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Physics

SENIOR CONTRIBUTING AUTHORS

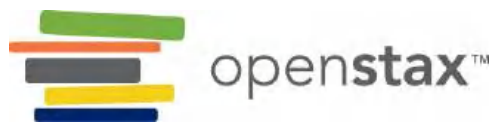**PAUL PETER URONE, CALIFORNIA STATE UNIVERSITY, SACRAMENTO**
ROGER HINRICHS, STATE UNIVERSITY OF NEW YORK, COLLEGE AT OSWEGO

CONTRIBUTING AUTHORS

FATIH GOZUACIK, PENNSYLVANIA DEPARTMENT OF EDUCATION
DENISE PATTISON, EAST CHAMBERS ISD
CATHERINE TABOR, NORTHWEST EARLY HIGH SCHOOL

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PREFACE

Welcome to *Physics*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

About OpenStax

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012 and our library has since scaled to over 25 books for college and AP® courses used by hundreds of thousands of students. OpenStax Tutor, our low-cost personalized learning tool, is being used in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

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All OpenStax textbooks undergo a rigorous review process. However, like any professional-grade textbook, errors sometimes occur. The good part is, since our books are web-based, we can make updates periodically. If you have a correction to suggest, submit it through our errata reporting tool. We will review your suggestion and make necessary changes.

Format

You can access this textbook for free in web view or PDF through OpenStax.org, and for a low cost in print.

About *Physics*

This instructional material was initially created through a Texas Education Agency (TEA) initiative to provide high-quality open-source instructional materials to districts free of charge. Funds were allocated by the 84th Texas Legislature (2015) for the creation of state-developed, open-source instructional materials with the request that advanced secondary courses supporting the study of science, technology, engineering, and mathematics should be prioritized.

Physics covers the scope and sequence requirements of a typical one-year physics course. The text provides comprehensive coverage of physical concepts, quantitative examples and skills, and interesting applications. High School Physics has been designed to meet and exceed the requirements of the relevant Texas Essential Knowledge and Skills (TEKS (http://ritter.tea.state.tx.us/rules/tac/chapter112/ch112c.html#112.39)), while allowing significant flexibility for instructors.

Qualified and experienced Texas faculty were involved throughout the development process, and the textbooks were reviewed extensively to ensure effectiveness and usability in each course. Reviewers considered each resource's clarity, accuracy, student support, assessment rigor and appropriateness, alignment to TEKS, and overall quality. Their invaluable suggestions provided the basis for continually improved material and helped to certify that the books are ready for use. The writers and reviewers also considered common course issues, effective teaching strategies, and student engagement to provide instructors and students with useful, supportive content and drive effective learning experiences.

Coverage and scope

Physics presents physical laws, research, concepts, and skills in a logical and engaging progression that should be familiar

to most physics faculty. The textbook begins with a general introduction to physics and scientific processes, which is followed by several chapters on motion and Newton's laws. After mechanics, the students will move through thermodynamics, waves and sound, and light and optics. Electricity and magnetism and nuclear physics complete the textbook.

- Chapter 1: What Is Physics?
- Chapter 2: Motion in One Dimension
- Chapter 3: Acceleration
- Chapter 4: Forces and Newton's Laws of Motion
- Chapter 5: Motion in Two Dimensions
- Chapter 6: Circular and Rotational Motion
- Chapter 7: Newton's Law of Gravitation
- Chapter 8: Momentum
- Chapter 9: Work, Energy, and Simple Machines
- Chapter 10: Special Relativity
- Chapter 11: Thermal Energy, Heat, and Work
- Chapter 12: Thermodynamics
- Chapter 13: Waves and Their Properties
- Chapter 14: Sound
- Chapter 15: Light
- Chapter 16: Mirrors and Lenses
- Chapter 17: Diffraction and Interference
- Chapter 18: Static Electricity
- Chapter 19: Electrical Circuits
- Chapter 20: Magnetism
- Chapter 21: The Quantum Nature of Light
- Chapter 22: The Atom
- Chapter 23: Particle Physics

Flexibility

Like any OpenStax content, this textbook can be modified as needed for use by the instructor depending on the needs of the students in the course. Each set of materials created by OpenStax is organized into units and chapters and can be used like a traditional textbook as the entire syllabus for each course. The materials can also be accessed in smaller chunks for more focused use with a single student or an entire class. Instructors are welcome to download and assign the PDF version of the textbook through a learning management system or can use their LMS to link students to specific chapters and sections of the book relevant to the concept being studied. The entire textbook will be available during the fall of 2020 in an editable Google document, and until then instructors are welcome to copy and paste content from the textbook to modify as needed prior to instruction.

Student-centered focus

Physics uses a friendly voice and exciting examples that appeal to a high school audience. The Chapter Openers, for example, include thought-provoking photographs and introductions that connect the content to experiences relevant to student's lives. The writing in our program has

been developed with universal design in mind to ensure students of all different backgrounds are reached. Content can be accessed through engaging text, informative visuals, hands-on activities, and online simulations. This diversity of learning media presents a wealth of reinforcement opportunities that allow students to review material in a new and fresh way.

Features

- *Snap Labs*: Give students the opportunity to experience physics through hands-on activities. The labs can be completed quickly and rely primarily on readily available materials so that students can do them at home as they read.
- *Worked Examples*: Promote both analytical and conceptual skills. In each example, the scenario/application is first introduced, followed by a description of the strategy used to solve the problem that emphasizes the concepts involved. These are followed by a fully worked mathematical solution and a discussion of the results.
- *Fun in Physics*: Features physics applications in various entertainment industries.
- *Work in Physics*: Students can explore careers in physics as well as other careers that routinely employ physics.
- *Boundless Physics*: Reveal frontiers in physical knowledge and descriptions of cutting-edge discoveries in physics.
- *Links to Physics*: Highlight connections of physics to other disciplines.
- *Watch Physics*: Support student's understanding of conceptual and computational skills using videos from Khan Academy.
- *Virtual Physics*: Provide inquiry and discovery-based learning by providing a virtual "sandbox" where students can experiment with simulated physics scenarios and equipment using the University of Colorado-developed PhET simulations.
- *Tips for Success*: Offer students advice on how to approach content or problems.

Practice and Assessment

- *Grasp Checks*: Formative assessments that review the comprehension of concepts and skills addressed through reading features, interactive features, and snap labs.
- *Practice Problems*: Challenge students to apply concepts and skills they have seen in a Worked Example to solve a problem.
- *Check Your Understanding*: Conceptual questions that, together with the practice problems, provide formative assessment on key topics in each section.
- *Performance Tasks*: Challenge students to apply the content and skills they have learned to find a solution to a practical situation.
- *Test Prep*: Helps prepare students to successfully respond to the format and rigor of standardized tests. The test prep includes multiple choice, short answer, and extended

response items.

Additional resources

Student and instructor resources

We've compiled additional resources for both students and instructors, including Getting Started Guides, PowerPoint slides, and an instructor answer guide. Instructor resources require a verified instructor account, which you can apply for when you log in or create your account on OpenStax.org. Take advantage of these resources to supplement your OpenStax book.

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About the Authors

Senior Contributing Authors

Paul Peter Urone, California State University, Sacramento
Roger Hinrichs, State University of New York, College at Oswego

Contributing Authors

Fatih Gozuacik (Director of Technology and Education) holds master's degrees in physics and atomic physics from Texas A&M Commerce and Sakarya University in Turkey. In addition to teaching AP® and College Physics, he has been a mentor for the Physics Bowl and the Science Olympiad, and has served as a college counselor for junior and senior students. Fatih was named the 2015 STEM teacher of the year by Educate Texas.

Denise Pattison (East Chambers ISD) was born and raised in Lumberton, Texas. She graduated from Lamar University - Beaumont in 2004 with a degree in science and has taught Physics, Pre AP® Physics, and IPC Physics. She loves teaching because it offers the opportunity to make a difference in a person's life.

Catherine Tabor (Northwest Early High School) holds Bachelors degrees in Mathematics and Physics, a Master's degree in Physics, and is working towards a PhD in Computer Science. She has taught for over twenty years, holding positions at a number of high schools in El Paso and colleges including Evergreen State and UT El Paso. At Northwest Early HS, she teaches Astronomy, AP® Physics, and AP® Computer Science.

Reviewers

- Bryan Callow, Lindenwold High School, Lindenwold, NJ
- John Boehringer, Prosper High School, Prosper, TX
- Wade Green, Stony Point High School, Round Rock, TX
- Susan Vogel, Rockwall-Heath High School, Heath TX
- Stan Hutto, Southwest Christian School, TX
- Richard Lines, Garland High School, Garland, TX
- Penny McCool, San Antonio ISD, Raise Up Texas Coach
- Loren Lykins, Science Department Chairman—Carlisle Independent School District, TX
- Janie Horn, Cleveland High School, Magnolia, TX
- Sean Oshman, Garland Independent School District, TX
- Cort Gillen, Cypress Ranch High School, TX

What is Physics?

Figure 1.1 Galaxies, such as the Andromeda galaxy pictured here, are immense in size. The small blue spots in this photo are also galaxies. The same physical laws apply to objects as large as galaxies or objects as small as atoms. The laws of physics are, therefore, surprisingly few in number. (NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics).

Chapter Outline

INTRODUCTION Take a look at the image above of the Andromeda Galaxy (Figure 1.1), which contains billions of stars. This galaxy is the nearest one to our own galaxy (the Milky Way) but is still a staggering 2.5 million light years from Earth. (A light year is a measurement of the distance light travels in a year.) Yet, the primary force that affects the movement of stars within Andromeda is the same force that we contend with here on Earth—namely, gravity.

You may soon realize that physics plays a much larger role in your life than you thought. This section introduces you to the realm of physics, and discusses applications of physics in other disciplines of study. It also describes the methods by which science is done, and how scientists communicate their results to each other.

1.1 Physics: Definitions and Applications

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the definition, aims, and branches of physics
- Describe and distinguish classical physics from modern physics and describe the importance of relativity, quantum mechanics, and relativistic quantum mechanics in modern physics
- Describe how aspects of physics are used in other sciences (e.g., biology, chemistry, geology, etc.) as well as in everyday technology

Section Key Terms

atom	classical physics	modern physics
physics	quantum mechanics	theory of relativity

What Physics Is

Think about all of the technological devices that you use on a regular basis. Computers, wireless internet, smart phones, tablets, global positioning system (GPS), MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above their tracks, *invisibility cloaks* that bend light around them, and microscopic robots that fight diseased cells in our bodies. All of these groundbreaking advancements rely on the principles of **physics**.

Physics is a branch of science. The word *science* comes from a Latin word that means *having knowledge*, and refers the knowledge of how the physical world operates, based on objective evidence determined through observation and experimentation. A key requirement of any scientific explanation of a natural phenomenon is that it must be testable; one must be able to devise and conduct an experimental investigation that either supports or refutes the explanation. It is important to note that some questions fall outside the realm of science precisely because they deal with phenomena that are not scientifically testable. This need for objective evidence helps define the investigative process scientists follow, which will be described later in this chapter.

Physics is the science aimed at describing the fundamental aspects of our universe. This includes what things are in it, what properties of those things are noticeable, and what processes those things or their properties undergo. In simpler terms, physics attempts to describe the basic mechanisms that make our universe behave the way it does. For example, consider a smart phone ([Figure 1.2](#)). Physics describes how electric current interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics relationships to determine the travel time from one location to another.

Figure 1.2 Physics describes the way that electric charge flows through the circuits of this device. Engineers use their knowledge of physics to construct a smartphone with features that consumers will enjoy, such as a GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (@gletham GIS, Social, Mobile Tech Images)

As our technology evolved over the centuries, physics expanded into many branches. Ancient peoples could only study things that they could see with the naked eye or otherwise experience without the aid of scientific equipment. This included the study of kinematics, which is the study of moving objects. For example, ancient people often studied the apparent motion of objects in the sky, such as the sun, moon, and stars. This is evident in the construction of prehistoric astronomical observatories, such as Stonehenge in England (shown in Figure 1.3).

Figure 1.3 Stonehenge is a monument located in England that was built between 3000 and 1000 B.C. It functions as an ancient astronomical observatory, with certain rocks in the monument aligning with the position of the sun during the summer and winter solstices. Other rocks align with the rising and setting of the moon during certain days of the year. (Citypeek, Wikimedia Commons)

Ancient people also studied statics and dynamics, which focus on how objects start moving, stop moving, and change speed and direction in response to forces that push or pull on the objects. This early interest in kinematics and dynamics allowed humans to invent simple machines, such as the lever, the pulley, the ramp, and the wheel. These simple machines were gradually

combined and integrated to produce more complicated machines, such as wagons and cranes. Machines allowed humans to gradually do more work more effectively in less time, allowing them to create larger and more complicated buildings and structures, many of which still exist today from ancient times.

As technology advanced, the branches of physics diversified even more. These include branches such as acoustics, the study of sound, and optics, the study of the light. In 1608, the invention of the telescope by a Germany spectacle maker, Hans Lippershey, led to huge breakthroughs in astronomy—the study of objects or phenomena in space. One year later, in 1609, Galileo Galilei began the first studies of the solar system and the universe using a telescope. During the Renaissance era, Isaac Newton used observations made by Galileo to construct his three laws of motion. These laws were the standard for studying kinematics and dynamics even today.

Another major branch of physics is thermodynamics, which includes the study of thermal energy and the transfer of heat. James Prescott Joule, an English physicist, studied the nature of heat and its relationship to work. Joule's work helped lay the foundation for the first of three laws of thermodynamics that describe how energy in our universe is transferred from one object to another or transformed from one form to another. Studies in thermodynamics were motivated by the need to make engines more efficient, keep people safe from the elements, and preserve food.

The 18th and 19th centuries also saw great strides in the study of electricity and magnetism. Electricity involves the study of electric charges and their movements. Magnetism had long ago been noticed as an attractive force between a magnetized object and a metal like iron, or between the opposite poles (North and South) of two magnetized objects. In 1820, Danish physicist Hans Christian Oersted showed that electric currents create magnetic fields. In 1831, English inventor Michael Faraday showed that moving a wire through a magnetic field could induce an electric current. These studies led to the inventions of the electric motor and electric generator, which revolutionized human life by bringing electricity and magnetism into our machines.

The end of the 19th century saw the discovery of radioactive substances by the French scientists Marie and Pierre Curie. Nuclear physics involves studying the nuclei of **atoms**, the source of nuclear radiation. In the 20th century, the study of nuclear physics eventually led to the ability to split the nucleus of an atom, a process called nuclear fission. This process is the basis for nuclear power plants and nuclear weapons. Also, the field of **quantum mechanics**, which involves the mechanics of atoms and molecules, saw great strides during the 20th century as our understanding of atoms and subatomic particles increased (see below).

Early in the 20th century, Albert Einstein revolutionized several branches of physics, especially relativity. Relativity revolutionized our understanding of motion and the universe in general as described further in this chapter. Now, in the 21st century, physicists continue to study these and many other branches of physics.

By studying the most important topics in physics, you will gain analytical abilities that will enable you to apply physics far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any career you choose to pursue.

Physics: Past and Present

The word physics is thought to come from the Greek word *phusis*, meaning nature. The study of nature later came to be called *natural philosophy*. From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, mathematics, and medicine. Over the last few centuries, the growth of scientific knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. Physics, as it developed from the Renaissance to the end of the 19th century, is called **classical physics**. Revolutionary discoveries starting at the beginning of the 20th century transformed physics from classical physics to **modern physics**.

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: (1) matter must be moving at speeds less than about 1 percent of the speed of light, (2) the objects dealt with must be large enough to be seen with the naked eye, and (3) only weak gravity, such as that generated by Earth, can be involved. Very small objects, such as atoms and molecules, cannot be adequately explained by classical physics. These three conditions apply to almost all of everyday experience. As a result, most aspects of classical physics should make sense on an intuitive level.

Many laws of classical physics have been modified during the 20th century, resulting in revolutionary changes in technology, society, and our view of the universe. As a result, many aspects of modern physics, which occur outside of the range of our everyday experience, may seem bizarre or unbelievable. So why is most of this textbook devoted to classical physics? There are

two main reasons. The first is that knowledge of classical physics is necessary to understand modern physics. The second reason is that classical physics still gives an accurate description of the universe under a wide range of everyday circumstances.

Modern physics includes two revolutionary theories: relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. The **theory of relativity** was developed by Albert Einstein in 1905. By examining how two observers moving relative to each other would see the same phenomena, Einstein devised radical new ideas about time and space. He came to the startling conclusion that the measured length of an object travelling at high speeds (greater than about one percent of the speed of light) is shorter than the same object measured at rest. Perhaps even more bizarre is the idea the time for the same process to occur is different depending on the motion of the observer. Time passes more slowly for an object travelling at high speeds. A trip to the nearest star system, Alpha Centauri, might take an astronaut 4.5 Earth years if the ship travels near the speed of light. However, because time is slowed at higher speeds, the astronaut would age only 0.5 years during the trip. Einstein's ideas of relativity were accepted after they were confirmed by numerous experiments.

Gravity, the force that holds us to Earth, can also affect time and space. For example, time passes more slowly on Earth's surface than for objects farther from the surface, such as a satellite in orbit. The very accurate clocks on global positioning satellites have to correct for this. They slowly keep getting ahead of clocks at Earth's surface. This is called time dilation, and it occurs because gravity, in essence, slows down time.

Large objects, like Earth, have strong enough gravity to distort space. To visualize this idea, think about a bowling ball placed on a trampoline. The bowling ball depresses or curves the surface of the trampoline. If you rolled a marble across the trampoline, it would follow the surface of the trampoline, roll into the depression caused by the bowling ball, and hit the ball. Similarly, the Earth curves space around it in the shape of a funnel. These curves in space due to the Earth cause objects to be attracted to Earth (i.e., gravity).

Because of the way gravity affects space and time, Einstein stated that gravity affects the space-time continuum, as illustrated in Figure 1.4. This is why time proceeds more slowly at Earth's surface than in orbit. In black holes, whose gravity is hundreds of times that of Earth, time passes so slowly that it would appear to a far-away observer to have stopped!
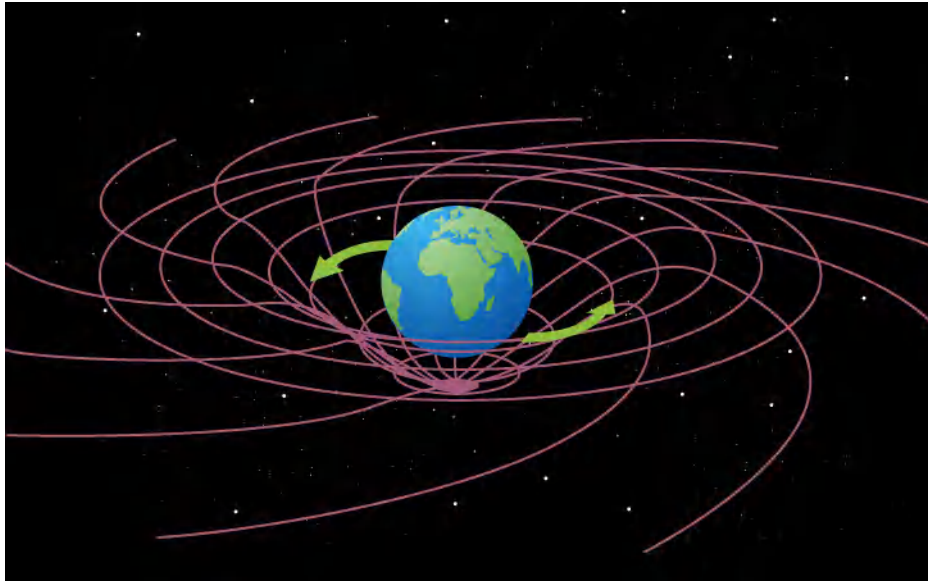
Figure 1.4 Einstein's theory of relativity describes space and time as an interweaved mesh. Large objects, such as a planet, distort space, causing objects to fall in toward the planet due to the action of gravity. Large objects also distort time, causing time to proceed at a slower rate near the surface of Earth compared with the area outside of the distorted region of space-time.

In summary, relativity says that in describing the universe, it is important to realize that time, space and speed are not absolute. Instead, they can appear different to different observers. Einstein's ability to reason out relativity is even more amazing because we cannot see the effects of relativity in our everyday lives.

Quantum mechanics is the second major theory of modern physics. Quantum mechanics deals with the very small, namely, the subatomic particles that make up atoms. Atoms (Figure 1.5) are the smallest units of elements. However, atoms themselves are constructed of even smaller subatomic particles, such as protons, neutrons and electrons. Quantum mechanics strives to

describe the properties and behavior of these and other subatomic particles. Often, these particles do not behave in the ways expected by classical physics. One reason for this is that they are small enough to travel at great speeds, near the speed of light.
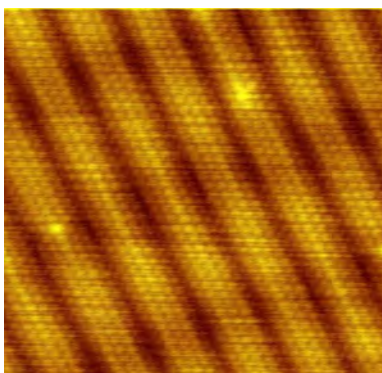
Figure 1.5 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (Erwinrossen)

At particle colliders (Figure 1.6), such as the Large Hadron Collider on the France-Swiss border, particle physicists can make subatomic particles travel at very high speeds within a 27 kilometers (17 miles) long superconducting tunnel. They can then study the properties of the particles at high speeds, as well as collide them with each other to see how they exchange energy. This has led to many intriguing discoveries such as the Higgs-Boson particle, which gives matter the property of mass, and antimatter, which causes a huge energy release when it comes in contact with matter.

Figure 1.6 Particle colliders such as the Large Hadron Collider in Switzerland or Fermilab in the United States (pictured here), have long tunnels that allows subatomic particles to be accelerated to near light speed. (Andrius.v)

Physicists are currently trying to unify the two theories of modern physics, relativity and quantum mechanics, into a single, comprehensive theory called relativistic quantum mechanics. Relating the behavior of subatomic particles to gravity, time, and space will allow us to explain how the universe works in a much more comprehensive way.

Application of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. For example, physics can help you understand why you shouldn't put metal in the microwave (Figure 1.7), why a black car radiator helps remove heat in a car engine, and why a white roof helps keep the inside of a house cool. The operation of a car's ignition system, as well as the transmission of electrical signals through our nervous system, are much easier to understand when you think about them in terms of the basic physics of electricity.

Figure 1.7 Why can't you put metal in the microwave? Microwaves are high-energy radiation that increases the movement of electrons in

metal. These moving electrons can create an electrical current, causing sparking that can lead to a fire. (= MoneyBlogNewz)

Physics is the foundation of many important scientific disciplines. For example, chemistry deals with the interactions of atoms and molecules. Not surprisingly, chemistry is rooted in atomic and molecular physics. Most branches of engineering are also applied physics. In architecture, physics is at the heart of determining structural stability, acoustics, heating, lighting, and cooling for buildings. Parts of geology, the study of nonliving parts of Earth, rely heavily on physics; including radioactive dating, earthquake analysis, and heat transfer across Earth's surface. Indeed, some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics also describes the chemical processes that power the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements (Figure 1.8). Medical therapy Physics also has many applications in biology, the study of life. For example, physics describes how cells can protect themselves using their cell walls and cell membranes (Figure 1.9). Medical therapy sometimes directly involves physics, such as in using X-rays to diagnose health conditions. Physics can also explain what we perceive with our senses, such as how the ears detect sound or the eye detects color.
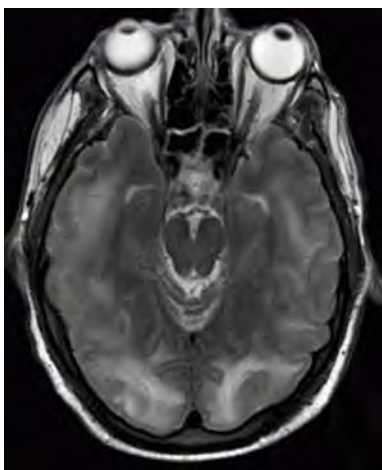
Figure 1.8 Magnetic resonance imaging (MRI) uses electromagnetic waves to yield an image of the brain, which doctors can use to find diseased regions. (Rashmi Chawla, Daniel Smith, and Paul E. Marik)
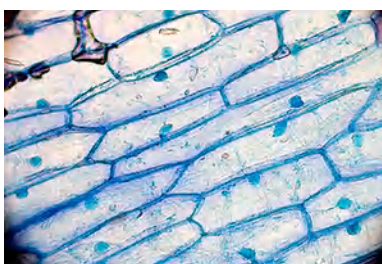
Figure 1.9 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (Umberto Salvagnin)

⚛ BOUNDLESS PHYSICS

The Physics of Landing on a Comet

On November 12, 2014, the European Space Agency's Rosetta spacecraft (shown in Figure 1.10) became the first ever to reach and orbit a comet. Shortly after, Rosetta's rover, Philae, landed on the comet, representing the first time humans have ever landed a space probe on a comet.

Figure 1.10 The Rosetta spacecraft, with its large and revolutionary solar panels, carried the Philae lander to a comet. The lander then detached and landed on the comet's surface. (European Space Agency)

After traveling 6.4 billion kilometers starting from its launch on Earth, Rosetta landed on the comet 67P/Churyumov-Gerasimenko, which is only 4 kilometers wide. Physics was needed to successfully plot the course to reach such a small, distant, and rapidly moving target. Rosetta's path to the comet was not straight forward. The probe first had to travel to Mars so that Mars's gravity could accelerate it and divert it in the exact direction of the comet.

This was not the first time humans used gravity to power our spaceships. Voyager 2, a space probe launched in 1977, used the gravity of Saturn to *slingshot* over to Uranus and Neptune (illustrated in Figure 1.11), providing the first pictures ever taken of these planets. Now, almost 40 years after its launch, Voyager 2 is at the very edge of our solar system and is about to enter interstellar space. Its sister ship, Voyager 1 (illustrated in Figure 1.11), which was also launched in 1977, is already there.

To listen to the sounds of interstellar space or see images that have been transmitted back from the Voyager I or to learn more about the Voyager mission, visit the Voyager's Mission website (https://openstax.org/l/28voyager) .
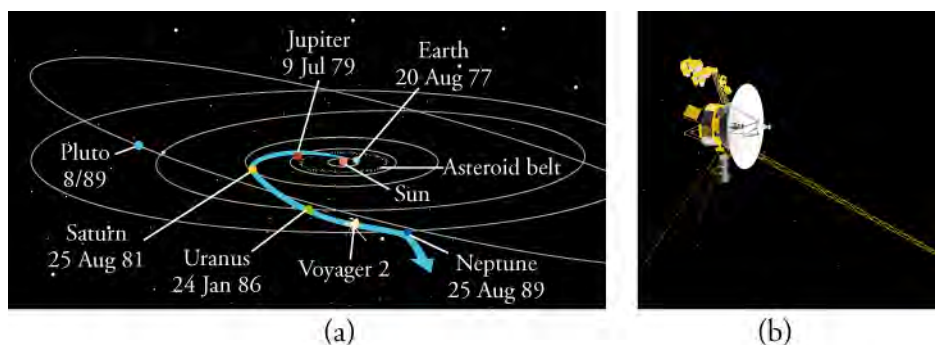
Figure 1.11 a) Voyager 2, launched in 1977, used the gravity of Saturn to slingshot over to Uranus and Neptune. NASA b) A rendering of Voyager 1, the first space probe to ever leave our solar system and enter interstellar space. NASA

Both Voyagers have electrical power generators based on the decay of radioisotopes. These generators have served them for almost 40 years. Rosetta, on the other hand, is solar-powered. In fact, Rosetta became the first space probe to travel beyond the asteroid belt by relying only on solar cells for power generation.

At 800 million kilometers from the sun, Rosetta receives sunlight that is only 4 percent as strong as on Earth. In addition, it is very cold in space. Therefore, a lot of physics went into developing Rosetta's low-intensity low-temperature solar cells.

In this sense, the Rosetta project nicely shows the huge range of topics encompassed by physics: from modeling the movement of gigantic planets over huge distances within our solar systems, to learning how to generate electric power from low-intensity light. Physics is, by far, the broadest field of science.

GRASP CHECK

What characteristics of the solar system would have to be known or calculated in order to send a probe to a distant planet, such as Jupiter?

a. the effects due to the light from the distant stars
b. the effects due to the air in the solar system
c. the effects due to the gravity from the other planets
d. the effects due to the cosmic microwave background radiation

In summary, physics studies many of the most basic aspects of science. A knowledge of physics is, therefore, necessary to understand all other sciences. This is because physics explains the most basic ways in which our universe works. However, it is not necessary to formally study all applications of physics. A knowledge of the basic laws of physics will be most useful to you, so that you can use them to solve some everyday problems. In this way, the study of physics can improve your problem-solving skills.

Check Your Understanding

1. Which of the following is *not* an essential feature of scientific explanations?
   a. They must be subject to testing.
   b. They strictly pertain to the physical world.
   c. Their validity is judged based on objective observations.
   d. Once supported by observation, they can be viewed as a fact.

2. Which of the following does *not* represent a question that can be answered by science?
   a. How much energy is released in a given nuclear chain reaction?
   b. Can a nuclear chain reaction be controlled?
   c. Should uncontrolled nuclear reactions be used for military applications?
   d. What is the half-life of a waste product of a nuclear reaction?

3. What are the three conditions under which classical physics provides an excellent description of our universe?
   a. 1. Matter is moving at speeds less than about 1 percent of the speed of light
      2. Objects dealt with must be large enough to be seen with the naked eye.
      3. Strong electromagnetic fields are involved.

   b. 1. Matter is moving at speeds less than about 1 percent of the speed of light.
      2. Objects dealt with must be large enough to be seen with the naked eye.
      3. Only weak gravitational fields are involved.

   c. 1. Matter is moving at great speeds, comparable to the speed of light.
      2. Objects dealt with are large enough to be seen with the naked eye.
      3. Strong gravitational fields are involved.

   d. 1. Matter is moving at great speeds, comparable to the speed of light.
      2. Objects are just large enough to be visible through the most powerful telescope.
      3. Only weak gravitational fields are involved.

4. Why is the Greek word for nature appropriate in describing the field of physics?
   a. Physics is a natural science that studies life and living organism on habitable planets like Earth.
   b. Physics is a natural science that studies the laws and principles of our universe.
   c. Physics is a physical science that studies the composition, structure, and changes of matter in our universe.
   d. Physics is a social science that studies the social behavior of living beings on habitable planets like Earth.

5. Which aspect of the universe is studied by quantum mechanics?
   a. objects at the galactic level
   b. objects at the classical level
   c. objects at the subatomic level
   d. objects at all levels, from subatomic to galactic

1.2 The Scientific Methods

Section Learning Objectives
By the end of this section, you will be able to do the following:

- Explain how the methods of science are used to make scientific discoveries
- Define a scientific model and describe examples of physical and mathematical models used in physics
- Compare and contrast hypothesis, theory, and law

Section Key Terms

experiment	hypothesis	model	observation	principle
scientific law	scientific methods	theory	universal	

Scientific Methods

Scientists often plan and carry out investigations to answer questions about the universe around us. Such laws are intrinsic to the universe, meaning that humans did not create them and cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. The cornerstone of discovering natural laws is observation. Science must describe the universe as it is, not as we imagine or wish it to be.

We all are curious to some extent. We look around, make generalizations, and try to understand what we see. For example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how data may be organized. We then formulate models, theories, and laws based on the data we have collected, and communicate those results with others. This, in a nutshell, describes the **scientific method** that scientists employ to decide scientific issues on the basis of evidence from observation and experiment.

An investigation often begins with a scientist making an **observation**. The scientist observes a pattern or trend within the natural world. Observation may generate questions that the scientist wishes to answer. Next, the scientist may perform some research about the topic and devise a **hypothesis**. A hypothesis is a testable statement that describes how something in the natural world works. In essence, a hypothesis is an educated guess that explains something about an observation.

Scientists may test the hypothesis by performing an **experiment**. During an experiment, the scientist collects data that will help them learn about the phenomenon they are studying. Then the scientists analyze the results of the experiment (that is, the data), often using statistical, mathematical, and/or graphical methods. From the data analysis, they draw conclusions. They may conclude that their experiment either supports or rejects their hypothesis. If the hypothesis is supported, the scientist usually goes on to test another hypothesis related to the first. If their hypothesis is rejected, they will often then test a new and different hypothesis in their effort to learn more about whatever they are studying.

Scientific processes can be applied to many situations. Let's say that you try to turn on your car, but it will not start. You have just made an observation! You ask yourself, "Why won't my car start?" You can now use scientific processes to answer this question. First, you generate a hypothesis such as, "The car won't start because it has no gasoline in the gas tank." To test this hypothesis, you put gasoline in the car and try to start it again. If the car starts, then your hypothesis is supported by the experiment. If the car does not start, then your hypothesis is rejected. You will then need to think up a new hypothesis to test such as, "My car won't start because the fuel pump is broken." Hopefully, your investigations lead you to discover why the car won't start and enable you to fix it.

Modeling

A **model** is a representation of something that is often too difficult (or impossible) to study directly. Models can take the form of physical models, equations, computer programs, or simulations—computer graphics/animations. Models are tools that are especially useful in modern physics because they let us visualize phenomena that we normally cannot observe with our senses, such as very small objects or objects that move at high speeds. For example, we can understand the structure of an atom using models, despite the fact that no one has ever seen an atom with their own eyes. Models are always approximate, so they are simpler to consider than the real situation; the more complete a model is, the more complicated it must be. Models put the

intangible or the extremely complex into human terms that we can visualize, discuss, and hypothesize about.

Scientific models are constructed based on the results of previous experiments. Even still, models often only describe a phenomenon partially or in a few limited situations. Some phenomena are so complex that they may be impossible to model them in their entirety, even using computers. An example is the electron cloud model of the atom in which electrons are moving around the atom's center in distinct clouds (Figure 1.12), that represent the likelihood of finding an electron in different places. This model helps us to visualize the structure of an atom. However, it does not show us exactly where an electron will be within its cloud at any one particular time.

Figure 1.12 The electron cloud model of the atom predicts the geometry and shape of areas where different electrons may be found in an atom. However, it cannot indicate exactly where an electron will be at any one time.

As mentioned previously, physicists use a variety of models including equations, physical models, computer simulations, etc. For example, three-dimensional models are often commonly used in chemistry and physics to model molecules. Properties other than appearance or location are usually modelled using mathematics, where functions are used to show how these properties relate to one another. Processes such as the formation of a star or the planets, can also be modelled using computer simulations. Once a simulation is correctly programmed based on actual experimental data, the simulation can allow us to view processes that happened in the past or happen too quickly or slowly for us to observe directly. In addition, scientists can also run virtual experiments using computer-based models. In a model of planet formation, for example, the scientist could alter the amount or type of rocks present in space and see how it affects planet formation.

Scientists use models and experimental results to construct explanations of observations or design solutions to problems. For example, one way to make a car more fuel efficient is to reduce the friction or drag caused by air flowing around the moving car. This can be done by designing the body shape of the car to be more aerodynamic, such as by using rounded corners instead of sharp ones. Engineers can then construct physical models of the car body, place them in a wind tunnel, and examine the flow of air around the model. This can also be done mathematically in a computer simulation. The air flow pattern can be analyzed for regions smooth air flow and for eddies that indicate drag. The model of the car body may have to be altered slightly to produce the smoothest pattern of air flow (i.e., the least drag). The pattern with the least drag may be the solution to increasing fuel efficiency of the car. This solution might then be incorporated into the car design.

Snap Lab

Using Models and the Scientific Processes
Be sure to secure loose items before opening the window or door.

In this activity, you will learn about scientific models by making a model of how air flows through your classroom or a room in your house.

- One room with at least one window or door that can be opened
- Piece of single-ply tissue paper
    1. Work with a group of four, as directed by your teacher. Close all of the windows and doors in the room you are working in. Your teacher may assign you a specific window or door to study.

2. Before opening any windows or doors, draw a to-scale diagram of your room. First, measure the length and width of your room using the tape measure. Then, transform the measurement using a scale that could fit on your paper, such as 5 centimeters = 1 meter.
3. Your teacher will assign you a specific window or door to study air flow. On your diagram, add arrows showing your hypothesis (before opening any windows or doors) of how air will flow through the room when your assigned window or door is opened. Use pencil so that you can easily make changes to your diagram.
4. On your diagram, mark four locations where you would like to test air flow in your room. To test for airflow, hold a strip of single ply tissue paper between the thumb and index finger. Note the direction that the paper moves when exposed to the airflow. Then, for each location, predict which way the paper will move if your air flow diagram is correct.
5. Now, each member of your group will stand in one of the four selected areas. Each member will test the airflow Agree upon an approximate height at which everyone will hold their papers.
6. When you teacher tells you to, open your assigned window and/or door. Each person should note the direction that their paper points immediately after the window or door was opened. Record your results on your diagram.
7. Did the airflow test data support or refute the hypothetical model of air flow shown in your diagram? Why or why not? Correct your model based on your experimental evidence.
8. With your group, discuss how accurate your model is. What limitations did it have? Write down the limitations that your group agreed upon.

GRASP CHECK
Your diagram is a model, based on experimental evidence, of how air flows through the room. Could you use your model to predict how air would flow through a new window or door placed in a different location in the classroom? Make a new diagram that predicts the room's airflow with the addition of a new window or door. Add a short explanation that describes how.

a. Yes, you could use your model to predict air flow through a new window. The earlier experiment of air flow would help you model the system more accurately.
b. Yes, you could use your model to predict air flow through a new window. The earlier experiment of air flow is not useful for modeling the new system.
c. No, you cannot model a system to predict the air flow through a new window. The earlier experiment of air flow would help you model the system more accurately.
d. No, you cannot model a system to predict the air flow through a new window. The earlier experiment of air flow is not useful for modeling the new system.

Scientific Laws and Theories

A **scientific law** is a description of a pattern in nature that is true in all circumstances that have been studied. That is, physical laws are meant to be **universal**, meaning that they apply throughout the known universe. Laws are often also concise, whereas theories are more complicated. A law can be expressed in the form of a single sentence or mathematical equation. For example, Newton's second law of motion, which relates the motion of an object to the force applied (F), the mass of the object (m), and the object's acceleration (a), is simply stated using the equation

$$F = ma.$$

Scientific ideas and explanations that are true in many, but not all situations in the universe are usually called **principles**. An example is Pascal's principle, which explains properties of liquids, but not solids or gases. However, the distinction between laws and principles is sometimes not carefully made in science.

A **theory** is an explanation for patterns in nature that is supported by much scientific evidence and verified multiple times by multiple researchers. While many people confuse theories with educated guesses or hypotheses, theories have withstood more rigorous testing and verification than hypotheses.

As a closing idea about scientific processes, we want to point out that scientific laws and theories, even those that have been supported by experiments for centuries, can still be changed by new discoveries. This is especially true when new technologies emerge that allow us to observe things that were formerly unobservable. Imagine how viewing previously invisible objects with a

microscope or viewing Earth for the first time from space may have instantly changed our scientific theories and laws! What discoveries still await us in the future? The constant retesting and perfecting of our scientific laws and theories allows our knowledge of nature to progress. For this reason, many scientists are reluctant to say that their studies *prove* anything. By saying *support* instead of *prove*, it keeps the door open for future discoveries, even if they won't occur for centuries or even millennia.

Check Your Understanding

6. Explain why scientists sometimes use a model rather than trying to analyze the behavior of the real system.
- a. Models are simpler to analyze.
- b. Models give more accurate results.
- c. Models provide more reliable predictions.
- d. Models do not require any computer calculations.

7. Describe the difference between a question, generated through observation, and a hypothesis.
- a. They are the same.
- b. A hypothesis has been thoroughly tested and found to be true.
- c. A hypothesis is a tentative assumption based on what is already known.
- d. A hypothesis is a broad explanation firmly supported by evidence.

8. What is a scientific model and how is it useful?
- a. A scientific model is a representation of something that can be easily studied directly. It is useful for studying things that can be easily analyzed by humans.
- b. A scientific model is a representation of something that is often too difficult to study directly. It is useful for studying a complex system or systems that humans cannot observe directly.
- c. A scientific model is a representation of scientific equipment. It is useful for studying working principles of scientific equipment.
- d. A scientific model is a representation of a laboratory where experiments are performed. It is useful for studying requirements needed inside the laboratory.

9. Which of the following statements is correct about the hypothesis?
- a. The hypothesis must be validated by scientific experiments.
- b. The hypothesis must not include any physical quantity.
- c. The hypothesis must be a short and concise statement.
- d. The hypothesis must apply to all the situations in the universe.

10. What is a scientific theory?
- a. A scientific theory is an explanation of natural phenomena that is supported by evidence.
- b. A scientific theory is an explanation of natural phenomena without the support of evidence.
- c. A scientific theory is an educated guess about the natural phenomena occurring in nature.
- d. A scientific theory is an uneducated guess about natural phenomena occurring in nature.

11. Compare and contrast a hypothesis and a scientific theory.
- a. A hypothesis is an explanation of the natural world with experimental support, while a scientific theory is an educated guess about a natural phenomenon.
- b. A hypothesis is an educated guess about natural phenomenon, while a scientific theory is an explanation of natural world with experimental support.
- c. A hypothesis is experimental evidence of a natural phenomenon, while a scientific theory is an explanation of the natural world with experimental support.
- d. A hypothesis is an explanation of the natural world with experimental support, while a scientific theory is experimental evidence of a natural phenomenon.

1.3 The Language of Physics: Physical Quantities and Units

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Associate physical quantities with their International System of Units (SI)and perform conversions among SI units using scientific notation
- Relate measurement uncertainty to significant figures and apply the rules for using significant figures in calculations
- Correctly create, label, and identify relationships in graphs using mathematical relationships (e.g., slope, *y*-intercept, inverse, quadratic and logarithmic)

Section Key Terms

accuracy	ampere	constant	conversion factor	dependent variable
derived units	English units	exponential relationship	fundamental physical units	independent variable
inverse relationship	inversely proportional	kilogram	linear relationship	logarithmic (log) scale
log-log plot	meter	method of adding percents	order of magnitude	precision
quadratic relationship	scientific notation	second	semi-log plot	SI units
significant figures	slope	uncertainty	variable	*y*-intercept

The Role of Units

Physicists, like other scientists, make observations and ask basic questions. For example, how big is an object? How much mass does it have? How far did it travel? To answer these questions, they make measurements with various instruments (e.g., meter stick, balance, stopwatch, etc.).

The measurements of physical quantities are expressed in terms of units, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in meters (for sprinters) or kilometers (for long distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way (Figure 1.13).
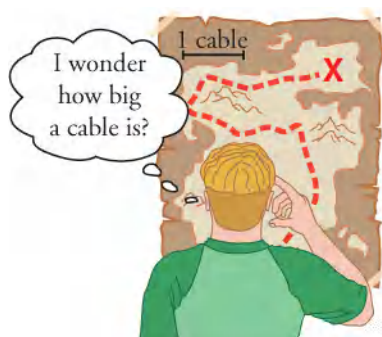
Figure 1.13 Distances given in unknown units are maddeningly useless.

All physical quantities in the International System of Units (SI) are expressed in terms of combinations of seven **fundamental**

physical units, which are units for: length, mass, time, electric current, temperature, amount of a substance, and luminous intensity.

SI Units: Fundamental and Derived Units

There are two major systems of units used in the world: **SI units** (acronym for the French Le Système International d'Unités, also known as the metric system), and **English units** (also known as the imperial system). English units were historically used in nations once ruled by the British Empire. Today, the United States is the only country that still uses English units extensively. Virtually every other country in the world now uses the metric system, which is the standard system agreed upon by scientists and mathematicians.

Some physical quantities are more fundamental than others. In physics, there are seven fundamental physical quantities that are measured in base or physical fundamental units: length, mass, time, electric current temperature, amount of substance, and luminous intensity. Units for other physical quantities (such as force, speed, and electric charge) described by mathematically combining these seven base units. In this course, we will mainly use five of these: length, mass, time, electric current and temperature. The units in which they are measured are the meter, kilogram, second, ampere, kelvin, mole, and candela (Table 1.1). All other units are made by mathematically combining the fundamental units. These are called **derived units**.

Quantity	Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	a
Temperature	Kelvin	k
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

Table 1.1 SI Base Units

The Meter

The SI unit for length is the **meter** (m). The definition of the meter has changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar. (The bar is now housed at the International Bureau of Weights and Meaures, near Paris). By 1960, some distances could be measured more precisely by comparing them to wavelengths of light. The meter was redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition as the distance light travels in a vacuum in 1/299,792,458 of a second (Figure 1.14).

Light travels a distance of 1 meter
in 1/299,792,458 seconds
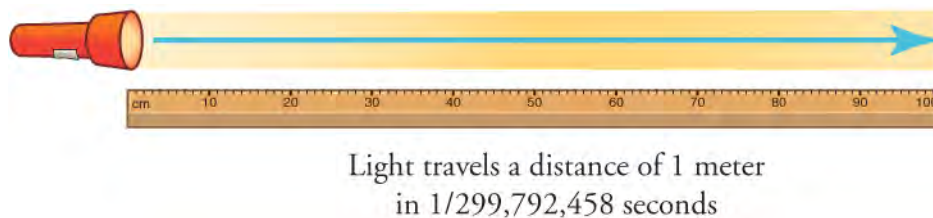
Figure 1.14 The meter is defined to be the distance light travels in 1/299,792,458 of a second through a vacuum. Distance traveled is speed multiplied by time.

The Kilogram

The SI unit for mass is the **kilogram** (kg). It is defined to be the mass of a platinum-iridium cylinder, housed at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram cylinder are kept in numerous locations throughout the world, such as the National Institute of Standards and Technology in Gaithersburg, Maryland. The determination of all other masses can be done by comparing them with one of these standard kilograms.

The Second

The SI unit for time, the **second** (s) also has a long history. For many years it was defined as 1/86,400 of an average solar day. However, the average solar day is actually very gradually getting longer due to gradual slowing of Earth's rotation. Accuracy in the fundamental units is essential, since all other measurements are derived from them. Therefore, a new standard was adopted to define the second in terms of a non-varying, or constant, physical phenomenon. One constant phenomenon is the very steady vibration of Cesium atoms, which can be observed and counted. This vibration forms the basis of the cesium atomic clock. In 1967, the second was redefined as the time required for 9,192,631,770 Cesium atom vibrations (Figure 1.15).
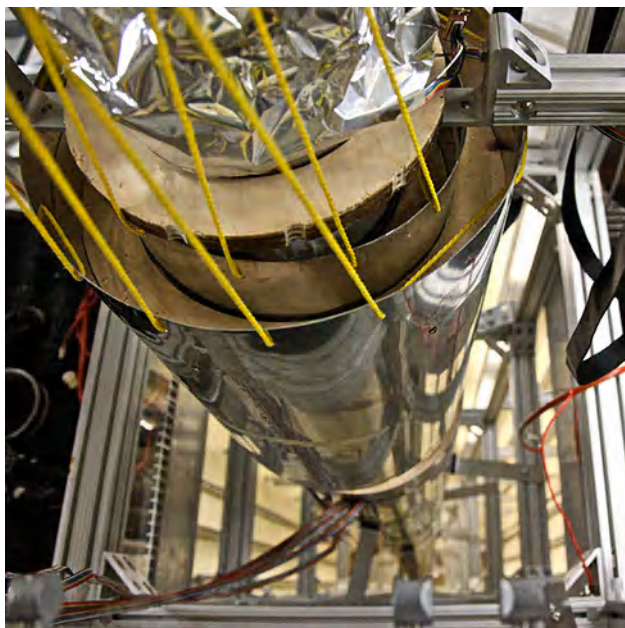
Figure 1.15 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of one microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic clock. (Steve Jurvetson/Flickr)

The Ampere

Electric current is measured in the **ampere** (A), named after Andre Ampere. You have probably heard of amperes, or *amps*, when people discuss electrical currents or electrical devices. Understanding an ampere requires a basic understanding of electricity and magnetism, something that will be explored in depth in later chapters of this book. Basically, two parallel wires with an electric current running through them will produce an attractive force on each other. One ampere is defined as the amount of electric current that will produce an attractive force of 2.7×10^{-7} newton per meter of separation between the two wires (the newton is the derived unit of force).

Kelvins

The SI unit of temperature is the **kelvin** (or kelvins, but not degrees kelvin). This scale is named after physicist William Thomson, Lord Kelvin, who was the first to call for an absolute temperature scale. The Kelvin scale is based on absolute zero. This is the point at which all thermal energy has been removed from all atoms or molecules in a system. This temperature, 0 K, is equal to $-273.15\,°C$ and $-459.67\,°F$. Conveniently, the Kelvin scale actually changes in the same way as the Celsius scale. For example, the freezing point (0 °C) and boiling points of water (100 °C) are 100 degrees apart on the Celsius scale. These two temperatures are also 100 kelvins apart (freezing point = 273.15 K; boiling point = 373.15 K).

Metric Prefixes

Physical objects or phenomena may vary widely. For example, the size of objects varies from something very small (like an atom)

to something very large (like a star). Yet the standard metric unit of length is the meter. So, the metric system includes many prefixes that can be attached to a unit. Each prefix is based on factors of 10 (10, 100, 1,000, etc., as well as 0.1, 0.01, 0.001, etc.). Table 1.2 gives the metric prefixes and symbols used to denote the different various factors of 10 in the metric system.

Prefix	Symbol	Value[1]	Example Name	Example Symbol	Example Value	Example Description
exa	E	10^{18}	Exameter	Em	10^{18} m	Distance light travels in a century
peta	P	10^{15}	Petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	Terawatt	TW	10^{12} W	Powerful laser output
giga	G	10^{9}	Gigahertz	GHz	10^{9} Hz	A microwave frequency
mega	M	10^{6}	Megacurie	MCi	10^{6} Ci	High radioactivity
kilo	k	10^{3}	Kilometer	km	10^{3} m	About 6/10 mile
hector	h	10^{2}	Hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	Dekagram	dag	10^{1} g	Teaspoon of butter
____	____	10^{0} (=1)				
deci	d	10^{-1}	Deciliter	dL	10^{-1} L	Less than half a soda
centi	c	10^{-2}	Centimeter	Cm	10^{-2} m	Fingertip thickness
milli	m	10^{-3}	Millimeter	Mm	10^{-3} m	Flea at its shoulder
micro	μ	10^{-6}	Micrometer	μm	10^{-6} m	Detail in microscope
nano	n	10^{-9}	Nanogram	Ng	10^{-9} g	Small speck of dust
pico	p	10^{-12}	Picofarad	pF	10^{-12} F	Small capacitor in radio
femto	f	10^{-15}	Femtometer	Fm	10^{-15} m	Size of a proton
atto	a	10^{-18}	Attosecond	as	10^{-18} s	Time light takes to cross an atom

Table 1.2 Metric Prefixes for Powers of 10 and Their Symbols   [1]See Appendix A for a discussion of powers of 10. Note—Some examples are approximate.

The metric system is convenient because conversions between metric units can be done simply by moving the decimal place of a number. This is because the metric prefixes are sequential powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as U.S. customary units, the relationships are less simple—there are 12 inches in a foot, 5,280 feet in a mile, 4 quarts in a gallon, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by switching to the most-appropriate metric prefix. For example, distances in meters are suitable for building construction, but kilometers are used to describe road construction. Therefore, with the metric system, there is no need to invent new units when measuring very small or very large objects—you just have to move the decimal

point (and use the appropriate prefix).

Known Ranges of Length, Mass, and Time

Table 1.3 lists known lengths, masses, and time measurements. You can see that scientists use a range of measurement units. This wide range demonstrates the vastness and complexity of the universe, as well as the breadth of phenomena physicists study. As you examine this table, note how the metric system allows us to discuss and compare an enormous range of phenomena, using one system of measurement (Figure 1.16 and Figure 1.17).

Length (m)	Phenomenon Measured	Mass (Kg)	Phenomenon Measured[1]	Time (s)	Phenomenon Measured[1]
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron (9.11×10^{-31} kg)	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom (1.67×10^{-27} kg)	10^{-22}	Mean life of an extremely unstable nucleus
10^{14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of a visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cell of living organism	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^2	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^3	Mass of a car	10^5	One day (8.64×10^4 s)
10^2	Length of a football field	10^8	Mass of a large ship	10^7	One year (3.16×10^7 s)
10^4	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^9	About half the life expectancy of a human
10^7	Diameter of Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from Earth to the sun	10^{23}	Mass of the moon (7.35×10^{22} kg)	10^{17}	Age of Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of Earth (5.97×10^{24} kg)	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way Galaxy	10^{30}	Mass of the Sun (1.99×10^{24} kg)		

Table 1.3 Approximate Values of Length, Mass, and Time   [1] More precise values are in parentheses.

Length (m)	Phenomenon Measured	Mass (Kg)	Phenomenon Measured[1]	Time (s)	Phenomenon Measured[1]
10^{22}	Distance from Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Table 1.3 Approximate Values of Length, Mass, and Time   [1] More precise values are in parentheses.
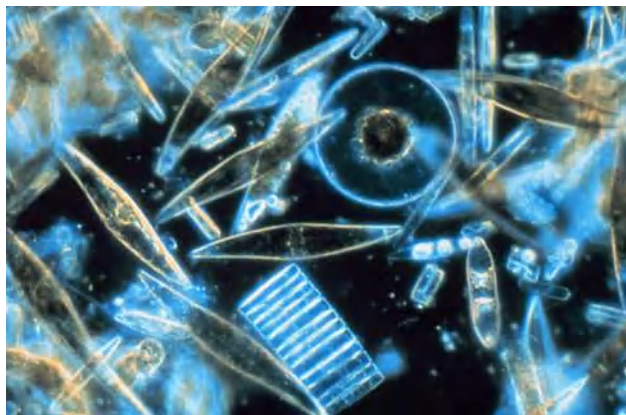
Figure 1.16 Tiny phytoplankton float among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

Figure 1.17 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Using Scientific Notation with Physical Measurements

Scientific notation is a way of writing numbers that are too large or small to be conveniently written as a decimal. For example, consider the number 840,000,000,000,000. It's a rather large number to write out. The scientific notation for this number is 8.40×10^{14}. Scientific notation follows this general format

$$x \ \times \ 10^{y}.$$

In this format x is the value of the measurement with all placeholder zeros removed. In the example above, x is 8.4. The x is multiplied by a factor, 10^{y}, which indicates the number of placeholder zeros in the measurement. Placeholder zeros are those at the end of a number that is 10 or greater, and at the beginning of a decimal number that is less than 1. In the example above, the factor is 10^{14}. This tells you that you should move the decimal point 14 positions to the right, filling in placeholder zeros as you go. In this case, moving the decimal point 14 places creates only 13 placeholder zeros, indicating that the actual measurement value is 840,000,000,000,000.

Numbers that are fractions can be indicated by scientific notation as well. Consider the number 0.0000045. Its scientific notation is 4.5×10^{-6}. Its scientific notation has the same format

$$x \ \times \ 10^y.$$

Here, x is 4.5. However, the value of y in the 10^y factor is negative, which indicates that the measurement is a fraction of 1. Therefore, we move the decimal place to the left, for a negative y. In our example of 4.5×10^{-6}, the decimal point would be moved to the left six times to yield the original number, which would be 0.0000045.

The term **order of magnitude** refers to the power of 10 when numbers are expressed in scientific notation. Quantities that have the same power of 10 when expressed in scientific notation, or come close to it, are said to be of the same order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Both numbers have the same value for y. Therefore, 800 and 450 are of the same order of magnitude. Similarly, 101 and 99 would be regarded as the same order of magnitude, 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-9} m, while the diameter of the sun is on the order of 10^9 m. These two values are 18 orders of magnitude apart.

Scientists make frequent use of scientific notation because of the vast range of physical measurements possible in the universe, such as the distance from Earth to the moon (Figure 1.18), or to the nearest star.

Figure 1.18 The distance from Earth to the moon may seem immense, but it is just a tiny fraction of the distance from Earth to our closest neighboring star. (NASA)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook in the United States, some quantities may be expressed in liters and you need to convert them to cups. A Canadian tourist driving through the United States might want to convert miles to kilometers, to have a sense of how far away his next destination is. A doctor in the United States might convert a patient's weight in pounds to kilograms.

Let's consider a simple example of how to convert units within the metric system. How can we want to convert 1 hour to seconds?

Next, we need to determine a **conversion factor** relating meters to kilometers. A **conversion factor** is a ratio expressing how many of one unit are equal to another unit. A conversion factor is simply a fraction which equals 1. You can multiply any number by 1 and get the same value. When you multiply a number by a conversion factor, you are simply multiplying it by one. For example, the following are conversion factors: (1 foot)/(12 inches) = 1 to convert inches to feet, (1 meter)/(100 centimeters) = 1 to convert centimeters to meters, (1 minute)/(60 seconds) = 1 to convert seconds to minutes. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor (1 km/1,000m) = 1, so we are simply multiplying 80m by 1:

$$1 \ \cancel{h} \times \frac{60 \ \cancel{min}}{1 \ \cancel{h}} \times \frac{60 \ s}{1 \ \cancel{min}} = 3600 \ s = 3.6 \times 10^2 \ s \qquad \boxed{1.1}$$

When there is a unit in the original number, and a unit in the denominator (bottom) of the conversion factor, the units cancel. In this case, hours and minutes cancel and the value in seconds remains.

You can use this method to convert between any types of unit, including between the U.S. customary system and metric system. Notice also that, although you can multiply and divide units algebraically, you cannot add or subtract different units. An expression like *10 km + 5 kg* makes no sense. Even adding two lengths in different units, such as *10 km + 20 m* does not make sense. You express both lengths in the same unit. See Appendix C for a more complete list of conversion factors.

✳ WORKED EXAMPLE

Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note—Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

2. Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0 \ \text{km}}{20.0 \ \text{min}} = 0.500 \ \frac{\text{km}}{\text{min}}$$

3. Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is $60 \ \text{min/1h}$. Thus,

$$\text{average speed} = 0.500 \ \frac{\text{km}}{\text{min}} \times \frac{60 \ \text{min}}{1 \ \text{h}} = 30.0 \ \frac{\text{km}}{\text{h}}.$$

Discussion for (a)

To check your answer, consider the following:

1. Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows

$$\frac{\text{km}}{\text{min}} \times \frac{1 \ \text{hr}}{60 \ \text{min}} = \frac{1}{60} \frac{\text{km·h}}{\text{min}^2},$$

which are obviously not the desired units of km/h.

2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

3. Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/h does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 min, so the precision of the conversion factor is perfect.

4. Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution (b)

There are several ways to convert the average speed into meters per second.

1. Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

2. Multiplying by these yields

$$\text{Average speed} = 30.0 \frac{\text{km}}{\text{h}} \times \frac{1 \ \text{h}}{3,600 \ \text{s}} \times \frac{1,000 \ \text{m}}{1 \ \text{km}}$$

$$\text{Average speed} = 8.33\,\frac{\text{m}}{\text{s}}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces?

✳ WORKED EXAMPLE

Using Physics to Evaluate Promotional Materials

A commemorative coin that is 2″ in diameter is advertised to be plated with 15 mg of gold. If the density of gold is 19.3 g/cc, and the amount of gold around the edge of the coin can be ignored, what is the thickness of the gold on the top and bottom faces of the coin?

Strategy

To solve this problem, the volume of the gold needs to be determined using the gold's mass and density. Half of that volume is distributed on each face of the coin, and, for each face, the gold can be represented as a cylinder that is 2″ in diameter with a height equal to the thickness. Use the volume formula for a cylinder to determine the thickness.

Solution

The mass of the gold is given by the formula $m = \rho V = 15 \times 10^{-3}$ g, where $\rho = 19.3$ g/cc and V is the volume. Solving for the volume gives $V = \dfrac{m}{\rho} = \dfrac{15 \times 10^{-3}\ \text{g}}{19.3\ \text{g/cc}} \cong 7.8 \times 10^{-4}$ cc.

If t is the thickness, the volume corresponding to half the gold is $\dfrac{1}{2}\left(7.8 \times 10^{-4}\right) = \pi r^2 t = \pi(2.54)^2 t$, where the 1″ radius has been converted to cm. Solving for the thickness gives $t = \dfrac{\left(3.9 \times 10^{-4}\right)}{\pi(2.54)^2} \cong 1.9 \times 10^{-5}$ cm = 0.00019 mm.

Discussion

The amount of gold used is stated to be 15 mg, which is equivalent to a thickness of about 0.00019 mm. The mass figure may make the amount of gold sound larger, both because the number is much bigger (15 versus 0.00019), and because people may have a more intuitive feel for how much a millimeter is than for how much a milligram is. A simple analysis of this sort can clarify the significance of claims made by advertisers.

Accuracy, Precision and Significant Figures

Science is based on experimentation that requires good measurements. The validity of a measurement can be described in terms of its accuracy and its precision (see Figure 1.19 and Figure 1.20). **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard piece of printer paper. The packaging in which you purchased the paper states that it is 11 inches long, and suppose this stated value is correct. You measure the length of the paper three times and obtain the following measurements: 11.1 inches, 11.2 inches, and 10.9 inches. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate. This is why measuring instruments are calibrated based on a known measurement. If the instrument consistently returns the correct value of the known measurement, it is safe for use in finding unknown values.

Figure 1.19 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The known masses are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (Serge Melki)

Figure 1.20 Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, some digital scales can measure the mass of an object up to the nearest thousandth of a gram. As in other measuring devices, the precision of a scale is limited to the last measured figures. This is the hundredths place in the scale pictured here. (Splarka, Wikimedia Commons)

Precision states how well repeated measurements of something generate the same or similar results. Therefore, the precision of measurements refers to how close together the measurements are when you measure the same thing several times. One way to analyze the precision of measurements would be to determine the range, or difference between the lowest and the highest measured values. In the case of the printer paper measurements, the lowest value was 10.9 inches and the highest value was 11.2 inches. Thus, the measured values deviated from each other by, at most, 0.3 inches. These measurements were reasonably precise because they varied by only a fraction of an inch. However, if the measured values had been 10.9 inches, 11.1 inches, and 11.9 inches, then the measurements would not be very precise because there is a lot of variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target. Then think of each GPS attempt to locate the restaurant as a black dot on the bull's eye.

In Figure 1.21, you can see that the GPS measurements are spread far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 1.22, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system. Finally, in Figure 1.23, the GPS is both precise and accurate, allowing the restaurant to be located.
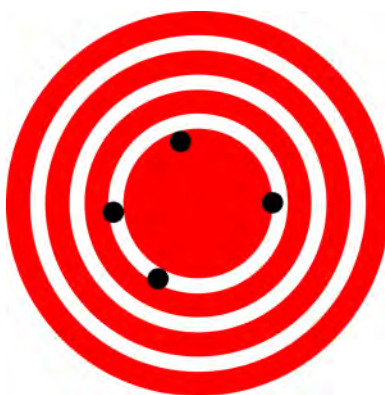
Figure 1.21 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (Dark Evil)

Figure 1.22 In this figure, the dots are concentrated close to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (Dark Evil)

Figure 1.23 In this figure, the dots are concentrated close to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (Dark Evil)

Uncertainty

The accuracy and precision of a measuring system determine the **uncertainty** of its measurements. Uncertainty is a way to describe how much your measured value deviates from the actual value that the object has. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 inches plus or minus 0.2 inches or 11.0 ± 0.2 inches. The uncertainty in a

measurement, A, is often denoted as δA ("delta A"),

The factors contributing to uncertainty in a measurement include the following:

1.   Limitations of the measuring device
2.   The skill of the person making the measurement
3.   Irregularities in the object being measured
4.   Any other factors that affect the outcome (highly dependent on the situation)

In the printer paper example uncertainty could be caused by: the fact that the smallest division on the ruler is 0.1 inches, the person using the ruler has bad eyesight, or uncertainty caused by the paper cutting machine (e.g., one side of the paper is slightly longer than the other.) It is good practice to carefully consider all possible sources of uncertainty in a measurement and reduce or eliminate them,

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement, A, is expressed with uncertainty, δA, the percent uncertainty is

$$\% \text{ uncertainty} = \frac{\delta A}{A} \times 100\%. \qquad \boxed{1.2}$$

✳ WORKED EXAMPLE

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: 4.8lb
- Week 2 weight: 5.3 lb
- Week 3 weight: 4.9lb
- Week 4 weight: 5.4 lb

You determine that the weight of the 5 lb bag has an uncertainty of ± 0.4 lb. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight

$$\% \text{ uncertainty} = \frac{\delta A}{A} \times 100\%.$$

Solution

Plug the known values into the equation

$$\% \text{ uncertainty} = \frac{0.4 \text{ lb}}{5 \text{ lb}} \times 100\% = 8\%.$$

Discussion

We can conclude that the weight of the apple bag is 5 lb $\pm$ 8 percent. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100 percent. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainty in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the both the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements in the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used. This method says that the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2 percent and 1

percent, respectively, then the area of the floor is 12.0 m² and has an uncertainty of 3 percent (expressed as an area this is 0.36 m², which we round to 0.4 m² since the area of the floor is given to a tenth of a square meter).

For a quick demonstration of the accuracy, precision, and uncertainty of measurements based upon the units of measurement, try this simulation (http://openstax.org/l/28precision) . You will have the opportunity to measure the length and weight of a desk, using milli- versus centi- units. Which do you think will provide greater accuracy, precision and uncertainty when measuring the desk and the notepad in the simulation? Consider how the nature of the hypothesis or research question might influence how precise of a measuring tool you need to collect data.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements is the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, consider measuring the thickness of a coin. A standard ruler can measure thickness to the nearest millimeter, while a micrometer can measure the thickness to the nearest 0.005 millimeter. The micrometer is a more precise measuring tool because it can measure extremely small differences in thickness. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool (such as the rulers shown in Figure 1.24). For example, if you use a standard ruler to measure the length of a stick, you may measure it with a decimeter ruler as 3.6 cm. You could not express this value as 3.65 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36 mm and 37 mm. He or she must estimate the value of the last digit. The rule is that the last digit written down in a measurement is the first digit with some uncertainty. For example, the last measured value 36.5 mm has three digits, or three significant figures. The number of **significant figures** in a measurement indicates the precision of the measuring tool. The more precise a measuring tool is, the greater the number of significant figures it can report.
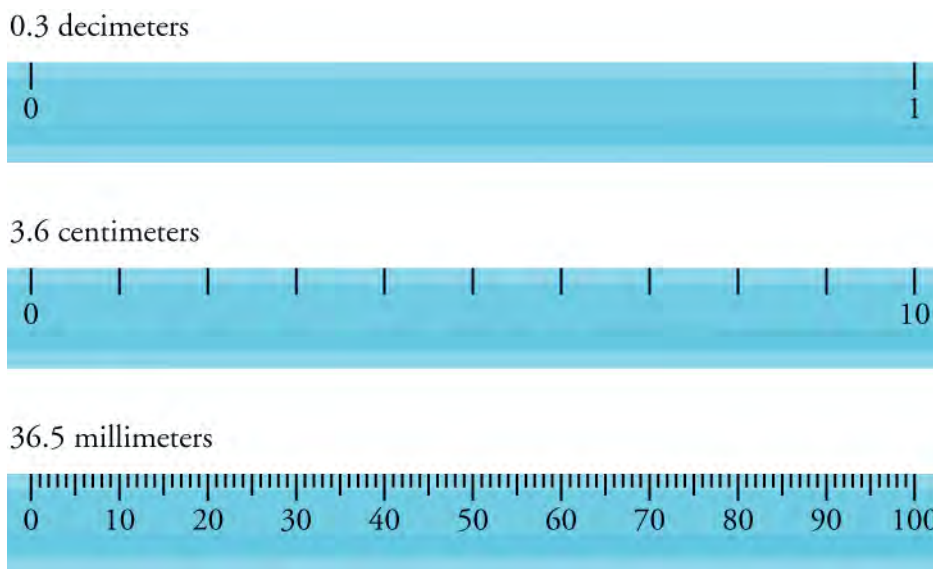
Figure 1.24 Three metric rulers are shown. The first ruler is in decimeters and can measure point three decimeters. The second ruler is in centimeters long and can measure three point six centimeters. The last ruler is in millimeters and can measure thirty-six point five millimeters.

Zeros

Special consideration is given to zeros when counting significant figures. For example, the zeros in 0.053 are not significant because they are only placeholders that locate the decimal point. There are two significant figures in 0.053—the 5 and the 3. However, if the zero occurs between other significant figures, the zeros are significant. For example, both zeros in 10.053 are significant, as these zeros were actually measured. Therefore, the 10.053 placeholder has five significant figures. The zeros in 1300 may or may not be significant, depending on the style of writing numbers. They could mean the number is known to the last zero, or the zeros could be placeholders. So 1300 could have two, three, or four significant figures. To avoid this ambiguity,

write 1300 in scientific notation as 1.3 × 10³. Only significant figures are given in the *x* factor for a number in scientific notation (in the form $x \times 10^y$). Therefore, we know that 1 and 3 are the only significant digits in this number. In summary, zeros are significant except when they serve only as placeholders. Table 1.4 provides examples of the number of significant figures in various numbers.

Number	Significant Figures	Rationale
1.657	4	There are no zeros and all non-zero numbers are always significant.
0.4578	4	The first zero is only a placeholder for the decimal point.
0.000458	3	The first four zeros are placeholders needed to report the data to the ten-thoudsandths place.
2000.56	6	The three zeros are significant here because they occur between other significant figures.
45,600	3	With no underlines or scientific notation, we assume that the last two zeros are placeholders and are not significant.
15895<u>00</u>0	7	The two underlined zeros are significant, while the last zero is not, as it is not underlined.
5.457 × 10¹³	4	In scientific notation, all numbers reported in front of the multiplication sign are significant
6.520 × 10⁻²³	4	In scientific notation, all numbers reported in front of the multiplication sign are significant, including zeros.

Table 1.4

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value. There are two different rules, one for multiplication and division and another rule for addition and subtraction, as discussed below.

1. **For multiplication and division:** The answer should have the same number of significant figures as the starting value with the fewest significant figures. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area will have if the radius has only two significant figures, for example, *r* = 2.0 m. Then, using a calculator that keeps eight significant figures, you would get

$$A = \pi r^2 = (3.1415927...) \times (2.0 \text{ m})^2 = 4.5238934 \text{ m}^2.$$

   But because the radius has only two significant figures, the area calculated is meaningful only to two significant figures or

$$A = 4.5 \text{ m}^2$$

   even though the value of π is meaningful to at least eight digits.

2. **For addition and subtraction**: The answer should have the same number places (e.g. tens place, ones place, tenths place, etc.) as the least-precise starting value. Suppose that you buy 7.56 kg of potatoes in a grocery store as measured with a scale having a precision of 0.01 kg. Then you drop off 6.052 kg of potatoes at your laboratory as measured by a scale with a precision of 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with a precision of 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$7.56 \quad \text{kg}$$
$$-6.052 \quad \text{kg}$$
$$\underline{+13.7 \quad \text{kg}}$$
$$15.208 \quad \text{kg}$$

The least precise measurement is 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer should be rounded to the tenths place, giving 15.2 kg. The same is true for non-decimal numbers. For example,

$$6527.23 + 2 = 6528.23 = 6528 \; .$$

We cannot report the decimal places in the answer because 2 has no decimal places that would be significant. Therefore, we can only report to the ones place.

It is a good idea to keep extra significant figures while calculating, and to round off to the correct number of significant figures only in the final answers. The reason is that small errors from rounding while calculating can sometimes produce significant errors in the final answer. As an example, try calculating $5,098 - (5.000) \times (1,010)$ to obtain a final answer to only two significant figures. Keeping all significant during the calculation gives 48. Rounding to two significant figures in the middle of the calculation changes it to $5,100 - (5.000) \times (1,000) = 100$, which is way off. You would similarly avoid rounding in the middle of the calculation in counting and in doing accounting, where many small numbers need to be added and subtracted accurately to give possibly much larger final numbers.

Significant Figures in this Text

In this textbook, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, such as optics, more than three significant figures will be used. Finally, if a number is exact, such as the 2 in the formula, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

⊛ **WORKED EXAMPLE**

Approximating Vast Numbers: a Trillion Dollars

The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks, like that shown in , and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Figure 1.25   A bank stack contains one hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (Andrew Magill)

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

1. Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is

$$\text{volume of stack} = \text{length} \times \text{width} \times \text{height},$$
$$\text{volume of stack} = 6 \text{ in.} \times 3 \text{ in.} \times 0.5 \text{ in.},$$
$$\text{volume of stack} = 9 \text{ in.}^3.$$

2. Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $100 bills is equal to $\$10,000$, or $\$1 \times 10^4$. The number of stacks you will have is

$$\$1 \times 10^{12} \text{ (a trillion dollars) / } \$1 \times 10^4 \text{ per stack} = 1 \times 10^8 \text{ stacks.}$$

   1.3

3. Calculate the area of a football field in square inches. The area of a football field is $100 \text{ yd} \times 50\text{yd}$, which gives $5,000 \text{ yd}^2$. Because we are working in inches, we need to convert square yards to square inches

$$\text{Area} = 5{,}000 \text{ yd}^2 \times \frac{3 \text{ ft}}{1\text{yd}} \times \frac{3 \text{ ft}}{1\text{yd}} \times \frac{12 \text{ in.}}{1 \text{ foot}} \times \frac{12 \text{ in.}}{1 \text{ foot}} = 6{,}480{,}000 \text{ in.}^2,$$
$$\text{Area} \approx 6 \times 10^6 \text{ in.}^2.$$

   This conversion gives us 6×10^6 in.^{2} for the area of the field. (Note that we are using only one significant figure in these calculations.)

4. Calculate the total volume of the bills. The volume of all the $100-bill stacks is
   $9 \text{ in.}^3 \text{ / stack} \times 10^8 \text{ stacks} = 9 \times 10^8 \text{ in.}^3$

5. Calculate the height. To determine the height of the bills, use the following equation

$$\text{volume of bills} \quad = \quad \text{area of field} \times \text{height of money}$$

$$\text{Height of money} \quad = \quad \frac{\text{volume of bills}}{\text{area of field}}$$

$$\text{Height of money} \quad = \quad \frac{9 \times 10^8 \text{ in.}^3}{6 \times 10^6 \text{ in.}^2} = 1.33 \times 10^2 \text{ in.}$$

$$\text{Height of money} \quad = \quad 1 \times 10^2 \text{ in.} = 100 \text{ in.}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100 \text{ in.} \times \frac{1 \text{ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft.}$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough *guesstimates* versus carefully calculated approximations?

In the example above, the final approximate value is much higher than the first friend's early estimate of 3 in. However, the other friend's early estimate of 10 ft. (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise suggest about the value of rough *guesstimates* versus carefully calculated approximations?

Graphing in Physics

Most results in science are presented in scientific journal articles using graphs. Graphs present data in a way that is easy to visualize for humans in general, especially someone unfamiliar with what is being studied. They are also useful for presenting large amounts of data or data with complicated trends in an easily-readable way.

One commonly-used graph in physics and other sciences is the line graph, probably because it is the best graph for showing how one quantity changes in response to the other. Let's build a line graph based on the data in Table 1.5, which shows the measured distance that a train travels from its station versus time. Our two **variables**, or things that change along the graph, are time in minutes, and distance from the station, in kilometers. Remember that measured data may not have perfect accuracy.

Time (min)	Distance from Station (km)
0	0
10	24
20	36
30	60
40	84
50	97
60	116
70	140

Table 1.5

1. Draw the two axes. The horizontal axis, or *x*-axis, shows the **independent variable**, which is the variable that is controlled or manipulated. The vertical axis, or *y*-axis, shows the **dependent variable**, the non-manipulated variable that changes with (or is dependent on) the value of the independent variable. In the data above, time is the independent variable and should be plotted on the *x*-axis. Distance from the station is the dependent variable and should be plotted on the *y*-axis.

2. Label each axes on the graph with the name of each variable, followed by the symbol for its units in parentheses. Be sure to leave room so that you can number each axis. In this example, use *Time (min)* as the label for the *x*-axis.
3. Next, you must determine the best scale to use for numbering each axis. Because the time values on the *x*-axis are taken every 10 minutes, we could easily number the *x*-axis from 0 to 70 minutes with a tick mark every 10 minutes. Likewise, the *y*-axis scale should start low enough and continue high enough to include all of the *distance from station* values. A scale from 0 km to 160 km should suffice, perhaps with a tick mark every 10 km.

    In general, you want to pick a scale for both axes that 1) shows all of your data, and 2) makes it easy to identify trends in your data. If you make your scale too large, it will be harder to see how your data change. Likewise, the smaller and more fine you make your scale, the more space you will need to make the graph. The number of significant figures in the axis values should be coarser than the number of significant figures in the measurements.

4. Now that your axes are ready, you can begin plotting your data. For the first data point, count along the *x*-axis until you find the 10 min tick mark. Then, count up from that point to the 10 km tick mark on the *y*-axis, and approximate where 22 km is along the *y*-axis. Place a dot at this location. Repeat for the other six data points (Figure 1.26).
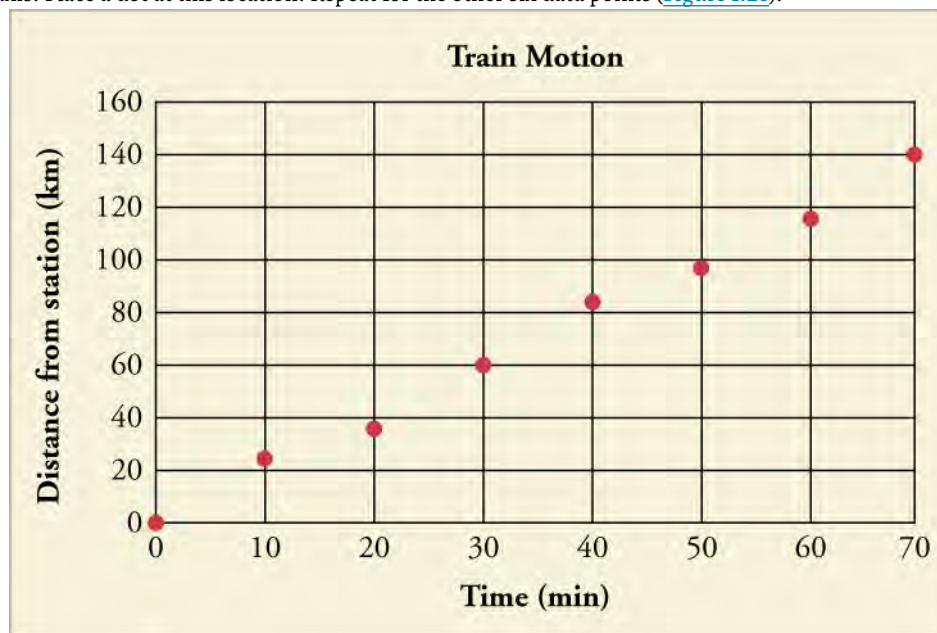
Figure 1.26 The graph of the train's distance from the station versus time from the exercise above.

5. Add a title to the top of the graph to state what the graph is describing, such as the *y*-axis parameter vs. the *x*-axis parameter. In the graph shown here, the title is *train motion*. It could also be titled distance of the train from the station vs. time.
6. Finally, with data points now on the graph, you should draw a trend line (Figure 1.27). The trend line represents the dependence you think the graph represents, so that the person who looks at your graph can see how close it is to the real data. In the present case, since the data points look like they ought to fall on a straight line, you would draw a straight line as the trend line. Draw it to come closest to all the points. Real data may have some inaccuracies, and the plotted points may not all fall on the trend line. In some cases, none of the data points fall exactly on the trend line.
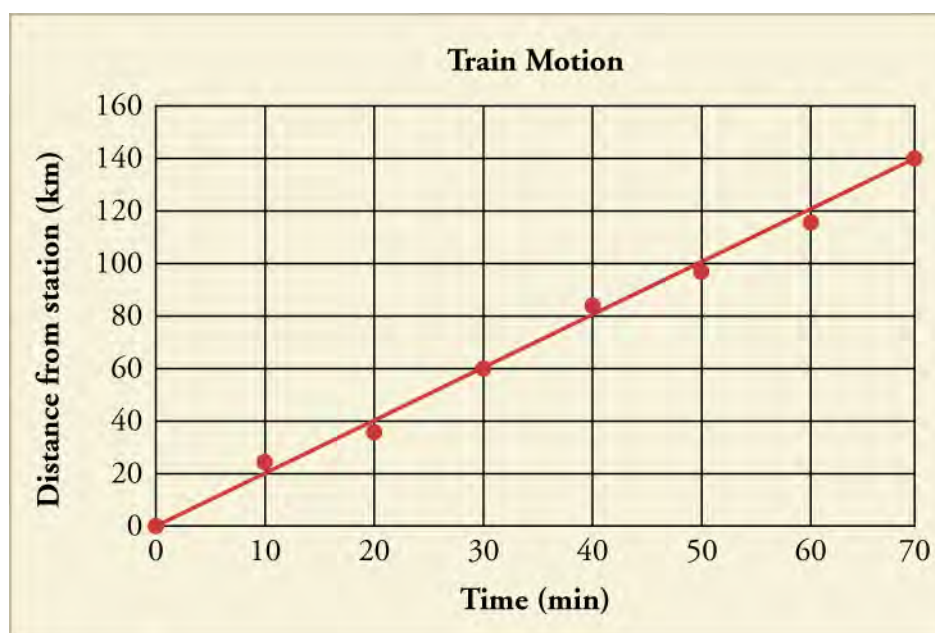
Figure 1.27 The completed graph with the trend line included.

Analyzing a Graph Using Its Equation

One way to get a quick snapshot of a dataset is to look at the equation of its trend line. If the graph produces a straight line, the equation of the trend line takes the form

$$y = mx + b.$$

The b in the equation is the y-intercept while the m in the equation is the **slope**. The y-intercept tells you at what y value the line intersects the y-axis. In the case of the graph above, the y-intercept occurs at 0, at the very beginning of the graph. The y-intercept, therefore, lets you know immediately where on the y-axis the plot line begins.

The m in the equation is the slope. This value describes how much the line on the graph moves up or down on the y-axis along the line's length. The slope is found using the following equation

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}.$$

In order to solve this equation, you need to pick two points on the line (preferably far apart on the line so the slope you calculate describes the line accurately). The quantities Y_2 and Y_1 represent the y-values from the two points on the line (not data points) that you picked, while X_2 and X_1 represent the two x-values of the those points.

What can the slope value tell you about the graph? The slope of a perfectly horizontal line will equal zero, while the slope of a perfectly vertical line will be undefined because you cannot divide by zero. A positive slope indicates that the line moves up the y-axis as the x-value increases while a negative slope means that the line moves down the y-axis. The more negative or positive the slope is, the steeper the line moves up or down, respectively. The slope of our graph in Figure 1.26 is calculated below based on the two endpoints of the line

$$
\begin{aligned}
m &= \frac{Y_2 - Y_1}{X_2 - X_1} \\
m &= \frac{(80\ \text{km}) - (20\ \text{km})}{(40\ \text{min}) - (10\ \text{min})} \\
m &= \frac{60\ \text{km}}{30\ \text{min}} \\
m &= 2.0\ \text{km/min.}
\end{aligned}
$$

Equation of line: $y = (2.0\ \text{km/min})\, x + 0$

Because the x axis is time in minutes, we would actually be more likely to use the time t as the independent (x-axis) variable and write the equation as

$$y = (2.0 \ \text{km/min}) \, t + 0.$$

<div style="float:right;border:1px solid;padding:2px">1.4</div>

The formula $y = mx + b$ only applies to **linear relationships**, or ones that produce a straight line. Another common type of line in physics is the **quadratic relationship**, which occurs when one of the variables is squared. One quadratic relationship in physics is the relation between the speed of an object its centripetal acceleration, which is used to determine the force needed to keep an object moving in a circle. Another common relationship in physics is the **inverse relationship**, in which one variable decreases whenever the other variable increases. An example in physics is Coulomb's law. As the distance between two charged objects increases, the electrical force between the two charged objects decreases. **Inverse proportionality**, such the relation between x and y in the equation

$$y = k/x,$$

for some number k, is one particular kind of inverse relationship. A third commonly-seen relationship is the **exponential relationship**, in which a change in the independent variable produces a proportional change in the dependent variable. As the value of the dependent variable gets larger, its rate of growth also increases. For example, bacteria often reproduce at an exponential rate when grown under ideal conditions. As each generation passes, there are more and more bacteria to reproduce. As a result, the growth rate of the bacterial population increases every generation (<u>Figure 1.28</u>).
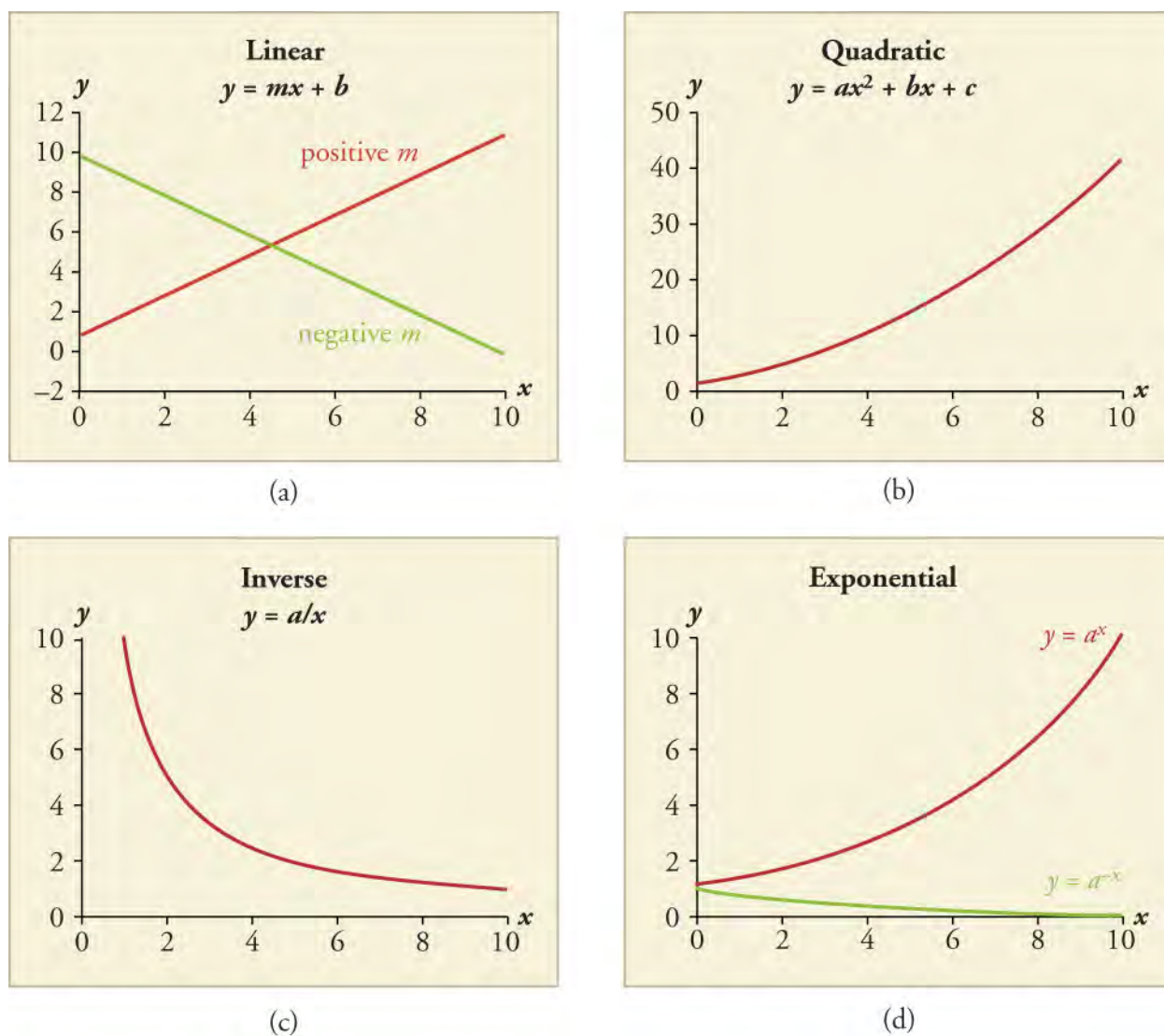
Figure 1.28 Examples of (a) linear, (b) quadratic, (c) inverse, and (d) exponential relationship graphs.

Using Logarithmic Scales in Graphing

Sometimes a variable can have a very large range of values. This presents a problem when you're trying to figure out the best scale to use for your graph's axes. One option is to use a **logarithmic (log) scale**. In a logarithmic scale, the value each mark labels

is the previous mark's value multiplied by some constant. For a log base 10 scale, each mark labels a value that is 10 times the value of the mark before it. Therefore, a base 10 logarithmic scale would be numbered: 0, 10, 100, 1,000, etc. You can see how the logarithmic scale covers a much larger range of values than the corresponding linear scale, in which the marks would label the values 0, 10, 20, 30, and so on.

If you use a logarithmic scale on one axis of the graph and a linear scale on the other axis, you are using a **semi-log plot**. The Richter scale, which measures the strength of earthquakes, uses a semi-log plot. The degree of ground movement is plotted on a logarithmic scale against the assigned intensity level of the earthquake, which ranges linearly from 1-10 (Figure 1.29 **(a)**).

If a graph has both axes in a logarithmic scale, then it is referred to as a **log-log plot**. The relationship between the wavelength and frequency of electromagnetic radiation such as light is usually shown as a log-log plot (Figure 1.29 **(b)**). Log-log plots are also commonly used to describe exponential functions, such as radioactive decay.

(a)   (b)
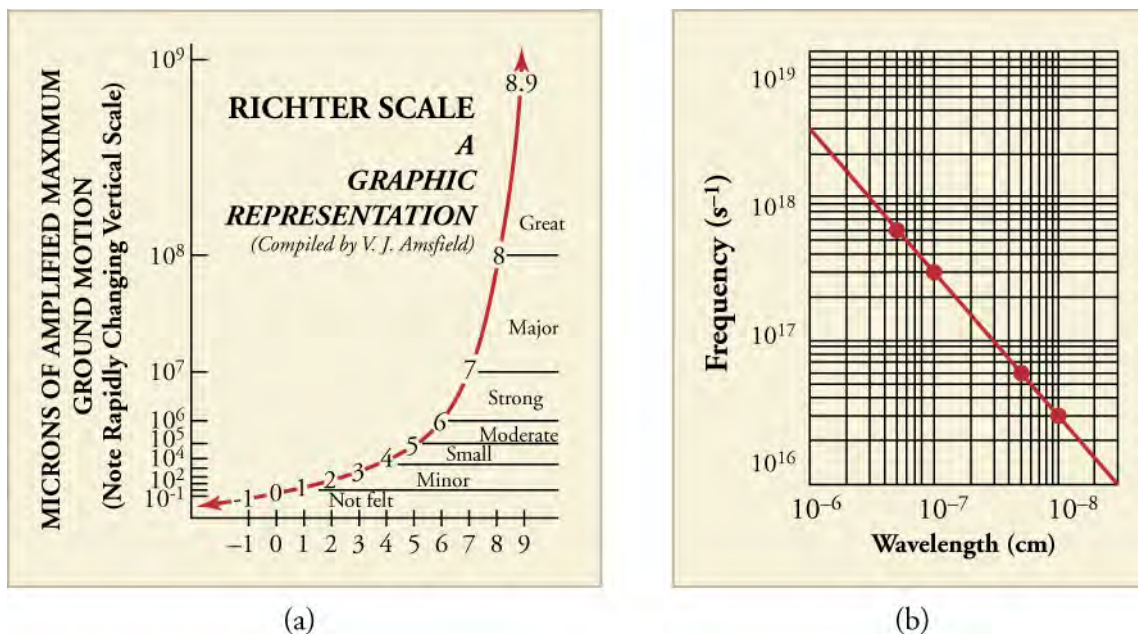
Figure 1.29 (a) The Richter scale uses a log base 10 scale on its *y-axis* (microns of amplified maximum ground motion). (b) The relationship between the frequency and wavelength of electromagnetic radiation can be plotted as a straight line if a log-log plot is used.

