $14 < 21$	12. $17 < 35$
13. $36 \geq 19$	14. $42 \geq 27$
15. $3n = 24$	16. $6n = 36$
17. $y - 1 > 6$	18. $y - 4 > 8$
19. $2 \leq 18 \div 6$	20. $3 \leq 20 \div 4$
21. $a \neq 7 \cdot 4$	22. $a \neq 1 \cdot 12$

Identify Expressions and Equations

In the following exercises, determine if each is an expression or an equation.

23. $9 \cdot 6 = 54$	24. $7 \cdot 9 = 63$
25. $5 \cdot 4 + 3$	26. $6 \cdot 3 + 5$
27. $x + 7$	28. $x + 9$
29. $y - 5 = 25$	30. $y - 8 = 32$

Simplify Expressions with Exponents

In the following exercises, write in exponential form.

31. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	32. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
33. $x \cdot x \cdot x \cdot x \cdot x$	34. $y \cdot y \cdot y \cdot y \cdot y \cdot y$

In the following exercises, write in expanded form.

35. 5^3	36. 8^3
37. 2^8	38. 10^5

Simplify Expressions Using the Order of Operations

In the following exercises, simplify.

39. a. $3+8\cdot 5$ b. $(3+8)\cdot 5$	40. a. $2+6\cdot 3$ b. $(2+6)\cdot 3$
41. $2^3-12\div(9-5)$	42. $3^2-18\div(11-5)$
43. $3\cdot 8+5\cdot 2$	44. $4\cdot 7+3\cdot 5$
45. $2+8\,(6+1)$	46. $4+6\,(3+6)$
47. $4\cdot 12/8$	48. $2\cdot 36/6$
49. $6+10/2+2$	50. $9+12/3+4$
51. $(6+10)\div(2+2)$	52. $(9+12)\div(3+4)$
53. $20\div 4+6\cdot 5$	54. $33\div 3+8\cdot 2$
55. $20\div(4+6)\cdot 5$	56. $33\div(3+8)\cdot 2$
57. 4^2+5^2	58. 3^2+7^2
59. $(4+5)^2$	60. $(3+7)^2$
61. $3\,(1+9\cdot 6)-4^2$	62. $5\,(2+8\cdot 4)-7^2$
63. $2\,[1+3\,(10-2)]$	64. $5\,[2+4\,(3-2)]$

Everyday Math

65. Basketball In the 2014 NBA playoffs, the San Antonio Spurs beat the Miami Heat. The table below shows the heights of the starters on each team. Use this table to fill in the appropriate symbol (= ,<, >).

Spurs	Height		Heat	Height
Tim Duncan	$83''$		Rashard Lewis	$82''$
Boris Diaw	$80''$		LeBron James	$80''$
Kawhi Leonard	$79''$		Chris Bosh	$83''$
Tony Parker	$74''$		Dwyane Wade	$76''$
Danny Green	$78''$		Ray Allen	$77''$

a. Height of Tim Duncan____Height of Rashard Lewis
b. Height of Boris Diaw____Height of LeBron James
c. Height of Kawhi Leonard____Height of Chris Bosh
d. Height of Tony Parker____Height of Dwyane Wade
e. Height of Danny Green____Height of Ray Allen

66. Elevation In Colorado there are more than 50 mountains with an elevation of over $14,000 \text{ feet}.$ The table shows the ten tallest. Use this table to fill in the appropriate inequality symbol.

Mountain	Elevation
Mt. Elbert	$14,433'$
Mt. Massive	$14,421'$
Mt. Harvard	$14,420'$
Blanca Peak	$14,345'$
La Plata Peak	$14,336'$
Uncompahgre Peak	$14,309'$
Crestone Peak	$14,294'$
Mt. Lincoln	$14,286'$
Grays Peak	$14,270'$
Mt. Antero	$14,269'$

a. Elevation of La Plata Peak____Elevation of Mt. Antero
b. Elevation of Blanca Peak____Elevation of Mt. Elbert
c. Elevation of Gray's Peak____Elevation of Mt. Lincoln
d. Elevation of Mt. Massive____Elevation of Crestone Peak
e. Elevation of Mt. Harvard____Elevation of Uncompahgre Peak

Writing Exercises

67.Explain the difference between an expression and an equation.

68. Why is it important to use the order of operations to simplify an expression?

Answers

1. 16 minus 9, the difference of sixteen and nine	3. 5 times 6, the product of five and six	5. 28 divided by 4, the quotient of twenty-eight and four
7. *x* plus 8, the sum of *x* and eight	9. 2 times 7, the product of two and seven	11. fourteen is less than twenty-one
13. thirty-six is greater than or equal to nineteen	15. 3 times n equals 24, the product of three and n equals twenty-four	17. *y* minus 1 is greater than 6, the difference of *y* and one is greater than six
19. 2 is less than or equal to 18 divided by 6; 2 is less than or equal to the quotient of eighteen and six	21. *a* is not equal to 7 times 4, *a* is not equal to the product of seven and four	23. equation
25. expression	27. expression	29. equation
31. 3^7	33. x^5	35. 125
37. 256	39. a. 43 b. 55	41. 5
43. 34	45. 58	47. 6
49. 13	51. 4	53. 35
55. 10	57. 41	59. 81
61. 149	63. 50	65. a. > b. = c. < d. < e. >
67. Answer may vary.		

Attributions

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1.3 Evaluate, Simplify, and Translate Expressions

Learning Objectives

By the end of this section, you will be able to:

- Evaluate algebraic expressions
- Identify terms, coefficients, and like terms
- Simplify expressions by combining like terms
- Translate word phrases to algebraic expressions

Evaluate Algebraic Expressions

In the last section, we simplified expressions using the order of operations. In this section, we'll evaluate expressions—again following the order of operations.

To evaluate an algebraic expression means to find the value of the expression when the variable is replaced by a given number. To evaluate an expression, we substitute the given number for the variable in the expression and then simplify the expression using the order of operations.

EXAMPLE 1

Evaluate $x + 7$ when

a. $x = 3$
b. $x = 12$

Solution
a. To evaluate, substitute 3 for x in the expression, and then simplify.

	$x+7$
Substitute.	$3+7$
Add.	10

When $x = 3$, the expression $x + 7$ has a value of 10.

b. To evaluate, substitute 12 for x in the expression, and then simplify.

	$x+7$
Substitute.	$12+7$
Add.	19

When $x = 12$, the expression $x + 7$ has a value of 19.

Notice that we got different results for parts a) and b) even though we started with the same expression. This is because the values used for x were different. When we evaluate an expression, the value varies depending on the value used for the variable.

TRY IT 1.1

Evaluate:

$y + 4$ when

a. $y = 6$
b. $y = 15$

Answer

a. 10
b. 19

TRY IT 1.2

Evaluate:

$a - 5$ when

a. $a = 9$
b. $a = 17$

Answer

a. 4
b. 12

EXAMPLE 2

Evaluate $9x - 2$, when

a. $x = 5$
b. $x = 1$

Solution

Remember ab means a times b, so $9x$ means 9 times x.

a. To evaluate the expression when $x = 5$, we substitute 5 for x, and then simplify.

	$9x - 2$
Substitute 5 for x.	$9 \cdot 5 - 2$
Multiply.	$45 - 2$
Subtract.	43

b. To evaluate the expression when $x = 1$, we substitute 1 for x, and then simplify.

	$9x - 2$
Substitute 1 for x.	$9(1) - 2$
Multiply.	$9 - 2$
Subtract.	7

Notice that in part a) that we wrote $9 \cdot 5$ and in part b) we wrote $9\,(1)$. Both the dot and the parentheses tell us to multiply.

TRY IT 2.1

Evaluate:

$8x - 3$, when

a. $x = 2$
b. $x = 1$

Answer

a. 13
b. 5

TRY IT 2.2

Evaluate:

$4y - 4$, when

a. $y = 3$
b. $y = 5$

Answer

a. 8
b. 16

EXAMPLE 3

Evaluate x^2 when $x = 10$.

Solution
We substitute 10 for x, and then simplify the expression.

	x^2
Substitute 10 for x.	10^2
Use the definition of exponent.	$10 \cdot 10$
Multiply.	100

When $x = 10$, the expression x^2 has a value of 100.

TRY IT 3.1

Evaluate:
x^2 when $x = 8$.
Answer
64

TRY IT 3.2

Evaluate:
x^3 when $x = 6$.
Answer
216

EXAMPLE 4

Evaluate 2^x when $x = 5$.

Solution
In this expression, the variable is an exponent.

	2^x
Substitute 5 for x.	2^5
Use the definition of exponent.	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
Multiply.	32

When $x = 5$, the expression 2^x has a value of 32.

TRY IT 4.1

Evaluate:

2^x when $x = 6$.

Answer

64

TRY IT 4.2

Evaluate:

3^x when $x = 4$.

Answer

81

EXAMPLE 5

Evaluate $3x + 4y - 6$ when $x = 10$ and $y = 2$.

Solution

This expression contains two variables, so we must make two substitutions.

	$3x + 4y - 6$
Substitute 10 for *x* and 2 for *y*.	$3(10) + 4(2) - 6$
Multiply.	$30 + 8 - 6$
Add and subtract left to right.	32

When $x = 10$ and $y = 2$, the expression $3x + 4y - 6$ has a value of 32.

TRY IT 5.1

Evaluate:

$2x + 5y - 4$ when $x = 11$ and $y = 3$

Answer

33

TRY IT 5.2

Evaluate:

$5x - 2y - 9$ when $x = 7$ and $y = 8$

Answer

10

EXAMPLE 6

Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Solution

We need to be careful when an expression has a variable with an exponent. In this expression, $2x^2$ means $2 \cdot x \cdot x$ and is different from the expression $(2x)^2$, which means $2x \cdot 2x$.

	$2x^2 + 3x + 8$
Substitute 4 for each x.	$2(4)^2 + 3(4) + 8$
Simplify 4^2.	$2(16) + 3(4) + 8$
Multiply.	$32 + 12 + 8$
Add.	52

TRY IT 6.1

Evaluate:

$3x^2 + 4x + 1$ when $x = 3$.

Answer

40

TRY IT 6.2

Evaluate:

$6x^2 - 4x - 7$ when $x = 2$.

Answer

9

Identify Terms, Coefficients, and Like Terms

Algebraic expressions are made up of *terms*. A term is a constant or the product of a constant and one or more variables. Some examples of terms are $7, y, 5x^2, 9a,$ and $13xy$.

The constant that multiplies the variable(s) in a term is called the coefficient. We can think of the coefficient as the number *in front of* the variable. The coefficient of the term $3x$ is 3. When we write x, the coefficient is 1, since $x = 1 \cdot x$. The table below gives the coefficients for each of the terms in the left column.

Term	**Coefficient**
7	7
$9a$	9
y	1
$5x^2$	5

An algebraic expression may consist of one or more terms added or subtracted. In this chapter, we will only work with terms that are added together. The table below gives some examples of algebraic expressions with various numbers of terms. Notice that we include the operation before a term with it.

Expression	**Terms**
7	7
y	y
$x+7$	$x, 7$
$2x+7y+4$	$2x, 7y, 4$
$3x^2+4x^2+5y+3$	$3x^2, 4x^2, 5y, 3$

EXAMPLE 7

Identify each term in the expression $9b+15x^2+a+6$. Then identify the coefficient of each term.

Solution

The expression has four terms. They are $9b$, $15x^2$, a, and 6.

The coefficient of $9b$ is 9.

The coefficient of $15x^2$ is 15.

Remember that if no number is written before a variable, the coefficient is 1. So the coefficient of a is 1.

The coefficient of a constant is the constant, so the coefficient of 6 is 6.

TRY IT 7.1

Identify all terms in the given expression, and their coefficients:

$4x+3b+2$

Answer

The terms are 4*x*, 3*b*, and 2. The coefficients are 4, 3, and 2

TRY IT 7.2

Identify all terms in the given expression, and their coefficients:

$9a+13a^2+a^3$

Answer
The terms are $9a$, $13a^2$, and a^3, The coefficients are 9, 13, and 1

Some terms share common traits. Look at the following terms. Which ones seem to have traits in common?

$5x, 7, n^2, 4, 3x, 9n^2$
Which of these terms are like terms?

- The terms 7 and 4 are both constant terms.
- The terms $5x$ and $3x$ are both terms with x.
- The terms n^2 and $9n^2$ both have n^2.

Terms are called like terms if they have the same variables and exponents. All constant terms are also like terms. So among the terms $5x, 7, n^2, 4, 3x, 9n^2$,
$7 \text{ and } 4 \text{ are like terms.}$
$5x \text{ and } 3x \text{ are like terms.}$
$n^2 \text{ and } 9n^2 \text{ are like terms.}$

Like Terms

Terms that are either constants or have the same variables with the same exponents are like terms.

EXAMPLE 8

Identify the like terms:

a. $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$
b. $4x^2 + 2x + 5x^2 + 6x + 40x + 8xy$

Solution
a. $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$

Look at the variables and exponents. The expression contains y^3, x^2, x, and constants.

The terms y^3 and $4y^3$ are like terms because they both have y^3.

The terms $7x^2$ and $5x^2$ are like terms because they both have x^2.

The terms 14 and 23 are like terms because they are both constants.

The term $9x$ does not have any like terms in this list since no other terms have the variable x raised to the power of 1.

b. $4x^2 + 2x + 5x^2 + 6x + 40x + 8xy$

Look at the variables and exponents. The expression contains the terms $4x^2, 2x, 5x^2, 6x, 40x, \text{and } 8xy$

The terms $4x^2$ and $5x^2$ are like terms because they both have x^2.

The terms $2x, 6x, \text{and } 40x$ are like terms because they all have x.

The term $8xy$ has no like terms in the given expression because no other terms contain the two variables xy.

TRY IT 8.1

Identify the like terms in the list or the expression:

$$9, 2x^3, y^2, 8x^3, 15, 9y, 11y^2$$

Answer

9, 15; $2x^3$ and $8x^3$, y^2, and $11y^2$

TRY IT 8.2

Identify the like terms in the list or the expression:

$$4x^3 + 8x^2 + 19 + 3x^2 + 24 + 6x^3$$

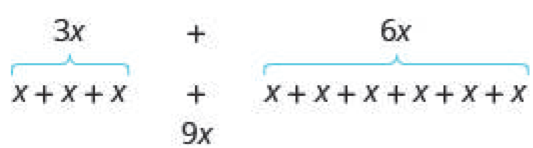
Answer

$4x^3$ and $6x^3$; $8x^2$ and $3x^2$; 19 and 24

Simplify Expressions by Combining Like Terms

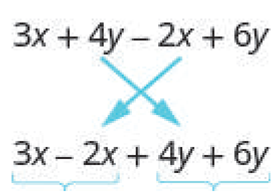
We can simplify an expression by combining the like terms. What do you think $3x + 6x$ would simplify to? If you thought $9x$, you would be right!

We can see why this works by writing both terms as addition problems.



Add the coefficients and keep the same variable. It doesn't matter what x is. If you have 3 of something and add 6 more of the same thing, the result is 9 of them. For example, 3 oranges plus 6 oranges is 9 oranges. We will discuss the mathematical properties behind this later.

The expression $3x + 6x$ has only two terms. When an expression contains more terms, it may be helpful to rearrange the terms so that like terms are together. The Commutative Property of Addition says that we can change the order of addends without changing the sum. So we could rearrange the following expression before combining like terms.



Now it is easier to see the like terms to be combined.

HOW TO: Combine like terms

1. Identify like terms.
2. Rearrange the expression so like terms are together.

3. Add the coefficients of the like terms.

EXAMPLE 9

Simplify the expression: $3x + 7 + 4x + 5$.

Solution

	$3x + 7 + 4x + 5$
Identify the like terms.	$3x + 7 + 4x + 5$
Rearrange the expression, so the like terms are together.	$3x + 4x + 7 + 5$
Add the coefficients of the like terms.	$\underbrace{3x + 4x}_{7x} + \underbrace{7 + 5}_{12}$
The original expression is simplified to…	$7x + 12$

TRY IT 9.1

Simplify:

$$7x + 9 + 9x + 8$$

Answer

$16x + 17$

TRY IT 9.2

Simplify:

$$5y + 2 + 8y + 4y + 5$$

Answer

$17y + 7$

EXAMPLE 10

Simplify the expression: $7x^2 + 8x + x^2 + 4x$.

Solution

	$7x^2 + 8x + x^2 + 4x$
Identify the like terms.	$7x^2 + 8x + x^2 + 4x$
Rearrange the expression so like terms are together.	$7x^2 + x^2 + 8x + 4x$
Add the coefficients of the like terms.	$8x^2 + 12x$

These are not like terms and cannot be combined. So $8x^2 + 12x$ is in simplest form.

TRY IT 10.1

Simplify:

$$3x^2 + 9x + x^2 + 5x$$

Answer

$4x^2 + 14x$

TRY IT 10.2

Simplify:

$$11y^2 + 8y + y^2 + 7y$$

Answer

$12y^2 + 15y$

Translate Words to Algebraic Expressions

In the previous section, we listed many operation symbols that are used in algebra, and then we translated expressions and equations into word phrases and sentences. Now we'll reverse the process and translate word phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. They are summarized in the table below.

Operation	**Phrase**	**Expression**
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a+b$
Subtraction	a minus b the difference of a and b b subtracted from a a decreased by b b less than a	$a-b$
Multiplication	a times b the product of a and b	$a\cdot b, ab, a\left(b\right), \left(a\right)\left(b\right)$
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$

Look closely at these phrases using the four operations:

- the sum *of* a *and* b
- the difference *of* a *and* b
- the product *of* a *and* b
- the quotient *of* a *and* b

Each phrase tells you to operate on two numbers. Look for the words ***of*** and ***and*** to find the numbers.

EXAMPLE 11

Translate each word phrase into an algebraic expression:

a. the difference of 20 and 4
b. the quotient of $10x$ and 3

Solution
a. The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.

$$\text{the difference of } 20 \textit{ and } 4$$
$$20 \text{ minus } 4$$
$$20-4$$

b. The key word is *quotient*, which tells us the operation is division.

$$\text{the quotient of } 10x \text{ and } 3$$
$$\text{divide } 10x \text{ by } 3$$
$$10x\div 3$$

This can also be written as $10x/3 \text{ or } \frac{10x}{3}$

TRY IT 11.1

Translate the given word phrase into an algebraic expression:

a. the difference of 47 and 41
b. the quotient of $5x$ and 2

Answer

a. 47 − 41
b. 5*x* ÷ 2

TRY IT 11.2

Translate the given word phrase into an algebraic expression:

a. the sum of 17 and 19
b. the product of 7 and x

Answer

a. 17 + 19
b. 7*x*

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight *more than* means eight added to your present age.

How old were you seven years ago? This is seven years less than your age now. You subtract 7 from your present age. Seven *less than* means seven subtracted from your present age.

EXAMPLE 12

Translate each word phrase into an algebraic expression:

a. Eight more than y
b. Seven less than $9z$

Solution
a. The key words are *more than*. They tell us the operation is addition. *More than* means “added to”.

$$\text{Eight more than } y$$
$$\text{Eight added to } y$$
$$y + 8$$

b. The key words are *less than*. They tell us the operation is subtraction. *Less than* means “subtracted from”.

$$\text{Seven less than } 9z$$
$$\text{Seven subtracted from } 9z$$
$$9z - 7$$

TRY IT 12.1

Translate each word phrase into an algebraic expression:

a. Eleven more than x
b. Fourteen less than $11a$

Answer

a. $x + 11$
b. $11a - 14$

TRY IT 12.2

Translate each word phrase into an algebraic expression:

a. 19 more than j
b. 21 less than $2x$

Answer

a. $j + 19$
b. $2x - 21$

EXAMPLE 13

Translate each word phrase into an algebraic expression:

a. five times the sum of m and n
b. the sum of five times m and n

Solution
a. There are two operation words: *times* tells us to multiply and *sum* tells us to add. Because we are multiplying 5 times the sum, we need parentheses around the sum of m and n.

five times the sum of m and n

$$5\left(m+n\right)$$

b. To take a sum, we look for the words *of* and *and* to see what is being added. Here we are taking the sum *of* five times m and n.

the sum of five times m and n

$$5m+n$$

Notice how the use of parentheses changes the result. In part a), we add first and in part b), we multiply first.

TRY IT 13.1

Translate the word phrase into an algebraic expression:

a. four times the sum of p and q
b. the sum of four times p and q

Answer

a. $4(p + q)$
b. $4p + q$

TRY IT 13.2

Translate the word phrase into an algebraic expression:

a. the difference of two times x and 8
b. two times the difference of x and 8

Answer

a. $2x - 8$
b. $2(x - 8)$

Later in this course, we'll apply our skills in algebra to solving equations. We'll usually start by translating a word phrase to an algebraic expression. We'll need to be clear about what the expression will represent. We'll see how to do this in the next two examples.

EXAMPLE 14

The height of a rectangular window is 6 inches less than the width. Let w represent the width of the window. Write an expression for the height of the window.

Solution

Write a phrase about the height.	6 less than the width
Substitute w for the width.	6 less than w
Rewrite 'less than' as 'subtracted from'.	6 subtracted from w
Translate the phrase into algebra.	$w - 6$

TRY IT 14.1

The length of a rectangle is 5 inches less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Answer

$w - 5$

TRY IT 14.2

The width of a rectangle is 2 metres greater than the length. Let l represent the length of the rectangle. Write an expression for the width of the rectangle.

Answer
$l + 2$

EXAMPLE 15

Blanca has dimes and quarters in her purse. The number of dimes is 2 less than 5 times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution

Write a phrase about the number of dimes.	two less than five times the number of quarters
Substitute q for the number of quarters.	2 less than five times q
Translate 5 *times* q.	2 less than $5q$
Translate the phrase into algebra.	$5q - 2$

TRY IT 15.1

Geoffrey has dimes and quarters in his pocket. The number of dimes is seven less than six times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Answer
$6q - 7$

TRY IT 15.2

Lauren has dimes and nickels in her purse. The number of dimes is eight more than four times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Answer
$4n + 8$

ACCESS ADDITIONAL ONLINE RESOURCES

- Algebraic Expression Vocabulary

Key Concepts

- **Combine like terms.**
    1. Identify like terms.
    2. Rearrange the expression so like terms are together.
    3. Add the coefficients of the like terms

Glossary

term
A term is a constant or the product of a constant and one or more variables.
coefficient
The constant that multiplies the variable(s) in a term is called the coefficient.
like terms
Terms that are either constants or have the same variables with the same exponents are like terms.
evaluate
To evaluate an algebraic expression means to find the value of the expression when the variable is replaced by a given number.

Practice Makes Perfect

Evaluate Algebraic Expressions

In the following exercises, evaluate the expression for the given value.

1. $7x+8$ when $x=2$	2. $9x+7$ when $x=3$
3. $5x-4$ when $x=6$	4. $8x-6$ when $x=7$
5. x^2 when $x=12$	6. x^3 when $x=5$
7. x^5 when $x=2$	8. x^4 when $x=3$
9. 3^x when $x=3$	10. 4^x when $x=2$
11. x^2+3x-7 when $x=4$	12. x^2+5x-8 when $x=6$
13. $2x+4y-5$ when $x=7, y=8$	14. $6x+3y-9$ when $x=6, y=9$
15. $(x-y)^2$ when $x=10, y=7$	16. $(x+y)^2$ when $x=6, y=9$
17. a^2+b^2 when $a=3, b=8$	18. r^2-s^2 when $r=12, s=5$
19. $2l+2w$ when $l=15, w=12$	20. $2l+2w$ when $l=18, w=14$

Identify Terms, Coefficients, and Like Terms

In the following exercises, list the terms in the given expression.

21. $15x^2 + 6x + 2$	22. $11x^2 + 8x + 5$
23. $10y^3 + y + 2$	24. $9y^3 + y + 5$

In the following exercises, identify the coefficient of the given term.

25. $8a$	26. $13m$
27. $5r^2$	28. $6x^3$

In the following exercises, identify all sets of like terms.

29. $x^3, 8x, 14, 8y, 5, 8x^3$	30. $6z, 3w^2, 1, 6z^2, 4z, w^2$
31. $9a, a^2, 16ab, 16b^2, 4ab, 9b^2$	32. $3, 25r^2, 10s, 10r, 4r^2, 3s$

Simplify Expressions by Combining Like Terms

In the following exercises, simplify the given expression by combining like terms.

33. $10x + 3x$	34. $15x + 4x$
35. $17a + 9a$	36. $18z + 9z$
37. $4c + 2c + c$	38. $6y + 4y + y$
39. $9x + 3x + 8$	40. $8a + 5a + 9$
41. $7u + 2 + 3u + 1$	42. $8d + 6 + 2d + 5$
43. $7p + 6 + 5p + 4$	44. $8x + 7 + 4x - 5$
45. $10a + 7 + 5a - 2 + 7a - 4$	46. $7c + 4 + 6c - 3 + 9c - 1$
47. $3x^2 + 12x + 11 + 14x^2 + 8x + 5$	48. $5b^2 + 9b + 10 + 2b^2 + 3b - 4$

Translate English Phrases into Algebraic Expressions

In the following exercises, translate the given word phrase into an algebraic expression.

49. The sum of 8 and 12	50. The sum of 9 and 1
51. The difference of 14 and 9	52. 8 less than 19
53. The product of 9 and 7	54. The product of 8 and 7
55. The quotient of 36 and 9	56. The quotient of 42 and 7
57. The difference of x and 4	58. 3 less than x
59. The product of 6 and y	60. The product of 9 and y
61. The sum of $8x$ and $3x$	62. The sum of $13x$ and $3x$
63. The quotient of y and 3	64. The quotient of y and 8
65. Eight times the difference of y and nine	66. Seven times the difference of y and one
67. Five times the sum of x and y	68. y times five less than twice x

In the following exercises, write an algebraic expression.

69. Adele bought a skirt and a blouse. The skirt cost $15 more than the blouse. Let b represent the cost of the blouse. Write an expression for the cost of the skirt.	70. Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let c represent the number of classical CDs. Write an expression for the number of rock CDs.
71. The number of girls in a second-grade class is 4 less than the number of boys. Let b represent the number of boys. Write an expression for the number of girls.	72. Marcella has 6 fewer male cousins than female cousins. Let f represent the number of female cousins. Write an expression for the number of boy cousins.
73. Greg has nickels and pennies in his pocket. The number of pennies is seven less than twice the number of nickels. Let n represent the number of nickels. Write an expression for the number of pennies.	74. Jeannette has $5 and $10 bills in her wallet. The number of fives is three more than six times the number of tens. Let t represent the number of tens. Write an expression for the number of fives.

Everyday Math

In the following exercises, use algebraic expressions to solve the problem.

75. **Car insurance** Justin's car insurance has a $750 deductible per incident. This means that he pays $750 and his insurance company will pay all costs beyond $750. If Justin files a claim for $2,100, how much will he pay, and how much will his insurance company pay?	76. **Home insurance** Pam and Armando's home insurance has a $2,500 deductible per incident. This means that they pay $2,500 and their insurance company will pay all costs beyond $2,500. If Pam and Armando file a claim for $19,400, how much will they pay, and how much will their insurance company pay?

Writing Exercises

77. Explain why "the sum of *x* and *y*" is the same as "the sum of *y* and *x*," but "the difference of *x* and *y*" is not the same as "the difference of *y* and *x*." Try substituting two random numbers for x and y to help you explain.	78. Explain the difference between "4 times the sum of x and y" and "the sum of 4 times x and y."

Answers

1. 22	3. 26	5. 144
7. 32	9. 27	11. 21
13. 41	15. 9	17. 73
19. 54	21. $15x^2$, $6x$, 2	23. $10y^3$, y, 2
25. 8	27. 5	29. x^3, $8x^3$ and 14, 5
31. $16ab$ and $4ab$; $16b^2$ and $9b^2$	33. $13x$	35. $26a$
37. $7c$	39. $12x + 8$	41. $10u + 3$
43. $12p + 10$	45. $22a + 1$	47. $17x^2 + 20x + 16$
49. $8 + 12$	51. $14 - 9$	53. $9 \cdot 7$
55. $36 \div 9$	57. $x - 4$	59. $6y$
61. $8x + 3x$	63. $\frac{y}{3}$	65. $8(y - 9)$
67. $5(x + y)$	69. $b + 15$	71. $b - 4$
73. $2n - 7$	75. He will pay $750. His insurance company will pay $1350.	77. Answers will vary.

Attributions

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1.4 Add and Subtract Integers

Learning Objectives

By the end of this section, you will be able to:

- Use negatives and opposites
- Simplify: expressions with absolute value
- Add integers
- Subtract integers

Use Negatives and Opposites

Our work so far has only included the counting numbers and the whole numbers. But if you have ever experienced a temperature below zero or accidentally overdrawn your checking account, you are already familiar with negative numbers. **Negative numbers** are numbers less than 0. The negative numbers are to the left of zero on the number line. See Figure 1.

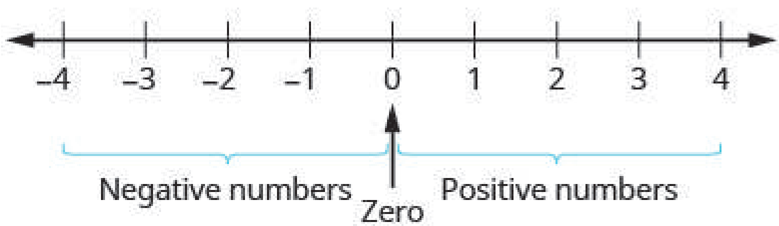


Figure 1 The number line shows the location of positive and negative numbers.

The arrows on the ends of the number line indicate that the numbers keep going forever. There is no biggest positive number, and there is no smallest negative number.

Is zero a positive or a negative number? Numbers larger than zero are positive, and numbers smaller than zero are negative. Zero is neither positive nor negative.

Consider how numbers are ordered on the number line. Going from left to right, the numbers increase in value. Going from right to left, the numbers decrease in value. See Figure 2.

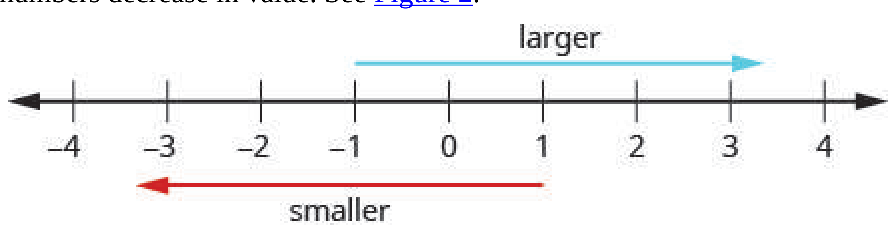


Figure 2 The numbers on a number line increase in value going from left to right and decrease in value going from right to left.

Remember that we use the notation:

a < *b* (read "a is less than b") when a is to the left of b on the number line.

a > *b* (read "*a* is greater than *b*") when *a* is to the right of *b* on the number line.

Now we need to extend the number line which showed the whole numbers to include negative numbers, too. The numbers marked by points in Figure 3 are called the integers. The integers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3\ldots$

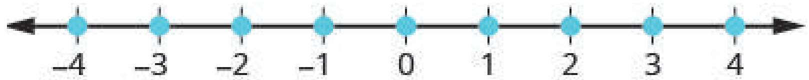
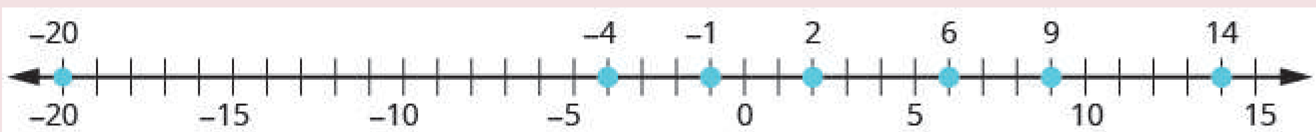


Figure 3 All the marked numbers are called integers.

EXAMPLE 1

Order each of the following pairs of numbers, using < or >: a) $14\_\_6$ b) $-1\_\_9$ c) $-1\_\_-4$ d) $2\_\_-20$.

Solution
It may be helpful to refer to the number line shown.



a) 14 is to the right of 6 on the number line.	$14\_\_6$ $14 > 6$
b) −1 is to the left of 9 on the number line.	$-1\_\_9$ $-1 < 9$
c) −1 is to the right of −4 on the number line.	$-1\_\_-4$
d) 2 is to the right of −20 on the number line.	$2\_\_-20$ $2 > -20$

TRY IT 1.1

Order each of the following pairs of numbers, using < or > : a) $15\_\_7$ b) $-2\_\_5$ c) $-3\_\_-7$ d) $5\_\_-17$.

Answer
a) > b) < c) > d) >

TRY IT 1.2

Order each of the following pairs of numbers, using < or > : a) $8\_\_13$ b) $3\_\_-4$ c) $-5\_\_-2$ d) $9\_\_-21$.

Answer
a) < b) > c) < d) >

You may have noticed that, on the number line, the negative numbers are a mirror image of the positive numbers, with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, they are called opposite**s**. The opposite of 2 is -2, and the opposite of -2 is 2

Opposite

The **opposite** of a number is the number that is the same distance from zero on the number line but on the opposite side of zero.

(Figure 4) illustrates the definition.

The opposite of 3 is -3.

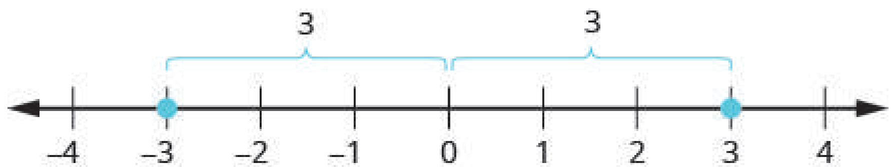


Figure 4

Sometimes in algebra the same symbol has different meanings. Just like some words in English, the specific meaning becomes clear by looking at how it is used. You have seen the symbol "–" used in three different ways.

$10-4$ Between two numbers, it indicates the operation of subtraction. We read $10-4$ as "10 minus 4."

-8 In front of a number, it indicates a negative number. We read -8 as "negative eight."

$-x$ In front of a variable, it indicates the opposite. We read $-x$ as "the opposite of x."

$-(-2)$ Here there are two "$-$" signs. The one in the parentheses tells us the number is negative 2. The one outside the parentheses tells us to take the opposite of -2. We read $-(-2)$ as "the opposite of negative two."

$10-4$	Between two numbers, it indicates the operation of *subtraction*. We read $10-4$ as "10 minus 4."
-8	In front of a number, it indicates a *negative* number. We read −8 as "negative eight."
$-x$	In front of a variable, it indicates the *opposite*. We read $-x$ as "the opposite of x."
$-(-2)$	Here there are two "–" signs. The one in the parentheses tells us the number is negative 2. The one outside the parentheses tells us to take the *opposite* of −2. We read $-(-2)$ as "the opposite of negative two."

Opposite Notation

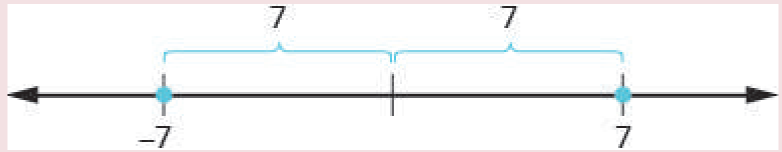
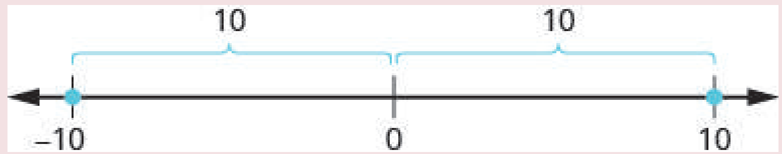
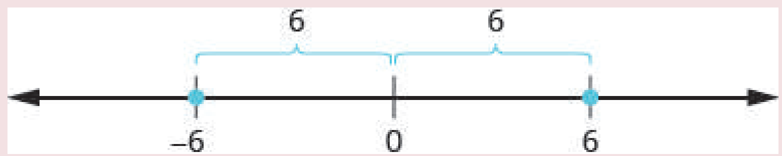
$-a$ means the opposite of the number *a*.

