

Springer Proceedings in Complexity

Pasquale Commendatore  
Ingrid Kubin  
Spiros Bougheas  
Alan Kirman  
Michael Kopel  
Gian Italo Bischi *Editors*

---

# The Economy as a Complex Spatial System

Macro, Meso and Micro Perspectives

 **cost**  
EUROPEAN COOPERATION  
IN SCIENCE & TECHNOLOGY

 Springer Open

[www.dbooks.org](http://www.dbooks.org)

# **Springer Proceedings in Complexity**

## Springer Complexity

Springer Complexity is an interdisciplinary program publishing the best research and academic-level teaching on both fundamental and applied aspects of complex systems—cutting across all traditional disciplines of the natural and life sciences, engineering, economics, medicine, neuroscience, social, and computer science.

Complex Systems are systems that comprise many interacting parts with the ability to generate a new quality of macroscopic collective behavior the manifestations of which are the spontaneous formation of distinctive temporal, spatial, or functional structures. Models of such systems can be successfully mapped onto quite diverse “real-life” situations like the climate, the coherent emission of light from lasers, chemical reaction—diffusion systems, biological cellular networks, the dynamics of stock markets and of the Internet, earthquake statistics and prediction, freeway traffic, the human brain, or the formation of opinions in social systems, to name just some of the popular applications.

Although their scope and methodologies overlap somewhat, one can distinguish the following main concepts and tools: self-organization, nonlinear dynamics, synergetics, turbulence, dynamical systems, catastrophes, instabilities, stochastic processes, chaos, graphs and networks, cellular automata, adaptive systems, genetic algorithms, and computational intelligence.

The three major book publication platforms of the Springer Complexity program are the monograph series “Understanding Complex Systems” focusing on the various applications of complexity, the “Springer Series in Synergetics”, which is devoted to the quantitative theoretical and methodological foundations, and the “SpringerBriefs in Complexity” which are concise and topical working reports, case-studies, surveys, essays, and lecture notes of relevance to the field. In addition to the books in these two core series, the program also incorporates individual titles ranging from textbooks to major reference works.

More information about this series at <http://www.springer.com/series/11637>

Pasquale Commendatore  
Ingrid Kubin · Spiros Bougheas  
Alan Kirman · Michael Kopel  
Gian Italo Bischi  
Editors

# The Economy as a Complex Spatial System

Macro, Meso and Micro Perspectives



### *Editors*

Pasquale Commendatore  
Department of Law  
University of Naples “Federico II”  
Naples  
Italy

Alan Kirman  
École des Hautes Études en Sciences Sociales  
CAMS - EHESS  
Paris  
France

Ingrid Kubin  
Department of Economics  
WU Vienna University of Economics  
and Business  
Vienna  
Austria

Michael Kopel  
Department of Organization and Economics  
of Institutions  
University of Graz  
Graz  
Austria

Spiros Bougheas  
School of Economics  
University of Nottingham  
Nottingham  
UK

Gian Italo Bischi  
DESP-Department of Economics, Society, Politics  
Università di Urbino “Carlo Bo”  
Urbino  
Italy



ISSN 2213-8684

ISSN 2213-8692 (electronic)

Springer Proceedings in Complexity

ISBN 978-3-319-65626-7

ISBN 978-3-319-65627-4 (eBook)

DOI 10.1007/978-3-319-65627-4

Library of Congress Control Number: 2017950272

© The Editor(s) (if applicable) and The Author(s) 2018. This book is an open access publication.

**Open Access** This book is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this book are included in the book's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the book's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature

The registered company is Springer International Publishing AG

The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

# About COST

COST (European Cooperation in Science and Technology) is a funding agency for research and innovation networks. Our actions help connect research initiatives across Europe and enable scientists to grow their ideas by sharing them with their peers. This boosts their research, career, and innovation. [www.cost.eu](http://www.cost.eu)



Funded by the Horizon 2020 Framework Programme of the European Union

# Acknowledgement

This book is based upon work from COST Action ISCH IS1104 “The EU in the new complex geography of economic systems: models, tools and policy evaluation,” supported by COST (European Cooperation in Science and Technology).

# Contents

<b>Introduction</b> . . . . .	1
Pasquale Commendatore, Ingrid Kubin, Spiros Bougheas, Alan Kirman, Michael Kopel, and Gian Italo Bischi	
<b>The Macro Perspective – Economic Geography</b>	
<b>Policy Issues in NEG Models: Established Results and Open Questions</b> . . . . .	13
Pasquale Commendatore, Christoph Hammer, Ingrid Kubin, and Carmelo Petraglia	
<b>Emerging Trade Patterns in a 3-Region Linear NEG Model: Three Examples</b> . . . . .	38
Pasquale Commendatore, Ingrid Kubin, and Iryna Sushko	
<b>Advances in Spatial Econometrics: Parametric vs. Semiparametric Spatial Autoregressive Models</b> . . . . .	81
Roberto Basile and Román Mínguez	
<b>Looking Ahead: Part I</b> . . . . .	107
Pasquale Commendatore and Ingrid Kubin	
<b>The Meso Perspective – Financial Markets</b>	
<b>Systemic Risk and Macroeconomic Fat Tails</b> . . . . .	119
Spiros Bougheas, David Harvey, and Alan Kirman	
<b>Market Interactions, Endogenous Dynamics and Stabilization Policies</b> . . . . .	137
Noemi Schmitt, Jan Tuinstra, and Frank Westerhoff	
<b>Looking Ahead: Part II</b> . . . . .	153
Spiros Bougheas and Alan Kirman	

**The Micro Perspective – Social and Industrial Interactions**

<b>A Dynamic Model of Firms' Strategic Location Choice . . . . .</b>	<b>159</b>
Luca Colombo and Herbert Dawid	

<b>Strategic Corporate Social Responsibility by a Local Firm Against a Multinational Enterprise . . . . .</b>	<b>178</b>
Michael Kopel, Constantine Manasakis, and Emmanuel Petrakis	

<b>Knowledge Spillovers, Congestion Effects, and Long-Run Location Patterns . . . . .</b>	<b>192</b>
Gian Italo Bischi, Michael Kopel, Fabio Lamantia, and Davide Radi	

<b>Looking Ahead: Part III . . . . .</b>	<b>216</b>
Gian Italo Bischi and Michael Kopel	

# Contributors

**Roberto Basile** Department of Economics, Università Della Campania “Luigi Vanvitelli”, Capua, CE, Italy

**Gian Italo Bischi** DESP - Department of Economics, Society, Politics, University of Urbino “Carlo Bo”, Urbino, Italy

**Spiros Bougheas** School of Economics, University of Nottingham, Nottingham, UK

**Luca Colombo** Department of Economics and Finance, Università Cattolica del Sacro Cuore, Milano, Italy

**Pasquale Commendatore** Department of Law, University of Naples “Federico II”, Naples, Italy

**Herbert Dawid** Department of Business Administration and Economics and Center for Mathematical Economics, Bielefeld University, Bielefeld, Germany

**Christoph Hammer** Department of Economics, Institute for International Economics and Development, WU Vienna University of Economics and Business, Vienna, Austria

**David Harvey** School of Economics, University of Nottingham, Nottingham, UK

**Alan Kirman** École des Hautes Études en Sciences Sociales, CAMS - EHESS, Paris, France

**Michael Kopel** Department of Organization and Economics of Institutions, University of Graz, Graz, Austria

**Ingrid Kubin** Department of Economics, Institute for International Economics and Development, WU Vienna University of Economics and Business, Vienna, Austria

**Fabio Lamantia** Department of Economics, Statistics and Finance, University of Calabria, Rende, CS, Italy; Economics – School of Social Sciences, The University of Manchester, Manchester, UK

**Constantine Manasakis** Department of Political Science, University of Crete, Rethymno, Crete, Greece

**Román Mínguez** Department of Economics, Universidad de Castilla-La Mancha, Cuenca, Spain

**Carmelo Petraglia** Department of Organization and Economics of Institutions, University of Basilicata, Potenza, Italy

**Emmanuel Petrakis** Department of Economics, University of Crete, Rethymno, Crete, Greece

**Davide Radi** School of Economics and Management, LIUC - Università Cattaneo, Castellanza, VA, Italy

**Noemi Schmitt** Department of Economics, University of Bamberg, Bamberg, Germany

**Iryna Sushko** Institute of Mathematics, NASU, Kiev, Ukraine

**Jan Tuinstra** Amsterdam School of Economics and CeNDEF, University of Amsterdam, Amsterdam, The Netherlands

**Frank Westerhoff** Department of Economics, University of Bamberg, Bamberg, Germany

# Introduction

Pasquale Commendatore<sup>1(✉)</sup>, Ingrid Kubin<sup>2</sup>, Spiros Bougheas<sup>3</sup>, Alan Kirman<sup>4</sup>,  
Michael Kopel<sup>5</sup>, and Gian Italo Bischi<sup>6</sup>

<sup>1</sup> Department of Law, University of Naples ‘Federico II’, Naples, Italy  
`commenda@unina.it`

<sup>2</sup> Department of Economics, Institute for International Economics and Development,  
WU Vienna University of Economics and Business, Vienna, Austria  
`Ingrid.Kubin@wu.ac.at`

<sup>3</sup> School of Economics, University of Nottingham, Nottingham, UK  
`Spiros.Bougheas@nottingham.ac.uk`

<sup>4</sup> École des Hautes Études en Sciences Sociales, CAMS - EHESS, Paris, France  
`alan.kirman@ehess.fr`

<sup>5</sup> Department of Organization and Economics of Institutions, University of Graz,  
Graz, Austria  
`michael.kopel@uni-graz.at`

<sup>6</sup> DESP - Department of Economics, Society, Politics,  
University of Urbino Carlo Bo, Urbino, Italy  
`gianitalo.bischi@gmail.com`

**Abstract.** This collected volume gives a concise account of the most relevant scientific results of the COST Action IS1104 “The EU in the new complex geography of economic systems: models, tools and policy evaluation”, a four-year project supported by COST (European Cooperation in Science and Technology). It is divided into three parts reflecting the different perspectives under which complex spatial economic systems have been studied: (i) the Macro perspective looks at the interactions among international or regional trading partners; (ii) the Meso perspective considers the functioning of (financial, labour) markets as social network structures; and, finally, (iii) the Micro perspective focuses on the strategic choices of single firms and households. This Volume points also at open issues to be addressed in future research.

**Keywords:** COST Action IS114 · European Union · Economic Geography · Financial Markets · Strategic Decisions and Interactions · Complexity

## 1 The COST Action IS1104 “The EU in the New Complex Geography of Economic Systems: Models, Tools and Policy Evaluation”

Inspired by the New Economic Geography (NEG) approach, initiated by Paul Krugman and Masahita Fujita in the 1990s, the main objective of the COST Action 1104 “The EU in the new complex geography of economic systems:

© The Author(s) 2018

P. Commendatore et al. (eds.), *The Economy as a Complex Spatial System*,  
Springer Proceedings in Complexity, DOI 10.1007/978-3-319-65627-4\_1



models, tools and policy evaluation (GeComplexity)” has been to approach the study of EU, more generally, economic systems from a multi-layered perspective featuring interconnected spatial structures. At each layer, different types of decisions and interactions take place: interactions among international or regional trading partners at the macro-level; the functioning of (financial, labour) markets as social network structures at the meso-level; and finally, the strategic choices of single firms and households at the micro-level. Within these structures, the spatial distribution of economic activities is evolving through time following complex patterns determined by economic, geographical, institutional and social factors. To study these structures, during its four years life time (March 2012–September 2016), the Action has built successfully an interdisciplinary approach. It has further developed advanced mathematical, computational and empirical methods and tools for analysing complex nonlinear systems, including macro and micro models, nonlinear dynamical systems, social networks, game theoretical models and agent based models.

This leads to the second objective of the Action: building multiregional models that integrate real world features; mapping the geography of the financial and banking networks; understanding firms’ strategic choices on location and R&D cooperation and competition. More generally highlighting the pervasiveness of networks structures at the various levels of aggregation.

To achieve these objectives, several activities have been performed. Twenty-three meetings of which four major events – including a Final Conference – and several workshops have taken place. Researches from twenty-six European countries and across the world and from an EU Commission research institution (the Joint Research Centre, JRC, Seville) have contributed with an interdisciplinary expertise: economists, regional economists, applied economists, mathematicians, physicists. These activities lead to several scientific outcomes including more than 140 articles in leading journals, Special Issues (four published and two as an expected outcome of the Final Conference), three collected volumes, multiple book chapters and working papers.

The Action has evolved into a very active network not only through several meetings but also via more than eighty Short Term Scientific Missions (STSMs). A high percentage of STSMs have been allocated to Early Stage Researchers (ESRs). Moreover, three Training School have been specifically designed to build the capacity of PhD students and young scholars. Dissemination has been conducted also via the Action Website <http://www.gecomplexity-cost.eu> and a Discussion Paper Series edited by COST Members and indexed in REPEC.

This book gives a concise account of the most relevant scientific results of the GeComplexity Cost Action and points at open issues to be addressed in future projects. The Volume is divided into three parts reflecting the different perspectives under which complex spatial economic systems have been studied.

## 2 The Macro Perspective - Economic Geography

The chapter by **Commendatore, Hammer, Kubin and Petraglia** provides a non-technical overview of new economic geography models dealing with policy

issues. The Chapter begins by describing the main ingredients of the NEG approach and illustrating the basic mechanism at work centred on the interplay of agglomeration and dispersion forces. As suggested by Krugman (1991), trade integration leads to spatial concentration of economic activity altering the balance in favour of the agglomeration forces. The authors consider various policy measures including alternative categories of public expenditure, international tax competition, unilateral actions of protection/liberalisation, and trade agreements. The implications of public intervention in two-region NEG models are discussed by unfolding the impact of policy measures on agglomeration/dispersion forces. Results are described and contrasted to those obtained in standard non-NEG theoretical models. Paradigmatic examples are the non-neutrality of home-bias public procurement for the determination of a country's pattern of specialization; the presence of taxable agglomeration rents leading to a "race to the top" rather than a "race to the bottom" within a tax competition game between countries; the impossibility to exploit a potential comparative advantage to avoid deindustrialization in poorer countries when agglomeration effects are at work.

The high degree of abstraction limits the applicability of NEG models to real world policy issues. The authors discuss in some detail two extensions of NEG models to reduce this applicability gap: the cases of multi-regional frameworks and firm heterogeneity.

The applicability gap is addressed by **Commendatore, Kubin and Sushko** by studying a three-region economy in a NEG model. By using linear (and not iso-elastic) demand functions, the model is able to account for trade patterns between pairs of regions allowing for unilateral, bilateral and no trade regimes. Thus, the proposed framework is suitable to study how changes in parameters that are typical for NEG models, such as trade costs and regional market size, not only shape the regional distribution of economic activity, but also at the same time determine the emergence of trade links between regions.

To focus the analysis, the authors study in more detail three specific trade patterns frequently found in the EU trade network. First, they consider three autarkic regions belonging to an economy at its first stages of development. For this set-up, instances of coexisting stable long-run equilibria (multi-stability) are found, which cannot occur in a two-region framework, stressing the importance of initial advantage in the process of industry concentration; then following the improvement of transport infrastructures between two regions, the possibility that these regions trade with each other (but only with each other) is introduced. As trade costs fall, the region with a larger initial endowment of the mobile factor starts exporting to the closer region and attracts the industrial sector. However, in contrast to the two-region case, some of the industry locates in the more remote region, finding shelter from competition; and, finally, even the remote region closes its distance with one of the other two regions, which increases its centrality in the trade network. However, centrality does not necessarily translate into a locational advantage.

More generally, the authors find a surprising plethora of long-run equilibria each involving a specific regional distribution of economic activity and a specific pattern of trade links. This implies that a variation in trade costs shapes simultaneously industry location and the configuration of the trade network.

**Basile and Mínguez** propose a critical review of parametric and semi-parametric spatial econometric models. They focus on the capability of each class of models to fit the main features of spatial data, namely spatial dependence (or weak cross sectional dependence, due to spatial spillover effects), strong cross-sectional dependence (due to unobserved common factors), spatial heterogeneity, nonlinearities, and time persistence. They also provide a brief discussion of the existent software developed to estimate most of the econometric models described in the chapter.

They start by summarizing the broad literature on parametric spatial autoregressive models, which is still the dominant paradigm in spatial econometrics. Within this literature, it is possible to distinguish between first-generation spatial econometric models (essentially developed to handle cross-sectional data) which focus on modeling spatial dependence through different alternative linear specifications, and second-generation spatial econometric models, developed during the last decade, more specifically, static and dynamic spatial panel data models which prove to be particularly useful to control for unobserved spatial heterogeneity and time persistence. A natural extension of the last class of models, proposed during the last few years, consists of the combination of spatial panel data models and common factor models in order to disentangle strong and weak (spatial) cross-section dependence.

In spite of these important advances in the literature, it is important to recognize that any parametric model is limited to specific forms of spatial variation of the parameters, while they are not able to capture more general forms of model mis-specifications, such as spatial parameter heterogeneity and nonlinearities. Thus, semiparametric spatial econometric models appear as more flexible estimation frameworks. The authors dedicate the second part of their Chapter to this category of models, distinguishing between (mixed)-GWR models based on kernel methods, and models based on penalized spline smoothers. Three recent contributions are of particular relevance in this context. First, combining kernel smoothing methods and standard spatial lag models, a new class of data generating processes (DGP) within the GWR literature (Mixed Geographically Weighted Regression Simultaneous AutoRegressive Models) provides an important framework to account for both spatial dependence and nonstationarity of the parameters. Second, combining penalized regression spline methods with standard cross-section spatial autoregressive models, another class of DGP allows the researchers to simultaneously control for spatial spillover effects, nonlinearities, spatially autocorrelated unobserved heterogeneity, and spatial nonstationarity of the parameters. Finally, semiparametric models for longitudinal data, which include a non-parametric spatio-temporal trend, a spatial lag of the dependent variable, and a time series autoregressive noise, represent a valid alternative to parametric methods aimed at disentangling strong and weak cross-sectional dependence. Natural directions in which these methods can be extended are specifications suitable for the analysis of dynamic frameworks.

The last chapter by **Commendatore and Kubin** concludes Part I of the Volume. The authors summarise the work carried out during the lifetime of the Action by the Working Group whose main task was to build multiregional NEG

models. The authors list the main results, point at the questions left open and suggest topics for future research. What emerges from the discussion is that the predictions of NEG models are highly sensitive to their specification: different assumptions concerning, for example, the geographical structure of the economy or even the functional form of consumer's preferences lead to different long-run spatial distributions.

### 3 The Meso Perspective - Financial Markets

**Bougheas, Harvey and Kirman** explore the relationship between systemic risk and the behavior of aggregate credit. The two most severe macroeconomic crises of the last 100 years, namely, the Great Depression of the 1930s and the Great Recession that commenced at the close of the first decade of the current century, were preceded by extreme events in financial markets in general and the banking system in particular. In a recent study, Schularick and Taylor (2012) have empirically identified a historical link between the level of aggregate credit in the economy and macroeconomic performance. They argue that aggregate credit can be a powerful predictor of economic crises, especially, rare catastrophic events.

Their aim is to provide a microfoundational explanation for the above relationship. In this work the focus is on the behavior of aggregate credit. In particular, the authors analyze the dynamics of aggregate bank credit in an economy where all financial transactions are intermediated through the banking system. Viewing the financial system as a network of banks that are connected through their financial obligations to each other, they examine how the impact of shocks on the asset side of the banking balance sheets may disrupt the supply of aggregate credit.

In their model, banks are unable to completely diversify their loan portfolios and thus they can become insolvent. This will be the case when the total loan repayments (from both entrepreneurs and other banks) are insufficient to cover their obligations to their depositors and other banks. In order to clear the banking system when some banks become insolvent the authors apply the method suggested by Eisenberg and Noe (2001). Insolvencies can propagate through the banking network. When one bank is unable to meet its obligations to another bank, the latter bank might itself become insolvent even if it would have remained solvent had its loans to the originally failed bank been repaid. The bankruptcy resolution process terminates when there are no insolvent banks left. The number of bank failures will depend on (a) the distribution of initial losses across the banking system, and (b) the structure of the financial network (see, for example, Acemoglu et al. 2015).

As long as the liquidation of assets held by insolvent institutions does not depress the market values of these assets the total systemic losses by the end of the resolution process will be equal to the initial losses due to the inability of entrepreneurs to repay their loans. However, as Shleifer and Vishny (1992) have argued during systemic episodes, exactly because there are many failing institutions, the market value (liquidation value) of the assets can drop below

their corresponding book values (fire sales). This drop in asset prices forces other institutions to reevaluate their own assets thus potentially causing new rounds of failures.

In the model, when the authors do not allow for fire sales, the value of aggregate credit provided by the banking network follows a random walk. This is because the capacity of the banking network to provide credit each period depends on the availability of reserves which in turn depends on the performance of aggregate loans the period before. Given that shocks are normally distributed each period it follows that aggregate lending activity follows a random walk. When they introduce fire sales, systemic losses can be much greater than initial losses thus introducing fat tails on the lower end of the distribution of aggregate credit. Under the supposition that aggregate credit is positively correlated with aggregate output their approach might be useful for explaining two features of business cycles: (a) the asymmetry in booms and busts (Acemoglu and Scott 1991), and (b) macroeconomic fat tails Acemoglu et al. 2017).

The chapter by **Schmitt, Tuinstra and Westerhoff** reviews the literature on market interactions and policy measures. In the wake of the financial crisis that hit the global economy almost ten years ago many economists and policy makers realized that the strong links between individual markets played an important role in allowing the crisis to spread globally, or may even have been at the core of the emergence of the crisis. This has spawned a literature that deals both with the effect that interactions between markets have on market stability, and with the policy measures that may be implemented to counter the instabilities that potentially arise from these interactions. In this chapter, the authors review a small part of that literature.

That individual markets may lead to instability has been recognized for some time already. Classic textbook examples are the cobweb model under naïve expectations (see Ezekiel 1938) or the Cournot oligopoly model under best reply dynamics (see Theocharis 1959). More recently the development of the theory of nonlinear dynamical systems has led to an increased attention for the possibility of market instability. Some early and important applications of this theory are Grandmont (1985) and Bullard (1994) on overlapping generations models, Chiarella (1998), Hommes (1994) and Brock and Hommes (1997) on cobweb markets, Day and Huang (1990), Lux (1995) and Brock and Hommes (1998) on financial markets and Puu (1991) and Kopel (1996) on Cournot duopoly models. Laboratory experiments with paid human subjects suggest that instability is indeed likely to occur in some of these market environments (see e.g. Hommes, Sonnemans, Tuinstra and van de Velden, 2005 and Heemeijer, Hommes, Sonnemans and Tuinstra 2009).

In the last decade, the interaction between markets has been identified as an additional route to market instability. Dieci and Westerhoff (2009, 2010), for example, find that two stable cobweb markets may become unstable when they are linked. Tuinstra, Wegener and Westerhoff (2014) show that this increased instability may result in counterintuitive policy prescriptions: increasing import tariffs between interacting markets may decrease allocative efficiency at the

steady state equilibrium, but may be welfare enhancing nevertheless. This is because they weaken the link between markets and thereby stabilize consumption and production patterns. Even in the absence of naïve price expectations and cobweb dynamics linking two markets may lead to instability, as demonstrated by Schmitt, Tuinstra and Westerhoff (2017a, 2017b). They study a stylized model of market interaction, where firms may migrate between two regions on the basis of profitability between these regions. If firms are sufficiently sensitive to these profit differences this may lead to unstable dynamics. Following the insights from Schmitt and Westerhoff (2015, 2017), the papers by Schmitt, Tuinstra and Westerhoff (2017a, 2017b) investigate how the introduction of profit taxes may dampen the profit differences between the two markets and thereby stabilize the dynamics. Schmitt, Tuinstra and Westerhoff (2017b) discuss the scenario where each region is overseen by an independent local government or regulatory authority. Optimally, these two regulators coordinate their profit taxes in such a way that markets are stable and total welfare is maximized. However, Schmitt, Tuinstra and Westerhoff (2017b) argue that, if regulators are only (or mainly) interested in welfare in their own region, each of them will have the incentive to decrease the profit tax, which can destabilizes markets.

The last chapter of Part II by **Bougheas and Kirman** offers a number of suggestions for future research, first, for exploring the link between the network structure of the banking system and aggregate credit and, second, the relationship between systemic risk in financial markets and macroeconomic fat tails.

## 4 The Micro Perspective - Strategic Decisions and Interactions

**Colombo and Dawid** offer an innovative approach to the issue of location decisions and R&D spillovers and provides ample opportunity for further developments. The authors assume that firms are forward looking and base their location decisions on sophisticated (Markov) strategies that determine their R&D investments. In particular, the authors consider a differential game based on a standard Cournot model with three firms. Firms 2 and 3 are located in an industrial cluster and at  $t = 0$ , firm 1 has the choice to either co-locate in the cluster or instead locate in isolation. The difference between these two choices is that each firm in the cluster receives knowledge spillovers from all the other firms in the cluster whereas if located in isolation, firm 1 receives no spillovers but also does not need to worry about outgoing spillovers. The firms choose production quantities and their R&D investments that increase their knowledge stocks. In the model, absorptive capacity plays a crucial role: a firm's current knowledge stock determines its absorptive capacity which, in turn, determines how much of incoming spillovers the firm can absorb to increase its own knowledge stock. The firms located in the cluster face fixed costs of congestion each period and all firms try to maximize their discounted profits. The authors characterize Markov-Perfect Equilibria of this game for various scenarios. One of their insight is that the relation between firm 1's location choice and firm 1's initial knowledge stock

depends on the characteristics of absorptive capacity (exogenous and constant versus endogenous and proportional to knowledge stock). In case of a constant absorptive capacity, firm 1 prefers to locate in the cluster if its initial knowledge stock is small. However, if absorptive capacity is endogenous, then firm 1 locates in the cluster if its initial knowledge stock is large. As a consequence, the authors conclude that a deeper understanding of firms' location decisions requires a thorough investigation of the associated characteristics of the spillover mechanisms.

The chapter by **Kopel, Manasakis, and Petrakis** studies competition between a local firm that invests in corporate social responsibility (CSR) and a multinational firm that enters the foreign market either via exports or foreign direct investments. As a modeling framework the authors use a multi-stage game which is more standard in the International Business literature. In stage 1, the government of the foreign country sets the tariff; in stage 2, the multinational home firm decides whether to serve the foreign country's market through exports or FDI; finally, in stage 3, the local firm invests in CSR and the two firms set their quantities for the markets. The game is solved by backward induction and the solution concept is Subgame Perfect Nash. The authors derive results on the interaction between the optimal mode of entry of the (multinational) enterprise and the local firm's investment in CSR. They further look at the effects on consumer welfare, firm profits, and total welfare. This paper brings together two important topics, namely the role of multinational enterprises in globalized markets and the impact of firms' corporate social responsibility. With the introduction of Directive 2014/95/EU on the disclosure of non-financial and diversity information, the European Commission expressed its view on CSR as an extended corporate governance policy. The Directive introduces enhanced reporting requirements on social, environmental and governance issues. During the process of the development of Directive 2014/95/EU, concerns were raised that due to this regulation European firms will have a competitive disadvantage against their international rivals. An important issue is, therefore, if CSR enhances or diminishes the competitiveness of European firms against their international rivals. This is particularly crucial, since the field of international trade and CSR policies is still under-researched (e.g. Kitzmuller and Shimshack 2012).

**Bischi, Kopel, Lamantia, and Radi** work within an evolutionary setting which is rarely used in the literature on location decisions of multinational firms. The authors study a population of firms that can either manufacture in their home country or off-shore production to a foreign country. The two locations are structurally different as the home country has higher unit production costs but higher internal knowledge spillovers whereas the foreign country has lower unit costs but also smaller internal knowledge spillovers. Additionally, (i) technology know-how developed in one country can potentially be transferred to the other country and be used for reducing costs, and (ii) firms located in the foreign country face congestion costs that are increasing in the manufacturing activity occurring in this country. The firms' location decisions are based on a simple (myopic) comparison of unit production costs taking into account current within-country spillovers, cross-border spillovers, and congestion costs. An evolutionary

choice mechanism based on an exponential replicator equation determines the share of firms that switch from one location to the other. From a modeling point of view, this chapter differs from the approaches taken in the previous two chapters of this section of the book since location strategies are spread in a population of firms via an evolutionary mechanism that captures some myopic decision rule instead of forward-looking rational decision calculus. The authors present a variety of scenarios and characterize long run location patterns for off-shoring and on-shoring and how these activities depend on spillovers and congestion costs. Such a framework can be used to generate robust predictions which can then be tested empirically.

Lastly, **Bischi and Kopel** briefly reviews the research activity carried out by the Working Group on ‘Social and Industrial Interactions’. The main focus of this group has been on the behaviour of economic agents at the micro-level. Concerning firms locational choices, and especially referring to multinationals, the authors notice that the trend that has lead many firms to off-shoring their main activities is, recently, reversing. Moreover, the authors stress that environmental, social and governance dimensions are becoming more and more relevant issues. Finally, they suggest as main avenues for future research the study on corporate social responsibility strategies of multinational enterprises and their global value chains.

## References

- Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A.: Systemic risk and stability in financial networks. *Am. Econ. Rev.* **105**, 564–608 (2015)
- Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A.: Microeconomic origins of macroeconomic tail risks. *Am. Econ. Rev.* **107**, 54–108 (2017)
- Acemoglu, D., Scott, A.: Asymmetric business cycles: theory and time-series evidence. *J. Monetary Econ.* **40**, 501–533 (1991)
- Brock, W., Hommes, C.: A rational route to randomness. *Econometrica* **65**, 1059–1095 (1997)
- Brock, W., Hommes, C.: Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *J. Econ. Dyn. Control* **22**, 1235–1274 (1998)
- Bullard, J.: Learning equilibria. *J. Econ. Theor.* **2**, 468–485 (1994)
- Chiarella, C.: The cobweb model - its instability and the onset of chaos. *Econ. Model.* **5**, 377–384 (1988)
- Day, R., Huang, W.: Bulls, bears and market sheep. *J. Econ. Behav. Organ.* **14**, 299–329 (1990)
- Dieci, R., Westerhoff, F.: Stability analysis of a cobweb model with market interactions. *Appl. Math. Comput.* **215**, 2011–2023 (2009)
- Dieci, R., Westerhoff, F.: Interacting cobweb markets. *J. Econ. Behav. Organ.* **75**, 461–481 (2010)
- Eisenberg, L., Noe, T.: Systemic risk in financial systems. *Manag. Sci.* **47**, 236–249 (2001)
- Ezekiel, M.: The cobweb theorem. *Q. J. Econ.* **52**, 255–280 (1938)
- Grandmont, J.-M.: On endogenous competitive business cycles. *Econometrica* **53**, 995–1045 (1985)



- Heemeijer, P., Hommes, C., Sonnemans, J., Tuinstra, J.: Price stability and volatility in markets with positive and negative expectations feedback: an experimental investigation. *J. Econ. Dyn. Control* **33**, 1052–1072 (2009)
- Hommes, C.: Dynamics of the cobweb model with adaptive expectations and nonlinear supply and demand. *J. Econ. Behav. Organ.* **24**, 315–335 (1994)
- Hommes, C., Sonnemans, J., Tuinstra, J., van de Velden, H.: Coordination of expectations in asset pricing experiments. *Rev. Financ. Stud.* **18**, 955–980 (2005)
- Kitzmueller, M., Shimshack, J.: Economic perspectives on corporate social responsibility. *J. Econ. Lit.* **50**, 51–84 (2012)
- Kopel, M.: Simple and complex adjustment dynamics in cournot duopoly models. *Chaos Solitons Fractals* **7**, 2031–2048 (1996)
- Krugman, P.: Increasing returns and economic geography. *J. Polit. Econ.* **99**, 483–499 (1991)
- Lux, T.: Herd behavior, bubbles and crashes. *Econ. J.* **105**, 881–896 (1995)
- Puu, T.: Chaos in duopoly pricing. *Chaos Solitons Fractals* **1**, 573–581 (1991)
- Schmitt, N., Tuinstra, J., Westerhoff, F.: Side effects of nonlinear profit taxes in a behavioral market entry model: abrupt changes, coexisting attractors and hysteresis problems. *J. Econ. Behav. Organ.* **135**, 15–38 (2017a)
- Schmitt, N., Tuinstra, J., Westerhoff, F.: Stability and welfare effects of profit taxes within an evolutionary market interaction model. *BERG Working Paper No. 122* (2017b)
- Schmitt, N., Westerhoff, F.: Managing rational routes to randomness. *J. Econ. Behav. Organ.* **116**, 157–173 (2015)
- Schmitt, N., Westerhoff, F.: Evolutionary competition and profit taxes: market stability versus tax burden. *Macroeconomic Dynamics* (2017), in press
- Schularick, M., Taylor, A.: Credit booms gone bust: monetary policy, leverage cycles, and financial crises, 1870–2008. *Am. Econ. Rev.* **102**, 1029–1061 (2012)
- Shleifer, A., Vishny, R.: Liquidation values and debt capacity: a market equilibrium approach. *J. Finance* **47**, 1343–1366 (1992)
- Theocharis, R.: On the stability of the Cournot solution to the oligopoly problem. *Rev. Econ. Stud.* **27**, 133–134 (1959)
- Tuinstra, J., Wegener, M., Westerhoff, F.: Positive welfare effects of trade barriers in a dynamic partial equilibrium model. *J. Econ. Dyn. Control* **48**, 246–264 (2014)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# **The Macro Perspective – Economic Geography**

# Policy Issues in NEG Models: Established Results and Open Questions

Pasquale Commendatore<sup>1</sup>, Christoph Hammer<sup>2</sup>, Ingrid Kubin<sup>2(✉)</sup>,  
and Carmelo Petraglia<sup>3</sup>

<sup>1</sup> Department of Law, University of Naples “Federico II”, Naples, Italy  
commenda@unina.it

<sup>2</sup> Department of Economics,  
Institute for International Economics and Development,  
WU Vienna University of Economics and Business, Vienna, Austria  
{Christoph.Hammer, Ingrid.Kubin}@wu.ac.at

<sup>3</sup> Department of Organization and Economics of Institutions,  
University of Basilicata, Potenza, Italy  
carmelo.petraglia@unibas.it

**Abstract.** This paper provides a non-technical overview of NEG models dealing with policy issues. Considered policy measures include alternative categories of public expenditure, international tax competition, unilateral actions of protection/liberalisation, and trade agreements. The implications of public intervention in two-region NEG models are discussed by unfolding the impact of policy measures on agglomeration/dispersion forces. Results are described in contrast with those obtained in standard non-NEG theoretical models. The high degree of abstraction limits the applicability of NEG models to real world policy issues. We discuss in some detail two extensions of NEG models to reduce this applicability gap: the cases of multi-regional frameworks and firm heterogeneity.

**Keywords:** New economic geography · Agglomeration · Economic integration · Tax competition · Government expenditure · Trade agreements

## 1 Policy Issues in NEG Models: A General Discussion

### 1.1 NEG Models in a Nutshell

In his seminal contribution, Krugman (1991) presented an intriguing and most stimulating argumentation: even if countries were identical, an uneven distribution of economic activity may emerge endogenously. The central mechanism is the following: Assume two identical regions that produce a homogenous agricultural commodity and differentiated manufacturing commodities. Markets for manufactured goods are monopolistically competitive (in the standard Dixit-Stiglitz set up with decreasing average costs in production and a love for variety utility function); trading commodities between regions is possible, but involves trade costs (that do not occur when selling commodities in the local market). In that environment, firms that have access to a larger local market have higher profits and are able to pay higher wages. Higher factor rewards, in turn, attract factor migration towards this region (or trigger more intense capital formation).

If migration is combined with a relocation of commodity demand, the local market size increases, which leads to even more migration – agglomeration is the result of a self-reinforcing factor migration process (or in some model variants of a self-reinforcing process of capital formation or firm entry). In models in which the mobile factor are workers, this so-called market size effect is enhanced by a price index effect. Variety loving consumers buy all commodity variants – irrespective of whether they are produced locally or imported. Therefore, the respective consumer price index is lower in the region in which more variants are produced locally (that do not involve transport costs), i.e. in the region with more firms. Workers thus migrate to the bigger region not only because firms are able to pay higher wages, but also because of the lower consumer price index. A competition effect works against these two agglomerative forces: with more firms in a market, the market niche of a single firm may shrink, even if the overall market size in that region has increased.

In more recent model variants, the competition effect may be enhanced via a variable mark-up – with more firms in the local market, the mark-up, which each single entrepreneur may charge, is reduced. Note that the last effect requires a departure from the standard Dixit-Stiglitz modelling of monopolistic competition. In these models, the CES utility function (giving rise to iso-elastic demand functions) is replaced by a log-linear utility function (that gives rise to linear demand functions implying varying mark-ups for monopoly pricing).

NEG models typically show that the competition effect dominates for high trade costs and an even regional distribution of economic activity emerges. Instead, agglomeration forces prevail for lower trade costs and a core-periphery pattern of economic activity appears. Given the symmetry of regions, agglomeration can appear in either of the two regions. In many models, both types of equilibrium co-exist as (locally) stable fixed points for intermediate levels of trade costs. In that case, small shocks may trigger a self-reinforcing process leading from a symmetric industry distribution to a core-periphery pattern. The direction of the small shock determines which region gets the core. In more technical terms: both types of equilibria have their own basin of attraction and small shocks can push the economy from one basin of attraction into the basin of attraction of another equilibrium. Thus, history matters for the long-run evolution of economic activity, which is an often-reiterated theme of NEG (see Krugman 1992). Another implication of the multiplicity of coexisting equilibria is that an economic evolution may not be reversible and hysteresis phenomena occur in the spatial pattern of economic activity.

The change in the locally available quantities of productive factors is at the core of each NEG model. Therefore, it is very common to classify models according to the mobility assumptions (see Baldwin et al. 2003; Brakman et al. 2001). We will use this classification also in this chapter:

**Core-periphery model (CP, see Krugman 1991):** Workers are mobile between regions but not between sectors. Since worker spend their income locally, demand is mobile as well.

**Footloose entrepreneur model (FE, see Forslid 1999; Ottaviano 2001; and Forslid and Ottaviano 2003):** Entrepreneurs (with their knowledge capital) are mobile between regions (but not between sectors). Entrepreneurs spend their income locally; therefore, demand is mobile as well.

Footloose capital model (FC, see Martin and Rogers 1995): Manufactured goods require entrepreneurial knowledge capital (blueprints). While capital is mobile between regions in search of the highest nominal reward, it is assumed that the capital earnings are remitted back to the owner who is regionally immobile. Hence, in that model class, factor mobility does not lead to a relocation of demand, which considerably simplifies the analytics, at the cost of losing some of the core features of a NEG model.

Constructed capital model (CC, see Baldwin 1999): There is no factor mobility between regions. However, capital is accumulated (constructed), possibly at different rates in both regions, which also may lead to agglomeration.

Vertical linkages model (VL, see Venables 1996; and Krugman and Venables 1995): This model class introduces input-output relations. Workers are not mobile between regions, but between sectors. They may move out of agriculture into newly created input firms, which may lead to a different regional industrial development and to agglomeration.

## 1.2 Policy in NEG Models: Fundamental Questions and the Applicability Gap

The highly abstract model construction with initially identical regions is the sparkling core of Krugman's argument. Even at this very abstract level, policy issues immediately emerge. Multiple equilibria with varying stability properties beg the question whether they are equivalent, viewed through the lens of a social welfare function. In addition, a typical NEG model involves several inefficiencies: most obvious, monopolistic price setting leads to socially inferior outcomes; in a context of imperfect competition, any change in the locally available amount of productive factors involves pecuniary externalities that are welfare relevant. There is a small strand of literature that addresses such questions explicitly: Ottaviano and Thisse (2001), Ottaviano et al. (2002), Ottaviano and Thisse (2002), Tabuchi and Thisse (2002), and more recently Pflüger and Südekum (2008) and Grafeneder-Weissteiner et al. (2015). We would like to point out two interesting results of this strand of literature: a first result relies on the possible multiplicity of equilibria. Those papers show the possibility of over-agglomeration, i.e. a situation in which decentralized market processes without policy intervention lead to agglomeration, while the symmetric equilibrium exhibits a higher social welfare. The papers take this as a basis for regional redistributive policy, either in the form of restrictions on factor mobility (in order to prevent agglomeration) or in the form of interregional transfers to compensate the periphery. A second result focusses on the stability properties: Given the monopolistic set up, the symmetric, stable equilibrium may quite well be inefficient and policy interventions can increase the social welfare (in the symmetric equilibrium). However – and this is the interesting result – these optimal policy interventions may change the stability properties of the symmetric equilibrium that becomes unstable; thus, an allegedly optimal policy intervention may lead to unintended agglomeration.

A related policy topic that already emerges at a very abstract level are distributive issues, which actually are pervasive in NEG models. The utility level of the immobile workers left behind in the periphery is lower than the utility level of the workers in the core region. In such a situation, using a social welfare function is not without problems.

Charlot et al. (2006) point out that a simple utilitarian social welfare function actually reflects indifference to inequality and the authors suggest using a more general CES specification that allows reflecting various attitudes towards inequality. Charlot et al. (2006) show that the attitude towards inequality heavily influences the result; however, over-agglomeration is a possibility also in their framework.

Leaving these fundamental policy questions aside, prototype NEG models with two initially identical regions are often used to assess the impact of various policy measures – including tariffs, free-trade agreements, customs unions, taxes, subsidies, and public expenditures on items such as infrastructure, transport systems, and R&D – on the regional distribution of economic activity. We review this literature in the following sections. Each section start with the analysis for two regions and, subsequently, we discuss whether the two regions’ results carry over to a multi-regional framework (for the latter see Commentatore et al. 2015).

Before proceeding, it is worth noting that the extreme degree of abstraction limits the applicability of NEG models to real world policy issues and that NEG models were extended in various directions to reduce this applicability gap: Many NEG models allow for different country sizes. The admittedly simplistic representation of geography in NEG is a highly disputed issue and several extension are found in the literature. Most recently, some NEG models include firm heterogeneity. We explicitly discuss the last two issues and start with extensions concerning geography.

It was Krugman (1993) who borrowed from Cronon’s famous book “Nature’s Metropolis: Chicago and the Great West” (see Cronon 1991) the notions of “first nature” and “second nature”. First nature regional differences are exogenous to economic activity, e.g. endowment with natural resources, geographical location – centrality, location at the sea, at a river, in the mountains – and geopolitical factors (see also Venables 2006; Roos 2005). First nature asymmetries can easily explain an asymmetric regional distribution of economic activity. However, the main achievement of the New Economic Geography is to show that even when starting with first nature identical regions, endogenous economic – i.e. second nature – processes may bring about a very uneven regional distribution of economic activity. In typical NEG models, first nature geography plays a minimal role. Regions are separated by (symmetric) transportation or trade costs (see also the discussion in Østbye 2010). In the unfolding of the NEG paradigm over the last decades, some contributions do bring back first nature differences and study their interaction with second nature agglomeration processes (for overviews see Fujita and Mori 2005 and Venables 2006). In the next paragraphs, we present two possibilities found in the literature for incorporating first nature differences into NEG models and discuss whether the basic NEG mechanisms are still relevant.

The first approach consists of assuming differences in trade costs, conveying first nature advantages to some regions. Krugman (1993) and Ago et al. (2006) analyse three (or more) regions located on a line, which gives a locational advantage to the middle region. Fujita and Mori (1996) also analyse a multi-region model in which some locations have a first nature locational advantage (e.g. a port functioning as transport hub). Østbye (2010) analyses a two region model in which one region has a first nature advantage because of trade links with an outside region. Differences in the trade costs change the strength of the various agglomeration and dispersion forces. A centrally

located region has an advantage in accessing markets. However, it is also subject to a stronger competition effect. The papers typically show that second nature forces may dominate first nature advantages. Krugman (1993) and Ago et al. (2006) show that agglomeration may not occur at the geographical center, instead intense competition in the middle region can force manufacturing to move out and agglomeration occurs in more peripheral regions. The typical NEG theme of irreversibility and regional hysteresis resonates in the results found in Fujita and Mori (1996): A region that became the core because of first nature advantages may continue to be core even if it had lost its first nature advantage. An example for this is a natural port that functioned as transport hub and that lost this first nature advantage due to innovations in the transport system.

A second approach to model first nature disparities is to specify productivity differences between the regions (possibly due to regional differences in the endowment with natural resources), i.e. to introduce Ricardian (comparative or absolute) advantages into a NEG framework.

NEG models typically show that for high trade costs both agglomeration (such as market access and price index effect) and dispersion forces (such as the competition effect) are strong. In addition, they typically show that for high trade costs dispersion forces dominate and that an equal regional distribution of economic activity emerges. Instead, for low trade costs, both forces are weaker and agglomeration forces dominate, which leads to a core-periphery pattern of economic activity. Comparative advantages add another dispersion force, since each region attracts industry that uses its specific comparative advantage, the strength of which does not vary with trade costs. Therefore, for low trade costs both NEG forces are weak and comparative advantage dominates, and this leads to a dispersed industry structure. Instead, for intermediate and high trade costs NEG forces dominate. In particular, for intermediate trade costs NEG agglomeration forces shape the regional distribution of industry and its regional specialization (and thus trading) pattern may not follow comparative advantages. For high trade costs, NEG dispersion forces lead to an equally distributed industry, perhaps biased towards comparative advantage (see e.g. Picard and Zeng 2010; Forslid and Wooton 2003, see also the related contributions by Ricci 1999; Bagoulla 2006; Matsuoka and Kikuchi 2012; Pflüger and Tabuchi 2016; and Commendatore et al. 2017). In an interesting related paper, Matsuyama and Takahashi (1998) analyse a two regions model that also includes Ricardian advantages into a NEG perspective. The authors show that allowing for factor migration may overturn the comparative advantage structure; in addition, they explicitly address the above mentioned welfare issues and show that in some cases agglomeration may be socially undesirable.

After having discussed the representation of geography in NEG models, we turn to another direction of increasing the degree of realism: The “new new economic geography” (NNEG, see Ottaviano 2011) loosens the symmetry assumption by introducing firm heterogeneity à la Melitz (2003), i.e. introducing firms that differ wrt productivity. This not only allows asking under which conditions firms or workers chose to agglomerate or disperse across space, but also if more or less productive firms behave differently in this process.

Baldwin and Okubo (2006) are the first to integrate part of the Melitz framework into the FC model. Firm heterogeneity is introduced by different marginal costs for each firm and these differences follow a Pareto distribution. Furthermore, switching

regions is associated with a quadratic cost function. While the regions are symmetric in tastes and technology, one region is assumed larger in terms of the number of firms and workers. As in the FC model, there is a tendency for firms to move towards the larger region, which is governed by agglomeration forces. Probably not surprising, the most productive firms move first. They can bear the costs associated with moving most easily. Therefore, there is a maximum marginal cost that drives the relocation of firms from the smaller to the larger market. As trade gets freer, more firms find it profitable to relocate to the larger region. The introduction of heterogeneity does not alter the break and sustain points derived in the standard FC model. However, measured in terms of the number of firms, relocation costs combined with differences in productivity act as a dispersion force as fewer firms are able to relocate to the larger market. This leads to a selection effect meaning that only particular firms agglomerate in the equilibrium.

While the FC model is rather simple but lacks some features of other NEG models, Okubo (2009) shows how firm heterogeneity works in the still tractable footloose capital vertical linkages model (FCVL). Symmetric regions additionally allow for the possibility of endogenous asymmetry. Similar to the former paper, Okubo (2009) finds that firm heterogeneity has a moderating effect on the relocation process. In this model, there is a stepwise agglomeration process. The most productive firms that already exported from the smaller region relocate first and stay (net-) exporters. Second, there are firms that sold only locally but become exporters once they relocate to the larger region. Finally, there are firms that only sold locally in the smaller market and that stay a local seller in the larger market. The existence of the last type of firm in the larger region lowers the strength of the agglomeration forces and increases the strength of the dispersion force, the agglomeration process is gradual and only partial. The author also shows that a decrease in the fixed costs that are associated with exporting strengthens the agglomeration forces making full agglomeration a possibility. What is more, he argues that in his model, trade integration leads to a divergence in welfare with the individuals in the larger market being better off.

In a linear setup, Okubo et al. (2010) focus on the role of competition and its relationship to location choice of differently productive firms. Their results indicate that efficient and inefficient firms move away from each other similar to the analysis discussed above. At some point of trade integration, all productive firms are in the larger market, while all less productive firms remain in the smaller market shielding themselves from competition. However, deeper market integration makes it harder for high-cost firms to avoid competition and the agglomeration advantages exceed the competition effect. So there might be a non-monotonic relation between trade integration and the sorting of firms.

By employing a version of the FE model, Ottaviano (2012) demonstrates how firm heterogeneity affects the relative strength of agglomeration and dispersion forces. Entrepreneurs first develop blue prints that are needed for production. Only after observing their productivity do firms decide whether to use them in order to produce. The blue prints depreciate at the end of the period. Similar to the papers before, the model exhibits partial agglomeration when trade barriers fall. The authors pay particular attention to the different effects that a variation of the scale and the shape parameters of the (Pareto) distribution has on the firm selection at the equilibrium.



Baldwin and Okubo (2014) update their previous work by including entry and exit of firms as well as fixed market entry costs, so-called “beachhead costs”. Firms are also able to relocate after their initial location choice or vice versa, they might enter or exit a market after they have relocated.

Incorporating firm heterogeneity into NEG models introduces new and interesting variations of the classic NEG mechanisms. There is still agglomeration and dispersion across space driven by the respective forces. However, agglomeration can be gradual and not every firm is equally likely to move to the larger markets. Thus, in equilibrium, not only the number of firms may be different between regions, but also the distribution of their productivities. This is relevant for welfare analyses. The presence of firm heterogeneity may therefore also alter some policy implications of NEG models.

## 2 Public Spending: Productivity and Demand Effects

NEG scholars have studied the impact of different categories of public expenditure on industrial location within different variants of NEG frameworks. The non-exhaustive survey covered in this section uses the typical distinction between unproductive and productive public spending to classify these models and elaborates on the main channels through which public intervention influences industrial location. In the first case, the government provides an additional channel to the “market-access” effect, via the so-called “demand” effect. Tax revenues are spent in consumption goods, thus financing an additional source of local demand for manufactured goods and magnifying the “market-access” effect. On the other hand, the so-called productive categories of public spending affect firms’ location via a “productivity” effect exerted on the supply side of the economy. Investment in R&D, transportation and other types of productivity-enhancing infrastructures affect production in the manufacturing sector via their positive impact on factors’ productivity.

The first group of works, which includes Trionfetti (1997 and 2001) and Brülhart and Trionfetti (2004), contradict the standard result obtained by Baldwin (1970, 1984) in a Heckscher-Ohlin framework according to which home-bias public procurement is neutral for the determination of a country’s pattern of specialisation. Trionfetti (1997) adds public expenditure to a standard two-country/two-region CP model. To focus on the “pure” demand effect of public expenditure, this author makes two important simplifying assumptions excluding other effects, which could arise from alternative uses of public resources or from taxation policies: The government spends all tax revenues on manufactured goods, which are destroyed after the purchase. Moreover, the income of the mobile factor, i.e. manufacturing workers’ income, is not taxed.

The additional public demand for manufactured goods increases local demand. This creates a new demand-linked effect in the model, which can, under certain conditions, dominate all the others and, acting as a dispersion force, may lead to a stable equilibrium with partial spatial concentration and no catastrophic agglomeration. A crucial result is that, by allocating a larger amount of public expenditure to the domestic manufacturing good, for a given level of the expenditure of the other government, a government can enlarge the share of the domestic industry: public expenditure exerts a “pull” effect on the location of industry. Trionfetti (1997) also briefly considers the

effects of different intergovernmental transfer schemes (pure transfers, tied aid and joint expenditure) showing that these effects depend on how public expenditure is allocated between sectors and domestic and foreign goods.

The above reported result of public expenditure favoring dispersion over agglomeration emerges independently from the employed variant of NEG framework. Trionfetti (2001) and Brülhart and Trionfetti (2004) accommodate for government procurement in the VL version of the CP model (Krugman and Venables 1995). The differentiated manufacturing good is used as an intermediate input by the manufacturing sector itself as well as by national governments for the provision of public services. When public procurement is home-biased, industrial agglomeration cannot occur. One or three long-run equilibria may exist depending on the parameter settings. If countries are identical, the central equilibrium is symmetric and can be either stable or unstable. In the latter case, the industrial sector partially agglomerates in one country (the one with the larger initial endowment of the mobile factor), with the second country keeping some industry: public expenditure exerts a “spread” effect on industry. In Trionfetti (2001), it is also shown that home-biased public procurement may reduce inequalities and, under specific circumstances, be welfare improving. Brülhart and Trionfetti (2004) put forward an empirical analysis aiming to validate the main predictions of their model concerning the “pull” and “spread” effects of home-biased public procurement.

The group of models dealing with the effects of productivity-enhancing public spending includes, among others, Brakman et al. (2002 and 2008) where the FE model is extended in order to include a government sector, which directly produces public goods that may enhance regional competitiveness. Public goods – produced under a constant returns of scale (CRS) technology that uses only variable human capital – are “productive” in that they reduce both fixed and variable costs of manufacturing firms<sup>1</sup>. Brakman et al. (2008) assume that the amount of public capital is fixed; whereas in Brakman et al. (2002), public capital is a share of the overall capital. Another difference between the two models is that in the latter one public goods also affects the individuals’ utility function. Two important features of these analyses are as follows. First, the provision of public goods is financed by a uniform income tax. A tax levied also on the mobile factor acts as a spreading force. Second, the effect of public goods on factors’ productivity is entirely local, reinforcing agglomeration. The main conclusions of these works are: When equal amounts of public goods are provided in the two regions, an increase of such provision perturbs the equilibrium between agglomeration and dispersion forces, thus fostering agglomeration if sufficiently large. When the provision of public goods in the two regions is asymmetric, numerical evidence shows that the attractiveness of locations is influenced by their endowments of public goods, confirming that the “pull” effect of public expenditure, as partial agglomeration occurs in the region with the larger endowment of public goods. However, if in one of the two regions public expenditure is more effective in reducing production costs or public capital is more efficient in the production of the public good, Brakman et al. (2002) show that even

---

<sup>1</sup> The fixed cost component embodies knowledge-intensive activities such as “R&D, marketing and management”; whereas both fixed and variable costs are affected by the quality of social and economic infrastructures.

if this region has a lower endowment of public or private inputs it could still attract the larger share of the manufacturing industry. Finally, when cost reduction is sufficiently large to offset the tax increase necessary to finance its production public expenditure may also have a positive impact on the overall number of firms of the economy.

The first group of models, focussing on the demand effect, deliberately abstract from the productivity effect: Typically, public expenditure affects industrial location through demand, while any impact on factors' productivity is neglected as public intervention is not assumed to affect the production function in the manufacturing sector. In the same manner, scholars dealing with the productivity effect deliberately abstract from the demand effect. The contributions by Commendatore et al. (2008, 2009 and 2010) are at the crossroads between these two groups of models as they suggest to jointly study the interplay between the demand and productivity effects of public policy in the same framework.

The framework chosen by Commendatore et al. (2008) is a two-region CC model with mobile capital, which extends the FC model allowing for the endogenous construction/depreciation of capital goods. In the model the asymmetric regional distribution of the immobile factor (labour) between the North (the "rich" or "advanced" region) and the South (the "poor" or "backward" region) translate into different market sizes. The overall economy is composed of four sectors: agriculture, manufacturing, investment sector and government sector. In the investment sector the mobile factor (capital) is produced under perfect competition with a CRS technology involving the use of labour. Capital is used in fixed amounts both by the government (as the only production factor with a CRS technology) and in manufacturing (coupled with the variable labour input). A central (national) government produces a local public good, which enhances labour productivity in the investment and manufacturing sectors.<sup>2</sup> In this framework the authors address two main questions revealing a possible trade-off between overall efficiency and regional equity: how the provision and the financing of productive public investments impact on the overall stock of private capital and on its distribution across the regions.

An increase in the provision of productivity-enhancing public capital does not always increase the overall private capital as the regional and sectoral distribution of the public investments matters. The "productivity" effect tends to increase private capital, while a "crowding-out" effect between public and private investments works in the opposite direction. Assuming that the investment sector agglomerates where its production is less costly and that the provision of the public good occurs in the other region, the overall capital stock will unambiguously decline due to the "crowding-out effect". On the other hand, if the provision of public goods occurs in the region where the investment good sector is located and the productivity gain in that sector is sufficiently large to overcome the "crowding out" effect, the overall capital stock will increase.

Concerning the effect on the allocation of private capital between the regions, the long-run outcome depends on the combined impact of the two possible effects of public

---

<sup>2</sup> As in Brakman et al. (2002) the overall capital stock is composed of public and private capital. On the other hand, differently from that contribution, local public goods (public expenditure in research, innovation and education) do not enter in agents' utility.

expenditure on manufacturing production: the “productivity effect” – a larger provision of public services in one region, by lowering the labour input requirement, attracts firms to that region; and the “demand effect” – the tax rise necessary to finance such provision, by contracting private expenditure for manufactures, induces a non-favorable change in the relative market size, driving away firms from that region. The relevant policy conclusion is that a central government whose aim is to reduce regional inequalities, can choose a suitable financing scheme for public expenditure that could weaken the demand effect inducing firms to relocate in the backward region. The demand effect will be negligible if also taxpayers residing in the advanced region contribute on the basis of their income to the financing. Finally, it is also possible that the provision of public goods in the South could, even in the presence of the “crowding-out” effect, increase the local level of private capital or could improve welfare in that region.

These results are in general confirmed in the much simpler FC setting without investment sector considered by Commendatore et al. (2009), where it is shown that the provision of a productivity-enhancing local public good may redistribute the industrial activity, counterbalancing differences in market sizes between regions. Furthermore, it is also confirmed that the process of agglomeration in the larger region following market integration could be slowed down or even overturned by increasing sufficiently the provision of productive public expenditure.

Commendatore et al. (2010) relax the assumption of pure local effects of productive public expenditure within a FC framework. Focusing on the case of R&D public expenditure, the authors explore how knowledge creation and diffusion across regions impinges upon the processes of spatial agglomeration and dispersion. The productivity-enhancing effect of public expenditure takes place by lowering fixed costs in the manufacturing sector, so that the number of firms may vary even if the number of capital units is given. Hence, public policy can impact on the endogenous number of firms as well. Two possible scenarios are studied. In the first scenario, knowledge spillovers – in the form of new ideas generated in public universities and research institutions – are (perfectly) global benefitting all firms independently of their location. In this scenario, public policy is able to affect the number of firms as a lower number of these units is required for each variety of the manufactured good: an increase in public expenditure unambiguously increase the total number of varieties. On the other hand, public policy is unable to affect the regional distribution of capital units. However, if the productivity effect is weak, the demand effect can reduce the dimension of the industrial sector in the region that finances public expenditure. In the second scenario, knowledge spillovers are (partially/perfectly) local. The impact of knowledge on productivity gains is spatially limited depending on its local specificity and on the absorbing capacity of firms, so that knowledge generated by R&D activities benefits firms, which are located in the region where public expenditures occurs. In this scenario, as in Commendatore et al. (2008 and 2009), both productivity and demand effects play a role in determining the equilibrium regional shares of capital as well as the number of varieties produced in each region. Moreover, it is shown that the relationship between the overall number of firms and public expenditure could be non-monotonic. These results hold in the dispersed equilibrium, when the at least some industry is located in both regions. In an agglomerated equilibrium, when industrial sector is concentrated only in one region, the total number of firms always increases

with public expenditure except in the case of perfectly local spillovers when industry is agglomerated in the region where the R&D activities do not occur. This conclusion reveals a trade-off between efficiency and equity, which typically emerges in models where the appropriability of knowledge has a spatial dimension.

Recently, so-called new economic geography and growth (NEGG) models introduce a R&D sector to explain the dynamic processes giving rise to spatial agglomeration and growth. This strand of NEG literature – which jointly considers the spatial and the temporal dimension of the economic development – is mainly based on the endogenous growth models of the Grossman and Helpman (1991) and Romer (1990) type. NEGG models abandon the typical NEG assumption of a fixed stock of resources, bringing to the fore the trade-growth relationship (Martin and Ottaviano 1999; Baldwin et al. 2003; Fujita and Thisse 2002; Dupont 2007; Cerina and Mureddu 2012; and for reviews Baldwin et al. 2003, Chap. 7; Baldwin and Martin 2004; Cerina and Pigliaru 2007; and Breinlich et al. 2014). The main engine of growth is technological change due to R&D investments and the spatial diffusion of knowledge plays a fundamental role. Moreover, once it is taken into account that knowledge spillovers may exhaust their effect with distance, NEGG models predict a potential trade-off between accumulation and territorial cohesion, giving rise to uneven growth. This extends to the very long-run the government dilemma over the choice between efficiency and equity.

A paradigmatic example of a NEGG model exploring how public policy may impact on agglomeration and growth is the dynamic version of the FC model put forward by Martin and Rogers (1995) and (1998, 1999) and reformulated in Baldwin et al. (2003, Chap. 17). As in Martin and Rogers (1995), in Martin (1998, 1999) public policy may improve intra-regional transport infrastructure – facilitating domestic trade (transport infrastructures, law and contract enforcement and network communications) – and inter-regional/international transport infrastructures – facilitating trade between regions (international communication and transportation system). Moreover, in the presence of partially local spillovers, better transport infrastructures may improve inter-regional knowledge transfer. Envisaging a more advanced region (North) where the investment sector is located and a backward region (South), it is possible to single out the effects of specific policies and study how they can mitigate/exacerbate the trade-off between equity and efficiency: (i) a persistent income transfer to the South reduces income inequality and favours dispersion of the economic activity at the cost of a lower growth rate of the overall economy; (ii) an investment in local (intra-regional) infrastructures in the South may reduce spatial equity at the cost of a larger north-south and capitalists-workers income inequality and a smaller rate of growth of the overall economy; (iii) an investment in (inter-regional) infrastructures favours agglomeration in the North but reduces income inequalities and enhances the growth of the overall economy. Finally, (iv) policies aiming to increase the spatial diffusion of knowledge (such as, investment in telecommunication infrastructures, in internet access or in human capital, and so on) does not suffer the trade-off between equity and efficiency. Indeed, such a policy may reduce both spatial and income inequalities in correspondence of a higher rate of growth of the whole economy.

A limit shared by all the above discussed analyses is that they only consider economies composed of two regions, not allowing for third-region (indirect) effects and complex feedbacks emerging in a multi-region framework or on the role of geography

(characterizing the accessibility of regions or their endowments of immobile factors or natural resources, and so on) shaping this framework. Indeed, in the words of Desmet and Rossi-Hansberg (2010, p. 44): “[t]he focus on a small number of locations does not allow this literature to capture the richness of the observed distribution of economic activity across space, thus restricting the way these models are able to connect with the data”. Thus, according to these authors, the assumption of a small number of regions represents the greatest weakness of NEG (and a fortiori NEG) models. In various contributions, these authors (see Desmet and Rossi-Hansberg 2009, 2010 and 2014) put forward a theory of spatial development in which space, and therefore the number of locations, is represented as a continuum. Behrens et al. (2007) put forward a study on a spatial network composed of nine regions with a specific geography (part of the network has a tree structure and another a circular structure), clarifying that it is not easy (excepts for specific network configurations) to derive clear-cut general analytic results and therefore to develop the welfare analysis necessary to formulate policy recommendations. In their specific set-up standard results related to preferential trade agreements (which have a global impact over the network) should be better specified taking into account changes in transport infrastructures (whose impact has a more local nature).

The richness of results delivered in multi-region frameworks dealing with the importance of different public policies can be exemplified by referring to the attempts made in the three regions case. Baldwin et al. (2003, Chap. 17) consider an economy where three regions are allocated equidistantly on a line representing a hub-and-spoke economy, where the central region is poor and the two at the borders are rich. They show that the result according to which an increase in interregional infrastructures favors agglomeration in the rich region (Martin and Rogers 1995; Martin 1998 and 1999) is overturned depending on the balancing between the “home market” effect favoring the two rich regions and the “central place” effect giving an advantage to the poor region.

Forslid (1994) confirms that results, which hold in a two-region context cannot always be applied when three regions are considered. This author extends the standard static FC model by introducing three (advanced, intermediate, and poor) regions with different size evaluating how different policies may counterbalance the effects of trade integration. With no policy intervention, at initial stages economic integration leads to deindustrialization in the smaller region (with both the larger and the indeterminate gaining industry) and at later stages to full agglomeration in the larger region (with the intermediate region losing all industry). Relocation of governmental agencies to the periphery, i.e. to one of the smaller regions, could counteract this process. However, if only public capital is transferred to the smaller region, if integration is not complete the reduction of private capital stock in the intermediate region is larger than the stock of governmental capital shifted to the smaller region, thus the net gain for the two peripheral regions following the policy is negative. Relocating public employees as well may advantage both the large and the intermediate region when integration is not too strong. Instead, relocating government capital in the intermediate region may lead to an equal capital distribution between the larger and the intermediate region. Investment in transport infrastructures reducing distance between the larger region and the other two strongly favours the first, which becomes a hub, whereas the smaller peripheral region suffers an acceleration of deindustrialization. The same would apply

to the smaller region by reducing its distance with the intermediate region. The latter gains all the industry with trade liberalization. Finally, regional subsidies to capital granted to the smaller region financed by a proportional nation-wide labour income tax. This may lead, with partial liberalization, to the disappearance of the industry in the smaller region. However, the smaller region may rapidly reindustrialize when integration progress further.

Such a paradoxical result on the effect of subsidies to poor regions also applies to the case of two asymmetric regions studied by Dupont and Martin (2006) in a static FC model. These authors study the effects of four types of subsidy schemes in a static FC model with two asymmetric regions (again labelled North and South), the first couple designed to encourage firms relocation to the poor region and the second couple to boost production in that region:

- (i) a subsidy proportional to (operating) profits granted to firms located in the South financed by a national income tax. This subsidy program determines a shift of the industry to the South (firms relocate where subsidies to profit are supplied) but a worsening of income distribution between workers and capitalists (profit rise in both regions but a larger number of capital owners lives the North);
- (ii) a subsidy to profits granted to firms located in the South financed by a local income tax. The signs of the effects just mentioned are the same but the magnitudes are different: the relocation of firms to the South is smaller and the effect upon and income inequality is larger;
- (iii) a subsidy to production (and thus to employment) in the South financed by an economy-wide taxation. Higher competition in the labour market induces a rise in wages, income inequalities are reduced and relocation to the South is more pronounced compared to the two previous schemes;
- (iv) a subsidy to production in the South financed by local taxation. Since taxes (and higher wages) are paid locally, there is no fiscal transfer from the North to the South. Higher profits are repatriated to the North reproducing the same effects of a subsidy to profits financed locally.

### 3 Tax Competition and Agglomeration

A key conclusion of the traditional tax competition literature is that tax competition leads to a loss of industrial capital to competing countries. This conclusion is based on the following mechanism. Given that capital owners are interested in their after-tax income, assuming that they are taxed according to source principle (i.e. according to the rules specific to the country where income is generated), the comparison of international tax rates will affect and distort international capital allocation.<sup>3</sup>

---

<sup>3</sup> A well-known result from the public finance literature is that capital income taxation according to the source principle may lead to a distortionary allocation of capital across countries. Thus, the residence principle, which does not affect capital owners location decisions, is preferable. However, as shown by Commendatore and Kubin (2016) in a NEG framework, once one allows for a different sectoral composition between private and public expenditures, the difference between the two taxation



Producers will move to whichever country has the lowest tax rates and countries will experience falling tax rates in the attempt to attract or hold on productive activities in order to raise local tax revenues. This is in a nutshell the well-known result of the “race to the bottom” of competing governments: countries that try to set a lower tax rate than their neighbors may end up taxing capital income not at all.<sup>4</sup>

As recalled by Andersson and Forslid (2003), key features of tax competition models are: the coexistence of immobile workers and mobile capital, and the question to what extent public goods can be financed by means of taxes on mobile capital. The “race to the bottom” conclusion will imply that taxation of the mobile factor will be distorted downwards compared with a situation where all factors are immobile, thus leading to an inefficient provision of public goods. In this perspective, co-ordination or harmonization of international tax policies is an issue. Typically, tax harmonization entails a shift from a non-cooperative tax game to a cooperative tax game, and Pareto improvement from the government’s perspective follows by definition (Baldwin and Krugman 2004).

NEG scholars have shown that this result does not hold in the presence of significant agglomeration economies and goods market integration, and have formally demonstrated that tax competition over mobile productive factors is affected by industrial agglomeration. The general argument rests on the result that increased economic integration leads imperfectly competitive firms (capital owners) to benefit from agglomeration rents that can be taxed. Baldwin et al. (2003) show that such rents arise in a wide range of NEG models. This will allow a country to raise its tax without losing capital and/or its industrial base, thus leading to a “race to the top” rather than a “race to the bottom”. As argued by Baldwin and Krugman (2004), wealthier countries offer capital favourable external economies and, within limits, this allows them to hold on to mobile factors of production even while levying higher tax rates than less advanced nations. However, should the tax rate get too high, the results could be catastrophic: “not only will capital move abroad, but because that movement undermines agglomeration economies it may be irreversible” (Baldwin and Krugman 2004, p. 2).

NEG models with public spending (see previous section) often assume some sort of taxation for the financing of public services; however, in that literature, governments typically do not engage in tax competition. In the following, we review models of tax competition involving typical NEG features such as imperfect competition, trade costs and agglomeration rents and thus departing from traditional tax competition analyses.

---

(Footnote 3 continued)

schemes is less clear-cut. Taxation on the basis of the residence principle is not neutral and may lead to capital relocation; whereas depending on how tax revenues are allocated between manufactured and agricultural goods, the corresponding change in the market size may reduce the distortionary effect of taxation according to the source principle.

<sup>4</sup> As summarized by Baldwin and Krugman (2004), the standard tax-competition literature works with a one period model featuring a single good produced by two factors. Labour is immobile between locations and capital is mobile. Trade costs are zero, firms face perfect competition and constant returns, so there is no trade among regions and capital faces smoothly diminishing returns. Typically, governments choose the capital tax rate in a Nash game. The standard approach is to compare equilibrium tax rates with no capital mobility and with perfect capital mobility. The standard result is such that equilibrium taxes are sub-optimally low.



While the tax competition and agglomeration literature shares the common assumption of mobile capital, NEG models with tax competition have originally explored the case of labour mobility. Accordingly, seminal models can be classified according to whether they work under the assumption of labour (Ludema and Wooton 2000, Andersson and Forslid 2003) or capital/entrepreneur mobility (Kind et al. 2000; Baldwin and Krugman 2004; Ottaviano and Ypersele 2005; Borck and Pflüger 2006). Regardless differences in the mobility assumption – and other technical aspects – a general feature, as opposed to traditional tax competition outcomes, is that results depend on the degree of production agglomeration. Typically, when the mobile factor is concentrated, the associated taxable rents imply that taxes will remain high without inducing relocation, provided that – in general – the international tax gap is smaller than the agglomeration rent.

Ludema and Wooton (2000) were the first to recognize the lack of theoretical work on the link between the two related issues arising from deeper economic integration: the erosion of fiscal autonomy experienced by integrating countries brought about by more mobile tax bases; and the spatial agglomeration of economic activity driven by divergent economic structures and incomes across the integrating countries. In motivating their interest in bringing together the two issues in a NEG framework, the authors explain the additional mechanism induced by international tax competition by which economic integration can have distributional consequences, thus providing the general rational at the core of the subsequent literature. That is, while agglomeration alters the distribution of income across countries, tax competition influences the distribution of income across factors. In Ludema and Wooton (2000) words, “if immobile factors compete to create or maintain a core by offering low (or negative) taxes to mobile manufacturing labor, they run the risk that much of the potential gain to having a core is dissipated in the process. If so, then agglomerative forces coupled with the tax competition may impoverish immobile factors, regardless of location” (Ludema and Wooton 2000, p. 333). For this reason, the question to be addressed is whether economic integration, by strengthening agglomerative forces, will intensify tax competition and will thus result in lower equilibrium taxes or not.

In order the answer to this question, Ludema and Wooton (2000) modify the Krugman (1991) model in two aspects. First, manufacturing firms are quantity-setting (Cournot) oligopolists, as opposed to the monopolistic competition approach commonly taken in NEG models. This departure allows obtaining closed-form solutions, while preserving all of the relevant characteristics of Krugman's model. Second, manufacturing workers are assumed to be imperfectly mobile. This assumption enables to study the effects of economic integration on the intensity of tax competition under two notions of international integration modelled as either an increase in factor mobility or as a reduction in trade costs on goods. They conclude that integration interpreted as decreasing trade costs reduces the intensity of tax competition, thus restoring rather than eroding fiscal autonomy. On the other hand, integration interpreted as increased labor mobility has mixed effects.

Andersson and Forslid (2003) use the Forslid and Ottaviano (1999) version of the Krugman (1991) model to study the effects of taxes on skilled and unskilled workers on the location of manufacturing production. In the model, proportional taxes on immobile unskilled workers and mobile skilled workers finance country-specific public goods.

The authors study the impact of taxes on the stability of a dispersed equilibrium and find that it depends on the relationship between taxes on mobile labor and taxes on immobile labor. In particular, a small tax redistribution from mobile to immobile workers makes the dispersed equilibrium unstable, causing a catastrophic agglomeration where all skilled workers end up in one country. In line with Ludema and Woodon (2000), once the agglomeration is established and the related agglomeration rents arise, taxes on mobile workers are not generally driven down to some minimum in the “core” country as predicted by traditional tax competition.

As opposed to Ludema and Wooton (2000) and Andersson and Forslid (2003), Kind et al. (2000) investigate how spatial agglomeration of economic activity affects the outcome of capital tax competition in a NEG framework with mobile capital, thus obtaining results directly comparable with those obtained in the traditional literature on capital tax competition. They show that the outcome of tax competition depends on the interaction between two forces of agglomeration: trade costs and pecuniary externalities. The analysis considers the two alternative scenarios of capital (and firms) either concentrated in one single location or evenly distributed between two countries. When capital is concentrated in one single country, a government may be able to exploit the locational rents created by agglomeration forces through a positive source tax and this in turn will lead to an increase in national welfare. This result is in line with the conclusion of Ludema and Woodon (2000) as it shows that increased economic integration achieved via lower trade costs allows a country to raise its tax on mobile capital without losing productive resources. Indeed, since agglomeration makes capital effectively immobile due to pecuniary externalities, the host country gains from setting its source-tax on capital above that of the other country, thus increasing its welfare per capita. In the opposite case of industrial activities evenly spread across countries, the equilibrium outcome is such that both countries provide a subsidy of equal size to capital.

Perhaps the most well-known contribution in this field is the one by Baldwin and Krugman (2004), which challenges the idea that the integrating nations should agree on common tax rates in order to avoid the “race to the bottom” and undermine their relatively generous welfare states. Its popularity is probably due to two reasons. First, the policy implication it carries on the desirability of tax harmonization. Second, the capability it has to explain observed corporate tax differentials in integrating regions such as European Union in terms of external economies due to agglomeration forces. The results of Baldwin and Krugman (2004) are obtained within the solvable variant of the model of Krugman (1991) due to Forslid (1999), where the two otherwise identical countries have different tax rates. Entrepreneurs are the mobile factor, while workers are immobile. Agglomeration forces imply that the real reward of entrepreneurs includes a location specific agglomeration rent. That is, entrepreneurs located in the country that initially has the core strictly prefer to locate there and would thus be willing to bear a higher tax in order to be there. There exists a bell-shaped relationship between agglomeration rents (and tax rates that entrepreneurs are willing to pay) and trade openness. Indeed, when trade is impossible, firms cannot serve both markets from a single location and agglomeration is not possible. At the opposite extreme, when trade is completely free, location is irrelevant and agglomeration is useless. Hence, the importance of agglomeration is greatest (the tax rate the entrepreneurs is higher)

at intermediate values of openness (where agglomeration is both feasible and useful). This implies that size of the locational rent is a bell-shaped function of the level of integration, so the tax gap first widens before narrowing as integration increases. From the policy perspective, the model delivers a result in sharp contrast with the traditional tax competition literature: harmonization makes one or both countries worse off when agglomeration forces are present.

Interestingly, Ottaviano and van Ypersele (2005) contribute to the tax competition and agglomeration literature by exploring three related issues. First, the distortions induced by tax competition on the international allocation of capital and, as a consequence, on the inefficient international specialization in production. Second, the subsequent inefficiencies caused to the pattern of international trade. Third, the impact of these inefficiencies on the gap in economic development among countries. According to the authors, the extant literature on tax competition and agglomeration – with the partial exception represented by Baldwin and Krugman (2004) – lacks a detailed welfare analysis able to reach a conclusion on the desirability of tax competition. Accordingly, they propose a full-fledge global welfare analysis in a general equilibrium model in which two countries compete for monopolistically competitive firms à la Ottaviano et al. (2002). Results are obtained using as a benchmark the free market outcome that yields a home market effect: the larger country hosts a more than proportionate share of firms. Unless trade costs are low enough, such an outcome is shown to be inefficient because it leads too many firms to concentrate in the larger country. In the tax competitive outcome, the location of firms is less concentrated than the free market one. When trade costs are large enough to make it inefficient for all firms to cluster in a single country, tax competition for mobile firms is efficiency-enhancing with respect to the free market outcome. This result is reversed as trade costs fall and clustering becomes efficient. Finally, under both free market and tax competition, the inefficiencies in international specialization and trade flows vanish when trade costs are low enough. Otherwise, only international tax coordination can implement the efficient spatial distribution of firms. All these results lead to conclude that “the policy attitude towards tax competition should depend on the degree of trade integration...at the initial stages of an integration process, forbidding tax competition without agreeing on tax coordination is a bad idea. It is much less so at later stages, when the free market and the harmonization outcomes tend to coincide” (Ottaviano and van Ypersele 2005, p. 45).

The models discussed above deal with variants of CP models with either symmetric equilibria or equilibria with complete agglomeration. Borck and Pflüger (2006) take one step forward and study whether the “race to the top” result suggested by previous work generalizes to a NEG framework, which features stable locational equilibria with only partial agglomeration of firms in one of two countries. The model draws on Pflüger (2004)<sup>5</sup> and shows that, in addition to the extreme outcome of complete agglomeration, a tax differential may arise as an equilibrium of the tax game between the two countries even when there is only partial agglomeration. In particular, in the

---

<sup>5</sup> Pflüger (2004) develops a model that deviates from the standard CP model in two respects. First, it is assumed that the fixed cost in the manufacturing sector consists of a separate internationally mobile factor (Forslid 1999; Forslid and Ottaviano 2003). Second, the Cobb-Douglas upper-tier utility is replaced by a logarithmic quasi-linear utility specification.

case with partial agglomeration, the partial core can maintain a positive tax gap even though no agglomeration rent accrues to the mobile factor.