Virtual Physics

Graphing Lines

In this simulation you will examine how changing the slope and *y*-intercept of an equation changes the appearance of a plotted line. Select slope-intercept form and drag the blue circles along the line to change the line's characteristics. Then, play the line game and see if you can determine the slope or *y*-intercept of a given line.

Click to view content (https://phet.colorado.edu/sims/html/graphing-lines/latest/graphing-lines_en.html)

GRASP CHECK
How would the following changes affect a line that is neither horizontal nor vertical and has a positive slope?
1. increase the slope but keeping the *y*-intercept constant
2. increase the *y*-intercept but keeping the slope constant
   a. Increasing the slope will cause the line to rotate clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move vertically up on the graph without changing the line's slope.
   b. Increasing the slope will cause the line to rotate counter-clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move vertically up on the graph without changing the line's slope.
   c. Increasing the slope will cause the line to rotate clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move horizontally right on the graph without changing the line's slope.

> d.   Increasing the slope will cause the line to rotate counter-clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move horizontally right on the graph without changing the line's slope.

Check Your Understanding

12. Identify some advantages of metric units.
    a.   Conversion between units is easier in metric units.
    b.   Comparison of physical quantities is easy in metric units.
    c.   Metric units are more modern than English units.
    d.   Metric units are based on powers of 2.

13. The length of an American football field is 100 yd, excluding the end zones. How long is the field in meters? Round to the nearest 0.1 m.
    a. 10.2 m
    b. 91.4 m
    c. 109.4 m
    d. 328.1 m

14. The speed limit on some interstate highways is roughly 100 km/h. How many miles per hour is this if 1.0 mile is about 1.609 km?
    a. 0.1 mi/h
    b. 27.8 mi/h
    c. 62 mi/h
    d. 160 mi/h

15. Briefly describe the target patterns for accuracy and precision and explain the differences between the two.
    a.   Precision states how much repeated measurements generate the same or closely similar results, while accuracy states how close a measurement is to the true value of the measurement.
    b.   Precision states how close a measurement is to the true value of the measurement, while accuracy states how much repeated measurements generate the same or closely similar result.
    c.   Precision and accuracy are the same thing. They state how much repeated measurements generate the same or closely similar results.
    d.   Precision and accuracy are the same thing. They state how close a measurement is to the true value of the measurement.

KEY TERMS

accuracy   how close a measurement is to the correct value for that measurement

ampere   the SI unit for electrical current

atom   smallest and most basic units of matter

classical physics   physics, as it developed from the Renaissance to the end of the nineteenth century

constant   a quantity that does not change

conversion factor   a ratio expressing how many of one unit are equal to another unit

dependent variable   the vertical, or y-axis, variable, which changes with (or is dependent on) the value of the independent variable

derived units   units that are derived by combining the fundamental physical units

English units   (also known as the customary or imperial system) system of measurement used in the United States; includes units of measurement such as feet, gallons, degrees Fahrenheit, and pounds

experiment   process involved with testing a hypothesis

exponential relationship   relation between variables in which a constant change in the independent variable is accompanied by change in the dependent variable that is proportional to the value it already had

fundamental physical units   the seven fundamental physical units in the SI system of units are length, mass, time, electric current, temperature, amount of a substance, and luminous intensity

hypothesis   testable statement that describes how something in the natural world works

independent variable   the horizontal, or x-axis, variable, which is not influence by the second variable on the graph, the dependent variable

inverse proportionality   a relation between two variables expressible by an equation of the form $y = k/x$ where k stays constant when x and y change; the special form of inverse relationship that satisfies this equation

inverse relationship   any relation between variables where one variable decreases as the other variable increases

kilogram   the SI unit for mass, abbreviated (kg)

linear relationships   relation between variables that produce a straight line when graphed

log-log plot   a plot that uses a logarithmic scale in both axes

logarithmic scale   a graphing scale in which each tick on an axis is the previous tick multiplied by some value

meter   the SI unit for length, abbreviated (m)

method of adding percents   calculating the percent uncertainty of a quantity in multiplication or division by adding the percent uncertainties in the quantities being added or divided

model   system that is analogous to the real system of interest in essential ways but more easily analyzed

modern physics   physics as developed from the twentieth century to the present, involving the theories of relativity and quantum mechanics

observation   step where a scientist observes a pattern or trend within the natural world

order of magnitude   the size of a quantity in terms of its power of 10 when expressed in scientific notation

physics   science aimed at describing the fundamental aspects of our universe—energy, matter, space, motion, and time

precision   how well repeated measurements generate the same or closely similar results

principle   description of nature that is true in many, but not all situations

quadratic relationship   relation between variables that can be expressed in the form $y = ax^2 + bx + c$, which produces a curved line when graphed

quantum mechanics   major theory of modern physics which describes the properties and nature of atoms and their subatomic particles

science   the study or knowledge of how the physical world operates, based on objective evidence determined through observation and experimentation

scientific law   pattern in nature that is true in all circumstances studied thus far

scientific methods   techniques and processes used in the constructing and testing of scientific hypotheses, laws, and theories, and in deciding issues on the basis of experiment and observation

scientific notation   way of writing numbers that are too large or small to be conveniently written in simple decimal form; the measurement is multiplied by a power of 10, which indicates the number of placeholder zeros in the measurement

second   the SI unit for time, abbreviated (s)

semi-log plot   A plot that uses a logarithmic scale on one axis of the graph and a linear scale on the other axis.

SI units   International System of Units (SI); the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams; also known as the metric system

significant figures   when writing a number, the digits, or number of digits, that express the precision of a measuring tool used to measure the number

slope   the ratio of the change of a graph on the y axis to the change along the x-axis, the value of m in the equation of a line, $y = mx + b$

theory   explanation of patterns in nature that is supported by much scientific evidence and verified multiple times by various groups of researchers

theory of relativity   theory constructed by Albert Einstein which describes how space, time and energy are different

for different observers in relative motion

uncertainty   a quantitative measure of how much measured values deviate from a standard or expected value

universal   applies throughout the known universe

y-intercept   the point where a plot line intersects the y-axis

SECTION SUMMARY

1.1 Physics: Definitions and Applications

- Physics is the most fundamental of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Modern physics involves the theory of relativity, which describes how time, space and gravity are not constant in our universe can be different for different observers, and quantum mechanics, which describes the behavior of subatomic particles.
- Physics is the basis for all other sciences, such as chemistry, biology and geology, because physics describes the fundamental way in which the universe functions.

1.2 The Scientific Methods

- Science seeks to discover and describe the underlying order and simplicity in nature.
- The processes of science include observation, hypothesis, experiment, and conclusion.
- Theories are scientific explanations that are supported by a large body experimental results.
- Scientific laws are concise descriptions of the universe that are universally true.

1.3 The Language of Physics: Physical Quantities and Units

- Physical quantities are a characteristic or property of an

object that can be measured or calculated from other measurements.
- The four fundamental units we will use in this textbook are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.
- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

KEY EQUATIONS

1.3 The Language of Physics: Physical Quantities and Units

slope intercept form	$y = mx + b$
quadratic formula	$y = ax^2 + bx + c$
positive exponential formula	$y = a^x$
negative exponential formula	$y = a^{-x}$

CHAPTER REVIEW

Concept Items

1.1 Physics: Definitions and Applications

1. Which statement best compares and contrasts the aims and topics of natural philosophy had versus physics?

   a. Natural philosophy included all aspects of nature including physics.
   b. Natural philosophy included all aspects of nature excluding physics.
   c. Natural philosophy and physics are different.
   d. Natural philosophy and physics are essentially the

same thing.

2. Which of the following is <u>not</u> an underlying assumption essential to scientific understanding?
   a. Characteristics of the physical universe can be perceived and objectively measured by human beings.
   b. Explanations of natural phenomena can be established with absolute certainty.
   c. Fundamental physical processes dictate how characteristics of the physical universe evolve.
   d. The fundamental processes of nature operate the same way everywhere and at all times.

3. Which of the following questions regarding a strain of genetically modified rice is <u>not</u> one that can be answered by science?
   a. How does the yield of the genetically modified rice compare with that of existing rice?
   b. Is the genetically modified rice more resistant to infestation than existing rice?
   c. How does the nutritional value of the genetically modified rice compare to that of existing rice?
   d. Should the genetically modified rice be grown commercially and sold in the marketplace?

4. What conditions imply that we can use classical physics without considering special relativity or quantum mechanics?
   a. 1. matter is moving at speeds of less than roughly 1 percent the speed of light,
      2. objects are large enough to be seen with the naked eye, and
      3. there is the involvement of a strong gravitational field.
   b. 1. matter is moving at speeds greater than roughly 1 percent the speed of light,
      2. objects are large enough to be seen with the naked eye, and
      3. there is the involvement of a strong gravitational field.
   c. 1. matter is moving at speeds of less than roughly 1 percent the speed of light,
      2. objects are too small to be seen with the naked eye, and
      3. there is the involvement of only a weak gravitational field.
   d. 1. matter is moving at speeds of less than roughly 1 percent the speed of light,
      2. objects are large enough to be seen with the naked eye, and
      3. there is the involvement of a weak gravitational field.

5. How could physics be useful in weather prediction?
   a. Physics helps in predicting how burning fossil fuel releases pollutants.
   b. Physics helps in predicting dynamics and movement of weather phenomena.
   c. Physics helps in predicting the motion of tectonic plates.
   d. Physics helps in predicting how the flowing water affects Earth's surface.

6. How do physical therapists use physics while on the job? Explain.
   a. Physical therapists do not require knowledge of physics because their job is mainly therapy and not physics.
   b. Physical therapists do not require knowledge of physics because their job is more social in nature and unscientific.
   c. Physical therapists require knowledge of physics know about muscle contraction and release of energy.
   d. Physical therapists require knowledge of physics to know about chemical reactions inside the body and make decisions accordingly.

7. What is meant when a physical law is said to be universal?
   a. The law can explain everything in the universe.
   b. The law is applicable to all physical phenomena.
   c. The law applies everywhere in the universe.
   d. The law is the most basic one and all laws are derived from it.

8. What subfield of physics could describe small objects traveling at high speeds or experiencing a strong gravitational field?
   a. general theory of relativity
   b. classical physics
   c. quantum relativity
   d. special theory of relativity

9. Why is Einstein's theory of relativity considered part of modern physics, as opposed to classical physics?
   a. Because it was considered less outstanding than the classics of physics, such as classical mechanics.
   b. Because it was popular physics enjoyed by average people today, instead of physics studied by the elite.
   c. Because the theory deals with very slow-moving objects and weak gravitational fields.
   d. Because it was among the new 19th-century discoveries that changed physics.

1.2 The Scientific Methods

10. Describe the difference between an observation and a hypothesis.

a. An observation is seeing what happens; a hypothesis is a testable, educated guess.
b. An observation is a hypothesis that has been confirmed.
c. Hypotheses and observations are independent of each other.
d. Hypotheses are conclusions based on some observations.

11. Describe how modeling is useful in studying the structure of the atom.
   a. Modeling replaces the real system by something similar but easier to examine.
   b. Modeling replaces the real system by something more interesting to examine.
   c. Modeling replaces the real system by something with more realistic properties.
   d. Modeling includes more details than are present in the real system.

12. How strongly is a hypothesis supported by evidence compared to a theory?
   a. A theory is supported by little evidence, if any, at first, while a hypothesis is supported by a large amount of available evidence.
   b. A hypothesis is supported by little evidence, if any, at first. A theory is supported by a large amount of available evidence.
   c. A hypothesis is supported by little evidence, if any, at first. A theory does not need any experiments in support.
   d. A theory is supported by little evidence, if any, at first. A hypothesis does not need any experiments in support.

1.3 The Language of Physics: Physical Quantities and Units

13. Which of the following does not contribute to the uncertainty?
   a. the limitations of the measuring device
   b. the skill of the person making the measurement
   c. the regularities in the object being measured
   d. other factors that affect the outcome (depending on the situation)

14. How does the independent variable in a graph differ from the dependent variable?
   a. The dependent variable varies linearly with the independent variable.
   b. The dependent variable depends on the scale of the axis chosen while independent variable does not.
   c. The independent variable is directly manipulated or controlled by the person doing the experiment, while dependent variable is the one that changes as

a result.
   d. The dependent and independent variables are fixed by a convention and hence they are the same.

15. What could you conclude about these two lines?
   1. Line A has a slope of -4.7
   2. Line B has a slope of 12.0

   a. Line A is a decreasing line while line B is an increasing line, with line A being much steeper than line B.
   b. Line A is a decreasing line while line B is an increasing line, with line B being much steeper than line A.
   c. Line B is a decreasing line while line A is an increasing line, with line A being much steeper than line B.
   d. Line B is a decreasing line while line A is an increasing line, with line B being much steeper than line A.

16. Velocity, or speed, is measured using the following formula: $v = \frac{d}{t}$, where v is velocity, d is the distance travelled, and t is the time the object took to travel the distance. If the velocity-time data are plotted on a graph, which variable will be on which axis? Why?
   a. Time would be on the x-axis and velocity on the y-axis, because time is an independent variable and velocity is a dependent variable.
   b. Velocity would be on the x-axis and time on the y-axis, because time is the independent variable and velocity is the dependent variable.
   c. Time would be on the x-axis and velocity on the y-axis, because time is a dependent variable and velocity is a independent variable.
   d. Velocity would be on x-axis and time on the y-axis, because time is a dependent variable and velocity is a independent variable.

17. The uncertainty of a triple-beam balance is 0.05 g. What is the percent uncertainty in a measurement of 0.445 kg?
   a. 0.011%
   b. 0.11%
   c. 1.1%
   d. 11%

18. What is the definition of uncertainty?
   a. Uncertainty is the number of assumptions made prior to the measurement of a physical quantity.
   b. Uncertainty is a measure of error in a measurement due to the use of a non-calibrated instrument.
   c. Uncertainty is a measure of deviation of the measured value from the standard value.
   d. Uncertainty is a measure of error in measurement

due to external factors like air friction and temperature.

Critical Thinking Items

1.1 Physics: Definitions and Applications

19. In what sense does Einstein's theory of relativity illustrate that physics describes fundamental aspects of our universe?
   a. It describes how speed affects different observers' measurements of time and space.
   b. It describes how different parts of the universe are far apart and do not affect each other.
   c. It describes how people think of other people's views from their own frame of reference.
   d. It describes how a frame of reference is necessary to describe position or motion.

20. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.
   a. No, because the satellite is moving at a speed much smaller than the speed of the light and is not in a strong gravitational field.
   b. No, because the satellite is moving at a speed much smaller than the speed of the light and is in a strong gravitational field.
   c. Yes, because the satellite is moving at a speed much smaller than the speed of the light and it is not in a strong gravitational field.
   d. Yes, because the satellite is moving at a speed much smaller than the speed of the light and is in a strong gravitational field.

21. What would be some ways in which physics was involved in building the features of the room you are in right now?
   a. Physics is involved in structural strength, dimensions, etc., of the room.
   b. Physics is involved in the air composition inside the room.
   c. Physics is involved in the desk arrangement inside the room.
   d. Physics is involved in the behavior of living beings inside the room.

22. What theory of modern physics describes the interrelationships between space, time, speed, and gravity?
   a. atomic theory
   b. nuclear physics
   c. quantum mechanics
   d. general relativity

23. According to Einstein's theory of relativity, how could you effectively travel many years into Earth's future, but not age very much yourself?
   a. by traveling at a speed equal to the speed of light
   b. by traveling at a speed faster than the speed of light
   c. by traveling at a speed much slower than the speed of light
   d. by traveling at a speed slightly slower than the speed of light

1.2 The Scientific Methods

24. You notice that the water level flowing in a stream near your house increases when it rains and the water turns brown. Which of these are the best hypothesis to explain why the water turns brown. Assume you have all of the means to test the contents of the stream water.
   a. The water in the stream turns brown because molecular forces between water molecules are stronger than mud molecules
   b. The water in the stream turns brown because of the breakage of a weak chemical bond with the hydrogen atom in the water molecule.
   c. The water in the stream turns brown because it picks up dirt from the bank as the water level increases when it rains.
   d. The water in the stream turns brown because the density of the water increases with increase in water level.

25. Light travels as waves at an approximate speed of 300,000,000 m/s (186,000 mi/s). Designers of devices that use mirrors and lenses model the traveling light by straight lines, or light rays. Describe why it would be useful to model the light as rays of light instead of describing them accurately as electromagnetic waves.
   a. A model can be constructed in such a way that the speed of light decreases.
   b. Studying a model makes it easier to analyze the path that the light follows.
   c. Studying a model will help us to visualize why light travels at such great speed.
   d. Modeling cannot be used to study traveling light as our eyes cannot track the motion of light.

26. A friend says that he doesn't trust scientific explanations because they are just theories, which are basically educated guesses. What could you say to convince him that scientific theories are different from the everyday use of the word theory?
   a. A theory is a scientific explanation that has been repeatedly tested and supported by many experiments.
   b. A theory is a hypothesis that has been tested and

supported by some experiments.

    c.   A theory is a set of educated guesses, but at least one of the guesses remain true in each experiment.

    d.   A theory is a set of scientific explanations that has at least one experiment in support of it.

27. Give an example of a hypothesis that cannot be tested experimentally.

    a.   The structure of any part of the broccoli is similar to the whole structure of the broccoli.

    b.   Ghosts are the souls of people who have died.

    c.   The average speed of air molecules increases with temperature.

    d.   A vegetarian is less likely to be affected by night blindness.

28. Would it be possible to scientifically prove that a supreme being exists or not? Briefly explain your answer.

    a.   It can be proved scientifically because it is a testable hypothesis.

    b.   It cannot be proved scientifically because it is not a testable hypothesis.

    c.   It can be proved scientifically because it is not a testable hypothesis.

    d.   It cannot be proved scientifically because it is a testable hypothesis.

1.3 The Language of Physics: Physical Quantities and Units

29. A marathon runner completes a 42.188 km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time.

    1.   Calculate the percent uncertainty in the distance.

    2.   Calculate the uncertainty in the elapsed time.

    3.   What is the average speed in meters per second?

    4.   What is the uncertainty in the average speed?

    a. $0.059\,\%, 0.01\,\%, 0.468\text{m/s}, 0.0003\text{m/s}$

    b. $0.059\,\%, 0.01\,\%, 0.468 \text{ m/s}, 0.07 \text{ m/s}$

    c. $0.59\,\%, 8.33\,\%, 4.681 \text{ m/s}, 0.003 \text{ m/s}$

    d. $0.059\,\%, 0.01\,\%, 4.681 \text{ m/s}, 0.003 \text{ m/s}$

30. A car engine moves a piston with a circular cross section of $7.500 \pm 0.002 \text{ cm}$ diameter a distance of $3.250 \pm 0.001 \text{ cm}$ to compress the gas in the cylinder. By what amount did the gas decrease in volume in cubic centimeters? Find the uncertainty in this volume.

    a. $143.6 \pm 0.002 \text{ cm}^3$

    b. $143.6 \pm 0.003 \text{ cm}^3$

    c. $143.6 \pm 0.005 \text{ cm}^3$

    d. $143.6 \pm 0.1 \text{ cm}^3$

31. What would be the slope for a line passing through the two points below?

Point 1: (1, 0.1) Point 2: (7, 26.8)

    a.   2.4

    b.   4.5

    c.   6.2

    d.   6.8

32. The sides of a small rectangular box are measured 1.80 cm and 2.05 cm long and 3.1 cm high. Calculate its volume and uncertainty in cubic centimeters. Assume the measuring device is accurate to $\pm 0.05 \text{ cm}$.

    a. $11.4 \pm 0.1 \text{ cm}^3$

    b. $11.4 \pm 0.6 \text{ cm}^3$

    c. $11.4 \pm 0.8 \text{ cm}^3$

    d. $11.4 \pm 0.10 \text{ cm}^3$

33. Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint—The mass of a hydrogen atom is on the order of 10^{-27} kg and the mass of a bacterium is on the order of 10^{-15} kg .)

    a. 10^{10} atoms

    b. 10^{11} atoms

    c. 10^{12} atoms

    d. 10^{13} atoms

Problems

1.3 The Language of Physics: Physical Quantities and Units

34. A commemorative coin that sells for $40 is advertised to be plated with 15 mg of gold. Suppose gold is worth about $1,300 per ounce. Which of the following best represents the value of the gold in the coin?

    a. $0.33

    b. $0.69

    c. $3.30

    d. $6.90

35. If a marathon runner runs 9.5 miles in one direction, 8.89 miles in another direction and 2.333 miles in a third direction, how much distance did the runner run? Be sure to report your answer using the proper number of significant figures.

    a.   20

    b.   20.7

    c.   20.72

d. 20.732

36. The speed limit on some interstate highways is roughly 80 km/h. What is this in meters per second? How many miles per hour is this?
    a. 62 m/s, 27.8 mi/h
    b. 22.2 m/s, 49.7 mi/h
    c. 62 m/s, 2.78 mi/h
    d. 2.78 m/s, 62 mi/h

Performance Task

1.3 The Language of Physics: Physical Quantities and Units

38. a. Create a new system of units to describe something that interests you. Your unit should be described using at least two subunits. For example, you can decide to measure the quality of songs using a new unit called *song awesomeness*. Song awesomeness

TEST PREP
Multiple Choice

1.1 Physics: Definitions and Applications

39. Modern physics could best be described as the combination of which theories?
    a. quantum mechanics and Einstein's theory of relativity
    b. quantum mechanics and classical physics
    c. Newton's laws of motion and classical physics
    d. Newton's laws of motion and Einstein's theory of relativity

40. Which of the following could be studied accurately using classical physics?
    a. the strength of gravity within a black hole
    b. the motion of a plane through the sky
    c. the collisions of subatomic particles
    d. the effect of gravity on the passage of time

41. Which of the following best describes why knowledge of physics is necessary to understand all other sciences?
    a. Physics explains how energy passes from one object to another.
    b. Physics explains how gravity works.
    c. Physics explains the motion of objects that can be seen with the naked eye.
    d. Physics explains the fundamental aspects of the universe.

42. What does radiation therapy, used to treat cancer patients, have to do with physics?
    a. Understanding how cells reproduce is mainly about

37. The length and width of a rectangular room are measured to be 3.955 ± 0.005 m by 3.050 ± 0.005 m. Calculate the area of the room and its uncertainty in square meters.
    a. $12.06 \pm 0.29 \, \text{m}^2$
    b. $12.06 \pm 0.01 \, \text{m}^2$
    c. $12.06 \pm 0.25 \, \text{m}^2$
    d. $12.06 \pm 0.04 \, \text{m}^2$

is measured by: the number of songs downloaded and the number of times the song was used in movies.
    b. Create an equation that shows how to calculate your unit. Then, using your equation, create a sample dataset that you could graph. Are your two subunits related linearly, quadratically, or inversely?

physics.
    b. Predictions of the side effects from the radiation therapy are based on physics.
    c. The devices used for generating some kinds of radiation are based on principles of physics.
    d. Predictions of the life expectancy of patients receiving radiation therapy are based on physics.

1.2 The Scientific Methods

43. The free-electron model of metals explains some of the important behaviors of metals by assuming the metal's electrons move freely through the metal without repelling one another. In what sense is the free-electron theory based on a model?
    a. Its use requires constructing replicas of the metal wire in the lab.
    b. It involves analyzing an imaginary system simpler than the real wire it resembles.
    c. It examines a model, or ideal, behavior that other metals should imitate.
    d. It attempts to examine the metal in a very realistic, or model, way.

44. A scientist wishes to study the motion of about 1,000 molecules of gas in a container by modeling them as tiny billiard balls bouncing randomly off one another. Which of the following is needed to calculate and store data on their detailed motion?
    a. a group of hypotheses that cannot be practically tested in real life

b.   a computer that can store and perform calculations on large data sets

c.   a large amount of experimental results on the molecules and their motion

d.   a collection of hypotheses that have not yet been tested regarding the molecules

45. When a large body of experimental evidence supports a hypothesis, what may the hypothesis eventually be considered?

a.   observation

b.   insight

c.   conclusion

d.   law

46. While watching some ants outside of your house, you notice that the worker ants gather in a specific area on your lawn. Which of the following is a testable hypothesis that attempts to explain why the ants gather in that specific area on the lawn.

a.   The worker thought it was a nice location.

b.   because ants may have to find a spot for the queen to lay eggs

c.   because there may be some food particles lying there

d.   because the worker ants are supposed to group together at a place.

1.3 The Language of Physics: Physical Quantities and Units

47. Which of the following would describe a length that is 2.0×10^{-3} of a meter?

a.   2.0 kilometers

b.   2.0 megameters

c.   2.0 millimeters

d.   2.0 micrometers

48. Suppose that a bathroom scale reads a person's mass as 65 kg with a 3 percent uncertainty. What is the uncertainty in their mass in kilograms?

a.   a. 2 kg

b.   b. 98 kg

c.   c. 5 kg

d.   d. 0

49. Which of the following best describes a variable?

a.   a trend that shows an exponential relationship

b.   something whose value can change over multiple measurements

c.   a measure of how much a plot line changes along the y-axis

d.   something that remains constant over multiple measurements

50. A high school track coach has just purchased a new stopwatch that has an uncertainty of ±0.05 s . Runners on the team regularly clock 100-m sprints in 12.49 s to 15.01 s . At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s . Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

a.   No, the uncertainty in the stopwatch is too large to effectively differentiate between the sprint times.

b.   No, the uncertainty in the stopwatch is too small to effectively differentiate between the sprint times.

c.   Yes, the uncertainty in the stopwatch is too large to effectively differentiate between the sprint times.

d.   Yes, the uncertainty in the stopwatch is too small to effectively differentiate between the sprint times.

Short Answer

1.1 Physics: Definitions and Applications

51. Describe the aims of physics.

a.   Physics aims to explain the fundamental aspects of our universe and how these aspects interact with one another.

b.   Physics aims to explain the biological aspects of our universe and how these aspects interact with one another.

c.   Physics aims to explain the composition, structure and changes in matter occurring in the universe.

d.   Physics aims to explain the social behavior of living beings in the universe.

52. Define the fields of magnetism and electricity and state how are they are related.

a.   Magnetism describes the attractive force between a magnetized object and a metal like iron. Electricity involves the study of electric charges and their movements. Magnetism is not related to the electricity.

b.   Magnetism describes the attractive force between a magnetized object and a metal like iron. Electricity involves the study of electric charges and their movements. Magnetism is produced by a flow electrical charges.

c.   Magnetism involves the study of electric charges and their movements. Electricity describes the attractive force between a magnetized object and a metal. Magnetism is not related to the electricity.

d.   Magnetism involves the study of electric charges and their movements. Electricity describes the attractive force between a magnetized object and a metal. Magnetism is produced by the flow electrical charges.

53. Describe what two topics physicists are trying to unify with relativistic quantum mechanics. How will this unification create a greater understanding of our universe?
   a. Relativistic quantum mechanics unifies quantum mechanics with Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it can explain objects of all sizes and masses.
   b. Relativistic quantum mechanics unifies classical mechanics with Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it can explain objects of all sizes and masses.
   c. Relativistic quantum mechanics unifies quantum mechanics with Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it is unable to explain objects of all sizes and masses.
   d. Relativistic quantum mechanics unifies classical mechanics with the Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it is unable to explain objects of all sizes and masses.

54. The findings of studies in quantum mechanics have been described as strange or weird compared to those of classical physics. Explain why this would be so.
   a. It is because the phenomena it explains are outside the normal range of human experience which deals with much larger objects.
   b. It is because the phenomena it explains can be perceived easily, namely, ordinary-sized objects.
   c. It is because the phenomena it explains are outside the normal range of human experience, namely, the very large and the very fast objects.
   d. It is because the phenomena it explains can be perceived easily, namely, the very large and the very fast objects.

55. How could knowledge of physics help you find a faster way to drive from your house to your school?
   a. Physics can explain the traffic on a particular street and help us know about the traffic in advance.
   b. Physics can explain about the ongoing construction of roads on a particular street and help us know about delays in the traffic in advance.
   c. Physics can explain distances, speed limits on a particular street and help us categorize faster routes.
   d. Physics can explain the closing of a particular street and help us categorize faster routes.

56. How could knowledge of physics help you build a sound and energy-efficient house?
   a. An understanding of force, pressure, heat, electricity, etc., which all involve physics, will help me design a sound and energy-efficient house.
   b. An understanding of the air composition, chemical composition of matter, etc., which all involves physics, will help me design a sound and energy-efficient house.
   c. An understanding of material cost and economic factors involving physics will help me design a sound and energy-efficient house.
   d. An understanding of geographical location and social environment which involves physics will help me design a sound and energy-efficient house.

57. What aspects of physics would a chemist likely study in trying to discover a new chemical reaction?
   a. Physics is involved in understanding whether the reactants and products dissolve in water.
   b. Physics is involved in understanding the amount of energy released or required in a chemical reaction.
   c. Physics is involved in what the products of the reaction will be.
   d. Physics is involved in understanding the types of ions produced in a chemical reaction.

1.2 The Scientific Methods

58. You notice that it takes more force to get a large box to start sliding across the floor than it takes to get the box sliding faster once it is already moving. Create a testable hypothesis that attempts to explain this observation.
   a. The floor has greater distortions of space-time for moving the sliding box faster than for the box at rest.
   b. The floor has greater distortions of space-time for the box at rest than for the sliding box.
   c. The resistance between the floor and the box is less when the box is sliding then when the box is at rest.
   d. The floor dislikes having objects move across it and therefore holds the box rigidly in place until it cannot resist the force.

59. Design an experiment that will test the following hypothesis: driving on a gravel road causes greater damage to a car than driving on a dirt road.
   a. To test the hypothesis, compare the damage to the car by driving it on a smooth road and a gravel road.
   b. To test the hypothesis, compare the damage to the car by driving it on a smooth road and a dirt road.
   c. To test the hypothesis, compare the damage to the car by driving it on a gravel road and the dirt road.
   d. This is not a testable hypothesis.

60. How is a physical model, such as a spherical mass held

in place by springs, used to represent an atom vibrating in a solid, similar to a computer-based model, such as that predicting how gravity affects the orbits of the planets?

  a. Both a physical model and a computer-based model should be built around a hypothesis and could be able to test the hypothesis.

  b. Both a physical model and a computer-based model should be built around a hypothesis but they cannot be used to test the hypothesis.

  c. Both a physical model and a computer-based model should be built around the results of scientific studies and could be used to make predictions about the system under study.

  d. Both a physical model and a computer-based model should be built around the results of scientific studies but cannot be used to make predictions about the system under study.

61. Explain the advantages and disadvantages of using a model to predict a life-or-death situation, such as whether or not an asteroid will strike Earth.

  a. The advantage of using a model is that it provides predictions quickly, but the disadvantage of using a model is that it could make erroneous predictions.

  b. The advantage of using a model is that it provides accurate predictions, but the disadvantage of using a model is that it takes a long time to make predictions.

  c. The advantage of using a model is that it provides predictions quickly without any error. There are no disadvantages of using a scientific model.

  d. The disadvantage of using models is that it takes longer time to make predictions and the predictions are inaccurate. There are no advantages to using a scientific model.

62. A friend tells you that a scientific law cannot be changed. State whether or not your friend is correct and then briefly explain your answer.

  a. Correct, because laws are theories that have been proved true.

  b. Correct, because theories are laws that have been proved true.

  c. Incorrect, because a law is changed if new evidence contradicts it.

  d. Incorrect, because a law is changed when a theory contradicts it.

63. How does a scientific law compare to a local law, such as that governing parking at your school, in terms of whether or not laws can be changed, and how universal a law is?

  a. A local law applies only in a specific area, but a

scientific law is applicable throughout the universe. Both the local law and the scientific law can change.

  b. A local law applies only in a specific area, but a scientific law is applicable throughout the universe. A local law can change, but a scientific law cannot be changed.

  c. A local law applies throughout the universe but a scientific law is applicable only in a specific area. Both the local and the scientific law can change.

  d. A local law applies throughout the universe, but a scientific law is applicable only in a specific area. A local law can change, but a scientific law cannot be changed.

64. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

  a. Models, theories and laws must be universally valid.

  b. Models, theories, and laws have only limited validity.

  c. Models have limited validity while theories and laws are universally valid.

  d. Models and theories have limited validity while laws are universally valid.

1.3 The Language of Physics: Physical Quantities and Units

65. The speed of sound is measured at 342 m/s on a certain day. What is this in km/h? Report your answer in scientific notation.

  a. 1.23×10^4 km/h

  b. 1.23×10^3 km/h

  c. 9.5×10^1 km/h

  d. 2.05×10^{-1} km/h

66. Describe the main difference between the metric system and the U.S. Customary System.

  a. In the metric system, unit changes are based on powers of 10, while in the U.S. customary system, each unit conversion has unrelated conversion factors.

  b. In the metric system, each unit conversion has unrelated conversion factors, while in the U.S. customary system, unit changes are based on powers of 10.

  c. In the metric system, unit changes are based on powers of 2, while in the U.S. customary system, each unit conversion has unrelated conversion factors.

  d. In the metric system, each unit conversion has unrelated conversion factors, while in the U.S. customary system, unit changes are based on

powers of 2.

67. An infant's pulse rate is measured to be
130 ± 5 beats/min. What is the percent uncertainty in this measurement?
   a. 2%
   b. 3%
   c. 4%
   d. 5%

68. Explain how the uncertainty of a measurement relates to the accuracy and precision of the measuring device. Include the definitions of accuracy and precision in your answer.
   a. A decrease in the precision of a measurement increases the uncertainty of the measurement, while a decrease in accuracy does not.
   b. A decrease in either the precision or accuracy of a measurement increases the uncertainty of the measurement.
   c. An increase in either the precision or accuracy of a measurement will increase the uncertainty of that measurement.
   d. An increase in the accuracy of a measurement will increase the uncertainty of that measurement, while an increase in precision will not.

69. Describe all of the characteristics that can be determined about a straight line with a slope of -3 and a y-intercept of 50 on a graph.
   a. Based on the information, the line has a negative slope. Because its y-intercept is 50 and its slope is negative, this line gradually rises on the graph as the x-value increases.
   b. Based on the information, the line has a negative slope. Because its y-intercept is 50 and its slope is negative, this line gradually moves downward on

the graph as the x-value increases.
   c. Based on the information, the line has a positive slope. Because its y-intercept is 50 and its slope is positive, this line gradually rises on the graph as the x-value increases.
   d. Based on the information, the line has a positive slope. Because its y-intercept is 50 and its slope is positive, this line gradually moves downward on the graph as the x-value increases.

70. The graph shows the temperature change over time of a heated cup of water.
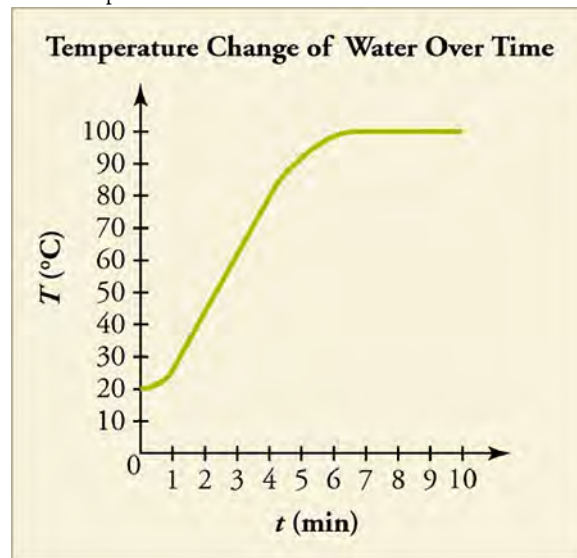
What is the slope of the graph between the time period 2 min and 5 min?
   a. $-15\ °C/min$
   b. $-0.07\ °C/min$
   c. $0.07\ °C/min$
   d. $15\ °C/min$

Extended Response
1.2 The Scientific Methods

71. You wish to perform an experiment on the stopping distance of your new car. Create a specific experiment to measure the distance. Be sure to specifically state how you will set up and take data during your experiment.
   a. Drive the car at exactly 50 mph and then press harder on the accelerator pedal until the velocity reaches the speed 60 mph and record the distance this takes.
   b. Drive the car at exactly 50 mph and then apply the brakes until it stops and record the distance this takes.
   c. Drive the car at exactly 50 mph and then apply the brakes until it stops and record the time it takes.

   d. Drive the car at exactly 50 mph and then apply the accelerator until it reaches the speed of 60 mph and record the time it takes.

72. You wish to make a model showing how traffic flows around your city or local area. Describe the steps you would take to construct your model as well as some hypotheses that your model could test and the model's limitations in terms of what could not be tested.
   a. 1. Testable hypotheses like the gravitational pull on each vehicle while in motion and the average speed of vehicles is 40 mph
      2. Non-testable hypotheses like the average number of vehicles passing is 935 per day and carbon emission from each of the moving vehicle

b. 1. Testable hypotheses like the average number of vehicles passing is 935 per day and the average speed of vehicles is 40 mph
    2. Non-testable hypotheses like the gravitational pull on each vehicle while in motion and the carbon emission from each of the moving vehicle

c. 1. Testable hypotheses like the average number of vehicles passing is 935 per day and the carbon emission from each of the moving vehicle
    2. Non-testable hypotheses like the gravitational pull on each vehicle while in motion and the average speed of the vehicles is 40 mph

d. 1. Testable hypotheses like the average number of vehicles passing is 935 per day and the gravitational pull on each vehicle while in motion
    2. Non-testable hypotheses like the average speed of vehicles is 40 mph and the carbon emission from each of the moving vehicle

73. What would play the most important role in leading to an experiment in the scientific world becoming a scientific law?
   a. Further testing would need to show it is a universally followed rule.
   b. The observation would have to be described in a published scientific article.
   c. The experiment would have to be repeated once or twice.
   d. The observer would need to be a well-known scientist whose authority was accepted.

1.3 The Language of Physics: Physical Quantities and Units

74. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of $4.0 \, \text{cm/year}$. What distance does it move in $1.0 \, \text{s}$ at this speed? What is its speed in kilometers per million years? Report all of your answers using scientific notation.
   a. 1.3×10^{-9} m; 4.0×10^{1} km/million years
   b. 1.3×10^{-6} m; 4.0×10^{1} km/million years
   c. 1.3×10^{-9} m; 4.0×10^{-11} km/million years
   d. 1.3×10^{-6} m; 4.0×10^{-11} km/million years

75. At x = 3, a function f(x) has a positive value, with a positive slope that is decreasing in magnitude with increasing x. Which option could correspond to f(x)?
   a. $y = 13x$
   b. $y = x^{2}$
   c. $y = 2x + 9$
   d. $y = \frac{x}{2} + 9$

CHAPTER 2
Motion in One Dimension

Figure 2.1 Shanghai Maglev. At this rate, a train traveling from Boston to Washington, DC, a distance of 439 miles, could make the trip in under an hour and a half. Presently, the fastest train on this route takes over six hours to cover this distance. (Alex Needham, Public Domain)

Chapter Outline

INTRODUCTION   Unless you have flown in an airplane, you have probably never traveled faster than 150 mph. Can you imagine traveling in a train like the one shown in Figure 2.1 that goes over 300 mph? Despite the high speed, the people riding in this train may not notice that they are moving at all unless they look out the window! This is because motion, even motion at 300 mph, is relative to the observer.

In this chapter, you will learn why it is important to identify a reference frame in order to clearly describe motion. For now, the motion you describe will be one-dimensional. Within this context, you will learn the difference between distance and displacement as well as the difference between speed and velocity. Then you will look at some graphing and problem-solving techniques.

2.1 Relative Motion, Distance, and Displacement

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe motion in different reference frames
- Define distance and displacement, and distinguish between the two
- Solve problems involving distance and displacement

Section Key Terms

displacement	distance	kinematics	magnitude
position	reference frame	scalar	vector

Defining Motion

Our study of physics opens with **kinematics**—the study of motion without considering its causes. Objects are in motion everywhere you look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects, atoms are always moving.

How do you know something is moving? The location of an object at any particular time is its **position**. More precisely, you need to specify its position relative to a convenient **reference frame**. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to Earth. To describe the position of a person in an airplane, for example, we use the airplane, not Earth, as the reference frame. (See Figure 2.2.) Thus, you can only know how fast and in what direction an object's position is changing against a background of something else that is either not moving or moving with a known speed and direction. The reference frame is the coordinate system from which the positions of objects are described.

Figure 2.2 Are clouds a useful reference frame for airplane passengers? Why or why not? (Paul Brennan, Public Domain)

Your classroom can be used as a reference frame. In the classroom, the walls are not moving. Your motion as you walk to the door, can be measured against the stationary background of the classroom walls. You can also tell if other things in the classroom are moving, such as your classmates entering the classroom or a book falling off a desk. You can also tell in what direction something is moving in the classroom. You might say, "The teacher is moving toward the door." Your reference frame allows you to determine not only that something is moving but also the direction of motion.

You could also serve as a reference frame for others' movement. If you remained seated as your classmates left the room, you would measure their movement away from your stationary location. If you and your classmates left the room together, then your perspective of their motion would be change. You, as the reference frame, would be moving in the same direction as your other moving classmates. As you will learn in the **Snap Lab**, your description of motion can be quite different when viewed from different reference frames.

Snap Lab

Looking at Motion from Two Reference Frames

In this activity you will look at motion from two reference frames. Which reference frame is correct?

- Choose an open location with lots of space to spread out so there is less chance of tripping or falling due to a collision and/or loose basketballs.

- 1 basketball

Procedure

1. Work with a partner. Stand a couple of meters away from your partner. Have your partner turn to the side so that you are looking at your partner's profile. Have your partner begin bouncing the basketball while standing in place. Describe the motion of the ball.
2. Next, have your partner again bounce the ball, but this time your partner should walk forward with the bouncing ball. You will remain stationary. Describe the ball's motion.
3. Again have your partner walk forward with the bouncing ball. This time, you should move alongside your partner while continuing to view your partner's profile. Describe the ball's motion.
4. Switch places with your partner, and repeat Steps 1–3.

GRASP CHECK

How do the different reference frames affect how you describe the motion of the ball?
a. The motion of the ball is independent of the reference frame and is same for different reference frames.
b. The motion of the ball is independent of the reference frame and is different for different reference frames.
c. The motion of the ball is dependent on the reference frame and is same for different reference frames.
d. The motion of the ball is dependent on the reference frames and is different for different reference frames.

🔗 LINKS TO PHYSICS

History: Galileo's Ship
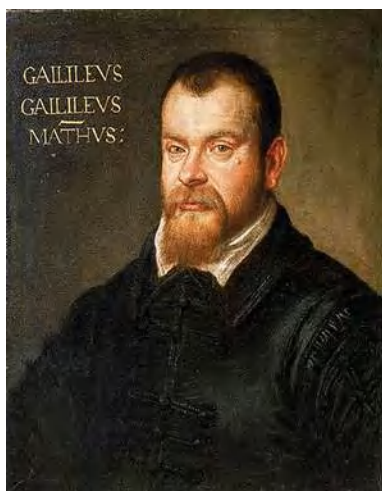
Figure 2.3 Galileo Galilei (1564–1642) studied motion and developed the concept of a reference frame. (Domenico Tintoretto)

The idea that a description of motion depends on the reference frame of the observer has been known for hundreds of years. The 17th-century astronomer Galileo Galilei (Figure 2.3) was one of the first scientists to explore this idea. Galileo suggested the following thought experiment: Imagine a windowless ship moving at a constant speed and direction along a perfectly calm sea. Is there a way that a person inside the ship can determine whether the ship is moving? You can extend this thought experiment

by also imagining a person standing on the shore. How can a person on the shore determine whether the ship is moving?

Galileo came to an amazing conclusion. Only by looking at each other can a person in the ship or a person on shore describe the motion of one relative to the other. In addition, their descriptions of motion would be identical. A person inside the ship would describe the person on the land as moving past the ship. The person on shore would describe the ship and the person inside it as moving past. Galileo realized that observers moving at a constant speed and direction relative to each other describe motion in the same way. Galileo had discovered that a description of motion is only meaningful if you specify a reference frame.

GRASP CHECK

Imagine standing on a platform watching a train pass by. According to Galileo's conclusions, how would your description of motion and the description of motion by a person riding on the train compare?
a. I would see the train as moving past me, and a person on the train would see me as stationary.
b. I would see the train as moving past me, and a person on the train would see me as moving past the train.
c. I would see the train as stationary, and a person on the train would see me as moving past the train.
d. I would see the train as stationary, and a person on the train would also see me as stationary.

Distance vs. Displacement

As we study the motion of objects, we must first be able to describe the object's position. Before your parent drives you to school, the car is sitting in your driveway. Your driveway is the starting position for the car. When you reach your high school, the car has changed position. Its new position is your school.
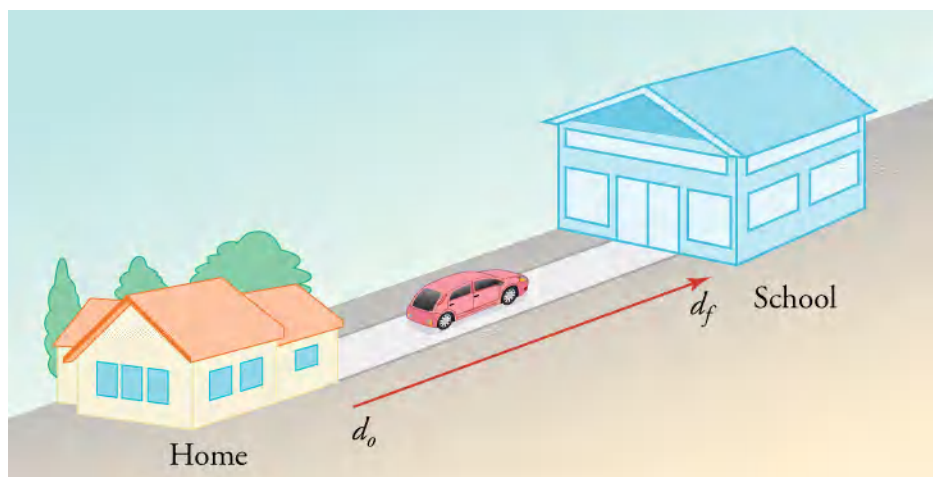
Figure 2.4 Your total change in position is measured from your house to your school.

Physicists use variables to represent terms. We will use $\mathbf{d}$ to represent car's position. We will use a subscript to differentiate between the initial position, $\mathbf{d}_o$, and the final position, $\mathbf{d}_f$. In addition, vectors, which we will discuss later, will be in bold or will have an arrow above the variable. Scalars will be italicized.

TIPS FOR SUCCESS

In some books, $\mathbf{x}$ or $\mathbf{s}$ is used instead of $\mathbf{d}$ to describe position. In $\mathbf{d}_o$, said *d naught*, the subscript o stands for *initial*. When we begin to talk about two-dimensional motion, sometimes other subscripts will be used to describe horizontal position, $\mathbf{d}_x$, or vertical position, $\mathbf{d}_y$. So, you might see references to $\mathbf{d}_{ox}$ and $\mathbf{d}_{fy}$.

Now imagine driving from your house to a friend's house located several kilometers away. How far would you drive? The **distance** an object moves is the length of the path between its initial position and its final position. The distance you drive to your friend's house depends on your path. As shown in Figure 2.5, distance is different from the length of a straight line between two points. The distance you drive to your friend's house is probably longer than the straight line between the two houses.

Figure 2.5 A short line separates the starting and ending points of this motion, but the distance along the path of motion is considerably longer.

We often want to be more precise when we talk about position. The description of an object's motion often includes more than just the distance it moves. For instance, if it is a five kilometer drive to school, the distance traveled is 5 kilometers. After dropping you off at school and driving back home, your parent will have traveled a total distance of 10 kilometers. The car and your parent will end up in the same starting position in space. The net change in position of an object is its **displacement**, or $\Delta\mathbf{d}$. The Greek letter delta, Δ, means *change in*.
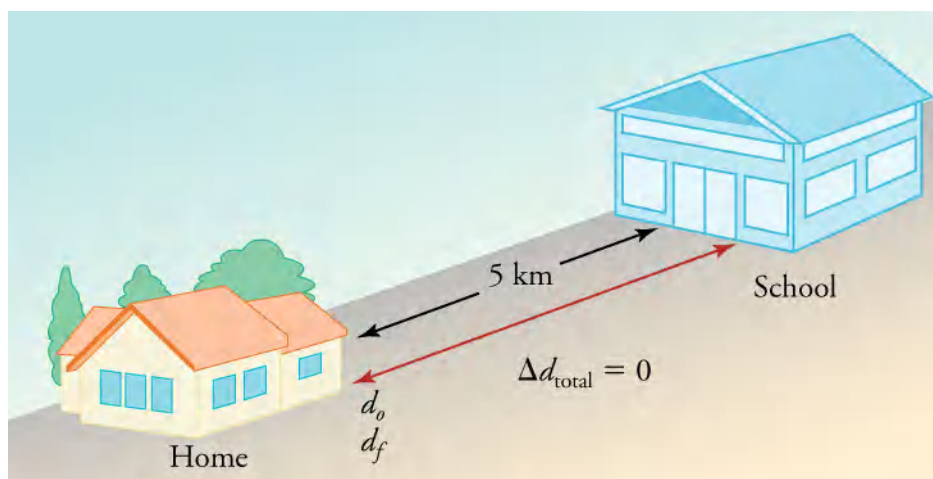
Figure 2.6 The total distance that your car travels is 10 km, but the total displacement is 0.

Snap Lab

Distance vs. Displacement

In this activity you will compare distance and displacement. Which term is more useful when making measurements?

- 1 recorded song available on a portable device
- 1 tape measure
- 3 pieces of masking tape
- A room (like a gym) with a wall that is large and clear enough for all pairs of students to walk back and forth without running into each other.

Procedure

1. One student from each pair should stand with their back to the longest wall in the classroom. Students should stand at least 0.5 meters away from each other. Mark this starting point with a piece of masking tape.
2. The second student from each pair should stand facing their partner, about two to three meters away. Mark this point

with a second piece of masking tape.

3. Student pairs line up at the starting point along the wall.
4. The teacher turns on the music. Each pair walks back and forth from the wall to the second marked point until the music stops playing. Keep count of the number of times you walk across the floor.
5. When the music stops, mark your ending position with the third piece of masking tape.
6. Measure from your starting, initial position to your ending, final position.
7. Measure the length of your path from the starting position to the second marked position. Multiply this measurement by the total number of times you walked across the floor. Then add this number to your measurement from step 6.
8. Compare the two measurements from steps 6 and 7.

GRASP CHECK

1. Which measurement is your total distance traveled?
2. Which measurement is your displacement?
3. When might you want to use one over the other?

a. Measurement of the total length of your path from the starting position to the final position gives the distance traveled, and the measurement from your initial position to your final position is the displacement. Use distance to describe the total path between starting and ending points,and use displacement to describe the shortest path between starting and ending points.
b. Measurement of the total length of your path from the starting position to the final position is distance traveled, and the measurement from your initial position to your final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.
c. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the total path between starting and ending points, and use displacement to describe the shortest path between starting and ending points.
d. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.

If you are describing only your drive to school, then the distance traveled and the displacement are the same—5 kilometers. When you are describing the entire round trip, distance and displacement are different. When you describe distance, you only include the **magnitude**, the size or amount, of the distance traveled. However, when you describe the displacement, you take into account both the magnitude of the change in position and the direction of movement.

In our previous example, the car travels a total of 10 kilometers, but it drives five of those kilometers forward toward school and five of those kilometers back in the opposite direction. If we ascribe the forward direction a positive (+) and the opposite direction a negative (–), then the two quantities will cancel each other out when added together.

A quantity, such as distance, that has magnitude (i.e., how big or how much) but does not take into account direction is called a **scalar**. A quantity, such as displacement, that has both magnitude and direction is called a **vector**.

👁 WATCH PHYSICS

Vectors & Scalars
This video (http://openstax.org/l/28vectorscalar) introduces and differentiates between vectors and scalars. It also introduces quantities that we will be working with during the study of kinematics.

Click to view content (https://www.khanacademy.org/embed_video?v=ihNZlp7iUHE)

Displacement Problems

Hopefully you now understand the conceptual difference between distance and displacement. Understanding concepts is half the battle in physics. The other half is math. A stumbling block to new physics students is trying to wade through the math of physics while also trying to understand the associated concepts. This struggle may lead to misconceptions and answers that make no sense. Once the concept is mastered, the math is far less confusing.

So let's review and see if we can make sense of displacement in terms of numbers and equations. You can calculate an object's displacement by subtracting its original position, d_o, from its final position d_f. In math terms that means

$$\Delta \mathbf{d} = \mathbf{d}_f - \mathbf{d}_0.$$

If the final position is the same as the initial position, then $\Delta \mathbf{d} = 0$.

To assign numbers and/or direction to these quantities, we need to define an axis with a positive and a negative direction. We also need to define an origin, or *O*. In Figure 2.6, the axis is in a straight line with home at zero and school in the positive direction. If we left home and drove the opposite way from school, motion would have been in the negative direction. We would have assigned it a negative value. In the round-trip drive, $\mathbf{d}_f$ and $\mathbf{d}_o$ were both at zero kilometers. In the one way trip to school, $\mathbf{d}_f$ was at 5 kilometers and $\mathbf{d}_o$ was at zero km. So, $\Delta \mathbf{d}$ was 5 kilometers.

TIPS FOR SUCCESS
You may place your origin wherever you would like. You have to make sure that you calculate all distances consistently from your zero and you define one direction as positive and the other as negative. Therefore, it makes sense to choose the easiest axis, direction, and zero. In the example above, we took home to be zero because it allowed us to avoid having to interpret a solution with a negative sign.

✳ WORKED EXAMPLE

Calculating Distance and Displacement

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?
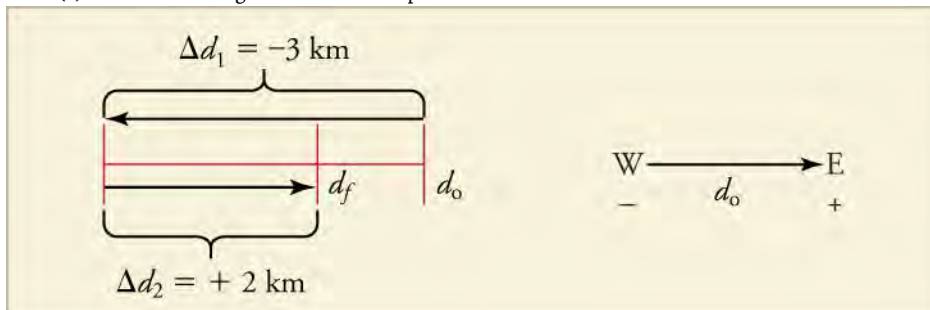
Strategy

To solve this problem, we need to find the difference between the final position and the initial position while taking care to note the direction on the axis. The final position is the sum of the two displacements, $\Delta \mathbf{d}_1$ and $\Delta \mathbf{d}_2$.

Solution

a.   Displacement: The rider's displacement is $\Delta \mathbf{d} = \mathbf{d}_f - \mathbf{d}_0 = -1$ km .

b.   Distance: The distance traveled is 3 km + 2 km = 5 km.

c.   The magnitude of the displacement is 1 km.

Discussion

The displacement is negative because we chose east to be positive and west to be negative. We could also have described the displacement as 1 km west. When calculating displacement, the direction mattered, but when calculating distance, the direction did not matter. The problem would work the same way if the problem were in the north–south or *y*-direction.

> **TIPS FOR SUCCESS**
>
> Physicists like to use standard units so it is easier to compare notes. The standard units for calculations are called *SI* units (International System of Units). SI units are based on the metric system. The SI unit for displacement is the meter (m), but sometimes you will see a problem with kilometers, miles, feet, or other units of length. If one unit in a problem is an SI unit and another is not, you will need to convert all of your quantities to the same system before you can carry out the calculation.

Practice Problems

1.   On an axis in which moving from right to left is positive, what is the displacement and distance of a student who walks 32 m to the right and then 17 m to the left?

   a.   Displacement is -15 m and distance is -49 *m*.

   b.   Displacement is -15 m and distance is 49 m.

   c.   Displacement is 15 m and distance is -49 m.

   d.   Displacement is 15 m and distance is 49 m.

2.   Tiana jogs 1.5 km along a straight path and then turns and jogs 2.4 km in the opposite direction. She then turns back and jogs 0.7 km in the original direction. Let Tiana's original direction be the positive direction. What are the displacement and distance she jogged?

   a.   Displacement is 4.6 km,and distance is -0.2 km.

   b.   Displacement is -0.2 km, and distance is 4.6 km.

   c.   Displacement is 4.6 km, and distance is +0.2 km.

   d.   Displacement is +0.2 km, and distance is 4.6 km.

WORK IN PHYSICS

Mars Probe Explosion
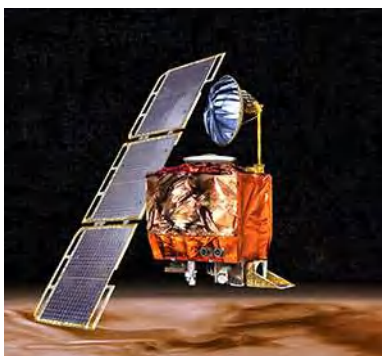
Figure 2.7 The Mars Climate Orbiter disaster illustrates the importance of using the correct calculations in physics. (NASA)

Physicists make calculations all the time, but they do not always get the right answers. In 1998, NASA, the National Aeronautics and Space Administration, launched the Mars Climate Orbiter, shown in Figure 2.7, a $125-million-dollar satellite designed to monitor the Martian atmosphere. It was supposed to orbit the planet and take readings from a safe distance. The American scientists made calculations in English units (feet, inches, pounds, etc.) and forgot to convert their answers to the standard metric SI units. This was a very costly mistake. Instead of orbiting the planet as planned, the Mars Climate Orbiter ended up flying into the Martian atmosphere. The probe disintegrated. It was one of the biggest embarrassments in NASA's history.

GRASP CHECK

In 1999 the Mars Climate Orbiter crashed because calculation were performed in English units instead of SI units. At one point the orbiter was just 187,000 feet above the surface, which was too close to stay in orbit. What was the height of the orbiter at this time in kilometers? (Assume 1 meter equals 3.281 feet.)

a.   16 km
b.   18 km
c.   57 km
d.   614 km

Check Your Understanding

3. What does it mean when motion is described as relative?
    a.   It means that motion of any object is described relative to the motion of Earth.
    b.   It means that motion of any object is described relative to the motion of any other object.
    c.   It means that motion is independent of the frame of reference.
    d.   It means that motion depends on the frame of reference selected.

4. If you and a friend are standing side-by-side watching a soccer game, would you both view the motion from the same reference frame?
    a.   Yes, we would both view the motion from the same reference point because both of us are at rest in Earth's frame of reference.
    b.   Yes, we would both view the motion from the same reference point because both of us are observing the motion from two points on the same straight line.
    c.   No, we would both view the motion from different reference points because motion is viewed from two different points; the reference frames are similar but not the same.
    d.   No, we would both view the motion from different reference points because response times may be different; so, the motion observed by both of us would be different.

5. What is the difference between distance and displacement?
    a.   Distance has both magnitude and direction, while displacement has magnitude but no direction.
    b.   Distance has magnitude but no direction, while displacement has both magnitude and direction.
    c.   Distance has magnitude but no direction, while displacement has only direction.
    d.   There is no difference. Both distance and displacement have magnitude and direction.

6. Which situation correctly identifies a race car's distance traveled and the magnitude of displacement during a one-lap car race?
    a.   The perimeter of the race track is the distance, and the shortest distance between the start line and the finish line is the magnitude of displacement.
    b.   The perimeter of the race track is the magnitude of displacement, and the shortest distance between the start and finish line is the distance.
    c.   The perimeter of the race track is both the distance and magnitude of displacement.
    d.   The shortest distance between the start line and the finish line is both the distance and magnitude of displacement.

7. Why is it important to specify a reference frame when describing motion?
    a.   Because Earth is continuously in motion; an object at rest on Earth will be in motion when viewed from outer space.
    b.   Because the position of a moving object can be defined only when there is a fixed reference frame.

c.   Because motion is a relative term; it appears differently when viewed from different reference frames.
d.   Because motion is always described in Earth's frame of reference; if another frame is used, it has to be specified with each situation.

2.2 Speed and Velocity

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Calculate the average speed of an object
- Relate displacement and average velocity

Section Key Terms

average speed	average velocity	instantaneous speed
instantaneous velocity	speed	velocity

Speed

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we will look at time, speed, and velocity to expand our understanding of motion.

A description of how fast or slow an object moves is its speed. **Speed** is the rate at which an object changes its location. Like distance, speed is a scalar because it has a magnitude but not a direction. Because speed is a rate, it depends on the time interval of motion. You can calculate the elapsed time or the change in time, Δt, of motion as the difference between the ending time and the beginning time

$$\Delta t = t_f - t_0.$$

The SI unit of time is the second (s), and the SI unit of speed is meters per second (m/s), but sometimes kilometers per hour (km/h), miles per hour (mph) or other units of speed are used.

When you describe an object's speed, you often describe the average over a time period. **Average speed**, v_{avg}, is the distance traveled divided by the time during which the motion occurs.

$$v_{avg} = \frac{\text{distance}}{\text{time}}$$

You can, of course, rearrange the equation to solve for either distance or time

$$\text{time} = \frac{\text{distance}}{v_{avg}}.$$

$$\text{distance} = v_{avg} \times \text{time}$$

Suppose, for example, a car travels 150 kilometers in 3.2 hours. Its average speed for the trip is

$$
\begin{aligned}
v_{avg} &= \frac{\text{distance}}{\text{time}} \\
&= \frac{150 \ \text{km}}{3.2 \ \text{h}} \\
&= 47 \ \text{km/h}.
\end{aligned}
$$

A car's speed would likely increase and decrease many times over a 3.2 hour trip. Its speed at a specific instant in time, however, is its **instantaneous speed**. A car's speedometer describes its instantaneous speed.
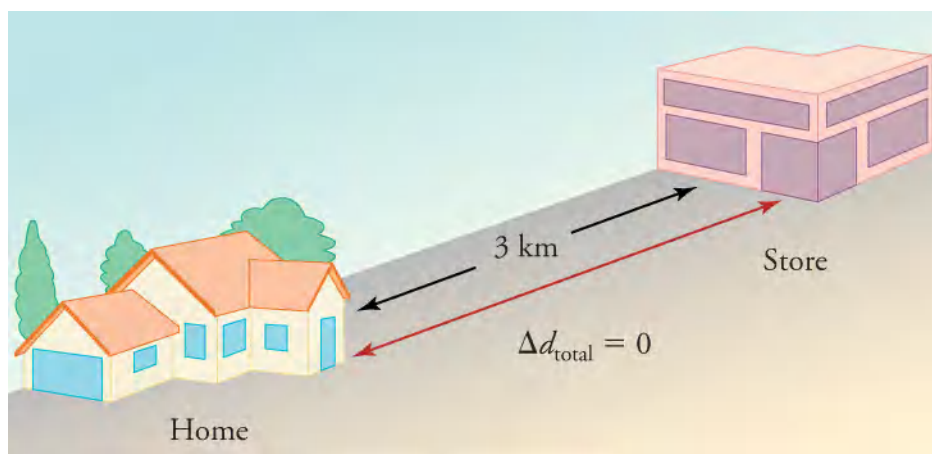
Figure 2.8 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, because there was no net change in position.

✳ WORKED EXAMPLE

Calculating Average Speed

A marble rolls 5.2 m in 1.8 s. What was the marble's average speed?

Strategy

We know the distance the marble travels, 5.2 m, and the time interval, 1.8 s. We can use these values in the average speed equation.

Solution

$$v_{avg} = \frac{\text{distance}}{\text{time}} = \frac{5.2 \text{ m}}{1.8 \text{ s}} = 2.9 \text{ m/s}$$

Discussion

Average speed is a scalar, so we do not include direction in the answer. We can check the reasonableness of the answer by estimating: 5 meters divided by 2 seconds is 2.5 m/s. Since 2.5 m/s is close to 2.9 m/s, the answer is reasonable. This is about the speed of a brisk walk, so it also makes sense.

Practice Problems

8. A pitcher throws a baseball from the pitcher's mound to home plate in 0.46 s. The distance is 18.4 m. What was the average speed of the baseball?
   a. 40 m/s
   b. - 40 m/s
   c. 0.03 m/s
   d. 8.5 m/s

9. Cassie walked to her friend's house with an average speed of 1.40 m/s. The distance between the houses is 205 m. How long did the trip take her?
   a. 146 s
   b. 0.01 s
   c. 2.50 min
   d. 287 s

Velocity

The vector version of speed is velocity. **Velocity** describes the speed and direction of an object. As with speed, it is useful to describe either the average velocity over a time period or the velocity at a specific moment. **Average velocity** is displacement divided by the time over which the displacement occurs.

$$v_{\text{avg}} = \frac{\text{distance}}{\text{time}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_\text{f} - \mathbf{d}_0}{t_\text{f} - t_0}$$

Velocity, like speed, has SI units of meters per second (m/s), but because it is a vector, you must also include a direction. Furthermore, the variable **v** for velocity is bold because it is a vector, which is in contrast to the variable *v* for speed which is italicized because it is a scalar quantity.

TIPS FOR SUCCESS

It is important to keep in mind that the average speed is not the same thing as the average velocity without its direction. Like we saw with displacement and distance in the last section, changes in direction over a time interval have a bigger effect on speed and velocity.

Suppose a passenger moved toward the back of a plane with an average velocity of −4 m/s. We cannot tell from the average velocity whether the passenger stopped momentarily or backed up before he got to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals such as those shown in Figure 2.9. If you consider infinitesimally small intervals, you can define **instantaneous velocity**, which is the velocity at a specific instant in time. Instantaneous velocity and average velocity are the same if the velocity is constant.
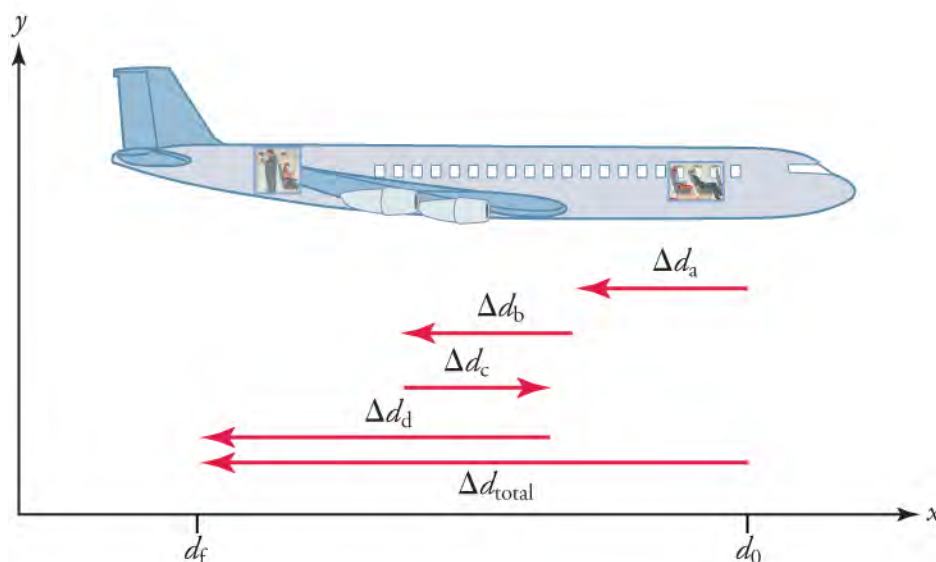
Figure 2.9 The diagram shows a more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

Earlier, you have read that distance traveled can be different than the magnitude of displacement. In the same way, speed can be different than the magnitude of velocity. For example, you drive to a store and return home in half an hour. If your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero because your displacement for the round trip is zero.

◉ WATCH PHYSICS

Calculating Average Velocity or Speed

This video (http://openstax.org/l/28avgvelocity) reviews vectors and scalars and describes how to calculate average velocity and average speed when you know displacement and change in time. The video also reviews how to convert km/h to m/s.

Click to view content (https://www.khanacademy.org/embed_video?v=MAS6mBRZZXA)

GRASP CHECK

Which of the following fully describes a vector and a scalar quantity and correctly provides an example of each?

a. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.

b. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.

c. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.

d. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.

✳ WORKED EXAMPLE

Calculating Average Velocity

A student has a displacement of 304 m north in 180 s. What was the student's average velocity?

Strategy

We know that the displacement is 304 m north and the time is 180 s. We can use the formula for average velocity to solve the problem.

Solution

$$\mathbf{v}_{avg} = \frac{\varDelta \mathbf{d}}{\varDelta t} = \frac{304 \ m}{180 \ s} = 1.7 \ m/s \ north$$

2.1

Discussion

Since average velocity is a vector quantity, you must include direction as well as magnitude in the answer. Notice, however, that the direction can be omitted until the end to avoid cluttering the problem. Pay attention to the significant figures in the problem. The distance 304 m has three significant figures, but the time interval 180 s has only two, so the quotient should have only two significant figures.

> **TIPS FOR SUCCESS**
>
> Note the way scalars and vectors are represented. In this book d represents distance and displacement. Similarly, v represents speed, and v represents velocity. A variable that is not bold indicates a scalar quantity, and a bold variable indicates a vector quantity. Vectors are sometimes represented by small arrows above the variable.

✳ WORKED EXAMPLE

Solving for Displacement when Average Velocity and Time are Known

Layla jogs with an average velocity of 2.4 m/s east. What is her displacement after 46 seconds?

Strategy

We know that Layla's average velocity is 2.4 m/s east, and the time interval is 46 seconds. We can rearrange the average velocity formula to solve for the displacement.

Solution

$$\begin{aligned}
\mathbf{v}_{avg} &= \frac{\varDelta \mathbf{d}}{\varDelta t} \\
\varDelta \mathbf{d} &= \mathbf{v}_{avg}\varDelta t \\
&= (2.4 \ m/s)(46 \ s) \\
&= 1.1 \times 10^2 \ m \ east
\end{aligned}$$

2.2

Discussion

The answer is about 110 m east, which is a reasonable displacement for slightly less than a minute of jogging. A calculator shows the answer as 110.4 m. We chose to write the answer using scientific notation because we wanted to make it clear that we only

used two significant figures.

✳ WORKED EXAMPLE

Solving for Time when Displacement and Average Velocity are Known

Phillip walks along a straight path from his house to his school. How long will it take him to get to school if he walks 428 m west with an average velocity of 1.7 m/s west?

Strategy

We know that Phillip's displacement is 428 m west, and his average velocity is 1.7 m/s west. We can calculate the time required for the trip by rearranging the average velocity equation.

Solution

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{d}}{\Delta t}$$

$$\Delta t = \frac{\Delta \mathbf{d}}{\mathbf{v}_{avg}}$$

$$= \frac{428 \ \text{m}}{1.7 \ \text{m/s}}$$

$$= 2.5 \times 10^2 \ \text{s}$$

2.3

Discussion

Here again we had to use scientific notation because the answer could only have two significant figures. Since time is a scalar, the answer includes only a magnitude and not a direction.

Practice Problems

10. A trucker drives along a straight highway for 0.25 h with a displacement of 16 km south. What is the trucker's average velocity?
    a. 4 km/h north
    b. 4 km/h south
    c. 64 km/h north
    d. 64 km/h south

11. A bird flies with an average velocity of 7.5 m/s east from one branch to another in 2.4 s. It then pauses before flying with an average velocity of 6.8 m/s east for 3.5 s to another branch. What is the bird's total displacement from its starting point?
    a. 42 m west
    b. 6 m west
    c. 6 m east
    d. 42 m east

Virtual Physics

The Walking Man

In this simulation you will put your cursor on the man and move him first in one direction and then in the opposite direction. Keep the *Introduction* tab active. You can use the *Charts* tab after you learn about graphing motion later in this chapter. Carefully watch the sign of the numbers in the position and velocity boxes. Ignore the acceleration box for now. See if you can make the man's position positive while the velocity is negative. Then see if you can do the opposite.

GRASP CHECK

Which situation correctly describes when the moving man's position was negative but his velocity was positive?

a. Man moving toward 0 from left of 0
b. Man moving toward 0 from right of 0
c. Man moving away from 0 from left of 0
d. Man moving away from 0 from right of 0

Check Your Understanding

12. Two runners travel along the same straight path. They start at the same time, and they end at the same time, but at the halfway mark, they have different instantaneous velocities. Is it possible for them to have the same average velocity for the trip?
    a. Yes, because average velocity depends on the net or total displacement.
    b. Yes, because average velocity depends on the total distance traveled.
    c. No, because the velocities of both runners must remain the exactly same throughout the journey.
    d. No, because the instantaneous velocities of the runners must remain same midway but can be different elsewhere.

13. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity, and under what circumstances are these two quantities the same?
    a. Average speed. Both are the same when the car is traveling at a constant speed and changing direction.
    b. Average speed. Both are the same when the speed is constant and the car does not change its direction.
    c. Magnitude of average velocity. Both are same when the car is traveling at a constant speed.
    d. Magnitude of average velocity. Both are same when the car does not change its direction.

14. Is it possible for average velocity to be negative?
    a. Yes, in cases when the net displacement is negative.
    b. Yes, if the body keeps changing its direction during motion.
    c. No, average velocity describes only magnitude and not the direction of motion.
    d. No, average velocity describes only the magnitude in the positive direction of motion.

2.3 Position vs. Time Graphs

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Explain the meaning of slope in position vs. time graphs
- Solve problems using position vs. time graphs

Section Key Terms

dependent variable   independent variable   tangent

Graphing Position as a Function of Time

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information, they also reveal relationships between physical quantities. In this section, we will investigate kinematics by analyzing graphs of position over time.

Graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against each other, the horizontal axis is usually considered the **independent variable**, and the vertical axis is the **dependent variable**. In algebra, you would have referred to the horizontal axis as the x-axis and the vertical axis as the y-axis. As in Figure 2.10, a straight-line graph has the general form $y = mx + b$.

Here *m* is the slope, defined as the rise divided by the run (as seen in the figure) of the straight line. The letter *b* is the *y*-intercept which is the point at which the line crosses the vertical, *y*-axis. In terms of a physical situation in the real world, these quantities will take on a specific significance, as we will see below. (Figure 2.10.)
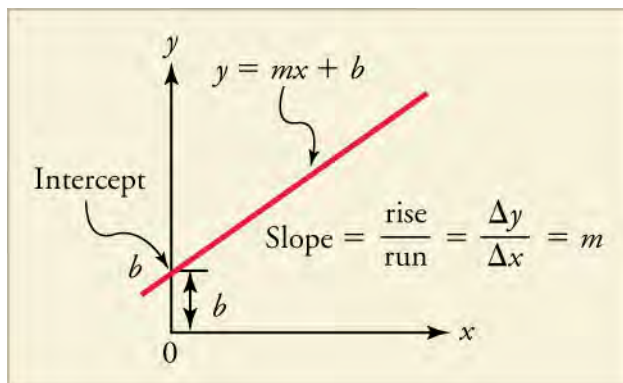
Figure 2.10 The diagram shows a straight-line graph. The equation for the straight line is *y* equals *mx* + *b*.

In physics, time is usually the independent variable. Other quantities, such as displacement, are said to depend upon it. A graph of position versus time, therefore, would have position on the vertical axis (dependent variable) and time on the horizontal axis (independent variable). In this case, to what would the slope and *y*-intercept refer? Let's look back at our original example when studying distance and displacement.

The drive to school was 5 km from home. Let's assume it took 10 minutes to make the drive and that your parent was driving at a constant velocity the whole time. The position versus time graph for this section of the trip would look like that shown in Figure 2.11.
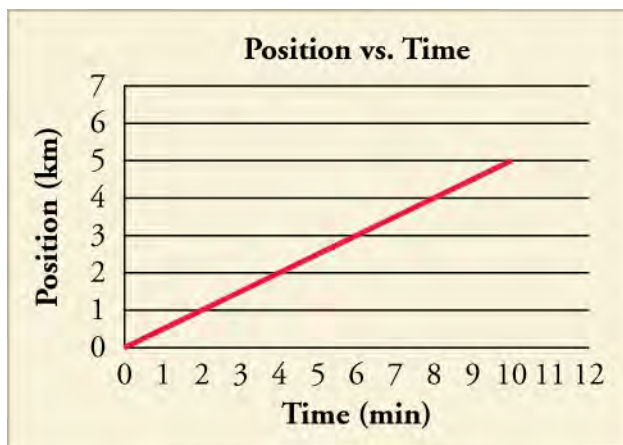
Figure 2.11 A graph of position versus time for the drive to school is shown. What would the graph look like if we added the return trip?

As we said before, $\mathbf{d}_0 = 0$ because we call home our *O* and start calculating from there. In Figure 2.11, the line starts at $\mathbf{d} = 0$, as well. This is the *b* in our equation for a straight line. Our initial position in a position versus time graph is always the place where the graph crosses the *x*-axis at *t* = 0. What is the slope? The *rise* is the change in position, (i.e., displacement) and the *run* is the change in time. This relationship can also be written

$$\frac{\Delta \mathbf{d}}{\Delta t}.$$

2.4

This relationship was how we defined average velocity. Therefore, the slope in a **d** versus *t* graph, is the average velocity.

TIPS FOR SUCCESS

Sometimes, as is the case where we graph both the trip to school and the return trip, the behavior of the graph looks different during different time intervals. If the graph looks like a series of straight lines, then you can calculate the average velocity for each time interval by looking at the slope. If you then want to calculate the average velocity for the entire trip, you can do a

weighted average.

Let's look at another example. Figure 2.12 shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.
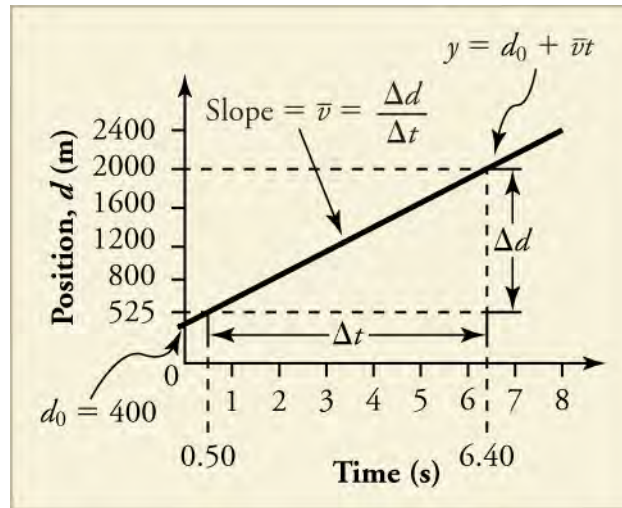
Figure 2.12 The diagram shows a graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph in Figure 2.12 is average velocity, $\mathbf{v}_{avg}$ and the intercept is displacement at time zero—that is, $\mathbf{d}_o$. Substituting these symbols into $y = mx + b$ gives

$$\mathbf{d} = \mathbf{v}t + \mathbf{d}_0$$

2.5

or

$$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t.$$

2.6

Thus a graph of position versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation. From the figure we can see that the car has a position of 400 m at $t = 0$ s, 650 m at $t = 1.0$ s, and so on. And we can learn about the object's velocity, as well.

Snap Lab

Graphing Motion
In this activity, you will release a ball down a ramp and graph the ball's displacement vs. time.

- Choose an open location with lots of space to spread out so there is less chance for tripping or falling due to rolling balls.

- 1 ball
- 1 board
- 2 or 3 books
- 1 stopwatch
- 1 tape measure
- 6 pieces of masking tape
- 1 piece of graph paper
- 1 pencil

Procedure
1. Build a ramp by placing one end of the board on top of the stack of books. Adjust location, as necessary, until there is no obstacle along the straight line path from the bottom of the ramp until at least the next 3 m.
2. Mark distances of 0.5 m, 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp. Write the distances on the tape.

3. Have one person take the role of the experimenter. This person will release the ball from the top of the ramp. If the ball does not reach the 3.0 m mark, then increase the incline of the ramp by adding another book. Repeat this Step as necessary.

4. Have the experimenter release the ball. Have a second person, the timer, begin timing the trial once the ball reaches the bottom of the ramp and stop the timing once the ball reaches 0.5 m. Have a third person, the recorder, record the time in a data table.

5. Repeat Step 4, stopping the times at the distances of 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp.

6. Use your measurements of time and the displacement to make a position vs. time graph of the ball's motion.

7. Repeat Steps 4 through 6, with different people taking on the roles of experimenter, timer, and recorder. Do you get the same measurement values regardless of who releases the ball, measures the time, or records the result? Discuss possible causes of discrepancies, if any.

GRASP CHECK
True or False: The average speed of the ball will be less than the average velocity of the ball.
a. True
b. False

Solving Problems Using Position vs. Time Graphs
So how do we use graphs to solve for things we want to know like velocity?

✳ WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car
Find the average velocity of the car whose position is graphed in .

Strategy

The slope of a graph of d vs. t is average velocity, since slope equals rise over run.

$$\text{slope} = \frac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v} \qquad \boxed{2.7}$$

Since the slope is constant here, any two points on the graph can be used to find the slope.

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the $\mathbf{d}$ and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$
\begin{aligned}
\mathbf{v} &= \frac{\Delta \mathbf{d}}{\Delta t} \\
&= \frac{2000 \ \text{m} - 525 \ \text{m}}{6.4 \ \text{s} - 0.50 \ \text{s}}, \\
&= 250 \ \text{m/s}
\end{aligned}
\qquad \boxed{2.8}
$$

Discussion

This is an impressively high land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 27 m/s or 96 km/h, but considerably shy of the record of 343 m/s or 1,234 km/h, set in 1997.

But what if the graph of the position is more complicated than a straight line? What if the object speeds up or turns around and goes backward? Can we figure out anything about its velocity from a graph of that kind of motion? Let's take another look at the jet-powered car. The graph in Figure 2.13 shows its motion as it is getting up to speed after starting at rest. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.
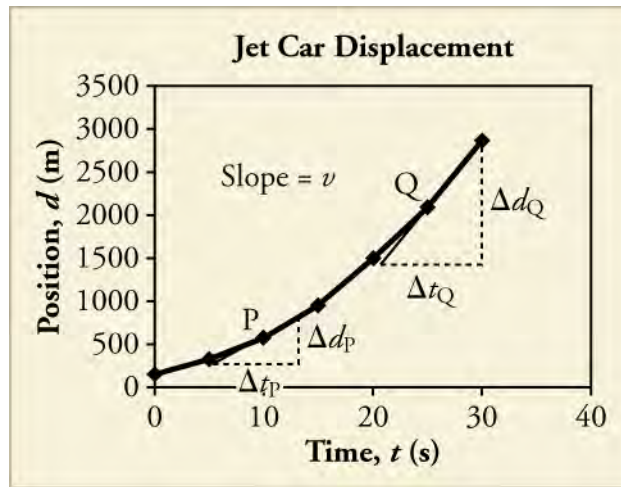
Figure 2.13 The diagram shows a graph of the position of a jet-powered car during the time span when it is speeding up. The slope of a distance versus time graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Figure 2.14 A U.S. Air Force jet car speeds down a track. (Matt Trostle, Flickr)

The graph of position versus time in Figure 2.13 is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line **tangent** to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 2.13. The average velocity is the net displacement divided by the time traveled.

✳ WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car, Take Two

Calculate the instantaneous velocity of the jet car at a time of 25 s by finding the slope of the tangent line at point Q in Figure 2.13.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point.

Solution

1. Find the tangent line to the curve at $t = 25$ s.
2. Determine the endpoints of the tangent. These correspond to a position of 1,300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, **v**.

$$\text{slope} = v_Q = \frac{\Delta d_Q}{\Delta t_Q}$$

$$= \frac{(3120-1300)\ \text{m}}{(32-19)\ \text{s}}$$

$$= \frac{1820\ \text{m}}{13\ \text{s}}$$

$$= 140\ \text{m/s}$$

2.9

Discussion

The entire graph of **v** versus t can be obtained in this fashion.

Practice Problems

15. Calculate the average velocity of the object shown in the graph below over the whole time interval.
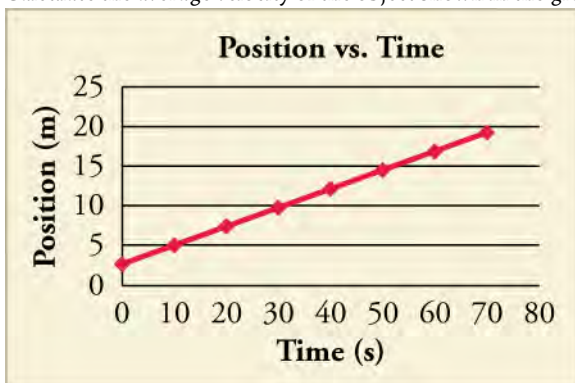
a. 0.25 m/s
b. 0.31 m/s
c. 3.2 m/s
d. 4.00 m/s

16. True or False: By taking the slope of the curve in the graph you can verify that the velocity of the jet car is 115 m/s at $t = 20$ s.
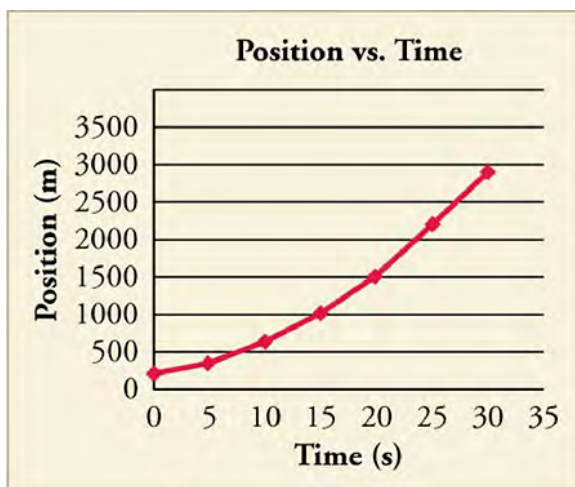
a. True
b. False

Check Your Understanding

17. Which of the following information about motion can be determined by looking at a position vs. time graph that is a straight line?

   a.   frame of reference
   b.   average acceleration
   c.   velocity
   d.   direction of force applied

18.   True or False: The position vs time graph of an object that is speeding up is a straight line.
   a.   True
   b.   False

2.4 Velocity vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope and area in velocity vs. time graphs
- Solve problems using velocity vs. time graphs

Section Key Terms

acceleration

Graphing Velocity as a Function of Time

Earlier, we examined graphs of position versus time. Now, we are going to build on that information as we look at graphs of velocity vs. time. Velocity is the rate of change of displacement. **Acceleration** is the rate of change of velocity; we will discuss acceleration more in another chapter. These concepts are all very interrelated.

> ### Virtual Physics
>
> **Maze Game**
>
> In this simulation you will use a vector diagram to manipulate a ball into a certain location without hitting a wall. You can manipulate the ball directly with position or by changing its velocity. Explore how these factors change the motion. If you would like, you can put it on the *a* setting, as well. This is acceleration, which measures the rate of change of velocity. We will explore acceleration in more detail later, but it might be interesting to take a look at it here.
>
> Click to view content (https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/)
>
> > ### GRASP CHECK
> > Click to view content (https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/#sim-maze-game)
> > a.   The ball can be easily manipulated with displacement because the arena is a position space.
> > b.   The ball can be easily manipulated with velocity because the arena is a position space.
> > c.   The ball can be easily manipulated with displacement because the arena is a velocity space.
> > d.   The ball can be easily manipulated with velocity because the arena is a velocity space.

What can we learn about motion by looking at velocity vs. time graphs? Let's return to our drive to school, and look at a graph of position versus time as shown in Figure 2.15.
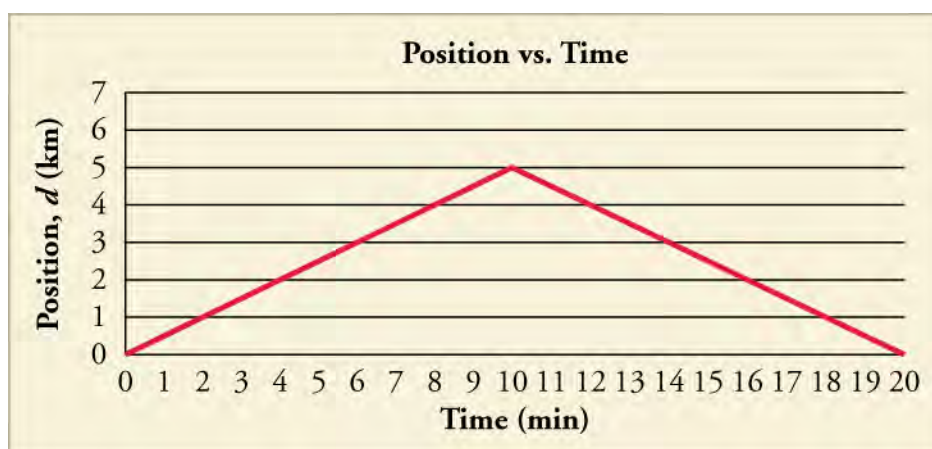
Figure 2.15 A graph of position versus time for the drive to and from school is shown.

We assumed for our original calculation that your parent drove with a constant velocity to and from school. We now know that the car could not have gone from rest to a constant velocity without speeding up. So the actual graph would be curved on either end, but let's make the same approximation as we did then, anyway.

TIPS FOR SUCCESS

It is common in physics, especially at the early learning stages, for certain things to be *neglected*, as we see here. This is because it makes the concept clearer or the calculation easier. Practicing physicists use these kinds of short-cuts, as well. It works out because usually the thing being *neglected* is small enough that it does not significantly affect the answer. In the earlier example, the amount of time it takes the car to speed up and reach its cruising velocity is very small compared to the total time traveled.

Looking at this graph, and given what we learned, we can see that there are two distinct periods to the car's motion—the way to school and the way back. The average velocity for the drive to school is 0.5 km/minute. We can see that the average velocity for the drive back is −0.5 km/minute. If we plot the data showing velocity versus time, we get another graph (Figure 2.16):
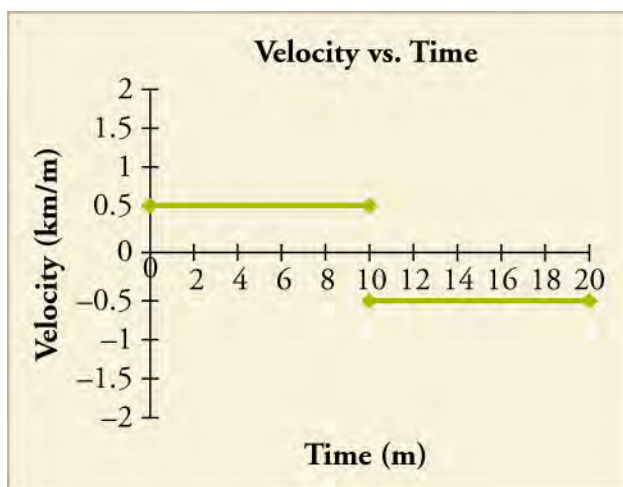
Figure 2.16 Graph of velocity versus time for the drive to and from school.