The notation $-a$ is read as "the opposite of *a*."

EXAMPLE 2

Find: a) the opposite of 7 b) the opposite of -10 c) $-(-6)$.

Solution

a) −7 is the same distance from 0 as 7, but on the opposite side of 0.	 The opposite of 7 is −7.
b) 10 is the same distance from 0 as −10, but on the opposite side of 0.	 The opposite of −10 is 10.
c) −(−6)	 The opposite of −(−6) is −6.

TRY IT 2.1

Find: a) the opposite of 4 b) the opposite of -3 c) $-(-1)$.

Answer
a)-4 b) 3 c) 1

TRY IT 2.2

Find: a) the opposite of 8 b) the opposite of -5 c) $-(-5)$.

Answer
a)-8 b) 5 c) 5

Our work with opposites gives us a way to define the integers.The whole numbers and their opposites are called the integers. The integers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3\ldots$

Integers

The whole numbers and their opposites are called the **integers**.

The integers are the numbers

$$\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$$

When evaluating the opposite of a variable, we must be very careful. Without knowing whether the variable represents a positive or negative number, we don't know whether $-x$ is positive or negative. We can see this in Example 3.

EXAMPLE 3

Evaluate a) $-x$, when $x = 8$ b) $-x$, when $x = -8$.

Solution

a.

To evaluate when *x* = 8 means to substitute 8 for *x*.	
	−*x*
Substitute 8 for *x*.	–(8)
Write the opposite of 8.	–8

b.

To evaluate when *x* = –8 means to substitute –8 for –*x*.	
	−*x*
Substitute –8 for *x*.	–(–8)
Write the opposite of −8.	8

TRY IT 3.1

Evaluate $-n$, when a) $n = 4$ b) $n = -4$.

Answer
a)-4 b) 4

TRY IT 3.2

Evaluate $-m$, when a) $m = 11$ b) $m = -11$.

Answer
a)-11 b) 11

Simplify: Expressions with Absolute Value

We saw that numbers such as $2 \text{ and } -2$ are opposites because they are the same distance from 0 on the number line. They are both two units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number.

Absolute Value

The absolute value of a number is its distance from 0 on the number line.

The absolute value of a number *n* is written as $|n|$.

For example,

- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.

Figure 5 illustrates this idea.

The integers 5 and are 5 units away from 0.

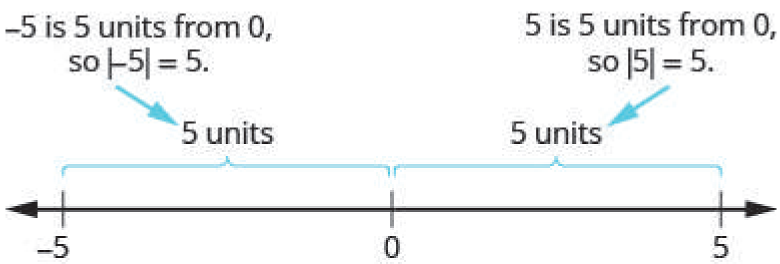


Figure 5

The absolute value of a number is never negative (because distance cannot be negative). The only number with absolute value equal to zero is the number zero itself, because the distance from 0 to 0 on the number line is zero units.

Property of Absolute Value

$|n| \geq 0$ for all numbers

Absolute values are always greater than or equal to zero!

Mathematicians say it more precisely, "absolute values are always non-negative." Non-negative means greater than or equal to zero.

EXAMPLE 4

Simplify: a) $|3|$ b) $|-44|$ c) $|0|$ d) $-|-44|$.

Solution
The absolute value of a number is the distance between the number and zero. Distance is never negative, so the absolute value is never negative.

a) $|3|$
3

b) $|-44|$
44

c) $|0|$
0

d) $|-44|$

We know that $|-44|=44$
Therefore, $-(44)=-44$

TRY IT 4.1

Simplify: a) $|4|$ b) $|-28|$ c) $|0|$ d) $-|-5|$.

Answer
a) 4 b) 28 c) 0 d) -5

TRY IT 4.2

Simplify: a) $|-13|$ b) $|47|$ c) $|0|$ d) $-|-17|$.

Answer
a) 13 b) 47 c) 0 d) -17

In the next example, we'll order expressions with absolute values. Remember, positive numbers are always greater than negative numbers!

EXAMPLE 5

Fill in <, >, $\text{or} \ =$ for each of the following pairs of numbers:

a) $|-5|$___$-|-5|$ b) 8___$-|-8|$c) -9___$-|-9|$ d) –(-16)___$-|-16|$

Solution

	$\lvert -5\rvert$___-$\lvert -5\rvert$
a) Simplify. Order.	5 ___ -5
	5 > -5
	$\lvert -5\rvert$ > -$\lvert -5\rvert$
b) Simplify. Order.	8 ___ -$\lvert -8\rvert$
	8 ___-8
	8 > -8
	8 > -$\lvert -8\rvert$
c) Simplify. Order.	-9 ___ -$\lvert -9\rvert$ -9 ___ -9 $-9 = -9$ -9 = -$\lvert -9\rvert$
d) Simplify. Order.	-(-16) ___ -$\lvert -16\rvert$ 16 ____ -16 16 > -16 -(-16) > -$\lvert -16\rvert$

TRY IT 5.1

Fill in <, >, or $=$ for each of the following pairs of numbers: a) $\lvert -9\rvert$___-$\lvert -9\rvert$ b) 2___-$\lvert -2\rvert$ c) -8___$\lvert -8\rvert$ d) (-9)___$-\lvert -9\rvert$.

Answer
a) > b) > c) < d) >

TRY IT 5.2

Fill in <, >, or $=$ for each of the following pairs of numbers: a) 7___-$\lvert -7\rvert$ b) $(-10$___-$\lvert -10\rvert$ c) $\lvert -4\rvert$___$-\lvert -4\rvert$ d) -1___$\lvert -1\rvert$.

Answer
a) > b) > c) > d) <

We now add absolute value bars to our list of grouping symbols. When we use the order of operations, first we simplify inside the absolute value bars as much as possible, then we take the absolute value of the resulting number.

Grouping Symbols

Parentheses	( )
Brackets	[ ]
Braces	{ }
Absolute value	| |

In the next example, we simplify the expressions inside absolute value bars first, just like we do with parentheses.

EXAMPLE 6

Simplify: $24 - \left|19 - 3\left(6 - 2\right)\right|$.

Solution

	$24 - \left|19 - 3\left(6 - 2\right)\right|$
Work inside parentheses first: subtract 2 from 6.	$24 - \left|19 - 3\left(4\right)\right|$
Multiply 3(4).	$24 - \left|19 - 12\right|$
Subtract inside the absolute value bars.	$24 - \left|7\right|$
Take the absolute value.	$24 - 7$
Subtract.	17

TRY IT 6.1

Simplify: $19 - \left|11 - 4\left(3 - 1\right)\right|$.

Answer
16

TRY IT 6.2

Simplify: $9 - \left|8 - 4\left(7 - 5\right)\right|$.

Answer
9

EXAMPLE 7

Evaluate: a) $|x|$ when $x = -35$ b) $|-y|$ when $y = -20$ c) $-|u|$ when $u = 12$ d) $-|p|$ when $p = -14$
.

Solution
a)$|x|$ when $x = -35$

	$\|x\|$
Substitute −35 for *x*.	$\|-35\|$
Take the absolute value.	35

b) $\|-y\|$ when $y = -20$

	$\|-y\|$
Substitute −20 for *y*.	$\|-(-20)\|$
Simplify.	$\|20\|$
Take the absolute value.	20

c) $-\|u\|$ when $u = 12$

	$-\|u\|$
Substitute 12 for *u*.	$-\|12\|$
Take the absolute value.	-12

d) $-\|p\|$ when $p = -14$

	$-\|p\|$
Substitute −14 for *p*.	$-\|-14\|$
Take the absolute value.	-14

TRY IT 7.1

Evaluate: a) $|x|$ when $x = -17$ b) $|-y|$ when $y = -39$ c) $-|m|$ when $m = 22$ d) $-|p|$ when $p = -11$.

Answer
a) 17 b) 39 c) -22 d) -11

TRY IT 7.2

Evaluate: a) $|y|$ when $y = -23$ b) $|-y|$ when $y = -21$ c) $-|n|$ when $n = 37$ d) $-|q|$ when $q = -49$.

Answer

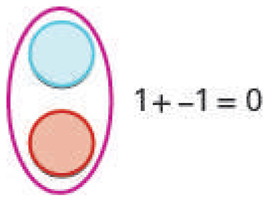
a) 23 b) 21 c) -37 d) -49

Add Integers

Most students are comfortable with the addition and subtraction facts for positive numbers. But doing addition or subtraction with both positive and negative numbers may be more challenging.

We will use two colour counters to model addition and subtraction of negatives so that you can visualize the procedures instead of memorizing the rules.

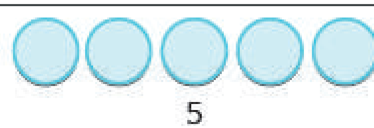
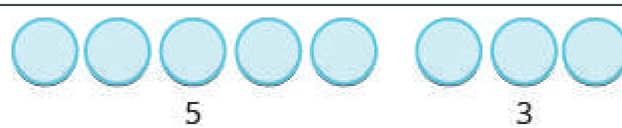
We let one colour (blue) represent positive. The other colour (red) will represent the negatives. If we have one positive counter and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero.



We will use the counters to show how to add the four addition facts using the numbers $5, -5$ and $3, -3$.

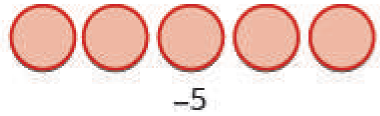
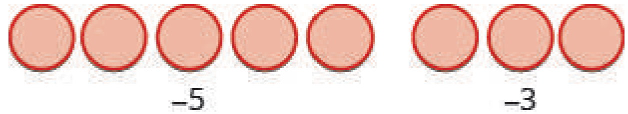
$5+3$ $\qquad -5+(-3)$ $\qquad -5+3$ $\qquad 5+(-3)$

To add $5+3$, we realize that $5+3$ means the sum of 5 and 3

We start with 5 positives.	
And then we add 3 positives.	
We now have 8 positives. The sum of 5 and 3 is 8.	

Now we will add $-5+(-3)$. Watch for similarities to the last example $5+3=8$.

To add $-5+(-3)$, we realize this means the sum of $-5 \text{ and } -3$.

We start with 5 negatives.	
And then we add 3 negatives.	
We now have 8 negatives. The sum of −5 and −3 is −8.	

In what ways were these first two examples similar?

- The first example adds 5 positives and 3 positives—both positives.
- The second example adds 5 negatives and 3 negatives—both negatives.

In each case we got 8—either 8 positives or 8 negatives.

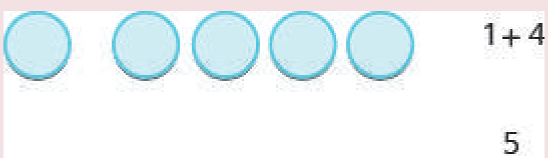
When the signs were the same, the counters were all the same color, and so we added them.

EXAMPLE 8

Add: a) $1+4$ b) $-1+(-4)$.

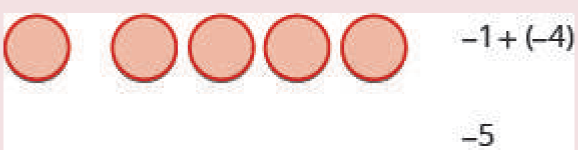
Solution

a)



1 positive plus 4 positives is 5 positives.

b)



1 negative plus 4 negatives is 5 negatives.

TRY IT 8.1

Add: a) $2+4$ b) $-2+(-4)$.

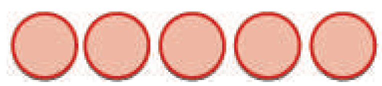
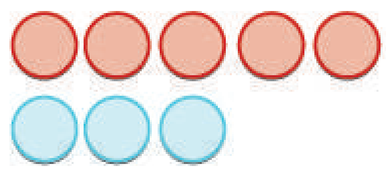
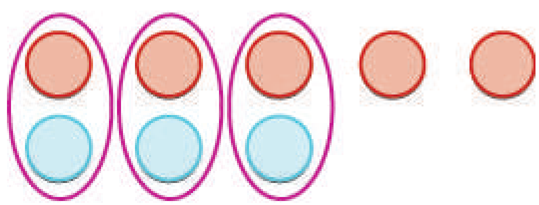
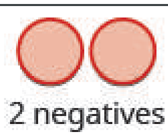
Answer
a) 6 b) -6

TRY IT 8.2

Add: a) $2+5$ b) $-2+(-5)$.

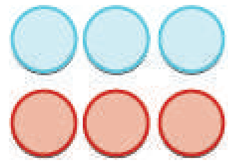
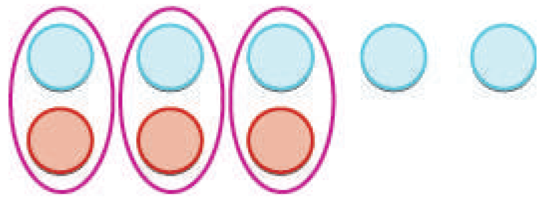
Answer
a) 7 b) -7

So what happens when the signs are different? Let's add $-5+3$. We realize this means the sum of -5 and 3. When the counters were the same color, we put them in a row. When the counters are a different color, we line them up under each other.

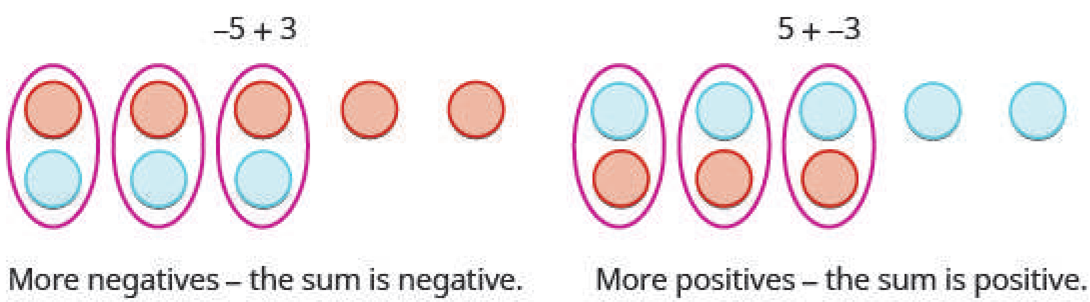
	−5 + 3 means the sum of −5 and 3.
We start with 5 negatives.	
And then we add 3 positives.	
We remove any neutral pairs.	
We have 2 negatives left.	
The sum of −5 and 3 is −2.	−5 + 3 = −2

Notice that there were more negatives than positives, so the result was negative.

Let's now add the last combination, $5+(-3)$.

	5 + (−3) means the sum of 5 and −3.
We start with 5 positives.	
And then we add 3 negatives.	
We remove any neutral pairs.	
We have 2 positives left.	
The sum of 5 and −3 is 2.	5 + (−3) = 2

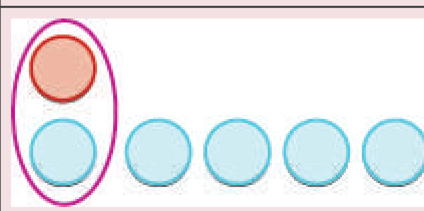
When we use counters to model addition of positive and negative integers, it is easy to see whether there are more positive or more negative counters. So we know whether the sum will be positive or negative.



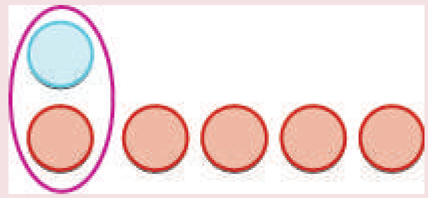
EXAMPLE 9

Add: a) $-1+5$ b) $1+(-5)$.

Solution
a)

	−1 + 5
	
There are more positives, so the sum is positive.	4

b)

	$1 + (-5)$
	
There are more negatives, so the sum is negative.	-4

TRY IT 9.1

Add: a) $-2 + 4$ b) $2 + (-4)$.

Answer
a) 2 b) -2

TRY IT 9.2

Add: a) $-2 + 5$ b) $2 + (-5)$.

Answer
a) 3 b) -3

Now that we have added small positive and negative integers with a model, we can visualize the model in our minds to simplify problems with any numbers.

When you need to add numbers such as $37 + (-53)$, you really don't want to have to count out 37 blue counters and 53 red counters. With the model in your mind, can you visualize what you would do to solve the problem?

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more red (negative) counters than blue (positive) counters, the sum would be *negative*. How many more red counters would there be? Because $53 - 37 = 16$, there are 16 more red counters.

Therefore, the sum of $37 + (-53)$ is -16.
$37 + (-53) = -16$
Let's try another one. We'll add $-74 + (-27)$. Again, imagine 74 red counters and 27 more red counters, so we'd have 101 red counters. This means the sum is -101.

$$-74 + (-27) = -101$$

Let's look again at the results of adding the different combinations of $5, -5$ and $3, -3$.

Addition of Positive and Negative Integers

$5+3$	$-5+(-3)$
8	-8
both positive, sum positive	both negative, sum negative

When the signs are the same, the counters would be all the same color, so add them.

$-5+3$	$5+(-3)$
-2	2
different signs, more negatives, sum negative	different signs, more positives, sum positive

When the signs are different, some of the counters would make neutral pairs, so subtract to see how many are left.

Visualize the model as you simplify the expressions in the following examples.

EXAMPLE 10

Simplify: a) $19+(-47)$ b) $-14+(-36)$.

Solution

a. Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

$$19+(-47)$$

Add. $\quad -28$

b. Since the signs are the same, we add. The answer will be negative because there are only negatives.

$$-14+(-36)$$

Add. $\quad -50$

TRY IT 10.1

Simplify: a) $-31+(-19)$ b) $15+(-32)$.

Answer
a)-50 b)-17

TRY IT 10.2

Simplify: a) $-42+(-28)$ b) $25+(-61)$.

Answer
a)-70 b)-36

The techniques used up to now extend to more complicated problems, like the ones we've seen before. Remember to follow the order of operations!

EXAMPLE 11

Simplify: $-5+3(-2+7)$.

Solution

	$-5+3(-2+7)$
Simplify inside the parentheses.	$-5+3(5)$
Multiply.	$-5+15$
Add left to right.	10

TRY IT 11.1

Simplify: $-2+5(-4+7)$.

Answer
13

TRY IT 11.2

Simplify: $-4+2(-3+5)$.

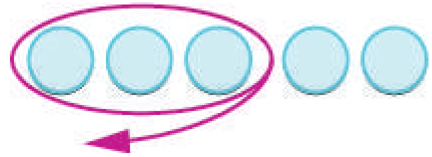
Answer
0

Subtract Integers

We will continue to use counters to model the subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative numbers.

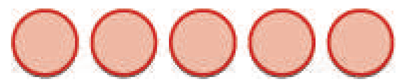
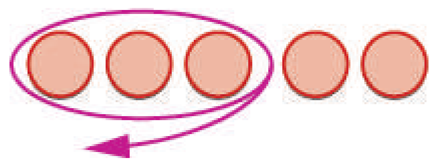
Perhaps when you were younger, you read "$5-3$" as "5 take away 3." When you use counters, you can think of subtraction the same way!

We will model the four subtraction facts using the numbers 5 and 3.
$5-3$ $\quad$ $-5-(-3)$ $\quad$ $-5-3$ $\quad$ $5-(-3)$
To subtract $5-3$, we restate the problem as "5 take away 3."

We start with 5 positives.	
We 'take away' 3 positives.	
We have 2 positives left.	
The difference of 5 and 3 is 2.	2

Now we will subtract $-5-(-3)$. Watch for similarities to the last example $5-3=2$.

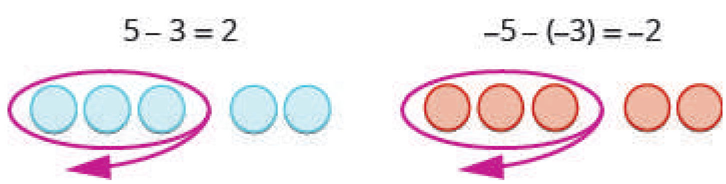
To subtract $-5-(-3)$, we restate this as "-5 take away -3"

We start with 5 negatives.	
We 'take away' 3 negatives.	
We have 2 negatives left.	
The difference of −5 and −3 is −2.	−2

Notice that these two examples are much alike: The first example, we subtract 3 positives from 5 positives and end up with 2 positives.

In the second example, we subtract 3 negatives from 5 negatives and end up with 2 negatives.

Each example used counters of only one color, and the "take away" model of subtraction was easy to apply.



EXAMPLE 12

Subtract: a) $7-5$ b) $-7-(-5)$.

Solution

a) Take 5 positive from 7 positives and get 2 positives.	$7-5$ 2
b) Take 5 negatives from 7 negatives and get 2 negatives.	$-7-(-5)$ -2

TRY IT 12.1

Subtract: a) $6-4$ b) $-6-(-4)$.

Answer
a) 2 b) -2

TRY IT 12.2

Subtract: a) $7-4$ b) $-7-(-4)$.

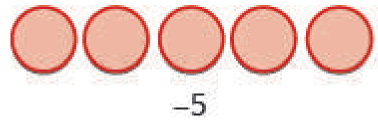
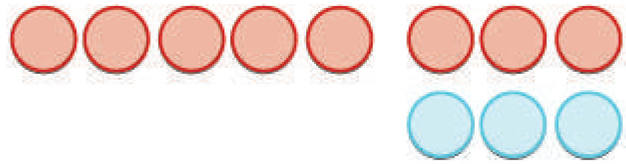
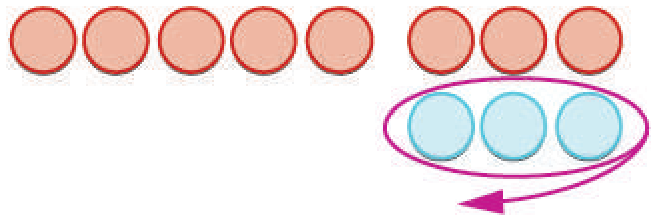
Answer
a) 3 b) -3

What happens when we have to subtract one positive and one negative number? We'll need to use both white and red counters as well as some neutral pairs. Adding a neutral pair does not change the value. It is like changing quarters to nickels—the value is the same, but it looks different.

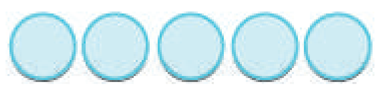
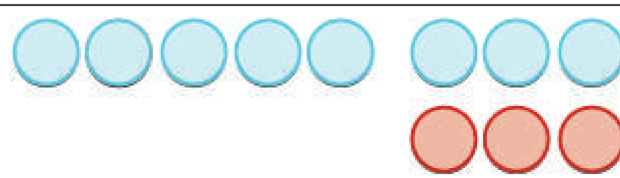
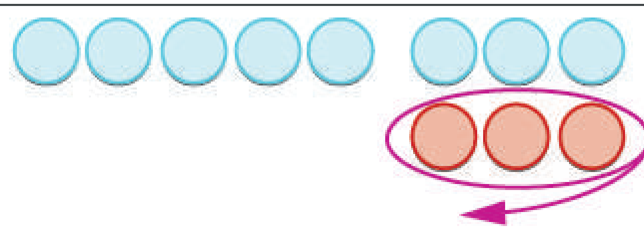
- To subtract $-5-3$, we restate it as -5 take away 3.

We start with 5 negatives. We need to take away 3 positives, but we do not have any positives to take away.

Remember, a neutral pair has value zero. If we add 0 to 5 its value is still 5. We add neutral pairs to the 5 negatives until we get 3 positives to take away.

	−5 − 3 means −5 take away 3.
We start with 5 negatives.	
We now add the neutrals needed to get 3 positives.	
We remove the 3 positives.	
We are left with 8 negatives.	
The difference of −5 and 3 is −8.	−5 − 3 = −8

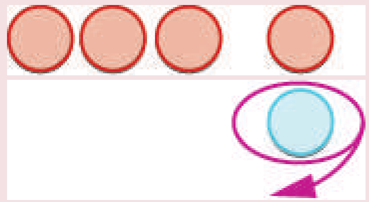
And now, the fourth case, $5-(-3)$. We start with 5 positives. We need to take away 3 negatives, but there are no negatives to take away. So we add neutral pairs until we have 3 negatives to take away.

	5 − (−3) means 5 take away −3.
We start with 5 positives.	
We now add the needed neutrals pairs.	
We remove the 3 negatives.	
We are left with 8 positives.	
The difference of 5 and −3 is 8.	5 − (−3) = 8

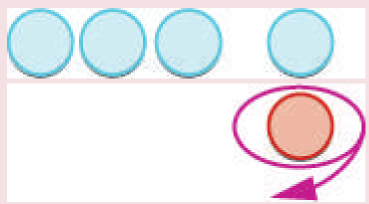
EXAMPLE 13

Subtract: a) $-3-1$ b) $3-(-1)$.

Solution
a)

Take 1 positive from the one added neutral pair.		−3 − 1 −4

b)

Take 1 negative from the one added neutral pair.		3 − (−1) 4

TRY IT 13.1

Subtract: a) $-6-4$ b) $6-(-4)$.

Answer
a)-10 b) 10

TRY IT 13.2

Subtract: a) $-7-4$ b) $7-(-4)$.

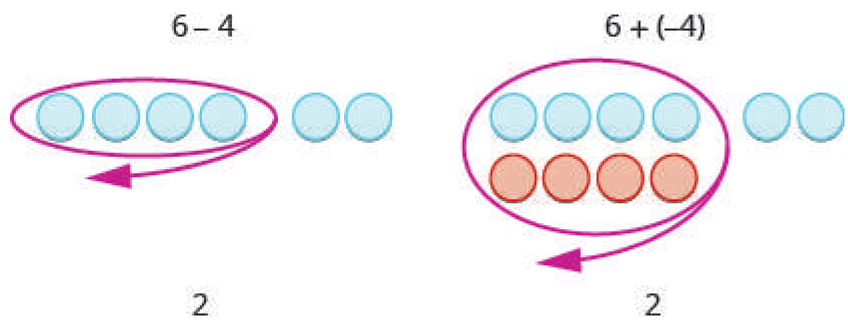
Answer
a)-11 b) 11

Have you noticed that *subtraction of signed numbers can be done by adding the opposite*? In Example 13, $-3-1$ is the same as $-3+(-1)$ and $3-(-1)$ is the same as $3+1$. You will often see this idea, the subtraction property, written as follows:

Subtraction Property

$a-b=a+(-b)$
Subtracting a number is the same as adding its opposite.

Look at these two examples.



$6 - 4$ gives the same answer as $6 + (-4)$.

Of course, when you have a subtraction problem that has only positive numbers, like $6 - 4$, you just do the subtraction. You already knew how to subtract $6 - 4$ long ago. But *knowing* that $6 - 4$ gives the same answer as $6 + (-4)$ helps when you are subtracting negative numbers. Make sure that you understand how $6 - 4$ and $6 + (-4)$ give the same results!

EXAMPLE 14

Simplify: a) $13 - 8$ and $13 + (-8)$ b) $-17 - 9$ and $-17 + (-9)$.

Solution

a) Subtract.	$13 - 8$ 5	$13 + (-8)$ 5
b) Subtract.	$-17 - 9$ -29	$-17 + (-9)$ -26

TRY IT 14.1

Simplify: a) $21 - 13$ and $21 + (-13)$ b) $-11 - 7$ and $-11 + (-7)$.

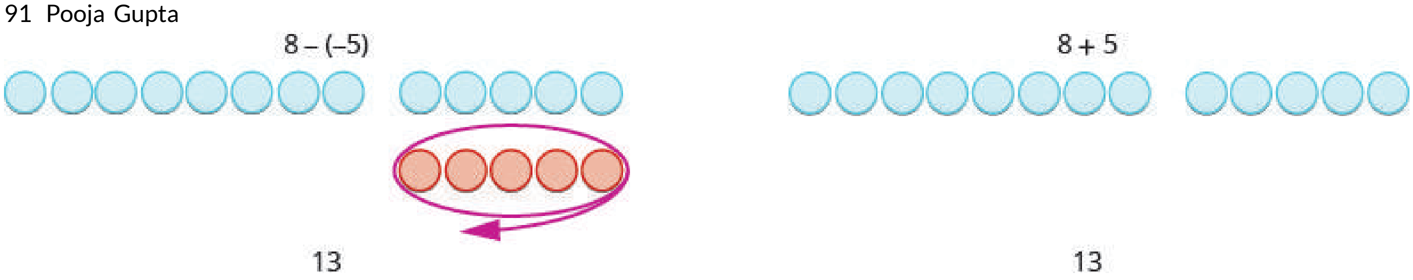
Answer
a) 8 b) -18

TRY IT 14.2

Simplify: a) $15 - 7$ and $15 + (-7)$ b) $-14 - 8$ and $-14 + (-8)$.

Answer
a) 8 b)-22

Look at what happens when we subtract a negative.



$8 - (-5)$ gives the same answer as $8 + 5$

Subtracting a negative number is like adding a positive!

You will often see this written as $a - (-b) = a + b$.

Does that work for other numbers, too? Let's do the following example and see.

EXAMPLE 15

Simplify: a) $9 - (-15)$ and $9 + 15$ b) $-7 - (-4)$ and $-7 + 4$.

Solution

a)

	$9 - (-15)$	$9 + 15$
Subtract.	24	24

b)

	$-7 - (-4)$	$-7 + 4$
Subtract.	-3	-3

a) Subtract.	$9 - (-15)$ 24	$9 + 15$ 24
b) Subtract.	$-7 - (-4)$ -3	$-7 + 4$ -3

TRY IT 15.1

Simplify: a) $6 - (-13)$ and $6 + 13$ b) $-5 - (-1)$ and $-5 + 1$.

Answer

a)19 b)-4

TRY IT 15.2

Simplify: a) $4 - (-19)$ and $4 + 19$ b) $-4 - (-7)$ and $-4 + 7$.

Answer

a) 23 b) 3

Let's look again at the results of subtracting the different combinations of $5, -5$ and $3, -3$.

Subtraction of Integers

$5 - 3$	$-5 - (-3)$
2	-2
5 positives take away 3 positives	5 negatives take away 3 negatives
2 positives	2 negatives

When there would be enough counters of the colour to take away, subtract.

$-5 - 3$	$5 - (-3)$
-8	8
5 negatives, want to take away 3 positives	5 positives, want to take away 3 negatives
need neutral pairs	need neutral pairs

When there would be not enough counters of the colour to take away, add.

What happens when there are more than three integers? We just use the order of operations as usual.

EXAMPLE 16

Simplify: $7 - (-4 - 3) - 9$.

Solution

	$7 - (-4 - 3) - 9$
Simplify inside the parentheses first.	$7 - (-7) - 9$
Subtract left to right.	$14 - 9$
Subtract.	5

TRY IT 16.1

Simplify: $8 - (-3 - 1) - 9$.

Answer
3

TRY IT 16.2

Simplify: $12 - (-9 - 6) - 14$.

Answer
13

Access these online resources for additional instruction and practice with adding and subtracting integers. You will need to enable Java in your web browser to use the applications.

- Add Colored Chip
- Subtract Colored Chip

Key Concepts

- **Addition of Positive and Negative Integers**

$5+3$ 8 both positive, sum positive	$-5+(-3)$ -8 both negative, sum negative
$-5+3$ -2 different signs, more negatives sum negative	$5+(-3)$ 2 different signs, more positives sum positive

- **Property of Absolute Value**: $|n| \geq 0$ for all numbers. Absolute values are always greater than or equal to zero!
- **Subtraction of Integers**

$5-3$ 2 5 positives take away 3 positives 2 positives	$-5-(-3)$ -2 5 negatives take away 3 negatives 2 negatives
$-5-3$ -8 5 negatives, want to subtract 3 positives need neutral pairs	$5-(-3)$ 8 5 positives, want to subtract 3 negatives need neutral pairs

- **Subtraction Property:** Subtracting a number is the same as adding its opposite.

Glossary

absolute value
The absolute value of a number is its distance from 0 on the number line. The absolute value of a number n is written as $|n|$.

integers
The whole numbers and their opposites are called the integers: …−3, −2, −1, 0, 1, 2, 3…

opposite
The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of

zero: $-a$ means the opposite of the number. The notation $-a$ is read “the opposite of a.”

Practice Makes Perfect

Use Negatives and Opposites of Integers

In the following exercises, order each of the following pairs of numbers, using < or >.

1. a) 9___4 b) -3___6 c) -8___-2 d) 1___-10	2. a) -7___3 b) -10___-5 c) 2___-6 d) 8___9

In the following exercises, find the opposite of each number.

3. a) 2 b) -6	4. a) 9 b) -4

In the following exercises, simplify.

5. $-(-4)$	6. $-(-8)$
7. $-(-15)$	8. $-(-11)$

In the following exercises, evaluate.

9. $-c$ when a) $c = 12$ b) $c = -12$	10. $-d$ when a) $d = 21$ b) $d = -21$

Simplify Expressions with Absolute Value

In the following exercises, simplify.

11. a) $\|-32\|$ b) $\|0\|$ c) $\|16\|$ d) $-\|-23\|$	12. a) $\|0\|$ b) $\|-40\|$ c) $\|22\|$ d)$-\|-34\|$

In the following exercises, fill in <, >, or $=$ for each of the following pairs of numbers.

13. a) -6___$\|-6\|$ b) $-\|-3\|$___-3	14. a) $\|-5\|$___$-\|-5\|$ b) 9___$-\|-9\|$

In the following exercises, simplify.

15. $-(-5)$ and $-\|-5\|$	16. $-\|-9\|$ and $-(-9)$
17. $8\|-7\|$	18. $5\|-5\|$
19. $\|15-7\|-\|14-6\|$	20. $\|17-8\|-\|13-4\|$
21. $18-\|2(8-3)\|$	22. $18-\|3(8-5)\|$

In the following exercises, evaluate.

23. a) $-\|p\|$ when $p=19$ b) $-\|q\|$ when $q=-33$	24. a) $-\|a\|$ when $a=60$ b) $-\|b\|$ when $b=-12$

Add Integers

In the following exercises, simplify each expression.

25. $-21+(-59)$	26. $-35+(-47)$
27. $48+(-16)$	28. $34+(-19)$
29. $-14+(-12)+4$	30. $-17+(-18)+6$
31. $135+(-110)+83$	32. 6 $-38+27+(-8)+126$
33. $19+2(-3+8)$	34. $24+3(-5+9)$

Subtract Integers

In the following exercises, simplify.

35. $8-2$	36. $-6-(-4)$
37. $-5-4$	38. $-7-2$
39. $8-(-4)$	40. $7-(-3)$
41. a) $44-28$ b) $44+(-28)$	42. a) $35-16$ b) $35+(-16)$
43. a) $27-(-18)$ b) $27+18$	44. a) $46-(-37)$ b) $46+37$

In the following exercises, simplify each expression.