## 4 Trade: Unilateral Protectionism and Trade Agreements

This section reviews trade policy implications of theoretical NEG models. We discuss unilateral actions of protection and liberalisation that aim at industrialisation, as well as trade agreements.

The most basic, symmetric NEG models yield a counter-intuitive result when it comes to unilateral protection. Import substitution in the form of unilateral protection always benefits the protecting region or nation (see for example Baldwin et al. 2003, Chap. 12).<sup>6</sup> This so-called “price-lowering-protection” (PLP) mechanism works because the price-lowering inflow of firms into the home market exceeds the more direct price-increasing effect of protectionism.<sup>7</sup> Therefore, it seems that unilateral protection is one strategy to gain welfare and attract industry. There are, however, some qualifications to be made.

The first one is the introduction of relocation costs. When it is costly for firms to change location, then unilateral protection has different impacts on the price index depending on (i) the costs of relocation, (ii) the trade openness of the respective nations and (iii) the relative strengths of agglomeration forces in the respective nations. The higher the costs for relocating, the more an unilateral protection will raise the price index because the offsetting inflow of firms does not take place. The level of relocation costs that prohibits the PLP is lower the easier it is to export to the foreign country and the stronger are the agglomeration forces in the home market. A second qualification to the PLP is the fact that a country has to be large enough for firms to have an incentive to relocate there. If the market size is too small, then unilateral protection will always raise the price index. Third, as long as factor endowments are assumed identical, there is no place for a comparative advantage (CA) story. Introducing such a Heckscher-Ohlin CA alters the PLP. Baldwin et al. (2003) assume different fixed costs across regions in order to model that. CA leads to a third effect of unilateral protection on the price index, the so-called “negative variety effect”. Asymmetric trade barriers now have a negative effect on the total number of firms. While there exists no analytical solution to the model anymore, the authors are able to derive results for a special case. They find that the PLP does not work when CA is strong enough.

As Baldwin et al. (2003, p. 297) point out, the PLP clearly is an artefact of the specific model assumptions: “The notion that unilateral protection always lowers the domestic price level by enticing industry to relocate is certainly one of the most outlandish policy implications of simple economic geography models.” They use the

---

<sup>6</sup> This chapter is in parts based on Puga and Venables (1997), Puga and Venables (1999) and Baldwin (1999). The political economy side of the discussion can for example be found in Baldwin and Robert-Nicoud (2007).

<sup>7</sup> Unilateral protection – represented as an increase in trade costs only in one direction – increases the price consumers have to pay for imported commodities, which involve a welfare loss similar to the one in standard models without factor mobility.

vertical linkages version of the FC model to show that unilateral liberalisation can lead to industrialisation. On the one hand, liberalisation may lower the costs of imported intermediate goods, which in turn makes the liberalising country more attractive for firms. On the other hand, if it decreases the costs of shipping final goods to this country, it would make it less attractive as a location for firms. Hence, in order to attract some industry, a country would have to liberalise the intermediate goods market without affecting the final goods market. However, they also show that an across-the-board liberalisation could have a similar effect if the liberalising country is small and the other markets are relatively large and open.

What about comparative advantage? It can be shown that in models of economic geography, smaller, poorer countries might well be without industry despite the possible CA due to lower wages compared to the richer country. The presence of agglomeration effects can dominate any trade or location patterns that would result from CA. If this is the case, the effect is stronger, (i) the stronger are agglomeration forces, (ii) the more expensive trade is, (iii) the smaller the market of the smaller country, and (iv) the higher possible trade barriers imposed by richer industrialised countries against manufacturing goods from smaller poorer countries. All of the analysis above is based on the FC or the FCVL model that do not feature an endogenous market size (because capital income is remitted back to the – regionally immobile – capital owners). Allowing for an endogenous market size and thus introducing circular causality strengthens the argumentation.

A further topic in this discussion is tariffs and quotas. While they might be equivalent in the simpler trade model, they are not in NEG models. Tariffs and quotas might lead to relocation into the protecting country due to the “price-lowering-protection” (PLP) effect. However, the locational equilibrium differs between quotas and tariffs, which is due to the different recipients of the trade rents. A tariff is collected by the protecting nation (and paid out to its residents) while a quota shifts the trade rent to the exporting firm. Thus, the operating profit for a firm exporting under a quote is larger than for a firm that exports under a tariff. This in turn leads to a situation where firms exporting to the protecting nation are more likely to stay where they are. This also means that firms in the protecting nation (and the other for that matter) are more likely to lobby for quotas than for tariffs. This is because higher profits from a tariff would be driven to zero by firms that enter the protecting nation. A quota, therefore, has a smaller effect on the PLP because of less movement of firms and an increasing effect on profits in both trading regions.

Melitz and Ottaviano (2008) develop a model with firm heterogeneity to focus on the interplay of market size, firm productivity and firm location. Unilateral liberalisation of a country leads to a decrease in competition in the liberalising country and an increase in the other country, which is driven by the cost cut-off that determines if a firm is productive enough to produce in a given country. The cut-off increases due to the liberalisation. The change in competition means a welfare loss for the liberalising country and a welfare gain in the other country. This result only holds in the long run when firms enter markets, while the number of firms is fixed in the short run. In the short run an unilateral liberalisation first leads to a decrease of the cost cut-off resulting in a pro-competitive effect. The increase in varieties that are exported dominates the

exit of local producers. To summarize, unilateral liberalisation first leads to an increase and then a decrease in welfare.

What the above discussion makes clear is that the welfare implications from more traditional trade models can be altered and even reversed when we consider NEG features such as the mobility of factors. While the price lowering protection effect is able to reverse the classical welfare implications, it very much depends on the specific model assumptions.

The second topic that we would like to review in this section on trade policy concerns the economic effects of preferential trade agreements. The standard analysis uses a Heckscher Ohlin model with two countries that form a union and a third, outside country. The union countries reduce mutual tariffs while maintaining the outside tariff, which leads to more specialisation within the union, creates additional trade between the countries of the union and diverts trade from the outside country to the countries inside the union. In a NEG perspective, the focus shifts from different countries (characterised by comparative advantages) to similar/identical countries and, in particular, factor mobility is possible. It turns out that – while the Heckscher-Ohlin results are still visible – NEG forces substantially alter the results. Note that for these questions multi-regional NEG models are indispensable. This stream of literature explicitly compares models with iso-elastic and with linear demand functions, where the latter – as pointed out above – introduces an additional dispersion force via a variable mark-up.

An early account is found in Baldwin et al. (2003, Chap. 14), which is mainly based and expands on Puga and Venables (1997). The authors present a multiregional footloose capital model in which demand functions are iso-elastic and capital is mobile between all countries. Reducing internal tariffs, i.e. reducing internal trade costs, improves market access for manufacturing firms inside the union and factor rewards increase, which in turn attracts factors from the outside country to move into the union. A production shifting effect occurs: the manufacturing sector inside the union grows (due to factor migration). If countries inside the union are of different size, NEG agglomerative processes may lead to a core-periphery pattern with all the industry ending up in the bigger country. In addition, the authors explicitly address welfare issues: they show that welfare in the union increases, whereas it declines in the outside country. Monfort and Nicolini (2000) construct a two-country core-periphery model each of which consists of two regions, in which factor mobility occurs only within countries. The authors show that also in this model set up deeper integration inside both countries may trigger agglomeration processes. Interestingly, the authors find similar results for external trade liberalisation (similar also Monfort and Van Ypersele 2003). In a related paper, Commendatore et al. (2016) also use a footloose capital model with two countries inside a union and one outside country and two production sectors: agriculture that produces a homogenous commodity and manufacturing that produces differentiated commodities. In contrast, the demand functions are linear; and factor mobility occurs only within the union. Reducing internal trade costs again increases manufacturing factor rewards inside the union (via a better market access), which leads to a higher employment in manufacturing sector within the union. It is interesting to note that the manufacturing sector grows even if firm migration from the outside country is not possible. It grows because it attracts additional workers from the agricultural sector in each of the union countries. In this sense, the union specialises in

manufacturing and – as a mirror image – the outside country specializes in agriculture. It leads to higher exports of the union, and to higher trade within the union. Integration, thus, leads to specialization, trade creation and trade diversion. These effects are similar to a Heckscher-Ohlin perspective. However, further reducing the trade costs may lead to a predominance of agglomeration forces within the union causing a core-periphery pattern of industrial activity. Competition is stronger in the newly formed core and – given the linear demand functions – firms react by reducing the price and further increasing the output. Specialization of the union in the manufacturing sector is reinforced, and trade creation and trade diversion become stronger. Thus, NEG forces corroborate and modify the static Heckscher Ohlin assessment of integration areas.

Another stream of papers shifts the focus from internal integration to external trade liberalisation, i.e. to a reduction of trade costs between the union and the outside country. These studies stress that the impact w.r.t. regional inequalities in the union depends upon model specifications (see Brühlhart 2011). A first group of models incorporates iso-elastic demand functions and thus constant mark-ups. The models typically assume that productive factors are mobile only within a country, but not between countries. A reduction of external trade costs increases market access for both countries within the union, agglomeration forces are strengthened, whereas competition from the other union country becomes less important. Thus, agglomeration force dominate and external trade liberalisation leads to internal agglomeration (see Paluzie 2001; Brühlhart et al. 2004; Crozet and Koenig Soubeyran 2004; Commendatore et al. 2014). Finally, in a quite comprehensive study Zeng and Zhao (2010) study the interaction of internal integration, external liberalization and the degree of international factor mobility. They show that international factor mobility may actually reverse the results obtained with internationally immobile productive factors. A second group of models introduces additional dispersion forces: Krugman and Elizondo (1996) take into account that factor migration leads to an increase in land rentals and commuting costs; Behrens (2011) uses linear demand functions, which imply lower mark-ups if more competitors operate in the markets. This additional dispersion force may actually reverse the result: External liberalisation leads to internal dispersion instead of agglomeration. In a related paper, Behrens et al. (2007) study the interaction of a reduction in international and intranational trade costs. They show that “lower intranational transport costs foster regional divergence when international trade costs are high enough, whereas lower international trade costs promote regional convergence when intranational transport costs are high enough” (Behrens et al. 2007, p. 1297).

## References

- Ago, T., Isono, I., Tabuchi, T.: Locational disadvantage of the hub. *Ann. Reg. Sci.* **40**, 819–848 (2006)
- Andersson, F., Forslid, R.: Tax competition and economic geography. *J. Public Econ. Theor.* **5**, 279–303 (2003)
- Bagoulla, C.: Localisation industrielle et spécialisation. *Rev. Économique* **57**, 705–726 (2006)
- Baldwin, R.: *Nontariff Distortions of International Trade*. Brookings Institution, Washington, DC (1970)



- Baldwin, R.: Trade policies in developed countries. In: Jones, R., Kenen, P. (eds.) *Handbook of International Economics*. Elsevier North-Holland (1984)
- Baldwin, R., Krugman, P.: Agglomeration, integration and tax harmonization. *Eur. Econ. Rev.* **48**, 1–23 (2004)
- Baldwin, R., Forslid, R., Martin, P., Ottaviano, G., Robert-Nicoud, F.: *Economic Geography and Public Policy*. Princeton University Press, Princeton (2003)
- Baldwin, R.E., Okubo, T.: International trade, offshoring and heterogeneous firms. *Rev. Int. Econ.* **22**, 59–72 (2014)
- Baldwin, R.E., Robert-Nicoud, F.: Entry and asymmetric lobbying: why governments pick losers. *J. Eur. Econ. Assoc.* **5**, 1064–1093 (2007)
- Baldwin, R.E., Martin, P.: Agglomeration and regional growth. In: Henderson, J.V., Thisse, J.F. (eds.) *Handbook of Regional and Urban Economics*. Cities and Geography, vol. 4. Elsevier North-Holland (2004)
- Baldwin, R.E., Okubo, T.: Heterogeneous firms, agglomeration and economic geography: spatial selection and sorting. *J. Econ. Geogr.* **6**, 323–346 (2006)
- Baldwin, R.E.: Agglomeration and endogenous capital. *Eur. Econ. Rev.* **43**, 253–280 (1999)
- Behrens, K.: International integration and regional inequalities: how important is national infrastructure? *Manchester School* **79**, 952–971 (2011)
- Behrens, K., Gaigné, C., Ottaviano, G.I.P., Thisse, J.-F.: Countries, regions and trade: on the welfare impacts of economic integration. *Eur. Econ. Rev.* **51**, 1277–1301 (2007)
- Borck, R., Pflüger, M.: Agglomeration and tax competition. *Eur. Econ. Rev.* **50**, 647–668 (2006)
- Brakman, S., Garretsen, H., Van Marrewijk, C.: *An Introduction to Geographical Economics*. Cambridge University Press, Cambridge (2001)
- Brakman, S., Garretsen, H., Van Marrewijk, C.: Location competition and agglomeration: the role of government spending. CESifo Working Paper No. 775, München (2002)
- Brakman, S., Garretsen, J.H., Marrewijk, J.G.M.: Agglomeration and government spending. In: Brakman, S., Garretsen, H. (eds.) *Foreign Direct Investment and the Multinational Enterprise*. MIT Press (2008)
- Breinlich, H., Ottaviano, G.I.P., Temple, J.R.W.: Regional growth and regional decline. In: Philippe, A., Steven, N.D. (eds.) *Handbook of Economic Growth*. Elsevier (2014)
- Brühlhart, M., Trionfetti, F.: Public expenditure, international specialisation, and agglomeration. *Eur. Econ. Rev.* **48**, 851–881 (2004)
- Brühlhart, M.: The spatial effects of trade openness: a survey. *Rev. World Econ.* **147**, 59–83 (2011)
- Brühlhart, M., Crozet, M., Koenig, P.: Enlargement and the EU periphery: the impact of changing market potential. *World Econ.* **27**, 853–875 (2004)
- Cerina, F., Pigliaru, F.: Agglomeration and growth: a critical assessment. In: Fingleton, B. (ed.) *New Directions in Economic Geography*. Edward Elgar, Cheltenham (2007)
- Cerina, F., Mureddu, F.: Agglomeration and growth with endogenous expenditure shares. *J. Reg. Sci.* **52**, 324–360 (2012)
- Charlot, S., Gaigné, C., Robert-Nicoud, F., Thisse, J.-F.: Agglomeration and welfare: the core–periphery model in the light of Bentham, Kaldor, and Rawls. *J. Publ. Econ.* **90**, 325–347 (2006)
- Commendatore, P., Kubin, I.: Source versus residence: a comparison from a new economic geography perspective. *Pap. Reg. Sci.* **95**, 201–223 (2016)
- Commendatore, P., Kubin, I., Mossay, P.: On the new economic geography of a multi-cone world. *Rev. Int. Econ.* (2017). doi:[10.1111/roie.12312](https://doi.org/10.1111/roie.12312), forthcoming
- Commendatore, P., Kubin, I., Sushko, I.: Trade agreements in a linear FE model. In: Bednar-Friedl, B., Kleinert, J. (eds.) *Dynamic Approaches to Global Economic Challenges*. Festschrift in Honor of Karl Farmer. Springer International Publishing (2016)



- Commendatore, P., Filoso, V., Grafeneder-Weisstener, T., Kubin, I.: Towards a multiregional neg framework: comparing alternative modelling strategies. In: Commendatore, P., Kayam, S., Kubin, I. (eds.) *Complexity and Geographical Economics, Dynamic Modeling and Econometrics in Economics and Finance*. Springer International Publishing, Cham (2015)
- Commendatore, P., Kubin, I., Petraglia, C.: Productive public expenditure in a new economic geography model. *Économie Int./Int. Econ.* **114**, 133–160 (2008)
- Commendatore, P., Kubin, I., Petraglia, C.: Footloose capital and productive public services. In: Salvadori, N., Commendatore, P., Tambari, M. (eds.) *Geography, Structural Change and Economic Development: Theory and Empirics*. Edward Elgar Publishing (2009)
- Commendatore, P., Kubin, I., Petraglia, C.: R&D public expenditure, knowledge spillovers and agglomeration: comparative statics and dynamics. In: Chiarella, C., Bischi, G.I., Gardini, L. (eds.) *Nonlinear Dynamics in Economics, Finance and the Social Sciences*. Springer, Heidelberg (2010)
- Commendatore, P., Kubin, I., Petraglia, C., Sushko, I.: Regional integration, international liberalisation and the dynamics of industrial agglomeration. *J. Econ. Dyn. Control* **48**, 265–287 (2014)
- Cronon, W.: *Nature's Metropolis: Chicago and the Great West*. WW Norton & Company (1991)
- Crozet, M., Koenig Soubeyran, P.: EU enlargement and the internal geography of countries. *J. Comp. Econ.* **32**, 265–279 (2004)
- Desmet, K., Rossi-Hansberg, E.: On spatial dynamics. *J. Reg. Sci.* **50**, 43–63 (2010)
- Desmet, K., Rossi-Hansberg, E.: Spatial development. *Am. Econ. Rev.* **104**, 1211–1243 (2014)
- Desmet, K., Rossi-Hansberg, E.: Spatial growth and industry age. *J. Econ. Theor.* **144**, 2477–2502 (2009)
- Dupont, V.: Do geographical agglomeration, growth and equity conflicts? *Pap. Reg. Sci.* **86**, 193–213 (2007)
- Dupont, V., Martin, P.: Subsidies to poor regions and inequalities: some unpleasant arithmetic. *J. Econ. Geogr.* **6**, 223–240 (2006)
- Forslid, R., Wooton, I.: Comparative advantage and the location of production. *Rev. Int. Econ.* **11**, 588–603 (2003)
- Forslid, R., Ottaviano, G.: An analytically solvable core-periphery model. *J. Econ. Geogr.* **3**, 229–240 (2003)
- Forslid, R.: Regional policy, integration and the location of industry in a mutiregion framework, CEPR discussion paper No 4630, London (1994)
- Forslid, R.: Agglomeration with human and physical capital: an analytically solvable case, CEPR Discussion Paper No. 2102, London (1999)
- Forslid, R., Ottaviano, G.I.P.: Trade and Agglomeration: An Analytically Solvable Case. Mimeo, Lund University (1999)
- Fujita, M., Thisse, J.-F.: *Economics of Agglomeration. Cities, Industrial Location and Regional Growth*. Cambridge University Press, Cambridge (2002)
- Fujita, M., Mori, T.: The role of ports in the making of major cities: self-agglomeration and hub-effect. *J. Dev. Econ.* **49**, 93–120 (1996)
- Fujita, M., Mori, T.: Frontiers of the new economic geography. *Pap. Reg. Sci.* **84**, 377–405 (2005)
- Grafeneder-Weisstener, T., Kubin, I., Prettnner, K.: Coping with inefficiencies in a new economic geography model: the unintended consequences of policy interventions. *Math. Soc. Sci.* **76**, 146–157 (2015)
- Grossman, G.M., Helpman, E.: Quality ladders in the theory of growth. *Rev. Econ. Stud.* **58**, 43–61 (1991)
- Kind, H., Midelfart-Knarvik, K., Schjelderup, G.: Competing for capital in a “lumpy” world. *J. Publ. Econ.* **78**, 253–274 (2000)

- Krugman, P., Elizondo, R.L.: Trade policy and the third world metropolis. *J. Dev. Econ. Increasing Returns Monopolistic Competition Econ. Dev.* **49**, 137–150 (1996)
- Krugman, P.: Increasing returns and economic geography. *J. Polit. Econ.* **99**, 483–499 (1991)
- Krugman, P.: *Geography and Trade*. MIT Press, Cambridge (1992)
- Krugman, P.: First nature, second nature, and metropolitan location. *J. Reg. Sci.* **33**, 129–144 (1993)
- Krugman, P.R., Venables, A.J.: Globalization and the inequality of nations. *Q. J. Econ.* **110**, 857–880 (1995)
- Ludema, R.D., Woodon, I.: Economic geography and the fiscal effects of regional integration. *J. Int. Econ.* **52**, 331–357 (2000)
- Martin, P., Rogers, C.A.: Industrial location and public infrastructure. *J. Int. Econ.* **39**, 335–351 (1995)
- Martin, P., Ottaviano, G.I.P.: Growing locations: industry location in a model of endogenous growth. *Eur. Econ. Rev.* **43**, 281–302 (1999)
- Martin, P.: Can regional policies affect growth and geography in Europe? *World Econ.* **21**, 757–774 (1998)
- Martin, P.: Public policies, regional inequalities and growth. *J. Public Econ.* **73**, 85–105 (1999)
- Matsuoka, Y., Kikuchi, T.: Footloose capital and comparative advantage. *Pac. Econ. Rev.* **17**, 677–686 (2012)
- Matsuyama, K., Takahashi, T.: Self-defeating regional concentration. *Rev. Econ. Stud.* **65**, 211–234 (1998)
- Melitz, M.J., Ottaviano, G.I.P.: Market size, trade, and productivity. *Rev. Econ. Stud.* **75**, 295–316 (2008)
- Melitz, M.J.: The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* **71**, 1695–1725 (2003)
- Monfort, P., Nicolini, R.: Regional convergence and international integration. *J. Urban Econ.* **48**, 286–306 (2000)
- Monfort, P., Van Ypersele, T.: *Integration, Regional Agglomeration and International Trade*, CEPR Discussion Paper No. 3752, London (2003)
- Okubo, T.: Trade liberalisation and agglomeration with firm heterogeneity: forward and backward linkages. *Reg. Sci. Urban Econ.* **39**, 530–541 (2009)
- Okubo, T., Picard, P.M., Thisse, J.-F.: The spatial selection of heterogeneous firms. *J. Int. Econ.* **82**, 230–237 (2010)
- Østbye, S.: Regional policy analysis in a simple general equilibrium model with vertical linkages. *J. Reg. Sci.* **50**, 756–775 (2010)
- Ottaviano, G.I.P., van Ypersele, T.: Market size and tax competition. *J. Int. Econ.* **67**, 25–47 (2005)
- Ottaviano, G.I.P., Tabuchi, T., Thisse, J.-F.: Agglomeration and trade revisited. *Int. Econ. Rev.* **43**, 409–435 (2002)
- Ottaviano, G.I.P., Thisse, J.-F.: On economic geography in economic theory: increasing returns and pecuniary externalities. *J. Econ. Geogr.* **1**, 153–179 (2001)
- Ottaviano, G.I.P., Thisse, J.-F.: Integration, agglomeration and the political economics of factor mobility. *J. Public Econ.* **83**, 429–456 (2002)
- Ottaviano, G.I.P.: Monopolistic competition, trade, and endogenous spatial fluctuations. *Reg. Sci. Urban Econ.* **31**, 55–77 (2001)
- Ottaviano, G.I.P.: “New” new economic geography: firm heterogeneity and agglomeration economies. *J. Econ. Geogr.* **11**, 231–240 (2011)
- Ottaviano, G.I.P.: Agglomeration, trade and selection. *Reg. Sci. Urban Econ.* **42**, 987–997 (2012)
- Paluzie, E.: Trade policy and regional inequalities. *Papers Reg. Sci.* **80**, 67–85 (2001)

- Pflüger, M., Tabuchi, T.: Comparative Advantage and Agglomeration of Economic Activity, IZA Discussion Paper No. 10273, Bonn (2016)
- Pflüger, M., Südekum, J.: Integration, agglomeration and welfare. *J. Urban Econ.* **63**, 544–566 (2008)
- Pflüger, M.: A simple, analytically solvable Chamberlinian agglomeration model. *Reg. Sci. Urban Econ.* **34**, 565–573 (2004)
- Picard, P.M., Zeng, D.-Z.: A harmonization of first and second natures. *J. Reg. Sci.* **50**, 973–994 (2010)
- Puga, D., Venables, A.J.: Preferential trading arrangements and industrial location. *J. Int. Econ.* **43**, 347–368 (1997)
- Puga, D., Venables, A.J.: Agglomeration and economic development: import substitution vs. trade liberalisation. *Econ. J.* **109**, 292–311 (1999)
- Ricci, A.L.: Economic geography and comparative advantage: agglomeration versus specialization. *Eur. Econ. Rev.* **43**, 357–377 (1999)
- Romer, P.M.: Endogenous technological change. *J. Polit. Econ.* **98**, S71–S102 (1990)
- Roos, M.W.M.: How important is geography for agglomeration? *J. Econ. Geogr.* **5**, 605–620 (2005)
- Tabuchi, T., Thisse, J.-F.: Taste heterogeneity, labor mobility and economic geography. *J. Dev. Econ.* **69**, 155–177 (2002)
- Trionfetti, F.: Public expenditure and economic geography. *Ann. d'Économie et de Stat.* **47**, 101–120 (1997)
- Trionfetti, F.: Public procurement, market integration, and income inequalities. *Rev. Int. Econ.* **9**, 29–41 (2001)
- Venables, A.J.: Shifts in economic geography and their causes. *Econ. Rev. Fed. Reserve Bank Kansas City* **91**, 61–85 (2006)
- Venables, A.J.: Equilibrium locations of vertically linked industries. *Int. Econ. Rev.* **37**, 341–359 (1996)
- Zeng, D.-Z., Zhao, L.: Globalization, interregional and international inequalities. *J. Urban Econ.* **67**, 352–361 (2010)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# Emerging Trade Patterns in a 3-Region Linear NEG Model: Three Examples

Pasquale Commendatore<sup>1</sup>(✉), Ingrid Kubin<sup>2</sup>, and Iryna Sushko<sup>3</sup>

<sup>1</sup> University of Naples 'Federico II', Naples, Italy  
`commenda@unina.it`

<sup>2</sup> Vienna University of Economics and Business Administration, Vienna, Austria  
`Ingrid.Kubin@wu.ac.at`

<sup>3</sup> Institute of Mathematics, NASU, Kiev, Ukraine  
`sushko@imath.kiev.ua`

**Abstract.** This chapter draws attention to a specific feature of a NEG model that uses linear (and not iso-elastic) demand functions, namely its ability to account for zero trade. Thus, it represents a suitable framework to study how changes in parameters that are typical for NEG models, such as trade costs and regional market size, not only shape the regional distribution of economic activity, but at the same time determine the emergence of additional trade links between formerly autarkic regions. We survey some related papers and present a three-region framework that potentially nests many possible trade patterns. To focus the analysis, we study in more detail three specific trade patterns frequently found in the EU trade network. We start with three autarkic regions; then we introduce the possibility that two regions trade with each other; and, finally, we allow for one region trading with the other two, but the latter are still not trading with each other. We find a surprising plethora of long-run equilibria each involving a specific regional distribution of economic activity and a specific pattern of trade links. We show how a reduction in trade costs shapes simultaneously industry location and the configuration of the trade network.

**Keywords:** Two-dimensional piecewise smooth map · Multistability · Basin of attraction · New Economic Geography model · Three-region models · Trade patterns

## 1 Introduction

In this chapter, we address the questions of what determines the creation of trade links between regions, and their direction, and how the presence of a trade link may interfere with the distribution of the manufacturing activity across space. Indeed, it is easy to infer that the presence of a trade link may affect the processes of agglomeration or dispersion of such activity and, in turn, the degree of spatial concentration of industry has a bearing on the formation of trade links by increasing/reducing the accessibility of a region for outside firms. As is well-known, the

interplay between agglomeration and dispersion forces and how it shapes the spatial economic landscape is at the core of the new economic geography (NEG) literature.

The NEG approach, originating from Krugman (1991) and Fujita, Krugman and Venables (1999) (see Baldwin *et al.* (2003), for a review), aims to explain what determines the distribution of industrial activities across space. Its main building blocks are Dixit-Stiglitz monopolistic competition, increasing returns, CES preferences over industrial varieties and isoelastic demands, multiplicative/iceberg trade costs and a dynamic mechanism based on factor migration (for example, in Krugman's seminal Core-Periphery model, this factor is undifferentiated labour; NEG model variants consider other factors such as physical capital or skilled workers/entrepreneurs). Mainly dealing with two-region economies, the predictions of NEG models depend on the balance between agglomeration and dispersion forces. Most NEG models involve two forces, related to market access and consumers' prices, that have agglomeration effects; and a third force, related to local competition, that has a dispersion effect. Which of these forces dominate, depends on trade costs: in a standard NEG set-up, high trade costs favours dispersion and low trade costs agglomeration. At intermediate trade costs, long-term dispersion or agglomeration of the industry may prevail, depending on its initial distribution of this sector between the regions. In the analytic perspective, this translates into coexistence of dispersion and agglomeration equilibria. A slight change in the initial distribution of economic activities may induce a catastrophic change from dispersion to agglomeration, switching from an equilibrium to another with completely different properties. For the present question it is important to note that – given the specific model set-up – firms always export, except when trade costs are infinite. This implication is at odds with empirical observations, where finding zero trade is quite common.

Ottaviano *et al.* (2002) propose an alternative model in which firms may not export even when trade costs are finite. Their model set up involves quadratic preferences, implying a linear demand (elasticity depends on prices and market fundamentals), segmented markets and additive trade costs. In Ottaviano *et al.* (2002) the local competition effect is reinforced by a variable mark-up. In a two-region framework, they show the following results: high trade costs imply dispersion; low trade costs imply agglomeration; and after a specific threshold for trade costs the economy may experience an abrupt change from dispersion to agglomeration. However, there is no coexistence of dispersion and agglomeration stable equilibria. Notice that Ottaviano *et al.* (2002) assume that trade costs are always sufficiently low to allow for bilateral trade between the regions.

Within the same framework, Behrens (2004) examines the opposite case by assuming trade costs sufficiently high that interregional trade never occurs. This author shows that even when regions are fully autarkic, agglomeration is still possible. Depending on the size of the immobile demand (fixing the size of the

demand that could shift),<sup>1</sup> dispersion, agglomeration or partial agglomeration are possible long-run outcomes. A large immobile local demand reinforces the dispersion forces and higher prices in autarky weakens the agglomeration ones, so that stable coexisting partial agglomeration equilibria are possible. Moreover, by reducing the size of immobile demand a smooth process from dispersion to agglomeration takes place. In a later paper, Behrens (2005) examines the case of intermediate trade costs, showing that, depending on the initial industry distribution across space and on trade costs, different trade patterns may emerge between two regions: autarky, unilateral or bilateral trade (alternatively, no trade, one-way trade or two-way trade). This author addresses also the interplay between patterns of trade and of regional distribution of economic activity. He shows that unilateral trade favours agglomeration forces over dispersion forces leading to the concentration of industry in the region that, in an initial stage, has the larger share of industry.<sup>2</sup>

Ago *et al.* (2006) consider an economy where three regions takes a specific hub-and-spoke configuration: they are aligned with the central region positioned at the same distance from the two peripheral ones. Dealing with the case of linear demand,<sup>3</sup> the authors show that in an economy with an intermediate size of the manufacturing a reduction in trade costs initially leads to a reduction in the share of the mobile factor located in the central region; subsequently, it leads to an increase; finally, when trade costs are sufficiently low, the mobile factor is fully agglomerated in the central region. Instead, when the size of the manufacturing sector (shifting demand) is relatively large compared to the agricultural sector (immobile demand) by reducing trade costs the share of industry in the central region shrinks and the mobile factor moves to the peripheral ones (finally agglomerating in one of them). In the linear framework, the competition effect is more intense, thus, firms have a move to the peripheral regions to

---

<sup>1</sup> More specifically, Behrens (2004) considers the size of the immobile factor, agricultural workers. However, taking into account that agricultural workers receive the same wage independently of their location, in our discussion we can safely refer to these agents demand.

<sup>2</sup> Okubo *et al.* (2014) consider the emergence of different trade patterns between two regions with different exogenous population sizes. They assume that the mobile factor is physical capital (separated from owner), this eliminates the agglomeration self-reinforcing effect linked to demand shifting.

<sup>3</sup> Indeed the main purpose of Ago *et al.* (2006) paper is to compare the two alternative NEG approaches. Their main conclusion is that moving from autarky to trade, the central region enjoys a locational advantage in the CES framework; whereas, in the linear demand framework, the apparently better location translates into a second nature disadvantage due to enhanced competition.

protect themselves from such competition.<sup>4</sup> Moreover, confining their analysis to a neighbourhood of the symmetric equilibrium, Ago *et al.* (2006) shows what patterns of trade may emerge, following a reduction in trade costs, given the specific geography assumed.

Finally, Commendatore *et al.* (2017) consider a three-region developing economy with poor infrastructure. A specific geography is assumed: two regions are relatively close to each other, whereas the third one is remote and difficult to access. Two stages of development are considered: in the first stage trade costs are so high that none of the regions trade; in the second stage, the first two regions improve their integration by lowering trade costs. Depending on the degree of integration and on the distribution of the industrial activity across the regions, different trade patterns may emerge (no trade, one-way trade or two way trade), but they only involve the two more integrated regions.

In this chapter we extend and/or integrate these contributions. We represent a small trade network, formed by three regions, aiming to: (i) highlight how distance and trade costs may determine the existence of a trade link and its direction; (ii) examine the long-term consequences of trade integration on the emergence/disappearance of trade links and on the distribution of economic activities across space; (iii) explore how the spatial distribution of economic activities and the presence/absence of trade links are interrelated. Given the large number of possible trade structures, we only consider three examples, representing three frequently realized trade patterns in the EU trade network (see Basile *et al.* (2016)): three regions in autarky; two regions engaging in trade with each other, with a third region in autarky; a hub and spoke economy but with a different structure from that suggested by Ago *et al.* (2006).

We structure the paper as follows. In Sect. 2, we present the basic economic framework, introducing three examples modelling trade network structures frequently realized in real world economies. In Sect. 3, we presents the short-run equilibrium solutions for these models. In Sect. 4, we move on to the discussion of the properties of the long-run equilibria and their economic interpretation. Section 5 concludes.

---

<sup>4</sup> We found few other contributions dealing with multiregional economies which employes Ottaviano *et al.* (2002) framework. Behrens (2011) and Commendatore *et al.* (2016) consider a two-country 3-region model where factor migration is limited to the two regions belonging to the same country. Tabuchi *et al.* (2005) focus on cities rather than regions, allowing for “urban costs”. Moreover, they do not study the emerging of different trade patterns (firms export to any city). Finally, Furusawa and Konishi (2007) explore the formation of free trade networks. They assume a given distribution of the industrial activity (confining their analysis to the short-run) and, in a network formation game setting, show how trade agreements are formed.

## 2 The Model

### 2.1 Basic Set-Up

The economy is composed of three regions (labelled  $r = 1, 2, 3$ ) distributed asymmetrically across space; two sectors, Agriculture ( $A$ ) and manufacturing ( $M$ ). The first characterized by perfect competition, constant returns and production of a homogeneous good and the second by monopolistic competition, increasing returns and production of a differentiated good; two factors of production, unskilled labour or simply labour ( $L$ ) and skilled labour or entrepreneurial activity ( $E$ ). Regions have the same endowment of labour. Workers are immobile, whereas entrepreneurs are allowed to migrate across regions.

### 2.2 Production

In the  $A$ -sector one unit of labour gives one unit of output. In the  $M$ -sector production requires use of one entrepreneur as fixed component plus  $\eta$  units of labour for each additional unit of output. The total cost of production ( $TC$ ) corresponds to:

$$TC = \pi_i + w\eta q_i,$$

where  $\pi_i$  represents the operating profits and the remuneration of an entrepreneur,  $w$  the wage rate,  $\eta$  the labour input requirement and  $q_i$  the quantity produced of variety  $i$ .

Due to increasing returns and absence of economies of scope, each firm produces a single variety. Following the assumption that only one entrepreneur is required to activate production, the number of entrepreneurs ( $E$ ) is equal to the number of firms and to the number of varieties ( $N$ ). Denoting by  $\lambda_r$  the share of entrepreneurs located in region  $r$ , the number of varieties produced in that region is:

$$N_r = \lambda_r N = \lambda_r E,$$

where  $0 \leq \lambda_r \leq 1$ ,  $\sum_{r=1}^3 \lambda_r = 1$  and  $r = 1, 2, 3$ .

### 2.3 Utility

The representative consumer's (unskilled or entrepreneur) preferences are quasi-linear (see Ottaviano *et al.* 2002), composed of a quadratic part defining the choice across the  $n$  varieties of the  $M$ -good and a linear component for the consumption of the  $A$ -good:

$$U = \alpha \sum_{i=1}^n c_i - \left( \frac{\beta - \delta}{2} \right) \sum_{i=1}^N c_i^2 - \frac{\delta}{2} \left( \sum_{i=1}^N c_i \right)^2 + C_A \quad (1)$$

where  $c_i$  is the consumption of variety  $i$  and  $C_A$  the consumption of the agricultural good. The parameters are interpreted as follows:  $\alpha > 0$  is the intensity of



preference over the  $M$ -varieties,  $\delta > 0$  the degree of substitutability across those varieties and the difference  $\beta - \delta$  measures the taste for variety; where  $\alpha > 0$  and  $\beta > \delta > 0$ .

The budget constraint is

$$\sum_{i=1}^n p_i c_i + p_A C_A = y + p_A \bar{C}_A \quad (2)$$

where  $p_i$  is the price of variety  $i$  including trade costs,  $p_A$  is the price of the agricultural good,  $y$  is the consumer's income and  $\bar{C}_A$  is her endowment of the agricultural good, sufficiently large to allow for positive consumption in equilibrium.

## 2.4 Trade Costs

The cost of trading varieties of the  $M$ -good from  $r$  to  $s$  (or in the opposite direction from  $s$  to  $r$ ) is  $T_{rs}$  ( $= T_{sr}$ );  $T_{rs} > 0$  for  $r \neq s$  and  $T_{rr} = 0$ , with  $r = 1, 2, 3$ . Trade costs separate the regions. Different combinations are possible, we will consider here three examples and leave to future research more general cases:

1. In the first example, all regions are in autarky: we assume that trade cost are sufficiently high that no trade can occur between the regions, independently of other factors (esp. the distribution of the economic activity across the regions). In this example, we assume that the cost of trading the  $M$ -varieties is very large and, for the sake of simplicity, it is the same for all the regions:  $T_{12} = T_{13} = T_{23} = T \gg 0$ . The specific value that  $T$  should take will be detailed below.
2. In the second example, only regions 1 and 2 may trade with each other, while region 3 is still in autarky. In this example, we assume  $T_{12} = T - \varepsilon$  and  $T_{13} = T_{23} = T$ , where  $0 < \varepsilon < T$  (this implies  $T_{12} < T_{13} = T_{23}$ ). One-way trade from 1 to 2 or from 2 to 1 or two-way trade between 1 and 2 can occur for  $\varepsilon$  sufficiently close to  $T$ .
3. In the third example, the three regions are positioned along a 'Hub and spoke' structure. Region 1 is the central and regions 2 and 3 are peripheral. In this example, we consider a special case: trade costs between region 1 and 2 and so small that these two regions are always engaged in bilateral trade independently of other factors (esp. the distribution of the economic activity across space); region 1 and 3 may trade as well; whereas no trade between 2 and 3 can occur. With respect to the other two examples, region 3 is getting closer to region 1 but is still too far from region 2. In this example, we assume  $T_{12} = \tau$ ,  $T_{13} = \psi$  and  $T_{23} = T$ , where  $0 < \psi < T$  and where  $\tau$  is sufficiently small to allow always for two-way trade between 1 and 2 (this implies  $T_{12} = \tau \leq T - \psi < T$ ). The specific value that  $\tau$  should take will be detailed below. One-way trade from 1 to 3 or from 3 to 1 or two-way trade between 1 and 2 can occur for  $\varepsilon$  sufficiently close to  $T$ .

### 3 Short-Run Equilibrium

In a short-run equilibrium, the distribution of entrepreneurs across space is given. All markets are in equilibrium. We choose the agricultural good as the numeraire. From perfect competition in the  $A$ -sector, it follows:  $p_A = w = 1$ .

To determine the short-run equilibrium solutions related to the  $M$ -sector, we proceed as follows.

Maximizing the utility (1) subject to the constraint (2), we obtain the first order conditions for  $i = 1, \dots, N$ :

$$\frac{\partial U}{\partial c_i} = \alpha - (\beta - \delta)c_i - \delta \sum_{i=1}^N c_i - p_i = 0$$

from which

$$p_i = \alpha - (\beta - \delta)c_i - \delta \sum_{i=1}^N c_i .$$

Solving this system for  $c_i$ , we obtain the individual linear demand function for each variety  $i$ :

$$c_i = \max[0, a - (b + cN)p_i + cP] \quad (3)$$

where  $P = \sum_{i=1}^N p_i$  is the price index and

$$a = \frac{\alpha}{(N-1)\delta + \beta}, \quad b = \frac{1}{(N-1)\delta + \beta}, \quad c = \frac{\delta}{(\beta - \gamma)[(N-1)\delta + \beta]} .$$

Moreover,  $c_i > 0$  for  $p_i < \tilde{p}_i = \frac{a+cP}{b+cN}$ ,  $\tilde{p}_i$  representing the cut-off price below which demand for variety  $i$  is positive.

The representative consumer's indirect utility corresponds to

$$V = S + y + \overline{C_A}$$

where  $S$  is the consumer's surplus:

$$S = U - \sum_{i=1}^N p_i c_i - C_A = \frac{a^2 N}{2b} + \frac{b + cN}{2} \sum_{i=1}^N p_i^2 - aP - \frac{c}{2} P^2$$

The consumer's demand originating from region  $s (= 1, 2, 3)$  for a good produced in region  $r (= 1, 2, 3)$ , dropping the subscript  $i$  because of symmetric firm behaviour, is:

$$c_{rs} = \max[0, a - (b + cN)p_{rs} + cP_s]$$

where  $p_{rs}$  is the price of a good produced in region  $r$  and consumed in region  $s$ ; and

$$P_s = \sum_{k=1}^3 N_k p_{ks} = \sum_{k=1}^3 \lambda_k E p_{ks} \quad (4)$$

is the price index in region  $s(= 1, 2, 3)$ . As before  $c_{rs} > 0$  if and only if  $p_{rs} < \tilde{p}_r = \frac{a+cP_s}{b+cN}$ , where  $\tilde{p}_r$  is the cut-off price above which demand is positive ( $r, s = 1, 2, 3$ ).

Taking into account that  $L_1 = L_2 = L_3 = \frac{L}{3}$ , with segmented markets, the operating profit of a firm located in  $r(= 1, 2, 3)$  is:

$$\pi_r = \sum_{s=1}^3 (p_{rs} - \eta - T_{rs}) q_{rs} \left( \frac{L}{3} + \lambda_s E \right) \quad (5)$$

In a short-run equilibrium, demand is equal to supply in each segmented market  $s(= 1, 2, 3)$ :  $q_{rs} = c_{rs}$ . From profit maximization, recalling that  $N = E$  and that firms consider the price index as given, the first order conditions for  $r, s = 1, 2, 3$  follow:

$$\frac{\partial \pi_r}{\partial p_{rs}} = [a + (\eta + T_{rs})(b + cE) + cP_s - 2p_{rs}(b + cE)] \left( \frac{L}{3} + \lambda_s E \right) = 0$$

Taking into account trade costs and letting  $\tilde{p}_r > \eta$  for  $r = 1, 2, 3$ , to allow for positive production in the local market, profit maximizing prices correspond to:

$$p_{rr} = \frac{a + cP_r + \eta(b + cE)}{2(b + cE)} = \frac{1}{2}(\tilde{p}_r + \eta) \quad (6)$$

which is the price that firms quote in the market where they are located; and to

$$p_{rs} = \begin{cases} \frac{a+cP_s+(\eta+T_{rs})(b+cE)}{2(b+cE)} = \frac{1}{2}(\tilde{p}_s + \eta + T_{rs}) & \text{if } T_{rs} < \tilde{p}_s - \eta \\ \tilde{p}_s & \text{if } T_{rs} \geq \tilde{p}_s - \eta \end{cases} \quad (7)$$

which is the price that a firm located in region  $r$  quotes in region  $s$ , with  $r, s = 1, 2, 3$  and  $r \neq s$ .

Using the demand and the price functions, we can write:

$$q_{rr} = (b + cE)(p_{rr} - \eta) \quad (8)$$

which is the quantity sold in the local market; and

$$q_{rs} = \begin{cases} (b + cE)(p_{rs} - \eta - T_{rs}) & \text{if } T_{rs} < \tilde{p}_s - \eta \\ 0 & \text{if } T_{rs} \geq \tilde{p}_s - \eta \end{cases} \quad (9)$$

which is the quantity that a firm located in region  $r$  sells in region  $s$ , with  $r, s = 1, 2, 3$  and  $r \neq s$ .

According to (7) and (9), if a firm located in  $r$  quotes in the market  $s$  a price larger than the reservation price consumers living in  $s$  are prepared to pay, the export from region  $r$  to region  $s$  is zero. The boundary conditions for trade, as reported in these expressions, are crucial in the following analysis to determine the patterns of trade between the regions.

The indirect utility for a  $r$ -entrepreneur is

$$V_r = S_r + \pi_r + \overline{C_A}$$

where

$$\pi_r = \sum_{s=1}^3 (p_{rs} - \eta - T_{rs}) q_{rs} \left( \frac{L}{3} + \lambda_s E \right)$$

is the equilibrium profit for a  $r$ -firm, and

$$S_r = \frac{a^2 E}{2b} + \frac{b + cE}{2} \sum_{s=1}^3 \lambda_s E p_{sr}^2 - aP_r - \frac{c}{2} P^2 \quad (10)$$

is the consumer's surplus of the entrepreneur.

### 3.1 Short-Run Solutions

#### 3.1.1 Model 1. All Autarkic Regions

In this set-up, we assume that the three regions are equidistant, with  $T_{12} = T_{13} = T_{23} = T$  and sufficiently far away in terms of trade costs that trade cannot take place for any of the possible distributions of economic activity. When no region is trading, the condition  $T_{rs} \geq \tilde{p}_s - \eta$  holds for all  $r, s$ , with  $r, s = 1, 2, 3$  and  $r \neq s$ . It corresponds to:

$$T_{rs} = T \geq \tilde{p}_s - \eta = \frac{2(a - \eta b)}{2b + c\lambda_s E}.$$

This expression can be alternatively written as

$$\lambda_s \geq \frac{2(a - \eta b - bT)}{cET} = \bar{\lambda}.$$

No trade occurs for any distribution of the economic activity across the regions when:

$$T \geq \frac{a}{b} - \eta \text{ or } \bar{\lambda} \leq 0,$$

which is what we assume in the first set-up.<sup>5</sup>

From (4)–(9), taking into account the linear demand non-negativity constraint, we derive the equilibrium short-run profit for a firm located in region  $r$ , which sells in the local market<sup>6</sup>:

$$\pi_r = \pi_r^{no} = (p_{rr}^{no} - \eta)^2 (b + cE) \left( \frac{L}{3} + \lambda_r E \right) = \pi_{rr}^{no}$$

Finally, taking also into account (10), we obtain the indirect utility of entrepreneur resident in  $r$ :

$$V_r = V_r^{no} = S_r^{no} + \pi_r^{no} + \overline{C_A} = S_r^{no} + \pi_{rr}^{no} + \overline{C_A}.$$

<sup>5</sup> For convenience, in the simulations we assume  $T = \frac{a}{b} - \eta$  corresponding to the minimum value of trade costs which ensures no trade for any distribution of the industrial activity between the regions,  $\lambda_r$ .

<sup>6</sup> More details on the short-run equilibrium solutions for the first and the second model can be found in Commendatore *et al.* (2017).

### 3.1.2 Model 2. Allowing Trade only Between Regions 1 and 2

In the second set-up, trade costs between regions 1 and 2 are reduced, so that trade between these two regions can take place. We assume  $T_{12} = T - \varepsilon$ , where  $0 < \varepsilon \leq T$ ,<sup>7</sup> then trade may occur when  $\varepsilon$  is sufficiently close to  $T$ . We also assume  $T_{13} = T_{23} = T \geq \frac{a}{b} - \eta$  so that trade between regions 1 and 2 as well as 2 and 3 is still too costly leaving region 3 in autarky.

There are four different scenarios depending on the ‘trade distance’ between regions 1 and 2, that is on  $\varepsilon$ .

In the first scenario, regions 1 and 2 still do not trade with each other: from (4), (6), and (7), the condition of no trade  $T_{rs} \geq \tilde{p}_s - \eta$  for  $r, s = 1, 2$  and  $r \neq s$  corresponds to:

$$T_{rs} = T - \varepsilon \geq \frac{2(a - \eta b)}{2b + c\lambda_s E}$$

This expression can alternatively be expressed as:

$$\lambda_r \geq \tilde{\lambda} \text{ for } r = 1, 2$$

where  $\tilde{\lambda} = \frac{2(a - \eta b - bT + \varepsilon b)}{cE(T - \varepsilon)}$ .

The same results as before apply, thus the equilibrium short-run profit for a firm located in region  $r$  is  $\pi_r = \pi_r^{no}$  and the indirect utility of an entrepreneur resident in  $r$  is  $V_r = V_r^{no}$ .

In the second and third scenarios, unilateral or one-way trade occurs (from 1 to 2 in the second and from 2 to 1 in the third scenario). We consider these two scenarios together. We have that  $T_{rs} < \tilde{p}_s - \eta$  and  $T_{rs} \geq \tilde{p}_s - \eta$  holds for  $r, s = 1, 2$  and  $r \neq s$  (i.e. for  $r = 1$  and  $s = 2$  in the second and for  $r = 2$  and  $s = 1$  in the third scenario). From (4), (6), and (7), the conditions for unidirectional trade from  $r$  to  $s$  are obtained:

$$T_{rs} = T - \varepsilon < \frac{2(a - \eta b)}{2b + c\lambda_s E} \quad \text{and} \quad T_{rs} = T - \varepsilon \geq \frac{2(a - \eta b)}{2b + c\lambda_r E}$$

or, alternatively:

$$\tilde{\lambda} \geq \lambda_r \quad \text{and} \quad \tilde{\lambda} < \lambda_s$$

with  $r = 1$  and  $s = 2$  in the second and  $r = 2$  and  $s = 1$  in the third scenario.

Letting  $r = 1$  and  $s = 2$  for the second and  $r = 2$  and  $s = 1$  for the third scenario and  $k = 3$  in both, from (4)–(9), the equilibrium short-run profits for the case of one-way trade from  $r$  to  $s$  are:

$$\begin{aligned} \pi_r &= \pi_r^{out} = (b + cE) \left[ (p_{rr}^{no} - \eta)^2 \left( \frac{L}{3} + \lambda_r E \right) + (p_{rs}^{out} - \eta - T_{rs})^2 \left( \frac{L}{3} + \lambda_s E \right) \right] \\ &= \pi_{rr}^{no} + \pi_{rs}^{out} \end{aligned}$$

<sup>7</sup> When  $\varepsilon = 0$ ,  $T_{12} = T$  and we are back to the previous set up. As before, in the simulation we fix  $T = \frac{a}{b} - \eta$ .

$$\pi_s = \pi_s^{in} = (b + cE)(p_{ss}^{in} - \eta)^2 \left( \frac{L}{3} + \lambda_s E \right) = \pi_{ss}^{in}$$

$$\pi_k = \pi_k^{no}$$

where  $\pi_r^{out}$  is the profit of a firm located in  $r$  which is composed of two parts: the first part  $\pi_{rr}^{no}$  is obtained by selling in the local market, which is not affected by its exports to  $s$  due to market segmentation, and the second part  $\pi_{rs}^{out}$  by selling in  $s$ ;  $\pi_s^{in}$  is the profit of a firm located in  $s$  obtained only by selling in the local market, which is affected by the competition from the firms located in  $r$ ; and  $\pi_k^{no}$  is the profit of a firm located in  $k$  obtained only by selling in the local market, which is not affected by competition from outside.

Finally, taking also into account (10), the indirect utilities for the case of one-way trade from  $r$  to  $s$  are:

$$\begin{aligned} V_r &= V_r^{out} = S_r^{out} + \pi_{rr}^{out} + \overline{C_A} = S_r^{no} + \pi_{rr}^{out} + \overline{C_A} \\ V_s &= V_s^{in} = S_s^{in} + \pi_s^{in} + \overline{C_A} = S_s^{in} + \pi_{ss}^{in} + \overline{C_A} \\ V_k &= V_k^{no} \end{aligned}$$

where  $V_r^{out}$  is the indirect utility of an entrepreneur located in  $r$ , whose profits are higher compared to autarky;  $V_r^{in}$  is the indirect utility of an entrepreneur located in  $s$ , whose profits are lower and the surplus,  $S_s^{in}$ , is higher compared to autarky; and  $V_k^{no}$  is the indirect utility of an entrepreneur located in  $k$ , which is the same as in the previous model.

In the fourth scenario bilateral or two-way trade occurs. In this scenario  $T_{rs} < \tilde{p}_s - \eta$  for  $r, s = 1, 2$  and  $r \neq s$ . From (4), (6), and (7), the conditions for unilateral trade between  $r$  and  $s$  are:

$$T_{rs} = T - \varepsilon < \frac{2(a - \eta b)}{2b + c\lambda_s E} \quad \text{for } r = 1, 2 \quad \text{and } r \neq s,$$

or, alternatively,

$$\lambda_r < \tilde{\lambda} \quad \text{for } r = 1, 2$$

From (4)–(9), the equilibrium profits for the case of bilateral trade between  $r$  and  $s$  are:

$$\begin{aligned} \pi_r &= \pi_r^{bil} = \pi_{rr}^{bil} + \pi_{rs}^{bil} \quad \text{for } r = 1, 2 \quad \text{and } r \neq s \\ \pi_k &= \pi_k^{no} \quad \text{for } k = 3 \end{aligned}$$

where  $\pi_r^{bil}$  is the profit of a firm located in  $r$ , which is composed of two parts: the first part  $\pi_{rr}^{bil}$  is obtained by selling in the local market, which is not affected by its export towards  $s$  but is affected by competition from the firms located in  $s$  and the second part  $\pi_{rs}^{bil}$  by selling in  $s$ , with  $r = 1, 2$  and  $r \neq s$ ; and  $\pi_k^{no}$ , with  $k = 3$ , is the profit of a firm located in  $k$ , which has the same meaning as before.

Finally, taking also into account 10, the indirect utilities for the case of two-way trade between  $r$  and  $s$  are:

$$\begin{aligned} V_r &= V_r^{bil} = S_r^{bil} + \pi_r^{bil} + \overline{C_A} = S_r^{bil} + \pi_{rr}^{bil} + \pi_{rs}^{bil} + \overline{C_A} \quad \text{for } r = 1, 2 \quad \text{and } r \neq s \\ V_k &= V_k^{no} \quad \text{for } k = 3 \end{aligned}$$

where  $V_r^{bil}$  is the indirect utility of an entrepreneur located in  $r$ . Even though her profits originated from the local market  $\pi_{rr}^{bil}$  are lower due to competition from firms located in  $s$ , she is enjoying profits by selling in region  $s$ ,  $\pi_{rs}^{bil}$ . Moreover, her surplus,  $S_r^{bil}$ , is higher compared to when only local goods are available for consumption, with  $r = 1, 2$  and  $r \neq s$ . As before, the indirect utility of an entrepreneur located in  $k$  is not changed, with  $k = 3$ .

### 3.1.3 Model 3. Hub and Spoke

In the third set-up, region 1 may trade with regions 2 and 3 but regions 2 and 3 do not trade with each other. We consider a special case of an ‘hub and spoke’ structure: that is, we assume that trade costs between regions 1 and 2 are sufficiently low that bilateral trade between these two regions always occur, that is, we set  $T_{12} = \tau < \frac{2(a-\eta b)}{2b+cE}$ . We also assume prohibitive trade costs between 2 and 3:

$$T_{23} = T \geq \frac{a}{b} - \eta$$

Moreover, letting  $T_{13} = T - \psi$  and considering that  $T_{12} \leq T_{13} \leq T_{23}$ , we have that  $\tau \leq T - \psi \leq T$ .

There are four possible scenarios, depending on  $\psi$ . In the first scenario, no trade between regions 1 and 3 occurs:  $T_{rk} \geq \tilde{p}_k - \eta$  for  $r, k = 1, 3$  and  $r \neq k$ . From (4), (6), and (7), the condition corresponding to no trade between 1 and 3 corresponds to:

$$T_{13} = T - \psi \geq \max \left( \frac{2(a - \eta b) + cE\lambda_2 T_{12}}{2b + c(\lambda_1 + \lambda_2)E}, \frac{2(a - \eta b)}{2b + c\lambda_3 E} \right)$$

or to

$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13} - T_{12}}{T_{13}} \lambda_2 \quad \text{and} \quad \lambda_2 \leq 1 - \bar{\lambda} - \lambda_1$$

where  $\bar{\lambda} = \frac{2(a-\eta b-bT_{13})}{cET_{13}} = \frac{2(a-\eta b-bT+\psi b)}{cET_{13}}$ .

The equilibrium profits and indirect utilities correspond to those in the fourth scenario of the second set-up. These are:  $\pi_r = \pi_r^{bil}$ ,  $\pi_k = \pi_k^{no}$ ,  $V_r = V_r^{bil}$  and  $V_k = V_k^{no}$  for  $r = 1, 2$ ,  $r \neq s$  and  $k = 3$ .

In the second and third scenarios. Unilateral trade occurs involving region 1 and 3 (from 1 to 3 in the second and from 3 to 1 in the third scenario). We consider these two scenarios together. We have that  $T_{rk} < \tilde{p}_k - \eta$  and  $T_{rk} \geq \tilde{p}_r - \eta$  holds for  $r, k = 1, 3$  and  $r \neq k$  (i.e. for  $r = 1$  and  $k = 3$  in the second and for  $r = 3$  and  $k = 1$  in the third scenario).

From (4), (6), and (7), one-way trade from region 1 to region 3 occurs when:

$$\frac{2(a - \eta b) + cE\lambda_2 T_{12}}{2b + c(\lambda_1 + \lambda_2)E} \leq T_{13} = T - \psi < \frac{2(a - \eta b)}{2b + c\lambda_3 E}$$

or when

$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13} - T_{12}}{T_{13}} \lambda_2 \quad \text{and} \quad \lambda_2 > 1 - \bar{\lambda} - \lambda_1$$

The short-run equilibrium profits for the case of one-way trade from region 1 to region 3 are:<sup>8</sup>

$$\begin{aligned}\pi_1 &= \pi_1^{bil, out} = (b + cE) \left[ (p_{11}^{bil} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) + (p_{12}^{bil} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_2 E \right) + \right. \\ &\quad \left. + (p_{13}^{out} - \eta - T_{13})^2 \left( \frac{L}{3} + \lambda_3 E \right) \right] = \pi_1^{bil} + \pi_{13}^{out} \\ \pi_2 &= \pi_2^{bil, out} = (b + cE) \left[ (p_{21}^{bil} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) + (p_{22}^{bil} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_2 E \right) \right] \\ &= \pi_2^{bil} \\ \pi_3 &= \pi_3^{bil, out} = (b + cE)(p_{33}^{bil, out} - \eta)^2 \left( \frac{L}{3} + \lambda_3 E \right) = \pi_{33}^{bil, out}\end{aligned}$$

where  $\pi_1^{bil, out}$  is the profit of a firm located in 1 which is composed of two parts: the first part corresponds to the profit obtained in the absence of the trade link from 1 to 3  $\pi_1^{bil}$  – which is itself composed of two parts (see above) – and the second part  $\pi_{13}^{out}$  is obtained by selling in 3, which is not affected by its exports to 3 due to market segmentation;  $\pi_2^{bil, out}$  is the profit of a firm located in region 2 which is equal to that in the fourth scenario of the second set-up; and  $\pi_3^{bil, out}$  is the profit of a firm located in 3 obtained only by selling in the local market, which is affected by the competition from the firms located in 1.

Finally, taking also into account (10), the indirect utilities for the case of one-way trade from region 1 to region 3 correspond to:

$$\begin{aligned}V_1 &= V_1^{bil, out} = V_1^{bil} + \pi_{13}^{out} \\ V_2 &= V_2^{bil, out} = V_2^{bil} \\ V_3 &= V_3^{bil, out} = V_3^{in} = S_3^{in} + \pi_{33}^{in} + \bar{C}_A\end{aligned}$$

where  $V_1^{bil, out}$  is the indirect utility of an entrepreneur located in 1, whose profits are higher compared to the case when there are no trade links between 1 and 3;  $V_2^{bil, out}$  is the indirect utility of an entrepreneur located in 2, which the same that the one obtained in the fourth scenario of the second set-up; and  $V_3^{bil, out}$  is the indirect utility of an entrepreneur located in 3, whose profits are lower and the surplus  $S_3^{in}$  is higher compared to the case of no trade links between 1 and 3.

From (4), (6), and (7), one-way trade from region 3 to region 1 occurs when:

$$\frac{2(a - \eta b)}{2b + c\lambda_3 E} \leq T_{13} = T - \psi < \frac{2(a - \eta b) + cE\lambda_2 T_{12}}{2b + c(\lambda_1 + \lambda_2)E}$$

or when

$$\lambda_1 < \bar{\lambda} - \frac{T_{13} - T_{12}}{T_{13}} \lambda_2 \quad \text{and} \quad \lambda_2 \leq 1 - \bar{\lambda} - \lambda_1.$$

<sup>8</sup> More details on the short-run equilibrium solutions for this model can be found in the Appendix.



The short-run equilibrium profits for the case of one-way trade from region 3 to region 1 are:

$$\begin{aligned}
 \pi_1 &= \pi_1^{bil, in} = (b + cE) \left[ (p_{11}^{bil, in} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) + (p_{12}^{out} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_2 E \right) \right] \\
 &= \pi_{11}^{bil, in} + \pi_{12}^{out} \\
 \pi_2 &= \pi_2^{bil, in} = (b + cE) \left[ (p_{21}^{bil, in} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) + (p_{22}^{out} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_2 E \right) \right] \\
 &= \pi_{21}^{bil, in} + \pi_{22}^{out} \\
 \pi_3 &= \pi_3^{bil, in} = (b + cE) \left[ (p_{31}^{bil, in} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) + (p_{33}^{no} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_3 E \right) \right] \\
 &= \pi_{31}^{bil, in} + \pi_{33}^{no}
 \end{aligned}$$

where  $\pi_1^{bil, in}$  is the profit of a firm located in 1 which is composed of two parts: the first part  $\pi_{11}^{bil, in}$  is obtained by selling in the local market, which is not affected by its exports towards 2 but is affected by the competition from the firms located both in 2 and in 3 and the second part  $\pi_{12}^{out}$  by selling in 2;  $\pi_2^{bil, in}$  is the profit of a firm located in region 2 which is composed of two parts: the first part  $\pi_{21}^{bil, in}$  is obtained by selling in 1 which is affected by the competition in that market not only from the local firms but also from the firms located in 3 and the second part  $\pi_{22}^{out}$  obtained by selling in the local market, which is not affected by the exports towards 1 but it is affected by competition from the firms located in 1; and  $\pi_3^{bil, in}$  is the profit of a firm located in 3 composed of two parts: the first part  $\pi_{31}^{bil, in}$  is obtained by selling in 1, which is affected by the competition in that market not only from the local firms but also from the firms located in 2 and the second part  $\pi_{33}^{no}$ , which is not affected by the exports towards 1.

Taking also into account (10), the indirect utilities for the case of one-way trade from region 3 to region 1 corresponds to:

$$\begin{aligned}
 V_1 &= V_1^{bil, in} = S_1^{bil, in} + \pi_1^{bil, in} + \overline{C_A} \\
 V_2 &= V_2^{bil, in} = S_2^{bil} + \pi_2^{bil, in} + \overline{C_A} \\
 V_3 &= V_3^{bil, in} = S_3^{no} + \pi_{31}^{bil, in} + \overline{C_A}
 \end{aligned}$$

where  $V_1^{bil, in}$  is the indirect utility of an entrepreneur located in 1, whose profits are lower and surplus is higher compared to when there are no trade links between 1 and 3;  $V_2^{bil, in}$  is the indirect utility of an entrepreneur located in 2, whose profits are lower compared to when there are no trade links between 1 and 3; and  $V_3^{bil, in}$  is the indirect utility of an entrepreneur located in 3, whose profits are higher compared to the case of no trade links between 1 and 3.

Finally, in the fourth scenario bilateral trade between regions 1 and 3 occurs. We have that  $T_{rk} < \tilde{p}_k - \eta$  for  $r, k = 1, 3$  and  $r \neq k$ . From (4), (6), and (7), bilateral trade between regions 1 and 3 occurs when:

$$T_{13} = T - \psi < \min \left( \frac{2(a - \eta b) + cE\lambda_2 T_{12}}{2b + c(\lambda_1 + \lambda_2)E}, \frac{2(a - \eta b)}{2b + c\lambda_3 E} \right)$$

or when

$$\lambda_1 < \bar{\lambda} - \frac{T_{13} - T_{12}}{T_{13}} \lambda_2 \quad \text{and} \quad \lambda_2 > 1 - \bar{\lambda} - \lambda_1.$$

The short-run equilibrium profits for the case of bilateral trade between regions 1 and 3 are:

$$\begin{aligned} \pi_1 &= \pi_1^{bil, bil} = (b + cE) \left[ (p_{11}^{bil, in} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) + (p_{12}^{out} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_2 E \right) \right. \\ &\quad \left. + (p_{13}^{out} - \eta - T_{13})^2 \left( \frac{L}{3} + \lambda_3 E \right) \right] = \pi_1^{bil, in} + \pi_{13}^{out} \\ \pi_2 &= \pi_2^{bil, bil} = (b + cE) \left[ (p_{21}^{bil, in} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) + (p_{22}^{out} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_2 E \right) \right] \\ &= \pi_2^{bil, in} \\ \pi_3 &= \pi_3^{bil, bil} = (b + cE) \left[ (p_{31}^{bil, in} - \eta)^2 \left( \frac{L}{3} + \lambda_1 E \right) \right. \\ &\quad \left. + (p_{33}^{bil, out} - \eta - T_{12})^2 \left( \frac{L}{3} + \lambda_3 E \right) \right] = \pi_{31}^{bil, in} + \pi_{33}^{bil, out} \end{aligned}$$

where  $\pi_1^{bil, bil}$  is the profit of a firm located in 1 which is composed of two parts: the first part  $\pi_1^{bil, in}$  is what is obtained in the third scenario and the second part  $\pi_{13}^{out}$  by selling in 3;  $\pi_2^{bil, bil}$  is the profit of a firm located in region 2 which is the same as in the third scenario; and  $\pi_3^{bil, bil}$  is the profit of a firm located in 3 composed of two parts: the first part  $\pi_{31}^{bil, in}$  is obtained by selling in 1, which is affected by the competition in that market not only from the local firms but also from the firms located in 2 and the second part  $\pi_{33}^{bil, out}$  is obtained by selling in the local market, which is affected by the competition from the firms located in 1.

Finally, taking also into account (10), the indirect utilities for the case of bilateral trade between 1 and 3 correspond to:

$$\begin{aligned} V_1 &= V_1^{bil, bil} = V_1^{bil, in} + \pi_{13}^{out} \\ V_2 &= V_2^{bil, bil} = V_2^{bil, in} \\ V_3 &= V_3^{bil, bil} = V_3^{bil, out} + \pi_{31}^{bil, in} \end{aligned}$$

where  $V_1^{bil, bil}$  is the indirect utility of an entrepreneur located in 1, which is equal to the indirect utility enjoyed in the third scenario plus the profits obtained by selling in 3;  $V_2^{bil, bil}$  is the indirect utility of an entrepreneur located in 2, which is the same as in the third scenario; and  $V_3^{bil, bil}$  is the indirect utility of an entrepreneur located in 3, which is equal to that enjoyed in the second scenario plus the profits obtained by selling in 1.

## 4 Definition of the Basic Dynamic Equations

The dynamics of the three NEG model variants presented above is described by a two-dimensional (2D) system of difference equations, or map,  $Z$ :

$$Z : \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \mapsto \begin{pmatrix} Z_1(\lambda_1, \lambda_2) \\ Z_2(\lambda_1, \lambda_2) \end{pmatrix}, \quad (11)$$

where

$$Z_i(\lambda_1, \lambda_2) = \begin{cases} 0 & \text{if } F_i \leq 0, \\ F_i & \text{if } F_i > 0, F_j > 0, F_i + F_j < 1, \\ \frac{F_i}{F_i + F_j} & \text{if } F_i > 0, F_j > 0, F_i + F_j \geq 1, \\ \frac{F_i}{1 - F_j} & \text{if } F_i > 0, F_j \leq 0, F_i + F_j < 1, \\ 1 & \text{if } F_i > 0, F_j \leq 0, F_i + F_j \geq 1, \end{cases}$$

with  $i = 1, j = 2$  for  $Z_1(\lambda_1, \lambda_2)$  and  $i = 2, j = 1$  for  $Z_2(\lambda_1, \lambda_2)$ ,

$$F_r(\lambda_1, \lambda_2) = \lambda_r(1 + \gamma\Omega_r(\lambda_1, \lambda_2)), \quad r = 1, 2,$$

$$\Omega_r(\lambda_1, \lambda_2) = \frac{V_r(\lambda_1, \lambda_2)}{\lambda_1 V_1(\lambda_1, \lambda_2) + \lambda_2 V_2(\lambda_1, \lambda_2) + (1 - \lambda_1 - \lambda_2) V_3(\lambda_1, \lambda_2)} - 1.$$

Due to the constraint on the regional shares of entrepreneurs, the map  $Z$  is piecewise smooth. In  $Z$ , the indirect utilities  $V_i(\lambda_1, \lambda_2)$ ,  $i = 1, 2, 3$ , of an entrepreneur in regions 1, 2 and 3, respectively, are defined according to the assumptions of the considered models.

Independently on the specification of the indirect utilities the following properties hold:

- All the relevant dynamics in the  $(\lambda_1, \lambda_2)$ -phase plane is trapped in a triangle denoted  $S$ , whose *borders*:

$$I_{b1} = \{(\lambda_1, \lambda_2) : \lambda_2 = 0\}, \quad I_{b2} = \{(\lambda_1, \lambda_2) : \lambda_1 = 0\}, \quad I_{b3} = \{(\lambda_1, \lambda_2) : \lambda_2 = 1 - \lambda_1\} \quad (12)$$

are *invariant lines*<sup>9</sup> of map  $Z$ .

- A fixed point of the map  $Z$ , which lies inside  $S$ , corresponds to a stationary long-run equilibrium of the economy; the vertices of  $S$  are *Core-periphery (CP)* fixed points/equilibria:

$$CP_0 : (\lambda_1, \lambda_2) = (0, 0), \quad CP_1 : (\lambda_1, \lambda_2) = (1, 0), \quad CP_2 : (\lambda_1, \lambda_2) = (0, 1), \quad (13)$$

characterised by full spatial agglomeration of the industrial activity, with all the entrepreneurs located in only one region.

- Any *interior fixed point* of  $Z$ , if it exists, is given by intersection of the curves

$$\Omega_1 = \{(\lambda_1, \lambda_2) : \Omega_1(\lambda_1, \lambda_2) = 0\} \text{ and } \Omega_2 = \{(\lambda_1, \lambda_2) : \Omega_2(\lambda_1, \lambda_2) = 0\}. \quad (14)$$

An interior (symmetric or asymmetric) equilibrium is characterised by positive shares of entrepreneurs in all regions (which can be equal or different).

<sup>9</sup> Recall that a set  $A$  is called invariant under a map  $F$  if  $F(A) = A$ .

- Any *border fixed point* belonging to  $I_{bi}$ ,  $i = 1, 2$ , if it exists, is an intersection point of  $\Omega_i$  and  $I_{bi}$ , while any border fixed point belonging to  $I_{b3}$  is an intersection point of  $\Omega_1$ ,  $\Omega_2$  and  $I_{b3}$ . A border (symmetric or asymmetric) equilibrium is characterised by positive shares of entrepreneurs in two regions and no entrepreneurs in the third one. In the regions where entrepreneurs are present they can be equally distributed (in a border symmetric equilibrium) or unevenly distributed (in a border asymmetric equilibrium).

## 5 Long-Run Equilibria Properties in Model 1

Under the assumptions associated with Model 1 (when all the regions are in autarky) the indirect utilities of an entrepreneur in regions 1, 2 and 3, respectively, correspond to:

$$V_r(\lambda_r) = V_r^{no}(\lambda_r), \quad r = 1, 2, \quad (15)$$

$$V_3(\lambda_1, \lambda_2) = V_3^{no}(\lambda_1, \lambda_2). \quad (16)$$

and are defined in Commendatore *et al.* (2017). This case generalizes the model studied in Behrens (2004), where two regions in full autarky are considered. We have examined in full detail the mathematical properties of the system (15)–(16) in Commendatore *et al.* (2017). In this paper, we will focus more on the economic meaning of some results.

First note that for Model 1 not only the borders of the trapping triangle  $S$  but also its medians:

$$I_{m1} = \{(\lambda_1, \lambda_2) : \lambda_2 = 1 - 2\lambda_1\}, \quad I_{m2} = \{(\lambda_1, \lambda_2) : \lambda_1 = 1 - 2\lambda_2\},$$

$$I_{m3} = \{(\lambda_1, \lambda_2) : \lambda_1 = \lambda_2\},$$

are invariant lines of map  $Z$ .

Besides the Core-periphery equilibria (see (13)), Model 1 can have also border symmetric/asymmetric equilibria, as well as interior symmetric/asymmetric equilibria.

In the first place, border symmetric (BS) equilibria always exist:

$$BS_1 : (\lambda_1, \lambda_2) = (1/2, 0), \quad BS_2 : (\lambda_1, \lambda_2) = (0, 1/2), \quad BS_3 : (\lambda_1, \lambda_2) = (1/2, 1/2).$$

Considering the border asymmetric (BA) equilibria

$$\begin{aligned} BA_1 : (\lambda_1, \lambda_2) &= (p, 0) \in I_{b1}, & BA'_1 : (\lambda_1, \lambda_2) &= (1 - p, 0) \in I_{b1}, \\ BA_2 : (\lambda_1, \lambda_2) &= (0, p) \in I_{b2}, & BA'_2 : (\lambda_1, \lambda_2) &= (0, 1 - p) \in I_{b2}, \\ BA_3 : (\lambda_1, \lambda_2) &= (p, 1 - p) \in I_{b3}, & BA'_3 : (\lambda_1, \lambda_2) &= (1 - p, p) \in I_{b3}, \end{aligned} \quad (17)$$

where

$$p = \frac{1}{2} - \frac{1}{6cE} \sqrt{(4b + cE)(36b + 3cE - 8cL)} \quad (18)$$

we can state that the equilibria  $BA_i$  and  $BA'_i$  exist for

$$BT_{CP} < L < PF_{BS} , \quad (19)$$

where

$$BT_{CP} = \frac{6b(3b + cE)}{c(4b + cE)} \quad \text{and} \quad PF_{BS} = \frac{36b + 3cE}{8c}. \quad (20)$$

It is easy to check that  $PF_{BS} - BT_{CP} > 0$ , thus the range (19) is nonempty.

Concerning the interior asymmetric (IA) equilibria

$$\begin{aligned} IA_1 : (\lambda_1, \lambda_2) &= (k_1, 1 - 2k_1) \in I_{m1}, \quad IA'_1 : (\lambda_1, \lambda_2) = (k_2, 1 - 2k_2) \in I_{m1}, \\ IA_2 : (\lambda_1, \lambda_2) &= (1 - 2k_1, k_1) \in I_{m2}, \quad IA'_2 : (\lambda_1, \lambda_2) = (1 - 2k_2, k_2) \in I_{m2}, \\ IA_3 : (\lambda_1, \lambda_2) &= (k_1, k_1) \in I_{m3}, \quad IA'_3 : (\lambda_1, \lambda_2) = (k_2, k_2) \in I_{m3}, \end{aligned} \quad (21)$$

where  $k_1$  and  $k_2$  are given by

$$k_{1,2} = \frac{1}{cE} \left( \frac{1}{6}Lc - b + \frac{1}{4}cE \mp \frac{1}{12} \sqrt{24b(42b - 10Lc + 9cE) - c^2(9E(4L - E) - 4L^2)} \right), \quad (22)$$

we can state that the interior asymmetric equilibria  $IA_i$ ,  $i = 1, 2, 3$ , exist for

$$BT_{CP} < L < F_{IA} \quad \text{and} \quad E > E_1 ,$$

and the interior asymmetric equilibria  $IA'_i$  exist for

$$BT_{BS} < L < F_{IA} \quad \text{if} \quad E > E_1 \quad \text{or} \quad BT_{BS} < L < BT_{CP} \quad \text{if} \quad 0 < E < E_1 ,$$

where

$$F_{IA} = \frac{3}{2c} \left( 20b + 3cE - 2\sqrt{2}(6b + cE) \right), \quad BT_{BS} = \frac{6b(cE + 6b)}{c(8b + cE)}, \quad E_1 = \frac{2b(7 - 5\sqrt{2})}{c(2\sqrt{2} - 3)}, \quad (23)$$

and where  $F_{IA} = BT_{CP}$  at  $E = E_1$ . Moreover, it is easy to check that  $F_{IA} - BT_{CP} > 0$  (for  $E \neq E_1$ ),  $F_{IA} - BT_{BS} > 0$ , and  $BT_{CP} - BT_{BS} > 0$ , thus, the existence ranges of  $IA_i$  and  $IA'_i$  are nonempty. In the  $(E, L)$ -parameter plane the curves defined by  $L = F_{IA}$  and  $L = BT_{CP}$  are tangent at  $E = E_1$ .

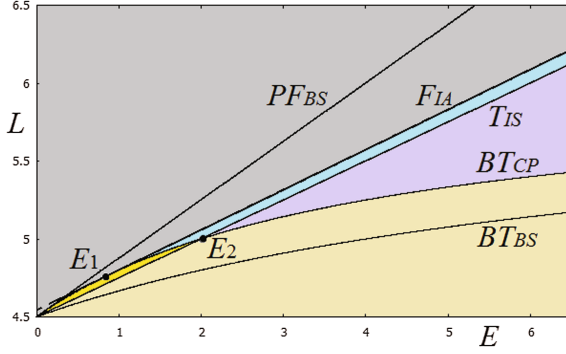
Finally, the interior symmetric (IS) equilibrium

$$IS : (\lambda_1, \lambda_2) = (1/3, 1/3)$$

always exists. Moreover, for a sufficiently small  $\gamma$  (namely, for  $\gamma \leq 1 + 6b/cE$ ) it is stable for any  $L > T_{IS} = \frac{18b+cE}{4c}$ .