We can learn a few things. First, we can derive a **v** versus *t* graph from a **d** versus *t* graph. Second, if we have a straight-line position–time graph that is positively or negatively sloped, it will yield a horizontal velocity graph. There are a few other interesting things to note. Just as we could use a position vs. time graph to determine velocity, we can use a velocity vs. time graph to determine position. We know that $v = d/t$. If we use a little algebra to re-arrange the equation, we see that $d = v \times t$. In Figure 2.16, we have velocity on the *y*-axis and time along the *x*-axis. Let's take just the first half of the motion. We get 0.5 km/minute × 10 minutes. The units for *minutes* cancel each other, and we get 5 km, which is the displacement for the trip to school. If we calculate the same for the return trip, we get −5 km. If we add them together, we see that the net displacement for the

whole trip is 0 km, which it should be because we started and ended at the same place.

TIPS FOR SUCCESS

You can treat units just like you treat numbers, so a km/km=1 (or, we say, it cancels out). This is good because it can tell us whether or not we have calculated everything with the correct units. For instance, if we end up with m × s for velocity instead of m/s, we know that something has gone wrong, and we need to check our math. This process is called dimensional analysis, and it is one of the best ways to check if your math makes sense in physics.

The area under a velocity curve represents the displacement. The velocity curve also tells us whether the car is speeding up. In our earlier example, we stated that the velocity was constant. So, the car is not speeding up. Graphically, you can see that the slope of these two lines is 0. This slope tells us that the car is not speeding up, or accelerating. We will do more with this information in a later chapter. For now, just remember that the area under the graph and the slope are the two important parts of the graph. Just like we could define a linear equation for the motion in a position vs. time graph, we can also define one for a velocity vs. time graph. As we said, the slope equals the acceleration, $\mathbf{a}$. And in this graph, the y-intercept is $\mathbf{v_0}$. Thus, $\mathbf{v} = \mathbf{v_0} + \mathbf{a}t$.

But what if the velocity is not constant? Let's look back at our jet-car example. At the beginning of the motion, as the car is speeding up, we saw that its position is a curve, as shown in Figure 2.17.
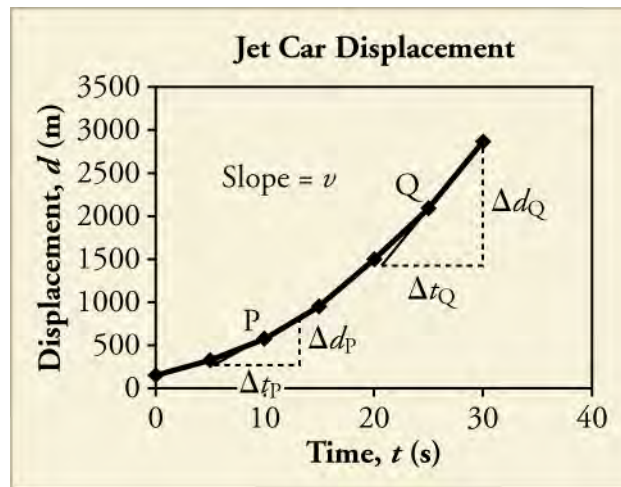
Figure 2.17 A graph is shown of the position of a jet-powered car during the time span when it is speeding up. The slope of a d vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

You do not have to do this, but you could, theoretically, take the instantaneous velocity at each point on this graph. If you did, you would get Figure 2.18, which is just a straight line with a positive slope.
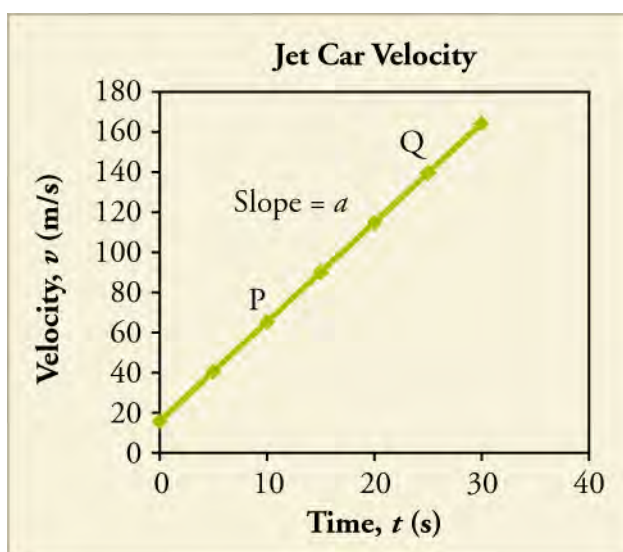
Figure 2.18 The graph shows the velocity of a jet-powered car during the time span when it is speeding up.

Again, if we take the slope of the velocity vs. time graph, we get the acceleration, the rate of change of the velocity. And, if we take the area under the slope, we get back to the displacement.

Solving Problems using Velocity–Time Graphs

Most velocity vs. time graphs will be straight lines. When this is the case, our calculations are fairly simple.

✳ WORKED EXAMPLE

Using Velocity Graph to Calculate Some Stuff: Jet Car

Use this figure to (a) find the displacement of the jet car over the time shown (b) calculate the rate of change (acceleration) of the velocity. (c) give the instantaneous velocity at 5 s, and (d) calculate the average velocity over the interval shown.

Strategy

a. The displacement is given by finding the area under the line in the velocity vs. time graph.
b. The acceleration is given by finding the slope of the velocity graph.
c. The instantaneous velocity can just be read off of the graph.
d. To find the average velocity, recall that $\mathbf{v}_{avg} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_f - \mathbf{d}_0}{t_f - t_0}$

Solution

a. 1. Analyze the shape of the area to be calculated. In this case, the area is made up of a rectangle between 0 and 20 m/s stretching to 30 s. The area of a rectangle is length × width. Therefore, the area of this piece is 600 m.
    2. Above that is a triangle whose base is 30 s and height is 140 m/s. The area of a triangle is 0.5 × length × width. The area of this piece, therefore, is 2,100 m.
    3. Add them together to get a net displacement of 2,700 m.

b. 1. Take two points on the velocity line. Say, $t = 5$ s and $t = 25$ s. At $t = 5$ s, the value of $\mathbf{v} = 40$ m/s. At $t = 25$ s, $\mathbf{v} = 140$ m/s.

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$$

    2. Find the slope. $= \frac{100 \ \text{m/s}}{20 \ \text{s}}$

$$= 5 \ \text{m/s}^2$$

c. The instantaneous velocity at $t = 5$ s, as we found in part (b) is just 40 m/s.
d. 1. Find the net displacement, which we found in part (a) was 2,700 m.
    2. Find the total time which for this case is 30 s.
    3. Divide 2,700 m/30 s = 90 m/s.

Discussion

The average velocity we calculated here makes sense if we look at the graph. 100m/s falls about halfway across the graph and since it is a straight line, we would expect about half the velocity to be above and half below.

TIPS FOR SUCCESS

You can have negative position, velocity, and acceleration on a graph that describes the way the object is moving. You should never see a graph with negative time on an axis. Why?

Most of the velocity vs. time graphs we will look at will be simple to interpret. Occasionally, we will look at curved graphs of velocity vs. time. More often, these curved graphs occur when something is speeding up, often from rest. Let's look back at a more realistic velocity vs. time graph of the jet car's motion that takes this *speeding up* stage into account.
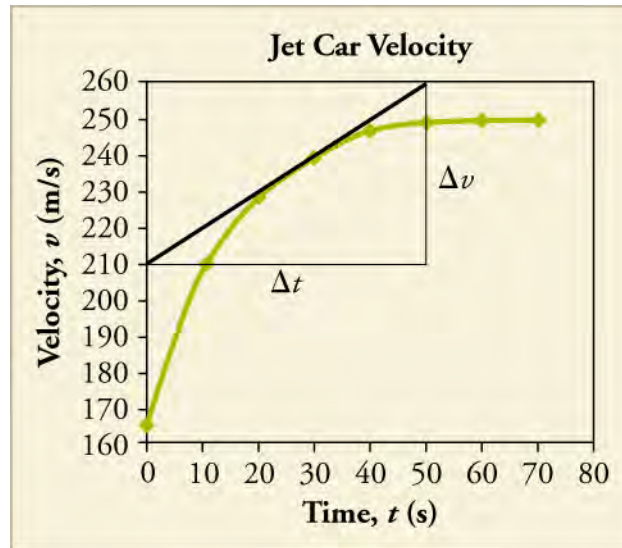
Figure 2.19 The graph shows a more accurate graph of the velocity of a jet-powered car during the time span when it is speeding up.

✳ WORKED EXAMPLE

Using Curvy Velocity Graph to Calculate Some Stuff: jet car, Take Two

Use Figure 2.19 to (a) find the approximate displacement of the jet car over the time shown, (b) calculate the instantaneous acceleration at $t = 30$ s, (c) find the instantaneous velocity at 30 s, and (d) calculate the approximate average velocity over the interval shown.

Strategy

a. Because this graph is an undefined curve, we have to estimate shapes over smaller intervals in order to find the areas.
b. Like when we were working with a curved displacement graph, we will need to take a tangent line at the instant we are interested and use that to calculate the instantaneous acceleration.
c. The instantaneous velocity can still be read off of the graph.
d. We will find the average velocity the same way we did in the previous example.

Solution

a. 1. This problem is more complicated than the last example. To get a good estimate, we should probably break the curve into four sections. $0 \rightarrow 10$ s, $10 \rightarrow 20$ s, $20 \rightarrow 40$ s, and $40 \rightarrow 70$ s.
    2. Calculate the bottom rectangle (common to all pieces). 165 m/s $\times$ 70 s = 11,550 m.
    3. Estimate a triangle at the top, and calculate the area for each section. Section 1 = 225 m; section 2 = 100 m + 450 m = 550 m; section 3 = 150 m + 1,300 m = 1,450 m; section 4 = 2,550 m.
    4. Add them together to get a net displacement of 16,325 m.

b. Using the tangent line given, we find that the slope is 1 m/s².

c.   The instantaneous velocity at $t = 30$ s, is 240 m/s.
d.   1.   Find the net displacement, which we found in part (a), was 16,325 m.
      2.   Find the total time, which for this case is 70 s.
      3.   Divide $\frac{16{,}325 \ \text{m}}{70 \ \text{s}} \sim 233$ m/s

Discussion

This is a much more complicated process than the first problem. If we were to use these estimates to come up with the average velocity over just the first 30 s we would get about 191 m/s. By approximating that curve with a line, we get an average velocity of 202.5 m/s. Depending on our purposes and how precise an answer we need, sometimes calling a curve a straight line is a worthwhile approximation.

Practice Problems

19.
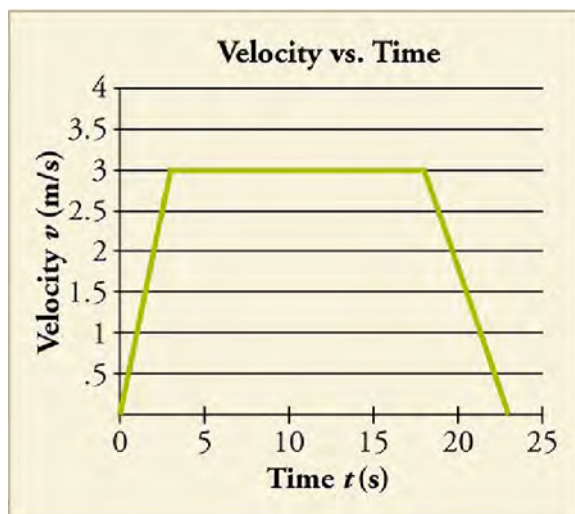
Figure 2.20

Consider the velocity vs. time graph shown below of a person in an elevator. Suppose the elevator is initially at rest. It then speeds up for 3 seconds, maintains that velocity for 15 seconds, then slows down for 5 seconds until it stops. Find the instantaneous velocity at $t = 10$ s and $t = 23$ s.
   a.   Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 0 m/s.
   b.   Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 3 m/s.
   c.   Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 0 m/s.
   d.   Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 1.5 m/s.

20.
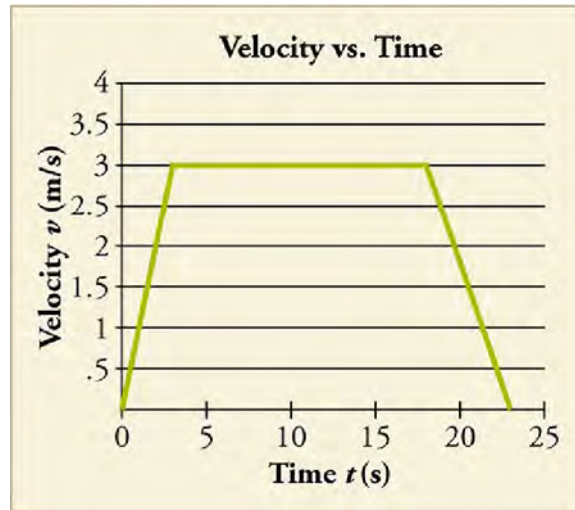
Figure 2.21

Calculate the net displacement and the average velocity of the elevator over the time interval shown.

   a.   Net displacement is 45 m and average velocity is 2.10 m/s.
   b.   Net displacement is 45 m and average velocity is 2.28 m/s.
   c.   Net displacement is 57 m and average velocity is 2.66 m/s.
   d.   Net displacement is 57 m and average velocity is 2.48 m/s.

Snap Lab

Graphing Motion, Take Two

In this activity, you will graph a moving ball's velocity vs. time.

- your graph from the earlier Graphing Motion Snap Lab!
- 1 piece of graph paper
- 1 pencil

Procedure

1. Take your graph from the earlier Graphing Motion Snap Lab! and use it to create a graph of velocity vs. time.
2. Use your graph to calculate the displacement.

GRASP CHECK
Describe the graph and explain what it means in terms of velocity and acceleration.

   a.   The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was not accelerating.
   b.   The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was accelerating.
   c.   The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was not accelerating.
   d.   The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was accelerating.

Check Your Understanding

21. What information could you obtain by looking at a velocity vs. time graph?
   a.   acceleration
   b.   direction of motion
   c.   reference frame of the motion

    d.   shortest path

22. How would you use a position vs. time graph to construct a velocity vs. time graph and vice versa?
   a. Slope of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
   b. Slope of position vs. time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.
   c. Area of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
   d. Area of position/time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.

KEY TERMS

acceleration   the rate at which velocity changes

average speed   distance traveled divided by time during which motion occurs

average velocity   displacement divided by time over which displacement occurs

dependent variable   the variable that changes as the independent variable changes

displacement   the change in position of an object against a fixed axis

distance   the length of the path actually traveled between an initial and a final position

independent variable   the variable, usually along the horizontal axis of a graph, that does not change based on human or experimental action; in physics this is usually time

instantaneous speed   speed at a specific instant in time

instantaneous velocity   velocity at a specific instant in time

kinematics   the study of motion without considering its causes

magnitude   size or amount

position   the location of an object at any particular time

reference frame   a coordinate system from which the positions of objects are described

scalar   a quantity that has magnitude but no direction

speed   rate at which an object changes its location

tangent   a line that touches another at exactly one point

vector   a quantity that has both magnitude and direction

velocity   the speed and direction of an object

SECTION SUMMARY

2.1 Relative Motion, Distance, and Displacement

- A description of motion depends on the reference frame from which it is described.
- The distance an object moves is the length of the path along which it moves.
- Displacement is the difference in the initial and final positions of an object.

2.2 Speed and Velocity

- Average speed is a scalar quantity that describes distance traveled divided by the time during which the motion occurs.
- Velocity is a vector quantity that describes the speed and direction of an object.
- Average velocity is displacement over the time period during which the displacement occurs. If the velocity is constant, then average velocity and instantaneous velocity are the same.

2.3 Position vs. Time Graphs

- Graphs can be used to analyze motion.
- The slope of a position vs. time graph is the velocity.
- For a straight line graph of position, the slope is the average velocity.
- To obtain the instantaneous velocity at a given moment for a curved graph, find the tangent line at that point and take its slope.

2.4 Velocity vs. Time Graphs

- The slope of a velocity vs. time graph is the acceleration.
- The area under a velocity vs. time curve is the displacement.
- Average velocity can be found in a velocity vs. time graph by taking the weighted average of all the velocities.

KEY EQUATIONS

2.1 Relative Motion, Distance, and Displacement

Displacement $\Delta \mathbf{d} = \mathbf{d_f} - \mathbf{d_0}$

2.2 Speed and Velocity

Average speed $v_{\text{avg}} = \dfrac{\text{distance}}{\text{time}}$

Average velocity $\mathbf{v}_{\text{avg}} = \dfrac{\Delta \mathbf{d}}{\Delta t} = \dfrac{\mathbf{d_f} - \mathbf{d_0}}{t_f - t_0}$

2.3 Position vs. Time Graphs

Displacement $\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t$.

2.4 Velocity vs. Time Graphs

Velocity $\mathbf{v} = \mathbf{v}_0 + at$

Acceleration $\mathbf{a} = \dfrac{\Delta \mathbf{v}}{\Delta t}$

CHAPTER REVIEW
Concept Items

2.1 Relative Motion, Distance, and Displacement

1. Can one-dimensional motion have zero distance but a nonzero displacement? What about zero displacement but a nonzero distance?
   a. One-dimensional motion can have zero distance with a nonzero displacement. Displacement has both magnitude and direction, and it can also have zero displacement with nonzero distance because distance has only magnitude.
   b. One-dimensional motion can have zero distance with a nonzero displacement. Displacement has both magnitude and direction, but it cannot have zero displacement with nonzero distance because distance has only magnitude.
   c. One-dimensional motion cannot have zero distance with a nonzero displacement. Displacement has both magnitude and direction, but it can have zero displacement with nonzero distance because distance has only magnitude and any motion will be the distance it moves.
   d. One-dimensional motion cannot have zero distance with a nonzero displacement. Displacement has both magnitude and direction, and it cannot have zero displacement with nonzero distance because distance has only magnitude.

2. In which example would you be correct in describing an object in motion while your friend would also be correct in describing that same object as being at rest?
   a. You are driving a car toward the east and your friend drives past you in the opposite direction with the same speed. In your frame of reference, you will be in motion. In your friend's frame of reference, you will be at rest.
   b. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, you will be in motion. In your friend's frame of reference, you will be at rest.
   c. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, your friend will be moving toward the west. In your friend's frame of reference, he will be at rest.
   d. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, your friend will be moving toward the east. In your friend's frame of reference, he will be at rest.

3. What does your car's odometer record?
   a. displacement
   b. distance
   c. both distance and displacement
   d. the sum of distance and displacement

2.2 Speed and Velocity

4. In the definition of velocity, what physical quantity is changing over time?
   a. speed
   b. distance
   c. magnitude of displacement
   d. position vector

5. Which of the following best describes the relationship between instantaneous velocity and instantaneous speed?
   a. Both instantaneous speed and instantaneous velocity are the same, even when there is a change in direction.
   b. Instantaneous speed and instantaneous velocity cannot be the same even if there is no change in direction of motion.
   c. Magnitude of instantaneous velocity is equal to instantaneous speed.
   d. Magnitude of instantaneous velocity is always greater than instantaneous speed.

2.3 Position vs. Time Graphs

6. Use the graph to describe what the runner's motion looks like.
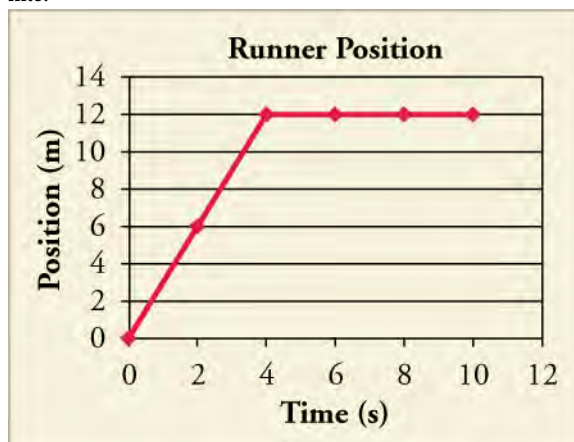
How are average velocity for only the first four seconds and instantaneous velocity related? What is the runner's net displacement over the time shown?
   a. The net displacement is 12 m and the average velocity is equal to the instantaneous velocity.
   b. The net displacement is 12 m and the average velocity

is two times the instantaneous velocity.

   c. The net displacement is 10 + 12 = 22 m and the average velocity is equal to the instantaneous velocity.

   d. The net displacement is 10 + 12 = 22 m and the average velocity is two times the instantaneous velocity.

7. A position vs. time graph of a frog swimming across a pond has two distinct straight-line sections. The slope of the first section is 1 m/s. The slope of the second section is 0 m/s. If each section lasts 1 second, then what is the frog's total average velocity?

   a. 0 m/s

   b. 2 m/s

   c. 0.5 m/s

   d. 1 m/s

2.4 Velocity vs. Time Graphs

8. A graph of velocity vs. time of a ship coming into a harbor is shown.
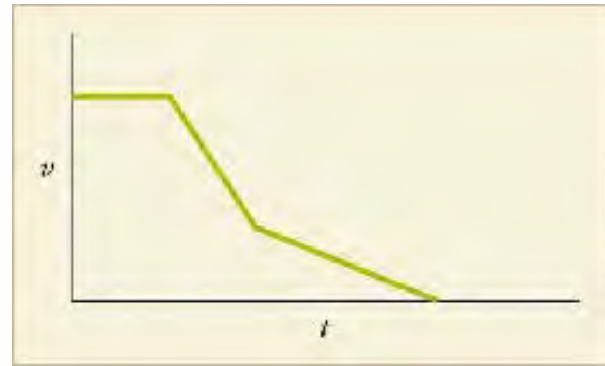
Describe the acceleration of the ship based on the graph.

   a. The ship is moving in the forward direction at a steady rate. Then it accelerates in the forward direction and then decelerates.

   b. The ship is moving in the forward direction at a steady rate. Then it turns around and starts decelerating, while traveling in the reverse direction. It then accelerates, but slowly.

   c. The ship is moving in the forward direction at a steady rate. Then it decelerates in the forward direction, and then continues to slow down in the forward direction, but with more deceleration.

   d. The ship is moving in the forward direction at a steady rate. Then it decelerates in the forward direction, and then continues to slow down in the forward direction, but with less deceleration.

Critical Thinking Items

2.1 Relative Motion, Distance, and Displacement

9. Boat A and Boat B are traveling at a constant speed in opposite directions when they pass each other. If a person in each boat describes motion based on the boat's own reference frame, will the description by a person in Boat A of Boat B's motion be the same as the description by a person in Boat B of Boat A's motion?

   a. Yes, both persons will describe the same motion because motion is independent of the frame of reference.

   b. Yes, both persons will describe the same motion because they will perceive the other as moving in the backward direction.

   c. No, the motion described by each of them will be different because motion is a relative term.

   d. No, the motion described by each of them will be different because the motion perceived by each will be opposite to each other.

10. Passenger A sits inside a moving train and throws a ball vertically upward. How would the motion of the ball be described by a fellow train passenger B and an observer C who is standing on the platform outside the train?

   a. Passenger B sees that the ball has vertical, but no horizontal, motion. Observer C sees the ball has vertical as well as horizontal motion.

   b. Passenger B sees the ball has vertical as well as horizontal motion. Observer C sees the ball has the vertical, but no horizontal, motion.

   c. Passenger B sees the ball has horizontal but no vertical motion. Observer C sees the ball has vertical as well as horizontal motion.

   d. Passenger B sees the ball has vertical as well as horizontal motion. Observer C sees the ball has horizontalbut no vertical motion.

2.2 Speed and Velocity

11. Is it possible to determine a car's instantaneous velocity from just the speedometer reading?

   a. No, it reflects speed but not the direction.

   b. No, it reflects the average speed of the car.

   c. Yes, it sometimes reflects instantaneous velocity of the car.

   d. Yes, it always reflects the instantaneous velocity of the car.

12. Terri, Aaron, and Jamal all walked along straight paths.

Terri walked 3.95 km north in 48 min. Aaron walked 2.65 km west in 31 min. Jamal walked 6.50 km south in 81 min. Which of the following correctly ranks the three boys in order from lowest to highest average speed?

a.   Jamal, Terri, Aaron
b.   Jamal, Aaron, Terri
c.   Terri, Jamal, Aaron
d.   Aaron, Terri, Jamal

13. Rhianna and Logan start at the same point and walk due north. Rhianna walks with an average velocity $v_{avg,R}$. Logan walks three times the distance in twice the time as Rhianna. Which of the following expresses Logan's average velocity in terms of $v_{avg,R}$?

a.   Logan's average velocity = $1.5 v_{avg,R}$.
b.   Logan's average velocity = $\frac{2}{3} v_{avg,R}$.
c.   Logan's average velocity = $3 v_{avg,R}$.
d.   Logan's average velocity = $\frac{1}{2} v_{avg,R}$.

2.3 Position vs. Time Graphs

14. Explain how you can use the graph of position vs. time to describe the change in velocity over time.
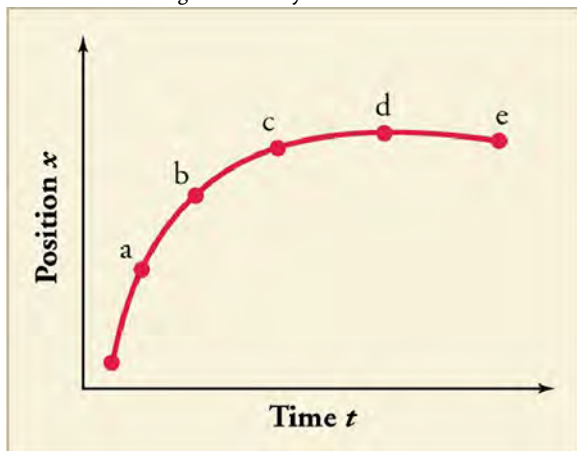
Problems

2.1 Relative Motion, Distance, and Displacement

16. In a coordinate system in which the direction to the right is positive, what are the distance and displacement of a person who walks 35 meters to the left, 18 meters to the right, and then 26 meters to the left?

a.   Distance is 79 m and displacement is −43 m.
b.   Distance is −79 m and displacement is 43 m.
c.   Distance is 43 m and displacement is −79 m.
d.   Distance is −43 m and displacement is 79 m.

17. Billy drops a ball from a height of 1 m. The ball bounces back to a height of 0.8 m, then bounces again to a height of 0.5 m, and bounces once more to a height of 0.2 m.

Identify the time (t_a, t_b, t_c, t_d, or t_e) at which at which the instantaneous velocity is greatest, the time at which it is zero, and the time at which it is negative.

2.4 Velocity vs. Time Graphs

15. Identify the time, or times, at which the instantaneous velocity is greatest, and the time, or times, at which it is negative. A sketch of velocity vs. time derived from the figure will aid in arriving at the correct answers.
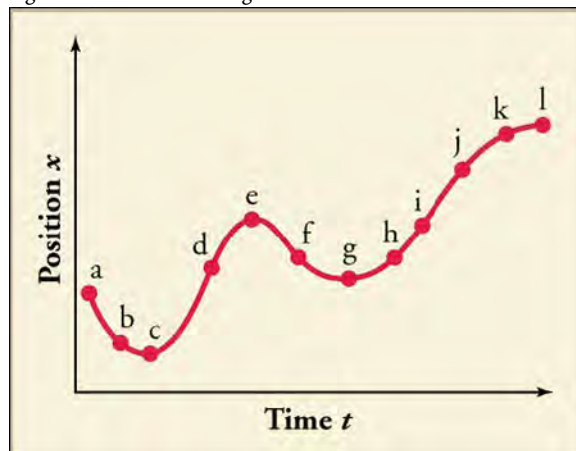
a.   The instantaneous velocity is greatest at *f*, and it is negative at *d*, *h*, *I*, *j*, and *k*.
b.   The instantaneous velocity is greatest at *e*, and it is negative at *a*, *b*, and *f*.
c.   The instantaneous velocity is greatest at *f*, and it is negative at *d*, *h*, *I*, *j*, and *k*
d.   The instantaneous velocity is greatest at *d*, and it is negative at *a*, *b*, and *f*.

Up is the positive direction. What are the total displacement of the ball and the total distance traveled by the ball?

a.   The displacement is equal to -4 m and the distance is equal to 4 m.
b.   The displacement is equal to 4 m and the distance is equal to 1 m.
c.   The displacement is equal to 4 m and the distance is equal to 1 m.
d.   The displacement is equal to -1 m and the distance is equal to 4 m.

2.2 Speed and Velocity

18. You sit in a car that is moving at an average speed of 86.4 km/h. During the 3.3 s that you glance out the window, how far has the car traveled?

a. 7.27 m
b. 79 m
c. 285 km
d. 1026 m

2.3 Position vs. Time Graphs

19. Using the graph, what is the average velocity for the whole 10 seconds?
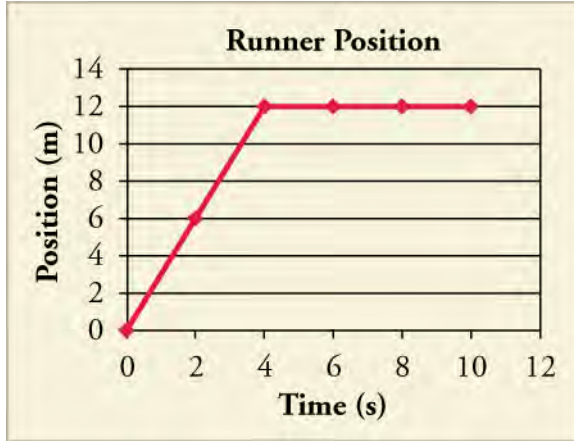
a. The total average velocity is 0 m/s.
b. The total average velocity is 1.2 m/s.
c. The total average velocity is 1.5 m/s.
d. The total average velocity is 3.0 m/s.

20. A train starts from rest and speeds up for 15 minutes until it reaches a constant velocity of 100 miles/hour. It stays at this speed for half an hour. Then it slows down for another 15 minutes until it is still. Which of the following correctly describes the position vs time graph of the train's journey?

a. The first 15 minutes is a curve that is concave upward, the middle portion is a straight line with slope 100 miles/hour, and the last portion is a concave downward curve.
b. The first 15 minutes is a curve that is concave downward, the middle portion is a straight line with slope 100 miles/hour, and the last portion is a concave upward curve.
c. The first 15 minutes is a curve that is concave upward, the middle portion is a straight line with slope zero, and the last portion is a concave downward curve.
d. The first 15 minutes is a curve that is concave downward, the middle portion is a straight line with slope zero, and the last portion is a concave upward curve.

2.4 Velocity vs. Time Graphs

21. You are characterizing the motion of an object by measuring the location of the object at discrete moments in time. What is the minimum number of data points you would need to estimate the average acceleration of the object?

a. 1
b. 2
c. 3
d. 4

22. Which option best describes the average acceleration from 40 to 70 s?
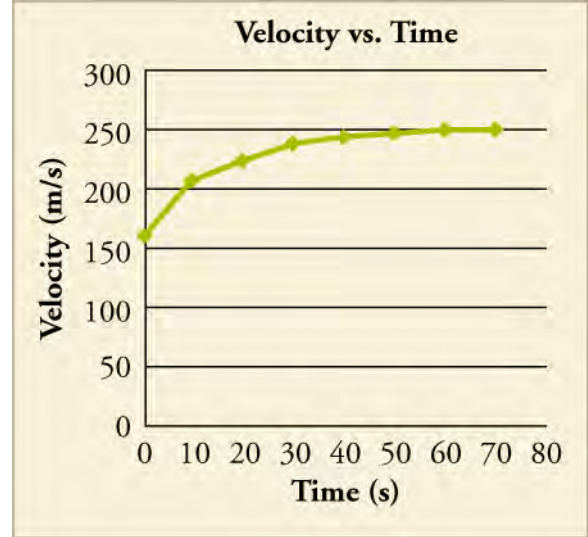
a. It is negative and smaller in magnitude than the initial acceleration.
b. It is negative and larger in magnitude than the initial acceleration.
c. It is positive and smaller in magnitude than the initial acceleration.
d. It is positive and larger in magnitude than the initial acceleration.

23. The graph shows velocity vs. time.
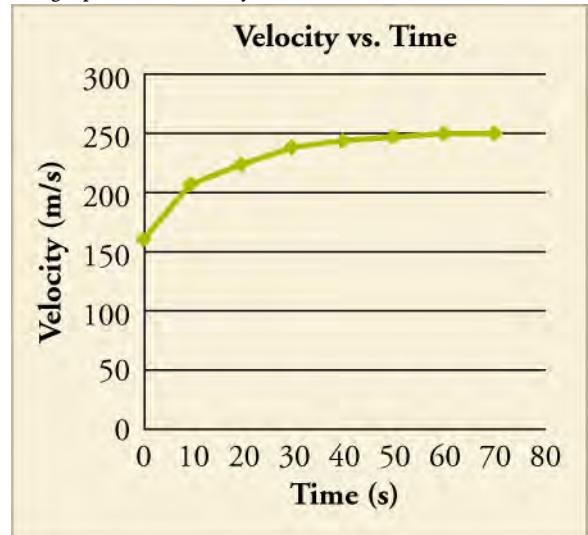
Calculate the net displacement using seven different divisions. Calculate it again using two divisions: $0 \rightarrow 40$ s

and 40 → 70 s . Compare. Using both, calculate the average velocity.

   a. Displacement and average velocity using seven divisions are 14,312.5 m and 204.5 m/s while with two divisions are 15,500 m and 221.4 m/s respectively.

   b. Displacement and average velocity using seven divisions are 15,500 m and 221.4 m/s while with two divisions are 14,312.5 m and 204.5 m/s respectively.

   c. Displacement and average velocity using seven divisions are 15,500 m and 204.5 m/s while with two divisions are 14,312.5 m and 221.4 m/s respectively.

   d. Displacement and average velocity using seven divisions are 14,312.5 m and 221.4 m/s while with two divisions are 15,500 m and 204.5 m/s respectively.

Performance Task

2.4 Velocity vs. Time Graphs

24. The National Mall in Washington, DC, is a national park containing most of the highly treasured memorials and museums of the United States. However, the National Mall also hosts many events and concerts. The map in shows the area for a benefit concert during which the president will speak. The concert stage is near the Lincoln Memorial. The seats and standing room for the crowd will stretch from the stage east to near the Washington Monument, as shown on the map. You are planning the logistics for the concert. Use the map scale to measure any distances needed to make the calculations below.
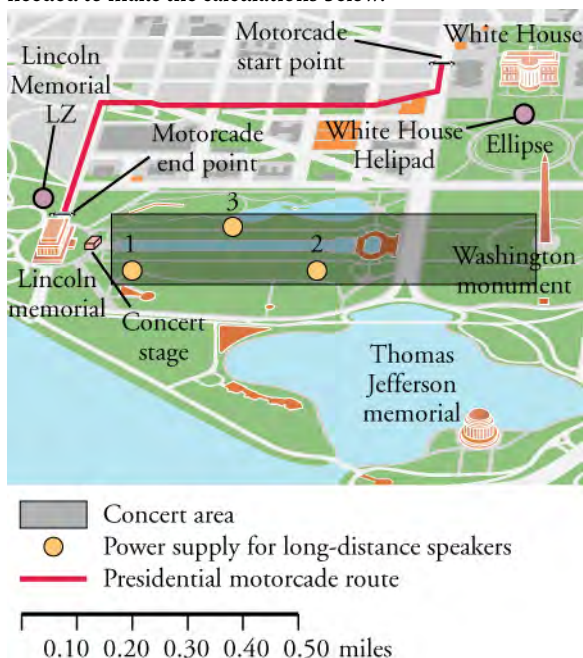
The park has three new long-distance speakers. They would like to use these speakers to broadcast the concert audio to other parts of the National Mall. The speakers can project sound up to 0.35 miles away but they must be connected to one of the power supplies within the concert area. What is the minimum amount of wire needed for each speaker, in miles, in order to project the audio to the following areas? Assume that wire cannot be placed over buildings or any memorials.

Part A. The center of the Jefferson Memorial using power supply 1 (This will involve an elevated wire that can travel over water.)

Part B. The center of the Ellipse using power supply 3 (This wire cannot travel over water.)

Part C. The president's motorcade will be traveling to the concert from the White House. To avoid concert traffic, the motorcade travels from the White House west down E Street and then turns south on 23rd Street to reach the Lincoln memorial. What minimum speed, in miles per hour to the nearest tenth, would the motorcade have to travel to make the trip in 5 minutes?

Part D. The president could also simply fly from the White House to the Lincoln Memorial using the presidential helicopter, Marine 1. How long would it take Marine 1, traveling slowly at 30 mph, to travel from directly above the White House landing zone (LZ) to directly above the Lincoln Memorial LZ? Disregard liftoff and landing times and report the travel time in minutes to the nearest minute.

TEST PREP

Multiple Choice

2.1 Relative Motion, Distance, and Displacement

25. Why should you specify a reference frame when describing motion?

   a. a description of motion depends on the reference frame

   b. motion appears the same in all reference frames

   c. reference frames affect the motion of an object

   d. you can see motion better from certain reference frames

26. Which of the following is true for the displacement of an object?

   a. It is always equal to the distance the object moved

between its initial and final positions.

b. It is both the straight line distance the object moved as well as the direction of its motion.

c. It is the direction the object moved between its initial and final positions.

d. It is the straight line distance the object moved between its initial and final positions.

27. If a biker rides west for 50 miles from his starting position, then turns and bikes back east 80 miles. What is his net displacement?

a. 130 miles

b. 30 miles east

c. 30 miles west

d. Cannot be determined from the information given

28. Suppose a train is moving along a track. Is there a single, correct reference frame from which to describe the train's motion?

a. Yes, there is a single, correct frame of reference because motion is a relative term.

b. Yes, there is a single, correct frame of reference which is in terms of Earth's position.

c. No, there is not a single, correct frame of reference because motion is a relative term.

d. No, there is not a single, correct frame of reference because motion is independent of frame of reference.

29. If a space shuttle orbits Earth once, what is the shuttle's distance traveled and displacement?

a. Distance and displacement both are zero.

b. Distance is circumference of the circular orbit while displacement is zero.

c. Distance is zero while the displacement is circumference of the circular orbit.

d. Distance and displacement both are equal to circumference of the circular orbit.

2.2 Speed and Velocity

30. Four bicyclists travel different distances and times along a straight path. Which cyclist traveled with the greatest average speed?

a. Cyclist 1 travels 95 m in 27 s.

b. Cyclist 2 travels 87 m in 22 s.

c. Cyclist 3 travels 106 m in 26 s.

d. Cyclist 4 travels 108 m in 24 s.

31. A car travels with an average velocity of 23 m/s for 82 s. Which of the following could NOT have been the car's displacement?

a. 1,700 m east

b. 1,900 m west

c. 1,600 m north

d. 1,500 m south

32. A bicyclist covers the first leg of a journey that is d_1 meters long in t_1 seconds, at a speed of v_1 m/s, and the second leg of d_2 meters in t_2 seconds, at a speed of v_2 m/s. If his average speed is equal to the average of v_1 and v_2, then which of the following is true?

a. $t_1 = t_2$

b. $t_1 \neq t_2$

c. $d_1 = d_2$

d. $d_1 \neq d_2$

33. A car is moving on a straight road at a constant speed in a single direction. Which of the following statements is true?

a. Average velocity is zero.

b. The magnitude of average velocity is equal to the average speed.

c. The magnitude of average velocity is greater than the average speed.

d. The magnitude of average velocity is less than the average speed.

2.3 Position vs. Time Graphs

34. What is the slope of a straight line graph of position vs. time?

a. Velocity

b. Displacement

c. Distance

d. Acceleration

35. Using the graph, what is the runner's velocity from 4 to 10 s?

a. −3 m/s

b. 0 m/s

c. 1.2 m/s

d. 3 m/s

2.4 Velocity vs. Time Graphs

36. What does the area under a velocity vs. time graph line represent?
    a. acceleration
    b. displacement
    c. distance
    d. instantaneous velocity

37. An object is moving along a straight path with constant

Short Answer

2.1 Relative Motion, Distance, and Displacement

38. While standing on a sidewalk facing the road, you see a bicyclist passing by toward your right. In the reference frame of the bicyclist, in which direction are you moving?
    a. in the same direction of motion as the bicyclist
    b. in the direction opposite the motion of the bicyclist
    c. stationary with respect to the bicyclist
    d. in the direction of velocity of the bicyclist

39. Maud sends her bowling ball straight down the center of the lane, getting a strike. The ball is brought back to the holder mechanically. What are the ball's net displacement and distance traveled?
    a. Displacement of the ball is twice the length of the lane, while the distance is zero.
    b. Displacement of the ball is zero, while the distance is twice the length of the lane.
    c. Both the displacement and distance for the ball are equal to zero.
    d. Both the displacement and distance for the ball are twice the length of the lane.

40. A fly buzzes four and a half times around Kit Yan's head. The fly ends up on the opposite side from where it started. If the diameter of his head is $14\,\text{cm}$, what is the total distance the fly travels and its total displacement?
    a. The distance is $63\pi\,\text{cm}$ with a displacement of zero.
    b. The distance is $7\,\text{cm}$ with a displacement of zero.
    c. The distance is $63\pi\,\text{cm}$ with a displacement of $14\,\text{cm}$.
    d. The distance is $7\,\text{cm}$ with a displacement of $63\pi\,\text{cm}$.

2.2 Speed and Velocity

41. Rob drove to the nearest hospital with an average speed of v m/s in t seconds. In terms of t, if he drives home on the same path, but with an average speed of 3v m/s, how

acceleration. A velocity vs. time graph starts at 0 and ends at 10 m/s, stretching over a time-span of 15 s. What is the object's net displacement?
    a. 75 m
    b. 130 m
    c. 150 m
    d. cannot be determined from the information given

long is the return trip home?
    a. t/6
    b. t/3
    c. 3t
    d. 6t

42. What can you infer from the statement, *Velocity of an object is zero*?
    a. Object is in linear motion with constant velocity.
    b. Object is moving at a constant speed.
    c. Object is either at rest or it returns to the initial point.
    d. Object is moving in a straight line without changing its direction.

43. An object has an average speed of 7.4 km/h. Which of the following describes two ways you could increase the average speed of the object to 14.8 km/h?
    a. Reduce the distance that the object travels by half, keeping the time constant, or keep the distance constant and double the time.
    b. Double the distance that the object travels, keeping the time constant, or keep the distance constant and reduce the time by half.
    c. Reduce the distance that the object travels to one-fourth, keeping the time constant, or keep the distance constant and increase the time by fourfold.
    d. Increase the distance by fourfold, keeping the time constant, or keep the distance constant and reduce the time by one-fourth.

44. Swimming one lap in a pool is defined as going across a pool and back again. If a swimmer swims 3 laps in 9 minutes, how can his average velocity be zero?
    a. His average velocity is zero because his total distance is zero.
    b. His average velocity is zero because his total displacement is zero.
    c. His average velocity is zero because the number of laps completed is an odd number.
    d. His average velocity is zero because the velocity of each successive lap is equal and opposite.

2.3 Position vs. Time Graphs

45. A hockey puck is shot down the arena in a straight line. Assume it does not slow until it is stopped by an opposing player who sends it back in the direction it came. The players are 20 m apart and it takes 1 s for the puck to go there and back. Which of the following describes the graph of the displacement over time? Consider the initial direction of the puck to be positive.
    a. The graph is an upward opening V.
    b. The graph is a downward opening V.
    c. The graph is an upward opening U.
    d. The graph is downward opening U.

46. A defensive player kicks a soccer ball 20 m back to her own goalie. It stops just as it reaches her. She sends it back to the player. Without knowing the time it takes, draw a rough sketch of the displacement over time. Does this graph look similar to the graph of the hockey puck from the previous question?
    a. Yes, the graph is similar to the graph of the hockey puck.
    b. No, the graph is not similar to the graph of the hockey puck.
    c. The graphs cannot be compared without knowing the time the soccer ball was rolling.

47. What are the net displacement, total distance traveled, and total average velocity in the previous two problems?
    a. net displacement = 0 m, total distance = 20 m, total average velocity = 20 m/s
    b. net displacement = 0 m, total distance = 40 m, total average velocity = 20 m/s
    c. net displacement = 0 m, total distance = 20 m, total average velocity = 0 m/s
    d. net displacement = 0 m, total distance = 40 m, total average velocity = 0 m/s

48. A bee flies straight at someone and then back to its hive along the same path. Assuming it takes no time for the bee to speed up or slow down, except at the moment it changes direction, how would the graph of position vs time look? Consider the initial direction to be positive.
    a. The graph will look like a downward opening V shape.
    b. The graph will look like an upward opening V shape.
    c. The graph will look like a downward opening parabola.
    d. The graph will look like an upward opening parabola.

2.4 Velocity vs. Time Graphs

49. What would the velocity vs. time graph of the object whose position is shown in the graph look like?

Position vs. Time
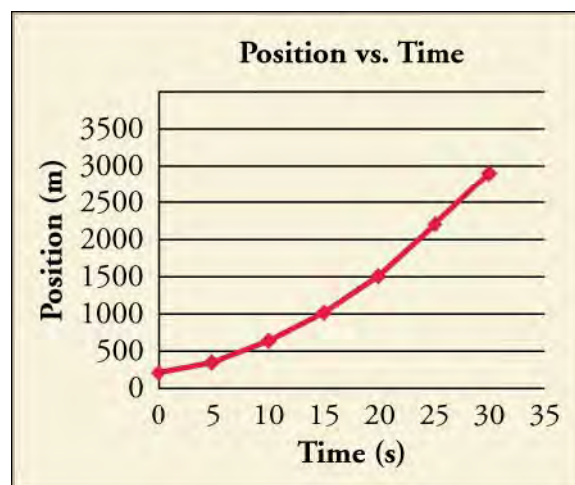
a. It is a straight line with negative slope.
b. It is a straight line with positive slope.
c. It is a horizontal line at some negative value.
d. It is a horizontal line at some positive value.

50. Which statement correctly describes the object's speed, as well as what a graph of acceleration vs. time would look like?
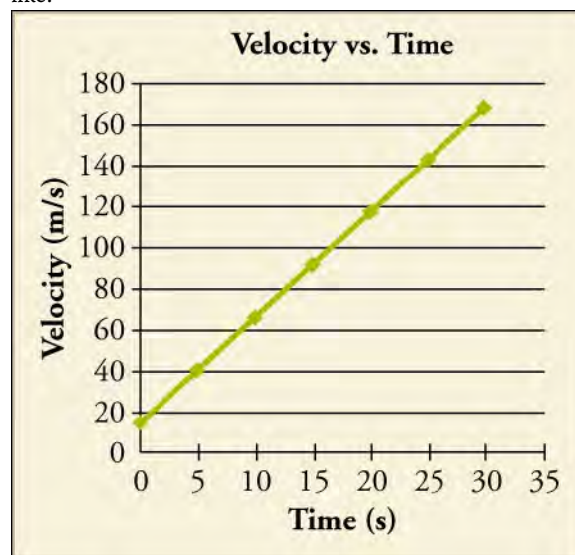
Velocity vs. Time

a. The object is not speeding up, and the acceleration vs. time graph is a horizontal line at some negative value.
b. The object is not speeding up, and the acceleration vs. time graph is a horizontal line at some positive value.
c. The object is speeding up, and the acceleration vs. time graph is a horizontal line at some negative value.
d. The object is speeding up, and the acceleration vs. time graph is a horizontal line at some positive value.

51. Calculate that object's net displacement over the time shown.

a.   540 m
b.   2,520 m
c.   2,790 m
d.   5,040 m

52. What is the object's average velocity?

a.   18 m/s
b.   84 m/s
c.   93 m/s
d.   168 m/s

Extended Response

2.1 Relative Motion, Distance, and Displacement

53. Find the distance traveled from the starting point for each path.
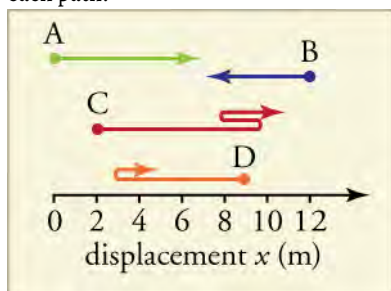
Which path has the maximum distance?
a.   The distance for Path A is 6 m, Path B is 4 m, Path C is 12 m and for Path D is 7 m. The net displacement for Path A is 7 m, Path B is −4m, Path C is 8 m and for Path D is −5m. Path C has maximum distance and it is equal to 12 meters.
b.   The distance for Path A is 6 m, Path B is 4 m, Path C is 8 m and for Path D is 7 m. The net displacement for Path A is 6 m, Path B is −4m, Path C is 12 m and for Path D is −5 m. Path A has maximum distance and it is equal to 6 meters.
c.   The distance for Path A is 6 m, Path B is 4 m, Path C is 12 m and for Path D is 7 m. The net displacement for Path A is 6 m, Path B is −4 m, Path C is 8 m and for Path D is −5 m. Path C has maximum distance
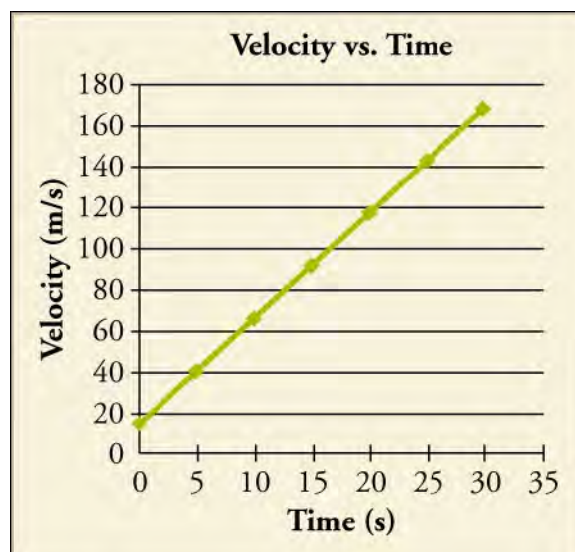
and it is equal to 12 meters.
d.   The distance for Path A is 6 m, Path B is −4 m, Path C is 12 m and for Path D is −5 m. The net displacement for Path A is 7 m, Path B is 4 m, Path C is 8 m and for Path D is 7 m. Path A has maximum distance and it is equal to 6 m.

54. Alan starts from his home and walks 1.3 km east to the library. He walks an additional 0.68 km east to a music store. From there, he walks 1.1 km north to a friend's house and an additional 0.42 km north to a grocery store before he finally returns home along the same path. What is his final displacement and total distance traveled?
a.   Displacement is 0 km and distance is 7 km.
b.   Displacement is 0 km and distance is 3.5 km.
c.   Displacement is 7 km towards west and distance is 7 km.
d.   Displacement is 3.5 km towards east and distance is 3.5 km.

2.2 Speed and Velocity

55. Two runners start at the same point and jog at a constant speed along a straight path. Runner A starts at time t = 0 s, and Runner B starts at time t = 2.5 s. The runners both reach a distance 64 m from the starting point at time t = 25 s. If the runners continue at the same speeds, how far from the starting point will each be at time t = 45 s?
a.   Runner A will be 72×10^3 m away and Runner B

will be 59.5×10^3 m away from the starting point.

   b.   Runner A will be 1.2×10^2 m away and runner B will be 1.1×10^2 m away from the starting point.
   c.   Runner A will be 1.2×10^2 m away and Runner B will be 1.3×10^2 m away from the starting point.
   d.   Runner A will be 7.2×10^2 m away and Runner B will be 1.3×10^2 m away from the starting point.

56.   A father and his daughter go to the bus stop that is located 75 m from their front door. The father walks in a straight line while his daughter runs along a varied path. Despite the different paths, they both end up at the bus stop at the same time. The father's average speed is 2.2 m/s, and his daughter's average speed is 3.5 m/s. (a) How long does it take the father and daughter to reach the bus stop? (b) What was the daughter's total distance traveled? (c) If the daughter maintained her same average speed and traveled in a straight line like her father, how far beyond the bus stop would she have traveled?

   a.   (a) 21.43 s (b) 75 m (c) 0 m
   b.   (a) 21.43 s (b) 119 m (c) 44 m
   c.   (a) 34 s (b) 75 m (c) 0 m
   d.   (a) 34 s (b) 119 m (c) 44 m

2.3 Position vs. Time Graphs

57.   What kind of motion would create a position graph like the one shown?

Position vs. Time
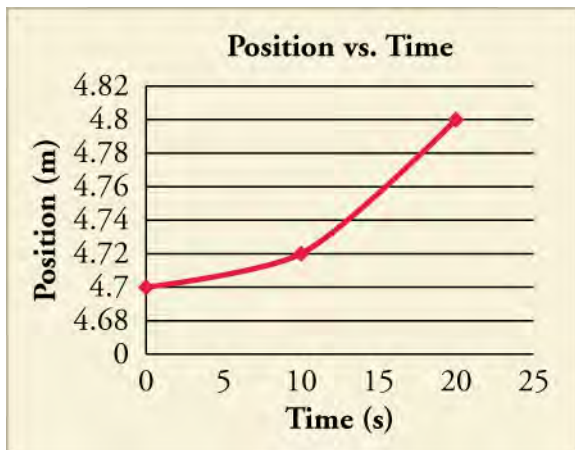
   a.   uniform motion
   b.   any motion that accelerates
   c.   motion that stops and then starts
   d.   motion that has constant velocity

58.   What is the average velocity for the whole time period shown in the graph?

Position vs. Time
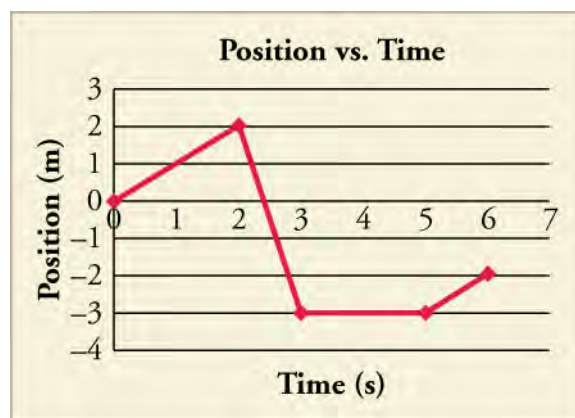
   a. $-\frac{1}{3}$ m/s
   b. $-\frac{3}{4}$ m/s
   c. $\frac{1}{3}$ m/s
   d. $\frac{3}{4}$ m/s

2.4 Velocity vs. Time Graphs

59.   Consider the motion of the object whose velocity is charted in the graph.
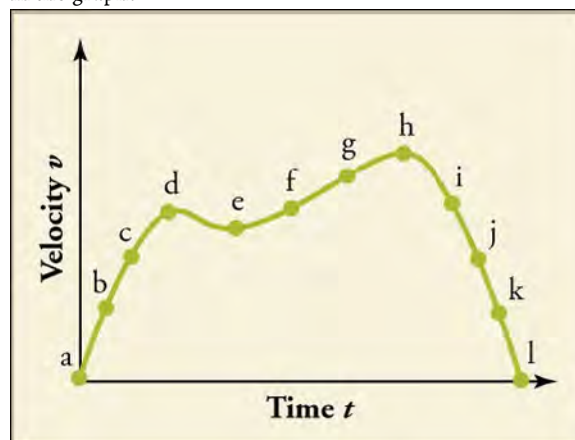
During which points is the object slowing down and speeding up?

   a.   It is slowing down between d and e. It is speeding up between a and d and e and h
   b.   It is slowing down between a and d and e and h. It is speeding up between d and e and then after i.
   c.   It is slowing down between d and e and then after h. It is speeding up between a and d and e and h.
   d.   It is slowing down between a and d and e and h. It is speeding up between d and e and then after i.

60.   Divide the graph into approximate sections, and use those sections to graph the velocity vs. time of the object.

Position vs. Time
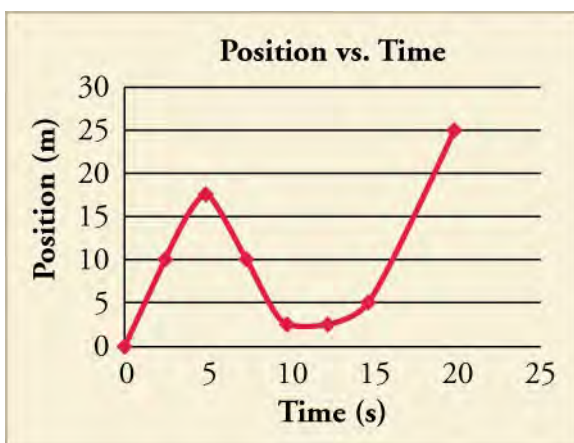
Then calculate the acceleration during each section, and calculate the approximate average velocity.

a. Acceleration is zero and average velocity is 1.25 m/s.

b. Acceleration is constant with some positive value and average velocity is 1.25 m/s.

c. Acceleration is zero and average velocity is 0.25 m/s.

d. Acceleration is constant with some positive value and average velocity is 0.25 m/s.

CHAPTER 3
Acceleration

Figure 3.1 A plane slows down as it comes in for landing in St. Maarten. Its acceleration is in the opposite direction of its velocity. (Steve Conry, Flickr)

Chapter Outline

INTRODUCTION You may have heard the term *accelerator*, referring to the gas pedal in a car. When the gas pedal is pushed down, the flow of gasoline to the engine increases, which increases the car's velocity. Pushing on the gas pedal results in acceleration because the velocity of the car increases, and acceleration is defined as a change in velocity. You need two quantities to define velocity: a speed and a direction. Changing either of these quantities, or both together, changes the velocity. You may be surprised to learn that pushing on the brake pedal or turning the steering wheel also causes acceleration. The first reduces the *speed* and so changes the velocity, and the second changes the *direction* and also changes the velocity.

In fact, any change in velocity—whether positive, negative, directional, or any combination of these—is called an acceleration in physics. The plane in the picture is said to be accelerating because its velocity is decreasing as it prepares to land. To begin our study of acceleration, we need to have a clear understanding of what acceleration means.

3.1 Acceleration

Section Learning Objectives
By the end of this section, you will be able to do the following:

- Explain acceleration and determine the direction and magnitude of acceleration in one dimension
- Analyze motion in one dimension using kinematic equations and graphic representations

Section Key Terms

average acceleration   instantaneous acceleration   negative acceleration

Defining Acceleration

Throughout this chapter we will use the following terms: *time*, *displacement*, *velocity*, and *acceleration*. Recall that each of these terms has a designated variable and SI unit of measurement as follows:

- Time: t, measured in seconds (s)
- Displacement: Δd, measured in meters (m)
- Velocity: v, measured in meters per second (m/s)
- Acceleration: a, measured in meters per second per second (m/s², also called meters per second squared)
- Also note the following:
  - Δ means *change in*
  - The subscript o refers to an initial value; sometimes subscript i is instead used to refer to initial value.
  - The subscript f refers to final value
  - A bar over a symbol, such as $\overline{a}$, means *average*

Acceleration is the change in velocity divided by a period of time during which the change occurs. The SI units of velocity are m/s and the SI units for time are s, so the SI units for acceleration are m/s². **Average acceleration** is given by

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

Average acceleration is distinguished from **instantaneous acceleration**, which is acceleration at a specific instant in time. The magnitude of acceleration is often not constant over time. For example, runners in a race accelerate at a greater rate in the first second of a race than during the following seconds. You do not need to know all the instantaneous accelerations at all times to calculate average acceleration. All you need to know is the change in velocity (i.e., the final velocity minus the initial velocity) and the change in time (i.e., the final time minus the initial time), as shown in the formula. Note that the average acceleration can be positive, negative, or zero. A **negative acceleration** is simply an acceleration in the negative direction.

Keep in mind that although acceleration points in the same direction as the *change* in velocity, it is not always in the direction of the velocity itself. When an object slows down, its acceleration is opposite to the direction of its velocity. In everyday language, this is called deceleration; but in physics, it is acceleration—whose direction happens to be opposite that of the velocity. For now, let us assume that motion to the right along the *x*-axis is *positive* and motion to the left is *negative*.

Figure 3.2 shows a car with positive acceleration in (a) and negative acceleration in (b). The arrows represent vectors showing both direction and magnitude of velocity and acceleration.
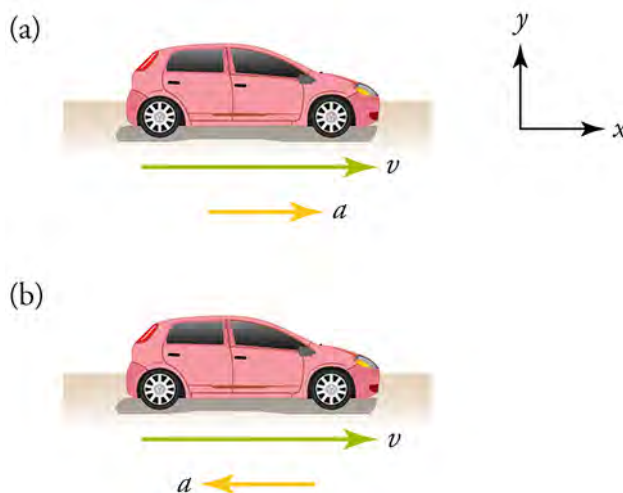
Figure 3.2 The car is speeding up in (a) and slowing down in (b).

Velocity and acceleration are both vector quantities. Recall that vectors have both magnitude and direction. An object traveling at a constant velocity—therefore having no acceleration—does accelerate if it changes direction. So, turning the steering wheel of a moving car makes the car accelerate because the velocity changes direction.

Virtual Physics

The Moving Man

With this animation in , you can produce both variations of acceleration and velocity shown in Figure 3.2, plus a few more variations. Vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Try changing acceleration from positive to negative while the man is moving. We will come back to this animation and look at the *Charts* view when we study graphical representation of motion.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK
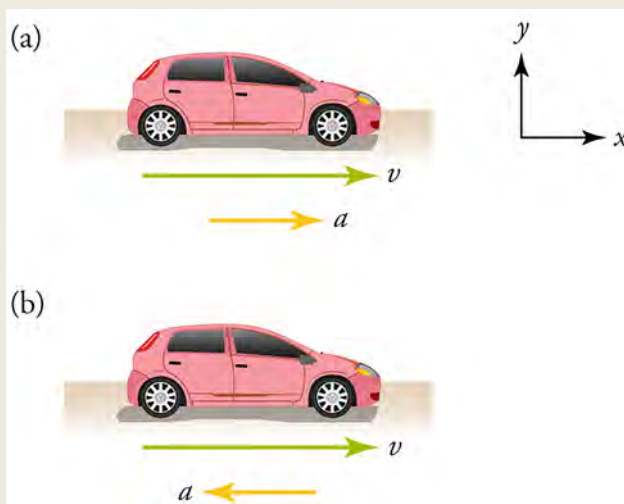
Figure 3.3

Which part, (a) or (b), is represented when the velocity vector is on the positive side of the scale and the acceleration vector is set on the negative side of the scale? What does the car's motion look like for the given scenario?

a. Part (a). The car is slowing down because the acceleration and the velocity vectors are acting in the opposite direction.
b. Part (a). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.
c. Part (b). The car is slowing down because the acceleration and velocity vectors are acting in the opposite directions.
d. Part (b). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.

Calculating Average Acceleration

Look back at the equation for average acceleration. You can see that the calculation of average acceleration involves three values: change in time, (Δt); change in velocity, (Δv); and acceleration (a).

Change in time is often stated as a time interval, and change in velocity can often be calculated by subtracting the initial velocity from the final velocity. Average acceleration is then simply change in velocity divided by change in time. Before you begin calculating, be sure that all distances and times have been converted to meters and seconds. Look at these examples of acceleration of a subway train.

✳ WORKED EXAMPLE

An Accelerating Subway Train

A subway train accelerates from rest to 30.0 km/h in 20.0 s. What is the average acceleration during that time interval?

Strategy

Start by making a simple sketch.
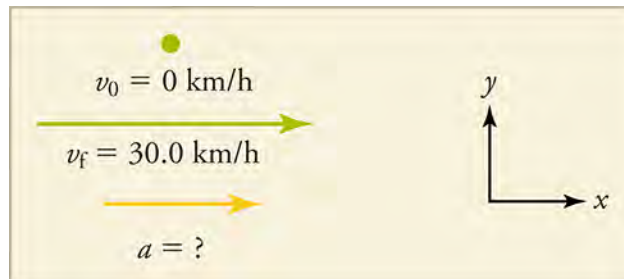
Figure 3.4

This problem involves four steps:

1.   Convert to units of meters and seconds.
2.   Determine the change in velocity.
3.   Determine the change in time.
4.   Use these values to calculate the average acceleration.

Solution

1.   Identify the knowns. Be sure to read the problem for given information, which may not *look* like numbers. When the problem states that the train starts from rest, you can write down that the initial velocity is 0 m/s. Therefore, $v_0 = 0$; $v_f = 30.0$ km/h; and $\Delta t = 20.0$ s.
2.   Convert the units.

$$\frac{30.0 \text{ km}}{\text{h}} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 8.333 \frac{\text{m}}{\text{s}} \qquad \boxed{3.1}$$

3.   Calculate change in velocity, $\Delta v = v_f - v_0 = 8.333$ m/s $- 0 = + 8.333$ m/s, where the plus sign means the change in velocity is to the right.
4.   We know Δt, so all we have to do is insert the known values into the formula for average acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8.333 \text{ m/s}}{20.00 \text{ s}} = +0.417 \frac{\text{m}}{\text{s}^2} \qquad \boxed{3.2}$$

Discussion

The plus sign in the answer means that acceleration is to the right. This is a reasonable conclusion because the train starts from rest and ends up with a velocity directed to the right (i.e., positive). So, acceleration is in the same direction as the *change* in velocity, as it should be.

✳ WORKED EXAMPLE

An Accelerating Subway Train

Now, suppose that at the end of its trip, the train slows to a stop in 8.00 s from a speed of 30.0 km/h. What is its average acceleration during this time?

Strategy

Again, make a simple sketch.
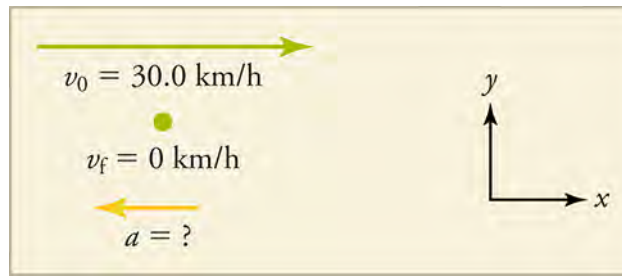
Figure 3.5

In this case, the train is decelerating and its acceleration is negative because it is pointing to the left. As in the previous example, we must find the change in velocity and change in time, then solve for acceleration.

Solution

1. Identify the knowns: $v_0 = 30.0$ km/h; $v_f = 0$; and $\Delta t = 8.00$ s.
2. Convert the units. From the first problem, we know that 30.0 km/h = 8.333 m/s.
3. Calculate change in velocity, $\Delta v = v_f - v_0 = 0 - 8.333$ m/s $= -8.333$ m/s, where the minus sign means that the change in velocity points to the left.
4. We know $\Delta t = 8.00$ s, so all we have to do is insert the known values into the equation for average acceleration.

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{-8.333 \text{ m/s}}{8.00 \text{ s}} = -1.04 \frac{\text{m}}{\text{s}^2}$$

3.3

Discussion

The minus sign indicates that acceleration is to the left. This is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would reduce the velocity. Again, acceleration is in the same direction as the *change* in velocity, which is negative in this case. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

> **TIPS FOR SUCCESS**
> - It is easier to get plus and minus signs correct if you always assume that motion is away from zero and toward positive values on the x-axis. This way *v* always starts off being positive and points to the right. If speed is increasing, then acceleration is positive and also points to the right. If speed is decreasing, then acceleration is negative and points to the left.
> - It is a good idea to carry two extra significant figures from step-to-step when making calculations. Do not round off with each step. When you arrive at the final answer, apply the rules of significant figures for the operations you carried out and round to the correct number of digits. Sometimes this will make your answer slightly more accurate.

Practice Problems

1. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?
   a. -0.23 m/s^2
   b. -4.29 m/s^2
   c. 0.23 m/s^2
   d. 4.29 m/s^2

2. A women backs her car out of her garage with an acceleration of 1.40 m/s^2. How long does it take her to reach a speed of 2.00 m/s?
   a. 0.70 s
   b. 1.43 s
   c. 2.80 s
   d. 3.40 s

👁 WATCH PHYSICS

Acceleration

This video shows the basic calculation of acceleration and some useful unit conversions.

Click to view content (https://www.khanacademy.org/embed_video?v=FOkQszg1-j8)

> **GRASP CHECK**
>
> Why is acceleration a vector quantity?
> a. It is a vector quantity because it has magnitude as well as direction.
> b. It is a vector quantity because it has magnitude but no direction.
> c. It is a vector quantity because it is calculated from distance and time.
> d. It is a vector quantity because it is calculated from speed and time.

> **GRASP CHECK**
>
> What will be the change in velocity each second if acceleration is 10 m/s/s?
> a. An acceleration of 10 m/s/s means that every second, the velocity increases by 10 m/s.
> b. An acceleration of 10 m/s/s means that every second, the velocity decreases by 10 m/s.
> c. An acceleration of 10 m/s/s means that every 10 seconds, the velocity increases by 10 m/s.
> d. An acceleration of 10 m/s/s means that every 10 seconds, the velocity decreases by 10 m/s.

Snap Lab

Measure the Acceleration of a Bicycle on a Slope

In this lab you will take measurements to determine if the acceleration of a moving bicycle is constant. If the acceleration is constant, then the following relationships hold: $\bar{v} = \frac{\Delta d}{\Delta t} = \frac{v_0 + v_f}{2}$ If $v_0 = 0$, then $v_f = 2\bar{v}$ and $\bar{a} = \frac{v_f}{\Delta t}$

You will work in pairs to measure and record data for a bicycle coasting down an incline on a smooth, gentle slope. The data will consist of distances traveled and elapsed times.

- Find an open area to minimize the risk of injury during this lab.

- stopwatch
- measuring tape
- bicycle

1. Find a gentle, paved slope, such as an incline on a bike path. The more gentle the slope, the more accurate your data will likely be.
2. Mark uniform distances along the slope, such as 5 m, 10 m, etc.
3. Determine the following roles: the bike rider, the timer, and the recorder. The recorder should create a data table to collect the distance and time data.
4. Have the rider at the starting point at rest on the bike. When the timer calls *Start*, the timer starts the stopwatch and the rider begins coasting down the slope on the bike without pedaling.
5. Have the timer call out the elapsed times as the bike passes each marked point. The recorder should record the times in the data table. It may be necessary to repeat the process to practice roles and make necessary adjustments.
6. Once acceptable data has been recorded, switch roles. Repeat Steps 3–5 to collect a second set of data.
7. Switch roles again to collect a third set of data.
8. Calculate average acceleration for each set of distance-time data. If your result for $\bar{a}$ is not the same for different pairs of Δv and Δt, then acceleration is not constant.
9. Interpret your results.

GRASP CHECK

If you graph the average velocity (*y*-axis) vs. the elapsed time (*x*-axis), what would the graph look like if acceleration is uniform?

a. a horizontal line on the graph
b. a diagonal line on the graph
c. an upward-facing parabola on the graph
d. a downward-facing parabola on the graph

Check Your Understanding

3. What are three ways an object can accelerate?

a. By speeding up, maintaining constant velocity, or changing direction
b. By speeding up, slowing down, or changing direction
c. By maintaining constant velocity, slowing down, or changing direction
d. By speeding up, slowing down, or maintaining constant velocity

4. What is the difference between average acceleration and instantaneous acceleration?

a. Average acceleration is the change in displacement divided by the elapsed time; instantaneous acceleration is the acceleration at a given point in time.
b. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in displacement divided by the elapsed time.
c. Average acceleration is the change in velocity divided by the elapsed time; instantaneous acceleration is acceleration at a given point in time.
d. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in velocity divided by the elapsed time.

5. What is the rate of change of velocity called?

a. Time
b. Displacement
c. Velocity
d. Acceleration

3.2 Representing Acceleration with Equations and Graphs

Section Learning Objectives
By the end of this section, you will be able to do the following:

• Explain the kinematic equations related to acceleration and illustrate them with graphs
• Apply the kinematic equations and related graphs to problems involving acceleration

Section Key Terms

acceleration due to gravity	kinematic equations	uniform acceleration

How the Kinematic Equations are Related to Acceleration

We are studying concepts related to motion: time, displacement, velocity, and especially acceleration. We are only concerned with motion in one dimension. The **kinematic equations** apply to conditions of constant acceleration and show how these concepts are related. **Constant acceleration** is acceleration that does not change over time. The first kinematic equation relates displacement d, average velocity $\bar{v}$, and time t.

$$d = d_0 + \bar{v}t$$

<div style="text-align:right">3.4</div>

The initial displacement d_0 is often 0, in which case the equation can be written as $\bar{v} = \frac{d}{t}$

This equation says that average velocity is displacement per unit time. We will express velocity in meters per second. If we graph displacement versus time, as in Figure 3.6, the slope will be the velocity. Whenever a rate, such as velocity, is represented graphically, time is usually taken to be the independent variable and is plotted along the x axis.
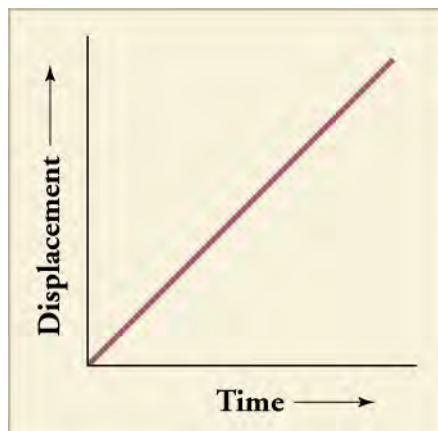
Figure 3.6 The slope of displacement versus time is velocity.

The second kinematic equation, another expression for average velocity $\bar{v}$, is simply the initial velocity plus the final velocity divided by two.

$$\bar{v} = \frac{v_0 + v_f}{2}$$

<div style="text-align: right">3.5</div>

Now we come to our main focus of this chapter; namely, the kinematic equations that describe motion with constant acceleration. In the third kinematic equation, acceleration is the rate at which velocity increases, so velocity at any point equals initial velocity plus acceleration multiplied by time

$$v = v_0 + at \text{ Also, if we start from rest } (v_0 = 0\ ), \text{ we can write } a = \frac{v}{t}$$

<div style="text-align: right">3.6</div>

Note that this third kinematic equation does not have displacement in it. Therefore, if you do not know the displacement and are not trying to solve for a displacement, this equation might be a good one to use.

The third kinematic equation is also represented by the graph in Figure 3.7.
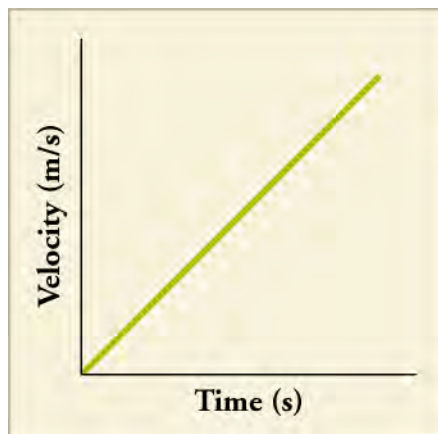
Figure 3.7 The slope of velocity versus time is acceleration.

The fourth kinematic equation shows how displacement is related to acceleration

$$d = d_0 + v_0 t + \frac{1}{2}at^2.$$

<div style="text-align: right">3.7</div>

When starting at the origin, $d_0 = 0$ and, when starting from rest, $v_0 = 0$, in which case the equation can be written as

$$a = \frac{2d}{t^2}.$$

This equation tells us that, for constant acceleration, the slope of a plot of $2d$ versus t^2 is acceleration, as shown in Figure 3.8.
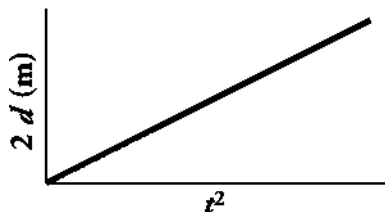
Figure 3.8 When acceleration is constant, the slope of $2d$ versus t^2 gives the acceleration.

The fifth kinematic equation relates velocity, acceleration, and displacement

$$v^2 = v_0^2 + 2a\,(d - d_0).$$ 3.8

This equation is useful for when we do not know, or do not need to know, the time.

When starting from rest, the fifth equation simplifies to

$$a = \frac{v^2}{2d}.$$

According to this equation, a graph of velocity squared versus twice the displacement will have a slope equal to acceleration.
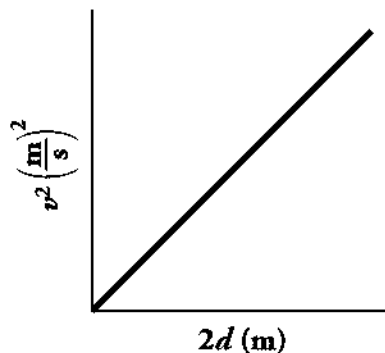
Figure 3.9

Note that, in reality, knowns and unknowns will vary. Sometimes you will want to rearrange a kinematic equation so that the knowns are the values on the axes and the unknown is the slope. Sometimes the intercept will not be at the origin (0,0). This will happen when v_0 or d_0 is not zero. This will be the case when the object of interest is already in motion, or the motion begins at some point other than at the origin of the coordinate system.

Virtual Physics

The Moving Man (Part 2)

Look at the Moving Man simulation again and this time use the *Charts* view. Again, vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Observe how the graphs of position, velocity, and acceleration vary with time. Note which are linear plots and which are not.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

On a velocity versus time plot, what does the slope represent?

a.   Acceleration

b.   Displacement

c.   Distance covered

d.   Instantaneous velocity

GRASP CHECK

On a position versus time plot, what does the slope represent?

a.   Acceleration

b.   Displacement

c.   Distance covered

d.   Instantaneous velocity

The kinematic equations are applicable when you have constant acceleration.

1. $d = d_0 + \bar{v}t$, or $\bar{v} = \frac{d}{t}$ when $d_\mathrm{o} = 0$

2. $\bar{v} = \frac{v_0 + v_\mathrm{f}}{2}$

3. $v = v_0 + at$, or $a = \frac{v}{t}$ when $v_\mathrm{o} = 0$

4. $d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_\mathrm{o} = 0$ and $v_\mathrm{o} = 0$

5. $v^2 = v_0^2 + 2a\,(d - d_0)$, or $a = \frac{2d}{t^2}$ when $d_\mathrm{o} = 0$ and $v_\mathrm{o} = 0$

Applying Kinematic Equations to Situations of Constant Acceleration

Problem-solving skills are essential to success in a science and life in general. The ability to apply broad physical principles, which are often represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Essential analytical skills will be developed by solving problems in this text and will be useful for understanding physics and science in general throughout your life.

Problem-Solving Steps

While no single step-by-step method works for every problem, the following general procedures facilitate problem solving and make the answers more meaningful. A certain amount of creativity and insight are required as well.

1. *Examine the situation to determine which physical principles are involved. It is vital to draw a simple sketch at the outset.* Decide which direction is positive and note that on your sketch.

2. *Identify the knowns: Make a list of what information is given or can be inferred from the problem statement.* Remember, not all given information will be in the form of a number with units in the problem. If something starts *from rest*, we know the initial velocity is zero. If something *stops*, we know the final velocity is zero.

3. *Identify the unknowns: Decide exactly what needs to be determined in the problem.*

4. *Find an equation or set of equations that can help you solve the problem.* Your list of knowns and unknowns can help here. For example, if time is not needed or not given, then the fifth kinematic equation, which does not include time, could be useful.

5. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made.

6. *Check the answer to see if it is reasonable: Does it make sense?* This final step is extremely important because the goal of physics is to accurately describe nature. To see if the answer is reasonable, check its magnitude, its sign, and its units. Are the significant figures correct?

Summary of Problem Solving

- Determine the knowns and unknowns.
- Find an equation that expresses the unknown in terms of the knowns. More than one unknown means more than one equation is needed.
- Solve the equation or equations.

- Be sure units and significant figures are correct.
- Check whether the answer is reasonable.

⛰ FUN IN PHYSICS

Drag Racing

Figure 3.10 Smoke rises from the tires of a dragster at the beginning of a drag race. (Lt. Col. William Thurmond. Photo courtesy of U.S. Army.)

The object of the sport of drag racing is acceleration. Period! The races take place from a standing start on a straight one-quarter-mile (402 m) track. Usually two cars race side by side, and the winner is the driver who gets the car past the quarter-mile point first. At the finish line, the cars may be going more than 300 miles per hour (134 m/s). The driver then deploys a parachute to bring the car to a stop because it is unsafe to brake at such high speeds. The cars, called dragsters, are capable of accelerating at 26 m/s². By comparison, a typical sports car that is available to the general public can accelerate at about 5 m/s².

Several measurements are taken during each drag race:

- Reaction time is the time between the starting signal and when the front of the car crosses the starting line.
- Elapsed time is the time from when the vehicle crosses the starting line to when it crosses the finish line. The record is a little over 3 s.
- Speed is the average speed during the last 20 m before the finish line. The record is a little under 400 mph.

The video shows a race between two dragsters powered by jet engines. The actual race lasts about four seconds and is near the end of the video (https://openstax.org/l/28dragsters) .

> **GRASP CHECK**
> A dragster crosses the finish line with a velocity of $140\,\text{m/s}$. Assuming the vehicle maintained a constant acceleration from start to finish, what was its average velocity for the race?
> a. $0\,\text{m/s}$
> b. $35\,\text{m/s}$
> c. $70\,\text{m/s}$
> d. $140\,\text{m/s}$

✳ WORKED EXAMPLE

Acceleration of a Dragster

The time it takes for a dragster to cross the finish line is unknown. The dragster accelerates from rest at 26 m/s² for a quarter mile (0.250 mi). What is the final velocity of the dragster?

Strategy

The equation $v^2 = v_0^2 + 2a\left(d - d_0\right)$ is ideally suited to this task because it gives the velocity from acceleration and displacement, without involving the time.

Solution

1. Convert miles to meters.

$$(0.250 \text{ mi}) \times \frac{1609 \text{ m}}{1 \text{ mi}} = 402 \text{ m} \qquad \boxed{3.9}$$

2. Identify the known values. We know that $v_o = 0$ since the dragster starts from rest, and we know that the distance traveled, $d - d_o$ is 402 m. Finally, the acceleration is constant at $a = 26.0$ m/s².

3. Insert the knowns into the equation $v^2 = v_0^2 + 2a\,(d - d_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0\frac{\text{m}}{\text{s}^2}\right)(402 \text{ m}) = 2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2} \qquad \boxed{3.10}$$

Taking the square root gives us $v = \sqrt{2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2}} = 145\frac{\text{m}}{\text{s}}$.

Discussion

145 m/s is about 522 km/hour or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values. We took the positive value because we know that the velocity must be in the same direction as the acceleration for the answer to make physical sense.

An examination of the equation $v^2 = v_0^2 + 2a\,(d - d_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on the magnitude of the acceleration and the distance over which it applies.
- For a given acceleration, a car that is going twice as fast does not stop in twice the distance—it goes much further before it stops. This is why, for example, we have reduced speed zones near schools.

Practice Problems

6. Dragsters can reach a top speed of 145 m/s in only 4.45 s. Calculate the average acceleration for such a dragster.
    a. −32.6 m/s²
    b. 0 m/s²
    c. 32.6 m/s²
    d. 145 m/s²

7. An Olympic-class sprinter starts a race with an acceleration of 4.50 m/s². Assuming she can maintain that acceleration, what is her speed 2.40 s later?
    a. 4.50 m/s
    b. 10.8 m/s
    c. 19.6 m/s
    d. 44.1 m/s

Constant Acceleration

In many cases, acceleration is not uniform because the force acting on the accelerating object is not constant over time. A situation that gives constant acceleration is the acceleration of falling objects. When air resistance is not a factor, all objects near Earth's surface fall with an acceleration of about 9.80 m/s². Although this value decreases slightly with increasing altitude, it may be assumed to be essentially constant. The value of 9.80 m/s² is labeled g and is referred to as **acceleration due to gravity**. Gravity is the force that causes nonsupported objects to accelerate downward—or, more precisely, toward the center of Earth. The magnitude of this force is called the weight of the object and is given by mg where m is the mass of the object (in kg). In places other than on Earth, such as the Moon or on other planets, g is not 9.80 m/s², but takes on other values. When using g for the acceleration a in a kinematic equation, it is usually given a negative sign because the acceleration due to gravity is downward.

🧰 WORK IN PHYSICS

Effects of Rapid Acceleration

Figure 3.11 Astronauts train using G Force Simulators. (NASA)

When in a vehicle that accelerates rapidly, you experience a force on your entire body that accelerates your body. You feel this force in automobiles and slightly more on amusement park rides. For example, when you ride in a car that turns, the car applies a force on your body to make you accelerate in the direction in which the car is turning. If enough force is applied, you will accelerate at 9.80 m/s². This is the same as the acceleration due to gravity, so this force is called one G.

One G is the force required to accelerate an object at the acceleration due to gravity at Earth's surface. Thus, one G for a paper cup is much less than one G for an elephant, because the elephant is much more massive and requires a greater force to make it accelerate at 9.80 m/s². For a person, a G of about 4 is so strong that his or her face will distort as the bones accelerate forward through the loose flesh. Other symptoms at extremely high Gs include changes in vision, loss of consciousness, and even death. The space shuttle produces about 3 Gs during takeoff and reentry. Some roller coasters and dragsters produce forces of around 4 Gs for their occupants. A fighter jet can produce up to 12 Gs during a sharp turn.

Astronauts and fighter pilots must undergo G-force training in simulators. The video (https://www.youtube.com/watch?v=n-8QHOUWECU) shows the experience of several people undergoing this training.

People, such as astronauts, who work with G forces must also be trained to experience zero G—also called free fall or weightlessness—which can cause queasiness. NASA has an aircraft that allows it occupants to experience about 25 s of free fall. The aircraft is nicknamed the *Vomit Comet*.

> **GRASP CHECK**
> A common way to describe acceleration is to express it in multiples of g, Earth's gravitational acceleration. If a dragster accelerates at a rate of 39.2 m/s², how many g's does the driver experience?
> a. 1.5 g
> b. 4.0 g
> c. 10.5 g
> d. 24.5 g

✳️ WORKED EXAMPLE

Falling Objects

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity v_0 of 13 m/s.
(a) Calculate the position and velocity of the rock at 1.00, 2.00, and 3.00 seconds after it is thrown. Ignore the effect of air resistance.

Strategy

Sketch the initial velocity and acceleration vectors and the axes.
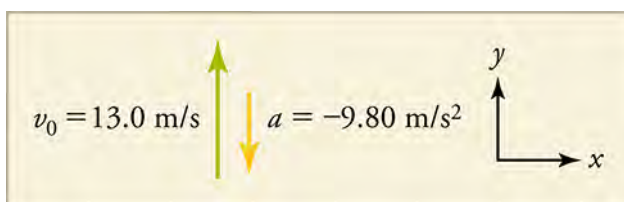
Figure 3.12   Initial conditions for rock thrown straight up.

List the knowns: time $t = 1.00$ s, 2.00 s, and 3.00 s; initial velocity $v_o = 13$ m/s; acceleration $a = g = -9.80$ m/s²; and position $y_o = 0$ m

List the unknowns: y_1, y_2, and y_3; v_1, v_2, and v_3—where 1, 2, 3 refer to times 1.00 s, 2.00 s, and 3.00 s

Choose the equations.

$$d = d_0 + v_0 t + \frac{1}{2}at^2 \text{ becomes } y = y_0 + v_0 t - \frac{1}{2}gt^2 \qquad \boxed{3.11}$$

$$v = v_0 + at \text{ becomes } v = v_0 + -gt \qquad \boxed{3.12}$$

These equations describe the unknowns in terms of knowns only.

Solution

$$y_1 = 0 + (13.0 \text{ m/s})(1.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(1.00 \text{ s})^2}{2} = 8.10 \text{ m}$$

$$y_2 = 0 + (13.0 \text{ m/s})(2.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(2.00 \text{ s})^2}{2} = 6.40 \text{ m}$$

$$y_3 = 0 + (13.0 \text{ m/s})(3.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(3.00 \text{ s})^2}{2} = -5.10 \text{ m}$$

$$v_1 = 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(1.00 \text{ s}) = 3.20 \text{ m/s}$$

$$v_2 = 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(2.00 \text{ s}) = -6.60 \text{ m/s}$$

$$v_3 = 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(3.00 \text{ s}) = -16.4 \text{ m/s}$$

Discussion

The first two positive values for y show that the rock is still above the edge of the cliff, and the third negative y value shows that it has passed the starting point and is below the cliff. Remember that we set *up* to be positive. Any position with a positive value is above the cliff, and any velocity with a positive value is an upward velocity. The first value for v is positive, so the rock is still on the way up. The second and third values for v are negative, so the rock is on its way down.

(b) Make graphs of position versus time, velocity versus time, and acceleration versus time. Use increments of 0.5 s in your graphs.

Strategy

Time is customarily plotted on the *x*-axis because it is the independent variable. Position versus time will not be linear, so calculate points for 0.50 s, 1.50 s, and 2.50 s. This will give a curve closer to the true, smooth shape.

Solution

The three graphs are shown in .
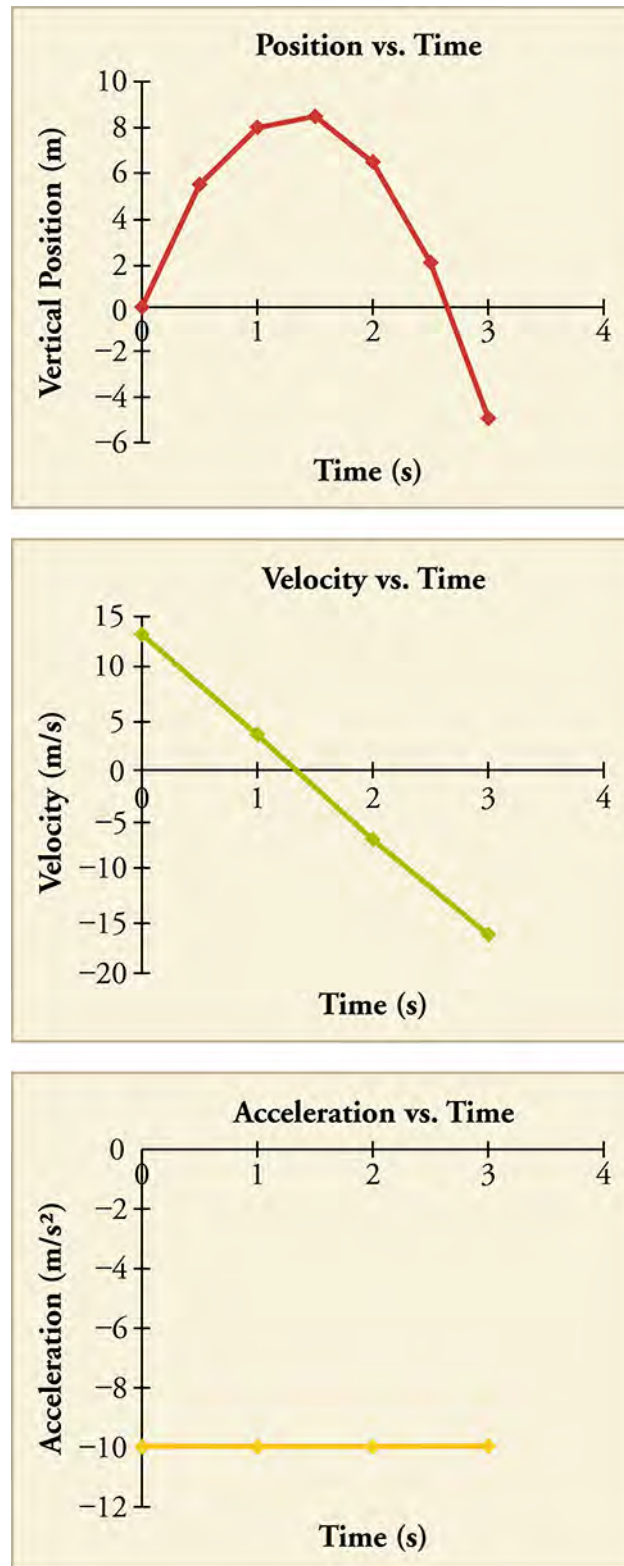
Figure 3.13

Discussion

- *y* vs. *t* does *not* represent the two-dimensional parabolic path of a trajectory. The path of the rock is straight up and straight down. The slope of a line tangent to the curve at any point on the curve equals the velocity at that point—i.e., the instantaneous velocity.