45. $15-(-12)$	46. $14-(-11)$
47. $48-87$	48. $45-69$
49. $-17-42$	50. $-19-46$
51. $-103-(-52)$	52. $-105-(-68)$
53. $-45-(54)$	54. $-58-(-67)$
55. $8-3-7$	56. $9-6-5$
57. $-5-4+7$	58. $-3-8+4$
59. $-14-(-27)+9$	60. $64+(-17)-9$
61. $(2-7)-(3-8)(2)$	62. $(1-8)-(2-9)$
63. $-(6-8)-(2-4)$	64. $-(4-5)-(7-8)$
65. $25-[10-(3-12)]$	66. $32-[5-(15-20)]$
67. $6.3-4.3-7.2$	68. $5.7-8.2-4.9$
69. 5^2-6^2	70. 6^2-7^2

Everyday Math

71. **Elevation** The highest elevation in North America is Mount McKinley, Alaska, at 20,320 feet above sea level. The lowest elevation is Death Valley, California, at 282 feet below sea level. Use integers to write the elevation of: a) Mount McKinley. b) Death Valley.	72. **Extreme temperatures** The highest recorded temperature on Earth was $58°$ Celsius, recorded in the Sahara Desert in 1922. The lowest recorded temperature was $90°$ below $0°$ Celsius, recorded in Antarctica in 1983 Use integers to write the: a) highest recorded temperature. b) lowest recorded temperature.
73. **Provincial budgets** For 2019 the province of Quebec estimated it would have a budget surplus of $5.6 million. That same year, Alberta estimated it would have a budget deficit of $7.5 million. Use integers to write the budget of: a) Quebec. b) Alberta.	74. **University enrolments**The number of international students enrolled in Canadian postsecondary institutions has been on the rise for two decades, with their numbers increasing at a higher rate than that of Canadian students. Enrolments of international students rose by 24,315 from 2015 to 2017. Meanwhile, there was a slight decline in the number of Canadian students, by 912 for the same fiscal years. Use integers to write the change: a) in International Student enrolment from Fall 2015 to Fall 2017. b) in Canadian student enrolment from Fall 2015 to Fall 2017.
75. **Stock Market** The week of September 15, 2008 was one of the most volatile weeks ever for the US stock market. The closing numbers of the Dow Jones Industrial Average each day were: Monday: -504 Tuesday: $+142$ Wednesday: -449 Thursday: $+410$ Friday: $+369$ What was the overall change for the week? Was it positive or negative?	76. **Stock Market** During the week of June 22, 2009, the closing numbers of the Dow Jones Industrial Average each day were: Monday: -201 Tuesday: -16 Wednesday: -23 Thursday: $+172$ Friday: -34 What was the overall change for the week? Was it positive or negative?

Writing Exercises

77. Give an example of a negative number from your life experience.	78. What are the three uses of the “ $-$ ” sign in algebra? Explain how they differ.
79. Explain why the sum of -8 and 2 is negative, but the sum of 8 and -2 is positive.	80. Give an example from your life experience of adding two negative numbers.

Answers

1. a) > b) < c) < d) >	3. a) -2 b) 6	5. 4
7. 15	9. a) -12 b) 12	11. a) 32 b) 0 c) 16 d) -23
13. a) < b) $=$	15. $5, -5$	17. 56
19. 0	21. 8	23. a) -19 b) -33
25. -80	27. 32	29. -22
31. 108	33. 29	35. 6
37. -9	39. 12	41. a) 16 b) 16
43. a) 45 b) 45	45. 27	47. -39
49. -59	51. -51	53. -99
55. -2	57. -2	59. 22
61. 5	63. 0	65. 6
67. -5.2	69. -11	71. a) 20,320 b) -282
73. a) $5.6 million b) $-\$7.5$ million	75. -32	77. Answers may vary
79. Answers may vary		

Attributions

1.5 Multiply and Divide Integers

Learning Objectives

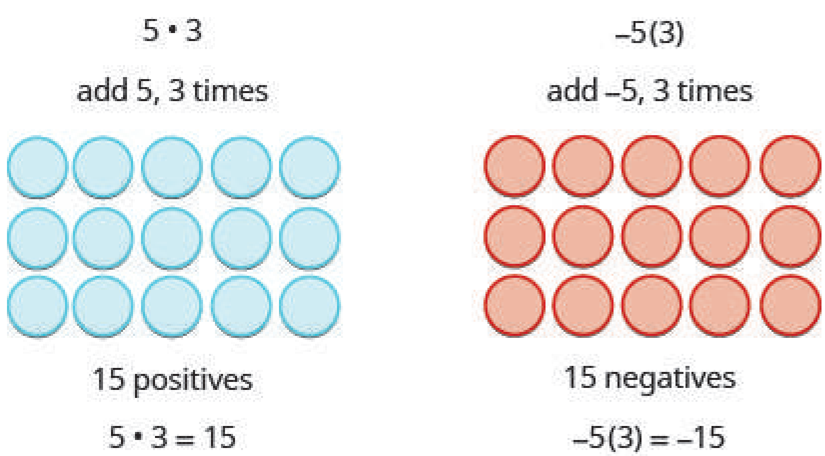
By the end of this section, you will be able to:

- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate English phrases to algebraic expressions
- Use integers in applications

Multiply Integers

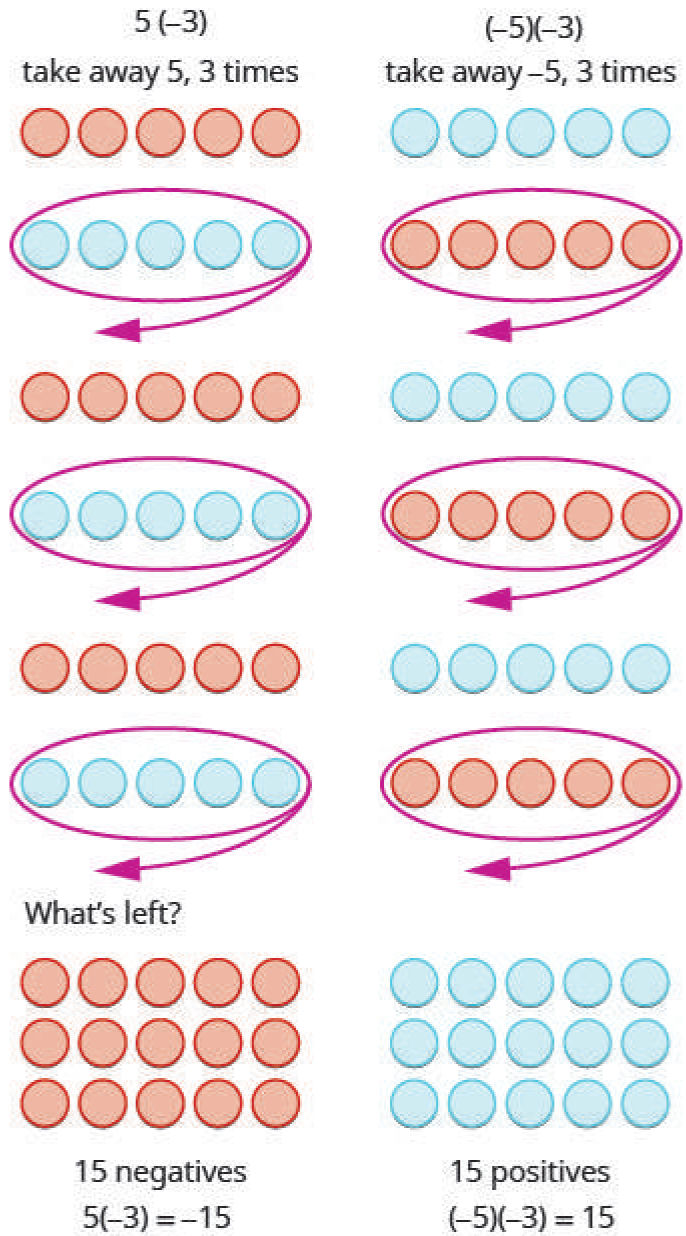
Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show multiplication of integers. Let's look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction. Here, we will use the model just to help us discover the pattern.

We remember that $a \cdot b$ means add *a*, *b* times. Here, we are using the model just to help us discover the pattern.



The next two examples are more interesting.

What does it mean to multiply 5 by -3? It means subtract 5, 3 times. Looking at subtraction as "taking away," it means to take away 5, 3 times. But there is nothing to take away, so we start by adding neutral pairs on the workspace. Then we take away 5 three times.



In summary:

$$\begin{aligned} 5\cdot 3 &= 15 & -5\,(3) &= -15 \\ 5\,(-3) &= -15 & (-5)\,(-3) &= 15 \end{aligned}$$

Notice that for multiplication of two signed numbers, when the:

- signs are the *same*, the product is *positive*.
- signs are *different*, the product is *negative*.

We'll put this all together in the chart below

Multiplication of Signed Numbers

For multiplication of two signed numbers:

Same signs	Product	Example
Two positives Two negatives	Positive Positive	$7 \cdot 4 = 28$ $-8(-6) = 48$

Different signs	Product	Example
Positive · Negative Negative · Positive	Negative Negative	$7(-9) = -63$ $-5 \cdot 10 = -50$

EXAMPLE 1

Multiply: a) $-9 \cdot 3$ b) $-2(-5)$ c) $4(-8)$ d) $7 \cdot 6$.

Solution

a) Multiply, noting that the signs are different so the product is negative.	$-9 \cdot 3$ -27
b) Multiply, noting that the signs are the same so the product is positive.	$-2(-5)$ 10
c) Multiply, with different signs.	$4(-8)$ -32
d) Multiply, with same signs.	$7 \cdot 6$ 42

TRY IT 1.1

Multiply: a) $-6 \cdot 8$ b) $-4(-7)$ c) $9(-7)$ d) $5 \cdot 12$.

Answer
a)-48 b) 28 c) -63 d) 60

TRY IT 1.2

Multiply: a) $-8 \cdot 7$ b) $-6(-9)$ c) $7(-4)$ d) $3 \cdot 13$.

Answer
a)-56 b) 54 c) -28 d) 39

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1? Let's multiply a positive number and then a negative number by -1 to see what we get.

Multiply.	$-1 \cdot 4$ -4 -4 is the opposite of 4.	$-1(-3)$ 3 3 is the opposite of -3.

Each time we multiply a number by -1, we get its opposite!

Multiplication by -1

$-1a =$ -a

Multiplying a number by -1 gives its opposite.

EXAMPLE 2

Multiply: a) $-1 \cdot 7$ b) $-1(-11)$.

Solution

a) Multiply, noting that the signs are different so the product is negative.	$-1 \cdot 7$ -7 -7 is the opposite of 7.
b) Multiply, noting that the signs are the same so the product is positive.	$-1(-11)$ 11 11 is the opposite of -11.

TRY IT 2.1

Multiply: a) $-1 \cdot 9$ b) $-1 \cdot (-17)$.

Answer
a)-9 b) 17

TRY IT 2.2

Multiply: a) $-1 \cdot 8$ b) $-1 \cdot (-16)$.

Answer
a)-8 b) 16

Divide Integers

What about division? Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $5 \cdot 3 = 15$. In words, this expression says that 15 can be divided into three groups of five each because adding five three times gives 15. Look at some examples of multiplying integers, to figure out the rules for dividing integers.

$$\begin{aligned} 5\cdot 3 &= 15 \text{ so } 15\div 3 &= 5 \qquad\qquad -5\,(3) &= -15 \text{ so } -15\div 3 &= -5 \\ (-5)\,(-3) &= 15 \text{ so } 15\div(-3) &= -5 \qquad\qquad 5\,(-3) &= -15 \text{ so } -15\div(-3) &= 5 \end{aligned}$$

Division follows the same rules as multiplication!

For division of two signed numbers, when the:

- signs are the *same*, the quotient is *positive*.
- signs are *different*, the quotient is *negative*.

And remember that we can always check the answer of a division problem by multiplying.

Multiplication and Division of Signed Numbers

For multiplication and division of two signed numbers:

- If the signs are the same, the result is positive.
- If the signs are different, the result is negative.

Same signs	**Result**
Two positives	Positive
Two negatives	Positive

If the signs are the same, the result is positive.

Different signs	**Result**
Positive and negative	Negative
Negative and positive	Negative

If the signs are different, the result is negative.

EXAMPLE 3

Divide: a) $-27\div 3$ b) $-100\div(-4)$.

Solution

a) Divide. With different signs, the quotient is negative.	$-27\div 3$ -9
b) Divide. With signs that are the same, the quotient is positive.	$-100\div(-4)$ 25

TRY IT 3.1

Divide: a) $-42 \div 6$ b) $-117 \div (-3)$.

Answer
a)-7 b) 39

TRY IT 3.2

Divide: a) $-63 \div 7$ b) $-115 \div (-5)$.

Answer
a)-9 b) 23

Simplify Expressions with Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember My Dear Aunt Sally?

Let's try some examples. We'll simplify expressions that use all four operations with integers—addition, subtraction, multiplication, and division. Remember to follow the order of operations.

EXAMPLE 4

Simplify: $7\left(-2\right)+4\left(-7\right)-6$.

Solution

	$7\left(-2\right)+4\left(-7\right)-6$
Multiply first.	$-14+\left(-28\right)-6$
Add.	$-42-6$
Subtract.	-48

TRY IT 4.1

Simplify: $8\left(-3\right)+5\left(-7\right)-4$.

Answer
-63

TRY IT 4.2

Simplify: $9\left(-3\right)+7\left(-8\right)-1$.

Answer
-84

EXAMPLE 5

Simplify: a) $(-2)^4$ b) -2^4.

Solution

a) Write in expanded form. Multiply. Multiply. Multiply.	$(-2)^4$ $(-2)(-2)(-2)(-2)$ $4(-2)(-2)$ $-8(-2)$ 16
b) Write in expanded form. We are asked to find the opposite of 2^4. Multiply. Multiply. Multiply.	-2^4 - $(2 \cdot 2 \cdot 2 \cdot 2)$ - $(4 \cdot 2 \cdot 2)$ - $(8 \cdot 2)$ -16

Notice the difference in parts a) and b). In part a), the exponent means to raise what is in the parentheses, the (-2) to the 4^{th} power. In part b), the exponent means to raise just the 2 to the 4^{th} power and then take the opposite.

TRY IT 5.1

Simplify: a) $(-3)^4$ b) -3^4.

Answer
a) 81 b) -81

TRY IT 5.2

Simplify: a) $(-7)^2$ b) -7^2.

Answer
a) 49 b) -49

The next example reminds us to simplify inside parentheses first.

EXAMPLE 6

Simplify: $12-3(9-12)$.

Solution

	$12 - 3(9 - 12)$
Subtract in parentheses first.	$12 - 3(-3)$
Multiply.	$12 - (-9)$
Subtract.	21

TRY IT 6.1

Simplify: $17 - 4(8 - 11)$.

Answer
29

TRY IT 6.2

Simplify: $16 - 6(7 - 13)$.

Answer
52

EXAMPLE 7

Simplify: $8(-9) \div (-2)^3$.

Solution

	$8(-9) \div (-2)^3$
Exponents first.	$8(-9) \div (-8)$
Multiply.	$-72 \div (-8)$
Divide.	9

TRY IT 7.1

Simplify: $12(-9) \div (-3)^3$.

Answer
4

TRY IT 7.2

Simplify: $18\left(-4\right) \div \left(-2\right)^3$.

Answer
9

EXAMPLE 8

Simplify: $-30 \div 2 + \left(-3\right)\left(-7\right)$.

Solution

	$-30 \div 2 + \left(-3\right)\left(-7\right)$
Multiply and divide left to right, so divide first.	$-15 + \left(-3\right)\left(-7\right)$
Multiply.	$-15 + 21$
Add.	6

TRY IT 8.1

Simplify: $-27 \div 3 + \left(-5\right)\left(-6\right)$.

Answer
21

TRY IT 8.2

Simplify: $-32 \div 4 + \left(-2\right)\left(-7\right)$.

Answer
6

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers.

EXAMPLE 9

When $n = -5$, evaluate: a) $n + 1$ b) -$n + 1$.

Solution
a)

	$n+1$
Substitute -5 for n.	$-5+1$
Simplify.	-4

b)

	$-n+1$
Substitute -5 for n.	$-(-5)+1$
Simplify.	$5+1$
Add.	6

TRY IT 9.1

When $n=-8$, evaluate a) $n+2$ b) -$n+2$.

Answer
a)-6 b) 10

TRY IT 9.2

When $y=-9$, evaluate a) $y+8$ b) -$y+8$.

Answer
a)-1 b) 17

EXAMPLE 10

Evaluate $(x+y)^2$ when $x=-18$ and $y=24$.

Solution

	$(x+y)^2$
Substitute -18 for x and 24 for y.	$(-18+24)^2$
Add inside parenthesis.	$(6)^2$
Simplify.	36

TRY IT 10.1

Evaluate $(x+y)^2$ when $x=-15$ and $y=29$.

Answer
196

TRY IT 10.2

Evaluate $(x+y)^3$ when $x=-8$ and $y=10$.

Answer
8

EXAMPLE 11

Evaluate $20-z$ when a) $z=12$ and b) $z=-12$.

Solution
a)

	20 – *z*
Substitute 12 for *z*.	20 – 12
Subtract.	8

b)

	20 – *z*
Substitute –12 for *z*.	20 – (–12)
Subtract.	32

TRY IT 11.1

Evaluate: $17-k$ when a) $k=19$ and b) $k=-19$.

Answer
a)-2 b) 36

TRY IT 11.2

Evaluate: $-5-b$ when a) $b=14$ and b) $b=-14$.

Answer

a) -19 b) 9

EXAMPLE 12

Evaluate: $2x^2 + 3x + 8$ when $x = 4$.

Solution
Substitute 4 for x. Use parentheses to show multiplication.

	$2x^2 + 3x + 8$
Substitute.	$2(4)^2 + 3(4) + 8$
Evaluate exponents.	$2(16) + 3(4) + 8$
Multiply.	$32 + 12 + 8$
Add.	52

TRY IT 12.1

Evaluate: $3x^2 - 2x + 6$ when $x = -3$.

Answer
39

TRY IT 12.2

Evaluate: $4x^2 - x - 5$ when $x = -2$.

Answer
13

Translate Phrases to Expressions with Integers

Our earlier work translating English to algebra also applies to phrases that include both positive and negative numbers.

EXAMPLE 13

Translate and simplify: the sum of 8 and -12, increased by 3

Solution

	the **sum** of 8 and -12, increased by 3.
Translate.	$[8+(-12)]+3$
Simplify. Be careful not to confuse the brackets with an absolute value sign.	$(-4)+3$
Add.	-1

TRY IT 13.1

Translate and simplify the sum of 9 and -16, increased by 4

Answer
$(9+(-16))+4-3$

TRY IT 13.2

Translate and simplify the sum of -8 and -12, increased by 7

Answer
$(-8+(-12))+7-13$

When we first introduced the operation symbols, we saw that the expression may be read in several ways. They are listed in the chart below.

$a-b$
a minus b the difference of a and b b subtracted from a b less than a

Be careful to get *a* and *b* in the right order!

EXAMPLE 14

Translate and then simplify a) the difference of 13 and -21 b) subtract 24 from -19.

Solution

a) Translate. Simplify.	the difference of 13 and -21 $13-(-21)$ 34
b) Translate. Remember, "subtract b from a means $a-b$. Simplify.	subtract 24 from -19 $-19-24$ -43

TRY IT 14.1

Translate and simplify a) the difference of 14 and -23 b) subtract 21 from -17.

Answer

a)$14-(-23)$; 37 b)$-17-21$; -38

TRY IT 14.2

Translate and simplify a) the difference of 11 and -19 b) subtract 18 from -11.

Answer

a)$11-(-19)$; 30 b)$-11-18$; -29

Once again, our prior work translating English to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the key word for multiplication is "product" and for division is "quotient."

EXAMPLE 15

Translate to an algebraic expression and simplify if possible: the product of -2 and 14

Solution

	the product of -2 and 14
Translate.	$(-2)(14)$
Simplify.	-28

TRY IT 15.1

Translate to an algebraic expression and simplify if possible: the product of -5 and 12

Answer

$-5(12)$; -60

TRY IT 15.2

Translate to an algebraic expression and simplify if possible: the product of 8 and -13.

Answer
$-8\,(13)\,;-104$

EXAMPLE 16

Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7.

Solution

	the quotient of -56 and -7
Translate.	$-56 \div (-7)$
Simplify.	8

TRY IT 16.1

Translate to an algebraic expression and simplify if possible: the quotient of -63 and -9.

Answer
$-63 \div (-9)\,;7$

TRY IT 16.2

Translate to an algebraic expression and simplify if possible: the quotient of -72 and -9.

Answer
$-72 \div (-9)\,;8$

Use Integers in Applications

We'll outline a plan to solve applications. It's hard to find something if we don't know what we're looking for or what to call it! So when we solve an application, we first need to determine what the problem is asking us to find. Then we'll write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

EXAMPLE 17

The temperature in Sparwood, British Columbia, one morning was 11 degrees. By mid-afternoon, the temperature had dropped to -9 degrees. What was the difference of the morning and afternoon temperatures?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the difference of the morning and afternoon temperatures
Step 3. Write a phrase the gives the information to find it.	the *difference of* 11 *and* –9
Step 4. Translate the phrase to an expression.	11 – (–9)
Step 5. Simplify the expression.	20
Step 6. Write a complete sentence that answers the question.	The difference in temperatures was 20 degrees.

TRY IT 17.1

The temperature in Whitehorse, Yukon, one morning was 15 degrees. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference in the morning and afternoon temperatures?

Answer
The difference in temperatures was 45 degrees.

TRY IT 17.2

The temperature in Quesnel, BC, was -6 degrees at lunchtime. By sunset the temperature had dropped to -15 degrees. What was the difference in the lunchtime and sunset temperatures?

Answer
The difference in temperatures was 9 degrees.

HOW TO: Apply a Strategy to Solve Applications with Integers

1. Read the problem. Make sure all the words and ideas are understood
2. Identify what we are asked to find.

3. Write a phrase that gives the information to find it.
4. Translate the phrase to an expression.
5. Simplify the expression.
6. Answer the question with a complete sentence.

EXAMPLE 18

The Mustangs football team received three penalties in the third quarter. Each penalty gave them a loss of fifteen yards. What is the number of yards lost?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the number of yards lost
Step 3. Write a phrase that gives the information to find it.	three times a 15-yard penalty
Step 4. Translate the phrase to an expression.	$3\left(-15\right)$
Step 5. Simplify the expression.	-45
Step 6. Answer the question with a complete sentence.	The team lost 45 yards.

TRY IT 18.1

The Bears played poorly and had seven penalties in the game. Each penalty resulted in a loss of 15 yards. What is the number of yards lost due to penalties?

Answer
The Bears lost 105 yards.

TRY IT 18.2

Bill uses the ATM on campus because it is convenient. However, each time he uses it he is charged a $2 fee. Last month he used the ATM eight times. How much was his total fee for using the ATM?

Answer
A $16 fee was deducted from his checking account.

Key Concepts

- **Multiplication and Division of Two Signed Numbers**
  - Same signs—Product is positive
  - Different signs—Product is negative
- **Strategy for Applications**

1. Identify what you are asked to find.
2. Write a phrase that gives the information to find it.
3. Translate the phrase to an expression.
4. Simplify the expression.
5. Answer the question with a complete sentence.

Practice Makes Perfect

Multiply Integers

In the following exercises, multiply.

1. $-4 \cdot 8$	2. $-3 \cdot 9$
3. $9\left(-7\right)$	4. $13\left(-5\right)$
5. $-1 \cdot 6$	6. $-1 \cdot 3$
7. $-1\left(-14\right)$	8. $-1\left(-19\right)$

Divide Integers

In the following exercises, divide.

9. $-24 \div 6$	10. $35 \div \left(-7\right)$
11. $-52 \div \left(-4\right)$	12. $-84 \div \left(-6\right)$
13. $-180 \div 15$	14. $-192 \div 12$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

15. $5(-6)+7(-2)-3$	16. $8(-4)+5(-4)-6$
17. $(-2)^6$	18. $(-3)^5$
19. -4^2	20. -6^2
21. $-3(-5)(6)$	22. $-4(-6)(3)$
23. $(8-11)(9-12)$	24. $(6-11)(8-13)$
25. $26-3(2-7)$	26. $23-2(4-6)$
27. $65\div(-5)+(-28)\div(-7)$	28. $52\div(-4)+(-32)\div(-8)$
29. $9-2[3-8(-2)]$	30. $11-3[7-4(-2)]$
31. $(-3)^2-24\div(8-2)$	32. $(-4)^2-32\div(12-4)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

33. $y+(-14)$ when a) $y=-33$ b) $y=30$	34. $x+(-21)$ when a) $x=-27$ b) $x=44$
35. a) $a+3$ when $a=-7$ b) $-a+3$ when $a=-7$	36. a) $d+(-9)$ when $d=-8$ b) $-d+(-9)$ when $d=-8$
37. $m+n$ when $m=-15, n=7$	38. $p+q$ when $p=-9, q=17$
39. $r+s$ when $r=-9, s=-7$	40. $t+u$ when $t=-6, u=-5$
41. $(x+y)^2$ when $x=-3, y=14$	42. $(y+z)^2$ when $y=-3, z=15$
43. $-2x+17$ when a) $x=8$ b) $x=-8$	44. $-5y+14$ when a) $y=9$ b) $y=-9$
45. $10-3m$ when a) $m=5$ b) $m=-5$	46. $18-4n$ when a) $n=3$ b) $n=-3$
47. $2w^2-3w+7$ when $w=-2$	48. $3u^2-4u+5$ when $u=-3$
49. $9a-2b-8$ when $a=-6$ and $b=-3$	50. $7m-4n-2$ when $m=-4$ and $n=-9$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

51. the sum of 3 and -15, increased by 7	52. the sum of -8 and -9, increased by 23
53. the difference of 10 and -18	54. subtract 11 from -25
55. the difference of -5 and -30	56. subtract -6 from -13
57. the product of -3 and 15	58. the product of -4 and 16
59. the quotient of -60 and -20	60. the quotient of -40 and -20
61. the quotient of -6 and the sum of *a* and *b*	62. the quotient of -7 and the sum of *m* and *n*
63. the product of -10 and the difference of $p \text{ and } q$	64. the product of -13 and the difference of $c \text{ and } d$

Use Integers in Applications

In the following exercises, solve.

65. **Temperature** On January 15, the high temperature in Lytton, British Columbia, was $84°$. That same day, the high temperature in Fort Nelson, British Columbia was $-12°$. What was the difference between the temperature in Lytton and the temperature in Embarrass?	66. **Temperature** On January 21, the high temperature in Palm Springs, California, was $89°$, and the high temperature in Whitefield, New Hampshire was $-31°$. What was the difference between the temperature in Palm Springs and the temperature in Whitefield?
67. **Football** At the first down, the Chargers had the ball on their 25 yard line. On the next three downs, they lost 6 yards, gained 10 yards, and lost 8 yards. What was the yard line at the end of the fourth down?	68. **Football** At the first down, the Steelers had the ball on their 30 yard line. On the next three downs, they gained 9 yards, lost 14 yards, and lost 2 yards. What was the yard line at the end of the fourth down?
69. **Checking Account** Ester has $124 in her checking account. She writes a check for $152. What is the new balance in her checking account?	70. **Checking Account** Selina has $165 in her checking account. She writes a check for $207. What is the new balance in her checking account?
71. **Checking Account** Kevin has a balance of $-\$38$ in his checking account. He deposits $225 to the account. What is the new balance?	72. **Checking Account** Reymonte has a balance of $-\$49$ in his checking account. He deposits $281 to the account. What is the new balance?

Everyday Math

73. **Stock market** Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped $12 per share. What was the total effect on Javier's portfolio?	74. **Weight loss** In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Writing Exercises

75. In your own words, state the rules for multiplying integers.	76. In your own words, state the rules for dividing integers.
77. Why is $-2^4 \neq (-2)^4$?	78. Why is $-4^3 = (-4)^3$?

Answers

1. -32	3. -63	5. -6
7. 14	9. -4	11. 13
13. -12	15. -47	17. 64
19. -16	21. 90	23. 9
25. 41	27. -9	29. -29
31. 5	33. a)-47 b) 16	35. a)-4 b) 10
37. -8	39. -16	41. 121
43. a) 1 b) 33	45. a)-5b) 25	47. 21
49. -56	51. $(3+(-15))+7;-5$	53. $10-(-18)\,;28$
55. $-5-(-30)\,;25$	57. $-3\cdot 15;-45$	59. $-60\div(-20)\,;3$
61. $\frac{-6}{a+b}$	63. $-10\,(p-q)$	65. $96°$
67. 21	69. -$\$28$	71. $187
73. -$\$3600$	75. Answers may vary	77. Answers may vary

Attributions

1.6 Chapter Review

Review Exercises

Use Place Value with Whole Number

In the following exercises find the place value of each digit.

1. 26,915 a) 1 b) 2 c) 9 d) 5 e) 6	2. 359,417 a) 9 b) 3 c) 4 d) 7 e) 1
3. 58,129,304 a) 5 b) 0 c) 1 d) 8 e) 2	4. 9,430,286,157 a) 6 b) 4 c) 9 d) 0 e) 5

In the following exercises, name each number.

5. 6,104	6. 493,068
7. 3,975,284	8. 85,620,435

In the following exercises, write each number as a whole number using digits.

9. three hundred fifteen	10. sixty-five thousand, nine hundred twelve
11. ninety million, four hundred twenty-five thousand, sixteen	12. one billion, forty-three million, nine hundred twenty-two thousand, three hundred eleven

In the following exercises, round to the indicated place value.

Round to the nearest ten. 13. a) 407 b) 8,564	Round to the nearest hundred. 14. a) 25,846 b) 25,864

In the following exercises, round each number to the nearest a) hundred b) thousand c) ten thousand.

15. 864,951	16. 3,972,849

Identify Multiples and Factors

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, by 3, by 5, by 6, and by 10

17. 168	18. 264
19. 375	20. 750
21. 1430	22. 1080

Find Prime Factorizations and Least Common Multiples

In the following exercises, find the prime factorization.

23. 420	24. 115
25. 225	26. 2475
27. 1560	28. 56
29. 72	30. 168
31. 252	32. 391

In the following exercises, find the least common multiple of the following numbers using the multiples method.

33. 6,15	34. 60, 75

In the following exercises, find the least common multiple of the following numbers using the prime factors method.

35. 24, 30	36. 70, 84

Use Variables and Algebraic Symbols

In the following exercises, translate the following from algebra to English.

37. $25 - 7$	38. $5 \cdot 6$
39. $45 \div 5$	40. $x + 8$
41. $42 \geq 27$	42. $3n = 24$
43. $3 \leq 20 \div 4$	44. $a \neq 7 \cdot 4$

In the following exercises, determine if each is an expression or an equation.

45. $6 \cdot 3 + 5$	46. $y - 8 = 32$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify each expression.

47. 3^5	48. 10^8

In the following exercises, simplify

49. 6 + 10/2 + 2	50. 9 + 12/3 + 4
51. 20 ÷ (4 + 6) · 5	52. 33 · (3 + 8) · 2
53. $4^2 + 5^2$	54. $(4 + 5)^2$

Evaluate an Expression

In the following exercises, evaluate the following expressions.

55. 9x + 7 when x = 3	56. 5x – 4 when x = 6
57. x^4 when x = 3	58. 3^x when x = 3
59. $x^2 + 5x – 8$ when x = 6	60. 2x + 4y – 5 when x = 7, y = 8

Simplify Expressions by Combining Like Terms

In the following exercises, identify the coefficient of each term.

61. 12n	62. $9x^2$

In the following exercises, identify the like terms.

63. 3n, n^2, 12, $12p^2$, 3, $3n^2$	64. 5, $18r^2$, 9s, 9r, $5r^2$, 5s

In the following exercises, identify the terms in each expression.

65. $11x^2 + 3x + 6$	66. $22y^3 + y + 15$

In the following exercises, simplify the following expressions by combining like terms.

67. 17a + 9a	68. 18z + 9z
69. 9x + 3x + 8	70. 8a + 5a + 9
71. 7p + 6 + 5p – 4	72. 8x + 7 + 4x – 5

Translate an English Phrase to an Algebraic Expression

In the following exercises, translate the following phrases into algebraic expressions.

73. the sum of 8 and 12	74. the sum of 9 and 1
75. the difference of x and 4	76. the difference of x and 3
77. the product of 6 and y	78. the product of 9 and y
79. Derek bought a skirt and a blouse. The skirt cost $15 more than the blouse. Let b represent the cost of the blouse. Write an expression for the cost of the skirt.	80. Marcella has 6 fewer boy cousins than girl cousins. Let g represent the number of girl cousins. Write an expression for the number of boy cousins.

Use Negatives and Opposites of Integers

In the following exercises, order each of the following pairs of numbers, using < or >.

81. a) 6___2 b) -7___4 c) -9___-1 d) 9___-3	82. a) -5___1 b) -4___-9 c) 6___10 d) 3___-8

In the following exercises,, find the opposite of each number.

83. a) -8 b) 1	84. a) -2 b) 6

In the following exercises, simplify.

85. (–19)	86. (–53)

In the following exercises, simplify.

87. −m when a) m = 3 b) m=-3	88. −p when a) p = 6 b) p = -6

Simplify Expressions with Absolute Value

In the following exercises,, simplify.

<table><tr><td>89. a) |7| b) |-25| c) |0|</td><td>90. a) |5| b) |0| c) |-19|</td></tr></table>

In the following exercises, fill in <, >, or = for each of the following pairs of numbers.

<table><tr><td>91.
a) – 8 ___ |–8|
b) – |–2|___ –2</td><td>92.
a) –3|___ – | –3|
b) 4 ___ – | –4|</td></tr></table>

In the following exercises, simplify.

93. |8 – 4|	94. |9 – 6|
95. 8 (14 – 2 |- 2|)	96. 6(13 – 4 |-2|)

In the following exercises, evaluate.

97. a) |x| when x = -28 b) |-x| when x =-15	98. a) |y| when y = -37 b) |-z| when z=-24

Add Integers

In the following exercises, simplify each expression.

99. -200 + 65	100. -150 + 45
101. 2 + (-8) + 6	102. 4 + (-9) + 7
103. 140 + (-75) + 67	104. -32 + 24 + (-6) + 10

Subtract Integers

In the following exercises, simplify.

105. 9 – 3	106. -5 – (-1)
107. a) 15 – 6 b) 15 + (-6)	108. a) 12 – 9 b) 12 + (-9)
109. a) 8 – (-9) b) 8 + 9	110. a) 4 – (-4) b) 4 + 4

In the following exercises, simplify each expression.

111. 10 – (-19)	112. 11 – (-18)
113. 31 – 79	114. 39 – 81
115. -31 – 11	116. -32 – 18
117. -15 – (-28) + 5	118. 71 + (-10) – 8
119. -16 – (-4 + 1) – 7	120. -15 – (-6 + 4) – 3

Multiply Integers

In the following exercises, multiply.

121. -5 (7)	122. -8 (6)
123. -18(-2)	124. -10 (-6)

Divide Integers

In the following exercises, divide.

125. -28 ÷ 7	126. 56 ÷ (-7)
127. -120 ÷(-20)	128. -200 ÷ 25

Simplify Expressions with Integers

In the following exercises, simplify each expression.

129. -8 (-2) -3 (-9)	130. -7 (-4) – 5(-3)
131. (-5)3	132. (-4)3
133. -4 · 2 · 11	134. -5 · 3 · 10
135. -10(-4) ÷ (-8)	136. -8(-6) ÷ (-4)
137. 31 – 4(3-9)	138. 24 – 3(2 – 10)

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

139. x + 8 when a) x = -26 b) x = -95	140. y + 9 when a) y = -29 b) y = -84
141. When b = -11, evaluate: a) b + 6 b) −b + 6	142. When c = -9, evaluate: a) c + (-4) b) −c + (-4)
143. $p^2 - 5p + 2$ when $p = -1$	144. $q^2 - 2q + 9$ when $q = -2$
145. $6x - 5y + 15$ when$x = 3andy = -1$	146. $3p - 2q + 9$ when $p = 8andq = -2$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

147. the sum of -4 and -17, increased by 32	148. a) the difference of 15 and -7 b) subtract 15 from -7
149. the quotient of -45 and -9	150. the product of -12 and the difference of c and d.