To illustrate the existence and stability properties of the long-run equilibria, we choose  $a$ ,  $b$ ,  $c$ ,  $\eta$  and  $E$  parameter values similar to those used in Behrens (2004):

$$a = 1/3, \quad b = 1/3, \quad c = 1/3, \quad \eta = 0, \quad E = 1.5 . \quad (24)$$



**Fig. 1.** Bifurcation curves related to fixed points of map  $Z$  in the  $(E, L)$ -parameter plane for  $a = 1/3$ ,  $b = 1/3$ ,  $c = 1/3$ .

We fix also

$$\gamma = 10, \quad \overline{C_A} = 1 \quad (25)$$

and vary  $L$  as in Commendatore *et al.* (2017).

Figure 1 summarizes the properties of the long-run equilibria. In this Figure we plotted the bifurcation curves  $L = BT_{BS}$ ,  $L = BT_{CP}$ ,  $L = T_{IS}$ ,  $L = F_{IA}$  and  $L = PF_{BS}$  in the  $(E, L)$ -parameter plane for the values of the other parameters as in (24) and (25). Crossing these curves the properties of the equilibria change.

In Fig. 1, the blue region is related to the coexisting attracting equilibria  $IS$  and  $IA_i$ ,  $i = 1, 2, 3$ , while for the parameter values belonging to the yellow region the attracting equilibrium  $IS$  coexists with the attracting equilibria  $CP_i$ . Note that the curve  $F_{IA}$  is meaningful for  $E > E_1$  and that the curve  $BT_{CP}$  intersects the curve  $T_{IS}$  at  $E = \frac{2b}{c} =: E_2$ .

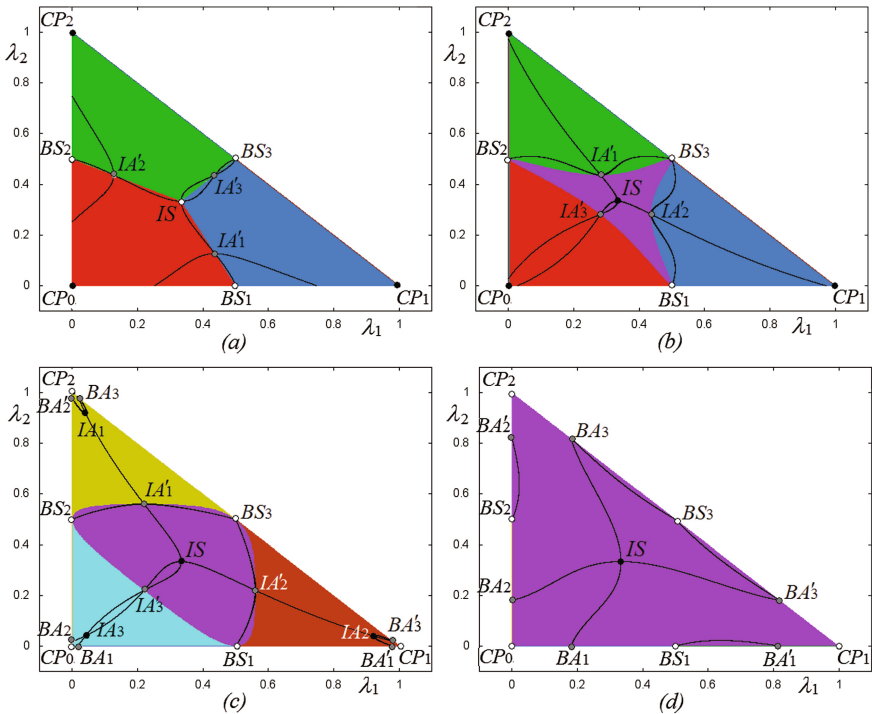
We now study how the properties of the equilibria of the dynamic system  $Z$  changes for a fixed  $E < E_2$ , for example, for  $E = 1.5$ , by increasing  $L$ :<sup>10</sup>

- For  $1 < L < BT_{BS}$  there are three coexisting attracting  $CP$  equilibria, and crossing  $L = BT_{BS}$  (when a transverse ‘border transcritical bifurcation’ of  $BS_i$ ,  $i = 1, 2, 3$ , occurs), the fixed points  $BS_i$  become repelling and the saddle fixed points  $IA'_i$  are born. Figure 2a (where the curves  $\Omega_1$  and  $\Omega_2$  given in (14) are also shown) depicts the case that holds for  $BT_{BS} < L < T_{IS}$ . In this Figure the basins of coexisting attracting  $CP$  equilibria – representing the set of initial conditions leading to a long-run equilibrium – are coloured red, blue and green (for  $CP_0$ ,  $CP_1$  and  $CP_2$ , respectively). For the interval  $1 < L < T_{IS}$ , the size of invariant local demand, represented by the number of immobile workers ( $L$ ), is ‘small’, compared with the size of demand that potentially could shift, represented by the number of entrepreneurs ( $E$ ). A small initial advantage of one region – i.e. an initial distribution of entrepreneurs in that

<sup>10</sup> In Commendatore *et al.* (2017) we have explored an alternative path considering the case  $E > E_2$  by setting  $E = 5$ .

region slightly larger than in the other two – leads to full agglomeration of the industrial activity in that region: the agglomeration forces overcome the dispersion forces.

- Crossing  $L = T_{IS}$  (where a ‘transcritical bifurcation’ of  $IS$  occurs at which  $IS = IA'_i$ ) the fixed point  $IS$  becomes attracting, so that, for  $T_{IS} < L < BT_{CP}$ , there are four coexisting attracting fixed points,  $IS$  and  $CP_i$  (see Fig. 2b, where the basin of attraction of  $IS$  is coloured in purple). For this interval, the increase in  $L$  is sufficient to make the interior equilibrium locally stable and attracting for initial distribution of the economic activity (i.e. for shares of entrepreneurs) not too unequal. This particular type of coexistence cannot occur in a two-region context and it represents a novel result with respect to Behrens (2004).
- Crossing  $L = BT_{CP}$  (where a ‘border transcritical bifurcation’ occurs) the  $CP$  equilibria become repelling while attracting fixed points  $IA_i$  and saddle fixed points  $BA_i$ ,  $BA'_i$  appear (see Fig. 2c where the basins of attraction of  $IA_1$ ,  $IA_2$  and  $IA_3$  are coloured in dark yellow, brown and light blue, respectively). Thus, for  $BT_{CP} < L < F_{IA}$ , four coexisting attracting equilibria



**Fig. 2.** Attracting fixed points of  $Z$  (Model 1) and their basins; curves  $\Omega_1$  and  $\Omega_2$  given in (14) are also shown. Here  $L = 4.8$  in (a),  $L = 4.9$  in (b),  $L = 4.92$  in (c),  $L = 5$  in (d), and the other parameters are fixed as in (24) and (25).

exist: the interior symmetric ( $IS$ ) equilibrium and three interior asymmetric equilibria ( $IA_1, IA_2, IA_3$ ). In this interval, the size of immobile local demand is sufficiently large so that all regions can keep the industrial sector, the relative dimension of which depends on the initial condition: it could be large for one region (as in an asymmetric equilibrium) or of equal size for all three (as in the symmetric equilibrium).

- Crossing  $L = F_{IA}$  (where a ‘fold bifurcation’ occurs and at which  $IA_i = IA'_i$ ) the fixed points  $IA_i$  and  $IA'_i$  disappear and the only attractor of map  $Z$  is  $IS$  (see Fig. 2d). Finally, at  $L = PF_{BS}$  a pitchfork bifurcation of  $BS_i$  occurs (at which  $BS_i = BA_i = BA'_i$ ); after this bifurcation the fixed points  $BS_i$  become saddles. Thus, for  $L > F_{IA}$  the only stable equilibrium is the symmetric equilibrium. The size of the immobile demand is sufficiently large, compared to the potentially shifting demand, so that there is scope for firms to locate in each region. Any initial condition involving positive shares of entrepreneurs leads to the complete dispersion of the economic activity across the three regions as dispersion forces overcome agglomeration forces.<sup>11</sup>

Note that this model does not involve trade between the regions and for this reason it represents a useful reference point to be compared with the other two models in order to isolate the effects of trade on the long-run distribution of the economic activity.

## 6 Long-Run Equilibria Properties in Model 2

Based on the results related to the full autarky case discussed in the previous section, we now reduce the trade costs between regions 1 and 2 allowing uni- and bilateral trade between these regions, but not with region 3. In such a case the indirect utilities  $V_i(\lambda_1, \lambda_2) =: V_i$ ,  $i = 1, 2, 3$ , of an entrepreneur in regions 1, 2 and 3, respectively, are defined as follows:

$$\begin{aligned}
 &\text{if } \lambda_1 \geq \tilde{\lambda}, \lambda_2 \geq \tilde{\lambda} \text{ then } V_1 = V_1^{no}(\lambda_1), V_2 = V_2^{no}(\lambda_2), \\
 &\text{if } \lambda_1 \geq \tilde{\lambda}, \lambda_2 < \tilde{\lambda} \text{ then } V_1 = V_1^{out}(\lambda_1, \lambda_2), V_2 = V_2^{in}(\lambda_1, \lambda_2), \\
 &\text{if } \lambda_1 < \tilde{\lambda}, \lambda_2 \geq \tilde{\lambda} \text{ then } V_1 = V_1^{in}(\lambda_1, \lambda_2), V_2 = V_2^{out}(\lambda_1, \lambda_2), \\
 &\text{if } \lambda_1 < \tilde{\lambda}, \lambda_2 < \tilde{\lambda} \text{ then } V_1 = V_1^{bil}(\lambda_1, \lambda_2), V_2 = V_2^{bil}(\lambda_1, \lambda_2), \\
 &V_3 = V_3^{no}(\lambda_1, \lambda_2),
 \end{aligned} \tag{26}$$

<sup>11</sup> A further condition required for the stability of  $IS$  is  $L < FI_{IS}$ , where

$$FI_{IS} = - \frac{E(a - b\eta)^2(b + cE)(3\gamma b(18b + cE) + 2(6b + cE)(9b + cE)) + 4\overline{CA}b(6b + cE)^3}{12b(a - b\eta)^2(b + cE)(6b + cE(1 - \gamma))}$$

At  $L = FI_{IS}$  the symmetric equilibrium loses stability via a ‘flip bifurcation’. Above  $L = FI_{IS}$  cycles of different order emerge and even complex behavior. If  $\gamma < 1 + \frac{6b}{cE}$  a flip bifurcation never occurs. The analysis of the case  $L > FI_{IS}$  is presented in Commendatore *et al.* (2017).



where  $V_r^{in}(\lambda_1, \lambda_2)$ ,  $V_r^{out}(\lambda_1, \lambda_2)$ ,  $V_r^{bil}(\lambda_1, \lambda_2)$  for  $r = 1, 2$  are given in Commendatore *et al.* (2017),  $V_1^{no}(\lambda_1)$ ,  $V_2^{no}(\lambda_2)$  and  $V_3^{no}(\lambda_1, \lambda_2)$  are defined as in Model 1. The dynamic system (or map)  $Z$  is modified accordingly.

We have examined in full detail the mathematical properties of the system  $Z$  modified to take into account 26 in Commendatore *et al.* (2017). In this paper, as for the previous model, we will focus more on the economic meaning of some results.

In our simulations, for convenience, we set  $T = \frac{a}{b} - \eta$ , that is, at the minimum value of trade costs which ensures no trade for any distribution of entrepreneurs across the regions, it follows

$$\tilde{\lambda} = \frac{2b^2\varepsilon}{cE(a - b\eta - b\varepsilon)} \Big|_{\eta=0, a=b=c} = \frac{2\varepsilon}{E(1 - \varepsilon)} . \quad (27)$$

Moreover, we fix the parameter values as in (24)–(25), and vary  $L$  and  $\varepsilon$  in the ranges

$$2 < L < 9, \quad 0 < \varepsilon < T = \frac{a}{b} - \eta \Big|_{a=b, \eta=0} = 1 .$$

Similarly to Model 1, the borders  $I_{bi}$ ,  $i = 1, 2, 3$  (see (12)) of the trapping triangle  $S$  are invariant lines of map  $Z$ , while among the medians of  $S$  only the main diagonal  $I_{m3} = \{(\lambda_1, \lambda_2) : \lambda_1 = \lambda_2\}$  is invariant.

The straight lines  $\lambda_1 = \tilde{\lambda}$  and  $\lambda_2 = \tilde{\lambda}$ , where  $\tilde{\lambda}$  is given in (27), separate the trapping triangle  $S$  into at most four regions, each of them characterized by a different trade pattern (see Fig. 3a):

- the region  $S_0 = \{\lambda_1 > \tilde{\lambda}, \lambda_2 > \tilde{\lambda}, \lambda_1 + \lambda_2 < 1\}$  corresponds to *no trade*;
- the regions  $S_1 = \{\lambda_1 > \tilde{\lambda}, 0 < \lambda_2 < \tilde{\lambda}, \lambda_1 + \lambda_2 < 1\}$  and  $S_2 = \{\lambda_2 > \tilde{\lambda}, 0 < \lambda_1 < \tilde{\lambda}, \lambda_1 + \lambda_2 < 1\}$  are related to *unilateral trade*;
- the region  $S_3 = \{0 < \lambda_1 < \tilde{\lambda}, 0 < \lambda_2 < \tilde{\lambda}\}$  is related to *bilateral trade*.

It is easy to see that in the trapping triangle  $S$  there are only regions  $S_1$ ,  $S_2$  and  $S_3$  (and there is no ‘no trade’ region  $S_0$ ) for

$$\frac{1}{2} < \tilde{\lambda} < 1 \quad \text{or} \quad \varepsilon_2 < \varepsilon < \varepsilon_3$$

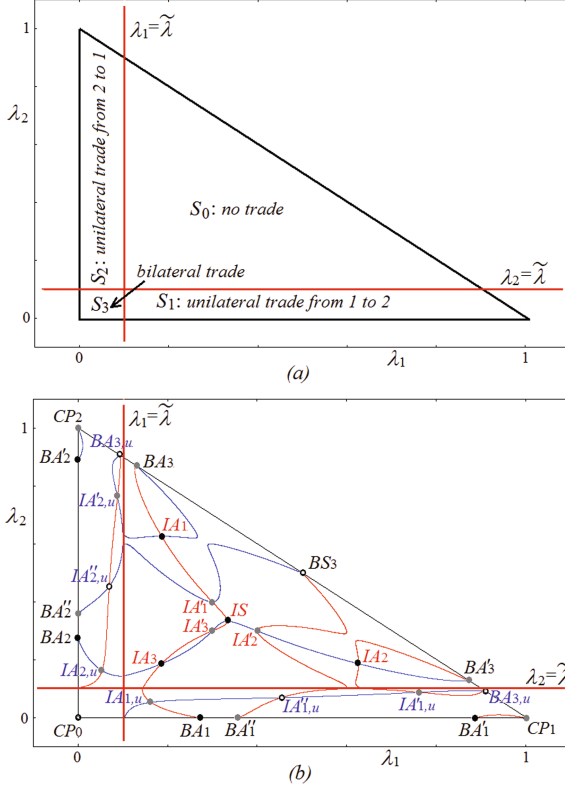
where

$$\varepsilon_2 = \frac{cE(a - b\eta)}{b(cE + 4b)}, \quad \varepsilon_3 = \frac{cE(a - b\eta)}{b(cE + 2b)}, \quad (28)$$

and  $S$  coincides to  $S_3$  for  $\tilde{\lambda} > 1$  or  $\varepsilon > \varepsilon_3$ , that is, the trapping triangle  $S$  is associated only with bilateral trade between regions 1 and 2.<sup>12</sup>

Compared with that associated with Model 1, the map  $Z$  considered in the present section can have more fixed points/equilibria. As an example, in Fig. 3b black, gray and white circles indicate attracting, saddle and repelling equilibria, respectively. Below we list different kinds of the equilibria of Model 2. Recall that for the region  $S_0$  the results obtained for the Model 1 are valid.

<sup>12</sup> We have obtained  $\varepsilon_2$  and  $\varepsilon_3$  by equating  $\tilde{\lambda}$  to  $1/2$  and to  $1$ , respectively.



**Fig. 3.** In (a): Partitioning of the trapping triangle  $S$  into the regions of no trade ( $S_0$ ), unilateral trade ( $S_1$  and  $S_2$ ) and bilateral trade ( $S_3$ ); In (b): Attracting, saddle and repelling fixed points of map  $Z$  are shown by black, gray and white circles, respectively; curves  $\Omega_1$  and  $\Omega_2$  are shown in red and blue, respectively. Here  $a = b = c = 1/3$ ,  $\eta = 0$ ,  $E = 5$ ,  $L = 5.8$ ,  $\varepsilon = 0.2$ .

The map  $Z$  keeps always all CP equilibria (see (13)) and the border symmetric equilibrium  $BS_3 : (\lambda_1, \lambda_2) = (1/2, 1/2)$ . Instead, the interior symmetric equilibrium,  $IS : (\lambda_1, \lambda_2) = (1/3, 1/3)$  is a fixed point of  $Z$  as long as  $IS \in S_0$ , that is, if  $\tilde{\lambda} < 1/3$ . This condition corresponds to  $\varepsilon < \varepsilon_1$ , where

$$\varepsilon_1 = \frac{cE(a - b\eta)}{b(cE + 6b)}. \quad (29)$$

As we shall see later, the equilibrium  $IS$  may disappear,<sup>13</sup> or it could move to region  $S_3$ . In the later case, we denote it as  $IA_{3,b}$ , where the index ‘ $b$ ’ refers to the bilateral trade region.

On the borders  $I_{b1}$  and  $I_{b2}$ , map  $Z$  can have also border asymmetric (BA) equilibria:

$$BA_1 : (\lambda_1, \lambda_2) = (p_1, 0), \quad BA_1'' : (\lambda_1, \lambda_2) = (p_2, 0), \quad BA_1' : (\lambda_1, \lambda_2) = (p_3, 0),$$

$$BA_2 : (\lambda_1, \lambda_2) = (0, p_1), \quad BA_2'' : (\lambda_1, \lambda_2) = (0, p_2), \quad BA_2' : (\lambda_1, \lambda_2) = (0, p_3),$$

where  $0 < p_1 < p_2 < p_3 < 1$  are solved numerically (see Commendatore *et al.* (2017)). The map  $Z$  can have, besides the  $CP$  equilibria, at most three equilibria on each border  $I_{bi}$ ,  $i = 1, 2$ . Note that for  $\varepsilon = 0$  it holds that  $BA_i' = BS_i$  with  $p_2 = 1/2$ .

We notice that the border asymmetric equilibria  $BA_i$ ,  $i = 1, 2$ , appear by increasing  $L$  (via a ‘border transcritical bifurcation’ of the equilibrium  $CP_0$ ) at

$$L = BT_{CP_0} = \frac{6bE(a - b\eta)^2(3b + cE)}{cE(4b + cE)(a - b\eta)^2 + (2b + cE)^2(b\varepsilon)^2}, \quad (30)$$

and the border asymmetric equilibria  $BA_i'$ ,  $i = 1, 2$ , appear by increasing  $L$  (via a ‘border transcritical bifurcation’ of the fixed points  $CP_i$ ) at

$$L = BT_{CP_{1,2}} = \frac{6bE(a - b\eta)^2(3b + cE)}{cE(4b + cE)(a - b\eta)^2 - 4b^2(b\varepsilon)^2}. \quad (31)$$

Note finally that for  $\varepsilon = 0$  it holds  $BT_{CP_0} = BT_{CP_{1,2}} = BT_{CP}$ , where  $BT_{CP}$  is defined in (20).

As we shall see below an important difference in the properties of the border asymmetric equilibria  $BA_i$  and  $BA_i'$ ,  $i = 1, 2$  compared to the previous model is that, depending on parameters, they become attracting.

On the border  $I_{b3}$ , besides the equilibrium  $BS_3$ , map  $Z$  can have four border asymmetric (BA) equilibria:

$$BA_3 : (\lambda_1, \lambda_2) = (p, 1 - p), \quad BA_3' : (\lambda_1, \lambda_2) = (1 - p, p),$$

$$BA_{3,u} : (\lambda_1, \lambda_2) = (l, 1 - l), \quad BA_{3,u}' : (\lambda_1, \lambda_2) = (1 - l, l),$$

where  $p$  is given in (18), the index ‘ $u$ ’ refers to the unilateral trade regions, and  $\lambda = l$ ,  $0 < l < \tilde{\lambda}$ , is solved numerically (see Commendatore *et al.* (2017)).

Map  $Z$  can have also the following interior asymmetric (IA) equilibria (see Fig. 3b):

- $IA_i$ ,  $IA_i'$ ,  $i = 1, 2, 3$ , belonging to  $S_0$  (see (21));

<sup>13</sup> The disappearance of the equilibrium  $IS$  after a collision with the border point  $(\lambda_1, \lambda_2) = (\tilde{\lambda}, \tilde{\lambda})$  at  $\varepsilon = \varepsilon_1$ , occurs via a so-called *fold border collision bifurcation* (fold BCB for short) at which  $IS$  merges with another equilibrium, also disappearing after the bifurcation.

- $IA_{i,u}$ ,  $IA'_{i,u}$ ,  $IA''_{i,u}$ , belonging to the unilateral trade regions  $S_i$ ,  $i = 1, 2$ ;
- $IA'_{3,b}$ ,  $IA''_{3,b}$  belonging to the bilateral trade region  $S_3$ .

Concerning the properties of these equilibria, we start noticing that, by increasing  $L$  and for  $\varepsilon < \varepsilon_t$ , the three couples,  $IA_1$ - $IA'_{2,u}$ ,  $IA_2$ - $IA'_{1,u}$  and  $IA_3$ - $IA'_{3,b}$ , appear simultaneously (via a ‘fold border collision bifurcation’) at

$$L = BC_{IA} = \frac{3b^2\varepsilon((4b - cE)(a - b\varepsilon - b\eta) + 4b^2\varepsilon) - 6b(cE + 3b)(a - b\varepsilon - b\eta)^2}{c(a - b\varepsilon - b\eta)(2b^2\varepsilon - (4b + cE)(a - b\varepsilon - b\eta))}, \quad (32)$$

where  $\varepsilon_t$  corresponds to the point where the curves  $L = BC_{IA}$  and  $L = F_{IA}$  are tangent (see Fig. 4b).<sup>14</sup>

Moreover, the equilibria  $IA_i$ ,  $i = 1, 2, 3$ , disappear merging with  $IA'_i$  (due to a ‘smooth fold bifurcation’) at  $L = F_{IA}$  defined in (23). It can be shown that for the considered parameter values  $F_{IA} - BC_{IA} < 0$ , thus, the existence range of the fixed points  $IA_i$  is not empty.

Note that for  $\varepsilon_t < \varepsilon < \varepsilon_1$  the curve  $L = BC_{IA}$  corresponds to the collision of the equilibria  $IA'_i$ ,  $i = 1, 2, 3$ , with the related borders. As  $L = BC_{IA}$  is crossed by increasing  $L$ , their stability properties are preserved, but not their location as  $IA'_1$  moves to  $S_2$ ,  $IA'_2$  to  $S_1$  and  $IA'_3$  to  $S_3$ .<sup>15</sup>

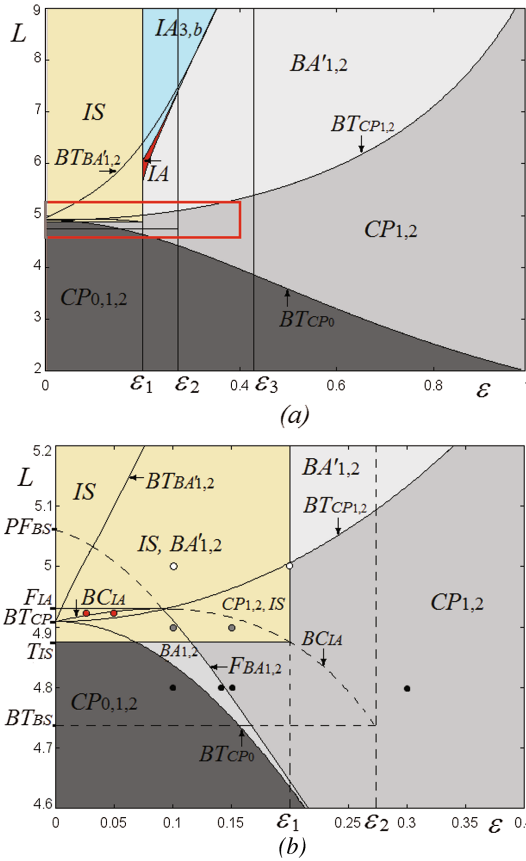
It can be also shown that, by increasing  $L$ , the interior fixed point  $IA''_{3,b}$  (belonging to the median  $I_{m3}$ ) appears at  $L = BT_{CP0}$  given in (30) (due to a ‘border transcritical bifurcation’ of  $CP_0$ ); the equilibrium  $IA'_3$  is born (due to a ‘border transcritical bifurcation’ of the border symmetric equilibrium  $BS_3$  at  $L = BT_{BS}$ ) for  $\varepsilon < \varepsilon_2$ . The other conditions related to the existence and/or stability of the equilibria mentioned above can be obtained numerically, as we discuss below.

To investigate the influence of decreasing trade cost  $T_{12}$  on the dynamics of the map, we begin with  $\varepsilon = 0$ , that corresponds to Model 1, and then we increase gradually the value of  $\varepsilon$ . Complexity of bifurcation sequences associated with equilibria in Model 2 is caused by multistability, when up to 8 attracting equilibria may coexist (as, e.g., in Fig. 3b), and each of them follows its own way to appear, disappear and interact with other equilibria.

In Fig. 4a, representing the  $(\varepsilon, L)$ -parameter plane for  $2 < L < 9$ ,  $0 < \varepsilon < 1$  and for the other parameters fixed as in (24), (25), we summarize the properties of the attracting equilibria. Figure 4b shows an enlargement of the rectangle indicated in Fig. 4a. In these figures the parameter regions associated with different coexisting attracting equilibria are colored differently, being separated by

<sup>14</sup> Note also that crossing  $L = BC_{IA}$  by decreasing  $L$  and considering the interval  $\varepsilon < \varepsilon_t$  the equilibria  $IA_1$  and  $IA'_{2,u}$  collide from the opposite sides with the border defined by  $\lambda_1 = \tilde{\lambda}$ , and similarly the couple  $IA_2$ - $IA'_{1,u}$  collides with the border defined by  $\lambda_2 = \tilde{\lambda}$ , while the couple  $IA_3$ - $IA'_{3,b}$ , belonging to the diagonal  $I_{m3}$ , collides with the border point  $(\lambda_1, \lambda_2) = (\tilde{\lambda}, \tilde{\lambda})$ . These collisions occur simultaneously because of the symmetry of  $Z$  with respect to the main diagonal.

<sup>15</sup> Details how the curve  $BC_{IA}$  is obtained are provided in Commendatore *et al.* (2017).



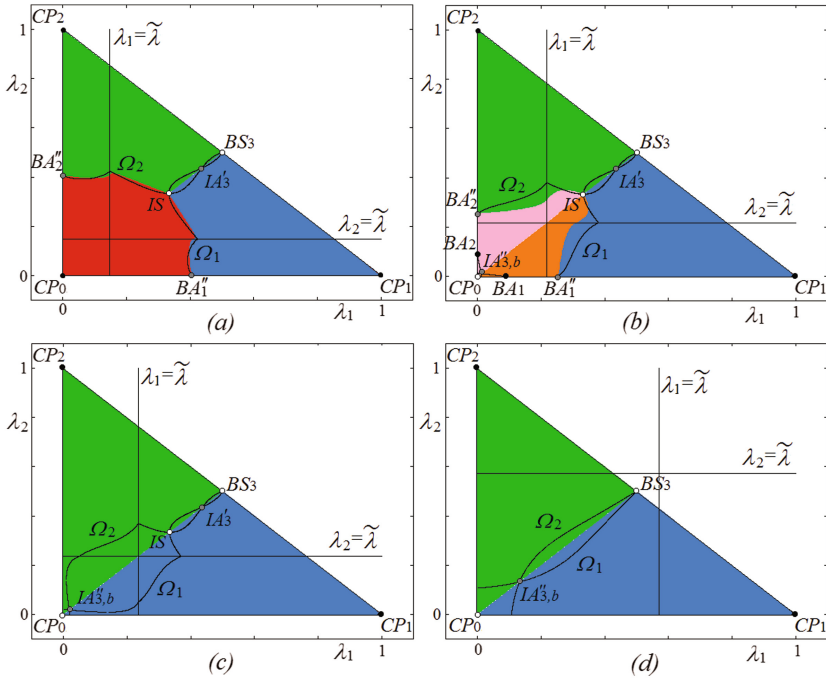
**Fig. 4.** (a) Bifurcation structure of the  $(\varepsilon, L)$ -parameter plane for  $0 < \varepsilon < 1$ ,  $2 < L < 9$ ; (b) an enlargement of the window indicated in (a) with regions of different coexisting attracting fixed points. The other parameters are fixed as in (24) and (25).

various bifurcation curves. In particular, we have drawn the bifurcation lines  $L = F_{IA}$ ,  $L = BT_{BS}$ ,  $L = T_{IS}$  associated with Model 1, the curves  $L = BT_{CP0}$ ,  $L = BT_{CP1,2}$ ,  $L = BC_{IA}$  defined in (30)–(32), and the straight vertical lines  $\varepsilon = \varepsilon_i$ ,  $i = 1, 2, 3$ , defined in (28), (29). The values  $L = BT_{CP}$ ,  $L = PF_{BS}$  valid for  $\varepsilon = 0$  are also marked on the vertical axis. Recall that the stability range  $T_{IS} < L < F_{LIS}$  of the equilibrium  $IS$  obtained for Model 1 is suitable for Model 2 for  $\varepsilon < \varepsilon_1$ ; the curve  $L = BT_{BS}$  is valid for  $\varepsilon < \varepsilon_2$  and corresponds to the transverse border transcritical bifurcation of the equilibrium  $BS_3$ ; the curve  $L = F_{IA}$  (related to the ‘smooth fold bifurcation’ at which  $IA_i = IA'_i$ ,  $i = 1, 2, 3$ ) is valid for  $0 < \varepsilon < \varepsilon_t$ , where  $(\varepsilon, L) = (\varepsilon_t, F_{IA})$  correspond to the point where the curves  $L = F_{IA}$  and  $L = BC_{IA}$  are tangent.

In Fig. 4b the bifurcation curves separating the regions of qualitatively different dynamics, besides those mentioned above, are obtained numerically, in

particular, the curve  $L = F_{BA1,2}$  (related to a ‘fold bifurcation’ leading, by increasing  $L$ , to the disappearance of the couples of the border equilibria  $BA_i - BA_i''$ ,  $i = 1, 2$ ); and the curve  $L = BT_{BA'1,2}$  (associated to a so-called ‘transverse border transcritical bifurcation’ of the border equilibria  $BA_i'$ ,  $i = 1, 2$ , leading, by increasing  $\varepsilon$ , to their stabilization and to the appearance of the saddle interior equilibrium  $IA'_{i,u}$ ). Accordingly, we have marked in the largest and more visible regions of Fig. 4b the associated attracting equilibria that coexist in those regions (for a description of the smaller regions of coexistence, as well as other bifurcation curves, we refer to Commendatore *et al.* (2017)).

Below, we present several examples for different values of  $L$  to illustrate the effects of lowering trade costs, determined by increasing  $\varepsilon$ , on the long-run properties of the equilibria and on the patterns of trade. Moreover, we also provide an economic interpretation of the results.



**Fig. 5.** Attracting fixed points of map  $Z$  (Model 2) and their basins for  $L = 4.8$  and  $\varepsilon = 0.1$  in (a),  $\varepsilon = 0.14$  in (b),  $\varepsilon = 0.15$  in (c),  $\varepsilon = 0.3$  in (d) (see the black circles in Fig. 4b). The other parameters are fixed as in (24) and (25).

One of the simplest scenarios is illustrated in Fig. 5 where  $L = 4.8$  and  $\varepsilon = 0.1$  in (a),  $\varepsilon = 0.14$  in (b),  $\varepsilon = 0.15$  in (c) and  $\varepsilon = 0.3$  in (d) (the corresponding points in the parameters space are indicated in Fig. 4b by black circles). For  $\varepsilon = 0$  there are three coexisting attracting  $CP$  equilibria (see Fig. 2a). When it is close to 0,

an increase of  $\varepsilon$  does not change abruptly the qualitative properties of the system  $Z$  (see Fig. 5a).<sup>16</sup> Its most relevant effects occur in the subregions of  $S$  related to unilateral trade,  $S_1$  and  $S_2$ ,<sup>17</sup> the basin of attraction of the equilibrium  $CP_0$  shrinks, whereas those of the equilibria  $CP_1$  (in  $S_2$ ) and  $CP_2$  (in  $S_2$ ) expand. This implies that when the initial distribution of entrepreneurs is sufficiently in favour of region 1 (in  $S_1$ ) or 2 (in  $S_2$ ), all the firms move in that region enjoying additional profits obtained by selling goods in the outside market (in region 2 or 1, respectively).

An abrupt change occurs when  $\varepsilon$  crosses the curve  $L = BT_{CP_0}$  (associated to a ‘border transcritical bifurcation’ of the equilibrium  $CP_0$ ) involving a loss of stability and the consequent disappearance of the basin of attraction of  $CP_0$ . This also leads to the appearance of the attracting border equilibria  $BA_i$ ,  $i = 1, 2$  and of the saddle interior equilibrium  $IA'_{3,b}$  in the subregion of  $S$  related to bilateral trade,  $S_3$ , so that map  $Z$  has now four attractors,  $CP_1$ ,  $CP_2$ ,  $BA_1$  and  $BA_2$  (see Fig. 5b). Compared with the equilibrium  $CP_0$ , in the equilibria  $BA_i$ ,  $i = 1, 2$ , trade costs are sufficiently low to allow for the location of some industry in region 1 (at  $BA_1$ ) or in region 2 (at  $BA_2$ ). Firms in region 1 (at  $BA_1$ ) and 2 (at  $BA_2$ ) are enjoying additional profits by selling goods in the outside market (in region 2 or 1, respectively). In region 3, the size of the local market is sufficiently large that a large share of firms still finds convenient to locate there. Finally, being on the borders, the equilibria  $BA_i$ ,  $i = 1, 2$  are obviously characterised by a unilateral trade pattern, even if along the transition path towards the equilibrium regions 1 and 2 may engage in bilateral trade for distributions of entrepreneurs positioned in  $S_3$ .<sup>18</sup>

Then, when the  $\varepsilon$  crosses the curve  $L = F_{BA_{1,2}}$  (and a ‘fold bifurcation’ occurs at which  $BA_i = BA''_i$ ) the equilibria  $BA_1$  and  $BA_2$  disappear, and the remaining attractors are the fixed point  $CP_1$  and  $CP_2$  (see Fig. 5c). Thus for  $\varepsilon$  sufficiently large (for trade costs sufficiently low) all initial distributions lead to an equilibrium in which the industrial sector is agglomerated in region 1 (or 2). Bilateral trade between regions 1 and 2 is possible along the transition path, however, only unilateral trade occurs in the long-run.

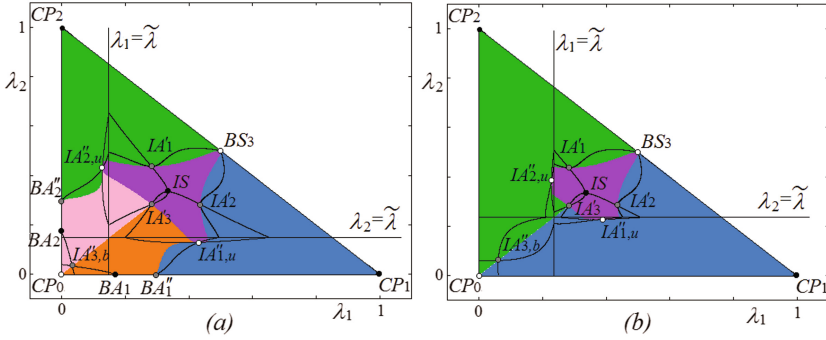
For larger values of  $\varepsilon$  the equilibria  $CP_1$  and  $CP_2$  continue to be the only attractors of map  $Z$  and the economic interpretation is the same (see Fig. 5d).<sup>19</sup>

<sup>16</sup> Notice that the basins of  $CP$  equilibria are bounded by the closure of the stable invariant sets of the saddle fixed points  $BA'_1$ ,  $BA'_2$  and  $IA'_3$ .

<sup>17</sup> In regions  $S_0$  and  $S_3$  the long-run results are qualitatively similar to those that apply for the case  $\varepsilon = 0$ . For initial conditions that start in region  $S_3$  bilateral trade occurs during the transition path towards the equilibrium  $CP_0$ .

<sup>18</sup> There are also initial conditions belonging to the region  $S_0$  which leads to  $BA_1$  or  $BA_2$ . Along the corresponding transitions path there is a switch from a no trade regime to a unilateral trade regime.

<sup>19</sup> Concerning the analytical properties of the map  $Z$ , when  $\varepsilon$  is increased, the repelling fixed point  $IS$ , after undergoing a ‘persistence border collision’, when the curve  $\varepsilon = \varepsilon_1$  is crossed, disappears together with the saddle fixed point  $IA'_3$  when the curve  $L = BC_{IA}$  is crossed undergoing a ‘fold BCB’ (see Fig. 5d).



**Fig. 6.** Attracting fixed points of map  $Z$  (Model 2) and their basins for  $L = 4.9$  and  $\varepsilon = 0.1$  in (a),  $\varepsilon = 0.15$  in (b) (see the gray circles in Fig. 4b). The other parameters are fixed as in (24) and (25).

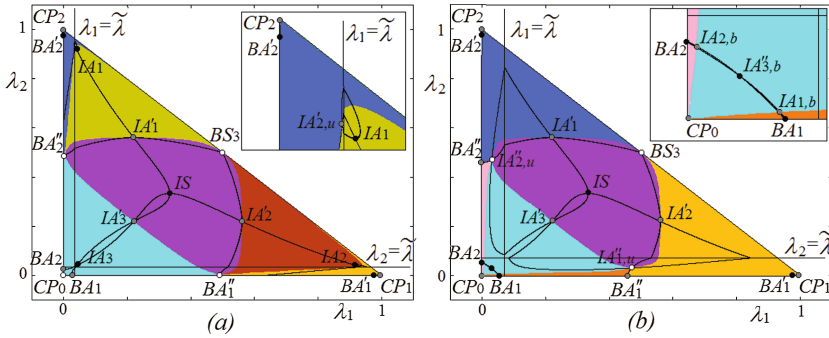
Let now set  $L = 4.9$  and consider the cases (a)  $\varepsilon = 0.1$  and (b)  $\varepsilon = 0.15$  (the corresponding points in the parameter space are indicated in Fig. 4b by grey circles). For  $\varepsilon = 0$  the attracting equilibrium  $IS$  coexists with three attracting  $CP$  equilibria (see Fig. 2b). Increasing  $\varepsilon$  at first the curve  $L = BT_{CP0}$  is crossed leading to the loss of stability of  $CP_0$  and to the appearance of the attracting border equilibria  $BA_1$  and  $BA_2$ . In Fig. 6a (where  $\varepsilon = 0.1$ ) these equilibria coexist with the attracting equilibria  $IS$ ,  $CP_1$  and  $CP_2$ . In  $S_0$  the results are qualitatively similar to those that apply when  $\varepsilon = 0$ :  $IS$  is still characterised by full autarky. However, there are initial distributions located in  $S_1$  and  $S_2$  (corresponding to short-run equilibria), leading to this equilibrium, for which unilateral trade may occur.

By further increasing  $\varepsilon$  the equilibria  $BA_1$  and  $BA_2$  disappear when the curve  $L = F_{BA1,2}$  is crossed (due to a ‘fold bifurcation’). After, map  $Z$  has three attractors,  $IS$ ,  $CP_1$  and  $CP_2$  (see Fig. 6b where  $\varepsilon = 0.15$ ). Then, the equilibrium  $IS$  disappears when the curve  $\varepsilon = \varepsilon_1$  is crossed (due to a ‘fold border collision bifurcation’),<sup>20</sup> so that the only attractors of map  $Z$  are the equilibria  $CP_1$  and  $CP_2$ . When  $CP_1$  and  $CP_2$  are the only stable equilibria, the properties of the system and their economic interpretation are analogous to those corresponding to Fig. 5d.

The change in the properties of the system observed when  $L = 4.92$  by increasing  $\varepsilon$  is a bit more complicated. To illustrate this case, we consider the examples: (a)  $\varepsilon = 0.025$  and (b)  $\varepsilon = 0.05$  (the corresponding points in the parameter space are indicated in Fig. 4b by red circles). For  $\varepsilon = 0$  (see Fig. 2c) the coexisting attracting equilibria are  $IS$  and  $IA_i$ ,  $i = 1, 2, 3$ . If  $\varepsilon$  is increased, the equilibria  $BA'_1$  and  $BA'_2$  gain stability (due to a ‘transverse border transcritical bifurcation’) when the curve  $L = BT_{BA'1,2}$  is crossed. After this bifurcation, map  $Z$  has six attracting equilibria,  $IS$ ,  $IA_i$ ,  $BA'_1$  and  $BA'_2$  (see Fig. 7a where

<sup>20</sup> Before, when the curve  $L = BC_{IA}$  is crossed, the saddle equilibria  $IA'_i$ ,  $i = 1, 2, 3$  undergo a ‘persistence border collision’.





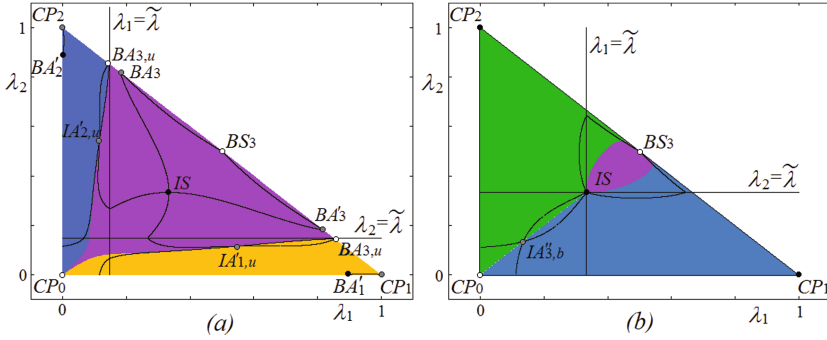
**Fig. 7.** Attracting fixed points of map  $Z$  (Model 2) and their basins for  $L = 4.92$  and  $\varepsilon = 0.025$  in (a),  $\varepsilon = 0.05$  in (b) (see the red circles in Fig. 4b). The other parameters are fixed as in (24) and (25). The insets show the related parts enlarged.

$\varepsilon = 0.025$ ). Again, the most important changes occur in the regions related to unilateral trade  $S_1$  and  $S_2$ , where the border equilibria  $BA'_1$  (in  $S_1$ ) and  $BA'_2$  ( $S_2$ ) are now attracting (the basins of attraction of  $BA'_1$  and  $BA'_2$  are colored in dark yellow and dark blue, respectively). Initial distributions that for  $\varepsilon = 0$  were in the basins of attraction of  $IA_2$  and  $IA_1$ , for  $\varepsilon = 0.025$  belong to the basins of  $BA'_1$  and  $BA'_2$ , respectively. Unilateral trade gives an incentive to entrepreneurs to locate in region 1 (for initial distributions in the basin of  $BA'_1$ ) or region 2 (for initial distributions in the basin of  $BA'_2$ ) because of the additional profits obtained by selling goods in the outside market in region 2 or region 1.

Then after a sequence of bifurcations (which includes a ‘fold border collision bifurcation’ of  $IA_i$  occurring when the curve  $L = BC_{IA}$  is crossed, and ‘transverse border transcritical bifurcations’ of  $BA_i$  and  $BA'_i$ ,  $i = 1, 2$ ) the attracting equilibria are  $IS$ ,  $BA'_i$ ,  $BA_i$  and  $IA''_{3,b}$  (see Fig. 7b where  $\varepsilon = 0.05$ ). The most relevant effects of the increase in  $\varepsilon$  is the appearance of the stable equilibrium  $IA''_{3,b}$  in the region related to bilateral trade,  $S_3$ , and the disappearance of the interior asymmetric equilibria  $IA_i$ ,  $i = 1, 2, 3$ . Initial distributions that were leading previously to  $IA_1$  or  $IA_2$  are now attracted respectively by  $BA'_2$  or  $BA'_1$ . Concerning trade patterns, it is interesting to notice that a large set of initial distributions characterised by full autarky may lead to a long-run equilibrium in which unilateral ( $BA'_2$  or  $BA'_1$ ) or bilateral ( $IA''_{3,b}$ ) trade prevails. Moreover, transition paths characterised by a switch from a unilateral to a bilateral trade regime are also possible.<sup>21</sup>

Further increasing  $\varepsilon$  leads to reverse pitchfork bifurcation of  $IA''_{3,b}$  which loses stability merging with saddles  $IS_{1,b}$  and  $IS_{2,b}$ , then the fixed points  $CP_1$  and  $CP_2$  are stabilized (due to a ‘border transcritical bifurcation’, when the

<sup>21</sup> It is also possible a transition path starting from a unilateral trade regime in which the share of entrepreneurs are positive in all regions ending to an equilibrium,  $BA_1$  or  $BA_2$ , in which unilateral trade still prevails because region 2 or region 1 is empty of entrepreneurs.



**Fig. 8.** Attracting fixed points of map  $Z$  (Model 2) and their basins for  $L = 5$  and  $\varepsilon = 0.1$  in (a),  $\varepsilon = 0.2$  in (b) (see the white circles in Fig. 4b). The other parameters are fixed as in (24) and (25).

curve  $L = BT_{CP1,2}$  is crossed), and the properties of the system and their interpretation become qualitatively similar to those holding for the case shown in Fig. 6a.

Further examples are shown in Fig. 8 where  $L = 5$  and (a)  $\varepsilon = 0.1$  and (b)  $\varepsilon = 0.2$  (the corresponding points in the parameter space are indicated in Fig. 4b by white circles). For  $\varepsilon = 0$ ,  $IS$  is the only stable equilibrium (see Fig. 2d). By increasing  $\varepsilon$  at first the curve  $L = BT_{BA'1,2}$  is crossed leading to the stabilization of the equilibria  $BA'_1$  and  $BA'_2$  (see Fig. 8a where  $\varepsilon = 0.1$ ).  $BA'_1$  and  $BA'_2$  belong to the regions of  $S$  related to unilateral trade,  $S_1$  and  $S_2$ , respectively, and are characterised by a large share of firms located in region 1 or 2 obtaining additional profit by selling in the outside market in region 2 or 1 and by a small share of firms located in region 3, attracted by a sufficiently large local market.

For this example, it is interesting to notice that transition paths are possible starting from a bilateral trade regime in  $S_3$  or from a unilateral trade regime in  $S_1$  or  $S_2$  ending to the equilibrium  $IS$  in  $S_0$  in which full autarky prevails. This is favoured by a relative large proportion of immobile local demand sufficient to countervail the effect of trade liberalization.

Crossing the curve  $L = BT_{CP1,2}$  leads to the disappearance of the equilibria  $BA'_1$  and  $BA'_2$  and the stabilization of the equilibria  $CP_1$  and  $CP_2$  (due to a ‘border transcritical bifurcation’). Then (due to a ‘border collision bifurcation’) at  $\varepsilon = \varepsilon_1$  the fixed point  $IS$  disappears (see Fig. 8b where  $\varepsilon = \varepsilon_1 = 0.2$ ) after which the only attractors of map  $Z$  are the fixed points  $CP_1$  and  $CP_2$  and the properties of the system and their interpretation become qualitatively similar to those holding for the case shown in Fig. 5d.

## 7 Long-Run Equilibria Properties in Model 3

The economy in Model 3 takes a specific ‘hub and spoke structure. Indeed, for the third set-up we assumed that trade costs between regions 1 and 2 are always

sufficiently low to establish bilateral trade (for this we let  $T_{12} < \frac{2(a-\eta b)}{2b+cE}$ ), while trade costs between regions 2 and 3 are always sufficiently high to impede trade (so that  $T_{23} \geq \frac{a}{b} - \eta$  holds). We set  $T_{12} = \tau$ ,  $T_{13} = T - \psi$  and  $T_{23} = T$ , with  $T = \frac{a}{b} - \eta$ .

The related indirect utilities  $V_i(\lambda_1, \lambda_2) =: V_i$ ,  $i = 1, 2, 3$ , of an entrepreneur in regions 1, 2 and 3, respectively, are defined as follows:

$$\begin{aligned} & \text{if } \lambda_1 \geq \bar{\Lambda}(\lambda_2), \lambda_2 \leq 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 = V_1^{bil}, \quad V_2 = V_2^{bil}, \quad V_3 = V_3^{bil}, \\ & \text{if } \lambda_1 \geq \bar{\Lambda}(\lambda_2), \lambda_2 > 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 = V_1^{bil, out}, \quad V_2 = V_2^{bil, out}, \quad V_3 = V_3^{bil, out}, \\ & \text{if } \lambda_1 < \bar{\Lambda}(\lambda_2), \lambda_2 \leq 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 = V_1^{bil, in}, \quad V_2 = V_2^{bil, in}, \quad V_3 = V_3^{bil, in}, \\ & \text{if } \lambda_1 < \bar{\Lambda}(\lambda_2), \lambda_2 > 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 = V_1^{bil, bil}, \quad V_2 = V_2^{bil, bil}, \quad V_3 = V_3^{bil, bil}, \end{aligned} \quad (33)$$

where

$$\begin{aligned} \bar{\lambda} &= \frac{2(a - \eta b - bT_{13})}{cET_{13}} = \frac{2b\psi}{cE\left(\frac{a}{b} - \eta - \psi\right)}, \\ \bar{\Lambda}(\lambda_2) &= \bar{\lambda} - \lambda_2 \frac{T_{13} - T_{12}}{T_{13}} = \bar{\lambda} - \lambda_2 \frac{\frac{a}{b} - \eta - \psi - \tau}{\frac{a}{b} - \eta - \psi}, \end{aligned}$$

and  $V_i^{bil}$ ,  $V_i^{bil, out}$ ,  $V_i^{bil, in}$ ,  $V_i^{bil, bil}$  for  $i = 1, 2, 3$  are defined in Commendatore *et al.* (2017).

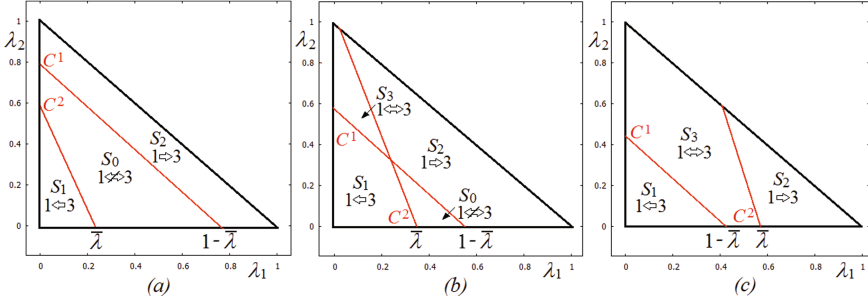
In the simulations we fix  $\tau = 0.5 < \frac{2(a-\eta b)}{2b+cE} \Big|_{(24)} = \frac{4}{7} \approx 0.571$ , and vary the values of parameters  $L$  and  $\psi$ , where  $0 < \psi < \tau = 0.5$ , keeping the other parameters fixed as in (24) and (25).

Let the straight lines  $\lambda_2 = 1 - \bar{\lambda} - \lambda_1$  and  $\lambda_1 = \bar{\Lambda}(\lambda_2)$  separating the regions associated with different indirect utilities be denoted as  $C^1$  and  $C^2$ , respectively. These lines can be written as

$$\begin{aligned} C^1 : \quad \lambda_2 &= -\lambda_1 + 1 - \frac{2b\psi}{cE\left(\frac{a}{b} - \eta - \psi\right)} \Big|_{(24)} = -\lambda_1 + 1 - \frac{2\psi}{E(1 - \psi)}, \\ C^2 : \quad \lambda_2 &= -\lambda_1 \frac{\frac{a}{b} - \eta - \psi}{\frac{a}{b} - \eta - \psi - \tau} + \frac{2b\psi}{cE\left(\frac{a}{b} - \eta - \psi - \tau\right)} \Big|_{(24)} \\ &= -\lambda_1 \frac{1 - \psi}{1 - \psi - \tau} + \frac{2\psi}{E(1 - \psi - \tau)}. \end{aligned}$$

Note that the slope of  $C^2$  in the  $(\lambda_1, \lambda_2)$ -phase plane is larger in modulus than the slope of  $C^1$ ; the lines  $C^1$  and  $C^2$  intersect the trapping triangle  $S$  if  $\psi < \frac{a}{b} - \eta \Big|_{(24)} = 1$  (related to  $\bar{\lambda} > 0$ ) and  $\psi < \left(\frac{a}{b} - \eta\right) \frac{cE}{2b+cE} \Big|_{(24)} = \frac{3}{7}$  (related to  $\bar{\lambda} < 1$ ).

Depending on the parameters the straight lines  $C^1$  and  $C^2$  may separate the trapping triangle  $S$  into at most four regions denoted  $S_i$ ,  $i = 0, 3$ , where region  $S_0$  is related to no trade between regions 1 and 3, regions  $S_1$ ,  $S_2$  correspond to unilateral trade from region 3 to region 1, and from region 1 to region 3, respectively, and region  $S_3$  is associated with bilateral trade between regions 1 and 3. In fact, there are five qualitatively different cases:



**Fig. 9.** Partitioning of the trapping triangle  $S$  into the regions  $S_0$  (no trade between regions 1 and 3),  $S_1$  (one-way trade from region 3 to region 1),  $S_2$  (one-way trade from region 1 to region 3) and  $S_3$  (two-way trade between regions 1 and 3). Here  $L = 4.8$ ,  $\tau = 0.5$  and  $\psi = 0.15$  in (a),  $\psi = 0.23$  in (b) and  $\psi = 0.3$  in (c). The other parameters are fixed as in (24) and (25).

- (1) if  $\bar{\lambda} = 0$ , that holds for  $\psi = 0$ , then  $S = S_0$ , i.e., the complete trapping triangle  $S$  is associated with no trade between the regions 1 and 3, that is, we are back to Model 2;
- (2) if  $0 < \bar{\lambda} \leq 1 - \lambda^*$  where  $\lambda^* = \frac{2b\psi}{cE(\frac{a}{b} - \eta - \psi - \tau)} \Big|_{(24)} = \frac{2\psi}{E(1 - \psi - \tau)}$ , then  $S = \cup_{j=0}^2 S_j$  (see Fig. 9a), that is, in  $S$  there are ‘no trade’ and ‘unilateral trade’ regions while bilateral trade is not possible; this holds for  $0 < \psi \leq \psi_1$ , where

$$\begin{aligned} \psi_1 &= \frac{1}{2(4b + cE)} \left( (2b + cE) \left( 2 \left( \frac{a}{b} - \eta \right) - \tau \right) - \right. \\ &\quad \left. - \sqrt{(2b + cE)^2 \left( 2 \left( \frac{a}{b} - \eta \right) - \tau \right)^2 - 4cE(4b + cE) \left( \frac{a}{b} - \eta \right) \left( \frac{a}{b} - \eta - \tau \right)} \right), \\ \psi_1|_{(24)} &= \frac{1}{2(4 + E)} \frac{(2 + E)(2 - \tau) - \sqrt{4(2 - \tau)^2 + E\tau^2(4 + E)}}{2(4 + E)} \Big|_{E=1.5, \tau=0.5} \approx 0.175; \end{aligned}$$

- (3) if  $1 - \lambda^* < \bar{\lambda} < \frac{1}{2}$ , then  $S = \cup_{j=0}^3 S_j$  (see Fig. 9b), that is, all four types of trade between regions 1 and 3 are possible; this occurs for  $\psi_1 < \psi < \psi_2$ , where

$$\psi_2 = \frac{cE(a - b\eta)}{b(4b + cE)} \Big|_{(24)} = \frac{3}{11} \approx 0.273;$$

- (4) if  $\frac{1}{2} \leq \bar{\lambda} < 1$ , then  $S = \cup_{j=1}^3 S_j$  (see Fig. 9c), that is, uni- and bilateral trade are possible, but there is no ‘no trade’ region; that holds for  $\psi_2 \leq \psi < \psi_3$ , where

$$\psi_3 = \frac{cE(a - b\eta)}{b(2b + cE)} \Big|_{(24)} = \frac{3}{7} \approx 0.429;$$

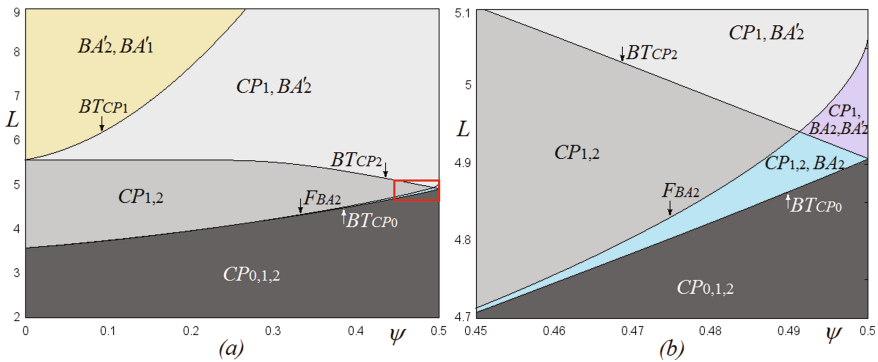
- (5) if  $\bar{\lambda} \geq 1$ , that holds for  $\psi_3 \leq \psi \leq \tau$ , then  $S = S_3$ .

Similar to Models 1 and 2, for Model 3 the borders  $I_{bi}$ ,  $i = 1, 2, 3$  (see (12)) of the trapping triangle  $S$  are invariant lines of map  $Z$ .<sup>22</sup> The border equilibria of  $Z$  are denoted  $BA_i \in I_{bi}$ , with additional upper index “'”, or “'''” to distinguish between different equilibria belonging to the same border.

Besides the  $CP$  equilibria (see (13)) and  $BA$  equilibria, map  $Z$  can have also interior equilibria which we denote  $IA_i \in S_i$ .

Figure 10 summarizes the properties of the long-run equilibria. In Fig. 10a we present the bifurcation structure of the  $(\psi, L)$ -parameter plane for  $0 < \psi < \tau = 0.5$ ,  $2 < L < 9$ , and other parameters fixed as in (24) and (25). In this figure the regions of different coexisting attracting equilibria are separated by the curves  $L = BT_{CP0}$ ,  $L = BT_{CP1}$ ,  $L = BT_{CP2}$  obtained numerically and related to border transcritical bifurcations of the equilibria  $CP_0$ ,  $CP_1$  and  $CP_2$ , respectively, as well as by the curve  $L = F_{BA2}$  associated with a fold bifurcation of the border equilibria belonging to  $I_{b2}$ . Figure 10b shows an enlargement of Fig. 10a.

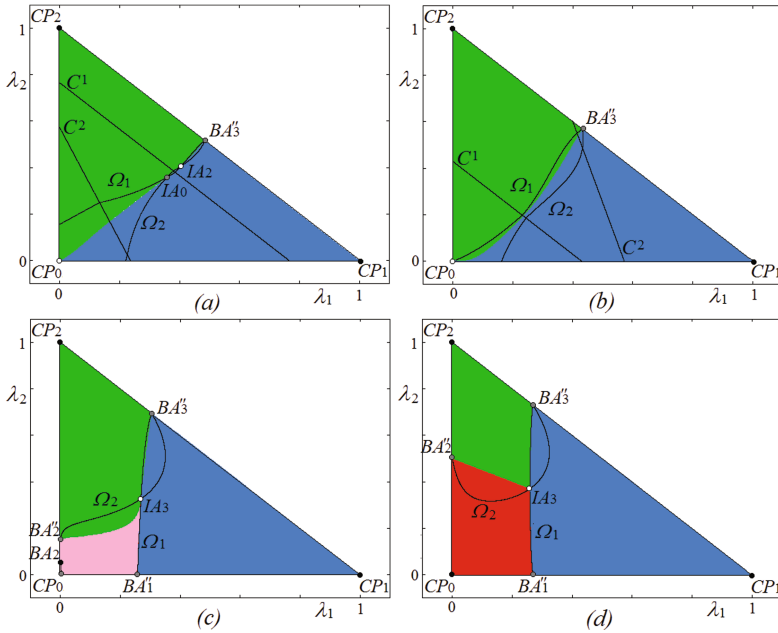
To study the influence of decreasing trade cost  $T_{13}$ , as a starting point we consider the bifurcation structure observed for  $\psi = 0$ , which is associated with Model 2 and corresponds to the cross-section at  $\varepsilon = 0.5$  of the  $(\varepsilon, L)$ -parameter plane shown in Fig. 4a (recall that for Model 3 it holds that  $T_{12} = \tau$ , while for Model 2  $T_{12} = \frac{a}{b} - \eta - \varepsilon$ , so  $\varepsilon = \frac{a}{b} - \eta - \tau$ ; for the considered parameter values  $\tau = 0.5$  corresponds to  $\varepsilon = 0.5$ ). Below we comment several transformations of the basins of the attracting equilibria of map  $Z$  observed for fixed  $L = 4.8$ ,  $L = 4.92$ ,  $L = 5$  and increasing  $\psi$ , when various bifurcation curves are crossed.



**Fig. 10.** (a) Bifurcation structure of the  $(\psi, L)$ -parameter plane for  $0 < \psi < 0.5$ ,  $2 < L < 9$ ,  $\tau = 0.5$ ; (b) an enlargement of the window indicated in (a) with regions of different coexisting attracting fixed points. The other parameters are fixed as in (24) and (25).

<sup>22</sup> However, for the third model map  $Z$  has no symmetry, and on each border  $I_{bi}$  it is reduced to a different 1D map. In Commendatore *et al.* (2017), we have seen that for model 1 and model 2 map  $Z$  is reduced on the borders  $I_{b1}$  and  $I_{b2}$  to the same 1D map, while on  $I_{b3}$  the 1D map is symmetric with respect to  $x = 1/2$ .

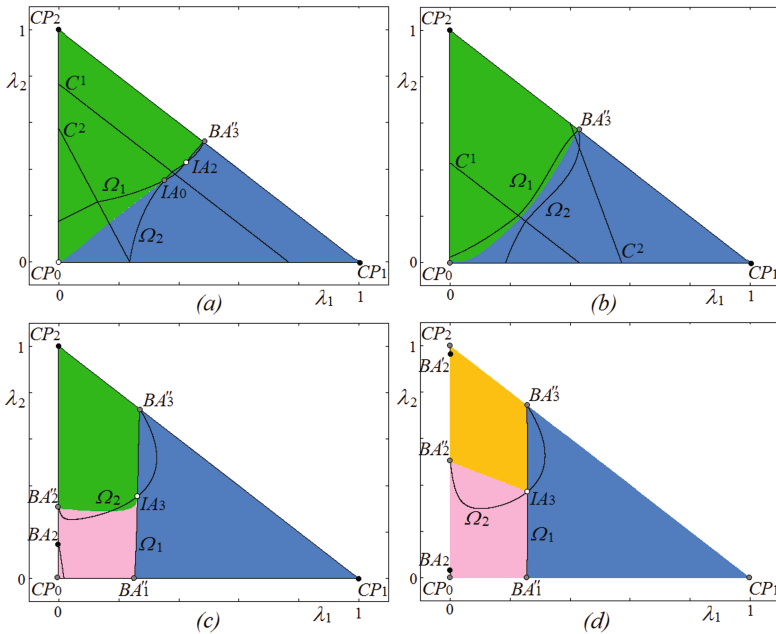
Let  $L = 4.8$ . For  $\psi = 0$  map  $Z$  has attracting equilibria  $CP_1$  and  $CP_2$  (see the parameter point  $(\varepsilon, L) = (0.5, 4.8)$  in Fig. 4a, which belongs to the region denoted  $CP_{1,2}$ ). For increasing  $\psi$  these equilibria at first remain the only attractors of map  $Z$ : in Fig. 11a (where  $\psi = 0.15$ ) we show the basins of  $CP_1$  and  $CP_2$  separated by the closure of stable invariant sets of the saddle fixed points  $IA_0 \in S_0$  and  $BA_3'' \in I_{b3}$ , and in Fig. 11b where  $\psi = 0.3$  the basins are separated by the closure of the stable invariant set of  $BA_3''$ . What can be noticed from Figs. 11a and b is the progressive enlargement of the basin of attraction of  $CP_1$ , and the shrinking of that of  $CP_2$ , compared with the case  $\psi = 0$ , as unilateral trade from 1 to 3 is allowed and firms in 1 could obtain additional profits by exporting goods in 3 exploiting their location. Indeed, in correspondence of  $CP_1$  one-way trade from 1 to 2 and from 1 to 3 occurs; instead, in  $CP_2$  only one way trade from 2 to 1 can take place. Interestingly, there are possible transition paths in which different trade patterns may occur, for example (looking at Fig. 11b) bilateral trade between 1 and 2 and 1 and 3 may occur for distributions of entrepreneurs in the partition  $S_3$ . However, at some point trade links are severed as all the industrial activity agglomerates in only one region.



**Fig. 11.** Attracting fixed points of map  $Z$  (Model 3) and their basins for  $L = 4.8$ ,  $\tau = 0.5$  and  $\psi = 0.15$  in (a),  $\psi = 0.3$  in (b),  $\psi = 0.47$  in (c),  $\psi = 0.5$  in (d). The other parameters are fixed as in (24) and (25).

A qualitative change is observed at  $\psi \approx 0.469$  as the curve  $L = F_{BA2}$  is crossed, see Fig. 10b (giving rise to a ‘fold bifurcation’) and a couple of border

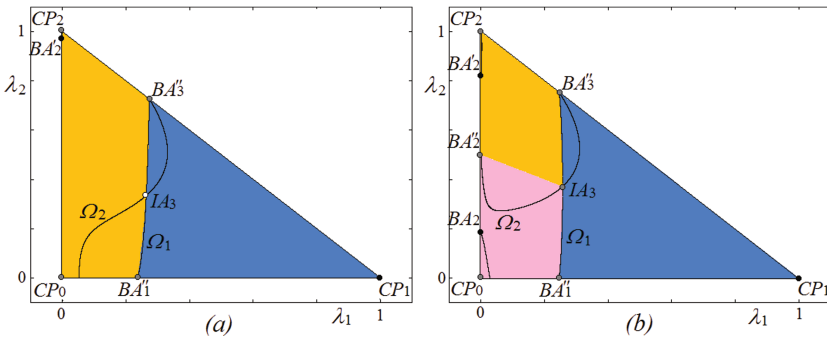
equilibria, attracting  $BA_2$  and saddle  $BA_2''$ , appear. These are shown in Fig. 11c where basins of the attracting equilibria  $CP_1$ ,  $CP_2$  and  $BA_2$  are plotted for  $\psi = 0.47$ . Recall that for  $\psi > \psi_3 \approx 0.43$ , it holds that  $S = S_3$ , that is, trade costs are sufficiently small that bilateral trade between 1 and 3 always occur for all distributions of entrepreneurs within  $S$ . The equilibrium  $BA_2$  is quite interesting, since it is characterized by trade from 2 to 1 and from 3 to 1. Thus, initial distributions of entrepreneurs that largely favour region 3 lead to a long-run equilibrium where the hub region has no manufacturing sector and it is importing from the two spoke regions. By further increasing  $\psi$  the curve  $L = BT_{CP_0}$  is crossed, and as a result the fixed point  $BA_2$  disappears merging with the fixed point  $CP_0$  that gains stability (due to a ‘border transcritical bifurcation’, see Fig. 11d where the basins of  $CP_1$ ,  $CP_2$  and  $CP_3$  are shown for  $\psi = 0.5$ ). The economic interpretation is that when trade costs between 1 and 3 become close to that between 1 and 2, regions 2 and 3 become more symmetric and the basins of attraction of the CP equilibria  $CP_2$  and  $CP_3$  are similar: initial distributions of entrepreneurs that favour region 2 or region 3, that is one of the spoke regions, lead to  $CP_2$  or  $CP_3$ , respectively. Notice also that the basin of attraction of  $CP_1$  is much larger being region 1 the hub.



**Fig. 12.** Attracting fixed points of map  $Z$  (Model 3) and their basins for  $L = 4.92$ ,  $\tau = 0.5$  and  $\psi = 0.15$  in (a),  $\psi = 0.3$  in (b),  $\psi = 0.49$  in (c),  $\psi = 0.5$  in (d). The other parameters are fixed as in (24) and (25).

For  $L = 4.92$  the sequence of bifurcations at first is similar to the one described above: Starting from the attracting fixed points  $CP_1$  and  $CP_2$  coexisting for  $\psi = 0$  (the parameter point  $(\varepsilon, L) = (0.5, 4.92)$  in Fig. 4a also belongs to the region  $CP_{1,2}$ , as in the previous example), by increasing  $\psi$  these fixed points remain the only attractors (see Fig. 12a and b where  $\psi = 0.15$  and  $\psi = 0.3$ , respectively). As before it is possible to observe a progressive change in the size of the basins of attraction of  $CP_1$  (enlarging) and  $CP_2$  (shrinking). By further increasing  $\psi$  (after a ‘fold bifurcation’) a border attracting equilibrium  $BA_2$  and a saddle equilibrium  $BA'_2$  are born (see Fig. 12c where  $\psi = 0.49$ ). Comparing Figs. 11c and 12c, we see that, by increasing the immobile local demand  $L$  (intensifying the dispersion forces),  $BA_2$  is characterised by a larger share of entrepreneurs located in region 2 (a smaller share located in region 3) and by a larger basin of attraction. Finally, by further increasing  $\psi$  the fixed point  $CP_2$  loses stability and the stable border equilibrium  $BA'_2$  is born (via a ‘border transcritical bifurcation’, see Fig. 12d where  $\psi = 0.5$ ). As before, by reducing trade costs between regions 1 and 3, regions 2 and 3 become more symmetric; differently from the previous case the larger immobile local demand allows for some entrepreneurs located in 2 (in  $BA_2$ ) or in 3 ( $BA'_2$ ). Both  $BA_2$  and  $BA'_2$  are characterised by one-way trade from the spokes to the hub.

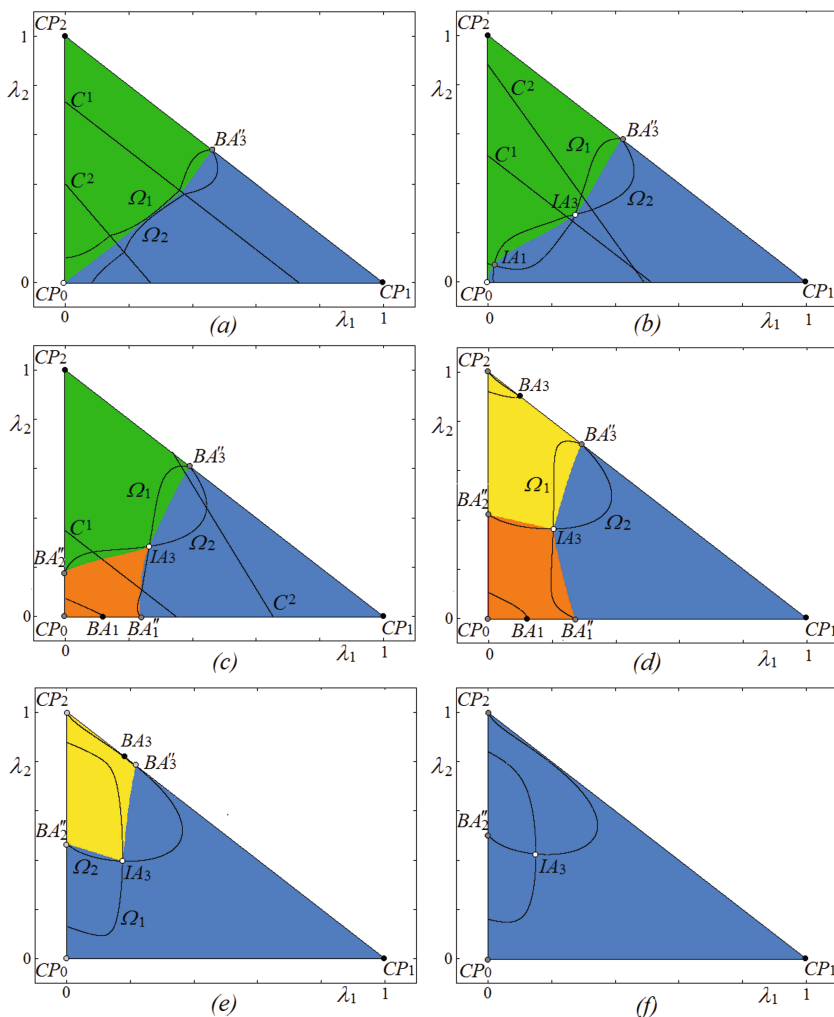
When  $L = 5$ , as for the previous cases, the attracting equilibria  $CP_1$  and  $CP_2$  coexist for  $\psi = 0$ . By increasing  $\psi$ , after crossing the curve  $L = BT_{CP_2}$  (see Fig. 10b), the fixed point  $CP_2$  loses stability and a border attracting equilibria  $BA'_2$  is born. Thus, as shown in Fig. 13a, where  $\psi = 0.48$ , compared with the previous case (see Fig. 12b) the increase in  $L$ , allows for the location of some entrepreneurs in region 3 ( $CP_2$  has lost stability in favour of  $BA'_2$ ). Further increasing  $\psi$ , leads (via a ‘fold bifurcation’) to a border attracting and a saddle equilibria  $BA_2$  and  $BA'_2$  (see Fig. 13b where  $\psi = 0.5$ ). Compared with the previous case, due to a larger  $L$ , this occurs for a lower value of  $\psi$ .



**Fig. 13.** Attracting fixed points of map  $Z$  (Model 3) and their basins for  $L = 5$ ,  $\tau = 0.5$  and  $\psi = 0.48$  in (a),  $\psi = 0.5$  in (b). The other parameters are fixed as in (24) and (25).



To clarify better the importance of the size of immobile local demand ( $L$ ) relative to the demand that could potentially shift ( $E$ ), let us comment one more sequence of transformations of the basins observed for  $E = 5$ . We fix  $\tau = 0.2 < \frac{2(a-\eta b)}{2b+cE} \Big| \approx 0.286$ ,  $L = 4.8$  and will increase  $\psi$ . As before, for  $\psi = 0$  map  $Z$  has two coexisting attracting equilibria  $CP_1$  and  $CP_2$ . At first these equilibria remain the only attractors of  $Z$  (see Fig. 14a and 14b where  $\psi = 0.4$  and  $\psi = 0.55$ , respectively), the qualitative behaviour of the system and its



**Fig. 14.** Attracting fixed points of map  $Z$  (Model 3) and their basins for  $E = 5$ ,  $L = 4.8$ ,  $\tau = 0.2$  and  $\psi = 0.4$  in (a),  $\psi = 0.55$  in (b),  $\psi = 0.62$  in (c),  $\psi = 0.75$  in (d),  $\psi = 0.779$  in (e),  $\psi = 0.8$  in (f). The other parameters are fixed as in (24) and (25).

economic interpretation are the same as before: compared with the case  $\psi = 0$ , firms in region 1 (the hub) take advantage of the trade opening towards region 3 (one of the spokes), this leads to higher indirect utilities for the entrepreneurs located in 1 and to the progressive enlargement of the basin of attraction of  $CP_1$  and the shrinking of that of  $CP_2$ . Then, by increasing  $\psi$ , an attracting and a saddle equilibria  $BA_1$  and  $BA'_1$  belonging to the border  $I_{b1}$  appear (via a ‘fold bifurcation’, see Fig. 14c where  $\psi = 0.62$ ). The fixed point  $BA_1$  is positioned in the partition  $S_1$ : the possibility of one-way trade from 3 to 1 allows for a long-run equilibrium where entrepreneurs are distributed between regions 1 and 3 and trade goes from 3 to 1 and from 1 to 2 (one-way trade to region 2 compensates for the stronger competition in the local market from firms located in 3, attracting some of the entrepreneurs to region 1).

If  $\psi$  is further increased, this leads to the appearance of a border attracting equilibrium  $BA_3 \in I_{b3}$  (via a ‘border transcritical bifurcation’ of  $CP_2$ , see Fig. 14d where  $\psi = 0.75$ ), which mirrors  $BA_1$ . Indeed, reducing trade costs between 1 and 3 makes regions 2 and 3 more symmetric (while partition  $S_3$  completely overlaps with  $S$ ): In  $BA_3$  (symmetrically with respect to  $BA_1$ ) entrepreneurs are distributed between regions 1 and 2 and trade goes from 2 to 1 and from 1 to 3.

By further increasing  $\psi$ , at first the couple  $BA_1 - BA'_1$  disappears (see Fig. 14e where  $\psi = 0.779$ ), and then the couple  $BA_3 - BA'_3$  disappears (both via a ‘fold bifurcation’). After these bifurcations the fixed point  $CP_1$  is a unique attractor of  $Z$  (see Fig. 14f where  $\psi = 0.8$ ). Due to the relatively small ratio  $L/E$  the agglomeration forces are much stronger than in the previous examples, thus due to the central position, region 1 attracts all entrepreneurs, with the exception of the initial conditions in the basin of attraction of  $BA_3$  in Fig. 14e or for all the initial conditions in the interior of  $S$  in Fig. 14f.

## 8 Final Remarks

In this paper we presented a basic analytic framework representing a small trade network whose main objectives were: (i) highlight how distance may affect the formation of trade links and their direction; (ii) examine the long-term consequences of trade integration on the emergence/disappearance of trade links and on the distribution of economic activities across space; (iii) explore how the spatial distribution of economic activities and the existence of trade links are interrelated. Given the large number of possible trade structures, we only considered three examples, representing three frequently realized patterns in the EU trade network (see Basile *et al.* (2016)) and we provided three respective models.

Some of our results are: For the first model, dealing with three autarkic regions, we found cases of coexistence of long-run equilibria which are absent in a two-region context (see Behrens (2004)); for the second model, when only region 1 and 2 trade with each other, we confirmed Behrens (2005) result that allowing for unilateral trade favours the region endowed with the higher initial

distribution of entrepreneurs, however given the presence of a third region, not necessarily all entrepreneurs agglomerate in the region with the better initial endowment. Finally, for the third model, we found that, notwithstanding the different geography assumed, for some parameter combinations, the result of Ago *et al.* (2006) according to which centrality could translate into a locational disadvantage, is confirmed.

**Acknowledgement.** This work has been prepared within the activities of the EU project COST Action IS1104 “The EU in the new complex geography of economic systems: models, tools and policy evaluation”. The authors are grateful for financial support.