- Note that the *v* vs. *t* line crosses the vertical axis at the initial velocity and crosses the horizontal axis at the time when the rock changes direction and begins to fall back to Earth. This plot is linear because acceleration is constant.
- The *a* vs. *t* plot also shows that acceleration is constant; that is, it does not change with time.

Practice Problems

8. A cliff diver pushes off horizontally from a cliff and lands in the ocean 2.00 s later. How fast was he going when he entered the water?
   a. 0 m/s
   b. 19.0 m/s
   c. 19.6 m/s
   d. 20.0 m/s

9. A girl drops a pebble from a high cliff into a lake far below. She sees the splash of the pebble hitting the water 2.00 s later. How fast was the pebble going when it hit the water?
   a. 9.80 m/s
   b. 10.0 m/s
   c. 19.6 m/s
   d. 20.0 m/s

Check Your Understanding

10. Identify the four variables found in the kinematic equations.
   a. Displacement, Force, Mass, and Time
   b. Acceleration, Displacement, Time, and Velocity
   c. Final Velocity, Force, Initial Velocity, and Mass
   d. Acceleration, Final Velocity, Force, and Initial Velocity

11. Which of the following steps is always required to solve a kinematics problem?
   a. Find the force acting on the body.
   b. Find the acceleration of a body.
   c. Find the initial velocity of a body.
   d. Find a suitable kinematic equation and then solve for the unknown quantity.

12. Which of the following provides a correct answer for a problem that can be solved using the kinematic equations?
   a. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 8 m/s.
   b. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 16 m/s.
   c. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 2 m/s^2.
   d. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 0.5 m/s^2.

KEY TERMS

acceleration due to gravity   acceleration of an object that is subject only to the force of gravity; near Earth's surface this acceleration is 9.80 m/s²

average acceleration   change in velocity divided by the time interval over which it changed

constant acceleration   acceleration that does not change with respect to time

instantaneous acceleration   rate of change of velocity at a specific instant in time

kinematic equations   the five equations that describe motion in terms of time, displacement, velocity, and acceleration

negative acceleration   acceleration in the negative direction

SECTION SUMMARY

3.1 Acceleration

- Acceleration is the rate of change of velocity and may be negative or positive.
- Average acceleration is expressed in m/s² and, in one dimension, can be calculated using $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$.

3.2 Representing Acceleration with Equations and Graphs

- The kinematic equations show how time, displacement, velocity, and acceleration are related for objects in motion.
- In general, kinematic problems can be solved by identifying the kinematic equation that expresses the unknown in terms of the knowns.
- Displacement, velocity, and acceleration may be displayed graphically versus time.

KEY EQUATIONS

3.1 Acceleration

Average acceleration $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$

Average velocity $\overline{v} = \frac{v_0 + v_f}{2}$

Velocity $v = v_0 + at$, or when $v_o = 0$

3.2 Representing Acceleration with Equations and Graphs

Average velocity $d = d_0 + \overline{v}t$, or $\overline{v} = \frac{d}{t}$ when $d_o = 0$

Displacement $d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_o = 0$ and $v_o = 0$

Acceleration $v^2 = v_0^2 + 2a(d - d_0)$, or $a = \frac{v^2}{2d}$ when $d_o = 0$ and $v_o = 0$

CHAPTER REVIEW

Concept Items

3.1 Acceleration

1. How can you use the definition of acceleration to explain the units in which acceleration is measured?
   a. Acceleration is the rate of change of velocity. Therefore, its unit is m/s².
   b. Acceleration is the rate of change of displacement. Therefore, its unit is m/s.
   c. Acceleration is the rate of change of velocity. Therefore, its unit is m²/s.
   d. Acceleration is the rate of change of displacement. Therefore, its unit is m²/s.

2. What are the SI units of acceleration?
   a. m^2/s
   b. cm^2/s
   c. m/s^2
   d. cm/s^2

3. Which of the following statements explains why a racecar going around a curve is accelerating, even if the speed is constant?
   a. The car is accelerating because the magnitude as well as the direction of velocity is changing.
   b. The car is accelerating because the magnitude of velocity is changing.
   c. The car is accelerating because the direction of velocity is changing.

d. The car is accelerating because neither the magnitude nor the direction of velocity is changing.

3.2 Representing Acceleration with Equations and Graphs

4. A student calculated the final velocity of a train that decelerated from 30.5 m/s and got an answer of −43.34 m/s. Which of the following might indicate that he made a mistake in his calculation?
  a. The sign of the final velocity is wrong.
  b. The magnitude of the answer is too small.
  c. There are too few significant digits in the answer.
  d. The units in the initial velocity are incorrect.

5. Create your own kinematics problem. Then, create a flow

Critical Thinking Items

3.1 Acceleration

7. Imagine that a car is traveling from your left to your right at a constant velocity. Which two actions could the driver take that may be represented as (a) a velocity vector and an acceleration vector both pointing to the right and then (b) changing so the velocity vector points to the right and the acceleration vector points to the left?
  a. (a) Push down on the accelerator and then (b) push down again on the accelerator a second time.
  b. (a) Push down on the accelerator and then (b) push down on the brakes.
  c. (a) Push down on the brakes and then (b) push down on the brakes a second time.
  d. (a) Push down on the brakes and then (b) push down on the accelerator.

8. A motorcycle moving at a constant velocity suddenly accelerates at a rate of 4.0 m/s^2 to a speed of 35 m/s in 5.0 s. What was the initial speed of the motorcycle?
  a. −34 m/s
  b. −15 m/s
  c. 15 m/s
  d. 34 m/s

3.2 Representing Acceleration with Equations and Graphs

9. A student is asked to solve a problem:
An object falls from a height for 2.0 s, at which point it is still 60 m above the ground. What will be the velocity of the object when it hits the ground?
Which of the following provides the correct order of kinematic equations that can be used to solve the problem?
  a. First use $v^2 = v_0^2 + 2a(d - d_0)$, then use

chart showing the steps someone would need to take to solve the problem.
  a. Acceleration
  b. Distance
  c. Displacement
  d. Force

6. Which kinematic equation would you use to find the velocity of a skydiver 2.0 s after she jumps from a plane and before she opens her parachute? Assume the positive direction is downward.
  a. $v = v_0 + at$
  b. $v = v_0 - at$
  c. $v^2 = v_0^2 + at$
  d. $v^2 = v_0^2 - at$

$v = v_0 + at$.
  b. First use $v = v_0 + at$, then use $v^2 = v_0^2 + 2a(d - d_0)$.
  c. First use $d = d_0 + v_0 t + \frac{1}{2}at^2$, then use $v = v_0 + at$.
  d. First use $v = v_0 + at$, then use $d - d_0 = v_0 t + \frac{1}{2}at^2$.

10. Skydivers are affected by acceleration due to gravity and by air resistance. Eventually, they reach a speed where the force of gravity is almost equal to the force of air resistance. As they approach that point, their acceleration decreases in magnitude to near zero.
Part A. Describe the shape of the graph of the magnitude of the acceleration versus time for a falling skydiver.
Part B. Describe the shape of the graph of the magnitude of the velocity versus time for a falling skydiver.
Part C. Describe the shape of the graph of the magnitude of the displacement versus time for a falling skydiver.
  a. Part A. Begins with a nonzero y-intercept with a downward slope that levels off at zero; Part B. Begins at zero with an upward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant
  b. Part A. Begins with a nonzero y-intercept with an upward slope that levels off at zero; Part B. Begins at zero with an upward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant
  c. Part A. Begins with a nonzero y-intercept with a downward slope that levels off at zero; Part B. Begins at zero with a downward slope that

decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant

d. Part A. Begins with a nonzero y-intercept with an upward slope that levels off at zero; Part B. Begins at zero with a downward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in

magnitude until it becomes a positive constant

11. Which graph in the previous problem has a positive slope?
a. Displacement versus time only
b. Acceleration versus time and velocity versus time
c. Velocity versus time and displacement versus time
d. Acceleration versus time and displacement versus time

Problems

3.1 Acceleration

12. The driver of a sports car traveling at 10.0 m/s steps down hard on the accelerator for 5.0 s and the velocity increases to 30.0 m/s. What was the average acceleration of the car during the 5.0 s time interval?
a. -1.0×10^2 m/s^2
b. -4.0 m/s^2
c. 4.0 m/s^2
d. 1.0×10^2 m/s^2

13. A girl rolls a basketball across a basketball court. The ball slowly decelerates at a rate of -0.20 m/s^2. If the initial velocity was 2.0 m/s and the ball rolled to a stop at 5.0 sec after 12:00 p.m., at what time did she start the ball rolling?
a. 0.1 seconds before noon
b. 0.1 seconds after noon
c. 5 seconds before noon
d. 5 seconds after noon

3.2 Representing Acceleration with Equations and Graphs

14. A swan on a lake gets airborne by flapping its wings and running on top of the water. If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of 0.350 m/s^2, how far will it travel before becoming airborne?
a. -8.60 m
b. 8.60 m
c. -51.4 m
d. 51.4 m

15. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s and her takeoff point is 8 m above the pool. How long are her feet in the air?
a. 0.408 s
b. 0.816 s
c. 1.34 s
d. 1.75 s
e. 1.28 s

Performance Task

3.2 Representing Acceleration with Equations and Graphs

16. Design an experiment to measure displacement and elapsed time. Use the data to calculate final velocity, average velocity, acceleration, and acceleration.

Materials

- a small marble or ball bearing
- a garden hose
- a measuring tape
- a stopwatch or stopwatch software download
- a sloping driveway or lawn as long as the garden

hose with a level area beyond

(a) How would you use the garden hose, stopwatch, marble, measuring tape, and slope to measure displacement and elapsed time? Hint—The marble is the accelerating object, and the length of the hose is total displacement.
(b) How would you use the displacement and time data to calculate velocity, average velocity, and acceleration? Which kinematic equations would you use?
(c) How would you use the materials, the measured and calculated data, and the flat area below the slope to determine the negative acceleration? What would you measure, and which kinematic equation would you use?

TEST PREP
Multiple Choice
3.1 Acceleration

17. Which variable represents displacement?
    a. a
    b. d
    c. t
    d. v

18. If a velocity increases from 0 to 20 m/s in 10 s, what is the average acceleration?
    a. 0.5 m/s^2
    b. 2 m/s^2
    c. 10 m/s^2
    d. 30 m/s^2

3.2 Representing Acceleration with Equations and Graphs

19. For the motion of a falling object, which graphs are

Short Answer
3.1 Acceleration

21. True or False—The vector for a negative acceleration points in the opposite direction when compared to the vector for a positive acceleration.
    a. True
    b. False

22. If a car decelerates from 20 m/s to 15 m/s in 5 s, what is Δv?
    a. -5 m/s
    b. -1 m/s
    c. 1 m/s
    d. 5 m/s

23. How is the vector arrow representing an acceleration of magnitude 3 m/s² different from the vector arrow representing a negative acceleration of magnitude 3 m/s²?
    a. They point in the same direction.
    b. They are perpendicular, forming a 90° angle between each other.
    c. They point in opposite directions.
    d. They are perpendicular, forming a 270° angle between each other.

24. How long does it take to accelerate from 8.0 m/s to 20.0 m/s at a rate of acceleration of 3.0 m/s²?
    a. 0.25 s
    b. 4.0 s
    c. 9.33 s

straight lines?
    a. Acceleration versus time only
    b. Displacement versus time only
    c. Displacement versus time and acceleration versus time
    d. Velocity versus time and acceleration versus time

20. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.30{\times}10^5$ m/s² for $8.10{\times}10^{-4}$ s. What is the bullet's final velocity when it leaves the barrel, commonly known as the muzzle velocity?
    a. 7.79 m/s
    b. 51.0 m/s
    c. 510 m/s
    d. 1020 m/s

    d. 36 s

3.2 Representing Acceleration with Equations and Graphs

25. If a plot of displacement versus time is linear, what can be said about the acceleration?
    a. Acceleration is 0.
    b. Acceleration is a non-zero constant.
    c. Acceleration is positive.
    d. Acceleration is negative.

26.
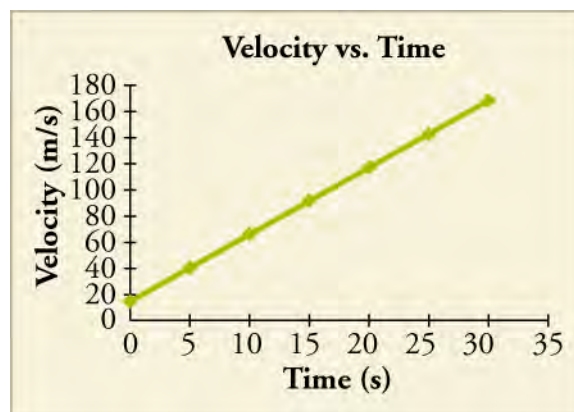
True or False: —The image shows a velocity vs. time graph for a jet car. If you take the slope at any point on the graph, the jet car's acceleration will be 5.0 m/s².
    a. True
    b. False

27. When plotted on the blank plots, which answer choice would show the motion of an object that has uniformly accelerated

from 2 m/s to 8 m/s in 3 s?
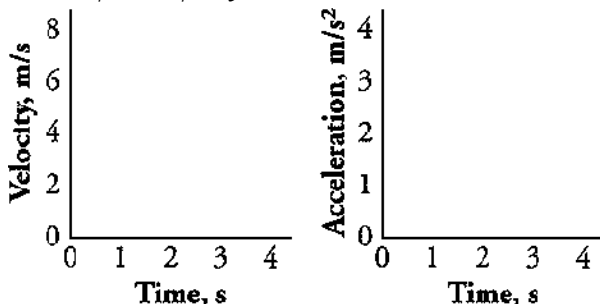
a. The plot on the left shows a line from (0,2) to (3,8) while the plot on the right shows a line from (0,2) to (3,2).
b. The plot on the left shows a line from (0,2) to (3,8) while the plot on the right shows a line from (0,3) to (3,3).
c. The plot on the left shows a line from (0,8) to (3,2) while

the plot on the right shows a line from (0,2) to (3,2).
d. The plot on the left shows a line from (0,8) to (3,2) while the plot on the right shows a line from (0,3) to (3,3).

28. When is a plot of velocity versus time a straight line and when is it a curved line?
a. It is a straight line when acceleration is changing and is a curved line when acceleration is constant.
b. It is a straight line when acceleration is constant and is a curved line when acceleration is changing.
c. It is a straight line when velocity is constant and is a curved line when velocity is changing.
d. It is a straight line when velocity is changing and is a curved line when velocity is constant.

Extended Response

3.1 Acceleration

29. A test car carrying a crash test dummy accelerates from 0 to 30 m/s and then crashes into a brick wall. Describe the direction of the initial acceleration vector and compare the initial acceleration vector's magnitude with respect to the acceleration magnitude at the moment of the crash.
a. The direction of the initial acceleration vector will point towards the wall, and its magnitude will be less than the acceleration vector of the crash.
b. The direction of the initial acceleration vector will point away from the wall, and its magnitude will be less than the vector of the crash.
c. The direction of the initial acceleration vector will point towards the wall, and its magnitude will be more than the acceleration vector of the crash.
d. The direction of the initial acceleration vector will point away from the wall, and its magnitude will be more than the acceleration vector of the crash.

30. A car accelerates from rest at a stop sign at a rate of 3.0 m/s² to a speed of 21.0 m/s, and then immediately begins to decelerate to a stop at the next stop sign at a rate of 4.0 m/s². How long did it take the car to travel

from the first stop sign to the second stop sign? Show your work.
a. 1.7 seconds
b. 5.3 seconds
c. 7.0 seconds
d. 12 seconds

3.2 Representing Acceleration with Equations and Graphs

31. True or False: Consider an object moving with constant acceleration. The plot of displacement versus time for such motion is a curved line while the plot of displacement versus time squared is a straight line.
a. True
b. False

32. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?
a. 0.574 s
b. 0.956 s
c. 1.53 s
d. 1.91 s

Forces and Newton's Laws of Motion

Figure 4.1 Newton's laws of motion describe the motion of the dolphin's path. (Credit: Jin Jang)

Chapter Outline

INTRODUCTION Isaac Newton (1642–1727) was a natural philosopher; a great thinker who combined science and philosophy to try to explain the workings of nature on Earth and in the universe. His laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance period of history to the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. Drawing upon earlier work by scientists Galileo Galilei and Johannes Kepler, Newton's laws of motion allowed motion on Earth and in space to be predicted mathematically. In this chapter you will learn about force as well as Newton's first, second, and third laws of motion.

4.1 Force

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Differentiate between force, net force, and dynamics
- Draw a free-body diagram

Section Key Terms

dynamics	external force	force
free-body diagram	net external force	net force

Defining Force and Dynamics

Force is the cause of motion, and motion draws our attention. Motion itself can be beautiful, such as a dolphin jumping out of the water, the flight of a bird, or the orbit of a satellite. The study of motion is called kinematics, but kinematics describes only the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws describe the way objects speed up, slow down, stay in motion, and interact with other objects. They are also universal laws: they apply everywhere on Earth as well as in space.

A force pushes or pulls an object. The object being moved by a force could be an inanimate object, a table, or an animate object, a person. The pushing or pulling may be done by a person, or even the gravitational pull of Earth. Forces have different magnitudes and directions; this means that some forces are stronger than others and can act in different directions. For example, a cannon exerts a strong force on the cannonball that is launched into the air. In contrast, a mosquito landing on your arm exerts only a small force on your arm.

When multiple forces act on an object, the forces combine. Adding together all of the forces acting on an object gives the total force, or **net force**. An **external force** is a force that acts on an object within the system *from outside* the system. This type of force is different than an internal force, which acts between two objects that are both within the system. **The net external force** combines these two definitions; it is the total combined external force. We discuss further details about net force, external force, and net external force in the coming sections.

In mathematical terms, two forces acting in opposite directions have opposite *signs* (positive or negative). By convention, the negative sign is assigned to any movement to the left or downward. If two forces pushing in opposite directions are added together, the larger force will be somewhat canceled out by the smaller force pushing in the opposite direction. It is important to be consistent with your chosen coordinate system within a problem; for example, if negative values are assigned to the downward direction for velocity, then distance, force, and acceleration should also be designated as being negative in the downward direction.

Free-Body Diagrams and Examples of Forces

For our first example of force, consider an object hanging from a rope. This example gives us the opportunity to introduce a useful tool known as a **free-body diagram**. A free-body diagram represents the object being acted upon—that is, the free body—as a single point. Only the forces acting *on* the body (that is, external forces) are shown and are represented by vectors (which are drawn as arrows). These forces are the only ones shown because only external forces acting on the body affect its motion. We can ignore any internal forces within the body because they cancel each other out, as explained in the section on Newton's third law of motion. Free-body diagrams are very useful for analyzing forces acting on an object.
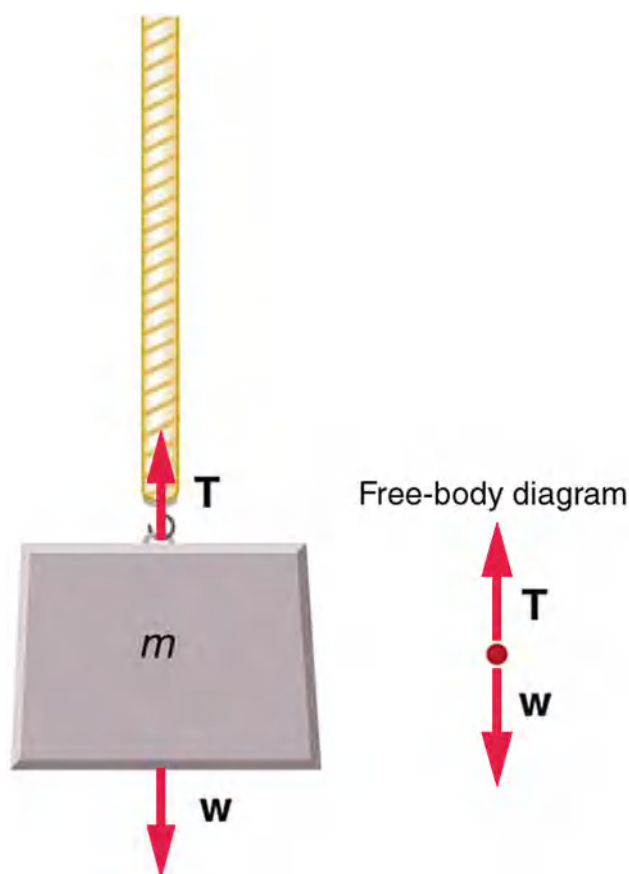
Figure 4.2 An object of mass, *m*, is held up by the force of tension.

Figure 4.2 shows the force of tension in the rope acting in the upward direction, opposite the force of gravity. The forces are indicated in the free-body diagram by an arrow pointing up, representing tension, and another arrow pointing down, representing gravity. In a free-body diagram, the lengths of the arrows show the relative magnitude (or strength) of the forces. Because forces are vectors, they add just like other vectors. Notice that the two arrows have equal lengths in Figure 4.2, which means that the forces of tension and weight are of equal magnitude. Because these forces of equal magnitude act in opposite directions, they are perfectly balanced, so they add together to give a net force of zero.

Not all forces are as noticeable as when you push or pull on an object. Some forces act without physical contact, such as the pull of a magnet (in the case of magnetic force) or the gravitational pull of Earth (in the case of gravitational force).

In the next three sections discussing Newton's laws of motion, we will learn about three specific types of forces: friction, the normal force, and the gravitational force. To analyze situations involving forces, we will create free-body diagrams to organize the framework of the mathematics for each individual situation.

TIPS FOR SUCCESS

Correctly drawing and labeling a free-body diagram is an important first step for solving a problem. It will help you visualize the problem and correctly apply the mathematics to solve the problem.

Check Your Understanding

1. What is kinematics?
    a. Kinematics is the study of motion.
    b. Kinematics is the study of the cause of motion.
    c. Kinematics is the study of dimensions.
    d. Kinematics is the study of atomic structures.

2. Do two bodies have to be in physical contact to exert a force upon one another?

    a.   No, the gravitational force is a field force and does not require physical contact to exert a force.

    b.   No, the gravitational force is a contact force and does not require physical contact to exert a force.

    c.   Yes, the gravitational force is a field force and requires physical contact to exert a force.

    d.   Yes, the gravitational force is a contact force and requires physical contact to exert a force.

3. What kind of physical quantity is force?

    a.   Force is a scalar quantity.

    b.   Force is a vector quantity.

    c.   Force is both a vector quantity and a scalar quantity.

    d.   Force is neither a vector nor a scalar quantity.

4. Which forces can be represented in a free-body diagram?

    a.   Internal forces

    b.   External forces

    c.   Both internal and external forces

    d.   A body that is not influenced by any force

4.2 Newton's First Law of Motion: Inertia

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's first law and friction, and
- Discuss the relationship between mass and inertia.

Section Key Terms

friction	inertia	law of inertia
mass	Newton's first law of motion	system

Newton's First Law and Friction

Newton's first law of motion states the following:

1. A body at rest tends to remain at rest.
2. A body in motion tends to remain in motion at a constant velocity unless acted on by a net external force. (Recall that *constant velocity* means that the body moves in a straight line and at a constant speed.)

At first glance, this law may seem to contradict your everyday experience. You have probably noticed that a moving object will usually slow down and stop unless some effort is made to keep it moving. The key to understanding why, for example, a sliding box slows down (seemingly on its own) is to first understand that a net external force acts on the box to make the box slow down. Without this net external force, the box would continue to slide at a constant velocity (as stated in Newton's first law of motion). What force acts on the box to slow it down? This force is called **friction**. Friction is an external force that acts opposite to the direction of motion (see Figure 4.3). Think of friction as a resistance to motion that slows things down.

Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it lifts the puck slightly, so the puck experiences very little friction as it moves over the surface. With friction almost eliminated, the puck glides along with very little change in speed. On a frictionless surface, the puck would experience no net external force (ignoring air resistance, which is also a form of friction). Additionally, if we know enough about friction, we can accurately predict how quickly objects will slow down.

Now let's think about another example. A man pushes a box across a floor at constant velocity by applying a force of +50 N. (The positive sign indicates that, by convention, the direction of motion is to the right.) What is the force of friction that opposes the motion? The force of friction must be –50 N. Why? According to Newton's first law of motion, any object moving at constant velocity has no net external force acting upon it, which means that the sum of the forces acting on the object must be zero. The mathematical way to say that no net external force acts on an object is $\mathbf{F}_{net} = 0$ or $\Sigma\mathbf{F} = 0$. So if the man applies +50 N of force, then the force of friction must be –50 N for the two forces to add up to zero (that is, for the two forces to *cancel* each

other). Whenever you encounter the phrase *at constant velocity*, Newton's first law tells you that the net external force is zero.
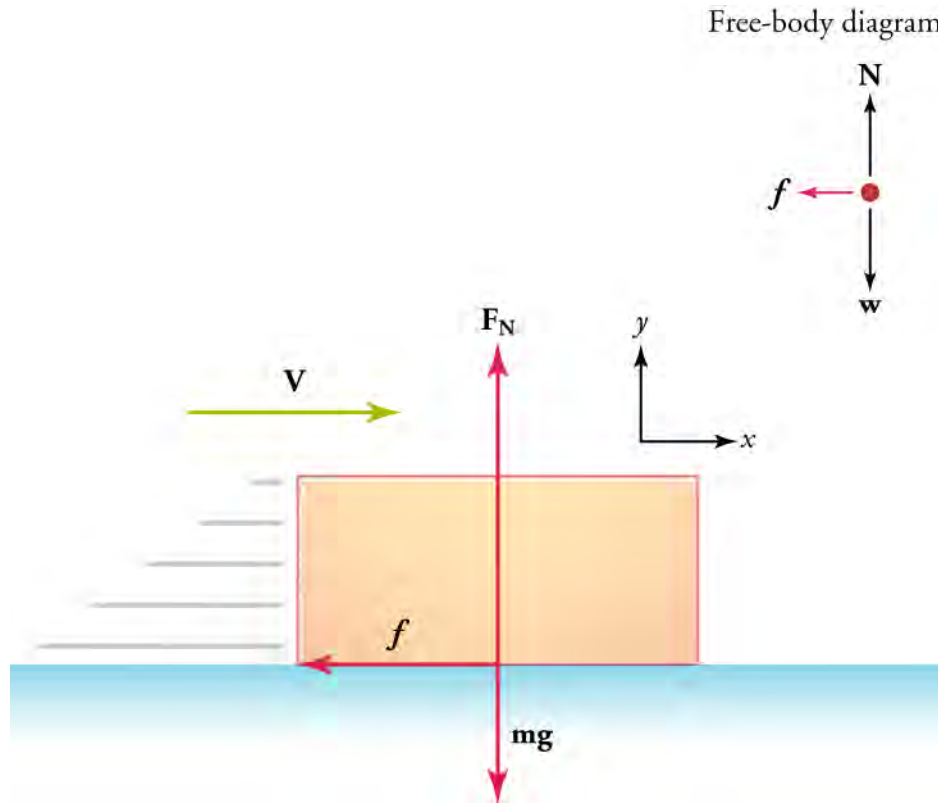
Figure 4.3 For a box sliding across a floor, friction acts in the direction opposite to the velocity.

The force of friction depends on two factors: the coefficient of friction and the normal force. For any two surfaces that are in contact with one another, the coefficient of friction is a constant that depends on the nature of the surfaces. The normal force is the force exerted by a surface that pushes on an object in response to gravity pulling the object down. In equation form, the force of friction is

$$\mathbf{f} = \mu\mathbf{N},$$

4.1

where μ is the coefficient of friction and $\mathbf{N}$ is the normal force. (The coefficient of friction is discussed in more detail in another chapter, and the normal force is discussed in more detail in the section *Newton's Third Law of Motion*.)

Recall from the section on Force that a net external force acts from outside on the object of interest. A more precise definition is that it acts on the **system** of interest. A system is one or more objects that you choose to study. It is important to define the system at the beginning of a problem to figure out which forces are external and need to be considered, and which are internal and can be ignored.

For example, in Figure 4.4 (a), two children push a third child in a wagon at a constant velocity. The system of interest is the wagon plus the small child, as shown in part (b) of the figure. The two children behind the wagon exert external forces on this system ($\mathbf{F}_1$, $\mathbf{F}_2$). Friction f acting at the axles of the wheels and at the surface where the wheels touch the ground two other external forces acting on the system. Two more external forces act on the system: the weight $\mathbf{W}$ of the system pulling down and the normal force $\mathbf{N}$ of the ground pushing up. Notice that the wagon is not accelerating vertically, so Newton's first law tells us that the normal force balances the weight. Because the wagon is moving forward at a constant velocity, the force of friction must have the same strength as the sum of the forces applied by the two children.
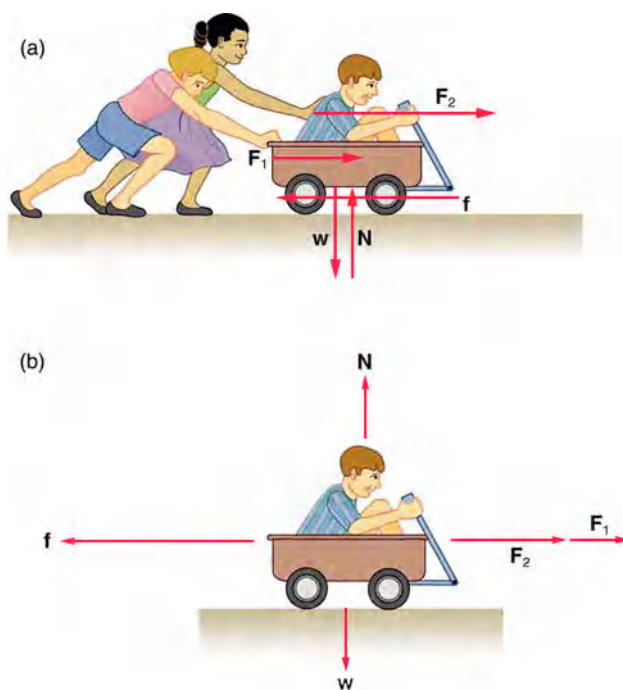
Figure 4.4 (a) The wagon and rider form a *system* that is acted on by external forces. (b) The two children pushing the wagon and child provide two external forces. Friction acting at the wheel axles and on the surface of the tires where they touch the ground provide an external force that act against the direction of motion. The weight **W** and the normal force **N** from the ground are two more external forces acting on the system. All external forces are represented in the figure by arrows. All of the external forces acting on the system add together, but because the wagon moves at a constant velocity, all of the forces must add up to zero.

Mass and Inertia

Inertia is the tendency for an object at rest to remain at rest, or for a moving object to remain in motion in a straight line with constant speed. This key property of objects was first described by Galileo. Later, Newton incorporated the concept of inertia into his first law, which is often referred to as the **law of inertia**.

As we know from experience, some objects have more inertia than others. For example, changing the motion of a large truck is more difficult than changing the motion of a toy truck. In fact, the inertia of an object is proportional to the mass of the object. **Mass** is a measure of the amount of matter (or *stuff*) in an object. The quantity or amount of matter in an object is determined by the number and types of atoms the object contains. Unlike weight (which changes if the gravitational force changes), mass does not depend on gravity. The mass of an object is the same on Earth, in orbit, or on the surface of the moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so mass is usually not determined this way. Instead, the mass of an object is determined by comparing it with the standard kilogram. Mass is therefore expressed in kilograms.

TIPS FOR SUCCESS
In everyday language, people often use the terms *weight* and *mass* interchangeably—but this is not correct. Weight is actually a force. (We cover this topic in more detail in the section *Newton's Second Law of Motion*.)

👁 WATCH PHYSICS

Newton's First Law of Motion
This video contrasts the way we thought about motion and force in the time before Galileo's concept of inertia and Newton's first law of motion with the way we understand force and motion now.

Click to view content (https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)

GRASP CHECK

Before we understood that objects have a tendency to maintain their velocity in a straight line unless acted upon by a net force, people thought that objects had a tendency to stop on their own. This happened because a specific force was not yet understood. What was that force?

a. Gravitational force
b. Electrostatic force
c. Nuclear force
d. Frictional force

Virtual Physics

Forces and Motion—Basics

In this simulation, you will first explore net force by placing blue people on the left side of a tug-of-war rope and red people on the right side of the rope (by clicking people and dragging them with your mouse). Experiment with changing the number and size of people on each side to see how it affects the outcome of the match and the net force. Hit the "Go!" button to start the match, and the "reset all" button to start over.

Next, click on the Friction tab. Try selecting different objects for the person to push. Slide the *applied force* button to the right to apply force to the right, and to the left to apply force to the left. The force will continue to be applied as long as you hold down the button. See the arrow representing friction change in magnitude and direction, depending on how much force you apply. Try increasing or decreasing the friction force to see how this change affects the motion.

Click to view content (https://phet.colorado.edu/sims/html/forces-and-motion-basics/latest/forces-and-motion-basics_en.html)

GRASP CHECK

Click on the tab for the *Acceleration Lab and check the Sum of Forces* option. Push the box to the right and then release. Notice which direction the sum of forces arrow points after the person stops pushing the box and lets it continue moving to the right on its own. At this point, in which direction is the net force, the sum of forces, pointing? Why?

a. The net force acts to the right because the applied external force acted to the right.
b. The net force acts to the left because the applied external force acted to the left.
c. The net force acts to the right because the frictional force acts to the right.
d. The net force acts to the left because the frictional force acts to the left.

Check Your Understanding

5. What does Newton's first law state?
   a. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant acceleration unless acted on by a net external force.
   b. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant velocity unless acted on by a net external force.
   c. The rate of change of momentum of a body is directly proportional to the external force applied to the body.
   d. The rate of change of momentum of a body is inversely proportional to the external force applied to the body.

6. According to Newton's first law, a body in motion tends to remain in motion at a constant velocity. However, when you slide an object across a surface, the object eventually slows down and stops. Why?
   a. The object experiences a frictional force exerted by the surface, which opposes its motion.
   b. The object experiences the gravitational force exerted by Earth, which opposes its motion
   c. The object experiences an internal force exerted by the body itself, which opposes its motion.
   d. The object experiences a pseudo-force from the body in motion, which opposes its motion.

7. What is inertia?
    a. Inertia is an object's tendency to maintain its mass.
    b. Inertia is an object's tendency to remain at rest.
    c. Inertia is an object's tendency to remain in motion
    d. Inertia is an object's tendency to remain at rest or, if moving, to remain in motion.

8. What is mass? What does it depend on?
    a. Mass is the weight of an object, and it depends on the gravitational force acting on the object.
    b. Mass is the weight of an object, and it depends on the number and types of atoms in the object.
    c. Mass is the quantity of matter contained in an object, and it depends on the gravitational force acting on the object.
    d. Mass is the quantity of matter contained in an object, and it depends on the number and types of atoms in the object.

4.3 Newton's Second Law of Motion

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe Newton's second law, both verbally and mathematically
- Use Newton's second law to solve problems

Section Key Terms

freefall   Newton's second law of motion   weight

Describing Newton's Second Law of Motion

Newton's first law considered bodies at rest or bodies in motion at a constant velocity. The other state of motion to consider is when an object is moving with a changing velocity, which means a change in the speed and/or the direction of motion. This type of motion is addressed by **Newton's second law of motion**, which states how force causes changes in motion. Newton's second law of motion is used to calculate what happens in situations involving forces and motion, and it shows the mathematical relationship between force, mass, and *acceleration*. Mathematically, the second law is most often written as

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \ \text{ or } \ \Sigma\mathbf{F} = m\mathbf{a}, \tag{4.2}$$

where $\mathbf{F}_{\text{net}}$ (or $\Sigma\mathbf{F}$) is the net external force, m is the mass of the system, and $\mathbf{a}$ is the acceleration. Note that $\mathbf{F}_{\text{net}}$ and $\Sigma\mathbf{F}$ are the same because the net external force is the sum of all the external forces acting on the system.

First, what do we mean by *a change in motion*? A change in motion is simply a change in velocity: the speed of an object can become slower or faster, the direction in which the object is moving can change, or both of these variables may change. A change in velocity means, by definition, that an acceleration has occurred. Newton's first law says that only a nonzero net external force can cause a change in motion, so a net external force must cause an acceleration. Note that acceleration can refer to slowing down or to speeding up. Acceleration can also refer to a change in the direction of motion with no change in speed, because acceleration is the change in velocity divided by the time it takes for that change to occur, *and* velocity is defined by speed *and* direction.

From the equation $\mathbf{F}_{\text{net}} = m\mathbf{a}$, we see that force is directly proportional to both mass and acceleration, which makes sense. To accelerate two objects from rest to the same velocity, you would expect more force to be required to accelerate the more massive object. Likewise, for two objects of the same mass, applying a greater force to one would accelerate it to a greater velocity.

Now, let's rearrange Newton's second law to solve for acceleration. We get

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m} \ \text{ or } \ \mathbf{a} = \frac{\Sigma\mathbf{F}}{m}. \tag{4.3}$$

In this form, we can see that acceleration is directly proportional to force, which we write as

$$\mathbf{a} \propto \mathbf{F}_{\text{net}}, \tag{4.4}$$

where the symbol $\propto$ means *proportional to*.

This proportionality mathematically states what we just said in words: acceleration is directly proportional to the net external

force. When two variables are directly proportional to each other, then if one variable doubles, the other variable must double. Likewise, if one variable is reduced by half, the other variable must also be reduced by half. In general, when one variable is multiplied by a number, the other variable is also multiplied by the same number. It seems reasonable that the acceleration of a system should be directly proportional to and in the same direction as the net external force acting on the system. An object experiences greater acceleration when acted on by a greater force.

It is also clear from the equation $\mathbf{a} = \mathbf{F}_{net}/m$ that acceleration is inversely proportional to mass, which we write as

$$\mathbf{a} \propto \frac{1}{m}.$$

<div style="text-align:right">4.5</div>

Inversely proportional means that if one variable is multiplied by a number, the other variable must be *divided* by the same number. Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. This relationship is illustrated in Figure 4.5, which shows that a given net external force applied to a basketball produces a much greater acceleration than when applied to a car.
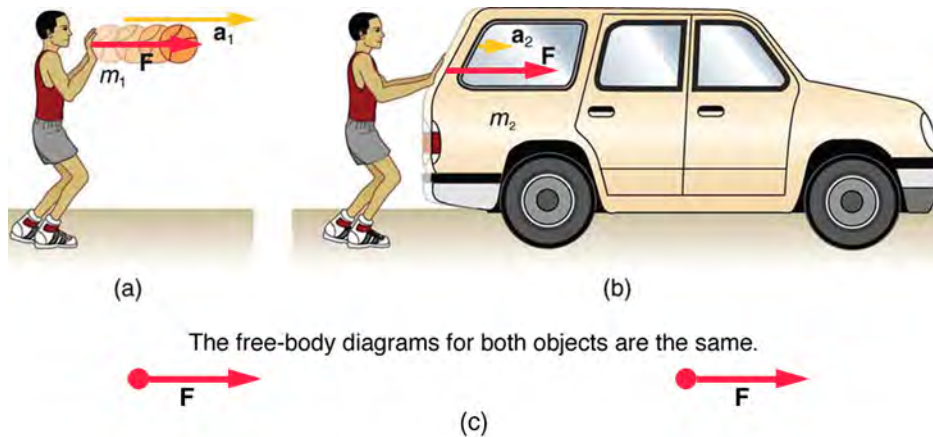
Figure 4.5 The same force exerted on systems of different masses produces different accelerations. (a) A boy pushes a basketball to make a pass. The effect of gravity on the ball is ignored. (b) The same boy pushing with identical force on a stalled car produces a far smaller acceleration (friction is negligible). Note that the free-body diagrams for the ball and for the car are identical, which allows us to compare the two situations.

Applying Newton's Second Law

Before putting Newton's second law into action, it is important to consider units. The equation $\mathbf{F}_{net} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units of mass, length, and time (recall that acceleration has units of length divided by time squared). The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of 1 m/s². That is, because $\mathbf{F}_{net} = m\mathbf{a}$, we have

$$1 \text{ N} = 1 \text{ kg} \times 1 \text{m/s}^2 = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}.$$

<div style="text-align:right">4.6</div>

One of the most important applications of Newton's second law is to calculate **weight** (also known as the gravitational force), which is usually represented mathematically as $\mathbf{W}$. When people talk about gravity, they don't always realize that it is an acceleration. When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that the net external force acting on an object is responsible for the acceleration of the object. If air resistance is negligible, the net external force on a falling object is only the gravitational force (i.e., the weight of the object).

Weight can be represented by a vector because it has a direction. Down is defined as the direction in which gravity pulls, so weight is normally considered a downward force. By using Newton's second law, we can figure out the equation for weight.

Consider an object with mass m falling toward Earth. It experiences only the force of gravity (i.e., the gravitational force or weight), which is represented by W. Newton's second law states that $\mathbf{F}_{net} = m\mathbf{a}$. Because the only force acting on the object is the gravitational force, we have $\mathbf{F}_{net} = \mathbf{W}$. We know that the acceleration of an object due to gravity is $\mathbf{g}$, so we have $\mathbf{a} = \mathbf{g}$. Substituting these two expressions into Newton's second law gives

$$\mathbf{W} = m\mathbf{g}.$$

<div style="text-align: right">4.7</div>

This is the equation for weight—the gravitational force on a mass m. On Earth, $\mathbf{g} = 9.80 \text{ m/s}^2$, so the weight (disregarding for now the direction of the weight) of a 1.0-kg object on Earth is

$$\mathbf{W} = m\mathbf{g} = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \ \text{N}.$$

<div style="text-align: right">4.8</div>

Although most of the world uses newtons as the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Recall that although gravity acts downward, it can be assigned a positive or negative value, depending on what the positive direction is in your chosen coordinate system. Be sure to take this into consideration when solving problems with weight. When the downward direction is taken to be negative, as is often the case, acceleration due to gravity becomes $\mathbf{g} = -9.8 \text{ m/s}^2$.

When the net external force on an object is its weight, we say that it is in **freefall**. In this case, the only force acting on the object is the force of gravity. On the surface of Earth, when objects fall downward toward Earth, they are never truly in freefall because there is always some upward force due to air resistance that acts on the object (and there is also the buoyancy force of air, which is similar to the buoyancy force in water that keeps boats afloat).

Gravity varies slightly over the surface of Earth, so the weight of an object depends very slightly on its location on Earth. Weight varies dramatically away from Earth's surface. On the moon, for example, the acceleration due to gravity is only 1.67 m/s^2. Because weight depends on the force of gravity, a 1.0-kg mass weighs 9.8 N on Earth and only about 1.7 N on the moon.

It is important to remember that weight and mass are very different, although they are closely related. Mass is the quantity of matter (how much *stuff*) in an object and does not vary, but weight is the gravitational force on an object and is proportional to the force of gravity. It is easy to confuse the two, because our experience is confined to Earth, and the weight of an object is essentially the same no matter where you are on Earth. Adding to the confusion, the terms mass and weight are often used interchangeably in everyday language; for example, our medical records often show our weight in kilograms, but never in the correct unit of newtons.

Snap Lab

Mass and Weight

In this activity, you will use a scale to investigate mass and weight.

- 1 bathroom scale
- 1 table

1. What do bathroom scales measure?
2. When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled.
3. The springs provide a measure of your weight (provided you are not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is now divided by 9.80 to give a reading in kilograms, which is a of mass. The scale detects weight but is calibrated to display mass.
4. If you went to the moon and stood on your scale, would it detect the same *mass* as it did on Earth?

GRASP CHECK
While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why?
a. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
b. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction opposite to your weight.
c. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
d. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on

you that acts in the direction opposite to your weight.

TIPS FOR SUCCESS
Only *net external force* impacts the acceleration of an object. If more than one force acts on an object and you calculate the acceleration by using only one of these forces, you will not get the correct acceleration for that object.

👁 WATCH PHYSICS

Newton's Second Law of Motion
This video reviews Newton's second law of motion and how net external force and acceleration relate to one another and to mass. It also covers units of force, mass, and acceleration, and reviews a worked-out example.

Click to view content (https://www.khanacademy.org/embed_video?v=ou9YMWlJgkE)

GRASP CHECK
True or False—If you want to reduce the acceleration of an object to half its original value, then you would need to reduce the net external force by half.
a. True
b. False

✳ WORKED EXAMPLE

What Acceleration Can a Person Produce when Pushing a Lawn Mower?
Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N parallel to the ground. The mass of the mower is 240 kg. What is its acceleration?

Figure 4.6

Strategy
Because $\mathbf{F_{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law: $\mathbf{F_{net}} = m\mathbf{a}$.

Solution
Solving Newton's second law for the acceleration, we find that the magnitude of the acceleration, $\mathbf{a}$, is $\mathbf{a} = \frac{\mathbf{F_{net}}}{m}$. Entering the given values for net external force and mass gives

$$\mathbf{a} = \frac{51 \ \text{N}}{240 \ \text{kg}}$$

4.9

Inserting the units $kg \cdot m/s^2$ for N yields

$$\mathbf{a} = \frac{51 \;\; kg \cdot m/s^2}{240 \;\; kg} = 0.21 \; m/s^2 .$$

Discussion

The acceleration is in the same direction as the net external force, which is parallel to the ground and to the right. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion, because we are given that the net external force is in the direction in which the person pushes. Also, the vertical forces must cancel if there is no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is reasonable for a person pushing a mower; the mower's speed must increase by 0.21 m/s every second, which is possible. The time during which the mower accelerates would not be very long because the person's top speed would soon be reached. At this point, the person could push a little less hard, because he only has to overcome friction.

✳ WORKED EXAMPLE

What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on humans at high accelerations. Rocket sleds consisted of a platform mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust, $\mathbf{T}$, for the four-rocket propulsion system shown below. The sled's initial acceleration is $49 \; m/s^2$, the mass of the system is 2,100 kg, and the force of friction opposing the motion is 650 N.
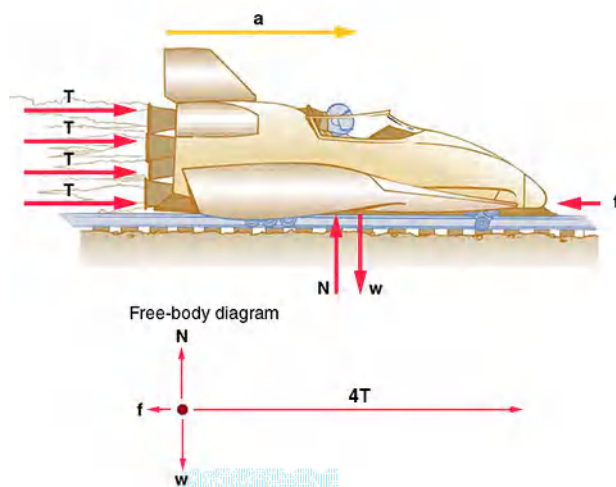
Figure 4.7

Strategy

The system of interest is the rocket sled. Although forces act vertically on the system, they must cancel because the system does not accelerate vertically. This leaves us with only horizontal forces to consider. We'll assign the direction to the right as the positive direction. See the free-body diagram in Figure 4.8.

Solution

We start with Newton's second law and look for ways to find the thrust $\mathbf{T}$ of the engines. Because all forces and acceleration are along a line, we need only consider the magnitudes of these quantities in the calculations. We begin with

$$\mathbf{F}_{net} = m\mathbf{a},$$

where $\mathbf{F}_{net}$ is the net external force in the horizontal direction. We can see from Figure 4.8 that the engine thrusts are in the same direction (which we call the positive direction), whereas friction opposes the thrust. In equation form, the net external force is

$$\mathbf{F}_{\text{net}} = 4\mathbf{T} - \mathbf{f}.$$

<div style="text-align: right;">4.12</div>

Newton's second law tells us that $\mathbf{F}_{\text{net}} = m\mathbf{a}$, so we get

$$m\mathbf{a} = 4\mathbf{T} - \mathbf{f}.$$

<div style="text-align: right;">4.13</div>

After a little algebra, we solve for the total thrust 4$\mathbf{T}$:

$$4\mathbf{T} = m\mathbf{a} + \mathbf{f},$$

<div style="text-align: right;">4.14</div>

which means that the individual thrust is

$$\mathbf{T} = \frac{m\mathbf{a} + \mathbf{f}}{4}.$$

<div style="text-align: right;">4.15</div>

Inserting the known values yields

$$\mathbf{T} = \frac{(2100 \text{ kg})(49 \text{ m/s}^2) + 650 \text{ N}}{4} = 2.6 \times 10^4 \text{ N}.$$

<div style="text-align: right;">4.16</div>

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and to test the apparatus designed to protect fighter pilots in emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 **g**. (Recall that g, the acceleration due to gravity, is 9.80 m/s^2. An acceleration of 45 **g** is $45 \times 9.80 \text{ m/s}^2$, which is approximately 440 m/s^2.) Living subjects are no longer used, and land speeds of 10,000 km/h have now been obtained with rocket sleds. In this example, as in the preceding example, the system of interest is clear. We will see in later examples that choosing the system of interest is crucial—and that the choice is not always obvious.

Practice Problems

9. If 1 N is equal to 0.225 lb, how many pounds is 5 N of force?
    a. 0.045 lb
    b. 1.125 lb
    c. 2.025 lb
    d. 5.000 lb

10. How much force needs to be applied to a 5-kg object for it to accelerate at 20 m/s²?
    a. 1 N
    b. 10 N
    c. 100 N
    d. 1,000 N

Check Your Understanding

11. What is the mathematical statement for Newton's second law of motion?
    a. $F = ma$
    b. $F = 2ma$
    c. $F = \frac{m}{a}$
    d. $F = ma^2$

12. Newton's second law describes the relationship between which quantities?
    a. Force, mass, and time
    b. Force, mass, and displacement
    c. Force, mass, and velocity
    d. Force, mass, and acceleration

13. What is acceleration?
    a. Acceleration is the rate at which displacement changes.
    b. Acceleration is the rate at which force changes.
    c. Acceleration is the rate at which velocity changes.

    d.   Acceleration is the rate at which mass changes.

4.4 Newton's Third Law of Motion

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe Newton's third law, both verbally and mathematically
- Use Newton's third law to solve problems

Section Key Terms

Newton's third law of motion   normal force   tension   thrust

Describing Newton's Third Law of Motion

If you have ever stubbed your toe, you have noticed that although your toe initiates the impact, the surface that you stub it on exerts a force back on your toe. Although the first thought that crosses your mind is probably "ouch, that hurt" rather than "this is a great example of Newton's third law," both statements are true.

This is exactly what happens whenever one object exerts a force on another—each object experiences a force that is the same strength as the force acting on the other object but that acts in the opposite direction. Everyday experiences, such as stubbing a toe or throwing a ball, are all perfect examples of Newton's third law in action.

Newton's third law of motion states that whenever a first object exerts a force on a second object, the first object experiences a force equal in magnitude but opposite in direction to the force that it exerts.

Newton's third law of motion tells us that forces always occur in pairs, and one object cannot exert a force on another without experiencing the same strength force in return. We sometimes refer to these force pairs as *action-reaction* pairs, where the force exerted is the action, and the force experienced in return is the reaction (although which is which depends on your point of view).

Newton's third law is useful for figuring out which forces are external to a system. Recall that identifying external forces is important when setting up a problem, because the external forces must be added together to find the net force.

We can see Newton's third law at work by looking at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in <u>Figure 4.8</u>. She pushes against the pool wall with her feet and accelerates in the direction opposite to her push. The wall has thus exerted on the swimmer a force of equal magnitude but in the direction opposite that of her push. You might think that two forces of equal magnitude but that act in opposite directions would cancel, *but they do not because they act on different systems.*

In this case, there are two different systems that we could choose to investigate: the swimmer or the wall. If we choose the swimmer to be the system of interest, as in the figure, then $F_{\text{wall on feet}}$ is an external force on the swimmer and affects her motion. Because acceleration is in the same direction as the net external force, the swimmer moves in the direction of $F_{\text{wall on feet}}$. Because the swimmer is our system (or object of interest) and not the wall, we do not need to consider the force $F_{\text{feet on wall}}$ because it originates *from* the swimmer rather than *acting on* the swimmer. Therefore, $F_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $F_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to the direction in which she wants to move.
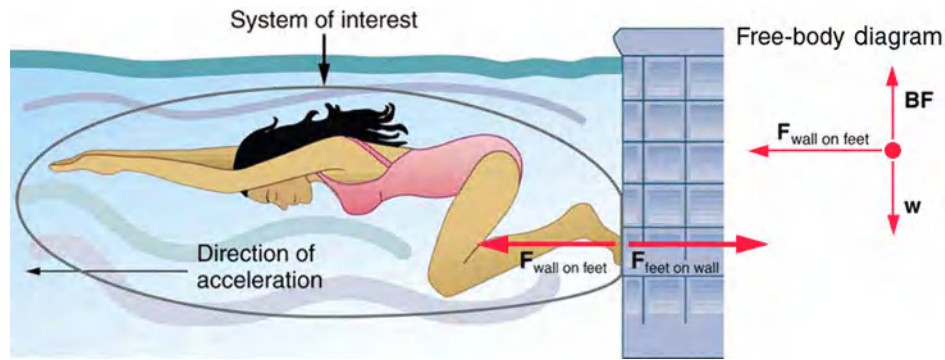
Figure 4.8 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means that the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition is the result of Newton's third law of motion, which dictates that the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on the swimmer that is equal in magnitude but that acts in the direction opposite to the force that the swimmer exerts on the wall.

Other examples of Newton's third law are easy to find. As a teacher paces in front of a whiteboard, he exerts a force backward on the floor. The floor exerts a reaction force in the forward direction on the teacher that causes him to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the car's wheels in reaction to the car's wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward.

Another example is the force of a baseball as it makes contact with the bat. Helicopters create lift by pushing air down, creating an upward reaction force. Birds fly by exerting force on air in the direction opposite that in which they wish to fly. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself forward in the water by ejecting water backward through a funnel in its body, which is similar to how a jet ski is propelled. In these examples, the octopus or jet ski push the water backward, and the water, in turn, pushes the octopus or jet ski forward.

Applying Newton's Third Law

Forces are classified and given names based on their source, how they are transmitted, or their effects. In previous sections, we discussed the forces called *push*, *weight*, and *friction*. In this section, applying Newton's third law of motion will allow us to explore three more forces: the **normal force**, **tension**, and **thrust**. However, because we haven't yet covered vectors in depth, we'll only consider one-dimensional situations in this chapter. Another chapter will consider forces acting in two dimensions.

The gravitational force (or weight) acts on objects at all times and everywhere on Earth. We know from Newton's second law that a net force produces an acceleration; so, why is everything not in a constant state of freefall toward the center of Earth? The answer is the normal force. The normal force is the force that a surface applies to an object to support the weight of that object; it acts perpendicular to the surface upon which the object rests. If an object on a flat surface is not accelerating, the net external force is zero, and the normal force has the same magnitude as the weight of the system but acts in the opposite direction. In equation form, we write that

$$\mathbf{N} = m\mathbf{g}.$$

<div style="text-align:right">4.17</div>

Note that this equation is only true for a horizontal surface.

The word *tension* comes from the Latin word meaning *to stretch*. Tension is the force along the length of a flexible connector, such as a string, rope, chain, or cable. Regardless of the type of connector attached to the object of interest, one must remember that the connector can only pull (or *exert tension*) in the direction *parallel* to its length. Tension is a pull that acts parallel to the connector, and that acts in opposite directions at the two ends of the connector. This is possible because a flexible connector is simply a long series of action-reaction forces, except at the two ends where outside objects provide one member of the action-reaction forces.

Consider a person holding a mass on a rope, as shown in Figure 4.9.
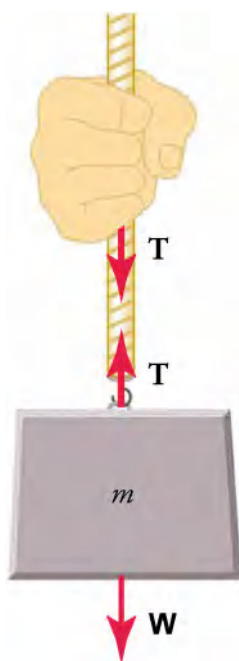
Figure 4.9 When a perfectly flexible connector (one requiring no force to bend it) such as a rope transmits a force **T**, this force must be parallel to the length of the rope, as shown. The pull that such a flexible connector exerts is a tension. Note that the rope pulls with equal magnitude force but in opposite directions to the hand and to the mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that transmits forces of equal magnitude between the two objects but that act in opposite directions.

Tension in the rope must equal the weight of the supported mass, as we can prove by using Newton's second law. If the 5.00 kg mass in the figure is stationary, then its acceleration is zero, so $\mathbf{F}_{net} = 0$. The only external forces acting on the mass are its weight **W** and the tension **T** supplied by the rope. Summing the external forces to find the net force, we obtain

$$\mathbf{F}_{net} = \mathbf{T} - \mathbf{W} = 0,$$

4.18

where **T** and **W** are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. By substituting $m\mathbf{g}$ for $\mathbf{F}_{net}$ and rearranging the equation, the tension equals the weight of the supported mass, just as you would expect

$$\mathbf{T} = \mathbf{W} = m\mathbf{g}.$$

4.19

For a 5.00-kg mass (neglecting the mass of the rope), we see that

$$\mathbf{T} = m\mathbf{g} = (5.00 \ \text{kg})(9.80 \ \text{m/s}^2) = 49.0 \ \text{N}.$$

4.20

Another example of Newton's third law in action is thrust. Rockets move forward by expelling gas backward at a high velocity. This means that the rocket exerts a large force backward on the gas in the rocket combustion chamber, and the gas, in turn, exerts a large force forward on the rocket in response. This reaction force is called *thrust*.

TIPS FOR SUCCESS
A common misconception is that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can expel exhaust gases more easily.

🔗 LINKS TO PHYSICS

Math: Problem-Solving Strategy for Newton's Laws of Motion
The basics of problem solving, presented earlier in this text, are followed here with specific strategies for applying Newton's laws of motion. These techniques also reinforce concepts that are useful in many other areas of physics.

First, identify the physical principles involved. If the problem involves forces, then Newton's laws of motion are involved, and it

is important to draw a careful sketch of the situation. An example of a sketch is shown in Figure 4.10. Next, as in Figure 4.10, use vectors to represent all forces. Label the forces carefully, and make sure that their lengths are proportional to the magnitude of the forces and that the arrows point in the direction in which the forces act.
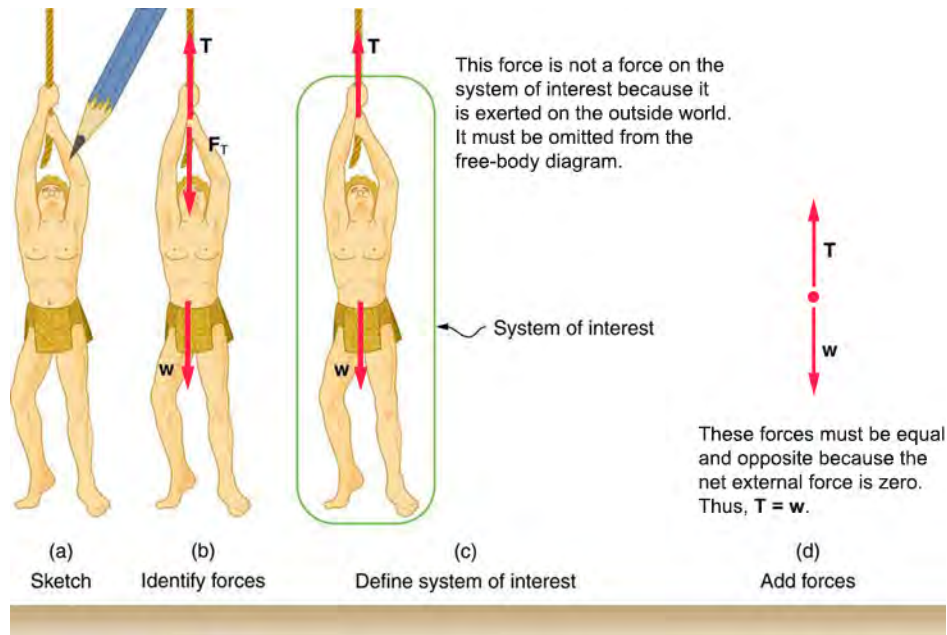
Figure 4.10 (a) A sketch of Tarzan hanging motionless from a vine. (b) Arrows are used to represent all forces. **T** is the tension exerted on Tarzan by the vine, $\mathbf{F}_T$ is the force exerted on the vine by Tarzan, and **W** is Tarzan's weight (i.e., the force exerted on Tarzan by Earth's gravity). All other forces, such as a nudge of a breeze, are assumed to be negligible. (c) Suppose we are given Tarzan's mass and asked to find the tension in the vine. We define the system of interest as shown and draw a free-body diagram, as shown in (d). $\mathbf{F}_T$ is no longer shown because it does not act on the system of interest; rather, $\mathbf{F}_T$ acts on the outside world. (d) The free-body diagram shows only the external forces acting on Tarzan. For these to sum to zero, we must have $\mathbf{T} = \mathbf{W}$.

Next, make a list of knowns and unknowns and assign variable names to the quantities given in the problem. Figure out which variables need to be calculated; these are the unknowns. Now carefully define the system: which objects are of interest for the problem. This decision is important, because Newton's second law involves only external forces. Once the system is identified, it's possible to see which forces are external and which are internal (see Figure 4.10).

If the system acts on an object outside the system, then you know that the outside object exerts a force of equal magnitude but in the opposite direction on the system.

A diagram showing the system of interest and all the external forces acting on it is called a free-body diagram. Only external forces are shown on free-body diagrams, not acceleration or velocity. Figure 4.10 shows a free-body diagram for the system of interest.

After drawing a free-body diagram, apply Newton's second law to solve the problem. This is done in Figure 4.10 for the case of Tarzan hanging from a vine. When external forces are clearly identified in the free-body diagram, translate the forces into equation form and solve for the unknowns. Note that forces acting in opposite directions have opposite signs. By convention, forces acting downward or to the left are usually negative.

GRASP CHECK
If a problem has more than one system of interest, more than one free-body diagram is required to describe the external forces acting on the different systems.
  a. True
  b. False

👁 WATCH PHYSICS

Newton's Third Law of Motion

This video explains Newton's third law of motion through examples involving push, normal force, and thrust (the force that propels a rocket or a jet).

Click to view content (https://www.openstax.org/l/astronaut)

> **GRASP CHECK**
> If the astronaut in the video wanted to move upward, in which direction should he throw the object? Why?
> a.   He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., upward).
> b.   He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., downward).
> c.   He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., upward).
> d.   He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., downward).

✳ WORKED EXAMPLE

An Accelerating Subway Train

A physics teacher pushes a cart of demonstration equipment to a classroom, as in Figure 4.11. Her mass is 65.0 kg, the cart's mass is 12.0 kg, and the equipment's mass is 7.0 kg. To push the cart forward, the teacher's foot applies a force of 150 N in the opposite direction (backward) on the floor. Calculate the acceleration produced by the teacher. The force of friction, which opposes the motion, is 24.0 N.
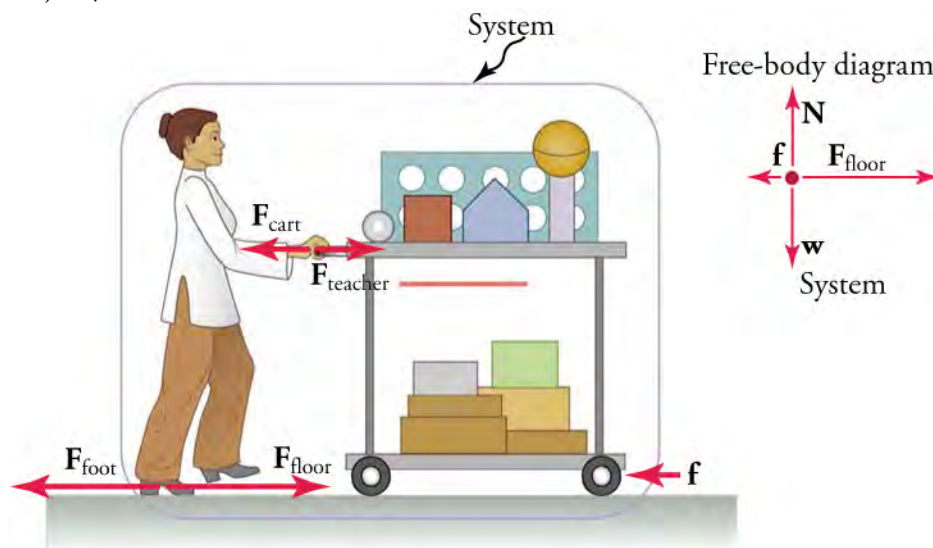
Figure 4.11

Strategy

Because they accelerate together, we define the system to be the teacher, the cart, and the equipment. The teacher pushes backward with a force $\mathbf{F}_{foot}$ of 150 N. According to Newton's third law, the floor exerts a forward force $\mathbf{F}_{floor}$ of 150 N on the system. Because all motion is horizontal, we can assume that no net force acts in the vertical direction, and the problem becomes one dimensional. As noted in the figure, the friction f opposes the motion and therefore acts opposite the direction of $\mathbf{F}_{floor}$.

We should not include the forces $\mathbf{F}_{\text{teacher}}$, $\mathbf{F}_{\text{cart}}$, or $\mathbf{F}_{\text{foot}}$ because these are exerted *by* the system, not *on* the system. We find the net external force by adding together the external forces acting on the system (see the free-body diagram in the figure) and then use Newton's second law to find the acceleration.

Solution

Newton's second law is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}.$$

4.21

The net external force on the system is the sum of the external forces: the force of the floor acting on the teacher, cart, and equipment (in the horizontal direction) and the force of friction. Because friction acts in the opposite direction, we assign it a negative value. Thus, for the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{F}_{\text{floor}} - \mathbf{f} = 150 \text{ N} - 24.0 \text{ N} = 126 \text{ N}.$$

4.22

The mass of the system is the sum of the mass of the teacher, cart, and equipment.

$$m = (65.0 + 12.0 + 7.0) \text{ kg} = 84 \text{ kg}$$

4.23

Insert these values of net F and m into Newton's second law to obtain the acceleration of the system.

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$$
$$a = \frac{126 \text{ N}}{84 \text{ kg}} = 1.5 \text{ m/s}^2$$

4.24

$$F_1 < F_2$$

4.25

Discussion

None of the forces between components of the system, such as between the teacher's hands and the cart, contribute to the net external force because they are internal to the system. Another way to look at this is to note that the forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the teacher on the cart is of equal magnitude but in the opposite direction of the force exerted by the cart on the teacher. In this case, both forces act on the same system, so they cancel. Defining the system was crucial to solving this problem.

Practice Problems

14. What is the equation for the normal force for a body with mass m that is at rest on a horizontal surface?
    a. $N = m$
    b. $N = mg$
    c. $N = mv$
    d. $N = g$

15. An object with mass m is at rest on the floor. What is the magnitude and direction of the normal force acting on it?
    a. $N = mv$ in upward direction
    b. $N = mg$ in upward direction
    c. $N = mv$ in downward direction
    d. $N = mg$ in downward direction

Check Your Understanding

16. What is Newton's third law of motion?
    a. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude and acts in the direction of the applied force.
    b. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude and acts in the direction of the applied force.
    c. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude but acts in the direction opposite the direction of the applied force.
    d. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude but

acts in the direction opposite the direction of the applied force.

17. Considering Newton's third law, why don't two equal and opposite forces cancel out each other?
    a. Because the two forces act in the same direction
    b. Because the two forces have different magnitudes
    c. Because the two forces act on different systems
    d. Because the two forces act in perpendicular directions

KEY TERMS

dynamics   the study of how forces affect the motion of objects and systems

external force   a force acting on an object or system that originates outside of the object or system

force   a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

free-body diagram   a diagram showing all external forces acting on a body

freefall   a situation in which the only force acting on an object is the force of gravity

friction   an external force that acts in the direction opposite to the direction of motion

inertia   the tendency of an object at rest to remain at rest, or for a moving object to remain in motion in a straight line and at a constant speed

law of inertia   Newton's first law of motion: a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

mass   the quantity of matter in a substance; measured in kilograms

net external force   the sum of all external forces acting on an object or system

net force   the sum of all forces acting on an object or system

Newton's first law of motion   a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion   the net external force, $\mathbf{F}_{net}$, on an object is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and also proportional to the object's mass, m; defined mathematically as $\mathbf{F}_{net} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$.

Newton's third law of motion   when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts

normal force   the force that a surface applies to an object; acts perpendicular and away from the surface with which the object is in contact

system   one or more objects of interest for which only the forces acting on them from the outside are considered, but not the forces acting between them or inside them

tension   a pulling force that acts along a connecting medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force exerted on the object by the rope is called tension

thrust   a force that pushes an object forward in response to the backward ejection of mass by the object; rockets and airplanes are pushed forward by a thrust reaction force in response to ejecting gases backward

weight   the force of gravity, $\mathbf{W}$, acting on an object of mass m; defined mathematically as $\mathbf{W} = m\mathbf{g}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

SECTION SUMMARY

4.1 Force

- Dynamics is the study of how forces affect the motion of objects and systems.
- Force is a push or pull that can be defined in terms of various standards. It is a vector and so has both magnitude and direction.
- External forces are any forces outside of a body that act on the body. A free-body diagram is a drawing of all external forces acting on a body.

4.2 Newton's First Law of Motion: Inertia

- Newton's first law states that a body at rest remains at rest or, if moving, remains in motion in a straight line at a constant speed, unless acted on by a net external force. This law is also known as the law of inertia.
- Inertia is the tendency of an object at rest to remain at rest or, if moving, to remain in motion at constant velocity. Inertia is related to an object's mass.

- Friction is a force that opposes motion and causes an object or system to slow down.
- Mass is the quantity of matter in a substance.

4.3 Newton's Second Law of Motion

- Acceleration is a change in velocity, meaning a change in speed, direction, or both.
- An external force acts on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to the system's mass.
- In equation form, Newton's second law of motion is $\mathbf{F}_{net} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$. This is sometimes written as $\mathbf{a} = \frac{\mathbf{F}_{net}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$.
- The weight of an object of mass m is the force of gravity that acts on it. From Newton's second law, weight is

given by $\mathbf{W} = m\mathbf{g}$.
- If the only force acting on an object is its weight, then the object is in freefall.

4.4 Newton's Third Law of Motion

- Newton's third law of motion states that when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.
- When an object rests on a surface, the surface applies a force on the object that opposes the weight of the object.

This force acts perpendicular to the surface and is called the normal force.
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension. When a rope supports the weight of an object at rest, the tension in the rope is equal to the weight of the object.
- Thrust is a force that pushes an object forward in response to the backward ejection of mass by the object. Rockets and airplanes are pushed forward by thrust.

KEY EQUATIONS

4.2 Newton's First Law of Motion: Inertia

Newton's first law of motion	$\mathbf{F}_{net} = 0$ or $\Sigma\mathbf{F} = 0$

4.3 Newton's Second Law of Motion

Newton's second law of motion	$\mathbf{F}_{net} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$
Newton's second law of motion to solve acceleration	$\mathbf{a} = \frac{\mathbf{F}_{net}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$

Newton's second law of motion to solve weight	$\mathbf{W} = m\mathbf{g}$

4.4 Newton's Third Law of Motion

normal force for a nonaccelerating horizontal surface	$\mathbf{N} = m\mathbf{g}$
tension for an object at rest	$\mathbf{T} = m\mathbf{g}$

CHAPTER REVIEW

Concept Items

4.1 Force

1. What is dynamics?
   a. Dynamics is the study of internal forces.
   b. Dynamics is the study of forces and their effect on motion.
   c. Dynamics describes the motion of points, bodies, and systems without consideration of the cause of motion.
   d. Dynamics describes the effect of forces on each other.

2. Two forces acting on an object are perpendicular to one another. How would you draw these in a free-body diagram?
   a. The two force arrows will be drawn at a right angle to one another.
   b. The two force arrows will be pointing in opposite directions.
   c. The two force arrows will be at a 45° angle to one another.

   d. The two force arrows will be at a 180° angle to one another.

3. A free-body diagram shows the forces acting on an object. How is that object represented in the diagram?
   a. A single point
   b. A square box
   c. A unit circle
   d. The object as it is

4.2 Newton's First Law of Motion: Inertia

4. A ball rolls along the ground, moving from north to south. What direction is the frictional force that acts on the ball?
   a. North to south
   b. South to north
   c. West to east
   d. East to west

5. The tires you choose to drive over icy roads will create more friction with the road than your summer tires. Give another example where more friction is desirable.

a. Children's slide

b. Air hockey table

c. Ice-skating rink

d. Jogging track

6. How do you express, mathematically, that no external force is acting on a body?

a. $F_{net} = -1$

b. $F_{net} = 0$

c. $F_{net} = 1$

d. $F_{net} = \infty$

4.3 Newton's Second Law of Motion

7. What does it mean for two quantities to be inversely proportional to each other?

a. When one variable increases, the other variable decreases by a greater amount.

b. When one variable increases, the other variable also increases.

c. When one variable increases, the other variable decreases by the same factor.

d. When one variable increases, the other variable also increases by the same factor.

Critical Thinking Items

4.1 Force

12. Only two forces are acting on an object: force A to the left and force B to the right. If force B is greater than force A, in which direction will the object move?

a. To the right

b. To the left

c. Upward

d. The object does not move

13. In a free-body diagram, the arrows representing tension and weight have the same length but point away from one another. What does this indicate?

a. They are equal in magnitude and act in the same direction.

b. They are equal in magnitude and act in opposite directions.

c. They are unequal in magnitude and act in the same direction.

d. They are unequal in magnitude and act in opposite directions.

14. An object is at rest. Two forces, X and Y, are acting on it. Force X has a magnitude of x and acts in the downward direction. What is the magnitude and direction of Y?

a. The magnitude is x and points in the upward direction.

b. The magnitude is $2x$ and points in the upward

8. True or False: Newton's second law can be interpreted based on Newton's first law.

a. True

b. False

4.4 Newton's Third Law of Motion

9. Which forces cause changes in the motion of a system?

a. internal forces

b. external forces

c. both internal and external forces

d. neither internal nor external forces

10. True or False—Newton's third law applies to the external forces acting on a system of interest.

a. True

b. False

11. A ball is dropped and hits the floor. What is the direction of the force exerted by the floor on the ball?

a. Upward

b. Downward

c. Right

d. Left

direction.

c. The magnitude is x and points in the downward direction.

d. The magnitude is $2x$ and points in the downward direction.

15. Three forces, A, B, and C, are acting on the same object with magnitudes a, b, and c, respectively. Force A acts to the right, force B acts to the left, and force C acts downward. What is a necessary condition for the object to move straight down?

a. The magnitude of force A must be greater than the magnitude of force B, so a > b.

b. The magnitude of force A must be equal to the magnitude of force B, so a = b.

c. The magnitude of force A must be greater than the magnitude of force C, so A > C.

d. The magnitude of force C must be greater than the magnitude of forces A or B, so A < C > B.

4.2 Newton's First Law of Motion: Inertia

16. Two people push a cart on a horizontal surface by applying forces F_1 and F_2 in the same direction. Is the magnitude of the net force acting on the cart, F_{net}, equal to, greater than, or less than $F_1 + F_2$? Why?

a. $F_{net} < F_1 + F_2$ because the net force will not include the frictional force.

b. $F_{net} = F_1 + F_2$ because the net force will not include

the frictional force

   c. $F_{net} < F_1 + F_2$ because the net force will include the component of frictional force

   d. $F_{net} = F_1 + F_2$ because the net force will include the frictional force

17. True or False: A book placed on a balance scale is balanced by a standard 1-kg iron weight placed on the opposite side of the balance. If these objects are taken to the moon and a similar exercise is performed, the balance is still level because gravity is uniform on the moon's surface as it is on Earth's surface.

   a. True

   b. False

4.3 Newton's Second Law of Motion

18. From the equation for Newton's second law, we see that F_{net} is directly proportional to **a** and that the constant of proportionality is **m**. What does this mean in a practical sense?

   a. An increase in applied force will cause an increase in acceleration if the mass is constant.

   b. An increase in applied force will cause a decrease in acceleration if the mass is constant.

   c. An increase in applied force will cause an increase in acceleration, even if the mass varies.

   d. An increase in applied force will cause an increase

Problems

4.3 Newton's Second Law of Motion

21. An object has a mass of 1 kg on Earth. What is its weight on the moon?

   a. 1 N

   b. 1.67 N

   c. 9.8 N

   d. 10 N

22. A bathroom scale shows your mass as 55 kg. What will it read on the moon?

   a. 9.4 kg

   b. 10.5 kg

Performance Task

4.4 Newton's Third Law of Motion

24. A car weighs 2,000 kg. It moves along a road by applying a force on the road with a parallel component of 560 N. There are two passengers in the car, each weighing 55 kg. If the magnitude of the force of friction

in acceleration and mass.

4.4 Newton's Third Law of Motion

19. True or False: A person accelerates while walking on the ground by exerting force. The ground in turn exerts force F_2 on the person. F_1 and F_2 are equal in magnitude but act in opposite directions. The person is able to walk because the two forces act on the different systems and the net force acting on the person is nonzero.

   a. True

   b. False

20. A helicopter pushes air down, which, in turn, pushes the helicopter up. Which force affects the helicopter's motion? Why?

   a. Air pushing upward affects the helicopter's motion because it is an internal force that acts on the helicopter.

   b. Air pushing upward affects the helicopter's motion because it is an external force that acts on the helicopter.

   c. The downward force applied by the blades of the helicopter affects its motion because it is an internal force that acts on the helicopter.

   d. The downward force applied by the blades of the helicopter affects its motion because it is an external force that acts on the helicopter.

   c. 55.0 kg

   d. 91.9 kg

4.4 Newton's Third Law of Motion

23. A person pushes an object of mass 5.0 kg along the floor by applying a force. If the object experiences a friction force of 10 N and accelerates at 18 m/s², what is the magnitude of the force exerted by the person?

   a. −90 N

   b. −80 N

   c. 90 N

   d. 100 N

experienced by the car is 45 N, what is the acceleration of the car?

   a. 0.244 m/s²

   b. 0.265 m/s²

   c. 4.00 m/s²

   d. 4.10 m/s²

TEST PREP
Multiple Choice
4.1 Force

25. Which of the following is a physical quantity that can be described by dynamics but not by kinematics?
   a. Velocity
   b. Acceleration
   c. Force

26. Which of the following is used to represent an object in a free-body diagram?
   a. A point
   b. A line
   c. A vector

4.2 Newton's First Law of Motion: Inertia

27. What kind of force is friction?
   a. External force
   b. Internal force
   c. Net force

28. What is another name for Newton's first law?
   a. Law of infinite motion
   b. Law of inertia
   c. Law of friction

29. True or False—A rocket is launched into space and escapes Earth's gravitational pull. It will continue to travel in a straight line until it is acted on by another force.
   a. True
   b. False

30. A 2,000-kg car is sitting at rest in a parking lot. A bike and rider with a total mass of 60 kg are traveling along a road at 10 km/h. Which system has more inertia? Why?
   a. The car has more inertia, as its mass is greater than the mass of the bike.
   b. The bike has more inertia, as its mass is greater than the mass of the car.
   c. The car has more inertia, as its mass is less than the mass of the bike.
   d. The bike has more inertia, as its mass is less than the mass of the car.

4.3 Newton's Second Law of Motion

31. In the equation for Newton's second law, what does F_{net} stand for?
   a. Internal force
   b. Net external force
   c. Frictional force

32. What is the SI unit of force?

   a. Kg
   b. dyn
   c. N

33. What is the net external force on an object in freefall on Earth if you were to neglect the effect of air?
   a. The net force is zero.
   b. The net force is upward with magnitude mg.
   c. The net force is downward with magnitude mg.
   d. The net force is downward with magnitude 9.8 N.

34. Two people push a 2,000-kg car to get it started. An acceleration of at least 5.0 m/s² is required to start the car. Assuming both people apply the same magnitude force, how much force will each need to apply if friction between the car and the road is 300 N?
   a. 4850 N
   b. 5150 N
   c. 97000 N
   d. 10300 N

4.4 Newton's Third Law of Motion

35. One object exerts a force of magnitude F_1 on another object and experiences a force of magnitude F_2 in return. What is true for F_1 and F_2?
   a. $F_1 > F_2$
   b. $F_1 < F_2$
   c. $F_1 = F_2$

36. A weight is suspended with a rope and hangs freely. In what direction is the tension on the rope?
   a. parallel to the rope
   b. perpendicular to the rope

37. A person weighing 55 kg walks by applying 160 N of force on the ground, while pushing a 10-kg object. If the person accelerates at 2 m/s², what is the force of friction experienced by the system consisting of the person and the object?
   a. 30 N
   b. 50 N
   c. 270 N
   d. 290 N

38. A 65-kg swimmer pushes on the pool wall and accelerates at 6 m/s². The friction experienced by the swimmer is 100 N. What is the magnitude of the force that the swimmer applies on the wall?
   a. −490 N
   b. −290 N
   c. 290 N
   d. 490 N

Short Answer

4.1 Force

39. True or False—An external force is defined as a force generated outside the system of interest that acts on an object inside the system.
   a.   True
   b.   False

40. By convention, which sign is assigned to an object moving downward?
   a.   A positive sign (+)
   b.   A negative sign (−)
   c.   Either a positive or negative sign (±)
   d.   No sign is assigned

41. A body is pushed downward by a force of 5 units and upward by a force of 2 units. How would you draw a free-body diagram to represent this?
   a.   Two force vectors acting at a point, both pointing up with lengths of 5 units and 2 units
   b.   Two force vectors acting at a point, both pointing down with lengths of 5 units and 2 units
   c.   Two force vectors acting at a point, one pointing up with a length of 5 units and the other pointing down with a length of 2 units
   d.   Two force vectors acting at a point, one pointing down with a length of 5 units and the other pointing up with a length of 2 units

42. A body is pushed eastward by a force of four units and southward by a force of three units. How would you draw a free-body diagram to represent this?
   a.   Two force vectors acting at a point, one pointing left with a length of 4 units and the other pointing down with a length of 3 units
   b.   Two force vectors acting at a point, one pointing left with a length of 4 units and the other pointing up with a length of 3 units
   c.   Two force vectors acting at a point, one pointing right with a length of 4 units and the other pointing down with a length of 3 units
   d.   Two force vectors acting at a point, one pointing right with a length of 4 units and the other pointing up with a length of 3 units

4.2 Newton's First Law of Motion: Inertia

43. A body with mass m is pushed along a horizontal surface by a force F and is opposed by a frictional force f. How would you draw a free-body diagram to represent this situation?
   a.   A dot with an arrow pointing right, labeled F, and an arrow pointing left, labeled f, that is of equal length or shorter than F

   b.   A dot with an arrow pointing right, labeled F, and an arrow pointing right, labeled f, that is of equal length or shorter than F
   c.   A dot with an arrow pointing right, labeled F, and a smaller arrow pointing up, labeled f, that is of equal length or longer than F
   d.   A dot with an arrow pointing right, labeled F, and a smaller arrow pointing down, labeled f, that is of equal length or longer than F

44. Two objects rest on a uniform surface. A person pushes both with equal force. If the first object starts to move faster than the second, what can be said about their masses?
   a.   The mass of the first object is less than that of the second object.
   b.   The mass of the first object is equal to the mass of the second object.
   c.   The mass of the first object is greater than that of the second object.
   d.   No inference can be made because mass and force are not related to each other.

45. Two similar boxes rest on a table. One is empty and the other is filled with pebbles. Without opening or lifting either, how can you tell which box is full? Why?
   a.   By applying an internal force; whichever box accelerates faster is lighter and so must be empty
   b.   By applying an internal force; whichever box accelerates faster is heavier and so the other box must be empty
   c.   By applying an external force; whichever box accelerates faster is lighter and so must be empty
   d.   By applying an external force; whichever box accelerates faster is heavier and so the other box must be empty

46. True or False—An external force is required to set a stationary object in motion in outer space away from all gravitational influences and atmospheric friction.
   a.   True
   b.   False

4.3 Newton's Second Law of Motion

47. A steadily rolling ball is pushed in the direction from east to west, which causes the ball to move faster in the same direction. What is the direction of the acceleration?
   a.   North to south
   b.   South to north
   c.   East to west
   d.   West to east

48. A ball travels from north to south at 60 km/h. After being hit by a bat, it travels from west to east at 60 km/

h. Is there a change in velocity?
    a. Yes, because velocity is a scalar.
    b. Yes, because velocity is a vector.
    c. No, because velocity is a scalar.
    d. No, because velocity is a vector

49. What is the weight of a 5-kg object on Earth and on the moon?
    a. On Earth the weight is 1.67 N, and on the moon the weight is 1.67 N.
    b. On Earth the weight is 5 N, and on the moon the weight is 5 N.
    c. On Earth the weight is 49 N, and on the moon the weight is 8.35 N.
    d. On Earth the weight is 8.35 N, and on the moon the weight is 49 N.

50. An object weighs 294 N on Earth. What is its weight on the moon?
    a. 50.1 N
    b. 30.0 N
    c. 249 N
    d. 1461 N

4.4 Newton's Third Law of Motion

51. A large truck with mass 30 m crashes into a small sedan with mass *m*. If the truck exerts a force F on the sedan, what force will the sedan exert on the truck?
    a. $\frac{F}{30}$

Extended Response

4.1 Force

55. True or False—When two unequal forces act on a body, the body will not move in the direction of the weaker force.
    a. True
    b. False

56. In the figure given, what is $F_{restore}$? What is its magnitude?

    (a)   (b)

    (c)

    a. $F_{restore}$ is the force exerted by the hand on the spring, and it pulls to the right.
    b. $F_{restore}$ is the force exerted by the spring on the hand, and it pulls to the left.

b. F
c. 2F
d. 30F

52. A fish pushes water backward with its fins. How does this propel the fish forward?
    a. The water exerts an internal force on the fish in the opposite direction, pushing the fish forward.
    b. The water exerts an external force on the fish in the opposite direction, pushing the fish forward.
    c. The water exerts an internal force on the fish in the same direction, pushing the fish forward.
    d. The water exerts an external force on the fish in the same direction, pushing the fish forward.

53. True or False—Tension is the result of opposite forces in a connector, such as a string, rope, chain or cable, that pulls each point of the connector apart in the direction parallel to the length of the connector. At the ends of the connector, the tension pulls toward the center of the connector.
    a. True
    b. False

54. True or False—Normal reaction is the force that opposes the force of gravity and acts in the direction of the force of gravity.
    a. True
    b. False

    c. $F_{restore}$ is the force exerted by the hand on the spring, and it pulls to the left.
    d. $F_{restore}$ is the force exerted by the spring on the hand, and it pulls to the right.

4.2 Newton's First Law of Motion: Inertia

57. Two people apply the same force to throw two identical balls in the air. Will the balls necessarily travel the same distance? Why or why not?
    a. No, the balls will not necessarily travel the same distance because the gravitational force acting on them is different.
    b. No, the balls will not necessarily travel the same distance because the angle at which they are thrown may differ.
    c. Yes, the balls will travel the same distance because the gravitational force acting on them is the same.
    d. Yes, the balls will travel the same distance because the angle at which they are thrown may differ.

58. A person pushes a box from left to right and then lets the box slide freely across the floor. The box slows down as it slides across the floor. When the box is sliding

freely, what is the direction of the net external force?

a. The net external force acts from left to right.
b. The net external force acts from right to left.
c. The net external force acts upward.
d. The net external force acts downward.

4.3 Newton's Second Law of Motion

59. A 55-kg lady stands on a bathroom scale inside an elevator. The scale reads 70 kg. What do you know about the motion of the elevator?

a. The elevator must be accelerating upward.
b. The elevator must be accelerating downward.
c. The elevator must be moving upward with a constant velocity.
d. The elevator must be moving downward with a constant velocity.

60. True or False—A skydiver initially accelerates in his jump. Later, he achieves a state of constant velocity called terminal velocity. Does this mean the skydiver becomes weightless?

a. Yes
b. No

4.4 Newton's Third Law of Motion

61. How do rockets propel themselves in space?

a. Rockets expel gas in the forward direction at high velocity, and the gas, which provides an internal force, pushes the rockets forward.
b. Rockets expel gas in the forward direction at high velocity, and the gas, which provides an external force, pushes the rockets forward.
c. Rockets expel gas in the backward direction at high velocity, and the gas, which is an internal force, pushes the rockets forward.
d. Rockets expel gas in the backward direction at high velocity, and the gas, which provides an external force, pushes the rockets forward.

62. Are rockets more efficient in Earth's atmosphere or in outer space? Why?

a. Rockets are more efficient in Earth's atmosphere than in outer space because the air in Earth's atmosphere helps to provide thrust for the rocket, and Earth has more air friction than outer space.
b. Rockets are more efficient in Earth's atmosphere than in outer space because the air in Earth's atmosphere helps to provide thrust to the rocket, and Earth has less air friction than the outer space.
c. Rockets are more efficient in outer space than in Earth's atmosphere because the air in Earth's atmosphere does not provide thrust but does create more air friction than in outer space.
d. Rockets are more efficient in outer space than in Earth's atmosphere because the air in Earth's atmosphere does not provide thrust but does create less air friction than in outer space.

CHAPTER 5
Motion in Two Dimensions

Figure 5.1 Billiard balls on a pool table are in motion after being hit with a cue stick. (Popperipopp, Wikimedia Commons)

Chapter Outline

5.1 Vector Addition and Subtraction: Graphical Methods

5.2 Vector Addition and Subtraction: Analytical Methods

5.3 Projectile Motion

5.4 Inclined Planes

5.5 Simple Harmonic Motion

INTRODUCTION In Chapter 2, we learned to distinguish between vectors and scalars; the difference being that a vector has magnitude and direction, whereas a scalar has only magnitude. We learned how to deal with vectors in physics by working straightforward one-dimensional vector problems, which may be treated mathematically in the same as scalars. In this chapter, we'll use vectors to expand our understanding of forces and motion into two dimensions. Most real-world physics problems (such as with the game of pool pictured here) are, after all, either two- or three-dimensional problems and physics is most useful when applied to real physical scenarios. We start by learning the practical skills of graphically adding and subtracting vectors (by using drawings) and analytically (with math). Once we're able to work with two-dimensional vectors, we apply these skills to problems of projectile motion, inclined planes, and harmonic motion.

5.1 Vector Addition and Subtraction: Graphical Methods

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe the graphical method of vector addition and subtraction
- Use the graphical method of vector addition and subtraction to solve physics problems

Section Key Terms

graphical method	head (of a vector)	head-to-tail method	resultant
resultant vector	tail	vector addition	vector subtraction

The Graphical Method of Vector Addition and Subtraction

Recall that a vector is a quantity that has magnitude and direction. For example, displacement, velocity, acceleration, and force are all vectors. In one-dimensional or straight-line motion, the direction of a vector can be given simply by a plus or minus sign. Motion that is forward, to the right, or upward is usually considered to be *positive* (+); and motion that is backward, to the left, or downward is usually considered to be *negative* (–).