Use Integers in Applications

In the following exercises, solve.

151. Temperature The high temperature one day in Miami Beach, Florida, was 76° F. That same day, the high temperature in Buffalo, New York was −8° F. What was the difference between the temperature in Miami Beach and the temperature in Buffalo?	152. CheckingAccount Adrianne has a balance of -$22 in her checking account. She deposits $301 to the account. What is the new balance?

Review Exercise Answers

1. a) tens b) ten thousands c) hundreds d) ones e) thousands	3. a) ten millions b) tens c) hundred thousands d) millions e) ten thousands	5. six thousand, one hundred four
7. three million, nine hundred seventy-five thousand, two hundred eighty-four	9. 315	11. 90,425,016
13. a)410b)8,560	15. a)865,000 b)865,000c)860,000	17. by 2,3,6
19. by 3,5	21. by 2,5,10	23. 2 · 2 · 3 · 5 · 7
25. 3 · 3 · 5 · 5	27. 2 · 2 · 2 · 3 · 5 · 13	29. 2 · 2 · 2 · 3 · 3
31. 2 · 2 · 3 · 3 · 7	33. 30	35. 120
37. 25 minus 7, the difference of twenty-five and seven	39. 45 divided by 5, the quotient of forty-five and five	41. forty-two is greater than or equal to twenty-seven
43. 3 is less than or equal to 20 divided by 4, three is less than or equal to the quotient of twenty and four	45. expression	47. 243
49. 13	51. 10	53. 41
55. 34	57. 81	59. 58
61. 12	63. 12 and 3, n2 and 3n2	65. 11×2, 3x, 6
67. 26a	69. 12x + 8	71. 12p + 2
73. 8 + 12	75. x – 4	77. 6y
79. b + 15	81. a) > b) < c) < d) >	83. a) 8 b) -1
85. 19	87. a) -3 b) 3	89. a) 7 b) 25 c) 0
91. a) < b) =	93. 4	95. 80
97. a) 28 b) 15	99. -135	101. 0
103. 132	105. 6	107. a) 9 b) 9
109. a) 17 b) 17	111. 29	113. -48
115. -42	117. 18	119. -20
121. -35	123. 36	125. -4
127. 6	129. 43	131. -125
133. -88	135. -5	137. 55
139. a) -18 b) -87	141. a) -5 b) 17	143. 8
145. 38	147. (-4 + (-17)) + 32; 11	149. $\frac{-45}{-9}$;5
151. 84 degrees F		

Practice Test

1. Write as a whole number using digits: two hundred five thousand, six hundred seventeen.	2. Find the prime factorization of 504.
3. Find the Least Common Multiple of 18 and 24.	4. Combine like terms: 5n + 8 + 2n – 1.

In the following exercises, evaluate.

5. −|x| when x = -2	6. 11 – a when a = -3
7. Translate to an algebraic expression and simplify: twenty less than negative 7.	8. Monique has a balance of −$18 in her checking account. She deposits $152 to the account. What is the new balance?
9. Round 677.1348 to the nearest hundredth.	10. Simplify expression $-6(-2) - 3 \cdot 4(-6)$
11. Simplify expression $4(-2) + 42 - (-3)^3$	12. Simplify expression $-8(-3)(-6)$
13.Simplify expression $21 - 5(2 - 7)$	14. Simplify expression $2 + 2(3 - 10) - (2)^3$

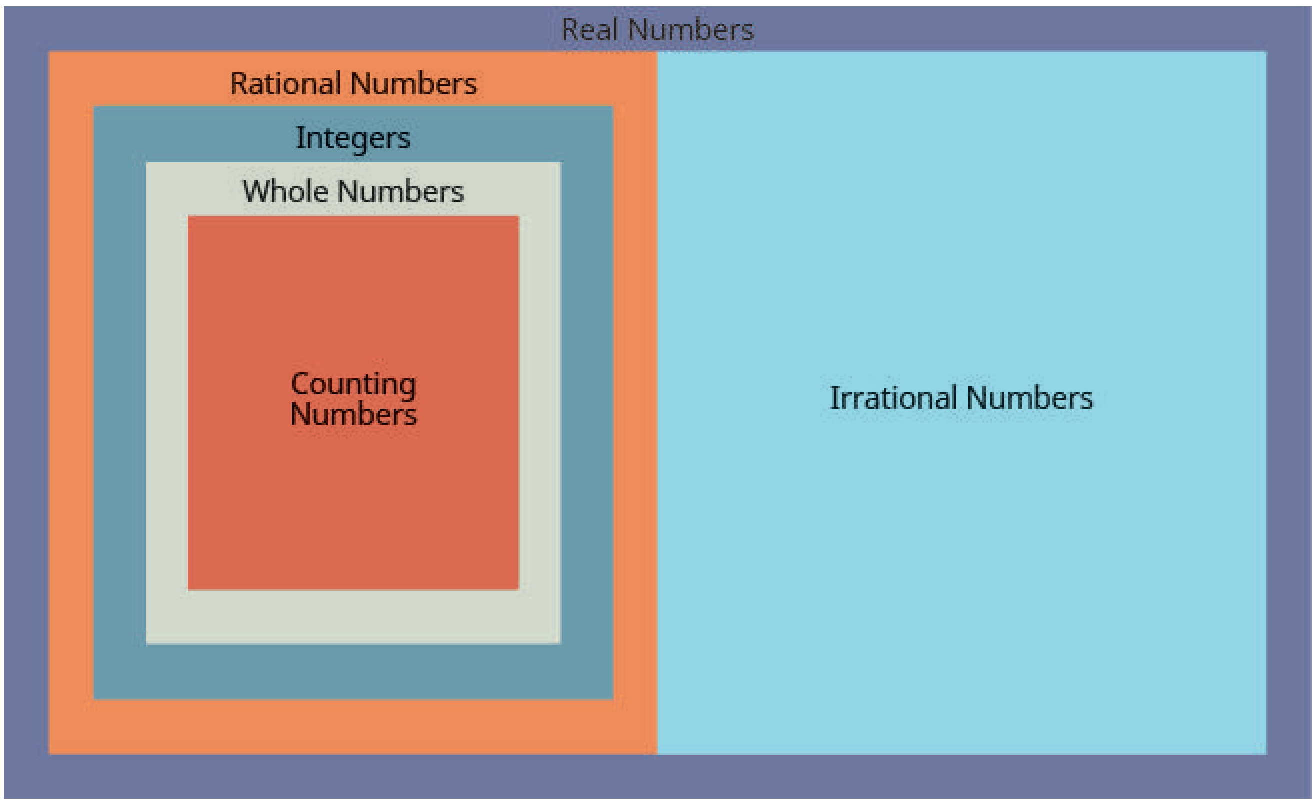
Practice Test Answers

1. 205,617	2. 2 · 2 · 2 · 3 · 3 · 7	3. 72
4. 7n + 7	5. -2	6. 14
7. -7 – 20; -27	8. $ 134	9. 677.13
10. 14	11. 27	12. -4
13. 46	14. -20	

II

CHAPTER 2 Operations with Rational Numbers and Introduction to Real Numbers

All the numbers we use in the intermediate algebra course are real numbers. The chart below shows us how the number sets we use in algebra fit together. In this chapter we will work with rational numbers, but you will be also introduced to irrational numbers. The set of rational numbers together with the set of irrational numbers make up the set of real numbers.



2.1 Visualize Fractions

Learning Objectives

By the end of this section, you will be able to:

- Find equivalent fractions
- Simplify fractions
- Multiply fractions
- Divide fractions
- Simplify expressions written with a fraction bar
- Translate phrases to expressions with fractions

Find Equivalent Fractions

Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts. See (Figure 1). The fraction $\frac{2}{3}$ represents two of three equal parts. In the fraction $\frac{2}{3}$, the 2 is called the numerator and the 3 is called the denominator.

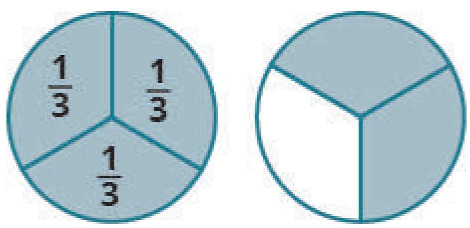


Figure 1.

The circle on the left has been divided into 3 equal parts. Each part is $\frac{1}{3}$ of the 3 equal parts. In the circle on the right, $\frac{2}{3}$ of the circle is shaded (2 of the 3 equal parts).

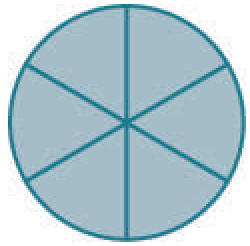
Fraction

A **fraction** is written $\frac{a}{b}$, where $b \neq 0$ and

- a is the **numerator** and b is the **denominator**.

A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

If a whole pie has been cut into 6 pieces and we eat all 6 pieces, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie.



So $\frac{6}{6} = 1$. This leads us to the property of one that tells us that any number, except zero, divided by itself is 1

Property of One

$\frac{a}{a} = 1 \qquad (a \neq 0)$

Any number, except zero, divided by itself is one.

If a pie was cut in 6 pieces and we ate all 6, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie. If the pie was cut into 8 pieces and we ate all 8, we ate $\frac{8}{8}$ pieces, or one whole pie. We ate the same amount—one whole pie.

The fractions $\frac{6}{6}$ and $\frac{8}{8}$ have the same value, 1, and so they are called equivalent fractions. **Equivalent fractions** are fractions that have the same value.

Let's think of pizzas this time. (Figure 2) shows two images: a single pizza on the left, cut into two equal pieces, and a second pizza of the same size, cut into eight pieces on the right. This is a way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. In other words, they are equivalent fractions.

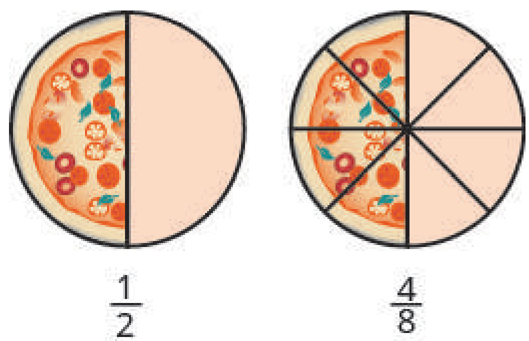


Figure 2.

Since the same amount is of each pizza is shaded, we see that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. They are equivalent fractions.

Equivalent Fractions

Equivalent fractions are fractions that have the same value.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could we take a pizza that is cut into 2 pieces and cut it into 8 pieces? We could cut each of the 2 larger pieces into 4 smaller pieces! The whole pizza would then be cut into 8 pieces instead of just 2. Mathematically, what we've described could be written like this as $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$. See (Figure 3).

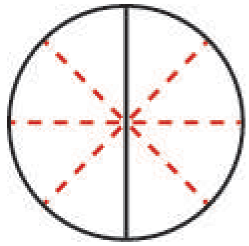


Figure 3.

Cutting each half of the pizza into 4 pieces, gives us pizza cut into 8 pieces: $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$.
This model leads to the following property:

Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0$, then
$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

If we had cut the pizza differently, we could get

$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4}$$

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{so} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

So, we say $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \text{ and } \frac{10}{20}$ are equivalent fractions.

EXAMPLE 1

Find three fractions equivalent to $\frac{2}{5}$.

Solution
To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number. We can choose any number, except for zero. Let's multiply them by 2, 3, and then 5.

$$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10} \qquad \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15} \qquad \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$$

So, $\frac{4}{10}, \frac{6}{15}, \text{ and } \frac{10}{25}$ are equivalent to $\frac{2}{5}$.

TRY IT 1.1

Find three fractions equivalent to $\frac{3}{5}$.

Answer
$\frac{6}{10}, \frac{9}{15}, \frac{12}{20}$; answers may vary

TRY IT 1.2

Find three fractions equivalent to $\frac{4}{5}$.

Answer
$\frac{8}{10}, \frac{12}{15}, \frac{16}{20}$; answers may vary

Simplify Fractions

A fraction is considered **simplified** if there are no common factors, other than 1, in its numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

Simplified Fraction

A fraction is considered simplified if there are no common factors in its numerator and denominator.

The phrase *reduce a fraction* means to simplify the fraction. We simplify, or reduce, a fraction by removing the common factors of the numerator and denominator. A fraction is not simplified until all common factors have been removed. If an expression has fractions, it is not completely simplified until the fractions are simplified.

In Example 1, we used the equivalent fractions property to find equivalent fractions. Now we'll use the equivalent fractions property in reverse to simplify fractions. We can rewrite the property to show both forms together.

Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0,$
$$\text{then} \quad \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

EXAMPLE 2

Simplify: $-\frac{32}{56}$.

Solution

	$-\frac{32}{56}$
Rewrite the numerator and denominator showing the common factors.	$-\frac{4 \cdot 8}{7 \cdot 8}$
Simplify using the equivalent fractions property.	$-\frac{4}{7}$

Notice that the fraction $-\frac{4}{7}$ is simplified because there are no more common factors.

TRY IT 2.1

Simplify: $-\frac{42}{54}$.

Answer
$-\frac{7}{9}$

TRY IT 2.2

Simplify: $-\frac{45}{81}$.

Answer
$-\frac{5}{9}$

Sometimes it may not be easy to find common factors of the numerator and denominator. When this happens, a good idea is to factor the numerator and the denominator into prime numbers. Then divide out the common factors using the equivalent fractions property.

EXAMPLE 3

How to Simplify a Fraction

Simplify: $-\frac{210}{385}$.

Solution

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.	Rewrite 210 and 385 as the product of the primes.	$-\frac{210}{385}$ $-\frac{2\cdot 3\cdot 5\cdot 7}{5\cdot 7\cdot 11}$
Step 2. Simplify using the equivalent fractions property by dividing out common factors.	Mark the common factors 5 and 7. Divide out the common factors.	$-\frac{2\cdot 3\cdot \cancel{5}\cdot \cancel{7}}{\cancel{5}\cdot \cancel{7}\cdot 11}$ $-\frac{2\cdot 3}{11}$
Step 3. Multiplify the remaining factors, if necessary.		$-\frac{6}{11}$

TRY IT 3.1

Simplify: $-\frac{69}{120}$.

Answer

$-\frac{23}{40}$

TRY IT 3.2

Simplify: $-\frac{120}{192}$.

Answer

$-\frac{5}{8}$

We now summarize the steps you should follow to simplify fractions.

HOW TO: Simplify a Fraction

1. Rewrite the numerator and denominator to show the common factors.
   If needed, factor the numerator and denominator into prime numbers first.
2. Simplify using the equivalent fractions property by dividing out common factors.
3. Multiply any remaining factors, if needed.

EXAMPLE 4

Simplify: $\frac{5x}{5y}$.

Solution

	$\frac{5x}{5y}$
Rewrite showing the common factors, then divide out the common factors.	$\frac{5x}{5y}$ Rewrite showing the common factors, then divide out the common factors. $\frac{\cancel{5} \cdot x}{\cancel{5} \cdot y}$ Simplify. $\frac{x}{y}$
Simplify.	$\frac{x}{y}$

TRY IT 4.1

Simplify: $\frac{7x}{7y}$.

Answer
$\frac{x}{y}$

TRY IT 4.2

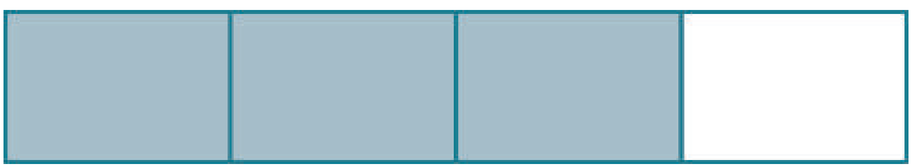
Simplify: $\frac{7x}{7y}$.

Answer
$\frac{x}{y}$

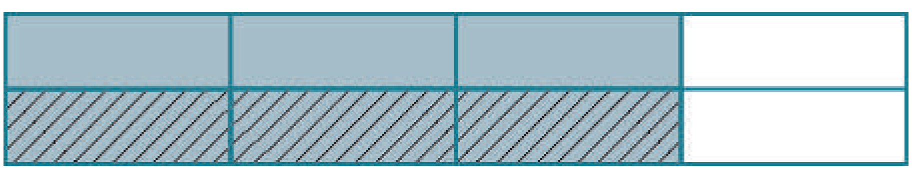
Multiply Fractions

Many people find multiplying and dividing fractions easier than adding and subtracting fractions. So we will start with fraction multiplication.

We'll use a model to show you how to multiply two fractions and to help you remember the procedure. Let's start with $\frac{3}{4}$.



Now we'll take $\frac{1}{2}$ of $\frac{3}{4}$.



Notice that now, the whole is divided into 8 equal parts. So $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

To multiply fractions, we multiply the numerators and multiply the denominators.

Fraction Multiplication

If a, b, c and d are numbers where $b \neq 0$ and $d \neq 0$, then
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
To multiply fractions, multiply the numerators and multiply the denominators.

When multiplying fractions, the properties of positive and negative numbers still apply, of course. It is a good idea to determine the sign of the product as the first step. In Example 5, we will multiply negative and a positive, so the product will be negative.

EXAMPLE 5

Multiply: $-\frac{11}{12} \cdot \frac{5}{7}$.

Solution

The first step is to find the sign of the product. Since the signs are the different, the product is negative.

	$-\frac{11}{12} \cdot \frac{5}{7}$
Determine the sign of the product; multiply.	$-\frac{11 \cdot 5}{12 \cdot 7}$
Are there any common factors in the numerator and the demoninator? No.	$-\frac{55}{84}$

TRY IT 5.1

Multiply: $-\frac{10}{28} \cdot \frac{8}{15}$.

Answer

$-\frac{4}{21}$

TRY IT 5.2

Multiply: $-\frac{9}{20} \cdot \frac{5}{12}$.

Answer

$-\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, *a*, can be written as $\frac{a}{1}$. So, for example, $3 = \frac{3}{1}$.

EXAMPLE 6

Multiply: $-\frac{12}{5} \cdot (-20x)$.

Solution

Determine the sign of the product. The signs are the same, so the product is positive.

	$-\frac{12}{5} \cdot (-20x)$
Write $20x$ as a fraction.	$\frac{12}{5} \cdot \left(\frac{20x}{1}\right)$
Multiply.	
Rewrite 20 to show the common factor 5 and divide it out.	$\frac{12 \cdot 4 \cdot \cancel{5}x}{\cancel{5} \cdot 1}$
Simplify.	$48x$

TRY IT 6.1

Multiply: $\frac{11}{3} \cdot (-9a)$.

Answer
$-33a$

TRY IT 6.2

Multiply: $\frac{13}{7} \cdot (-14b)$.

Answer
$-26b$

Divide Fractions

Now that we know how to multiply fractions, we are almost ready to divide. Before we can do that, that we need some vocabulary.

The reciprocal of a fraction is found by inverting the fraction, placing the numerator in the denominator and the denominator in the numerator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Notice that $\frac{2}{3} \cdot \frac{3}{2} = 1$. A number and its reciprocal multiply to 1.

To get a product of positive 1 when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

The reciprocal of $-\frac{10}{7}$ is $-\frac{7}{10}$, since $-\frac{10}{7} \cdot \left(-\frac{7}{10}\right) = 1$.

Reciprocal

The **reciprocal** of $\frac{a}{b}$ is $\frac{b}{a}$.

A number and its reciprocal multiply to one $\frac{a}{b} \cdot \frac{b}{a} = 1$.

To divide fractions, we multiply the first fraction by the reciprocal of the second.

Fraction Division

If a, b, c and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero!

EXAMPLE 7

Divide: $-\frac{2}{3} \div \frac{n}{5}$.

Solution

	$-\frac{2}{3} \div \frac{n}{5}$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{2}{3} \cdot \frac{5}{n}$
Multiply.	$-\frac{10}{3n}$

TRY IT 7.1

Divide: $-\frac{3}{5} \div \frac{p}{7}$.

Answer
$-\frac{21}{5p}$

TRY IT 7.2

Divide: $-\frac{5}{8} \div \frac{q}{3}$.

Answer
$-\frac{15}{8q}$

EXAMPLE 8

Find the quotient: $-\frac{7}{18} \div \left(-\frac{14}{27}\right)$.

Solution

	$-\frac{7}{18} \div \left(-\frac{14}{27}\right)$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{7}{18} \cdot -\frac{27}{14}$
Determine the sign of the product, and then multiply..	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{\cancel{7} \cdot \cancel{9} \cdot 3}{\cancel{9} \cdot 2 \cdot \cancel{7} \cdot 2}$
Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

TRY IT 8.1

Find the quotient: $-\frac{7}{27} \div \left(-\frac{35}{36}\right)$.

Answer
$\frac{4}{15}$

TRY IT 8.2

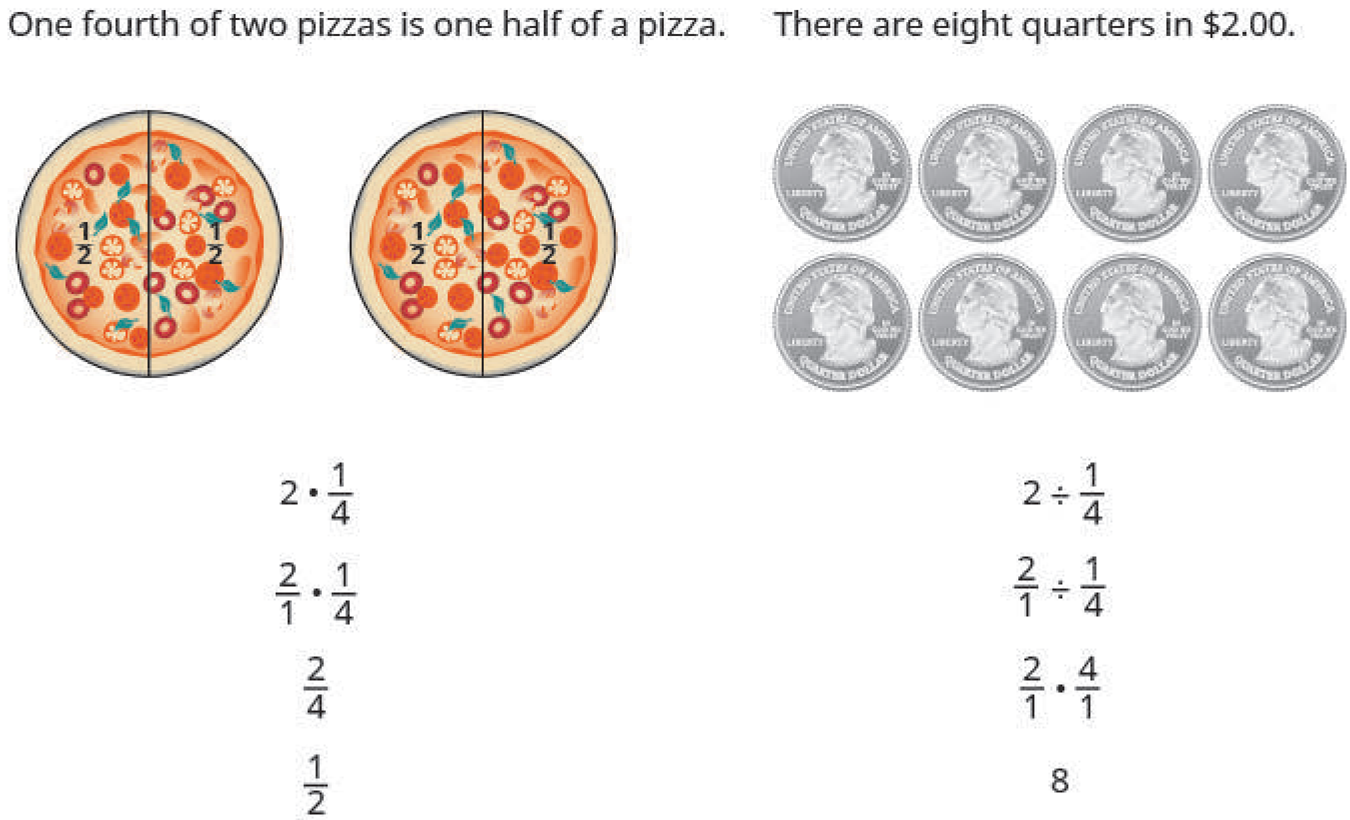
Find the quotient: $-\frac{5}{14} \div \left(-\frac{15}{28}\right)$.

Answer
$\frac{2}{3}$

There are several ways to remember which steps to take to multiply or divide fractions. One way is to repeat the call outs to yourself. If you do this each time you do an exercise, you will have the steps memorized.

- "To multiply fractions, multiply the numerators and multiply the denominators."
- "To divide fractions, multiply the first fraction by the reciprocal of the second."

Another way is to keep two examples in mind:



The numerators or denominators of some fractions contain fractions themselves. A fraction in which the numerator or the denominator is a fraction is called a **complex fraction**.

Complex Fraction

A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:

$\frac{\frac{6}{7}}{3}$ $\frac{\frac{3}{4}}{\frac{5}{8}}$ $\frac{\frac{x}{2}}{\frac{5}{6}}$

To simplify a complex fraction, we remember that the fraction bar means division. For example, the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ means $\frac{3}{4} \div \frac{5}{8}$.

EXAMPLE 9

Simplify: $\frac{\frac{3}{4}}{\frac{5}{8}}$.

Solution

	$\frac{\frac{3}{4}}{\frac{5}{8}}$
Rewrite as division.	$\frac{3}{4} \div \frac{5}{8}$
Multiply the first fraction by the reciprocal of the second.	$\frac{3}{4} \cdot \frac{8}{5}$
Multiply.	$\frac{3 \cdot 8}{4 \cdot 5}$
Look for common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 5}$
Divide out common factors and simplify.	$\frac{6}{5}$

TRY IT 9.1

Simplify: $\frac{\frac{2}{3}}{\frac{5}{6}}$.

Answer
$\frac{4}{5}$

TRY IT 9.2

Simplify: $\frac{\frac{3}{7}}{\frac{6}{11}}$.

Answer
$\frac{11}{14}$

EXAMPLE 10

Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$.

Solution

	$\frac{\frac{x}{2}}{\frac{xy}{6}}$
Rewrite as division.	$\frac{x}{2} \div \frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2} \cdot \frac{6}{xy}$
Multiply.	$\frac{x \cdot 6}{2 \cdot xy}$
Look for common factors.	$\frac{\cancel{x} \cdot 3 \cdot \cancel{2}}{\cancel{2} \cdot \cancel{x} \cdot y}$
Divide out common factors and simplify.	$\frac{3}{y}$

TRY IT 10.1

Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$.

Answer
$\frac{3}{4b}$

TRY IT 10.2

Simplify: $\frac{\frac{p}{2}}{\frac{pq}{8}}$.

Answer
$\frac{4}{q}$

Simplify Expressions with a Fraction Bar

The line that separates the numerator from the denominator in a fraction is called a fraction bar. A fraction bar acts as grouping symbol. The order of operations then tells us to simplify the numerator and then the denominator. Then we divide.

To simplify the expression $\frac{5-3}{7+1}$, we first simplify the numerator and the denominator separately. Then we divide.

$\frac{5-3}{7+1}$

$\frac{2}{8}$

$\frac{1}{4}$

HOW TO: Simplify an Expression with a Fraction Bar

1. Simplify the expression in the numerator. Simplify the expression in the denominator.
2. Simplify the fraction.

EXAMPLE 11

Simplify: $\frac{4-2(3)}{2^2+2}$.

Solution

	$\frac{4-2(3)}{2^2+2}$
Use the order of operations to simpliy the numerator and the denominator.	$\frac{4-6}{4+2}$
Simplify the numerator and the denominator.	$\frac{-2}{6}$
Simplify. A negative divided by a positive is negative.	$-\frac{1}{3}$

TRY IT 11.1

Simplify: $\frac{6-3(5)}{3^2+3}$.

Answer
$-\frac{3}{4}$

TRY IT 11.2

Simplify: $\frac{4-4(6)}{3^2+3}$.

Answer
$-\frac{5}{3}$

Where does the negative sign go in a fraction? Usually the negative sign is in front of the fraction, but you will sometimes see a fraction with a negative numerator, or sometimes with a negative denominator. Remember that fractions represent division. When the numerator and denominator have different signs, the quotient is negative.

$\frac{-1}{3}=-\frac{1}{3}$ $\qquad$ $\frac{\text{negative}}{\text{positive}}=\text{negative}$

$\frac{1}{-3}=-\frac{1}{3}$ $\qquad$ $\frac{\text{positive}}{\text{negative}}=\text{negative}$

Placement of Negative Sign in a Fraction

For any positive numbers *a* and *b*,

$$\frac{-a}{b}=\frac{a}{-b}=\ -\frac{a}{b}$$

EXAMPLE 12

Simplify: $\frac{4(-3)+6(-2)}{-3(2)-2}$.

Solution

	$\frac{4(-3)+6(-2)}{-3(2)-2}$
Multiply.	$\frac{-12+(-12)}{-6-2}$
Simplify.	$\frac{-24}{-8}$
Divide.	3

TRY IT 12.1

Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$.

Answer
4

TRY IT 12.2

Simplify: $\frac{7(-1)+9(-3)}{-5(3)-2}$.

Answer
2

Translate Phrases to Expressions with Fractions

Now that we have done some work with fractions, we are ready to translate phrases that would result in expressions with fractions.

The English words quotient and ratio are often used to describe fractions. Remember that “quotient” means division. The quotient of a and b is the result we get from dividing a by b, or $\frac{a}{b}$.

EXAMPLE 13

Translate the English phrase into an algebraic expression: the quotient of the difference of *m* and *n*, and *p*.

Solution
We are looking for the *quotient of* the difference of *m* and *n*, *and p*. This means we want to divide the difference of m and n by p.
$\frac{m-n}{p}$

TRY IT 13.1

Translate the English phrase into an algebraic expression: the quotient of the difference of *a* and *b*, and *cd*.

Answer
$\frac{a-b}{cd}$

TRY IT 13.2

Translate the English phrase into an algebraic expression: the quotient of the sum of p and q, and r

Answer
$\frac{p+q}{r}$

Key Concepts

- **Equivalent Fractions Property:** If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.
- **Fraction Division:** If a, b, c and d are numbers where $b \neq 0, c \neq 0,$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. To divide fractions, multiply the first fraction by the reciprocal of the second.
- **Fraction Multiplication:** If a, b, c and d are numbers where $b \neq 0,$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. To multiply fractions, multiply the numerators and multiply the denominators.
- **Placement of Negative Sign in a Fraction:** For any positive numbers a and b, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.
- **Property of One:** $\frac{a}{a} = 1$; Any number, except zero, divided by itself is one.
- **Simplify a Fraction**
    1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.
    2. Simplify using the equivalent fractions property by dividing out common factors.
    3. Multiply any remaining factors.
- **Simplify an Expression with a Fraction Bar**
    1. Simplify the expression in the numerator. Simplify the expression in the denominator.
    2. Simplify the fraction.

Glossary

complex fraction
A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

denominator
The denominator is the value on the bottom part of the fraction that indicates the number of equal parts into which the whole has been divided.

equivalent fractions
Equivalent fractions are fractions that have the same value.

fraction
A fraction is written $\frac{a}{b}$, where $b \neq 0$ a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

numerator
The numerator is the value on the top part of the fraction that indicates how many parts of the whole are included.

reciprocal
The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. A number and its reciprocal multiply to one: $\frac{a}{b} \cdot \frac{b}{a} = 1.$

simplified fraction
A fraction is considered simplified if there are no common factors in its numerator and denominator.

Practice Makes Perfect

Find Equivalent Fractions

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

1. $\frac{3}{8}$	2. $\frac{5}{8}$
3. $\frac{5}{9}$	4. $\frac{1}{8}$

Simplify Fractions

In the following exercises, simplify.

5. $-\frac{40}{88}$	6. $-\frac{63}{99}$
7. $-\frac{108}{63}$	8. $-\frac{104}{48}$
9. $\frac{120}{252}$	10. $\frac{182}{294}$
11. $-\frac{3x}{12y}$	12. $-\frac{4x}{32y}$
13. $\frac{14x^2}{21y}$	14. $\frac{24a}{32b^2}$

Multiply Fractions

In the following exercises, multiply.

15. $\frac{3}{4} \cdot \frac{9}{10}$	16. $\frac{4}{5} \cdot \frac{2}{7}$
17. $-\frac{2}{3} \cdot \left(-\frac{3}{8}\right)$	18. $-\frac{3}{4} \cdot \left(-\frac{4}{9}\right)$
19. $-\frac{5}{9} \cdot \frac{3}{10}$	20. $-\frac{3}{8} \cdot \frac{4}{15}$
21. $\left(-\frac{14}{15}\right) \cdot \left(\frac{9}{20}\right)$	22. $\left(-\frac{9}{10}\right) \cdot \left(\frac{25}{33}\right)$
23. $\left(-\frac{63}{84}\right) \cdot \left(-\frac{44}{90}\right)$	24. $\left(-\frac{63}{60}\right) \cdot \left(-\frac{40}{88}\right)$
25. $4 \cdot \frac{5}{11}$	26. $5 \cdot \frac{8}{3}$
27. $\frac{3}{7} \cdot 21n$	28. $\frac{5}{6} \cdot 30m$
29. $-8 \cdot \left(\frac{17}{4}\right)$	30. $(-1) \cdot \left(-\frac{6}{7}\right)$

Divide Fractions

In the following exercises, divide.

31. $\frac{3}{4} \div \frac{2}{3}$	32. $\frac{4}{5} \div \frac{3}{4}$
33. $-\frac{7}{9} \div \left(-\frac{7}{4}\right)$	34. $-\frac{5}{6} \div \left(-\frac{5}{6}\right)$
35. $\frac{3}{4} \div \frac{x}{11}$	36. $\frac{2}{5} \div \frac{y}{9}$
37. $\frac{5}{18} \div \left(-\frac{15}{24}\right)$	38. $\frac{7}{18} \div \left(-\frac{14}{27}\right)$
39. $\frac{8u}{15} \div \frac{12v}{25}$	40. $\frac{12r}{25} \div \frac{18s}{35}$
41. $-5 \div \frac{1}{2}$	42. $-3 \div \frac{1}{4}$
43. $\frac{3}{4} \div (-12)$	44. $-15 \div \left(-\frac{5}{3}\right)$

In the following exercises, simplify.

45. $\frac{-\frac{8}{21}}{\frac{12}{35}}$	46. $\frac{-\frac{9}{16}}{\frac{33}{40}}$
47. $\frac{-\frac{4}{5}}{2}$	48. $\frac{5}{\frac{3}{10}}$
49. $\frac{\frac{m}{3}}{\frac{n}{2}}$	50. $\frac{-\frac{3}{8}}{-\frac{y}{12}}$

Simplify Expressions Written with a Fraction Bar

In the following exercises, simplify.