## Appendix

For Model 3 the indirect utilities are defined as follows:

$$\begin{aligned} \text{if } \lambda_1 \geq \bar{A}(\lambda_2), \lambda_2 \leq 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 &= V_1^{bil}, V_2 = V_2^{bil}, V_3 = V_3^{bil}, \\ \text{if } \lambda_1 \geq \bar{A}(\lambda_2), \lambda_2 > 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 &= V_1^{bil, out}, V_2 = V_2^{bil, out}, V_3 = V_3^{bil, out}, \\ \text{if } \lambda_1 < \bar{A}(\lambda_2), \lambda_2 \leq 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 &= V_1^{bil, in}, V_2 = V_2^{bil, in}, V_3 = V_3^{bil, in}, \\ \text{if } \lambda_1 < \bar{A}(\lambda_2), \lambda_2 > 1 - \bar{\lambda} - \lambda_1, \text{ then } V_1 &= V_1^{bil, bil}, V_2 = V_2^{bil, bil}, V_3 = V_3^{bil, bil}, \end{aligned}$$

where

$$\begin{aligned} \bar{\lambda} &= \frac{2(a - \eta b - bT_{13})}{cET_{13}}, \quad \bar{A}(\lambda_2) = \bar{\lambda} - \frac{T_{13} - T_{12}}{T_{13}}\lambda_2, \\ V_1^{bil} &= (\theta_1 T_{12} + \psi_1)T_{12} + \omega + \bar{C}_A, \quad V_2^{bil} = (\theta_2 T_{12} + \psi_2)T_{12} + \omega + \bar{C}_A, \\ \theta_1 &= \frac{(b + cE) \left( 4 [c\lambda_2 E(2b + c\lambda_2 E) + 2b^2] L + 3\lambda_2 E \Delta_1 \right)}{24 [2b + cE(\lambda_1 + \lambda_2)]^2}, \\ \Delta_1 &= cE[cE(\lambda_1 + \lambda_2)(\lambda_1 + 2\lambda_2) + 4b(\lambda_1 + 3\lambda_2)] + 12b^2, \\ \psi_1 &= -\frac{(a - \eta b)(b + cE) [3\lambda_2 E(3b + 2c\lambda_2 E) + 2bL]}{3 [2b + cE(\lambda_1 + \lambda_2)]^2}, \\ \psi_2 &= -\frac{(a - \eta b)(b + cE) [3\lambda_1 E(3b + 2c\lambda_1 E) + 2bL]}{3 [2b + cE(\lambda_1 + \lambda_2)]^2}, \\ \omega &= \frac{(a - \eta b)^2(b + cE) (4bL + 3E(\lambda_1 + \lambda_2)[3b + cE(\lambda_1 + \lambda_2)])}{6b [2b + cE(\lambda_1 + \lambda_2)]^2}, \\ \theta_2 &= \frac{(b + cE) \left( 4 [c\lambda_1 E(2b + c\lambda_1 E) + 2b^2] L + 3\lambda_1 E \Delta_2 \right)}{24 [2b + cE(\lambda_1 + \lambda_2)]^2}, \\ \Delta_2 &= cE[cE(\lambda_1 + \lambda_2)(\lambda_2 + 2\lambda_1) + 4b(\lambda_2 + 3\lambda_1)] + 12b^2, \\ V_3^{bil} &= V_3^{no} = \frac{(a - \eta)^2(b + cE) [3\lambda_3 E(3b + c\lambda_3 E) + 2bL]}{6b(2b + c\lambda_3 E)^2} + \bar{C}_A, \\ V_1^{bil, out} &= V_1^{bil} + \Pi_{13}^{out}, \end{aligned}$$

$$\Pi_{13}^{out} = (b + cE) \left[ \frac{a - \eta b - [cE(1 - \lambda_1 - \lambda_2) + 2b] \frac{T_{13}}{2}}{2b + c(1 - \lambda_2)E} \right]^2 \left[ \frac{L}{3} + (1 - \lambda_1 - \lambda_2)E \right],$$

$$V_2^{bil, out} = V_2^{bil},$$

$$V_3^{bil, out} = (f_3 T_{13} + g_3) T_{13} + h_3 + \overline{C}_A,$$

$$f_3 = \frac{\lambda_1 E(b + cE) (cE [12b(1 - \lambda_2) + 2c\lambda_1 L + 3cE(1 - \lambda_1 - \lambda_2)(1 + 2\lambda_1 - \lambda_2)] + 12b^2)}{24 [2b + cE(1 - \lambda_2)]^2},$$

$$g_3 = \frac{3\lambda_1 E(a - \eta b)(b + cE)(c\frac{L}{3} - b - c\lambda_1 E)}{3 [2b + cE(1 - \lambda_2)]^2},$$

$$h_3 = \frac{3(a - \eta b)^2(b + cE) (cE^2(1 - \lambda_2)^2 + b[2\frac{L}{3} + E(3 - 2\lambda_1 - 3\lambda_2)])}{6b [2b + cE(1 - \lambda_2)]^2},$$

$$V_1^{bil, in} = S_1^{bil, in} + \Pi_{11}^{bil, in} + \Pi_{12}^{bil, in} + \overline{C}_A,$$

$$S_1^{bil, in} = \frac{\lambda_2 Q_1 T_{12} + Q_2(1 - \lambda_1 - \lambda_2) T_{13} + Q_3}{8(2b + cE)^2} (b + cE)E,$$

$$Q_1 = T_{12} (cE[cE(1 - \lambda_2) + 4b] + 4b^2) - 8(b + cE)(a - \eta b) - 2c^2 E^2(1 - \lambda_1 - \lambda_2) T_{13},$$

$$Q_2 = [c^2 E^2(\lambda_1 + \lambda_2) + 4b(b + cE)] T_{13} - 8(b + cE)(a - \eta b), \quad Q_3 = \frac{4(a - \eta b)^2(b + cE)}{b},$$

$$\Pi_{11}^{bil, in} = (b + cE) \left[ \frac{a - \eta b + [(1 - \lambda_1 - \lambda_2) \frac{T_{13}}{2} + \frac{T_{12}}{2} \lambda_2] cE}{2b + cE} \right]^2 \left( \frac{L}{3} + \lambda_1 E \right),$$

$$\Pi_{12}^{bil, in} = (b + cE) \left[ \frac{a - \eta b - \frac{T_{12}}{2} (2b + c\lambda_2 E)}{2b + c(\lambda_1 + \lambda_2)E} \right]^2 \left( \frac{L}{3} + \lambda_2 E \right),$$

$$V_2^{bil, in} = S_2^{bil} + \Pi_{21}^{bil, in} + \Pi_{22}^{bil} + \overline{C}_A,$$

$$S_2^{bil} = \frac{\lambda_1 \Omega T_{12} + \Psi}{8[2b + cE(\lambda_1 + \lambda_2)]^2} (b + cE)E,$$

$$\Omega = T_{12} \{ cE(\lambda_1 + \lambda_2)[cE\lambda_2 + 4b] + 4b^2 \} - 8[b + cE(\lambda_1 + \lambda_2)](a - \eta b),$$

$$\Psi = \frac{4(a - \eta b)^2(\lambda_1 + \lambda_2)[b + cE(\lambda_1 + \lambda_2)]}{b},$$

$$\Pi_{21}^{bil, in} = (b + cE) \left[ \frac{a - \eta b + cE(1 - \lambda_1 - \lambda_2) \frac{T_{13}}{2} - \frac{T_{12}}{2} [cE(1 - \lambda_2) + 2b]}{2b + cE} \right]^2 \left( \frac{L}{3} + \lambda_1 E \right),$$

$$\Pi_{22}^{bil} = (b + cE) \left[ \frac{a - \eta b + \frac{T_{12}}{2} c\lambda_1 E}{2b + c(\lambda_1 + \lambda_2)E} \right]^2 \left( \frac{L}{3} + \lambda_2 E \right),$$

$$\begin{aligned}
 V_3^{bil, in} &= V_3^{no} + \Pi_{31}^{bil, in}, \\
 \Pi_{31}^{bil, in} &= (b + cE) \left[ \frac{a - \eta b + cE\lambda_2 \frac{T_{12}}{2} - \frac{T_{13}}{2} [cE(\lambda_1 + \lambda_2) + 2b]}{2b + cE} \right]^2 \left( \frac{L}{3} + \lambda_1 E \right), \\
 V_1^{bil, bil} &= V_1^{bil, in} + \Pi_{13}^{out}, \quad V_2^{bil, bil} = V_2^{bil, in}, \quad V_3^{bil, bil} = V_3^{bil, out} + \Pi_{31}^{bil, in}.
 \end{aligned}$$

## References

- Ago, T., Isono, I., Tabuchi, T.: Locational disadvantage of the hub. *Ann. Reg. Sci.* **40**(4), 819–848 (2006)
- Baldwin, R.E., Forslid, R., Martin, P., Ottaviano, G.I.P., Robert-Nicoud, F.: *Economic Geography and Public Policy*. Princeton University Press, Princeton (2003)
- Basile, R., Commendatore, P., De Benedictis, L., Kubin, I.: An Investigation of Interregional Trade Network Structures. In: Commendatore, P., Matilla-Garcia, M., Varela, L.M., Canovas, J. (eds.) *Complex Networks and Dynamics. Social Economic Interactions*, pp. 123–172. Springer (2016)
- Behrens, K.: Agglomeration without trade: how non-traded goods shape the space economy. *J. Urban Econ.* **55**, 68–92 (2004)
- Behrens, K.: How endogenous asymmetries in interregional market access trigger regional divergence. *Reg. Sci. Urban Econ.* **35**, 471–492 (2005)
- Behrens, K.: International integration and regional inequalities: how important is national infrastructure? *Manchester Sch.* **79**(5), 952–971 (2011)
- Commendatore, P., Kubin I., Sushko, I.: Trade agreements in a linear FE model. In: Bednar-Friedl, B., Kleinert, J. (eds.) *Dynamic Approaches to Global Economic Challenges. Festschrift in Honor of Karl Farmer*. Springer International Publishing (2016)
- Commendatore, P., Kubin, I., Sushko, I.: The dynamics of a three-region linear NEG model. *Communications in Nonlinear Science and Numerical Simulation* (2017, to appear)
- Fujita, M., Krugman, P., Venables, A.J.: *The Spatial Economy Cities, Regions, and International Trade*. The MIT Press, Cambridge (1999)
- Furusawa, T., Konishi, H.: Free trade networks. *J. Int. Econ.* **72**(2), 310–335 (2007)
- Krugman, P.R.: Increasing returns and economic geography. *J. Polit. Econ.* **99**, 483–499 (1991)
- Okubo, T., Picard, P.M., Thisse, J.F.: On the impact of competition on trade and firm location. *J. Reg. Sci.* **54**(5), 731–754 (2014)
- Ottaviano, G.I., Tabuchi, T., Thisse, J.F.: Agglomeration and trade revisited. *Int. Econ. Rev.* **43**, 409–436 (2002)
- Tabuchi, T., Thisse, J.F., Zeng, D.Z.: On the number and size of cities. *J. Econ. Geogr.* **5**(4), 423–448 (2005)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# Advances in Spatial Econometrics: Parametric vs. Semiparametric Spatial Autoregressive Models

Roberto Basile<sup>1(✉)</sup> and Román Mínguez<sup>2</sup>

<sup>1</sup> Department of Economics, Università della Campania “Luigi Vanvitelli”,  
Corso Gran Priorato di Malta, 1, 81043 Capua, CE, Italy  
`roberto.basile@unina2.it`

<sup>2</sup> Universidad de Castilla-La Mancha, Cuenca, Spain  
`roman.minguez@uclm.es`

**Abstract.** In this Chapter we provide a critical review of parametric and semi-parametric spatial econometric approaches. We focus on the capability of each class of models to fit the main features of spatial data (such as strong and weak cross-sectional dependence, spatial heterogeneity, nonlinearities, and time persistence), leaving aside the technicalities related to the estimation methods. We also provide a brief discussion of the existent software developed to estimate most of the econometric models exposed in this Chapter.

**Keywords:** Spatial econometrics · Semiparametric models

## 1 Introduction and Motivation

Nowadays, the dominant paradigm in spatial econometrics is still a parametric one. The *first generation* of spatial econometric models (essentially developed to handle cross-sectional data) focused on modeling spatial dependence (or spatial spillover effects) through different alternative linear specifications, such as the Spatial Lag or Spatial Autoregressive Model (SAR), the Spatial Error Model (SEM), the Spatial Durbin Model (SDM), the Spatial Autoregressive in  $X$ -variables Model (SLX), and a mix of SAR and SEM (SAC or SARAR) (Anselin 1988; LeSage and Pace 2009). We may call this collection of econometric tools as “econometrics of interaction”, since they can be applied to any kind of network relationship among different sample units.

During the last decade, these models have been extended to handle spatial panel data (or spatio-temporal data), that is data containing time series observations of a number of geographical units. Elhorst (2014b) defines them *second generation* spatial econometric models. By including a regional specific fixed or random effect, these models prove to be particularly useful to control for unobserved spatial heterogeneity, that is a fundamental task in empirical economic analyses, as failing to do so can introduce omitted-variable biases and preclude causal inference. Moreover, spatial dependence may simply be the consequence of (spatially correlated) omitted variables rather than being the result of spillovers. Thus, controlling both for spatial dependence (through

spatial lag terms) and spatial heterogeneity (through fixed or random effects) is a primary task when dealing with spatial data. More recent developments concern dynamic spatial panel data models and spatial VAR (Vector-Autoregressive) models, which allow to control for time persistence and reverse causality problems.

Notwithstanding these important advances in the literature, it is worth noting that any parametric model is limited to specific forms of spatial variation of the parameters, such as spatial regimes. They are not suitable for more general forms of spatial heterogeneity of model parameters, i.e. when the variation of parameters is continuous (smooth) over space and depends on coordinates, and when the functional form of the relationship between the dependent variable and the regressor is unknown (potentially non-monotonic). Moving away from the parametric approach, another strand of the spatial econometric literature has proposed semiparametric methods as more flexible estimation frameworks, thus following the recommendations of McMillen (2012) of using smoother techniques in order to remove spatial heterogeneity while considering other potential nonlinearities.

First, following Brunsdon et al. (1996); Cho et al. (2010) have proposed an approach that combines geographically weighted regression (GWR) and spatial autoregression (SEM) methods, called GWR-SEM. The spatial autoregressive error term should allay spatial dependency, while GWR addresses spatial heterogeneity by allowing the coefficients to vary across observations. In the same vein, Páez et al. (2002) propose an estimation method for cross-sectional data in which the covariance is locally varying and that can handle spatial autocorrelation of the error terms. Another notable contribution accounting for both spatial autocorrelation and nonstationarity of the parameters has been made by Pace and LeSage (2004): they propose a spatial autoregressive local estimation based on a recursive approach for maximum-likelihood estimation of SAR that implies estimates on subsamples related to a neighboring of each observation. More recently, combining kernel smoothing methods and standard spatial lag models, Geniaux and Martinetti (2017) have introduced a new class of data generating processes, called MGWR-SAR (Mixed Geographically Weighted Regression Simultaneous Autoregressive Model), in which the regression parameters and the spatial dependence coefficient can vary over space. The advantage of the last class of models is that it allows to consider the mixed case in which some parameters are constant over space and others are spatially varying.

Second, Basile et al. (2014); Montero et al. (2012) have combined penalized regression spline (PS) methods (Eilers et al. 2015) with standard cross-section spatial autoregressive models (such as SAR, SEM, SDM and SLX). An important feature of PS-SAR, PS-SEM, PS-SDM and PS-SLX models is the possibility to include within the same specification (i) spatial autoregressive terms to capture spatial interaction or network effects (thus avoiding *spatial dependence bias*), (ii) parametric and nonparametric (smooth) terms to identify nonlinear relationships between the response variable and the covariates (thus avoiding *functional form bias*), (iii) a geoadditive term, that is a smooth function of the spatial coordinates, to capture a spatial trend effect, that is to capture spatially autocorrelated unobserved heterogeneity (thus avoiding *spatial heterogeneity bias*), and (iv) the interaction between the geoadditive term and a covariate of particular interest to identify spatially varying effects of *X*-variables.



Third, Mínguez et al. (2017) have proposed an extension of the PS-SAR to spatio-temporal data when both a large cross-section and a large time series dimensions are available. With this kind of data it is possible to estimate not only spatial trends, but also spatio-temporal trends in a nonparametric way (Lee and Durbán 2011), so as to capture region-specific nonlinear time trends net of the effect of spatial autocorrelation. In other words, this approach allows to answer questions like: How do unobserved time-related factors (i.e. common factors), such as economic-wide technological or demand shocks, heterogeneously affect long term dynamics of all units in the sample? And how does their inclusion in the model affect the estimation of spatial interaction effects? In this sense, the PS-SAR model with spatio-temporal trend represents an alternative to parametric methods aimed at disentangling *common factors* effects (such as common business cycle effects) and *spatial dependence* effects (local interactions between spatial units generating spillover effects), where the former is sometimes regarded as ‘strong’ cross-sectional dependence, and the latter as ‘weak’ cross-sectional dependence (Chudik et al. 2011).

In this paper, we propose a critical review of parametric and semiparametric spatial econometric approaches trying to highlight their pros and cons. We will focus on the capability of each class of models to fit the main features of spatial data (such as strong and weak spatial dependence, spatial heterogeneity, nonlinearities, and time persistence) leaving the estimation techniques on backstage. The plan of the paper is as follows. Section 2 summarizes the huge literature on parametric spatial autoregressive models. Section 3 is dedicated to the broad category of semiparametric spatial autoregressive models, disentangling GWR (or MGWR) models based on kernel methods and models based on penalized spline smoothers. Section 4 provides a brief discussion of the software available for the practitioners to apply all these models. Finally, Sect. 5 concludes.

## 2 Parametric Spatial Autoregressive Models

### 2.1 Modeling Spatial Interaction Effects: Spatial Autoregressive Models for Cross-Sectional Data

Unlike time dependence, spatial dependence is a difficult concept to grasp, some people find. Let us start from a generic notion of “*interdependence*” and, then, return to the specific concept of spatial dependence. To introduce the concept of “*interdependence*”, let us consider a simple example. Imagine we want to model the scientific productivity ( $SP$ ) of a sample of researchers connected among each other in a network of co-authorships.  $SP$  can be measured, for example, in terms of number of publications or better in terms of a continuous outcome variable such as an evaluation score whose distribution is assumed to be normal. For simplicity, we assume that this score depends only on investments in human capital (such as number of books read, number of new courses attended, number and length of academic visits abroad, and so on). To model  $y_i = SP_i$  for each individual researcher  $i$ , we start from the classical linear regression model:

$$y_i = \alpha + \sum_k \beta_k x_{ik} + \varepsilon_i \quad i = 1, \dots, N \quad \varepsilon_i \sim iid \mathcal{N}(0, \sigma_\varepsilon^2) \quad (1)$$

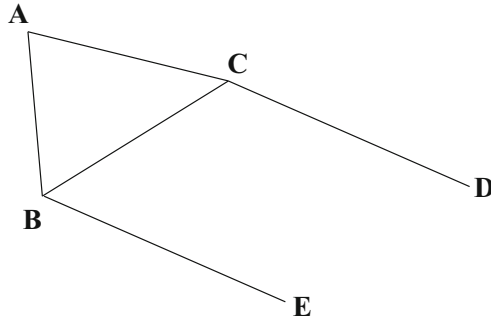
where  $x_{ik}$  indicates a measure of human capital investment. This model imposes a strong assumption of independence. First, the assumptions on the error term ( $\varepsilon_i$ ) exclude any type of covariance. Second, the partial derivatives exclude any kind of indirect (interaction or spillover) effect, i.e. an investment in human capital by a researcher  $i$  will affect only his/her own scientific productivity ( $y_i$ ), but not the productivity of any other researcher ( $y_j$ ):

$$\partial E[y_i]/\partial x_{ik} = \hat{\beta}_k \quad \partial E[y_j]/\partial x_{ik} = \partial E[y_i]/\partial x_{jk} = 0 \quad i, j = 1, \dots, N$$

We can write this model in matrix form as

$$\mathbf{y} = \mathbf{I}_N \alpha + \mathbf{X} \beta + \varepsilon \quad E[\varepsilon] = 0 \quad E[\varepsilon \varepsilon'] = \sigma^2 \mathbf{I}_N \quad (2)$$

The independence assumption is quite unrealistic, however. In fact, we cannot evaluate the scientific performance of this sample of individuals without taking into account the possibility of knowledge spillovers among them. Suppose that our sample is composed of only five researchers (identified by the letters A, B, C, D, E). Scientific collaborations (co-authorship relations) will determine a *network* or *connectivity scheme* such as the one shown in Fig. 1:



**Fig. 1.** A network scheme of scientific collaborations (co-authorship relations)

Researcher A has a co-authorship (that is a direct link) only with individuals B and C. Researcher B has a co-authorship only with individuals A, C and E; and so on. This network scheme can be translated into a symmetric  $5 \times 5$  binary matrix  $\mathbf{W}^*$ :

$$\mathbf{W}^* = \begin{bmatrix} & A & B & C & D & E \\ A & 0 & 1 & 1 & 0 & 0 \\ B & 1 & 0 & 1 & 0 & 1 \\ C & 1 & 1 & 0 & 1 & 0 \\ D & 0 & 0 & 1 & 0 & 0 \\ E & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

with  $w_{ij}^* = 1$  if  $i$  and  $j$  are classified as co-authors, and  $w_{ij}^* = 0$  otherwise. This binary matrix can be row-standardized so as  $w_{ij} = w_{ij}^* / \sum_j w_{ij}^*$  s.t.  $\sum_j w_{ij}^* = 1$ :

$$\mathbf{W} = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Now, we can multiply  $\mathbf{W}$  by the vector  $\mathbf{y}$ :

$$\mathbf{W}\mathbf{y} = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1/2y_2 + 1/2y_3 \\ 1/3y_1 + 1/3y_3 + 1/3y_5 \\ 1/3y_1 + 1/3y_2 + 1/3y_4 \\ y_3 \\ y_2 \end{bmatrix}$$

Each element of the vector  $\mathbf{W}\mathbf{y}$  measures the weighted average of the scientific productivity of the co-authors of each individual. We can also compute  $\mathbf{W}\mathbf{X}$  and  $\mathbf{W}\boldsymbol{\varepsilon}$ . These three terms can be used to extend model (2). For example, we can include  $\mathbf{W}\mathbf{y}$  on the r.h.s. of (2):

$$\mathbf{y} = \mathbf{I}_N \boldsymbol{\alpha} + \rho \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad \boldsymbol{\varepsilon} \sim iid \mathcal{N}(0, \sigma_{\boldsymbol{\varepsilon}}^2 \mathbf{I}_N) \quad (3)$$

The reduced form of this model is:

$$\mathbf{y} = (\mathbf{I}_N - \rho \mathbf{W})^{-1} (\mathbf{I}_N \boldsymbol{\alpha} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}) \quad (\mathbf{I}_N - \rho \mathbf{W})^{-1} = \mathbf{I}_N + \rho \mathbf{W} + \rho^2 \mathbf{W}^2 + \dots$$

To ensure that  $\mathbf{I}_N - \rho \mathbf{W}$  is invertible, one needs to impose some restrictions on the parameter  $\rho$ , which for a row-normalized interaction matrix  $\mathbf{W}$  correspond to take use of a compact set of  $(1/\omega_{min}, 1)$ , where  $\omega_{min}$  is the minimum eigenvalue of  $\mathbf{W}$  matrix. Once this restriction is satisfied, using the estimated parameters of the model ( $\hat{\rho}$  and  $\hat{\boldsymbol{\beta}}_k$ ), we can compute the impacts of a change in the  $k$ -th explanatory variable, i.e. the partial derivatives of the expected value of the dependent variable  $\mathbf{y}$  with respect to the concerned variable,  $\mathbf{x}_k$ :

$$\Xi_y^{x_k} = \partial E[\mathbf{y}] / \partial \mathbf{x}_k = (\mathbf{I}_N - \hat{\rho} \mathbf{W})^{-1} \hat{\boldsymbol{\beta}}_k \quad (4)$$

Unlike to what we observe for the traditional classical linear regression model, diagonal elements of (4) are different from each other, off diagonal elements differ from zero and the matrix itself is not symmetric. In particular, diagonal elements of (4) represent own-partial derivatives, meaning the impact of a change in the  $k$ -th variable in unit  $i$  on the expected value of the dependent variable in this unit. They are formally written as

$$\partial E[y_i] / \partial x_{ik} = [\Xi_y^{x_k}]_{ii} \quad i = 1, \dots, N \quad (5)$$

These own-partial derivatives are labeled *direct impacts* and include feedback loop effects that arise as a result of impacts passing through interacting units  $j$  and back

to unit  $i$ . As the set of interacting units is different for each unit, the feedback will be heterogeneous by nature, giving birth to the notion of *interactive heterogeneity*. This interactive heterogeneity should not be confused with *parameter heterogeneity*, which refers to instability of parameters (structural breaks, clubs) or heteroskedasticity.

Off-diagonal elements of (4) represent the effects of a change in the  $k$ -th explanatory variable in unit  $j$  on the dependent variable in unit  $i$ . As matrix (4) is asymmetric, this further imply that this impact will not be the same as the one caused by a change in unit  $i$  on unit  $j$ . Formally,

$$\partial E[y_i] / \partial x_{jk} = [\Xi_y^{x_k}]_{ij} \neq \partial E[y_j] / \partial x_{ik} = [\Xi_y^{x_k}]_{ji} \quad (6)$$

These cross-derivative elements are thus labeled *indirect effects*. Using expressions (5) and (6), we can for example say that an investment in human capital by individual A (i.e. an idiosyncratic shock in a  $x_k$  variable) will affect not only the scientific productivity of A (*direct effect*), but also the scientific productivity of his/her own co-authors (individual A will transmit part of the new knowledge to his/her own co-authors B and C), the co-authors of his/her co-authors and so on (*spillover* or *indirect effect*). Thus, we can say that there is a global diffusion of the idiosyncratic shock. Given the stability condition  $|\rho| < 1$ , the intensity of these knowledge spillovers decreases with the increase in the order of co-authorship relations. Since the matrix  $(\mathbf{I}_N - \hat{\rho}\mathbf{W})^{-1}$  pre-multiplies also the error term, we can also say that there is a global diffusion of shocks in the unobserved term.

Eventually, we may introduce both  $\mathbf{W}\mathbf{y}$  and  $\mathbf{W}\mathbf{X}$  on the r.h.s. of Eq. (1):

$$\mathbf{y} = \iota_N \alpha + \rho \mathbf{W}\mathbf{y} + \mathbf{X}\beta + \mathbf{W}\mathbf{X}\delta + \varepsilon \quad \varepsilon \sim iid.\mathcal{N}(0, \sigma_\varepsilon^2 \mathbf{I}_N) \quad (7)$$

Again, the reduced form of this model implies a global diffusion of both observed and unobserved shocks. The matrix of partial derivatives of  $\mathbf{y}$  with respect to the  $k$ -th explanatory variable, presented in (8) and computed from the reduced form of model (7), contains the additional term  $\mathbf{W}\delta_k$ .

$$\Xi_y^{x_k} = \partial E[\mathbf{y}] / \partial \mathbf{x}_k = (\mathbf{I}_N - \hat{\rho}\mathbf{W})^{-1} (\mathbf{I}_N \hat{\beta}_k + \mathbf{W}\hat{\delta}_k) \quad (8)$$

Alternatively, we can leave the systematic part of model (2) unchanged and introduce the assumption of spatial autocorrelation in the error term:

$$\begin{aligned} \mathbf{y} &= \iota_N \alpha + \mathbf{X}\beta + \varepsilon & \varepsilon &= \lambda \mathbf{W}\varepsilon + \mathbf{u} \\ |\lambda| < 1 & & \mathbf{u} &\sim iid.\mathcal{N}(0, \sigma_u^2 \mathbf{I}_N) \end{aligned} \quad (9)$$

The reduced form of this model

$$\mathbf{y} = \iota_N \alpha + \mathbf{X}\beta + (\mathbf{I}_N - \lambda \mathbf{W})^{-1} \mathbf{u}$$

implies a global diffusion of random shocks, but not spillovers of idiosyncratic shocks in an observed variable. Thus, using model (9), in our example, we would exclude knowledge spillovers from observed changes in human capital investments of researcher A; only spillovers from unobserved factors would take place.

Finally, we may extend model (2) by introducing on the r.h.s. only  $\mathbf{WX}$ :

$$\mathbf{y} = \iota_N \alpha + \mathbf{X}\beta + \mathbf{WX}\delta + \varepsilon \quad \varepsilon \sim iid \mathcal{N}(0, \sigma_\varepsilon^2 \mathbf{I}_N) \quad (10)$$

This model implies only *local spillovers*: an investment in new knowledge by individual A will spill over only to his/her own co-authors, and *vice-versa*.

Now, let's turn to a *spatial* context and imagine that the network structure depicted in Fig. 1 represents a *spatial* network, identifying direct neighborhood links (i.e. direct proximity relationships) between regions or firms in space. In spatial statistics and spatial econometrics,  $\mathbf{W}^*$  and  $\mathbf{W}$  are called the spatial weights matrix and the row-standardized spatial weights matrix, respectively.  $\mathbf{Wy}$  is called the spatial lag operator; it works to produce a weighted average of the neighboring observations. In spatial econometrics, model (3) is called the *Spatial Lag Model* or *Spatial Autoregressive Model* (SAR), model (7) is known as the *Spatial Durbin Model* (SDM), model (9) is known as the *Spatial Error Model* (SEM), and model (10) is known as the *Spatial in X-variable Model* (SLX). Each of them allows us to capture a different spatial spillover effect.

For example, using cross-regional data, one may estimate a SDM version of the so-called *knowledge production function*, according to which the knowledge produced in a region ( $K_i$ ) (approximated by the number of patents per capita or by the total factor productivity) is an increasing function of both internal and external cumulative research and development ( $R\&D_i$ ,  $\sum_{j \neq i} w_{ij} \ln R\&D_j$ ), and both internal and external human capital stocks:

$$\begin{aligned} \ln K_i = & \alpha + \beta_1 \ln R\&D_i + \beta_2 \sum_{j \neq i} w_{ij} \ln R\&D_j \\ & + \beta_3 \ln H_i + \beta_4 \sum_{j \neq i} w_{ij} \ln H_j + \rho \sum_{j \neq i} w_{ij} \ln K_j + \varepsilon_i \end{aligned} \quad (11)$$

Technological spillovers among regions may be assumed to be driven by inter-regional trade relations, as suggested by the endogenous growth theory. Thus, if inter-regional trade data are available for the regional sample used in the analysis, a researcher may use them to build a  $\mathbf{W}$  matrix. Alternatively, spatial proximity measures (such binary contiguity measures or inverse distance) can be used.

It is worth noticing that a more parsimonious version of (11) is often estimated, which imposes zero values to parameters  $\beta_4$  and  $\rho$ , thus assuming only local spillovers from R&D investments carried out by direct neighboring regions and excluding global spillovers captured by a spatial multiplier mechanism. A natural way to proceed is to estimate model (11) and then test these restrictions on parameters  $\beta_4$  and  $\rho$ .

The term  $\mathbf{Wy}$  that appears on the r.h.s. of (3) and (7) is correlated with the error term,  $\text{Cov}[\mathbf{Wy}; \varepsilon] \neq \mathbf{0}$ , so that ordinary least squares (OLS) estimates are biased and inconsistent. Consistent and efficient estimates can be obtained by maximum likelihood (ML) or quasi-maximum likelihood estimates (QML) (Lee 2004). Two-Stage Least Squares (2SLS) estimates adapt well to the case of (3) because higher orders of spatial lags of the  $\mathbf{X}$  variables are natural candidates to be used as instrumental variables (Kelejian and Prucha 1997). A more efficient estimator is the method of moments estimator (MM) (Kelejian and Prucha 2001). Lee (2004) generalized the MM approach into a

fully generalized method of moments (GMM) estimator for the case of the SDM model (7), while Liu et al. (2007) proposed a GMM estimator for a SDM with dependent structures in the error term. The GMM estimator may have, under general conditions, the same limiting distribution as the ML or QML estimators. Moreover, the 2SLS and the GMM estimators allow the researcher to take into account any endogeneity problems in the r.h.s., different from the spatial lag of  $\mathbf{y}$ .

As mentioned above, direct, indirect and total marginal effects change across spatial units. Specifically, they depend on the specific position of the region within the spatial proximity network. Thus, in order to summarize the results, it could be easier to compute average measures of direct, indirect and total effects. In the case of Eq. 3, the average total marginal effect is computed as  $N^{-1}\mathbf{i}'_N \left[ (\mathbf{I}_N - \rho \mathbf{W})^{-1} \mathbf{I}_N \beta_k \right] \mathbf{i}_N$  (see Table 1). The average direct impact is  $N^{-1}tr \left[ (\mathbf{I}_N - \rho \mathbf{W})^{-1} \mathbf{I}_N \beta_k \right]$ , while the average indirect (spatial spillover) impact is the difference between average total and average indirect effects. In order to draw inference regarding the statistical significance of average direct and indirect effects, LeSage and Pace (2009, p. 39) suggest simulating the distribution of these effects using the variance-covariance matrix implied by the ML estimates. Efficient simulation approaches can be used to produce an empirical distribution of the parameters  $\alpha, \beta, \theta, \rho, \sigma^2$  that are needed to calculate the scalar summary measures. This distribution can be constructed using a large number of simulated parameters drawn from the multivariate distribution of the parameters implied by the ML estimates.

## 2.2 Modeling Spatial Spillovers and Unobserved Spatial Heterogeneity: Spatial Autoregressive Models for Panel Data

### 2.2.1 Static Spatial Panel Data Models

Recently, spatial econometric models have been extended to deal with spatial panel data, that is data with both a spatial and a temporal dimension (Elhorst 2014b). The two-dimensional structure of the data allows us to control for unobserved spatial and time heterogeneity by including individual (spatial) and time effects on the r.h.s. of the model. Thus, for example, the static panel data SAR model can be written in vector form for a cross-section of observations at time  $t$  ( $t = 1, 2, \dots, T$ ) as:

**Table 1.** Average total (ATE), direct (ADE), and indirect (AIE) marginal effects

Model	ADE	AIE	ATE
Linear	$\beta_k$	0	$\beta_k$
SAR	$\frac{tr \left[ (\mathbf{I}_N - \rho \mathbf{W})^{-1} \mathbf{I}_N \beta_k \right]}{N}$	Total-Direct	$\frac{\mathbf{i}'_N \left[ (\mathbf{I}_N - \rho \mathbf{W})^{-1} \mathbf{I}_N \beta_k \right] \mathbf{i}_N}{N}$
SEM	$\beta_k$	0	$\beta_k$
SDM	$\frac{tr \left[ (\mathbf{I}_N - \rho \mathbf{W})^{-1} (\mathbf{I}_N \beta_k + \mathbf{W} \theta_k) \right]}{N}$	Total-Direct	$\frac{\mathbf{i}'_N \left[ (\mathbf{I}_N - \rho \mathbf{W})^{-1} (\mathbf{I}_N \beta_k + \mathbf{W} \theta_k) \right] \mathbf{i}_N}{N}$
SLX	$\beta_k$	$\theta_k$	$\beta_k + \theta_k$

$$\begin{aligned} \mathbf{y}_t &= \rho \mathbf{W} \mathbf{y}_t + \alpha + \iota_N \tau_t + \mathbf{X}_t \beta + \varepsilon_t \\ E(\varepsilon_t) &= 0 \quad E(\varepsilon_t \varepsilon_t) = \sigma^2 \mathbf{I}_N \end{aligned} \quad (12)$$

where, again,  $\mathbf{W}$  is a row-standardized  $N \times N$  spatial weights matrix whose diagonal elements  $w_{ii}$  are 0;  $\rho$  is the spatial spillover parameter satisfying the usual stability conditions, and  $\rho \sum_{j=1}^N w_{ij} y_{jt}$  captures the spatial spillover effects net of the unobserved heterogeneity effects filtered out by the spatial fixed effects,  $\alpha_i$ , and time fixed effects,  $\tau_t$ .

Similarly, the static panel SEM can be expressed as:

$$\begin{aligned} \mathbf{y}_t &= \alpha + \iota_N \tau_t + \mathbf{X}_t \beta + \varphi_t \quad \varphi_t = \lambda \mathbf{W} \varphi_t + \varepsilon_t \\ E(\varepsilon_t) &= 0 \quad E(\varepsilon_t \varepsilon_t) = \sigma^2 \mathbf{I}_N \end{aligned} \quad (13)$$

And, the static panel data SDM as:

$$\begin{aligned} \mathbf{y}_t &= \rho \mathbf{W} \mathbf{y}_t + \alpha + \iota_N \tau_t + \mathbf{X}_t \beta + \mathbf{W} \mathbf{X}_t \theta + \varepsilon_t \\ E(\varepsilon_t) &= 0 \quad E(\varepsilon_t \varepsilon_t) = \sigma^2 \mathbf{I}_N \end{aligned} \quad (14)$$

For example, the static panel version of the spatial Durbin knowledge production function (11) reads as:

$$\begin{aligned} \ln K_{it} &= \beta_1 \ln R \& D_{it} + \beta_2 \sum_{j \neq i} w_{ij} \ln R \& D_{jt} \\ &+ \beta_3 \ln H_{it} + \beta_4 \sum_{j \neq i} w_{ij} \ln H_{jt} + \rho \sum_{j \neq i} w_{ij} \ln K_{jt} + \alpha_i + \tau_t + \varepsilon_{it} \end{aligned} \quad (15)$$

Depending on the assumptions about individual and time effects, these models will be estimated using *fixed effects* (FE) or *random effects* (RE). The latter, more efficient, is adequate when the effects (individual and temporal) are independent from all regressors included in the specification and are traditionally assumed normally distributed. When this hypothesis of independence is rejected, either on the basis of a test statistic (Hausman, Lagrange multiplier (LM) or likelihood ratio (LR)) or from economic insights, the fixed effects specification should be preferred. Even though these two estimation procedures are different, they both consist in first transforming the data (either applying the within operator for the fixed effects or a quasi-within transformation when the random effects estimation is used) and then applying standard spatial econometrics techniques (for example, the QML estimator; Lee and Yu 2010a) on these transformed data to obtain the estimated parameters.

It should be stressed that the spatial fixed effects can only be estimated consistently when  $T$  is sufficiently large, because the number of observations available for the estimation of each  $\hat{\alpha}_i$  is  $T$ . Importantly, sampling more observations in the cross-sectional domain is not a solution for insufficient observations in the time domain, since the number of unknown parameters increases as  $N$  increases, a situation known as the *incidental parameters problem*. Fortunately, the inconsistency of  $\hat{\alpha}_i$  is not transmitted to the estimator of the slope coefficients  $\hat{\beta}$  in the demeaned equation, since this estimator is not a function of the estimated  $\hat{\alpha}_i$ . Consequently, the incidental parameters problem does

not matter when  $\hat{\beta}$  are the coefficients of interest and the spatial fixed effects  $\hat{\alpha}_i$  are not, which is the case in many empirical studies.

Finally, it is important to recognize that, apart from the control for unobserved heterogeneity, the economic interpretation of static spatial autoregressive models is the same as the one for cross-sectional data. Impacts measures implied by a spatial static panel data model are indeed the same as those in a spatial autoregressive model for cross-sectional data, as soon as the interaction matrix and the parameters of interest of the former are assumed constant across time. Different is the case of spatial dynamic panel data models, which give rise to the possibility of evaluating the effects of transitory and permanent shocks both in the short-run and in the long-run equilibrium.

### 2.2.2 Dynamic Spatial Panel Data Models

In order to simultaneously deal with time persistence and spatial interdependence along with spatial and temporal heterogeneity, a dynamic spatial panel data model with fixed spatial and time effects is needed. The spatial econometric literature provides several alternative specifications of spatial dynamic models. A very general one includes time lags of both the dependent and independent variables, contemporaneous spatial lags of both, and lagged spatial lags of both. However, as Elhorst (2014b) have pointed out, this generalized model suffers from identification problems, and is thus not useful for empirical research. A more parsimonious model (written in vector form for a cross-section of observations at time  $t$ ) can be expressed as:

$$\mathbf{y}_t = \tau \mathbf{y}_{t-1} + \rho \mathbf{W} \mathbf{y}_t + \eta \mathbf{W} \mathbf{y}_{t-1} + \mathbf{X}_t \beta + \mathbf{W} \mathbf{X}_t \theta + \alpha + \lambda_t \mathbf{1}_N + \varepsilon_t \quad (16)$$

$$\varepsilon_t \sim iid \mathcal{N}(0, \sigma_\varepsilon^2 \mathbf{I}_N)$$

Lee and Yu (2010b); Yu et al. (2008) have proposed bias corrected QML estimators for a dynamic model with spatial and time fixed effects. However, these estimators are based on the assumption of only exogenous covariates except for the time and spatial lag terms. Kukenova and Monteiro (2008) have suggested to use System-GMM estimator Blundell and Bond (1998) for dynamic spatial panel model with several endogenous variables. More specifically, they have investigated the finite sample properties of different estimators for spatial dynamic panel models (namely, spatial ML, spatial dynamic ML, least-square-dummy-variable, Diff-GMM and System-GMM) and concluded that, in order to account for the endogeneity of several covariates, spatial dynamic panel models should be estimated using System-GMM.

The stationarity conditions on the spatial and temporal parameters in a dynamic spatial panel data model like (16) go beyond the standard condition  $|\tau| < 1$  in serial models, and the standard condition  $1/\omega_{min} < \rho < 1$  in spatial models. Indeed, to achieve stationarity in the dynamic spatial panel data model (16), the characteristic roots of the matrix  $(\mathbf{I}_N - \rho \mathbf{W})^{-1}(\tau \mathbf{I}_N + \eta \mathbf{W})$  should lie within the unit circle (Debarsy et al. 2012) which is the case when

$$\begin{aligned} \tau + (\rho + \eta)\omega_{max} &< 1 & \text{if } \rho + \eta &\geq 0 \\ \tau + (\rho + \eta)\omega_{min} &< 1 & \text{if } \rho + \eta < 0 \\ \tau - (\rho - \eta)\omega_{max} &> -1 & \text{if } \rho - \eta &\geq 0 \\ \tau - (\rho - \eta)\omega_{min} &> -1 & \text{if } \rho - \eta < 0 \end{aligned}$$



Assuming that the matrix  $(\mathbf{I}_N - \rho \mathbf{W})^{-1}$  is invertible, the reduced form of model (16) can be re-written as

$$\mathbf{y}_t = (\mathbf{I}_N - \rho \mathbf{W} \mathbf{y}_t)^{-1} (\tau \mathbf{I}_N + \eta \mathbf{W}) \mathbf{y}_{t-1} + (\mathbf{I}_N - \rho \mathbf{W} \mathbf{y}_t)^{-1} (\mathbf{X}_t \beta + \mathbf{W} \mathbf{X}_t \theta + \alpha + \lambda_t \mathbf{I}_N + \varepsilon_t)$$

Taking the partial derivatives of the expected value of  $\mathbf{y}$  with respect to each  $k$ -th variable in  $\mathbf{X}$  in each unit  $i$  at each time  $t$ , we then obtain the so-called impacts matrices in the *short run*:

$$\left[ \frac{\partial E(\mathbf{y})}{\partial x_{k1}} \dots \frac{\partial E(\mathbf{y})}{\partial x_{kN}} \right]_t = (\mathbf{I}_N - \hat{\rho} \mathbf{W})^{-1} (\hat{\beta}_k \mathbf{I}_N + \mathbf{W}_t \hat{\theta}_k)$$

and in the *long run*:

$$\left[ \frac{\partial E(\mathbf{y})}{\partial x_{k1}} \dots \frac{\partial E(\mathbf{y})}{\partial x_{kN}} \right] = [(1 - \hat{\tau}) \mathbf{I}_N - (\hat{\rho} + \hat{\eta}) \mathbf{W}]^{-1} (\hat{\beta}_k \mathbf{I}_N + \mathbf{W}_t \hat{\theta}_k)$$

The diagonal elements of both matrices give a measure of the so-called *direct effect*. The off-diagonal elements of the matrices give a measure of the so-called *indirect* or *spillover effect* (Table 2).

**Table 2.** Average total, direct, and indirect short-term and long marginal effects in dynamic spatial panels.  $\bar{d}$ : operator that calculates the mean diagonal element of a matrix.  $\overline{rsum}$ : operator that calculates the mean row sum of the non-diagonal elements.

Time horizon	ADE
Short-term	$[(\mathbf{I}_N - \hat{\rho} \mathbf{W})^{-1} (\hat{\beta}_k \mathbf{I}_N + \hat{\theta}_k \mathbf{W})]_{\bar{d}}$
Long-term	$[(1 - \hat{\tau}) \mathbf{I}_N - (\hat{\rho} + \hat{\eta}) \mathbf{W}]^{-1} (\hat{\beta}_k \mathbf{I}_N + \hat{\theta}_k \mathbf{W})_{\bar{d}}$
	AIE
Short-term	$[(\mathbf{I}_N - \hat{\rho} \mathbf{W})^{-1} (\hat{\beta}_k \mathbf{I}_N + \hat{\theta}_k \mathbf{W})]_{\overline{rsum}}$
Long-term	$[(1 - \hat{\tau}) \mathbf{I}_N - (\hat{\rho} + \hat{\eta}) \mathbf{W}]^{-1} (\hat{\beta}_k \mathbf{I}_N + \hat{\theta}_k \mathbf{W})_{\overline{rsum}}$

Moreover, Debarsy et al. (2012) derive the algorithms to calculate partial derivatives that can quantify the magnitude and timing of dependent variable responses in each region at various time horizons  $t + T$  to changes in the explanatory variables at time  $t$ . They also distinguish between two different interpretative scenarios, one where the change in explanatory variables represents a *permanent* or sustained change in the level and the other where we have a *transitory* (or one-period) change.

In particular, the  $T$ -period-ahead (cumulative) impact arising from a *permanent change* at time  $t$  in the  $k$ -th variable is<sup>1</sup>:

$$\partial \mathbf{Y}_{t+T} / \partial \mathbf{X}^k = \sum_{s=0}^T \mathbf{D}_s [\mathbf{I}_N \beta_k + \mathbf{W} \theta_k] \quad (17)$$

<sup>1</sup> By a permanent change at time  $t$  they mean that:  $\partial \mathbf{X}^k = (\mathbf{x}_t + \delta, \mathbf{x}_{t+1} + \delta, \dots, \mathbf{x}_T + \delta)$ , so the values increase to a new level and remain there in future time periods.

where  $D^s = (-1)^s(\mathbf{B}^{-1} + \mathbf{C})^s\mathbf{B}^{-1}$ , with  $s = 0, \dots, T-1$ ,  $\mathbf{B} = (\mathbf{I}_N - \rho\mathbf{W})$ , and  $\mathbf{C} = -(\tau\mathbf{I}_N + \eta\mathbf{W})$ .

The main diagonal elements of the  $N \times N$  matrix sums in (17) for time horizon  $T$  represent (cumulative) own-region impacts that arise from both time and spatial dependence. The sum of off-diagonal elements of this matrix reflect both *spillovers* measuring contemporaneous cross-partial derivatives, and *diffusion* measuring cross-partial derivatives that involve different time periods.<sup>2</sup>

The  $T$ -horizon impulse response to a *transitory* change in the  $k$ -th explanatory variable at time  $t$  would be given by the main and off-diagonal elements of:

$$\partial \mathbf{Y}_{t+T} / \partial \mathbf{X}^k = \mathbf{D}_T [\mathbf{I}_N \beta_k + \mathbf{W} \theta_k] \quad (18)$$

where  $\mathbf{D}_T = (-1)^T (\mathbf{B}^{-1} \mathbf{C})^T \mathbf{B}^{-1}$ .

Getting back to the example of the knowledge production function, the spatial dynamic version of (15) would be:

$$\begin{aligned} \ln K_{it} = & \beta_1 \ln R \& D_{it} + \beta_2 \sum_{j \neq i} w_{ij} \ln R \& D_{jt} + \beta_3 \ln H_{it} + \beta_4 \sum_{j \neq i} w_{ij} \ln H_{jt} \\ & + \tau \ln K_{i,t-1} + \rho \sum_{j \neq i} w_{ij} \ln K_{jt} + \eta \sum_{j \neq i} w_{ij} \ln K_{j,t-1} + \alpha_i + \tau_t + \varepsilon_{it} \end{aligned} \quad (19)$$

The estimation of this model would allow us to compute not only spatial (contemporaneous) R&D spillovers, but also spatio-temporal diffusion processes of R&D shocks originating in a region (or a country).

### 2.3 Modeling Spatial Dependence, Spatial Heterogeneity and Common Factors: Spatial Autoregressive Models for Large Panel Data

When spatial panel data have both a large cross-sectional and a large time series dimension, it becomes important to distinguish between spatial spillover effects and common factors. As discussed above, spatial spillovers are due to unobserved idiosyncratic shocks which propagate to all other regions with a distance-decay mechanism driven by network relationships. Instead, common factors are unobserved time-related factors which influence all regions (probably heterogeneously). Both determine cross-sectional correlation in the residuals and make it difficult to get unbiased and efficient estimates.

On the one hand, spatial spillover effects can be analyzed by using, for example, the spatial autoregressive model with fixed effects, described above. On the other hand, *strong* cross-sectional dependence can be accommodated by the Common Correlated Effects Pooled (CCEP) estimator proposed by Pesaran (2006). Suppose that  $y_{it}$  is generated by the following DGP with a multifactor error structure:

$$\begin{aligned} y_{it} &= \alpha_i + \mathbf{x}'_{it} \beta + \varepsilon_{it} \\ \mathbf{x}_{it} &= \gamma'_i \mathbf{f}_t + v_{it} \end{aligned} \quad (20)$$

<sup>2</sup> The term *spillover* is referred to contemporaneous cross-partial derivatives, those that involve the same time period. These cross-partial derivatives involving different time periods are referred to as diffusion effects, since diffusion takes time.

where  $\mathbf{f}_t$  is a  $m \times 1$  vector of common factors (introduced to allow for unobserved cross-sectional dependence), and  $\gamma_t$  the corresponding heterogeneous response.  $\mathbf{f}_t$  are allowed to be correlated with  $\mathbf{x}_{it}$ , while the idiosyncratic errors,  $\varepsilon_{it}$ , are assumed to be independently distributed over  $\mathbf{x}_{it}$ . Pesaran (2006) shows that, for sufficiently large  $N$ , it is valid to use cross-sectional averages of  $y_{it}$  and  $\mathbf{x}_{it}$  as observable proxies for  $\mathbf{f}_t$ . Thus, consistent  $\beta$  parameters can be estimated using the so-called CCEP estimator, which can be viewed as a generalized fixed effects estimator<sup>3</sup>:

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\beta + \delta_i\bar{\mathbf{x}}_t + \eta_i\bar{y}_t + \varepsilon_{it} \quad (21)$$

where  $\bar{\mathbf{x}}_t = N^{-1} \sum_{i=1}^N \mathbf{x}_{it}$  and  $\bar{y}_t = N^{-1} \sum_{i=1}^N y_{it}$ .

The CCEP approach has been proved to be valid in presence of both strong and weak (or semi-strong and semi-weak) cross dependence (Chudik et al. 2011; Pesaran and Tosetti 2011). Thus, it can easily collect even the pure spatial spillover effects. However, economic analyses often requires the assessment of the different forms of cross dependence, or better still, they require the assessment of spatial network effects, net of the effects of common factors. A natural way to deal with this problem is to combine the two approaches.

Using slightly different frameworks, Bai and Li (2015); Bailey et al. (2016); Shi and Lee (2016); Vega and Elhorst (2016) consider a joint modeling of spatial interaction effects and common-shocks effects:

$$y_{it} = \alpha_i + \rho \sum_{j=1}^N w_{ij,N} y_{jt} + \mathbf{x}'_{it}\beta + \gamma'_t \mathbf{f}_t + \varepsilon_{it} \quad (22)$$

This model (we may call it SAR-CCEP model) allows one to test which type of effects (common shocks,  $\gamma'_t \mathbf{f}_t$ , and/or spatial spillovers,  $\rho \sum_{j=1}^N w_{ij,N} y_{jt}$ ) is responsible for the cross-sectional dependence. Bai and Li (2015); Shi and Lee (2016) use principle components to estimate common factors, while Bailey et al. (2016); Vega and Elhorst (2016) follow Pesaran (2006) in using cross-sectional averages of  $y_{it}$  and  $\mathbf{x}_{it}$  as observable proxies for  $\mathbf{f}_t$ . Bailey et al. (2016) propose a two-stage estimation and inference strategy, whereby in the first step strong cross-sectional dependence is modeled by means of a factor model. Residuals from such factor models, referred to as de-factored observations, are then used to model the remaining weak cross dependencies, making use of spatial econometrics techniques. Vega and Elhorst (2016), instead, suggest to model common factors and spatial dependence simultaneously in a single-step procedure. All these authors show that the QMLE is an effective way of estimating this model.

Getting back to the example of the knowledge production function, the SAR-CCEP version of (15) would be:

$$\begin{aligned} \ln K_{it} = & \beta_1 \ln R\&D_{it} + \beta_2 \sum_{j \neq i} w_{ij} \ln R\&D_{jt} + \beta_3 \ln H_{it} + \beta_4 \sum_{j \neq i} w_{ij} \ln H_{jt} \\ & + \rho \sum_{j \neq i} w_{ij} \ln K_{jt} + \alpha_i + \gamma'_t \mathbf{f}_t + \varepsilon_{it} \end{aligned} \quad (23)$$

<sup>3</sup> The assumption of fixed  $\beta$  parameters can be relaxed, and a random coefficient specification can be assumed:  $\beta_i = \beta + u_i$ , with  $u_i \sim i.i.d.(\mathbf{0}, \Omega_u)$ . In this case the estimator proposed by Pesaran (2006) is the common correlated effects mean group (CCEMG) estimator.

Strong cross-sectional dependence in the errors of a knowledge production function may arise as a result of unobserved common factors, including, for instance, aggregate technological shocks, national policies intended to raise the level of technology or oil price shocks that may influence TFP through their effects on product costs. The heterogeneous effects of these factors may be the result, for instance, of country-specific technological constraints (Ertur and Musolesi 2016). Cross-sectional dependence in the errors of a knowledge production function can also be regarded as a result of spatial effects. Thus, a SAR-CCEP version of the knowledge production function seems to be a natural choice when the panel data is large enough.

Some drawbacks of this approach are worth noticing. First, there is a large number of incidental parameters under the joint modeling. Admittedly, this is not a serious problem as long as the model is linear, since inconsistency in the estimation of the incidental parameters is not transmitted to the estimation of the slope parameters of interest ( $\beta$ ); but, it may create a problem when nonlinear terms are considered. Second, the ability of the SAR-CCEP method to capture strong cross-sectional dependence and to disentangle spatial spillover effects and common factor effects is crucially affected by the set of covariates included in the model. On the one hand, if the estimated model contains one or only a few regressors, the CCEP estimator may not fully control for cross-sectional correlation (few regressors implies few cross-sectional averages as proxies for unobserved common factors); on the other hand, if the model includes many regressors, the resulting large number of cross-sectional averages hardly leave space for residual spatial spillovers. In Sect. 3.3, we review an alternative semiparametric approach to filter common-factor (or time-related) effects and, thus, to assess the presence of “residual” spatial dependence effects which adequately addresses these problems.

### 3 Semiparametric Spatial Autoregressive Models

Parametric spatial econometric frameworks described above are unfeasible in the simultaneous presence of different sources of model misspecification, such as substantial spatial dependence, nonlinear relationship of spatially correlated independent variables, unobserved spatial heterogeneity, spatially varying relationships, and common factors. Nonlinearities, spatial heterogeneity and time-related factors can cause spatial (or, more generally, cross-sectional) dependence and the reverse is also true. Studies that consider simultaneously spatial dependence, spatial heterogeneity, nonlinearities and common factors are still scarce in spatial econometrics literature. The recent contributions of Basile et al. (2014); Geniaux and Martinetti (2017); Mínguez et al. (2017) represent some attempts to promote more flexible estimation frameworks to address this problem.

#### 3.1 Modeling Spatial Heterogeneity and Spatial Dependence: MGWR-SAR

What are the economic motivations underlying the specification of a spatially-varying coefficient model? First, one can argue that models which only consider spatial autocorrelation are not capable of correcting all the problems related to non-observable spatial heterogeneity. This has pushed several authors to consider a non-stationary intercept term amongst the regression variables, for example by means of a smooth interaction of

the spatial coordinates, known as spatial trend (Wood 2006).<sup>4</sup> Nevertheless, this argument can be extended to consider a model with spatially-varying slope coefficients. It is also possible to consider a non-stationary spatial autocorrelation parameter. Indeed, when the spatial weight matrix  $\mathbf{W}$  is unknown and spatial locations are irregularly distributed over space, the choice of a neighboring scheme based only on distance or first nearest neighbors can be tricky. Choosing one weighting scheme instead of the other can lead to a spatial interaction matrix that is too dense or too dispersed in the heterogeneous parts of the space, resulting in under or overestimation of the parameters. Hence, the use of a non-stationary spatial autocorrelation parameter could mitigate the effect of the spatial weight matrix misspecification.

Very recently, Geniaux and Martinetti (2017) have introduced a new class of models, called MGWR-SAR (Mixed Geographically Weighted Regression Simultaneous AutoRegressive models), where the regression parameters and the spatial dependence coefficient can vary over space. In its most general form, the MGWR-SAR is specified as:

$$\mathbf{y} = \rho(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}; h) \mathbf{W} \mathbf{y} + \mathbf{X}^* \boldsymbol{\beta}^* + \boldsymbol{\beta}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}; h) \mathbf{X} + \boldsymbol{\varepsilon} \quad (24)$$

where  $\mathbf{y}$  is the  $N$ -vector of the continuous dependent variable,  $\mathbf{X}^*$  is a matrix of  $k_1$  exogenous explanatory variables entering the model linearly (i.e. with spatially stationary coefficients  $\boldsymbol{\beta}^*$ ), while  $\mathbf{X}$  is a matrix of  $k_2$  exogenous explanatory variables with non-stationary coefficients  $\boldsymbol{\beta}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}; h)$ ,  $\mathbf{x}_{s_1}, \mathbf{x}_{s_2}$  are spatial coordinates,  $\mathbf{W}$  is the spatial weights matrix,  $\rho$  the spatial spillover parameter,  $\boldsymbol{\varepsilon}$  is an i.i.d. error vector.

Thus, Geniaux and Martinetti (2017) relax one of the main hypothesis generally adopted by existing estimators of SAR models, i.e. the spatial parameter  $\rho$  and the regression parameters  $\boldsymbol{\beta}$  are constant over the coordinates space. In fact, in equation (24) the value of  $\rho$  and  $\boldsymbol{\beta}$  depends on the coordinates. The parameters  $\rho(\mathbf{x}_{s_1}, \mathbf{x}_{s_2})$  and  $\boldsymbol{\beta}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2})$  are only required to be spatially smoothed. The degree of smoothness depends on the bandwidth parameter  $h$  which allows to define the local sub-sample around the coordinates of each point  $(\mathbf{x}_{s_1}, \mathbf{x}_{s_2})$  using a given kernel function.

Because of the presence of the endogenous spatial lag term ( $\mathbf{W} \mathbf{y}$ ) on the r.h.s. of Eq. (24), the marginal effects of a change in  $\mathbf{X}^*$  or in  $\mathbf{X}$  must be computed starting from the reduced form of the model. Specifically, the marginal effect of a change in  $\mathbf{X}^*$  is:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{X}^*} = [\mathbf{I}_N - \rho(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}; h) \mathbf{W}]^{-1} \boldsymbol{\beta}^* \quad (25)$$

while the marginal effect of a change in  $\mathbf{X}$  is:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{X}} = [\mathbf{I}_N - \rho(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}; h) \mathbf{W}]^{-1} \boldsymbol{\beta}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}; h) \quad (26)$$

For the estimation of these new models, Geniaux and Martinetti (2017) resort to the Spatial Two-Stage Least Squares (S2SLS) technique. In particular, they use a 5-step approach, a local linear estimator (a variant of the GWR) and Cross Validation for the selection of the bandwidth parameter.

<sup>4</sup> It is worth noting that the spatial trend term can be included even in a model for cross-sectional data, while spatial fixed or random effects can be included in a model to control for spatial unobserved heterogeneity only when spatial panel data are available.

Using cross-regional data, one may for example estimate a *knowledge production function* with heterogeneous parameters:

$$\ln K_i = \alpha(\mathbf{x}_{s_1,i}, \mathbf{x}_{s_2,i}) + \beta_1(\mathbf{x}_{s_1,i}, \mathbf{x}_{s_2,i}) \ln R\&D_i + \beta_2(\mathbf{x}_{s_1,i}, \mathbf{x}_{s_2,i}) \ln H_i + \rho(\mathbf{x}_{s_1,i}, \mathbf{x}_{s_2,i}) \sum_{j \neq i} w_{ij} \ln K_j + \varepsilon_i \quad (27)$$

The regional learning process of generating and transferring knowledge may be affected by local social capital, i.e. the institutional and cultural context of local networks, trust and conventions. Therefore, heterogeneous region-specific conditions are a source of spatial heterogeneity in intra-regional knowledge creation. In addition, heterogeneous region-specific conditions are related with the regional capacity of exploiting external knowledge sources. Thus, model 27 would allow a researcher to assess the spatial stationarity (homogeneity) of the parameters associated to R&D investments and to human capital investments, as well as the spatial stationarity of the spatial knowledge spillover parameter ( $\rho$ ). Nonstationarity may be evident by inspection of basic maps, and can be formally tested. For example, Kang and Dallerba (2016) have investigated the spatial heterogeneity in the marginal effects of a regional knowledge production function by using nonparametric local modeling approaches such as GWR and mixed GWR with two distinct samples of the US Metropolitan Statistical Area (MSA) and non-MSA counties. The results indicate a high degree of spatial heterogeneity in the marginal effects of the knowledge input variables, more specifically for the local and distant spillovers of private knowledge measured across MSA counties. On the other hand, local academic knowledge spillovers are found to display spatially homogeneous elasticities in both MSA and non-MSA counties.

A characteristic of this approach is that it only considers spatial parameter heterogeneity (i.e. parameter heterogeneity over the coordinates space), while neglecting the possibility of pure nonlinearities (i.e. parameter heterogeneity over the domain of the explanatory variable). Nevertheless, it remains very important to assess the existence of pure nonlinearities in the relationship between the response variable and the covariates. In fact, regional and urban economic development literature often predicts threshold effects (for example in growth theory) or monotonic relationships (for example in urban economics). Moreover, keeping the spatial autocorrelation parameter ( $\rho$ ) constant over space is a valid option: in that case, the feedback effects of spatial autocorrelation have a clearer definition and the interpretation of direct and indirect effects is easier.

### 3.2 Modeling Spatial Dependence, Spatial Heterogeneity and Nonlinearities: P-Spline Models for Cross-Sectional Data and Short Panels

Another recent strand of the spatial econometric literature has proposed *Spatial Autoregressive Semiparametric Geoadditive Models* as a means of simultaneously dealing with different critical issues typically encountered when using spatial economic data; namely, spatial dependence, spatial heterogeneity and unknown functional form (Basile et al. 2014; Montero et al. 2012). This approach combines penalized regression spline (PS) methods (Eilers et al. 2015) with standard spatial autoregressive models (such as SAR, SEM, SDM and SLX). An important feature of these models is that they make it

possible to include within the same specification: (i) spatial autoregressive terms to capture spatial interaction or network effects; (ii) parametric and nonparametric (smooth) terms to identify nonlinear relationships between the response variable and the covariates; and (iii) a geoadditive term, i.e. a smooth function of the spatial coordinates, to capture a spatial trend effect, that is, to capture spatially autocorrelated unobserved heterogeneity.

The structural form of the Penalized-Spline Spatial Lag model (PS-SAR) is:

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}^* \boldsymbol{\beta}^* + f_1(\mathbf{x}_1) + f_2(\mathbf{x}_2) + f_3(\mathbf{x}_3, \mathbf{x}_4) + f_4(\mathbf{x}_1) \mathbf{z} + \dots + h(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}) + \varepsilon \quad (28)$$

where  $\mathbf{y}$  is a continuous univariate output variable,  $\mathbf{W}\mathbf{y}$  its spatial lag,  $\mathbf{X}^* \boldsymbol{\beta}^*$  is the linear predictor for any strictly parametric component (including the intercept, all categorical covariates and eventually a set of continuous covariates).  $f_k(\cdot)$  are unknown smooth functions of univariate continuous covariates or bivariate interaction surfaces of continuous covariates, capturing nonlinear effects of exogenous variables. Which of the explanatory variables enter the model parametrically or non-parametrically may depend on theoretical priors or can be suggested by the results of model specification tests (Kneib et al. 2009).  $f_4(\mathbf{x}_1) \mathbf{z}$  is a varying coefficient term, where  $\mathbf{z}$  is either a continuous or a binary covariate. The term  $h(\mathbf{x}_{s_1}, \mathbf{x}_{s_2})$  is a smooth spatial trend surface, i.e. a smooth interaction between latitude and longitude. It allows us to control for unobserved spatial heterogeneity, which is a primary task when dealing with spatial data. When the term  $h(\mathbf{x}_{s_1}, \mathbf{x}_{s_2})$  is interacted with one of the explanatory variables (e.g.,  $h(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}) \mathbf{x}_1$ ), it allows us to estimate spatially varying coefficients (like in the GWR model). Finally,  $\varepsilon$  are *iid* normally distributed random shocks.<sup>5</sup>

This model reflects the notion of spatial dependence made of two parts: (i) a spatial trend due to unobserved regional characteristics, which is modeled by the smooth function of the coordinates, and (ii) global spatial spillover effects, which are modeled by including the spatial lag of the dependent variable. The introduction of the spatial lags of the exogenous ( $\mathbf{X}$ ) variables results in what can be called the Penalized-Spline Geoadditive Spatial Durbin Model (PS-SDM).

When the  $\rho$  parameter is not statistically different from zero, i.e. in the case of a simpler semiparametric geoadditive model without the spatial lag of the dependent variable (*PS* model), if all regressors are manipulated independently of the errors,  $\hat{f}_k(x_k)$  can be interpreted as the conditional expectation of  $y$  given  $x_k$  (net of the effect of the other regressors). Blundell and Powell (2003) use the term Average Structural Function (ASF) with reference to this function. Instead, when  $\rho$  is different from zero, the estimated smooth functions —  $\hat{f}_k(x_k)$  — cannot be interpreted as ASF. Taking advantage of the results obtained for parametric SAR, we can compute the total smooth effect (total-ASF) of  $x_k$  as

$$\hat{f}_k^T(x_k) = \Sigma_q [\mathbf{I}_n - \hat{\rho} \mathbf{W}_n]_{ij}^{-1} b_{kq}(x_k) \hat{\beta}_{kq} \quad (29)$$

<sup>5</sup> This assumption can be relaxed by a more general specification, such as  $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2 \Lambda)$  being  $\Lambda$  a covariance matrix reflecting cross-sectional dependence in the errors as, for example, in Pinheiro and Bates (2000).

where  $b_{kq}(x_k)$  are P-spline basis functions, and  $\hat{\beta}_{kq}$  the corresponding estimated parameters.

We can also compute direct and indirect (or spillover) effects of smooth terms in the PS-SAR case as:

$$\hat{f}_k^D(x_k) = \Sigma_q [\mathbf{I}_n - \hat{\rho} \mathbf{W}_n]_{ii}^{-1} b_{kq}(x_k) \hat{\beta}_{kq} \quad (30)$$

$$\hat{f}_k^I(x_k) = \hat{f}_k^T(x_k) - \hat{f}_k^D(x_k) \quad (31)$$

Similar expressions can be provided for the direct, indirect and total effects of the PS-SDM (Table 3).

**Table 3.** Total, direct, and indirect smooth effects

Model	Direct Smooth Effect: $\hat{f}_k^{de_k}(x_k)$
PS model	$b_{kq}(x_k) \hat{\beta}_{kq}$
PS-SLM	$\Sigma_q [\mathbf{I}_N - \hat{\rho} \mathbf{W}_N]_{ii}^{-1} b_{kq}(x_k) \hat{\beta}_{kq}$
PS-SEM	$b_{kq}(x_k) \hat{\beta}_{kq}$
PS-SDM	$\Sigma_q [\mathbf{I}_N - \hat{\rho} \mathbf{W}_N]_{ii}^{-1} [b_{kq}(x_k) \hat{\beta}_{kq} + b_{kg}(\mathbf{W}_N x_k) \hat{\beta}_{kg}]$
PS-SLX	$b_{kq}(x_k) \hat{\beta}_{kq}$
Model	Indirect Smooth Effect: $\hat{f}_k^{ie_k}(x_k)$
PS model	0
PS-SLM	$\hat{f}_k^{te_k}(x_k) - \hat{f}_k^{de_k}(x_k)$
PS-SEM	0
PS-SDM	$\hat{f}_k^{te_k}(x_k) - \hat{f}_k^{de_k}(x_k)$
PS-SLX	$b_{kg}(\mathbf{W}_N x_k) \hat{\beta}_{kg}$
Model	Total Smooth Effect: $\hat{f}_k^{te_k}(x_k)$
PS model	$b_{kq}(x_k) \hat{\beta}_{kq}$
PS-SLM	$\Sigma_q [\mathbf{I}_N - \hat{\rho} \mathbf{W}_N]_{ij}^{-1} b_{kq}(x_k) \hat{\beta}_{kq}$
PS-SEM	$b_{kq}(x_k) \hat{\beta}_{kq}$
PS-SDM	$\Sigma_q [\mathbf{I}_N - \hat{\rho} \mathbf{W}_N]_{ij}^{-1} [b_{kq}(x_k) \hat{\beta}_{kq} + b_{kg}(\mathbf{W}_N x_k) \hat{\beta}_{kg}]$
PS-SLX	$b_{kq}(x_k) \hat{\beta}_{kq} + b_{kg}(\mathbf{W}_N x_k) \hat{\beta}_{kg}$

The Spatial Error Geoadditive Model (PS-SEM) proposed by Mínguez et al. (2012) augments the PS model by including a spatial autoregressive error term, while leaving the systematic part unchanged:

$$\begin{aligned} \mathbf{y} &= \mathbf{X}^* \boldsymbol{\beta}^* + f_1(\mathbf{x}_1) + f_2(\mathbf{x}_2) + f_3(\mathbf{x}_3, \mathbf{x}_4) \\ &\quad + f_4(\mathbf{x}_1) \mathbf{z} + \dots + h(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}) + \mathbf{u} \\ \mathbf{u} &= \lambda \mathbf{W} \mathbf{u} + \boldsymbol{\varepsilon} \quad \boldsymbol{\varepsilon} \sim iid \mathcal{N}(0, \sigma_\varepsilon^2) \end{aligned} \quad (32)$$



where  $\lambda$  is a spatial autoregressive parameter. As in the case of the pure PS model, if all regressors are exogenous,  $\hat{f}_k(x_k) = \Sigma_q b_{kq}(x_k) \hat{\beta}_{kq}$  can be directly interpreted as the conditional expectation of  $y$  given  $x_k$  (ASF).

Getting back to the example of the knowledge production function, the PS-SAR counterpart of model (15) for a short panel data can be for example specified as:

$$\ln K_{it} = \alpha + f(\ln R\&D_{it}, \ln H_{it}) + \rho \sum_{j \neq i} w_{ij} \ln K_{jt} + h(x_{s1,i}, x_{s2,i}) + \varepsilon_{it} \quad (33)$$

The nonparametric part of model 33 relaxes the standard assumptions of linearity and additivity regarding the effect of R&D and human capital. Charlot et al. (2015) use a similar specification to analyze the genesis of innovation in the regions of the European Union. Their results unveil nonlinearities, threshold effects, complex interactions and shadow effects that cannot be uncovered by standard parametric formulations.

### 3.3 Modeling Spatial Spillovers, Spatial Heterogeneity, Nonlinearities and Time-Related Factors: Spatio-Temporal Semiparametric Autoregressive Models for Large Panel Data

In this section we propose a class of spatio-temporal models for large spatial panel data which represent a generalization of the *Spatial Autoregressive Semiparametric Geoad-ditive Models* discussed in Sect. 3.2. They are a flexible alternative to the parametric models presented in Sect. 2.3 for modeling spatial panel data as long as the spatio-temporal heterogeneity is smoothly distributed (a very common case, one may say, in empirical economic analyses), so that we can approximate it with smooth nonparametric functions.

The general model proposed is written as:

$$\mathbf{y} = \tilde{f}(\mathbf{x}_{s1}, \mathbf{x}_{s2}, \mathbf{x}_t) + \rho \mathbf{W}\mathbf{y} + \sum_{\delta=1}^k g_{\delta}(\mathbf{x}_{\delta}) + \varepsilon \quad (34)$$

where  $\tilde{f}(\mathbf{x}_{s1}, \mathbf{x}_{s2}, \mathbf{x}_t)$  is a smooth spatio-temporal trend, i.e. a three-dimensional smooth function of the spatial coordinates  $(\mathbf{x}_{s1}, \mathbf{x}_{s2})$ , and of the time component  $\mathbf{x}_t$ ;  $g_{\delta}(\cdot)$ ,  $\delta = 1, \dots, k$ , are also smooth functions of the covariates  $x_{\delta, it}$  (they can be linear, or can accommodate varying coefficient terms, smooth interactions between covariates, factor-by-smooth curves, and so on);  $\mathbf{W}$  is the spatial weights matrix,  $\rho$  the spatial spillover parameter, and  $\varepsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$  where  $\mathbf{R}$  can be multiple of the identity (if errors are independent), or include a temporal correlation structure.

In many situations the spatio-temporal trend to be estimated by  $\tilde{f}$  can be complex, and the use of a multidimensional smooth function may not be flexible enough to capture the structure in the data. To solve this problem, Lee and Durbán (2011) proposed an ANOVA-type decomposition of  $\tilde{f}(\mathbf{x}_{s1}, \mathbf{x}_{s2}, \mathbf{x}_t)$  where spatial and temporal main effects, and second- and third-order interactions between them can be identified:

$$\begin{aligned} \tilde{f}(\mathbf{x}_{s1}, \mathbf{x}_{s2}, \mathbf{x}_t) = & f_1(\mathbf{x}_{s1}) + f_2(\mathbf{x}_{s2}) + f_t(\mathbf{x}_t) + f_{1,2}(\mathbf{x}_{s1}, \mathbf{x}_{s2}) \\ & + f_{1,t}(\mathbf{x}_{s1}, \mathbf{x}_t) + f_{2,3}(\mathbf{x}_{s2}, \mathbf{x}_t) + f_{1,2,3}(\mathbf{x}_{s1}, \mathbf{x}_{s2}, \mathbf{x}_t) \end{aligned}$$

Thus, model (34) can be written as:

$$\begin{aligned} \mathbf{y} = & f_1(\mathbf{x}_{s_1}) + f_2(\mathbf{x}_{s_2}) + f_t(\mathbf{x}_t) + f_{1,2}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}) + f_{1,t}(\mathbf{x}_{s_1}, \mathbf{x}_t) \\ & + f_{2,3}(\mathbf{x}_{s_2}, \mathbf{x}_t) + f_{1,2,3}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}, \mathbf{x}_t) + \rho \mathbf{W}_N \mathbf{y} + \sum_{\delta=1}^k g_{\delta}(\mathbf{x}_{\delta}) + \varepsilon \end{aligned} \quad (35)$$

We will refer to it as the PS-ANOVA-SAR(AR1) model. It is flexible enough to simultaneously control for different sources of bias: *spatial heterogeneity bias*, *spatial dependence bias*, *omitted-time related factors bias*, and *functional form bias*.

First, as already pointed out in Basile et al. (2014), the geoaddivitive terms given by  $f_1(\mathbf{x}_{s_1})$ ,  $f_2(\mathbf{x}_{s_2})$  and  $f_{1,2}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2})$  work as control functions to filter the spatial trend out of the residuals, and transfer it to the mean response in a model specification. Thus, they allow to capture the shape of the spatial distribution of  $\mathbf{y}$ , eventually conditional on the determinants included in the model. These control functions also isolate stochastic spatial dependence in the residuals, that is spatially autocorrelated unobserved heterogeneity. Thus, they can be regarded as an alternative to individual regional dummies to capture unobserved heterogeneity as long as the latter is smoothly distributed over space. Regional dummies peak significantly higher and lower levels of the mean response variable. If these peaks are smoothly distributed over a two-dimensional surface (i.e., if unobserved heterogeneity is spatially autocorrelated), the smooth spatial trend is able to capture them.

Second, the smooth time trend,  $f_t(\mathbf{x}_t)$ , and the smooth interactions between space and time -  $f_{1,t}(\mathbf{x}_{s_1}, \mathbf{x}_t)$ ,  $f_{2,t}(\mathbf{x}_{s_2}, \mathbf{x}_t)$ , and  $f_{1,2,t}(\mathbf{x}_{s_1}, \mathbf{x}_{s_2}, \mathbf{x}_t)$  - work as control functions to capture the heterogeneous effect of common shocks. Thus, the PS-ANOVA-SAR model works as an alternative to the models proposed by Bai and Li (2015); Bailey et al. (2016); Pesaran and Tosetti (2011); Shi and Lee (2016); Vega and Elhorst (2016) based on extensions of common factor models to accommodate both strong cross-sectional dependence (through the estimation of the spatio-temporal trend) and weak cross-sectional dependence (through the estimation of the  $\rho$  parameter). The advantage of the PS-ANOVA-SAR model lies in the fact that its ability to fully control for the residual cross-sectional dependence and to assess the presence of network effects net of common factor effects, is not crucially affected by the set of covariates included in the model.

Furthermore, this framework is also flexible enough to control for the linear and nonlinear functional relationships between the dependent variable and the covariates.