In two dimensions, a vector describes motion in two perpendicular directions, such as vertical and horizontal. For vertical and horizontal motion, each vector is made up of vertical and horizontal components. In a one-dimensional problem, one of the components simply has a value of zero. For two-dimensional vectors, we work with vectors by using a frame of reference such as a coordinate system. Just as with one-dimensional vectors, we graphically represent vectors with an arrow having a length proportional to the vector's magnitude and pointing in the direction that the vector points.

Figure 5.2 shows a graphical representation of a vector; the total displacement for a person walking in a city. The person first walks nine blocks east and then five blocks north. Her total displacement does not match her path to her final destination. The displacement simply connects her starting point with her ending point using a straight line, which is the shortest distance. We use the notation that a boldface symbol, such as **D**, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction is given by an angle represented by the symbol θ. Note that her displacement would be the same if she had begun by first walking five blocks north and then walking nine blocks east.

> **TIPS FOR SUCCESS**
> In this text, we represent a vector with a boldface variable. For example, we represent a force with the vector **F**, which has both magnitude and direction. The magnitude of the vector is represented by the variable in italics, F, and the direction of the variable is given by the angle θ.
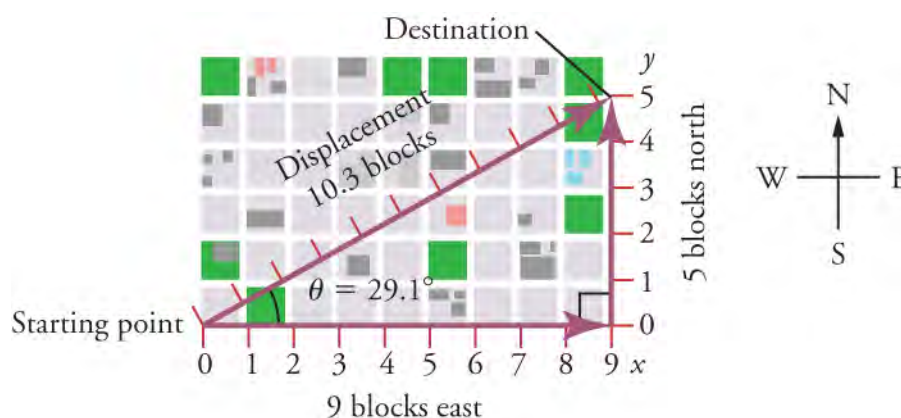
Figure 5.2 A person walks nine blocks east and five blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.

The **head-to-tail method** is a **graphical** way to add vectors. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the pointed end of the arrow. The following steps describe how to use the head-to-tail method for graphical **vector addition**.

1. Let the *x*-axis represent the east-west direction. Using a ruler and protractor, draw an arrow to represent the first vector (nine blocks to the east), as shown in Figure 5.3(a).
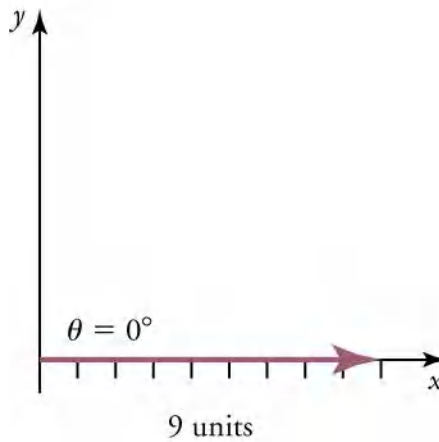
(a)

Figure 5.3 The diagram shows a vector with a magnitude of nine units and a direction of 0°.

2. Let the *y*-axis represent the north-south direction. Draw an arrow to represent the second vector (five blocks to the north). Place the tail of the second vector at the head of the first vector, as shown in Figure 5.4(b).
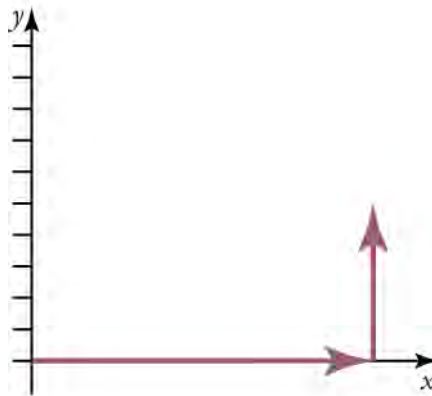
(b)

Figure 5.4 A vertical vector is added.

3. If there are more than two vectors, continue to add the vectors head-to-tail as described in step 2. In this example, we have only two vectors, so we have finished placing arrows tip to tail.

4. Draw an arrow from the tail of the first vector to the head of the last vector, as shown in Figure 5.5(c). This is the **resultant**, or the sum, of the vectors.
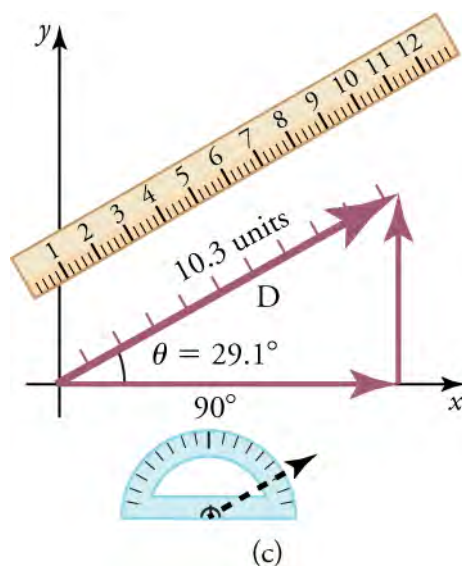
Figure 5.5 The diagram shows the resultant vector, a ruler, and protractor.

5. To find the magnitude of the resultant, measure its length with a ruler. When we deal with vectors analytically in the next section, the magnitude will be calculated by using the Pythagorean theorem.
6. To find the direction of the resultant, use a protractor to measure the angle it makes with the reference direction (in this case, the x-axis). When we deal with vectors analytically in the next section, the direction will be calculated by using trigonometry to find the angle.

WATCH PHYSICS

Visualizing Vector Addition Examples

This video shows four graphical representations of vector addition and matches them to the correct vector addition formula.

Click to view content (https://openstax.org/l/02addvector)

> **GRASP CHECK**
>
> There are two vectors $\vec{a}$ and $\vec{b}$. The head of vector $\vec{a}$ touches the tail of vector $\vec{b}$. The addition of vectors $\vec{a}$ and $\vec{b}$ gives a resultant vector $\vec{c}$. Can the addition of these two vectors can be represented by the following two equations? $\vec{a} + \vec{b} = \vec{c}$; $\vec{b} + \vec{a} = \vec{c}$
>
> a.   Yes, if we add the same two vectors in a different order it will still give the same resultant vector.
> b.   No, the resultant vector will change if we add the same vectors in a different order.

Vector subtraction is done in the same way as vector addition with one small change. We add the first vector to the negative of the vector that needs to be subtracted. A negative vector has the same magnitude as the original vector, but points in the opposite direction (as shown in Figure 5.6). Subtracting the vector **B** from the vector **A**, which is written as **A** − **B**, is the same as **A** + (−**B**). Since it does not matter in what order vectors are added, **A** − **B** is also equal to (−**B**) + **A**. This is true for scalars as well as vectors. For example, 5 − 2 = 5 + (−2) = (−2) + 5.
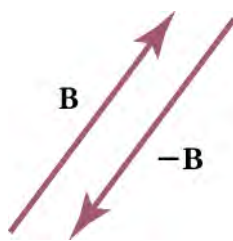
Figure 5.6 The diagram shows a vector, B, and the negative of this vector, −B.

Global angles are calculated in the counterclockwise direction. The clockwise direction is considered negative. For example, an angle of $30°$ south of west is the same as the global angle $210°$, which can also be expressed as $−150°$ from the positive x-axis.

Using the Graphical Method of Vector Addition and Subtraction to Solve Physics Problems

Now that we have the skills to work with vectors in two dimensions, we can apply vector addition to graphically determine the **resultant vector**, which represents the total force. Consider an example of force involving two ice skaters pushing a third as seen in Figure 5.7.
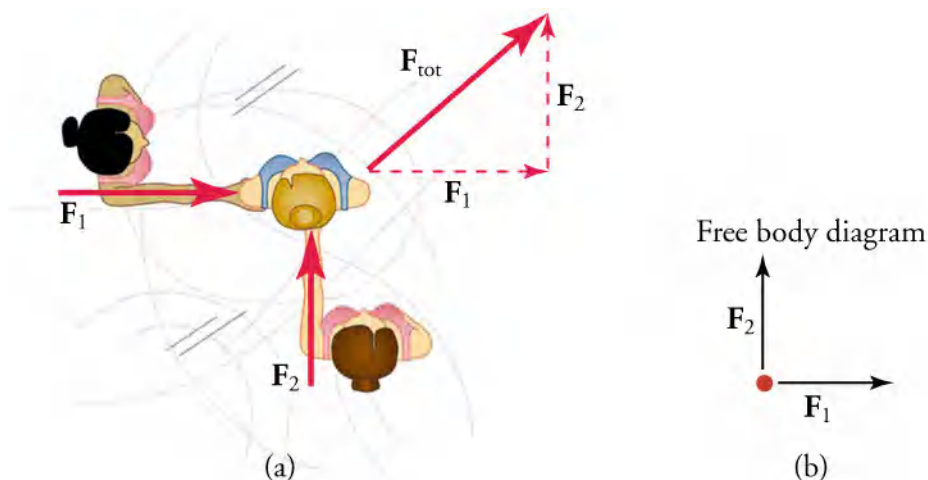
Figure 5.7 Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

In problems where variables such as force are already known, the forces can be represented by making the length of the vectors proportional to the magnitudes of the forces. For this, you need to create a scale. For example, each centimeter of vector length could represent 50 N worth of force. Once you have the initial vectors drawn to scale, you can then use the head-to-tail method to draw the resultant vector. The length of the resultant can then be measured and converted back to the original units using the scale you created.

You can tell by looking at the vectors in the free-body diagram in Figure 5.7 that the two skaters are pushing on the third skater with equal-magnitude forces, since the length of their force vectors are the same. Note, however, that the forces are not equal because they act in different directions. If, for example, each force had a magnitude of 400 N, then we would find the magnitude of the total external force acting on the third skater by finding the magnitude of the resultant vector. Since the forces act at a right angle to one another, we can use the Pythagorean theorem. For a triangle with sides a, b, and c, the Pythagorean theorem tells us that

$$a^2 + b^2 = c^2$$
$$c = \sqrt{a^2 + b^2}.$$

Applying this theorem to the triangle made by $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_{tot}$ in Figure 5.7, we get

$$\mathbf{F}^2_{tot} = \sqrt{\mathbf{F}^2_1 + \mathbf{F}^2_1},$$

or

$$\mathbf{F}_{tot} = \sqrt{(400 \text{ N})^2 + (400 \text{ N})^2} = 566 \text{ N}.$$

Note that, if the vectors were not at a right angle to each other ($90°$ to one another), we would not be able to use the Pythagorean theorem to find the magnitude of the resultant vector. Another scenario where adding two-dimensional vectors is necessary is for velocity, where the direction may not be purely east-west or north-south, but some combination of these two directions. In the next section, we cover how to solve this type of problem analytically. For now let's consider the problem graphically.

✳ WORKED EXAMPLE

Adding Vectors Graphically by Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, he walks 25 m in a direction $49°$ north of east. Then, he walks 23 m heading $15°$ north of east. Finally, he turns and walks 32 m in a direction $68°$ south of east.

Strategy

Graphically represent each displacement vector with an arrow, labeling the first **A**, the second **B**, and the third **C**. Make the lengths proportional to the distance of the given displacement and orient the arrows as specified relative to an east-west line. Use the head-to-tail method outlined above to determine the magnitude and direction of the resultant displacement, which we'll call **R**.

Solution

(1) Draw the three displacement vectors, creating a convenient scale (such as 1 cm of vector length on paper equals 1 m in the problem), as shown in Figure 5.8.
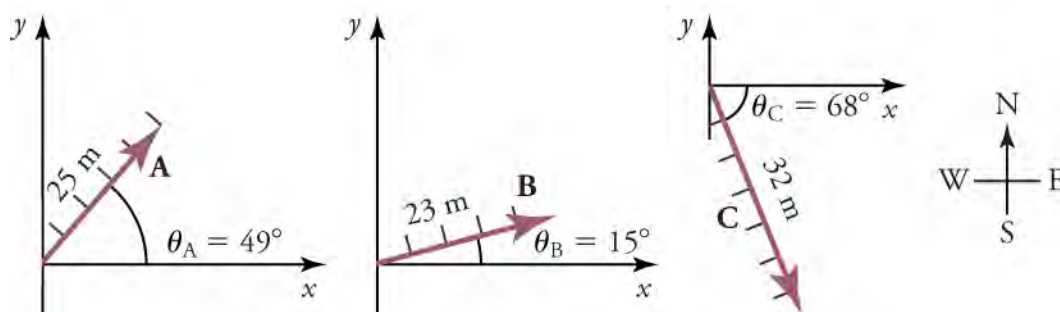
Figure 5.8 The three displacement vectors are drawn first.

(2) Place the vectors head to tail, making sure not to change their magnitude or direction, as shown in Figure 5.9.
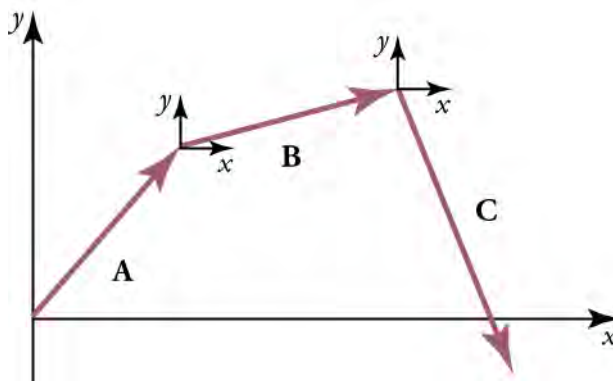
Figure 5.9 Next, the vectors are placed head to tail.

(3) Draw the **resultant vector R** from the tail of the first vector to the head of the last vector, as shown in Figure 5.10.
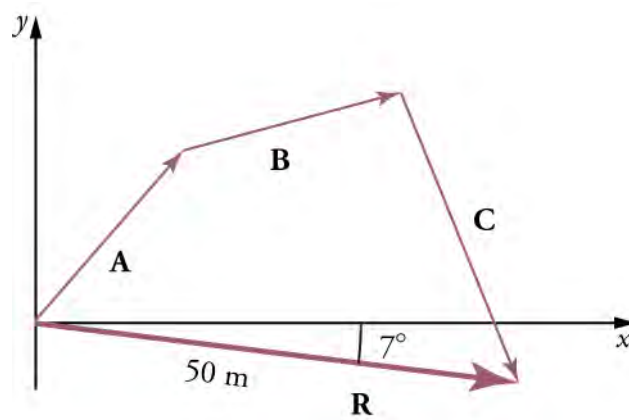
Figure 5.10 The resultant vector is drawn .

(4) Use a ruler to measure the magnitude of **R**, remembering to convert back to the units of meters using the scale. Use a protractor to measure the direction of **R**. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since **R** is south of the eastward pointing axis (the *x*-axis), we flip the protractor upside down and measure the angle between the eastward axis and the vector, as illustrated in Figure 5.11.
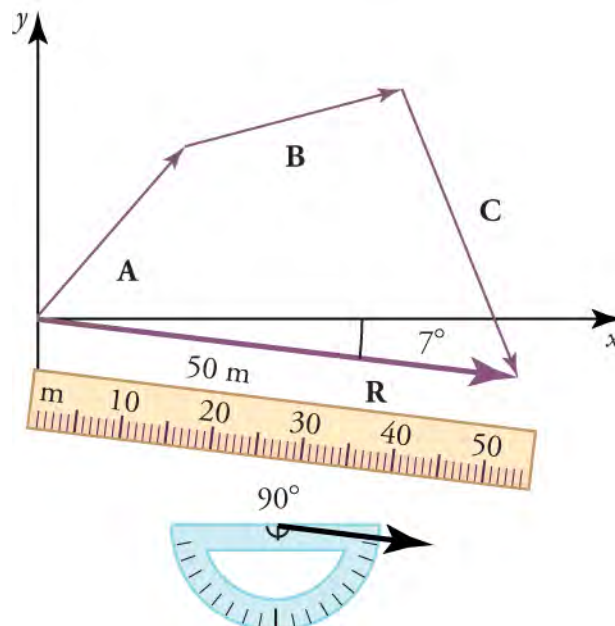
Figure 5.11 A ruler is used to measure the magnitude of **R**, and a protractor is used to measure the direction of **R**.

In this case, the total displacement **R** has a magnitude of 50 m and points 7° south of east. Using its magnitude and direction, this vector can be expressed as

$$R = 50 \text{ m}$$

<div align="right">5.1</div>

and

$$\theta = 7° \text{ south of east.}$$

<div align="right">5.2</div>

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that it does not matter in what order the vectors are added. Changing the order does not change the resultant. For example, we could add the vectors as shown in Figure 5.12, and we would still get the same solution.
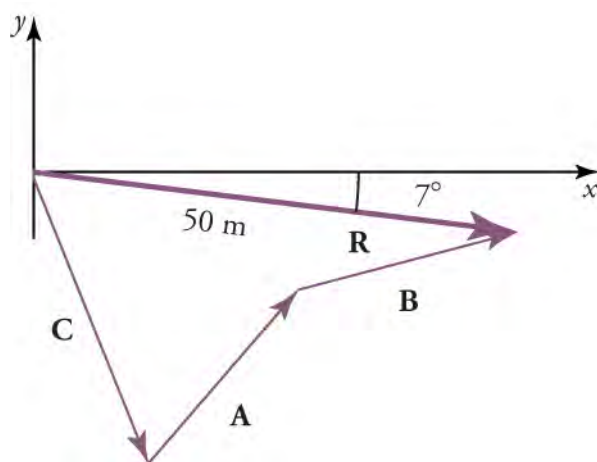
Figure 5.12 Vectors can be added in any order to get the same result.

✳ WORKED EXAMPLE

Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction $66.0°$ north of east from her current location, and then travel 30.0 m in a direction $112°$ north of east (or $22.0°$ west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? The two legs of the woman's trip are illustrated in Figure 5.13.
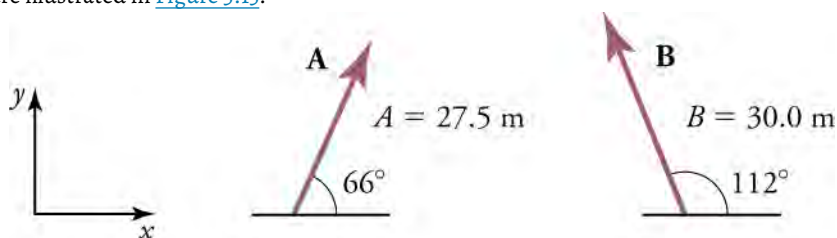
Figure 5.13   In the diagram, the first leg of the trip is represented by vector A and the second leg is represented by vector B.

Strategy

We can represent the first leg of the trip with a vector **A**, and the second leg of the trip that she was *supposed to* take with a vector **B**. Since the woman mistakenly travels in the opposite direction for the second leg of the journey, the vector for second leg of the trip she *actually* takes is −**B**. Therefore, she will end up at a location **A** + (−**B**), or **A** − **B**. Note that −**B** has the same magnitude as **B** (30.0 m), but is in the opposite direction, $68°(180° − 112°)$ south of east, as illustrated in Figure 5.14.
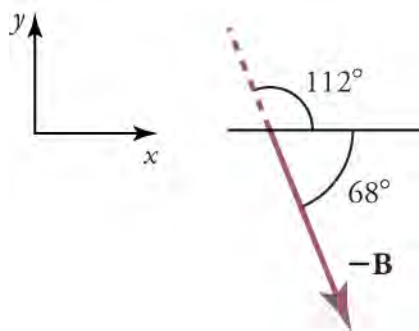
Figure 5.14   Vector −B represents traveling in the opposite direction of vector B.

We use graphical vector addition to find where the woman arrives **A** + (−**B**).

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors **A** and –**B**.

(2) Place the vectors head to tail.

(3) Draw the resultant vector **R**.

(4) Use a ruler and protractor to measure the magnitude and direction of **R**.

These steps are demonstrated in Figure 5.15.
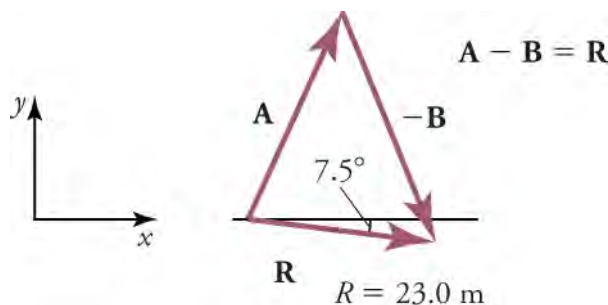
Figure 5.15 The vectors are placed head to tail.

In this case

$$R = 23.0 \text{ m}$$

<div style="float:right">5.3</div>

and

$$\theta = 7.5° \text{south of east.}$$

<div style="float:right">5.4</div>

Discussion

Because subtraction of a vector is the same as addition of the same vector with the opposite direction, the graphical method for subtracting vectors works the same as for adding vectors.

✳ WORKED EXAMPLE

Adding Velocities: A Boat on a River

A boat attempts to travel straight across a river at a speed of 3.8 m/s. The river current flows at a speed v_{river} of 6.1 m/s to the right. What is the total velocity and direction of the boat? You can represent each meter per second of velocity as one centimeter of vector length in your drawing.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river. Because the boat is directed straight toward the other shore, its velocity is perpendicular to the velocity of the river. We draw the two vectors, $\mathbf{v}_{boat}$ and $\mathbf{v}_{river}$, as shown in Figure 5.16.

Using the head-to-tail method, we draw the resulting total velocity vector from the tail of $\mathbf{v}_{boat}$ to the head of $\mathbf{v}_{river}$.
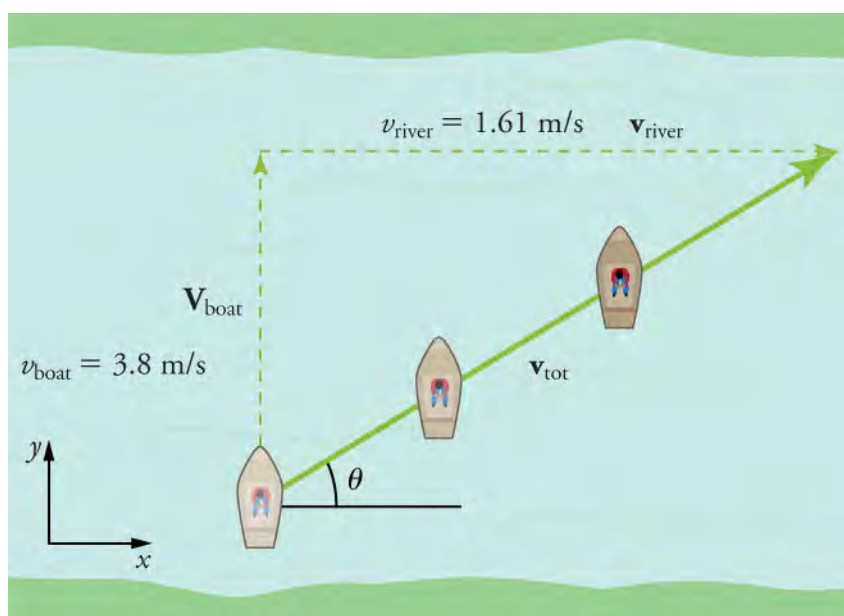
Figure 5.16   A boat attempts to travel across a river. What is the total velocity and direction of the boat?

Solution

By using a ruler, we find that the length of the resultant vector is 7.2 cm, which means that the magnitude of the total velocity is

$$v_{tot} = 7.2 \ \text{m/s}.$$

<div style="text-align:right">5.5</div>

By using a protractor to measure the angle, we find $\theta = 32.0°$.

Discussion

If the velocity of the boat and river were equal, then the direction of the total velocity would have been 45°. However, since the velocity of the river is greater than that of the boat, the direction is less than 45° with respect to the shore, or x axis.

Practice Problems

1. Vector $\overrightarrow{A}$, having magnitude 2.5 m, pointing 37° south of east and vector $\overrightarrow{B}$ having magnitude 3.5 m, pointing 20° north of east are added. What is the magnitude of the resultant vector?
    a.   1.0 m
    b.   5.3 m
    c.   5.9 m
    d.   6.0 m

2. A person walks 32° north of west for 94 m and 35° east of south for 122 m. What is the magnitude of his displacement?
    a.   28 m
    b.   51 m
    c.   180 m
    d.   216 m

> ## Virtual Physics
>
> ### Vector Addition
>
> In this simulation (https://archive.cnx.org/specials/d218bf9b-e50e-4d50-9a6c-b3db4dad0816/vector-addition/) , you will experiment with adding vectors graphically. Click and drag the red vectors from the Grab One basket onto the graph in the middle of the screen. These red vectors can be rotated, stretched, or repositioned by clicking and dragging with your mouse. Check the Show Sum box to display the resultant vector (in green), which is the sum of all of the red vectors placed on the

graph. To remove a red vector, drag it to the trash or click the Clear All button if you wish to start over. Notice that, if you click on any of the vectors, the $|\mathbf{R}|$ is its magnitude, θ is its direction with respect to the positive x-axis, $\mathbf{R_x}$ is its horizontal component, and R_y is its vertical component. You can check the resultant by lining up the vectors so that the head of the first vector touches the tail of the second. Continue until all of the vectors are aligned together head-to-tail. You will see that the resultant magnitude and angle is the same as the arrow drawn from the tail of the first vector to the head of the last vector. Rearrange the vectors in any order head-to-tail and compare. The resultant will always be the same.

Click to view content (https://archive.cnx.org/specials/d218bf9b-e50e-4d50-9a6c-b3db4dad0816/vector-addition/)

GRASP CHECK
True or False—The more long, red vectors you put on the graph, rotated in any direction, the greater the magnitude of the resultant green vector.
a. True
b. False

Check Your Understanding

3. While there is no single correct choice for the sign of axes, which of the following are conventionally considered positive?
    a. backward and to the left
    b. backward and to the right
    c. forward and to the right
    d. forward and to the left

4. True or False—A person walks 2 blocks east and 5 blocks north. Another person walks 5 blocks north and then two blocks east. The displacement of the first person will be more than the displacement of the second person.
    a. True
    b. False

5.2 Vector Addition and Subtraction: Analytical Methods

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Define components of vectors
- Describe the analytical method of vector addition and subtraction
- Use the analytical method of vector addition and subtraction to solve problems

Section Key Terms

analytical method   component (of a two-dimensional vector)

Components of Vectors

For the **analytical method** of vector addition and subtraction, we use some simple geometry and trigonometry, instead of using a ruler and protractor as we did for graphical methods. However, the graphical method will still come in handy to visualize the problem by drawing vectors using the head-to-tail method. The analytical method is more accurate than the graphical method, which is limited by the precision of the drawing. For a refresher on the definitions of the sine, cosine, and tangent of an angle, see Figure 5.17.
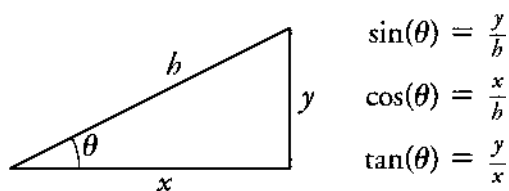
Figure 5.17 For a right triangle, the sine, cosine, and tangent of θ are defined in terms of the adjacent side, the opposite side, or the hypotenuse. In this figure, x is the adjacent side, y is the opposite side, and h is the hypotenuse.

Since, by definition, $\cos\theta = x/h$, we can find the length x if we know h and θ by using $x = h\cos\theta$. Similarly, we can find the length of y by using $y = h\sin\theta$. These trigonometric relationships are useful for adding vectors.

When a vector acts in more than one dimension, it is useful to break it down into its x and y components. For a two-dimensional vector, a **component** is a piece of a vector that points in either the x- or y-direction. Every 2-d vector can be expressed as a sum of its x and y components.

For example, given a vector like $\mathbf{A}$ in Figure 5.18, we may want to find what two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it. In this example, $\mathbf{A}_x$ and $\mathbf{A}_y$ form a right triangle, meaning that the angle between them is 90 degrees. This is a common situation in physics and happens to be the least complicated situation trigonometrically.
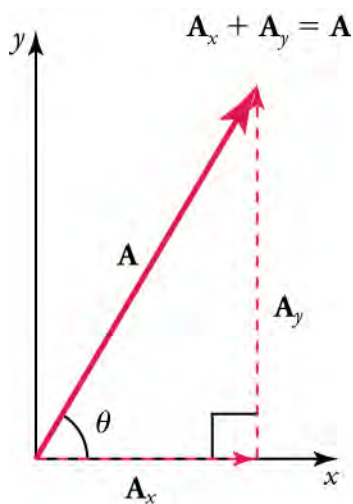
Figure 5.18 The vector $\mathbf{A}$, with its tail at the origin of an x- y-coordinate system, is shown together with its x- and y-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the x- and y-axes. The three vectors, $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$, form a right triangle.

$$\mathbf{A_x} + \mathbf{A_y} = \mathbf{A}$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its x- and y-components, we use the following relationships for a right triangle:

$$A_x = A\cos\theta$$

and

$$A_y = A\sin\theta,$$

where A_x is the magnitude of $\mathbf{A}$ in the x-direction, A_y is the magnitude of $\mathbf{A}$ in the y-direction, and θ is the angle of the resultant with respect to the x-axis, as shown in Figure 5.19.
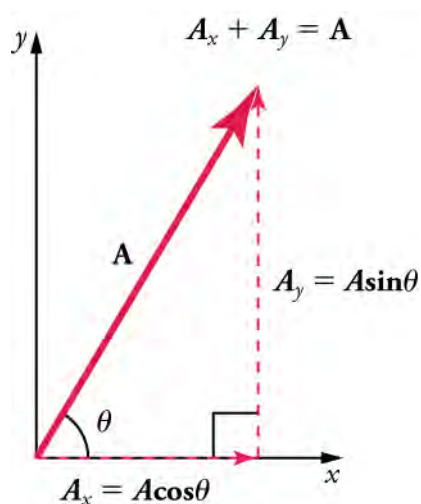
Figure 5.19 The magnitudes of the vector components A_x and A_y can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city, as illustrated in Figure 5.20.
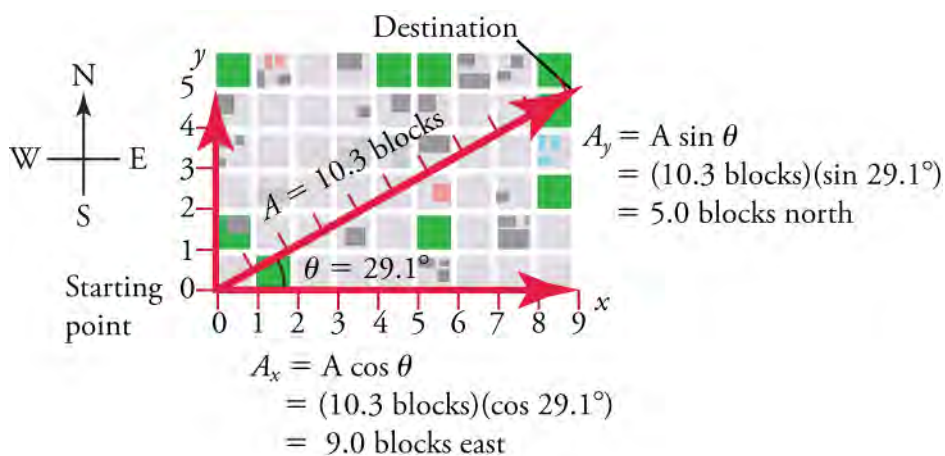
Figure 5.20 We can use the relationships $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then A = 10.3 blocks and $\theta = 29.1°$, so that

$$
\begin{aligned}
A_x &= A\cos\theta \\
&= (10.3 \ \text{blocks})(\cos 29.1°) \\
&= (10.3 \ \text{blocks})(0.874) \\
&= 9.0 \ \text{blocks}.
\end{aligned}
$$

5.6

This magnitude indicates that the walker has traveled 9 blocks to the east—in other words, a 9-block eastward displacement. Similarly,

$$
\begin{aligned}
A_y &= A\sin\theta \\
&= (10.3 \ \text{blocks})(\sin 29.1°) \\
&= (10.3 \ \text{blocks})(0.846) \\
&= 5.0 \ \text{blocks},
\end{aligned}
$$

5.7

indicating that the walker has traveled 5 blocks to the north—a 5-block northward displacement.

Analytical Method of Vector Addition and Subtraction

Calculating a resultant vector (or vector addition) is the reverse of breaking the resultant down into its components. If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then we can find $\mathbf{A}$ analytically. How do we do this? Since, by definition,

$$\tan\theta = y/x \text{ (or in this case } \tan\theta = A_y/A_x\text{)},$$

we solve for θ to find the direction of the resultant.

$$\theta = \tan^{-1}(A_y/A_x)$$

Since this is a right triangle, the Pythagorean theorem ($x^2 + y^2 = h^2$) for finding the hypotenuse applies. In this case, it becomes

$$A^2 = A_x^2 + A_y^2.$$

Solving for A gives

$$A = \sqrt{A_x{}^2 + A_y{}^2}.$$

In summary, to find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, as illustrated in Figure 5.21, we use the following relationships:

$$A = \sqrt{A_x{}^2 + A_y{}^2}$$
$$\theta = \tan^{-1}(A_y/A_x)$$
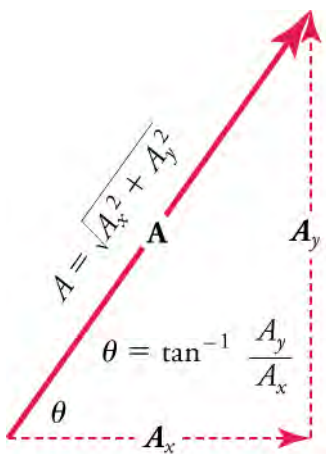
Figure 5.21 The magnitude and direction of the resultant vector $\mathbf{A}$ can be determined once the horizontal components $\mathbf{A}_x$ and $\mathbf{A}_y$ have been determined.

Sometimes, the vectors added are not perfectly perpendicular to one another. An example of this is the case below, where the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$, as illustrated in Figure 5.22.
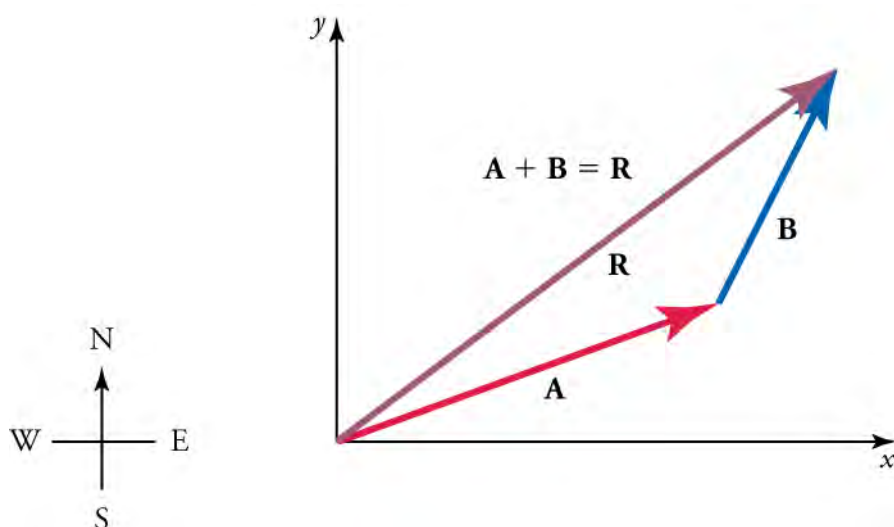
Figure 5.22 Vectors **A** and **B** are two legs of a walk, and **R** is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of **R**.

If **A** and **B** represent two legs of a walk (two displacements), then **R** is the total displacement. The person taking the walk ends up at the tip of **R**. There are many ways to arrive at the same point. The person could have walked straight ahead first in the x-direction and then in the y-direction. Those paths are the x- and y-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $R = \sqrt{R_x{}^2 + R_y{}^2}$ and $\theta = tan^{-1}(R_y/R_x)$.

1. Draw in the x and y components of each vector (including the resultant) with a dashed line. Use the equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to find the components. In Figure 5.23, these components are A_x, A_y, B_x, and B_y. Vector **A** makes an angle of θ_A with the x-axis, and vector **B** makes and angle of θ_B with its own x-axis (which is slightly above the x-axis used by vector **A**).
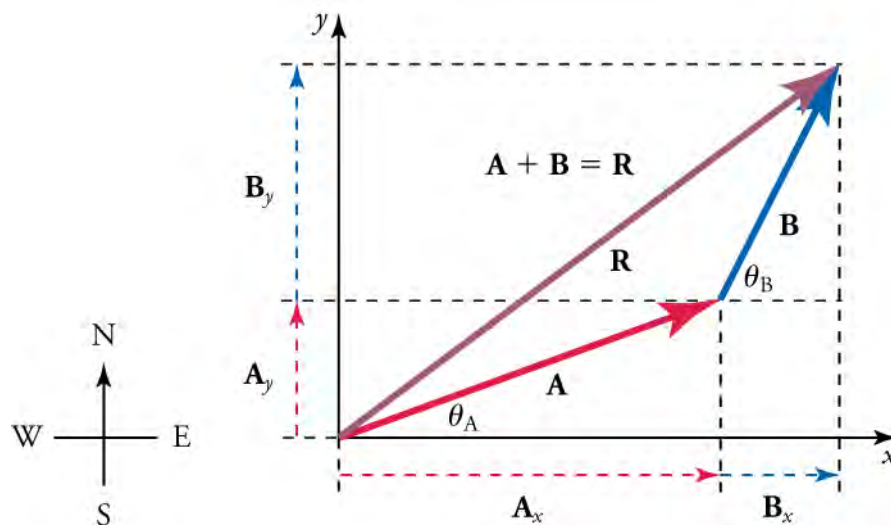
Figure 5.23 To add vectors **A** and **B**, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$ $\mathbf{B}_y$ shown in the image.

2. Find the x component of the resultant by adding the x component of the vectors
$$R_x = A_x + B_x$$

and find the y component of the resultant (as illustrated in Figure 5.24) by adding the y component of the vectors.

$$R_y = A_y + B_y.$$
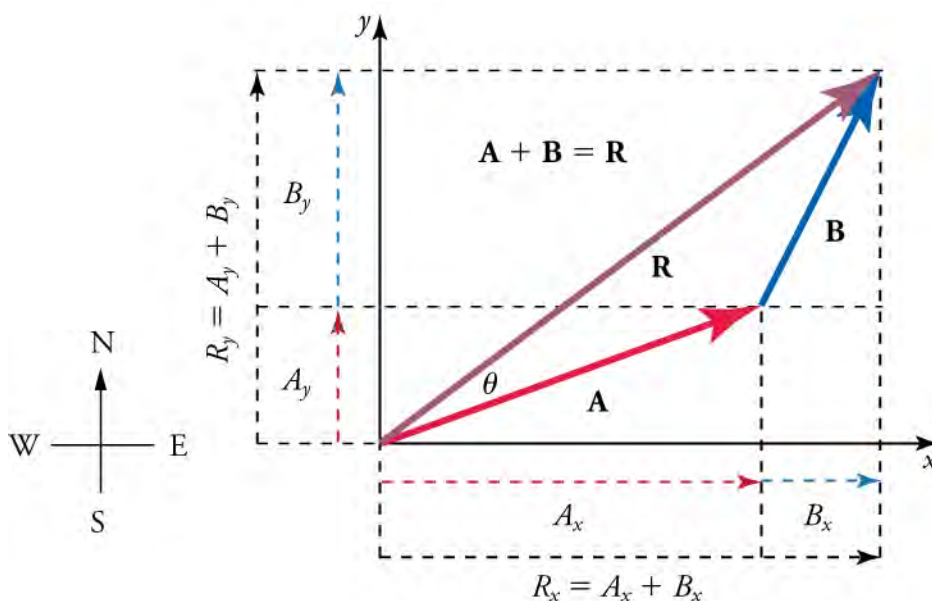
Figure 5.24 The vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude of the resultant vector in the horizontal direction, R_x. Similarly, the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude of the resultant vector in the vertical direction, R_y.

Now that we know the components of $\mathbf{R}$, we can find its magnitude and direction.

3. To get the magnitude of the resultant R, use the Pythagorean theorem.

$$R = \sqrt{R_x^2 + R_y^2}$$

4. To get the direction of the resultant

$$\theta = \tan^{-1}(R_y/R_x).$$

WATCH PHYSICS

Classifying Vectors and Quantities Example

This video contrasts and compares three vectors in terms of their magnitudes, positions, and directions.

Click to view content (https://www.youtube.com/embed/YpoEhcVBxNU)

> **GRASP CHECK**
> Three vectors, $\vec{u}$, $\vec{v}$, and $\vec{w}$, have the same magnitude of 5 units. Vector $\vec{v}$ points to the northeast. Vector $\vec{w}$ points to the southwest exactly opposite to vector $\vec{u}$. Vector $\vec{u}$ points in the northwest. If the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ were added together, what would be the magnitude of the resultant vector? Why?
> a. 0 units. All of them will cancel each other out.
> b. 5 units. Two of them will cancel each other out.
> c. 10 units. Two of them will add together to give the resultant.
> d. 15 units. All of them will add together to give the resultant.

> ## TIPS FOR SUCCESS
> In the video, the vectors were represented with an arrow above them rather than in bold. This is a common notation in math classes.

Using the Analytical Method of Vector Addition and Subtraction to Solve Problems

Figure 5.25 uses the analytical method to add vectors.

✳ WORKED EXAMPLE

An Accelerating Subway Train

Add the vector **A** to the vector **B** shown in Figure 5.25, using the steps above. The x-axis is along the east–west direction, and the y-axis is along the north–south directions. A person first walks 53.0 m in a direction $20.0°$ north of east, represented by vector **A**. The person then walks 34.0 m in a direction $63.0°$ north of east, represented by vector **B**.
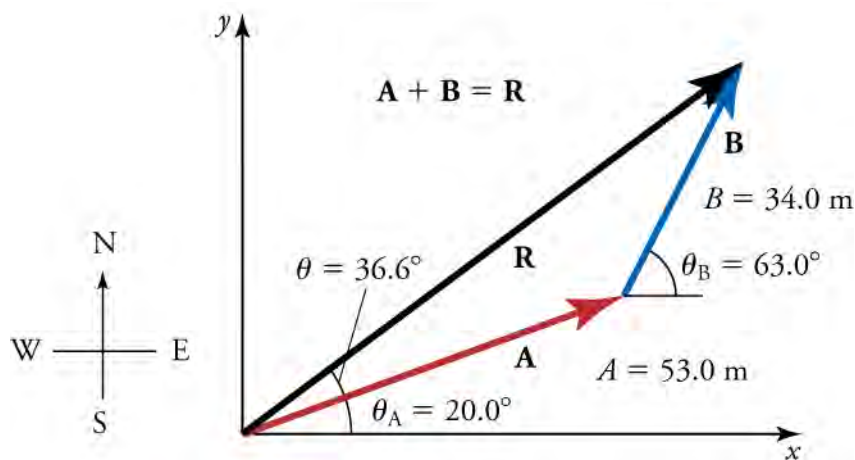
Figure 5.25   You can use analytical models to add vectors.

Strategy

The components of **A** and **B** along the x- and y-axes represent walking due east and due north to get to the same ending point. We will solve for these components and then add them in the x-direction and y-direction to find the resultant.

Solution

First, we find the components of **A** and **B** along the x- and y-axes. From the problem, we know that $A = 53.0$ m, $\theta_A = 20.0°$, $B = 34.0$ m, and $\theta_B = 63.0°$. We find the x-components by using $A_x = A \cos\theta$, which gives

$$
\begin{aligned}
A_x &= A \cos\theta_A = (53.0\ \text{m})(\cos 20.0°) \\
&= (53.0\ \text{m})(0.940) = 49.8\ \text{m}
\end{aligned}
$$

and

$$
\begin{aligned}
B_x &= B \cos\theta_B = (34.0\ \text{m})(\cos 63.0°) \\
&= (34.0\ \text{m})(0.454) = 15.4\ \text{m.}
\end{aligned}
$$

Similarly, the y-components are found using $A_y = A \sin\theta_A$

$$
\begin{aligned}
A_y &= A \sin\theta_A = (53.0\ \text{m})(\sin 20.0°) \\
&= (53.0\ \text{m})(0.342) = 18.1\ \text{m}
\end{aligned}
$$

and

$$
\begin{aligned}
B_y &= B \sin\theta_B = (34.0\ \text{m})(\sin 63.0°) \\
&= (34.0\ \text{m})(0.891) = 30.3\ \text{m.}
\end{aligned}
$$

The x- and y-components of the resultant are

$$
R_x = A_x + B_x = 49.8\ \text{m} + 15.4\ \text{m} = 65.2\ \text{m}
$$

and

$$R_y = A_y + B_y = 18.1 \ \text{m} + 30.3 \ \text{m} = 48.4 \ \text{m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2} \ \text{m} \qquad 5.8$$

so that

$$R = \sqrt{6601 \ \text{m}} = 81.2 \ \text{m}.$$

Finally, we find the direction of the resultant

$$\theta = \tan^{-1}(R_y/R_x) = +\tan^{-1}(48.4/65.2).$$

This is

$$\theta = \tan^{-1}(0.742) = 36.6°.$$

Discussion

This example shows vector addition using the analytical method. Vector subtraction using the analytical method is very similar. It is just the addition of a negative vector. That is, $A - B \equiv A + (-B)$. The components of $-B$ are the negatives of the components of B. Therefore, the x- and y-components of the resultant $A - B = R$ are

$$R_x = A_x + -B_x$$

and

$$R_y = A_y + -B_y$$

and the rest of the method outlined above is identical to that for addition.

Practice Problems

5. What is the magnitude of a vector whose x-component is 4 cm and whose y-component is 3 cm?
    a. 1 cm
    b. 5 cm
    c. 7 cm
    d. 25 cm

6. What is the magnitude of a vector that makes an angle of 30° to the horizontal and whose x-component is 3 units?
    a. 2.61 units
    b. 3.00 units
    c. 3.46 units
    d. 6.00 units

🔗 LINKS TO PHYSICS

Atmospheric Science

Figure 5.26 This picture shows Bert Foord during a television Weather Forecast from the Meteorological Office in 1963. (BBC TV)

Atmospheric science is a physical science, meaning that it is a science based heavily on physics. Atmospheric science includes meteorology (the study of weather) and climatology (the study of climate). Climate is basically the average weather over a longer time scale. Weather changes quickly over time, whereas the climate changes more gradually.

The movement of air, water and heat is vitally important to climatology and meteorology. Since motion is such a major factor in weather and climate, this field uses vectors for much of its math.

Vectors are used to represent currents in the ocean, wind velocity and forces acting on a parcel of air. You have probably seen a weather map using vectors to show the strength (magnitude) and direction of the wind.

Vectors used in atmospheric science are often three-dimensional. We won't cover three-dimensional motion in this text, but to go from two-dimensions to three-dimensions, you simply add a third vector component. Three-dimensional motion is represented as a combination of x-, y- and z components, where z is the altitude.

Vector calculus combines vector math with calculus, and is often used to find the rates of change in temperature, pressure or wind speed over time or distance. This is useful information, since atmospheric motion is driven by changes in pressure or temperature. The greater the variation in pressure over a given distance, the stronger the wind to try to correct that imbalance. Cold air tends to be more dense and therefore has higher pressure than warm air. Higher pressure air rushes into a region of lower pressure and gets deflected by the spinning of the Earth, and friction slows the wind at Earth's surface.

Finding how wind changes over distance and multiplying vectors lets meteorologists, like the one shown in Figure 5.26, figure out how much rotation (spin) there is in the atmosphere at any given time and location. This is an important tool for tornado prediction. Conditions with greater rotation are more likely to produce tornadoes.

> **GRASP CHECK**
> Why are vectors used so frequently in atmospheric science?
> a. Vectors have magnitude as well as direction and can be quickly solved through scalar algebraic operations.
> b. Vectors have magnitude but no direction, so it becomes easy to express physical quantities involved in the atmospheric science.
> c. Vectors can be solved very accurately through geometry, which helps to make better predictions in atmospheric science.
> d. Vectors have magnitude as well as direction and are used in equations that describe the three dimensional motion of the atmosphere.

Check Your Understanding

7. Between the analytical and graphical methods of vector additions, which is more accurate? Why?
   a. The analytical method is less accurate than the graphical method, because the former involves geometry and

trigonometry.

   b.   The analytical method is more accurate than the graphical method, because the latter involves some extensive calculations.

   c.   The analytical method is less accurate than the graphical method, because the former includes drawing all figures to the right scale.

   d.   The analytical method is more accurate than the graphical method, because the latter is limited by the precision of the drawing.

8. What is a component of a two dimensional vector?

   a.   A component is a piece of a vector that points in either the x or y direction.

   b.   A component is a piece of a vector that has half of the magnitude of the original vector.

   c.   A component is a piece of a vector that points in the direction opposite to the original vector.

   d.   A component is a piece of a vector that points in the same direction as original vector but with double of its magnitude.

9. How can we determine the global angle θ (measured counter-clockwise from positive x) if we know A_x and A_y?

   a. $\theta = \cos^{-1} \frac{A_y}{A_x}$

   b. $\theta = \cot^{-1} \frac{A_y}{A_x}$

   c. $\theta = \sin^{-1} \frac{A_y}{A_x}$

   d. $\theta = \tan^{-1} \frac{A_y}{A_x}$

10. How can we determine the magnitude of a vector if we know the magnitudes of its components?

   a. $\left|\vec{A}\right| = A_x + A_y$

   b. $\left|\vec{A}\right| = A_x{}^2 + A_y{}^2$

   c. $\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2}$

   d. $\left|\vec{A}\right| = \left(A_x{}^2 + A_y{}^2\right)^2$

5.3 Projectile Motion

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe the properties of projectile motion
- Apply kinematic equations and vectors to solve problems involving projectile motion

Section Key Terms

air resistance	maximum height (of a projectile)	projectile
projectile motion	range	trajectory

Properties of Projectile Motion

Projectile motion is the motion of an object thrown (projected) into the air. After the initial force that launches the object, it only experiences the force of gravity. The object is called a **projectile**, and its path is called its **trajectory**. As an object travels through the air, it encounters a frictional force that slows its motion called **air resistance**. Air resistance does significantly alter trajectory motion, but due to the difficulty in calculation, it is ignored in introductory physics.

The most important concept in projectile motion is that *horizontal and vertical motions are independent*, meaning that they don't influence one another. Figure 5.27 compares a cannonball in free fall (in blue) to a cannonball launched horizontally in projectile motion (in red). You can see that the cannonball in free fall falls at the same rate as the cannonball in projectile motion. Keep in mind that if the cannon launched the ball with any vertical component to the velocity, the vertical displacements would not line up perfectly.

Since vertical and horizontal motions are independent, we can analyze them separately, along perpendicular axes. To do this, we separate projectile motion into the two components of its motion, one along the horizontal axis and the other along the vertical.
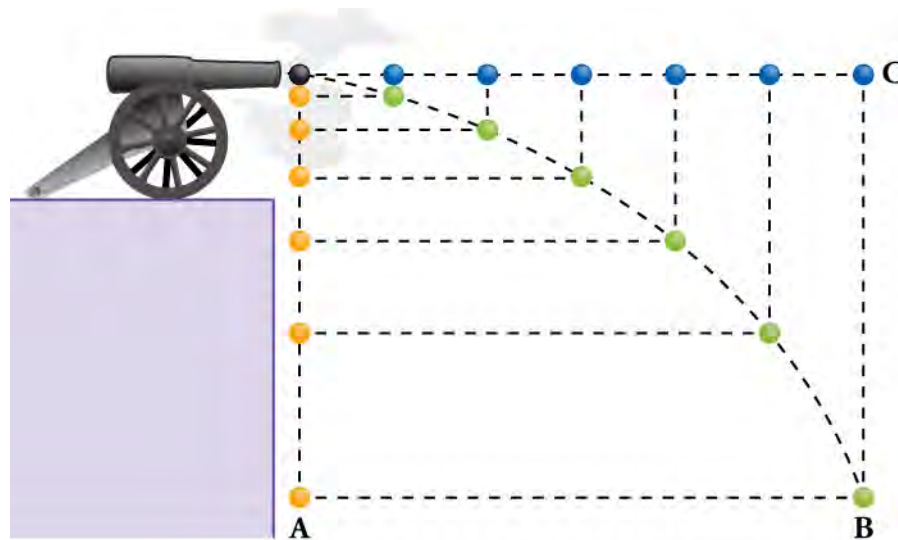
Figure 5.27 The diagram shows the projectile motion of a cannonball shot at a horizontal angle versus one dropped with no horizontal velocity. Note that both cannonballs have the same vertical position over time.

We'll call the horizontal axis the x-axis and the vertical axis the y-axis. For notation, $\mathbf{d}$ is the total displacement, and $\mathbf{x}$ and $\mathbf{y}$ are its components along the horizontal and vertical axes. The magnitudes of these vectors are x and y, as illustrated in Figure 5.28.
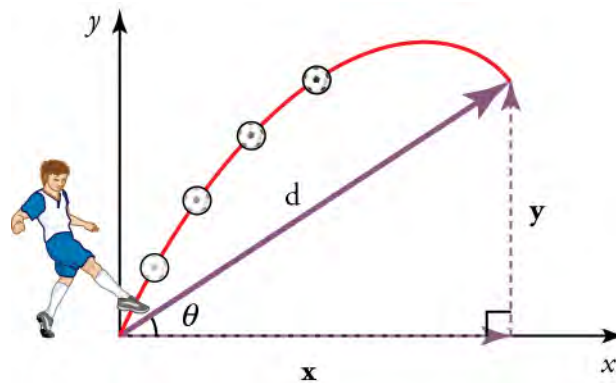
Figure 5.28 A boy kicks a ball at angle θ, and it is displaced a distance of $\mathbf{s}$ along its trajectory.

As usual, we use velocity, acceleration, and displacement to describe motion. We must also find the components of these variables along the x- and y-axes. The components of acceleration are then very simple $\mathbf{a_y} = -\mathbf{g} = -9.80$ m/s². Note that this definition defines the upwards direction as positive. Because gravity is vertical, $\mathbf{a_x} = 0$. Both accelerations are constant, so we can use the kinematic equations. For review, the kinematic equations from a previous chapter are summarized in Table 5.1.

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{v_{avg}}\,t \text{ (when } \mathbf{a}=0\text{)}$$
$$\mathbf{v_{avg}} = \frac{\mathbf{v}_0+\mathbf{v}}{2} \text{ (when } \mathbf{a}=0\text{)}$$
$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$
$$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$
$$\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_0)$$

Table 5.1 Summary of Kinematic Equations (constant a)

Where $\mathbf{x}$ is position, $\mathbf{x}_0$ is initial position, $\mathbf{v}$ is velocity, $\mathbf{v_{avg}}$ is average velocity, t is time and a is acceleration.

Solve Problems Involving Projectile Motion

The following steps are used to analyze projectile motion:

1. Separate the motion into horizontal and vertical components along the x- and y-axes. These axes are perpendicular, so $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used. The magnitudes of the displacement **s** along x- and y-axes are called x and y. The magnitudes of the components of the velocity **v** are $v_x = v \quad \cos\theta$ and $v_y = v \quad \sin\theta$, where v is the magnitude of the velocity and θ is its direction. Initial values are denoted with a subscript 0.

2. Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical. The kinematic equations for horizontal and vertical motion take the following forms

$$\text{Horizontal Motion}(\mathbf{a}_x = 0)$$

$$x = x_0 + v_x t$$

$$v_x = v_{0x} = \mathbf{v}_x = \text{velocity is a constant.}$$

Vertical motion (assuming positive is up $\mathbf{a}_y = -\mathbf{g} = -9.80 \text{ m/s}^2$)

$$y = y_0 + \tfrac{1}{2}(v_{0y} + v_y)t$$

$$v_y = v_{0y} - \mathbf{g}t$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}\mathbf{g}t^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

3. Solve for the unknowns in the two separate motions (one horizontal and one vertical). Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional kinematics.

4. Recombine the two motions to find the total displacement **s** and velocity **v**. We can use the analytical method of vector addition, which uses $A = \sqrt{A_x{}^2 + A_y{}^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ to find the magnitude and direction of the total displacement and velocity.

$$\text{Displacement}$$

$$\mathbf{d} = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$\text{Velocity}$$

$$\mathbf{v} = \sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2}$$

$$\theta_v = \tan^{-1}(\mathbf{v}_y/\mathbf{v}_x)$$

θ is the direction of the displacement **d**, and θ_v is the direction of the velocity **v**. (See Figure 5.29)
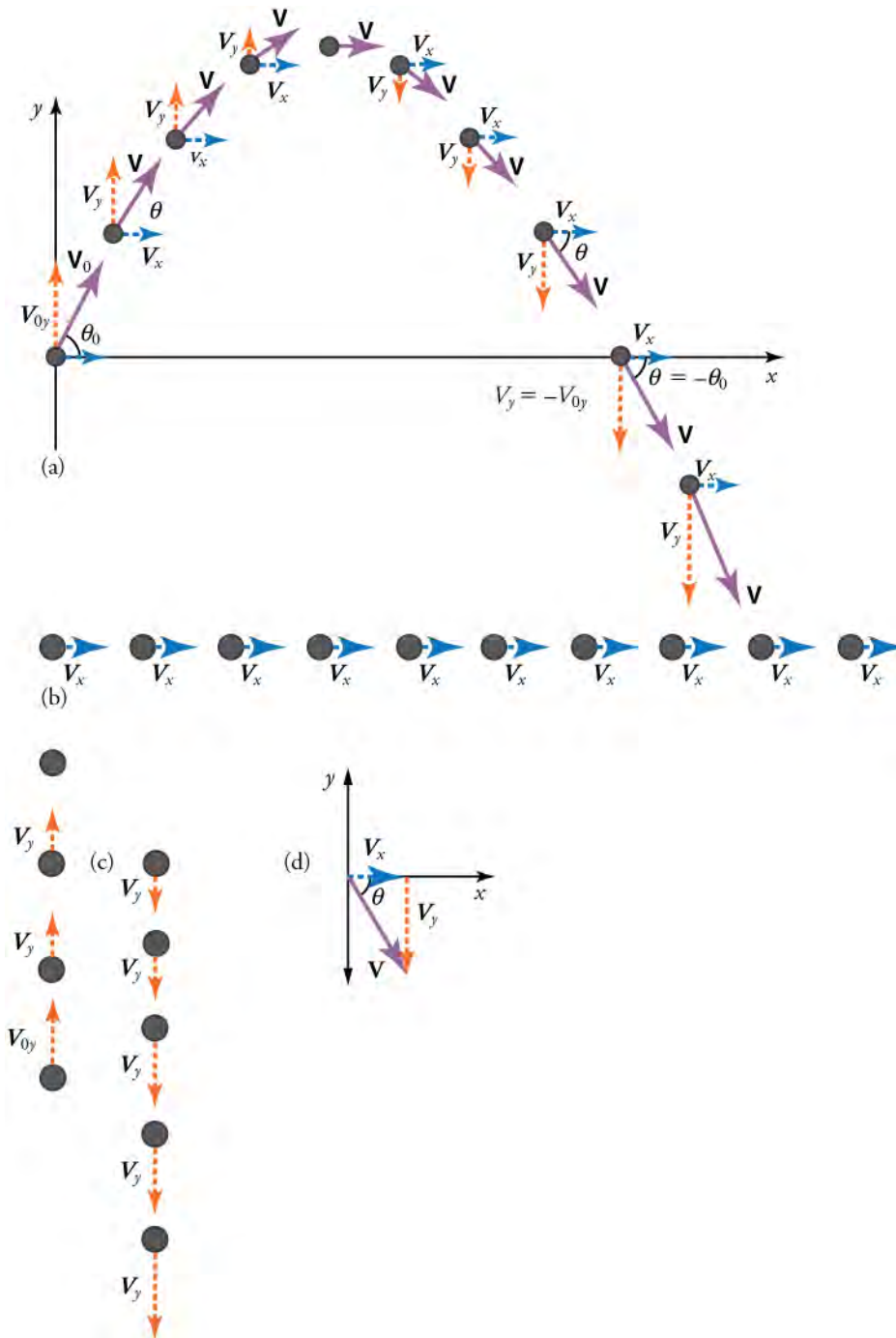
Figure 5.29 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The *x*- and *y*-motions are recombined to give the total velocity at any given point on the trajectory.

TIPS FOR SUCCESS

For problems of projectile motion, it is important to set up a coordinate system. The first step is to choose an initial position for **x** and **y** . Usually, it is simplest to set the initial position of the object so that $x_0 = 0$ and $y_0 = 0$.

WATCH PHYSICS

Projectile at an Angle

This video presents an example of finding the displacement (or range) of a projectile launched at an angle. It also reviews basic trigonometry for finding the sine, cosine and tangent of an angle.

Click to view content (https://www.khanacademy.org/embed_video?v=ZZ3901rAZWY)

> **GRASP CHECK**
> Assume the ground is uniformly level. If the horizontal component a projectile's velocity is doubled, but the vertical component is unchanged, what is the effect on the time of flight?
> a. The time to reach the ground would remain the same since the vertical component is unchanged.
> b. The time to reach the ground would remain the same since the vertical component of the velocity also gets doubled.
> c. The time to reach the ground would be halved since the horizontal component of the velocity is doubled.
> d. The time to reach the ground would be doubled since the horizontal component of the velocity is doubled.

WORKED EXAMPLE

A Fireworks Projectile Explodes High and Away

During a fireworks display like the one illustrated in Figure 5.30, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75° above the horizontal. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?
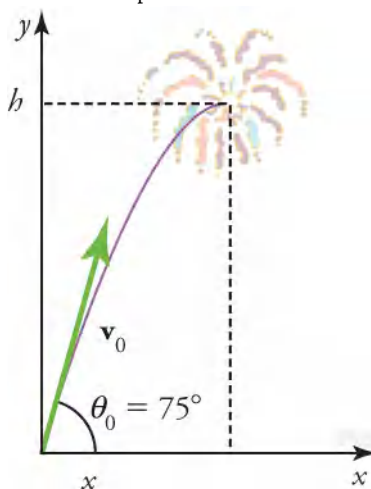
Figure 5.30   The diagram shows the trajectory of a fireworks shell.

Strategy

The motion can be broken into horizontal and vertical motions in which $\mathbf{a}_x = 0$ and $\mathbf{a}_y = \mathbf{g}$. We can then define $\mathbf{x}_0$ and $\mathbf{y}_0$ to be zero and solve for the **maximum height**.

Solution for (a)

By height we mean the altitude or vertical position $\mathbf{y}$ above the starting point. The highest point in any trajectory, the maximum height, is reached when $\mathbf{v}_y = 0$; this is the moment when the vertical velocity switches from positive (upwards) to negative (downwards). Since we know the initial velocity, initial position, and the value of $\mathbf{v}_y$ when the firework reaches its maximum height, we use the following equation to find $\mathbf{y}$

$$\mathbf{v}_y^2 = \mathbf{v}_{0y}^2 - 2\mathbf{g}(\mathbf{y} - y_0).$$

Because $\mathbf{y}_0$ and $\mathbf{v}_y$ are both zero, the equation simplifies to

$$0 = \mathbf{v}_{0y}^2 - 2\mathbf{gy}.$$

Solving for $\mathbf{y}$ gives

$$\mathbf{y} = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}.$$

Now we must find $\mathbf{v}_{0y}$, the component of the initial velocity in the y-direction. It is given by $\mathbf{v}_{0y} = \mathbf{v}_0 \sin \theta$, where $\mathbf{v}_{0y}$ is the initial velocity of 70.0 m/s, and $\theta = 75°$ is the initial angle. Thus,

$$\mathbf{v}_{0y} = \mathbf{v}_0 \sin \theta_0 = (70.0 \ \text{m/s})(\sin 75°) = 67.6 \ \text{m/s}$$

and $\mathbf{y}$ is

$$\mathbf{y} = \frac{(67.6 \ \text{m/s})^2}{2(9.80 \ \text{m/s}^2),}$$

so that

$$\mathbf{y} = 233 \ \text{m}.$$

Discussion for (a)

Since up is positive, the initial velocity and maximum height are positive, but the acceleration due to gravity is negative. The maximum height depends only on the vertical component of the initial velocity. The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding.

Solution for (b)

There is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $\mathbf{y} = \mathbf{y}_0 + \frac{1}{2}(\mathbf{v}_{0y} + \mathbf{v}_y)t$. Because y_0 is zero, this equation reduces to

$$\mathbf{y} = \frac{1}{2}(\mathbf{v}_{0y} + \mathbf{v}_y)t.$$

Note that the final vertical velocity, $\mathbf{v}_y$, at the highest point is zero. Therefore,

$$t = \frac{2\mathbf{y}}{(\mathbf{v}_{0y} + \mathbf{v}_y)} = \frac{2(233 \ \text{m})}{(67.6 \ \text{m/s})}$$
$$= 6.90 \ \text{s}.$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. Another way of finding the time is by using $\mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0y}t - \frac{1}{2}\mathbf{g}t^2$, and solving the quadratic equation for t.

Solution for (c)

Because air resistance is negligible, $\mathbf{a}_x = 0$ and the horizontal velocity is constant. The horizontal displacement is horizontal velocity multiplied by time as given by $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_x t$, where $\mathbf{x}_0$ is equal to zero

$$\mathbf{x} = \mathbf{v}_x t,$$

where $\mathbf{v}_x$ is the x-component of the velocity, which is given by $\mathbf{v}_x = \mathbf{v}_0 \cos \theta_0$. Now,

$$\mathbf{v}_x = \mathbf{v}_0 \cos \theta_0 = (70.0 \ \text{m/s})(\cos 75°) = 18.1 \ \text{m/s}.$$

The time t for both motions is the same, and so $\mathbf{x}$ is

$$\mathbf{x} = (18.1 \ \text{m/s})(6.90 \ \text{s}) = 125 \ \text{m}.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below, while some of the fragments may now have a velocity in the –x direction due to the

forces of the explosion.

The expression we found for **y** while solving part (a) of the previous problem works for any projectile motion problem where air resistance is negligible. Call the maximum height **y** = *h*; then,

$$h = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}.$$

This equation defines the **maximum height of a projectile**. The maximum height depends only on the vertical component of the initial velocity.

✳ WORKED EXAMPLE

Calculating Projectile Motion: Hot Rock Projectile

Suppose a large rock is ejected from a volcano, as illustrated in Figure 5.31, with a speed of 25.0 m/s and at an angle 35° above the horizontal. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path.
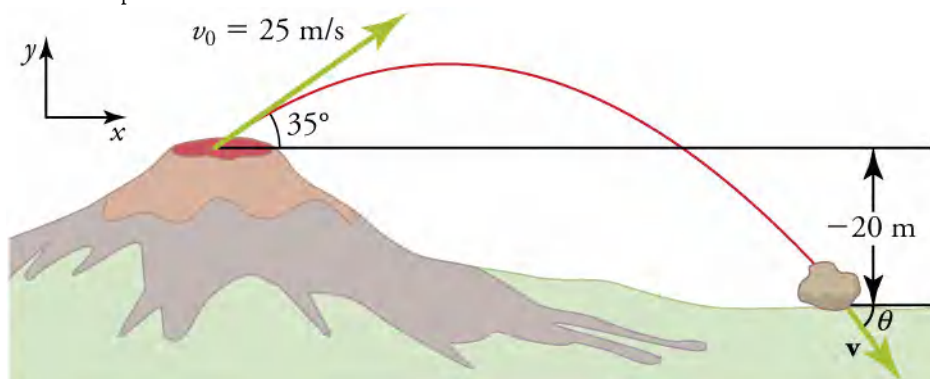
Figure 5.31   The diagram shows the projectile motion of a large rock from a volcano.

Strategy

Breaking this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the time. The time a projectile is in the air depends only on its vertical motion.

Solution

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$\mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0y}t - \frac{1}{2}\mathbf{g}t^2.$$

If we take the initial position $\mathbf{y}_0$ to be zero, then the final position is $\mathbf{y} = -20.0$ m. Now the initial vertical velocity is the vertical component of the initial velocity, found from

$$\mathbf{v}_{0y} = \mathbf{v}_0\sin\theta_0 = (25.0 \text{ m/s})(\sin 35°) = 14.3 \text{ m/s}. \qquad 5.9$$

Substituting known values yields

$$-20.0 \text{ m} = (14.3 \text{ m/s})t - \left(4.90 \text{ m/s}^2\right)t^2.$$

Rearranging terms gives a quadratic equation in *t*

$$\left(4.90 \text{ m/s}^2\right)t^2 - (14.3 \text{ m/s})t - (20.0 \text{ m}) = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are *a* = 4.90, *b* = −14.3, and *c* = −20.0. Its solutions are given by the quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This equation yields two solutions $t = 3.96$ and $t = -1.03$. You may verify these solutions as an exercise. The time is $t = 3.96$ s or -1.03 s. The negative value of time implies an event before the start of motion, so we discard it. Therefore,

$$t = 3.96 \ \text{s}.$$

Discussion

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Practice Problems

11. If an object is thrown horizontally, travels with an average x-component of its velocity equal to 5 m/s, and does not hit the ground, what will be the x-component of the displacement after 20 s?
    a. -100 m
    b. -4 m
    c. 4 m
    d. 100 m

12. If a ball is thrown straight up with an initial velocity of 20 m/s upward, what is the maximum height it will reach?
    a. -20.4 m
    b. -1.02 m
    c. 1.02 m
    d. 20.4 m

The fact that vertical and horizontal motions are independent of each other lets us predict the range of a projectile. The **range** is the horizontal distance **R** traveled by a projectile on level ground, as illustrated in Figure 5.32. Throughout history, people have been interested in finding the range of projectiles for practical purposes, such as aiming cannons.
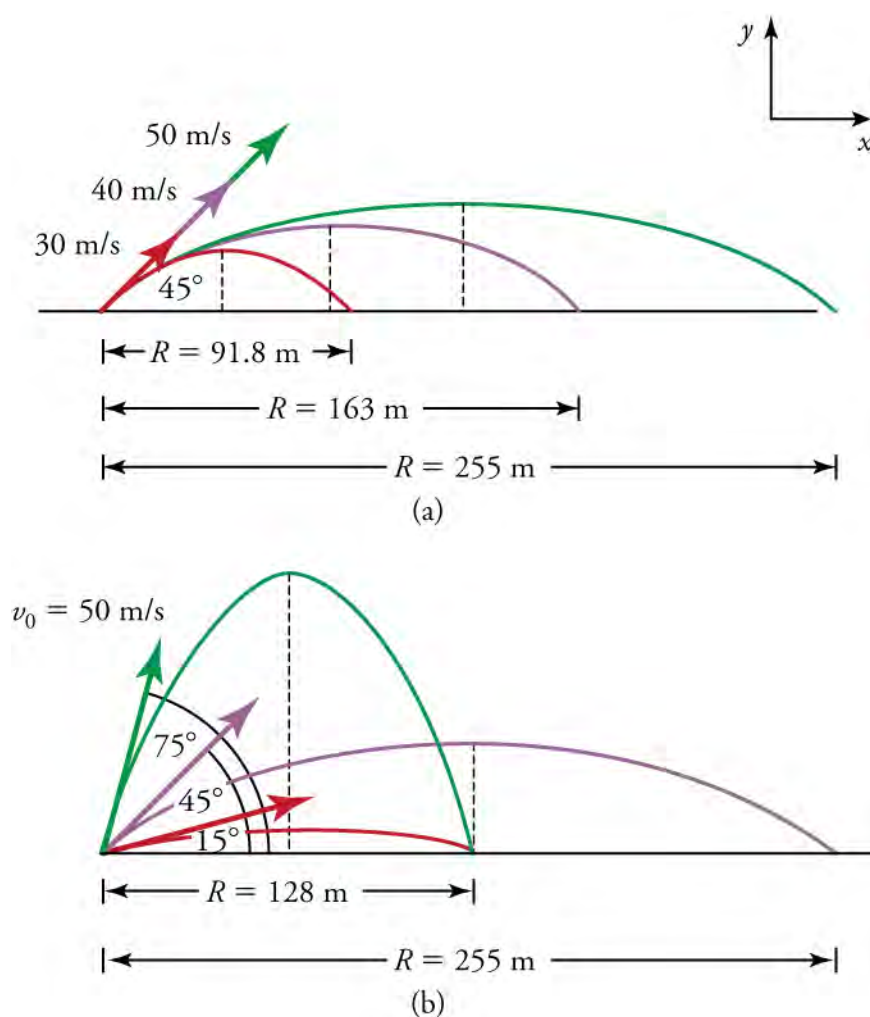
Figure 5.32 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that any combination of trajectories that add to 90 degrees will have the same range in the absence of air resistance, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in the figure above. The initial angle θ_0 also has a dramatic effect on the range. When air resistance is negligible, the range R of a projectile on *level ground* is

$$R = \frac{v_0^2 \sin 2\theta_0}{g},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. It is important to note that the range doesn't apply to problems where the initial and final y position are different, or to cases where the object is launched perfectly horizontally.

Virtual Physics

Projectile Motion
In this simulation you will learn about projectile motion by blasting objects out of a cannon. You can choose between objects such as a tank shell, a golf ball or even a Buick. Experiment with changing the angle, initial speed, and mass, and adding in air resistance. Make a game out of this simulation by trying to hit the target.

Click to view content (https://archive.cnx.org/specials/317dbd00-8e61-4065-b3eb-f2b80db9b7ed/projectile-motion/)

GRASP CHECK

If a projectile is launched on level ground, what launch angle maximizes the range of the projectile?

a. 0°
b. 30°
c. 45°
d. 60°

Check Your Understanding

13. What is projectile motion?
    a. Projectile motion is the motion of an object projected into the air, which moves under the influence of gravity.
    b. Projectile motion is the motion of an object projected into the air which moves independently of gravity.
    c. Projectile motion is the motion of an object projected vertically upward into the air which moves under the influence of gravity.
    d. Projectile motion is the motion of an object projected horizontally into the air which moves independently of gravity.

14. What is the force experienced by a projectile after the initial force that launched it into the air in the absence of air resistance?
    a. The nuclear force
    b. The gravitational force
    c. The electromagnetic force
    d. The contact force

5.4 Inclined Planes

Section Learning Objectives
By the end of this section, you will be able to do the following:

- Distinguish between static friction and kinetic friction
- Solve problems involving inclined planes

Section Key Terms

kinetic friction   static friction

Static Friction and Kinetic Friction

Recall from the previous chapter that friction is a force that opposes motion, and is around us all the time. Friction allows us to move, which you have discovered if you have ever tried to walk on ice.

There are different types of friction—kinetic and static. **Kinetic friction** acts on an object in motion, while **static friction** acts on an object or system at rest. The maximum static friction is usually greater than the kinetic friction between the objects.

Imagine, for example, trying to slide a heavy crate across a concrete floor. You may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion, it is easier to keep it in motion than it was to get it started because the kinetic friction force is less than the static friction force. If you were to add mass to the crate, (for example, by placing a box on top of it) you would need to push even harder to get it started and also to keep it moving. If, on the other hand, you oiled the concrete you would find it easier to get the crate started and keep it going.

Figure 5.33 shows how friction occurs at the interface between two objects. Magnifying these surfaces shows that they are rough on the microscopic level. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them.
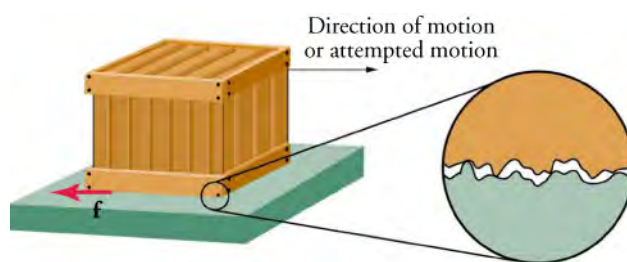
Figure 5.33 Frictional forces, such as **f**, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view.

The magnitude of the frictional force has two forms: one for static friction, the other for kinetic friction. When there is no motion between the objects, the magnitude of static friction $\mathbf{f_s}$ is

$$\mathbf{f}_s \leq \mu_s \mathbf{N}_s,$$

where μ_s is the coefficient of static friction and **N** is the magnitude of the normal force. Recall that the normal force opposes the force of gravity and acts perpendicular to the surface in this example, but not always.

Since the symbol $\leq$ means less than or equal to, this equation says that static friction can have a maximum value of $\mu_s \mathbf{N}$. That is,

$$\mathbf{f}_s(\max) = \mu_s \mathbf{N}.$$

Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds **f**s(max), the object will move. Once an object is moving, the magnitude of kinetic friction $\mathbf{f}_k$ is given by

$$\mathbf{f}_k = \mu_k \mathbf{N}.$$

where μ_k is the coefficient of kinetic friction.

Friction varies from surface to surface because different substances are rougher than others. Table 5.2 compares values of static and kinetic friction for different surfaces. The coefficient of the friction depends on the two surfaces that are in contact.

System	Static Friction μ_s	Kinetic Friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

Table 5.2 Coefficients of Static and Kinetic Friction

System	Static Friction μ_s	Kinetic Friction μ_k
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Table 5.2 Coefficients of Static and Kinetic Friction

Since the direction of friction is always opposite to the direction of motion, friction runs parallel to the surface between objects and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight

$$\mathbf{W} = m\mathbf{g} = (100 \text{ kg})(9.80 \text{m/s}^2) = 980 \ \text{N},$$

perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

$$\mathbf{f}_s(\text{max}) = \mu_s \mathbf{N} = (0.45)(980 \ \text{N}) = 440 \ \text{N}$$

to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N

$$\mathbf{f}_k = \mu_k \mathbf{N} = (0.30)(980 \ \text{N}) = 290 \ \text{N}$$

would keep it moving at a constant speed. If the floor were lubricated, both coefficients would be much smaller than they would be without lubrication. The coefficient of friction is unitless and is a number usually between 0 and 1.0.

Working with Inclined Planes

We discussed previously that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Up until now, we dealt only with normal force in one dimension, with gravity and normal force acting perpendicular to the surface in opposing directions (gravity downward, and normal force upward). Now that you have the skills to work with forces in two dimensions, we can explore what happens to weight and the normal force on a tilted surface such as an inclined plane. For inclined plane problems, it is easier breaking down the forces into their components if we rotate the coordinate system, as illustrated in Figure 5.34. The first step when setting up the problem is to break down the force of weight into components.

$$w_{\parallel} = w \sin(\theta) = mg \sin(\theta)$$
$$w_{\perp} = w \cos(\theta) = mg \cos(\theta)$$
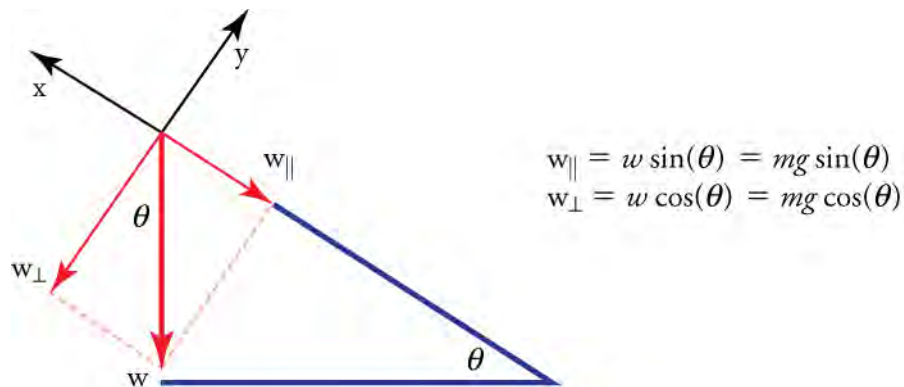
Figure 5.34 The diagram shows perpendicular and horizontal components of weight on an inclined plane.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: A force acting perpendicular to the plane, $\mathbf{w}_{\perp}$, and a force acting parallel to the plane, $\mathbf{w}_{\parallel}$. The perpendicular force of weight, $\mathbf{w}_{\perp}$, is typically equal in magnitude and opposite in direction to the normal force, $\mathbf{N}$. The force acting parallel to the plane, $\mathbf{w}_{\parallel}$, causes the object to accelerate down the incline. The force of friction, $\mathbf{f}$, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$\mathbf{w}_{\parallel} = \mathbf{w}sin(\theta) = m\mathbf{g}sin(\theta) \text{ and}$$

$$\mathbf{w}_{\perp} = \mathbf{w}cos(\theta) = m\mathbf{g}cos(\theta).$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components

$$cos(\theta) = \frac{\mathbf{w}_{\perp}}{\mathbf{w}}$$

$$\mathbf{w}_{\perp} = \mathbf{w}cos(\theta) = m\mathbf{g}cos(\theta)$$

$$sin(\theta) = \frac{\mathbf{w}_{\parallel}}{\mathbf{w}}$$

$$\mathbf{w}_{\parallel} = \mathbf{w}sin(\theta) = m\mathbf{g}sin(\theta).$$

👁 WATCH PHYSICS

Inclined Plane Force Components

This video (https://www.khanacademy.org/embed_video?v=TC23wD34C7k) shows how the weight of an object on an inclined plane is broken down into components perpendicular and parallel to the surface of the plane. It explains the geometry for finding the angle in more detail.

> **GRASP CHECK**
> Click to view content (https://www.youtube.com/embed/TC23wD34C7k)
> This video shows how the weight of an object on an inclined plane is broken down into components perpendicular and parallel to the surface of the plane. It explains the geometry for finding the angle in more detail.
> When the surface is flat, you could say that one of the components of the gravitational force is zero; Which one? As the angle of the incline gets larger, what happens to the magnitudes of the perpendicular and parallel components of gravitational force?
>
> a. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component decreases and the perpendicular component increases. This is because the cosine of the angle shrinks while the sine of the angle increases.
> b. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component decreases and the perpendicular component increases. This is because the cosine of the angle increases while the sine of the angle shrinks.
> c. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component increases and the perpendicular component decreases. This is because the cosine of the angle shrinks while the sine of the angle increases.
> d. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component increases and the perpendicular component decreases. This is because the cosine of the angle increases while the sine of the angle shrinks.

> **TIPS FOR SUCCESS**
> Normal force is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. It is important to tell apart these symbols, especially since the units for normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force, $\mathbf{N}$, that the floor exerts on a chair might be $\mathbf{N} = 100$ N. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations!

To review, the process for solving inclined plane problems is as follows:

1. Draw a sketch of the problem.
2. Identify known and unknown quantities, and identify the system of interest.
3. Draw a free-body diagram (which is a sketch showing all of the forces acting on an object) with the coordinate system rotated at the same angle as the inclined plane. Resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
4. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x-direction) then $\mathbf{F}$net $x = 0$. If the object does accelerate in that direction, $\mathbf{F}$net $x = m\mathbf{a}$.
5. Check your answer. Is the answer reasonable? Are the units correct?

✳ WORKED EXAMPLE

Finding the Coefficient of Kinetic Friction on an Inclined Plane

A skier, illustrated in Figure 5.35(a), with a mass of 62 kg is sliding down a snowy slope at an angle of 25 degrees. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.
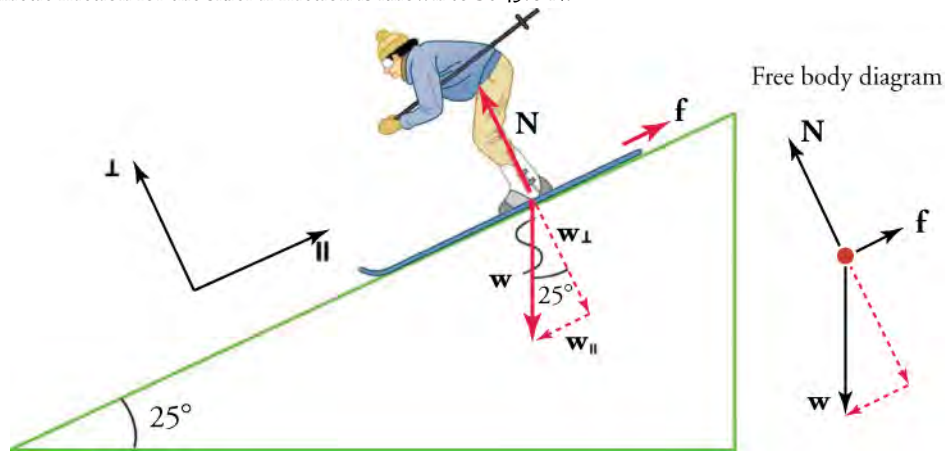
Figure 5.35   Use the diagram to help find the coefficient of kinetic friction for the skier.

Strategy

The magnitude of kinetic friction was given as 45.0 N. Kinetic friction is related to the normal force $\mathbf{N}$ as $\mathbf{f}_k = \mu_k \mathbf{N}$. Therefore, we can find the coefficient of kinetic friction by first finding the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope.

That is,

$$\mathbf{N} = \mathbf{w}_\perp = \mathbf{w} \cos(25°) = m\mathbf{g} \cos(25°).$$

Substituting this into our expression for kinetic friction, we get

$$\mathbf{f}_k = \mu_k m\mathbf{g} \cos 25°,$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{\mathbf{f}_k}{\mathbf{w} \cos 25°} = \frac{\mathbf{f}_k}{m\mathbf{g} \cos 25°}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0 \ \text{N}}{(62 \ \text{kg})(9.80 \ \text{m/s}^2)(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in Table 5.2 for waxed wood on snow, but it is still reasonable since values

of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $\mathbf{f}_k = \mu_k m\mathbf{g}\cos\theta$.

✳ WORKED EXAMPLE

Weight on an Incline, a Two-Dimensional Problem

The skier's mass, including equipment, is 60.0 kg. (See Figure 5.36(b).) (a) What is her acceleration if friction is negligible? (b) What is her acceleration if the frictional force is 45.0 N?
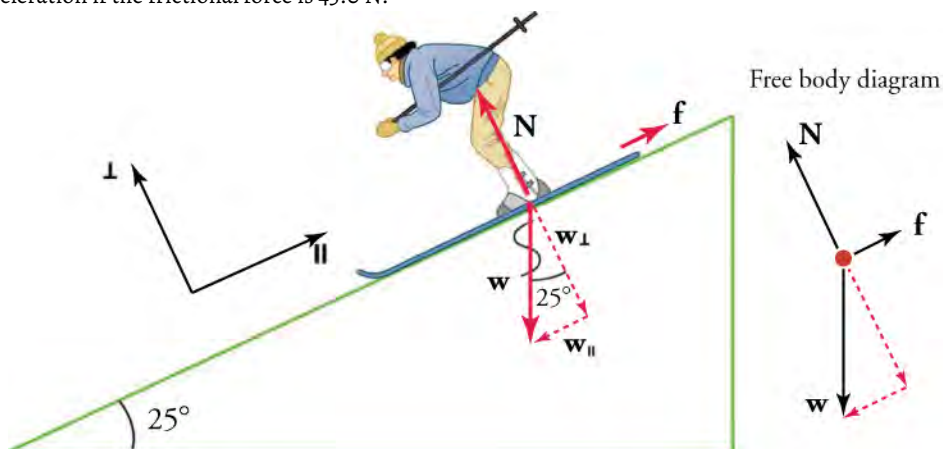
Figure 5.36   Now use the diagram to help find the skier's acceleration if friction is negligible and if the frictional force is 45.0 N.

Strategy

The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. Remember that motions along perpendicular axes are independent. We use the symbol $\perp$ to mean perpendicular, and $\|$ to mean parallel.

The only external forces acting on the system are the skier's weight, friction, and the normal force exerted by the ski slope, labeled $\mathbf{w}$, $\mathbf{f}$, and $\mathbf{N}$ in the free-body diagram. $\mathbf{N}$ is always perpendicular to the slope and $\mathbf{f}$ is parallel to it. But $\mathbf{w}$ is not in the direction of either axis, so we must break it down into components along the chosen axes. We define $\mathbf{w}_{\|}$ to be the component of weight parallel to the slope and $\mathbf{w}_{\perp}$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $\mathbf{w}_{\|} = \mathbf{w}\sin(25°) = m\mathbf{g}\sin(25°)$, and the magnitude of the component of the weight perpendicular to the slope is $\mathbf{w}_{\perp} = \mathbf{w}\cos(25°) = m\mathbf{g}\cos(25°\ )$.

(a) Neglecting friction: Since the acceleration is parallel to the slope, we only need to consider forces parallel to the slope. Forces perpendicular to the slope add to zero, since there is no acceleration in that direction. The forces parallel to the slope are the amount of the skier's weight parallel to the slope $\mathbf{w}_{\|}$ and friction $\mathbf{f}$. Assuming no friction, by Newton's second law the acceleration parallel to the slope is

$$\mathbf{a}_{\|} = \frac{\mathbf{F}_{\text{net }\|}}{m},$$

Where the net force parallel to the slope $\mathbf{F}_{\text{net }\|} = \mathbf{w}_{\|} = m\mathbf{g}\sin(25°)$, so that

$$
\begin{aligned}
\mathbf{a}_{\|} &= \frac{\mathbf{F}_{\text{net }\|}}{m} = \frac{m\mathbf{g}\sin(25°)}{m} = \mathbf{g}\sin(25°) \\
&= (9.80\ \text{m/s}^2)(0.423) = 4.14\ \text{m/s}^2
\end{aligned}
$$

is the acceleration.

(b) Including friction: Here we now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$\mathbf{F}_{\text{net }\parallel} = \mathbf{w}_{\parallel} - \mathbf{f},$$

and substituting this into Newton's second law, $a_{\parallel} = \frac{F_{\text{net }\parallel}}{m}$ gives

$$\mathbf{a}_{\parallel} = \frac{\mathbf{F}_{\text{net }\parallel}}{m} = \frac{\mathbf{w}_{\parallel} - \mathbf{f}}{m} = \frac{m\mathbf{g}\sin(25°) - \mathbf{f}}{m}.$$

We substitute known values to get

$$\mathbf{a}_{\parallel} = \frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(0.423) - 45.0 \text{ N}}{60.0 \text{ kg}},$$

or

$$\mathbf{a}_{\parallel} = 3.39 \text{ m/s}^2,$$

which is the acceleration parallel to the incline when there is 45 N opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is not.

Practice Problems

15. When an object sits on an inclined plane that makes an angle θ with the horizontal, what is the expression for the component of the objects weight force that is parallel to the incline?
    a. $w_{\parallel} = w\cos\theta$
    b. $w_{\parallel} = w\sin\theta$
    c. $w_{\parallel} = w\sin\theta - \cos\theta$
    d. $w_{\parallel} = w\cos\theta - \sin\theta$

16. An object with a mass of 5 kg rests on a plane inclined 30° from horizontal. What is the component of the weight force that is parallel to the incline?
    a. 4.33 N
    b. 5.0 N
    c. 24.5 N
    d. 42.43 N

Snap Lab

Friction at an Angle: Sliding a Coin

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in the first Worked Example, the kinetic friction on a slope $\mathbf{f}_k = \mu_k m\mathbf{g}\cos\theta$, and the component of the weight down the slope is equal to $m\mathbf{g}\sin\theta$. These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out

$$\mathbf{f}_k = \mathbf{Fg}_x$$
$$\mu_k m\mathbf{g}\cos\theta = m\mathbf{g}\sin\theta.$$

Solving for μ_k, since $\tan\theta = \sin\theta/\cos\theta$ we find that

$$\mu_k = \frac{m\mathbf{g}\sin\theta}{m\mathbf{g}\cos\theta} = \tan\theta. \qquad \boxed{5.10}$$

- 1 coin
- 1 book
- 1 protractor
    1. Put a coin flat on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move.