51. $\frac{22+3}{10}$	52. $\frac{19-4}{6}$
53. $\frac{48}{24-15}$	54. $\frac{46}{4+4}$
55. $\frac{-6+6}{8+4}$	56. $\frac{-6+3}{17-8}$
57. $\frac{4\cdot 3}{6\cdot 6}$	58. $\frac{6\cdot 6}{9\cdot 2}$
59. $\frac{4^2-1}{25}$	60. $\frac{7^2+1}{60}$
61. $\frac{8\cdot 3+2\cdot 9}{14+3}$	62. $\frac{9\cdot 6-4\cdot 7}{22+3}$
63. $\frac{5\cdot 6-3\cdot 4}{4\cdot 5-2\cdot 3}$	64. $\frac{8\cdot 9-7\cdot 6}{5\cdot 6-9\cdot 2}$
65. $\frac{5^2-3^2}{3-5}$	66. $\frac{6^2-4^2}{4-6}$
67. $\frac{7\cdot 4-2(8-5)}{9\cdot 3-3\cdot 5}$	68. $\frac{9\cdot 7-3(12-8)}{8\cdot 7-6\cdot 6}$
69. $\frac{9(8-2)-3(15-7)}{6(7-1)-3(17-9)}$	70. $\frac{8(9-2)-4(14-9)}{7(8-3)-3(16-9)}$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

71. the quotient of *r* and the sum of *s* and 10	72. the quotient of *A* and the difference of 3 and *B*
73. the quotient of the difference of $x \text{ and } y, \text{ and } -3$	74. the quotient of the sum of $m \text{ and } n, \text{ and } 4q$

Everyday Math

75. **Baking.** A recipe for chocolate chip cookies calls for $\frac{3}{4}$ cup brown sugar. Imelda wants to double the recipe. a) How much brown sugar will Imelda need? Show your calculation. b) Measuring cups usually come in sets of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \text{ and } 1$ cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the cookie recipe.	76. **Baking.** Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk. a) How much condensed milk will Nina need? Show your calculation. b) Measuring cups usually come in sets of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \text{ and } 1$ cup. Draw a diagram to show two different ways that Nina could measure the condensed milk needed for 4 pans of fudge.
77. **Portions** Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?	78. **Portions** Kristen has $\frac{3}{4}$ yards of ribbon that she wants to cut into 6 equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Writing Exercises

79. Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.	80. Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.
81. Explain how you find the reciprocal of a fraction.	82. Explain how you find the reciprocal of a negative number.

Answers

1. $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$ answers may vary	3. $\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$ answers may vary	5. $-\frac{5}{11}$
7. $-\frac{12}{7}$	9. $\frac{10}{21}$	11. $-\frac{x}{4y}$
13. $\frac{2x^2}{3y}$	15. $\frac{27}{40}$	17. $\frac{1}{4}$
19. $-\frac{1}{6}$	21. $-\frac{21}{50}$	23. $\frac{11}{30}$
25. $\frac{20}{11}$	27. 9*n*	29. -34
31. $\frac{9}{8}$	33. 1	35. $\frac{33}{4x}$
37. $-\frac{4}{9}$	39. $\frac{10u}{9v}$	41. -10
43. $-\frac{1}{16}$	45. $-\frac{10}{9}$	47. $-\frac{2}{5}$
49. $\frac{2m}{3n}$	51. $\frac{5}{2}$	53. $\frac{16}{3}$
55. 0	57. $\frac{1}{3}$	59. $\frac{3}{5}$
61. $2\frac{8}{17}$	63. $\frac{3}{5}$	65. -8
67. $\frac{11}{6}$	69. $\frac{5}{2}$	71. $\frac{r}{s+10}$
73. $\frac{x-y}{-3}$	75. a) $1\frac{1}{2}$ cups b) answers will vary	77. 20 bags
79. Answers may vary.	81. Answers may vary.	

Attributions

2.2 Add and Subtract Fractions

Learning Objectives

By the end of this section, you will be able to:

- Add or subtract fractions with a common denominator
- Add or subtract fractions with different denominators
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

Add or Subtract Fractions with a Common Denominator

When we multiplied fractions, we just multiplied the numerators and multiplied the denominators right straight across. To add or subtract fractions, they must have a common denominator.

Fraction Addition and Subtraction

If $a, b,$ and c are numbers where $c \neq 0$, then

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \text{and} \quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

EXAMPLE 1

Find the sum: $\frac{x}{3} + \frac{2}{3}$.

Solution

	$\frac{x}{3} + \frac{2}{3}$
Add the numerators and place the sum over the common denominator.	$\frac{x+2}{3}$

TRY IT 1.1

Find the sum: $\frac{x}{4} + \frac{3}{4}$.

Answer
$\frac{x+3}{4}$

TRY IT 1.2

Find the sum: $\frac{y}{8}+\frac{5}{8}$.

Answer
$\frac{y+5}{8}$

EXAMPLE 2

Find the difference: $-\frac{23}{24}-\frac{13}{24}$.

Solution

	$-\frac{23}{24}-\frac{13}{24}$
Subtract the numerators and place the difference over the common denominator.	$\frac{-23-13}{24}$
Simplify.	$\frac{-36}{24}$
Simplify. Remember, $-\frac{a}{b}=\frac{-a}{b}$.	$-\frac{3}{2}$

TRY IT 2.1

Find the difference: $-\frac{19}{28}-\frac{7}{28}$.

Answer
$-\frac{26}{28}=-\frac{13}{14}$

TRY IT 2.2

Find the difference: $-\frac{27}{32}-\frac{1}{32}$.

Answer
$-\frac{7}{8}$

EXAMPLE 3

Simplify: $-\frac{10}{x}-\frac{4}{x}$.

Solution

	$-\frac{10}{x}-\frac{4}{x}$
Subtract the numerators and place the difference over the common denominator.	$\frac{-14}{x}$
Rewrite with the sign in front of the fraction.	$-\frac{14}{x}$

TRY IT 3.1

Find the difference: $-\frac{9}{x}-\frac{7}{x}$.

Answer
$-\frac{16}{x}$

TRY IT 3.2

Find the difference: $-\frac{17}{a}-\frac{5}{a}$.

Answer
$-\frac{22}{a}$

Now we will do an example that has both addition and subtraction.

EXAMPLE 4

Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$.

Solution

Add and subtract fractions—do they have a common denominator? Yes.	$\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$
Add and subtract the numerators and place the difference over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify left to right.	$\frac{-2-1}{8}$
Simplify.	$-\frac{3}{8}$

TRY IT 4.1

Simplify: $\frac{2}{5}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Answer
$-\frac{37}{45}$

TRY IT 4.2

Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Answer
$-\frac{2}{3}$

Add or Subtract Fractions with Different Denominators

As we have seen, to add or subtract fractions, their denominators must be the same. The least common denominator (LCD) of two fractions is the smallest number that can be used as a common denominator of the fractions. The LCD of the two fractions is the least common multiple (LCM) of their denominators.

Least Common Denominator

The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

After we find the least common denominator of two fractions, we convert the fractions to equivalent fractions with the LCD. Putting these steps together allows us to add and subtract fractions because their denominators will be the same!

EXAMPLE 5

Add: $\frac{7}{12}+\frac{5}{18}$.

Solution

Step 1. Do they have a common denominator?	No.	$12 = 2\cdot 2\cdot 3$ $18 = 2\cdot \quad 3\cdot 3$
No—rewrite each fraction with the LCD (least common denominator).	Find the LCD of 12, 18.	$\text{LCD} = 2\cdot 2\cdot 3\cdot 3$ $\text{LCD} = 36$
	Change into equivalent fractions with the LCD, 36.	$\frac{7}{12}+\frac{5}{18}$ $\frac{7\cdot 3}{12\cdot 3}+\frac{5\cdot 2}{18\cdot 2}$
	Do not simplify the equivalent fractions! If you do, you'll get back to the original fractions and lose the common denominator!	$\frac{21}{36}+\frac{10}{36}$

Step 2. Add or subtract the fractions.	Add.	$\frac{31}{36}$
Step 3. Simplify, if possible.	Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.	

TRY IT 5.1

Add: $\frac{7}{12} + \frac{11}{15}$.

Answer
$\frac{79}{60}$

TRY IT 5.2

Add: $\frac{13}{15} + \frac{17}{20}$.

Answer
$\frac{103}{60}$

HOW TO: Add or Subtract Fractions

1. Do they have a common denominator?
   - Yes—go to step 2.
   - No—rewrite each fraction with the LCD (least common denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.
2. Add or subtract the fractions.
3. Simplify, if possible.

When finding the equivalent fractions needed to create the common denominators, there is a quick way to find the number we need to multiply both the numerator and denominator. This method works if we found the LCD by factoring into primes.

Look at the factors of the LCD and then at each column above those factors. The “missing” factors of each denominator are the numbers we need.

missing factors

$$12 = 2 \cdot 2 \cdot 3$$
$$18 = 2 \cdot \quad 3 \cdot 3$$
$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3$$
$$\text{LCD} = 36$$

In (Example 5), the LCD, 36, has two factors of 2 and two factors of 3.

The numerator 12 has two factors of 2 but only one of 3—so it is “missing” one 3—we multiply the numerator and denominator by 3

The numerator 18 is missing one factor of 2—so we multiply the numerator and denominator by 2

We will apply this method as we subtract the fractions in (Example 6).

EXAMPLE 6

Subtract: $\frac{7}{15} - \frac{19}{24}$.

Solution
Do the fractions have a common denominator? No, so we need to find the LCD.

Find the LCD. $\frac{7}{15} - \frac{19}{24}$; $15 = 3 \cdot 5$; $24 = 2 \cdot 2 \cdot 2 \cdot 3$; $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$; $\text{LCD} = 120$	
Notice, 15 is “missing” three factors of 2 and 24 is “missing” the 5 from the factors of the LCD. So we multiply 8 in the first fraction and 5 in the second fraction to get the LCD.	
Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 8}{15 \cdot 8} - \frac{19 \cdot 5}{24 \cdot 5}$
Simplify.	$\frac{56}{120} - \frac{95}{120}$
Subtract.	$-\frac{39}{120}$
Check to see if the answer can be simplified.	$-\frac{13 \cdot 3}{40 \cdot 3}$
Both 39 and 120 have a factor of 3.	
Simplify.	$-\frac{13}{40}$

Do not simplify the equivalent fractions! If you do, you’ll get back to the original fractions and lose the common denominator!

TRY IT 6.1

Subtract: $\frac{13}{24} - \frac{17}{32}$.

Answer
$\frac{1}{96}$

TRY IT 6.2

Subtract: $\frac{21}{32} - \frac{9}{28}$.

Answer
$\frac{75}{224}$

In the next example, one of the fractions has a variable in its numerator. Notice that we do the same steps as when both numerators are numbers.

EXAMPLE 7

Add: $\frac{3}{5} + \frac{x}{8}$.

Solution
The fractions have different denominators.

		$\frac{3}{5} + \frac{x}{8}$
Find the LCD. $\begin{array}{l} 5 = 5 \\ 8 = 2 \cdot 2 \cdot 2 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 5 \\ \text{LCD} = 40 \end{array}$		
Rewrite as equivalent fractions with the LCD.		$\frac{3 \cdot 8}{5 \cdot 8} + \frac{x \cdot 5}{8 \cdot 5}$
Simplify.		$\frac{24}{40} + \frac{5x}{40}$
Add.		$\frac{24 + 5x}{40}$

TRY IT 7.1

Add: $\frac{y}{6} + \frac{7}{9}$.

Answer

$\frac{9y+42}{54}$

TRY IT 7.2

Add: $\frac{x}{6}+\frac{7}{15}$.

Answer
$\frac{15x+42}{90}$

We now have all four operations for fractions. The table below summarizes fraction operations.

Summary of Fraction Operations

Fraction Operation	**Sample Equation**	**What to Do**
Fraction multiplication	$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$	Multiply the numerators and multiply the denominators
Fraction division	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$	Multiply the first fraction by the reciprocal of the second.
Fraction addition	$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$	Add the numerators and place the sum over the common denominator.
Fraction subtraction	$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$	Subtract the numerators and place the difference over the common denominator.

To multiply or divide fractions, an LCD is NOT needed. To add or subtract fractions, an LCD is needed.

EXAMPLE 8

Simplify: a) $\frac{5x}{6}-\frac{3}{10}$ b) $\frac{5x}{6}\cdot\frac{3}{10}$.

Solution
First ask, “What is the operation?” Once we identify the operation that will determine whether we need a common denominator. Remember, we need a common denominator to add or subtract, but not to multiply or divide.

a) **What is the operation? The operation is subtraction.**	
Do the fractions have a common denominator? No.	$\frac{5x}{6}-\frac{3}{10}$
Rewrite each fraction as an equivalent fraction with the LCD.	$\frac{5x\cdot5}{6\cdot5}-\frac{3\cdot3}{10\cdot3}$ $\frac{25x}{30}-\frac{9}{30}$
Subtract the numerators and place the difference over the common denominators.	$\frac{25x-9}{30}$
Simplify, if possible.	There are no common factors. The fraction is simplified.

b) **What is the operation? Multiplication.**	$\frac{5x}{6} \cdot \frac{3}{10}$
To multiply fractions, multiply the numerators and multiply the denominators.	$\frac{5x \cdot 3}{6 \cdot 10}$
Rewrite, showing common factors. Remove common factors.	$\frac{\cancel{5}x \cdot \cancel{3}}{2 \cdot \cancel{3} \cdot 2 \cdot \cancel{5}}$
Simplify.	$\frac{x}{4}$

Notice we needed an LCD to add $\frac{5x}{6} - \frac{3}{10}$, but not to multiply $\frac{5x}{6} \cdot \frac{3}{10}$.

TRY IT 8.1

Simplify. a) $\frac{27a-32}{36}$ b) $\frac{2a}{3}$

Answer
a) $\frac{27a-32}{36}$ b) $\frac{2a}{3}$

TRY IT 8.2

Simplify: a) $\frac{4k}{5} - \frac{1}{6}$ b) $\frac{4k}{5} \cdot \frac{1}{6}$.

Answer
a) $\frac{24k-5}{30}$ b) $\frac{2k}{15}$

Use the Order of Operations to Simplify Complex Fractions

We have seen that a complex fraction is a fraction in which the numerator or denominator contains a fraction. The fraction bar indicates division. We simplified the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ by dividing $\frac{3}{4}$ by $\frac{5}{8}$.

Now we'll look at complex fractions where the numerator or denominator contains an expression that can be simplified. So we first must completely simplify the numerator and denominator separately using the order of operations. Then we divide the numerator by the denominator.

EXAMPLE 9

Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Solution

Step 1. Simplify the numerator. * Remember, $\left(\frac{1}{2}\right)^2$ means $\frac{1}{2}\cdot\frac{1}{2}$.	$\dfrac{\left(\frac{1}{2}\right)^2}{4+3^2}$ $\dfrac{\frac{1}{4}}{4+3^2}$
Step 2. Simplify the denominator.	$\dfrac{\frac{1}{4}}{4+9}$ $\dfrac{\frac{1}{4}}{13}$
Step 3. Divide the numerator by the denominator. Simplify if possible. * Remember, $13=\frac{13}{1}$	$\frac{1}{4}\div 13$ $\frac{1}{4}\cdot\frac{1}{13}$ $\frac{1}{52}$

TRY IT 9.1

Simplify: $\frac{\left(\frac{1}{3}\right)^2}{2^3+2}$.

Answer
$\frac{1}{90}$

TRY IT 9.2

Simplify: $\frac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Answer
272

HOW TO: Simplify Complex Fractions

1. Simplify the numerator.
2. Simplify the denominator.
3. Divide the numerator by the denominator. Simplify if possible.

EXAMPLE 10

Simplify: $\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution
It may help to put parentheses around the numerator and the denominator.

	$\frac{\left(\frac{1}{2}+\frac{2}{3}\right)}{\left(\frac{3}{4}-\frac{1}{6}\right)}$
Simplify the numerator (LCD = 6) and simplify the denominator (LCD = 12).	$\frac{\left(\frac{3}{6}+\frac{4}{6}\right)}{\left(\frac{9}{12}-\frac{2}{12}\right)}$
Simplify.	$\frac{\left(\frac{7}{6}\right)}{\left(\frac{7}{12}\right)}$
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$
Simplify.	$\frac{7}{6} \cdot \frac{12}{7}$
Divide out common factors.	$\frac{7 \cdot 6 \cdot 2}{6 \cdot 7}$
Simplify.	2

TRY IT 10.1

Simplify: $\frac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$.

Answer
2

TRY IT 10.2

Simplify: $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Answer
$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

EXAMPLE 11

Evaluate $x+\frac{1}{3}$ when a) $x=-\frac{1}{3}$ b) $x=-\frac{3}{4}$.

Solution

a. To evaluate $x+\frac{1}{3}$ when $x=-\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Simplify.	0

b. To evaluate $x+\frac{1}{3}$ when $x=-\frac{3}{4}$, we substitute $-\frac{3}{4}$ for *x* in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{3}{4}$ for x.	$-\frac{3}{4}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot 3}{4\cdot 3}+\frac{1\cdot 4}{3\cdot 4}$
Simplify.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

TRY IT 11.1

Evaluate $x+\frac{3}{4}$ when a) $x=-\frac{7}{4}$ b) $x=-\frac{5}{4}$.

Answer
a) -1 b) $-\frac{1}{2}$

TRY IT 11.2

Evaluate $y+\frac{1}{2}$ when a) $y=\frac{2}{3}$ b) $y=-\frac{3}{4}$.

Answer
a) $\frac{7}{6}$ b) $-\frac{1}{4}$

EXAMPLE 12

Evaluate $-\frac{5}{6}-y$ when $y=-\frac{2}{3}$.

Solution

	$-\frac{5}{6}-y$
Substitute $-\frac{2}{3}$ for y.	$-\frac{5}{6}-\left(-\frac{2}{3}\right)$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{5}{6}-\left(-\frac{4}{6}\right)$
Subtract.	$\frac{-5-(-4)}{6}$
Simplify.	$-\frac{1}{6}$

TRY IT 12.1

Evaluate $-\frac{1}{2}-y$ when $y=-\frac{1}{4}$.

Answer
$-\frac{1}{4}$

TRY IT 12.2

Evaluate $-\frac{3}{8}-y$ when $y=-\frac{5}{2}$.

Answer
$\frac{17}{8}$

EXAMPLE 13

Evaluate $2x^2y$ when $x=\frac{1}{4}$ and $y=-\frac{2}{3}$.

Solution
Substitute the values into the expression.

	$2x^2y$
Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply. Divide out the common factors. Notice we write 16 as $2 \cdot 2 \cdot 4$ to make it easy to remove common factors.	$-\frac{\not{2}\cdot 1\cdot\not{2}}{\not{2}\cdot\not{2}\cdot 4\cdot 3}$
Simplify.	$-\frac{1}{12}$

TRY IT 13.1

Evaluate $3ab^2$ when $a = -\frac{2}{3}$ and $b = -\frac{1}{2}$.

Answer
$-\frac{1}{2}$

TRY IT 13.2

Evaluate $4c^3d$ when $c = -\frac{1}{2}$ and $d = -\frac{4}{3}$.

Answer
$\frac{2}{3}$

The next example will have only variables, no constants.

EXAMPLE 14

Evaluate $\frac{p+q}{r}$ when $p = -4, q = -2, \text{ and } r = 8$.

Solution
To evaluate $\frac{p+q}{r}$ when $p = -4, q = -2, \text{ and } r = 8$, we substitute the values into the expression.

	$\frac{p+q}{r}$
Substitute -4 for p, -2 for q and 8 for r.	$\frac{-4+(-2)}{8}$
Add in the numerator first.	$\frac{-6}{8}$
Simplify.	$-\frac{3}{4}$

TRY IT 14.1

Evaluate $\frac{a+b}{c}$ when $a = -8, b = -7, \text{ and } c = 6.$

Answer

$-\frac{5}{2}$

TRY IT 14.2

Evaluate $\frac{x+y}{z}$ when $x = 9, y = -18, \text{ and } z = -6.$

Answer

$\frac{3}{2}$

Key Concepts

- **Fraction Addition and Subtraction:** If $a, b, \text{ and } c$ are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.
  To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.
- **Strategy for Adding or Subtracting Fractions**
  1. Do they have a common denominator?
     Yes—go to step 2.
     No—Rewrite each fraction with the LCD (Least Common Denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.
  2. Add or subtract the fractions.
  3. Simplify, if possible. To multiply or divide fractions, an LCD IS NOT needed. To add or subtract fractions, an LCD IS needed.
- **Simplify Complex Fractions**
  1. Simplify the numerator.
  2. Simplify the denominator.
  3. Divide the numerator by the denominator. Simplify if possible.

Glossary

least common denominator
The least common denominator (LCD) of two fractions is the Least common multiple (LCM) of their denominators.

Practice Makes Perfect

Add and Subtract Fractions with a Common Denominator

In the following exercises, add.

1. $\frac{6}{13}+\frac{5}{13}$	2. $\frac{4}{15}+\frac{7}{15}$
3. $\frac{x}{4}+\frac{3}{4}$	4. $\frac{8}{q}+\frac{6}{q}$
5. $-\frac{3}{16}+\left(-\frac{7}{16}\right)$	6. $-\frac{5}{16}+\left(-\frac{9}{16}\right)$
7. $-\frac{8}{17}+\frac{15}{17}$	8. $-\frac{9}{19}+\frac{17}{19}$
9. $\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$	10. $\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$

In the following exercises, subtract.

11. $\frac{11}{15}-\frac{7}{15}$	12. $\frac{9}{13}-\frac{4}{13}$
13. $\frac{11}{12}-\frac{5}{12}$	14. $\frac{7}{12}-\frac{5}{12}$
15. $\frac{19}{21}-\frac{4}{21}$	16. $\frac{17}{21}-\frac{8}{21}$
17. $\frac{5y}{8}-\frac{7}{8}$	18. $\frac{11z}{13}-\frac{8}{13}$
19. $-\frac{23}{u}-\frac{15}{u}$	20. $-\frac{29}{v}-\frac{26}{v}$
21. $-\frac{3}{5}-\left(-\frac{4}{5}\right)$	22. $-\frac{3}{7}-\left(-\frac{5}{7}\right)$
23. $-\frac{7}{9}-\left(-\frac{5}{9}\right)$	24. $-\frac{8}{11}-\left(-\frac{5}{11}\right)$

Mixed Practice

In the following exercises, simplify.

25. $-\frac{5}{18}\cdot\frac{9}{10}$	26. $-\frac{3}{14}\cdot\frac{7}{12}$
27. $\frac{n}{5}-\frac{4}{5}$	28. $\frac{6}{11}-\frac{s}{11}$
29. $-\frac{7}{24}+\frac{2}{24}$	30. $-\frac{5}{18}+\frac{1}{18}$
31. $\frac{8}{15}\div\frac{12}{5}$	32. $\frac{7}{12}\div\frac{9}{28}$

Add or Subtract Fractions with Different Denominators

In the following exercises, add or subtract.

33. $\frac{1}{2}+\frac{1}{7}$	34. $\frac{1}{3}+\frac{1}{8}$
35. $\frac{1}{3}-\left(-\frac{1}{9}\right)$	36. $\frac{1}{4}-\left(-\frac{1}{8}\right)$
37. $\frac{7}{12}+\frac{5}{8}$	38. $\frac{5}{12}+\frac{3}{8}$
39. $\frac{7}{12}-\frac{9}{16}$	40. $\frac{7}{16}-\frac{5}{12}$
41. $\frac{2}{3}-\frac{3}{8}$	42. $\frac{5}{6}-\frac{3}{4}$
43. $-\frac{11}{30}+\frac{27}{40}$	44. $-\frac{9}{20}+\frac{17}{30}$
45. $-\frac{13}{30}+\frac{25}{42}$	46. $-\frac{23}{30}+\frac{5}{48}$
47. $-\frac{39}{56}-\frac{22}{35}$	48. $-\frac{33}{49}-\frac{18}{35}$
49. $-\frac{2}{3}-\left(-\frac{3}{4}\right)$	50. $-\frac{3}{4}-\left(-\frac{4}{5}\right)$
51. $1+\frac{7}{8}$	52. $1-\frac{3}{10}$
53. $\frac{x}{3}+\frac{1}{4}$	54. $\frac{y}{2}+\frac{2}{3}$
55. $\frac{y}{4}-\frac{3}{5}$	56. $\frac{x}{5}-\frac{1}{4}$

Mixed Practice

In the following exercises, simplify.

57. a) $\frac{2}{3}+\frac{1}{6}$ b) $\frac{2}{3}\div\frac{1}{6}$	58. a) $-\frac{2}{5}-\frac{1}{8}$ b) $-\frac{2}{5}\cdot\frac{1}{8}$
59. a) $\frac{5n}{6}\div\frac{8}{15}$ b) $\frac{5n}{6}-\frac{8}{15}$	60. a) $\frac{3a}{8}\div\frac{7}{12}$ b) $\frac{3a}{8}-\frac{7}{12}$
61. $-\frac{3}{8}\div\left(-\frac{3}{10}\right)$	62. $-\frac{5}{12}\div\left(-\frac{5}{9}\right)$
63. $-\frac{3}{8}+\frac{5}{12}$	64. $-\frac{1}{8}+\frac{7}{12}$
65. $\frac{5}{6}-\frac{1}{9}$	66. $\frac{5}{9}-\frac{1}{6}$
67. $-\frac{7}{15}-\frac{y}{4}$	68. $-\frac{3}{8}-\frac{x}{11}$
69. $\frac{11}{12a}\cdot\frac{9a}{16}$	70. $\frac{10y}{13}\cdot\frac{8}{15y}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

71. $\frac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$	72. $\frac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$
73. $\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$	74. $\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$
75. $\frac{2}{\frac{1}{3}+\frac{1}{5}}$	76. $\frac{5}{\frac{1}{4}+\frac{1}{3}}$
77. $\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$	78. $\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$
79. $\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$	80. $\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$
81. $1-\frac{3}{5}\div\frac{1}{10}$	82. $1-\frac{5}{6}\div\frac{1}{12}$
83. $\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$	84. $\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$
85. $\frac{3}{8}-\frac{1}{6}+\frac{3}{4}$	86. $\frac{2}{5}+\frac{5}{8}-\frac{3}{4}$
87. $12\left(\frac{9}{20}-\frac{4}{15}\right)$	88. $8\left(\frac{15}{16}-\frac{5}{6}\right)$
89. $\frac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$	90. $\frac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$
91. $\left(\frac{5}{9}+\frac{1}{6}\right)\div\left(\frac{2}{3}-\frac{1}{2}\right)$	92. $\left(\frac{3}{4}+\frac{1}{6}\right)\div\left(\frac{5}{8}-\frac{1}{3}\right)$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

93. $x+\left(-\frac{5}{6}\right)$ when a) $x=\frac{1}{3}$ b) $x=-\frac{1}{6}$	94. $x+\left(-\frac{11}{12}\right)$ when a) $x=\frac{11}{12}$ b) $x=\frac{3}{4}$
95. $x-\frac{2}{5}$ when a) $x=\frac{3}{5}$ b) $x=-\frac{3}{5}$	96. $x-\frac{1}{3}$ when a) $x=\frac{2}{3}$ b) $x=-\frac{2}{3}$
97. $\frac{7}{10}-w$ when a) $w=\frac{1}{2}$ b) $w=-\frac{1}{2}$	98. $\frac{5}{12}-w$ when a) $w=\frac{1}{4}$ b) $w=-\frac{1}{4}$
99. $2x^2y^3$ when $x=-\frac{2}{3}$ and $y=-\frac{1}{2}$	100. $8u^2v^3$ when $u=-\frac{3}{4}$ and $v=-\frac{1}{2}$
101. $\frac{a+b}{a-b}$ when $a=-3, b=8$	102. $\frac{r-s}{r+s}$ when $r=10, s=-5$

Everyday Math

103. **Decorating** Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{1}{2}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?	104. **Baking** Samuel is baking chocolate chip cookies and oatmeal cookies. He needs $\frac{1}{2}$ cup of sugar for the chocolate chip cookies and $\frac{1}{4}$ of sugar for the oatmeal cookies. How much sugar does he need altogether?

Answers

1. $\frac{11}{13}$	3. $\frac{x+3}{4}$	5. $-\frac{5}{8}$
7. $\frac{7}{17}$	9. $-\frac{16}{13}$	11. $\frac{4}{15}$
13. $\frac{1}{2}$	15. $\frac{5}{7}$	17. $\frac{5y-7}{8}$
19. $-\frac{38}{u}$	21. $\frac{1}{5}$	23. $-\frac{2}{9}$
25. $-\frac{1}{4}$	27. $\frac{n-4}{5}$	29. $-\frac{5}{24}$
31. $\frac{2}{9}$	33. $\frac{9}{14}$	35. $\frac{4}{9}$
37. $\frac{29}{24}$	39. $\frac{1}{48}$	41. $\frac{7}{24}$
43. $\frac{37}{120}$	45. $\frac{17}{105}$	47. $-\frac{53}{40}$
49. $\frac{1}{12}$	51. $\frac{15}{8}$	53. $\frac{4x+3}{12}$
55. $\frac{4y-12}{20}$	57. a) $\frac{5}{6}$ b) 4	59. a) $\frac{25n}{16}$ b) $\frac{25n-16}{30}$
61. $\frac{5}{4}$	63. $\frac{1}{24}$	65. $\frac{13}{18}$
67. $\frac{-28-15y}{60}$	69. $\frac{33}{64}$	71. 54
73. $\frac{49}{25}$	75. $\frac{15}{4}$	77. $\frac{5}{21}$
79. $\frac{7}{9}$	81. -5	83. $\frac{19}{12}$
85. $\frac{23}{24}$	87. $\frac{11}{5}$	89. 1
91. $\frac{13}{3}$	93. a) $-\frac{1}{2}$ b) -1	95. a) $\frac{1}{5}$ b) -1
97. a) $\frac{1}{5}$ b) $\frac{6}{5}$	99. $-\frac{1}{9}$	101. $-\frac{5}{11}$
103. $\frac{7}{8}$ yard		

Attributions

2.3 Decimals

Learning Objectives

By the end of this section, you will be able to:

- Name and write decimals
- Round decimals
- Add and subtract decimals
- Multiply and divide decimals
- Convert decimals, fractions, and percent

Name and Write Decimals

Decimals are another way of writing fractions whose denominators are powers of 10.

$$\begin{aligned} 0.1 &= \tfrac{1}{10} && 0.1 \text{ is "one tenth"} \\ 0.01 &= \tfrac{1}{100} && 0.01 \text{ is "one hundredth"} \\ 0.001 &= \tfrac{1}{1{,}000} && 0.001 \text{ is "one thousandth"} \\ 0.0001 &= \tfrac{1}{10{,}000} && 0.0001 \text{ is "one ten-thousandth"} \end{aligned}$$

Notice that “ten thousand” is a number larger than one, but “one ten-thousand**th**” is a number smaller than one. The “th” at the end of the name tells you that the number is smaller than one.

When we name a whole number, the name corresponds to the place value based on the powers of ten. We read 10,000 as “ten thousand” and 10,000,000 as “ten million.” Likewise, the names of the decimal places correspond to their fraction values. Figure 1 shows the names of the place values to the left and right of the decimal point.
Place value of decimal numbers are shown to the left and right of the decimal point.

Place Value											
Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
						•					

Figure 1

EXAMPLE 1

Name the decimal 4.3

Solution

Step 1. Name the number to the left of the decimal point.	4 is to the left of the decimal point.	4.3 four ______
Step 2. Write 'and' for the decimal point.		four and ______
Step 3. Name the 'number' part to the right of the decimal point as if it were a whole number.	3 is to the right of the decimal point.	four and three ______
Step 4. Name the decimal place.		four and three tenths

TRY IT 1.1

Name the decimal: 6.7.

Answer
six and seven tenths

TRY IT 1.2

Name the decimal: 5.8.

Answer
five and eight tenths

We summarize the steps needed to name a decimal below.

HOW TO: Name a Decimal

1. Name the number to the left of the decimal point.
2. Write "and" for the decimal point.
3. Name the "number" part to the right of the decimal point as if it were a whole number.
4. Name the decimal place of the last digit.

EXAMPLE 2

Name the decimal: -15.571.

Solution

	-15.571
Name the number to the left of the decimal point.	negative fifteen ________________________________
Write "and" for the decimal point.	negative fifteen and ____________________________
Name the number to the right of the decimal point.	negative fifteen and five hundred seventy-one _________
The 1 is in the thousandths place.	negative fifteen and five hundred seventy-one thousandths

TRY IT 2.1

Name the decimal: -13.461.

Answer
negative thirteen and four hundred sixty-one thousandths

TRY IT 2.2

Name the decimal: -2.053.

Answer
negative two and fifty-three thousandths

When we write a check we write both the numerals and the name of the number. Let's see how to write the decimal from the name.

EXAMPLE 3

Write "fourteen and twenty-four thousandths" as a decimal.

Solution

Step 1. Look for the word 'and'; it locates the decimal point. Place a decimal point under the word 'and'. Translate the words before 'and' into the whole number and place to the left of the decimal point.		fourteen and twenty-four thousandths fourteen and twenty-four thousandths _____ . _________ 14 . _________
Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.	The last word is 'thousandths'.	14 . _____ _____ _____ tenths hundredths thousandths

Step 3. Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces – putting the final digit in the last place.		14 . __ 2 4
Step 4. Fill in zeros for empty place holders as needed.	Zeros are needed in the tenths place.	14 . 0 2 4 Fourteen and twenty-four thousandths is written 14.024.

TRY IT 3.1

Write as a decimal: thirteen and sixty-eight thousandths.

Answer
13.068

TRY IT 3.2

Write as a decimal: five and ninety-four thousandths.

Answer
5.094

We summarize the steps to writing a decimal.

HOW TO: Write a Decimal

1. Look for the word “and”—it locates the decimal point.
    - Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
    - If there is no “and,” write a “0” with a decimal point to its right.
2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
3. Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
4. Fill in zeros for place holders as needed.

Round Decimals

Rounding decimals is very much like rounding whole numbers. We will round decimals with a method based on the one we used to round whole numbers.

EXAMPLE 4

Round 18.379 to the nearest hundredth.

Solution

Step 1. Locate the given place value and mark it with an arrow.		hundredths place ↓ 18.379
Step 2. Underline the digit to the right of the given place value.		hundredths place ↓ 18.37<u>9</u>
Step 3. Is this digit greater than or equal to 5? **Yes:** Add 1 to the digit in the given place value. **No:** Do <u>not</u> change the digit in the given place value.	Because 9 is greater than or equal to 5, add 1 to the 7.	18.37 9 add 1 delete
Step 4. Rewrite the number, removing all digits to the right of the rounding digit.		18.38 18.38 is 18.379 rounded to the nearest hundredth.

TRY IT 4.1

Round to the nearest hundredth: 1.047.

Answer
1.05

TRY IT 4.2

Round to the nearest hundredth: 9.173.

Answer
9.17

We summarize the steps for rounding a decimal here.

HOW TO: Round Decimals

1. Locate the given place value and mark it with an arrow.

2. Underline the digit to the right of the place value.
3. Is this digit greater than or equal to 5?
    - Yes—add 1 to the digit in the given place value.
    - No—do <u>not</u> change the digit in the given place value.
4. Rewrite the number, deleting all digits to the right of the rounding digit.

EXAMPLE 5

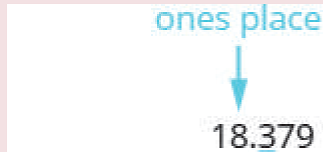
Round 18.379 to the nearest a) tenth b) whole number.

Solution
Round 18.379

a) to the nearest tenth

Locate the tenths place with an arrow.	
Underline the digit to the right of the given place value.	
Because 7 is greater than or equal to 5, add 1 to the 3.	
Rewrite the number, deleting all digits to the right of the rounding digit.	18.4
Notice that the deleted digits were NOT replaced with zeros.	So, 18.379 rounded to the nearest tenth is 18.4.

b) to the nearest whole number

Locate the ones place with an arrow.	
Underline the digit to the right of the given place value.	
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	
Rewrite the number, deleting all digits to the right of the rounding digit.	18
	So, 18.379 rounded to the nearest whole number is 18.

TRY IT 5.1

Round 6.582 to the nearest a) hundredth b) tenth c) whole number.

Answer
a) 6.58 b) 6.6 c) 7

TRY IT 5.2

Round 15.2175 to the nearest a) thousandth b) hundredth c) tenth.

Answer
a) 15.218 b) 15.22 c) 15.2

Add and Subtract Decimals

To add or subtract decimals, we line up the decimal points. By lining up the decimal points this way, we can add or subtract the corresponding place values. We then add or subtract the numbers as if they were whole numbers and then place the decimal point in the sum.

HOW TO: Add or Subtract Decimals

1. Write the numbers so the decimal points line up vertically.
2. Use zeros as place holders, as needed.
3. Add or subtract the numbers as if they were whole numbers. Then place the decimal point in the answer under the decimal points in the given numbers.