Getting back to the example of the knowledge production function, the PS-ANOVA-SAR version of (33) for a panel data with a long time series would be:

$$\begin{aligned} \ln K_{it} = & f(\ln R\&D_{it}, \ln H_{it}) + \rho \sum_{j \neq i} w_{ij} \ln K_{jt} \\ & + f_1(x_{s_1,i}) + f_2(x_{s_2,i}) + f_t(x_t) + f_{1,2}(x_{s_1,i}, x_{s_2,i}) + f_{1,t}(x_{s_1,i}, x_t) \\ & + f_{2,3}(x_{s_2,i}, x_t) + f_{1,2,3}(x_{s_1,i}, x_{s_2,i}, x_t) + \varepsilon_{it} \end{aligned} \quad (36)$$

## 4 Software

Nowadays there is a wide range of software allowing to estimate most of the econometric models exposed in this Chapter. Some of them, like *GeoDa* (Anselin et al. 2006),

use a menu interface which permits the user to perform spatial exploratory analysis, and to estimate parametric spatial econometric models for cross-sectional data without the need to learn new commands. Nevertheless, other well-known software alternatives require some skills in the corresponding programming language to deal with the spatial data. This is the case of some specialized packages in *R* (R Core Team 2016), the library *PySAL* (Rey and Anselin 2007) written in *Python* (Van Rossum 1995), the toolbox for spatial econometric models written by LeSage (2009) in *MATLAB* (MATLAB 2017), some functions, also in *MATLAB*, to estimate static and dynamic spatial panel data models developed by Elhorst (Elhorst et al. 2013), and a suite of commands for spatial data in *SAS* (SAS Institute Inc. 2013) or *Stata* (StataCorp. 2015). Bivand and Piras (2015) compare the results obtained by using different software alternatives and conclude that all of them provide similar results.

In this overview we focus on *R*, for the following reasons<sup>6</sup>:

- it is a well-tested free software with a growing number of packages in all statistical fields (spatial analysis included);
- it has a huge community of users;
- the possibility to combine functional programming with object-oriented programming (Chambers 2016) allows the developers to build new packages making use of the existing ones;
- it allows to estimate most of the spatial econometric models exposed in this chapter including both parametric models (for cross-sectional and static panel data) and semiparametric models.

The *R* packages **spdep** (Bivand 2013) and **sp** (Bivand et al. 2013; Pebesma and Bivand 2005) facilitate the creation, transformation and manipulation of spatial objects, neighborhood matrices and the computation of descriptive measures of spatial autocorrelation. Moreover, the package **spdep** allows researchers to estimate the whole set of cross-sectional spatial autoregressive models exposed in Sect. 2.1 including SAR, SEM, SDM, SLX and SAC models using either ML or GMM estimation in an efficient way. Furthermore, this package also permits us to compute the marginal effects and make inference on their values. To extend the range of standard spatial models considered, Piras (2010) created the **sphet** package for estimating and testing parametric spatial models with heteroskedastic innovations using estimation procedures based on GMM.

To deal with the static spatial panel data models discussed in Sect. 2.2.1, Millo and Piras (2012) have developed the **splm** package. It includes a set of functions able to estimate a full range of static spatial panel data models including fixed or random effects; spatial lags for the error term or dependent variable and, possibly, serial correlation in the noise of the model. Millo (2014) provides an extensive overview of these models including algorithms to estimate them using MLE. These packages can also be used to estimate the SAR-CCEP model discussed in Sect. 2.3. Unfortunately, there is not a freely available *R* package for the estimation and inference of dynamic spatial panel data models, revised in Sect. 2.2.2, while some functions are available in *MATLAB* (Elhorst 2014a; Elhorst et al. 2013).

<sup>6</sup> Two recent references of the use of *R* for spatial statistical and econometric analysis are Arbia (2014); Brunsdon and Comber (2015). A more classical reference is given by Bivand et al. (2013).

Focusing on semiparametric spatial data models (Sect. 3), McMillen (2013) has written the **McSpatial** package which includes routines to estimate nonparametric and conditionally parametric versions of spatial linear regression and spatial models with binary dependent variable. It mainly uses kernel techniques to perform the nonparametric estimations. Moreover, the package **GWmodel** (Gollini et al. 2015; Lu et al. 2014) deals with geographical weighted (GW) models, and includes functions for computation of GW summary statistics and regression, GW principal components analysis, and GW discriminant analysis. The techniques to estimate MGWR-SAR models discussed in Sect. 3.1 are already included in the forthcoming *R* package **gwrsar** (Geniaux and Martinetti 2017).

Finally, considering semiparametric regression models that include spatial or spatio-temporal trends, both packages **mgcv** (Wood 2006) and **R2BayesX** (Belitz et al. 2016; Umlauf et al. 2015) include some functions to estimate models including complex spatial and spatio-temporal trends, parametric and non-parametric covariates and interactions between them. Both packages have the possibility to choose P-spline methodology or the combination of other type of spline bases with penalty matrices for the nonparametric terms. The full class of models are usually estimated either by restricted maximum likelihood (REML) or bayesian methods. The techniques to estimate PS-SAR and PS-ANOVA-SAR models (Mínguez et al. 2017) discussed in Sect. 3.2 will also be included in a forthcoming *R* package.

## 5 Conclusions

Spatial econometrics is commonly conceived as a powerful method for capturing spatial spillover (or spatial interaction) effects. It is based on the assumption that, when an idiosyncratic shock hits a specific spatial unit (a country, a region, a firm, etc.), then its effects propagate to all other spatial units in the sample with a distance-decay mechanism. For example, in estimating a regional knowledge production function using a simple cross-section of regional data, we must be able to assess the impact of the investment in R&D in a region on both its own productivity outcome (*TFP*) and on the outcome of all other regions in the sample. Spatial econometricians have also derived statistical measures of direct and indirect (spillover) marginal impacts to quantify this phenomenon (LeSage and Pace 2009).

Nevertheless, is also important to recognize that the evidence of spatial spillovers might (at least partially) mask other specification errors, such as wrong functional form, unobserved spatial heterogeneity, heteroskedasticity, unobserved common factors, time persistence, and so on. Without a proper control for these sources of bias, the estimated spatial spillover effect often appears very (unrealistically) strong. For example, in estimating a regional knowledge production function using a simple cross-section of regional data without any control for nonlinearities and spatial unobserved heterogeneity, one may find evidence of an average indirect (spillover) impact of R&D on *TFP* similar to the corresponding average direct marginal effect. This is obviously unreasonable.

In this Chapter we have reviewed different parametric and semiparametric approaches recently developed to mitigate this problem. Not surprisingly, parametric spatial panel models received most attention in the literature. In particular, dynamic spatial panel data models and spatial panel autoregressive models with common factors turn

to be very important tools for simultaneously control for spatial spillovers, unobserved spatial heterogeneity, unobserved common factor and time persistence. However, in the Chapter we have also pointed out that spatial autoregressive semiparametric geoadditive models (PS-SAR models; Basile et al. 2014) may play a prominent role in those context in which the theory suggests the existence of spatial interdependence and heterogeneous behavior of the spatial units. These methods represent indeed some flexible approaches which are able to address simultaneously spatial dependence, heterogeneity and nonlinearity. Moreover, we have reviewed more recently developed semiparametric models for longitudinal data including a non-parametric spatio-temporal trend, a spatial lag of the dependent variable, and a time series autoregressive noise (PS-ANOVA-SAR-AR1) which represent a valid alternative to parametric methods aimed at disentangling strong and weak cross-sectional dependence (Mínguez et al. 2017). Natural directions in which these methods can be extended are a specification for a dynamic framework.

**Acknowledgements.** The work of Román Mínguez was supported by the research project MTM2014-52184 and ECO2015-65826-P from the Spanish Ministry of Economy and Competitiveness.

## References

- Anselin, L.: Spatial Econometrics: Methods and Models, vol. 4. Kluwer Academic Pub., Dordrecht (1988)
- Anselin, L., Syabri, I., Kho, Y.: GeoDa: an introduction to spatial data analysis. *Geograph. Anal.* **38**(1), 5–22 (2006)
- Arbia, G.: A primer for spatial econometrics. With Applications in R. Palgrave Texts in Econometrics: Palgrave Macmillan UK, UK (2014)
- Bai, J., Li, K.: Dynamic Spatial Panel Data Models with Common Shocks. Manuscript: Columbia University (2015)
- Bailey, N., Holly, S., Pesaran, M.H.: A two-stage approach to spatio-temporal analysis with strong and weak cross-sectional dependence. *J. Appl. Econom.* **31**(1), 249–280 (2016)
- Basile, R., Durbán, M., Mínguez, R., Montero, J.M., Mur, J.: Modeling regional economic dynamics: spatial dependence, spatial heterogeneity and nonlinearities. *J. Econ. Dyn. Control* **48**, 229–245 (2014)
- Belitz, C., Brezger, A., Kneib, T., Lang, S., Umlauf, N.: BayesX: Software for Bayesian Inference in Structured Additive Regression Models. Version 1.1 (2016)
- Bivand, R.: spdep: spatial dependence, weighting schemes, statistics and models. R package version 0.5-56 (2013). <http://CRAN.R-project.org/package=spdep>
- Bivand, R., Piras, G.: Comparing implementations of estimation methods for spatial econometrics. *J. Stat. Softw.* **63**(1), 1–36 (2015)
- Bivand, R.S., Pebesma, E., Gómez-Rubio, V.: Applied Spatial Data Analysis with R. Use R!, 2nd edn. Springer, New York (2013)
- Blundell, R., Bond, S.: Initial conditions and moment restrictions in dynamic panel data models. *J. econ.* **87**(1), 115–143 (1998)
- Blundell, R., Powell, J.: Endogeneity in nonparametric and semiparametric regression models. In: *Advances in Economics and Econometrics Theory and Application*. Cambridge University Press (2003)
- Brunsdon, C., Comber, L.: An Introduction to R for Spatial Analysis and Mapping. SAGE Publications Ltd., London (2015)

- Brunsdon, C., Fotheringham, A.S., Charlton, M.E.: Geographically weighted regression: a method for exploring spatial nonstationarity. *Geograph. Anal.* **28**(4), 281–298 (1996)
- Chambers, J.M.: *Extending R. The R Series*. Chapman and Hall/CRC, Boca Raton (2016)
- Charlot, S., Crescenzi, R., Musolesi, A.: Econometric modelling of the regional knowledge production function in Europe. *J. Econ. Geogr.* **15**(6), 1227–1259 (2015)
- Cho, S.-H., Lambert, D.M., Roberts, R.K., Kim, S.G.: Moderating urban sprawl: is there a balance between shared open space and housing parcel size? *J. Econ. Geogr.* **10**(5), 763–783 (2010)
- Chudik, A., Pesaran, M.H., Tosetti, E.: Weak and strong cross-section dependence and estimation of large panels. *Econom. J.* **14**(1), 45–90 (2011)
- Debarys, N., Ertur, C., LeSage, J.P.: Interpreting dynamic space-time panel data models. *Stat. Methodol.* **9**(1), 158–171 (2012)
- Eilers, P.H., Marx, B.D., Durbán, M.: Twenty years of P-splines. *SORT-Stat. Oper. Res. Trans.* **39**(2), 149–186 (2015)
- Elhorst, J.: Matlab software for spatial panels. *Int. Reg. Sci. Rev.* **37**(3), 389–405 (2014a)
- Elhorst, J.: *Spatial Econometrics. From Cross-Sectional Data to Spatial Panels*. SpringerBriefs in Regional Science. Springer, New York (2014b)
- Elhorst, J., Zandberg, E., de Haan, J.: The impact of interaction effects among neighbouring countries on financial liberalization and reform: a dynamic spatial panel data approach. *Spat. Econ. Anal.* **8**(3), 293–313 (2013)
- Ertur, C., Musolesi, A.: Weak and strong cross-sectional dependence: a panel data analysis of international technology diffusion. *J. Appl. Econom.* **32**, 477–503 (2016)
- Geniaux, G., Martinetti, D.: A new method for dealing simultaneously with spatial autocorrelation and spatial heterogeneity in regression models. *Reg. Sci. Urban Econ.* (2017). Forthcoming
- Gollini, I., Lu, B., Charlton, M., Brunsdon, C., Harris, P.: GWmodel: an R package for exploring spatial heterogeneity using geographically weighted models. *J. Stat. Softw.* **63**(17), 1–50 (2015)
- Kang, D., Dallerba, S.: Exploring the spatially varying innovation capacity of the US counties in the framework of Griliches knowledge production function: a mixed GWR approach. *J. Geogr. Syst.* **18**(2), 125–157 (2016)
- Kelejian, H., Prucha, I.: Estimation of spatial regression models with autoregressive errors by two-stage least squares procedures: a serious problem. *Int. Reg. Sci. Rev.* **20**, 103–111 (1997)
- Kelejian, H., Prucha, I.: On the asymptotic distribution of moran I test statistic with applications. *J. Econom.* **104**, 219–257 (2001)
- Kneib, T., Hothorn, T., Tutz, G.: Variable selection and model choice in geoadditive regression models. *Biometrics* **65**(2), 626–634 (2009)
- Kukenova, M., Monteiro, J.A.: Spatial dynamic panel model and system GMM: a Monte Carlo investigation (2008). Unpublished article
- Lee, D., Durbán, M.: P-spline ANOVA type interaction models for spatio-temporal smoothing. *Stat. Modell.* **11**, 49–69 (2011)
- Lee, L.: Asymptotic distribution of quasi-maximum likelihood estimators for spatial econometric models. *Econometrica* **72**, 1899–1926 (2004)
- Lee, L.-F., Yu, J.: Estimation of spatial autoregressive panel data models with fixed effects. *J. Econom.* **154**(2), 165–185 (2010a)
- Lee, L.-F., Yu, J.: A spatial dynamic panel data model with both time and individual fixed effects. *Econom. Theory* **26**(02), 564–597 (2010b)
- LeSage, J., Pace, K.: *Introduction to Spatial Econometrics*. CRC Press, Boca Raton (2009)
- LeSage, J.P.: *Econometrics Toolbox for MATLAB* (2009). [www.spatial-econometrics.com](http://www.spatial-econometrics.com)
- Liu, X., Lee, L., Bollinger, C.: An efficient GMM of spatial autoregressive models. *J. Econom.* **159**, 303–319 (2007)

- Lu, B., Harris, P., Charlton, M., Brunsdon, C.: The GWmodel R package: further topics for exploring spatial heterogeneity using geographically weighted models. *Geo-Spat. Inf. Sci.* **17**(2), 85–101 (2014)
- MATLAB: version 9.2 (R2017a), Natick, Massachusetts (2017)
- McMillen, D.: McSpatial: nonparametric spatial data analysis. R package version 2.0 (2013)
- McMillen, D.P.: Perspectives on spatial econometrics: linear smoothing with structured models. *J. Reg. Sci.* **52**(2), 192–209 (2012)
- Millo, G.: Maximum likelihood estimation of spatially and serially correlated panels with random effects. *Comput. Stat. Data Anal.* **71**, 914–933 (2014)
- Millo, G., Piras, G.: splm: spatial panel data models in R. *J. Stat. Softw.* **47**(1), 1–38 (2012)
- Mínguez, R., Basile, R., Durbán, M.: Spatio-temporal autoregressive semiparametric models for the analysis of regional economic data (2017)
- Mínguez, R., Durbán, M., Montero, J., Lee, D.: Competing Spatial Parametric and Non-Parametric Specifications. *Mimeo*, New York (2012)
- Montero, J., Mínguez, R., Durbán, M.: SAR models with nonparametric spatial trends. A P-spline approach. *Estadística Española* **54**(177), 89–111 (2012)
- Pace, R.K., LeSage, J.P.: Spatial autoregressive local estimation. *Spat. Econom. Spat. Stat.*, 31–51 (2004)
- Páez, A., Uchida, T., Miyamoto, K.: A general framework for estimation and inference of geographically weighted regression models: 1. Location-specific kernel bandwidths and a test for locational heterogeneity. *Environ. Plann. A* **34**(4), 733–754 (2002)
- Pebesma, E.J., Bivand, R.S.: Classes and methods for spatial data in R. *R News* **5**(2), 9–13 (2005)
- Pesaran, M.H.: Estimation and inference in large heterogeneous panels with a multifactor error structure. *Econometrica* **74**(4), 967–1012 (2006)
- Pesaran, M.H., Tosetti, E.: Large panels with common factors and spatial correlation. *J. Econom.* **161**(2), 182–202 (2011)
- Pinheiro, J., Bates, D.: *Mixed-Effects Models in S and S-PLUS*. Springer, New York (2000)
- Piras, G.: sphet: spatial models with heteroskedastic innovations in R. *J. Stat. Softw.* **35**(1), 1–21 (2010)
- R Core Team: *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria (2016)
- Rey, S.J., Anselin, L.: PySAL: a python library of spatial analytical methods. *Rev. Reg. Stud.* **37**(1), 5–27 (2007)
- SAS Institute Inc.: *SAS/IML(R) 13.1 User's Guide*. SAS Institute Inc., Cary (2013)
- Shi, W., Lee, L.: *A Spatial Panel Data Model with Time Varying Endogenous Weights Matrices and Common Factors* (2016)
- StataCorp.: *Stata Statistical Software: Release 14* College Station. StataCorp LP, TX (2015)
- Umlauf, N., Adler, D., Kneib, T., Lang, S., Zeileis, A.: Structured additive regression models: an R interface to BayesX. *J. Stat. Softw.* **63**(21), 1–46 (2015)
- Van Rossum, G.: *Python Reference Manual*. CWI report CS-R9524 (1995)
- Vega, S.H., Elhorst, J.P.: A regional unemployment model simultaneously accounting for serial dynamics, spatial dependence and common factors. *Reg. Sci. Urban Econ.* **60**, 85–95 (2016)
- Wood, S.: *Generalized Additive Models. An Introduction with R*. Chapman and Hall, London (2006)
- Yu, J., De Jong, R., Lee, L.-F.: Quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects when both  $n$  and  $T$  are large. *J. Econom.* **146**(1), 118–134 (2008)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.





# Looking Ahead: Part I

Pasquale Commendatore<sup>1</sup>(✉) and Ingrid Kubin<sup>2</sup>

<sup>1</sup> Department of Law, University of Naples “Federico II”, Naples, Italy  
commenda@unina.it

<sup>2</sup> Department of Economics, Institute for International Economics  
and Development, WU Vienna University of Economics and Business,  
Vienna, Austria  
Ingrid.Kubin@wu.ac.at

**Abstract.** This Chapter summarises the work carried out during the lifetime of the Action by Working Group I whose main task was to build multiregional NEG models. The main results are briefly presented and some of the questions left open are pointed at. Finally, topics for future research are suggested.

**Keywords:** Multiregional NEG models · Social network analysis · Regional trade flows · Policy issue in NEG models · European regions · Trade agreements

## 1 Introduction

Within the activities of the COST Action IS1104 ‘The EU in the new complex geography of economic systems: models, tools and policy evaluation’ the main objective of the Working Group on ‘Economic Geography modelling’ has been to provide a modelling strategy to represent the ‘EU as a multi-level complex evolving system’. We started from the observation that the process of integration did not fully deliver its promise leaving the economic activity unevenly spread across European regions.

As a modelling strategy, we opted for the New Economic Geography (NEG) approach – originated from Krugman’s (1991) contribution – which has proven to be quite fruitful in describing the basic economic mechanisms behind the spatial distribution of economic activity. Briefly, the standard set-up of a NEG model includes an economy with two regions (or countries), two sectors and two factors of production, Dixit-Stiglitz monopolistic competition, CES preferences over the varieties of the manufactured good, factor mobility between the regions, iceberg/multiplicative trade costs. The spatial distribution of the economic activity is the result of the interplay between agglomeration forces – market-access and cost-of-living effects – and dispersion forces – competition and local market size effects (for a more detailed description of the NEG approach, please refer to Chaps. 1 and 2 of this book).

In the following sections, we summarise the research carried out by Working Group I on its main objective: multiregional NEG modelling; then we present some of the main results and pointed at questions left open; finally, we suggest topics for future research.

## 2 Summary of the Research Conducted Within the Action

Our main research focus has been multiregional New Economic Geography (NEG) modelling. The aim of NEG models is to explain the uneven distribution of economic activity across space driven by factor migration (labour, physical or human capital) governed by interplay of agglomeration and dispersion forces.

In Commendatore et al. (2015a), we reviewed systematically multiregional NEG models recently developed in the literature and compared different modelling strategies. In that survey, we presented a taxonomy of the literature that groups the contributions into two classes: the first category of models follows the standard NEG approach: they adopt CES consumer's preferences leading to an isoelastic demand for each variety and to a fixed mark-up. In this type of models, trade liberalization or integration – modelled as a reduction of 'iceberg' trade costs proportional to the manufactured commodity price – favours the agglomeration forces over the dispersion forces. Conversely, the second class of models suggests that trade integration – modelled as a reduction of trade costs which are added to the price – may lead to the prevalence of dispersion forces over agglomeration forces. This feature is obtained by Ottaviano et al. (2002) by simply introducing quadratic consumer's preferences over the manufactured varieties generating linear demands for these varieties and variable mark-ups. In a 'linear' NEG model (see Baldwin et al. 2003) prices fall depending on the number of (local and outside) firms which compete in the local market. This adds a further dispersion force to those operating in NEG models which manifests itself especially in a multiregional context. Indeed, Ago et al. (2006) and Behrens (2011) show in models with a hub-and-spoke structure that centrality delivers a locational advantage to the hub when competition is not too strong (as in standard NEG models) and a locational disadvantage when competition is fiercer (as in linear NEG models). This translates to completely different firms' location patterns and regional policy recommendations. In Commendatore et al. (2015a), we also sketched the analytic structure of a general multi-regional model and we showed how simpler cases can be derived from that general framework.

In other contributions, we put forward simpler standard NEG models (see Commendatore and Kubin 2013; Commendatore et al. 2014, 2015b, c) and linear NEG models (Commendatore et al. 2016, 2017a, b) where the number of regions is assumed to be small (but larger than two) to obtain as much as possible analytic results and numerical simulations easy to interpret. A distinguishing feature of these analyses is the discrete time dynamic framework, which is a set-up able to generate a larger variety of long-term behaviours compared to the continuous time version. We mainly used tools from the mathematical theory of dynamical systems to study the qualitative properties of such multiregional discrete time models.

Behrens (2004, 2005) and Ago et al. (2006) showed that, when demands are linear, there are circumstances – linked to trade costs and local competition – in which one region does not have a sufficient incentive to export the manufactured good to another region. So that different trade patterns may emerge: no trade, unilateral trade and bilateral trade. Commendatore et al. (2017a, b) extend/integrate these contributions. They represent a small trade network composed of three identical regions. In these contributions, three examples have been discussed that can be interpreted as three

stages of development. In the first stage, since trade costs are very high throughout the economy, all regions are fully autarkic; in the second stage two regions begin to integrate and may engage in unilateral or bilateral trade, while the third region – geographically more remote – is still in autarky; finally, in the third stage, when the first two regions are fully integrated, the trade costs between the remote region and one of the other two shrink, so that the remote region and the now more central region engage in trade. The three-region economy takes the shape of a hub-and-spoke trade structure. Within this framework, Commendatore et al. (2017a, b) clarify: how distance and trade costs are related to the existence and direction of a trade link; how trade integration affects the long-term distribution of economic activities and which trade pattern characterises each spatial equilibrium; and, finally, how trade patterns and the spatial distribution of the economic activity are interrelated.

In recent efforts, we applied Social Network Analysis to investigate the statistical properties of the network of trade flows between European regions at the NUTS 2 level and put forward first attempts to provide theoretical underpinnings of such a structure (see Basile et al. 2016, 2017). In Basile et al. (2016), we used a new set of data on regional trade flows and exploited their binary structure and their relative weights to visualize the European regional trade network. Given the limits of the data (partially inferred from other data aggregated at the national and international levels), to reduce the density of the network we used a meaningful threshold cutting off negligible links. This allowed us to detect the higher order statistics via clustering analysis and the main triadic structures via the triad census of the interregional trade links. The latter methodology is typically used to study the local properties of a network – which is a structure composed of nodes (in our case, regions) and relationships linking one node to another node (in our case, trade flows: one in the outward direction, exports; and one in the inward direction, imports). An important property is, for example, the ‘third region effect’ according to which the existence/absence of a trade link between two regions is contingent upon the presence of (at least) another pre-existing link between one of these two regions and a third region. A triad is a fundamental unit of analysis composed of three nodes and by the possibility of presence/absence of links relating pairwise these nodes. There are 64 possible types of triad that without specific differentiations (related for example to regional sizes or geographical distances) can be grouped into 16 isomorphism classes. In Basile et al. 2016, by using a specific cut off threshold, we calculated for the European trade network the frequencies of these classes and drew interesting insights on the interregional EU trade network. Moreover, we put forward a three-region linear NEG model which is more general compared to those existing in the literature. We assumed that the distance between the regions is not necessarily the same so, differently from other contributions (see for example Ago et al. 2006; and Behrens 2011), we did not impose any specific geography on the possible network structures. Thus, focusing on the short-run with no factor migration, we have been able to derive the conditions, expressed in terms of different combinations of trade costs and distributions of the economic activity, corresponding to each of the 16 possible network structures.

In Basile et al. (2017), we focussed on the role of trade costs in determining the topological structure of the EU network. In dealing with the empirical analysis, to better approximate their broad theoretical meaning, we considered two dimensions of trade

costs: geographical distance – we created different sub-graphs (i.e. sub-networks) composed of groups of regions differentiated on the basis of regional bilateral distances – and the presence of a border effect – we created different sub-graphs distinguishing between regions involved in intra-national or international regional trade. The theoretical part extends Basile et al. (2016) by explicitly differentiating the distances between the regions and exploring how trade costs impact on the frequency of the 64 triadic configurations, examining the likelihood of each configuration. We found correspondence, with some exceptions, between empirical and theoretical results.

### 3 Main Results and Open Questions

#### 3.1 Main Results

We focus here on results concerning agglomeration and dispersion patterns characterising the stationary long-term equilibria of some of the multiregional NEG models we have examined during the life time of the Action. Other results are reviewed in Chaps. 1 and 2 of this book. Moreover, readers interested in the mathematical properties of these models can refer to Kubin et al. (2016). In that contribution we stressed that the possible long-term states of the economy are by all means not limited to stationary equilibria.

Concerning three-region models, we found that the presence of a third region matters. In Commendatore et al. (2014), we provided a paradigmatic example: we considered a 3-region footloose-entrepreneur new economic geography model with standard CES preferences. The three regions have a specific geographical arrangement: two of them are symmetric and form an economically integrated area (the Union), the third one is an outside trade partner. Entrepreneurs can freely migrate within the Union, but no factor mobility is allowed between the Union and the third region. Depending on the skill endowment, the market size of the outside region and the different degrees of trade liberalisation – explored in its two aspects of regional integration and globalisation – we found that stationary long-run interior equilibria may exist characterised by an industrial sector unevenly distributed across all regions. Trade integration may lead to agglomeration of industry in only one region via a smooth transition (in contrast to the NEG typical catastrophic scenario).

In a second paper (see Commendatore et al. 2016), we adopted the same geographical arrangement but assumed a quadratic utility function and linear demand functions (for a similar approach see Behrens 2011). This set-up allowed us to compare the NEG and the Heckscher-Ohlin perspectives. According to the standard predictions of the Heckscher-Ohlin framework trade integration leads to specialization, trade diversion and trade creation; however, with factor mobility, it may also lead to agglomeration within the Union, which agrees with the standard NEG result, strengthening the specialization and trade effects of integration. Finally, given the simpler analytical structure following the assumption on preferences, only a stable symmetric equilibrium or full agglomeration (Core-Periphery) equilibria exist.

In other two works, we increased the number of regions to four in a standard NEG model. In the first contribution (Commendatore et al. 2015c), four regions of equal size

are located along a line, the two regions on the left and those on the right form two countries (or trade blocs) sharing a common border passing through the two regions in the middle. Again, the mobile factor (entrepreneurs) can only migrate within the same country. Due to various geographical impediments to trade the two inland regions (at the extremes of the line) can only access the national market and international trade can only occur between the two bordering regions (at the centre of the line). The specific geography adopted bears on the relative strength of agglomeration and dispersion forces: local firms in the peripheral region face only national competition from firms located in the adjacent central region, whereas the latter face both national and international competition, respectively originating from the peripheral region in the same country and from the bordering region in the other country. On the other hand, a firm located in a central region has access to a larger market selling goods both to the national market (both locally and to the adjacent peripheral region) and to the international market (i.e. to the foreign bordering region). Moreover, in a central region a larger variety of goods are accessible to local consumers. Thus, the market size effect, the price index effect and the competition effect are all stronger in a central region with the strength depending on the size of the manufacturing sector in the bordering region in the other country. The usual NEG sequence (as depicted in the standard tomahawk diagram) occurs with trade liberalization: low trade costs bring dispersion and high trade costs agglomeration. However, a larger variety of patterns can emerge compared with the standard two-region set up: with low trade costs, dispersion involves a larger share of industry in the two central regions; with high trade costs agglomeration could be symmetric – industry agglomerates in the two central regions, thus firms enjoy the full extent of the market size effect – or asymmetric – industry agglomerates in a central region in one country and in a peripheral region in the other country, thus firms in the periphery are sheltered from competition –; with intermediate trade costs, partial agglomeration may occur: industry is agglomerated in one country and dispersed in the other where firms, as in the previous case, find shelter from foreign competition.

In the second contribution (Commendatore et al. 2017c), we differentiated the regions on the basis of their size according to the sequence: small, big, small, big. This set-up allowed us to study the interplay between centrality and local market size effects – two different manifestations of the market access effect – which is not possible in other 4-region NEG models. Thus, each country is composed of a small region and a big region. In one country, the small region is peripheral and the large region is central and in the other country, on the contrary, the small region is central and the large region is peripheral. As before, factor mobility is only allowed between regions in the same country. Instead, differently from the previous contribution, we allow for both direct and indirect trade (i.e. between adjacent and non-adjacent regions). Taking into account that additional trade costs are incurred crossing the international border, we differentiated between national and international trade costs. Confining here our discussion to the long-run full agglomeration outcomes (Core-Periphery or CP equilibria), we have been able to find that by varying national and international trade costs, the stability properties of CP equilibria follow patterns which depend on the interplay of three effects: centrality, local market size and competition. Our model includes the standard two-region asymmetric NEG model as a special case (when trade costs between countries are prohibitive). In the two-region set-up, when trade costs take intermediate values,

agglomeration can only occur in the bigger region (the local market size effect is strong); and when trade costs are low, depending on initial conditions, industry may agglomerate in the smaller region as well (the local market size effect is weak). In our more general four-region set-up, matters are more complicated. Given that agglomeration can only occur within each country, because factors cannot migrate across countries, what can occur is that or all firms end up in the two bigger regions, or in the two smaller regions, or in the two central regions or in the two peripheral regions. The following scenarios are possible: (i) when trade costs are high between countries and low within countries all CP equilibria are stable and are possible long-run outcomes, replicating the standard result which applies to the case of two asymmetric regions; (ii) when we increase national trade costs (without reducing too much international trade costs) simultaneous agglomeration in the two small regions is not possible, again as it was the case in the standard two-region set-up, because of the market size effect; (iii) when we further increase trade costs within countries (or decrease trade costs between countries) simultaneous agglomeration in the two remote regions is not possible. Centrality gains importance. Firms may still end up in the larger remote region which keeps the advantage of a larger size; (iv) further reducing trade costs within countries and letting international trade costs sufficiently high leads to agglomeration only in the bigger regions, as the local market size effect becomes prevalent; (v) instead the same reduction of national trade costs, but in correspondence of low international trade costs leads to agglomeration in the central regions as centrality becomes the prevailing force.

### 3.2 Open Questions

There are a few questions left open deserving to be addressed in future projects:

- Number of regions. We limited our analysis to economies composed of a small number of regions. Given the complicated analytical structure of the NEG approach with a CES utility function, also for this ‘simpler’ models, it was not easy to interpret all the results.
- Trade Patterns. In the standard model, with finite trade costs, only a trade pattern is possible corresponding to two-way trade between any two regions. By introducing a quadratic utility function leading to linear demand functions, the analysis is made simpler and more analytical results can be derived. Moreover, other issues can be explored given that, for example, with a linear demand, differentiated patterns of trade can emerge (no trade, one-way trade, two-way trade). In fact, considering two regions four trade patterns are possible; when three regions are considered the possible patterns of trade are 64 and so on. We believe that this apparently minor difference represents a powerful tool of analysis.
- Policy issues. We examined a variety of policy issues (tax competition, government expenditure, trade integration, and so on) but, following the large part of the current NEG literature, only considering two or little more regions (for a review on how policy issues are treated in this literature, see Chap. 1 of this book).

## 4 Suggested Topics for Future Research

Following the previous section on open issues, we would like to draw a few lines for future research:

- (i) Multi-regional NEG modelling. We plan to extend the standard NEG model to many regions arranged on a square lattice (for a similar framework see Stelder 2005; Ikeda and Murota 2014; Ikeda et al. 2017). To reconcile computing simplification and real world resemblance, the mobile factor migration process is modelled as a stationary markovian stochastic process. One of the objective of this project is to put forward a platform for simulating multiregional NEG models by using a user-friendly programming language, as for example Python.
- (ii) Multiregional NEG modelling. We also plan to extend the linear NEG model to many regions. Given that this type of NEG models is much less treated in the literature more gaps should be filled. In Basile et al. (2016, 2017), we put forward a three-region linear NEG model and explored both theoretically and empirically the probability of the emerging of different trade patterns, depending on trade costs and the degree of competition for a given distribution of the industrial activity (by fixing the number of entrepreneurs). The next step would be allowing for factor mobility and explore the simultaneous evolution through time of industrial location and trade patterns. This would lead to more general cases compared to those discussed in Chap. 2 of this book. A final more ambitious step would be to consider a large number of regions and to study the effects of local and global shocks, affecting trade costs, on the trade network structure in the short run and on the distribution of economic activities and in the endogenous formation of trade network structures in the long run. Notice that a complementary problem has been studied in the literature: Countries may have an economic incentive – fixed in the short-run – to create bilateral trade agreements, this leading to the formation of a free trade agreement network (see Furusawa and Konishi 2007). It would be interesting to verify the evolution of such network once the economic incentive is allowed to vary.
- (iii) Market structure. We plan to strengthen the link between IO modelling and spatial issues by using linear demand spatial models and departing from the standard monopolistic competition set-up generally adopted in NEG modelling.
- (iv) Policy issues. Very much connected with the other three lines of research, another project would address specific policy issues: impact of creation/resolution/modification of trade agreements; impact of EU policies; impact of local government policies; regulations related to environmental issues, and so on. As an example, a three-region linear NEG model could be fruitfully used as tool of analysis to clarify the possible effects of Brexit (the withdrawal of the UK from the EU) on trade flows between the most important economic areas involved – Britain, the EU and United States – and on their citizens welfare. The model would be an extension of the linear NEG model with two regions of asymmetric sizes developed by Okubo et al. (2014). In our set-up, two large regions (the USA and the EU) may engage in trade between each other and with a third small region (the UK). Alternative trade agreements resulting from EU and UK negotiations translate into

trade costs of different magnitudes, reshaping trade patterns and long-run distribution of the economic activity between the three economic areas.

## References

- Ago, T., Isono, I., Tabuchi, T.: Locational disadvantage of the hub. *Ann. Reg. Sci.* **40**(4), 819–848 (2006)
- Baldwin, R.E., Forslid, R., Martin, P., Ottaviano, G.I.P., Robert-Nicoud, F.: *Economic Geography and Public Policy*. Princeton University Press, Princeton (2003)
- Basile, R., Commendatore, P., De Benedictis, L., Kubin, I.: An investigation of interregional trade network structures. In: Commendatore, P., Matilla-Garcia, M., Varela, L.M., Cánovas, J. S. (eds.) *Complex Networks and Dynamics Social and Economic Interaction*. Springer, Heidelberg (2016)
- Basile, R., Commendatore, P., De Benedictis, L., Kubin, I.: The impact of trade costs on the european regional trade network: an empirical and theoretical analysis. *Rev. Int. Econ.* (2017). doi:[10.1111/roie.12314](https://doi.org/10.1111/roie.12314), forthcoming
- Behrens, K.: Agglomeration without trade: how non-traded goods shape the space economy. *J. Urban Econ.* **55**, 68–92 (2004)
- Behrens, K.: How endogenous asymmetries in interregional market access trigger regional divergence. *Reg. Sci. Urban Econ.* **35**, 471–492 (2005)
- Behrens, K.: International Integration and Regional Inequalities: How Important Is National Infrastructure? *Manchester Sch.* **79**, 952–971 (2011)
- Commendatore, P., Kubin, I.: A three-region new economic geography model in discrete time: preliminary results on global dynamics. In: Bischi, G.-I., Chiarella, C., Sushko, I. (eds.) *Global Analysis of Dynamic Models for Economics, Finance and Social Sciences*. Springer, Heidelberg (2013)
- Commendatore, P., Filoso, V., Grafeneder-Weisteiner, T., Kubin, I.: Towards a Multiregional NEG Framework: Comparing Alternative Modelling Strategies. In: Commendatore, P., Kayam, S., Kubin, I. (eds.) *Complexity and Geographical Economics, Dynamic Modeling and Econometrics in Economics and Finance*. Springer, Cham (2015a)
- Commendatore, P., Kubin, I., Sushko, I.: Typical bifurcation scenario in a three-region identical new economic geography model. *Math. Comput. Simul.* **108**, 63–80 (2015b)
- Commendatore, P., Kubin, I., Sushko, I.: Trade agreements and regional disparities. In: Bednar-Friedl, B., Kleinert, J. (eds.) *Dynamic Approaches to Global Economic Challenges*. Springer, Cham (2016)
- Commendatore, P., Kubin, I., Sushko, I.: Emerging trade patterns in a 3-Region linear NEG model: three examples. In: Commendatore, P., Kubin, I., Bougheas, S., Kirman, A., Kopel M., Bischi, G.-I. (eds.) *The Economy as a Complex Spatial System: Macro, Meso and Micro perspectives*. Springer, Cham (2017b)
- Commendatore, P., Kubin, I., Sushko, I.: The dynamics of a three-region model linear NEG model. *Commun. Nonlinear Sci. Numer. Simul.* (2017a). doi:[10.1016/j.cnsns.2017.04.006](https://doi.org/10.1016/j.cnsns.2017.04.006), forthcoming
- Commendatore, P., Kubin, I., Mossay, P., Sushko, I.: The role of centrality and market size in a 4-region asymmetric new economic geography model. *J. Evol. Econ.* (2017c). forthcoming
- Commendatore, P., Kubin, I., Mossay, P., Sushko, I.: Dynamic agglomeration patterns in a two-country new economic geography model with four regions. *Chaos, Solitons Fractals* **79**, 2–17 (2015c)



- Commendatore, P., Kubin, I., Petraglia, C., Sushko, I.: Regional integration, international liberalisation and the dynamics of industrial agglomeration. *J. Econ. Dyn. Control* **48**, 265–287 (2014)
- Furusawa, T., Konishi, H.: Free Trade Networks. *J. Int. Econ.* **72**, 310–335 (2007)
- Ikeda, K., Murota, K.: Bifurcation theory for hexagonal agglomeration in economic geography. Springer, Japan (2014)
- Ikeda, K., Onda, M., Takayama, Y.: Bifurcation theory of a square lattice economy: Racetrack economy analogy in an economic geography model, MPRA Paper No. 78120 (2017)
- Krugman, P.: Increasing returns and economic geography. *J. Polit. Econ.* **99**, 483–499 (1991)
- Kubin, I., Commendatore, P., Sushko, I.: Some dynamical models in regional economics: economic structure and analytic tools. In: Bischi, G.-I., Panchuk, A., Radi, D. (eds.) *Qualitative Theory of Dynamical Systems, Tools and Applications for Economic Modelling*. Springer, Cham (2016)
- Ottaviano, G.I.P., Tabuchi, T., Thisse, J.-F.: Agglomeration and Trade Revisited. *Int. Econ. Rev.* **43**, 409–435 (2002)
- Okubo, T., Picard, P.M., Thisse, J.F.: On the impact of competition on trade and firm location. *J. Reg. Sci.* **54**(5), 731–754 (2014)
- Stelder, D.: Where do Cities Form? A Geographical agglomeration model for Europe. *J. Reg. Sci.* **45**, 657–679 (2005)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# **The Meso Perspective – Financial Markets**

# Systemic Risk and Macroeconomic Fat Tails

Spiros Bougheas<sup>1(✉)</sup>, David Harvey<sup>1</sup>, and Alan Kirman<sup>2</sup>

<sup>1</sup> School of Economics, University of Nottingham, Nottingham NG7 2RD, UK  
[spiros.bougheas@nottingham.ac.uk](mailto:spiros.bougheas@nottingham.ac.uk)

<sup>2</sup> École des Hautes Études en Sciences Sociales, CAMS - EHESS, Paris, France

**Abstract.** We propose a mechanism for shock amplification that potentially can account for fat tails in the distribution of the growth rate of national output. We argue that extreme macroeconomic events, such as the Great Depression and the Great Recession, were preceded by significant turmoil in the banking system. We have developed a model of bank network formation and presented numerical simulations that show that, for the benchmark case, aggregate credit follows a random walk. When we introduce fire sales the model does not only produce larger variations in the growth of aggregate credit but also shows that there is an asymmetry between booms and busts that is also consistent with empirical evidence.

**Keywords:** Systemic risk · Banking system · Aggregate risk · Financial network · Fat tails

## 1 Introduction

The two most severe macroeconomic crises of the last 100 years, namely, the Great Depression of the 1930s and the Great Recession that commenced at the close of the first decade of the current century, were preceded by extreme events in financial markets in general and the banking system in particular. In a recent study, Schularick and Taylor (2012) have empirically identified a historical link between the level aggregate credit in the economy and macroeconomic performance. They argue that aggregate credit can be a powerful predictor of economic crises, especially, rare catastrophic events.

Our aim is to provide a microfoundational explanation for the above relationship. In this work we focus on the behavior of aggregate credit. In particular, we analyze the dynamics of aggregate bank credit in an economy where all financial transactions are intermediated through the banking system. Viewing the financial system as a network of banks that are connected through their financial obligations to each other, we examine how the impact of shocks on the asset side of the banking balance sheets may disrupt the supply of aggregate credit.

Each period the capacity of a bank to finance new projects depends on the size of its balance sheet which, in turn, depends on the success rate of the projects that it financed the period before. Each period, the total capacity of the banking system to finance new projects entirely depends on the aggregate

liquidity available within the system. At the beginning of each period each bank's available liquidity (reserves) is equal to the sum of its household deposits plus its equity.

Projects are allocated sequentially and randomly across banks. A bank that faces a demand for funds but has run out of liquidity can borrow from other banks. This process creates a network of banks where its links, that reflect inter-bank exposures, are both directed and weighted.

As banks are unable to completely diversify their loan portfolios they can become insolvent. This will be the case when the total loan repayments (from both entrepreneurs and other banks) are insufficient to cover their obligations to their depositors and other banks. In order to clear the banking system when some banks become insolvent we apply the method suggested by Eisenberg and Noe (2001). Insolvencies can propagate through the banking network. When one bank is unable to meet its obligations to another bank, the latter bank might itself become insolvent even if it would have remained solvent had its loans to the originally failed bank been repaid. The bankruptcy resolution process terminated when there are no insolvent banks left. The number of bank failures will depend on (a) the distribution of initial losses across the banking system, and (b) the structure of the financial network (see, for example, Acemoglu et al. 2015).

As long as the liquidation of assets held by insolvent institutions does not depress the market values of these assets the total systemic losses by the end of the resolution process will be equal to the initial losses due to the inability of entrepreneurs to repay their loans. However, as Shleifer and Vishny (1992) have argued during systemic episodes, exactly because there are many failing institutions, the market value (liquidation value) of the assets can drop below their corresponding book values (fire sales). These drops in asset prices forces other institutions to reevaluate their own assets thus potentially causing new rounds of failures.<sup>1</sup>

In our model, when we do not allow for fire sales, the value of aggregate credit provided by the banking network follows a random walk. This is because the capacity of the banking network to provide credit each period depends on the availability of reserves which in turn depends on the performance of aggregate loans the period before. Given that shocks are normally distributed each period it follows that aggregate lending activity follows a random walk. When we introduce fire sales we observe that systemic losses can be much greater than initial losses thus introducing fat tails on the lower end of the distribution of aggregate credit. Under the supposition that aggregate credit is positively correlated with aggregate output our approach might be useful for accounting two features of business cycles: (a) the asymmetry in booms and busts (Acemoglu and Scott, 1991), and (b) macroeconomic fat tails Acemoglu et al. 2017a).

Our work is related to many strands of the economics literature. Our main premise is that bank leverage can be the source of systemic risk which in turn can lead to fat tails in the distributions of many macroeconomic aggregates.

---

<sup>1</sup> For models of fire sales see Diamond and Rajan (2011) and Caballero and Simpssek (2013). For a review of the literature on fire sales, see Shleifer and Vishny (2011).

The relationship between leverage in financial markets and fat tails is also addressed by Thurner et al. (2012).

In recent years a number there have been some attempts to build network models of the economy that can account for the fat tails in the distribution of the growth rate of national output. This literature has been motivated by the inability of traditional DSGE models to account for such fat tails (see Ascari et al. 2015). Acemoglu et al. (2012, 2017a) have analyzed production networks where the aggregate effects of idiosyncratic shocks depend on both the initial distribution of shocks and the structure of the network.<sup>2</sup> Anthonissen (2016) considers dynamic versions of similar economies. Thurner et al. (2012) show how leverage, through bankruptcies, can exacerbate volatility.

There is also a growing related literature that develops agent-based models to study the relationship between financial markets and the macroeconomy. Ashraf et al. (2017) integrate a banking sector with an agent-based economy where the source of turmoil is the market for goods. In contrast, in our work we view the financial sector as the one being responsible for the amplification effects on shocks. Battiston et al. (2007) consider the propagation of bankruptcies in production networks while the source of system risk in Geanakoplos et al. (2012) is the housing market. For an empirical investigation of the relationship between the macroeconomy and systemic risk, see Giglio et al. (2016).

Lastly, our work is related to a very large literature that uses network analysis to address issues related to systemic risk in banking systems. The interested reader is referred to the literature reviews on this subject by Acemoglu et al. (2017b), Babus and Allen (2009), Bougheas and Kirman (2015), and Glasserman and Young (2016).

## 2 The Model Without Fire Sales

Time is discrete ( $t = 0, 1, \dots$ ); each period  $t$  is divided into sub-periods ( $\tau = 0, 1, \dots$ ). There is a set of  $n$  banks with a typical element  $b_i$ , where ( $i = 1, \dots, n$ ). Table 1 shows the general form of a bank balance sheet:

**Table 1.** Bank balance sheet

Assets	Liabilities
Reserves: $R_i$	Deposits from Households: $D_i^H$
Loans to Households: $L_i^H$	Deposits from other Banks: $D_i^B$
Loans to Banks: $L_i^B$	Equity: $E_i$

There is a single divisible good that banks can (a) hold as reserves, (b) lend it to households to finance projects, and (c) lend it to other banks. Banks accept

<sup>2</sup> See Carvalho (2014) for an overview of this approach.

deposits from households and from other banks (loans from other banks). We assume bank equity is given by:

$$E_i \equiv R_i + L_i^H + L_i^B - D_i^H - D_i^B \quad (1)$$

Further, the following condition must hold for any closed bank network:

$$\sum_{i=1}^n L_i^B = \sum_{i=1}^n D_i^B$$

We set the net interest rate on household deposits  $r_D$  equal to 1, the net interest rate on household loans  $r_L$  equal to  $\frac{1}{\theta}$  (given (a) limited liability debt contracts, and (b) zero-profit condition for risk-neutral banks), and the net interest rate on interbank loans  $r_B$  equal to  $\frac{1+\theta}{2\theta}$  (nash-bargaining splits the difference between the interest rates on households loans and household deposits between the two banks).<sup>3</sup>

At the beginning of each period  $t$  ( $\tau = 0$ ) the balance sheet of  $b_i$  is shown on Table 2 (all entries are endogenously determined):

**Table 2.** Bank balance sheet at ( $\tau = 0$ )

Assets	Liabilities
$R_{i0} \geq 0$	$D_{i0}^H \geq 0$
$L_{i0}^H = 0$	$D_{i0}^B = 0$
$L_{i0}^B = 0$	$E_{i0} \geq 0$

**Projects.** Projects require 1 unit of investment, last for one period and yield a stochastic return. With probability  $\theta$  they yield a gross return  $Z$  and with probability  $1 - \theta$  they fail yielding nothing, where  $\theta Z \geq 1$ . Projects returns are independently distributed.

**Network Formation.** The demand for project financing is infinitely elastic. Projects are financed sequentially and the allocation of projects to banks is random. As long as there exists at least one bank with reserves greater or equal to unity the banking system will keep financing new projects. Thus, the aggregate credit provided each period will be approximately equal to aggregate reserves.<sup>4</sup> Suppose that  $b_i$  is allocated a project. If  $R_i \geq 1$ ,  $b_i$  offers a loan and the following two changes take place on its balance sheet  $\Delta L_i^H = +1$  and  $\Delta R_i = -1$ . If  $R_i < 1$  then  $b_i$  randomly selects another bank, say  $b_j$  and request an interbank loan.

<sup>3</sup> It will become clear below that our qualitative results are not sensitive to the processes by which the three interest rates are determined.

<sup>4</sup> To keep the program simple we do not allow projects to be financed by multiple banks. This means that aggregate lending might be less than aggregate reserves. However, given that the number of banks is small relative to the amount of aggregate reserves this simplification will not have any qualitative influence on our results.

If  $R_j \geq 1$ ,  $b_j$  offers an interbank loan to  $b_i$  and the latter finances the project. The balance sheet changes for the two banks are given by:  $\Delta D_i^B = +1$ ,  $\Delta L_i^H = +1$ ,  $\Delta R_j = -1$  and  $\Delta L_j^B = +1$ . If  $R_j < 1$  then  $b_i$  repeats the process by randomly selecting one of the remaining banks. The whole process terminates when no bank has reserves greater or equal to unity. At the end of the network formation process ( $\tau = 1$ ) the balance sheet of  $b_i$  is shown on Table 3:

**Table 3.** Bank balance sheet at ( $\tau = 1$ )

Assets	Liabilities
$R_{i1} < 1$	$D_{i1}^H \geq 0$
$L_{i1}^H \geq 0$	$D_{i1}^B \geq 0$
$L_{i1}^B \geq 0$	$E_{i1} \geq 0$

**Household Loan Repayments.** Close to the end of period  $t$  project returns are realized and loans granted for successful projects are repaid. Despite the fact that project returns are independently distributed the finiteness of a bank’s loan portfolio implies that bank equity can take negative values, that is banks can become insolvent. Now, balance sheets also reflect the interest payments due. Table 4 shows the bank balance sheet after loans are repaid or written off but before the interbank market clears ( $\tau = 2$ ):

**Table 4.** Bank balance sheet at ( $\tau = 2$ )

Assets	Liabilities
$R_{i2} \geq 0$	$D_{i2}^H = D_{i1}^H$
$L_{i2}^H = 0$	$D_{i2}^B = \frac{1+\theta}{2\theta} D_{i1}^B$
$L_{i2}^B = \frac{1+\theta}{2\theta} L_{i1}^B$	$E_{i0} \geq 0$

Reserves have been augmented by loan repayments. As projects last only one period all loans to households are either repaid or written off. Identity (1) is used for the calculation of bank equity.

**Bankruptcy Resolution Process (No Fire Sales).** All banks with negative equity are insolvent and they will be liquidated. The proceeds of the liquidation process will be distributed *pro rata* to all the liability holders. In this section, we assume that the market value of liquidated assets are equal to their book values. That is, for the moment, we assume no fire sales. Below we extend our model by including fire sales. We follow the method proposed by Eisenberg and Noe (2001) for clearing the banking network. This procedure, at least for the case with

no fire sales, is independent of the order that insolvent banks are liquidated.<sup>5</sup> This is because interbank loans are only settled after the bankruptcy resolution process is completed. Completion implies either that all remaining banks are solvent or that no banks are left. Below we describe the algorithm that we used to implement the procedure.

If there are any banks with negative equity choose one randomly, say  $b_i$ . Let  $\lambda \equiv \frac{D_{i2}^H}{D_{i2}^H + D_{i2}^B}$ ; the ratio of household deposits to total deposits. Let  $d_i^j$  denote the liabilities of  $b_i$  to  $b_j$  (interbank loan from  $b_j$  to  $b_i$ ). Let  $l_i^k \equiv d_k^i$  denote an interbank loan from  $b_i$  to  $b_k$ .<sup>6</sup> Then, the assets of  $b_i$  will be distributed to its creditors *pro rata* as follows<sup>7</sup>: The depositors of  $b_i$  will receive a fraction  $\lambda$  of (a)  $R_{i2}$ , and (b)  $l_i^k$  for all  $k$  ( $1, \dots, n$ ).<sup>8</sup> Each bank  $b_j$  such that  $d_i^j > 0$  will receive a fraction  $\frac{d_i^j}{D_{i2}^H + D_{i2}^B}$  of (a)  $R_{i2}$ , and (b)  $l_i^k$  for all  $k$  ( $1, \dots, n$ ). Given that  $\sum_{j=1}^n d_i^j = D_{i2}^B$  the above process will redistribute all the assets of  $b_i$  to its two classes of creditholders.

Next consider changes on the balance sheets of all other banks following the resolution of  $b_i$ . Consider any bank  $b_j$  such that  $d_i^j > 0$  and any bank  $b_k$  such that  $d_k^i > 0$ . Then,

1.  $b_k$  will set its liabilities to  $b_i$  equal to zero:  $\Delta d_k^i = -d_k^i$ ;
2.  $b_k$  will increase its household deposits:  $\Delta D_{k2}^H = \lambda d_k^i$ ;
3.  $b_k$  will increase its deposits by bank  $b_j$ :  $\Delta d_k^j = \frac{d_i^j}{D_{i2}^H + D_{i2}^B} d_k^i$ ;
4.  $b_j$  will set its loan to  $b_i$  equal to zero:  $\Delta l_j^i = -l_j^i$ ;
5.  $b_j$  will increase its loans to bank  $b_k$ :  $\Delta l_j^k = \frac{d_i^j}{D_{i2}^H + D_{i2}^B} d_k^i$ .

This will end the bankruptcy procedure for  $b_i$ . Notice that given that some of the creditors of  $b_i$  were not fully repaid it is possible that some banks that before the above process were solvent now they are insolvent. If there are any banks insolvent then by choosing one of them randomly the whole procedure repeats itself. If there are no more insolvent banks the bankruptcy resolution process terminates. After the resolution process all the assets of insolvent banks would have been distributed between their depositors and the remaining banks.

<sup>5</sup> Uniqueness is achieved under very mild conditions. In the Appendix we provide a numerical example.

<sup>6</sup> According to the network formation process of our model it is possible that both  $L_{i2}^H$  and  $D_{i2}^H$  to be positive. However, you cannot have a pair of banks where each one has offered a loan to the other.

<sup>7</sup> In this work we have followed Eisenberg and Noe (2001) and have assumed that all creditors are treated equal. As Acemoglu et al. (2015) have shown the clearing process can easily be modified to allow a class of creditholders (e.g. depositors) to have a priority claim over the bank's assets. There is an ongoing debate over the design of optimal priority rules for banks (for a review of the relevant literature see Bougheas and Kirman 2016).

<sup>8</sup> Clearly  $l_i^i = 0$ , and  $l_i^k = 0$  if  $b_i$  has not offered a loan to  $b_k$ .



**Interbank Market Clearing.** Given that all remaining banks are solvent all outstanding loans in the interbank market can be settled. After the interbank market is cleared the balance sheets of solvent banks will have exactly the same form as those at  $\tau - 0$ .

**Dynamics.** In order to complete the description of the model we need to specify how the initial balance sheets are formed, what happens to the banking systems after some banks failed and what happens to the depositors of failed banks.

At  $t = 0$ , each bank is randomly allocated reserves,  $R$ , drawn from a uniform distribution with support  $[0, \bar{R}]$ . Household deposits are set equal to a fixed ratio  $\delta$  of reserves. Thus, bank equity is equal to  $(1 - \delta)R$ .

At the end of each period  $t$ , new banks enter to replace the ones that have been insolvent. We make this assumption to ensure that the banking system does not vanish. It would then seem natural to have households whose original banks became insolvent to deposit their funds at the new banks. However, there is a problem. On one hand, new banks, on average, would be smaller in size as their depositors have suffered losses. On the other hand, given that the allocation of projects to banks is random these new banks would be disproportionately highly indebted to other banks and given that their equity levels would also be low they would fail again with a high probability. Put differently, the dynamics of the system would be such that in the long-run the banking system would artificially become very highly concentrated. There are two possible ways to avoid this problem. The first one, and the one that we have followed in this paper, is to have all reserves randomly redistributed in the system. While this approach is much simpler and, as a result, our main results very easy to interpret, it destroys some interesting dynamic interactions. The second approach is to allow households who had deposits at failed institution to have them now depositing their funds at the new banks but now change the network formation process so that banks with higher reserves have a higher probability of being allocated a project. This second approach is more realistic, however, when we introduce fire sales, it makes it more difficult to assess the exact mechanism that produces the distribution of aggregate credit shocks.

Lastly, we adjust equity and deposits at each bank so that at the beginning of  $t + 1$  the ratio of deposits to reserves is equal to  $\delta$ .<sup>9</sup>

### 3 Results Without Fire Sales

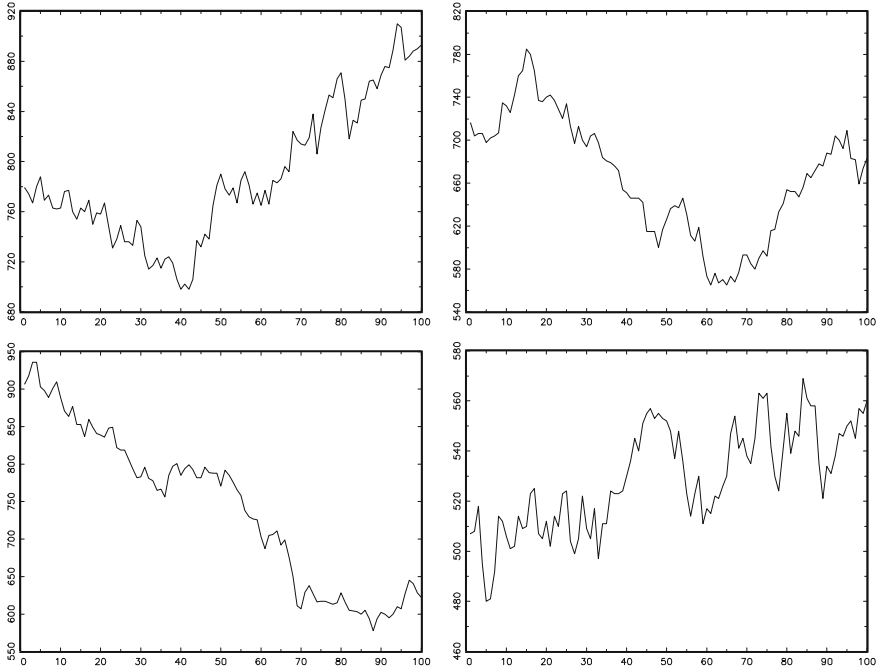
The number of banks that will become insolvent each period would depend on three factors. The first factor is the realized proportion of successful projects. Even if each project fails with probability  $1 - \theta$ , the economy is finite and the law of large numbers does not hold. The resulting aggregate uncertainty implies that the realized proportion of successful projects will vary over time.

---

<sup>9</sup> The recapitalization is clearly necessary for all new banks that otherwise would begin with no equity.

The second factor is the distribution of failed projects across the banking system. Other things equal, the more uneven this distribution is the higher the numbers of insolent banks will be. The third factor is the structure of the banking network. Acemoglu et al. (2015) have shown that as long as the initial losses are not too large a more connected banking system can provide a buffer against contagion as losses are distributed across many banks. In contrast, when initial losses are large a less connected system might prevent such contagion.

Even if it is difficult to predict the number of banks that will fail following an aggregate shock, the level aggregate of credit provided by the banking system follows a well defined pattern. At the beginning of each period  $t$  aggregate credit is approximately equal to aggregate reserves,  $\hat{R}_t$ .<sup>10</sup> Projects that are successful boost reserves and equity of the banks that financed them. The losses of the projects that failed are initially absorbed by the equity of the banks that financed them. If this equity is not large enough to absorb the losses then the creditholders of the bank absorb the losses, that is its depositors and other banks.



**Fig. 1.** Aggregate credit, no fire sales

<sup>10</sup> The approximation qualification is due to the fact that we do allow banks to co-finance projects and thus aggregate credit is less than or equal to aggregate reserves but more than or equal to aggregate reserves minus the number of banks. Given that the number of banks is relative to the level of aggregate reserves is relatively small in what follows we ignore this approximation.

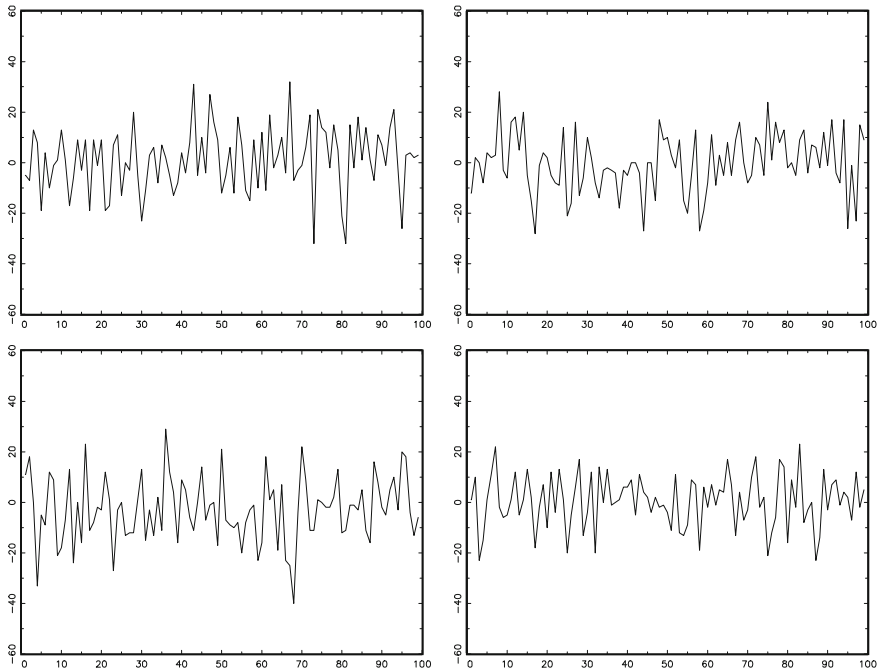
Thus when the process ends all losses have been absorbed either by the depositors or equityholders. Given that at the end of the period all liquidation proceeds are redeposited in the banking system aggregate credit follows a random walk (with drift if  $\theta Z > 1$ ).

$$\hat{R}_{t+1} = \hat{R}_t + \varepsilon_t$$

Notice that the error term depends on the realized number of successful projects and thus it is binomially distributed, however, the fact that the number of projects is large implies that the distribution is approximately normal.

The introduction of fire sales below complicates significantly the dynamic behavior of the model and we will have to use calibrations. Below we present calibration results for the benchmark case when fire sales are set equal to zero.

**Numerical Results.** For our calibration exercise we set the following parameter values:  $n = 20$ ,  $\bar{R} = 100$ ,  $\theta = 0.8$ ,  $Z = 1.25$ , and  $\delta = 0.8$ . In this particular case there is no growth as  $\theta Z = 1$ . The four panels of Fig. 1 show four of aggregate credit for 100 periods, while the four panels of Fig. 2 show the corresponding runs of the first differences of aggregate credit activity. These examples just verify our assertion that without fire sales the dynamic path of aggregate credit not inconsistent with a random walk process.



**Fig. 2.** First differences of aggregate credit, no fire sales

## 4 The Model with Fire Sales

In our model the only assets that banks hold when they get liquidated are reserves and loans to other banks. In reality banks would hold many other assets including outstanding loans to households. When banks are forced to liquidate any assets, either loans to households (firms) or any other liquid assets that they hold as reserves, the prices of these assets can fall below their book values. Such a fall in prices can be the result of either (a) asymmetric information problems arising from allocating assets, such as loans to firms, to new creditors, or (b) price externalities generated when many institutions are attempting to sell their assets at the same time. The first to consider fire sales within a financial equilibrium framework was Shleifer and Vishny (1992). More closely relate to our work Caballero and Simsek (2013) have considered fire sales in a banking network.<sup>11</sup>

As we have demonstrated in the section above, without introducing liquidation costs the aggregate performance of the banking system is completely determined by the realized distribution of initial shocks. Put differently, the network structure only affects the distribution of gains and losses across the system but not their aggregate value.

When we analyzed above the bankruptcy resolution process for the case without liquidation costs we noted that insolvent banks distribute their reserves  $R_{i2}$  *pro rata* among their creditors. The presence of liquidation costs implies that now the value of reserves (liquid assets) distributed  $R_{i3}$  will be lower than  $R_{i2}$ .

Below we consider two alternative amplification mechanisms for aggregate shocks.

**Linear Liquidation Costs.** Suppose that when a bank's assets (reserves) are liquidated they lose a fraction  $f$  of their book value. Thus, we have

$$R_{i3} = (1 - f)R_{i2}$$

The linearity restriction refers to the fact that  $f$  is independent of the number of banks that become insolvent,  $\hat{n} \leq n$ . The availability of aggregate credit at the beginning of the next period will depend on how the following three factors affect  $\hat{n}$ : (a) the aggregate value of loan repayments (b) the distribution of loan repayments across the network, and (c) the structure (topology),  $g$ , of the network.

Without liquidation costs the evolution of aggregate credit depended only on the aggregate value of loan repayments. However, with the introduction of liquidity costs the performance of aggregate credit will now depend on the other

---

<sup>11</sup> For assets that are not liquid traditional 'mark to market' accounting evaluation methods tend to exacerbate such problems. This is because such asset revaluations might also affect institutions that are not directly connected with institutions that have become insolvent. In our work we are unable to capture such effects given that we do not explicitly allow for illiquid assets.

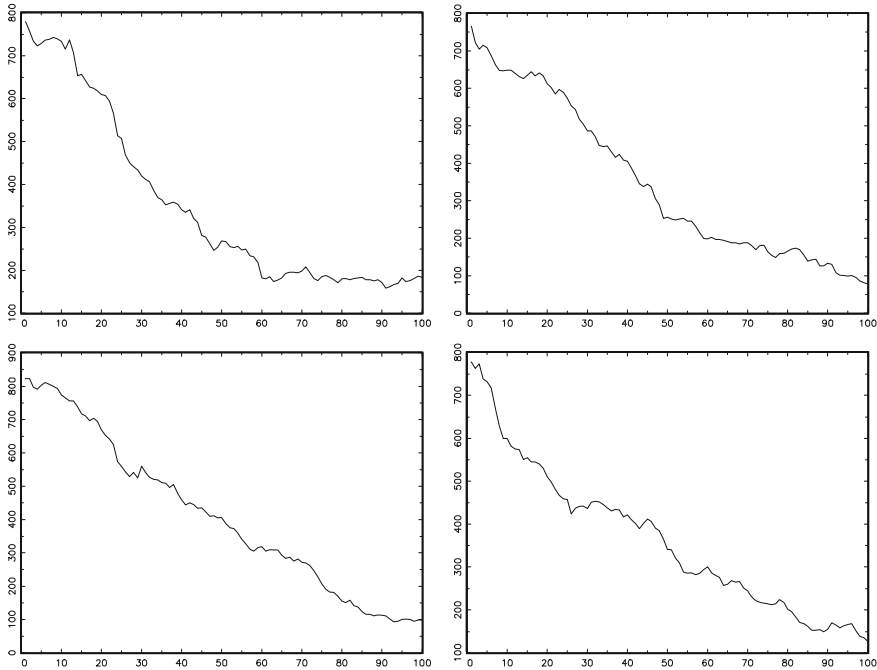
two factors.<sup>12</sup> For a given network structure, the total losses would depend on how connected the affected banks are. For example, isolated banks do not impose any external effects. But the total losses would also depend on the structure of the network itself (see Acemoglu et al. 2015).

Thus, whether or not the linear case is sufficient to produce fat tails in the distribution of aggregate credit would depend on how the last two factors affect  $\hat{n}$ .

**Non-Linear Liquidation Costs.** In this case we allow for the liquidation cost to depend on the number of banks that become insolvent. Suppose that  $b_i$  is the  $j$ th bank that is liquidated. Then, we let

$$R_{i3} = (1 - f)^j R_{i2}$$

The idea here is that as the number of banks that become insolvent increase the more depressed asset values become.



**Fig. 3.** Aggregate credit, fire sales

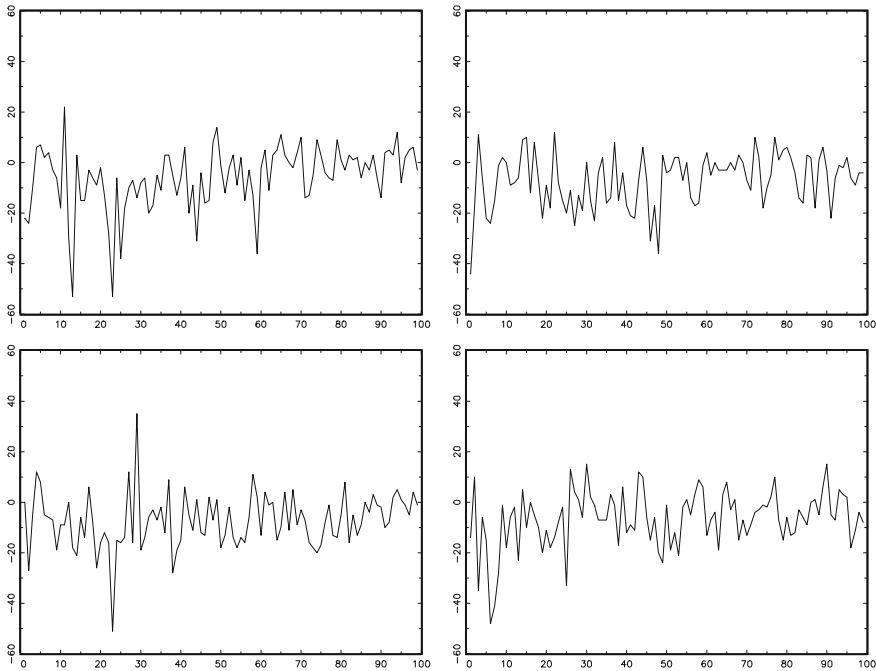
<sup>12</sup> The amplification mechanism is asymmetric as it affects only losses. However as we will see below, even if the initial aggregate shock is positive, that is the number of successful projects is greater than  $\theta$ , depending on the distribution of failed projects across the banking system it is still possible that aggregate credit declines.

## 5 Results with Fire Sales

We use the same parameters as for the case without fire sales and we also set  $f = 0.05$ .

There is one additional complication when we introduce fire sales. We have noted above that for the case without liquidation costs the final outcome of the bankruptcy resolution process does not depend on the exact sequence by which insolvent banks are liquidated. Unfortunately, this is not the case anymore. We control for this complication by allowing for multiple randomizations after we reach that stage.

The four panels of Fig. 3 show four of aggregate credit while the four panels of Fig. 4 show the corresponding runs of the first differences of aggregate credit activity. Comparing these figures with the corresponding figures obtained in the case without fire sales we make the following observations. From Fig. 3 we find that aggregate credit almost vanishes. This is because the asymmetry of the amplification mechanism. While there is nothing to boost the performance of institutions that are unaffected by bankruptcies those that are affected are suffering from additional losses due to fire sales. Without compensating by allowing for growth, that is  $\theta Z > 1$ , the distribution is not stationary. Nevertheless, Fig. 4, at least in the early periods when the size of the banking system is still relatively large, clearly shows that the magnitude of negative shocks has increased.



**Fig. 4.** First differences of aggregate credit, fire sales

These preliminary observations provide hints that our proposed mechanism might account for both the fat tails and the asymmetric nature of the distribution of aggregate economic activity over time.

## 6 Conclusion

We have suggested that some of the properties of the time series of aggregate output might be accounted by the behavior of aggregate credit activity. Our work provides a microfoundations explanation for the relationship between the provision of aggregate credit and macroeconomic crises observed by Schularick and Taylor (2012). In particular, we have argued that by analyzing the causes of systemic risk in the financial system can potentially help us understand both the presence of fat macroeconomic tail risk and the asymmetry between booms and busts along the business cycle.

We have captured systemic risk through an interbank network with links representing interbank exposures. Each period banks borrow and lend to each other so that they can finance loans to households that are randomly allocated across the banking system. Given that households loans are risky and lack of complete diversification the loan portfolio of each bank is also risky and banks can become insolvent. By applying standard methods for the bankruptcy resolution process we have captured the process of contagion across the banking system as failing banks put at risk their own creditors and thus providing a measure of systemic risk.

We have presented some preliminary numerical results where we compare two versions of our model, namely, one with and one without fire sales, for the case when there is no economic growth. For the case without fire sales we have shown that aggregate credit follows a random walk. The introduction of fire sales has significantly amplified negative shocks on aggregate credit. Without a stabilizing mechanism at the limit aggregate credit vanishes. Below we discuss our plans for extending the present work.

Our first priority is to allow for growth by setting  $\theta Z > 1$  so that the distribution of the growth rate of aggregate credit becomes stationary. We will be able then to compare the moments of the distribution for the two versions of our model and thus provide a quantitative assessment of the ability of our model to produce both fat tails and asymmetries in the distribution of aggregate credit.

As we explained in Sect. 2, unless we introduce a mechanism for rebalancing the system we would end up with some small-size banks being heavily indebted and thus repeatedly failing. In order to avoid this from happening we have, at the end of each period, redistributed deposits across the banking system. Doing so has allowed us to keep the network formation process completely random. We plan to explore an alternative method where larger banks face a higher demand for household loans. This is a more natural way to model the banking system, however, the disadvantage is that we introduce another potential source of variation in the model, mainly tails in the distribution of the size of banks, that could also affect systemic risk. However, comparing the two methods we should be able to disentangle these effects.

Another important task would be to try to improve our understanding of the relationship between network structure and systemic risk. We can do that by producing estimates for network measures (e.g. average degree, centrality) for each period of our model and then by checking how these measures are correlated with the corresponding growth rates of aggregate credit at the end of the period and the aggregate shock at the beginning of the period.

Lastly, our plan is to embed the whole banking structure in an agent-based model of the whole economy so that we can assess how fluctuations in the supply of credit are related to booms and busts of the economy. At the minimum, we would hope that our model would account for the fat tails in the growth rate of aggregate economic activity and the asymmetry in the patterns between booms and busts. However, by also allowing for an endogenous growth process we would hope that the more general framework would also provide an account not only for the long-term growth patterns of economic activity but also for the persistence in aggregate shocks.

**Acknowledgement.** We would like to acknowledge financial support from COST Action IS1104 “The EU in the new economic complex geography: models, tools and policy analysis”.

## A Appendix: Numerical Example

We present an example that demonstrates how the bankruptcy resolution process works and why the outcome is independent of the order of bank resolutions. There are three banks:  $b_1$ ,  $b_2$  and  $b_3$ . For ease of exposition we have set all net interest rates equal to 0. The balance sheets of the three banks at the beginning of the period are given by (Table 5):

**Table 5.** A1: Initial balance sheets

	$b_1$	$b_2$	$b_3$
$R$	1	1	1
$L^H$	0	2	2
$L^B$	$1(b_2)$	$1(b_3)$	0
<i>Assets</i>	2	4	3
$D^H$	1	2	1
$D^B$	0	$1(b_1)$	$1(b_2)$
$E$	1	1	1
<i>Liabilities</i>	2	4	3

Suppose that all projects failed. Then the three balance sheets after the writing off of losses are given by (Table 6):



**Table 6.** A2: Balance sheets adjusted for losses

	$b_1$	$b_2$	$b_3$
$R$	1	1	1
$L^H$	0	0	0
$L^B$	$1(b_2)$	$1(b_3)$	0
<i>Assets</i>	2	2	1
$D^H$	1	2	1
$D^B$	0	$1(b_1)$	$1(b_2)$
$E$	1	-1	-1
<i>Liabilities</i>	2	2	1

Banks  $b_2$  and  $b_3$  are insolvent.<sup>13</sup> We need to consider two cases:

**Bankruptcy Resolution Process Begins with  $b_3$ .** The reserves of  $b_3$  will be divided equally between the depositors of  $b_3$  and  $b_2$ . After this step the balance sheets are given by (Table 7):

**Table 7.** A3: Balance sheets after the resolution of  $b_3$ 

	$b_1$	$b_2$	$b_3$
$R$	1	1.5	0.5
$L^H$	0	0	0
$L^B$	$1(b_2)$	0	0
<i>Assets</i>	2	1.5	0.5
$D^H$	1	2	0.5
$D^B$	0	$1(b_1)$	0
$E$	1	-1.5	0
<i>Liabilities</i>	2	3	0.5

The reserves of  $b_2$  will be divided pro-rata between the depositors of  $b_2$  and  $b_1$ . After this step the balance sheets are given by (Table 8):

<sup>13</sup> After a bank is liquidated it ceases to exist. However, to keep track of what happened to its depositors we assume that another bank (with the same name) has replaced it where households can deposit their liquidation proceeds.

**Table 8.** A4: Balance sheets after the resolution of  $b_2$

	$b_1$	$b_2$	$b_3$
$R$	1.5	1	0.5
$L^H$	0	0	0
$L^B$	0	0	0
<i>Assets</i>	1.5	1	0.5
$D^H$	1	1	0.5
$D^B$	0	0	0
$E$	0.5	0	0
<i>Liabilities</i>	1.5	1	0.5

The process has been completed.

**Bankruptcy Resolution Process Begins with  $b_2$ .** The reserves and deposits in  $b_3$  of  $b_2$  will be divided pro-rata between the depositors of  $b_2$  and  $b_1$ . The depositors of  $b_2$  will receive  $2/3$  in reserves (keep them as deposits) and  $2/3$  in deposits in  $b_3$ .  $b_1$  will receive  $1/3$  in reserves and  $1/3$  in deposits in  $b_3$ . After this step the balance sheets are given by (Table 9):

**Table 9.** A5: Balance sheets after the resolution of  $b_2$

	$b_1$	$b_2$	$b_3$
$R$	$4/3$	$2/3$	1
$L^H$	0	0	0
$L^B$	$1/3(b_3)$	0	0
<i>Assets</i>	$5/3$	$2/3$	1
$D^H$	1	$2/3$	$5/3$
$D^B$	0	0	$1/3(b_1)$
$E$	$2/3$	0	-1
<i>Liabilities</i>	$5/3$	$2/3$	1

The reserves of  $b_3$  will be divided *pro rata* between the depositors of  $b_2$  depositors of  $b_3$  and  $b_1$ . Depositors of  $b_3$  will receive  $1/2$  in reserves, depositors of  $b_2$  will receive  $1/3$  in reserves and  $b_1$  will receive  $1/6$  in reserves. After this step the balance sheets are given by (Table 10):

**Table 10.** A6: Balance sheets after the resolution of  $b_3$ 

	$b_1$	$b_2$	$b_3$
$R$	1.5	1	0.5
$L^H$	0	0	0
$L^B$	0	0	0
<i>Assets</i>	1.5	1	0.5
$D^H$	1	1	0.5
$D^B$	0	0	0
$E$	0.5	0	0
<i>Liabilities</i>	1.5	1	0.5

The process has been completed. The results for the two cases are identical.