2.   Measure the angle of tilt relative to the horizontal and find μ_k .

> **GRASP CHECK**
> True or False—If only the angles of two vectors are known, we can find the angle of their resultant addition vector.
> a.   True
> b.   False

Check Your Understanding

17. What is friction?
   a.   Friction is an internal force that opposes the relative motion of an object.
   b.   Friction is an internal force that accelerates an object's relative motion.
   c.   Friction is an external force that opposes the relative motion of an object.
   d.   Friction is an external force that increases the velocity of the relative motion of an object.

18. What are the two varieties of friction? What does each one act upon?
   a.   Kinetic and static friction both act on an object in motion.
   b.   Kinetic friction acts on an object in motion, while static friction acts on an object at rest.
   c.   Kinetic friction acts on an object at rest, while static friction acts on an object in motion.
   d.   Kinetic and static friction both act on an object at rest.

19. Between static and kinetic friction between two surfaces, which has a greater value? Why?
   a.   The kinetic friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
   b.   The static friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
   c.   The kinetic friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.
   d.   The static friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.

5.5 Simple Harmonic Motion

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe Hooke's law and Simple Harmonic Motion
- Describe periodic motion, oscillations, amplitude, frequency, and period
- Solve problems in simple harmonic motion involving springs and pendulums

Section Key Terms

amplitude	deformation	equilibrium position	frequency
Hooke's law	oscillate	period	periodic motion
restoring force	simple harmonic motion	simple pendulum	

Hooke's Law and Simple Harmonic Motion

Imagine a car parked against a wall. If a bulldozer pushes the car into the wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important things can happen. First, unlike the car and bulldozer example, the object returns to its original shape when the force is removed. Second, the size of the deformation is proportional to the force. This

second property is known as **Hooke's law**. In equation form, Hooke's law is

$$F = -kx,$$

where **x** is the amount of deformation (the change in length, for example) produced by the restoring force **F**, and **k** is a constant that depends on the shape and composition of the object. The restoring force is the force that brings the object back to its equilibrium position; the minus sign is there because the restoring force acts in the direction opposite to the displacement. Note that the restoring force is proportional to the deformation **x**. The deformation can also be thought of as a displacement from equilibrium. It is a change in position due to a force. In the absence of force, the object would rest at its equilibrium position. The force constant **k** is related to the stiffness of a system. The larger the force constant, the stiffer the system. A stiffer system is more difficult to deform and requires a greater restoring force. The units of **k** are newtons per meter (N/m). One of the most common uses of Hooke's law is solving problems involving springs and pendulums, which we will cover at the end of this section.

Oscillations and Periodic Motion

What do an ocean buoy, a child in a swing, a guitar, and the beating of hearts all have in common? They all **oscillate**. That is, they move back and forth between two points, like the ruler illustrated in Figure 5.37. All oscillations involve force. For example, you push a child in a swing to get the motion started.
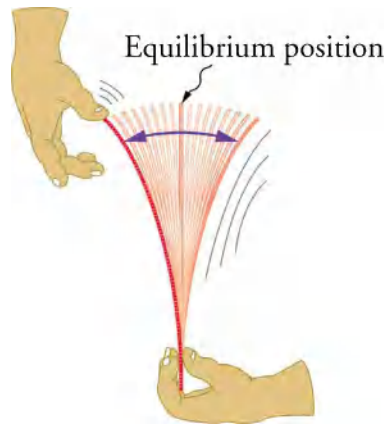
Figure 5.37 A ruler is displaced from its equilibrium position.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in Figure 5.38. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until it gradually loses all of its energy. The simplest oscillations occur when the restoring force is directly proportional to displacement. Recall that Hooke's law describes this situation with the equation **F** = –**kx**. Therefore, Hooke's law describes and applies to the simplest case of oscillation, known as **simple harmonic motion**.
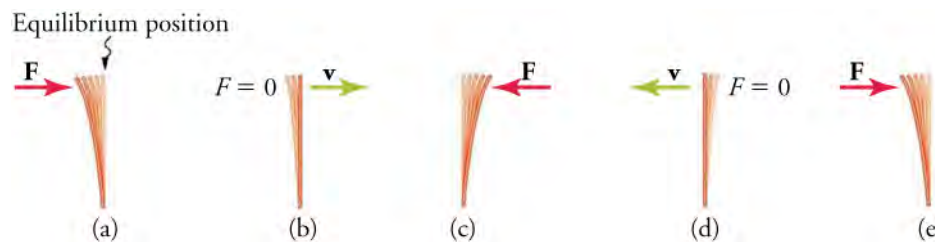
Figure 5.38 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each vibration of the string takes the same time as the previous one. **Periodic motion** is a motion that repeats itself at regular time intervals, such as with an object bobbing up and down on a spring or a pendulum swinging back and forth. The time to complete one oscillation (a complete cycle of motion) remains constant and is called the **period** T. Its units are usually seconds.

Frequency f is the number of oscillations per unit time. The SI unit for frequency is the hertz (Hz), defined as the number of oscillations per second. The relationship between frequency and period is

$$f = 1/T.$$

As you can see from the equation, frequency and period are different ways of expressing the same concept. For example, if you get a paycheck twice a month, you could say that the frequency of payment is two per month, or that the period between checks is half a month.

If there is no friction to slow it down, then an object in simple motion will oscillate forever with equal displacement on either side of the equilibrium position. The **equilibrium position** is where the object would naturally rest in the absence of force. The maximum displacement from equilibrium is called the **amplitude X**. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, shown in Figure 5.39, the units of amplitude and displacement are meters.
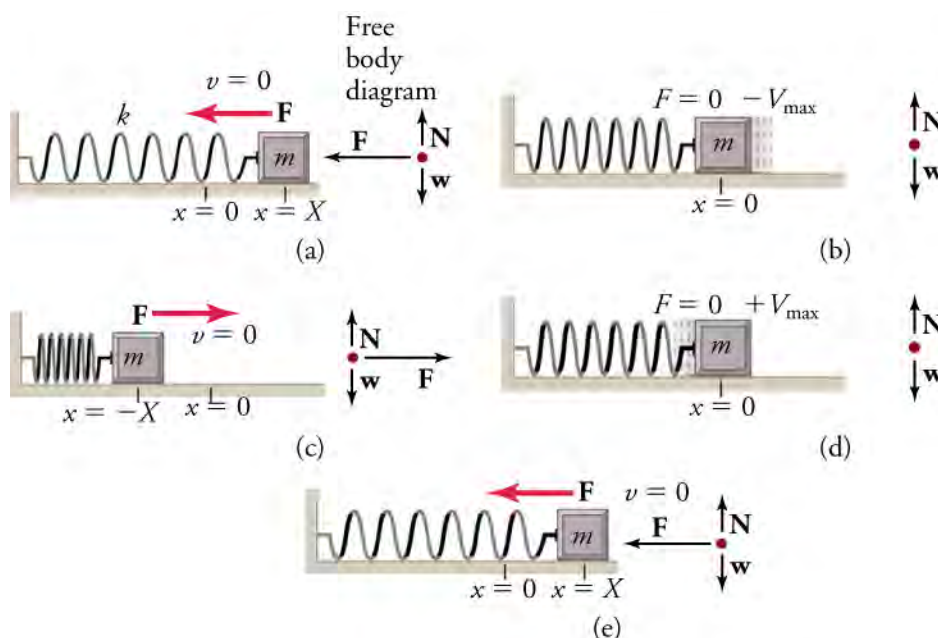
Figure 5.39 An object attached to a spring sliding on a frictionless surface is a simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude **X** and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

The mass m and the force constant **k** are the *only* factors that affect the period and frequency of simple harmonic motion. The period of a simple harmonic oscillator is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and, because $f = 1/T$, the frequency of a simple harmonic oscillator is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

👁 WATCH PHYSICS

Introduction to Harmonic Motion

This video shows how to graph the displacement of a spring in the x-direction over time, based on the period. Watch the first 10 minutes of the video (you can stop when the narrator begins to cover calculus).

Click to view content (https://www.khanacademy.org/embed_video?v=Nk2q-_jkJVs)

> **GRASP CHECK**
>
> If the amplitude of the displacement of a spring were larger, how would this affect the graph of displacement over time? What would happen to the graph if the period was longer?
> a. Larger amplitude would result in taller peaks and troughs and a longer period would result in greater separation in time between peaks.
> b. Larger amplitude would result in smaller peaks and troughs and a longer period would result in greater distance between peaks.
> c. Larger amplitude would result in taller peaks and troughs and a longer period would result in shorter distance between peaks.
> d. Larger amplitude would result in smaller peaks and troughs and a longer period would result in shorter distance between peaks.

Solving Spring and Pendulum Problems with Simple Harmonic Motion

Before solving problems with springs and pendulums, it is important to first get an understanding of how a pendulum works. Figure 5.40 provides a useful illustration of a simple pendulum.
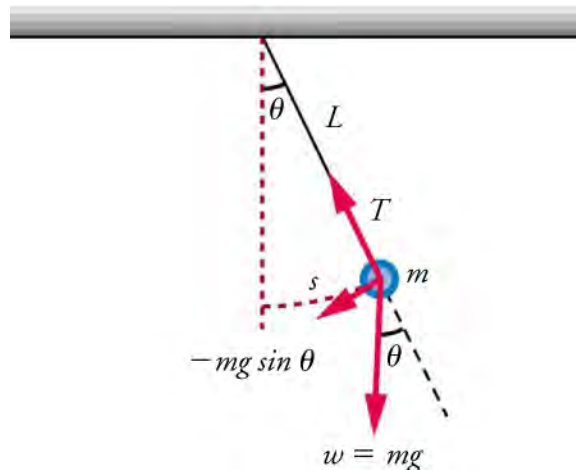
Figure 5.40 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg\sin\theta$ toward the equilibrium position—that is, a restoring force.

Everyday examples of pendulums include old-fashioned clocks, a child's swing, or the sinker on a fishing line. For small displacements of less than 15 degrees, a pendulum experiences simple harmonic oscillation, meaning that its restoring force is directly proportional to its displacement. A pendulum in simple harmonic motion is called a **simple pendulum**. A pendulum has an object with a small mass, also known as the pendulum bob, which hangs from a light wire or string. The equilibrium position for a pendulum is where the angle θ is zero (that is, when the pendulum is hanging straight down). It makes sense that without any force applied, this is where the pendulum bob would rest.

The displacement of the pendulum bob is the arc length s. The weight mg has components $mg \cos\theta$ along the string and $mg \sin\theta$ tangent to the arc. Tension in the string exactly cancels the component $mg \cos\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position that runs tangent to the arc and equals $-mg \sin\theta$.

For a simple pendulum, The period is $T = 2\pi\sqrt{\frac{L}{g}}$.

The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass or amplitude. However, note that T does depend on **g**. This means that if we know the length of a pendulum, we can actually use it to measure gravity! This will come in useful in .

> **TIPS FOR SUCCESS**
> Tension is represented by the variable **T**, and period is represented by the variable T. It is important not to confuse the two, since tension is a force and period is a length of time.

✳ WORKED EXAMPLE

Measuring Acceleration due to Gravity: The Period of a Pendulum
What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find **g** given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for **g**, assuming that the angle of deflection is less than 15 degrees. Recall that when the angle of deflection is less than 15 degrees, the pendulum is considered to be in simple harmonic motion, allowing us to use this equation.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g.

$$\mathbf{g} = 4\pi^2 \frac{L}{T^2}$$

2. Substitute known values into the new equation.

$$\mathbf{g} = 4\pi^2 \frac{0.75000 \ \text{m}}{(1.7357 \ \text{s})^2}$$

3. Calculate to find **g**.

$$\mathbf{g} = \ 9.8281 \ \text{m/s}^2$$

Discussion

This method for determining **g** can be very accurate. This is why length and period are given to five digits in this example.

✳ WORKED EXAMPLE

Hooke's Law: How Stiff Are Car Springs?
What is the force constant for the suspension system of a car, like that shown in , that settles 1.20 cm when an 80.0-kg person gets in?

Figure 5.41 A car in a parking lot. (exfordy, Flickr)

Strategy

Consider the car to be in its equilibrium position $x = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x = -1.20 \times 10^{-2}$ m.

At that point, the springs supply a restoring force F equal to the person's weight

$w = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = 784$ N. We take this force to be F in Hooke's law.

Knowing F and x, we can then solve for the force constant k.

Solution

Solve Hooke's law, $F = -kx$, for k.

$$k = \frac{F}{x}$$

Substitute known values and solve for k.

$$k = \frac{-784 \text{ N}}{-1.20 \times 10^{-2} \text{ m}}$$
$$= 6.53 \times 10^4 \text{ N/m}$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in, if it were not for the shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Practice Problems

20. A force of 70 N applied to a spring causes it to be displaced by 0.3 m. What is the force constant of the spring?
   a. -233 N/m
   b. -21 N/m
   c. 21 N/m
   d. 233 N/m

21. What is the force constant for the suspension system of a car that settles 3.3 cm when a 65 kg person gets in?
   a. 1.93×10^4 N/m
   b. 1.97×10^3 N/m

c. 1.93×10^2 N/m
d. 1.97×10^1 N/m

Snap Lab

Finding Gravity Using a Simple Pendulum

Use a simple pendulum to find the acceleration due to gravity **g** in your home or classroom.

- 1 string
- 1 stopwatch
- 1 small dense object

1. Cut a piece of a string or dental floss so that it is about 1 m long.
2. Attach a small object of high density to the end of the string (for example, a metal nut or a car key).
3. Starting at an angle of less than 10 degrees, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch.
4. Calculate **g**.

GRASP CHECK

How accurate is this measurement for g? How might it be improved?
a. Accuracy for value of g will increase with an increase in the mass of a dense object.
b. Accuracy for the value of g will increase with increase in the length of the pendulum.
c. The value of g will be more accurate if the angle of deflection is more than 15°.
d. The value of g will be more accurate if it maintains simple harmonic motion.

Check Your Understanding

22. What is deformation?
    a. Deformation is the magnitude of the restoring force.
    b. Deformation is the change in shape due to the application of force.
    c. Deformation is the maximum force that can be applied on a spring.
    d. Deformation is regaining the original shape upon the removal of an external force.

23. According to Hooke's law, what is deformation proportional to?
    a. Force
    b. Velocity
    c. Displacement
    d. Force constant

24. What are oscillations?
    a. Motion resulting in small displacements
    b. Motion which repeats itself periodically
    c. Periodic, repetitive motion between two points
    d. motion that is the opposite to the direction of the restoring force

25. True or False—Oscillations can occur without force.
    a. True
    b. False

KEY TERMS

air resistance   a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

amplitude   the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

analytical method   the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

component (of a 2-dimensional vector)   a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

deformation   displacement from equilibrium, or change in shape due to the application of force

equilibrium position   where an object would naturally rest in the absence of force

frequency   number of events per unit of time

graphical method   drawing vectors on a graph to add them using the head-to-tail method

head (of a vector)   the end point of a vector; the location of the vector's arrow; also referred to as the tip

head-to-tail method   a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

Hooke's law   proportional relationship between the force **F** on a material and the deformation ΔL it causes,
$$\mathbf{F} = \mathbf{k}\Delta L$$

kinetic friction   a force that opposes the motion of two systems that are in contact and moving relative to one another

maximum height (of a projectile)   the highest altitude, or maximum displacement in the vertical position reached in the path of a projectile

oscillate   moving back and forth regularly between two points

period   time it takes to complete one oscillation

periodic motion   motion that repeats itself at regular time intervals

projectile   an object that travels through the air and experiences only acceleration due to gravity

projectile motion   the motion of an object that is subject only to the acceleration of gravity

range   the maximum horizontal distance that a projectile travels

restoring force   force acting in opposition to the force caused by a deformation

resultant   the sum of the a collection of vectors

resultant vector   the vector sum of two or more vectors

simple harmonic motion   the oscillatory motion in a system where the net force can be described by Hooke's law

simple pendulum   an object with a small mass suspended from a light wire or string

static friction   a force that opposes the motion of two systems that are in contact and are not moving relative to one another

tail   the starting point of a vector; the point opposite to the head or tip of the arrow

trajectory   the path of a projectile through the air

vector addition   adding together two or more vectors

SECTION SUMMARY

5.1 Vector Addition and Subtraction: Graphical Methods

- The graphical method of adding vectors **A** and **B** involves drawing vectors on a graph and adding them by using the head-to-tail method. The resultant vector **R** is defined such that **A** + **B** = **R**. The magnitude and direction of **R** are then determined with a ruler and protractor.
- The graphical method of subtracting vectors **A** and **B** involves adding the opposite of vector **B**, which is defined as –**B**. In this case,
  A – **B** = **A** + (–**B**) = **R**. Next, use the head-to-tail method as for vector addition to obtain the resultant vector **R** .
- Addition of vectors is independent of the order in which they are added; **A** + **B** = **B** + **A**.
- The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- Variables in physics problems, such as force or velocity, can be represented with vectors by making the length of the vector proportional to the magnitude of the force or velocity.
- Problems involving displacement, force, or velocity may be solved graphically by measuring the resultant vector's magnitude with a ruler and measuring the direction with a protractor.

5.2 Vector Addition and Subtraction: Analytical Methods

- The analytical method of vector addition and subtraction uses the Pythagorean theorem and trigonometric identities to determine the magnitude

and direction of a resultant vector.

- The steps to add vectors A and B using the analytical method are as follows:

1. Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

$$A_x = A\cos\theta$$
$$B_x = B\cos\theta$$

and

$$A_y = A\sin\theta$$
$$B_y = B\sin\theta.$$

2. Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, R.

$$R_x = A_x + B_x$$

and

$$R_y = A_y + B_y.$$

3. Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector R.

$$R = \sqrt{R_x^2 + R_y^2}$$

4. Use a trigonometric identity to determine the direction, θ, of R.

$$\theta = \tan^{-1}(R_y/R_x)$$

5.3 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- Projectile motion in the horizontal and vertical directions are independent of one another.
- The maximum height of an projectile is the highest altitude, or maximum displacement in the vertical position reached in the path of a projectile.
- The range is the maximum horizontal distance traveled by a projectile.

- To solve projectile problems: choose a coordinate system; analyze the motion in the vertical and horizontal direction separately; then, recombine the horizontal and vertical components using vector addition equations.

5.4 Inclined Planes

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force **N** pushing the systems together. A normal force is always perpendicular to the contact surface between systems. Friction depends on both of the materials involved.
- μ_s is the coefficient of static friction, which depends on both of the materials.
- μ_k is the coefficient of kinetic friction, which also depends on both materials.
- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be broken into components that act perpendicular ($\mathbf{w}_\perp$) and parallel ($\mathbf{w}_\parallel$) to the surface of the plane.

5.5 Simple Harmonic Motion

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations are related to systems that can be described by Hooke's law.
- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency
- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15 degrees.

KEY EQUATIONS

5.2 Vector Addition and Subtraction: Analytical Methods

resultant magnitude	$R = \sqrt{R_x^2 + R_y^2}$
resultant direction	$\theta = \tan^{-1}(R_y/R_x)$
x-component of a vector A (when an angle is given relative to the horizontal)	$A_x = A\cos\theta$
y-component of a vector A (when an angle is given relative to the horizontal)	$A_y = A\sin\theta$
addition of vectors	$\mathbf{A_x} + \mathbf{A_y} = \mathbf{A}$

5.3 Projectile Motion

angle of displacement	$\theta = \tan^{-1}(\mathbf{y/x})$

velocity	$\mathbf{v} = \sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2}$

angle of velocity $\quad\quad \theta_v = \tan^{-1}(\mathbf{v}_y/\mathbf{v}_x)$

maximum height $\quad\quad h = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}$

range $\quad\quad R = \frac{\mathbf{v}_0^2 \sin 2\theta_0}{\mathbf{g}}$

5.4 Inclined Planes

force of static friction $\quad \mathbf{f}_s \leq \mu_s \mathbf{N}$

force of kinetic friction $\quad \mathbf{f}_k = \mu_k \mathbf{N}$

perpendicular component of weight on an inclined plane $\quad \mathbf{w}_\perp = \mathbf{w}\cos(\theta) = m\mathbf{g}\cos(\theta)$

parallel component of weight on an inclined plane $\quad \mathbf{w}_{\parallel} = \mathbf{w}\sin(\theta) = m\mathbf{g}\sin(\theta)$

5.5 Simple Harmonic Motion

Hooke's law $\quad\quad \mathbf{F} = -\mathbf{k}\mathbf{x}$

period in simple harmonic motion $\quad T = 2\pi\sqrt{\frac{m}{\mathbf{k}}}$

frequency in simple harmonic motion $\quad f = \frac{1}{2\pi}\sqrt{\frac{\mathbf{k}}{m}}$

period of a simple pendulum $\quad T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$

CHAPTER REVIEW
Concept Items

5.1 Vector Addition and Subtraction: Graphical Methods

1. There is a vector $\overrightarrow{A}$, with magnitude 5 units pointing towards west and vector $\overrightarrow{B}$, with magnitude 3 units, pointing towards south. Using vector addition, calculate the magnitude of the resultant vector.
   a. 4.0
   b. 5.8
   c. 6.3
   d. 8.0

2. If you draw two vectors using the head-to-tail method, how can you then draw the resultant vector?
   a. By joining the head of the first vector to the head of the last
   b. By joining the head of the first vector with the tail of the last
   c. By joining the tail of the first vector to the head of the last
   d. By joining the tail of the first vector with the tail of the last

3. What is the global angle of $20°$ south of west?
   a. 110°
   b. 160°
   c. 200°
   d. 290°

5.2 Vector Addition and Subtraction: Analytical Methods

4. What is the angle between the x and y components of a vector?
   a. 0°
   b. 45°
   c. 90°
   d. 180°

5. Two vectors are equal in magnitude and opposite in direction. What is the magnitude of their resultant vector?
   a. The magnitude of the resultant vector will be zero.
   b. The magnitude of resultant vector will be twice the magnitude of the original vector.
   c. The magnitude of resultant vector will be same as magnitude of the original vector.
   d. The magnitude of resultant vector will be half the magnitude of the original vector.

6. How can we express the x and y-components of a vector in terms of its magnitude, A, and direction, global angle θ?
   a. $A_x = A\cos\theta\, A_y = A\sin\theta$
   b. $A_x = A\cos\theta\, A_y = A\cos\theta$
   c. $A_x = A\sin\theta\, A_y = A\cos\theta$

d. $A_x = A \sin \theta \, A_y = A \sin \theta$

7. True or False—Every 2-D vector can be expressed as the product of its x and y-components.
    a.   True
    b.   False

5.3 Projectile Motion

8. Horizontal and vertical motions of a projectile are independent of each other. What is meant by this?
    a.   Any object in projectile motion falls at the same rate as an object in freefall, regardless of its horizontal velocity.
    b.   All objects in projectile motion fall at different rates, regardless of their initial horizontal velocities.
    c.   Any object in projectile motion falls at the same rate as its initial vertical velocity, regardless of its initial horizontal velocity.
    d.   All objects in projectile motion fall at different rates and the rate of fall of the object is independent of the initial velocity.

9. Using the conventional choice for positive and negative axes described in the text, what is the y-component of the acceleration of an object experiencing projectile motion?
    a. -9.8 m/s
    b. -9.8 m/s^2
    c. 9.8 m/s
    d. 9.8 m/s^2

5.4 Inclined Planes

10. True or False—Kinetic friction is less than the limiting static friction because once an object is moving, there are fewer points of contact, and the friction is reduced. For this reason, more force is needed to start moving an object than to keep it in motion.
    a.   True
    b.   False

11. When there is no motion between objects, what is the relationship between the magnitude of the static friction f_s and the normal force N?
    a. $f_s \leq N$
    b. $f_s \leq \mu_s N$

c. $f_s \geq N$
d. $f_s \geq \mu_s N$

12. What equation gives the magnitude of kinetic friction?
    a. $f_k = \mu_s N$
    b. $f_k = \mu_k N$
    c. $f_k \leq \mu_s N$
    d. $f_k \leq \mu_k N$

5.5 Simple Harmonic Motion

13. Why is there a negative sign in the equation for Hooke's law?
    a.   The negative sign indicates that displacement decreases with increasing force.
    b.   The negative sign indicates that the direction of the applied force is opposite to that of displacement.
    c.   The negative sign indicates that the direction of the restoring force is opposite to that of displacement.
    d.   The negative sign indicates that the force constant must be negative.

14. With reference to simple harmonic motion, what is the equilibrium position?
    a.   The position where velocity is the minimum
    b.   The position where the displacement is maximum
    c.   The position where the restoring force is the maximum
    d.   The position where the object rests in the absence of force

15. What is Hooke's law?
    a.   Restoring force is directly proportional to the displacement from the mean position and acts in the the opposite direction of the displacement.
    b.   Restoring force is directly proportional to the displacement from the mean position and acts in the same direction as the displacement.
    c.   Restoring force is directly proportional to the square of the displacement from the mean position and acts in the opposite direction of the displacement.
    d.   Restoring force is directly proportional to the square of the displacement from the mean position and acts in the same direction as the displacement.

Critical Thinking Items

5.1 Vector Addition and Subtraction: Graphical Methods

16. True or False—A person is following a set of directions. He has to walk 2 km east and then 1 km north. He takes a wrong turn and walks in the opposite direction for the second leg of the trip. The magnitude of his total

displacement will be the same as it would have been had he followed directions correctly.
    a.   True
    b.   False

5.2 Vector Addition and Subtraction: Analytical Methods

17. What is the magnitude of a vector whose x-component is 2 units and whose angle is $60°$?
  a. 1.0 units
  b. 2.0 units
  c. 2.3 units
  d. 4.0 units

18. Vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude and opposite in direction. Does $\vec{A} - \vec{B}$ have the same direction as vector $\vec{A}$ or $\vec{B}$?
  a. $\vec{A}$
  b. $\vec{B}$

5.3 Projectile Motion

19. Two identical items, object 1 and object 2, are dropped from the top of a 50.0 m building. Object 1 is dropped with an initial velocity of 0 m/s, while object 2 is thrown straight downward with an initial velocity of 13.0 m/s. What is the difference in time, in seconds rounded to the nearest tenth, between when the two objects hit the ground?
  a.   Object 1 will hit the ground 3.2 s after object 2.
  b.   Object 1 will hit the ground 2.1 s after object 2.
  c.   Object 1 will hit the ground at the same time as object 2.
  d.   Object 1 will hit the ground 1.1 s after object 2.

20. An object is launched into the air. If the y-component of its acceleration is 9.8 m/s², which direction is defined as positive?
  a.   Vertically upward in the coordinate system
  b.   Vertically downward in the coordinate system
  c.   Horizontally to the right side of the coordinate system
  d.   Horizontally to the left side of the coordinate system

5.4 Inclined Planes

21. A box weighing 500 N is at rest on the floor. A person

Problems

5.1 Vector Addition and Subtraction: Graphical Methods

25. A person attempts to cross a river in a straight line by navigating a boat at 15 m/s. If the river flows at 5.0 m/s from his left to right, what would be the magnitude of the boat's resultant velocity? In what direction would the boat go, relative to the straight line

pushes against it and it starts moving when 100 N force is applied to it. What can be said about the coefficient of kinetic friction between the box and the floor?
  a. $\mu_k = 0$
  b. $\mu_k = 0.2$
  c. $\mu_k < 0.2$
  d. $\mu_k > 0.2$

22. The component of the weight parallel to an inclined plane of an object resting on an incline that makes an angle of $70.0°$ with the horizontal is 100.0 N. What is the object's mass?
  a. 10.9 kg
  b. 29.8 kg
  c. 106 kg
  d. 292 kg

5.5 Simple Harmonic Motion

23. Two springs are attached to two hooks. Spring A has a greater force constant than spring B. Equal weights are suspended from both. Which of the following statements is true?
  a.   Spring A will have more extension than spring B.
  b.   Spring B will have more extension than spring A.
  c.   Both springs will have equal extension.
  d.   Both springs are equally stiff.

24. Two simple harmonic oscillators are constructed by attaching similar objects to two different springs. The force constant of the spring on the left is 5 N/m and that of the spring on the right is 4 N/m. If the same force is applied to both, which of the following statements is true?
  a.   The spring on the left will oscillate faster than spring on the right.
  b.   The spring on the right will oscillate faster than the spring on the left.
  c.   Both the springs will oscillate at the same rate.
  d.   The rate of oscillation is independent of the force constant.

across it?
  a.   The resultant velocity of the boat will be 10.0 m/s. The boat will go toward his right at an angle of $26.6°$ to a line drawn across the river.
  b.   The resultant velocity of the boat will be 10.0 m/s. The boat will go toward his left at an angle of $26.6°$ to a line drawn across the river.
  c.   The resultant velocity of the boat will be 15.8 m/s. The boat will go toward his right at an angle of

18.4° to a line drawn across the river.

d. The resultant velocity of the boat will be 15.8 m/s. The boat will go toward his left at an angle of 18.4° to a line drawn across the river.

26. A river flows in a direction from south west to north east at a velocity of 7.1 m/s. A boat captain wants to cross this river to reach a point on the opposite shore due east of the boat's current position. The boat moves at 13 m/s. Which direction should it head towards if the resultant velocity is 19.74 m/s?

   a. It should head in a direction 22.6° east of south.

   b. It should head in a direction 22.6° south of east.

   c. It should head in a direction 45.0° east of south.

   d. It should head in a direction 45.0° south of east.

5.2 Vector Addition and Subtraction: Analytical Methods

27. A person walks 10.0 m north and then 2.00 m east. Solving analytically, what is the resultant displacement of the person?

   a. $\vec{R}$ = 10.2 m, θ = 78.7° east of north

   b. $\vec{R}$ = 10.2 m, θ = 78.7° north of east

   c. $\vec{R}$ = 12.0 m, θ = 78.7° east of north

   d. $\vec{R}$ = 12.0 m, θ = 78.7° north of east

28. A person walks 12.0° north of west for 55.0 m and 63.0° south of west for 25.0 m. What is the magnitude of his displacement? Solve analytically.

   a. 10.84 m

   b. 65.1 m

   c. 66.04 m

   d. 80.00 m

5.3 Projectile Motion

29. A water balloon cannon is fired at 30 m/s at an angle of 50° above the horizontal. How far away will it fall?

   a. 2.35 m

   b. 3.01 m

   c. 70.35 m

   d. 90.44 m

Performance Task

5.5 Simple Harmonic Motion

35. Construct a seconds pendulum (pendulum with time

30. A person wants to fire a water balloon cannon such that it hits a target 100 m away. If the cannon can only be launched at 45° above the horizontal, what should be the initial speed at which it is launched?

   a. 31.3 m/s

   b. 37.2 m/s

   c. 980.0 m/s

   d. 1,385.9 m/s

5.4 Inclined Planes

31. A coin is sliding down an inclined plane at constant velocity. If the angle of the plane is 10° to the horizontal, what is the coefficient of kinetic friction?

   a. $\mu_k = 0$

   b. $\mu_k = 0.18$

   c. $\mu_k = 5.88$

   d. $\mu_k = \infty$

32. A skier with a mass of 55 kg is skiing down a snowy slope that has an incline of 30°. Find the coefficient of kinetic friction for the skier if friction is known to be 25 N.

   a. $\mu k = 0$

   b. $\mu k = 0.05$

   c. $\mu k = 0.09$

   d. $\mu k = \infty$

5.5 Simple Harmonic Motion

33. What is the time period of a 6 cm long pendulum on earth?

   a. 0.08 s

   b. 0.49 s

   c. 4.9 s

   d. 80 s

34. A simple harmonic oscillator has time period 4 s. If the mass of the system is 2 kg, what is the force constant of the spring used?

   a. 0.125 N/m

   b. 0.202 N/m

   c. 0.81 N/m

   d. 4.93 N/m

period 2 seconds).

TEST PREP
Multiple Choice

5.1 Vector Addition and Subtraction: Graphical Methods

36. True or False—We can use Pythagorean theorem to calculate the length of the resultant vector obtained from the addition of two vectors which are at right angles to each other.
    a. True
    b. False

37. True or False—The direction of the resultant vector depends on both the magnitude and direction of added vectors.
    a. True
    b. False

38. A plane flies north at 200 m/s with a headwind blowing from the north at 70 m/s. What is the resultant velocity of the plane?
    a. 130 m/s north
    b. 130 m/s south
    c. 270 m/s north
    d. 270 m/s south

39. Two hikers take different routes to reach the same spot. The first one goes 255 m southeast, then turns and goes 82 m at $14°$ south of east. The second hiker goes 200 m south. How far and in which direction must the second hiker travel now, in order to reach the first hiker's location destination?
    a. 200 m east
    b. 200 m south
    c. 260 m east
    d. 260 m south

5.2 Vector Addition and Subtraction: Analytical Methods

40. When will the x-component of a vector with angle θ be greater than its y-component?
    a. $0° < \theta < 45°$
    b. $\theta = 45°$
    c. $45° < \theta < 60°$
    d. $60° < \theta < 90°$

41. The resultant vector of the addition of vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}$. The magnitudes of $\vec{a}$, $\vec{b}$, and $\vec{r}$ are A, B, and R, respectively. Which of the following is true?
    a. $R_x + R_y = 0$
    b. $A_x + A_y = \vec{A}$
    c. $A_x + B_y = B_x + A_y$
    d. $A_x + B_x = R_x$

42. What is the dimensionality of vectors used in the study of atmospheric sciences?
    a. One-dimensional
    b. Two-dimensional
    c. Three-dimensional

5.3 Projectile Motion

43. After a projectile is launched in the air, in which direction does it experience constant, non-zero acceleration, ignoring air resistance?
    a. The x direction
    b. The y direction
    c. Both the x and y directions
    d. Neither direction

44. Which is true when the height of a projectile is at its maximum?
    a. $v_y = 0$
    b. $v_y = \text{maximum}$
    c. $v_x = \text{maximum}$

45. A ball is thrown in the air at an angle of 40°. If the maximum height it reaches is 10 m, what must be its initial speed?
    a. 17.46 m/s
    b. 21.78 m/s
    c. 304.92 m/s
    d. 474.37 m/s

46. A large rock is ejected from a volcano with a speed of 30 m/s and at an angle $60°$ above the horizontal. The rock strikes the side of the volcano at an altitude of 10.0 m lower than its starting point. Calculate the horizontal displacement of the rock.
    a. 84.90 m
    b. 96.59 m
    c. 169.80 m
    d. 193.20 m

5.4 Inclined Planes

47. For objects of identical masses but made of different materials, which of the following experiences the most static friction?
    a. Shoes on ice
    b. Metal on wood
    c. Teflon on steel

48. If an object sits on an inclined plane and no other object makes contact with the object, what is typically equal in magnitude to the component of the weight perpendicular to the plane?

a.   The normal force
b.   The total weight
c.   The parallel force of weight

49. A 5 kg box is at rest on the floor. The coefficient of static friction between the box and the floor is 0.4. A horizontal force of 50 N is applied to the box. Will it move?
a.   No, because the applied force is less than the maximum limiting static friction.
b.   No, because the applied force is more than the maximum limiting static friction.
c.   Yes, because the applied force is less than the maximum limiting static friction.
d.   Yes, because the applied force is more than the maximum limiting static friction.

50. A skier with a mass of 67 kg is skiing down a snowy slope with an incline of 37°. Find the friction if the coefficient of kinetic friction is 0.07.
a.   27.66 N
b.   34.70 N
c.   36.71 N
d.   45.96 N

5.5 Simple Harmonic Motion

51. A change in which of the following is an example of deformation?
a.   Velocity
b.   Length
c.   Mass
d.   Weight

Short Answer

5.1 Vector Addition and Subtraction: Graphical Methods

56. Find $\vec{A} - \vec{B}$ for the following vectors:
$\vec{A} = (122\,cm, \angle\,145°)\ \vec{B} = (110\,cm, \angle\,270°)$
a.   108 cm, $\theta = 119.0°$
b.   108 cm, $\theta = 125.0°$
c.   206 cm, $\theta = 119.0°$
d.   206 cm, $\theta = 125.0°$

57. Find $\vec{A} + \vec{B}$ for the following vectors:
$\vec{A} = (122\,cm, \angle\,145°)\ \vec{B} = (110\,cm, \angle\,270°)$
a.   108 cm, $\theta = 119.1°$
b.   108 cm, $\theta = 201.8°$
c.   232 cm, $\theta = 119.1°$
d.   232 cm, $\theta = 201.8°$

58. Consider six vectors of 2 cm each, joined from head to tail making a hexagon. What would be the magnitude of

52. The units of amplitude are the same as those for which of the following measurements?
a.   Speed
b.   Displacement
c.   Acceleration
d.   Force

53. Up to approximately what angle is simple harmonic motion a good model for a pendulum?
a.   15°
b.   45°
c.   75°
d.   90°

54. How would simple harmonic motion be different in the absence of friction?
a.   Oscillation will not happen in the absence of friction.
b.   Oscillation will continue forever in the absence of friction.
c.   Oscillation will have changing amplitude in the absence of friction.
d.   Oscillation will cease after a certain amount of time in the absence of friction.

55. What mass needs to be attached to a spring with a force constant of 7 N/m in order to make a simple harmonic oscillator oscillate with a time period of 3 s?
a.   0.03 kg
b.   1.60 kg
c.   30.7 kg
d.   63.0 kg

the addition of these vectors?
a.   Zero
b.   Six
c.   Eight
d.   Twelve

59. Two people pull on ropes tied to a trolley, each applying 44 N of force. The angle the ropes form with each other is 39.5°. What is the magnitude of the net force exerted on the trolley?
a.   0.0 N
b.   79.6 N
c.   82.8 N
d.   88.0 N

5.2 Vector Addition and Subtraction: Analytical Methods

60. True or False—A vector can form the shape of a right angle triangle with its x and y components.
a.   True

b. False

61. True or False—All vectors have positive x and y components.
   a. True
   b. False

62. Consider $\vec{A} - \vec{B} = \vec{R}$. What is R_x in terms of A_x and B_x?
   a. $R_x = \frac{A_x}{B_x}$
   b. $R_x = \frac{B_x}{A_x}$
   c. $R_x = A_x + B_x$
   d. $R_x = A_x - B_x$

63. Consider $\vec{A} - \vec{B} = \vec{R}$. What is R_y in terms of A_y and B_y?
   a. $R_y = \frac{A_y}{B_y}$
   b. $R_y = \frac{B_y}{A_y}$
   c. $R_y = A_y + B_y$
   d. $R_y = A_y - B_y$

64. When a three dimensional vector is used in the study of atmospheric sciences, what is z?
   a. Altitude
   b. Heat
   c. Temperature
   d. Wind speed

65. Which method is not an application of vector calculus?
   a. To find the rate of change in atmospheric temperature
   b. To study changes in wind speed and direction
   c. To predict changes in atmospheric pressure
   d. To measure changes in average rainfall

5.3 Projectile Motion

66. How can you express the velocity, $\vec{v}$, of a projectile in terms of its initial velocity, $\vec{v_0}$, acceleration, $\vec{a}$, and time, t?
   a. $\vec{v} = \vec{a}\,t$
   b. $\vec{v} = \vec{v_0} + \vec{a}\,t$
   c. $\vec{v} + \vec{v_0} = \vec{a}\,t$
   d. $\vec{v_0} + \vec{v} + \vec{a}\,t$

67. In the equation for the maximum height of a projectile, what does v_{0y} stand for? $h = \dfrac{v_{0y}{}^2}{2g}$
   a. Initial velocity in the x direction
   b. Initial velocity in the y direction
   c. Final velocity in the x direction
   d. Final velocity in the y direction

68. True or False—Range is defined as the maximum vertical distance travelled by a projectile.

   a. True
   b. False

69. For what angle of a projectile is its range equal to zero?
   a. 0° or 30°
   b. 0° or 45°
   c. 90° or 0°
   d. 90° or 45°

5.4 Inclined Planes

70. What are the units of the coefficient of friction?
   a. N
   b. m/s
   c. m/s^2
   d. unitless

71. Two surfaces in contact are moving slowly past each other. As the relative speed between the two surfaces in contact increases, what happens to the magnitude of their coefficient of kinetic friction?
   a. It increases with the increase in the relative motion.
   b. It decreases with the increase in the relative motion.
   c. It remains constant and is independent of the relative motion.

72. When will an object slide down an inclined plane at constant velocity?
   a. When the magnitude of the component of the weight along the slope is equal to the magnitude of the frictional force.
   b. When the magnitude of the component of the weight along the slope is greater than the magnitude of the frictional force.
   c. When the magnitude of the component of the weight perpendicular to the slope is less than the magnitude of the frictional force.
   d. When the magnitude of the component of the weight perpendicular to the slope is equal to the magnitude of the frictional force.

73. A box is sitting on an inclined plane. At what angle of incline is the perpendicular component of the box's weight at its maximum?
   a. 0°
   b. 30°
   c. 60°
   d. 90°

5.5 Simple Harmonic Motion

74. What is the term used for changes in shape due to the application of force?
   a. Amplitude

b.   Deformation

c.   Displacement

d.   Restoring force

75. What is the restoring force?

a.   The normal force on the surface of an object

b.   The weight of a mass attached to an object

c.   Force which is applied to deform an object from its original shape

d.   Force which brings an object back to its equilibrium position

76. For a given oscillator, what are the factors that affect its period and frequency?

a.   Mass only

b.   Force constant only

c.   Applied force and mass

d.   Mass and force constant

77. For an object in simple harmonic motion, when does the

Extended Response

5.1 Vector Addition and Subtraction: Graphical Methods

80. True or False—For vectors the order of addition is important.

a.   True

b.   False

81. Consider five vectors *a, b, c, d,* and *e.* Is it true or false that their addition always results in a vector with a greater magnitude than if only two of the vectors were added?

a.   True

b.   False

5.2 Vector Addition and Subtraction: Analytical Methods

82. For what angle of a vector is it possible that its magnitude will be equal to its y-component?

a. $\theta = 0°$

b. $\theta = 45°$

c. $\theta = 60°$

d. $\theta = 90°$

83. True or False—If only the angles of two vectors are known, we can find the angle of their resultant addition vector.

a.   True

b.   False

84. True or false—We can find the magnitude and direction of the resultant vector if we know the angles of two vectors and the magnitude of one.

maximum speed occur?

a.   At the extreme positions

b.   At the equilibrium position

c.   At the moment when the applied force is removed

d.   Midway between the extreme and equilibrium positions

78. What is the equilibrium position of a pendulum?

a.   When the tension in the string is zero

b.   When the pendulum is hanging straight down

c.   When the tension in the string is maximum

d.   When the weight of the mass attached is minimum

79. If a pendulum is displaced by an angle θ, what is the net restoring force it experiences?

a. $mg\sin\theta$

b. $mg\cos\theta$

c. $-mg\sin\theta$

d. $-mg\cos\theta$

a.   True

b.   False

5.3 Projectile Motion

85. Ignoring drag, what is the x-component of the acceleration of a projectile? Why?

a.   The x-component of the acceleration of a projectile is 0 because acceleration of a projectile is due to gravity, which acts in the y direction.

b.   The x component of the acceleration of a projectile is g because acceleration of a projectile is due to gravity, which acts in the x direction.

c.   The x-component of the acceleration of a projectile is 0 because acceleration of a projectile is due to gravity, which acts in the x direction.

d.   The x-component of the acceleration of a projectile is g because acceleration of a projectile is due to gravity, which acts in the y direction.

86. What is the optimum angle at which a projectile should be launched in order to cover the maximum distance?

a.   0°

b.   45°

c.   60°

d.   90°

5.4 Inclined Planes

87. True or False—Friction varies from surface to surface because different substances have different degrees of roughness or smoothness.

a.   True

b.   False

88. As the angle of the incline gets larger, what happens to

the magnitudes of the perpendicular and parallel components of gravitational force?

a.   Both the perpendicular and the parallel component will decrease.

b.   The perpendicular component will decrease and the parallel component will increase.

c.   The perpendicular component will increase and the parallel component will decrease.

d.   Both the perpendicular and the parallel component will increase.

5.5 Simple Harmonic Motion

89.   What physical characteristic of a system is its force constant related to?

a.   The force constant k is related to the stiffness of a system: The larger the force constant, the stiffer the system.

b.   The force constant k is related to the stiffness of a system: The larger the force constant, the looser the system.

c.   The force constant k is related to the friction in the system: The larger the force constant, the greater the friction in the system.

d.   The force constant k is related to the friction in the system: The larger the force constant, the lower the friction in the system.

90.   How or why does a pendulum oscillate?

a.   A pendulum oscillates due to applied force.

b.   A pendulum oscillates due to the elastic nature of the string.

c.   A pendulum oscillates due to restoring force arising from gravity.

d.   A pendulum oscillates due to restoring force arising from tension in the string.

91.   If a pendulum from earth is taken to the moon, will its frequency increase or decrease? Why?

a.   It will increase because g on the Moon is less than g on Earth.

b.   It will decrease because g on the Moon is less than g on Earth.

c.   It will increase because g on the Moon is greater than g on Earth.

d.   It will decrease because g on the Moon is greater than g on Earth.

CHAPTER 6
Circular and Rotational Motion

Figure 6.1 This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly. The same physical principles are involved in both of these motions. (Richard Munckton).

Chapter Outline

INTRODUCTION You may recall learning about various aspects of motion along a straight line: kinematics (where we learned about displacement, velocity, and acceleration), projectile motion (a special case of two-dimensional kinematics), force, and Newton's laws of motion. In some ways, this chapter is a continuation of Newton's laws of motion. Recall that Newton's first law tells us that objects move along a straight line at constant speed unless a net external force acts on them. Therefore, if an object moves along a circular path, such as the car in the photo, it must be experiencing an external force. In this chapter, we explore both circular motion and rotational motion.

6.1 Angle of Rotation and Angular Velocity

Section Learning Objectives
By the end of this section, you will be able to do the following:

- Describe the angle of rotation and relate it to its linear counterpart
- Describe angular velocity and relate it to its linear counterpart
- Solve problems involving angle of rotation and angular velocity

Section Key Terms

angle of rotation	angular velocity	arc length	circular motion
radius of curvature	rotational motion	spin	tangential velocity

Angle of Rotation

What exactly do we mean by *circular motion* or *rotation*? **Rotational motion** is the circular motion of an object about an axis of rotation. We will discuss specifically circular motion and spin. **Circular motion** is when an object moves in a circular path. Examples of circular motion include a race car speeding around a circular curve, a toy attached to a string swinging in a circle around your head, or the circular *loop-the-loop* on a roller coaster. **Spin** is rotation about an axis that goes through the center of mass of the object, such as Earth rotating on its axis, a wheel turning on its axle, the spin of a tornado on its path of destruction, or a figure skater spinning during a performance at the Olympics. Sometimes, objects will be spinning while in circular motion, like the Earth spinning on its axis while revolving around the Sun, but we will focus on these two motions separately.

When solving problems involving rotational motion, we use variables that are similar to linear variables (distance, velocity, acceleration, and force) but take into account the curvature or rotation of the motion. Here, we define the **angle of rotation**, which is the angular equivalence of distance; and **angular velocity**, which is the angular equivalence of linear velocity.

When objects rotate about some axis—for example, when the CD in Figure 6.2 rotates about its center—each point in the object follows a circular path.
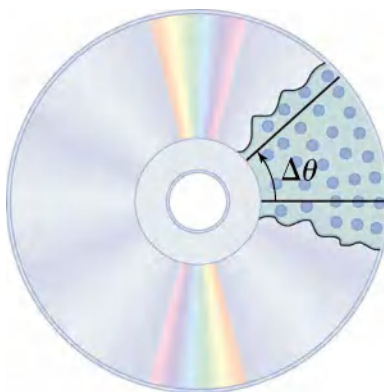
Figure 6.2 All points on a CD travel in circular paths. The pits (dots) along a line from the center to the edge all move through the same angle $\Delta\theta$ in time Δt .

The **arc length,** , is the distance traveled along a circular path. The **radius of curvature**, **r**, is the radius of the circular path. Both are shown in Figure 6.3.
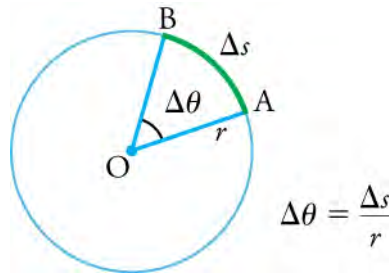
Figure 6.3 The radius (r) of a circle is rotated through an angle $\Delta\theta$. The arc length, Δs, is the distance covered along the circumference.

Consider a line from the center of the CD to its edge. In a given time, each *pit* (used to record information) on this line moves through the same angle. The angle of rotation is the amount of rotation and is the angular analog of distance. The angle of rotation $\Delta\theta$ is the arc length divided by the radius of curvature.

$$\Delta\theta = \frac{\Delta s}{r}$$

The angle of rotation is often measured by using a unit called the radian. (Radians are actually dimensionless, because a radian is defined as the ratio of two distances, radius and arc length.) A revolution is one complete rotation, where every point on the circle returns to its original position. One revolution covers 2π radians (or 360 degrees), and therefore has an angle of rotation of 2π radians, and an arc length that is the same as the circumference of the circle. We can convert between radians, revolutions, and degrees using the relationship

1 revolution = 2π rad = 360°. See Table 6.1 for the conversion of degrees to radians for some common angles.

$$2\pi \text{ rad} = 360°$$
$$1 \text{ rad} = \frac{360°}{2\pi} \approx 57.3°$$

6.1

Degree Measures	Radian Measures
30°	$\frac{\pi}{6}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	π

Table 6.1 Commonly Used Angles in Terms of Degrees and Radians

Angular Velocity

How fast is an object rotating? We can answer this question by using the concept of angular velocity. Consider first the angular speed (ω) is the rate at which the angle of rotation changes. In equation form, the angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t},$$

6.2

which means that an angular rotation ($\Delta\theta$) occurs in a time, Δt. If an object rotates through a greater angle of rotation in a given time, it has a greater angular speed. The units for angular speed are radians per second (rad/s).

Now let's consider the direction of the angular speed, which means we now must call it the angular velocity. The direction of the

angular velocity is along the axis of rotation. For an object rotating clockwise, the angular velocity points away from you along the axis of rotation. For an object rotating counterclockwise, the angular velocity points toward you along the axis of rotation.

Angular velocity (ω) is the angular version of linear velocity $\mathbf{v}$. **Tangential velocity** is the instantaneous linear velocity of an object in rotational motion. To get the precise relationship between angular velocity and tangential velocity, consider again a pit on the rotating CD. This pit moves through an arc length (Δs) in a *short* time (Δt) so its tangential *speed* is

$$v = \frac{\Delta s}{\Delta t}.$$

<div align="right">6.3</div>

From the definition of the angle of rotation, $\Delta\theta = \frac{\Delta s}{r}$, we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega.$$

The equation $v = r\omega$ says that the tangential speed v is proportional to the distance r from the center of rotation. Consequently, tangential speed is greater for a point on the outer edge of the CD (with larger r) than for a point closer to the center of the CD (with smaller r). This makes sense because a point farther out from the center has to cover a longer arc length in the same amount of time as a point closer to the center. Note that both points will still have the same angular speed, regardless of their distance from the center of rotation. See Figure 6.4.
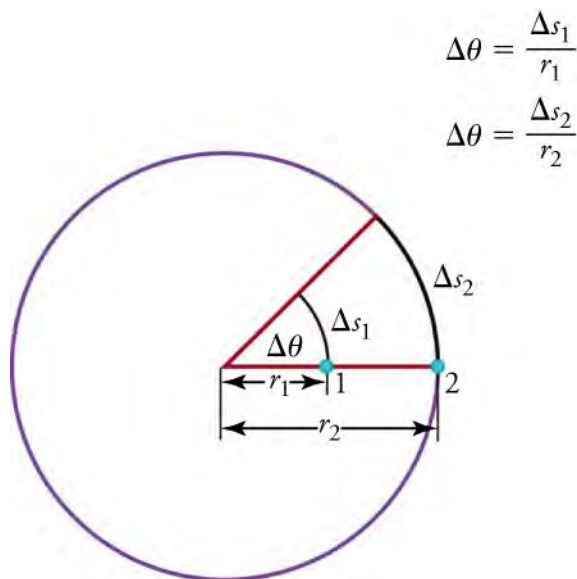
Figure 6.4 Points 1 and 2 rotate through the same angle ($\Delta\theta$), but point 2 moves through a greater arc length (Δs_2) because it is farther from the center of rotation.

Now, consider another example: the tire of a moving car (see Figure 6.5). The faster the tire spins, the faster the car moves—large ω means large v because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity, $\mathbf{\omega}$, will produce a greater linear (tangential) velocity, $\mathbf{v}$, for the car. This is because a larger radius means a longer arc length must contact the road, so the car must move farther in the same amount of time.
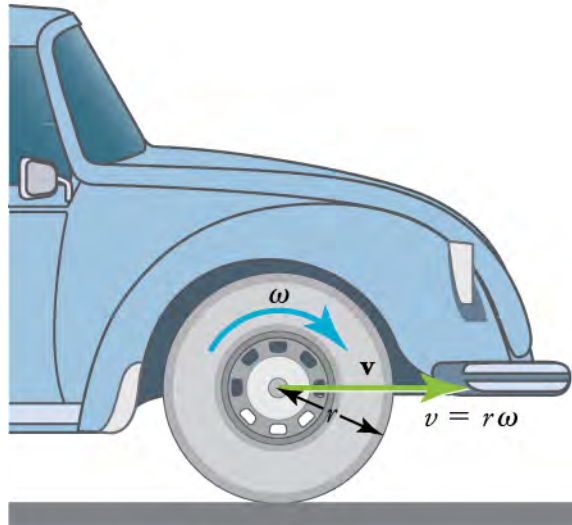
Figure 6.5 A car moving at a velocity, **v,** to the right has a tire rotating with angular velocity **ω** . The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up and the wheels spinning without touching the road. Directly below the axle, where the tire touches the road, the tire tread moves backward with respect to the axle with tangential velocity $v = r\omega$, where r is the tire radius. Because the road is stationary with respect to this point of the tire, the car must move forward at the linear velocity **v**. A larger angular velocity for the tire means a greater linear velocity for the car.

However, there are cases where linear velocity and tangential velocity are not equivalent, such as a car spinning its tires on ice. In this case, the linear velocity will be less than the tangential velocity. Due to the lack of friction under the tires of a car on ice, the arc length through which the tire treads move is greater than the linear distance through which the car moves. It's similar to running on a treadmill or pedaling a stationary bike; you are literally going nowhere fast.

TIPS FOR SUCCESS

Angular velocity **ω** and tangential velocity **v** are vectors, so we must include magnitude and direction. The direction of the angular velocity is along the axis of rotation, and points away from you for an object rotating clockwise, and toward you for an object rotating counterclockwise. In mathematics this is described by the right-hand rule. Tangential velocity is usually described as up, down, left, right, north, south, east, or west, as shown in Figure 6.6.
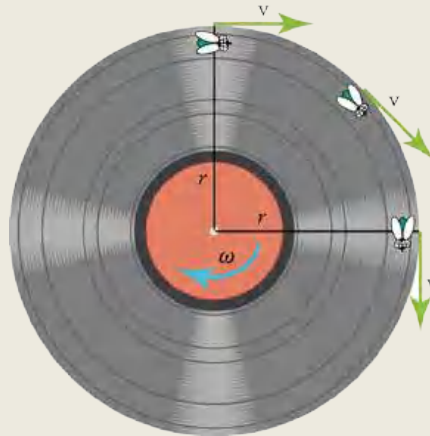
Figure 6.6 As the fly on the edge of an old-fashioned vinyl record moves in a circle, its instantaneous velocity is always at a tangent to the circle. The direction of the angular velocity is into the page this case.

👁 WATCH PHYSICS

Relationship between Angular Velocity and Speed

This video reviews the definition and units of angular velocity and relates it to linear speed. It also shows how to convert between revolutions and radians.

Click to view content (https://www.youtube.com/embed/zAx61CO5mDw)

> **GRASP CHECK**
> For an object traveling in a circular path at a constant angular speed, would the linear speed of the object change if the radius of the path increases?
> a.   Yes, because tangential speed is independent of the radius.
> b.   Yes, because tangential speed depends on the radius.
> c.   No, because tangential speed is independent of the radius.
> d.   No, because tangential speed depends on the radius.

Solving Problems Involving Angle of Rotation and Angular Velocity

Snap Lab

Measuring Angular Speed

In this activity, you will create and measure uniform circular motion and then contrast it with circular motions with different radii.

- One string (1 m long)
- One object (two-hole rubber stopper) to tie to the end
- One timer

Procedure
1.   Tie an object to the end of a string.
2.   Swing the object around in a horizontal circle above your head (swing from your wrist). It is important that the circle be horizontal!
3.   Maintain the object at uniform speed as it swings.
4.   Measure the angular speed of the object in this manner. Measure the time it takes in seconds for the object to travel 10 revolutions. Divide that time by 10 to get the angular speed in revolutions per second, which you can convert to radians per second.
5.   What is the approximate linear speed of the object?
6.   Move your hand up the string so that the length of the string is 90 cm. Repeat steps 2–5.
7.   Move your hand up the string so that its length is 80 cm. Repeat steps 2–5.
8.   Move your hand up the string so that its length is 70 cm. Repeat steps 2–5.
9.   Move your hand up the string so that its length is 60 cm. Repeat steps 2–5
10.   Move your hand up the string so that its length is 50 cm. Repeat steps 2–5
11.   Make graphs of angular speed vs. radius (i.e. string length) and linear speed vs. radius. Describe what each graph looks like.

> **GRASP CHECK**
> If you swing an object slowly, it may rotate at less than one revolution per second. What would be the revolutions per second for an object that makes one revolution in five seconds? What would be its angular speed in radians per second?
> a.   The object would spin at $\frac{1}{5}$ rev/s. The angular speed of the object would be $\frac{2\pi}{5}$ rad/s.
> b.   The object would spin at $\frac{1}{5}$ rev/s. The angular speed of the object would be $\frac{\pi}{5}$ rad/s.

c.   The object would spin at 5 rev/s. The angular speed of the object would be 10π rad/s.

d.   The object would spin at 5 rev/s. The angular speed of the object would be 5π rad/s.

Now that we have an understanding of the concepts of angle of rotation and angular velocity, we'll apply them to the real-world situations of a clock tower and a spinning tire.

✳ WORKED EXAMPLE

Angle of rotation at a Clock Tower

The clock on a clock tower has a radius of 1.0 m. (a) What angle of rotation does the hour hand of the clock travel through when it moves from 12 p.m. to 3 p.m.? (b) What's the arc length along the outermost edge of the clock between the hour hand at these two times?

Strategy

We can figure out the angle of rotation by multiplying a full revolution (2π radians) by the fraction of the 12 hours covered by the hour hand in going from 12 to 3. Once we have the angle of rotation, we can solve for the arc length by rearranging the equation $\Delta\theta = \frac{\Delta s}{r}$ since the radius is given.

Solution to (a)

In going from 12 to 3, the hour hand covers 1/4 of the 12 hours needed to make a complete revolution. Therefore, the angle between the hour hand at 12 and at 3 is $\frac{1}{4} \times 2\ \pi \text{rad} = \frac{\pi}{2}$ (i.e., 90 degrees).

Solution to (b)

Rearranging the equation

$$\Delta\theta = \frac{\Delta s}{r},$$
<div style="text-align:right">6.4</div>

we get

$$\Delta s = r\Delta\theta.$$
<div style="text-align:right">6.5</div>

Inserting the known values gives an arc length of

$$\begin{aligned}\Delta s\ &=\ (1.0\text{ m})\left(\tfrac{\pi}{2}\text{rad}\right)\\ &=\ 1.6\text{ m}\end{aligned}$$
<div style="text-align:right">6.6</div>

Discussion

We were able to drop the radians from the final solution to part (b) because radians are actually dimensionless. This is because the radian is defined as the ratio of two distances (radius and arc length). Thus, the formula gives an answer in units of meters, as expected for an arc length.

✳ WORKED EXAMPLE

How Fast Does a Car Tire Spin?

Calculate the angular speed of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h). See <u>Figure 6.5</u>.

Strategy

In this case, the speed of the tire tread with respect to the tire axle is the same as the speed of the car with respect to the road, so we have $v = 15.0$ m/s. The radius of the tire is $r = 0.300$ m. Since we know v and r, we can rearrange the equation $v = r\omega$, to get $\omega = \frac{v}{r}$ and find the angular speed.

Solution

To find the angular speed, we use the relationship: $\omega = \frac{v}{r}$.

Inserting the known quantities gives

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}}$$
$$= 50.0 \text{ rad/s.}$$

6.7

Discussion
When we cancel units in the above calculation, we get 50.0/s (i.e., 50.0 per second, which is usually written as 50.0 s^{-1}). But the angular speed must have units of rad/s. Because radians are dimensionless, we can insert them into the answer for the angular speed because we know that the motion is circular. Also note that, if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular speed of

$$\omega = \frac{15.0 \text{ m/s}}{1.20 \text{ m}}$$
$$= 12.5 \text{ rad/s}$$

6.8

Practice Problems

1. What is the angle in degrees between the hour hand and the minute hand of a clock showing 9:00 a.m.?
   a. 0°
   b. 90°
   c. 180°
   d. 360°

2. What is the approximate value of the arc length between the hour hand and the minute hand of a clock showing 10:00 a.m if the radius of the clock is 0.2 m?
   a. 0.1 m
   b. 0.2 m
   c. 0.3 m
   d. 0.6 m

Check Your Understanding

3. What is circular motion?
   a. Circular motion is the motion of an object when it follows a linear path.
   b. Circular motion is the motion of an object when it follows a zigzag path.
   c. Circular motion is the motion of an object when it follows a circular path.
   d. Circular motion is the movement of an object along the circumference of a circle or rotation along a circular path.

4. What is meant by radius of curvature when describing rotational motion?
   a. The radius of curvature is the radius of a circular path.
   b. The radius of curvature is the diameter of a circular path.
   c. The radius of curvature is the circumference of a circular path.
   d. The radius of curvature is the area of a circular path.

5. What is angular velocity?
   a. Angular velocity is the rate of change of the diameter of the circular path.
   b. Angular velocity is the rate of change of the angle subtended by the circular path.
   c. Angular velocity is the rate of change of the area of the circular path.
   d. Angular velocity is the rate of change of the radius of the circular path.

6. What equation defines angular velocity, ω? Take that r is the radius of curvature, θ is the angle, and t is time.
   a. $\omega = \frac{\Delta\theta}{\Delta t}$
   b. $\omega = \frac{\Delta t}{\Delta\theta}$
   c. $\omega = \frac{\Delta r}{\Delta t}$
   d. $\omega = \frac{\Delta t}{\Delta r}$

7. Identify three examples of an object in circular motion.

a. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a top spinning on its axis
b. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
c. Earth spinning on its own axis, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
d. Earth spinning on its own axis, blades of a working ceiling fan, and a top spinning on its own axis

8. What is the relative orientation of the radius and tangential velocity vectors of an object in uniform circular motion?
a. Tangential velocity vector is always parallel to the radius of the circular path along which the object moves.
b. Tangential velocity vector is always perpendicular to the radius of the circular path along which the object moves.
c. Tangential velocity vector is always at an acute angle to the radius of the circular path along which the object moves.
d. Tangential velocity vector is always at an obtuse angle to the radius of the circular path along which the object moves.

6.2 Uniform Circular Motion

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe centripetal acceleration and relate it to linear acceleration
- Describe centripetal force and relate it to linear force
- Solve problems involving centripetal acceleration and centripetal force

Section Key Terms

centrifugal force   centripetal acceleration   centripetal force   uniform circular motion

Centripetal Acceleration

In the previous section, we defined circular motion. The simplest case of circular motion is **uniform circular motion**, where an object travels a circular path *at a constant speed.* Note that, unlike speed, the linear velocity of an object in circular motion is constantly changing because it is always changing direction. We know from kinematics that acceleration is a change in velocity, either in magnitude or in direction or both. Therefore, an object undergoing uniform circular motion is always accelerating, even though the magnitude of its velocity is constant.

You experience this acceleration yourself every time you ride in a car while it turns a corner. If you hold the steering wheel steady during the turn and move at a constant speed, you are executing uniform circular motion. What you notice is a feeling of sliding (or being flung, depending on the speed) away from the center of the turn. This isn't an actual force that is acting on you—it only happens because your body wants to continue moving in a straight line (as per Newton's first law) whereas the car is turning off this straight-line path. Inside the car it appears as if you are forced away from the center of the turn. This fictitious force is known as the **centrifugal force**. The sharper the curve and the greater your speed, the more noticeable this effect becomes.

Figure 6.7 shows an object moving in a circular path at constant speed. The direction of the instantaneous tangential velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity; in this case it points roughly toward the center of rotation. (The center of rotation is at the center of the circular path). If we imagine Δs becoming smaller and smaller, then the acceleration would point *exactly* toward the center of rotation, but this case is hard to draw. We call the acceleration of an object moving in uniform circular motion the **centripetal acceleration a_c** because centripetal means *center seeking*.
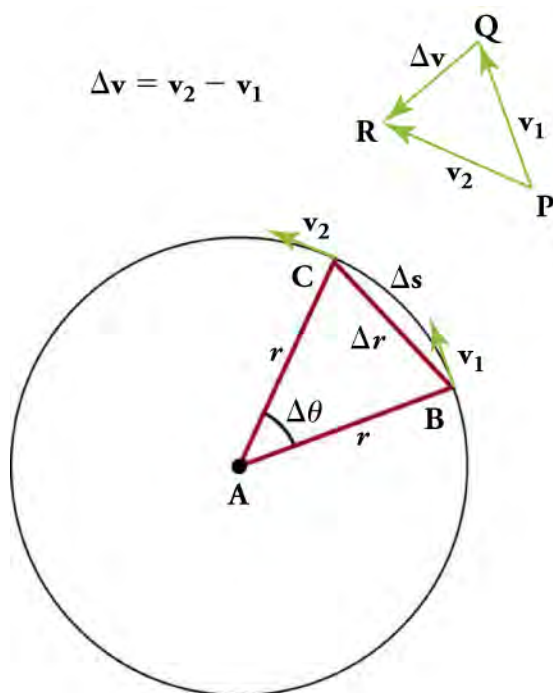
Figure 6.7 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta\mathbf{v}$ is seen to point approximately toward the center of curvature (see small inset). For an extremely small value of Δs, $\Delta\mathbf{v}$ points exactly toward the center of the circle (but this is hard to draw). Because $\mathbf{a}_c = \Delta\mathbf{v}/\Delta t$, the acceleration is also toward the center, so $\mathbf{a}_c$ is called centripetal acceleration.

Now that we know that the direction of centripetal acceleration is toward the center of rotation, let's discuss the magnitude of centripetal acceleration. For an object traveling at speed v in a circular path with radius r, the magnitude of centripetal acceleration is

$$\mathbf{a}_c = \frac{v^2}{r}.$$

Centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you may have noticed when driving a car, because the car actually pushes you toward the center of the turn. But it is a bit surprising that $\mathbf{a}_c$ is proportional to the speed squared. This means, for example, that the acceleration is four times greater when you take a curve at 100 km/h than at 50 km/h.

We can also express $\mathbf{a}_c$ in terms of the magnitude of angular velocity. Substituting $v = r\omega$ into the equation above, we get $a_c = \frac{(r\omega)^2}{r} = r\omega^2$. Therefore, the magnitude of centripetal acceleration in terms of the magnitude of angular velocity is

$$\mathbf{a}_c = r\omega^2. \qquad \boxed{6.9}$$

TIPS FOR SUCCESS

The equation expressed in the form $a_c = r\omega^2$ is useful for solving problems where you know the angular velocity rather than the tangential velocity.

Virtual Physics

Ladybug Motion in 2D

In this simulation, you experiment with the position, velocity, and acceleration of a ladybug in circular and elliptical motion. Switch the type of motion from linear to circular and observe the velocity and acceleration vectors. Next, try elliptical motion and notice how the velocity and acceleration vectors differ from those in circular motion.

Click to view content (https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/)

GRASP CHECK

In uniform circular motion, what is the angle between the acceleration and the velocity? What type of acceleration does a body experience in the uniform circular motion?

a.   The angle between acceleration and velocity is 0°, and the body experiences linear acceleration.

b.   The angle between acceleration and velocity is 0°, and the body experiences centripetal acceleration.

c.   The angle between acceleration and velocity is 90°, and the body experiences linear acceleration.

d.   The angle between acceleration and velocity is 90°, and the body experiences centripetal acceleration.

Centripetal Force

Because an object in uniform circular motion undergoes constant acceleration (by changing direction), we know from Newton's second law of motion that there must be a constant net external force acting on the object.

Any force or combination of forces can cause a centripetal acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, the friction between a road and the tires of a car as it goes around a curve, or the normal force of a roller coaster track on the cart during a loop-the-loop.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of rotation, the same as for centripetal acceleration. According to Newton's second law of motion, a net force causes the acceleration of mass according to $\mathbf{F}_{net} = m\mathbf{a}$. For uniform circular motion, the acceleration is centripetal acceleration: $\mathbf{a} = \mathbf{a}_c$. Therefore, the magnitude of centripetal force, $\mathbf{F}_c$, is $\mathbf{F}_c = m\mathbf{a}_c$.

By using the two different forms of the equation for the magnitude of centripetal acceleration, $\mathbf{a}_c = v^2/r$ and $\mathbf{a}_c = r\omega^2$, we get two expressions involving the magnitude of the centripetal force $\mathbf{F}_c$. The first expression is in terms of tangential speed, the second is in terms of angular speed: $\mathbf{F}_c = m\frac{v^2}{r}$ and $\mathbf{F}_c = mr\omega^2$.

Both forms of the equation depend on mass, velocity, and the radius of the circular path. You may use whichever expression for centripetal force is more convenient. Newton's second law also states that the object will accelerate in the same direction as the net force. By definition, the centripetal force is directed towards the center of rotation, so the object will also accelerate towards the center. A straight line drawn from the circular path to the center of the circle will always be perpendicular to the tangential velocity. Note that, if you solve the first expression for r, you get

$$r = \frac{mv^2}{\mathbf{F}_c}.$$

From this expression, we see that, for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.
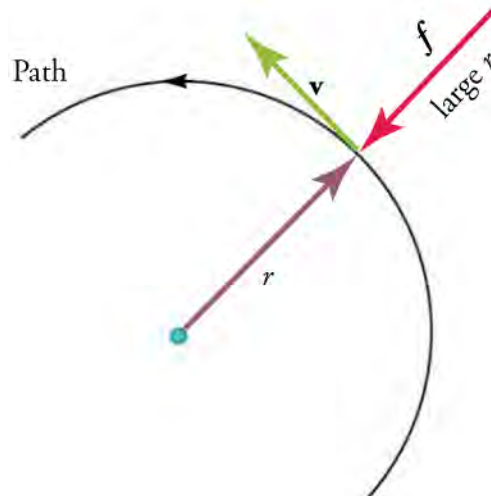
$$f = \mathbf{F_c} \text{ is parallel to } \mathbf{a_c} \text{ since } \mathbf{F_c} = m\mathbf{a_\infty}$$
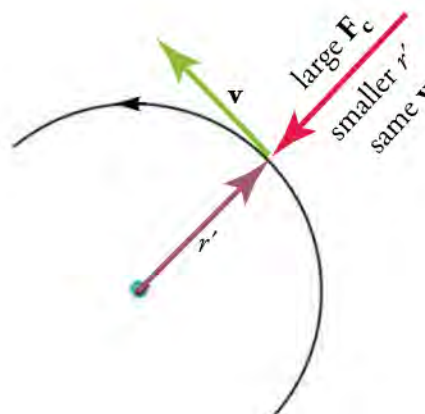
Figure 6.8 In this figure, the frictional force f serves as the centripetal force $\mathbf{F_c}$. Centripetal force is perpendicular to tangential velocity and causes uniform circular motion. The larger the centripetal force $\mathbf{F_c}$, the smaller is the radius of curvature r and the sharper is the curve. The lower curve has the same velocity $\mathbf{v}$, but a larger centripetal force $\mathbf{F_c}$ produces a smaller radius r' .

👁 WATCH PHYSICS

Centripetal Force and Acceleration Intuition
This video explains why a centripetal force creates centripetal acceleration and uniform circular motion. It also covers the difference between speed and velocity and shows examples of uniform circular motion.

Click to view content (https://www.youtube.com/embed/vZOk8NnjILg)

> **GRASP CHECK**
> Imagine that you are swinging a yoyo in a vertical clockwise circle in front of you, perpendicular to the direction you are facing. Now, imagine that the string breaks just as the yoyo reaches its bottommost position, nearest the floor. Which of the following describes the path of the yoyo after the string breaks?
> a.   The yoyo will fly upward in the direction of the centripetal force.
> b.   The yoyo will fly downward in the direction of the centripetal force.

c. The yoyo will fly to the left in the direction of the tangential velocity.

d. The yoyo will fly to the right in the direction of the tangential velocity.

Solving Centripetal Acceleration and Centripetal Force Problems

To get a feel for the typical magnitudes of centripetal acceleration, we'll do a lab estimating the centripetal acceleration of a tennis racket and then, in our first Worked Example, compare the centripetal acceleration of a car rounding a curve to gravitational acceleration. For the second Worked Example, we'll calculate the force required to make a car round a curve.

Snap Lab

Estimating Centripetal Acceleration

In this activity, you will measure the swing of a golf club or tennis racket to estimate the centripetal acceleration of the end of the club or racket. You may choose to do this in slow motion. Recall that the equation for centripetal acceleration is $\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$.

- One tennis racket or golf club
- One timer
- One ruler or tape measure

Procedure

1. Work with a partner. Stand a safe distance away from your partner as he or she swings the golf club or tennis racket.
2. Describe the motion of the swing—is this uniform circular motion? Why or why not?
3. Try to get the swing as close to uniform circular motion as possible. What adjustments did your partner need to make?
4. Measure the radius of curvature. What did you physically measure?
5. By using the timer, find either the linear or angular velocity, depending on which equation you decide to use.
6. What is the approximate centripetal acceleration based on these measurements? How accurate do you think they are? Why? How might you and your partner make these measurements more accurate?

GRASP CHECK

Was it more useful to use the equation $a_c = \frac{v^2}{r}$ or $a_c = r\omega^2$ in this activity? Why?

a. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be easier.

b. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be easier.

c. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be difficult.

d. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be difficult.

✳ WORKED EXAMPLE

Comparing Centripetal Acceleration of a Car Rounding a Curve with Acceleration Due to Gravity

A car follows a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h). What is the magnitude of the car's centripetal acceleration? Compare the centripetal acceleration for this fairly gentle curve taken at highway speed with acceleration due to gravity (g).
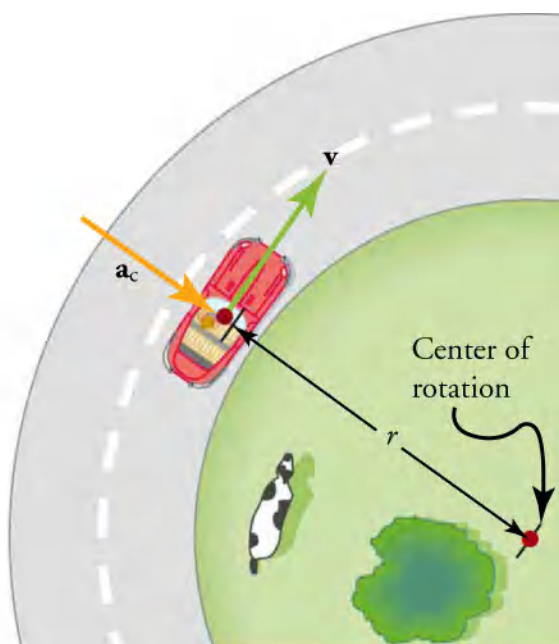
Car around corner

Strategy

Because linear rather than angular speed is given, it is most convenient to use the expression $\mathbf{a}_c = \frac{v^2}{r}$ to find the magnitude of the centripetal acceleration.

Solution

Entering the given values of $v = 25.0$ m/s and $r = 500$ m into the expression for $\mathbf{a}_c$ gives

$$
\begin{aligned}
\mathbf{a}_c &= \frac{v^2}{r} \\
&= \frac{(25.0 \ \text{m/s})^2}{500 \ \text{m}} \\
&= 1.25 \text{m/s}^2 .
\end{aligned}
$$

Discussion

To compare this with the acceleration due to gravity ($g = 9.80$ m/s²), we take the ratio
$\mathbf{a}_c/g = (1.25 \ \text{m/s}^2)/(9.80 \text{m/s}^2) = 0.128$. Therefore, $\mathbf{a}_c = 0.128g$, which means that the centripetal acceleration is about one tenth the acceleration due to gravity.

✳ WORKED EXAMPLE

Frictional Force on Car Tires Rounding a Curve

a. Calculate the centripetal force exerted on a 900 kg car that rounds a 600-m-radius curve on horizontal ground at 25.0 m/s.
b. Static friction prevents the car from slipping. Find the magnitude of the frictional force between the tires and the road that allows the car to round the curve without sliding off in a straight line.

Free-body diagram
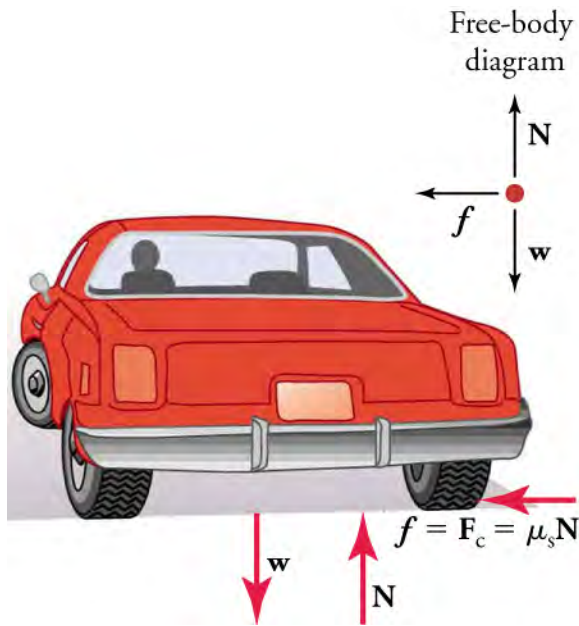
Strategy and Solution for (a)

We know that $\mathbf{F}_c = m\frac{v^2}{r}$. Therefore,

$$\mathbf{F}_c = m\frac{v^2}{r}$$
$$= \frac{(900 \ \text{kg})(25.0 \ \text{m/s})^2}{600 \ \text{m}}$$
$$= 938 \ \text{N}.$$

Strategy and Solution for (b)

The image above shows the forces acting on the car while rounding the curve. In this diagram, the car is traveling into the page as shown and is turning to the left. Friction acts toward the left, accelerating the car toward the center of the curve. Because friction is the only horizontal force acting on the car, it provides all of the centripetal force in this case. Therefore, the force of friction is the centripetal force in this situation and points toward the center of the curve.

$$f = \mathbf{F}_c = 938 \ \text{N}$$

Discussion

Since we found the force of friction in part (b), we could also solve for the coefficient of friction, since $f = \mu_s N = \mu_s mg$.

Practice Problems

9. What is the centripetal acceleration of an object with speed 12 m/s going along a path of radius 2.0 m?
   a. 6 m/s
   b. 72 m/s
   c. 6 m/s^2
   d. 72 m/s^2

10. Calculate the centripetal acceleration of an object following a path with a radius of a curvature of 0.2 m and at an angular velocity of 5 rad/s.
    a. 1 m/s
    b. 5 m/s
    c. 1 m/s^2
    d. 5 m/s^2

Check Your Understanding

11. What is uniform circular motion?

   a.   Uniform circular motion is when an object accelerates on a circular path at a constantly increasing velocity.

   b.   Uniform circular motion is when an object travels on a circular path at a variable acceleration.

   c.   Uniform circular motion is when an object travels on a circular path at a constant speed.

   d.   Uniform circular motion is when an object travels on a circular path at a variable speed.

12. What is centripetal acceleration?

   a.   The acceleration of an object moving in a circular path and directed radially toward the center of the circular orbit

   b.   The acceleration of an object moving in a circular path and directed tangentially along the circular path

   c.   The acceleration of an object moving in a linear path and directed in the direction of motion of the object

   d.   The acceleration of an object moving in a linear path and directed in the direction opposite to the motion of the object

13. Is there a net force acting on an object in uniform circular motion?

   a.   Yes, the object is accelerating, so a net force must be acting on it.

   b.   Yes, because there is no acceleration.

   c.   No, because there is acceleration.

   d.   No, because there is no acceleration.

14. Identify two examples of forces that can cause centripetal acceleration.

   a.   The force of Earth's gravity on the moon and the normal force

   b.   The force of Earth's gravity on the moon and the tension in the rope on an orbiting tetherball

   c.   The normal force and the force of friction acting on a moving car

   d.   The normal force and the tension in the rope on a tetherball

6.3 Rotational Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe rotational kinematic variables and equations and relate them to their linear counterparts
- Describe torque and lever arm
- Solve problems involving torque and rotational kinematics

Section Key Terms

angular acceleration	kinematics of rotational motion	lever arm
tangential acceleration	torque	

Rotational Kinematics

In the section on uniform circular motion, we discussed motion in a circle at constant speed and, therefore, constant angular velocity. However, there are times when angular velocity is not constant—rotational motion can speed up, slow down, or reverse directions. Angular velocity is not constant when a spinning skater pulls in her arms, when a child pushes a merry-go-round to make it rotate, or when a CD slows to a halt when switched off. In all these cases, **angular acceleration** occurs because the angular velocity ω changes. The faster the change occurs, the greater is the angular acceleration. Angular acceleration α is the rate of change of angular velocity. In equation form, angular acceleration is

$$\alpha = \frac{\Delta \omega}{\Delta t},$$

where $\Delta \omega$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2. If ω increases, then α is positive. If ω decreases, then α is negative. Keep in mind that, by convention, counterclockwise is the positive direction and clockwise is the negative direction. For example, the skater in Figure 6.9 is rotating counterclockwise as seen from above, so her angular velocity is positive. Acceleration would be negative, for example, when an object that is rotating counterclockwise slows down. It would be positive when an object that is rotating counterclockwise speeds up.

Figure 6.9 A figure skater spins in the counterclockwise direction, so her angular velocity is normally considered to be positive. (Luu, Wikimedia Commons)

The relationship between the magnitudes of **tangential acceleration**, **a**, and angular acceleration,

$$\boldsymbol{\alpha}, \text{is} \, \mathbf{a} = r\boldsymbol{\alpha} \, \text{or} \, \boldsymbol{\alpha} = \frac{\mathbf{a}}{r}.$$

<div style="float:right">6.10</div>

These equations mean that the magnitudes of tangential acceleration and angular acceleration are directly proportional to each other. The greater the angular acceleration, the larger the change in tangential acceleration, and vice versa. For example, consider riders in their pods on a Ferris wheel at rest. A Ferris wheel with greater angular acceleration will give the riders greater tangential acceleration because, as the Ferris wheel increases its rate of spinning, it also increases its tangential velocity. Note that the radius of the spinning object also matters. For example, for a given angular acceleration $\boldsymbol{\alpha}$, a smaller Ferris wheel leads to a smaller tangential acceleration for the riders.

TIPS FOR SUCCESS

Tangential acceleration is sometimes denoted $\mathbf{a_t}$. It is a linear acceleration in a direction tangent to the circle at the point of interest in circular or rotational motion. Remember that tangential acceleration is parallel to the tangential velocity (either in the same direction or in the opposite direction.) Centripetal acceleration is always perpendicular to the tangential velocity.

So far, we have defined three rotational variables: θ, $\boldsymbol{\omega}$, and $\boldsymbol{\alpha}$. These are the angular versions of the linear variables x, $\mathbf{v}$, and $\mathbf{a}$. Table 6.2 shows how they are related.

Rotational	Linear	Relationship
θ	x	$\theta = \frac{x}{r}$

Table 6.2 Rotational and Linear Variables

Rotational	Linear	Relationship
ω	$\mathbf{v}$	$\omega = \frac{\mathbf{v}}{r}$
α	$\mathbf{a}$	$\alpha = \frac{\mathbf{a}}{r}$

Table 6.2 Rotational and Linear Variables

We can now begin to see how rotational quantities like θ, ω, and α are related to each other. For example, if a motorcycle wheel that starts at rest has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. Putting this in terms of the variables, if the wheel's angular acceleration α is large for a long period of time t, then the final angular velocity ω and angle of rotation θ are large. In the case of linear motion, if an object starts at rest and undergoes a large linear acceleration, then it has a large final velocity and will have traveled a large distance.

The **kinematics of rotational motion** describes the relationships between the angle of rotation, angular velocity, angular acceleration, and time. It only *describes* motion—it does not include any forces or masses that may affect rotation (these are part of dynamics). Recall the kinematics equation for linear motion: $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ (constant $\mathbf{a}$).

As in linear kinematics, we assume $\mathbf{a}$ is constant, which means that angular acceleration α is also a constant, because $\mathbf{a} = r\alpha$. The equation for the kinematics relationship between ω, α, and t is

$$\omega = \omega_0 + \alpha t \, (\text{constant} \, \alpha),$$

where ω_0 is the initial angular velocity. Notice that the equation is identical to the linear version, except with angular analogs of the linear variables. In fact, all of the linear kinematics equations have rotational analogs, which are given in [Table 6.3](#). These equations can be used to solve rotational or linear kinematics problem in which $\mathbf{a}$ and α are constant.

Rotational	Linear	
$\theta = \overline{\omega}t$	$x = \overline{\mathbf{v}}t$	
$\omega = \omega_0 + \alpha t$	$\mathbf{v} = \mathbf{v}_0 + \alpha t$	constant α, $\mathbf{a}$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = \mathbf{v}_0 t + \frac{1}{2}\alpha t^2$	constant α, $\mathbf{a}$
$\omega^2 = \omega_0{}^2 + 2\alpha\theta$	$\mathbf{v}^2 = \mathbf{v}_0{}^2 + 2\alpha x$	constant α, $\mathbf{a}$

Table 6.3 Equations for Rotational Kinematics

In these equations, ω_0 and $\mathbf{v}_0$ are initial values, t_0 is zero, and the average angular velocity $\overline{\omega}$ and average velocity $\overline{\mathbf{v}}$ are

$$\overline{\omega} = \frac{\omega_0 + \omega}{2} \, \text{and} \, \overline{\mathbf{v}} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}. \qquad \boxed{6.11}$$

![icon] FUN IN PHYSICS

Storm Chasing

Figure 6.10 Tornadoes descend from clouds in funnel-like shapes that spin violently. (Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

Storm chasers tend to fall into one of three groups: Amateurs chasing tornadoes as a hobby, atmospheric scientists gathering data for research, weather watchers for news media, or scientists having fun under the guise of work. Storm chasing is a dangerous pastime because tornadoes can change course rapidly with little warning. Since storm chasers follow in the wake of the destruction left by tornadoes, changing flat tires due to debris left on the highway is common. The most active part of the world for tornadoes, called *tornado alley*, is in the central United States, between the Rocky Mountains and Appalachian Mountains.

Tornadoes are perfect examples of rotational motion in action in nature. They come out of severe thunderstorms called supercells, which have a column of air rotating around a horizontal axis, usually about four miles across. The difference in wind speeds between the strong cold winds higher up in the atmosphere in the jet stream and weaker winds traveling north from the Gulf of Mexico causes the column of rotating air to shift so that it spins around a vertical axis, creating a tornado.

Tornadoes produce wind speeds as high as 500 km/h (approximately 300 miles/h), particularly at the bottom where the funnel is narrowest because the rate of rotation increases as the radius decreases. They blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw.

> **GRASP CHECK**
> What is the physics term for the eye of the storm? Why would winds be weaker at the eye of the tornado than at its outermost edge?
> a. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to radius of curvature.
> b. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to radius of curvature.
> c. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to the square of the radius of curvature.
> d. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to the square of the radius of curvature.

Torque

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity. The farther the force is applied from the pivot point (or fulcrum), the greater the angular acceleration. For example, a door opens slowly if you push too close to its hinge, but opens easily if you push far from the hinges. Furthermore, we know that the more

massive the door is, the more slowly it opens; this is because angular acceleration is inversely proportional to mass. These relationships are very similar to the relationships between force, mass, and acceleration from Newton's second law of motion. Since we have already covered the angular versions of distance, velocity and time, you may wonder what the angular version of force is, and how it relates to linear force.

The angular version of force is **torque τ**, which is the turning effectiveness of a force. See Figure 6.11. The equation for the magnitude of torque is

$$\tau = r\mathbf{F}\sin\theta,$$

where r is the magnitude of the **lever arm**, $\mathbf{F}$ is the magnitude of the linear force, and θ is the angle between the lever arm and the force. The lever arm is the vector from the point of rotation (pivot point or fulcrum) to the location where force is applied. Since the magnitude of the lever arm is a distance, its units are in meters, and torque has units of N·m. Torque is a vector quantity and has the same direction as the angular acceleration that it produces.
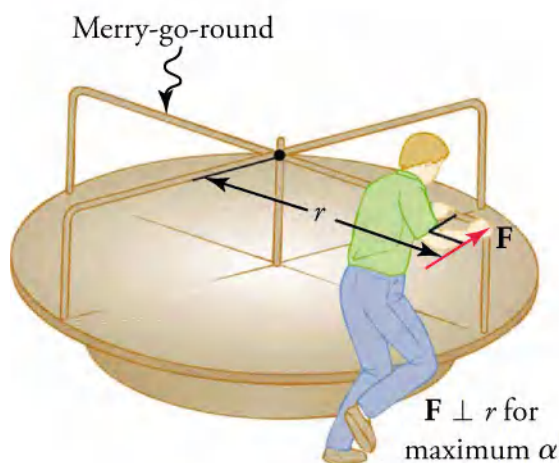
Figure 6.11 A man pushes a merry-go-round at its edge and perpendicular to the lever arm to achieve maximum torque.

Applying a stronger torque will produce a greater angular acceleration. For example, the harder the man pushes the merry-go-round in Figure 6.11, the faster it accelerates. Furthermore, the more massive the merry-go-round is, the slower it accelerates for the same torque. If the man wants to maximize the effect of his force on the merry-go-round, he should push as far from the center as possible to get the largest lever arm and, therefore, the greatest torque and angular acceleration. Torque is also maximized when the force is applied perpendicular to the lever arm.

Solving Rotational Kinematics and Torque Problems

Just as linear forces can balance to produce zero net force and no linear acceleration, the same is true of rotational motion. When two torques of equal magnitude act in opposing directions, there is no net torque and no angular acceleration, as you can see in the following video. If zero net torque acts on a system spinning at a constant angular velocity, the system will continue to spin at the same angular velocity.

👁 WATCH PHYSICS

Introduction to Torque
This video (https://www.khanacademy.org/science/physics/torque-angular-momentum/torque-tutorial/v/introduction-to-torque) defines torque in terms of moment arm (which is the same as lever arm). It also covers a problem with forces acting in opposing directions about a pivot point. (At this stage, you can ignore Sal's references to work and mechanical advantage.)