EXAMPLE 6

Add: $23.5 + 41.38$.

Solution

Write the numbers so the decimal points line up vertically.	$\begin{array}{r} 23.5 \\ +41.38 \\ \hline \end{array}$
Put 0 as a placeholder after the 5 in 23.5. Remember, $\frac{5}{10} = \frac{50}{100}$ so $0.5 = 0.50$.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array}$
Add the numbers as if they were whole numbers. Then place the decimal point in the sum.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline 64.88 \end{array}$

TRY IT 6.1

Add: $4.8 + 11.69$.

Answer
16.49

TRY IT 6.2

Add: $5.123 + 18.47$.

Answer
23.593

EXAMPLE 7

Subtract: $20 - 14.65$.

Solution

	$20 - 14.65$
Write the numbers so the decimal points line up vertically. Remember, 20 is a whole number, so place the decimal point after the 0.	$\begin{array}{r} 20. \\ -14.65 \\ \hline \end{array}$
Put in zeros to the right as placeholders.	$\begin{array}{r} 20.00 \\ \underline{-14.65} \end{array}$
Subtract and place the decimal point in the answer.	$\begin{array}{r} 20.00 \\ -14.65 \\ \hline 5.35 \end{array}$

TRY IT 7.1

Subtract: $10 - 9.58$.

Answer
0.42

TRY IT 7.2

Subtract: $50 - 37.42$.

Answer
12.58

Multiply and Divide Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first convert them to fractions and then multiply.

So let’s see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side. Look for a pattern!

	$\underbrace{(0.3)}_{\text{1 place}}\ \underbrace{(0.7)}_{\text{1 place}}$	$\underbrace{(0.2)}_{\text{1 place}}\ \underbrace{(0.46)}_{\text{2 places}}$
Convert to fractions.	$\frac{3}{10} \cdot \frac{7}{10}$	$\frac{2}{10} \cdot \frac{46}{100}$
Multiply.	$\frac{21}{100}$	$\frac{92}{1000}$
Convert to decimals.	$0.\underbrace{21}_{\text{2 places}}$	$0.\underbrace{092}_{\text{3 places}}$

Notice, in the first example, we multiplied two numbers that each had one digit after the decimal point and the product had two decimal places. In the second example, we multiplied a number with one decimal place by a number with two decimal places and the product had three decimal places.

We multiply the numbers just as we do whole numbers, temporarily ignoring the decimal point. We then count the number of decimal points in the factors and that sum tells us the number of decimal places in the product.

The rules for multiplying positive and negative numbers apply to decimals, too, of course!

When *multiplying* two numbers,

- if their signs are the *same* the product is *positive*.
- if their signs are *different* the product is *negative*.

When we multiply signed decimals, first we determine the sign of the product and then multiply as if the numbers were both positive. Finally, we write the product with the appropriate sign.

HOW TO: Multiply Decimals

1. Determine the sign of the product.
2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
3. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors.
4. Write the product with the appropriate sign.

EXAMPLE 8

Multiply: $(-3.9)\,(4.075)$.

Solution

	(−3.9)(4.075)
The signs are different. The product will be negative.	
Write in vertical format, lining up the numbers on the right.	4.075 × 3.9
Multiply.	4.075 × 3.9 36675 12225 158925
Add the number of decimal places in the factors (1 + 3). (−3.9) (4.075) 1 place 3 places Place the decimal point 4 places from the right.	4.075 × 3.9 36675 12225 15.8925 4 places
The signs are different, so the product is negative.	(−3.9)(4.075) = −15.8925

TRY IT 8.1

Multiply: $-4.5\,(6.107)$.

Answer
-27.4815

TRY IT 8.2

Multiply: $-10.79\,(8.12)$.

Answer
-87.6148

In many of your other classes, especially in the sciences, you will multiply decimals by powers of 10 (10, 100, 1000, etc.). If you multiply a few products on paper, you may notice a pattern relating the number of zeros in the power of 10 to number of decimal places we move the decimal point to the right to get the product.

HOW TO: Multiply a Decimal by a Power of Ten

1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
2. Add zeros at the end of the number as needed.

EXAMPLE 9

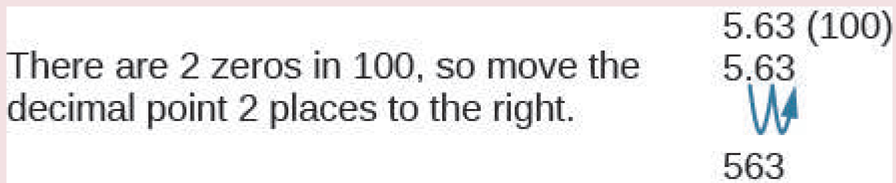
Multiply 5.63 a) by 10 b) by 100 c) by 1,000.

Solution
By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

a)

	5.63(10)
There is 1 zero in 10, so move the decimal point 1 place to the right.	

b)

	5.63(100)
There are 2 zeros in 100, so move the decimal point 2 places to the right.	

c)

	5.63(1,000)
There are 3 zeros in 1,000, so move the decimal point 3 places to the right.	
A zero must be added at the end.	5,630

TRY IT 9.1

Multiply 2.58 a) by 10 b) by 100 c) by 1,000.

Answer
a) 25.8 b) 258 c) 2,580

TRY IT 9.2

Multiply 14.2 a) by 10 b) by 100 c) by 1,000.

Answer
a) 142 b) 1,420 c) 14,200

Just as with multiplication, division of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To divide decimals, determine what power of 10 to multiply the denominator by to make it a whole number. Then multiply the numerator by that same power of 10. Because of the equivalent fractions property, we haven't changed the value of the fraction! The effect is to move the decimal points in the numerator and denominator the same number of places to the right. For example:

$$\frac{0.8}{0.4}$$
$$\frac{0.8(10)}{0.4(10)}$$
$$\frac{8}{4}$$

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign.

We review the notation and vocabulary for division:

$$\underset{\text{dividend}}{a} \div \underset{\text{divisor}}{b} = \underset{\text{quotient}}{\text{c}} \qquad \underset{\text{divisor}}{b}\overset{\underset{\text{quotient}}{\text{c}}}{\overline{\big)\,\underset{\text{dividend}}{a}}}$$

We'll write the steps to take when dividing decimals, for easy reference.

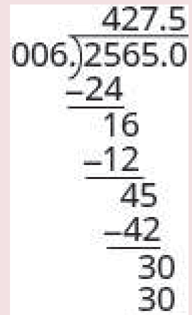
HOW TO: Divide Decimals

1. Determine the sign of the quotient.
2. Make the divisor a whole number by "moving" the decimal point all the way to the right. "Move" the decimal point in the dividend the same number of places—adding zeros as needed.
3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
4. Write the quotient with the appropriate sign.

EXAMPLE 10

Divide: $-25.56 \div (-0.06)$.

Solution
Remember, you can "move" the decimals in the divisor and dividend because of the Equivalent Fractions Property.

	$-25.65 \div (-0.06)$
The signs are the same.	The quotient is positive.
Make the divisor a whole number by “moving” the decimal point all the way to the right.	
“Move” the decimal point in the dividend the same number of places.	
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	
Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

TRY IT 10.1

Divide: $-23.492 \div (-0.04)$.

Answer
587.3

TRY IT 10.2

Divide: $-4.11 \div (-0.12)$.

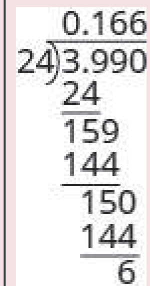
Answer
34.25

A common application of dividing whole numbers into decimals is when we want to find the price of one item that is sold as part of a multi-pack. For example, suppose a case of 24 water bottles costs $3.99. To find the price of one water bottle, we would divide $3.99 by 24. We show this division in Example 11. In calculations with money, we will round the answer to the nearest cent (hundredth).

EXAMPLE 11

Divide: $\$3.99 \div 24$.

Solution

	$\$3.99 \div 24$
Place the decimal point in the quotient above the decimal point in the dividend.	
Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth.) To do this, we must carry the division to the thousandths place.	
Round to the nearest cent.	$\$0.166 \approx \$0.17$ $\$3.99 \div 24 \approx \$0.17$

TRY IT 11.1

Divide: $\$6.99 \div 36$.

Answer
$0.19

TRY IT 11.2

Divide: $\$4.99 \div 12$.

Answer
$0.42

Convert Decimals and Fractions

We convert decimals into fractions by identifying the place value of the last (farthest right) digit. In the decimal 0.03 the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03

$$0\;0.03 = \frac{3}{100}$$

Notice, when the number to the left of the decimal is zero, we get a fraction whose numerator is less than its denominator. Fractions like this are called proper fractions.

The steps to take to convert a decimal to a fraction are summarized in the procedure box.

HOW TO: Covert a Decimal to a Proper Fraction

1. Determine the place value of the final digit.

2. Write the fraction.
    - numerator—the “numbers” to the right of the decimal point
    - denominator—the place value corresponding to the final digit

EXAMPLE 12

Write 0.374 as a fraction.

Solution

	0.374
Determine the place value of the final digit.	0.3 7 4 tenths hundredths thousandths
Write the fraction for 0.374: • The numerator is 374. • The denominator is 1,000.	$\frac{374}{1000}$
Simplify the fraction.	$\frac{2 \cdot 187}{2 \cdot 500}$
Divide out the common factors.	$\frac{187}{500}$ so, $0.374 = \frac{187}{500}$

Did you notice that the number of zeros in the denominator of $\frac{374}{1,000}$ is the same as the number of decimal places in 0.374?

TRY IT 12.1

Write 0.234 as a fraction.

Answer
$\frac{117}{500}$

TRY IT 12.2

Write 0.024 as a fraction.

Answer
$\frac{3}{125}$

We’ve learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar means division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\overline{)4}$. This leads to the following method for converting a fraction to a decimal.

HOW TO: Covert a Fraction to a Decimal

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

EXAMPLE 13

Write $-\frac{5}{8}$ as a decimal.

Solution

Since a fraction bar means division, we begin by writing $\frac{5}{8}$ as $8\overline{)5}$. Now divide.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

so, $-\frac{5}{8} = -0.625$

TRY IT 13.1

Write $-\frac{7}{8}$ as a decimal.

Answer
-0.875

TRY IT 13.2

Write $-\frac{3}{8}$ as a decimal.

Answer
-0.375

When we divide, we will not always get a zero remainder. Sometimes the quotient ends up with a decimal that repeats. A repeating decimal is a decimal in which the last digit or group of digits repeats endlessly. A bar is placed over the repeating block of digits to indicate it repeats.

Repeating Decimal

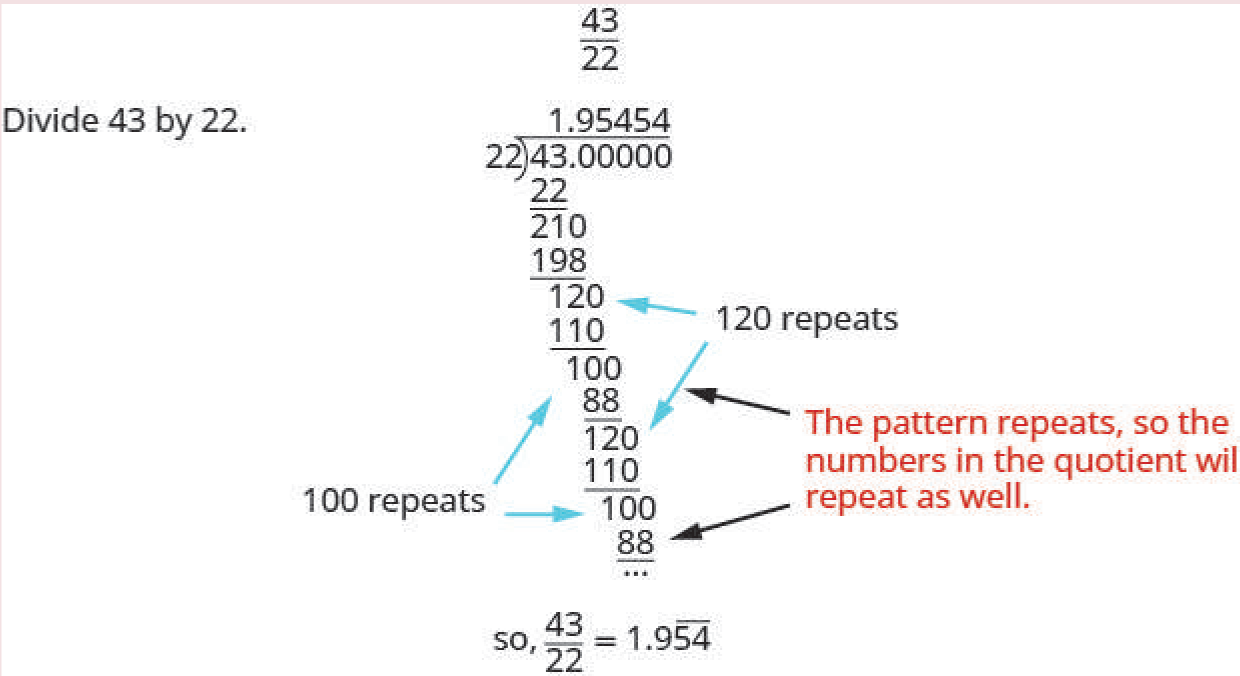
A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly.

A bar is placed over the repeating block of digits to indicate it repeats.

EXAMPLE 14

Write $\frac{43}{22}$ as a decimal.

Solution



TRY IT 14.1

Write $\frac{27}{11}$ as a decimal.

Answer

$2.\overline{45}$

TRY IT 14.2

Write $\frac{51}{22}$ as a decimal.

Answer

$2.3\overline{18}$

Sometimes we may have to simplify expressions with fractions and decimals together.

EXAMPLE 15

Simplify: $\frac{7}{8} + 6.4.$

Solution

First we must change one number so both numbers are in the same form. We can change the fraction to a decimal, or change the decimal to a fraction. Usually it is easier to change the fraction to a decimal.

		$\frac{7}{8}+6.4$
Change $\frac{7}{8}$ to a decimal.	$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$	
Add.		$0.875+6.4$
		7.275
		So, $\frac{7}{8}+6.4=7.275$

TRY IT 15.1

Simplify: $\frac{3}{8}+4.9$.

Answer
5.275

TRY IT 15.2

Simplify: $5.7+\frac{13}{20}$.

Answer
6.35

Key Concepts

- **Name a Decimal**
  1. Name the number to the left of the decimal point.
  2. Write ”and” for the decimal point.
  3. Name the “number” part to the right of the decimal point as if it were a whole number.
  4. Name the decimal place of the last digit.
- **Write a Decimal**
  1. Look for the word ‘and’—it locates the decimal point. Place a decimal point under the word ‘and.’ Translate the words before ‘and’ into the whole number and place it to the left of the decimal point.

If there is no “and,” write a “0” with a decimal point to its right.

2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
3. Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
4. Fill in zeros for place holders as needed.

- **Round a Decimal**

1. Locate the given place value and mark it with an arrow.
2. Underline the digit to the right of the place value.
3. Is this digit greater than or equal to 5? Yes—add 1 to the digit in the given place value. No—do not change the digit in the given place value.
4. Rewrite the number, deleting all digits to the right of the rounding digit.

- **Add or Subtract Decimals**

1. Write the numbers so the decimal points line up vertically.
2. Use zeros as place holders, as needed.
3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

- **Multiply Decimals**

1. Determine the sign of the product.
2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
3. Place the decimal point. The number of decimal places in the product is the sum of the decimal places in the factors.
4. Write the product with the appropriate sign.

- **Multiply a Decimal by a Power of Ten**

1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
2. Add zeros at the end of the number as needed.

- **Divide Decimals**

1. Determine the sign of the quotient.
2. Make the divisor a whole number by “moving” the decimal point all the way to the right. “Move” the decimal point in the dividend the same number of places – adding zeros as needed.
3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
4. Write the quotient with the appropriate sign.

- **Convert a Decimal to a Proper Fraction**

1. Determine the place value of the final digit.

2. Write the fraction: numerator—the 'numbers' to the right of the decimal point; denominator—the place value corresponding to the final digit.

- **Convert a Fraction to a Decimal** Divide the numerator of the fraction by the denominator.

Practice Makes Perfect

Name and Write Decimals

In the following exercises, write as a decimal.

1. Twenty-nine and eighty-one hundredths	2. Sixty-one and seventy-four hundredths
3. Seven tenths	4. Six tenths
5. Twenty-nine thousandth	6. Thirty-five thousandths
7. Negative eleven and nine ten-thousandths	8. Negative fifty-nine and two ten-thousandths

In the following exercises, name each decimal.

9. 5.5	10. 14.02
11. 8.71	12. 2.64
13. 0.002	14. 0.479
15. -17.9	16. -31.4

Round Decimals

In the following exercises, round each number to the nearest tenth.

17. 0.67	18. 0.49
19. 2.84	20. 4.63

In the following exercises, round each number to the nearest hundredth.

21. 0.845	22. 0.761
23. 0.299	24. 0.697
25. 4.098	26. 7.096

In the following exercises, round each number to the nearest a) hundredth b) tenth c) whole number.

27. 5.781	28. 1.6381
29. 63.479	30. 84.281

Add and Subtract Decimals

In the following exercises, add or subtract.

31. $16.92 + 7.56$	32. $248.25 - 91.29$
33. $21.76 - 30.99$	34. $38.6 + 13.67$
35. $-16.53 - 24.38$	36. $-19.47 - 32.58$
37. $-38.69 + 31.47$	38. $29.83 + 19.76$
39. $72.5 - 100$	40. $86.2 - 100$
41. $15 + 0.73$	42. $27 + 0.87$
43. $91.95 - (-10.462)$	44. $94.69 - (-12.678)$
45. $55.01 - 3.7$	46. $59.08 - 4.6$
47. $2.51 - 7.4$	48. $3.84 - 6.1$

Multiply and Divide Decimals

In the following exercises, multiply.

49. $(0.24)(0.6)$	50. $(0.81)(0.3)$
51. $(5.9)(7.12)$	52. $(2.3)(9.41)$
53. $(-4.3)(2.71)$	54. $(-8.5)(1.69)$
55. $(-5.18)(-65.23)$	56. $(-9.16)(-68.34)$
57. $(0.06)(21.75)$	58. $(0.08)(52.45)$
59. $(9.24)(10)$	60. $(6.531)(10)$
61. $(55.2)(1000)$	62. $(99.4)(1000)$

In the following exercises, divide.

63. $4.75 \div 25$	64. $12.04 \div 43$
65. $\$117.25 \div 48$	66. $\$109.24 \div 36$
67. $0.6 \div 0.2$	68. $0.8 \div 0.4$
69. $1.44 \div (-0.3)$	70. $1.25 \div (-0.5)$
71. $-1.75 \div (-0.05)$	72. $-1.15 \div (-0.05)$
73. $5.2 \div 2.5$	74. $6.5 \div 3.25$
75. $11 \div 0.55$	76. $14 \div 0.35$

Convert Decimals and Fractions

In the following exercises, write each decimal as a fraction.

77. 0.04	78. 0.19
79. 0.52	80. 0.78
81. 1.25	82. 1.35
83. 0.375	84. 0.464
85. 0.095	86. 0.085

In the following exercises, convert each fraction to a decimal.

87. $\frac{17}{20}$	88. $\frac{13}{20}$
89. $\frac{11}{4}$	90. $\frac{17}{4}$
91. $-\frac{310}{25}$	92. $-\frac{284}{25}$
93. $\frac{15}{11}$	94. $\frac{18}{11}$
95. $\frac{15}{111}$	96. $\frac{25}{111}$
97. $2.4+\frac{5}{8}$	98. $3.9+\frac{9}{20}$

Everyday Math

99. **Salary Increase** Danny got a raise and now makes \$58,965.95 a year. Round this number to the nearest a) dollar b) thousand dollars c) ten thousand dollars.	100. **New Car Purchase** Selena's new car cost \$23,795.95. Round this number to the nearest a) dollar b) thousand dollars c) ten thousand dollars.
101. **Sales Tax** Hyo Jin lives in Vancouver. She bought a refrigerator for \$1,624.99 and when the clerk calculated the sales tax it came out to exactly \$142.186625. Round the sales tax to the nearest a) penny and b) dollar.	102. **Sales Tax** Jennifer bought a \$1,038.99 dining room set for her home in Burnaby. She calculated the sales tax to be exactly \$67.53435. Round the sales tax to the nearest a) penny and b) dollar.
103. **Paycheck** Annie has two jobs. She gets paid \$14.04 per hour for tutoring at Community College and \$8.75 per hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours. a) How much did she earn? b) If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?	104. **Paycheck** Jake has two jobs. He gets paid \$7.95 per hour at the college cafeteria and \$20.25 at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery. a) How much did he earn? b) If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Writing Exercises

105. How does knowing about Canadian money help you learn about decimals?	106. Explain how you write "three and nine hundredths" as a decimal.

Glossary

decimal
A decimal is another way of writing a fraction whose denominator is a power of ten.
percent
A percent is a ratio whose denominator is 100.
repeating decimal
A repeating decimal is a decimal in which the last digit or group of digits repeats endlessly.

Answers

1. 29.81	3. 0.7	5. 0.029
7. -11.0009	9. five and five tenths	11. eight and seventy-one hundredths
13. two thousandths	15. negative seventeen and nine tenths	17. 0.7
19. 2.8	21. 0.85	23. 0.30
25. 4.10	27. a) 5.78 b) 5.8 c) 6	29. a) 63.48 b) 63.5 c) 63
31. 24.48	33. -9.23	35. -40.91
37. -7.22	39. -27.5	41. 15.73
43. 102.212	45. 51.31	47. -4.89
49. 0.144	51. 42.008	53. -11.653
55. 337.8914	57. 1.305	59. 92.4
61. 55,200	63. 0.19	65. $2.44
67. 3	69. -4.8	71. 35
73. 2.08	75. 20	77. $\frac{1}{25}$
79. $\frac{13}{25}$	81. $\frac{5}{4}$	83. $\frac{3}{8}$
85. $\frac{19}{200}$	87. 0.85	89. 2.75
91. -12.4	93. $1.\overline{36}$	95. $0.\overline{135}$
97. 3.025	99. a) $58,966 b) $59,000 c) $60,000	101. a) $142.19; b) $142
103. a) $243.57 b) $79.35	105. Answers may vary.	107. Answers may vary.

Attributions

2.4 Introduction to the Real Numbers

Learning Objectives

By the end of this section, you will be able to:

- Identify integers, rational numbers, irrational numbers, and real numbers
- Locate fractions on the number line
- Locate decimals on the number line

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

We have already described numbers as *counting numbers*, *whole numbers*, and *integers*. What is the difference between these types of numbers?

$$\begin{array}{ll} \text{Counting numbers} & 1, 2, 3, 4, \ldots \\ \text{Whole numbers} & 0, 1, 2, 3, 4, \ldots \\ \text{Integers} & \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \end{array}$$

What type of numbers would we get if we started with all the integers and then included all the fractions? The numbers we would have form the set of rational numbers. A rational number is a number that can be written as a ratio of two integers.

Rational Number

A **rational number** is a number of the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

A rational number can be written as the ratio of two integers.

All signed fractions, such as $\frac{4}{5}, -\frac{7}{8}, \frac{13}{4}, -\frac{20}{3}$ are rational numbers. Each numerator and each denominator is an integer.

Are integers rational numbers? To decide if an integer is a rational number, we try to write it as a ratio of two integers. Each integer can be written as a ratio of integers in many ways. For example, 3 is equivalent to $\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5} \ldots$

An easy way to write an integer as a ratio of integers is to write it as a fraction with denominator one.

$$3 = \frac{3}{1} \qquad -8 = -\frac{8}{1} \qquad 0 = \frac{0}{1}$$

Since any integer can be written as the ratio of two integers, *all integers are rational numbers*! Remember that the counting numbers and the whole numbers are also integers, and so they, too, are rational.

What about decimals? Are they rational? Let's look at a few to see if we can write each of them as the ratio of two integers.

We've already seen that integers are rational numbers. The integer -8 could be written as the decimal -8.0. So, clearly, some decimals are rational.

Think about the decimal 7.3. Can we write it as a ratio of two integers? Because 7.3 means $7\frac{3}{10}$, we can write it as an improper fraction, $\frac{73}{10}$. So 7.3 is the ratio of the integers 73 and 10. It is a rational number.

In general, any decimal that ends after a number of digits (such as 7.3 or -1.2684 is a rational number. We can use the place value of the last digit as the denominator when writing the decimal as a fraction.

EXAMPLE 1

Write as the ratio of two integers: a) -27 b) 7.31

Solution

a) Write it as a fraction with denominator 1.	-27 $\frac{-27}{1}$
b) Write it as a mixed number. Remember, 7 is the whole number and the decimal part, 0.31, indicates hundredths. Convert to an improper fraction.	7.31 $7\frac{31}{100}$ $\frac{731}{100}$

So we see that -27 and 7.31 are both rational numbers, since they can be written as the ratio of two integers.

TRY IT 1.1

Write as the ratio of two integers: a) -24 b) 3.57

Answer
a) $\frac{-24}{1}$ b) $\frac{357}{100}$

TRY IT 1.2

Write as the ratio of two integers: a) -19 b) 8.41

Answer
a) $\frac{-19}{1}$ b) $\frac{841}{100}$

Let's look at the decimal form of the numbers we know are rational.

We have seen that *every integer is a rational number*, since $a = \frac{a}{1}$ for any integer, *a*. We can also change any integer to a decimal by adding a decimal point and a zero.

Integer	-2	-1	0	1	2	3
Decimal form	-2.0	-1.0	0.0	1.0	2.0	3.0
These decimal numbers stop.						

We have also seen that *every fraction is a rational number*. Look at the decimal form of the fractions we considered above.

Ratio of integers	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$
The decimal form	0.8	-0.875	3.25	$-6.666..$
These decimals either stop or repeat.				

What do these examples tell us?

Every rational number can be written both as a ratio of integers, $\frac{p}{q}$,*where p and q are integers and* $q \neq 0$,*and as a decimal that either stops or repeats.*

Here are the numbers we looked at above expressed as a ratio of integers and as a decimal:

	Fractions				**Integers**					
Number	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$	-2	-1	0	1	2	3
Ratio of Integers	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$	$-\frac{2}{1}$	$-\frac{1}{1}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$
Decimal Form	0.8	-0.875	3.25	$-6.\overline{6}$	-2.0	-1.0	0.0	1.0	2.0	3.0

Rational Number

A **rational number** is a number of the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

Its decimal form stops or repeats.

Are there any decimals that do not stop or repeat? Yes!

The number π (the Greek letter *pi*, pronounced “pie”), which is very important in describing circles, has a decimal form that does not stop or repeat.

$$\pi = 3.141592654...$$

We can even create a decimal pattern that does not stop or repeat, such as

$$2.01001000100001\ldots$$

Numbers whose decimal form does not stop or repeat cannot be written as a fraction of integers. We call these numbers irrational. More on irrational numbers later on is this course.

Irrational Number

An irrational number is a number that cannot be written as the ratio of two integers.

Its decimal form does not stop and does not repeat.

Let's summarize a method we can use to determine whether a number is rational or irrational.

Rational or Irrational?

If the decimal form of a number

- *repeats or stops*, the number is **rational**.
- *does not repeat and does not stop*, the number is irrational

EXAMPLE 2

Given the numbers $0.58\bar{3}, 0.47, 3.605551275...$ list the a) rational numbers b) irrational numbers.

Solution

a) Look for decimals that repeat or stop.	The 3 repeats in $0.58\bar{3}$. The decimal 0.47 stops after the 7. So $0.58\bar{3}$ and 0.47 are rational.
b) Look for decimals that neither stop nor repeat.	$3.605551275\ldots$ has no repeating block of digits and it does not stop. So $3.605551275\ldots$ is irrational.

TRY IT 2.1

For the given numbers list the a) rational numbers b) irrational numbers: $0.29, 0.81\bar{6}, 2.515115111\ldots$.

Answer

a) $0.29, 0.81\bar{6}$ b) $2.515115111\ldots$

TRY IT 2.2

For the given numbers list the a) rational numbers b) irrational numbers: $2.6\bar{3}, 0.125, 0.418302\ldots$

Answer

a) $2.6\bar{3}, 0.125$ b) $0.418302\ldots$

We have seen that all counting numbers are whole numbers, all whole numbers are integers, and all integers are rational numbers. The irrational numbers are numbers whose decimal form does not stop and does not repeat. When we put together the rational numbers and the irrational numbers, we get the set of real numbers.

Real Number

A **real number** is a number that is either rational or irrational.

All the numbers we use in algebra are real numbers. Figure 1 illustrates how the number sets we've discussed in this section fit together.

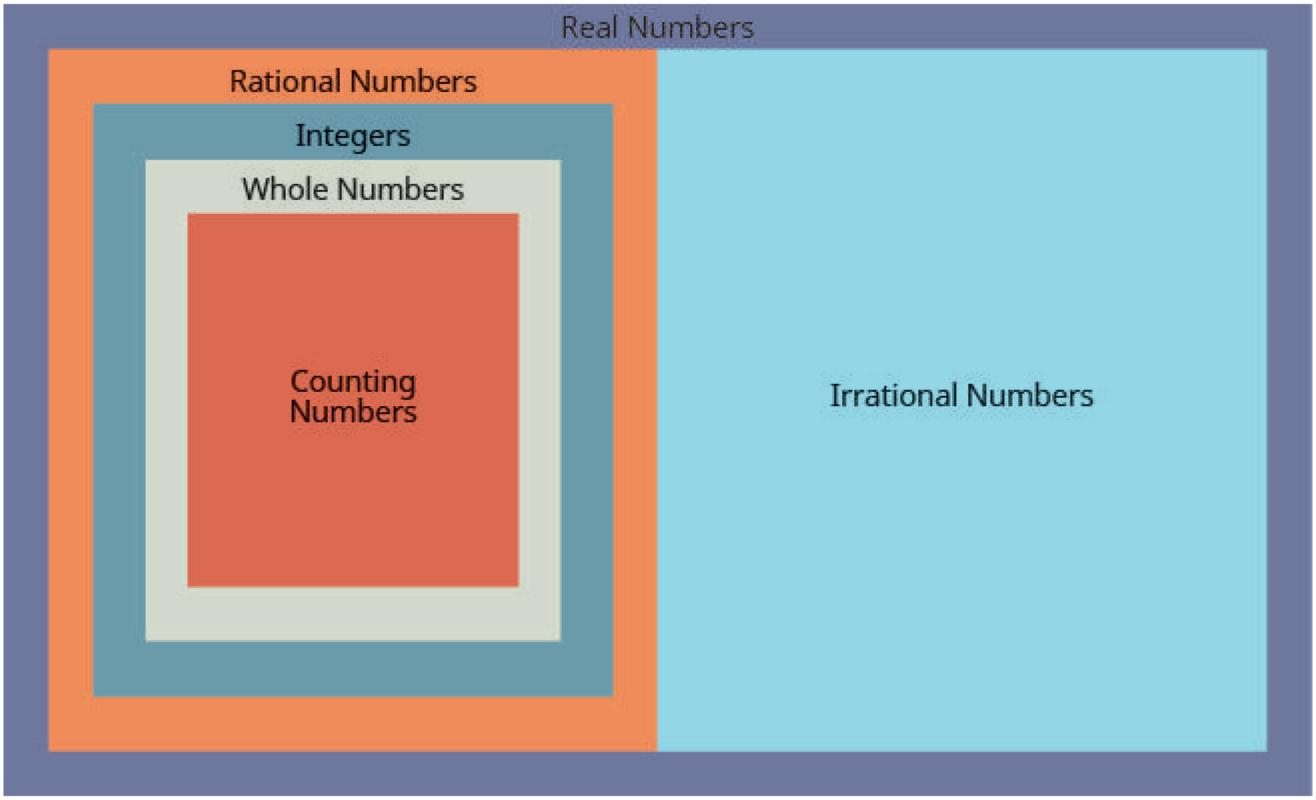


Figure 1 This chart shows the number sets that make up the set of real numbers. Does the term "real numbers" seem strange to you? Are there any numbers that are not "real," and, if so, what could they be?

EXAMPLE 3

Given the numbers -7, $\frac{14}{5}$, $8, 0, 5.9$, $-6.457...$, list the a) whole numbers b) integers c) rational numbers d) irrational numbers e) real numbers.

Solution

a) Remember, the whole numbers are 0, 1, 2, 3, … So 0 and 8 are the only whole numbers given.

b) The integers are the whole numbers, their opposites, and 0. So the whole numbers 0 and 8 are integers, and -7 is the opposite of a whole number so it is an integer,too. So the integers are $-7, 0, and 8$.

c) Since all integers are rational, then $-7, 0, 8$, are rational. Rational numbers also include fractions and decimals that repeat or stop, so $\frac{14}{5}$ and 5.9 are rational. So the list of rational numbers is $-7, 0$, $\frac{14}{5}$, $8, 5.9$,

d) Remember that 6.457… is a decimal that *does not repeat and does not stop* , so 6.457… is irrational.

e) All the numbers listed are real numbers.

TRY IT 3.1

For the given numbers, list the a) whole numbers b) integers c) rational numbers d) irrational numbers e) real numbers:
-3, -$\sqrt{2.97294...}$, $\frac{0}{5}\ \bar{3}$, $\frac{9}{5}$, 4, $\sqrt{49}$.

Answer

a) $4, \sqrt{49}$ b) $-3, 4, \sqrt{49}$ c) $-3, 0.\ \overline{3}, \frac{9}{5}, 4, \sqrt{49}$ d) $-\sqrt{2}$e)$-3, -\sqrt{2}, 0.\ \overline{3}, \frac{9}{5}, 4, \sqrt{49}$

TRY IT 3.2

For the given numbers, list the a) whole numbers b) integers c) rational numbers d) irrational numbers e) real numbers:
-$0.25, -\ \frac{3}{8}, -1, 6, 2.041975...$

Answer
a) $6, \sqrt{121}$ b) $-\sqrt{25}, -1, 6, \sqrt{121}$ c) $-\sqrt{25}, -\ \frac{3}{8}, -1, 6, \sqrt{121}$ d) $2.041975...$e) $-\sqrt{25}, -\ \frac{3}{8}, -1, 6, \sqrt{121}, 2.041975...$

Locate Fractions on the Number Line

The last time we looked at the number line, it only had positive and negative integers on it. We now want to include fractions and decimals on it.

Let's start with fractions and locate $\frac{1}{5}, -\ \frac{4}{5}, 3, \frac{7}{4}, -\ \frac{9}{2}, -5,\ \text{and}\ \frac{8}{3}$ on the number line.

We'll start with the whole numbers 3 and -5. because they are the easiest to plot. See Figure 2.

The proper fractions listed are $\frac{1}{5}\ \text{and}\ -\ \frac{4}{5}$. We know the proper fraction $\frac{1}{5}$ has value less than one and so would be located between $0\ \text{and}\ 1.$ The denominator is 5, so we divide the unit from 0 to 1 into 5 equal parts $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$. We plot $\frac{1}{5}$. See Figure 2.

Similarly, $-\ \frac{4}{5}$ is between 0 and -1. After dividing the unit into 5 equal parts we plot $-\ \frac{4}{5}$. See Figure 2.

Finally, look at the improper fractions $\frac{7}{4}, -\ \frac{9}{2}, \frac{8}{3}$. These are fractions in which the numerator is greater than the denominator. Locating these points may be easier if you change each of them to a mixed number. See Figure 2.

$\frac{7}{4} = 1\frac{3}{4} \qquad -\ \frac{9}{2} = -4\frac{1}{2} \qquad \frac{8}{3} = 2\frac{2}{3}$

Figure 2 shows the number line with all the points plotted.