## References

- Acemoglu, D., Carvalho, V., Ozdaglar, A., Tahbaz-Salehi, A.: The network origins of aggregate fluctuations. *Econometrica* **80**, 1977–2016 (2012)
- Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A.: Systemic risk and stability in financial networks. *Am. Econ. Rev.* **105**, 564–608 (2015)
- Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A.: Microeconomic origins of macroeconomic tail risks. *Am. Econ. Rev.* **107**, 54–108 (2017a)
- Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A.: Networks, shocks, and systemic risk. In: Bramouille, Y., Galeotti, A., Rogers, B. (eds.) *The Oxford Handbook on the Economics of Networks*. Oxford University Press, NY (2017b)
- Acemoglu, D., Scott, A.: Asymmetric business cycles: theory and time-series evidence. *J. Monetary Econ.* **40**, 501–533 (1991)
- Anthonissen, N.: Microeconomic shocks and macroeconomic fluctuations in a dynamic network economy. *J. Macroecon.* **47**, 233–254 (2016)
- Ascari, G., Fagiolo, G., Roventini, A.: Fat-tail distributions and business cycle models. *Macroecon. Dyn.* **19**, 415–476 (2015)
- Ashraf, Q., Gershman, B., Howitt, P.: Banks, market organization, and macroeconomic performance: an agent-based computational analysis. *J. Econ. Behav. Organ.* **135**, 143–180 (2017)
- Babus, A., Allen, F.: Networks in finance. In: Kleindorfer, P., Wind, J. (eds.) *Network-Based Strategies and Competencies*, pp. 367–382. Wharton School Publishing, Upper Saddle River (2009)
- Battiston, S., Delli Gatti, D., Gallegati, M., Greenwald, B., Stiglitz, J.: Credit chains and bankruptcy propagation in production networks. *J. Econ. Dyn. Control* **31**, 2061–2084 (2007)
- Bougheas, S., Kirman, A.: Complex financial networks and systemic risk: a review. In: Commendatore, P., Kayam, S., Kubin, I. (eds.) *Complexity and Geographical Economics: Topics and Tools*. Springer, Heidelberg (2015)
- Bougheas, S., Kirman, A.: Bank insolvencies, priority claims and systemic risk. In: Commendatore, P., Matilla-Garcia, M., Varela, L., Canovas, J. (eds.) *Complex Networks and Dynamics: Social Economic Interactions*. Springer, Heidelberg (2016)

- Caballero, R., Simsek, A.: Fire sales in a model of complexity. *J. Finance* **68**, 2549–2587 (2013)
- Carvalho, V.: From micro to macro via production networks. *J. Econ. Perspect.* **28**, 23–248 (2014)
- Diamond, D., Rajan, R.: Fear of fire sales, illiquidity seeking, and credit freezes. *Q. J. Econ.* **126**, 557–591 (2011)
- Eisenberg, L., Noe, T.: Systemic risk in financial systems. *Manag. Sci.* **47**, 236–249 (2001)
- Geanakoplos, J., Axtell, R., Farmer, D., Howitt, P., Conlee, B., Goldstein, J., Hendrey, M., Palmer, N., Yang, C.-Y.: Getting at systemic risk via an agent-based model of the housing market. *Am. Econ. Rev. Papers Proc.* **102**, 53–58 (2012)
- Giglio, S., Kelly, B., Pruitt, S.: Systemic risk and the macroeconomy: an empirical evaluation. *J. Financ. Econ.* **119**, 457–471 (2016)
- Glasserman, P., Young, P.: Contagion in financial networks. *J. Econ. Lit.* **54**, 779–831 (2016)
- Schularick, M., Taylor, A.: Credit booms gone bust: monetary policy, leverage cycles, and financial crises 1870–2008. *Am. Econ. Rev.* **102**, 1029–1061 (2012)
- Shleifer, A., Vishny, R.: Liquidation values and debt capacity: a market equilibrium approach. *J. Finance* **47**, 1343–1366 (1992)
- Shleifer, A., Vishny, R.: Fire sales in finance and macroeconomics. *J. Econ. Perspect.* **25**, 29–48 (2011)
- Thurner, S., Farmer, D., Geanakoplos, J.: Leverage causes fat tails and clustered volatility. *Quant. Finance* **12**, 695–707 (2012)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# Market Interactions, Endogenous Dynamics and Stabilization Policies

Noemi Schmitt<sup>1</sup>, Jan Tuinstra<sup>2</sup>, and Frank Westerhoff<sup>1</sup>(✉)

<sup>1</sup> Department of Economics, University of Bamberg, Bamberg, Germany  
frank.westerhoff@uni-bamberg.de

<sup>2</sup> Amsterdam School of Economics and CeNDEF, University of Amsterdam,  
Amsterdam, The Netherlands

**Abstract.** We review a recent literature that shows that interactions between markets, created by the market entry and exit behavior of boundedly rational firms, may cause complex endogenous dynamics. In particular, these models predict that welfare decreases if firms rapidly switch between markets. Against this background, we show that policy makers have the opportunity to stabilize markets and thus to enhance welfare by regulating interacting markets. For instance, imposing profit taxes reduces the markets' profit differentials and thus slows down the firms' market entry and exit behavior. However, these stabilization policies may also lead to undesirable side effects, such as coexistence of attractors, hysteresis effects and, in a multi-region setting, failure of policy makers to coordinate on the globally optimal policy. Moreover, regulation may be subject to the lobbying efforts of special interest groups and thus not be optimal.

**Keywords:** Market interactions · Endogenous dynamics · Welfare effects · Stabilization policies

**JEL classification:** D83 · E30 · H20

## 1 Introduction and Outline

In the wake of the financial crisis that hit the global economy by the end of the noughties many economists and policy makers realized that the strong links between individual markets played an important role in allowing the crisis to spread globally, or may even have been at the core of the emergence of the crisis (see, amongst others, Karras and Song 1996, Bordo et al. 2001 and Shiller 2015). This has spawned a literature that deals both with the effect that interactions between markets have on market stability, and with the policy measures that may be implemented to counter the instabilities that potentially arise from these interactions. While some policy measures indeed stabilize markets and thereby improve welfare, other policy measures may yield surprising and unwanted side effects. In this chapter we will review a small part of that literature.

That individual markets may lead to instability has been recognized for some time already. Classic textbook examples are the cobweb model under naïve expectations (see Ezekiel 1938) or the Cournot oligopoly model under best reply dynamics (see

Theocharis 1959). More recently the development of the theory of nonlinear dynamical systems has led to an increased attention for the possibility of market instability. Some early and important applications of this theory are Grandmont (1985) and Bullard (1994) on overlapping generations models, Chiarella (1988), Hommes (1994) and Brock and Hommes (1997) on cobweb markets, Day and Huang (1990), Lux (1995) and Brock and Hommes (1998) on financial markets and Puu (1991) and Kopel (1996) on Cournot duopoly models. Laboratory experiments with paid human subjects suggest that instability is indeed likely to occur in some of these market environments (see e.g. Hommes et al. 2005 and Heemeijer et al. 2009).

In the last decade the interaction between markets has been identified as an additional route to market instability. Dieci and Westerhoff (2009, 2010), for example, find that two stable cobweb markets may become unstable when they are linked. Tuinstra et al. (2014) show that this increased instability may result in the counterintuitive policy prescription that under certain circumstances strictly positive import tariffs are welfare enhancing. Even in the absence of naïve price expectations and cobweb dynamics, linking two markets may lead to instability, as demonstrated by Schmitt et al. (2017a, b). If firms are sufficiently sensitive to profit differences between markets this may lead to unstable dynamics. Following the insights from Schmitt and Westerhoff (2015, 2017), the papers by Schmitt et al. (2017a, b) investigate how the introduction of profit taxes may dampen the profit differences between the two markets and thereby stabilize the dynamics and increase welfare. However, these profit taxes may also induce undesirable side effects, such as coexistence of attractors and hysteresis effects (Schmitt et al. 2017a), or it may turn out to be difficult for regulators to coordinate on a globally optimal profit tax policy (Schmitt et al. 2017b).

The remainder of this chapter is organized as follows. In Sect. 2 we provide a brief review of the literature on market interactions, and Sect. 3 discusses some contributions that analyze the stabilizing or destabilizing effect that regulatory policies may have on the dynamics of interacting markets. Section 4 concludes.

## 2 Market Interactions

The benefits of a non-regulated market economy depend crucially on the assumption that markets are stable and prices attain their equilibrium values. In this way a (Pareto) efficient allocation of scarce resources will be established, and governments have no reason to interfere in the market process (other than for, typically subjective, distributional concerns). Market stability therefore has been, and continues to be, an important field of research in economics. One of the simplest and intuitive models to study this issue is by means of the so-called cobweb model, initially introduced by Ezekiel (1938). This model represents a market where suppliers face a production lag. That is, it takes one period for their (non-storable) product to be produced, implying that the producers have to make a supply decision one period in advance – this setting is relevant, for example, for many agricultural markets. The optimal supply decision consequently depends upon the producers' expectation of the market clearing price in the next period. The actual market price is the price that clears the market, that is, the price that equates consumer demand for the produced commodity with the aggregate

supply that was determined, on the basis of producers' price expectations, in the previous period. If the number of producers on the market and the consumer demand schedule for the produced commodity is constant over time this gives rise to a unique 'fundamental' steady state price. If the producers have *rational expectations* (that is: they know the full market structure, including consumer demand and cost functions of all producers), and if in addition there is *common knowledge of rationality* (that is: every producer knows that every producer is rational, and knows that every producer knows that every producer is rational, and so on) they will coordinate on this fundamental steady state price (and correctly predict it).

However, these assumptions are very strong, and less demanding forms of expectation formation have been suggested in the literature. In particular, at the other extreme we can find naïve expectations, as discussed in the original model of Ezekiel (1938), where producers predict next period's price to be equal to the last observed price (that is, the price in this period). Introducing naïve expectations for all producers implies that prices evolve according to a first order difference equation which, depending on the relative slopes of the consumer demand and producer supply functions, as well as on the number of producers, may lead to oscillating converge to the unique steady state, to diverging oscillations, or to a period two cycle, where market clearing prices jump back and forth between a low and a high level.

More complicated unstable dynamics may emerge in the cobweb model for more sophisticated, but still boundedly rational, prediction strategies. For example, Chiarella (1988) and Hommes (1994, 1998) consider adaptive expectations, where the price expectation is adapted in the direction of the last observed price. They show that this expectation mechanism can lead to more erratic dynamics in prices and predictions. Another promising avenue to study the role of expectations in the stability of the cobweb market was advanced by the influential work of Brock and Hommes (1997). They assume that producers in the cobweb model form expectations either in a rational or in a naïve manner, and switch between these 'forecasting heuristics' on the basis of past forecasting accuracy. The interaction between cobweb dynamics and endogenous switching provides an intuitive mechanism that leads to endogenous fluctuations, both in market clearing prices and in the distribution of producers over the different expectation rules. For other contributions in this direction, see for example, Goeree and Hommes (2000), Branch (2002) and Chiarella and He (2003). That the cobweb model has a tendency to result in complicated dynamics is also confirmed by laboratory experiments (see Hommes et al. 2007) and strategy experiments (Sonnemans et al. 2004).

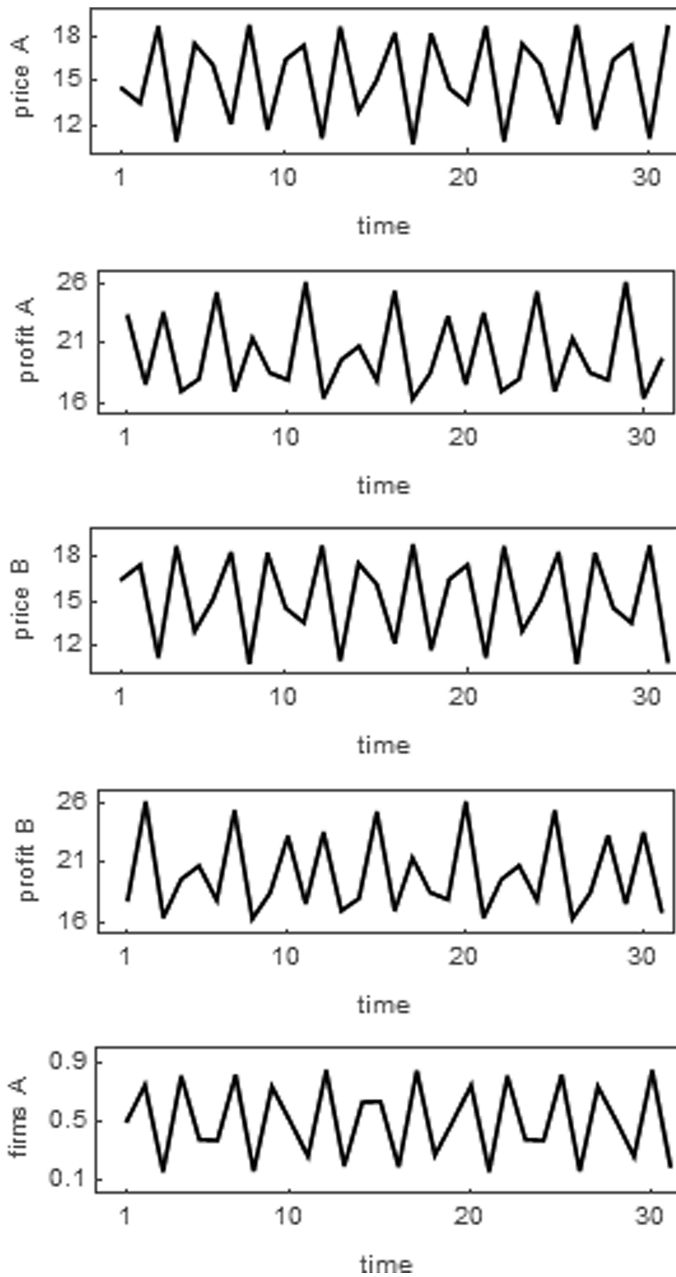
Note that, if producers switch between the stabilizing rational and the destabilizing naïve forecasting heuristic according to a performance measure that also includes past realized profits, policy makers may have an opportunity to stabilize markets. Indeed, Schmitt and Westerhoff (2015, 2017) show that policy makers may set profit taxes such that the more stabilizing forecasting heuristic gains in popularity, thereby calming down complex dynamics. For instance, rational expectations usually require some kind of information costs and may thus be less profitable than naïve expectations – at least at the steady state where predictions of rational and naïve expectations are the same. In such an environment, policy makers should increase profit taxes to reduce the forecasting heuristics' profitability. In doing so, they promote the use of rational expectations.

The literature discussed so far has focused on the dynamics of a single market. However, another route to complicated dynamics may arise when different markets are coupled and producers can switch between these markets, for example – similar to the mechanism used in the literature on switching between different expectation rules – based upon past (relative) profitability of being active on these two markets. Note that there might be different interpretations for the two markets. They may, for example, correspond to different regional markets for the same homogeneous commodity. Switching then refers to firms specializing on exporting their product to the other region, or even migrating their full production facility to that region. The markets may also relate to two different products that are produced and consumed in the same region, and that can be produced by slight (and cheap) modifications in the production technology. In this case switching means that firms use their productive assets for producing a different commodity.

Dieci and Westerhoff (2009, 2010) are among the first to investigate this switching of firms between markets, which gives a very intuitive explanation for complicated dynamics. To appreciate the mechanism it is important to understand that stability of the cobweb dynamics on an individual market depends upon the sensitivity of the aggregate supply function with respect to the expected price. Suppose aggregate supply responds aggressively to a change in the expected price. In that case a small change in the actual price generates, through producers' naïve expectations, a strong response from the producers and a price correction that tends to be larger than the initial price change. Now, if individual supply functions are the same for all producers, it follows immediately that an increase in the number of producers active on an individual market increases aggregate supply for each value of the expected price and, by the argument given above, tends to destabilize the price dynamics.

To see how the mechanism works consider two interacting markets, market A and market B, and suppose that the distribution of firms over these two markets is such that market A is more profitable than market B. This will attract firms from market B to market A. The resulting increased aggregate supply sensitivity on market A may very well destabilize that market, generating volatility in market clearing prices. In turn, this decreases profits on market A and there will be a tendency for firms to leave market A and enter market B, which may stabilize the dynamics on market A, but simultaneously destabilizes market B, after which the whole cycle repeats again. Note that we may have a scenario where in the absence of switching both markets are stable under cobweb dynamics, but connecting them by allowing firms to move between markets, makes both of them unstable. This is illustrated by Fig. 1, which considers a setting where, when considered in isolation, both cobweb markets are indeed stable under naïve expectations. The time series in Fig. 1 show that interaction destabilizes the markets. In period 1 firms are almost equally distributed between markets, and although markets are symmetric, profits in market A are slightly higher in period 1. Since the price in market A is below the fundamental value, under naïve expectations we would expect each firm in that market to have a relatively low supply, and that the market clearing price therefore goes up. However, the difference in profits in period 1 leads to an increase in the number of firms in market A in period 2, which *increases* aggregate supply in that market, and the market clearing price actually decreases and moves further *away* from the steady state. Similarly, although individual supply under naïve





**Fig. 1.** Interacting cobweb markets. The panels show the time evolution of prices in market A, profits in market A, prices in market B, profits in market B and the fraction of firms active in market A, respectively. Parameter setting as in Dieci and Westerhoff (2010).

expectations goes up in market B (because of the relatively high price in period 1), aggregate supply *decreases* because a number of firms move to market A. Also for this market the market clearing price moves away from the steady state. The net effect of these changes is that in period 2 market B is the most profitable and firms move back to market B, and so on. It is important to note that this type of dynamics may even hold if firms only respond to differences in past profits slowly (see also Tuinstra et al. 2014).

Although complicated behavior in interacting markets arises quite naturally in the case of cobweb dynamics (that is, when producers face a one period production lag and have naïve expectations), this type of behavior occurs more generally. First, also if firms have adaptive expectations the same mechanism may apply. Moreover, even in the extreme case of rational expectations, or alternatively when there is no production lag and the market clearing price is established immediately, the interaction between markets may lead to complicated dynamics, see Schmitt et al. (2017a, b). The dynamics in this case depend upon how responsive firms are with respect to past profits generated by the two regions. Consider, for example, the scenario where there are two markets that, given the number of firms active on the market, are always in equilibrium. That is, firms either have perfect foresight about the equilibrium price *after* they enter the market, or prices adjust instantaneously and firms do not need to predict the price, but observe it when they make their supply decision. The market equilibrium prices obviously depend upon the distribution of firms over the two markets. If that distribution is such that profits in market A are higher than in market B, firms active in the latter will move to the former. If they respond slowly to the profit differential the distribution of firms will gradually adjust such that in the end profits in the two markets are equalized. However, if firms are sensitive to the profit differential the number of firms moving to market A may be so large that the resulting equilibrium profits in that market become lower than that of market B and firms consequently move back. Through this process of *overshooting* complicated dynamics may emerge again.

Complicated dynamics also emerge in more elaborate general equilibrium environments. A number of models use a New Economic Geography perspective to model market interactions, for example, Agliari et al. (2011, 2014) and Commendatore et al. (2014, 2015). These contributions show that endogenous fluctuations may emerge from market interactions between different economic regions. Another interesting aspect of these contributions is that they investigate the tools that policy makers have to stabilize fluctuations, for example trading costs. In the next section we review the effect of different types of regulatory policies that may stabilize dynamics in market interaction models with a partial equilibrium flavor as discussed above. As we will see, some of these regulatory policies may also give rise to surprising side effects.

### 3 Stabilization Policies

Different types of regulatory policies can be implemented to stabilize the erratic dynamics generated by interacting markets. We will briefly discuss two of them here: the imposition of import tariffs between regions (see Tuinstra et al. 2014) and the use of profit taxes (Schmitt et al. 2017a, b).

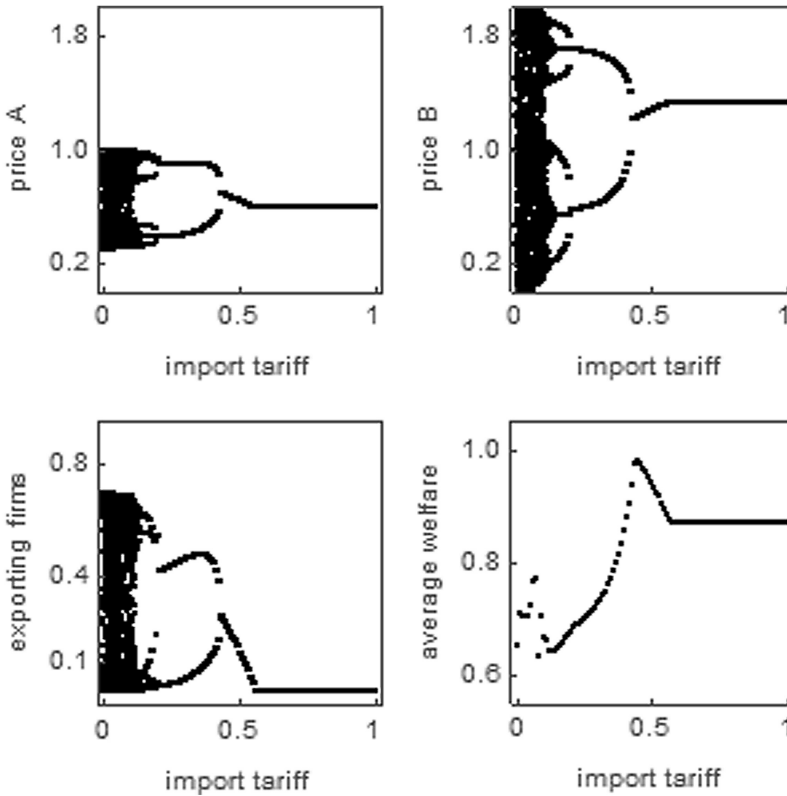
### 3.1 Optimal Trade Barriers

Tuinstra et al. (2014) extend the interacting cobweb markets, advanced by Dieci and Westerhoff (2009, 2010), by introducing trade barriers for firms in one region (say region A) that want to export their product to the other region (region B). The size of these trade barriers determines the level of interaction between the markets. On one hand, a prohibitively high trade barrier takes away any incentive for firms in one region to export to the other region, and consequently implies that markets function in isolation. On the other hand, in the absence of any trade barriers there will be free trade and no restrictions on exports. From Dieci and Westerhoff (2009, 2010) we know that the resulting unrestricted interaction between markets may lead to instability, even if markets are stable in isolation. If this is the case it follows that increasing trade barriers sufficiently (for example in the form of import tariffs) may stabilize markets.

However, trade barriers may also come at substantial economic costs. In particular, suppose that firms in region A are more productive, are more numerous and/or face relatively smaller consumer demand, relative to region B. Then, when comparing steady states with and without trade barriers, it follows straightforwardly that although consumers in region A and producers in region B may suffer from free trade, aggregate welfare in each of the regions (and thereby total welfare over the regions) increases. This conclusion is only valid, however, if markets remain stable. The trade-off between increased efficiency at the steady state that follows from diminishing trade barriers, versus the resulting increased instability means that the level of trade barriers that maximizes total aggregate welfare may be strictly positive. This contradicts conventional economic wisdom, which is based on comparing steady state allocations only. Figure 2 illustrates the main point. The figure shows bifurcation diagrams for increasing import tariffs. Starting with a high level of import tariffs it will not be profitable for firms in region A to export their product to region B, although market prices in region B are higher. If the import tariffs decrease firms from region A will start to export to region B, see the lower left panel of Fig. 2. The market clearing price in the latter will decrease, because of increasing supply, and the market clearing price in region A increases (because of decreasing supply). However, if import tariffs decrease too much, the inflow of supply to region B destabilizes that market and fluctuations in prices in both regions and in the export level emerge endogenously. The lower right panel shows aggregate welfare as a function of the import tariff. If the government objective can be characterized by the aim to maximize total aggregate welfare, it follows that the government will choose that level of the import tariff such that market dynamics are ‘just’ stable – that is, any further decrease in trade barriers would lead to welfare decreasing fluctuations. In Fig. 2 this corresponds to an import tariff of about 0.43.<sup>1</sup> Note that, if the government is incompletely informed, or if the economic environment is subject to regular (demand and/or supply) shocks, this optimal policy may (occasionally) lead to instability as well. Instability may furthermore occur if special interest groups, for example representing consumers and producers from the

---

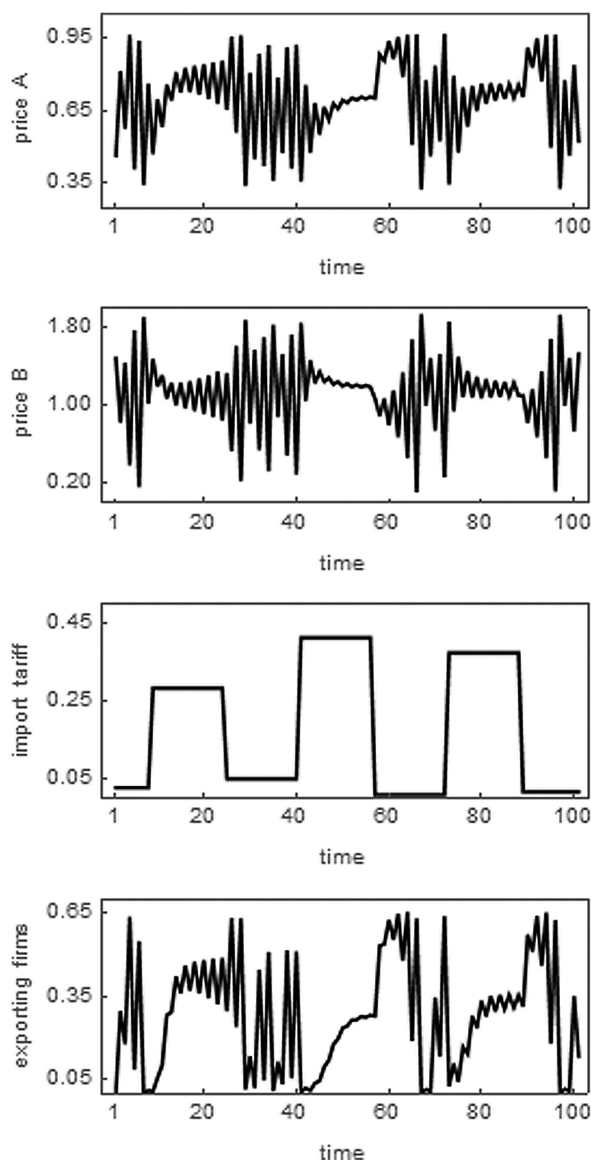
<sup>1</sup> Note that in general it is not apparent that fluctuations always reduce welfare. Indeed several contributions to the literature have shown that welfare-improving fluctuations are possible in some environments, see for example Dawid and Kopel (1999), Matsumoto (2003) and Huang (2008).



**Fig. 2.** Optimal barriers to entry. The panels show prices in market A, prices in market B, the fraction of exporting firms and average welfare versus import tariffs, respectively. Parameter setting as in Tuinstra et al. (2014).

different regions, lobby for a decrease or increase in trade barriers. Tuinstra et al. (2014) also present a model where trade barriers are endogenously determined by the lobbying efforts of these special interest groups. These efforts depend on the fluctuations in consumer and producer welfare, implying that trade barriers will vary over time. Figure 3 shows a simulation with this model, where import tariffs are revised every sixteen periods (that is, once in a political cycle of four years, if we interpret one period in the model as a quarter). Although the government of region B is in principle able to set an import tariff such that the markets are stable, the effect of lobbying is that the dynamics of prices and export levels becomes highly complicated.

There have been some other contributions to the literature that have similarly identified a potential trade-off between allocative efficiency on the one hand, and stability on the other. For example, Commendatore and Kubin (2009) show that deregulating labor and product markets may lead to instability and endogenous fluctuations, although it would increase steady state employment. Moreover, recent contributions in the field of New Economic Geography also analyze the effect of trade



**Fig. 3.** Import tariffs and lobbying efforts of special interest groups. The panels show the time evolution of prices in market A, prices in market B, import tariffs, and the fraction of exporting firms, respectively. Parameter setting as in Tuinstra et al. (2014).

costs on stability and volatility. Reducing trade barriers, by cutting trade costs, may lead to an increase in instability (see e.g. Agliari et al. 2011, 2014 and Commendatore et al. 2014, 2015). There is also some empirical evidence suggesting a relation between trade openness and volatility. A positive correlation between the two was found by

Karras and Song (1996). Moreover, Bordo et al. (2001) argue that the increased incidence of financial and economic crises is due to an increase in deregulation.

### 3.2 Profit Taxes

As discussed above, Schmitt and Westerhoff (2015, 2017) show that fluctuations on an individual market can be dampened or even fully stabilized by an appropriate level of profit taxes. The intuition is that, in an environment where firms switch strategies on the basis of their past profitability, and instability occurs because firms respond strongly to these profit differences, profit taxes curb the sensitivity with respect to profits. Schmitt et al. (2017a, b) analyze whether profit taxes can have a similar stabilizing effect when there are interacting markets. To that end, they consider a model where there is a large number of firms, and each period all firms have to decide simultaneously on which of the two markets to supply their good. For convenience it is assumed that, after entry decisions have been made, markets immediately adjust such that the market clearing price is established. Therefore there is no production lag and firms are not required to form price expectations. This implies that cobweb dynamics are absent. Equilibrium profits on each market decrease in the number of firms on that market, so firms have to solve a nontrivial coordination problem. If firms are quite sensitive with respect to profit differences the number of firms entering the market that was more profitable in the previous period tends to overshoot the steady state number of firms for that market, and complicated dynamics may emerge. Profit taxes may be introduced to mitigate the response of the firms, thereby slowing down the dynamics and eventually stabilizing it altogether. Schmitt et al. (2017a) and (b) each focus on two potential complications that may come to the fore when regulators contemplate using profit taxes to stabilize interacting markets.

A first complication derives from the observation that profit taxes typically only apply to strictly positive profits. That is, net profits (as a function of gross profits) have a kink at zero, with a slope equal to 1 for negative profits, and a slope equal to  $1 - \tau$  for positive profits, where  $0 \leq \tau < 1$  is the level of profit taxes. This implies that the evolutionary model where firms make entry decisions on the basis of past profits becomes a piecewise (one-dimensional) nonlinear map. These types of maps have been studied extensively in recent years (see Avrutin et al. 2018, for a general introduction and Commendatore et al. 2014, 2015, and Tramontana et al. 2010, 2013 for economic applications) and it turns out they typically give rise to an even richer set of complicated behaviors than smooth nonlinear maps already do.

Schmitt et al. (2017a) study the complications that emerge through the kink in the profit function. For convenience they focus on a stylized setting where there is one market and a safe outside option. Profits associated with the outside option are independent of the number of firms choosing that option. Firms have strictly positive fixed costs for supplying in the market implying that profits become negative if too many firms enter the market, and the kink in the profit function then becomes relevant.

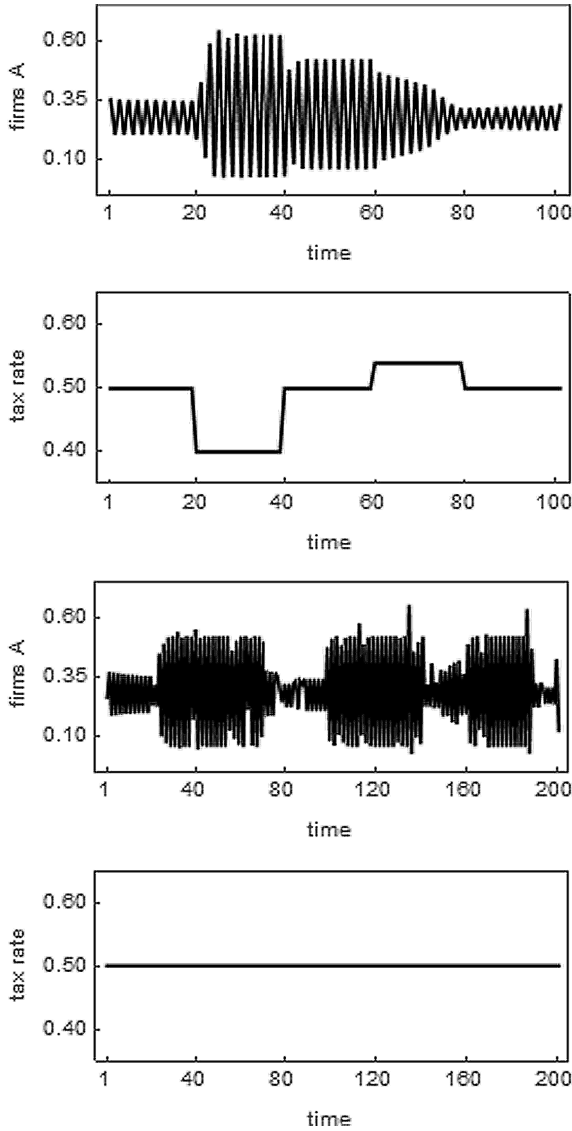
As long as profits remain strictly positive an increase in the sensitivity of firms with respect to profit differences will destabilize the dynamics through a so-called period doubling bifurcation and the fraction of entrants oscillates between a high and a low value. However, due to the kink in the profit function it turns out that a high-amplitude

period two cycle already exists when the steady state is locally stable. For some parameter values this high-amplitude period two cycle coexists with the locally stable steady state, and for other parameter values it coexists with a low-amplitude period two cycle. This coexistence of attractors leads to a number of intriguing dynamical phenomena. For example, a small decrease in the profit tax rate may lead to abrupt changes in the dynamics, because suddenly the low-amplitude period two cycle ceases to exist and the dynamics is attracted to the high-amplitude period two cycle. This is illustrated by the upper two panels in Fig. 4, which show that a decrease of the profit tax rate from 0.5 to 0.4 drives the dynamics to a high-amplitude period two cycle. It may prove difficult for the government to correct for this. Increasing the profit tax rate to its initial value typically does not suffice, as is confirmed by period 41–60 in the second panel of Fig. 4 and the corresponding dynamics in the first panel. A much higher increase in the profit tax rate may be required. This hysteresis effect is due to the kink in the profit function and turns out to be very robust to changes in the model. It imposes some important restrictions on stabilizing markets through profit taxes that regulators should be aware of. Even if the profit tax rate is constant over time interesting dynamical phenomena may emerge. The lower two panels of Fig. 4 show the dynamics when the tax rate is constant at 0.5 but where stochastic shocks hit the dynamical system occasionally. These shocks may move the dynamics from the basin of attraction of one of the coexisting attractors to that of the other attractor. The fluctuations that may emerge can be quite complicated, as illustrated by the third panel of Fig. 4.

Schmitt et al. (2017b) consider a variation of the model used by Schmitt et al. (2017a) with two important changes. First, the outside option is explicitly modelled as a different market, and second, firms have no fixed costs. The implication of the second adjustment is that firms will never make losses and therefore there is no kink in the profit function. This makes the model more suitable for understanding the effect of profit taxes in the two different markets (or regions) on the stabilization of complicated dynamics and on (the distribution of) welfare in the two different markets, which is exactly the aim of that paper.

By imposing a profit tax in both regions volatile market dynamics can be stabilized, see the first two lines of Fig. 5. The first line shows the effect of the so-called *intensity of choice*, a parameter that measures how strongly firms respond to profit differentials. The profit tax rates are zero in both regions, and an increase in the intensity of choice induces instability (left panel), which decreases total average welfare (right panel). The second line shows that this instability can be reversed by increasing profit tax rates in both regions (note that the intensity of choice parameter used in the second line corresponds to the maximum value in the panels on the first line).

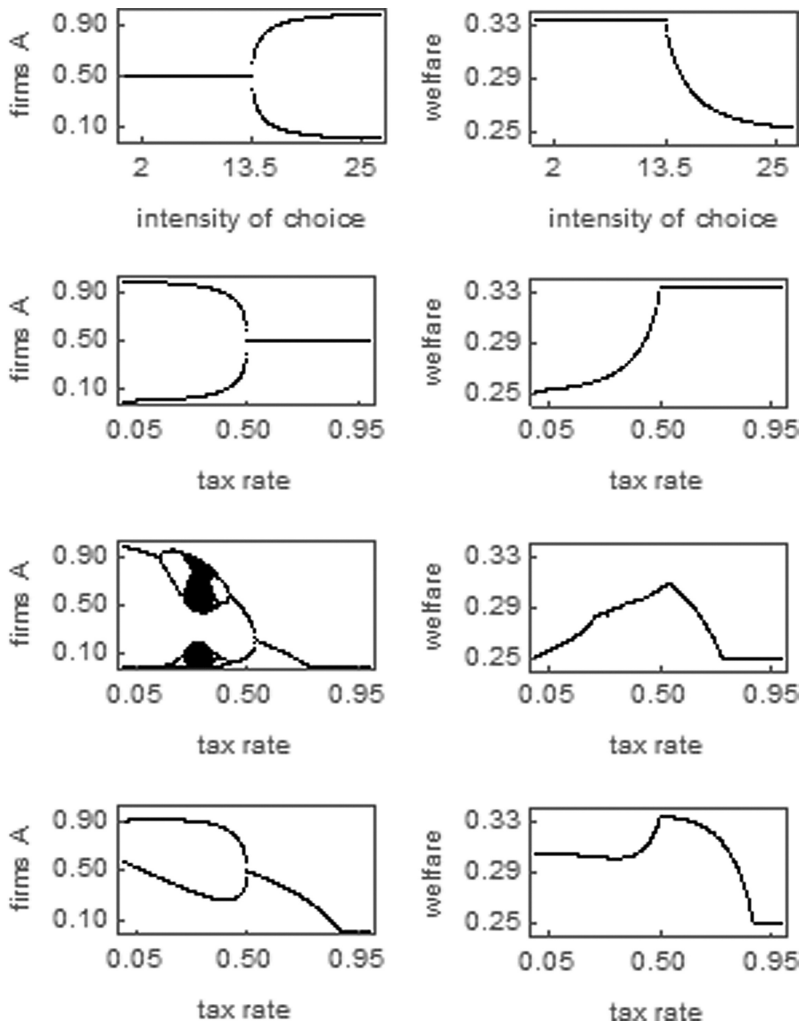
It turns out that, in the symmetric setting of the model, in order to stabilize the dynamics it will be sufficient to introduce a profit tax in only one of the two regions. The third line of Fig. 5 shows the case where only region A imposes a profit tax rate (again with intensity of choice equal to 27). Although the profit tax rate in region A that is required to stabilize the dynamics is higher than before, stabilization of the dynamics is still possible. However, because the difference in profit tax rates distorts the steady state distribution of firms between regions total aggregate welfare is not maximized in this way. If the aim of the regulators is to maximize average welfare aggregated over the two regions it is better to coordinate on profit taxes and set them at the same



**Fig. 4.** Time evolution of firms active in market A and the profit tax rate in market A. The first two panels illustrate the hysteresis effect, the latter two panels show the dynamics with occasional exogenous noise, with constant tax rates. Parameter setting as in Schmitt et al. (2017a).

(sufficiently high) level (see the fourth line of Fig. 5, where total average welfare is maximized when both regions set a tax rate of 0.5). Such a coordinated tax policy may be difficult to achieve in practice, since regulators in each region will have the incentive to reduce the profit tax and thereby attract more firms to their region. Although this may lead to instability, it will also increase tax revenues for the consumers in that region and





**Fig. 5.** The first two lines show bifurcation diagrams for the fraction of firms in region A and average welfare as a function of the intensity of choice (profit tax rate is zero), and as a function of the profit tax rate, taken to be equal in both regions. For the third (fourth) line the profit tax rate in region B is zero (0.5) and the profit tax rate in region A varies. The intensity of choice parameter equals 27 in all graphs (except for those on the first line, where this parameter varies from 0 to 27).

the net effect may be beneficial. In this way, a coordinated tax policy could easily unravel and the regions then become trapped into a regime with volatile markets and low welfare levels.

## 4 Conclusions and Outlook

In recent years the interaction between individual markets has received attention for its role in reinforcing, or even creating, instability and complicated endogenous dynamics. In this chapter we have reviewed some of the mechanisms that have been identified, as well as the effect of stabilization policies that have been put forward.

Tuinstra et al. (2014), for example, show that market interaction may imply that, although import tariffs between markets may decrease allocative efficiency at the steady state equilibrium, such tariffs may be welfare enhancing nevertheless. This is because they weaken the link between markets and thereby stabilize consumption and production patterns. Schmitt et al. (2017a, b) investigate how the introduction of profit taxes may stabilize interacting markets where firms responds strongly to past profit differences. Schmitt et al. (2017a) argue that profit taxes result in an additional nonlinearity in the dynamics (a kink of the profit function at zero), which introduces complicated dynamics such as coexistence of attractors and hysteresis. Schmitt et al. (2017b) discuss the scenario where each region is overseen by an independent local government or regulatory authority. Optimally, these two regulators coordinate their profit taxes in such a way that markets are stable and total welfare is maximized. However, Schmitt et al. (2017b) argue that, if regulators are only (or mainly) interested in welfare in their own region, each of them will have the incentive to decrease the profit tax, which can destabilize markets.

We conclude this chapter by briefly discussing several possible extensions to the work discussed here. One extension is to present the tax competition discussed in Schmitt et al. (2017b) as a game between the regulators of the two regions, where each regulator independently sets the profit tax rate for its own region with the goal to maximize average welfare in that region. The question then is under which conditions the Nash equilibria of this game are characterized by unstable market dynamics and suboptimal global welfare levels. Other interesting extensions naturally follow from the observation that the models that we discussed in this chapter are quite stylized. One simplifying assumption has been to consider a partial equilibrium framework. An obvious question is whether the main insights discussed in this chapter will also be valid in a full-fledged general equilibrium model. Moreover, thus far we have only looked at interaction between markets on the supply side. New insights may be obtained if markets are also connected through consumer demand. Similarly, it makes sense to consider models that depart from the assumption that producers can migrate free of any costs between markets or regions and that consumers are fully immobile. Relaxing these assumptions may to a certain extent mitigate some of the adverse effects of stabilization policies on stability, although we believe the basic mechanism will survive the generalizations discussed here.

## References

- Agliari, A., Commendatore, P., Foroni, I., Kubin, I.: Border collision bifurcations in a footloose capital model with first nature firms. *Comput. Econ.* **38**, 349–366 (2011)
- Agliari, A., Commendatore, P., Foroni, I., Kubin, I.: Expectations and industry location: a discrete time dynamical analysis. *Decis. Econ. Finance* **37**, 3–26 (2014)

- Avrutin, V., Gardini, L., Schanz, M., Sushko, I., Tramontana, F.: Continuous and Discontinuous Piecewise-Smooth One-Dimensional Maps: Invariant Sets and Bifurcation Structures. World Scientific, Singapore (2018)
- Bordo, M., Eichengreen, B., Klingebiel, D., Soledad, M., Martinez-Peria, M.S., Rose, A.K.: Is the crisis problem growing more severe? *Econ. Policy* **16**, 51–82 (2001)
- Branch, W.A.: Local convergence properties of a cobweb model with rationally heterogeneous expectations. *J. Econ. Dyn. Control* **27**, 63–85 (2002)
- Brock, W., Hommes, C.: A rational route to randomness. *Econometrica* **65**, 1059–1095 (1997)
- Brock, W., Hommes, C.: Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *J. Econ. Dyn. Control* **22**, 1235–1274 (1998)
- Bullard, J.: Learning equilibria. *J. Econ. Theory* **2**, 468–485 (1994)
- Chiarella, C.: The cobweb model – its instability and the onset of chaos. *Econ. Model.* **5**, 377–384 (1988)
- Chiarella, C., He, X.Z.: Dynamics of beliefs and learning under a(L)-processes – the heterogeneous case. *J. Econ. Dyn. Control* **27**, 503–531 (2003)
- Commendatore, P., Kubin, I.: Dynamic effects of regulation and deregulation in goods and labor markets. *Oxford Econ. Papers* **61**, 517–537 (2009)
- Commendatore, P., Kubin, I., Petraglia, C., Sushko, I.: Regional integration, international liberalisation and the dynamics of industrial agglomeration. *J. Econ. Dyn. Control* **48**, 265–287 (2014)
- Commendatore, P., Kubin, I., Sushko, I.: Typical bifurcation scenario in a three region symmetric New Economic Geography model. *Math. Comput. Simul.* **108**, 63–80 (2015)
- Dawid, H., Kopel, M.: On optimal cycles in dynamic programming models with convex return function. *Econ. Theory* **13**, 309–327 (1999)
- Day, R.H., Huang, W.H.: Bulls, bears and market sheep. *J. Econ. Behav. Organ.* **14**, 299–329 (1990)
- Dieci, R., Westerhoff, F.: Stability analysis of a cobweb model with market interactions. *Appl. Math. Comput.* **215**, 2011–2023 (2009)
- Dieci, R., Westerhoff, F.: Interacting cobweb markets. *J. Econ. Behav. Organ.* **75**, 461–481 (2010)
- Ezekiel, M.: The cobweb theorem. *Q. J. Econ.* **52**, 255–280 (1938)
- Goeree, J.K., Hommes, C.H.: Heterogeneous beliefs and the non-linear cobweb model. *J. Econ. Dyn. Control* **24**, 761–798 (2000)
- Grandmont, J.M.: On endogenous competitive business cycles. *Econometrica* **53**, 995–1045 (1985)
- Heemeijer, P., Hommes, C., Sonnemans, J., Tuinstra, J.: Price stability and volatility in markets with positive and negative expectations feedback: an experimental investigation. *J. Econ. Dyn. Control* **33**, 1052–1072 (2009)
- Hommes, C.H.: Dynamics of the cobweb model with adaptive expectations and nonlinear supply and demand. *J. Econ. Behav. Organ.* **24**, 315–335 (1994)
- Hommes, C.H.: On the consistency of backward-looking expectations: the case of the cobweb. *J. Econ. Behav. Organ.* **33**, 333–362 (1998)
- Hommes, C., Sonnemans, J., Tuinstra, J., van de Velden, H.: Coordination of expectations in asset pricing experiments. *Rev. Finan. Stud.* **18**, 955–980 (2005)
- Hommes, C., Sonnemans, J., Tuinstra, J., van de Velden, H.: Learning in cobweb experiments. *Macroecon. Dyn.* **11**, 8–33 (2007)
- Huang, W.: The long-run benefits of chaos to oligopolistic firms. *J. Econ. Dyn. Control* **32**, 1332–1355 (2008)
- Karras, G., Song, F.: Sources of business-cycle volatility: an exploratory study on a sample of OECD countries. *J. Macroecon.* **18**, 621–637 (1996)

- Kopel, M.: Simple and complex adjustment dynamics in Cournot duopoly models. *Chaos, Solitons Fractals* **7**, 2031–2048 (1996)
- Lux, T.: Herd behavior, bubbles and crashes. *Econ. J.* **105**, 881–896 (1995)
- Matsumoto, A.: Let it be: chaotic price instability can be beneficial. *Chaos, Solitons Fractals* **18**, 745–758 (2003)
- Puu, T.: Chaos in duopoly pricing. *Chaos, Solitons Fractals* **1**, 573–581 (1991)
- Schmitt, N., Tuinstra, J., Westerhoff, F.: Side effects of nonlinear profit taxes in a behavioral market entry model: abrupt changes, coexisting attractors and hysteresis problems. *J. Econ. Behav. Organ.* **135**, 15–38 (2017a)
- Schmitt, N., Tuinstra, J., Westerhoff, F.: Stability and welfare effects of profit taxes within an evolutionary market interaction model. *Review of International Economics*, forthcoming (2017b)
- Schmitt, N., Westerhoff, F.: Managing rational routes to randomness. *J. Econ. Behav. Organ.* **116**, 157–173 (2015)
- Schmitt, N., Westerhoff, F.: Evolutionary competition and profit taxes: market stability versus tax burden. *Macroeconomic Dynamics* (2017). in press
- Shiller, R.: *Irrational Exuberance*. Princeton University Press, Princeton (2015)
- Sonnemans, J., Hommes, C., Tuinstra, J., van de Velden, H.: The instability of a heterogeneous cobweb economy: a strategy experiment on expectation formation. *J. Econ. Behav. Organ.* **54**, 453–481 (2004)
- Theocharis, R.D.: On the stability of the Cournot solution to the oligopoly problem. *Rev. Econ. Stud.* **27**, 133–134 (1959)
- Tramontana, F., Westerhoff, F., Gardini, L.: On the complicated price dynamics of a simple one-dimensional discontinuous financial market model with heterogeneous interacting traders. *J. Econ. Behav. Organ.* **74**, 187–205 (2010)
- Tramontana, F., Westerhoff, F., Gardini, L.: The bull and bear market model of Huang and Day: some extensions and new results. *J. Econ. Dyn. Control* **37**, 2351–2370 (2013)
- Tuinstra, J., Wegener, M., Westerhoff, F.: Positive welfare effects of trade barriers in a dynamic partial equilibrium model. *J. Econ. Dyn. Control* **48**, 246–264 (2014)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# Looking Ahead: Part II

Spiros Bougheas<sup>1</sup>(✉) and Alan Kirman<sup>2</sup>

<sup>1</sup> School of Economics, University of Nottingham, Nottingham NG7 2RD, UK  
[spiros.bougheas@nottingham.ac.uk](mailto:spiros.bougheas@nottingham.ac.uk)

<sup>2</sup> École des Hautes Études en Sciences Sociales, CAMS - EHESS, Paris, France

**Abstract.** We propose a number of possibilities for future research on issues related to systemic risk in financial markets and its relationship to the macroeconomy. Some of the proposals aim to improve our understanding of the behavior of aggregate credit. Other proposals involve the development or richer agent-based models that potentially can help us understand the time-series properties of national output.

**Keywords:** Aggregate credit · Systemic risk · Banks, business cycles · Fat tails

## 1 Future Challenges

Traditional dynamic stochastic general equilibrium (DSGE) models of the macroeconomy view fluctuations of GDP around its trend as the product of aggregate shocks drawn from a lognormal distribution. The investment function of these models produces the observed autocorrelation in error terms (GDP follows an AR process) while a number of alternative mechanisms have been proposed to account for the increasing secular trend. However, there are other features of the behavior of the aggregate output trend that these models cannot explain. For example, persistence, that is the tendency for output shocks in one period to be correlated with those in the period before, is usually captured by directly imposing such pattern on the structure of macroeconomic shocks (error terms are serially correlated). It is also well known that there is an asymmetry in the behavior of aggregate output between expansions and slumps, whereby the former are smooth and the latter are sharp. Acemoglu and Scott (1991) have demonstrated how the last two features, namely persistence and asymmetry, can be accounted by allowing heterogeneity among producers and some form of increasing returns at the firm level. Lastly, as Ascari et al. (2015) have shown DSGE models cannot explain the observation that the tails of the distribution of aggregate shocks are ‘fat’; that is are inconsistent with normal draws. This last observation has recently motivated researchers to consider alternative approaches, such as network theory (e.g. Acemoglu et al. 2012, 2017; Carvalho 2014), agent-based modeling (e.g. Ashraf et al. 2017) and also frameworks that combine these two approaches (e.g. Battiston et al. 2007). In all these papers the amplification mechanism is produced within the production

sector. The financial sector either is non-existent or when it exists, as in Ashraf et al. (2017), is not responsible for the excess volatility in the system.

In Chap. 20 of this volume, we have suggested an alternative mechanism according to which the excess volatility is created by network effects in the banking system. Support for our approach is offered by Schularick and Taylor (2012) who have empirically identified a historical link between the level aggregate credit in the economy and macroeconomic performance. The authors argue that aggregate credit can predict economic crises, especially, rare catastrophic events, like the Great Depression and the Great Recession. These were the two more severe macroeconomic crises of the last 100 years, and both were preceded by extreme events in the banking system. For example, in the period 1930–1933 approx. 10,000 banks suspended operations or failed. The corresponding losses from the more recent crisis have been well documented.

Our central feature of our model was the presence of ‘fire sales’ (see, Shleifer and Vishny 2011). This is a price externality (see, Caballero and Simsek 2013) imposed by failing banks on other banks. As failing banks sale their assets, the prices of these assets drop causing other banks to reevaluate their assets at these new lower prices potentially leading to their own failure. Our work thus far has suggested that this mechanism is a good candidate for accounting for macroeconomic fat tails. Our aim is to firmly establish that this is indeed the case but also within a more general model that potentially can account for the other features of aggregate output described above.

Thus, looking ahead we are planning a number of extensions of our benchmark model. The first two are direct extension of the benchmark model focusing on the banking sector. The following two will be addressing issues related to the impact of the financial system on the rest of the economy.

**Network structure, fire sales and systemic risk.** Our first task is to try to understand how the interplay between the distribution of idiosyncratic shocks and network structure generate the amplification mechanism. We can do that by producing estimates of network measures (e.g. average degree, centrality) for each period of our model and then by checking how these measures are correlated with the corresponding growth rates of aggregate credit at the end of the period and the aggregate shock (aggregate of idiosyncratic shocks) at the beginning of the period.

**Core-periphery network structure, distribution of bank size and systemic risk.** We can experiment with alternative network formation specifications where banks that perform better face a higher demand for loans from firms. Actual banking networks have a core-periphery structure where banks in the core are responsible for a higher proportion of lending activity. Within such a framework we should be able to address important policy issues related to institutions that are either ‘too big’ or ‘too connected’ to fail.

The above issues have to varying degrees already been addressed in the literature. What is novel in our approach, is the introduction of fire sales in the general model. This is important because we will be able to get a measure of systemic losses. In models without fire sales, as we have already shown in Chap. 5,

the total losses are equal to the initial losses and thus these models can only address issues related to the total number of institutions being affected following a shock (more likely providing a minimum estimate as fire sales would more likely generate more failures) but not the impact of such failures on economic losses.

**An agent-based model of the macroeconomy with a bank network.** In its present form the model captures the dynamics of aggregate credit as generated by a dynamic interbank model. We plan to extend the model by introducing consumers (depositors) and producers (firms) in the model. By taking into account the saving decisions of agents and by explicitly introducing production technologies along with externalities we hope that our model will be able to account for (a) the empirical relationship between growth patterns and cyclical fluctuations, (b) the persistence in GDP growth movements, and (c) the asymmetry between booms and busts.

**Inequality and Business Cycles.** Recently, there has been a lot research on understanding global inequality trends (see, Piketty 2013). One issue that has been overlooked are the cyclical patterns of inequality. Our agent-based model will naturally produce inequality variations along the economic cycle and this can help us identify economic groups that are more likely to suffer from economic downturns.

As we argued in Chap. 5 the asymmetry between booms and busts can be naturally captured by fire sales. By introducing some externality at the firm level, following Acemoglu and Scott (1991), we can also explain persistence. For example, firms that succeed in one period are more likely to succeed in the following period. The additional advantage of such specification would be that potentially can account for the observed size distribution of firms (Gabaix 2011). Within such a generalized framework we could then compare alternative sources of macroeconomic stability (financial versus technological).

## References

- Acemoglu, D., Carvalho, V., Ozdaglar, A., Tahbaz-Salehi, A.: The network origins of aggregate fluctuations. *Econometrica* **80**, 1977–2016 (2012)
- Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A.: Microeconomic origins of macroeconomic tail risks. *Am. Econ. Rev.* **107**, 54–108 (2017)
- Acemoglu, D., Scott, A.: Asymmetric business cycles: theory and time-series evidence. *J. Monetary Econ.* **40**, 501–533 (1991)
- Ascari, G., Fagiolo, G., Roventini, A.: Fat-tail distributions and business cycle models. *Macroecon. Dyn.* **19**, 415–476 (2015)
- Ashraf, Q., Gershman, B., Howitt, P.: Banks, market organization, and macroeconomic performance: an agent-based computational analysis. *J. Econ. Behav. Organ.* **135**, 143–180 (2017)
- Battiston, S., Delli Gatti, D., Gallegati, M., Greenwald, B., Stiglitz, J.: Credit chains and bankruptcy propagation in production networks. *J. Econ. Dyn. Control* **31**, 2061–2084 (2007)

- Caballero, R., Simsek, A.: Fire sales in a model of complexity. *J. Financ.* **68**, 2549–2587 (2013)
- Carvalho, V.: From micro to macro via production networks. *J. Econ. Perspect.* **28**, 23–48 (2014)
- Gabaix, X.: The granular origins of aggregate fluctuations. *Econometrica* **79**, 733–772 (2011)
- Piketty, T.: *Capital*. Harvard University Press, Cambridge (2013)
- Schularick, M., Taylor, A.: Credit booms gone bust: monetary policy, leverage cycles, and financial crises, 1870–2008. *Am. Econ. Rev.* **102**, 1029–1061 (2012)
- Shleifer, A., Vishny, R.: Fire sales in finance and macroeconomics. *J. Econ. Perspect.* **25**, 29–48 (2011)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.





# **The Micro Perspective – Social and Industrial Interactions**

# A Dynamic Model of Firms' Strategic Location Choice

Luca Colombo<sup>1</sup> and Herbert Dawid<sup>2</sup>(✉)

<sup>1</sup> Department of Economics and Finance, Università Cattolica del Sacro Cuore,  
Largo Gemelli 1, 20123 Milano, Italy

lucava.colombo@unicatt.it

<sup>2</sup> Department of Business Administration and Economics  
and Center for Mathematical Economics, Bielefeld University,

P.O. Box 100131, 33501 Bielefeld, Germany

hdawid@wiwi.uni-bielefeld.de

**Abstract.** This paper analyzes the optimal location choice of a firm in a dynamic Cournot framework, in which firms' absorptive capacities may depend on their knowledge stock. The firm decides whether to locate irreversibly in a cluster or in isolation. In the cluster the firm benefits from inward spillovers from its competitors, but also generates outward spillovers. If the firm chooses to locate in isolation no knowledge flows occur. All firms' production costs are determined by their knowledge stocks, which evolve over time due to own R&D investments and potentially inward spillovers. It is shown that, if absorptive capacity is constant, the incentive to locate in the cluster decreases with respect to the firm's knowledge stock. Conversely, if absorptive capacity depends positively on knowledge stock, the firm's incentive to join the cluster is larger the more knowledge it has. It is also shown that qualitative properties of the equilibrium paths of R&D investments and knowledge stocks differ substantially depending on whether absorptive capacities are constant or knowledge dependent.

## 1 Introduction

Firms' location choices are affected by several factors. In particular, the pertinent literature – often focusing on the choices of multinational firms – has highlighted the role of the proximity to sales and factor markets, that of the local institutions and regulations, and that of local labor markets, especially in relation to availability of the appropriate skill mix (see e.g. Almazan et al. 2007; Lee and Mansfield 1996; Henisz 2000; De Beule and Duanmu 2012). Somewhat surprisingly, the effects of inward and (especially) outward knowledge spillovers on the location choices of firms have not received much attention, although such spillovers are known to have a large impact on firms' decisions. Given that knowledge is in many cases tacit and localized, a firm's ability to benefit from

---

Financial support under COST Action IS1104 “The EU in the new economic complex geography: models, tools and policy evaluation” is gratefully acknowledged.

knowledge flows and the associated spillovers is likely to depend to a large extent on geographical proximity. This establishes an immediate link between the relevance of knowledge spillovers and firms' strategic location choices, which is confirmed by the findings of a relatively recent (and mainly) empirical literature on the borders between Management and Industrial Organization. Alcacer and Chung (2007) find that technologically advanced firms tend to avoid locations with strong industrial activity in an attempt to distance themselves from competitors, favoring instead areas characterized by high levels of academic activity. Knowledge spillovers – more precisely, the consideration of the gains from inward spillovers (the opportunity for knowledge sourcing from other firms in the same location) vs. the costs of outward spillovers – are argued to lie at the hearth of the observed location choices. Along the same lines, Leiponen and Helfat (2011) show that firms mainly involved with imitative innovation tend to choose multiple locations for their R&D activities, while firms mainly dealing with 'new to the market' innovations do not. The heterogeneity between the location choices of the two types of firms can again be easily related to the differential impact of knowledge spillovers. The relevance of knowledge spillovers in clusters has been the object of a large literature (see e.g. Griliches 1992; Jaffe et al. 1993) mainly emphasizing the positive externalities – in terms of knowledge generation dynamics – stemming from firms' agglomerations (Head et al. 1995). The importance of outward spillovers – obviously a negative externality of local agglomerations – has been instead substantially underweighted in the literature, at least until Alcacer and Chung (2007).<sup>1</sup>

As already noted, despite the abundance of empirical and anecdotal evidence about the importance of knowledge spillovers for firms' location choices, relatively little theoretical work has been done but for a few notable exceptions. For instance, Gersbach and Schmutzler (1999) focus on the effects of internal and external knowledge spillovers on the location of production and innovative activities in a Bertrand duopoly. Knowledge spillovers play a role also in Piga and Poyago-Theotoky (2005) that focus on a Hotelling-type oligopoly where firms choose their locations, as well as their R&D efforts and prices. In their framework, however, firms' choices depend crucially on transportation costs that play no role at all in our setup.<sup>2</sup> Quite a few papers investigate the location choices of multinational firms. Among them, Gersbach and Schmutzler (2011) focus on the location of the production and R&D activities of multinational firms in the presence of knowledge sourcing. Belderbos et al. (2008) build on Gersbach and Schmutzler's (1999) setup to investigate the strategic location of R&D by two

<sup>1</sup> More recently, Mariotti et al. (2010) have stressed the negative role of technological leakages in the location decisions of multi-national firms, qualitatively confirming Alcacer and Chung's (2007) key insights. Furthermore, Belderbos et al. (2008) have shown that technological leaders are more attracted than followers by countries endowed with better intellectual property right protection mechanisms, indirectly confirming that firms are afraid of possible outward spillovers.

<sup>2</sup> Note that in Piga and Poyago-Theotoky (2005) firms choose from a continuum of locations, while here – as in most of the literature – we focus on a binary choice only: either firms locate in a cluster or in isolation.

multinational firms (a technological leader and a laggard) competing in their home markets and abroad, showing that the fraction of R&D located abroad depends on the extent of inward and outward spillovers, on product market competition, as well as on the gap between the technological leader and the laggard.<sup>3</sup>

A fundamental difference between all these papers and our contribution is that they focus on essentially static frameworks, so that they cannot investigate how the dynamics of knowledge and profits affect firms' location decisions, as well as the differences between their short run and long run implications. In Colombo and Dawid (2014) we make a first attempt at modeling firms' strategic location choices – considering the binary choice of clustering vs. isolation – in a dynamic game-theoretic framework by putting the role of inward and outward spillovers on the front stage. That paper develops a differential game setup studying the conditions under which it is optimal for a firm to locate inside or outside an R&D cluster, focusing also on how firms' incentives depend on the intensity of knowledge spillovers in the cluster, on the degree of competition in the industry, and on firms' planning horizons. Furthermore, it characterizes the effects of firms' location choices on social welfare in the long run, providing a normative base on which to ground the evaluation of alternative policies.<sup>4</sup> To be more specific, we consider a differential game with  $n$  firms producing at each time  $t$  a homogeneous good and competing in a common market. Firms' production costs are decreasing in their knowledge stock, which can be improved through R&D. The industry is characterized by the presence of a cluster of firms and each firm can either decide to locate in the cluster, or in isolation. By locating in the cluster, a firm can benefit from knowledge spillovers that depend on the overall knowledge stock of all the firms belonging to the cluster, while by locating in isolation a firm cannot benefit from inward spillovers neither suffer from outward spillovers. We solve the game by first characterizing the feedback strategies of the firms in the cluster and in isolation with respect to their R&D investments in a Markov perfect equilibrium for an arbitrary location pattern of firms. We then investigate the incentives to locate in the cluster or in isolation by comparing the numerically determined value functions under the Markov perfect equilibrium of the differential game for the two location scenarios.

We focus in particular on the location choice of a firm – a technological leader – that is either more efficient in performing R&D than its competitors, because of a structural advantage, or has only an initial advantage with respect to the size of its knowledge stock. Under the assumption that the technological leader enjoys a structural advantage, we show that it adopts a 'threshold'

---

<sup>3</sup> The location of R&D by multinational firms in the presence of spillovers is also the focus e.g. of Petit and Sanna-Randaccio (2000); Ekholm and Hakkala (2007); Alcacer and Zhao (2012), although their research questions differ substantially from the one we focus on here.

<sup>4</sup> Also Alcacer et al. (2013) highlight the role of dynamic strategic interactions on (multinational) firms' location choices, although they concentrate on the effects of firms' decisions on an industry competitive environment.

strategy, choosing to locate outside the industry cluster whenever its advantage over the competitors is sufficiently large. Two major forces are at play in our dynamic setup. In the short run, the leader benefits from locating in the cluster as its investment there is lower than in isolation. This follows immediately from the observation that, when in the cluster, the leader's R&D efforts end up reducing the future costs faced by competitors. In the long run, however, the leader is worse off in the cluster than in isolation (i.e. it has a lower market share), due to the faster catching-up of the followers that benefit from the outward spillovers generated by the presence of the technological leader in the cluster. The relative strength of the short run investment effect vs. that of the long run market share effect determines whether – for given intensity of the knowledge spillovers – the technological leader chooses to locate in the cluster or in isolation. The threshold at which the location choice of the leader switches from cluster to isolation is shown to be increasing in the discount rate – indicating that the more myopic the leader is the larger are its incentives to join the cluster, as well as in the dispersion of the industry and in the intensity of spillovers – suggesting that the leader's incentives to join the cluster are larger the higher the number of competitors is and the more pervasive spillovers are.

The implications of the model are entirely different when focusing on the case in which the technological leader enjoys an initial knowledge advantage only, as it is often the case for market pioneers in new sub-markets within an industry. Although the leader's location choice between the cluster and isolation is still determined by a threshold strategy, both the long run profits and the knowledge stock of the leader are larger when locating in the cluster rather than in isolation (exactly the opposite of what happens under a structural advantage). This follows immediately from the observations that, under a temporary knowledge advantage, in the long run the balance between outward and inward spillovers is the same for both the leader and its competitors (whereas under a structural advantage the wedge between the two is systematically larger for the leader than for the competitors), and that by locating in the cluster the leader can benefit from the spillovers generated by the competitors, which would instead be absent were it to locate in isolation. Similar arguments explain the opposite results we find in this case with respect to the structural advantage one in terms of comparative statics, with a negative relationship between the leader's location threshold and firms' discount rate.

Interestingly, both under a structural and a temporary R&D advantage of the leader, our numerical simulations show the existence of scenarios in which, although the technological leader chooses to locate in isolation, both total industry profits and consumers' surplus would be larger if it were to locate in the cluster. Hence, the leader's optimal choice contrasts with the socially optimal one. This inefficiency result is easily understood by noting that locating in the cluster the leader would strengthen competition, hence reducing market price and correspondingly increasing consumers' surplus; an effect that the leader has no incentives to take into account.

A clear limitation of the analysis of Colombo and Dawid (2014) is that all firms are assumed to be equally able to assimilate and exploit the knowledge generated by other firms. Cohen and Levinthal (1989), and the literature originating from their seminal contribution, emphasize instead that a firm's ability to effectively use external information depends also on its own R&D. Indeed, besides generating innovations, R&D is argued to enhance a firm's ability to "identify, assimilate, and exploit knowledge from the environment .... encompassing a firm's ability to imitate new process or innovations [but also] outside knowledge of a more intermediate sort" (Cohen and Levinthal 1989, p. 369); what the authors label as absorptive capacity. As R&D contributes to a firm's absorptive capacity, the incentives to spend in R&D are affected by the firm's available knowledge base and by the ease of learning external technological and scientific knowledge. In our terminology, this translates into the observation that the ability of a firm to benefit from inward spillovers depends on its absorptive capacity, while the magnitudes of the outward spillovers it generates depend on that of competitors.

The main purpose of this chapter is to explicitly account for the effects of absorptive capacities on knowledge spillovers and to investigate their impact on firms' location choices. We do so by augmenting the framework in Colombo and Dawid (2014), a detailed summary of which has been provided above, by allowing for differences in absorptive capacities across firms. In particular, we extend the setting of Colombo and Dawid (2014) by considering in addition to the case of constant absorptive capacity also that in which absorptive capacity is proportional to the firm's knowledge stock. In the latter case, the resulting differential game does not have a linear-quadratic structure and we rely on numerical methods to approximately solve the Hamilton-Jacobi-Bellman equations characterizing the value functions corresponding to Markov Perfect Equilibria of the game for different location choices. Interestingly, the insight in Colombo and Dawid (2014) that a technological leader has higher incentives than a laggard to locate in isolation is reversed if the absorptive capacity is proportional to the knowledge stock. In particular, in the latter case it is optimal for the firm to locate in the cluster only if its knowledge stock is substantially larger than that of its competitors. Furthermore, we show that a knowledge dependent absorptive capacity also gives rise to different strategic effects, as well as different dynamic patterns of R&D investment and knowledge accumulation, compared to the case of constant absorptive capacity. Specifically, the strategic implications of locating in the cluster are reversed. Under constant absorptive capacity, the other firms in the cluster increase their R&D investments if an additional firm enters, whereas under knowledge dependent absorptive capacity the opposite occurs. Also, under knowledge dependent absorptive capacity, a technological leader is able to keep its advantage for an extended time window even when entering the cluster, which makes this location choice more attractive. Conversely, for a firm with a small initial knowledge stock locating in the cluster becomes less attractive if the absorptive capacity depends on knowledge, because at least initially this firm can hardly profit from inward spillovers.

The chapter is organized as follows. Section 2 introduces our model. Section 3 characterizes Markov-perfect equilibria and describes the usage of collocation methods in our numerical analysis. Section 4 discusses the incentives of firms to locate in an industrial cluster rather than in isolation, as a function of knowledge spillovers and of firms absorptive capacity. Section 5 concludes.

## 2 The Model

We investigate the location choice of a firm competing in a dynamic Cournot oligopoly market consisting of three firms, which at each point in time  $t \geq 0$  simultaneously choose the quantities of a homogeneous good. Following Colombo and Dawid (2014), we focus on an economy with a representative consumer whose preferences are described by a quadratic Dixit-Stiglitz utility function yielding an inverse demand function of the form

$$p(t) = a - b \sum_{j=1}^3 q_j(t), \quad a, b > 0, \quad (1)$$

where  $q_j(t) \geq 0$  denotes the quantity of firm  $j$  and  $p(t)$  is the price of the good at time  $t$ .

We assume that all firms have constant marginal costs. More specifically, the level of marginal costs of firm  $i$  at time  $t$ ,  $c_i(t)$ , depends in a linear way on its stock of (cost-reducing) knowledge  $k_i(t)$  at time  $t$ , i.e.

$$c_i(t) = \bar{c} - \gamma k_i(t), \quad \bar{c}, \gamma > 0. \quad (2)$$

This formulation implies that in the absence of any cost-reducing knowledge all firms have identical marginal costs. To avoid negative marginal costs, we add the state constraint

$$k_i(t) \leq \frac{\bar{c}}{\gamma} \quad i = 1, \dots, 3. \quad (3)$$

We concentrate on the location decision of firm  $i = 1$  that, at time  $t = 0$ , chooses whether to locate in an industrial cluster or in isolation. By locating in the cluster the firm is exposed to inward and outward knowledge spillovers, affecting the dynamics both of its own knowledge stock and of that of its competitors. We assume that the two competitors of firm 1, firms  $i = 2, 3$  are located in the industrial cluster.<sup>5</sup> Each firm in the cluster receives spillovers from all other firms in the cluster and transfers (a fraction of) its own knowledge to the other firms in the cluster. The intensity according to which firm  $i$  can transform incoming spillovers into own knowledge depends on the firm's absorptive capacity,  $\kappa_i(k_i)$ . In what follows we consider two specifications. In the first, we assume that absorptive capacity is independent from the firms' knowledge stock, i.e.  $\kappa_i(k_i) = \kappa^{const} \forall k_i \geq 0$  and we normalize  $\kappa^{const}$  to 1. In the second,

<sup>5</sup> We focus on three firms as it is the lowest number of firms such that spillovers arise in the cluster even if one of the firms decides to locate outside the cluster.

we consider a scenario in which absorptive capacity is proportional to the firm's knowledge stock, i.e.  $\kappa_i(k_i) = \kappa^{lin}(k_i) := \xi k_i$ .

Rather than locating in the cluster, firm 1 can decide to locate in isolation, where it is not affected by inward or outward spillovers. To keep our analysis as simple as possible, we assume that there are no direct costs associated with either of the two location choices and that no relocation is possible at any point in time  $t > 0$ .<sup>6</sup> However, in order to capture congestion in the cluster (e.g. in terms of higher rental costs for production facilities), we assume that the firms located in the cluster suffer a fixed cost  $F > 0$  per period, whereas the corresponding cost is normalized to zero if a firm locates in isolation.

Formally, firm 1 chooses its location  $l_1 \in \{C, I\}$  at  $t = 0$ . Following again Colombo and Dawid (2014), we denote with  $N_C \subseteq \{1, \dots, 3\}$  the set of firms in the cluster, where  $|N_C| = n_C \leq 3$ . The overall number of firms located in the cluster is then  $n_C$ . Having assumed that the competitors of firm 1 are in the cluster implies that  $l_2 = l_3 = C$ . The knowledge stock of firm  $i$  depends positively on its R&D effort,  $x_i$ , and negatively on knowledge depreciation, which is assumed to occur at the same rate  $\delta > 0$  for all firms. Furthermore, firms in the cluster benefit from inward spillovers. Formally, we have

$$\dot{k}_i(t) = \begin{cases} x_i(t) + \beta \kappa_i(k_i) \sum_{j \in N_C \setminus \{i\}} k_j(t) - \delta k_i(t) & l_i = C \\ x_i(t) - \delta k_i(t) & l_i = I \end{cases} \quad (4)$$

The parameter  $\beta > 0$  captures the general intensity of spillover flows in the cluster – that may depend on the characteristics of the key technology in the considered industry, or on the institutional properties of the cluster – and  $\kappa_i$  denotes the specific absorptive capacity of firm  $i$ . This formulation is fully consistent with the observation, well established empirically, that a firm typically acquire knowledge by interacting with the other firms in its proximity (see e.g. Jaffe et al. (1993); Saxenian (1994)).

The R&D activities of firm  $i$ ,  $i = 1, \dots, 3$ , are associated to the quadratic cost function

$$g_i(x_i) = \frac{\eta_i}{2} x_i^2.$$

whereas in principle we allow for heterogeneous R&D cost functions, in the numerical analysis below we will only consider scenarios in which all firms are symmetric in this respect, i.e.  $\eta_1 = \eta_2 = \eta_3$ .

All firms are assumed to maximize their discounted profits. Hence, the decision problem of the generic firm  $i$  is given by

$$J_i = \int_0^\infty e^{-rt} [(p(t) - c_i(t)) q_i(t) - g_i(x_i(t))] dt, \quad (5)$$

subject to (1), (2), (4) the constraints (3),  $(q_i(t), x_i(t)) \geq 0$ , and  $k_i(0) = k_i^{ini}$ , for a given distribution of initial knowledge  $(k_1^{ini}, k_2^{ini}, k_3^{ini})$ .

<sup>6</sup> The decision to relocate often implies substantial transaction costs and therefore it is typically a long run decision.



Recalling that in our setup firm 1 is the only one having the possibility to choose where to locate, its choice variables are the initial location choice,  $l_1$ , and – at each point in time  $t \geq 0$  – the R&D effort,  $x_1(t)$ , and the output quantity,  $q_1(t)$ . Instead, firms 2 and 3 – being both located in the cluster – only choose  $x_i(t)$  and  $q_i(t)$ ,  $i = 2, 3$ , at each point in time  $t \geq 0$ .

### 3 Markov Perfect Equilibria

We investigate the optimal location choice of firm 1 under the assumption that for any given profile of location choices the two dynamic decision variables of each firm  $(x_i(t), q_i(t))$ ,  $i = 1, \dots, 3$  are chosen according to a Markov Perfect Equilibrium (MPE) of the underlying differential game. Taking into account that the quantity choices of firms have no intertemporal effects, it follows that at each point in time in equilibrium firms choose Cournot quantities given the current profile of marginal costs. More precisely, the equilibrium quantities are given by

$$q_i^*(k_1, \dots, k_3) = \frac{a - \bar{c} + \gamma \left( 3k_i - \sum_{j \neq i} k_j \right)}{4b} \quad (6)$$

and the resulting instantaneous profits at each point in time read

$$\pi_i^*(x_i, k_1, \dots, k_3) = \frac{\left( a - \bar{c} + \gamma \left( 3k_i - \sum_{j \neq i} k_j \right) \right)^2}{16b} - g_i(x_i). \quad (7)$$

Hence, we can rewrite (5) as

$$J_i = \int_0^\infty e^{-rt} \pi_i^*(x_i, k_1, \dots, k_3) dt. \quad (8)$$

Concerning R&D investments, a Markovian feedback strategy of firm  $i$  takes the form

$$\phi_i(k_1, \dots, k_3), i = 1, \dots, 3 \text{ with } \phi_i : \left[ 0, \frac{\bar{c}}{\gamma} \right]^3 \rightarrow [0, \infty).$$

A profile  $(\phi_1, \dots, \phi_3)$  of feedback strategies constitutes a Markov Perfect Equilibrium if, for each firm  $i$ , the strategy  $\phi_i$  maximizes (8) subject to (4) as well as  $x_i \geq 0$  and the initial conditions, given that the other firms use  $\phi_j$ ,  $j \neq i$ . Although in general the existence and uniqueness of a MPE cannot be guaranteed, our numerical procedure always yields a unique MPE profile for each parameter setting and location choice. We denote the MPE profile resulting from firm 1 locating in the cluster as  $(x_1^C, \dots, x_3^C)$ , whereas we refer to the MPE with firm 1 locating in isolation as  $(x_1^I, \dots, x_3^I)$ . The following proposition characterizes for both location scenarios firms' equilibrium investment strategies in terms of the corresponding value functions  $V_i^C$  and  $V_i^I$ , respectively.

**Proposition 1.** *For a given location choice of firm 1,  $l_1 \in \{C, I\}$ , any profile of MPE investment strategies has to satisfy*

$$x_i^l = \frac{1}{\eta_i} \frac{\partial V_i^l}{\partial k_i}, i = 1, \dots, 3, \quad (9)$$

where the value functions  $V_i^l$  solve the following Hamilton Jacobi Bellman (HJB) equations:

$$\begin{aligned} rV_i^C &= \max_{x_i \geq 0} \left[ \pi_i^*(x_i, k_1, \dots, k_3) - F + \left( x_i + \beta \kappa_i(k_i) \sum_{j \neq i} k_j - \delta k_i \right) \frac{\partial V_i^C}{\partial k_i} \right. \\ &\quad \left. + \sum_{j \neq i} \left( x_j^C + \beta \kappa_j(k_j) \sum_{m \neq j} k_m - \delta k_j \right) \frac{\partial V_j^C}{\partial k_j} \right], \\ rV_1^I &= \max_{x_1 \geq 0} \left[ \pi_1^*(x_1, k_1, \dots, k_3) + (x_1 - \delta k_1) \frac{\partial V_1^I}{\partial k_1} \right. \\ &\quad \left. + (x_2^I + \beta \kappa_2(k_2)k_3 - \delta k_2) \frac{\partial V_1^I}{\partial k_2} + (x_3^I + \beta \kappa_3(k_3)k_2 - \delta k_3) \frac{\partial V_1^I}{\partial k_3} \right], \\ rV_2^I &= \max_{x_2 \geq 0} \left[ \pi_2^*(x_2, k_1, \dots, k_3) - F + (x_1^I - \delta k_1) \frac{\partial V_2^I}{\partial k_1} \right. \\ &\quad \left. + (x_2 + \beta \kappa_2(k_2)k_3 - \delta k_2) \frac{\partial V_2^I}{\partial k_2} + (x_3^I + \beta \kappa_3(k_3)k_2 - \delta k_3) \frac{\partial V_2^I}{\partial k_3} \right], \\ rV_3^I &= \max_{x_3 \geq 0} \left[ \pi_3^*(x_3, k_1, \dots, k_3) - F + (x_1^I - \delta k_1) \frac{\partial V_3^I}{\partial k_1} \right. \\ &\quad \left. + (x_2^I + \beta \kappa_2(k_2)k_3 - \delta k_2) \frac{\partial V_3^I}{\partial k_2} + (x_3 + \beta \kappa_3(k_3)k_2 - \delta k_3) \frac{\partial V_3^I}{\partial k_3} \right]. \end{aligned} \quad (10)$$

*Proof.* The structure of the HJB equations follows from standard characterizations of Markov Perfect Equilibria (see e.g. Dockner et al. (2000)) and the expression (9) for the optimal investment is immediately obtained from the first order conditions of the right hand side of (10).

If  $\kappa_i = \kappa^{const}$ , then the considered game corresponds to that analyzed by Colombo and Dawid (2014) and it has a linear-quadratic structure. In this case, the MPE under the different location choices of firm 1 give rise to value functions that are quadratic in the state variables and the set of HJB equations (10) can be easily solved taking this into account.

For  $\kappa_i = \kappa^{lin}$  the right hand side of the HJB equations (10) includes terms where the state derivative of the value function is multiplied by an expression that is non-linear in the state variables. This implies that no polynomial value functions exist for the problem and we cannot provide an analytical characterization of the value functions. Hence, we rely on numerical methods to solve the HJB equations and to characterize the equilibrium investment functions. In particular, we employ a collocation method using Chebychev polynomials (see e.g. Dawid et al. (2017)) in order to obtain an approximate solution to the HJB equations (10). We determine polynomial approximations  $\hat{V}_i^C$ , respectively  $\hat{V}_i^I$ , such that (10), after substituting (9) for the optimal investment, is satisfied on a finite set of nodes in the state-space and it exhibits small differences between the left- and the right-hand side of the equations for all other points in the state space. To this end, we generate a set of  $n_i$  Chebychev nodes  $\mathcal{N}_{k_i}$  in  $[0, \bar{k}]$  for  $i = 1, \dots, 3$  and some  $\bar{k} < \frac{\bar{e}}{\gamma}$  (see e.g. Judd (1998) for the definition of Chebychev

nodes and Chebychev polynomials) and we define the set of interpolation nodes in the state space  $[0, \bar{k}]^3$  as

$$\mathcal{N} = \{(k_1, k_2, k_3) | k_i \in \mathcal{N}_{k_i}\}.$$

As a set of basis functions for the polynomial approximation of the value function, we use  $\mathcal{B} = \{B_{j,k,l}, j = 1, \dots, n_1, k = 1, \dots, n_2, l = 1, \dots, n_3\}$  with

$$B_{j,k,l}(k_1, k_2, k_3) = T_{j-1} \left( -1 + \frac{2k_1}{\bar{k}} \right) T_{k-1} \left( -1 + \frac{2K_2}{\bar{k}} \right) T_{l-1} \left( -1 + \frac{2K_3}{\bar{k}} \right),$$

where  $T_j(x)$  denotes the  $j$ -th Chebychev polynomial. Since Chebychev polynomials are defined on  $[-1, 1]$ , the state variables have to be transformed accordingly.

The value function is approximated by

$$V_i^h(k_1, k_2, k_3) \approx \hat{V}(k_1, k_2, k_3) = \sum_{j=1}^{n_1} \sum_{k=1}^{n_2} \sum_{l=1}^{n_3} C_{j,k,l}^h B_{j,k,l}(k_1, k_2, k_3),$$

$$k_1, k_2, k_3 \in [0, \bar{k}]; i = 1, \dots, 3; h = C, I, \quad (11)$$

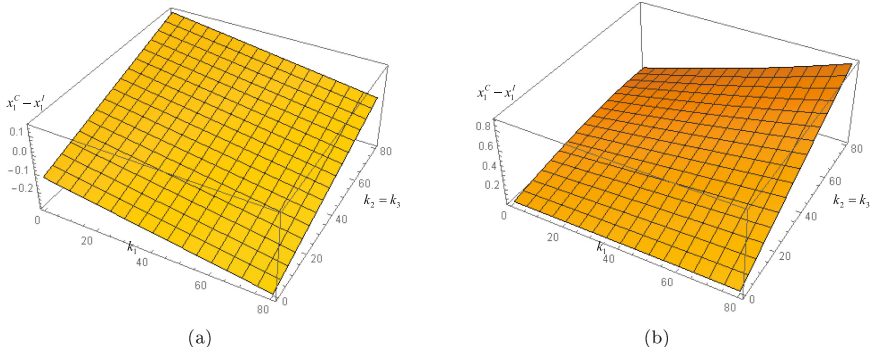
where  $C^h = \{C_{j,k,l}^h\}$  with  $j = 1, \dots, n_1, k = 1, \dots, n_2, l = 1, \dots, n_3$  and  $h = C, I$  is the set of  $n_1 n_2 n_3$  coefficients to be determined for each location choice  $h = C, I$ . To calculate these coefficients we solve the system of non-linear equations derived from the condition that  $\hat{V}_i^C$  satisfies the HJB equation (10) on the set of interpolation nodes  $\mathcal{N}$ . To this end, an initial guess  $\tilde{C}^{h,0} = (C_{j,k,l}^{h,0})_{j,k,l=1,\dots,n_1,n_2,n_3}$  of the coefficients is chosen, and in iteration  $m \geq 1$  the coefficients  $\tilde{C}^{h,m-1}$  are used to calculate approximations of the value functions and their partial derivatives at each node in  $\mathcal{N}$ . These approximations are inserted for all terms that occur in (10), after insertion of (9), where the value function or its derivatives appear in a non-linear form. Inserting the approximation (11) with  $C^h$  replaced by  $\tilde{C}^{h,m}$  for all terms in (10), where the value function and its derivatives occur in a linear way, yields a linear system of equations for the coefficients  $\tilde{C}^{h,m}$  that can be solved efficiently using standard methods, even for large values of  $n_i, i = 1, \dots, 3$ , as long as the coefficient matrix is well conditioned. The solution of this linear system gives the new set of coefficient values  $\tilde{C}^{h,m}$ . To complete the iteration, the new approximations of the value functions and their derivatives are inserted into all (including the non-linear) corresponding terms in (10) and (9), and the resulting absolute value of the difference between left and right hand side of this equation relative to the corresponding value function is determined for all nodes in  $\mathcal{N}$ . If the maximum of this relative error is below a given threshold  $\epsilon$  the algorithm is stopped. It is then checked that the absolute value of the difference between the left and the right hand side of (10) is sufficiently small on the entire state space, and if this is satisfied we set  $C^h = \tilde{C}^{h,m}$  and the current approximation of the value function is used to calculate the equilibrium feedback function according to (9). If the error outside  $\mathcal{N}$  is too large, the number of considered nodes or the size of the state space are adjusted and the procedure is repeated.

As a final step, it is checked that the considered state space  $[0, \bar{k}]^3$  is invariant under the state dynamics induced by the equilibrium feedback functions determined by the numerical method. In the following numerical calculations, the parameters of the collocation procedure have been set to  $n_1 = n_2 = n_3 = 6$ ,  $\bar{k} = 80$  and  $\epsilon = 10^{-6}$ . The rationale underlying this parameter configuration is the aim to keep the number of considered nodes as small as possible while keeping the error small. With respect to the model parameters we stay as close as possible to Colombo and Dawid (2014) in order to ensure the comparability of our findings across the two settings, and use

$$a = 100, b = 1, \gamma = 0.22, \bar{c} = 60, \beta = 0.01, \delta = 0.1, r = 0.05, \xi = 0.025, \\ \eta_i = \eta = 10, i = 1, \dots, 3, F = 10.$$

## 4 Economic Analysis

In what follows, we compare the equilibrium outcomes and the induced location choice of firm 1 in the benchmark case in which absorptive capacity is independent from the firm's knowledge stock ( $\kappa_i = \kappa^{const}$ ) with a situation in which it depends positively on the firm's knowledge stock ( $\kappa_i = \kappa^{lin}$ ).

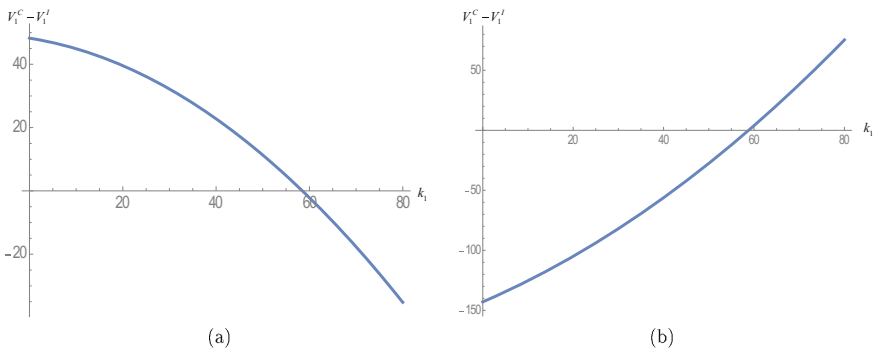


**Fig. 1.** Difference in equilibrium R&D investments if firm 1 locates in the cluster and in isolation, for (a)  $\kappa_i = \kappa^{const}$  and (b)  $\kappa_i = \kappa^{lin}$

Although our main focus is on the optimal location decision of firm 1, it is instructive to characterize the incentives of firm 1 to invest in R&D depending on whether it is in the cluster or in isolation, for  $\kappa_i = \kappa^{const}$  and  $\kappa_i = \kappa^{lin}$ , respectively. Figure 1 displays the difference between firm 1's equilibrium investment when it locates in the cluster and in isolation under the assumption that the two competitors have the same knowledge stock (i.e.  $k_2 = k_3$ ). Panel (a) shows that under constant absorptive capacity firm 1 invests more in the cluster than in isolation if its own knowledge stock is substantially smaller than that of its competitors. In such a situation, firm 1 expects to catch-up quickly in

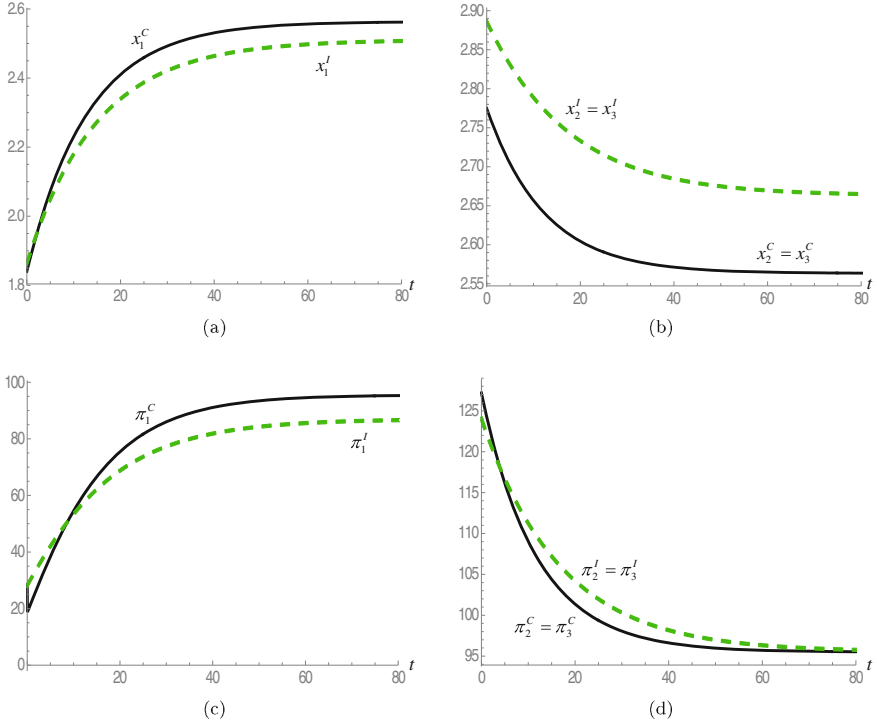
terms of its knowledge stock if it locates in the cluster, whereas the gap to its competitors will be closed at a much slower pace if it locates in isolation. Since a gap in the knowledge stock translates into higher marginal costs and a lower market share, the incentives to invest in (unit cost reducing) innovation is lower if firm 1 locates in isolation. A similar rationale explains why under constant absorptive capacity firm 1 invests less in the cluster than in isolation if it is a technological leader (i.e. if  $k_1$  is larger than  $k_2 = k_3$ ). Furthermore, it should be noted that for a given level of  $k_2 = k_3$  the difference  $x_1^C - x_1^I$  becomes smaller as the knowledge stock of firm 1 increases. To obtain an intuitive understanding for this relationship observe that a marginal increase of the knowledge stock of firm 1 induces a positive externality on the competitors' knowledge stock if firm 1 is in the cluster. Hence, the positive effects on the future market share of firm 1 are smaller compared to the scenario in which the firm is in isolation. This implies that the positive effect of an increase of  $k_1$  on R&D incentives is larger if firm 1 locates in isolation rather than in the cluster.

As it can be seen in panel (b), if absorptive capacity depends on the firm's knowledge stock we reach completely different conclusions. Indeed, for  $\kappa_i = \kappa^{lin}$  firm 1 invests more if located in the cluster rather than in isolation regardless of the (relative) size of the firms' knowledge stock. Also, the difference between  $x_1^C$  and  $x_1^I$  is always increasing in  $k_1$ , with a slope that becomes larger the larger the competitors' knowledge stock is. Intuitively, if absorptive capacity depends on knowledge stock, then increasing  $k_1$  has a positive impact on the size of future inward spillovers, which increases the future market share of firm 1 if it locates in the cluster. Clearly, this effect is absent if firm 1 is in isolation, which explains why for a knowledge dependent absorptive capacity R&D incentives grow faster with  $k_1$  if the firm locates in the cluster.



**Fig. 2.** Difference in value functions if firm 1 locates in the cluster and in isolation depending on the initial knowledge stock of firm 1, for (a)  $\kappa_i = \kappa^{const}$  and (b)  $\kappa_i = \kappa^{lin}$

Having studied firm 1's optimal R&D investment as a function of location, we now turn to the analysis of its optimal location choice. Figure 2 shows the difference in the value function of firm 1 between locating in the cluster and

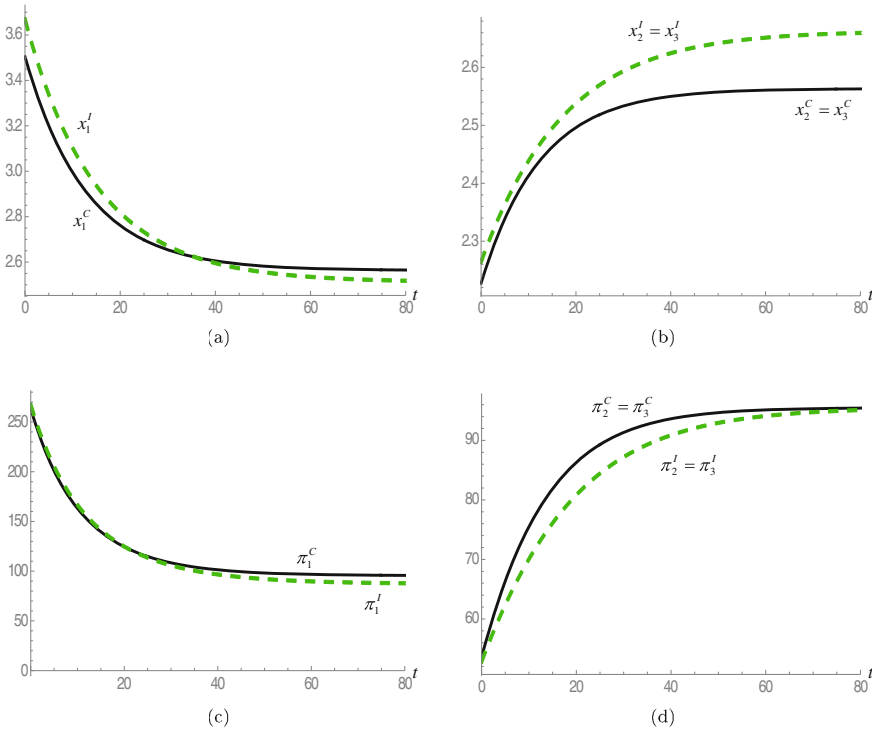


**Fig. 3.** Evolution of the R&D investment of firm 1 (a) and firms 2 and 3 (b), as well as of the instantaneous profits of firm 1 (c) and firms 2 and 3 (d), for  $\kappa_i = \kappa^{const}$  and  $k_1(0) = 0$ . The solid line depicts the case in which firm 1 locates in the cluster, the dashed line corresponds to the case in which the firm locates in isolation

in isolation for different levels of the firm's knowledge stock. In Fig. 2, and also in the examination of the model dynamics, we assume the (initial) knowledge stock of the competitors to be  $k_2 = k_3 = k^{*h}$ , where  $k^{*h}, h \in \{const, lin\}$ , is their steady state knowledge stock under the two specifications for absorptive capacity and assuming that firm 1 locates in isolation.<sup>7</sup> Panel (a) refers to the case of constant absorptive capacity. Consistently with the results in Colombo and Dawid (2014), the incentive to locate in the cluster decreases the larger  $k_1$  is. If absorptive capacity depends on the firm's knowledge stock, the effect of  $k_1$  on the incentives to locate in the cluster is exactly the opposite. Panel (b) of Fig. 2 highlights that the difference in the value functions of firm 1 when locating in the cluster and in isolation grows larger the more knowledge the firm has. In particular, for the level of cluster fixed cost  $F$  underlying Fig. 2, the optimal location choice of firm 1 depends crucially on whether its absorptive capacity is

<sup>7</sup> It should be noted that the steady state knowledge stock differs between the case in which  $\kappa_i = \kappa^{const}$  and that in which  $\kappa_i = \kappa^{lin}$ . In particular, for our parametrization we have that  $k^{*const} = 29.59$  and  $k^{*lin} = 30.85$ .

a function of its knowledge stock. When the value of  $k_1$  is not too large compared to the steady state level  $k^*$ , then it is optimal for firm 1 to locate in isolation if its absorptive capacity depends on the knowledge stock, and in the cluster if its absorptive capacity is constant. In scenarios in which firm 1 has a large knowledge advantage compared to its competitors, the optimal location decision again differs between the cases of  $\kappa_i = \kappa^{const}$  and  $\kappa_i = \kappa^{lin}$ . In particular, as a strong knowledge leader, firm 1 locates in the cluster only if its absorptive capacity depends on  $k_1$ .



**Fig. 4.** Evolution of the R&D investment of firm 1 (a) and firms 2 and 3 (b), as well as of the instantaneous profits of firm 1 (c) and firms 2 and 3 (d), for  $\kappa_i = \kappa^{const}$  and  $k_1(0) = 70$ . The solid line depicts the case in which firm 1 locates in the cluster, the dashed line corresponds to the case in which the firm locates in isolation

In order to obtain a clear understanding of the mechanisms driving the insights of Fig. 2 we first focus on the case of constant absorptive capacity. Figures 3 and 4 show the dynamics of the R&D investments and of the instantaneous profits of all firms for the two scenarios in which  $k_1(0) = 0$  and  $k_1(0) = 70$ , respectively. Figure 2 shows that it is optimal for firm 1 to locate in the cluster if  $k_1(0) = 0$ , and to locate in isolation if  $k_1(0) = 70$ . It can be clearly seen that, regardless of the initial knowledge stock of firm 1, its competitors invest less in

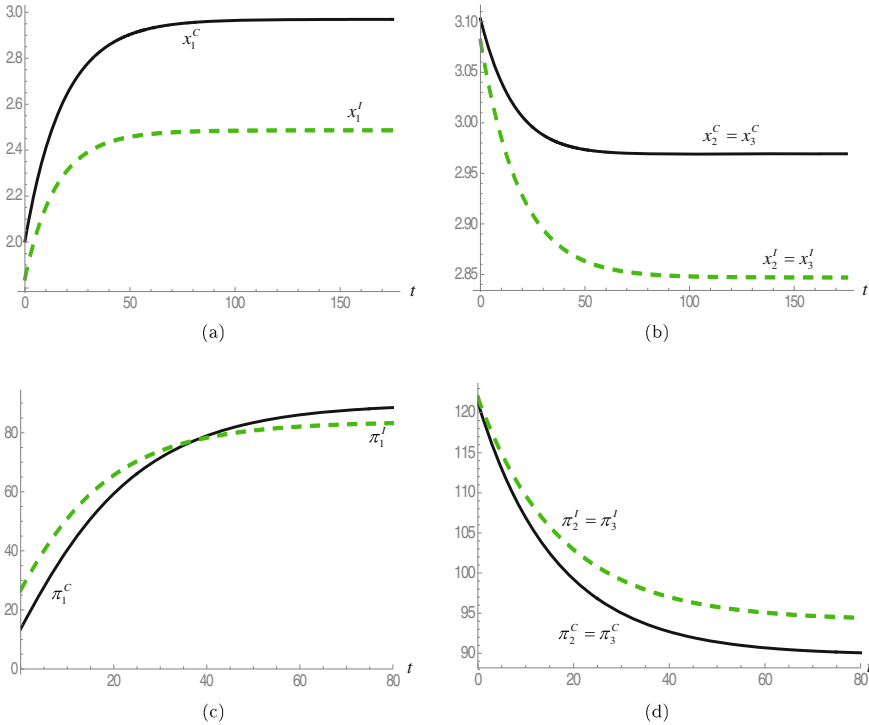
R&D if it locates in the cluster. Intuitively, for constant absorptive capacity, the crucial factor determining R&D investment is the expected trajectory of firm sales. If firm 1 locates in the cluster, it will benefit from a higher knowledge stock in the long run compared to the scenarios in which it locates in isolation. By knowing this, firm 1's competitors anticipate that their own market share will become smaller. Hence, they invest less in R&D if firm 1 locates in the cluster. This strategic effect has positive implications for the profits of firm 1 and it is the dominant effect as long as the initial knowledge stock of firm 1 is not too large. In particular, panel (c) of Fig. 3 shows that, after a short initial phase, the fixed costs incurred in the cluster are more than outweighed by the strategic effect we just discussed.

Focusing now on Fig. 4, it can be seen that the initial impact of the location choice of firm 1 on the competitors' R&D investment is much smaller if  $k_1$  is large compared to the case in which  $k_1(0) = 0$ . Hence, the strategic incentive for firm 1 to locate in the cluster is much weaker when  $k_1(0) = 70$ . Furthermore, if for  $k_1(0) = 70$  firm 1 locates in the cluster, its competitors will profit substantially from inward spillovers reducing their unit costs. Clearly, this also has negative implications for firm 1's profits. Overall, these effects imply that the initial time interval for which the instantaneous profits of firm 1 are larger if it locates in isolation rather than in the cluster is much longer for  $k_1(0) = 70$  than for  $k_1(0) = 0$  (cf. panel (c) of Figs. 3 and 4). Considering the instantaneous profits of firms 2 and 3, we observe that they benefit from firm 1 locating in the cluster if the latter is a knowledge leader ( $k_1(0) = 70$ ), while their profits are negatively affected if firm 1 joins the cluster with no initial knowledge.

Finally, we turn to the case of knowledge dependent absorptive capacity that is illustrated in Figs. 5 and 6 for the two situations in which  $k_1(0) = 0$  and  $k_1(0) = 70$ , respectively. The two panels (b) of these figures highlight that for knowledge dependent absorptive capacity the R&D investment of firms 2 and 3 is larger if firm 1 locates in the cluster rather than in isolation. This is exactly the opposite of what we observe in the case of constant absorptive capacity (see the corresponding panels in Figs. 3 and 4). Accordingly, if absorptive capacity depends on knowledge stock, the strategic effect on the opponents' behavior increases the incentive for firm 1 to locate in isolation. Furthermore, if the initial knowledge stock of firm 1 is small (Fig. 5) the firm cannot absorb substantial inward spillovers, at least initially. However, its own investment in R&D generates knowledge flows to its competitors, which have a substantially larger knowledge stock of  $k^{*lin} = 30.85$ . This effect weakens the competitiveness of firm 1 relative to the other firms. Hence, as it is shown in panel (c) of Fig. 5, firm 1's instantaneous profits remain larger if it locates in isolation for a long initial time window. Only in the long run the cost reductions resulting from the spillovers that emerge in the cluster become sufficiently strong to make being in the cluster more profitable. Therefore, as it can be seen in Fig. 2(b), it is optimal for firm 1 to locate in isolation if  $k_1(0)$  is small.

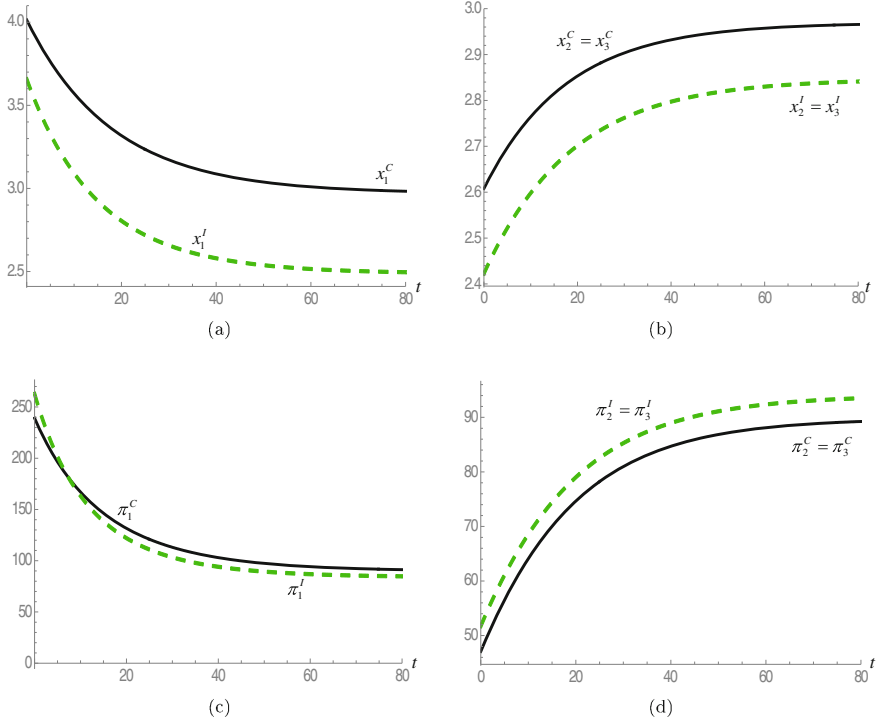
Different conclusions are reached if focusing instead on the case in which  $k_1(0) = 70$ , corresponding to a scenario (illustrated in Fig. 6) where firm 1





**Fig. 5.** Evolution of the R&D investment of firm 1 (a) and firms 2 and 3 (b), as well as of the instantaneous profits of firm 1 (c) and firms 2 and 3 (d), for  $\kappa_i = \kappa^{lin}$  and  $k_1(0) = 0$ . The solid line depicts the case in which firm 1 locates in the cluster, the dashed line corresponds to the case in which the firm locates in isolation

has an initial knowledge advantage compared to its competitors (recall that  $k_2(0) = k_3(0) = 30.85$ ). In this case, firm 1 is able to greatly benefit from inward spillovers if it locates in the cluster. Due to the fact that the absorptive capacity of firm 1 is larger than that of firms 2 and 3, it is able to sustain its initial knowledge advantage over time. Taking this into account – as well as the investment incentive resulting from firm 1’s goal to preserve its (high) absorptive capacity when locating in the cluster – implies that for  $\kappa_i = \kappa^{lin}$  the R&D investment of firm 1 is consistently larger if it locates in the cluster rather than in isolation. Also in this respect the scenario discussed here is qualitatively different from the one with constant absorptive capacity (see Fig. 4(a)). Overall, if firm 1 is a technological leader, the instantaneous profits it can achieve in the cluster become quickly larger than those it can obtain by locating in isolation, and it is therefore optimal for the firm to locate in the cluster (Fig. 2(b)). Finally, panels (d) of Figs. 5 and 6 show that if the absorptive capacity depends on the knowledge stock, then the instantaneous profits of firms 2 and 3 are always larger if firm 1 locates in isolation regardless of its initial knowledge stock.



**Fig. 6.** Evolution of the R&D investment of firm 1 (a) and firms 2 and 3 (b), as well as of the instantaneous profits of firm 1 (c) and firms 2 and 3 (d), for  $\kappa_i = \kappa^{lin}$  and  $k_1(0) = 70$ . The solid line depicts the case in which firm 1 locates in the cluster, the dashed line corresponds to the case in which the firm locates in isolation

## 5 Concluding Remarks

In this chapter, we augment the analysis of firms' optimal location choice in Colombo and Dawid (2014) by explicitly considering the implications of an endogenously determined absorptive capacity. In particular, we compare the benchmark of constant absorptive capacity with a scenario in which absorptive capacity is assumed to be an increasing function of the knowledge stock accumulated by a firm. We consider the differential games emerging for different location choices of a firm (cluster vs. isolation) and, by applying appropriate numerical methods, we are able to characterize the feedback strategies and the value functions associated with the Markov Perfect Equilibria of these games. We find that under endogenous absorptive capacity the relationship between the initial knowledge stock of a firm and its optimal location decision is exactly the opposite with respect to the one emerging under constant absorptive capacity. In particular, our analysis shows that for constant absorptive capacity a firm chooses to locate in an industrial cluster rather than in isolation if its initial knowledge stock is relatively small, whereas under endogenous absorptive

capacity the initial knowledge stock has to be large (relative to that of competitors) for the firm to locate in the cluster.

Our findings highlight that a full understanding of firms' optimal location choices in different industries requires a careful examination of the characteristics of the spillovers that are associated to these choices. In particular, it is important to investigate how the ability to benefit from spillovers depends on the knowledge of the absorbing firm.

Several extensions are possible that would further enrich our analysis. First, in this study we focus on the investigation of the location choice of only one firm in a market with three competitors. Although this setting already allows us to capture the main mechanisms that are responsible for firms' location decisions, it would be interesting to develop a framework with an arbitrary number of firms, all initially choosing their locations. Second, some preliminary findings for the setting with endogenous absorptive capacity presented here suggest that, even in a model with symmetric firms, (stable) steady states exist in which firms have asymmetric knowledge stocks. This raises the question of how the long term market outcome depends on firms' initial knowledge stocks. Third, more general relationships between the knowledge stock and the absorptive capacity of a firm could be explored. Finally, more fundamental extensions of the work reported here involve the consideration of multiple locations, as well as the option for firms to relocate their activities. Exploring these issues in more details is left to future work.

## References

- Alcácer, J., Chung, W.: Location strategies and knowledge spillovers. *Manage. Sci.* **53**, 760–776 (2007)
- Alcácer, J., Zhao, M.: Local R&D strategies and multilocation firms: the role of internal linkages. *Manage. Sci.* **58**, 734–753 (2012)
- Alcácer, J., Dezsó, C.L., Zhao, M.: Firm rivalry, knowledge accumulation, and MNE location choices. *J. Int. Bus. Stud.* **44**, 504–520 (2013)
- Almazan, A., De Motta, A., Titman, S.: Firm location and the creation and utilization of human capital. *Rev. Econ. Stud.* **74**, 1305–1327 (2007)
- Belderbos, R., Lykogianni, E., Veugelers, R.: Strategic R&D location by multinational firms: spillovers, technology sourcing, and competition. *J. Econ. Manage. Strategy* **17**, 759–779 (2008)
- Cohen, W.M., Levinthal, D.A.: Innovation and learning: the two faces of R&D. *Econ. J.* **99**, 569–596 (1989)
- Colombo, L., Dawid, H.: Strategic location choice under dynamic oligopolistic competition and spillovers. *J. Econ. Dyn. Control* **48**, 288–307 (2014)
- Dawid, H., Keoula, M.Y., Kort, P.M.: Numerical analysis of markov-perfect equilibria with multiple stable steady states: a duopoly application with innovative firms. *Dyn. Games Appl.* (2017). Forthcoming
- Dockner, E.J., Jorgensen, S., Van Long, N., Sorger, G.: *Differential Games in Economics and Management Science*. Cambridge University Press, Cambridge (2000)
- De Beule, F., Duanmu, J.-L.: Locational determinants of internationalization: a firm-level analysis of chinese and indian acquisitions. *Eur. Manag. J.* **30**, 264–277 (2012)

- Ekholm, K., Hakkala, K.: Location of R&D and high-tech production by vertically integrated multinationals. *Econ. J.* **117**, 512–543 (2007)
- Gersbach, H., Schmutzler, A.: External spillovers, internal spillovers and the geography of production and innovation. *Reg. Sci. Urban Econ.* **29**, 679–696 (1999)
- Gersbach, H., Schmutzler, A.: Foreign direct investment and R&D-offshoring. *Oxford Econ. Pap.* **63**, 134–157 (2011)
- Griliches, Z.: The search for R&D spillovers. *Scand. J. Econ.* **94**, 29–47 (1992)
- Head, K., Ries, J., Swenson, D.: Agglomeration benefits and location choice: evidence from Japanese manufacturing investments in the United States. *J. Int. Econ.* **38**, 223–247 (1995)
- Henisz, W.J.: The institutional environment for multinational investment. *J. Law Econ. Organ.* **16**, 334–364 (2000)
- Jaffe, A., Trajtenberg, M., Henderson, R.: Geographic localization of knowledge spillovers as evidenced by patent citations. *Q. J. Econ.* **108**, 577–598 (1993)
- Judd, K.: *Numerical Methods in Economics*. MIT Press, Cambridge (1998)
- Lee, J.Y., Mansfield, E.: Intellectual property protection and US foreign direct investment. *Rev. Econ. Stat.* **78**, 181–186 (1996)
- Leiponen, A., Helfat, C.E.: Location, decentralization, and knowledge sources for innovation. *Organ. Sci.* **22**, 641–658 (2011)
- Mariotti, S., Piscitello, L., Elia, S.: Spatial agglomeration of multinational enterprises: the role of information externalities and knowledge spillovers. *J. Econ. Geogr.* **10**, 519–538 (2010)
- Petit, M.-L., Sanna-Randaccio, F.: Endogenous R&D and foreign direct investment in international oligopolies. *Int. J. Ind. Organ.* **18**, 339–367 (2000)
- Piga, C., Poyago-Theotoky, J.: Endogenous R&D spillovers and locational choice. *Reg. Sci. Urban Econ.* **35**, 127–139 (2005)
- Saxenian, A.: *Regional Advantage: Culture and Competition in Silicon Valley and Route 128*. Harvard University Press, Cambridge (1994)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# Strategic Corporate Social Responsibility by a Local Firm Against a Multinational Enterprise

Michael Kopel<sup>1</sup>, Constantine Manasakis<sup>2</sup>(✉), and Emmanuel Petrakis<sup>3</sup>

<sup>1</sup> Institute of Organization and Economics of Institutions, University of Graz,  
Universitätsstrasse 15/E4, Graz, Austria

`michael.kopel@uni-graz.at`

<sup>2</sup> Department of Political Science, University of Crete,  
Gallos Campus, 741 00 Rethymno, Crete, Greece

`manasakis@uoc.gr`

<sup>3</sup> Department of Economics, University of Crete,  
Gallos Campus, 741 00 Rethymno, Crete, Greece

`petrakis@uoc.gr`

**Abstract.** The present work considers competition between a local firm and a multinational enterprise (MNE). The MNE has a competitive advantage in terms of lower unit costs and plans to enter the local firm's market either through exports or through FDI. The local firm may strategically become "socially responsible" and follow a "doing well by doing good" strategy by investing in socially responsible activities along its value chain. Investments in corporate social responsibility (CSR) increase the responsible firm's equilibrium output and profit as well as consumer surplus and total welfare in its country. The multinational firm's incentives to serve the foreign country through FDI are mitigated in the average consumer's valuation for CSR in the responsible firm's country implying that CSR investments by local firms give space for inward FDI by low-cost multinationals targeting consumers without environmental and social responsibility consciousness. Policy suggestions are also discussed.

**Keywords:** Corporate social responsibility · Multinational firms · Foreign direct investment · Exports · Import tariffs

## 1 Introduction

The core role of multinational enterprises on globalization, mainly through foreign direct investments and international trade (United Nations 2014a; 2014b), gives increasing attention to their practices as well as their resulting market and societal effects. In parallel with the expansion of multinational enterprises, stylized facts and evidence suggest that consumers are increasingly aware of the social and environmental responsibility and footprint of their local firms (Ioannou and Serafeim 2015). In this spirit, Manasakis et al. (2013) cite evidence

suggesting that consumers express a willingness to pay a premium for goods and services produced by socially responsible firms in manufacturing industries, tourism services and agriculture. Within a policy context, the promotion of corporate social responsibility (CSR) has become a top priority in the agenda for sustainable development in many countries and international organizations. Interestingly, when CSR started to become more widespread, its further encouragement became a central policy objective in both the U.S. and the E.U., aiming at the promotion of sustainable growth and competitiveness (European Commission 2001; 2006).<sup>1</sup> Despite its importance, the statement of Benabou and Tirole (2010) according to which “... little is known about the economics of individual and corporate social responsibility”, is still valid. Kitzmueller and Shimshack (2012) further suggest that the field of international CSR warrants greater attention while the preferences and politics that motivate CSR differ substantively across countries.

The present work has been motivated by the apparent interdependence between the increasing trend to invest in CSR and multinational firms’ expansion to foreign countries. In this context, we consider two firms located in two different countries, a home country and a foreign country. The firm in the home country plans to become “multinational” and, besides serving its home country’s market, serve the foreign country’s market too, either through exports or through foreign direct investment (FDI), i.e. establishing a subsidiary in the foreign country. If the multinational firm exports, the exported quantity is subject to a tariff set by the foreign country’s government. Alternatively, if the multinational firm chooses FDI, it incurs a fixed set-up cost. The local firm in the foreign country operates under higher unit costs than the multinational firm, but has the option to become “socially responsible”. By following such a “doing well by doing good” -strategy it invests in CSR activities along its value chain (Porter and Kramer 2006; 2011) and integrates social and environmental concerns in its business operations “above and beyond” that mandated by its government (Campbell et al. 2012). This strategy meets the preferences of socially conscious consumers for *responsible* goods whose production processes comply with criteria

---

<sup>1</sup> The OECD Guidelines for multinational enterprises (OECD 2011) offer government-backed recommendations covering business conduct in a wide variety of areas, including employment and industrial relations, human rights, disclosure of financial and non-financial information, environmental issues. The United Nations Global Compact principles (United Nations 2014c) acknowledge the importance of communicating with stakeholders when supporting a precautionary approach to environmental challenges and encourage enterprises to develop sustainability indicators and measure, track, and report progress in incorporating sustainability principles into business practices.

for social and environmental sustainability (Becchetti et al. 2011).<sup>2</sup> The products of the local foreign firm and the multinational home firm combine horizontal and vertical differentiation aspects (Häckner 2000; Garella and Petrakis 2008). In particular, socially and environmentally *responsible* attributes are perceived by socially conscious consumers as a “quality improvement” (see also Manasakis et al. 2013; Liu et al. 2015). In this context, our work addresses the following questions: How do CSR investments by the local firm affect market and societal outcomes in both countries under two different modes of entry, exports and FDI? How does consumers’ consciousness for the responsible local firm’s product affect the latter outcomes as well as the multinational firm’s decision to serve the foreign country through exports or FDI? How does the multinational firm’s mode of entry in the foreign country affect consumer surplus and total welfare in the home country and the foreign country?

Our main finding is that the local foreign firm competing against the multinational enterprise seeks to obtain a competitive advantage by strategically engaging in CSR activities that meet the socially conscious consumers’ demands. CSR investments increase the responsible local firm’s equilibrium output and profit as well as consumer surplus and total welfare in its country, while they decrease the multinational firm’s equilibrium output and profits. Under exports, CSR investments reduce total welfare in the multinational firm’s home country. We also find that the average consumer’s consciousness for the local firm’s responsible product increases the tariff, the local firm’s CSR effort, output and profit, as well as consumer surplus and total welfare in the foreign country. Yet, it decreases the multinational firm’s output and profit, as well as total welfare in the multinational firm’s home country. This implies that CSR investments are not welfare-enhancing per se. Interestingly, the aforementioned consciousness of consumers reduces the multinational firm’s maximum affordable set-up cost for FDI in the foreign country and gives space for attracting inward FDI by low-cost multinationals that target consumers with low preference for environmental and social responsibility. Moreover, consumer surplus and total welfare in the foreign country are always higher under FDI than under exports. The opposite holds for total welfare in the multinational firm’s home country where consumer surplus is independent from the multinational’s mode of entry into the foreign country. Assuming that within each country, the firm, consumers and the policy maker are the related stakeholders, our findings suggest that the stakeholders’ preferences for the mode of entry of the multinational firm in the foreign country are not aligned.

The literature on international trade has studied the question of a multinational firm’s optimal mode of entry and its drivers. For example, Ishikawa and

---

<sup>2</sup> In the terminology of Porter and Kramer (2011), CSR activities connect company success with social progress and constitute a profit center for firms while creating value, and for society, by addressing needs and challenges of the firm’s stakeholders, such as its employees (investments in health and safety in the workplace), suppliers (support to local suppliers rather than cheaper alternative sources), and the environment (reduction on emissions of pollutants; use of environmentally friendly technologies).

Horiuchi (2012) and Glass and Saggi (2005) relate it to vertically related markets and local sourcing respectively. Sinha (2010) demonstrates the impact of technology licensing on the mode of entry. Mukherjee (2008) and Ma and Zhou (2016) show that FDI and exports can coexist under certain circumstances. None of these papers considers the impact of a local foreign firm's investment in social responsibility, which is the main focus of our paper. Graf and Wirl (2014) investigate if CSR can be used as a strategy to deter entry of a potential competitor. However, they do not deal with a setting of international competition. More specifically, the present work contributes to the scant literature on the interaction between CSR activities and multinational firms. Wang et al. (2012) extend Brander and Spencer (1985), where two foreign firms export a homogenous good in a third country's market. Each firm has the option for a consumer-friendly initiative which is captured by firms pursuing a mixed objective that includes a firm's own profit and consumer surplus. Likewise, Chang et al. (2014) extend Brander and Spencer (1984), where one domestic and one foreign firm produce a homogenous product in the domestic market. Becchetti et al. (2011) consider a standard Hotelling approach where a not-for-profit *fair* trader enters into a market, with heterogeneous consumers regarding their preferences on social responsibility, where a profit-maximizing producer is already active. We depart from these papers since we formalize the local foreign firm's CSR effort as a particular for-profit investment strategy to compete against a multinational low-cost firm that enters the local foreign market.

The rest of the work is organized as follows. Section 2 presents the model and in Sect. 3 we analyze the multinational firm's possible modes of entry in the foreign country's market. Section 4 studies the multinational's decision between exports and FDI. Finally, Sect. 5 concludes.

## 2 The Model

We consider two firms, denoted 1 and 2. Firm 1 is based in the home country  $h$  and firm 2 resides in the foreign country  $f$ . Firm 1, besides serving the home country  $h$ , plans to become "multinational" and serve the foreign country  $f$ , either through exports or by FDI, i.e. through establishing a production facility in country  $f$ . Firm 2 serves country  $f$ 's market solely. We will refer to firm 1 as the multinational firm and Firm 2 as the local firm. If the multinational firm 1 exports to country  $f$ , the exported quantity is subject to a tariff  $t$ , set by the importing country's government to maximize country  $f$ 's welfare. Following the terminology of Motta and Norman (1996),  $t$  is an inverse measure of "market accessibility" and policy changes that increase  $t$  "heighten the asymmetry" between the rival firms.<sup>3</sup> In case of FDI, the multinational firm 1 incurs a fixed

<sup>3</sup> Moreover, the commodity exported is subject to a constant transportation unit cost, which, following Fumagalli (2003) and without loss of generality, is normalized to zero. This assumption allows us to economize with the parameters of the model that create unnecessary analytical complications without qualitatively altering our results.



set-up cost  $T$ , which contains the transaction and construction costs necessary to open a subsidiary in the foreign country (Naylor and Santoni 2003). We assume that the home firm and the multinational firm are both endowed with identical constant returns to scale production technologies. Following Eicher and Kang (2005), Hao and Lahiri (2009), and Mukherjee and Suetrong (2009) we assume that the local firm in country  $f$  produces with constant marginal cost denoted by  $c$ . The multinational firm has a competitive advantage in its production technology, i.e., it is more cost-efficient, with its marginal cost being  $c(1 - z)$ , and  $0 < z < 1$ .

We consider that the response of the local firm in country  $f$  to the multinational firm's entrance into the foreign market is to become "socially responsible". By following such a "doing well by doing good" strategy, the local firm addresses the preferences of socially conscious consumers for *responsible* production in country  $f$ 's market. This strategy is executed by the local firm through investing in CSR activities along its value chain (Porter and Kramer 2006; 2011) and integrating social and environmental concerns in its business operations. Such concerns include, e.g., purchasing inputs from local suppliers and improving working conditions for employees and point to activities that are "above and beyond" that mandated by government (Campbell et al. 2012; Chambolle and Giraud-Héraud 2005).

On the demand side, following Häckner (2000), Garella and Petrakis (2008), Manasakis et al. (2013) and Liu et al. (2015), the utility function of a representative consumer in country  $i$  for  $i = h, f$  is:

$$U_h = aq_{h_1} - \frac{q_{h_1}^2}{2} + e_h \quad (1)$$

and

$$U_f = aq_{f_1} + (a + k_f s_f)q_{f_2} - \frac{q_{f_1}^2 + q_{f_2}^2 + 2\gamma q_{f_1} q_{f_2}}{2} + e_f \quad (2)$$

where  $q_{h_1}, q_{f_1}, q_{f_2}$  represent the quantities of firms 1 and 2 bought by the representative consumer in the markets of country  $h, f$ . The quantities of the "composite good" in country  $h, f$  are denoted by  $e_h, e_f$ . The composite good's quantity and price are normalized to unity. The parameter  $\gamma \in (0, 1]$  is a measure of the degree of substitutability between the products offered in the market in country  $f$ , with  $\gamma \rightarrow 0$  ( $\gamma = 1$ ) corresponding to the case of almost independent (homogeneous) goods. Alternatively,  $\gamma$  may be interpreted as the intensity of market competition with a higher  $\gamma$  representing a higher level of competition.

The variable  $s_f \geq 0$  represents the level of CSR investments undertaken by the local firm. Investing  $s_f$  in CSR are assumed to increase the consumers' valuation for the responsible product of the local firm by  $k_f s_f$ , where  $k_f \in [0, 1]$  represents the increase of the average consumer's willingness to pay for the product per unit of CSR investment.

Maximization of (1) and (2) with respect to  $q_{I_i}$ , gives the inverse demand functions:

$$\begin{aligned} P_{h_1} &= a - q_{h_1} \\ P_{f_1} &= a - q_{f_1} - \gamma q_{f_2}; \quad P_{f_2} = a + k_f s_f - q_{f_2} - \gamma q_{f_1}. \end{aligned} \quad (3)$$

The local firm's CSR investments increase the demand and the average consumer's valuation for its product. Moreover, the local firm's production cost is given by  $C_f(q_{f_2}, s_f) = c(1 + s_f^2)q_{f_2}$ , implying that a higher CSR effort level increases its marginal cost at an increasing rate (Manasakis et al. 2013).

If the multinational firm serves the foreign country through exports ( $e$ ), its profit function is:

$$\Pi_1^e = \Pi_{h_1} + \Pi_{f_1}^e = [P_{h_1} - c(1 - z)] q_{h_1} + [P_{f_1} - c(1 - z) - t] q_{f_1}. \quad (4)$$

If the multinational serves the foreign country through FDI ( $d$ ), its profit function is:

$$\Pi_1^d = \Pi_{h_1} + \Pi_{f_1}^d = [P_{h_1} - c(1 - z)] q_{h_1} + [P_{f_1} - c(1 - z)] q_{f_1} - T. \quad (5)$$

The corresponding profit function for the local firm is given by:

$$\Pi_2 = [P_{f_2} - c(1 + s_f^2)] q_{f_2} \quad (6)$$

To guarantee interior solutions under all circumstances, we make the following assumptions regarding the tariff  $t$  set by the foreign country's government and the fixed set-up cost  $T$  in case of FDI:

Assumption 1:  $t < t_c := (2 - \gamma)(a - cz) + \frac{\gamma k_f^2}{4c}$ ,

Assumption 2:  $T < T_c := \frac{[4c(\gamma - 2)(a - 2cz) + \gamma k_f^2]^2}{[4c(\gamma^2 - 4)]^2}$ .

In this context, we consider the following game with observable actions. In the first stage of the game, the government of country  $f$  sets the tariff. In the second stage, the multinational firm decides whether to serve country  $f$ 's market through exports or FDI. In the third stage, the local firm invests in CSR and the two firms set their quantities for the markets. We solve the game by backward induction and employ Subgame Perfect Nash Equilibrium (SPNE) as a solution concept.

### 3 The Multinational Firm's Modes of Entry

**The Multinational Firm Exports to the Foreign Country.** We first consider the case where the multinational firm produces in country  $h$  and exports its product to the foreign country. In the third stage of the game, firms choose their quantities  $q_{h_1}, q_{f_1}, q_{f_2}$  such that their corresponding profits given by (4) and (6) are maximized. Simultaneously, the local firm invests in CSR.

From the first-order conditions, the firms' output reaction functions for country  $f$ 's market can be derived:

$$R_{f_1}^e(q_{f_2}) = \frac{1}{2} [a - c(1 - z) - \gamma q_{f_2} - t] \quad (7)$$

$$R_{f_2}^e(q_{f_1}) = \frac{1}{2} [a - c + s_f(k_f - cs_f) - \gamma q_{f_1}]. \quad (8)$$

The term  $s_f(k_f - cs_f)$  in (8) captures the two opposing effects of CSR investments. On the one hand, investment  $s_f$  in CSR increases the local firm's demand by  $k_f s_f$  and its unit cost by  $cs_f^2$ . Moreover, as  $R_{f_2}^e/\partial s_f = k_f/2 - cs_f$ , the local firm's best response output has an inverted  $U$ -shaped relation with its CSR efforts, with the maximum attained at  $s_f = k_f/2c$ . This suggests that the local firm's profit-maximizing level of CSR investments is  $s_f = k_f/2c$ . This level increases in the average consumer's willingness to pay for this firm's product as well as in the efficiency of the CSR (and output) "production technology" (captured by a lower  $c$ ). Intuitively, for a relatively low level of CSR efforts, the positive demand effect dominates the negative unit cost effect and  $R_{f_2}^e$  shifts outwards.

Using the optimal level of the local firm's CSR investment and solving the system of first order conditions, we obtain the firm's outputs in country  $f$ :

$$q_{f_1}^e = \frac{(a - 2cz)(2 - \gamma) - \frac{\gamma k_f^2}{4c} - 2t}{4 - \gamma^2} \quad (9)$$

$$q_{f_2}^e = \frac{a(2 - \gamma) - c[2 - \gamma(1 - z)] + \frac{k_f^2}{2c} + \gamma t}{4 - \gamma^2}. \quad (10)$$

These output levels highlight the impact of firms' relative comparative advantages in country  $f$ . More specifically,  $\frac{dq_{f_1}^e}{dz} > 0$  and  $\frac{dq_{f_1}^e}{dt} < 0$  suggest that the multinational firm's output increases in its relative technological superiority, i.e., as  $z \rightarrow 1$ , and decreases in the tariff rate  $t$ . The opposites hold for the local firm.

Given that the multinational firm serves the foreign country through exports, in the first stage of the game the foreign country's government determines its tariff rate  $t$  so as to maximize national total welfare given by the sum of consumer surplus, the local firm's profit, and tariff payments:

$$TW_f^e(t) = CS_f^e(t) + \Pi_{f_2}^e(t) + tq_{f_1}^e(t). \quad (11)$$

The first-order condition determines the socially optimal tariff rate  $t = \frac{1}{3} [a - c(1 - z)]$ . Note that  $m_f(\frac{k_f}{a}, \frac{c}{a}) = k_f/\sqrt{c(a - c)} = (k_f/a)/\sqrt{c/a(1 - c/a)}$  is a measure of the average consumer's valuation for CSR activities per unit of market size (adjusted for unit cost relative to market size,  $\frac{c}{a}$ ). Moreover,  $m_f$  is increasing in  $\frac{k_f}{a}$  and it is U-shaped in  $\frac{c}{a}$  reaching its minimum value  $\frac{2k_f}{a}$  at  $c = \frac{a}{2}$ . Its maximum value is equal to 1.  $v = zc/(a - c)$  is a measure of the multinational firm's cost advantage adjusted with market size minus unit cost.

Hence, the equilibrium tariff rate is  $t = \frac{1}{3}(a - c)(1 + v)$ . Using  $t$ , we obtain firm  $i$ 's equilibrium output  $q_{fi}^e$  and profits  $\Pi_{fi}^e$ , as well as consumer surplus  $CS_f^e$  and total welfare  $TW_f^e$  in the foreign country. The resulting equilibrium outcomes in the home country are  $q_{hi}^e$ ,  $\Pi_{hi}^e$ ,  $CS_h^e$  and  $TW_h^e$ . The expressions are provided in Appendix 1a.

### The Multinational Firm Establishes a FDI in the Foreign Country.

We now consider the alternative case where the multinational establishes a subsidiary in the foreign country. In this case, in the third stage of the game, each firm chooses its output to maximize its profits given by (5) and (6) respectively and the local firm invests in CSR. From the first-order conditions, the firms' output reaction functions for the market in country  $f$  can be obtained:

$$R_{f1}^d(q_{f2}) = \frac{1}{2}[a - c(1 - z) - \gamma q_{f2}] \quad (12)$$

$$R_{f2}^d(q_{f1}) = a - c + s_f(k_f - cs_f) - \gamma q_{f1}. \quad (13)$$

As in the previous case, as long as  $s_f < k_f/2c$ , the positive demand effect of CSR dominates its negative unit cost effect and the local firm's output reaction function shifts outwards. Using the optimal level of CSR investments,  $s_f = k_f/2c$ , and solving the system of first order conditions, we obtain each firm's output in the foreign country as

$$q_{f1}^d = \frac{(a - 2cz)(2 - \gamma) - \frac{\gamma k_f^2}{4c}}{4 - \gamma^2} \quad (14)$$

$$q_{f2}^d = \frac{a(2 - \gamma) - c[2 - \gamma(1 - z)] + \frac{k_f^2}{2c}}{4 - \gamma^2}. \quad (15)$$

Compared to the previous case of exports, if the multinational firm selects FDI, its firm's cost advantage is further strengthened because it does not face a tariff. Therefore, in case of FDI market competition in the foreign country is fiercer than under exports. The local firm's equilibrium CSR investments are identical to the case of exports. Using  $s_f$ , we obtain firm  $i$ 's equilibrium profits  $\Pi_{fi}^d$ , as well as consumer surplus  $CS_f^d$  and total welfare  $TW_f^d$  in the foreign country (expressions are provided in Appendix 1b). Regarding the home country, output and profits are identical to those obtained in case of exports. Consumer surplus and total welfare in the home country are  $CS_h^d$  and  $TW_h^d$  respectively.

## 4 Comparing FDI and Exports

In this section, we compare the results for the case where the multinational firm exports to the foreign country with the results under the assumption that the multinational firm chooses FDI. The following Lemma summarizes the market and societal effects of CSR investments for each mode of entry by the multinational firm in the foreign country.<sup>4</sup>

<sup>4</sup> Due to space constraints, the full analysis of the benchmark scenario, where no firm invests in CSR, is not given here but available from the authors upon request.

**Lemma 1**

- Under both modes of entry, exports and FDI, CSR investments: (a) increase (decrease) the local firm's (multinational firm's) equilibrium output and profit; (b) increase consumer surplus and total welfare in the foreign country.
- When the multinational firm serves the foreign country's market through exports (FDI), total welfare in the multinational firm's home country is lower when the local firm invests in CSR rather than when it does not (is not affected by the local firm's CSR investments).

The following Proposition summarizes the effects of the firms' relative comparative advantages on the market and societal outcomes for each mode of entry by the multinational firm into the market of the foreign country.

**Proposition 1**

- The average consumer's willingness to pay for the local firm's product (higher  $m_f$ ): (a) increases the tariff, the CSR effort, the output and the profit of the local firm, as well as consumer surplus and total welfare in the foreign country; (b) decreases the multinational firm's output and profit, as well as total welfare in the multinational firm's home country.
- The extent of the multinational firm's cost advantage (higher  $v$ ) increases the tariff, the multinational firm's output and profit as well as consumer surplus and total welfare in both countries.

We can also observe that the multinational firm's quantity in the foreign country is always higher in case of FDI than under exports, i.e.,  $q_{f1}^d > q_{f1}^e$ . Recall that market competition in country  $f$  is fiercer in case of FDI than under exports due to the sunk cost character of FDI investments. In the FDI scenario, the multinational firm avoids the tariff and, hence, has lower marginal production costs. As quantities are in our case strategic substitutes, we can conclude that  $q_{f2}^e > q_{f2}^d$ .

Turning our attention to the second stage of the game, we find that the multinational firm will choose to serve the foreign country's market through FDI if  $T < T^c = \frac{[20(1+v)-3\gamma(4+m_f^2)](1+v)(a-c)^2}{9(4-\gamma^2)^2}$ . Regarding this critical level of sunk costs, the following observations are in order. First,  $\frac{dT^c}{dm_f} < 0$  suggests that an increase in consumers' valuation for the local firm's responsible product reduces the multinational firm's output and profit in country  $f$ . This, in turn, reduces the maximum affordable set-up cost that the multinational firm can pay for FDI in country  $f$ , i.e., such an increase in consumers' valuation mitigates the firm's incentive for FDI. Interestingly,  $\frac{dT^c}{dm_f} < 0$  further suggests that the multinational firm's maximum affordable set-up cost for FDI in country  $f$  is relatively lower than the corresponding level if the local firm does not invest in CSR. Hence, consumers' consciousness for responsible products gives space for attracting inward FDI by low-cost multinationals that target consumers with low willingness to pay for responsible products. Second,  $\frac{dT^c}{dv} > 0$  suggests that the multinational firm's cost advantage increases its maximum affordable set-up cost

for FDI in country  $f$ . Third, we find that the local firm prefers the multinational firm to export rather than entering through FDI, i.e.,  $\Pi_{f_2}^e > \Pi_{f_2}^d$  always holds.

The following Proposition summarizes:

### Proposition 2

- *The multinational firm will choose to serve the foreign country's market through FDI, if and only if the sunk cost for establishing a production plant in the foreign country is sufficiently low, i.e.,  $T < T^c$ .*
- *The multinational firm's incentives to serve the foreign country through FDI are strengthened in this firm's cost advantage (higher  $v$  or higher  $z$ ) and mitigated in the average consumer's valuation for CSR (higher  $m_f$ ) in the foreign country.*

Let us now focus on the relative welfare effects of the multinational firm's choice to serve the foreign country's market. First of all,  $CS_h^d = CS_h^e$  always holds. That is, the multinational firm's mode of entry in country  $f$  does not affect country  $h$ 's consumer welfare. Yet, total welfare in country  $h$  is always higher under exports than under FDI, i.e.,  $TW_h^e > TW_h^d$ . The reason is that under FDI the MNE's profit in the foreign country only counts for the foreign country's welfare. This suggests that the home country  $h$ 's policy maker could introduce an industrial policy subsidizing exports.

By contrast, consumer surplus in country  $f$  is always higher under FDI, i.e.,  $CS_f^d > CS_f^e$  always holds. This happens because market competition in the former case is fiercer than in the latter and hence, total quantity and consumer surplus in country  $f$  are relatively higher under FDI. Moreover, total welfare in country  $f$  is also higher under FDI than under exports, i.e.,  $TW_f^e < TW_f^d$  always holds. Intuitively, besides consumer surplus which is relatively higher under FDI, the multinational firm's profit in country  $f$  in case of FDI exceed the tariff revenues in case of exports to this country, i.e.,  $\Pi_{f_1}^d > tq_{f_1}^e$ . Although the local firm's profits are always higher in case of exports than under FDI, the above two positive effects dominate and total welfare in country  $f$  is relatively higher under FDI. This suggests that country  $f$ 's policy maker could take measures to attract inward FDIs by multinational firms. That is, country  $f$ 's policy maker can provide incentives through an industrial policy subsidizing inward FDIs, with the minimum subsidy being equal to the sunk cost needed for the establishment of the multinational firm's production plant in country  $f$ . The following Proposition summarizes our findings.

### Proposition 3

- *Consumer surplus in the multinational firm's country is independent of the multinational firm's mode of entry in the foreign country. Total welfare in the multinational firm's country is always higher in case of exports than under FDI.*
- *Consumer surplus and total welfare in the foreign country are always higher in case of FDI than under exports.*

The above analysis leads us to two further observations. First, independently of the multinational firm's mode of entry in the foreign country, CSR is welfare-enhancing and policy makers should take measures to promote CSR activities. The policy of the European Commission (2011) is in line with this finding and argues that "...the Commission will step up its cooperation with Member States, partner countries and relevant international fora to promote respect for internationally recognised principles and guidelines, and to foster consistency between them. This approach also requires EU enterprises to renew their efforts to respect such principles and guidelines." See also European Union (2014). Second, it should be noted, however, that the stakeholders' preferences for the multinational firm's mode of entry in the foreign country are not aligned. More specifically, assuming that within each country, the firm, the consumers, and the policy maker are the related stakeholders, we find that the local firm and the multinational country's policy maker would prefer the multinational firm to serve the local firm's country through exports. On the contrary, consumers and the policy maker in the local firm's country would always prefer FDI. These observations reveal that there is space for lobbying over trade and/or industrial policies affecting the mode of entry of multinational firms in foreign countries.

## 5 Conclusion

The present work contributes to the scant literature on the interface between CSR activities to improve the quality or "greenness" of products and the mode of entry of multinational firms into the markets of foreign countries. In our setting, a local firm has the option to become "responsible" and follow a "doing well by doing good" strategy through investing in CSR activities along its value chain. By doing CSR, it tries to achieve a competitive advantage against a multinational rival firm that operates with lower unit costs and plans to enter the local firm's market either through exports or through FDI.

We find that the average consumer's consciousness for the local firm's responsible product increases the tariff, the local firm's CSR effort, output and profit, as well as consumer surplus and total welfare in the foreign country. Yet, it decreases the multinational firm's output and profit, as well as total welfare in the multinational firm's country. Interestingly, the aforementioned consciousness reduces the multinational firm's maximum affordable set-up cost for FDI in the foreign country and gives space for attracting inward FDI by low-cost multinationals that target consumers with low willingness to pay for responsible products.

We also find that there is misalignment of preferences between the stakeholders of the two countries over the multinational firm's mode of entry in the foreign country. This leaves space for lobbying about the relevant trade/industrial policies, an issue that we leave for future research.

### Disclosure Statement

No potential conflict of interest was reported by the authors.

**Acknowledgement.** Part of this work was supported by COST Action IS1104 “The EU in the new economic complex geography: models, tools and policy evaluation”. Manasakis acknowledges financial support from the University of Crete, Special Account for Research. Full responsibility for all shortcomings is ours.

## Appendix

### 1a: The Multinational Firm Exports to the Foreign Country

$$\begin{aligned}
 q_{h_1}^e &= \frac{1}{2} (\alpha - c) (1 + v); \Pi_{h_1}^e = (q_{h_1}^e)^2 \\
 q_{f_1}^e &= \frac{[16(1 + v) - 3\gamma(4 + m_f^2)] (a - c)}{12(4 - \gamma^2)}; \Pi_{f_1}^e = (q_{f_1}^e)^2 \\
 q_{f_2}^e &= \frac{[3(4 + m_f^2) - 4\gamma(1 + v)] (a - c)}{6(4 - \gamma^2)}; \Pi_{f_2}^e = (q_{f_2}^e)^2 \\
 CS_h^e &= \frac{1}{2} (q_{h_1}^e)^2; TW_h^e = CS_h^e + \Pi_{h_1}^e + \Pi_{f_1}^e \\
 CS_f^e &= \frac{1}{2} [(q_{f_1}^e)^2 + (q_{f_2}^e)^2 + 2\gamma q_{f_1}^e q_{f_2}^e]; TW_f^e = CS_f^e + \Pi_{f_2}^e + tq_{f_1}^e
 \end{aligned}$$

### 1b: The Multinational Firm Establishes FDI in the Foreign Country

$$\begin{aligned}
 q_{h_1}^d &= \frac{1}{2} (\alpha - c) (1 + v); \Pi_{h_1}^d = (q_{h_1}^d)^2 \\
 q_{f_1}^d &= \frac{[8(1 + v) - \gamma(4 + m_f^2)] (a - c)}{4(4 - \gamma^2)}; \Pi_{f_1}^d = (q_{f_1}^d)^2 - T \\
 q_{f_2}^d &= \frac{[4 + m_f^2 - 2\gamma(1 + v)] (a - c)}{2(4 - \gamma^2)}; \Pi_{f_2}^d = (q_{f_2}^d)^2 \\
 CS_h^d &= \frac{1}{2} (q_{h_1}^d)^2; TW_h^d = CS_h^d + \Pi_{h_1}^d \\
 CS_f^d &= \frac{1}{2} [(q_{f_1}^d)^2 + (q_{f_2}^d)^2 + 2\gamma q_{f_1}^d q_{f_2}^d]; TW_f^d = CS_f^d + \Pi_{f_1}^d + \Pi_{f_2}^d
 \end{aligned}$$

## References

- Becchetti, L., Federico, G., Nazaria, S.: What to do in globalised economies if global governance is missing? The vicarious role of competition in social responsibility. *Int. Rev. Econ.* **58**(2), 185–211 (2011)
- Benabou, R., Tirole, J.: Individual and corporate social responsibility. *Economica* **77**, 1–19 (2010)



- Brander, J.A., Spencer, B.J.: Tariff protection and imperfect competition. In: Kierzkowski, H. (ed.) *Monopolistic competition and international trade*. Oxford University Press, Oxford (1984)
- Brander, J.A., Spencer, B.J.: Export subsidies and international market share rivalry. *J. Int. Econ.* **18**(1–2), 83–100 (1985)
- Campbell, T.J., Eden, L., Miller, S.R.: Multinationals and corporate social responsibility in host countries: does distance matter? *J. Int. Bus. Stud.* **43**, 84–106 (2012)
- Chambolle, C., Giraud-Héraud, E.: Certification of origin as a non-tariff barrier. *Rev. Int. Econ.* **13**(3), 461–471 (2005)
- Chang, Y.M., Chen, H.Y., Wang, L.F.S., Wu, S.J.: Corporate social responsibility and international competition: a welfare analysis. *Rev. Int. Econ.* **22**(3), 625–638 (2014)
- Eicher, T., Kang, J.W.: Trade, foreign direct investment or acquisition: optimal entry modes for multinationals. *J. Dev. Econ.* **77**(1), 207–228 (2005)
- European Commission, Promoting a European framework for corporate social responsibility. Green Paper and COM, vol. 366, Brussels (2001)
- European Commission, Implementing the partnership for growth and jobs: making Europe a pole of excellence on corporate social responsibility. COM, vol. 136, Brussels (2006)
- European Commission, A renewed EU strategy 2011–2014 for corporate social responsibility. COM, vol. 681, Brussels (2011)
- European Union, Directive 2014/95/EU of the European Parliament and of the Council of 22 October 2014 amending Directive 2013/34/EU as regards disclosure of non-financial and diversity information by certain large undertakings and groups. Official Journal of the European Union, vol. L 330/1, 15 November 2014 (2014)
- Fumagalli, C.: On the welfare effects of competition for foreign direct investments. *Eur. Econ. Rev.* **47**(6), 963–983 (2003)
- Garella, P.G., Petrakis, E.: Minimum quality standards and consumers information. *Econ. Theor.* **36**(2), 283–302 (2008)
- Häckner, J.: A note on price and quantity competition in differentiated oligopolies. *J. Econ. Theor.* **93**(2), 233–239 (2000)
- Glass, A.J., Saggi, K.: Exporting versus direct investment under local sourcing. *Rev. World Econ.* **141**(4), 627–647 (2005)
- Graf, C., Wirl, F.: Corporate social responsibility: a strategic and profitable response to entry? *J. Bus. Econ.* **84**, 917–927 (2014)
- Hao, Q., Lahiri, S.: Competition for foreign direct investment: the role of technology and market structure. *Int. Rev. Econ. Finance* **18**, 680–690 (2009)
- Ioannou, I., Serafeim, G.: Impact of corporate social responsibility on investment recommendations: analysts' perceptions and shifting institutional logics. *Strateg. Manag. J.* **36**(7), 1053–1081 (2015)
- Ishikawa, J., Horiuchi, E.: Strategic foreign direct investment in vertically related markets. *Econ. Rec.* **88**(281), 229–242 (2012)
- Kitzmueller, M., Shimshack, J.: Economic perspectives on corporate social responsibility. *J. Econ. Lit.* **50**(1), 51–84 (2012)
- Liu, C.-C., Wang, L.F.S., Lee, S.-H.: Strategic environmental corporate social responsibility in a differentiated duopoly market. *Econ. Lett.* **129**, 108–111 (2015)
- Ma, J., Zhou, J.: A note on foreign direct investment and exports in vertically differentiated industries. *Bull. Econ. Res.* (2016). doi:[10.1111/boer.12087](https://doi.org/10.1111/boer.12087)
- Manasakis, C., Mitrokostas, E., Petrakis, E.: Certification of corporate social responsibility activities in oligopolistic markets. *Can. J. Econ.* **46**(1), 282–309 (2013)
- Motta, M., Norman, G.: Does economic integration cause foreign direct investment? *Int. Econ. Rev.* **37**, 757–783 (1996)

- Mukherjee, A.: Unionised labour market and strategic decision of a multinational. *Econ. J.* **118**, 1621–1639 (2008)
- Mukherjee, A., Suetrong, K.: Privatization, strategic foreign direct investment and host-country welfare. *Eur. Econ. Rev.* **53**, 75–785 (2009)
- Naylor, R., Santoni, M.: Foreign direct investment and wage bargaining. *J. Int. Trade Econ. Dev. Int. Comp. Rev.* **12**(1), 1–18 (2003)
- OECD, OECD Guidelines for Multinational Enterprises. OECD Publishing (2011)
- Porter, M.E., Kramer, M.R.: Strategy and society: the link between competitive advantage and corporate social responsibility. *Harvard Bus. Rev.* **84**, 76–92 (2006)
- Porter, M.E., Kramer, M.R.: Creating shared value. *Harvard Bus. Rev.* **89**, 62–77 (2011)
- Sinha, U.B.: Strategic licensing, exports FDI, and host country welfare. *Oxford Econ. Pap.* **62**, 114–131 (2010)
- United Nations, Trade and development report, Report by the secretariat of the United Nations Conference on Trade and Development, New York and Geneva (2014a)
- United Nations, World investment report 2014: Investing in the sustainable development goals (SDGs): An action plan, New York and Geneva (2014b)
- United Nations, United Nations Global Compact, New York and Geneva (2014c)
- Wang, L.F.S., Wang, Y.C., Zhao, L.: Tariff policy and welfare in an international duopoly with consumer-friendly initiative. *Bull. Econ. Res.* **64**(1), 56–64 (2012)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# Knowledge Spillovers, Congestion Effects, and Long-Run Location Patterns

Gian Italo Bischi<sup>1</sup>, Michael Kopel<sup>2</sup>, Fabio Lamantia<sup>3,4</sup>, and Davide Radi<sup>5</sup>(✉)

<sup>1</sup> DESP-Department of Economics, Society, Politics,  
University of Urbino Carlo Bo, 42 Via Saffi, 61029 Urbino, PU, Italy  
[gian.bischi@uniurb.it](mailto:gian.bischi@uniurb.it)

<sup>2</sup> Institute of Organization and Economics of Institutions,  
University of Graz, Graz, Austria  
[michael.kopel@uni-graz.at](mailto:michael.kopel@uni-graz.at)

<sup>3</sup> Department of Economics, Statistics and Finance, University of Calabria,  
3C Via P. Bucci, 87036 Rende, CS, Italy  
[fabio.lamantia@unical.it](mailto:fabio.lamantia@unical.it)

<sup>4</sup> Economics – School of Social Sciences, The University of Manchester,  
Arthur Lewis Building, Manchester, UK

<sup>5</sup> School of Economics and Management, LIUC - Università Cattaneo,  
22 C.so Matteotti, 21053 Castellanza, VA, Italy  
[dradi@liuc.it](mailto:dradi@liuc.it)

**Abstract.** We introduce an evolutionary two-country model to characterize long run location patterns of the manufacturing activities of competing multinational enterprises. Firms located in country 1 can decide to offshore their manufacturing activities to country 2. The profitability of production in a country depends on several factors: unitary costs of production, the number of firms that are located in each country, within-country spillovers, and cross-border spillovers. Furthermore, profits in country 2 are influenced by congestion costs. Country 1 is assumed to be technologically advanced and has an advantage in terms of internal spillovers. In contrast, country 2 offers lower production unit cost which, however, may be offset by congestion costs. The firms' (re)location choices are based on a simple comparison of current production costs obtained in the two countries and the dynamics of switching is modeled by a simple replicator dynamics. The global analysis of the resulting one-dimensional dynamical system reveals that a large advantage in terms of unitary production costs encourages the firms to off-shore manufacturing activities to country 2. This off-shoring process stops when congestion costs offset this advantage of country 2, even though congestion costs do not cause all manufacturing activities to be re-shored to country 1. The re-shoring process can be accelerated by an increase of within-country spillovers in country 1, while cross-border spillovers tend to favor a geographic dispersion of manufacturing activities and make location patterns that lead to suboptimal long run outcomes less likely.

**Keywords:** Long run location patterns · Off-shoring · Re-shoring · Knowledge spillovers · Congestion costs · Exponential replicator dynamics · Global dynamics

## 1 Introduction

The off-shoring of manufacturing activities to emerging economies has offered an opportunity to reduce manufacturing costs and has therefore been a common practice for many Multinational Enterprises (MNEs). This phenomenon has provoked massive changes in the organization of industrial activity and in the labor market of developed economies. As a consequence, it has attracted political attention. Governments of the OECD economies have focused on developing incentive-schemes designed to reduce job losses by providing incentives for MNEs to re-shore their manufacturing activities. An increase in re-shoring activity has been observed in recent years, with many MNEs increasingly bringing manufacturing activity back home. Empirical evidence documents the increase in re-shoring activity, but also indicates that in comparison to firms' off-shoring activities the process is still limited, see Backer et al. (2016).

The identification of drivers that cause firms to choose particular locations for their activities still attracts increasing attention from scholars. The literature on International Business has identified a variety of drivers that guide the location decision of MNEs and has provided empirical tests. Following the literature (Alcácer et al. (2013)), location drivers can be gathered in two broad categories: *endowment* drivers and *agglomeration* drivers. The stream of literature based on endowment drivers tries to explain the location patterns of manufacturing activity in terms of location traits such as physical infrastructure, quality of the labor force, and cultural distance, see, e.g., Coughlin et al. (1991) and Flores and Aguilera (2007). In more recent studies, focus has been put on institutional features such as contractual hazards and appropriation concerns (Henisz (2000)) and on the enforcement of property rights (Lee and Mansfield (1996)). Differently, the stream of literature based on agglomeration drivers takes into account positive externalities that derive from the geographical clustering of manufacturing activities, for example, due to technological spillovers, access to specialized labor, and access to specialized intermediate inputs, see Marshall (1982). Then, according to this approach, the presence of knowledge spillovers is country-specific, but also depends on the location choice of firms. In other words, it has an endogenous component that depends on firms observing the economic environment and choosing the location strategically. More recent literature puts increased attention to strategic interaction between MNEs in oligopolistic markets and the impact on agglomeration phenomena; see, e.g., Alcácer and Chung (2007). These agglomeration economies have been further studied with formal models, e.g., in David and Rosenbloom (1990), Krugman (1991), Bischi et al. (2003a) and Bischi et al. (2003b), and have been empirically documented in global and international settings, see, e.g., Carlton (1983) and Mariotti et al. (2010).