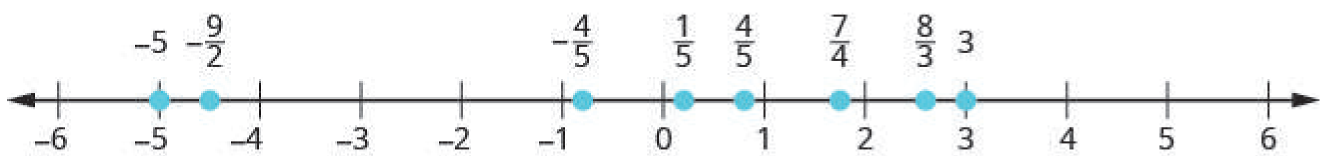


Figure 2

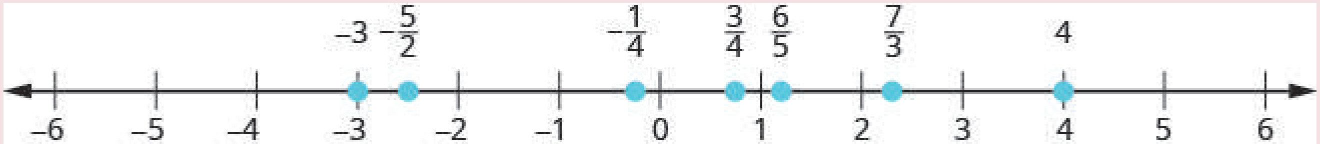
EXAMPLE 4

Locate and label the following on a number line: $4, \frac{3}{4}, -\frac{1}{4}, -3, \frac{6}{5}, -\frac{5}{2}$, and $\frac{7}{3}$.

Solution
Locate and plot the integers, $4, -3$.

Locate the proper fraction $\frac{3}{4}$ first. The fraction $\frac{3}{4}$ is between 0 and 1. Divide the distance between 0 and 1 into four equal parts then, we plot $\frac{3}{4}$. Similarly plot $-\frac{1}{4}$.

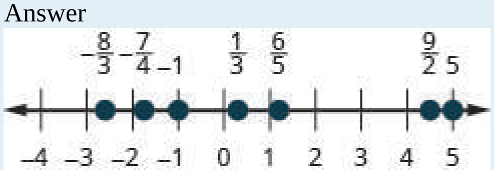
Now locate the improper fractions $\frac{6}{5}, -\frac{5}{2}, \frac{7}{3}$. It is easier to plot them if we convert them to mixed numbers and then plot them as described above: $\frac{6}{5} = 1\frac{1}{5}, -\frac{5}{2} = -2\frac{1}{2}, \frac{7}{3} = 2\frac{1}{3}$.



TRY IT 4.1

Locate and label the following on a number line: $-1, \frac{1}{3}, \frac{6}{5}, -\frac{7}{4}, \frac{9}{2}, 5, -\frac{8}{3}$.

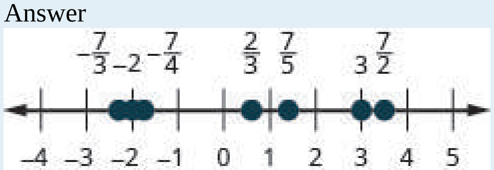
Answer



TRY IT 4.2

Locate and label the following on a number line: $-2, \frac{2}{3}, \frac{7}{5}, -\frac{7}{4}, \frac{7}{2}, 3, -\frac{7}{3}$.

Answer



In Example 5, we'll use the inequality symbols to order fractions. In previous chapters we used the number line to order numbers.

- $a < b$ "*a* is less than *b*" when *a* is to the left of *b* on the number line
- $a > b$ "*a* is greater than *b*" when *a* is to the right of *b* on the number line

As we move from left to right on a number line, the values increase.

EXAMPLE 5

Order each of the following pairs of numbers, using < or >. It may be helpful to refer Figure 3.

a) $-\frac{2}{3}$___-1 b) -$3\frac{1}{2}$___-3 c) $\frac{3}{4}$___$-\frac{1}{4}$ d) -2___$-\frac{8}{3}$

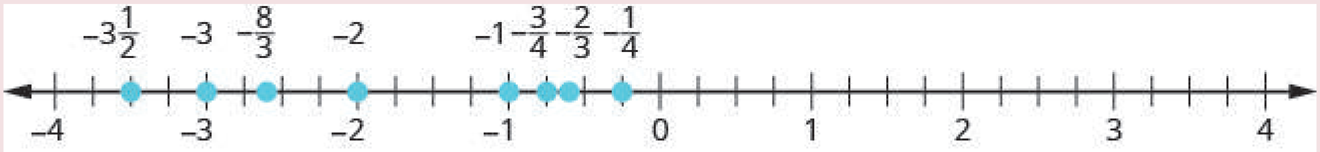


Figure 3

-$3\frac{1}{2}$___ -3

$-3\frac{1}{2} < -3$

Solution

a) $-\frac{2}{3}$ is to the right of -1 on the number line.	$-\frac{2}{3}$___-1 $-\frac{2}{3}$ > -1
b) -$3\frac{1}{2}$ is to the right of -3 on the number line.	$-3\frac{1}{2}$___ -3 $-3\frac{1}{2} < -3$
c) $-\frac{3}{4}$ is to the right of $-\frac{1}{4}$ on the number line.	$-\frac{3}{4}$___ $-\frac{1}{4}$ $-\frac{3}{4} < -\frac{1}{4}$
d) -2 is to the right of $-\frac{8}{3}$ on the number line.	-2___ $-\frac{8}{3}$ $-2 > -\frac{8}{3}$

TRY IT 5.1

Order each of the following pairs of numbers, using < or >:

a) $-\frac{1}{3}$___ -1 b) $-1\frac{1}{2}$___ -2 c) $-\frac{2}{3}$___ $-\frac{1}{3}$ d) -3___ $-\frac{7}{3}$.

Answer

a) > b) > c) < d) <

TRY IT 5.2

Order each of the following pairs of numbers, using < or >:

a) -1___ $-\frac{2}{3}$ b) $-2\frac{1}{4}$___ -2 c) $-\frac{3}{5}$___ $-\frac{4}{5}$ d) -4___ $-\frac{10}{3}$.

Answer

a) < b) < c) > d) <

Locate Decimals on the Number Line

Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

EXAMPLE 6

Locate 0.4 on the number line.

Solution

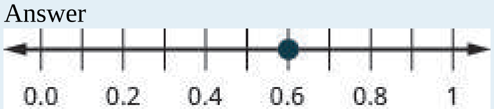
A proper fraction has value less than one. The decimal number 0.4 is equivalent to $\frac{4}{10}$, a proper fraction, so 0.4 is located between 0 and 1. On a number line, divide the interval between 0 and 1 into 10 equal parts. Now label the parts 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. We write 0 as 0.0 and 1 and 1.0, so that the numbers are consistently in tenths. Finally, mark 0.4 on the number line. See Figure 4.

Figure 4

TRY IT 6.1

Locate on the number line: 0.6

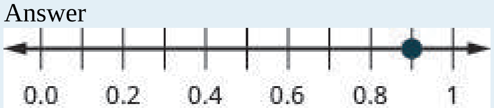
Answer



TRY IT 6.2

Locate on the number line: 0.9

Answer



EXAMPLE 7

Locate -0.74 on the number line.

Solution

The decimal -0.74 is equivalent to $-\frac{74}{100}$, so it is located between 0 and -1. On a number line, mark off and label the hundredths in the interval between 0 and -1. See Figure 5.

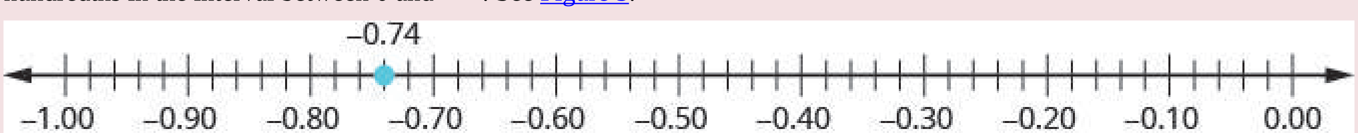


Figure 5

TRY IT 7.1

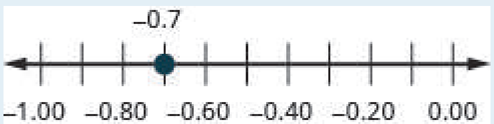
Locate on the number line: -0.6.

Answer

TRY IT 7.2

Locate on the number line: -0.7.

Answer



Which is larger, 0.04 or 0.40? If you think of this as money, you know that ?0.40 (forty cents) is greater than ?0.04 (four cents). So,

$$0.40 > 0.04$$

Again, we can use the number line to order numbers.

- *a* < *b* "*a* is less than *b*" when *a* is to the left of *b* on the number line
- *a* > *b* "*a* is greater than *b*" when *a* is to the right of *b* on the number line

Where are 0.04 and 0.40 located on the number line? See Figure 6.

Figure 6

We see that 0.40 is to the right of 0.04 on the number line. This is another way to demonstrate that 0.40 > 0.04

How does 0.31 compare to 0.308? This doesn't translate into money to make it easy to compare. But if we convert 0.31 and 0.308 into fractions, we can tell which is larger.

	0.31	0.308
Convert to fractions.	$\frac{31}{100}$	$\frac{308}{1000}$
We need a common denominator to compare them.	$\frac{31 \cdot 10}{100 \cdot 10}$	$\frac{308}{1000}$
	$\frac{310}{1000}$	$\frac{308}{1000}$

Because 310 > 308, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, 0.31 > 0.308

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives 0.310. So 0.31 is equivalent to 0.310. Writing zeros at the end of a decimal does not change its value!

$$\frac{31}{100} = \frac{310}{1000} \quad \text{and} \quad 0.31 = 0.310$$

We say 0.31 and 0.310 are equivalent decimals.

Equivalent Decimals

Two decimals are equivalent if they convert to equivalent fractions.

We use equivalent decimals when we order decimals.

The steps we take to order decimals are summarized here.

HOW TO: Order Decimals.

1. Write the numbers one under the other, lining up the decimal points.
2. Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
3. Compare the numbers as if they were whole numbers.
4. Order the numbers using the appropriate inequality sign.

EXAMPLE 8

Order $0.64\_\_\_\_0.6$ using < or >.

Solution

Write the numbers one under the other, lining up the decimal points.	0.64 $\ 0.6$
Add a zero to 0.6 to make it a decimal with 2 decimal places. Now they are both hundredths.	0.64 0.60
64 is greater than 60.	$64 > 60$
64 hundredths is greater than 60 hundredths.	$0.64 > 0.60$
	$0.64 > 0.6$

TRY IT 8.1

Order each of the following pairs of numbers, using $<$ or $>$: $0.42\_\_\_\_0.4$.

Answer
>

TRY IT 8.2

Order each of the following pairs of numbers, using $<$ or $>$: 0.18____0.1.

Answer

>

EXAMPLE 9

Order 0.83____0.803 using < or >.

Solution

	0.83____0.803
Write the numbers one under the other, lining up the decimals.	0.83 0.803
They do not have the same number of digits. Write one zero at the end of 0.83.	0.830 0.803
Since $830 > 803$, 830 thousandths is greater than 803 thousandths.	$0.830 > 0.803$
	$0.83 > 0.803$

TRY IT 9.1

Order the following pair of numbers, using $<$ or $>$: 0.76____0.706.

Answer

>

TRY IT 9.2

Order the following pair of numbers, using < or > : 0.305____0.35.

Answer

<

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because -2 lies to the right of -3 on the number line, we know that $-2 > -3$. Similarly, smaller numbers lie to the left on the number line. For example, because -9 lies to the left of -6 on the number line, we know that $-9 < -6$. See Figure 7.

Figure 7

If we zoomed in on the interval between 0 and -1, as shown in Example 10, we would see in the same way that $-0.2 > -0.3$ and $-0.9 < -0.6$.

EXAMPLE 10

Use < or > to order $-0.1\_\_\_\_\_ -0.8.$

Solution

	$-0.1\_\_\_\_\_ -0.8$
Write the numbers one under the other, lining up the decimal points. They have the same number of digits.	-0.1 -0.8
Since $-1 > -8$, −1 tenth is greater than −8 tenths.	$-0.1 > -0.8$

TRY IT 10.1

Order the following pair of numbers, using < or >: $-0.3\_\_\_\_\_ -0.5.$

Answer
>

TRY IT 10.2

Order the following pair of numbers, using < or >: $-0.6\_\_\_\_\_ -0.7.$

Answer
>

Key Concepts

- **Order Decimals**
  1. Write the numbers one under the other, lining up the decimal points.
  2. Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
  3. Compare the numbers as if they were whole numbers.
  4. Order the numbers using the appropriate inequality sign.

Glossary

equivalent decimals
Two decimals are equivalent if they convert to equivalent fractions.

irrational number
An irrational number is a number that cannot be written as the ratio of two integers. Its decimal form does not stop and does not repeat.

rational number

A rational number is a number of the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$. A rational number can be written as the ratio of two integers. Its decimal form stops or repeats.

real number

A real number is a number that is either rational or irrational.

Practice Makes Perfect

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

In the following exercises, write as the ratio of two integers.

1. a) 5 b) 3.19	2. a) 8 b) 1.61
3. a) -12 b) 9.279	4. a) -16 b) 4.399

In the following exercises, list the a) rational numbers, b) irrational numbers

5. $0.75, 0.22\overline{3}, 1.39174$	6. $0.36, 0.94729\ldots, 2.52\overline{8}$
7. $0.4\overline{5}, 1.919293\ldots, 3.59$	8. $0.1\overline{3}, 0.42982\ldots, 1.875$

In the following exercises, list the a) whole numbers, b) integers, c) rational numbers, d) irrational numbers, e) real numbers for each set of numbers.

9. $-8, 0, 1.95286\ldots, \frac{12}{5}, 9$	10. $-9, -3\frac{4}{9}, 0.40\overline{9}, \frac{11}{6}, 7$
11. $-7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$	12. $-6, -\frac{5}{2}, 0, 0.\overline{714285}, 2\frac{1}{5}$

Locate Fractions on the Number Line

In the following exercises, locate the numbers on a number line.

13. $\frac{3}{4}, \frac{8}{5}, \frac{10}{3}$	14. $\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$
15. $\frac{3}{10}, \frac{7}{2}, \frac{11}{6}, 4$	16. $\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$
17. $\frac{2}{5}, -\frac{2}{5}$	18. $\frac{3}{4}, -\frac{3}{4}$
19. $\frac{3}{4}, -\frac{3}{4}, 1\frac{2}{3}, -1\frac{2}{3}, \frac{5}{2}, -\frac{5}{2}$	20. $\frac{1}{5}, -\frac{2}{5}, 1\frac{3}{4}, -1\frac{3}{4}, \frac{8}{3}, -\frac{8}{3}$

In the following exercises, order each of the pairs of numbers, using < or >.

21. -1_____ $-\frac{1}{4}$	22. -1_____ $-\frac{1}{3}$
23. $-2\frac{1}{2}$_____ -3	24. $-1\frac{3}{4}$_____ -2
25. $-\frac{5}{12}$_____ $-\frac{7}{12}$	26. $-\frac{9}{10}$_____ $-\frac{3}{10}$
27. -3_____ $-\frac{13}{5}$	28. -4_____ $-\frac{23}{6}$

Locate Decimals on the Number Line In the following exercises, locate the number on the number line.

29. 0.8	30. -0.9
31. -1.6	32. 3.1

In the following exercises, order each pair of numbers, using < or >.

33. 0.37_____0.63	34. 0.86_____0.69
35. 0.91_____0.901	36. 0.415_____0.41
37. -0.5_____ -0.3	38. -0.1_____ -0.4
39. -0.62_____ -0.619	40. -7.31_____ -7.3

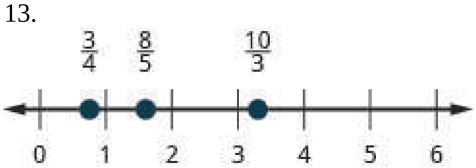
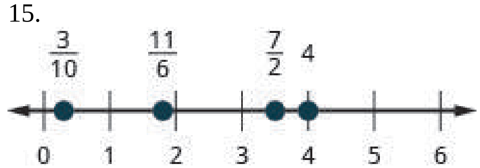
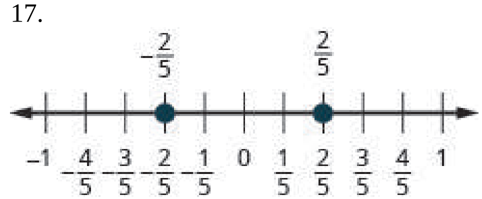
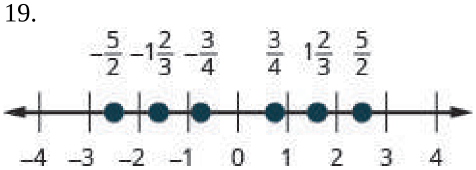
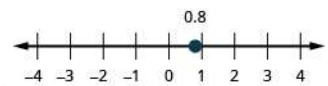
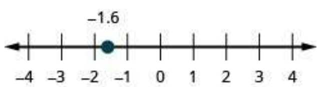
Everyday Math

41. **Field trip.** All the 5th graders at Lord Selkirk Elementary School will go on a field trip to the science museum. Counting all the children, teachers, and chaperones, there will be 147 people. Each bus holds 44 people. a) How many buses will be needed? b) Why must the answer be a whole number? c) Why shouldn't you round the answer the usual way, by choosing the whole number closest to the exact answer?	42. **Child care.** Serena wants to open a licensed child care center. Her state requires there be no more than 12 children for each teacher. She would like her child care centre to serve 40 children. a) How many teachers will be needed? b) Why must the answer be a whole number? c) Why shouldn't you round the answer the usual way, by choosing the whole number closest to the exact answer?

Writing Exercises

43. In your own words, explain the difference between a rational number and an irrational number.	44. Explain how the sets of numbers (counting, whole, integer, rational, irrationals, reals) are related to each other.

Answers

1. a) $\frac{5}{1}$ b) $\frac{319}{100}$	3. a) $\frac{-12}{1}$ b) $\frac{9297}{1000}$	5. a) $0.75, 0.22\overline{3}$ b) $1.39174\ldots$
7. a) $0.4\overline{5}, 3.59$ b) $1.919293\ldots$	9. a) $0, 9$ b) $-8, 9$ c) $-8, 0, \frac{12}{5}, 9$ d) $1.95286\ldots$ e) $-8, 0, 1.95286\ldots, \frac{12}{5}, 9$	11. a) none b) $-7, -1$ c) -7, $-\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$ d) none e) $-7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$
13. 	15. 	17. 
19. 	21. <	23. >
25. >	27. <	29. 
31. 	33. <	35. >
37. <	39. <	41. a) 4 buses b) answers may vary c) answers may vary
43. Answers may vary.		

Attributions

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2.5 Properties of Real Numbers

Learning Objectives

By the end of this section, you will be able to:

- Use the commutative and associative properties
- Use the identity and inverse properties of addition and multiplication
- Use the properties of zero
- Simplify expressions using the distributive property

Use the Commutative and Associative Properties

Think about adding two numbers, say 5 and 3. The order we add them doesn't affect the result, does it?

$5+3 \qquad 3+5$

$8 \qquad 8$

$5+3=3+5$

The results are the same.

As we can see, the order in which we add does not matter!

What about multiplying $5 \text{ and } 3?$

$5\cdot 3 \qquad 3\cdot 5$

$15 \qquad 15$

$5\cdot 3=3\cdot 5$

Again, the results are the same!

The order in which we multiply does not matter!

These examples illustrate the commutative property. When adding or multiplying, changing the *order* gives the same result.

Commutative Property

of Addition	If a, b are real numbers, then	$a+b$	$=$	$b+a$
of Multiplication	If a, b are real numbers, then	$a\cdot b$	$=$	$b\cdot a$

When adding or multiplying, changing the *order* gives the same result.

The commutative property has to do with order. If you change the order of the numbers when adding or multiplying, the result is the same.

What about subtraction? Does order matter when we subtract numbers? Does $7-3$ give the same result as $3-7?$

$7-3 \qquad 3-7$

$4 \qquad -4$

$$4 \neq -4$$
$$7-3 \neq 3-7$$

The results are not the same.

Since changing the order of the subtraction did not give the same result, we know that *subtraction is not commutative*.

Let’s see what happens when we divide two numbers. Is division commutative?

$12 \div 4 \qquad 4 \div 12$

$\frac{12}{4} \qquad \frac{4}{12}$

$3 \qquad \frac{1}{3}$

$$3 \neq \frac{1}{3}$$
$$12 \div 4 \neq 4 \div 12$$

The results are not the same.

Since changing the order of the division did not give the same result, *division is not commutative*. The commutative properties only apply to addition and multiplication!

- Addition and multiplication *are* commutative.
- Subtraction and Division *are not* commutative.

If you were asked to simplify this expression, how would you do it and what would your answer be?

$7+8+2$

Some people would think $7+8$ is 15 and then $15+2$ is 17. Others might start with $8+2$ makes 10 and then $7+10$ makes 17.

Either way gives the same result. Remember, we use parentheses as grouping symbols to indicate which operation should be done first.

Add $7+8$. Add.	$(7+8)+2$ $15+2$ 17
Add $8+2$. Add.	$7+(8+2)$ $7+10$ 17
	$(7+8)+2=7+(8+2)$

When adding three numbers, changing the grouping of the numbers gives the same result.

This is true for multiplication, too.

Multiply. $5 \cdot \frac{1}{3}$ Multiply.	$\left(5 \cdot \frac{1}{3}\right) \cdot 3$ $\frac{5}{3} \cdot 3$ 5
Multiply. $\frac{1}{3} \cdot 3$. Multiply.	$5 \cdot \left(\frac{1}{3} \cdot 3\right)$ $5 \cdot 1$ 5
	$\left(5 \cdot \frac{1}{3}\right) \cdot 3 = 5 \cdot \left(\frac{1}{3} \cdot 3\right)$

When multiplying three numbers, changing the grouping of the numbers gives the same result.

You probably know this, but the terminology may be new to you. These examples illustrate the associative property.

Associative Property

of Addition	If a, b, c are real numbers, then $(a+b)+c = a+(b+c)$
of Multiplication	If a, b, c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

When adding or multiplying, changing the *grouping* gives the same result.

Let's think again about multiplying $5 \cdot \frac{1}{3} \cdot 3$. We got the same result both ways, but which way was easier? Multiplying $\frac{1}{3}$ and 3 first, as shown above on the right side, eliminates the fraction in the first step. Using the associative property can make the math easier!

The associative property has to do with grouping. If we change how the numbers are grouped, the result will be the same. Notice it is the same three numbers in the same order—the only difference is the grouping.

We saw that subtraction and division were not commutative. They are not associative either.

When simplifying an expression, it is always a good idea to plan what the steps will be. In order to combine like terms in the next example, we will use the commutative property of addition to write the like terms together.

EXAMPLE 1

Simplify: $18p + 6q + 15p + 5q$.

Solution

	$18p + 6q + 15p + 5q$
Use the commutative property of addition to re-order so that like terms are together.	$18p + 15p + 6q + 5q$
Add like terms.	$33p + 11q$

TRY IT 1.1

Simplify: $23r + 14s + 9r + 15s$.

Answer
$32r + 29s$

TRY IT 1.2

Simplify: $37m + 21n + 4m - 15n$.

Answer
$41m + 6n$

When we have to simplify algebraic expressions, we can often make the work easier by applying the commutative or associative property first, instead of automatically following the order of operations. When adding or subtracting fractions, combine those with a common denominator first.

EXAMPLE 2

Simplify: $\left(\frac{5}{13} + \frac{3}{4}\right) + \frac{1}{4}$.

Solution

	$\left(\frac{5}{13} + \frac{3}{4}\right) + \frac{1}{4}$
Notice that the last 2 terms have a common denominator, so change the grouping.	$\frac{5}{13} + \left(\frac{3}{4} + \frac{1}{4}\right)$
Add in parentheses first.	$\frac{5}{13} + \left(\frac{4}{4}\right)$
Simplify the fraction.	$\frac{5}{13} + 1$
Add.	$1\frac{5}{13}$
Convert to an improper fraction.	$\frac{18}{13}$

TRY IT 2.1

Simplify: $\left(\frac{7}{15} + \frac{5}{8}\right) + \frac{3}{8}$.

Answer
$1\frac{7}{15}$

TRY IT 2.2

Simplify: $\left(\frac{2}{9} + \frac{7}{12}\right) + \frac{5}{12}$.

Answer
$1\frac{2}{9}$

EXAMPLE 3

Use the associative property to simplify $6\,(3x)$.

Solution

	$6\,(3x)$
Change the grouping.	$(6\cdot3)\,x$
Multiply in the parentheses.	$18x$

Notice that we can multiply $6\cdot3$ but we could not multiply 3*x* without having a value for *x*.

TRY 3.1

Use the associative property to simplify 8(4*x*).

Answer
32*x*

TRY IT 3.2

Use the associative property to simplify $-9\,(7y)$.

Answer
$-63y$

Use the Identity and Inverse Properties of Addition and Multiplication

What happens when we add 0 to any number? Adding 0 doesn't change the value. For this reason, we call 0 the additive identity.

For example,

$$\begin{array}{ccc} 13+0 & -14+0 & 0+(-8) \\ 13 & -14 & -8 \end{array}$$

These examples illustrate the Identity Property of Addition that states that for any real number a, $a+0=a$ and $0+a=a$.

What happens when we multiply any number by one? Multiplying by 1 doesn't change the value. So we call 1 the multiplicative identity.

For example,

$$\begin{array}{ccc} 43\cdot1 & -27\cdot1 & 1\cdot\frac{3}{5} \\ 43 & -27 & \frac{3}{5} \end{array}$$

These examples illustrate the Identity Property of Multiplication that states that for any real number a, $a \cdot 1 = a$ and $1 \cdot a = a$.

We summarize the Identity Properties below.

Identity Property

of addition For any real number a: 0 is the additive identity	$a + 0 = a$	$0 + a = a$
of multiplication For any real number a: 1 is the multiplicative identity	$a \cdot 1 = a$	$1 \cdot a = a$

What number added to 5 gives the additive identity, 0?

$5 + \_\_\_\_ = 0$ We know $5 + (-5) = 0$

What number added to –6 gives the additive identity, 0?

$-6 + \_\_\_\_ = 0$ We know $-6 + 6 = 0$

Notice that in each case, the missing number was the opposite of the number!

We call $-a$. the additive inverse of *a*. *The opposite of a number is its additive inverse.* A number and its opposite add to zero, which is the additive identity. This leads to the Inverse Property of Addition that states for any real number $a, a + (-a) = 0$. Remember, a number and its opposite add to zero.

What number multiplied by $\frac{2}{3}$ gives the multiplicative identity, 1? In other words, $\frac{2}{3}$ times what results in 1?

$$\frac{2}{3} \cdot \_\_\_ = 1 \qquad \text{We know } \frac{2}{3} \cdot \frac{3}{2} = 1$$

What number multiplied by 2 gives the multiplicative identity, 1? In other words 2 times what results in 1?

$$2 \cdot \_\_\_ = 1 \qquad \text{We know } 2 \cdot \frac{1}{2} = 1$$

Notice that in each case, the missing number was the reciprocal of the number!

We call $\frac{1}{a}$ the multiplicative inverse of *a*. *The reciprocal of a number is its multiplicative inverse.* A number and its reciprocal multiply to one, which is the multiplicative identity. This leads to the Inverse Property of Multiplication that states that for any real number $a, a \neq 0, a \cdot \frac{1}{a} = 1$.

We'll formally state the inverse properties here:

Inverse Property

of addition	For any real number a, $-a$ is the **additive inverse** of a. A number and its opposite add to zero.	$a+(-a)=0$
of multiplication	For any real number a, $\frac{1}{a}$ is the **multiplicative inverse** of a. A number and its reciprocal multiply to one.	$a\cdot\frac{1}{a}=1$

EXAMPLE 4

Find the additive inverse of a) $\frac{5}{8}$ b) 0.6 c) -8 d) $-\frac{4}{3}$.

Solution
To find the additive inverse, we find the opposite.

a. The additive inverse of $\frac{5}{8}$ is the opposite of $\frac{5}{8}$. The additive inverse of $\frac{5}{8}$ is $-\frac{5}{8}$.
b. The additive inverse of 0.6 is the opposite of 0.6. The additive inverse of 0.6 is -0.6.
c. The additive inverse of -8 is the opposite of -8. We write the opposite of -8 as $-(-8)$, and then simplify it to 8. Therefore, the additive inverse of -8 is 8.
d. The additive inverse of $-\frac{4}{3}$ is the opposite of $-\frac{4}{3}$. We write this as $-\left(-\frac{4}{3}\right)$, and then simplify to $\frac{4}{3}$. Thus, the additive inverse of $-\frac{4}{3}$ is $\frac{4}{3}$.

TRY IT 4.1

Find the additive inverse of: a) $\frac{7}{9}$ b) 1.2 c) -14 d) $-\frac{9}{4}$.

Answer
a) $-\frac{7}{9}$ b) -1.2 c)14 d) $\frac{9}{4}$

Exercises

Find the additive inverse of: a) $\frac{7}{13}$ b) 8.4 c) -46 d) $-\frac{5}{2}$.

Answer
a) $-\frac{7}{13}$ b) -8.4 c) 46 d) $\frac{5}{2}$

EXAMPLE 5

Find the multiplicative inverse of a) 9 b) $-\frac{1}{9}$ c) 0.9.

Solution

To find the multiplicative inverse, we find the reciprocal.

a. The multiplicative inverse of 9 is the reciprocal of 9, which is $\frac{1}{9}$. Therefore, the multiplicative inverse of 9 is $\frac{1}{9}$.
b. The multiplicative inverse of $-\frac{1}{9}$ is the reciprocal of $-\frac{1}{9}$, which is -9. Thus, the multiplicative inverse of $-\frac{1}{9}$ is -9.
c. To find the multiplicative inverse of 0.9, we first convert 0.9 to a fraction, $\frac{9}{10}$. Then we find the reciprocal of the fraction. The reciprocal of $\frac{9}{10}$ is $\frac{10}{9}$. So the multiplicative inverse of 0.9 is $\frac{10}{9}$.

TRY IT 5.1

Find the multiplicative inverse of a) 4 b) $-\frac{1}{7}$ c) 0.3

Answer
a) $\frac{1}{4}$ b) -7 c) $\frac{10}{3}$

TRY IT 5.2

Find the multiplicative inverse of a) 18 b) $-\frac{4}{5}$ c) 0.6.

Answer
a) $\frac{1}{18}$ b) $-\frac{5}{4}$ c) $\frac{5}{3}$

Use the Properties of Zero

The identity property of addition says that when we add 0 to any number, the result is that same number. What happens when we multiply a number by 0? Multiplying by 0 makes the product equal zero.

Multiplication by Zero

For any real number *a*.

$a \cdot 0 = 0$ $\qquad$ $0 \cdot a = 0$

The product of any real number and 0 is 0.

What about division involving zero? What is $0 \div 3?$ Think about a real example: If there are no cookies in the cookie jar and 3 people are to share them, how many cookies does each person get? There are no cookies to share, so each person gets 0 cookies. So,

$0 \div 3 = 0$

We can check division with the related multiplication fact.

$12 \div 6 = 2 \text{ because } 2 \cdot 6 = 12.$

So we know $0 \div 3 = 0$ because $0 \cdot 3 = 0$.

Division of Zero

For any real number *a*, except $0, \frac{0}{a} = 0$ and $0 \div a = 0$.

Zero divided by any real number except zero is zero.

Now think about dividing *by* zero. What is the result of dividing 4 by 0? Think about the related multiplication fact: $4 \div 0 = ?$ means $? \cdot 0 = 4$. Is there a number that multiplied by 0 gives 4? Since any real number multiplied by 0 gives 0, there is no real number that can be multiplied by 0 to obtain 4

We conclude that there is no answer to $4 \div 0$ and so we say that division by 0 is undefined.

Division by Zero

For any real number *a*, except 0, $\frac{a}{0}$ and $a \div 0$ are undefined.

Division by zero is undefined.

We summarize the properties of zero below.

Properties of Zero

Multiplication by Zero: For any real number *a*,

$a \cdot 0 = 0 \quad 0 \cdot a = 0$	The product of any number and 0 is 0.

Division of Zero, Division by Zero: For any real number $a, a \neq 0$

$\frac{0}{a} = 0$	Zero divided by any real number except itself is zero.
$\frac{a}{0}$ is undefined	Division by zero is undefined.

EXAMPLE 6

Simplify: a) $-8 \cdot 0$ b) $\frac{0}{-2}$ c) $\frac{-32}{0}$.

Solution

a) The product of any real number and 0 is 0.	$-8 \cdot 0$ 0
b) Zero divided by any real number except itself is zero.	$\frac{0}{-2}$ 0
c) Division by 0 is undefined.	$\frac{-32}{0}$ Undefined

TRY IT 6.1

Simplify: a) $-14 \cdot 0$ b) $\frac{0}{-6}$ c) $\frac{-2}{0}$.

Answer
a) 0 b) 0 c) undefined

TRY IT 6.2

Simplify: a) $0\left(-17\right)$ b) $\frac{0}{-10}$ c) $\frac{-5}{0}$.

Answer
a) 0 b) 0 c) undefined

We will now practice using the properties of identities, inverses, and zero to simplify expressions.

EXAMPLE 7

Simplify: a) $\frac{0}{n+5}$, where $n \neq -5$ b) $\frac{10-3p}{0}$, where $10 - 3p \neq 0$.

Solution

a) Zero divided by any real number except itself is 0.	$\frac{0}{n+5}$ 0
b) Division by 0 is undefined.	$\frac{10-3p}{0}$ Undefined

TRY IT 7.1

Simplify: a) $\frac{0}{m+7}$, where $m \neq -7$ b) $\frac{18-6c}{0}$, where $18 - 6c \neq 0$.

Answer

a) 0 b) undefined

TRY IT 7.2

Simplify: a) $\frac{0}{d-4}$, where $d \neq 4$ b) $\frac{15-4q}{0}$, where $15 - 4q \neq 0$.

Answer
a) 0 b) undefined

EXAMPLE 8

Simplify: $-84n + (-73n) + 84n$.

Solution

	$-84n + (-73n) + 84n$
Notice that the first and third terms are opposites; use the commutative property of addition to re-order the terms.	$-84n + 84n + (-73n)$
Add left to right.	$0 + (-73)$
Add.	$-73n$

TRY IT 8.1

Simplify: $-27a + (-48a) + 27a$.

Answer
$-48a$

TRY IT 8.2

Simplify: $39x + (-92x) + (-39x)$.

Answer
$-92x$

Now we will see how recognizing reciprocals is helpful. Before multiplying left to right, look for reciprocals—their product is 1

EXAMPLE 9

Simplify: $\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$.

Solution

	$\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$
Notice that the first and third terms are reciprocals, so use the commutative property of multiplication to re-order the factors.	$\frac{7}{15} \cdot \frac{15}{7} \cdot \frac{8}{23}$
Multiply left to right.	$1 \cdot \frac{8}{23}$
Multiply.	$\frac{8}{23}$

TRY IT 9.1

Simplify: $\frac{9}{16} \cdot \frac{5}{49} \cdot \frac{16}{9}$.

Answer
$\frac{5}{49}$

TRY IT 9.2

Simplify: $\frac{6}{17} \cdot \frac{11}{25} \cdot \frac{17}{6}$.

Answer
$\frac{11}{25}$

EXAMPLE 10

Simplify: $\frac{3}{4} \cdot \frac{4}{3}\,(6x + 12)$.

Solution

	$\frac{3}{4} \cdot \frac{4}{3}\,(6x + 12)$
There is nothing to do in the parentheses, so multiply the two fractions first—notice, they are reciprocals.	$1\,(6x + 12)$
Simplify by recognizing the multiplicative identity.	$6x + 12$

TRY IT 10.1

Simplify: $\frac{2}{5} \cdot \frac{5}{2}\,(20y + 50)$.