In the current work, we develop a dynamic model that considers a combination of endowment drivers and agglomeration drivers. The model describes the repeated manufacturing location decisions of firms that choose between off-shoring and re-shoring their activities to benefit from a reduction in manufacturing costs taking into account knowledge spillovers between firms. Firms are based

in a developed country, called country 1, and have the option to off-shore manufacturing activity to an emerging country, called country 2. The unit costs of production in country 1 are higher but are decreased by cost-reducing externalities, to which we refer as *within-country spillovers*. These within-country spillovers depend on the share of manufacturing activity in country 1. More specifically, the higher the level of manufacturing activity, the higher are the spillovers and the cost-reducing externalities. In this case, the level of spillovers co-evolves over time with the share of manufacturing activity and crucially influences the strategic location choices of firms. Manufacturing costs can also be reduced by *cross-border spillovers* that capture the benefit of knowledge developed in country  $i$  that spills over to the other country  $j$  and reduces the costs of production there. In line with empirical evidence which reveals that knowledge spillovers are geographically localized and that knowledge transfer decreases with distance even within the same firm (see, e.g., Adams and Jaffe (1996, Rosenthal and Strange 2003 and Jaffe et al. 1993)), we assume that cross-border spillovers per unit of production are smaller than (or equal to) within-country spillovers. In our framework, knowledge spillovers can also be developed within country 2, but the related cost-reducing externalities might differ from country 1. This assumption is motivated by empirical evidence which highlights that technological spillovers and innovation activity vary across locations due to differences in initial endowments, the actions of actors engaged in R&D (governments, universities and firms), the links among those actors, and the differences in educational systems and regulation, see, e.g., Nelson (1993) and Furman et al. (2002). The effect of endowment drivers are simply captured by an additional linear cost component – representing congestion costs – that depends on the level of activity in country 2. Congestion costs measure the level, the quality, and the (in)efficiencies of the infrastructure and facilities that a country can offer. It is an essential feature of a manufacturing site that has a crucial impact on the location choice of MNEs. Moreover, this feature is country-specific as it cannot be easily transferred from country to country and it cannot be modified in the short run. It depends on the level of education, the structural investments undertaken in the past, the resources invested in research in the last decades and the quality of the institutions. For the sake of simplicity, and without loss of generality, the congestion costs are assumed to be zero for country 1.

The cost-reducing effect due to internal (within-country) and external (cross-border) spillovers are modeled as in Bischi et al. (2003a) and Bischi et al. (2003b) and we adopt their functional form for the unit costs of manufacturing. The novel element in our model is that we introduce congestion costs. The firms' location decisions are based on the relative performance of producing at home in country 1 and producing in the foreign country 2. The switching between decisions is described by a replicator dynamics, see, e.g., Hofbauer and Sigmund (2003). The evolutionary framework proposed in this chapter links firm's location choices with internal and external knowledge spillovers and congestion costs. We illustrate how such a highly stylized dynamical model can be used to derive policy implications. We further shed light on aspects such as the combined impact of

knowledge spillovers and congestion costs on the long run location patterns of manufacturing activity. In particular, we try to improve our understanding of which combinations of within-country spillovers and congestion costs lead to re-shoring of manufacturing activity, the effects of cross-border spillovers on the relocation choice, the existence of suboptimal location patterns and the parameter settings that lead to optimal (minimum production costs) outcomes. We also try to identify parameters that have the biggest impact on location patterns and, hence, might be suitable for economic policy.

As the literature reports that over time off-shoring of manufacturing has lead to a substantial increase in production costs in the foreign countries, we focus our analysis on the effect of congestion costs on the global dynamics of location patterns that result from our evolutionary model. The investigation starts with the global analysis in absence of cross-border spillovers and considers three different configurations of unit costs of production. For each configuration, three cases of different within-country spillovers are considered. Hence, our analysis considers nine different scenarios ranging from the benchmark case in which the two countries differ only in congestion costs to an extreme situation in which country 2 offers a rather large cost advantage while country 1 offers large cost-reducing within-country spillovers. Then, the analysis is completed considering cross-border spillovers and asymmetric cases in which the firms benefit from cross-border spillovers only operating in one of the two countries.

The investigation reveals that the particular combination of within-country and cross-border spillovers crucially shape the spatial distribution of the industrial activities between the two countries. In particular, if there is no difference in unit production cost, the manufacturing activity tends to be polarized in one of the two countries. In this case, congestion costs reduce off-shoring of manufacturing activity to country 2. In contrast, the possibility to benefit from a large advantage in unit production costs if manufacturing is located in country 2 accelerates off-shoring activities and, for sufficiently high congestion costs, leads to a location pattern in which the manufacturing activity is spread between the two countries. It is worth observing that large congestion costs increase re-shoring activities, but do not completely eliminate manufacturing activity in country 2. In fact, depending on the strategic location choices of firms, congestion costs are negligible if the level of manufacturing activity in country 2 is low. In this context, the presence of cross-border spillovers reduces the polarization of the manufacturing activity in a single country. The optimality of the geographic allocation of the manufacturing activity in terms of resulting costs of manufacturing is another important aspect. Our investigation reveals that congestion costs and asymmetric within-country spillovers can lead to equilibrium location patterns that are suboptimal. This occurs if firms concentrate their manufacturing activities in a location that does not ensure the lowest production costs. The presence of cross-border spillovers reduces the likelihood of long run location patterns that lead to such suboptimal situations, while a counter-intuitive effect is observed in presence of asymmetric cross-border spillovers. In particular, when firms can absorb the knowledge spillovers coming from country 2 only,

a reduction of within-country spillovers in country 2 contributes to an increase in off-shoring. This finding highlights that the conventional wisdom put forward by the International Business literature that firms will be attracted to locations with larger knowledge activity, see, e.g., Alcácer and Chung (2007), does not hold in general and, specifically, does not hold in case of asymmetric outflows of spillovers.

The structure of the chapter is as follows. Section 2 introduces the two-country model. Section 3 identifies the equilibria of the model, their local stability and their optimality. Section 4 investigates the global dynamics of the location patterns for different configurations of the parameters and studies the effect of the congestion costs. Section 5 concludes. All proofs are given in the Appendix.

## 2 The Model

Let us consider a population of firms that manufacture a homogenous product. There are two countries indexed by  $i = 1, 2$ . Total production is sold at a positive price in a global market characterized by a given demand function. Total production quantity is normalized to one and we are interested in the share of total production that is manufactured in country  $i$ . The firms choose whether to produce in country 1 or to locate production in country 2. At each discrete time  $t \in \mathbb{N}$ , the firms' choices determine the fraction  $x(t)$  of production in country 1 and the complementary fraction  $1 - x(t)$  in country 2. The current allocation determines production costs in each country. In particular, we assume that producing in country 1 and country 2 implies the following costs,

$$\begin{aligned} C_1(x(t)) &= \frac{c_1}{1 + \beta_1 x(t) + \gamma_{12}(1 - x(t))} \quad \text{and} \\ C_2(x(t)) &= \frac{c_2}{1 + \beta_2(1 - x(t)) + \gamma_{21}x(t)} + k(1 - x(t)). \end{aligned} \quad (1)$$

The unit production costs in country  $i = 1, 2$  are denoted by  $c_i$  and  $\beta_i$  is the coefficient of within-country spillovers in country  $i$ . We assume that country 1 is more efficient. In particular, we consider country 1 to be a technology leader (a developed economy) and country 2 to be a technology laggard (a developing economy). Since production costs in a technologically advanced country are typically higher (e.g. due to higher wages) than the costs of production in country 2 that is a technology laggard, we consider  $c_1 \geq c_2 \geq 0$ . The cost-reducing externalities related to internal spillovers are higher in country 1 than in country 2,  $\beta_1 \geq \beta_2$ . Cross-border spillovers are represented by  $\gamma_{ji}$  which capture cost-reducing externalities in country  $j$  related to knowledge spillovers coming from country  $i \neq j$ , see similarly Bischi and Lamantia (2002), Bischi et al. (2003a) and Bischi et al. (2003b). Knowledge spillovers are geographically constrained, see, e.g., Rosenthal and Strange (2003) and Jaffe et al. (1993), and the benefits of knowledge transfer decreases with distance even within the same firm, see Adams and Jaffe (1996) and Alcácer and Chung (2007). It follows that the benefits from cost-reducing spillovers are assumed to be higher in the country in

which (manufacturing) know-how is developed. In other words,  $\beta_i \geq \gamma_{ji}$ . Finally,  $k$  is a coefficient that measures the effect of congestion, a cost-increasing externality related to overcrowding which increasingly becomes important if the level of manufacturing activity in country 2 increases. A firm operating in country 1 does not suffer from congestion costs due to, e.g., better infrastructure. These arguments yield the following parameter constraints.

**Assumption 1.** *We assume that  $c_1 \geq c_2 > 0$ ,  $\beta_1 \geq \beta_2 \geq 0$ ,  $\beta_2 \geq \gamma_{12} \geq 0$ ,  $\beta_1 \geq \gamma_{21} \geq 0$ ,  $k \geq 0$ .*

Note that cost function  $C_1$  is monotonically decreasing with respect to  $x$ , the fraction of manufacturing activity located in country 1, while the cost function  $C_2$  is U-shaped. We assume that the evolution of the share of manufacturing activity located in country 1, i.e.  $x(t) \in [0, 1]$ , is given by the following exponential replicator equation (see Cabrales and Sobel (1992)):

$$x_i(t+1) = T(x(t)) = (1-\alpha)x(t) + \alpha x(t) \frac{1}{x(t) + (1-x(t))e^{\beta(C_1(x(t)) - C_2(x(t)))}}. \quad (2)$$

This model proposes that the share of manufacturing activity in country 1 increases (decreases) from period  $t$  to period  $t+1$  if country 1's production costs in period  $t$  are lower (higher) than country 2's production costs. The parameter  $\beta > 0$  measures the intensity of switching between the locate-production-at-home strategy and the off-shore-production strategy. It determines the fraction of firms that move production from one country to the other in their search for lower costs of production. The firms are affected by inertia which is related to difficulties in changing the location choice. This inertia is captured by parameter  $\alpha \in [0, 1]$ . The replicator Equation (2) captures an evolutionary selection mechanism by which better strategies spread in a population of firms. Differently from the classical replicator dynamics, see, e.g., Hofbauer and Sigmund (2003) and references therein, it has the convenient property that the state space  $[0, 1]$  is invariant under the dynamics of (2). Thus, long run location patterns of the firms are obtained as the steady states of this evolutionary model.

The goal of our work is to analyze the possible long run location patterns of the manufacturing activity and to identify the possible measures that a policymaker can take to influence these location patterns. In this regard, we are interested to understand the role that is played by spillovers and congestion costs if firms choose to produce in country 2 to take advantage of lower production costs and under which circumstances country 1 becomes attractive.

### 3 Equilibrium Location Patterns: Local Stability and Efficiency

The off-shoring of the entire manufacturing activity to country 2 represents an equilibrium of the model (2) as well as concentrating manufacturing in country 1. These are only two of several possible long-run location patterns and the



possibility to converge to one of these equilibria depends on the initial share of firms that manufacture in country 1 and on the particular parameter values. In the next section, we analyze this aspect by so-called bifurcation diagrams which show the changes of such equilibria and their basins of attraction as congestion costs vary. Our analysis is performed for various configurations of the parameters of the cost functions. The numerical analysis is supported by and combined with analytical results which shed some light on the possible long-run location patterns. In particular, it is possible to prove the following stability properties for the long-run location patterns.

**Theorem 1.** *The model (2), given assumption 1 holds, has the two border equilibria, i.e.  $x = 1$  and  $x = 0$ , and at most two more equilibria in  $(0, 1)$ . Let us define*

$$k_{T_1} = \frac{c_1}{1 + \gamma_{12}} - \frac{c_2}{1 + \beta_2} \quad \text{and} \quad k_{T_2} = \frac{c_1}{1 + \beta_1} \left( \frac{\beta_1 - \gamma_{12}}{1 + \beta_1} - \frac{\gamma_{21} - \beta_2}{1 + \gamma_{21}} \right) \quad (3)$$

Then,

- if  $C_1(1) < C_2(1)$ , i.e.  $\frac{c_1}{c_2} < \frac{1+\beta_1}{1+\gamma_{21}}$ , equilibria  $x = 1$  and  $x = 0$  are both locally asymptotically stable and a repeller exists in  $(0, 1)$  for  $k < k_{T_1}$ ; at  $k = k_{T_1}$  equilibrium  $x = 0$  loses stability through either a transcritical or a pitchfork bifurcation and, for  $k > k_{T_1}$ , equilibrium  $x = 1$  is stable while  $x = 0$  is unstable.
- if  $C_1(1) > C_2(1)$ , i.e.  $\frac{c_1}{c_2} > \frac{1+\beta_1}{1+\gamma_{21}}$ ,  $x = 1$  is unstable and  $x = 0$  is locally asymptotically stable for  $k < k_{T_1}$ ,  $x = 0$  loses stability at  $k = k_{T_1}$  and for  $k > k_{T_1}$  equilibria  $x = 1$  and  $x = 0$  are both unstable and at least one internal fixed point exists.
- if  $C_1(1) = C_2(1)$ , i.e.  $\frac{c_1}{c_2} = \frac{1+\beta_1}{1+\gamma_{21}}$ ,  $x = 0$  is local asymptotically stable with basin of attraction given by  $\mathcal{B}(0) = [0, 1)$  and  $x = 1$  is unstable, at  $k = k_{T_2} < k < k_{T_1}$  both  $x = 1$  and  $x = 0$  are asymptotically stable, at  $k = k_{T_1}$  a transcritical bifurcation occurs and for  $k > k_{T_1}$  equilibrium  $x = 0$  is unstable and equilibrium  $x = 1$  is locally asymptotically stable with  $\mathcal{B}(1) = [0, 1)$ .

The Theorem reveals that it is feasible to produce in the technologically advanced country 1 if the higher unit costs of production are offset by higher technological spillovers, while the congestion costs do not affect the stability of this long-run location pattern. After all, these costs are related to the concentration of manufacturing activity in the less technologically advanced country and the cost effect is marginal when production activity is mostly located in country 1. On the contrary, large congestion costs make off-shoring of manufacturing activity infeasible.

Although some insights on the long-run location patterns can be gained from analytical derivations, the complicated form of the cost functions makes it difficult to identify all equilibria of the model and the type of bifurcations that are responsible for their existence or stability. Nevertheless, it is possible to consider

some benchmark cases that allow us to develop insights on the changes in the more complicated scenarios. We start by abstracting from congestion costs. In this case, it is possible to prove the following result.

**Theorem 2 (No congestion costs).** *Let  $k = 0$ . Then,*

- *when  $C_1(0) > C_2(0)$ , i.e. i.e.  $\frac{c_1}{c_2} > \frac{1+\gamma_{12}}{1+\beta_2}$ , and either  $\beta_1 > \gamma_{12}$  or  $\beta_2 \neq \gamma_{21}$ , the long-run location pattern  $x = 0$  is always at least locally stable. Specifically, for  $C_1(1) > C_2(1)$ , we have  $\mathcal{B}(0) = [0, 1]$ , for  $C_1(1) = C_2(1)$  a transcritical bifurcation occurs through which the equilibrium  $x^*$ , given by*

$$x^* = \frac{c_1(1 + \beta_2) - c_2(1 + \gamma_{12})}{c_1(\beta_2 - \gamma_{21}) + c_2(\beta_1 - \gamma_{12})} \quad (4)$$

*merges with the equilibrium  $x = 1$  and becomes feasible, i.e.  $x^* \in [0, 1]$ . For  $C_1(1) < C_2(1)$ , the equilibria  $x = 1$  and  $x = 0$  are both locally asymptotically stable with basin of attraction given by  $\mathcal{B}(1) = (x^*, 1]$  and  $\mathcal{B}(0) = [0, x^*)$ , respectively. The fixed point  $x^*$  is a repellor. For the special case  $c_1 = c_2$ ,  $\beta_1 = \beta_2$  and  $\gamma_{12} = \gamma_{21} = 0$ ,  $x^* = \frac{1}{2}$ , we have that equilibria  $x = 0$  and  $x = 1$  are both asymptotically stable with  $\mathcal{B}(0) = [0, \frac{1}{2})$  and  $\mathcal{B}(1) = (\frac{1}{2}, 1]$ .*

- *when  $C_1(0) = C_2(0)$  (which implies  $\beta_2 = \gamma_{12}$  and  $c_1 = c_2$  under assumption 1) and either  $\beta_1 > \gamma_{12}$  or  $\beta_2 \neq \gamma_{21}$ , it follows that  $\mathcal{B}(1) = (0, 1]$  for  $C_1(1) < C_2(1)$ , the fixed points of model (2) fill the region  $[0, 1]$  when  $C_1(1) = C_2(1)$  and  $\mathcal{B}(0) = [0, 1]$  when  $C_1(1) > C_2(1)$ .*
- *in case of total absorption of cross-border spillovers, i.e.  $\beta_1 = \gamma_{12}$  and  $\beta_2 = \gamma_{21}$ , when the production costs are symmetric, i.e.  $c_1 = c_2$ , the fixed points of model (2) fill the region  $[0, 1]$ , otherwise the long-run outsourcing location pattern 0 is the only locally stable equilibrium such that  $\mathcal{B}(0) = [0, 1]$ .*

The Theorem underlines that in the absence of congestion costs, off-shoring of manufacturing represents a stable long-run location pattern which may or may not coexist with the equilibrium where manufacturing occurs only in the home country 1. Moreover, if knowledge is transferable between countries without losses (i.e. internal and cross-border spillovers are identical), then it is attractive to off-shore manufacturing to country 2. In this case, the absence of congestion cost and the presence of even a small production cost advantage offered by country 2, makes off-shoring of manufacturing the unique long-run location pattern. The scenario changes when production in country 2 implies congestion costs. Assuming that cost-reducing know-how is perfectly transferable between countries, it is possible to prove the following results.

**Theorem 3.** *Let  $\beta_1 = \gamma_{21}$ ,  $\beta_2 = \gamma_{12}$  and  $k > 0$ . If  $c_1 = c_2$ , then equilibrium  $x = 1$  is asymptotically stable with basin of attraction given by  $\mathcal{B}(1) = (0, 1]$ , otherwise equilibrium  $x = 1$  is never asymptotically stable and,*

- *when  $\beta_1 > 1 + 2\beta_2$  and  $\beta_1 \neq \beta_2$ , for  $k < k_F$ , where*

$$k_F = \frac{4(\beta_1 - \beta_2)(c_1 - c_2)}{(\beta_1 + 1)^2 + 2\beta_1\beta_2} \quad (5)$$

the equilibrium  $x = 0$  is locally stable with basin of attraction given by  $\mathcal{B}(0) = [0, 1)$ . At  $k = k_F$ , two fixed points appear through a fold bifurcation:

$$x_{1,2}^* = \frac{(1 - \beta_1 + 2\beta_2)k \pm \sqrt{k[(\beta_1 + 1)^2 k + 2\beta_1\beta_2 k + 4(\beta_2 - \beta_1)(c_1 - c_2)]}}{2k(\beta_2 - \beta_1)}$$

for  $k_F < k < k_{T_1}$ , where

$$k_{T_1} = \frac{c_1 - c_2}{1 + \beta_2}, \quad (6)$$

the fixed point  $x = 0$  is asymptotically stable with basin of attraction given by  $\mathcal{B}(0) = [0, x_1^*)$ , while the fixed point  $x_1^*$  is either stable with basin of attraction  $\mathcal{B}(x_1^*) = (x_2^*, 1)$  or it loses stability through a period doubling bifurcation. Moreover, at  $k = k_{T_1}$  the equilibrium  $x_2^*$  merges with equilibrium  $x = 0$  which loses its stability through a transcritical bifurcation. For  $k > k_{T_1}$ , both equilibria  $x = 0$  and  $x = 1$  are unstable and the internal fixed point is either stable with basin of attraction  $\mathcal{B}(x_1^*) = (0, 1)$ , or it loses stability through a flip bifurcation.

- when  $\beta_1 < 1 + 2\beta_2$  and  $\beta_1 \neq \beta_2$ , for  $k < k_{T_1}$  the fixed point  $x = 0$  is asymptotically stable with basin of attraction given by  $\mathcal{B}(0) = [0, 1)$ , while at  $k = k_{T_1}$  the fixed point  $x = 0$  loses stability through a transcritical bifurcation and the internal fixed point, i.e.

$$x^* = \frac{(1 - \beta_1 + 2\beta_2)k + \sqrt{(1 - \beta_1 + 2\beta_2)^2 k^2 + 4k(\beta_1 - \beta_2)(c_2 - c_1 + (1 + \beta_2)k)}}{2k(\beta_2 - \beta_1)} \quad (7)$$

appears which, for  $k > k_{T_1}$ , is either locally asymptotically stable, with basin of attraction  $\mathcal{B}(x^*) = (0, 1)$ , or it loses stability through a flip bifurcation.

- when  $\beta_1 = \beta_2$ , for  $k < \frac{c_1 - c_2}{1 + \beta_1}$  the fixed point 0 is locally asymptotically stable with basin of attraction given by  $\mathcal{B}(0) = [0, 1)$ , while at  $k = \frac{c_1 - c_2}{1 + \beta_1}$  the equilibrium loses stability through a transcritical bifurcation and the internal fixed point

$$x^* = 1 - \frac{c_1 - c_2}{k(1 + \beta_1)} \quad (8)$$

appears and for  $k > \frac{c_1 - c_2}{1 + \beta_1}$  it is locally asymptotically stable with basin of attraction  $\mathcal{B}(x^*) = (0, 1)$ .

This analytical result underlines that if cross-border spillovers equal within-country spillovers, the firm will choose not to locate the entire manufacturing activity in the country that has a production cost disadvantage. Intuitively, the possibility to benefit from spillovers independently of where know-how is generated eliminates the unique strength of the technologically advanced country, i.e. the additional cost reduction due to higher within-country spillovers. Thus, country 2 benefits from outgoing spillovers and manufacturing in this country

becomes the unique long-run location pattern. However, Theorem 3 also points out that high congestion costs tend to offset the production cost advantage of country 2 and makes a mixed-location strategy the unique long-run location pattern. A deeper investigation for more general scenarios requires a numerical analysis which is developed in the next section. Before doing that, it is worth addressing the optimality of the long run location patterns. Indeed, our evolutionary model allows for long-run location patterns which are suboptimal (in terms of manufacturing costs) as specified in the following Theorem.

**Theorem 4.** *Off-shoring the entire production volume to country 2 ensures lower equilibrium production cost whenever  $k < \hat{k}$ , while locating production in country 1 does so whenever  $k > \hat{k}$ , where*

$$\hat{k} = \frac{c_1 - c_2 + c_1\beta_2 - c_2\beta_1}{(1 + \beta_1)(1 + \beta_2)} \quad (9)$$

Theorem 4 underlines that spreading the production over the two countries is always a suboptimal choice in terms of production costs. Moreover, it implies the following result.

**Corollary 1.** *Off-shoring the entire production volume to country 2 is a suboptimal equilibrium choice whenever  $k > \hat{k}$ , while locating production in country 1 is so whenever  $k < \hat{k}$ .*

## 4 Location Patterns: The Role of Knowledge Spillovers and Congestion Costs

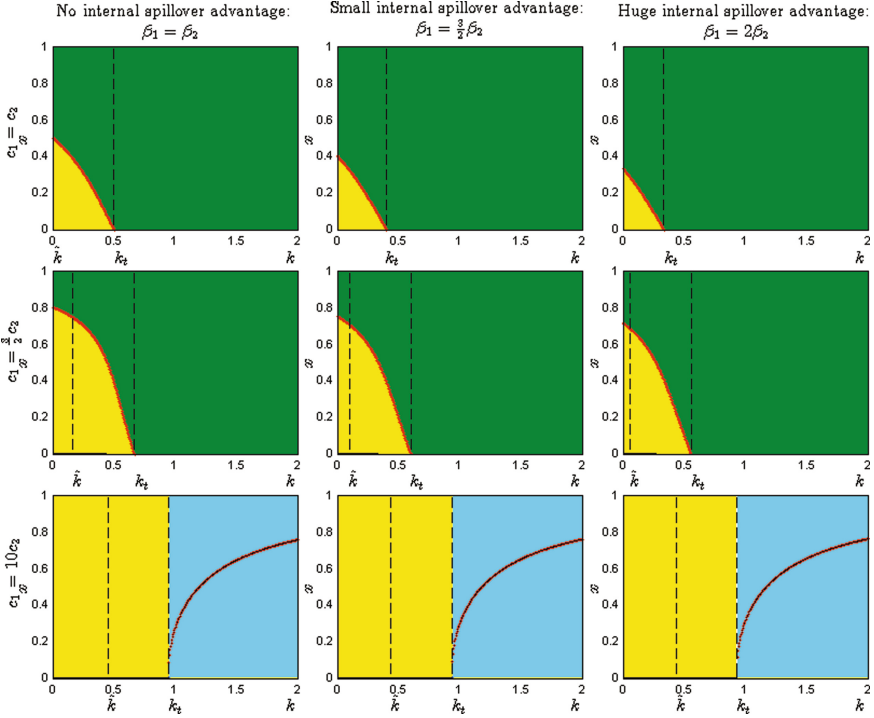
The aim of this section is to study the effects that congestion costs and internal and external knowledge spillovers have on the long run location patterns. In order to do so, we investigate the global dynamics of model (2) by numerical analysis. In particular, we employ bifurcation diagrams that show the dependence of long run location patterns, i.e. the long run share of firms  $x$  that locate production in country 1, if congestion costs,  $k$ , vary from a situation of no congestion costs,  $k = 0$ , to  $k = 2$ . Bifurcation diagrams are helpful as they illustrate the equilibrium location patterns and their basins of attraction for each value of the congestion costs. The investigation is conducted by normalizing unit costs in country, i.e.  $c_1 = 1$ , and considering three values of unit costs in country 2,  $c_2$ . The first case is  $c_1 = c_2$ , i.e. no cost advantage from producing in country 2. The second case is  $c_1 = \frac{3}{2}c_2$ , i.e. there is a small cost advantage if firms produce in country 2. The third case is  $c_1 = 10c_2$ , i.e. there is a large cost advantage if firms produce in country 2. For each of these three cases, three different levels of internal spillovers are considered: symmetric within-country spillovers, i.e.  $\beta_1 = \beta_2$ ; a small advantage of country 1 regarding internal spillovers, specifically  $\beta_1 = \frac{3}{2}\beta_2$ ; and a large advantage of country 1 regarding internal spillovers, i.e.  $\beta_1 = 2\beta_2$ .

Our first numerical results we report describe the location patterns without cross-border spillovers ( $\gamma_{12} = \gamma_{21} = 0$ ), see Figs. 1 and 2. The bifurcation diagrams in Fig. 1 reveal that the congestion costs in country 2 have a significant

impact on the location decisions of firms. In particular, when the costs of production in the two countries are identical, i.e.  $c_1 = c_2$ , and internal spillovers are identical as well, i.e.  $\beta_1 = \beta_2$ , the congestion costs reduce the chances that firms locate production in country 2. The yellow region in this figure represents the basin of attraction of the fixed point  $x = 0$  (i.e. all production occurs in country 2), that first shrinks and eventually disappears when congestion costs keep increasing. This highlights that the set of initial allocation of manufacturing activity which eventually lead to location patterns where the entire manufacturing occurs in country 2 shrinks and then disappears. On the contrary, the dark-green region represents the basin of attraction of the fixed point  $x = 1$  (i.e. all production occurs in country 1). This region grows and the fixed point 1 eventually becomes the only asymptotically stable attractor with basin of attraction given by  $\mathcal{B}(1) = (0, 1]$  when congestion costs increase. This effect is further pronounced when firms that locate production in country 1 also benefit from higher within-country spillovers, i.e.  $\beta_1 > \beta_2$ . This can be seen in the first row of Fig. 1 which shows the change in the basins of  $x = 0$  (yellow region) and in the basin of  $x = 1$  (dark-green region) for increasing internal spillovers. The numerical findings confirm the analytical results in Theorem 1 and, in addition, illustrate that manufacturing activity tends to be polarized in one of the two countries. It is worth pointing out that the coexistence of the yellow and dark-green regions indicates that for a range of congestions costs  $k$  the long run location pattern is path dependent, i.e. the long-run outcome of the evolutionary switching process depends on the initial share  $x_0$  of firms that manufacture in country 1. The model also admits an internal mixed equilibrium  $x^* \in (0, 1)$  which represents a pattern where parts of the manufacturing activity occurs in country 1 while the remaining share of firms manufacture in country 2. Such an equilibrium is a repeller and lies on the border of the basins of attraction of the two asymptotically stable fixed points  $x = 0$  and  $x = 1$ . More formally,  $\mathcal{B}(1) = [0, x^*)$  and  $\mathcal{B}(1) = (x^*, 1]$ . Increasing congestion costs causes the interior fixed point to merge with the fixed point  $x = 0$  and then to disappear through a transcritical bifurcation. Due to this bifurcation, the fixed point  $x = 0$  becomes unstable. Hence, production will not be located in country 2 any longer for sufficiently large congestion costs.

Identical unit costs,  $c_1 = c_2$ , do not lead to a long run location pattern with a positive share of manufacturing occurring in both countries. The only possible long-run location patterns are that either all manufacturing is done in country 1 or all production is carried out in country 2. A similar result is obtained if  $c_2$  is only slightly lower than  $c_1$ ; see the bifurcation diagrams in the second row of Fig. 1. Even in this case, as congestion costs increase, production in country 2 becomes less appealing and it shrinks the set of initial conditions which lead to manufacturing activity solely occurring in country 2 in the long run. For sufficiently high congestion costs, all production activity is located in country 1.

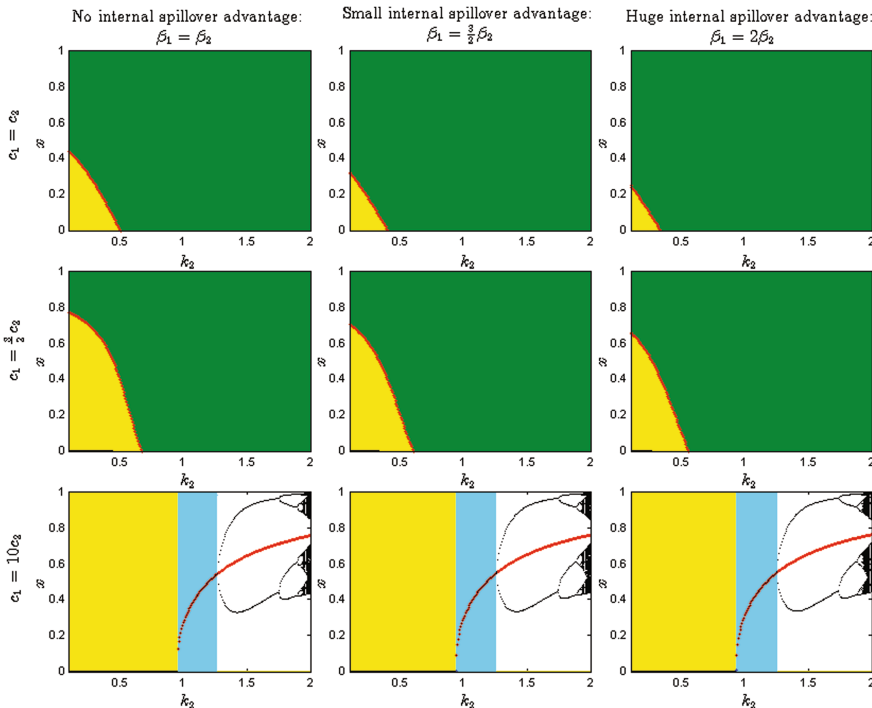
The situation is different when country 2 offers a large cost advantage. This situation is arguably akin to the situation MNEs faced when they started offshoring their activities in the 1990s. The last row of Fig. 1 shows a scenario where



**Fig. 1.** One-dimensional bifurcation diagrams where congestion costs in country 2, i.e.  $k$ , is the bifurcation parameter and varies between 0 and 2. The bifurcation diagrams are reported in three different rows. In the first row, the  $\frac{c_1}{c_2}$  ratio is equal to 1, which means that there is no cost advantage in country 2. In the second row, the ratio is  $\frac{3}{2}$ , which means a relative cost advantage of country 2. In the last row, unit cost of production (excluding congestion costs and spillovers) in country 1 is ten times the unit cost in country 2. For each row there are three bifurcation diagrams. The first column represents the situation of no internal spillover advantage,  $\beta_1 = \beta_2$ . The second column represents the situation of small internal spillover advantage,  $\frac{\beta_1}{\beta_2} = \frac{3}{2}$ . The last column represents the situation of a large internal spillover advantage,  $\frac{\beta_1}{\beta_2} = 2$ . The bifurcation diagram includes the basins of attraction as well. In particular, the basin of attraction of equilibrium  $x = 0$  is depicted in yellow, the basin of equilibrium  $x = 1$  is depicted in dark green, and the basin of the stable internal fixed point is in cyan. The values of the remaining parameters are as follows:  $c_1 = 1$ ,  $\beta = 1$ ,  $\beta_1 = 1$ ,  $\alpha = 1$ ,  $\gamma_{12} = \gamma_{21} = 0$ .

the cost of production (excluding congestion costs and spillovers) in country 1 is ten times the cost of production in country 2. Here, a slight change in congestion costs might trigger a completely different location pattern. More specifically, if the congestion costs increase and cross the bifurcation value  $k_{T_1} \approx 1$ , the long run location patterns changes from a situation in which the firm chooses to produce solely in country 2 to a scenario in which production only partially occurs in country 2. Formally, for  $k = k_{T_1}$ , see (5), an internal fixed point appears through

a transcritical bifurcation. This equilibrium is an attractor and is marked in red. In this scenario, if congestion costs are sufficiently large, the firms' total manufacturing activity is spread geographically between the two countries. In this case, the internal equilibrium is the only asymptotically stable fixed point and its basin of attraction is represented by the cyan region. Increasing the intensity of choice, which measures the firms' intensity of switching between the two location strategies, has the only effect of destabilizing the internal equilibrium for increasing congestion costs. In particular, the internal equilibrium undergoes a sequence of period-doubling bifurcations which lead to a chaotic long-run location pattern. See the last row of Fig. 2.



**Fig. 2.** Same parameter setting as in Fig. 1, except  $\beta = 10$  (higher intensity of switching).

The evolutionary process of switching between locations in the search for cost reductions can lead to suboptimal equilibrium allocations of production activities in the long run. This is the case represented by the left-top bifurcation diagram in Fig. 1. Here, being  $\hat{k} = 0$ , locating the entire production in country 2 is a suboptimal equilibrium whenever congestion costs are positive, as we have specified in Corollary 1. In fact, due to congestion, firms have to bear higher costs in country 2 than in country 1. Despite the fact that the fixed

point  $x = 0$  is a suboptimal equilibrium, it is asymptotically stable and its basin of attraction (the yellow region) represents a so-called trapping region. The existence of suboptimal and stable equilibria is typical of evolutionary games where agents choose the strategy based on best-relative performance as this might lead to lower absolute profits in the long run; see, e.g., Schaffer (1989).

Summarizing, the analysis reveals that in the absence of cross-border (external) spillovers and if the two countries are characterized by similar costs of production, i.e.  $c_1$  is either equal or slightly larger than  $c_2$ , industrial production tends to be located either in country 1 or in country 2. In particular, the probability to locate production in country 2 decreases as congestion costs in country 2 increase. On the contrary, a huge difference in production costs, for example if  $c_1$  is ten times  $c_2$ , may lead to a spread of industrial production between the two countries, see last row of Fig. 1. This occurs if congestion costs are sufficiently high.

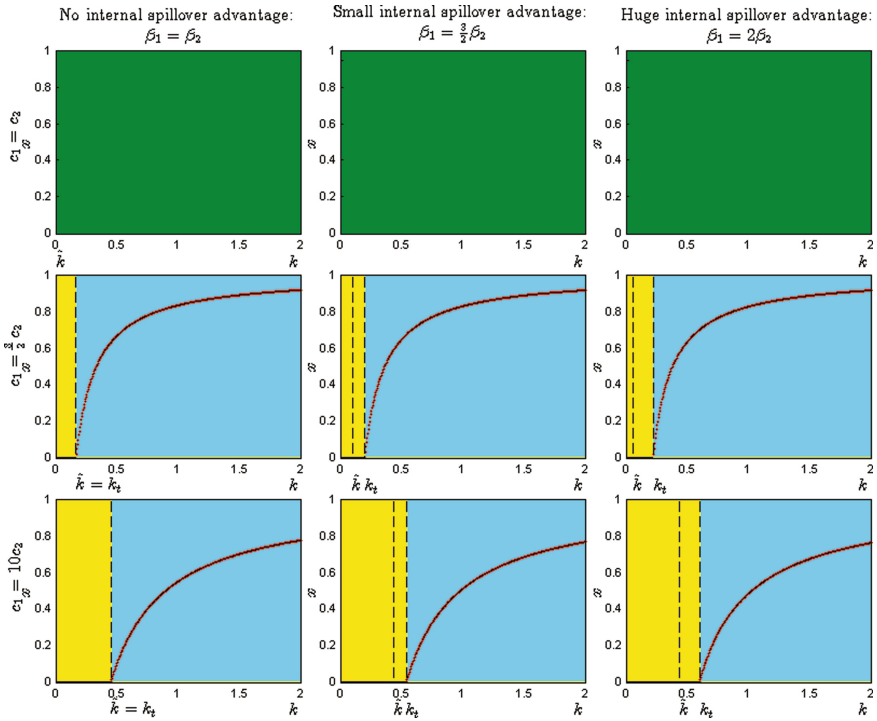
Cross-border spillovers also have a remarkable impact on the geographic allocation of industrial activities. If we assume that cross-border transfer of know-how is frictionless, i.e.  $\beta_1 = \gamma_{12}$  and  $\beta_2 = \gamma_{21}$ , the numerical investigation reveals that the attractiveness of country 2 decreases. This occurs if country 2 does not offer a production cost advantage, i.e.  $c_1 = c_2$ . In fact, relocating production from country 2 to country 1, firms avoid the costs of congestion and (due to cross-border spillovers) firms benefit from cost reduction gained by manufacturing activity in country 2. Hence, firms do not have an incentive to locate its production in country 2. In terms of policy implications, it follows that it is in the interest of the policy maker of country 1 that firms try to develop the capability to exploit inter-country flows of know-how due to external spillovers. On the contrary, the policy maker of country 2 where congestion costs matter, does not find it equally beneficial that firms can transfer know-how between countries. To increase the rate of industrial activity in its country 2, the policy maker has to aim at reducing the costs of congestion. Moreover, comparing the first row of Fig. 1, which shows the location patterns when there are no external spillovers ( $\gamma_{12} = \gamma_{21} = 0$ ), with the first row of Fig. 3, which shows the location patterns when there are external spillovers, it also becomes apparent that external spillovers eliminate the occurrence of inefficient off-shoring of manufacturing activity that may occur when unit production costs are identical, i.e.  $c_1 = c_2$ . Indeed, the comparison shows that the set of initial conditions that leads to location patterns with inefficient off-shoring of manufacturing to country 2 (see the yellow region in Fig. 1) disappears if there are cross-border spillovers. In other words, the presence of cross-border spillovers contributes to eliminate inefficient location patterns and favor the location of manufacturing activity in country 1 if congestion costs arise only in country 2. Hence, the policy maker of country 2 which is a suboptimal manufacturing location has an incentive to limit external spillovers. In contrast, the policy maker of country 1 which is the more efficient manufacturing location has an incentive to boost external spillovers since in this case firms move their manufacturing activity to the most efficient country without losing the cost reduction due to know-how developed in the less efficient country.



The situation changes significantly when country 2 has an advantage in terms of unit costs, i.e.  $c_2 < c_1$ ; see the second and third row of Fig. 3. In this case, firms that manufacture in country 2 benefit from both smaller unit costs of production and cross-border spillovers coming from country 1. Nevertheless, when congestion costs increase, the competitive advantage from producing in country 2 due to the smaller unit costs of production is diminished by an increasing congestion effect and, consequently, country 1 becomes increasingly attractive. Notwithstanding, keeping some production activity in country 2 is still beneficial. In fact, in this case the effect of the congestion costs is reduced since the concentration of production activity in country 2 is decreased. The evolutionary process of location choice leads to a situation where part of the manufacturing activity occurs in country 1 and the remaining part is carried out in the off-shore country 2. Mathematically, this is reflected by the presence of the asymptotically stable internal fixed point  $x^* \in (0, 1)$ , which is marked in red in the second row of Fig. 3. The presence of asymmetric unit costs results in location patterns that are represented by a suboptimal equilibrium. Specifically, the asymptotically stable fixed point  $x^*$  is never an optimal equilibrium as specified in Theorem 4. Then, in case of a limited cost advantage for country 2, the economic policy implications related to external spillovers change.

Comparing the second row of Fig. 1 and the second row of Fig. 3, we observe that for country 2 it is advantageous if external spillovers exist, while it is not beneficial for country 1. Indeed, as long as congestion costs are small, the presence of external spillovers drives firms to off-shore the entire production activity to country 2. In fact, the fixed point  $x = 0$  is the only locally asymptotically stable equilibrium as indicated by the yellow region in Fig. 3. Moreover, if congestion costs increase, country 2 loses its attractiveness but the firms still benefit from having some part of the production activity there. However, there is a trade-off between external spillovers and local unit costs of production. In fact, if country 2 has much smaller unit costs of production (excluding internal and external spillovers) than country 1, for example  $c_1$  is ten times  $c_2$ , then the cost reduction due to internal and external spillovers is limited in country 2 since the base unit costs are already relatively low compared to country 1. In this situation, country 1 benefits more from external spillovers. This effect becomes obvious by comparing the last row in Fig. 3 and the last row of Fig. 1. A similar scenario is observable if the capability of firms to absorb external spillovers reduces.

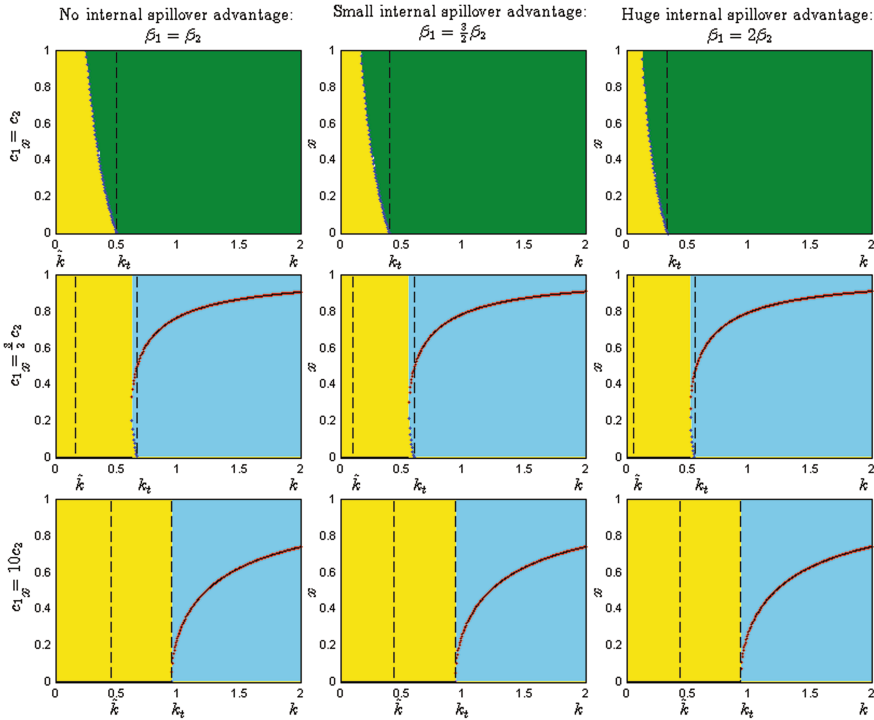
In case of asymmetries between internal and external spillovers, the location patterns and the economic implications are different. We start the analysis by considering a situation where know-how developed in country 1 can be perfectly transferred to reduce costs in country 2. Conversely, know-how developed in country 2 cannot (or can only be partially) transferred to country 1. Such asymmetric external spillovers obviously favor off-shoring manufacturing activity to country 2. Figure 4 depicts a scenario where a firm producing in country 2 benefits from external spillovers while a firm producing in country 1 does not ( $\gamma_{12} = 0$  while  $\gamma_{21} = \beta_1$ ). Comparing it with Figs. 1 and 3, we observe that the effect of such asymmetric external spillovers is to increase the yellow region,



**Fig. 3.** Parameter setting as in Fig. 1, except  $\gamma_{21} = \beta_1$  and  $\gamma_{12} = \beta_2$ .

i.e. the set of initial conditions that lead to long run off-shoring of production to country 2. The expansion of the yellow region occurs even when  $k > \hat{k}$ , i.e. when off-shoring the production to country 2 is a suboptimal location strategy as specified in Corollary 1.

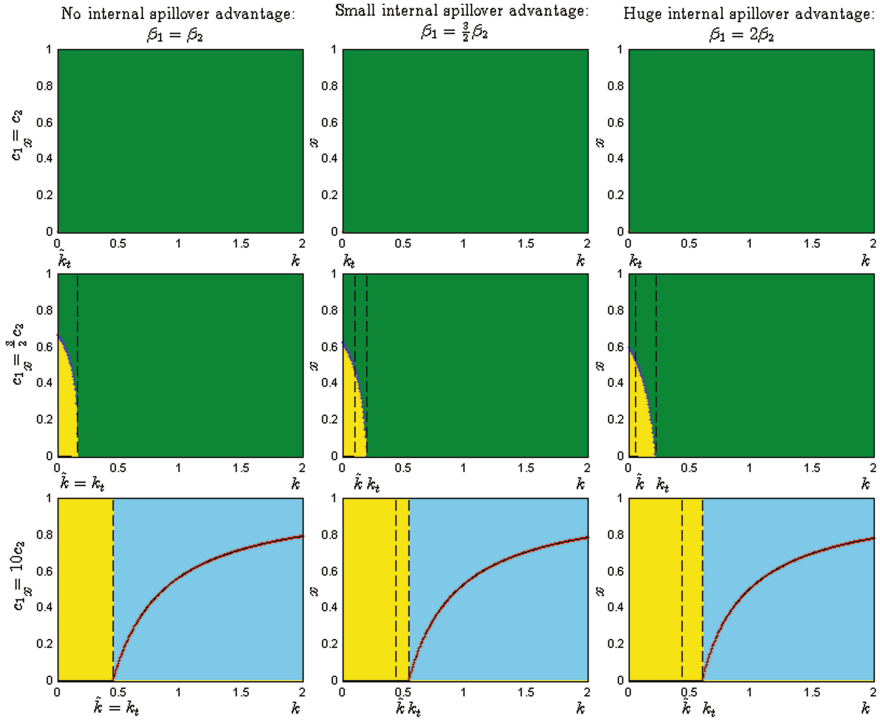
Hence, it is possible to conclude that asymmetric external spillovers that favor off-shoring to country 2 cause long run suboptimal location patterns. It is also worth highlighting that the effects of these asymmetric external spillovers vanish when country 2 has a large cost advantage in comparison to country 1. In this case, country 2's cost advantage is further increased by cost-reducing externalities coming from cross-border spillovers. In this case, the location patterns are mainly affected by the congestion costs rather than by the production costs. Comparing the bifurcation diagrams in Figs. 4 and 3, we observe that the presence of an asymmetry between internal and external spillovers in country 2 gives rise to a qualitative change of the bifurcation structure of the location patterns. Here, when congestion costs increase and the value of  $k$  crosses the bifurcation value  $k_F \approx 1$ , we move from a situation in which the firm chooses to off-shore production to country 2 to a scenario in which the production is only partially off-shored to country 2. In particular, for  $k = k_F$ , see (5), two internal fixed points appear through a fold bifurcation. One of these two equi-



**Fig. 4.** Parameter setting as in Fig. 1, except  $\gamma_{12} = 0$  and  $\gamma_{21} = \beta_1$ .

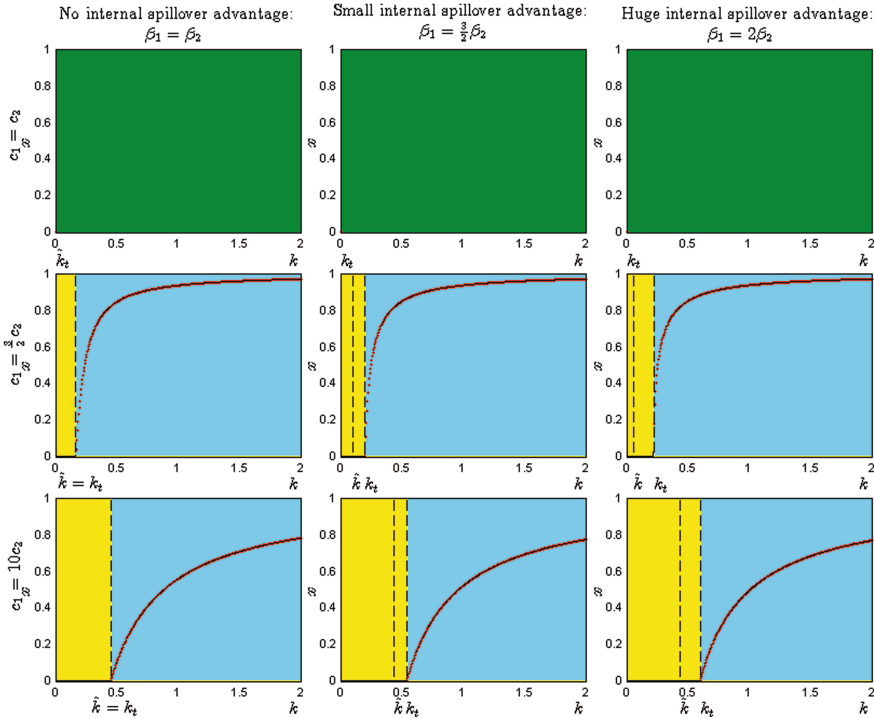
libria is a repellor and is marked in blue, while the other one is an attractor and is marked in red. The repellor lies at the border of the basins of attraction of the fixed point  $x = 0$  and of the other internal fixed point  $x^*$ . In this case, production will be off-shored to country 2 only if the majority of firms have already off-shored their production activity. Otherwise, total production activity will be spread between the two countries in the long run. Although the dynamic scenario just described is interesting as it is characterized by bi-stability and, hence, path dependence, it is, however, not robust with regard to variations in congestion costs. More precisely, immediately after the value for  $k$  (representing congestion costs) crosses the threshold value where the fold bifurcation occurs, it crosses another bifurcation value, namely the transcritical bifurcation value  $k_{T_1}$ . Through this bifurcation, the unstable internal fixed point merges with the fixed point  $x = 0$  and disappears while the fixed point  $x = 0$  becomes unstable. For further increasing congestion costs, total production activity is spread between the two countries and the internal equilibrium is the only asymptotically stable fixed point. Its basin of attraction is represented by the cyan region. Numerical simulations, which are not reported here, reveal that the qualitative effect of asymmetric external spillovers in favor of country 2 is the same as the one just described whatever the magnitude of the asymmetry.

An interesting effect can be observed when know-how can be transferred from country 2 to country 1, but cannot be transferred (or at least can only be partially transferred) from country 1 to country 2 ( $\gamma_{12} = \beta_2$  and  $\gamma_{21} = 0$ ). In the presence of such asymmetric external spillovers that work in favor of country 1 and whenever country 2 has a cost advantage over country 1, the attractiveness of country 2 increases when the internal spillovers  $\beta_2$  in country 2 decreases. This argument can be understood by observing the changes in the yellow region depicted in the second and third rows of Figs. 5 and 6. Moving from the left column (where  $\beta_2 = \beta_1 = 1$  to the right column (where  $\beta_2 = \beta_1/2 = 1/2$ ) shows that the yellow region increases when  $\beta_2$  decreases. As a consequence, in terms of implications for economic policy, the policy maker of country 2 can decrease internal spillovers in its territory to increase the likelihood of attracting industrial production.



**Fig. 5.** Same parameter setting as in Fig. 1, except  $\gamma_{12} = \beta_2$  and  $\gamma_{21} = 0$ . (Asymmetric spillovers in favor of country 1)

To conclude our analysis, we point out that our investigation revealed that whenever country 2 has an advantage in terms of unit production cost over country 1, congestion costs make production in country 2 less profitable and reduce



**Fig. 6.** Same parameter setting as in Fig. 1, except  $\gamma_{12} = \beta_2$  and  $\gamma_{21} = \frac{1}{2}\beta_1$ . (Asymmetric spillovers in favor of country 1)

the off-shoring activities of firms. Our analysis also shows, however, that the presence of congestion costs does not eliminate off-shoring activity completely.

## 5 Conclusions

In this paper, an evolutionary mechanism for choosing the location of manufacturing is proposed. The model is employed to study the question how the long-run off-shoring and back-shoring patterns of a population of firms is affected by differences in unit production costs, within-country spillovers, cross-border spillovers, and congestion costs. The investigation reveals that a manufacturing site that offers lower unit production costs becomes more attractive and causes an off-shoring wave. This competitive advantage can be eroded by asymmetric cross-border spillovers, by positive congestion costs, or by a combination of the two. Although these factors influence off-shoring decisions, they rarely lead to total re-shoring of production activities to the home country if it suffers a rather large disadvantage in terms of unit production costs. This underlines that unit production costs are indeed a key factor which drives location choices for MNEs. A fundamental role is played by cross-border spillovers as well. These spillovers

tend to reduce the occurrence of location patterns that lead to suboptimal equilibria and favor the geographic dispersion of manufacturing activity.

These findings are obtained by analyzing the endogenous location patterns that emerge in the long run if a population of firms choose manufacturing locations based on relative production costs. The logic of the process that governs the dynamic evolution of location choices of firms is intentionally simple: if a location currently offers a production cost advantage, then the share of firms which move production activities to this country will increase. We also took into account that there might be some inertia in this process of switching between manufacturing sites as moving production activities from one country to another is costly and takes time. What we do not take into account in our model is that MNEs are typically active in oligopolistic markets and, therefore, strategic interactions and intertemporal objectives play a crucial role. Further, MNEs are interconnected through the supply side as they share common suppliers which is a further source for strategic linkages. In this respect, our contribution differs from Alcácer and Chung (2007) and Alcacer et al. (2015), where the location choices of firms are analyzed assuming an oligopolistic industry. Papers along this promising line of research explore how rivalry and differential knowledge accumulation among competitors is affected by the multinational enterprises' geographic expansion across countries. The location choice of the firms impacts the accessibility of certain markets and firms take location decisions also to enhance and protect their relative competitive positions. The analysis focuses on how rivalry and competition influences the location decisions of multinational enterprises. In this way, these contributions integrate rivalry and location choices in multi-stage, dynamic settings. Another aspect we do not pursue in our contribution is the location of research and development activities of firms. This aspect has been investigated in the International Business literature, see, e.g., Belderbos et al. (2008). These elements, as well as the introduction of an outside option as in Bischi et al. (2003a), are key elements that can have a strong impact on the off-shoring process. The introduction of these further elements in our setup is left for future research.

## Disclosure Statement

No potential conflict of interest was reported by the authors.

**Acknowledgements.** The authors thank the participants at the GeComplexity Conference in Crete 2016 for valuable comments and suggestions on an earlier draft of this paper. Special thanks go to J. Tuinstra (University of Amsterdam) for hosting the first meeting of the Working Group 3 of the ISCH COST Action IS1104, from which the current work started. Special thanks also go to Mario Pezzino for his valuable suggestions. Financial support was provided by COST Action “The EU in the new complex geography of economic systems: models, tools and policy evaluation” and in the framework of the research project on “Dynamic models for behavioral economics” financed by DESP-University of Urbino.

## Appendix

*Proof (Proof of Theorem 1).* The fixed points of model (2) solve the equation

$$x^* = (1 - \alpha)x^* + \alpha x^* \frac{1}{x^* + (1 - x^*)e^{\beta(C_1(x^*) - C_2(x^*))}} \Leftrightarrow \begin{cases} x^* = 0 \\ x^* = 1 \\ C_1(x^*) = C_2(x^*) \end{cases}$$

Thus, the equilibria of the model are 0, 1 and the roots, in  $(0, 1)$ , of the following polynomial

$$p(x) = C_1(x) - C_2(x) \quad (10)$$

where  $C_1(x) - C_2(x)$  are decreasing and convex functions in  $[0, 1]$ , hence they have at most two intersections in  $[0, 1]$ . The numerator of  $p(x)$  is a third degree polynomial.

which are at most three. The eigenvalue associated to model (2) is

$$\lambda(x) = 1 - \alpha + \alpha \frac{1}{x + (1 - x)e^{\beta p(x)}} - \alpha x \frac{1 - e^{\beta p(x)} + (1 - x)\beta \frac{\partial p(x)}{\partial x} e^{\beta p(x)}}{(x + (1 - x)e^{\beta p(x)})^2} \quad (11)$$

It follows that the eigenvalue associated to equilibrium  $x = 0$  and the one associated to equilibrium  $x = 1$  are, respectively

$$\lambda(0) = 1 - \alpha + \alpha e^{\beta(k - k_{T_1})} \quad \text{and} \quad \lambda(1) = 1 - \alpha + \alpha e^{\beta \frac{c_1 - c_2 + c_1 \gamma_{21} - c_2 \beta_1}{(1 + \beta_1)(1 + \gamma_{21})}} \quad (12)$$

where  $k_{T_1}$  is given in (3). Then, since  $\lambda(0), \lambda(1) > 0$ , imposing the classical stability conditions, we have that the fixed point  $x^* = 0$  is locally asymptotically stable when  $k < k_{T_1}$ , it undergoes a bifurcation of eigenvalue 1 at  $k = k_{T_1}$ , and it is unstable for  $k > k_{T_1}$ . Similarly, the fixed point  $x^* = 1$  is locally asymptotically stable when  $\frac{c_1}{c_2} < \frac{1 + \beta_1}{1 + \gamma_{21}}$ , it undergoes a bifurcation of eigenvalue 1 at  $\frac{c_1}{c_2} = \frac{1 + \beta_1}{1 + \gamma_{21}}$ , and it is unstable for  $\frac{c_1}{c_2} > \frac{1 + \beta_1}{1 + \gamma_{21}}$ . Being  $[0, 1]$  an invariant region for model (2), when fixed points 0 and 1 are both either stable or unstable at least an internal fixed point has to exist.

For  $\frac{c_1}{c_2} = \frac{1 + \beta_1}{1 + \gamma_{21}}$ , one root of  $p(x)$  is 1, i.e.  $p(x) = (x - 1)g(x)$ ,  $\lambda(1) = 1$  and  $T''(1) = \alpha\beta(k - k_{T_2})$ . By imposing  $T''(1) > 0$ , we have that  $x = 1$  is semi-asymptotically stable from the left when  $k > k_{T_2}$ . Analyzing the dynamics of the model w.r.t. the invariant region  $[0, 1]$ , the condition  $k > k_{T_2}$  implies the local stability of  $x = 1$ . The remainder of the Theorem follows by classical eigenvalue analysis.

*Proof (Proof of Theorem 2).* For  $k = 0$ ,  $C_1(0) > C_2(0)$ , and either  $\beta_1 > \gamma_{12}$  or  $\beta_2 \neq \gamma_{21}$ , the numerator of  $p(x)$  becomes a polynomial of degree one since  $A = B = 0$ . Its unique root is  $x^*$  as given in (4), which is then the unique possible equilibrium of the model in  $(0, 1)$ . By straightforward algebra it follows that  $0 < x^* < 1$  if and only if  $C_1(1) < C_2(1)$ . Since  $p(x) = C_1(x) - C_2(x)$

is a polynomial of degree one with its root  $x^*$  in  $(0, 1)$  for  $C_1(1) < C_2(1)$ , it follows that  $C_1(x) < C_2(x)$  for  $x \in (x^*, 1]$  and  $C_1(x) > C_2(x)$  for  $x \in [0, x^*)$ . From which it follows that  $x^* \in (0, 1)$  is always a repeller. Imposing the classical stability condition to eigenvalues in (12), the first part of the Theorem follows. For  $k = 0$ ,  $C_1(0) = C_2(0)$ , and either  $\beta_1 > \gamma_{12}$  or  $\beta_2 \neq \gamma_{21}$ , let us note that  $\forall x$  it holds  $p(x) > 0$  when  $C_1(1) > C_2(1)$ , the numerator of  $p(x) = 0$  when  $C_1(1) = C_2(1)$  and  $p(x) < 0$  when  $C_1(1) < C_2(1)$ . Then, the second part of the Theorem follows. Let  $k = 0$ ,  $\beta_1 = \gamma_{12}$ ,  $\beta_2 = \gamma_{21}$ ,  $\beta_1 = \beta_2$ . Then  $p(x) = 0 \forall x$  when  $c_1 = c_2$  and  $p(x) > 0$  when  $c_1 > c_2$ . Thus the third part of the Theorem follows.

*Proof (Proof of Theorem 3).* For  $\beta_1 = \gamma_{21}$ ,  $\beta_2 = \gamma_{12}$  and  $c_1 = c_2$ , we have  $p(x) < 0 \forall x \in (0, 1)$ , thus  $\mathcal{B}(1) = (0, 1]$ . For  $\beta_1 = \gamma_{21}$ ,  $\beta_2 = \gamma_{12}$  and  $c_1 > c_2$ , it becomes a polynomial of degree less than three. In particular, for  $\beta_1 \neq \beta_2$ ,  $p(x)$  becomes a polynomial of degree two, the roots of which are given by:

$$x_{1,2}^* = \frac{(1 - \beta_1 + 2\beta_2)k \pm \sqrt{(1 - \beta_1 + 2\beta_2)^2 k^2 + 4k(\beta_1 - \beta_2)(c_2 - c_1 + (1 + \beta_2)k)}}{2k(\beta_2 - \beta_1)} \quad (13)$$

Since  $0 < \frac{(1 - \beta_1 + 2\beta_2)}{2(\beta_2 - \beta_1)} < 1$  if and only if  $\beta_1 > 1 + 2\beta_2$  and  $\Delta = 0$  if and only if  $k = k_F$ , where  $k_F$  is given in (5), it follows that, for  $\beta_1 > 1 + 2\beta_2$ , at  $k = k_F$  two internal fixed points, i.e.  $x_{1,2}^*$  in (13), appear through a fold bifurcation. Since  $p(1) = C_1(1) - C_2(1) > 0$ , by continuity of  $p$  it has to be  $p(x) = C_1(x) - C_2(x) > 0$  for  $x \in (x_1^*, 1)$ ,  $p(x) < 0$  for  $x \in (x_2^*, x_1^*)$  and  $p(x) > 0$  for  $x \in (0, x_2^*)$ . Moreover, since the function that defines the dynamics of the model is increasing when  $p(x) < 0$  and vice versa, it follows that  $x_1^*$  is either stable or loses stability through a period-doubling bifurcation and  $x_2^*$  is locally unstable. Moreover, we have that  $x_1^*$  increases as  $k$  increases but it never reaches the value 1, while  $x_2^*$  decreases as  $k$  increases and it undergoes a transcritical bifurcation, it merges with equilibrium 0, for  $k = k_{T_1} > k_F$ . For  $k > k_{T_1}$ , the fixed point 0 is unstable and  $x_2^*$  becomes unfeasible (it exits the invariant region  $[0, 1]$ ).

On the contrary, for  $\beta_1 < 1 + 2\beta_2$ , we have  $\frac{(1 - \beta_1 + 2\beta_2)}{2(\beta_2 - \beta_1)} < 0$  and the internal fixed point can be at most one, i.e.

$$x^* = \frac{(1 - \beta_1 + 2\beta_2)k + \sqrt{(1 - \beta_1 + 2\beta_2)^2 k^2 + 4k(\beta_1 - \beta_2)(c_2 - c_1 + (1 + \beta_2)k)}}{2k(\beta_2 - \beta_1)} \quad (14)$$

For,  $\beta_1 = \beta_2$ , then there is at most one internal fixed point

$$x^* = 1 - \frac{c_1 - c_2}{k(1 + \beta_1)} \quad (15)$$

which is locally asymptotically stable. It appears through transcritical bifurcation at  $k = \frac{c_1 - c_2}{1 + \beta_1}$ .



*Proof (Proof of Theorem 4).* Since  $C_1(1) = \frac{c_1}{1+\beta_1}$  and  $C_2(0) = \frac{c_2}{1+\beta_2} + k$ , it follows that  $C_1(1) \leq C_2(0)$  if and only if  $k \geq \hat{k}$ . Moreover, setting  $\phi = \gamma_{12} - \beta_1$ , by assumption 1  $\phi < 0$  from which  $C_1(x^*) = \frac{c_1}{1+\beta_1+\phi(1-x^*)} = C_2(x^*) > C_1(1)$  whenever  $x^* \in (0, 1)$  is a general internal equilibrium. Then, being 0 and 1 two equilibria always, the statement of the Theorem follows.

## References

- Adams, J.D., Jaffe, A.B.: Bounding the effects of r&d: an investigation using matched establishment-firm data. *RAND J. Econ.* **27**(4), 700–721 (1996)
- Alcácer, J., Chung, W.: Location strategies and knowledge spillovers. *Manag. Sci.* **53**(5), 760–776 (2007)
- Alcácer, J., Dezső, C.L., Zhao, M.: Firm rivalry, knowledge accumulation, and mne location choices. *J. Int. Bus. Stud.* **44**, 504–520 (2013)
- Alcacer, J., Dezsó, C., Zhao, M.: Location choices under strategic interactions. *Strateg. Manag. J.* **36**, 197–215 (2015)
- Backer, K.D., Menon, C., Desnoyers-James, I., Moussiegt, L.: ‘Reshoring: Myth or reality?’. OCED Science, Technology and Industry Policy Papers (27) (2016)
- Belderbos, R., Lykogianni, E., Veugelers, R.: Strategic r&d location by multinational firms: spillovers, technology sourcing and competition. *J. Econ. Manag. Strategy* **17**(3), 759–779 (2008)
- Bischi, G.-I., Dawid, H., Kopel, M.: Gaining the competitive edge using internal and external spillovers: a dynamic analysis. *J. Econ. Dyn. Control* **27**(11), 2171–2193 (2003a)
- Bischi, G.-I., Dawid, H., Kopel, M.: Spillover effects and the evolution of firm clusters. *J. Econ. Behav. Organ.* **50**(1), 47–75 (2003b)
- Bischi, G.-I., Lamantia, F.: Chaos synchronization and intermittency in a duopoly game with spillover effects. In: Puu, T., Sushko, I. (eds.) *Oligopoly Dynamics: Models and Tools*, pp. 195–217. Springer (2002)
- Cabrales, A., Sobel, J.: On the limit points of discrete selection dynamics. *J. Econ. Theor.* **57**(2), 407–419 (1992)
- Carlton, D.W.: The location and employment choices of new firms: an econometric model with discrete and continuous endogenous variable. *Rev. Econ. Stat.* **65**(3), 440–450 (1983)
- Coughlin, C.C., Terza, J.V., Arromdee, V.: State characteristics and the location of foreign direct investment within the United States. *Rev. Econ. Stat.* **73**(4), 675–683 (1991)
- David, P.A., Rosenbloom, J.L.: Marshallian factor market externalities and the dynamics of industrial location. *J. Urban Econ.* **28**(3), 349–370 (1990)
- Flores, R., Aguilera, R.V.: Globalization and location choice: an analyses of multinational firms in 1980 and 2000. *J. Int. Bus. Stud.* **38**(7), 1187–1210 (2007)
- Furman, J.L., Porter, M.E., Stern, S.: The determinants of national innovative capacity. *Res. Policy* **31**(6), 899–933 (2002)
- Henisz, W.J.: The institutional environment for multinational investment. *J. Law Econ. Organ.* **16**(2), 334–364 (2000)
- Hofbauer, J., Sigmund, K.: Evolutionary game dynamics. *Bull. (New Series) Am. Math. Soc.* **40**(4), 479–519 (2003)
- Jaffe, A.B., Trajtenberg, M., Henderson, R.: Geographic localization of knowledge spillovers as evidence by patent citations. *Q. J. Econ.* **108**(3), 577–598 (1993)

- Krugman, P.: Increasing returns and economic geography. *J. Polit. Econ.* **99**(3), 483–500 (1991)
- Lee, J.Y., Mansfield, E.: Intellectual property protection and US foreign direct investment. *Rev. Econ. Stat.* **78**(2), 181–186 (1996)
- Mariotti, S., Piscitello, L., Elia, S.: Spatial agglomeration of multinational enterprises: the role of information externalities and knowledge spillovers. *J. Econ. Geogr.* **10**(4), 519–538 (2010)
- Marshall, A.: *Elements of Economics of Industry*. Macmillan, London (1982)
- Nelson, R.: *National Innovation Systems: A Comparative Analysis*. Oxford University Press, New York (1993)
- Rosenthal, S.S., Strange, W.: Geography, industrial organization, and agglomeration. *Rev. Econ. Stat.* **85**(2), 377–393 (2003)
- Schaffer, M.E.: Are profit-maximisers the best survivors?: a Darwinian model of economic natural selection. *J. Econ. Behav. Organ.* **12**, 29–45 (1989)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.



# Looking Ahead: Part III

Gian Italo Bischi<sup>1</sup> and Michael Kopel<sup>2</sup>(✉)

<sup>1</sup> DESP-Department of Economics, Society, Politics, University of Urbino Carlo Bo,  
42 Via Saffi, 61029 Urbino, PU, Italy  
[gian.bischi@uniurb.it](mailto:gian.bischi@uniurb.it)

<sup>2</sup> Institute of Organization and Economics of Institutions, University of Graz,  
Universitätsstrasse 15/E4, Graz, Austria  
[michael.kopel@uni-graz.at](mailto:michael.kopel@uni-graz.at)

**Abstract.** In the past decades, manufacturing firms have increasingly off-shored main activities along their value chain to emerging economies in order to take advantage of lower costs. More recently, however, the trend of re-shoring has gained increasing attention. We argue that more research is needed to fully understand firms' motives for bringing their activities back home. Furthermore, multinational firms need to evaluate their activities along environmental, social and governance dimensions. Research on corporate social responsibility strategies of multinational enterprises and their global value chains is still scarce, however.

**Keywords:** Off-shoring · Re-shoring · Corporate social responsibility

## 1 Conclusions and Future Challenges

For the European Union (EU), the uneven geographical distribution of economic activity among and even within member countries is a huge challenge. These spatial inequalities evolve over time and follow complex patterns which are influenced by economic, geographic, institutional and social factors. The main question that arises is “What are the drivers for such empirically observed heterogeneity of economic activity?” Clustering might take place for a variety of reasons, such as to be closer to large markets, to exploit scale economies and knowledge spillovers, and to benefit from lower transportation costs. Institutions at different territorial levels (e.g. at the EU, national, regional or local level) obviously have a strong impact as they set the regulatory framework that guides the decisions of economic actors and, thereby, foster growth and reduce regional disparities. Consequently, it is important to increase the quality of governance and to develop a better understanding of the linkages and the coordination issues between central and local governments.

The European market offers a wide array of examples of geographic areas where groups of firms are interconnected by horizontal and vertical relations and locate and interact. The International Business literature groups the drivers for location decisions into two broad categories (e.g. Alcácer et al. 2013). First, *endowment drivers* include location traits such as physical infrastructure, labor

force quality, cultural distance, and institutional features like political risk and the enforcement of property rights. Second, *agglomeration drivers* include inter-firm technological spillovers and access to specialized labor and specialized intermediate inputs. In addition to these two established categories, McIvor (2013) argues more recently that an analysis based on the Transaction Cost approach and the Capability perspective can contribute to an improved understanding of what drives firms' location activities. Finally, a growing stream of papers has demonstrated that the behavior of MNEs – which are typically active in oligopolistic markets – is crucially influenced by *strategic interaction with various stakeholders* like rival firms, input suppliers, and regulators (e.g. Kopel et al. 2015; Head et al. 2002; Leahy and Pavelin 2008; Leahy and Montagna 2009; Alcácer et al. 2013, 2015; Sanna-Randaccio and Veugelers 2007; Belderbos et al. 2008; Wu and Zhang 2014).

The objective of the working group on social and industrial interactions was to improve our understanding of the emergence of (behavioral) heterogeneity in the European Union by looking at the EU as a complex, multi-level, evolving geographical system and by taking into account the dynamic processes that occur within such a system. The main focus of the working group was on the role of economic agents at the micro-level (e.g. firms, suppliers, consumers) in spatial economic systems. In particular, multinational enterprises (MNEs) frequently have to make vital decisions concerning the geographic location of activities along their value chain such as retail, production, research & development (R&D), distribution, and services. These location choices of firms interact with and are crucially influenced by their market, social, and institutional environment. Hence, emphasis was also given to the development of improved strategies for EU regional policies. The majority of research carried out by the working group on social and industrial interactions is representative of the game-theoretic approach. Hence, the common element is the assumption that players in the game (e.g. firms, suppliers, regulators) are taking into account that they are interacting with rational players and, hence, that all players are *reacting strategically* to the choices taken by the other players. From the work that has been done by this group of researchers over the last years, several open issues have been identified that are promising for future research activities. We expect that pursuing these topics will improve our knowledge about the drivers of homogeneity or heterogeneity of firms' (location) choices.

With regard to future research, we notice that the dynamics of firms' strategies are evident. Trade liberalization and innovations in information and communication technologies have provided incentives for MNEs to move their production abroad in order to exploit cost advantages. These off-shoring and outsourcing activities were oftentimes solely based on the lower cost of labor in foreign developing countries. However, more recently firms are reconsidering their previous decisions and are starting to 're-shore' or 'near-shore' their activities (Ellram 2013; Porter and Rivkin 2012). Accounting for the hidden costs of outsourcing and off-shoring, three reasons are advanced for explaining this trend (e.g. Booth 2013; Barbieri and Stentoft 2016; Stentoft et al. 2016; de Treville

and Trigeorgis 2010). First, the labor cost advantage of foreign locations in developing countries has been significantly reduced and the costs of transportation are rising sharply. Instead of cheap labor, multinationals are now considering direct investments abroad to take an advantageous position in conquering fast growing foreign markets like China. Second, firms are realizing that keeping activities like manufacturing and R&D at distant locations can have negative consequences on innovation and quality since knowledge spillovers are localized. Moreover, large distances disproportionately increase the risk of supply chain disruptions. Given such a globalized channel structure, firms might damage their reputation as they are increasingly held responsible if their foreign suppliers disregard national codes of conduct or norms. Third and finally, companies are starting to re-shore to be able to customize their products to preferences in local markets and to be able to respond to customs and preferences quickly. Although some work on strategic re-location and near-shoring decisions of multinational firms has been done, we believe that more research is needed to understand the strategic motives of MNEs for choosing particular channel structures and locations.

Location decisions and the choice of channel structures and their interaction with the Corporate Social Responsibility (CSR) strategies of multinational enterprises also deserve more attention. According to Kitzmüller and Shimshack (2012), the field of international trade and CSR policies is still under-researched. With the introduction of Directive 2014/95/EU on the disclosure of non-financial and diversity information, the European Commission expressed its view on CSR as an extended corporate governance policy. The Directive introduces enhanced reporting requirements on environmental, social, and governance (ESG) issues. It affects not only corporate reporting, but also measurement of performance on ESG dimensions. As a consequence, it is expected to have an impact on internal firm processes, compensation issues, and incentives of top management. Relating to the reputation loss of manufacturers mentioned above, if their suppliers do not meet CSR standards the question arises how MNEs can control their global supply chains and ensure compliance of their partners. Additionally, an MNE's mode of entry into a foreign country might crucially interact with its investments in corporate social responsibility (see, e.g., Manasakis et al. 2017). During the process of the development of Directive 2014/95/EU, concerns were raised that due to this new regulation European firms will have a competitive disadvantage against their international rivals. An important issue is, therefore, if CSR enhances or diminishes the competitiveness of European firms against their international rivals. Furthermore, the question might be raised whether investments in CSR threaten the long run survival of MNEs and lead to the dominance of firms that only narrowly seek to achieve profits. Again, some preliminary work on the impact of social and environmental objectives in oligopolistic markets has been done, but we need more research to fully understand the conditions that make it possible for firms to consider non-profit objectives and still do well. Taken together, the brief review hopefully demonstrates that there are ample research opportunities that might lead to novel insights on the drivers that motivate multinational firms' decisions under competition and strategic interaction.

## References

- Alcácer, J., Dezső, C.L., Zhao, M.: Firm rivalry, knowledge accumulation, and MNE location choices. *J. Int. Bus. Stud.* **44**, 504–520 (2013)
- Alcácer, J., Dezső, C.L., Zhao, M.: Location choices under strategic interactions. *Strateg. Manag. J.* **36**, 197–215 (2015)
- Barbieri, P., Stentoft, J.: Guest editorial, special issue on ‘Reshoring: a supply chain innovation perspective’. *Oper. Manag. Res.* **9**(3), 49–52 (2016)
- Belderbos, R., Lykogianni, E., Veugelers, R.: Strategic R&D location by multinational firms: spillovers, technology sourcing, and competition. *J. Econ. Manag. Strategy* **17**(3), 759–779 (2008)
- Booth, T.: Here, there and everywhere. In: *The Economist, Special Report: Outsourcing and Offshoring*, 19 January 2013
- de Treville, S., Trigeorgis, L.: It may be cheaper to manufacture at home. *Harvard Bus. Rev.* **88**(10), 84–87 (2010)
- Ellram, L.M.: Offshoring, reshoring and the manufacturing location decision. *J. Supply Chain Manag.* **49**(2), 3–5 (2013)
- Head, K., Mayer, T., Ries, J.: Revisiting oligopolistic reaction: are decisions on Foreign direct investment strategic complements? *J. Econ. Manag. Strategy* **11**(3), 453–472 (2002)
- Kitzmueller, M., Shimshack, J.: Economic perspectives on corporate social responsibility. *J. Econ. Lit.* **50**(1), 51–84 (2012)
- Kopel, M., Pezzino, M., Brand, B.: Strategic location choice, R&D, and sourcing strategies. In: Commendatore, P., Kayam, S., Kubin, I. (eds.) *Complexity and Geographical Economics. Topics and Tools. Springer Book Series on Dynamic Modeling and Econometrics in Economics and Finance* (2015)
- Leahy, D., Pavelin, S.: Playing away to win at home. *J. Econ. Bus.* **60**(5), 455–468 (2008)
- Leahy, D., Montagna, C.: Outsourcing vs. FDI in oligopoly equilibrium. *Spat. Econ. Anal.* **4**(2), 149–166 (2009)
- Manasakis, C., Mitrokostas, E., Petrakis, E.: Strategic Corporate Social Responsibility by a Multinational Firm, Discussion Paper No. 246, Düsseldorf Institute for Competition Economics (2017)
- McIvor, R.: Understanding the manufacturing location decision: the case for the transaction cost and capability perspectives. *J. Supply Chain Manag.* **49**(2), 23–25 (2013)
- Porter, M., Rivkin, R.W.: Choosing the United States. *Harvard Bus. Rev.* **90**(10), 80–93 (2012)
- Sanna-Randaccio, F., Veugelers, R.: Multinational knowledge spillovers with decentralised R&D: a game-theoretic approach. *J. Int. Bus. Stud.* **38**, 47–63 (2007)
- Stentoft, J., Olhager, J., Heikkilä, J., Thomas, L.: Manufacturing backshoring: a systematic literature review. *Oper. Manag. Res.* **9**(3), 53–61 (2016)
- Wu, X., Zhang, F.: Home or overseas? An analysis of sourcing strategies under competition. *Manag. Sci.* **60**(5), 1223–1240 (2014)

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