Answer
$20y + 50$

TRY IT 10.2

Simplify: $\frac{3}{8} \cdot \frac{8}{3}(12z + 16)$.

Answer

$12z + 16$

Simplify Expressions Using the Distributive Property

Suppose that three friends are going to the movies. They each need $9.25—that's 9 dollars and 1 quarter—to pay for their tickets. How much money do they need all together?

You can think about the dollars separately from the quarters. They need 3 times $9 so $27, and 3 times 1 quarter, so 75 cents. In total, they need $27.75. If you think about doing the math in this way, you are using the **distributive property**.

Distributive Property

If a, b, c are real numbers, then $\quad a(b + c) = ab + ac$

Also, $\quad (b + c)a = ba + ca$

$a(b - c) = ab - ac$

$(b - c)a = ba - ca$

Back to our friends at the movies, we could find the total amount of money they need like this:

3(9.25)
3(9 + 0.25)
3(9) + 3(0.25)
27 + 0.75
27.75

In algebra, we use the distributive property to remove parentheses as we simplify expressions.

For example, if we are asked to simplify the expression $3(x + 4)$, the order of operations says to work in the parentheses first. But we cannot add *x* and 4, since they are not like terms. So we use the distributive property, as shown in (Example 11).

EXAMPLE 11

Simplify: $3(x + 4)$.

Solution

	$3\left(x+4\right)$
Distribute.	$3\cdot x+3\cdot 4$
Multiply.	$3x+12$

TRY IT 11.1

Simplify: $4\left(x+2\right)$.

Answer
$4x+8$

TRY IT 11.2

Simplify: $6\left(x+7\right)$.

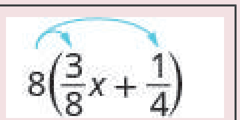
Answer
$6x+42$

Some students find it helpful to draw in arrows to remind them how to use the distributive property. Then the first step in (Example 11) would look like this:

EXAMPLE 12

Simplify: $8\left(\frac{3}{8}x+\frac{1}{4}\right)$.

Solution

	
Distribute.	$8\cdot\frac{3}{8}x+8\cdot\frac{1}{4}$
Multiply.	$3x+2$

TRY IT 12.1

Simplify: $6\left(\frac{5}{6}y+\frac{1}{2}\right)$.

Answer
$5y+3$

TRY IT 12.2

Simplify: $12\left(\frac{1}{3}n+\frac{3}{4}\right)$.

Answer
$4n+9$

Using the distributive property as shown in (Example 13) will be very useful when we solve money applications in later chapters.

EXAMPLE 13

Simplify: $100\left(0.3+0.25q\right)$.

Solution

	$100(0.3+0.25q)$
Distribute.	$100(0.3)+100(0.25q)$
Multiply.	$30+25q$

TRY IT 13.1

Simplify: $100\left(0.7+0.15p\right)$.

Answer
$70+15p$

TRY IT 13.2

Simplify: $100\left(0.04+0.35d\right)$.

Answer
$4+35d$

When we distribute a negative number, we need to be extra careful to get the signs correct!

EXAMPLE 14

Simplify: $-2\left(4y+1\right)$.

Solution

	$-2(4y+1)$
Distribute.	$-2 \cdot 4y + (-2) \cdot 1$
Multiply.	$-8y - 2$

TRY IT 14.1

Simplify: $-3\left(6m+5\right)$.

Answer
$-18m-15$

TRY IT 14.2

Simplify: $-6\left(8n+11\right)$.

Answer
$-48n-66$

EXAMPLE 15

Simplify: $-11\left(4-3a\right)$.

Solution

Distribute.	$-11(4-3a)$
Multiply.	$-11 \cdot 4 - (-11) \cdot 3a$ $-44 - (-33a)$
Simplify.	$-44 + 33a$

Notice that you could also write the result as $33a-44$. Do you know why?

TRY IT 15.1

Simplify: $-5\left(2-3a\right)$.

Answer
$-10+15a$

TRY IT 15.2

Simplify: $-7\left(8-15y\right)$.

Answer
$-56+105y$

(Example 16) will show how to use the distributive property to find the opposite of an expression.

EXAMPLE 16

Simplify: $-\left(y+5\right)$.

Solution

	$\left(y+5\right)$
Multiplying by −1 results in the opposite.	$-1\left(y+5\right)$
Distribute.	$-1\cdot y+\left(-1\right)\cdot 5$
Simplify.	$-y+\left(-5\right)$
	$-y-5$

TRY IT 16.1

Simplify: $-\left(z-11\right)$.

Answer
$-z+11$

TRY IT 16.2

Simplify: $-\left(x-4\right)$.

Answer
$-x+4$

There will be times when we'll need to use the distributive property as part of the order of operations. Start by looking at the parentheses. If the expression inside the parentheses cannot be simplified, the next step would be multiply using the distributive property, which removes the parentheses. The next two examples will illustrate this.

EXAMPLE 17

Simplify: $8 - 2\,(x + 3)$.

Be sure to follow the order of operations. Multiplication comes before subtraction, so we will distribute the 2 first and then subtract.

Solution

	$8 - 2\,(x + 3)$
Distribute.	$8 - 2 \cdot x - 2 \cdot 3$
Multiply.	$8 - 2x - 6$
Combine like terms.	$-2x + 2$

TRY IT 17.1

Simplify: $9 - 3\,(x + 2)$.

Answer
$3 - 3x$

TRY IT 17.2

Simplify: $7x - 5\,(x + 4)$.

Answer
$2x - 20$

EXAMPLE 18

Simplify: $4\,(x - 8) - (x + 3)$.

Solution

	$4\,(x - 8) - (x + 3)$
Distribute.	$4x - 32 - x - 3$
Combine like terms.	$3x - 35$

TRY IT 18.1

Simplify: $6\,(x - 9) - (x + 12)$.

Answer

$5x-66$

TRY IT 18.2

Simplify: $8(x-1)-(x+5)$.

Answer
$7x-13$

All the properties of real numbers we have used in this chapter are summarized in the table below.

Commutative Property	**of addition** If a, b are real numbers, then	$a+b=b+a$
	of multiplication If a, b are real numbers, then	$a \cdot b=b \cdot a$
Associative Property	**of addition** If a, b, c are real numbers, then	$(a+b)+c=a+(b+c)$
	of multiplication If a, b, c are real numbers, then	$(a \cdot b) \cdot c=a \cdot(b \cdot c)$
Distributive Property	If a, b, c are real numbers, then	$a(b+c)=ab+ac$
Identity Property	**of addition** For any real number a: 0 is the **additive identity**	$a+0=a$ $0+a=a$
	of multiplication For any real number a: 1 is the **multiplicative identity**	$a \cdot 1=a$ $1 \cdot a=a$
Inverse Property	**of addition** For any real number a, $-a$ is the **additive inverse** of a	$a+(-a)=0$
	of multiplication For any real number $a, a \neq 0$ $\frac{1}{a}$ is the **multiplicative inverse** of a.	$a \cdot \frac{1}{a}=1$
Properties of Zero	For any real number *a*, For any real number $a, a \neq 0$ For any real number $a, a \neq 0$	$a \cdot 0=0$ $0 \cdot a=0$ $\frac{0}{a}=0$ $\frac{a}{0}$ is undefined

Key Concepts

- **Commutative Property of**
  - **Addition:** If a, b are real numbers, then $a+b=b+a$.
  - **Multiplication:** If a, b are real numbers, then $a \cdot b=b \cdot a$. When adding or multiplying, changing the *order* gives the same result.

- **Associative Property of**
    - **Addition:** If a, b, c are real numbers, then $(a+b)+c=a+(b+c)$.
    - **Multiplication:** If a, b, c are real numbers, then $(a \cdot b) \cdot c=a \cdot (b \cdot c)$.
      When adding or multiplying, changing the *grouping* gives the same result.
- **Distributive Property:** If a, b, c are real numbers, then
    - $a\left(b+c\right)=ab+ac$
    - $\left(b+c\right)a=ba+ca$
    - $a\left(b-c\right)=ab-ac$
    - $\left(b-c\right)a=ba-ca$
- **Identity Property**
    - **of Addition:** For any real number a: $a+0=a \;\; 0+a=a$
      0 is the **additive identity**
    - **of Multiplication:** For any real number a: $a \cdot 1=a \;\; 1 \cdot a=a$
      1 is the **multiplicative identity**
- **Inverse Property**
    - **of Addition:** For any real number a, $a+(-a)=0$. A number and its *opposite* add to zero. $-a$ is the **additive inverse** of a.
    - **of Multiplication:** For any real number a, $(a \neq 0)$ $a \cdot \frac{1}{a}=1$. A number and its *reciprocal* multiply to one. $\frac{1}{a}$ is the **multiplicative inverse** of a.
- **Properties of Zero**
    - For any real number a,
      $a \cdot 0=0 \;\; 0 \cdot a=0$ – The product of any real number and 0 is 0.
    - $\frac{0}{a}=0$ for $a \neq 0$ – Zero divided by any real number except zero is zero.
    - $\frac{a}{0}$ is undefined – Division by zero is undefined.

Glossary

additive identity
The additive identity is the number 0; adding 0 to any number does not change its value.
additive inverse
The opposite of a number is its additive inverse. A number and it additive inverse add to 0.
multiplicative identity
The multiplicative identity is the number 1; multiplying 1 by any number does not change the value of the number.
multiplicative inverse
The reciprocal of a number is its multiplicative inverse. A number and its multiplicative inverse multiply to one.

Practice Makes Perfect

Use the Commutative and Associative Properties

In the following exercises, use the associative property to simplify.

1. 3(4x)	2. 4(7m)
3. $(y+12)+28$	4. $(n+17)+33$

In the following exercises, simplify.

5. $\frac{1}{2}+\frac{7}{8}+\left(-\frac{1}{2}\right)$	6. $\frac{2}{5}+\frac{5}{12}+\left(-\frac{2}{5}\right)$
7. $\frac{3}{20}\cdot\frac{49}{11}\cdot\frac{20}{3}$	8. $\frac{13}{18}\cdot\frac{25}{7}\cdot\frac{18}{13}$
9. $-24\cdot 7\cdot\frac{3}{8}$	10. $-36\cdot 11\cdot\frac{4}{9}$
11. $\left(\frac{5}{6}+\frac{8}{15}\right)+\frac{7}{15}$	12. $\left(\frac{11}{12}+\frac{4}{9}\right)+\frac{5}{9}$
13. 17(0.25)(4)	14. 36(0.2)(5)
15. [2.48(12)](0.5)	16. [9.731(4)](0.75)
17. 7(4a)	18. 9(8w)
19. $-15\,(5m)$	20. $-23\,(2n)$
21. $12\left(\frac{5}{6}p\right)$	22. $20\left(\frac{3}{5}q\right)$
23. $43m+(-12n)+(-16m)+(-9n)$	24. $-22p+17q+(-35p)+(-27q)$
25. $\frac{3}{8}g+\frac{1}{12}h+\frac{7}{8}g+\frac{5}{12}h$	26. $\frac{5}{6}a+\frac{3}{10}b+\frac{1}{6}a+\frac{9}{10}b$
27. $6.8p+9.14q+(-4.37p)+(-0.88q)$	28. $9.6m+7.22n+(-2.19m)+(-0.65n)$

Use the Identity and Inverse Properties of Addition and Multiplication

In the following exercises, find the additive inverse of each number.

29. a) $\frac{2}{5}$ b) 4.3 c) -8 d) $-\frac{10}{3}$	30. a) $\frac{5}{9}$ b) 2.1 c) -3 d) $-\frac{9}{5}$
31. a) $-\frac{7}{6}$ b) -0.075 c) 23 d) $\frac{1}{4}$	32. a) $-\frac{8}{3}$ b) -0.019 c) 52 d) $\frac{5}{6}$

In the following exercises, find the multiplicative inverse of each number.

33. a) 6 b) $-\frac{3}{4}$ c) 0.7	34. a) 12 b) $-\frac{9}{2}$ c) 0.13
35. a) $\frac{11}{12}$ b) -1.1 c) -4	36. a) $\frac{17}{20}$ b) -1.5 c) -3

Use the Properties of Zero

In the following exercises, simplify.

37. $\frac{0}{6}$	38. $\frac{3}{0}$
39. $0 \div \frac{11}{12}$	40. $\frac{6}{0}$
41. $\frac{0}{3}$	42. $0 \cdot \frac{8}{15}$
43. $(-3.14)\,(0)$	44. $\frac{\frac{1}{10}}{0}$

Mixed Practice

In the following exercises, simplify.

45. $19a + 44 - 19a$	46. $27c + 16 - 27c$
47. $10\,(0.1d)$	48. $100\,(0.01p)$
49. $\frac{0}{u-4.99}$, where $u \neq 4.99$	50. $\frac{0}{v-65.1}$, where $v \neq 65.1$
51. $0 \div \left(x - \ \frac{1}{2}\right)$, where $x \neq \frac{1}{2}$	52. $0 \div \left(y - \ \frac{1}{6}\right)$, where $x \neq \frac{1}{6}$
53. $\frac{32-5a}{0}$, where $32 - 5a \neq 0$	54. $\frac{28-9b}{0}$, where $28 - 9b \neq 0$
55. $\left(\frac{3}{4} + \frac{9}{10}m\right) \div 0$ where $\frac{3}{4} + \frac{9}{10}m \neq 0$	56. $\left(\frac{5}{16}n - \ \frac{3}{7}\right) \div 0$ where $\frac{5}{16}n - \ \frac{3}{7} \neq 0$
57. $15 \cdot \frac{3}{5}\,(4d + 10)$	58. $18 \cdot \frac{5}{6}\,(15h + 24)$

Simplify Expressions Using the Distributive Property

In the following exercises, simplify using the distributive property.

59. $8(4y+9)$	60. $9(3w+7)$
61. $6(c-13)$	62. $7(y-13)$
63. $\frac{1}{4}(3q+12)$	64. $\frac{1}{5}(4m+20)$
65. $9\left(\frac{5}{9}y-\ \frac{1}{3}\right)$	66. $10\left(\frac{3}{10}x-\ \frac{2}{5}\right)$
67. $12\left(\frac{1}{4}+\frac{2}{3}r\right)$	68. $12\left(\frac{1}{6}+\frac{3}{4}s\right)$
69. $r(s-18)$	70. $u(v-10)$
71. $(y+4)p$	72. $(a+7)x$
73. $-7(4p+1)$	74. $-9(9a+4)$
75. $-3(x-6)$	76. $-4(q-7)$
77. $-(3x-7)$	78. $-(5p-4)$
79. $16-3(y+8)$	80. $18-4(x+2)$
81. $4-11(3c-2)$	82. $9-6(7n-5)$
83. $22-(a+3)$	84. $8-(r-7)$
85. $(5m-3)-(m+7)$	86. $(4y-1)-(y-2)$
87. $5(2n+9)+12(n-3)$	88. $9(5u+8)+2(u-6)$
89. $9(8x-3)-(-2)$	90. $4(6x-1)-(-8)$
91. $14(c-1)-8(c-6)$	92. $11(n-7)-5(n-1)$
93. $6(7y+8)-(30y-15)$	94. $7(3n+9)-(4n-13)$

Everyday Math

95. **Insurance copayment** Carrie had to have 5 fillings done. Each filling cost \$80. Her dental insurance required her to pay 20% of the cost as a copay. Calculate Carrie's copay: a) First, by multiplying 0.20 by 80 to find her copay for each filling and then multiplying your answer by 5 to find her total copay for 5 fillings. b) Next, by multiplying [5(0.20)](80) c) Which of the properties of real numbers says that your answers to parts (a), where you multiplied 5[(0.20)(80)] and (b), where you multiplied [5(0.20)](80), should be equal?	96. **Cooking time** Matt bought a 24-pound turkey for his family's Thanksgiving dinner and wants to know what time to put the turkey in to the oven. He wants to allow 20 minutes per pound cooking time. Calculate the length of time needed to roast the turkey: a) First, by multiplying $24 \cdot 20$ to find the total number of minutes and then multiplying the answer by $\frac{1}{60}$ to convert minutes into hours. b) Next, by multiplying $24\left(20 \cdot \frac{1}{60}\right)$. c) Which of the properties of real numbers says that your answers to parts (a), where you multiplied $(24 \cdot 20)\,\frac{1}{60}$, and (b), where you multiplied $24\left(20 \cdot \frac{1}{60}\right)$, should be equal?
97. **Buying by the case.** Trader Joe's grocery stores sold a can of Coke Zero for \$1.99. They sold a case of 12 cans for \$23.88. To find the cost of 12 cans at \$1.99, notice that 1.99 is $2 - 0.01$. a) Multiply 12(1.99) by using the distributive property to multiply $12\,(2 - 0.01)$. b) Was it a bargain to buy Coke Zero by the case?	98. **Multi-pack purchase.** Adele's shampoo sells for \$3.99 per bottle at the grocery store. At the warehouse store, the same shampoo is sold as a 3 pack for \$10.49. To find the cost of 3 bottles at \$3.99, notice that 3.99 is $4 - 0.01$. a) Multiply 3(3.99) by using the distributive property to multiply $3\,(4 - 0.01)$. b) How much would Adele save by buying 3 bottles at the warehouse store instead of at the grocery store?

Writing Exercises

99. In your own words, state the commutative property of addition.	100. What is the difference between the additive inverse and the multiplicative inverse of a number?
101. Simplify $8\left(x - \frac{1}{4}\right)$ using the distributive property and explain each step.	102. Explain how you can multiply 4(\$5.97) without paper or calculator by thinking of \$5.97 as $6 - 0.03$ and then using the distributive property.

Answers

1. $12x$	3. $y + 40$	5. $\frac{7}{8}$
7. $\frac{49}{11}$	9. -63	11. $1\frac{5}{6}$
13. 17	15. 14.88	17. $28a$
19. $-75m$	21. $10p$	23. $27m + (-21n)$
25. $\frac{5}{4}g + \frac{1}{2}h$	27. $2.43p + 8.26q$	29. a) $-\frac{2}{5}$ b) -4.3 c) 8 d) $\frac{10}{3}$
31. a) $\frac{7}{6}$ b) 0.075 c) -23 d) $-\frac{1}{4}$	33. a) $\frac{1}{6}$ b) $-\frac{4}{3}$ c) $\frac{10}{7}$	35. a) $\frac{12}{11}$ b) $-\frac{10}{11}$ c) $-\frac{1}{4}$
37. 0	39. 0	41. 0
43. 0	45. 44	47. d
49. 0	51. 0	53. undefined
55. undefined	57. $36d + 90$	59. $32y + 72$
61. $6c - 78$	63. $\frac{3}{4}q + 3$	65. $5y - 3$
67. $3 + 8r$	69. $rs - 18r$	71. $yp + 4p$
73. $-28p - 7$	75. $-3x + 18$	77. $-3x + 7$
79. $-3y - 8$	81. $-33c + 26$	83. $-a + 19$
85. $4m - 10$	87. $22n + 9$	89. $72x - 25$
91. $6c + 34$	93. $12y + 63$	95. a) \$80 b) \$80 c) answers will vary
97. a) \$23.88 b) no, the price is the same	99. Answers may vary	101. Answers may vary

Attributions

2.6 Chapter Review

Review Exercises

Find Equivalent Fractions

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

1. $\frac{1}{4}$	2. $\frac{1}{3}$
3. $\frac{5}{6}$	4. $\frac{2}{7}$

Simplify Fractions

In the following exercises, simplify.

5. $\frac{7}{21}$	6. $\frac{8}{24}$
7. $\frac{15}{20}$	8. $\frac{12}{18}$
9. $-\frac{168}{192}$	10. $-\frac{140}{224}$
11. $\frac{11x}{11y}$	12. $\frac{15a}{15b}$

Multiply Fractions

In the following exercises, multiply.

13. $\frac{2}{5} \cdot \frac{1}{3}$	14. $\frac{1}{2} \cdot \frac{3}{8}$
15. $\frac{7}{12}(-\frac{8}{21})$	16. $\frac{5}{12}(-\frac{8}{15})$
17. $-28p(-\frac{1}{4})$	18. $-51q(-\frac{1}{3})$
19. $\frac{14}{5}(-15)$	20. $-1(-\frac{3}{8})$

Divide Fractions

In the following exercises, divide.

21. $\frac{1}{2} \div \frac{1}{4}$	22. $\frac{1}{2} \div \frac{1}{8}$
23. $-\frac{4}{5} \div \frac{4}{7}$	24. $-\frac{3}{4} \div \frac{3}{5}$
25. $\frac{5}{8} \div \frac{a}{10}$	26. $\frac{5}{6} \div \frac{c}{15}$
27. $\frac{7p}{12} \div \frac{21p}{8}$	28. $\frac{5q}{12} \div \frac{15q}{8}$
29. $\frac{2}{5} \div (-10)$	30. $-18 \div -\left(\frac{9}{2}\right)$

In the following exercises, simplify.

31. $\frac{\frac{2}{3}}{\frac{8}{9}}$	32. $\frac{\frac{4}{5}}{\frac{8}{15}}$
33. $\frac{-\frac{9}{10}}{3}$	34. $\frac{2}{\frac{5}{8}}$
35. $\frac{\frac{r}{5}}{\frac{s}{3}}$	36. $\frac{-\frac{x}{6}}{-\frac{8}{9}}$

Simplify Expressions Written with a Fraction Bar

In the following exercises, simplify.

37. $\frac{4+11}{8}$	38. $\frac{9+3}{7}$
39. $\frac{30}{7-12}$	40. $\frac{15}{4-9}$
41. $\frac{22-14}{19-13}$	42. $\frac{15+9}{18+12}$
43. $\frac{5\cdot 8}{-10}$	44. $\frac{3\cdot 4}{-24}$
45. $\frac{15\cdot 5-5^2}{2\cdot 10}$	46. $\frac{12\cdot 9-3^2}{3\cdot 18}$
47. $\frac{2+4(3)}{-3-2^2}$	48. $\frac{7+3(5)}{-2-3^2}$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

49. the quotient of *c* and the sum of *d* and 9.	50. the quotient of the difference of *h* and *k*, and -5.

Add and Subtract Fractions with a Common Denominator

In the following exercises, add.

51. $\frac{4}{9}+\frac{1}{9}$	52. $\frac{2}{9}+\frac{5}{9}$
53. $\frac{y}{3}+\frac{2}{3}$	54. $\frac{7}{p}+\frac{9}{p}$
55. $-\frac{1}{8}+\left(-\frac{3}{8}\right)$	56. $-\frac{1}{8}+\left(-\frac{5}{8}\right)$

In the following exercises, subtract.

57. $\frac{4}{5}-\frac{1}{5}$	58. $\frac{4}{5}-\frac{3}{5}$
59. $\frac{y}{17}-\frac{9}{17}$	60. $\frac{x}{19}-\frac{8}{19}$
61. $-\frac{8}{d}-\frac{3}{d}$	62. $-\frac{7}{c}-\frac{7}{c}$

Add or Subtract Fractions with Different Denominators

In the following exercises, add or subtract.

63. $\frac{1}{3}+\frac{1}{5}$	64. $\frac{1}{4}+\frac{1}{5}$
65. $\frac{1}{5}-\left(-\frac{1}{10}\right)$	66. $\frac{1}{2}-\left(-\frac{1}{6}\right)$
67. $\frac{2}{3}+\frac{3}{4}$	68. $\frac{3}{4}+\frac{2}{5}$
69. $\frac{11}{12}-\frac{3}{8}$	70. $\frac{5}{8}-\frac{7}{12}$
71. $-\frac{9}{16}-\left(-\frac{4}{5}\right)$	72. $-\frac{7}{20}-\left(-\frac{5}{8}\right)$
73. $1+\frac{5}{6}$	74. $1-\frac{5}{9}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

75. $\frac{\left(\frac{1}{5}\right)^2}{2+3^2}$	76. $\frac{\left(\frac{1}{3}\right)^2}{5+2^2}$
77. $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{2}{3}}$	78. $\frac{\frac{3}{4}+\frac{1}{2}}{\frac{5}{6}-\frac{2}{3}}$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

79. $x+\frac{1}{2}$ when a) $x=-\frac{1}{8}$ b) $x=-\frac{1}{2}$	80. $x+\frac{2}{3}$ when a) $x=-\frac{1}{6}$ b) $x=-\frac{5}{3}$
81. $4p^2q$ when $p=-\frac{1}{2}$ and $q=\frac{5}{9}$	82. $5m^2n$ when $m=-\frac{2}{5}$ and $n=\frac{1}{3}$
83. $\frac{u+v}{w}$ when $u=-4, v=-8, w=2$	84. $\frac{m+n}{p}$ when $m=-6, n=-2, p=4$

Name and Write Decimals

In the following exercises, write as a decimal.

85. Eight and three hundredths	86. Nine and seven hundredths
87. One thousandth	88. Nine thousandths

In the following exercises, name each decimal.

89. 7.8	90. 5.01
91. 0.005	92. 0.381

Round Decimals

In the following exercises, round each number to the nearest a) hundredth b) tenth c) whole number.

93. 5.7932	94. 3.6284
95. 12.4768	96. 25.8449

Add and Subtract Decimals

In the following exercises, add or subtract.

97. $18.37+9.36$	98. $256.37-85.49$
99. $15.35-20.88$	100. $37.5+12.23$
101. $-4.2+(-9.3)$	102. $-8.6+(-8.6)$
103. $100-64.2$	104. $100-65.83$
105. $2.51+40$	106. $9.38+60$

Multiply and Divide Decimals

In the following exercises, multiply.

107. $(0.3)(0.4)$	108. $(0.6)(0.7)$
109. $(8.52)(3.14)$	110. $(5.32)(4.86)$
111. $(0.09)(24.78)$	112. $(0.04)(36.89)$

In the following exercises, divide.

113. 0.15 ÷ 5	114. 0.27 ÷ 3
115. $8.49 ÷ 12	116. $16.99 ÷ 9
117. 12 ÷ 0.08	118. 5 ÷ 0.04

Convert Decimals and Fractions

In the following exercises, write each decimal as a fraction.

119. 0.08	120. 0.17
121. 0.425	122. 0.184
123. 1.75	124. 0.035

In the following exercises, convert each fraction to a decimal.

125. $\frac{2}{5}$	126. $\frac{4}{5}$
127. $-\frac{3}{8}$	128. $-\frac{5}{8}$
129. $\frac{5}{9}$	130. $\frac{2}{9}$
131. $\frac{1}{2}+6.5$	132. $\frac{1}{4}+10.75$

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

In the following exercises, write as the ratio of two integers.

133. a) 9 b) 8.47	134. a) -15b) 3.591

In the following exercises, list the a) rational numbers, b) irrational numbers.

135. $0.84, 0.79132..., 1.\overline{3}$	136. $2.3\overline{8}, 0.572, 4.93814...$

In the following exercises, list the a) whole numbers, b) integers, c) rational numbers, d) irrational numbers, e) real numbers for each set of numbers.

137. $-4, 0, \frac{5}{6}, 17, 5.2537...$	138. $-2, 0.\overline{36}, \frac{13}{3}, 6.9152..., 10\frac{1}{2}$

Locate Fractions on the Number Line

In the following exercises, locate the numbers on a number line.

139. $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$	140. $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$
141. $2\frac{1}{3}, -2\frac{1}{3}$	142. $1\frac{3}{5}, -1\frac{3}{5}$

In the following exercises, order each of the following pairs of numbers, using < or >.

143. -1 ___ $-\frac{1}{8}$	144. $-3\frac{1}{4}$ ___ -4
145. $-\frac{7}{9}$ ___ $-\frac{4}{9}$	146. -2 ___ $-\frac{19}{8}$

Locate Decimals on the Number Line

In the following exercises, locate on the number line.

147. 0.3	148. -0.2
149. -2.5	150. 2.7

In the following exercises, order each of the following pairs of numbers, using < or >.

151. 0.9 ___ 0.6	152. 0.7 ___ 0.8
153. -0.6 ___ -0.59	154. -0.27 ___ -0.3

Use the Commutative and Associative Properties

In the following exercises, use the Associative Property to simplify.

155. $-12\,(4m)$	156. $30\left(\frac{5}{6}q\right)$
157. $(a+16)+31$	158. $(c+0.2)+0.7$

In the following exercises, simplify.

159. $6y+37+(-6y)$	160. $\frac{1}{4}+\frac{11}{15}+\left(-\frac{1}{4}\right)$
161. $\frac{14}{11}\cdot\frac{35}{9}\cdot\frac{14}{11}$	162. $-18\cdot 15\cdot\frac{2}{9}$
163. $\left(\frac{7}{12}+\frac{4}{5}\right)+\frac{1}{5}$	164. $(3.98d+0.75d)+1.25d$
165. $11x+8y+16x+15y$	166. $52m+(-20n)+(-18m)+(-5n)$

Use the Identity and Inverse Properties of Addition and Multiplication

In the following exercises, find the additive inverse of each number.

167. a) $\frac{1}{3}$ b) 5.1 c) -14 d) $-\frac{8}{5}$	168. a) $-\frac{7}{8}$ b) -0.03 c) 17 d) $\frac{12}{5}$

In the following exercises, find the multiplicative inverse of each number.

169. a) 10 b) $-\frac{4}{9}$ c) 0.6	170. a) $-\frac{9}{2}$ b) -7 c) 2.1

Use the Properties of Zero

In the following exercises, simplify.

171. $83\cdot 0$	172. $\frac{0}{9}$
173. $\frac{5}{0}$	174. $\frac{0}{\left(\frac{2}{3}\right)}$

In the following exercises, simplify.

175. $43+39+(-43)$	176. $(n+6.75)+0.25$
177. $\frac{5}{13}\cdot 57\cdot \frac{13}{5}$	178. $\frac{1}{6}\cdot 17\cdot 12$
179. $\frac{2}{3}\cdot 28\cdot \frac{3}{7}$	180. $9\left(6x-11\right)+15$

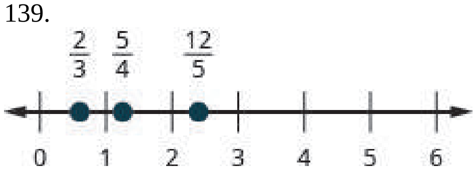
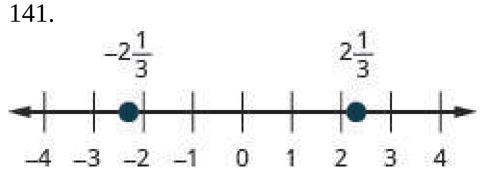
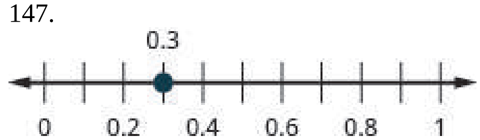
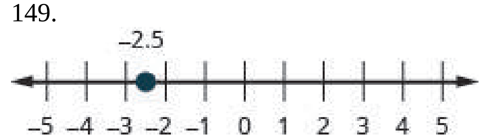
Simplify Expressions Using the Distributive Property

In the following exercises, simplify using the Distributive Property.

181. $7\left(x+9\right)$	182. $9\left(u-4\right)$
183. $-3\left(6m-1\right)$	184. $-8\left(-7a-12\right)$
185. $\frac{1}{3}\left(15n-6\right)$	186. $(y+10)\cdot p$
187. $(a-4)-(6a+9)$	188. $4\left(x+3\right)-8\left(x-7\right)$

Review Exercise Answers

1. $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}$ answers may vary	3. $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}$ answers may vary	5. $\frac{1}{3}$
7. $\frac{3}{4}$	9. $-\frac{7}{8}$	11. $\frac{x}{y}$
13. $\frac{2}{15}$	15. $-\frac{2}{9}$	17. $7p$
19. -42	21. 2	23. $-\frac{7}{5}$
25. $\frac{25}{4a}$	27. $\frac{2}{9}$	29. $-\frac{1}{25}$
31. $\frac{3}{4}$	33. $-\frac{3}{10}$	35. $\frac{3r}{5s}$
37. $\frac{15}{8}$	39. -6	41. $\frac{4}{3}$
43. -4	45. $\frac{5}{21}$	47. -2
49. $\frac{c}{d+9}$	51. $\frac{5}{9}$	53. $\frac{y+2}{3}$
55. $-\frac{1}{2}$	57. $\frac{3}{5}$	59. $\frac{y-9}{17}$
61. $-\frac{11}{d}$	63. $\frac{8}{15}$	65. $\frac{3}{10}$
67. $\frac{17}{12}$	69. $\frac{13}{24}$	71. $\frac{19}{80}$
73. $\frac{11}{6}$	75. $\frac{1}{275}$	77. 14
79. a) $\frac{3}{8}$ b) 0	81. $\frac{5}{9}$	83. -6
85. 8.03	87. 0.001	89. seven and eight tenths
91. five thousandths	93. a) 5.79 b) 5.8 c) 6	95. a) 12.48 b) 12.5 c) 12
97. 27.73	99. −5.53	101. −13.5
103. 35.8	105. 42.51	107. 0.12
109. 26.7528	111. 2.2302	113. 0.03
115. $0.71	117. 150	119. $\frac{2}{25}$
121. $\frac{17}{40}$	123. $\frac{7}{4}$	125. 0.4

127. -0.375	129. $0.\bar{5}$	131. 7
133. a) $\frac{9}{1}$ b) $\frac{847}{100}$	135. a) $0.84, 1.\bar{3}$ b) $0.79132\ldots,$	137. a) 0, 17 b) -4,0,17 c) $-4, 0, \frac{5}{6}, 17$ d) $5.2537\ldots$ e) $-4, 0, 17, \frac{5}{6}, 5.2537\ldots$
139. 	141. 	143. <
145. >	147. 	149. 
151. >	153. >	155. $-48m$
157. $a+47$	159. 37	161. $\frac{35}{9}$
163. $1\frac{7}{12}$	165. $27x+23y$	167. a) $-\frac{1}{3}$ b) -5.1 c) 14 d) $\frac{8}{5}$
169. a) $\frac{1}{10}$ b) $-\frac{9}{4}$ c) $\frac{5}{3}$	171. 0	173. undefined
175. 39	177. 57	179. 8
181. $7x+63$	183. $-18m+3$	185. $5n-2$
187. $-5a-13$		

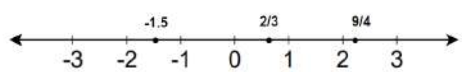
Practice Test

1. Convert 1.85 to a fraction and simplify.	2. Locate $\frac{2}{3}, -1.5,$ and $\frac{9}{4}$ on a number line.

In the following exercises, simplify each expression.

3. $4+10(3+9)-5^2$	4. $-85+42$
5. $-19-25$	6. $(-2)^4$
7. $-5(-9)\div 15$	8. $\frac{3}{8}\cdot\frac{11}{12}$
9. $\frac{4}{5}\div\frac{9}{20}$	10. $\frac{12+3\cdot 5}{15-6}$
11. $\frac{m}{7}+\frac{10}{7}$	12. $\frac{7}{12}-\frac{3}{8}$
13. $-5.8+(-4.7)$	14. $100-64.25$
15. $(0.07)(31.95)$	16. 9 ÷ 0.05
17. $-14\left(\frac{5}{7}p\right)$	18. $(u+8)-9$
19. $6x+(-4y)+9x+8y$	20. $\frac{0}{23}$
21. $\frac{75}{0}$	22. $-2(13q-5)$

Practice Test Answers

1. $\frac{37}{20}$		3. 99
4. -43	5. -44	6. 16
7. 3	8. $\frac{11}{32}$	9. $\frac{16}{9}$
10 3	11. $\frac{m+10}{7}$	12. $\frac{5}{24}$
13. -10.5	14. 35.75	15. 2.2365
16. 180	17. $-10p$	18. $u-1$
19. $15x+4y$	20. 0	21. undefined
22. $-26q+10$		