> **GRASP CHECK**
> Click to view content (https://www.openstax.org/l/28torque)
> If the net torque acting on the ruler from the example was positive instead of zero, what would this say about the angular

acceleration? What would happen to the ruler over time?

a. The ruler is in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.

b. The ruler is not in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.

c. The ruler is not in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.

d. The ruler is in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.

Now let's look at examples applying rotational kinematics to a fishing reel and the concept of torque to a merry-go-round.

✳ WORKED EXAMPLE

Calculating the Time for a Fishing Reel to Stop Spinning

A deep-sea fisherman uses a fishing rod with a reel of radius 4.50 cm. A big fish takes the bait and swims away from the boat, pulling the fishing line from his fishing reel. As the fishing line unwinds from the reel, the reel spins at an angular velocity of 220 rad/s. The fisherman applies a brake to the spinning reel, creating an angular acceleration of -300 rad/s². How long does it take the reel to come to a stop?
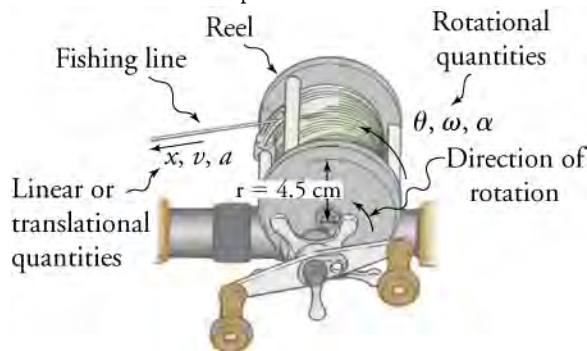
Strategy

We are asked to find the time t for the reel to come to a stop. The magnitude of the initial angular velocity is $\omega_0 = 220$ rad/s, and the magnitude of the final angular velocity $\omega = 0$. The signed magnitude of the angular acceleration is $\alpha = -300$ rad/s², where the minus sign indicates that it acts in the direction opposite to the angular velocity. Looking at the rotational kinematic equations, we see all quantities but t are known in the equation $\omega = \omega_0 + \alpha t$, making it the easiest equation to use for this problem.

Solution

The equation to use is $\omega = \omega_0 + \alpha t$.

We solve the equation algebraically for t, and then insert the known values.

$$
\begin{aligned}
t &= \frac{\omega - \omega_0}{\alpha} \\
&= \frac{0 - 220 \ \text{rad/s}}{-300 \text{rad/s}^2} \\
&= 0.733 \ \text{s}
\end{aligned}
$$

6.12

Discussion

The time to stop the reel is fairly small because the acceleration is fairly large. Fishing lines sometimes snap because of the forces involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration and therefore a smaller force.

✳ WORKED EXAMPLE

Calculating the Torque on a Merry-Go-Round

Consider the man pushing the playground merry-go-round in Figure 6.11. He exerts a force of 250 N at the edge of the merry-go-round and perpendicular to the radius, which is 1.50 m. How much torque does he produce? Assume that friction acting on the merry-go-round is negligible.

Strategy

To find the torque, note that the applied force is perpendicular to the radius and that friction is negligible.

Solution

$$
\begin{aligned}
\tau &= r\mathbf{F}\sin\theta \\
&= (1.50 \text{ m})(250 \text{ N})\sin\left(\tfrac{\pi}{2}\right). \\
&= 375 \text{ N} \cdot \text{m}
\end{aligned}
$$

6.13

Discussion

The man maximizes the torque by applying force perpendicular to the lever arm, so that $\theta = \frac{\pi}{2}$ and $\sin\theta = 1$. The man also maximizes his torque by pushing at the outer edge of the merry-go-round, so that he gets the largest-possible lever arm.

Practice Problems

15. How much torque does a person produce if he applies a $12\,\text{N}$ force $1.0\,\text{m}$ away from the pivot point, perpendicularly to the lever arm?
    a. $\frac{1}{144}$ N-m
    b. $\frac{1}{12}$ N-m
    c. $12\,\text{N-m}$
    d. $144\,\text{N-m}$

16. An object's angular velocity changes from 3 rad/s clockwise to 8 rad/s clockwise in 5 s. What is its angular acceleration?
    a. 0.6 rad/s²
    b. 1.6 rad/s²
    c. 1 rad/s²
    d. 5 rad/s²

Check Your Understanding

17. What is angular acceleration?
    a. Angular acceleration is the rate of change of the angular displacement.
    b. Angular acceleration is the rate of change of the angular velocity.
    c. Angular acceleration is the rate of change of the linear displacement.
    d. Angular acceleration is the rate of change of the linear velocity.

18. What is the equation for angular acceleration, α? Assume θ is the angle, ω is the angular velocity, and t is time.
    a. $\alpha = \frac{\Delta\omega}{\Delta t}$
    b. $\alpha = \Delta\omega\Delta t$
    c. $\alpha = \frac{\Delta\theta}{\Delta t}$
    d. $\alpha = \Delta\theta\Delta t$

19. Which of the following best describes torque?
    a. It is the rotational equivalent of a force.
    b. It is the force that affects linear motion.
    c. It is the rotational equivalent of acceleration.
    d. It is the acceleration that affects linear motion.

20. What is the equation for torque?

a. $\tau = F \cos\theta \, r$

b. $\tau = \frac{F \sin\theta}{r}$

c. $\tau = rF\cos\theta$

d. $\tau = rF\sin\theta$

KEY TERMS

angle of rotation   the ratio of the arc length to the radius of curvature of a circular path

angular acceleration   the rate of change of angular velocity with time

angular velocity   (ω) the rate of change in the angular position of an object following a circular path

arc length   (Δs) the distance traveled by an object along a circular path

centrifugal force   a fictitious force that acts in the direction opposite the centripetal acceleration

centripetal acceleration   the acceleration of an object moving in a circle, directed toward the center of the circle

centripetal force   any force causing uniform circular motion

circular motion   the motion of an object along a circular path

kinematics of rotational motion   the relationships between rotation angle, angular velocity, angular acceleration, and

time

lever arm   the distance between the point of rotation (pivot point) and the location where force is applied

radius of curvature   the distance between the center of a circular path and the path

rotational motion   the circular motion of an object about an axis of rotation

spin   rotation about an axis that goes through the center of mass of the object

tangential acceleration   the acceleration in a direction tangent to the circular path of motion and in the same direction or opposite direction as the tangential velocity

tangential velocity   the instantaneous linear velocity of an object in circular or rotational motion

torque   the effectiveness of a force to change the rotational speed of an object

uniform circular motion   the motion of an object in a circular path at constant speed

SECTION SUMMARY

6.1 Angle of Rotation and Angular Velocity

- Circular motion is motion in a circular path.
- The angle of rotation $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature.
- The arc length Δs is the distance traveled along a circular path and r is the radius of curvature of the circular path.
- The angle of rotation $\Delta\theta$ is measured in units of radians (rad), where $2\ \pi\mathrm{rad} = 360° = 1$ revolution.
- Angular velocity ω is the rate of change of an angle, where a rotation $\Delta\theta$ occurs in a time Δt .
- The units of angular velocity are radians per second (rad/s).
- Tangential speed v and angular speed ω are related by $v = r\omega$, and tangential velocity has units of m/s.
- The direction of angular velocity is along the axis of rotation, toward (away) from you for clockwise (counterclockwise) motion.

6.2 Uniform Circular Motion

- Centripetal acceleration $\mathbf{a}_c$ is the acceleration experienced while in uniform circular motion.
- Centripetal acceleration force is a *center-seeking* force

that always points toward the center of rotation, perpendicular to the linear velocity, in the same direction as the net force, and in the direction opposite that of the radius vector.
- The standard unit for centripetal acceleration is m/s².
- Centripetal force $\mathbf{F}_c$ is any net force causing uniform circular motion.

6.3 Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.
- Torque is the effectiveness of a force to change the rotational speed of an object. Torque is the rotational analog of force.
- The lever arm is the distance between the point of rotation (pivot point) and the location where force is applied.
- Torque is maximized by applying force perpendicular to the lever arm and at a point as far as possible from the pivot point or fulcrum. If torque is zero, angular acceleration is zero.

KEY EQUATIONS

6.1 Angle of Rotation and Angular Velocity

Angle of rotation	$\Delta\theta = \frac{\Delta s}{r}$
Angular speed:	$\omega = \frac{\Delta\theta}{\Delta t}$
Tangential speed:	$v = r\omega$

6.2 Uniform Circular Motion

Centripetal acceleration	$\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$
Centripetal force	$\mathbf{F}_c = m\mathbf{a}_c, \mathbf{F}_c = m\frac{v^2}{r},$ $\mathbf{F}_c = mr\omega^2$

6.3 Rotational Motion

Angular acceleration	$\boldsymbol{\alpha} = \frac{\Delta\boldsymbol{\omega}}{\Delta t}$
Rotational kinematic equations	$\theta = \omega t, \omega = \omega_0 + \alpha t,$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2,$ $\omega^2 = \omega_0{}^2 + 2\alpha\theta$
Tangential (linear) acceleration	$\mathbf{a} = r\boldsymbol{\alpha}$
Torque	$\boldsymbol{\tau} = r\mathbf{F}\sin\theta$

CHAPTER REVIEW

Concept Items

6.1 Angle of Rotation and Angular Velocity

1. One revolution is equal to how many radians? Degrees?
   a. $1 \text{ rev} = \pi \text{ rad} = 180°$
   b. $1 \text{ rev} = \pi \text{ rad} = 360°$
   c. $1 \text{ rev} = 2\pi \text{ rad} = 180°$
   d. $1 \text{ rev} = 2\pi \text{ rad} = 360°$

2. What is tangential velocity?
   a. Tangential velocity is the average linear velocity of an object in a circular motion.
   b. Tangential velocity is the instantaneous linear velocity of an object undergoing rotational motion.
   c. Tangential velocity is the average angular velocity of an object in a circular motion.
   d. Tangential velocity is the instantaneous angular velocity of an object in a circular motion.

3. What kind of motion is called *spin*?
   a. Spin is rotational motion of an object about an axis parallel to the axis of the object.
   b. Spin is translational motion of an object about an axis parallel to the axis of the object.
   c. Spin is the rotational motion of an object about its center of mass.
   d. Spin is translational motion of an object about its own axis.

6.2 Uniform Circular Motion

4. What is the equation for centripetal acceleration in terms of angular velocity and the radius?
   a. $a_c = \frac{\omega^2}{r}$
   b. $a_c = \frac{\omega}{r}$
   c. $a_c = r\omega^2$
   d. $a_c = r\omega$

5. How can you express centripetal force in terms of centripetal acceleration?
   a. $F_c = \frac{a_c^2}{m}$
   b. $F_c = \frac{a_c}{m}$
   c. $F_c = ma_c^2$
   d. $F_c = ma_c$

6. What is meant by the word centripetal?
   a. center-seeking
   b. center-avoiding
   c. central force
   d. central acceleration

6.3 Rotational Motion

7. Conventionally, for which direction of rotation of an object is angular acceleration considered positive?
   a. the positive x direction of the coordinate system
   b. the negative x direction of the coordinate system
   c. the counterclockwise direction
   d. the clockwise direction

8. When you push a door closer to the hinges, why does it open more slowly?
    a.   It opens slowly, because the lever arm is shorter so the torque is large.
    b.   It opens slowly because the lever arm is longer so the torque is large.
    c.   It opens slowly, because the lever arm is shorter so the torque is less.
    d.   It opens slowly, because the lever arm is longer so the torque is less.

9. When is angular acceleration negative?
    a.   Angular acceleration is the rate of change of the displacement and is negative when ω increases.
    b.   Angular acceleration is the rate of change of the displacement and is negative when ω decreases.
    c.   Angular acceleration is the rate of change of angular velocity and is negative when ω increases.
    d.   Angular acceleration is the rate of change of angular velocity and is negative when ω decreases.

Critical Thinking Items

6.1 Angle of Rotation and Angular Velocity

10. When the radius of the circular path of rotational motion increases, what happens to the arc length for a given angle of rotation?
    a.   The arc length is directly proportional to the radius of the circular path, and it increases with the radius.
    b.   The arc length is inversely proportional to the radius of the circular path, and it decreases with the radius.
    c.   The arc length is directly proportional to the radius of the circular path, and it decreases with the radius.
    d.   The arc length is inversely proportional to the radius of the circular path, and it increases with the radius.

11. Consider a CD spinning clockwise. What is the sum of the instantaneous velocities of two points on both ends of its diameter?
    a. $2v$
    b. $\frac{v}{2}$
    c. $-v$
    d. 0

6.2 Uniform Circular Motion

12. What are the directions of the velocity and acceleration of an object in uniform circular motion?
    a.   Velocity is tangential, and acceleration is radially outward.
    b.   Velocity is tangential, and acceleration is radially inward.
    c.   Velocity is radially outward, and acceleration is tangential.
    d.   Velocity is radially inward, and acceleration is tangential.

13. Suppose you have an object tied to a rope and are rotating it over your head in uniform circular motion. If you increase the length of the rope, would you have to apply more or less force to maintain the same speed?
    a.   More force is required, because the force is inversely proportional to the radius of the circular orbit.
    b.   More force is required because the force is directly proportional to the radius of the circular orbit.
    c.   Less force is required because the force is inversely proportional to the radius of the circular orbit.
    d.   Less force is required because the force is directly proportional to the radius of the circular orbit.

6.3 Rotational Motion

14. Consider two spinning tops with different radii. Both have the same linear instantaneous velocities at their edges. Which top has a higher angular velocity?
    a.   the top with the smaller radius because the radius of curvature is inversely proportional to the angular velocity
    b.   the top with the smaller radius because the radius of curvature is directly proportional to the angular velocity
    c.   the top with the larger radius because the radius of curvature is inversely proportional to the angular velocity
    d.   The top with the larger radius because the radius of curvature is directly proportional to the angular velocity

15. A person tries to lift a stone by using a lever. If the lever arm is constant and the mass of the stone increases, what is true of the torque necessary to lift it?
    a.   It increases, because the torque is directly proportional to the mass of the body.
    b.   It increases because the torque is inversely proportional to the mass of the body.
    c.   It decreases because the torque is directly proportional to the mass of the body.
    d.   It decreases, because the torque is inversely proportional to the mass of the body.

Problems

6.1 Angle of Rotation and Angular Velocity

16. What is the angle of rotation (in degrees) between two hands of a clock, if the radius of the clock is 0.70 m and the arc length separating the two hands is 1.0 m?
   a. $40°$
   b. $80°$
   c. $81°$
   d. $163°$

17. A clock has radius of 0.5 m. The outermost point on its minute hand travels along the edge. What is its tangential speed?
   a. 9×10^{-4} m/s
   b. 3.4×10^{-3} m/s
   c. 8.5×10^{-4} m/s
   d. 1.3×10^{1} m/s

6.2 Uniform Circular Motion

18. What is the centripetal force exerted on a 1,600 kg car that rounds a 100 m radius curve at 12 m/s?
   a. 192 N
   b. 1, 111 N
   c. 2, 300 N

Performance Task

6.3 Rotational Motion

22. Design a lever arm capable of lifting a 0.5 kg object such as a stone. The force for lifting should be provided by

TEST PREP

Multiple Choice

6.1 Angle of Rotation and Angular Velocity

23. What is 1 radian approximately in degrees?
   a. $57.3°$
   b. $360°$
   c. $\pi°$
   d. $2\pi°$

24. If the following objects are spinning at the same angular velocities, the edge of which one would have the highest speed?
   a. Mini CD
   b. Regular CD
   c. Vinyl record

25. What are possible units for tangential velocity?
   a. m/s
   b. rad/s

   d. 13, 333 N

19. Find the frictional force between the tires and the road that allows a 1,000 kg car traveling at 30 m/s to round a 20 m radius curve.
   a. 22 N
   b. 667 N
   c. 1, 500 N
   d. 45, 000 N

6.3 Rotational Motion

20. An object's angular acceleration is 36 rad/s². If it were initially spinning with a velocity of 6.0 m/s, what would its angular velocity be after 5.0 s?
   a. 186 rad/s
   b. 190 rad/s²
   c. −174 rad/s
   d. −174 rad/s²

21. When a fan is switched on, it undergoes an angular acceleration of 150 rad/s². How long will it take to achieve its maximum angular velocity of 50 rad/s?
   a. −0.3 s
   b. 0.3 s
   c. 3.0 s

placing coins on the other end of the lever. How many coins would you need? What happens if you shorten or lengthen the lever arm? What does this say about torque?

   c. °/s

26. What is $30°$ in radians?
   a. $\frac{\pi}{12}$
   b. $\frac{\pi}{9}$
   c. $\frac{\pi}{6}$
   d. $\frac{\pi}{3}$

27. For a given object, what happens to the arc length as the angle of rotation increases?
   a. The arc length is directly proportional to the angle of rotation, so it increases with the angle of rotation.
   b. The arc length is inversely proportional to the angle of rotation, so it decreases with the angle of rotation.
   c. The arc length is directly proportional to the angle of rotation, so it decreases with the angle of rotation.

d.   The arc length is inversely proportional to the angle of rotation, so it increases with the angle of rotation.

6.2 Uniform Circular Motion

28. Which of these quantities is constant in uniform circular motion?
     a.   Speed
     b.   Velocity
     c.   Acceleration
     d.   Displacement

29. Which of these quantities impact centripetal force?
     a.   Mass and speed only
     b.   Mass and radius only
     c.   Speed and radius only
     d.   Mass, speed, and radius all impact centripetal force

30. An increase in the magnitude of which of these quantities causes a reduction in centripetal force?
     a.   Mass
     b.   Radius of curvature
     c.   Speed

31. What happens to centripetal acceleration as the radius of curvature decreases and the speed is constant, and why?
     a.   It increases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
     b.   It increases, because the centripetal acceleration is directly proportional to the radius of curvature.
     c.   It decreases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
     d.   It decreases, because the centripetal acceleration is directly proportional to the radius of the curvature.

32. Why do we experience more sideways acceleration while driving around sharper curves?

Short Answer

6.1 Angle of Rotation and Angular Velocity

37. What is the rotational analog of linear velocity?
     a.   Angular displacement
     b.   Angular velocity
     c.   Angular acceleration
     d.   Angular momentum

38. What is the rotational analog of distance?
     a.   Rotational angle
     b.   Torque
     c.   Angular velocity
     d.   Angular momentum

a.   Centripetal acceleration is inversely proportional to the radius of curvature, so it increases as the radius of curvature decreases.
b.   Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature decreases.
c.   Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature increases.
d.   Centripetal acceleration is directly proportional to the radius of curvature, so it increases as the radius of curvature increases.

6.3 Rotational Motion

33. Which of these quantities is not described by the kinematics of rotational motion?
     a.   Rotation angle
     b.   Angular acceleration
     c.   Centripetal force
     d.   Angular velocity

34. In the equation $\tau = rF\sin\theta$, what is F?
     a.   Linear force
     b.   Centripetal force
     c.   Angular force

35. What happens when two torques act equally in opposite directions?
     a.   Angular velocity is zero.
     b.   Angular acceleration is zero.

36. What is the mathematical relationship between angular and linear accelerations?
     a. $a = r\alpha$
     b. $a = \frac{\alpha}{r}$
     c. $a = r^2\alpha$
     d. $a = \frac{\alpha}{r^2}$

39. What is the equation that relates the linear speed of a point on a rotating object with the object's angular quantities?
     a. $v = \frac{\omega}{r}$
     b. $v = r\omega$
     c. $v = \frac{\alpha}{r}$
     d. $v = r\alpha$

40. As the angular velocity of an object increases, what happens to the linear velocity of a point on that object?
     a.   It increases, because linear velocity is directly proportional to angular velocity.
     b.   It increases, because linear velocity is inversely proportional to angular velocity.

c. It decreases because linear velocity is directly proportional to angular velocity.

d. It decreases because linear velocity is inversely proportional to angular velocity.

41. What is angular speed in terms of tangential speed and the radius?

a. $\omega = \frac{v^2}{r}$

b. $\omega = \frac{v}{r}$

c. $\omega = rv$

d. $\omega = rv^2$

42. Why are radians dimensionless?

a. Radians are dimensionless, because they are defined as a ratio of distances. They are defined as the ratio of the arc length to the radius of the circle.

b. Radians are dimensionless because they are defined as a ratio of distances. They are defined as the ratio of the area to the radius of the circle.

c. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the arc length to the radius of the circle.

d. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the area to the radius of the circle.

6.2 Uniform Circular Motion

43. What type of quantity is centripetal acceleration?

a. Scalar quantity; centripetal acceleration has magnitude only but no direction

b. Scalar quantity; centripetal acceleration has magnitude as well as direction

c. Vector quantity; centripetal acceleration has magnitude only but no direction

d. Vector quantity; centripetal acceleration has magnitude as well as direction

44. What are the standard units for centripetal acceleration?

a. m/s

b. m/s^2

c. m^2/s

d. m^2/s^2

45. What is the angle formed between the vectors of tangential velocity and centripetal force?

a. 0°

b. 30°

c. 90°

d. 180°

46. What is the angle formed between the vectors of centripetal acceleration and centripetal force?

a. 0°

b. 30°

c. 90°

d. 180°

47. What are the standard units for centripetal force?

a. m

b. m/s

c. m/s^2

d. newtons

48. As the mass of an object in uniform circular motion increases, what happens to the centripetal force required to keep it moving at the same speed?

a. It increases, because the centripetal force is directly proportional to the mass of the rotating body.

b. It increases, because the centripetal force is inversely proportional to the mass of the rotating body.

c. It decreases, because the centripetal force is directly proportional to the mass of the rotating body.

d. It decreases, because the centripetal force is inversely proportional to the mass of the rotating body.

6.3 Rotational Motion

49. The relationships between which variables are described by the kinematics of rotational motion?

a. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, and angular acceleration.

b. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and angular momentum.

c. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.

d. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, torque, and time.

50. What is the kinematics relationship between ω, α, and t?

a. $\omega = \alpha t$

b. $\omega = \omega_0 - \alpha t$

c. $\omega = \omega_0 + \alpha t$

d. $\omega = \omega_0 + \frac{1}{2}\alpha t$

51. What kind of quantity is torque?

a. Scalar

b. Vector

c. Dimensionless
d. Fundamental quantity

52. If a linear force is applied to a lever arm farther away from the pivot point, what happens to the resultant torque?
a. It decreases.
b. It increases.
c. It remains the same.
d. It changes the direction.

53. How can the same force applied to a lever produce different torques?
a. By applying the force at different points of the lever

Extended Response

6.1 Angle of Rotation and Angular Velocity

54. Consider two pits on a CD, one close to the center and one close to the outer edge. When the CD makes one full rotation, which pit would have gone through a greater angle of rotation? Which one would have covered a greater arc length?
a. The one close to the center would go through the greater angle of rotation. The one near the outer edge would trace a greater arc length.
b. The one close to the center would go through the greater angle of rotation. The one near the center would trace a greater arc length.
c. Both would go through the same angle of rotation. The one near the outer edge would trace a greater arc length.
d. Both would go through the same angle of rotation. The one near the center would trace a greater arc length.

55. Consider two pits on a CD, one close to the center and one close to the outer edge. For a given angular velocity of the CD, which pit has a higher angular velocity? Which has a higher tangential velocity?
a. The point near the center would have the greater angular velocity and the point near the outer edge would have the higher linear velocity.
b. The point near the edge would have the greater angular velocity and the point near the center would have the higher linear velocity.
c. Both have the same angular velocity and the point near the outer edge would have the higher linear velocity.
d. Both have the same angular velocity and the point near the center would have the higher linear velocity.

56. What happens to tangential velocity as the radius of an object increases provided the angular velocity remains

the same?
a. It increases because tangential velocity is directly proportional to the radius.
b. It increases because tangential velocity is inversely proportional to the radius.
c. It decreases because tangential velocity is directly proportional to the radius.
d. It decreases because tangential velocity is inversely proportional to the radius.

6.2 Uniform Circular Motion

57. Is an object in uniform circular motion accelerating? Why or why not?
a. Yes, because the velocity is not constant.
b. No, because the velocity is not constant.
c. Yes, because the velocity is constant.
d. No, because the velocity is constant.

58. An object is in uniform circular motion. Suppose the centripetal force was removed. In which direction would the object now travel?
a. In the direction of the centripetal force
b. In the direction opposite to the direction of the centripetal force
c. In the direction of the tangential velocity
d. In the direction opposite to the direction of the tangential velocity

59. An object undergoes uniform circular motion. If the radius of curvature and mass of the object are constant, what is the centripetal force proportional to?
a. $F_c \propto \frac{1}{v}$
b. $F_c \propto \frac{1}{v^2}$
c. $F_c \propto v$
d. $F_c \propto v^2$

6.3 Rotational Motion

60. Why do tornadoes produce more wind speed at the

arm along the length of the lever or by changing the angle between the lever arm and the applied force.
b. By applying the force at the same point of the lever arm along the length of the lever or by changing the angle between the lever arm and the applied force.
c. By applying the force at different points of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.
d. By applying the force at the same point of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.

bottom of the funnel?

a. Wind speed is greater at the bottom because rate of rotation increases as the radius increases.

b. Wind speed is greater at the bottom because rate of rotation increases as the radius decreases.

c. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.

d. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.

61. How can you maximize the torque applied to a given lever arm without applying more force?

a. The force should be applied perpendicularly to the lever arm as close as possible from the pivot point.

b. The force should be applied perpendicularly to the lever arm as far as possible from the pivot point.

c. The force should be applied parallel to the lever arm as far as possible from the pivot point.

d. The force should be applied parallel to the lever arm as close as possible from the pivot point.

62. When will an object continue spinning at the same angular velocity?

a. When net torque acting on it is zero

b. When net torque acting on it is non zero

c. When angular acceleration is positive

d. When angular acceleration is negative

CHAPTER 7
Newton's Law of Gravitation

Figure 7.1 Johannes Kepler (left) showed how the planets move, and Isaac Newton (right) discovered that gravitational force caused them to move that way. ((left) unknown, Public Domain; (right) Sir Godfrey Kneller, Public Domain)

Chapter Outline

INTRODUCTION What do a falling apple and the orbit of the moon have in common? You will learn in this chapter that each is caused by gravitational force. The motion of all celestial objects, in fact, is determined by the gravitational force, which depends on their mass and separation.

Johannes Kepler discovered three laws of planetary motion that all orbiting planets and moons follow. Years later, Isaac Newton found these laws useful in developing his law of universal gravitation. This law relates gravitational force to the masses of objects and the distance between them. Many years later still, Albert Einstein showed there was a little more to the gravitation story when he published his theory of general relativity.

7.1 Kepler's Laws of Planetary Motion

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Explain Kepler's three laws of planetary motion
- Apply Kepler's laws to calculate characteristics of orbits

Section Key Terms

aphelion	Copernican model	eccentricity
Kepler's laws of planetary motion	perihelion	Ptolemaic model

Concepts Related to Kepler's Laws of Planetary Motion

Examples of orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The moon's orbit around Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets around the sun are no less interesting. If we look farther, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force. The orbital motions of objects in our own solar system are simple enough to describe with a few fairly simple laws. The orbits of planets and moons satisfy the following two conditions:

- The mass of the orbiting object, m, is small compared to the mass of the object it orbits, M.
- The system is isolated from other massive objects.

Based on the motion of the planets about the sun, Kepler devised a set of three classical laws, called **Kepler's laws of planetary motion**, that describe the orbits of all bodies satisfying these two conditions:

1. The orbit of each planet around the sun is an ellipse with the sun at one focus.
2. Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times.
3. The ratio of the squares of the periods of any two planets about the sun is equal to the ratio of the cubes of their average distances from the sun.

These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630). He devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed. Let's look closer at each of these laws.

Kepler's First Law

The orbit of each planet about the sun is an ellipse with the sun at one focus, as shown in . The planet's closest approach to the sun is called **aphelion** and its farthest distance from the sun is called **perihelion**.

(a)   (b)

Aphelion (r_a)   Perihelion (r_p)
(c)
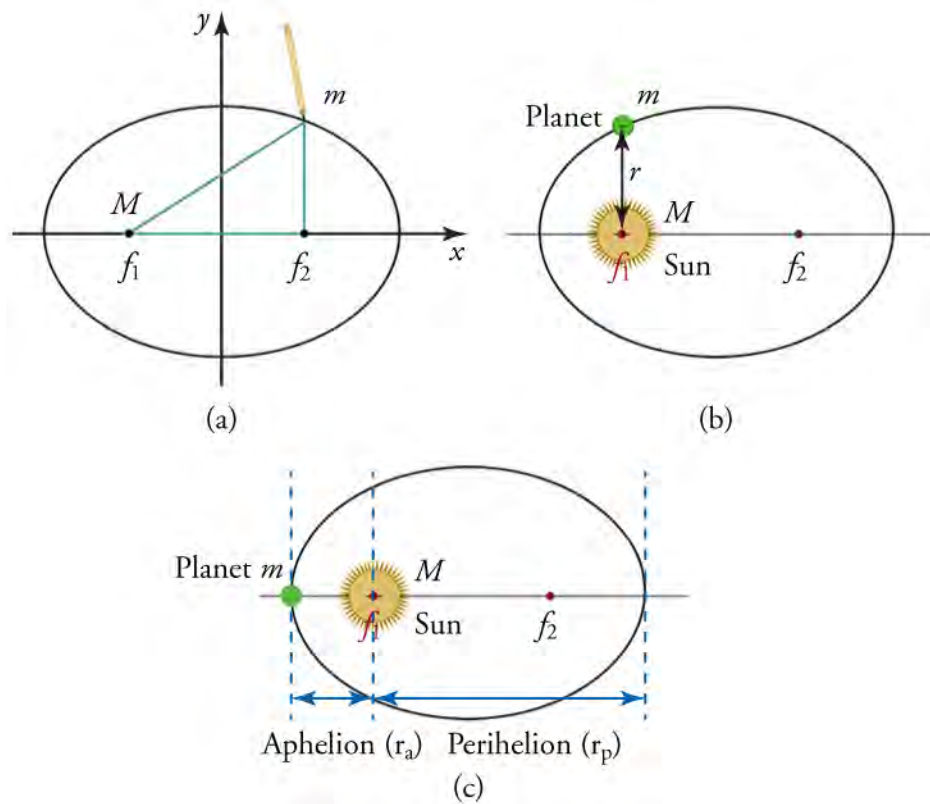
Figure 7.2 (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is constant. (b) For any closed orbit, m follows an elliptical path with M at one focus. (c) The aphelion (ra) is the closest distance between the planet and the sun, while the perihelion (rp) is the farthest distance from the sun.

If you know the aphelion (r_a) and perihelion (r_p) distances, then you can calculate the semi-major axis (a) and semi-minor axis (b).

$$a = \frac{(r_a + r_p)}{2}$$
$$b = \sqrt{r_a r_p}$$

Figure 7.3 You can draw an ellipse as shown by putting a pin at each focus, and then placing a loop of string around a pen and the pins and tracing a line on the paper.

Kepler's Second Law

Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times, as shown in Figure 7.4.
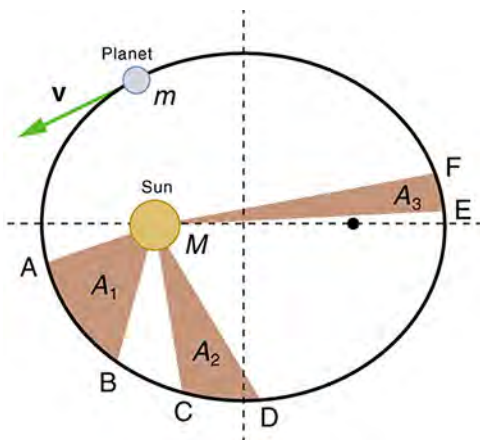
Figure 7.4 The shaded regions have equal areas. The time for m to go from A to B is the same as the time to go from C to D and from E to F. The mass m moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the sun, but it has broader validity.

TIPS FOR SUCCESS

Note that while, for historical reasons, Kepler's laws are stated for planets orbiting the sun, they are actually valid for all bodies satisfying the two previously stated conditions.

Kepler's Third Law

The ratio of the periods squared of any two planets around the sun is equal to the ratio of their average distances from the sun cubed. In equation form, this is

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

where T is the period (time for one orbit) and r is the average distance (also called orbital radius). This equation is valid only for comparing two small masses orbiting a single large mass. Most importantly, this is only a descriptive equation; it gives no information about the cause of the equality.

🔗 LINKS TO PHYSICS

History: Ptolemy vs. Copernicus

Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in Figure 7.5 **(a)**. This is called the **Ptolemaic model**, named for the Greek philosopher Ptolemy who lived in the second century AD. The Ptolemaic model is characterized by a list of facts for the motions of planets, with no explanation of cause and effect. There tended to be a different rule for each heavenly body and a general lack of simplicity.

Figure 7.5 **(b)** represents the modern or **Copernican model**. In this model, a small set of rules and a single underlying force explain not only all planetary motion in the solar system, but also all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling.
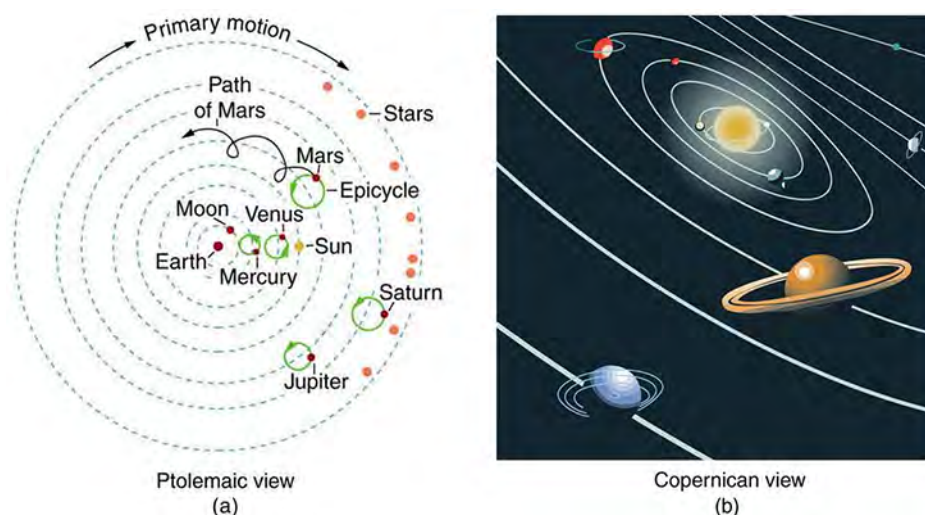
Figure 7.5 (a) The Ptolemaic model of the universe has Earth at the center with the moon, the planets, the sun, and the stars revolving about it in complex circular paths. This geocentric (Earth-centered) model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints about the causes of these motions. (b) The Copernican heliocentric (sun-centered) model is a simpler and more accurate model.

Nicolaus Copernicus (1473–1543) first had the idea that the planets circle the sun, in about 1514. It took him almost 20 years to work out the mathematical details for his model. He waited another 10 years or so to publish his work. It is thought he hesitated because he was afraid people would make fun of his theory. Actually, the reaction of many people was more one of fear and anger. Many people felt the Copernican model threatened their basic belief system. About 100 years later, the astronomer Galileo was put under house arrest for providing evidence that planets, including Earth, orbited the sun. In all, it took almost 300 years for everyone to admit that Copernicus had been right all along.

GRASP CHECK
Explain why Earth does actually appear to be the center of the solar system.
   a.   Earth appears to be the center of the solar system because Earth is at the center of the universe, and everything revolves around it in a circular orbit.
   b.   Earth appears to be the center of the solar system because, in the reference frame of Earth, the sun, moon, and planets all appear to move across the sky as if they were circling Earth.
   c.   Earth appears to be at the center of the solar system because Earth is at the center of the solar system and all the heavenly bodies revolve around it.
   d.   Earth appears to be at the center of the solar system because Earth is located at one of the foci of the elliptical orbit of the sun, moon, and other planets.

Virtual Physics

Acceleration
This simulation allows you to create your own solar system so that you can see how changing distances and masses determines the orbits of planets. Click *Help* for instructions.

Click to view content (https://archive.cnx.org/specials/ee816dff-0b5f-4e6f-8250-f9fb9e39d716/my-solar-system/)

GRASP CHECK
When the central object is off center, how does the speed of the orbiting object vary?
   a.   The orbiting object moves fastest when it is closest to the central object and slowest when it is farthest away.
   b.   The orbiting object moves slowest when it is closest to the central object and fastest when it is farthest away.

c. The orbiting object moves with the same speed at every point on the circumference of the elliptical orbit.

d. There is no relationship between the speed of the object and the location of the planet on the circumference of the orbit.

Calculations Related to Kepler's Laws of Planetary Motion

Kepler's First Law

Refer back to Figure 7.2 **(a)**. Notice which distances are constant. The foci are fixed, so distance $\overline{f_1 f_2}$ is a constant. The definition of an ellipse states that the sum of the distances $\overline{f_1 m} + \overline{mf_2}$ is also constant. These two facts taken together mean that the perimeter of triangle $\Delta f_1 mf_2$ must also be constant. Knowledge of these constants will help you determine positions and distances of objects in a system that includes one object orbiting another.

Kepler's Second Law

Refer back to Figure 7.4. The second law says that the segments have equal area and that it takes equal time to sweep through each segment. That is, the time it takes to travel from A to B equals the time it takes to travel from C to D, and so forth. Velocity **v** equals distance d divided by time t: $\mathbf{v} = d/t$. Then, $t = d/\mathbf{v}$, so distance divided by velocity is also a constant. For example, if we know the average velocity of Earth on June 21 and December 21, we can compare the distance Earth travels on those days.

The degree of elongation of an elliptical orbit is called its **eccentricity** (e). Eccentricity is calculated by dividing the distance f from the center of an ellipse to one of the foci by half the long axis a.

$$(e) = f/a$$

<div style="text-align:right">7.1</div>

When $e = 0$, the ellipse is a circle.

The area of an ellipse is given by $A = \pi ab$, where b is half the short axis. If you know the axes of Earth's orbit and the area Earth sweeps out in a given period of time, you can calculate the fraction of the year that has elapsed.

✳ WORKED EXAMPLE

Kepler's First Law

At its closest approach, a moon comes within 200,000 km of the planet it orbits. At that point, the moon is 300,000 km from the other focus of its orbit, f_2. The planet is focus f_1 of the moon's elliptical orbit. How far is the moon from the planet when it is 260,000 km from f_2?

Strategy

Show and label the ellipse that is the orbit in your solution. Picture the triangle $f_1 mf_2$ collapsed along the major axis and add up the lengths of the three sides. Find the length of the unknown side of the triangle when the moon is 260,000 km from f_2.

Solution

Perimeter of $f_1 mf_2 = 200,000 \text{ km} + 100,000 \text{ km} + 300,000 \text{ km} = 600,000 \text{ km}.$

$mf_1 = 600,000 \text{ km} - (100,000 \text{ km} + 200,000 \text{ km}) = 240,000 \text{ km}.$

Discussion

The perimeter of triangle $f_1 mf_2$ must be constant because the distance between the foci does not change and Kepler's first law says the orbit is an ellipse. For any ellipse, the sum of the two sides of the triangle, which are $f_1 m$ and mf_2, is constant.

✳ WORKED EXAMPLE

Kepler's Second Law

Figure 7.6 shows the major and minor axes of an ellipse. The semi-major and semi-minor axes are half of these, respectively.
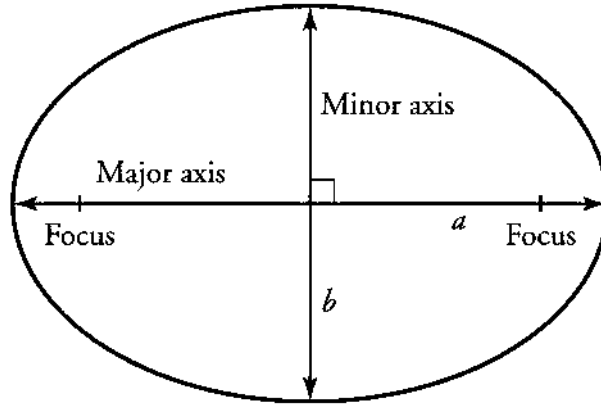
Figure 7.6   The major axis is the length of the ellipse, and the minor axis is the width of the ellipse. The semi-major axis is half the major axis, and the semi-minor axis is half the minor axis.

Earth's orbit is slightly elliptical, with a semi-major axis of 152 million km and a semi-minor axis of 147 million km. If Earth's period is 365.26 days, what area does an Earth-to-sun line sweep past in one day?

Strategy

Each day, Earth sweeps past an equal-sized area, so we divide the total area by the number of days in a year to find the area swept past in one day. For total area use $A = \pi a b$. Calculate A, the area inside Earth's orbit and divide by the number of days in a year (i.e., its period).

Solution

$$
\begin{aligned}
\text{area per day} &= \frac{\text{total area}}{\text{total number of days}} \\
&= \frac{\pi a b}{365 \text{ d}} \\
&= \frac{\pi (1.47 \times 10^8 \text{ km})(1.52 \times 10^3 \text{ km})}{365 \text{ d}} \\
&= 1.92 \times 10^{14} \text{ km}^2/\text{d}
\end{aligned}
$$

7.2

The area swept out in one day is thus $1.92 \times 10^{14} \text{ km}^2$.

Discussion

The answer is based on Kepler's law, which states that a line from a planet to the sun sweeps out equal areas in equal times.

Kepler's Third Law

Kepler's third law states that the ratio of the squares of the periods of any two planets (T_1, T_2) is equal to the ratio of the cubes of their average orbital distance from the sun (r_1, r_2). Mathematically, this is represented by

$$
\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}.
$$

From this equation, it follows that the ratio r^3/T^2 is the same for all planets in the solar system. Later we will see how the work of Newton leads to a value for this constant.

✳ WORKED EXAMPLE

Kepler's Third Law

Given that the moon orbits Earth each 27.3 days and that it is an average distance of $3.84 \times 10^8 \text{ m}$ from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1,500 km above Earth's surface.

Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form by

$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$. Let us use the subscript 1 for the moon and the subscript 2 for the satellite. We are asked to find T_2. The given information tells us that the orbital radius of the moon is $r_1 = 3.84 \times 10^8 \, \text{m}$, and that the period of the moon is $T_1 = 27.3 \, \text{days}$. The height of the artificial satellite above Earth's surface is given, so to get the distance r_2 from the center of Earth we must add the height to the radius of Earth (6380 km). This gives $r_2 = 1500 \, \text{km} + 6380 \, \text{km} = 7880 \, \text{km}$. Now all quantities are known, so T_2 can be found.

Solution

To solve for T_2, we cross-multiply and take the square root, yielding

$$T_2^2 = T_1^2 \left(\frac{r_2}{r_1} \right)^3 ; \; T_2 = T_1 \left(\frac{r_2}{r_1} \right)^{\frac{3}{2}}$$

$$T_2 = (27.3 \, \text{d}) \left(\tfrac{24.0 \, \text{h}}{\text{d}} \right) \left(\tfrac{7880 \, \text{km}}{3.84 \times 10^5 \, \text{km}} \right)^{\frac{3}{2}} = 1.93 \, \text{h}.$$

7.3

Discussion

This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will complete one orbit in the same amount of time.

Practice Problems

1. A planet with no axial tilt is located in another solar system. It circles its sun in a very elliptical orbit so that the temperature varies greatly throughout the year. If the year there has 612 days and the inhabitants celebrate the coldest day on day 1 of their calendar, when is the warmest day?
   a. Day 1
   b. Day 153
   c. Day 306
   d. Day 459

2. A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). The ratio $\frac{r^3}{T^2}$ for the moon is $1.01 \times 10^{18} \, \frac{\text{km}^3}{\text{y}^2}$. Calculate the radius of the orbit of such a satellite.
   a. $2.75 \times 10^3 \, \text{km}$
   b. $1.96 \times 10^4 \, \text{km}$
   c. $1.40 \times 10^5 \, \text{km}$
   d. $1.00 \times 10^6 \, \text{km}$

Check Your Understanding

3. Are Kepler's laws purely descriptive, or do they contain causal information?
   a. Kepler's laws are purely descriptive.
   b. Kepler's laws are purely causal.
   c. Kepler's laws are descriptive as well as causal.
   d. Kepler's laws are neither descriptive nor causal.

4. True or false—According to Kepler's laws of planetary motion, a satellite increases its speed as it approaches its parent body and decreases its speed as it moves away from the parent body.
   a. True
   b. False

5. Identify the locations of the foci of an elliptical orbit.
   a. One focus is the parent body, and the other is located at the opposite end of the ellipse, at the same distance from the center as the parent body.
   b. One focus is the parent body, and the other is located at the opposite end of the ellipse, at half the distance from the center as the parent body.

c. One focus is the parent body and the other is located outside of the elliptical orbit, on the line on which is the semi-major axis of the ellipse.

d. One focus is on the line containing the semi-major axis of the ellipse, and the other is located anywhere on the elliptical orbit of the satellite.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain Newton's law of universal gravitation and compare it to Einstein's theory of general relativity
- Perform calculations using Newton's law of universal gravitation

Section Key Terms

Einstein's theory of general relativity   gravitational constant   Newton's universal law of gravitation

Concepts Related to Newton's Law of Universal Gravitation

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure 7.7. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner, Galileo Galilei, had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph. It had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose an explanation of the mechanism that caused them to follow these paths and not others.

Figure 7.7 The popular legend that Newton suddenly discovered the law of universal gravitation when an apple fell from a tree and hit him on the head has an element of truth in it. A more probable account is that he was walking through an orchard and wondered why all the apples fell in the same direction with the same acceleration. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance

between them. Expressed in modern language, **Newton's universal law of gravitation** states that every object in the universe attracts every other object with a force that is directed along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This attraction is illustrated by .
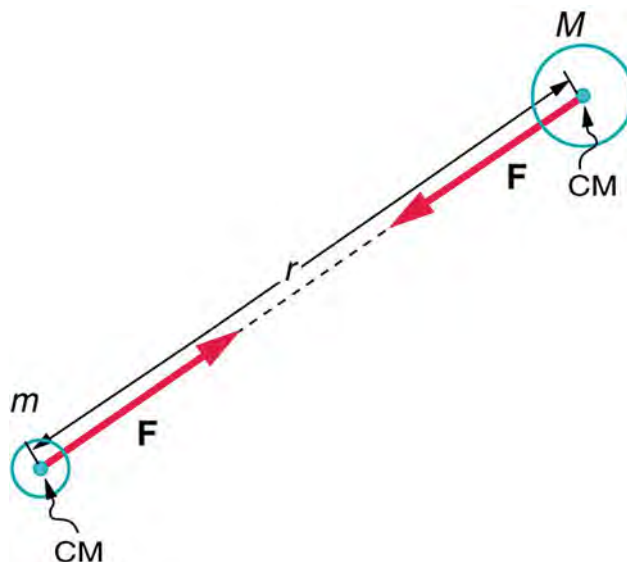
Figure 7.8 Gravitational attraction is along a line joining the centers of mass (CM) of the two bodies. The magnitude of the force on each body is the same, consistent with Newton's third law (action-reaction).

For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is

$$\mathbf{F} = G\frac{mM}{r^2}$$

where $\mathbf{F}$ is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal constant, meaning that it is thought to be the same everywhere in the universe. It has been measured experimentally to be $G = 6.673 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$.

If a person has a mass of 60.0 kg, what would be the force of gravitational attraction on him at Earth's surface? G is given above, Earth's mass M is 5.97×10^{24} kg, and the radius r of Earth is 6.38×10^6 m. Putting these values into Newton's universal law of gravitation gives

$$\mathbf{F} = G\frac{mM}{r^2} = \left(6.673 \times 10^{-11}\,\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\left(\frac{(60.0\ \text{kg})\,(5.97 \times 10^{24}\ \text{kg})}{(6.38 \times 10^6\ \text{m})^2}\right) = 584\ \text{N}$$

We can check this result with the relationship: $\mathbf{F} = m\mathbf{g} = (60\ \text{kg})(9.8\text{m/s}^2) = 588\ \text{N}$

You may remember that $\mathbf{g}$, the acceleration due to gravity, is another important constant related to gravity. By substituting $\mathbf{g}$ for $\mathbf{a}$ in the equation for Newton's second law of motion we get $\mathbf{F} = m\mathbf{g}$. Combining this with the equation for universal gravitation gives

$$m\mathbf{g} = G\frac{mM}{r^2}$$

Cancelling the mass m on both sides of the equation and filling in the values for the gravitational constant and mass and radius of the Earth, gives the value of g, which may look familiar.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\,\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\left(\frac{5.98 \times 10^{24}\text{kg}}{(6.38 \times 10^6\text{m})^2}\right) = 9.80\text{m/s}^2$$

This is a good point to recall the difference between mass and weight. Mass is the amount of matter in an object; weight is the

force of attraction between the mass within two objects. Weight can change because g is different on every moon and planet. An object's mass m does not change but its weight $m\mathbf{g}$ can.

Virtual Physics

Gravity and Orbits

Move the sun, Earth, moon and space station in this simulation to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies. Turn off gravity to see what would happen without it!

[Click to view content (https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)](https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)

GRASP CHECK
Why doesn't the Moon travel in a smooth circle around the Sun?
a. The Moon is not affected by the gravitational field of the Sun.
b. The Moon is not affected by the gravitational field of the Earth.
c. The Moon is affected by the gravitational fields of both the Earth and the Sun, which are always additive.
d. The moon is affected by the gravitational fields of both the Earth and the Sun, which are sometimes additive and sometimes opposite.

Snap Lab

Take-Home Experiment: Falling Objects

In this activity you will study the effects of mass and air resistance on the acceleration of falling objects. Make predictions (hypotheses) about the outcome of this experiment. Write them down to compare later with results.

- Four sheets of $8\text{-}1/2 \times 11$ -inch paper

Procedure
- Take four identical pieces of paper.
  - Crumple one up into a small ball.
  - Leave one uncrumpled.
  - Take the other two and crumple them up together, so that they make a ball of exactly twice the mass of the other crumpled ball.
  - Now compare which ball of paper lands first when dropped simultaneously from the same height.
    1. Compare crumpled one-paper ball with crumpled two-paper ball.
    2. Compare crumpled one-paper ball with uncrumpled paper.

GRASP CHECK
Why do some objects fall faster than others near the surface of the earth if all mass is attracted equally by the force of gravity?
a. Some objects fall faster because of air resistance, which acts in the direction of the motion of the object and exerts more force on objects with less surface area.
b. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with less surface area.
c. Some objects fall faster because of air resistance, which acts in the direction of motion of the object and exerts more force on objects with more surface area.
d. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with more surface area.

It is possible to derive Kepler's third law from Newton's law of universal gravitation. Applying Newton's second law of motion to

angular motion gives an expression for centripetal force, which can be equated to the expression for force in the universal gravitation equation. This expression can be manipulated to produce the equation for Kepler's third law. We saw earlier that the expression r^3/T^2 is a constant for satellites orbiting the same massive object. The derivation of Kepler's third law from Newton's law of universal gravitation and Newton's second law of motion yields that constant:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

where M is the mass of the central body about which the satellites orbit (for example, the sun in our solar system). The usefulness of this equation will be seen later.

The universal gravitational constant G is determined experimentally. This definition was first done accurately in 1798 by English scientist Henry Cavendish (1731–1810), more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most) by using an apparatus like that in Figure 7.9. Remarkably, his value for G differs by less than 1% from the modern value.
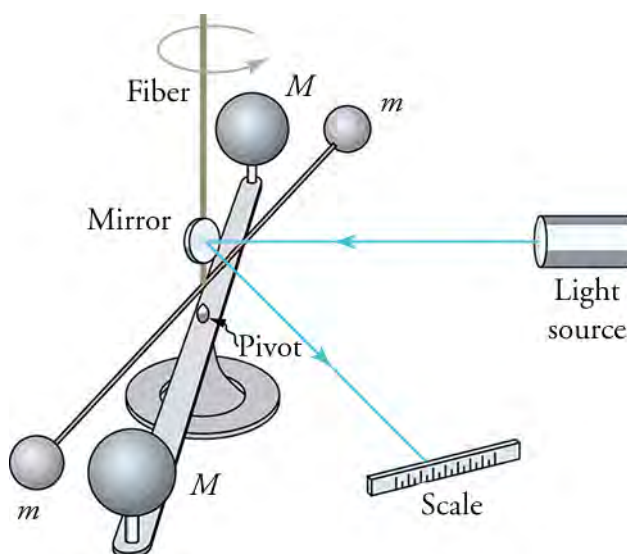
Figure 7.9 Cavendish used an apparatus like this to measure the gravitational attraction between two suspended spheres (*m*) and two spheres on a stand (*M*) by observing the amount of torsion (twisting) created in the fiber. The distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Einstein's Theory of General Relativity

Einstein's theory of general relativity explained some interesting properties of gravity not covered by Newton's theory. Einstein based his theory on the postulate that acceleration and gravity have the same effect and cannot be distinguished from each other. He concluded that light must fall in both a gravitational field and in an accelerating reference frame. Figure 7.10 shows this effect (greatly exaggerated) in an accelerating elevator. In Figure 7.10(a), the elevator accelerates upward in zero gravity. In Figure 7.10(b), the room is not accelerating but is subject to gravity. The effect on light is the same: it "falls" downward in both situations. The person in the elevator cannot tell whether the elevator is accelerating in zero gravity or is stationary and subject to gravity. Thus, gravity affects the path of light, even though we think of gravity as acting between masses, while photons are massless.
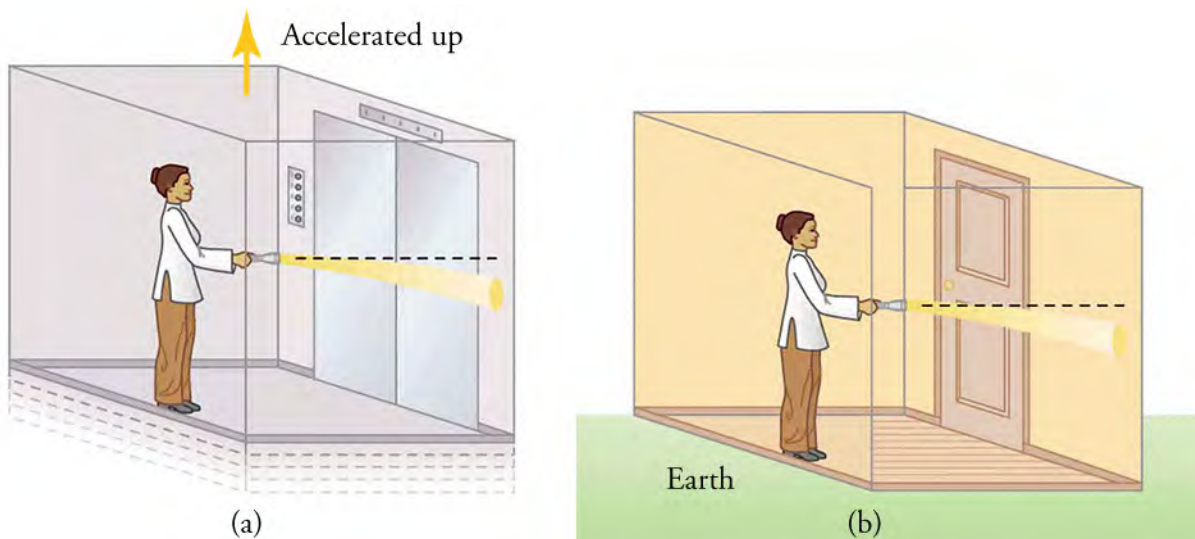
Figure 7.10 (a) A beam of light emerges from a flashlight in an upward-accelerating elevator. Since the elevator moves up during the time the light takes to reach the wall, the beam strikes lower than it would if the elevator were not accelerated. (b) Gravity must have the same effect on light, since it is not possible to tell whether the elevator is accelerating upward or is stationary and acted upon by gravity.

Einstein's theory of general relativity got its first verification in 1919 when starlight passing near the sun was observed during a solar eclipse. (See Figure 7.11.) During an eclipse, the sky is darkened and we can briefly see stars. Those on a line of sight nearest the sun should have a shift in their apparent positions. Not only was this shift observed, but it agreed with Einstein's predictions well within experimental uncertainties. This discovery created a scientific and public sensation. Einstein was now a folk hero as well as a very great scientist. The bending of light by matter is equivalent to a bending of space itself, with light following the curve. This is another radical change in our concept of space and time. It is also another connection that any particle with mass *or* energy (e.g., massless photons) is affected by gravity.
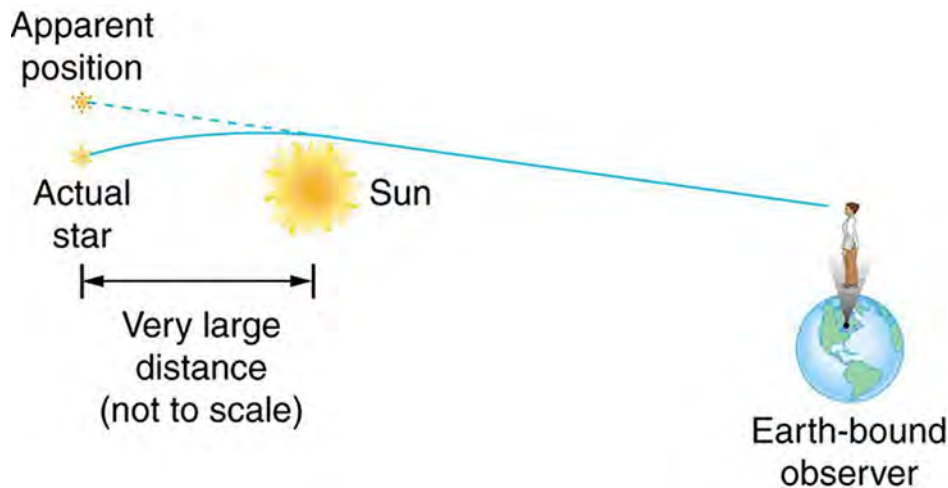
Figure 7.11 This schematic shows how light passing near a massive body like the sun is curved toward it. The light that reaches the Earth then seems to be coming from different locations than the known positions of the originating stars. Not only was this effect observed, but the amount of bending was precisely what Einstein predicted in his general theory of relativity.

To summarize the two views of gravity, Newton envisioned gravity as a tug of war along the line connecting any two objects in the universe. In contrast, Einstein envisioned gravity as a bending of space-time by mass.

🔬 **BOUNDLESS PHYSICS**

NASA gravity probe B

NASA's Gravity Probe B (GP-B) mission has confirmed two key predictions derived from Albert Einstein's general theory of relativity. The probe, shown in <u>Figure 7.12</u> was launched in 2004. It carried four ultra-precise gyroscopes designed to measure two effects hypothesized by Einstein's theory:

- The geodetic effect, which is the warping of space and time by the gravitational field of a massive body (in this case, Earth)
- The frame-dragging effect, which is the amount by which a spinning object pulls space and time with it as it rotates

Figure 7.12 Artist concept of Gravity Probe B spacecraft in orbit around the Earth. (credit: NASA/MSFC)

Both effects were measured with unprecedented precision. This was done by pointing the gyroscopes at a single star while orbiting Earth in a polar orbit. As predicted by relativity theory, the gyroscopes experienced very small, but measureable, changes in the direction of their spin caused by the pull of Earth's gravity.

The principle investigator suggested imagining Earth spinning in honey. As Earth rotates it drags space and time with it as it would a surrounding sea of honey.

GRASP CHECK

According to the general theory of relativity, a gravitational field bends light. What does this have to do with time and space?
a. Gravity has no effect on the space-time continuum, and gravity only affects the motion of light.
b. The space-time continuum is distorted by gravity, and gravity has no effect on the motion of light.
c. Gravity has no effect on either the space-time continuum or on the motion of light.
d. The space-time continuum is distorted by gravity, and gravity affects the motion of light.

Calculations Based on Newton's Law of Universal Gravitation

TIPS FOR SUCCESS

When performing calculations using the equations in this chapter, use units of kilograms for mass, meters for distances, newtons for force, and seconds for time.

The mass of an object is constant, but its weight varies with the strength of the gravitational field. This means the value of $\mathbf{g}$ varies from place to place in the universe. The relationship between force, mass, and acceleration from the second law of motion can be written in terms of $\mathbf{g}$.

$$\mathbf{F} = m\mathbf{a} = m\mathbf{g}$$

In this case, the force is the weight of the object, which is caused by the gravitational attraction of the planet or moon on which the object is located. We can use this expression to compare weights of an object on different moons and planets.

👁 WATCH PHYSICS

Mass and Weight Clarification

This video shows the mathematical basis of the relationship between mass and weight. The distinction between mass and weight are clearly explained. The mathematical relationship between mass and weight are shown mathematically in terms of the equation for Newton's law of universal gravitation and in terms of his second law of motion.

Click to view content (https://www.khanacademy.org/embed_video?v=IuBoeDihLUc)

> **GRASP CHECK**
> Would you have the same mass on the moon as you do on Earth? Would you have the same weight?
> a. You would weigh more on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
> b. You would weigh less on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.
> c. You would weigh less on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
> d. You would weigh more on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.

Two equations involving the gravitational constant, G, are often useful. The first is Newton's equation, $\mathbf{F} = G\frac{mM}{r^2}$. Several of the values in this equation are either constants or easily obtainable. $\mathbf{F}$ is often the weight of an object on the surface of a large object with mass M, which is usually known. The mass of the smaller object, m, is often known, and G is a universal constant with the same value anywhere in the universe. This equation can be used to solve problems involving an object on or orbiting Earth or other massive celestial object. Sometimes it is helpful to equate the right-hand side of the equation to $m\mathbf{g}$ and cancel the m on both sides.

The equation $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ is also useful for problems involving objects in orbit. Note that there is no need to know the mass of the object. Often, we know the radius r or the period T and want to find the other. If these are both known, we can use the equation to calculate the mass of a planet or star.

👁 WATCH PHYSICS

Mass and Weight Clarification

This video demonstrates calculations involving Newton's universal law of gravitation.

Click to view content (https://www.khanacademy.org/embed_video?v=391txUI76gM)

> **GRASP CHECK**
> Identify the constants g and G.
> a. g and G are both the acceleration due to gravity
> b. g is acceleration due to gravity on Earth and G is the universal gravitational constant.
> c. g is the gravitational constant and G is the acceleration due to gravity on Earth.
> d. g and G are both the universal gravitational constant.

✳ WORKED EXAMPLE

Change in g

The value of $\mathbf{g}$ on the planet Mars is 3.71 m/s². If you have a mass of 60.0 kg on Earth, what would be your mass on Mars? What would be your weight on Mars?

Strategy

Weight equals acceleration due to gravity times mass: $\mathbf{W} = m\mathbf{g}$. An object's mass is constant. Call acceleration due to gravity on Mars $\mathbf{g}_M$ and weight on Mars $\mathbf{W}_M$.

Solution

Mass on Mars would be the same, 60 kg.

$$\mathbf{W}_M = m\mathbf{g}_M = (60.0 \text{ kg})\left(3.71\text{m/s}^2\right) = 223 \text{ N} \qquad \boxed{7.4}$$

Discussion

The value of **g** on any planet depends on the mass of the planet and the distance from its center. If the material below the surface varies from point to point, the value of **g** will also vary slightly.

✳ WORKED EXAMPLE

Earth's g at the Moon

Find the acceleration due to Earth's gravity at the distance of the moon.

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \text{ /kg}^2$$

$$\text{Earth-moon distance} = 3.84 \times 10^8 \text{ m} \qquad \boxed{7.5}$$

$$\text{Earth's mass} = 5.98 \times 10^{24} \text{ kg}$$

Express the force of gravity in terms of *g*.

$$\text{F} = \text{W} = ma = mg \qquad \boxed{7.6}$$

Combine with the equation for universal gravitation.

$$m\mathbf{g} = mG\frac{M}{r^2} \qquad \boxed{7.7}$$

Solution

Cancel *m* and substitute.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\left(\frac{5.98 \times 10^{24} \text{ kg}}{\left(3.84 \times 10^8 \text{ m}\right)^2}\right) = 2.70 \times 10^{-3}\text{m/s}^2 \qquad \boxed{7.8}$$

Discussion

The value of **g** for the moon is 1.62 m/s². Comparing this value to the answer, we see that Earth's gravitational influence on an object on the moon's surface would be insignificant.

Practice Problems

6. What is the mass of a person who weighs 600 N?
   a. 6.00 kg
   b. 61.2 kg
   c. 600 kg
   d. 610 kg

7. Calculate Earth's mass given that the acceleration due to gravity at the North Pole is 9.830m/s² and the radius of the Earth is 6371 km from pole to center.
   a. 5.94×10^{17} kg
   b. 5.94×10^{24} kg
   c. 9.36×10^{17} kg
   d. 9.36×10^{24} kg

Check Your Understanding

8. Some of Newton's predecessors and contemporaries also studied gravity and proposed theories. What important advance did Newton make in the study of gravity that the other scientists had failed to do?
   a. He gave an exact mathematical form for the theory.

b. He added a correction term to a previously existing formula.
c. Newton found the value of the universal gravitational constant.
d. Newton showed that gravitational force is always attractive.

9. State the law of universal gravitation in words only.
    a. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the square of the distance between them.
    b. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
    c. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the distance between them.
    d. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the distance between them.

10. Newton's law of universal gravitation explains the paths of what?
    a. A charged particle
    b. A ball rolling on a plane surface
    c. A planet moving around the sun
    d. A stone tied to a string and whirled at constant speed in a horizontal circle

KEY TERMS

aphelion   closest distance between a planet and the sun (called apoapsis for other celestial bodies)

Copernican model   the model of the solar system where the sun is at the center of the solar system and all the planets orbit around it; this is also called the heliocentric model

eccentricity   a measure of the separation of the foci of an ellipse

Einstein's theory of general relativity   the theory that gravitational force results from the bending of spacetime by an object's mass

gravitational constant   the proportionality constant in Newton's law of universal gravitation

Kepler's laws of planetary motion   three laws derived by Johannes Kepler that describe the properties of all orbiting satellites

Newton's universal law of gravitation   states that gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

perihelion   farthest distance between a planet and the sun (called periapsis for other celestial bodies)

Ptolemaic model   the model of the solar system where Earth is at the center of the solar system and the sun and all the planets orbit around it; this is also called the geocentric model

SECTION SUMMARY

7.1 Kepler's Laws of Planetary Motion

- All satellites follow elliptical orbits.
- The line from the satellite to the parent body sweeps out equal areas in equal time.
- The radius cubed divided by the period squared is a constant for all satellites orbiting the same parent body.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

- Newton's law of universal gravitation provides a mathematical basis for gravitational force and Kepler's laws of planetary motion.
- Einstein's theory of general relativity shows that gravitational fields change the path of light and warp space and time.
- An object's mass is constant, but its weight changes when acceleration due to gravity, **g**, changes.

KEY EQUATIONS

7.1 Kepler's Laws of Planetary Motion

Kepler's third law	$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$
eccentricity	$e = \frac{f}{a}$
area of an ellipse	$A = \pi a b$
semi-major axis of an ellipse	$a = \left(r_a + r_p\right)/2$
semi-minor axis of an ellipse	$b = \sqrt{r_a r_p}$

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

Newton's second law of motion	$\mathbf{F} = m\mathbf{a} = m\mathbf{g}$
Newton's universal law of gravitation	$\mathbf{F} = G\frac{mM}{r^2}$
acceleration due to gravity	$\mathbf{g} = G\frac{M}{r^2}$
constant for satellites orbiting the same massive object	$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

CHAPTER REVIEW

Concept Items

7.1 Kepler's Laws of Planetary Motion

1.   A circle is a special case of an ellipse. Explain how a circle is different from other ellipses.

   a.   The foci of a circle are at the same point and are located at the center of the circle.

   b.   The foci of a circle are at the same point and are

located at the circumference of the circle.

   c. The foci of a circle are at the same point and are located outside of the circle.

   d. The foci of a circle are at the same point and are located anywhere on the diameter, except on its midpoint.

2. Comets have very elongated elliptical orbits with the sun at one focus. Using Kepler's Law, explain why a comet travels much faster near the sun than it does at the other end of the orbit.

   a. Because the satellite sweeps out equal areas in equal times

   b. Because the satellite sweeps out unequal areas in equal times

   c. Because the satellite is at the other focus of the ellipse

   d. Because the square of the period of the satellite is proportional to the cube of its average distance from the sun

3. True or False—A planet-satellite system must be isolated from other massive objects to follow Kepler's laws of planetary motion.

   a. True

   b. False

4. Explain why the string, pins, and pencil method works for drawing an ellipse.

   a. The string, pins, and pencil method works because the length of the two sides of the triangle remains constant as you are drawing the ellipse.

   b. The string, pins, and pencil method works because the area of the triangle remains constant as you are drawing the ellipse.

   c. The string, pins, and pencil method works because the perimeter of the triangle remains constant as you are drawing the ellipse.

   d. The string, pins, and pencil method works because the volume of the triangle remains constant as you are drawing the ellipse.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

5. Describe the postulate on which Einstein based the theory of general relativity and describe an everyday experience that illustrates this postulate.

   a. Gravity and velocity have the same effect and cannot be distinguished from each other. An acceptable illustration of this is any description of the feeling of constant velocity in a situation where no outside frame of reference is considered.

   b. Gravity and velocity have different effects and can be distinguished from each other. An acceptable

illustration of this is any description of the feeling of constant velocity in a situation where no outside frame of reference is considered.

   c. Gravity and acceleration have the same effect and cannot be distinguished from each other. An acceptable illustration of this is any description of the feeling of acceleration in a situation where no outside frame of reference is considered.

   d. Gravity and acceleration have different effects and can be distinguished from each other. An acceptable illustration of this is any description of the feeling of acceleration in a situation where no outside frame of reference is considered.

6. Titan, with a radius of 2.58×10^6 m, is the largest moon of the planet Saturn. If the mass of Titan is 1.35×10^{23} kg, what is the acceleration due to gravity on the surface of this moon?

   a. 1.35 m/s^2

   b. 3.49 m/s^2

   c. 3.49×10^6 m/s^2

   d. 1.35×10^6 m/s^2

7. Saturn's moon Titan has an orbital period of 15.9 days. If Saturn has a mass of 5.68×10^{23} kg, what is the average distance from Titan to the center of Saturn?

   a. 1.22×10^6 m

   b. 4.26×10^7 m

   c. 5.25×10^4 km

   d. 4.26×10^{10} km

8. Explain why doubling the mass of an object doubles its weight, but doubling its distance from the center of Earth reduces its weight fourfold.

   a. The weight is two times the gravitational force between the object and Earth.

   b. The weight is half the gravitational force between the object and Earth.

   c. The weight is equal to the gravitational force between the object and Earth, and the gravitational force is inversely proportional to the distance squared between the object and Earth.

   d. The weight is directly proportional to the square of the gravitational force between the object and Earth.

9. Explain why a star on the other side of the Sun might appear to be in a location that is not its true location.

   a. It can be explained by using the concept of atmospheric refraction.

   b. It can be explained by using the concept of the special theory of relativity.

   c. It can be explained by using the concept of the general theory of relativity.

   d. It can be explained by using the concept of light

scattering in the atmosphere.

10. The Cavendish experiment marked a milestone in the study of gravity.
Part A. What important value did the experiment determine?
Part B. Why was this so difficult in terms of the masses used in the apparatus and the strength of the gravitational force?

   a. Part A. The experiment measured the acceleration due to gravity, g. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between very massive objects.

   b. Part A. The experiment measured the gravitational constant, G. Part B. Gravity is a very weak force but, despite this limitation, Cavendish was able to measure the attraction between very massive objects.

   c. Part A. The experiment measured the acceleration due to gravity, g. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between less massive objects.

   d. Part A. The experiment measured the gravitational constant, G. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between less massive objects.

Critical Thinking Items

7.1 Kepler's Laws of Planetary Motion

11. In the figure, the time it takes for the planet to go from A to B, C to D, and E to F is the same.
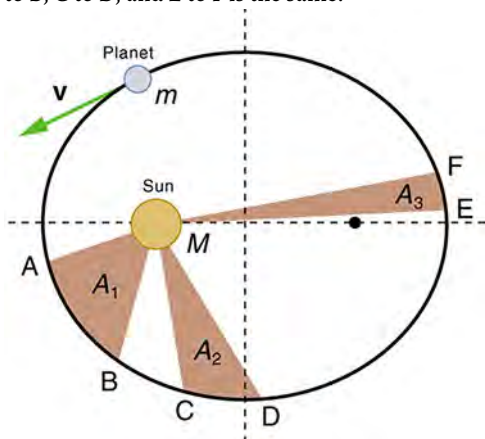
Compare the areas A_1, A_2, and A_3 in terms of size.
   a. $A_1 \neq A_2 \neq A_3$
   b. $A_1 = A_2 = A_3$
   c. $A_1 = A_2 > A_3$
   d. $A_1 > A_2 = A_3$

12. A moon orbits a planet in an elliptical orbit. The foci of the ellipse are 50, 000 km apart. The closest approach of the moon to the planet is 400, 000 km. What is the length of the major axis of the orbit?
   a. 400, 000 km
   b. 450, 000, km
   c. 800, 000 km
   d. 850, 000 km

13. In this figure, if f_1 represents the parent body, which set of statements holds true?
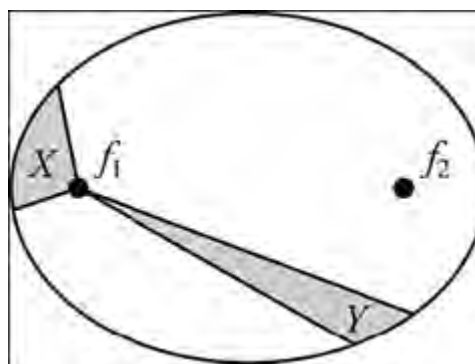
   a. Area X < Area Y; the speed is greater for area X.
   b. Area X > Area Y; the speed is greater for area Y.
   c. Area X = Area Y; the speed is greater for area X.
   d. Area X = Area Y; the speed is greater for area Y.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

14. Rhea, with a radius of 7.63×10^5 m, is the second-largest moon of the planet Saturn. If the mass of Rhea is 2.31×10^{21} kg, what is the acceleration due to gravity on the surface of this moon?
   a. 2.65×10^{-1} m/s
   b. 2.02×10^5 m/s
   c. 2.65×10^{-1} m/s^2
   d. 2.02×10^5 m/s^2

15. Earth has a mass of 5.971×10^{24} kg and a radius of 6.371×10^6 m. Use the data to check the value of the gravitational constant.
   a. $6.66 \times 10^{-11} \ \frac{\text{N} \cdot \text{m}}{\text{kg}^2}$, it matches the value of the gravitational constant G.
   b. $1.05 \times 10^{-17} \ \frac{\text{N} \cdot \text{m}}{\text{kg}^2}$, it matches the value of the gravitational constant G.
   c. $6.66 \times 10^{-11} \ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$, it matches the value of the gravitational constant G.

d. $1.05 \times 10^{-17} \ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$, it matches the value of the gravitational constant G.

16. The orbit of the planet Mercury has a period of 88.0 days and an average radius of 5.791×10^{10} m. What is the mass of the sun?

a. 3.43×10^{19} kg
b. 1.99×10^{30} kg
c. 2.56×10^{29} kg
d. 1.48×10^{40} kg

Problems

7.1 Kepler's Laws of Planetary Motion

17. The closest Earth comes to the sun is 1.47×10^8 km, and Earth's farthest distance from the sun is 1.52×10^8 km. What is the area inside Earth's orbit?
    a. 2.23×10^{16} km^2
    b. 6.79×10^{16} km^2
    c. 7.02×10^{16} km^2
    d. 7.26×10^{16} km^2

18. Earth is 1.496×10^8 km from the sun, and Neptune is 4.490×10^9 km from the sun. What best represents the number of Earth years it takes for Neptune to complete one orbit around the sun?
    a. 10 years
    b. 30 years
    c. 160 years
    d. 900 years

Performance Task

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

19. Design an experiment to test whether magnetic force is inversely proportional to the square of distance. Gravitational, magnetic, and electrical fields all act at a distance, but do they all follow the inverse square law? One difference in the forces related to these fields is that gravity is only attractive, but the other two can repel as well. In general, the inverse square law says that force F equals a constant C divided by the distance between objects, d, squared: $F = C/d^2$.
    Incorporate these materials into your design:

- Two strong, permanent bar magnets
- A spring scale that can measure small forces
- A short ruler calibrated in millimeters

Use the magnets to study the relationship between attractive force and distance.
    a. What will be the independent variable?
    b. What will be the dependent variable?
    c. How will you measure each of these variables?
    d. If you plot the independent variable versus the dependent variable and the inverse square law is upheld, will the plot be a straight line? Explain.
    e. Which plot would be a straight line if the inverse square law were upheld?

TEST PREP

Multiple Choice

7.1 Kepler's Laws of Planetary Motion

20. A planet of mass m circles a sun of mass M. Which distance changes throughout the planet's orbit?
    a. $\overline{f_1 f_2}$
    b. $\overline{mM}$
    c. $\overline{M f_2}$
    d. $\overline{M f_1}$

21. The focal point of the elliptical orbit of a moon is $50,000$ km from the center of the orbit. If the eccentricity of the orbit is 0.25, what is the length of the semi-major axis?
    a. $12,500$ km
    b. $100,000$ km
    c. $200,000$ km
    d. $400,000$ km

22. An artificial satellite orbits the Earth at a distance of 1.45×10^4 km from Earth's center. The moon orbits the Earth at a distance of 3.84×10^5 km once every 27.3 days. How long does it take the satellite to orbit the Earth?
    a. 0.200 days
    b. 3.07 days
    c. 243 days
    d. 3721 days

23. Earth is 1.496×10^8 km from the sun, and Venus is 1.08×10^8 km from the sun. One day on Venus is 243 Earth days long. What best represents the number of Venusian days in a Venusian year?
    a. 0.78 days
    b. 0.92 days
    c. 1.08 days
    d. 1.21 days

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

24. What did the Cavendish experiment measure?
    a. The mass of Earth
    b. The gravitational constant
    c. Acceleration due to gravity
    d. The eccentricity of Earth's orbit

25. You have a mass of $55\,\text{kg}$ and you have just landed on one of the moons of Jupiter where you have a weight of $67.9\,\text{N}$. What is the acceleration due to gravity, g, on the moon you are visiting?
    a. $.810\text{m/s}^2$
    b. 1.23m/s^2
    c. 539m/s^2
    d. $3735\,\text{m/s}^2$

26. A person is in an elevator that suddenly begins to descend. The person knows, intuitively, that the feeling of suddenly becoming lighter is because the elevator is accelerating downward. What other change would produce the same feeling? How does this demonstrate Einstein's postulate on which he based the theory of general relativity?
    a. It would feel the same if the force of gravity suddenly became weaker. This illustrates Einstein's postulates that gravity and acceleration are indistinguishable.
    b. It would feel the same if the force of gravity suddenly became stronger. This illustrates Einstein's postulates that gravity and acceleration are indistinguishable.
    c. It would feel the same if the force of gravity suddenly became weaker. This illustrates Einstein's postulates that gravity and acceleration are distinguishable.
    d. It would feel the same if the force of gravity suddenly became stronger. This illustrates Einstein's postulates that gravity and acceleration are distinguishable.

Short Answer

7.1 Kepler's Laws of Planetary Motion

27. Explain how the masses of a satellite and its parent body must compare in order to apply Kepler's laws of planetary motion.
    a. The mass of the parent body must be much less than that of the satellite.
    b. The mass of the parent body must be much greater than that of the satellite.
    c. The mass of the parent body must be equal to the mass of the satellite.
    d. There is no specific relationship between the masses for applying Kepler's laws of planetary motion.

28. Hyperion is a moon of the planet Saturn. Its orbit has an eccentricity of 0.123 and a semi-major axis of 1.48×10^6 km. How far is the center of the orbit from the center of Saturn?
    a. 1.82×10^5 km
    b. 3.64×10^5 km
    c. 1.20×10^7 km
    d. 2.41×10^7 km

29. The orbits of satellites are elliptical. Define an ellipse.
    a. An ellipse is an open curve wherein the sum of the distance from the foci to any point on the curve is constant.
    b. An ellipse is a closed curve wherein the sum of the distance from the foci to any point on the curve is constant.
    c. An ellipse is an open curve wherein the distances from the two foci to any point on the curve are equal.
    d. An ellipse is a closed curve wherein the distances from the two foci to any point on the curve are equal.

30. Mars has two moons, Deimos and Phobos. The orbit of Deimos has a period of $1.26\,\text{days}$ and an average radius of 2.35×10^3 km. The average radius of the orbit of Phobos is 9.374×10^3 km. According to Kepler's third law of planetary motion, what is the period of Phobos?
    a. $0.16\,\text{d}$
    b. $0.50\,\text{d}$
    c. $3.17\,\text{d}$
    d. $10.0\,\text{d}$

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

31. Newton's third law of motion says that, for every action force, there is a reaction force equal in magnitude but that acts in the opposite direction. Apply this law to gravitational forces acting between the Washington Monument and Earth.
    a. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with a force equal to Earth's weight. The situation can be represented with two force vectors of unequal magnitude and pointing in the same direction.

   b. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with a force equal to Earth's weight. The situation can be represented with two force vectors of unequal magnitude but pointing in opposite directions.

   c. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with an equal force. The situation can be represented with two force vectors of equal magnitude and pointing in the same direction.

   d. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with an equal force. The situation can be represented with two force vectors of equal magnitude but pointing in opposite directions.

32. True or false—Gravitational force is the attraction of the mass of one object to the mass of another. Light, either as a particle or a wave, has no rest mass. Despite this fact gravity bends a beam of light.

   a. True

   b. False

33. The average radius of Earth is 6.37×10^6 m. What is Earth's mass?

   a. 9.35×10^{17} kg

   b. 5.96×10^{24} kg

   c. 3.79×10^{31} kg

   d. 2.42×10^{38} kg

34. What is the gravitational force between two 60.0 kg people sitting 100 m apart?

   a. 2.4×10^{-11} N

   b. 2.4×10^{-9} N

   c. 3.6×10^{-1} N

   d. 3.6×10^1 N

Extended Response

7.1 Kepler's Laws of Planetary Motion

35. The orbit of Halley''s Comet has an eccentricity of 0.967 and stretches to the edge of the solar system.
Part A. Describe the shape of the comet's orbit.
Part B. Compare the distance traveled per day when it is near the sun to the distance traveled per day when it is at the edge of the solar system.
Part C. Describe variations in the comet's speed as it completes an orbit. Explain the variations in terms of Kepler's second law of planetary motion.

   a. Part A. The orbit is circular, with the sun at the center. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet decelerates as it approaches the sun and accelerates as it leaves the sun.

   b. Part A. The orbit is circular, with the sun at the center. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet accelerates as it approaches the sun and decelerates as it leaves the sun.

   c. Part A. The orbit is very elongated, with the sun near one end. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet decelerates as it approaches the sun and accelerates as it moves away from the sun.

36. For convenience, astronomers often use astronomical units (AU) to measure distances within the solar system. One AU equals the average distance from Earth to the sun. Halley's Comet returns once every 75.3 years. What is the average radius of the orbit of Halley's Comet in AU?

   a. 0.002 AU

   b. 0.056 AU

   c. 17.8 AU

   d. 653 AU

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

37. It took scientists a long time to arrive at the understanding of gravity as explained by Galileo and Newton. They were hindered by two ideas that seemed like common sense but were serious misconceptions. First was the fact that heavier things fall faster than light things. Second, it was believed impossible that forces could act at a distance. Explain why these ideas persisted and why they prevented advances.

   a. Heavier things fall faster than light things if they have less surface area and greater mass density. In the Renaissance and before, forces that acted at a distance were considered impossible, so people were skeptical about scientific theories that invoked such forces.

   b. Heavier things fall faster than light things because they have greater surface area and less mass density. In the Renaissance and before, forces that act at a distance were considered impossible, so people were skeptical about scientific theories that invoked such forces.

   c. Heavier things fall faster than light things because they have less surface area and greater mass density. In the Renaissance and before, forces that

act at a distance were considered impossible, so people were quick to accept scientific theories that invoked such forces.

d. Heavier things fall faster than light things because they have larger surface area and less mass density. In the Renaissance and before, forces that act at a distance were considered impossible because of people's faith in scientific theories.

38. The masses of Earth and the moon are 5.97×10^{24} kg and 7.35×10^{22} kg, respectively. The distance from Earth to the moon is 3.80×10^{5} km. At what point between the Earth and the moon are the opposing gravitational forces equal? (Use subscripts e and m to represent Earth and moon.)

a. 3.42×10^{5} km from the center of Earth
b. 3.80×10^{5} km from the center of Earth
c. 3.42×10^{6} km from the center of Earth
d. 3.10×10^{7} km from the center of Earth

CHAPTER 8
Momentum

Figure 8.1 NFC defensive backs Ronde Barber and Roy Williams along with linebacker Jeremiah Trotter gang tackle AFC running back LaDainian Tomlinson during the 2006 Pro Bowl in Hawaii. (United States Marine Corps)

Chapter Outline

INTRODUCTION We know from everyday use of the word *momentum* that it is a tendency to continue on course in the same direction. Newscasters speak of sports teams or politicians gaining, losing, or maintaining the momentum to win. As we learned when studying about inertia, which is Newton's first law of motion, every object or system has inertia—that is, a tendency for an object in motion to remain in motion or an object at rest to remain at rest. Mass is a useful variable that lets us quantify inertia. Momentum is mass in motion.

Momentum is important because it is conserved in isolated systems; this fact is convenient for solving problems where objects collide. The magnitude of momentum grows with greater mass and/or speed. For example, look at the football players in the photograph (Figure 8.1). They collide and fall to the ground. During their collisions, momentum will play a large part. In this chapter, we will learn about momentum, the different types of collisions, and how to use momentum equations to solve collision problems.

8.1 Linear Momentum, Force, and Impulse

Section Learning Objectives
By the end of this section, you will be able to do the following:

- Describe momentum, what can change momentum, impulse, and the impulse-momentum theorem
- Describe Newton's second law in terms of momentum
- Solve problems using the impulse-momentum theorem

Section Key Terms

change in momentum	impulse	impulse–momentum theorem	linear momentum

Momentum, Impulse, and the Impulse-Momentum Theorem

Linear momentum is the product of a system's mass and its velocity. In equation form, linear momentum $\mathbf{p}$ is

$$\mathbf{p} = m\mathbf{v}.$$

You can see from the equation that momentum is directly proportional to the object's mass (m) and velocity ($\mathbf{v}$). Therefore, the greater an object's mass or the greater its velocity, the greater its momentum. A large, fast-moving object has greater momentum than a smaller, slower object.

Momentum is a vector and has the same direction as velocity $\mathbf{v}$. Since mass is a scalar, when velocity is in a negative direction (i.e., opposite the direction of motion), the momentum will also be in a negative direction; and when velocity is in a positive direction, momentum will likewise be in a positive direction. The SI unit for momentum is kg m/s.

Momentum is so important for understanding motion that it was called the *quantity of motion* by physicists such as Newton. Force influences momentum, and we can rearrange Newton's second law of motion to show the relationship between force and momentum.

Recall our study of Newton's second law of motion ($\mathbf{F}_{net} = m\mathbf{a}$). Newton actually stated his second law of motion in terms of momentum: The net external force equals the **change in momentum** of a system divided by the time over which it changes. The change in momentum is the difference between the final and initial values of momentum.

In equation form, this law is

$$\mathbf{F}_{net} = \frac{\Delta\mathbf{p}}{\Delta t},$$

where $\mathbf{F}_{net}$ is the net external force, $\Delta\mathbf{p}$ is the change in momentum, and Δt is the change in time.

We can solve for $\Delta\mathbf{p}$ by rearranging the equation

$$\mathbf{F}_{net} = \frac{\Delta\mathbf{p}}{\Delta t}$$

to be

$$\Delta\mathbf{p} = \mathbf{F}_{net}\Delta t.$$

$\mathbf{F}_{net}\Delta t$ is known as **impulse** and this equation is known as the **impulse-momentum theorem**. From the equation, we see that the impulse equals the average net external force multiplied by the time this force acts. It is equal to the change in momentum. *The effect of a force on an object depends on how long it acts, as well as the strength of the force.* Impulse is a useful concept because it quantifies the effect of a force. A very large force acting for a short time can have a great effect on the momentum of an object, such as the force of a racket hitting a tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time.

Newton's Second Law in Terms of Momentum

When Newton's second law is expressed in terms of momentum, it can be used for solving problems where mass varies, since $\Delta\mathbf{p} = \Delta(m\mathbf{v})$. In the more traditional form of the law that you are used to working with, mass is assumed to be constant. In fact, this traditional form is a special case of the law, where mass is constant. $\mathbf{F}_{net} = m\mathbf{a}$ is actually derived from the equation:

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$$

For the sake of understanding the relationship between Newton's second law in its two forms, let's recreate the derivation of $\mathbf{F}_{net} = m\mathbf{a}$ from

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$$

by substituting the definitions of acceleration and momentum.

The change in momentum $\Delta \mathbf{p}$ is given by

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}).$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta \mathbf{v}.$$

By substituting $m\Delta \mathbf{v}$ for $\Delta \mathbf{p}$, Newton's second law of motion becomes

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}$$

for a constant mass.

Because

$$\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a},$$

we can substitute to get the familiar equation

$$\mathbf{F}_{net} = m\mathbf{a}$$

when the mass of the system is constant.

TIPS FOR SUCCESS

We just showed how $\mathbf{F}_{net} = m\mathbf{a}$ applies only when the mass of the system is constant. An example of when this formula would not apply would be a moving rocket that burns enough fuel to significantly change the mass of the rocket. In this case, you would need to use Newton's second law expressed in terms of momentum to account for the changing mass.

Snap Lab

Hand Movement and Impulse

In this activity you will experiment with different types of hand motions to gain an intuitive understanding of the relationship between force, time, and impulse.

- one ball
- one tub filled with water

Procedure:

1. Try catching a ball while *giving* with the ball, pulling your hands toward your body.
2. Next, try catching a ball while keeping your hands still.
3. Hit water in a tub with your full palm. Your full palm represents a swimmer doing a belly flop.
4. After the water has settled, hit the water again by diving your hand with your fingers first into the water. Your diving hand represents a swimmer doing a dive.
5. Explain what happens in each case and why.

GRASP CHECK

What are some other examples of motions that impulse affects?

a.   a football player colliding with another, or a car moving at a constant velocity
b.   a car moving at a constant velocity, or an object moving in the projectile motion
c.   a car moving at a constant velocity, or a racket hitting a ball
d.   a football player colliding with another, or a racket hitting a ball

🔗 LINKS TO PHYSICS

Engineering: Saving Lives Using the Concept of Impulse

Cars during the past several decades have gotten much safer. Seat belts play a major role in automobile safety by preventing people from flying into the windshield in the event of a crash. Other safety features, such as airbags, are less visible or obvious, but are also effective at making auto crashes less deadly (see Figure 8.2). Many of these safety features make use of the concept of impulse from physics. Recall that impulse is the net force multiplied by the duration of time of the impact. This was expressed mathematically as $\Delta \mathbf{p} = \mathbf{F}_{net} \Delta t$.

Figure 8.2 Vehicles have safety features like airbags and seat belts installed.

Airbags allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant whether an airbag is deployed or not. But the force that brings the occupant to a stop will be much less if it acts over a larger time. By rearranging the equation for impulse to solve for force $\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$, you can see how increasing Δt while $\Delta \mathbf{p}$ stays the same will decrease $\mathbf{F}_{net}$. This is another example of an inverse relationship. Similarly, a padded dashboard increases the time over which the force of impact acts, thereby reducing the force of impact.

Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the occupants of the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

GRASP CHECK
You may have heard the advice to bend your knees when jumping. In this example, a friend dares you to jump off of a park bench onto the ground without bending your knees. You, of course, refuse. Explain to your friend why this would be a foolish thing. Show it using the impulse-momentum theorem.
a.   Bending your knees increases the time of the impact, thus decreasing the force.
b.   Bending your knees decreases the time of the impact, thus decreasing the force.
c.   Bending your knees increases the time of the impact, thus increasing the force.
d.   Bending your knees decreases the time of the impact, thus increasing the force.

Solving Problems Using the Impulse-Momentum Theorem

✳ **WORKED EXAMPLE**

Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110 kg football player running at 8 m/s. (b) Compare the player's momentum with the momentum of a 0.410 kg football thrown hard at a speed of 25 m/s.

Strategy

No information is given about the direction of the football player or the football, so we can calculate only the magnitude of the momentum, p. (A symbol in italics represents magnitude.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum:

$$\mathbf{p} = m\mathbf{v}$$

Solution for (a)

To find the player's momentum, substitute the known values for the player's mass and speed into the equation.

$$\mathbf{p}_{\text{player}} = (110 \ \text{kg})(8 \ \text{m/s}) = 880 \ \text{kg} \cdot \text{m/s}$$

Solution for (b)

To find the ball's momentum, substitute the known values for the ball's mass and speed into the equation.

$$\mathbf{p}_{\text{ball}} = (0.410 \ \text{kg})(25 \ \text{m/s}) = 10.25 \ \text{kg} \cdot \text{m/s}$$

The ratio of the player's momentum to the ball's momentum is

$$\frac{\mathbf{p}_{\text{player}}}{\mathbf{p}_{\text{ball}}} = \frac{880}{10.3} = 85.9 \ .$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Therefore, the momentum of the player is about 86 times greater than the momentum of the football.

✳ **WORKED EXAMPLE**

Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams (Figure 8.3) hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What was the average force exerted on the 0.057 kg tennis ball by Williams' racquet? Assume that the ball's speed just after impact was 58 m/s, the horizontal velocity before impact is negligible, and that the ball remained in contact with the racquet for 5 ms (milliseconds).

Figure 8.3   Venus Williams playing in the 2013 US Open (Edwin Martinez, Flickr)

Strategy

Recall that Newton's second law stated in terms of momentum is

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta \mathbf{p} = m\Delta \mathbf{v} = m(\mathbf{v}_f - \mathbf{v}_i),$$

where $\mathbf{v}_f$ is the final velocity and $\mathbf{v}_i$ is the initial velocity. In this example, the velocity just after impact and the change in time are given, so after we solve for $\Delta \mathbf{p}$, we can use $\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$ to find the force.

Solution

To determine the change in momentum, substitute the values for mass and the initial and final velocities into the equation above.

$$
\begin{aligned}
\Delta \mathbf{p} &= m(\mathbf{v}_f - \mathbf{v}_i) \\
&= (0.057 \text{ kg})(58 \text{ m/s} - 0 \text{ m/s}) \\
&= 3.306 \text{ kg·m/s} \approx 3.3 \text{ kg·m/s}
\end{aligned}
$$

8.1

Now we can find the magnitude of the net external force using $\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$

$$
\begin{aligned}
\mathbf{F}_{net} &= \frac{\Delta \mathbf{p}}{\Delta t} = \frac{3.306}{5 \times 10^{-3}} \\
&= 661 \text{ N} \\
&\approx 660 \text{ N}.
\end{aligned}
$$

8.2

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact. This problem could also be solved by first finding the acceleration and then using $\mathbf{F}_{net} = m\mathbf{a}$, but we would have had to do one more step. In this case, using momentum was a shortcut.

Practice Problems

1. What is the momentum of a bowling ball with mass 5 kg and velocity 10 m/s?
    a. $0.5 \text{ kg} \cdot \text{m/s}$
    b. $2 \text{ kg} \cdot \text{m/s}$
    c. $15 \text{ kg} \cdot \text{m/s}$
    d. $50 \text{ kg} \cdot \text{m/s}$

2. What will be the change in momentum caused by a net force of 120 N acting on an object for 2 seconds?
    a. $60 \text{ kg} \cdot \text{m/s}$
    b. $118 \text{ kg} \cdot \text{m/s}$
    c. $122 \text{ kg} \cdot \text{m/s}$
    d. $240 \text{ kg} \cdot \text{m/s}$

Check Your Understanding

3. What is linear momentum?
    a. the sum of a system's mass and its velocity
    b. the ratio of a system's mass to its velocity
    c. the product of a system's mass and its velocity
    d. the product of a system's moment of inertia and its velocity

4. If an object's mass is constant, what is its momentum proportional to?
    a. Its velocity
    b. Its weight
    c. Its displacement
    d. Its moment of inertia

5. What is the equation for Newton's second law of motion, in terms of mass, velocity, and time, when the mass of the system is

constant?

a. $\mathbf{F}_{net} = \frac{\Delta \mathbf{v}}{\Delta m \Delta t}$

b. $\mathbf{F}_{net} = \frac{m \Delta t}{\Delta \mathbf{v}}$

c. $\mathbf{F}_{net} = \frac{m \Delta \mathbf{v}}{\Delta t}$

d. $\mathbf{F}_{net} = \frac{\Delta m \Delta \mathbf{v}}{\Delta t}$

6. Give an example of a system whose mass is not constant.

a. A spinning top

b. A baseball flying through the air

c. A rocket launched from Earth

d. A block sliding on a frictionless inclined plane

8.2 Conservation of Momentum

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the law of conservation of momentum verbally and mathematically

Section Key Terms

angular momentum   isolated system   law of conservation of momentum

Conservation of Momentum

It is important we realize that momentum is conserved during collisions, explosions, and other events involving objects in motion. To say that a quantity is conserved means that it is constant throughout the event. In the case of conservation of momentum, the total momentum in the system remains the same before and after the collision.

You may have noticed that momentum was *not* conserved in some of the examples previously presented in this chapter. where forces acting on the objects produced large changes in momentum. Why is this? The systems of interest considered in those problems were not inclusive enough. If the systems were expanded to include more objects, then momentum would in fact be conserved in those sample problems. It is always possible to find a larger system where momentum is conserved, even though momentum changes for individual objects within the system.

For example, if a football player runs into the goalpost in the end zone, a force will cause him to bounce backward. His momentum is obviously greatly changed, and considering only the football player, we would find that momentum is not conserved. However, the system can be expanded to contain the entire Earth. Surprisingly, Earth also recoils—conserving momentum—because of the force applied to it through the goalpost. The effect on Earth is not noticeable because it is so much more massive than the player, but the effect is real.

Next, consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—in the example shown in Figure 8.4 of one car bumping into another. Both cars are coasting in the same direction when the lead car, labeled m_2, is bumped by the trailing car, labeled m_1. The only unbalanced force on each car is the force of the collision, assuming that the effects due to friction are negligible. Car m_1 slows down as a result of the collision, losing some momentum, while car m_2 speeds up and gains some momentum. If we choose the system to include both cars and assume that friction is negligible, then the momentum of the two-car system should remain constant. Now we will prove that the total momentum of the two-car system does in fact remain constant, and is therefore conserved.
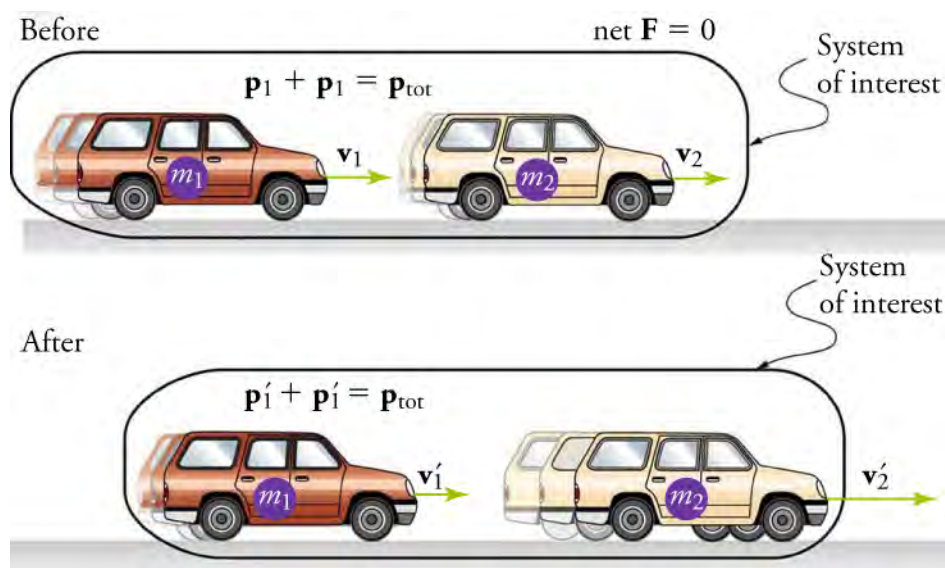
Figure 8.4 Car of mass m_1 moving with a velocity of $\mathbf{v}_1$ bumps into another car of mass m_2 and velocity $\mathbf{v}_2$. As a result, the first car slows down to a velocity of $\mathbf{v'}_1$ and the second speeds up to a velocity of $\mathbf{v'}_2$. The momentum of each car is changed, but the total momentum $\mathbf{p}_{tot}$ of the two cars is the same before and after the collision if you assume friction is negligible.

Using the impulse-momentum theorem, the change in momentum of car 1 is given by

$$\Delta \mathbf{p}_1 = \mathbf{F}_1 \Delta t,$$

where $\mathbf{F}_1$ is the force on car 1 due to car 2, and Δt is the time the force acts, or the duration of the collision.

Similarly, the change in momentum of car 2 is $\Delta \mathbf{p}_2 = \mathbf{F}_2 \Delta t$ where $\mathbf{F}_2$ is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law of motion that $\mathbf{F}_2 = -\mathbf{F}_1$, and so
$\Delta \mathbf{p}_2 = -\mathbf{F}_1 \Delta t = -\Delta \mathbf{p}_1$.

Therefore, the changes in momentum are equal and opposite, and $\Delta \mathbf{p}_1 + \Delta \mathbf{p}_2 = 0$.

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$$

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p'}_1 + \mathbf{p'}_2,$$

where $\mathbf{p'}_1$ and $\mathbf{p'}_2$ are the momenta of cars 1 and 2 after the collision.

This result that momentum is conserved is true not only for this example involving the two cars, but for any system where the net external force is zero, which is known as an **isolated system**. The **law of conservation of momentum** states that for an isolated system with any number of objects in it, the total momentum is conserved. In equation form, the law of conservation of momentum for an isolated system is written as

$$\mathbf{p}_{tot} = \text{constant}$$

or

$$\mathbf{p}_{tot} = \mathbf{p'}_{tot},$$

where $\mathbf{p}_{tot}$ is the total momentum, or the sum of the momenta of the individual objects in the system at a given time, and $\mathbf{p'}_{tot}$ is the total momentum some time later.

The conservation of momentum principle can be applied to systems as diverse as a comet striking the Earth or a gas containing huge numbers of atoms and molecules. Conservation of momentum appears to be violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

TIPS FOR SUCCESS

Momenta is the plural form of the word momentum. One object is said to have momentum, but two or more objects are said to have *momenta*.

FUN IN PHYSICS

Angular Momentum in Figure Skating

So far we have covered linear momentum, which describes the inertia of objects traveling in a straight line. But we know that many objects in nature have a curved or circular path. Just as linear motion has linear momentum to describe its tendency to move forward, circular motion has the equivalent **angular momentum** to describe how rotational motion is carried forward.

This is similar to how torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass or inertia. You may recall learning that the quantity mr^2 is called the rotational inertia or moment of inertia of a point mass m at a distance r from the center of rotation.

We already know the equation for linear momentum, $\mathbf{p} = m\mathbf{v}$. Since angular momentum is analogous to linear momentum, the moment of inertia (I) is analogous to mass, and angular velocity ($\boldsymbol{\omega}$) is analogous to linear velocity, it makes sense that angular momentum ($\mathbf{L}$) is defined as

$$\mathbf{L} = I\boldsymbol{\omega}$$

Angular momentum is conserved when the net external torque ($\boldsymbol{\tau}$) is zero, just as linear momentum is conserved when the net external force is zero.

Figure skaters take advantage of the conservation of angular momentum, likely without even realizing it. In Figure 8.5, a figure skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice, and because the friction is exerted very close to the pivot point. Both $\mathbf{F}$ and r are small, and so $\boldsymbol{\tau}$ is negligibly small.
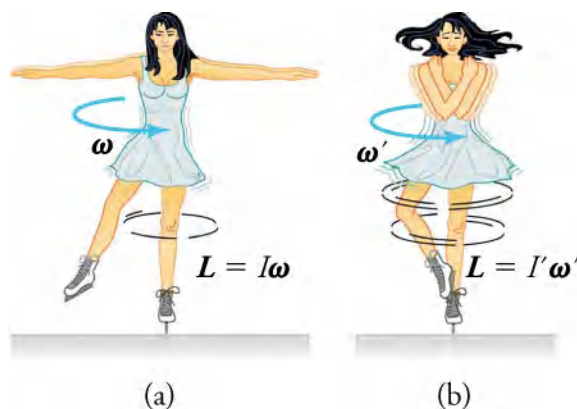
Figure 8.5 (a) An ice skater is spinning on the tip of her skate with her arms extended. In the next image, (b), her rate of spin increases greatly when she pulls in her arms.

Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that $\mathbf{L} = \mathbf{L}'$.

Expressing this equation in terms of the moment of inertia,

$$I\boldsymbol{\omega} = I'\boldsymbol{\omega}',$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity $\boldsymbol{\omega}'$ must increase to keep the angular momentum constant. This allows her to spin much faster without exerting any extra torque.

A video (http://openstax.org/l/28figureskater) is also available that shows a real figure skater executing a spin. It discusses the physics of spins in figure skating.

> **GRASP CHECK**
>
> Based on the equation $\mathbf{L} = I\omega$, how would you expect the moment of inertia of an object to affect angular momentum? How would angular velocity affect angular momentum?
>
> a. Large moment of inertia implies large angular momentum, and large angular velocity implies large angular momentum.
>
> b. Large moment of inertia implies small angular momentum, and large angular velocity implies small angular momentum.
>
> c. Large moment of inertia implies large angular momentum, and large angular velocity implies small angular momentum.
>
> d. Large moment of inertia implies small angular momentum, and large angular velocity implies large angular momentum.

Check Your Understanding

7. When is momentum said to be conserved?
   a. When momentum is changing during an event
   b. When momentum is increasing during an event
   c. When momentum is decreasing during an event
   d. When momentum is constant throughout an event

8. A ball is hit by a racket and its momentum changes. How is momentum conserved in this case?
   a. Momentum of the system can never be conserved in this case.
   b. Momentum of the system is conserved if the momentum of the racket is not considered.
   c. Momentum of the system is conserved if the momentum of the racket is also considered.
   d. Momentum of the system is conserved if the momenta of the racket and the player are also considered.

9. State the law of conservation of momentum.
   a. Momentum is conserved for an isolated system with any number of objects in it.
   b. Momentum is conserved for an isolated system with an even number of objects in it.
   c. Momentum is conserved for an interacting system with any number of objects in it.
   d. Momentum is conserved for an interacting system with an even number of objects in it.

8.3 Elastic and Inelastic Collisions

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Distinguish between elastic and inelastic collisions
- Solve collision problems by applying the law of conservation of momentum

Section Key Terms

elastic collision   inelastic collision   point masses   recoil

Elastic and Inelastic Collisions

When objects collide, they can either stick together or bounce off one another, remaining separate. In this section, we'll cover these two different types of collisions, first in one dimension and then in two dimensions.

In an **elastic collision**, the objects separate after impact and don't lose any of their kinetic energy. Kinetic energy is the energy of motion and is covered in detail elsewhere. The law of conservation of momentum is very useful here, and it can be used whenever the net external force on a system is zero. Figure 8.6 shows an elastic collision where momentum is conserved.
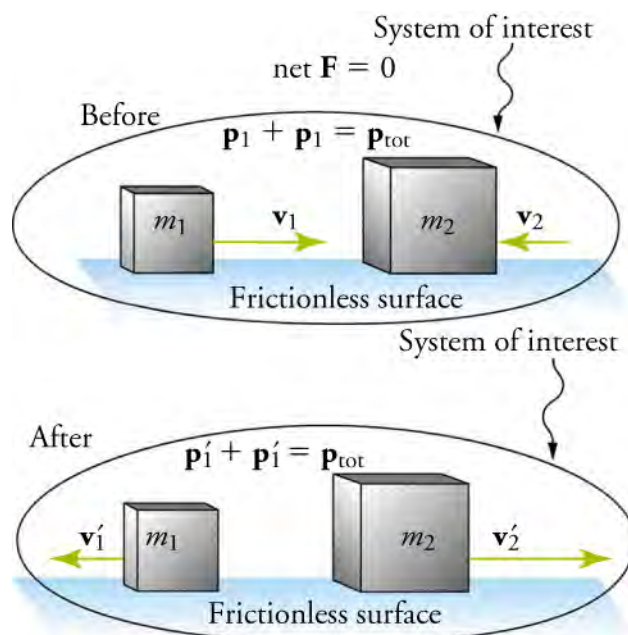
Figure 8.6 The diagram shows a one-dimensional elastic collision between two objects.

An animation of an elastic collision between balls can be seen by watching this video (http://openstax.org/l/28elasticball) . It replicates the elastic collisions between balls of varying masses.

Perfectly elastic collisions can happen only with subatomic particles. Everyday observable examples of perfectly elastic collisions don't exist—some kinetic energy is always lost, as it is converted into heat transfer due to friction. However, collisions between everyday objects are almost perfectly elastic when they occur with objects and surfaces that are nearly frictionless, such as with two steel blocks on ice.

Now, to solve problems involving one-dimensional elastic collisions between two objects, we can use the equation for conservation of momentum. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2 \, (\mathbf{F}_{net} = 0).$$

Substituting the definition of momentum $\mathbf{p} = m\mathbf{v}$ for each initial and final momentum, we get

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2,$$

where the primes (') indicate values after the collision; In some texts, you may see i for initial (before collision) and f for final (after collision). The equation assumes that the mass of each object does not change during the collision.

👁 WATCH PHYSICS

Momentum: Ice Skater Throws a Ball
This video covers an elastic collision problem in which we find the **recoil** velocity of an ice skater who throws a ball straight forward. To clarify, Sal is using the equation

$$m_{ball} \mathbf{V}_{ball} + m_{skater} \mathbf{V}_{skater} = m_{ball} \mathbf{v}'_{ball} + m_{skater} \mathbf{v}'_{skater} \, .$$

Click to view content (https://www.khanacademy.org/embed_video?v=vPkkCOlGND4)

> **GRASP CHECK**
>
> The resultant vector of the addition of vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}$. The magnitudes of $\vec{a}$, $\vec{b}$, and $\vec{r}$ are A, B, and R, respectively. Which of the following is true?
> a. $R_x + R_y = 0$
> b. $A_x + A_y = \vec{A}$

c. $A_x + B_y = B_x + A_y$
d. $A_x + B_x = R_x$

Now, let us turn to the second type of collision. An **inelastic collision** is one in which objects stick together after impact, and kinetic energy is *not* conserved. This lack of conservation means that the forces between colliding objects may convert kinetic energy to other forms of energy, such as potential energy or thermal energy. The concepts of energy are discussed more thoroughly elsewhere. For inelastic collisions, kinetic energy may be lost in the form of heat. Figure 8.7 shows an example of an inelastic collision. Two objects that have equal masses head toward each other at equal speeds and then stick together. The two objects come to rest after sticking together, conserving momentum but not kinetic energy after they collide. Some of the energy of motion gets converted to thermal energy, or heat.
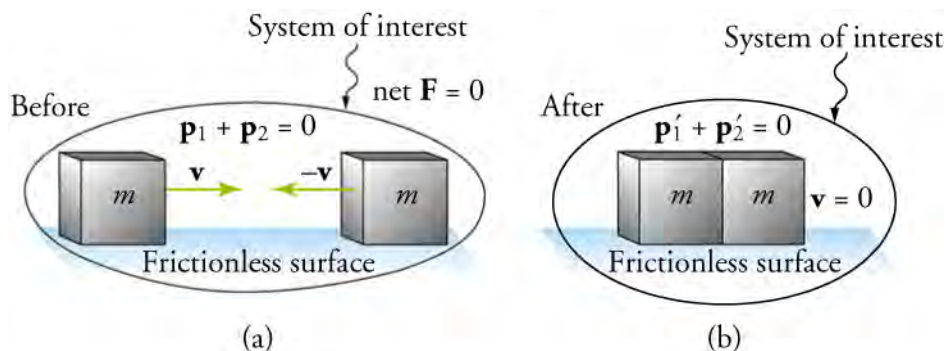
Figure 8.7 A one-dimensional inelastic collision between two objects. Momentum is conserved, but kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward each other at the same speed. (b) The objects stick together, creating a perfectly inelastic collision. In the case shown in this figure, the combined objects stop; This is not true for all inelastic collisions.

Since the two objects stick together after colliding, they move together at the same speed. This lets us simplify the conservation of momentum equation from

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

to

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2) \mathbf{v}'$$

for inelastic collisions, where **v'** is the final velocity for both objects as they are stuck together, either in motion or at rest.

👁 WATCH PHYSICS

Introduction to Momentum

This video reviews the definitions of momentum and impulse. It also covers an example of using conservation of momentum to solve a problem involving an inelastic collision between a car with constant velocity and a stationary truck. Note that Sal accidentally gives the unit for impulse as Joules; it is actually N · s or k · gm/s.

Click to view content (https://www.khanacademy.org/embed_video?v=XFhntPxowoU)

> **GRASP CHECK**
> How would the final velocity of the car-plus-truck system change if the truck had some initial velocity moving in the same direction as the car? What if the truck were moving in the opposite direction of the car initially? Why?
> a.   If the truck was initially moving in the same direction as the car, the final velocity would be greater. If the truck was initially moving in the opposite direction of the car, the final velocity would be smaller.
> b.   If the truck was initially moving in the same direction as the car, the final velocity would be smaller. If the truck was initially moving in the opposite direction of the car, the final velocity would be greater.
> c.   The direction in which the truck was initially moving would not matter. If the truck was initially moving in either

direction, the final velocity would be smaller.

  d.   The direction in which the truck was initially moving would not matter. If the truck was initially moving in either direction, the final velocity would be greater.

Snap Lab

Ice Cubes and Elastic Collisions

In this activity, you will observe an elastic collision by sliding an ice cube into another ice cube on a smooth surface, so that a negligible amount of energy is converted to heat.

  •   Several ice cubes (The ice must be in the form of cubes.)
  •   A smooth surface

Procedure
  1.   Find a few ice cubes that are about the same size and a smooth kitchen tabletop or a table with a glass top.
  2.   Place the ice cubes on the surface several centimeters away from each other.
  3.   Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes.
  4.   Explain the speeds and directions of the ice cubes using momentum.

GRASP CHECK
Was the collision elastic or inelastic?
  a.   perfectly elastic
  b.   perfectly inelastic
  c.   Nearly perfect elastic
  d.   Nearly perfect inelastic

TIPS FOR SUCCESS
Here's a trick for remembering which collisions are elastic and which are inelastic: Elastic is a bouncy material, so when objects *bounce* off one another in the collision and separate, it is an elastic collision. When they don't, the collision is inelastic.

Solving Collision Problems

The Khan Academy videos referenced in this section show examples of elastic and inelastic collisions in one dimension. In one-dimensional collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and just as we did with two-dimensional forces, we will solve these problems by first choosing a coordinate system and separating the motion into its x and y components.

One complication with two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass each other, they will spin in circles. We will not consider such rotation until later, and so for now, we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{net} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in Figure 8.8. Because momentum is conserved, the components of momentum along the x- and y-axes, displayed as $\mathbf{p}_x$ and $\mathbf{p}_y$, will also be conserved. With the chosen coordinate system, $\mathbf{p}_y$ is initially zero and $\mathbf{p}_x$ is the momentum of the incoming particle.
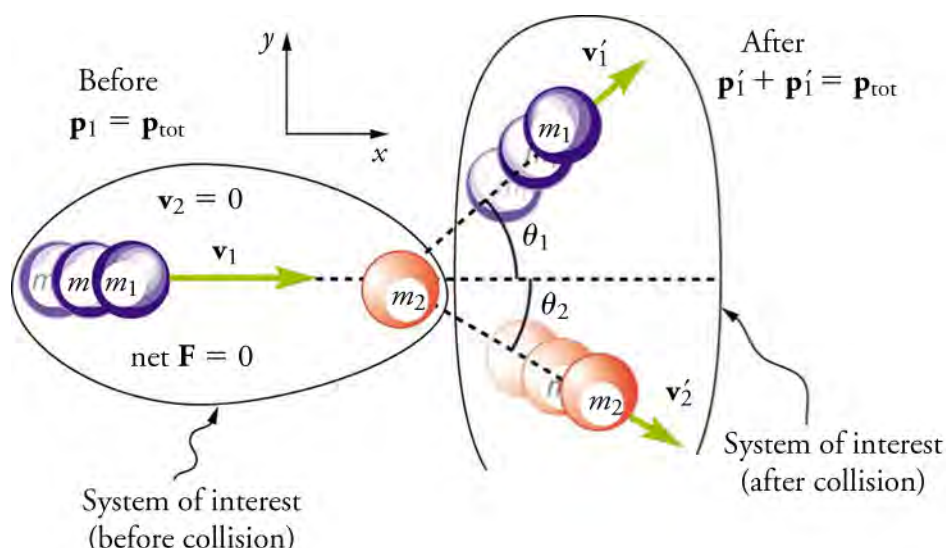
Figure 8.8 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and $\mathbf{v}_1$ is parallel to the x-axis.

Now, we will take the conservation of momentum equation, $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$ and break it into its x and y components.

Along the x-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1x} + \mathbf{p}_{2x} = \mathbf{p}'_{1x} + \mathbf{p}'_{2x}.$$

In terms of masses and velocities, this equation is

$$m_1\mathbf{v}_{1\ x} + m_2\mathbf{v}_{2\ x} = m_1\mathbf{v}'_{1\ x} + m_2\mathbf{v}'_{2\ x}. \qquad \boxed{8.3}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1\mathbf{v}_{1\ x} = m_1\mathbf{v}'_{1\ x} + m_2\mathbf{v}'_{2\ x}. \qquad \boxed{8.4}$$

The components of the velocities along the x-axis have the form $\mathbf{v}\cos\theta$. Because particle 1 initially moves along the x-axis, we find $\mathbf{v}_{1x} = \mathbf{v}_1$. Conservation of momentum along the x-axis gives the equation

$$m_1\mathbf{v}_1 = m_1\mathbf{v}'_1\cos\theta_1 + m_2\mathbf{v}'_2\cos\theta_2,$$

where θ_1 and θ_2 are as shown in Figure 8.8.

Along the y-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1\ y} + \mathbf{p}_{2\ y} = \mathbf{p}'_{1\ y} + \mathbf{p}'_{2\ y}, \qquad \boxed{8.5}$$

or

$$m_1\mathbf{v}_{1\ y} + m_2\mathbf{v}_{2\ y} = m_1\mathbf{v}'_{1\ y} + m_2\mathbf{v}'_{2\ y}. \qquad \boxed{8.6}$$

But $\mathbf{v}_1y$ is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, $\mathbf{v}_2y$ is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1\mathbf{v}'_1y + m_2\mathbf{v}'_2y. \qquad \boxed{8.7}$$

The components of the velocities along the y-axis have the form $\mathbf{v}\sin\theta$. Therefore, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1\mathbf{v}'_1\sin\theta_1 + m_2\mathbf{v}'_2\sin\theta_2$$

Virtual Physics

Collision Lab

In this simulation, you will investigate collisions on an air hockey table. Place checkmarks next to the momentum vectors

and momenta diagram options. Experiment with changing the masses of the balls and the initial speed of ball 1. How does this affect the momentum of each ball? What about the total momentum? Next, experiment with changing the elasticity of the collision. You will notice that collisions have varying degrees of elasticity, ranging from perfectly elastic to perfectly inelastic.

Click to view content (https://archive.cnx.org/specials/2c7acb3c-2fbd-11e5-b2d9-e7f92291703c/collision-lab/)

GRASP CHECK

If you wanted to maximize the velocity of ball 2 after impact, how would you change the settings for the masses of the balls, the initial speed of ball 1, and the elasticity setting? Why? Hint—Placing a checkmark next to the velocity vectors and removing the momentum vectors will help you visualize the velocity of ball 2, and pressing the More Data button will let you take readings.

a.   Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 50 percent.
b.   Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 100 percent.
c.   Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 100 percent.
d.   Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 50 percent.

✳ WORKED EXAMPLE

Calculating Velocity: Inelastic Collision of a Puck and a Goalie

Find the recoil velocity of a 70 kg ice hockey goalie who catches a 0.150-kg hockey puck slapped at him at a velocity of 35 m/s. Assume that the goalie is at rest before catching the puck, and friction between the ice and the puck-goalie system is negligible (see Figure 8.9).
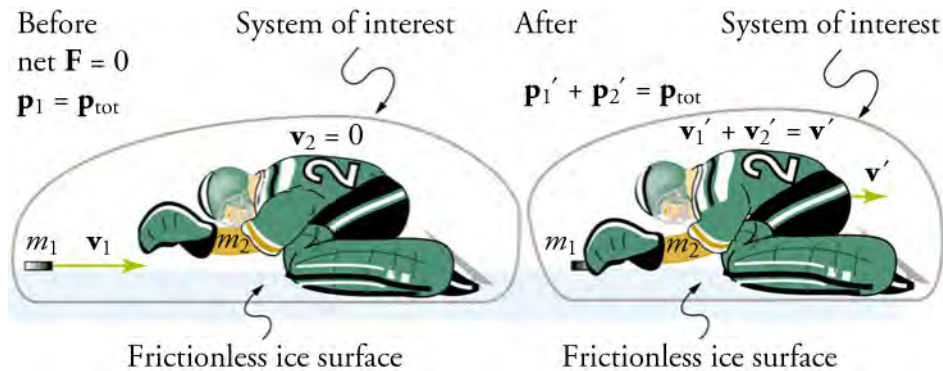
Figure 8.9   An ice hockey goalie catches a hockey puck and recoils backward in an inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. Therefore, we can use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same.

Solution

For an inelastic collision, conservation of momentum is

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2) \mathbf{v}', \qquad \qquad \boxed{8.8}$$

where $\mathbf{v}'$ is the velocity of both the goalie and the puck after impact. Because the goalie is initially at rest, we know $\mathbf{v}_2 = 0$. This simplifies the equation to

$$m_1 \mathbf{v}_1 = (m_1 + m_2) \mathbf{v}'. \qquad \qquad \boxed{8.9}$$

Solving for $\mathbf{v}'$ yields

$$\mathbf{v}' = \left(\frac{m_1}{m_1 + m_2}\right)\mathbf{v}_1. \qquad \text{8.10}$$

Entering known values in this equation, we get

$$\mathbf{v}' = \left(\frac{0.150 \text{ kg}}{70.0 \text{ kg}+0.150 \text{ kg}}\right)(35 \text{ m/s})$$

$$= 7.48 \times 10^{-2}\text{m/s}. \qquad \text{8.11}$$

Discussion
This recoil velocity is small and in the same direction as the puck's original velocity.

✳ WORKED EXAMPLE

Calculating Final Velocity: Elastic Collision of Two Carts

Two hard, steel carts collide head-on and then ricochet off each other in opposite directions on a frictionless surface (see Figure 8.10). Cart 1 has a mass of 0.350 kg and an initial velocity of 2 m/s. Cart 2 has a mass of 0.500 kg and an initial velocity of –0.500 m/s. After the collision, cart 1 recoils with a velocity of –4 m/s. What is the final velocity of cart 2?
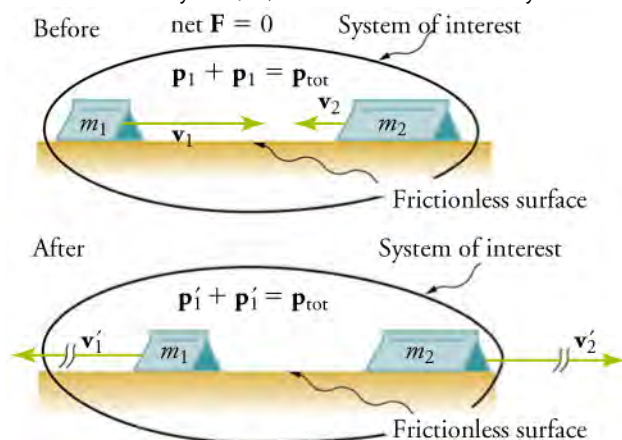
Figure 8.10   Two carts collide with each other in an elastic collision.

Strategy
Since the track is frictionless, $\mathbf{F}_{net} = 0$ and we can use conservation of momentum to find the final velocity of cart 2.

Solution
As before, the equation for conservation of momentum for a one-dimensional elastic collision in a two-object system is

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2. \qquad \text{8.12}$$

The only unknown in this equation is $\mathbf{v}'_2$. Solving for $\mathbf{v}'_2$ and substituting known values into the previous equation yields

$$\mathbf{v}'_2 = \frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2-m_1\mathbf{v}'_1}{m_2}$$

$$= \frac{(0.350\text{kg})(2.00\text{m/s})+(0.500 \text{ kg})(-0.500 \text{ m/s})-(0.350\text{kg})(-4.00\text{m/s})}{0.500\text{kg}} \qquad \text{8.13}$$

$$= 3.70\text{m/s}.$$

Discussion
The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision.

✳ WORKED EXAMPLE

Calculating Final Velocity in a Two-Dimensional Collision

Suppose the following experiment is performed (Figure 8.11). An object of mass 0.250 kg (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object of mass 0.400 kg (m_2). The 0.250 kg object emerges from the room at an angle of 45° with its incoming direction. The speed of the 0.250 kg object is originally 2 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (v'_2 and θ_2) of the 0.400 kg object after the collision.
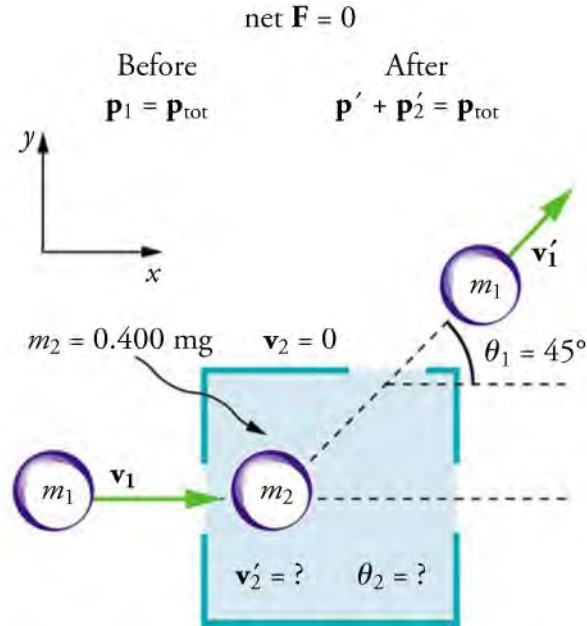
Figure 8.11   The incoming object of mass m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which the object of mass m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Strategy

Momentum is conserved because the surface is frictionless. We chose the coordinate system so that the initial velocity is parallel to the x-axis, and conservation of momentum along the x- and y-axes applies.

Everything is known in these equations except $\mathbf{v}'_2$ and θ_2, which we need to find. We can find two unknowns because we have two independent equations—the equations describing the conservation of momentum in the x and y directions.

Solution

First, we'll solve both conservation of momentum equations ($m_1\mathbf{v}_1 = m_1\mathbf{v}'_1 \cos\theta_1 + m_2\mathbf{v}'_2 \cos\theta_2$ and $0 = m_1\mathbf{v}'_1 \sin\theta_1 + m_2\mathbf{v}'_2 \sin\theta_2$) for $\mathbf{v}'_2 \sin\theta_2$.

For conservation of momentum along x-axis, let's substitute $\sin\theta_2 / \tan\theta_2$ for $\cos\theta_2$ so that terms may cancel out later on. This comes from rearranging the definition of the trigonometric identity $\tan\theta = \sin\theta / \cos\theta$. This gives us

$$m_1\mathbf{v}_1 = m_1\mathbf{v}'_1 \cos\theta_1 + m_2\mathbf{v}'_2 \frac{\sin\theta_2}{\tan\theta_2}.$$

<div align="right">8.14</div>

Solving for $\mathbf{v}'_2 \sin\theta_2$ yields

$$\mathbf{v}'_2 \sin\theta_2 = \frac{(m_1\mathbf{v}_1 - m_1\mathbf{v}'_1 \cos\theta_1)(\tan\theta_2)}{m_2}.$$

<div align="right">8.15</div>

For conservation of momentum along y-axis, solving for $\mathbf{v}'_2 \sin\theta_2$ yields

$$\mathbf{v}'_2 \sin\theta_2 = \frac{-(m_1\mathbf{v}'_1 \sin\theta_1)}{m_2}.$$

<div align="right">8.16</div>

Since both equations equal $\mathbf{v}'_2 \sin \theta_2$, we can set them equal to one another, yielding

$$\frac{(m_1 \mathbf{v}_1 - m_1 \mathbf{v}'_1 \cos \theta_1)(\tan \theta_2)}{m_2} = \frac{-(m_1 \mathbf{v}'_1 \sin \theta_1)}{m_2}.$$

<div style="text-align:right">8.17</div>

Solving this equation for $\tan \theta_2$, we get

$$\tan \theta_2 = \frac{\mathbf{v}'_1 \sin \theta_1}{\mathbf{v}'_1 \cos \theta_1 - \mathbf{v}_1}.$$

<div style="text-align:right">8.18</div>

Entering known values into the previous equation gives

$$\tan \theta_2 = \frac{(1.50)(0.707)}{(1.50)(0.707) - 2.00} = -1.129.$$

<div style="text-align:right">8.19</div>

Therefore,

$$\theta_2 = \tan^{-1}(-1.129) = 312^0.$$

<div style="text-align:right">8.20</div>

Since angles are defined as positive in the counterclockwise direction, m_2 is scattered to the right.

We'll use the conservation of momentum along the y-axis equation to solve for $\mathbf{v}'_2$.

$$\mathbf{v}'_2 = -\frac{m_1}{m_2} \mathbf{v}'_1 \frac{\sin \theta_1}{\sin \theta_2}$$

<div style="text-align:right">8.21</div>

Entering known values into this equation gives

$$\mathbf{v}'_2 = -\frac{(0.250)}{(0.400)}(1.50)\left(\frac{0.7071}{-0.7485}\right).$$

<div style="text-align:right">8.22</div>

Therefore,

$$\mathbf{v}'_2 = 0.886 \ \text{m/s}.$$

<div style="text-align:right">8.23</div>

Discussion

Either equation for the x- or y-axis could have been used to solve for $\mathbf{v}'_2$, but the equation for the y-axis is easier because it has fewer terms.

Practice Problems

10. In an elastic collision, an object with momentum $25 \ \text{kg} \cdot \text{m/s}$ collides with another object moving to the right that has a momentum $35 \ \text{kg} \cdot \text{m/s}$. After the collision, both objects are still moving to the right, but the first object's momentum changes to $10 \ \text{kg} \cdot \text{m/s}$. What is the final momentum of the second object?
    a. $10 \ \text{kg} \cdot \text{m/s}$
    b. $20 \ \text{kg} \cdot \text{m/s}$
    c. $35 \ \text{kg} \cdot \text{m/s}$
    d. $50 \ \text{kg} \cdot \text{m/s}$

11. In an elastic collision, an object with momentum $25 \ \text{kg} \cdot \text{m/s}$ collides with another that has a momentum $35 \ \text{kg} \cdot \text{m/s}$. The first object's momentum changes to $10 \ \text{kg} \cdot \text{m/s}$. What is the final momentum of the second object?
    a. $10 \ \text{kg} \cdot \text{m/s}$
    b. $20 \ \text{kg} \cdot \text{m/s}$
    c. $35 \ \text{kg} \cdot \text{m/s}$
    d. $50 \ \text{kg} \cdot \text{m/s}$

Check Your Understanding

12. What is an elastic collision?
    a.   An elastic collision is one in which the objects after impact are deformed permanently.
    b.   An elastic collision is one in which the objects after impact lose some of their internal kinetic energy.

    c.   An elastic collision is one in which the objects after impact do not lose any of their internal kinetic energy.

    d.   An elastic collision is one in which the objects after impact become stuck together and move with a common velocity.

13. Are perfectly elastic collisions possible?

    a.   Perfectly elastic collisions are not possible.

    b.   Perfectly elastic collisions are possible only with subatomic particles.

    c.   Perfectly elastic collisions are possible only when the objects stick together after impact.

    d.   Perfectly elastic collisions are possible if the objects and surfaces are nearly frictionless.

14. What is the equation for conservation of momentum for two objects in a one-dimensional collision?

    a. $\mathbf{p}_1 + \mathbf{p}_1' = \mathbf{p}_2 + \mathbf{p}_2'$

    b. $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_1' + \mathbf{p}_2'$

    c. $\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{p}_1' - \mathbf{p}_2'$

    d. $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_1' + \mathbf{p}_2' = 0$

KEY TERMS

angular momentum the product of the moment of inertia and angular velocity

change in momentum the difference between the final and initial values of momentum; the mass times the change in velocity

elastic collision collision in which objects separate after impact and kinetic energy is conserved

impulse average net external force multiplied by the time the force acts; equal to the change in momentum

impulse–momentum theorem the impulse, or change in momentum, is the product of the net external force and the time over which the force acts

inelastic collision collision in which objects stick together

after impact and kinetic energy is not conserved

isolated system system in which the net external force is zero

law of conservation of momentum when the net external force is zero, the total momentum of the system is conserved or constant

linear momentum the product of a system's mass and velocity

point masses structureless particles that cannot rotate or spin

recoil backward movement of an object caused by the transfer of momentum from another object in a collision

SECTION SUMMARY

8.1 Linear Momentum, Force, and Impulse

- Linear momentum, often referenced as *momentum* for short, is defined as the product of a system's mass multiplied by its velocity, $\mathbf{p} = m\mathbf{v}$.
- The SI unit for momentum is kg m/s.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes, $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$.
- Impulse is the average net external force multiplied by the time this force acts, and impulse equals the change in momentum, $\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$.
- Forces are usually not constant over a period of time, so we use the average of the force over the time it acts.

8.2 Conservation of Momentum

- The law of conservation of momentum is written $\mathbf{p}_{\text{tot}} =$ constant or $\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}$ (isolated system), where $\mathbf{p}_{\text{tot}}$ is the initial total momentum and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later.

- In an isolated system, the net external force is zero.
- Conservation of momentum applies only when the net external force is zero, within the defined system.

8.3 Elastic and Inelastic Collisions

- If objects separate after impact, the collision is elastic; If they stick together, the collision is inelastic.
- Kinetic energy is conserved in an elastic collision, but not in an inelastic collision.
- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the *x*-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses, where mass 2 is initially at rest, conserve momentum along the initial direction of mass 1, or the *x*-axis, and along the direction perpendicular to the initial direction, or the *y*-axis.
- Point masses are structureless particles that cannot spin.

KEY EQUATIONS

8.1 Linear Momentum, Force, and Impulse

impulse	$\mathbf{F}_{\text{net}} \Delta t$
impulse–momentum theorem	$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$
linear momentum	$\mathbf{p} = m\mathbf{v}$

Newton's second law in terms of momentum	$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$

8.2 Conservation of Momentum

law of conservation of momentum	$\mathbf{p}_{\text{tot}} =$ constant, or $\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}$

conservation of momentum for two objects	$\mathbf{p}_1 + \mathbf{p}_2 = $ constant, or $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$
angular momentum	$\mathbf{L} = I\,\boldsymbol{\omega}$

8.3 Elastic and Inelastic Collisions

conservation of momentum in an elastic collision	$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2,$
conservation of momentum in an inelastic collision	$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\,\mathbf{v}'$

conservation of momentum along x-axis for 2D collisions	$m_1\mathbf{v}_1 = m_1\mathbf{v}'_1 \cos\theta_1 + m_2\mathbf{v}'_2 \cos\theta_2$
conservation of momentum along y-axis for 2D collisions	$0 = m_1\mathbf{v}'_1 \sin\theta_1 + m_2\mathbf{v}'_2 \sin\theta_2$

CHAPTER REVIEW

Concept Items

8.1 Linear Momentum, Force, and Impulse

1. What is impulse?
   a. Change in velocity
   b. Change in momentum
   c. Rate of change of velocity
   d. Rate of change of momentum

2. In which equation of Newton's second law is mass assumed to be constant?
   a. $\mathbf{F} = ma$
   b. $\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t}$
   c. $\mathbf{F} = \Delta\mathbf{p}\Delta t$
   d. $\mathbf{F} = \frac{\Delta m}{\Delta a}$

3. What is the SI unit of momentum?
   a. N
   b. $kg \cdot m$
   c. $kg \cdot m/s$
   d. $kg \cdot m/s^2$

4. What is the equation for linear momentum?
   a. $\mathbf{p} = m\mathbf{v}$
   b. $\mathbf{p} = m/\mathbf{v}$
   c. $\mathbf{p} = m\mathbf{v}^2$
   d. $\mathbf{p} = \frac{1}{2}m\mathbf{v}^2$

8.2 Conservation of Momentum

5. What is angular momentum?
   a. The sum of moment of inertia and angular velocity
   b. The ratio of moment of inertia to angular velocity
   c. The product of moment of inertia and angular velocity
   d. Half the product of moment of inertia and square of angular velocity

6. What is an isolated system?
   a. A system in which the net internal force is zero
   b. A system in which the net external force is zero
   c. A system in which the net internal force is a nonzero constant
   d. A system in which the net external force is a nonzero constant

8.3 Elastic and Inelastic Collisions

7. In the equation $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$ for the collision of two objects, what is the assumption made regarding the friction acting on the objects?
   a. Friction is zero.
   b. Friction is nearly zero.
   c. Friction acts constantly.
   d. Friction before and after the impact remains the same.

8. What is an inelastic collision?

a. when objects stick together after impact, and their internal energy is not conserved
b. when objects stick together after impact, and their internal energy is conserved

c. when objects stick together after impact, and always come to rest instantaneously after collision
d. when objects stick together after impact, and their internal energy increases

a. It will decrease.
b. It will increase.
c. It will become zero.
d. It will remain the same.

Critical Thinking Items

8.1 Linear Momentum, Force, and Impulse

9. Consider two objects of the same mass. If a force of $100\,N$ acts on the first for a duration of $1\,s$ and on the other for a duration of $2\,s$, which of the following statements is true?
   a. The first object will acquire more momentum.
   b. The second object will acquire more momentum.
   c. Both objects will acquire the same momentum.
   d. Neither object will experience a change in momentum.

10. Cars these days have parts that can crumple or collapse in the event of an accident. How does this help protect the passengers?
    a. It reduces injury to the passengers by increasing the time of impact.
    b. It reduces injury to the passengers by decreasing the time of impact.
    c. It reduces injury to the passengers by increasing the change in momentum.
    d. It reduces injury to the passengers by decreasing the change in momentum.

11. How much force would be needed to cause a $17\,kg \cdot m/s$ change in the momentum of an object, if the force acted for 5 seconds?
    a. 3.4 N
    b. 12 N
    c. 22 N
    d. 85 N

8.2 Conservation of Momentum

12. A billiards ball rolling on the table has momentum p_1. It hits another stationary ball, which then starts rolling. Considering friction to be negligible, what will happen to the momentum of the first ball?

13. A ball rolling on the floor with momentum p_1 collides with a stationary ball and sets it in motion. The momentum of the first ball becomes p'_1, and that of the second becomes p'_2. Compare the magnitudes of p_1 and p'_2.
    a. Momenta p_1 and p'_2 are the same in magnitude.
    b. The sum of the magnitudes of p_1 and p'_2 is zero.
    c. The magnitude of p_1 is greater than that of p'_2.
    d. The magnitude of p'_2 is greater than that of p_1.

14. Two cars are moving in the same direction. One car with momentum p_1 collides with another, which has momentum p_2. Their momenta become p'_1 and p'_2 respectively. Considering frictional losses, compare (p'_1 + p'_2) with (p_1 + p_2).
    a. The value of (p'_1 + p'_2) is zero.
    b. The values of (p_1 + p_2) and (p'_1 + p'_2) are equal.
    c. The value of (p_1 + p_2) will be greater than (p'_1 + p'_2).
    d. The value of (p'_1 + p'_2) will be greater than (p_1 + p_2).

8.3 Elastic and Inelastic Collisions

15. Two people, who have the same mass, throw two different objects at the same velocity. If the first object is heavier than the second, compare the velocities gained by the two people as a result of recoil.
    a. The first person will gain more velocity as a result of recoil.
    b. The second person will gain more velocity as a result of recoil.
    c. Both people will gain the same velocity as a result of recoil.
    d. The velocity of both people will be zero as a result of recoil.

Problems

8.1 Linear Momentum, Force, and Impulse

16. If a force of $50\,N$ is applied to an object for $0.2\,s$, and it changes its velocity by $10\,m/s$, what could be the mass of the object?
    a. 1 kg
    b. 2 kg

c. 5 kg
d. 250 kg

17. For how long should a force of 130 N be applied to an object of mass 50 kg to change its speed from 20 m/s to 60 m/s?
    a. 0.031 s
    b. 0.065 s
    c. 15.4 s

d.   40 s

8.3 Elastic and Inelastic Collisions

18. If a man with mass 70 kg, standing still, throws an object with mass 5 kg at 50 m/s, what will be the recoil velocity of the man, assuming he is standing on a frictionless surface?
     a.   −3.6 m/s
     b.   0 m/s
     c.   3.6 m/s

Performance Task

8.3 Elastic and Inelastic Collisions

20. You will need the following:
     - balls of different weights
     - a ruler or wooden strip
     - some books
     - a paper cup

     Make an inclined plane by resting one end of a ruler on a stack of books. Place a paper cup on the other end. Roll

TEST PREP

Multiple Choice

8.1 Linear Momentum, Force, and Impulse

21. What kind of quantity is momentum?
     a.   Scalar
     b.   Vector

22. When does the net force on an object increase?
     a.   When $\Delta\mathbf{p}$ decreases
     b.   When Δt increases
     c.   When Δt decreases

23. In the equation $\Delta\mathbf{p} = m(\mathbf{v}_f - \mathbf{v}_i)$, which quantity is considered to be constant?
     a.   Initial velocity
     b.   Final velocity
     c.   Mass
     d.   Momentum

24. For how long should a force of 50 N be applied to change the momentum of an object by 12 kg · m/s?
     a.   0.24 s
     b.   4.15 s
     c.   62 s
     d.   600 s

8.2 Conservation of Momentum

25. In the equation $\mathbf{L} = I\boldsymbol{\omega}$, what is I?

d.   50.0 m/s

19. Find the recoil velocity of a 65 kg ice hockey goalie who catches a 0.15 kg hockey puck slapped at him at a velocity of 50 m/s. Assume that the goalie is at rest before catching the puck, and friction between the ice and the puck-goalie system is negligible.
     a.   −0.12 m/s
     b.   0 m/s
     c.   0.12 m/s
     d.   7.5 m/s

a ball from the top of the ruler so that it hits the paper cup. Measure the displacement of the paper cup due to the collision. Now use increasingly heavier balls for this activity and see how that affects the displacement of the cup. Plot a graph of mass vs. displacement. Now repeat the same activity, but this time, instead of using different balls, change the incline of the ruler by varying the height of the stack of books. This will give you different velocities of the ball. See how this affects the displacement of the paper cup.

     a.   Linear momentum
     b.   Angular momentum
     c.   Torque
     d.   Moment of inertia

26. Give an example of an isolated system.
     a.   A cyclist moving along a rough road
     b.   A figure skater gliding in a straight line on an ice rink
     c.   A baseball player hitting a home run
     d.   A man drawing water from a well

8.3 Elastic and Inelastic Collisions

27. In which type of collision is kinetic energy conserved?
     a.   Elastic
     b.   Inelastic

28. In physics, what are structureless particles that cannot rotate or spin called?
     a.   Elastic particles
     b.   Point masses
     c.   Rigid masses

29. Two objects having equal masses and velocities collide with each other and come to a rest. What type of a collision is this and why?
     a.   Elastic collision, because internal kinetic energy is conserved

b.   Inelastic collision, because internal kinetic energy is not conserved

c.   Elastic collision, because internal kinetic energy is not conserved

d.   Inelastic collision, because internal kinetic energy is conserved

Short Answer

8.1 Linear Momentum, Force, and Impulse

31. If an object's velocity is constant, what is its momentum proportional to?
a.   Its shape
b.   Its mass
c.   Its length
d.   Its breadth

32. If both mass and velocity of an object are constant, what can you tell about its impulse?
a.   Its impulse would be constant.
b.   Its impulse would be zero.
c.   Its impulse would be increasing.
d.   Its impulse would be decreasing.

33. When the momentum of an object increases with respect to time, what is true of the net force acting on it?
a.   It is zero, because the net force is equal to the rate of change of the momentum.
b.   It is zero, because the net force is equal to the product of the momentum and the time interval.
c.   It is nonzero, because the net force is equal to the rate of change of the momentum.
d.   It is nonzero, because the net force is equal to the product of the momentum and the time interval.

34. How can you express impulse in terms of mass and velocity when neither of those are constant?
a. $\Delta \mathbf{p} = \Delta(m\mathbf{v})$
b. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{\Delta(m\mathbf{v})}{\Delta t}$
c. $\Delta \mathbf{p} = \Delta(\frac{m}{\mathbf{v}})$
d. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{1}{\Delta t} \cdot \Delta(m\mathbf{v})$

35. How can you express impulse in terms of mass and initial and final velocities?
a. $\Delta \mathbf{p} = m(\mathbf{v}_f - \mathbf{v}_i)$
b. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{m(\mathbf{v}_f - \mathbf{v}_i)}{\Delta t}$
c. $\Delta \mathbf{p} = \frac{(\mathbf{v}_f - \mathbf{v}_i)}{m}$
d. $\frac{\Delta \mathbf{p}}{\Delta t} = \frac{1}{m} \frac{(\mathbf{v}_f - \mathbf{v}_i)}{\Delta t}$

36. Why do we use average force while solving momentum problems? How is net force related to the momentum of the object?
a.   Forces are usually constant over a period of time,

and net force acting on the object is equal to the rate of change of the momentum.

b.   Forces are usually not constant over a period of time, and net force acting on the object is equal to the product of the momentum and the time interval.

c.   Forces are usually constant over a period of time, and net force acting on the object is equal to the product of the momentum and the time interval.

d.   Forces are usually not constant over a period of time, and net force acting on the object is equal to the rate of change of the momentum.

8.2 Conservation of Momentum

37. Under what condition(s) is the angular momentum of a system conserved?
a.   When net torque is zero
b.   When net torque is not zero
c.   When moment of inertia is constant
d.   When both moment of inertia and angular momentum are constant

38. If the moment of inertia of an isolated system increases, what happens to its angular velocity?
a.   It increases.
b.   It decreases.
c.   It stays constant.
d.   It becomes zero.

39. If both the moment of inertia and the angular velocity of a system increase, what must be true of the force acting on the system?
a.   Force is zero.
b.   Force is not zero.
c.   Force is constant.
d.   Force is decreasing.

8.3 Elastic and Inelastic Collisions

40. Two objects collide with each other and come to a rest. How can you use the equation of conservation of momentum to describe this situation?
a. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$
b. $m_1\mathbf{v}_1 - m_2\mathbf{v}_2 = 0$
c. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1'$
d. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_2$

30. Two objects having equal masses and velocities collide with each other and come to a rest. Is momentum conserved in this case?
a.   Yes
b.   No

41. What is the difference between momentum and impulse?
    a. Momentum is the sum of mass and velocity. Impulse is the change in momentum.
    b. Momentum is the sum of mass and velocity. Impulse is the rate of change in momentum.
    c. Momentum is the product of mass and velocity. Impulse is the change in momentum.
    d. Momentum is the product of mass and velocity. Impulse is the rate of change in momentum.

42. What is the equation for conservation of momentum along the x-axis for 2D collisions in terms of mass and velocity, where one of the particles is initially at rest?

    a. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1$
    b. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1 + m_2\mathbf{v}_2'\cos\theta_2$
    c. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1 - m_2\mathbf{v}_2'\cos\theta_2$
    d. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\sin\theta_1 + m_2\mathbf{v}_2'\sin\theta_2$

43. What is the equation for conservation of momentum along the y-axis for 2D collisions in terms of mass and velocity, where one of the particles is initially at rest?
    a. $0 = m_1\mathbf{v}_1'\sin\theta_1$
    b. $0 = m_1\mathbf{v}_1'\sin\theta_1 + m_2\mathbf{v}_2'\sin\theta_2$
    c. $0 = m_1\mathbf{v}_1'\sin\theta_1 - m_2\mathbf{v}_2'\sin\theta_2$
    d. $0 = m_1\mathbf{v}_1'\cos\theta_1 + m_2\mathbf{v}_2'\cos\theta_2$

Extended Response

8.1 Linear Momentum, Force, and Impulse

44. Can a lighter object have more momentum than a heavier one? How?
    a. No, because momentum is independent of the velocity of the object.
    b. No, because momentum is independent of the mass of the object.
    c. Yes, if the lighter object's velocity is considerably high.
    d. Yes, if the lighter object's velocity is considerably low.

45. Why does it hurt less when you fall on a softer surface?
    a. The softer surface increases the duration of the impact, thereby reducing the effect of the force.
    b. The softer surface decreases the duration of the impact, thereby reducing the effect of the force.
    c. The softer surface increases the duration of the impact, thereby increasing the effect of the force.
    d. The softer surface decreases the duration of the impact, thereby increasing the effect of the force.

46. Can we use the equation $\mathrm{F}_{net} = \frac{\Delta p}{\Delta t}$ when the mass is constant?
    a. No, because the given equation is applicable for the variable mass only.
    b. No, because the given equation is not applicable for the constant mass.
    c. Yes, and the resultant equation is $\mathrm{F} = m\mathbf{v}$
    d. Yes, and the resultant equation is $\mathrm{F} = ma$

8.2 Conservation of Momentum

47. Why does a figure skater spin faster if he pulls his arms and legs in?
    a. Due to an increase in moment of inertia
    b. Due to an increase in angular momentum
    c. Due to conservation of linear momentum
    d. Due to conservation of angular momentum

8.3 Elastic and Inelastic Collisions

48. A driver sees another car approaching him from behind. He fears it is going to collide with his car. Should he speed up or slow down in order to reduce damage?
    a. He should speed up.
    b. He should slow down.
    c. He should speed up and then slow down just before the collision.
    d. He should slow down and then speed up just before the collision.

49. What approach would you use to solve problems involving 2D collisions?
    a. Break the momenta into components and then choose a coordinate system.
    b. Choose a coordinate system and then break the momenta into components.
    c. Find the total momenta in the x and y directions, and then equate them to solve for the unknown.
    d. Find the sum of the momenta in the x and y directions, and then equate it to zero to solve for the unknown.

CHAPTER 9
Work, Energy, and Simple Machines

Figure 9.1 People on a roller coaster experience thrills caused by changes in types of energy. (Jonrev, Wikimedia Commons)

Chapter Outline

INTRODUCTION Roller coasters have provided thrills for daring riders around the world since the nineteenth century. Inventors of roller coasters used simple physics to build the earliest examples using railroad tracks on mountainsides and old mines. Modern roller coaster designers use the same basic laws of physics to create the latest amusement park favorites. Physics principles are used to engineer the machines that do the work to lift a roller coaster car up its first big incline before it is set loose to roll. Engineers also have to understand the changes in the car's energy that keep it speeding over hills, through twists, turns, and even loops.

What exactly is energy? How can changes in force, energy, and simple machines move objects like roller coaster cars? How can machines help us do work? In this chapter, you will discover the answer to this question and many more, as you learn about

work, energy, and simple machines.

9.1 Work, Power, and the Work–Energy Theorem

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe and apply the work–energy theorem
- Describe and calculate work and power

Section Key Terms

energy	gravitational potential energy	joule	kinetic energy	mechanical energy
potential energy	power		watt	work	work–energy theorem

The Work–Energy Theorem

In physics, the term **work** has a very specific definition. Work is application of force, $\mathbf{f}$, to move an object over a distance, d, in the direction that the force is applied. Work, W, is described by the equation

$$W = \mathbf{f}d.$$

Some things that we typically consider to be work are not work in the scientific sense of the term. Let's consider a few examples. Think about why each of the following statements is true.

- Homework *is not* work.
- Lifting a rock upwards off the ground *is* work.
- Carrying a rock in a straight path across the lawn at a constant speed *is not* work.

The first two examples are fairly simple. Homework is not work because objects are not being moved over a distance. Lifting a rock up off the ground is work because the rock is moving in the direction that force is applied. The last example is less obvious. Recall from the laws of motion that force is *not* required to move an object at constant velocity. Therefore, while some force may be applied to keep the rock up off the ground, no net force is applied to keep the rock moving forward at constant velocity.

Work and **energy** are closely related. When you do work to move an object, you change the object's energy. You (or an object) also expend energy to do work. In fact, energy can be defined as the ability to do work. Energy can take a variety of different forms, and one form of energy can transform to another. In this chapter we will be concerned with **mechanical energy**, which comes in two forms: **kinetic energy** and **potential energy**.

- Kinetic energy is also called energy of motion. A moving object has kinetic energy.
- Potential energy, sometimes called stored energy, comes in several forms. **Gravitational potential energy** is the stored energy an object has as a result of its position above Earth's surface (or another object in space). A roller coaster car at the top of a hill has gravitational potential energy.

Let's examine how doing work on an object changes the object's energy. If we apply force to lift a rock off the ground, we increase the rock's potential energy, PE. If we drop the rock, the force of gravity increases the rock's kinetic energy as the rock moves downward until it hits the ground.

The force we exert to lift the rock is equal to its weight, w, which is equal to its mass, m, multiplied by acceleration due to gravity, $\mathbf{g}$.

$$\mathbf{f} = w = m\mathbf{g}$$

The work we do on the rock equals the force we exert multiplied by the distance, d, that we lift the rock. The work we do on the rock also equals the rock's gain in gravitational potential energy, PE_e.

$$W = PE_e = \mathbf{f}m\mathbf{g}$$

Kinetic energy depends on the mass of an object and its velocity, $\mathbf{v}$.

$$KE = \frac{1}{2}m\mathbf{v}^2$$

When we drop the rock the force of gravity causes the rock to fall, giving the rock kinetic energy. When work done on an object increases only its kinetic energy, then the net work equals the change in the value of the quantity $\frac{1}{2}m\mathbf{v}^2$. This is a statement of the **work–energy theorem**, which is expressed mathematically as

$$W = \Delta KE = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2.$$

The subscripts $_2$ and $_1$ indicate the final and initial velocity, respectively. This theorem was proposed and successfully tested by James Joule, shown in Figure 9.2.

Does the name Joule sound familiar? The **joule** (J) is the metric unit of measurement for both work and energy. The measurement of work and energy with the same unit reinforces the idea that work and energy are related and can be converted into one another. 1.0 J = 1.0 N•m, the units of force multiplied by distance. 1.0 N = 1.0 k•m/s², so 1.0 J = 1.0 k•m²/s². Analyzing the units of the term $(1/2)m\mathbf{v}^2$ will produce the same units for joules.

Figure 9.2 The joule is named after physicist James Joule (1818–1889). (C. H. Jeens, Wikimedia Commons)

👁 WATCH PHYSICS

Work and Energy
This video explains the work energy theorem and discusses how work done on an object increases the object's KE.

Click to view content (https://www.khanacademy.org/embed_video?v=2WS1sG9fhOk)

> **GRASP CHECK**
> True or false—The energy increase of an object acted on only by a gravitational force is equal to the product of the object's weight and the distance the object falls.
> a. True
> b. False

Calculations Involving Work and Power

In applications that involve work, we are often interested in how fast the work is done. For example, in roller coaster design, the amount of time it takes to lift a roller coaster car to the top of the first hill is an important consideration. Taking a half hour on the ascent will surely irritate riders and decrease ticket sales. Let's take a look at how to calculate the time it takes to do work.

Recall that a rate can be used to describe a quantity, such as work, over a period of time. **Power** is the rate at which work is done. In this case, rate means *per unit of time*. Power is calculated by dividing the work done by the time it took to do the work.

$$P = \frac{W}{t}$$

Let's consider an example that can help illustrate the differences among work, force, and power. Suppose the woman in Figure 9.3 lifting the TV with a pulley gets the TV to the fourth floor in two minutes, and the man carrying the TV up the stairs takes five

minutes to arrive at the same place. They have done the same amount of work ($\mathbf{f}d$) on the TV, because they have moved the same mass over the same vertical distance, which requires the same amount of upward force. However, the woman using the pulley has generated more power. This is because she did the work in a shorter amount of time, so the denominator of the power formula, t, is smaller. (For simplicity's sake, we will leave aside for now the fact that the man climbing the stairs has also done work on himself.)
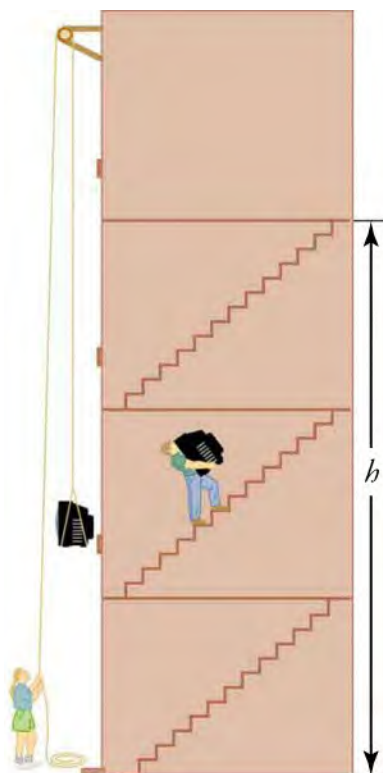
Figure 9.3 No matter how you move a TV to the fourth floor, the amount of work performed and the potential energy gain are the same.

Power can be expressed in units of **watts** (W). This unit can be used to measure power related to any form of energy or work. You have most likely heard the term used in relation to electrical devices, especially light bulbs. Multiplying power by time gives the amount of energy. Electricity is sold in kilowatt-hours because that equals the amount of electrical energy consumed.

The watt unit was named after James Watt (1736–1819) (see Figure 9.4). He was a Scottish engineer and inventor who discovered how to coax more power out of steam engines.

Figure 9.4 Is James Watt thinking about watts? (Carl Frederik von Breda, Wikimedia Commons)

🔗 LINKS TO PHYSICS

Watt's Steam Engine

James Watt did not invent the steam engine, but by the time he was finished tinkering with it, it was more useful. The first steam engines were not only inefficient, they only produced a back and forth, or reciprocal, motion. This was natural because pistons move in and out as the pressure in the chamber changes. This limitation was okay for simple tasks like pumping water or mashing potatoes, but did not work so well for moving a train. Watt was able build a steam engine that converted reciprocal motion to circular motion. With that one innovation, the industrial revolution was off and running. The world would never be the same. One of Watt's steam engines is shown in Figure 9.5. The video that follows the figure explains the importance of the steam engine in the industrial revolution.
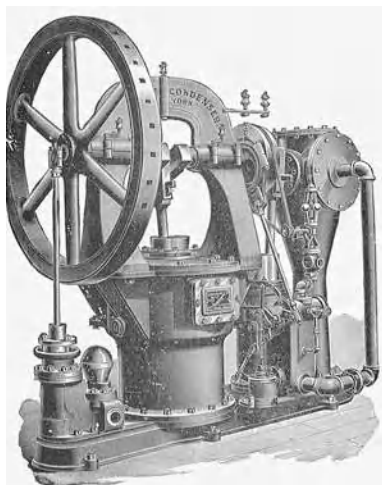
Figure 9.5 A late version of the Watt steam engine. (Nehemiah Hawkins, Wikimedia Commons)

👁 WATCH PHYSICS

Watt's Role in the Industrial Revolution

This video demonstrates how the watts that resulted from Watt's inventions helped make the industrial revolution possible and allowed England to enter a new historical era.

Click to view content (https://www.youtube.com/embed/zhL5DCizj5c)

> **GRASP CHECK**
> Which form of mechanical energy does the steam engine generate?
> a. Potential energy
> b. Kinetic energy
> c. Nuclear energy
> d. Solar energy

Before proceeding, be sure you understand the distinctions among force, work, energy, and power. Force exerted on an object over a distance does work. Work can increase energy, and energy can do work. Power is the rate at which work is done.

✳ WORKED EXAMPLE

Applying the Work–Energy Theorem

An ice skater with a mass of 50 kg is gliding across the ice at a speed of 8 m/s when her friend comes up from behind and gives her a push, causing her speed to increase to 12 m/s. How much work did the friend do on the skater?

Strategy

The work–energy theorem can be applied to the problem. Write the equation for the theorem and simplify it if possible.

$$W = \Delta\text{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$$

$$\text{Simplify to } W = \frac{1}{2}m(\mathbf{v}_2^2 - \mathbf{v}_1^2)$$

Solution

Identify the variables. $m = 50$ kg,

$$\mathbf{v}_2 = 12\frac{\text{m}}{\text{s}}, \text{ and } \mathbf{v}_1 = 8\frac{\text{m}}{\text{s}}$$

9.1

Substitute.

$$W = \frac{1}{2}50(12^2 - 8^2) = 2,000 \text{ J}$$

9.2

Discussion

Work done on an object or system increases its energy. In this case, the increase is to the skater's kinetic energy. It follows that the increase in energy must be the difference in KE before and after the push.

> **TIPS FOR SUCCESS**
>
> This problem illustrates a general technique for approaching problems that require you to apply formulas: Identify the unknown and the known variables, express the unknown variables in terms of the known variables, and then enter all the known values.

Practice Problems

1. How much work is done when a weightlifter lifts a 200 N barbell from the floor to a height of 2 m?
   a. 0 J
   b. 100 J
   c. 200 J
   d. 400 J

2. Identify which of the following actions generates more power. Show your work.

   - carrying a 100 N TV to the second floor in 50 s or
   - carrying a 24 N watermelon to the second floor in 10 s?

   a. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as work done times the time interval.
   b. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as the ratio of work done to the time interval.
   c. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as work done times the time interval.
   d. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as the ratio of work done and the time interval.

Check Your Understanding

3. Identify two properties that are expressed in units of joules.
   a. work and force
   b. energy and weight
   c. work and energy
   d. weight and force

4. When a coconut falls from a tree, work W is done on it as it falls to the beach. This work is described by the equation

$$W = Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2.$$

9.3

Identify the quantities F, d, m, v_1, and v_2 in this event.
   a. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
   b. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
   c. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.
   d. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.

9.2 Mechanical Energy and Conservation of Energy

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Explain the law of conservation of energy in terms of kinetic and potential energy
- Perform calculations related to kinetic and potential energy. Apply the law of conservation of energy

Section Key Terms

law of conservation of energy

Mechanical Energy and Conservation of Energy

We saw earlier that mechanical energy can be either potential or kinetic. In this section we will see how energy is transformed from one of these forms to the other. We will also see that, in a closed system, the sum of these forms of energy remains constant.

Quite a bit of potential energy is gained by a roller coaster car and its passengers when they are raised to the top of the first hill. Remember that the *potential* part of the term means that energy has been stored and can be used at another time. You will see that this stored energy can either be used to do work or can be transformed into kinetic energy. For example, when an object that has gravitational potential energy falls, its energy is converted to kinetic energy. Remember that both work and energy are expressed in joules.

Refer back to . The amount of work required to raise the TV from point A to point B is equal to the amount of gravitational potential energy the TV gains from its height above the ground. This is generally true for any object raised above the ground. If all the work done on an object is used to raise the object above the ground, the amount work equals the object's gain in gravitational potential energy. However, note that because of the work done by friction, these energy–work transformations are never perfect. Friction causes the loss of some useful energy. In the discussions to follow, we will use the approximation that transformations are frictionless.

Now, let's look at the roller coaster in [Figure 9.6](#). Work was done on the roller coaster to get it to the top of the first rise; at this point, the roller coaster has gravitational potential energy. It is moving slowly, so it also has a small amount of kinetic energy. As the car descends the first slope, its *PE* is converted to *KE*. At the low point much of the original *PE* has been transformed to *KE*, and speed is at a maximum. As the car moves up the next slope, some of the *KE* is transformed back into *PE* and the car slows down.

Figure 9.6 During this roller coaster ride, there are conversions between potential and kinetic energy.

Virtual Physics

Energy Skate Park Basics

This simulation shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in *KE* and *PE* by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation.

Click to view content (http://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html)

GRASP CHECK

This simulation (http://phet.colorado.edu/en/simulation/energy-skate-park-basics (http://phet.colorado.edu/en/simulation/energy-skate-park-basics)) shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in KE and PE by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation. The bar graphs show how KE and PE are transformed back and forth. Which statement best explains what happens to the mechanical energy of the system as speed is increasing?

a. The mechanical energy of the system increases, provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
b. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
c. The mechanical energy of the system increases provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.
d. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.

On an actual roller coaster, there are many ups and downs, and each of these is accompanied by transitions between kinetic and potential energy. Assume that no energy is lost to friction. At any point in the ride, the total mechanical energy is the same, and it is equal to the energy the car had at the top of the first rise. This is a result of the **law of conservation of energy**, which says that, in a closed system, total energy is conserved—that is, it is constant. Using subscripts 1 and 2 to represent initial and final energy, this law is expressed as

$$KE_1 + PE_1 = KE_2 + PE_2.$$

Either side equals the total mechanical energy. The phrase *in a closed system* means we are assuming no energy is lost to the surroundings due to friction and air resistance. If we are making calculations on dense falling objects, this is a good assumption. For the roller coaster, this assumption introduces some inaccuracy to the calculation.

Calculations involving Mechanical Energy and Conservation of Energy

TIPS FOR SUCCESS

When calculating work or energy, use units of meters for distance, newtons for force, kilograms for mass, and seconds for time. This will assure that the result is expressed in joules.

WATCH PHYSICS

Conservation of Energy

This video discusses conversion of *PE* to *KE* and conservation of energy. The scenario is very similar to the roller coaster and the skate park. It is also a good explanation of the energy changes studied in the snap lab.

Click to view content (https://www.khanacademy.org/embed_video?v=kw_4Loo1HR4)

GRASP CHECK

Did you expect the speed at the bottom of the slope to be the same as when the object fell straight down? Which statement best explains why this is not exactly the case in real-life situations?
a. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
b. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
c. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.
d. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.

WORKED EXAMPLE

Applying the Law of Conservation of Energy

A 10 kg rock falls from a 20 m cliff. What is the kinetic and potential energy when the rock has fallen 10 m?

Strategy

Choose the equation.

$$KE_1 + PE_1 = KE_2 + PE_2$$ 9.4

$$KE = \tfrac{1}{2}m\mathbf{v}^2; \quad PE = m\mathbf{g}h$$ 9.5

$$\frac{1}{2}m\mathbf{v}_1^2 + m\mathbf{g}h_1 = \frac{1}{2}m\mathbf{v}_2^2 + m\mathbf{g}h_2$$ 9.6

List the knowns.

$m = 10$ kg, $\mathbf{v}_1 = 0$, $\mathbf{g} = 9.80$

$$\frac{\text{m}}{\text{s}^2},$$ 9.7

$h_1 = 20$ m, $h_2 = 10$ m

Identify the unknowns.

KE_2 and PE_2

Substitute the known values into the equation and solve for the unknown variables.

Solution

$$PE_2 = m\mathbf{g}h_2 = 10\,(9.80)\,10 = 980\ \text{J} \qquad \boxed{9.8}$$

$$KE_2 = PE_2 - (KE_1 + PE_1) = 980 - \{[0 - [10\,(9.80)\,20]]\} = 980\ \text{J} \qquad \boxed{9.9}$$

Discussion

Alternatively, conservation of energy equation could be solved for $\mathbf{v}_2$ and KE_2 could be calculated. Note that m could also be eliminated.

TIPS FOR SUCCESS

Note that we can solve many problems involving conversion between KE and PE without knowing the mass of the object in question. This is because kinetic and potential energy are both proportional to the mass of the object. In a situation where $KE = PE$, we know that $m\mathbf{g}h = (1/2)m\mathbf{v}^2$.

Dividing both sides by m and rearranging, we have the relationship

$2\mathbf{g}h = \mathbf{v}^2$.

Practice Problems

5. A child slides down a playground slide. If the slide is 3 m high and the child weighs 300 N, how much potential energy does the child have at the top of the slide? (Round g to $10\ m/s^2$.)

   a.   0 J

   b.   100 J

   c.   300 J

   d.   900 J

6. A 0.2 kg apple on an apple tree has a potential energy of 10 J. It falls to the ground, converting all of its PE to kinetic energy. What is the velocity of the apple just before it hits the ground?

   a.   0 m/s

   b.   2 m/s

   c.   10 m/s

   d.   50 m/s

Snap Lab

Converting Potential Energy to Kinetic Energy

In this activity, you will calculate the potential energy of an object and predict the object's speed when all that potential energy has been converted to kinetic energy. You will then check your prediction.

You will be dropping objects from a height. Be sure to stay a safe distance from the edge. Don't lean over the railing too far. Make sure that you do not drop objects into an area where people or vehicles pass by. Make sure that dropping objects will not cause damage.

You will need the following:

Materials for each pair of students:
- Four marbles (or similar small, dense objects)
- Stopwatch

Materials for class:
- Metric measuring tape long enough to measure the chosen height
- A scale

Instructions

Procedure

1. Work with a partner. Find and record the mass of four small, dense objects per group.
2. Choose a location where the objects can be safely dropped from a height of at least 15 meters. A bridge over water with a safe pedestrian walkway will work well.
3. Measure the distance the object will fall.
4. Calculate the potential energy of the object before you drop it using $PE = m\mathbf{g}h = (9.80)mh$.
5. Predict the kinetic energy and velocity of the object when it lands using $PE = KE$ and so,
    $$m\mathbf{g}h = \frac{m\mathbf{v}^2}{2}; \; \mathbf{v} = \sqrt{2(9.80)h} = 4.43\sqrt{h}.$$
6. One partner drops the object while the other measures the time it takes to fall.
7. Take turns being the dropper and the timer until you have made four measurements.
8. Average your drop multiplied by and calculate the velocity of the object when it landed using $\mathbf{v} = \mathbf{a}t = \mathbf{g}t = (9.80)t$.
9. Compare your results to your prediction.

GRASP CHECK

Galileo's experiments proved that, contrary to popular belief, heavy objects do not fall faster than light objects. How do the equations you used support this fact?

a. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
b. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
c. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.
d. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.

Check Your Understanding

7. Describe the transformation between forms of mechanical energy that is happening to a falling skydiver before his parachute opens.
    a. Kinetic energy is being transformed into potential energy.
    b. Potential energy is being transformed into kinetic energy.
    c. Work is being transformed into kinetic energy.
    d. Kinetic energy is being transformed into work.

8. True or false—If a rock is thrown into the air, the increase in the height would increase the rock's kinetic energy, and then the increase in the velocity as it falls to the ground would increase its potential energy.
    a. True
    b. False

9. Identify equivalent terms for *stored energy* and *energy of motion*.
    a. Stored energy is potential energy, and energy of motion is kinetic energy.
    b. Energy of motion is potential energy, and stored energy is kinetic energy.
    c. Stored energy is the potential as well as the kinetic energy of the system.
    d. Energy of motion is the potential as well as the kinetic energy of the system.

9.3 Simple Machines

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe simple and complex machines
- Calculate mechanical advantage and efficiency of simple and complex machines

Section Key Terms

complex machine	efficiency output	ideal mechanical advantage	inclined plane	input work
lever	mechanical advantage	output work	pulley	screw
simple machine	wedge	wheel and axle		

Simple Machines

Simple machines make work easier, but they do not decrease the amount of work you have to do. Why can't simple machines change the amount of work that you do? Recall that in closed systems the total amount of energy is conserved. A machine cannot increase the amount of energy you put into it. So, why is a simple machine useful? Although it cannot change the amount of work you do, a simple machine can change the amount of force you must apply to an object, and the distance over which you apply the force. In most cases, a simple machine is used to reduce the amount of force you must exert to do work. The down side is that you must exert the force over a greater distance, because the product of force and distance, **f**d, (which equals work) does not change.

Let's examine how this works in practice. In Figure 9.7(a), the worker uses a type of **lever** to exert a small force over a large distance, while the pry bar pulls up on the nail with a large force over a small distance. Figure 9.7(b) shows the how a lever works mathematically. The effort force, applied at $\mathbf{F_e}$, lifts the load (the resistance force) which is pushing down at $\mathbf{F_r}$. The triangular pivot is called the fulcrum; the part of the lever between the fulcrum and $\mathbf{F_e}$ is the effort arm, L_e; and the part to the left is the resistance arm, L_r. The **mechanical advantage** is a number that tells us how many times a simple machine multiplies the effort force. The **ideal mechanical advantage**, *IMA*, is the mechanical advantage of a perfect machine with no loss of useful work caused by friction between moving parts. The equation for *IMA* is shown in Figure 9.7(b).
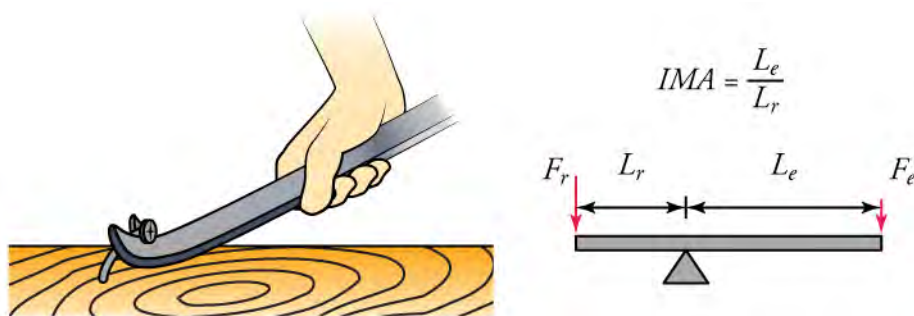
Figure 9.7 (a) A pry bar is a type of lever. (b) The ideal mechanical advantage equals the length of the effort arm divided by the length of the resistance arm of a lever.

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Getting back to conservation of energy, for any simple machine, the work put into the machine, W_i, equals the work the machine puts out, W_o. Combining this with the information in the paragraphs above, we can write

$$W_i = W_o$$
$$\mathbf{F}_e d_e = \mathbf{F}_r d_r$$
$$\text{If } \mathbf{F}_e < \mathbf{F}_r, \text{ then } d_e > d_r.$$

The equations show how a simple machine can output the same amount of work while reducing the amount of effort force by increasing the distance over which the effort force is applied.

👁 **WATCH PHYSICS**

Introduction to Mechanical Advantage

This video shows how to calculate the *IMA* of a lever by three different methods: (1) from effort force and resistance force; (2) from the lengths of the lever arms, and; (3) from the distance over which the force is applied and the distance the load moves.

Click to view content (https://www.youtube.com/embed/pfzJ-z5Ij48)

> **GRASP CHECK**
>
> Two children of different weights are riding a seesaw. How do they position themselves with respect to the pivot point (the fulcrum) so that they are balanced?
> a. The heavier child sits closer to the fulcrum.
> b. The heavier child sits farther from the fulcrum.
> c. Both children sit at equal distance from the fulcrum.
> d. Since both have different weights, they will never be in balance.

Some levers exert a large force to a short effort arm. This results in a smaller force acting over a greater distance at the end of the resistance arm. Examples of this type of lever are baseball bats, hammers, and golf clubs. In another type of lever, the fulcrum is at the end of the lever and the load is in the middle, as in the design of a wheelbarrow.

The simple machine shown in Figure 9.8 is called a **wheel and axle.** It is actually a form of lever. The difference is that the effort arm can rotate in a complete circle around the fulcrum, which is the center of the axle. Force applied to the outside of the wheel causes a greater force to be applied to the rope that is wrapped around the axle. As shown in the figure, the ideal mechanical advantage is calculated by dividing the radius of the wheel by the radius of the axle. Any crank-operated device is an example of a wheel and axle.
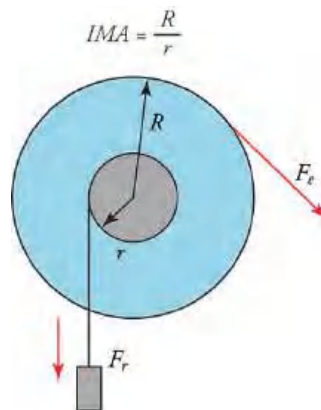
Figure 9.8 Force applied to a wheel exerts a force on its axle.

An **inclined plane** and a **wedge** are two forms of the same simple machine. A wedge is simply two inclined planes back to back. Figure 9.9 shows the simple formulas for calculating the *IMA*s of these machines. All sloping, paved surfaces for walking or driving are inclined planes. Knives and axe heads are examples of wedges.

$$IMA = \frac{L}{h} \qquad\qquad IMA = \frac{L}{t}$$
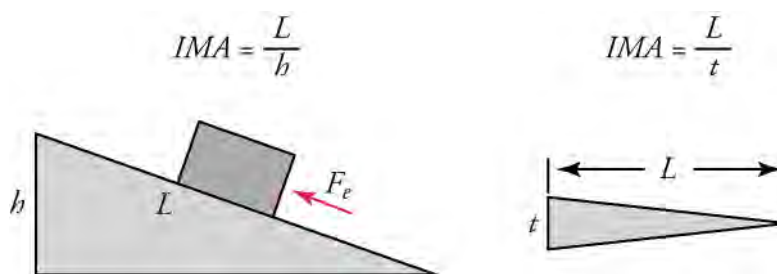
Figure 9.9 An inclined plane is shown on the left, and a wedge is shown on the right.

The **screw** shown in Figure 9.10 is actually a lever attached to a circular inclined plane. Wood screws (of course) are also examples of screws. The lever part of these screws is a screw driver. In the formula for IMA, the distance between screw threads is called *pitch* and has the symbol P.

$$IMA = \frac{2\pi L}{P}$$
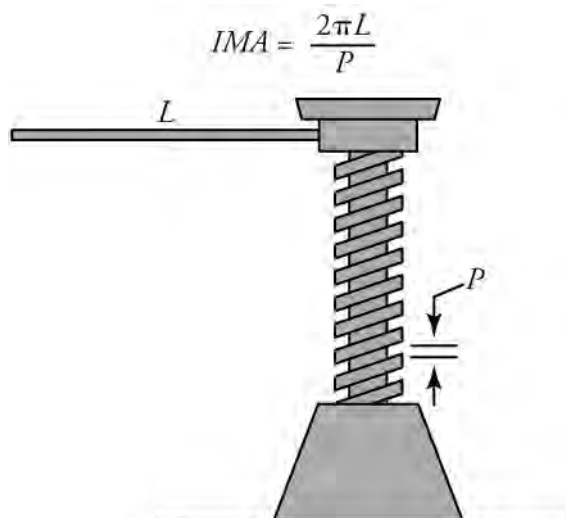
Figure 9.10 The screw shown here is used to lift very heavy objects, like the corner of a car or a house a short distance.

Figure 9.11 shows three different **pulley** systems. Of all simple machines, mechanical advantage is easiest to calculate for pulleys. Simply count the number of ropes supporting the load. That is the IMA. Once again we have to exert force over a longer distance to multiply force. To raise a load 1 meter with a pulley system you have to pull N meters of rope. Pulley systems are often used to raise flags and window blinds and are part of the mechanism of construction cranes.

$$IMA = N$$
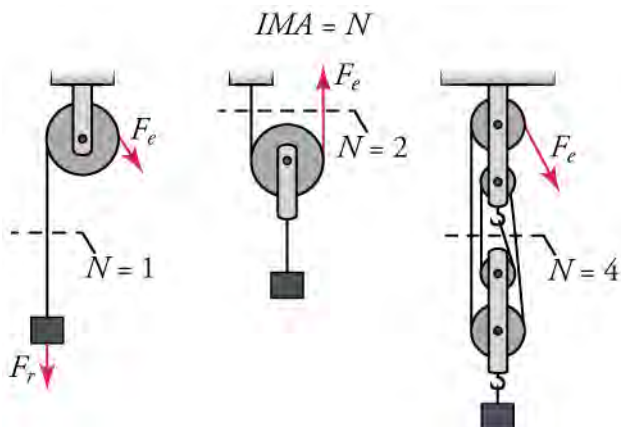
Figure 9.11 Three pulley systems are shown here.

👁 WATCH PHYSICS

Mechanical Advantage of Inclined Planes and Pulleys
The first part of this video shows how to calculate the *IMA* of pulley systems. The last part shows how to calculate the *IMA* of an inclined plane.

Click to view content (https://www.khanacademy.org/embed_video?v=vSsK7Rfa3yA)

> **GRASP CHECK**
> How could you use a pulley system to lift a light load to great height?
> a. Reduce the radius of the pulley.
> b. Increase the number of pulleys.
> c. Decrease the number of ropes supporting the load.
> d. Increase the number of ropes supporting the load.

A **complex machine** is a combination of two or more simple machines. The wire cutters in Figure 9.12 combine two levers and two wedges. Bicycles include wheel and axles, levers, screws, and pulleys. Cars and other vehicles are combinations of many machines.

Figure 9.12 Wire cutters are a common complex machine.

Calculating Mechanical Advantage and Efficiency of Simple Machines

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Refer back to the discussions of each simple machine for the specific equations for the *IMA* for each type of machine.

No simple or complex machines have the actual mechanical advantages calculated by the *IMA* equations. In real life, some of the applied work always ends up as wasted heat due to friction between moving parts. Both the **input work** (W_i) and **output work** (W_o) are the result of a force, $\mathbf{F}$, acting over a distance, d.

$$W_i = \mathbf{F}_i d_i \text{ and } W_o = \mathbf{F}_o d_o$$

The **efficiency output** of a machine is simply the output work divided by the input work, and is usually multiplied by 100 so that it is expressed as a percent.

$$\% \text{ efficiency} = \frac{W_o}{W_i} \times 100$$

Look back at the pictures of the simple machines and think about which would have the highest efficiency. Efficiency is related to friction, and friction depends on the smoothness of surfaces and on the area of the surfaces in contact. How would lubrication affect the efficiency of a simple machine?

✳ WORKED EXAMPLE

Efficiency of a Lever
The input force of 11 N acting on the effort arm of a lever moves 0.4 m, which lifts a 40 N weight resting on the resistance arm a

distance of 0.1 m. What is the efficiency of the machine?

Strategy

State the equation for efficiency of a simple machine, $\% \text{ efficiency} = \frac{W_o}{W_i} \times 100$, and calculate W_o and W_i. Both work values are the product Fd.

Solution

$W_i = \mathbf{F}_i d_i$ = (11)(0.4) = 4.4 J and $W_o = \mathbf{F}_o d_o$ = (40)(0.1) = 4.0 J, then $\% \text{ efficiency} = \frac{W_o}{W_i} \times 100 = \frac{4.0}{4.4} \times 100 = 91\%$

Discussion

Efficiency in real machines will always be less than 100 percent because of work that is converted to unavailable heat by friction and air resistance. W_o and W_i can always be calculated as a force multiplied by a distance, although these quantities are not always as obvious as they are in the case of a lever.

Practice Problems

10. What is the IMA of an inclined plane that is 5 m long and 2 m high?
    a. 0.4
    b. 2.5
    c. 0.4 m
    d. 2.5 m

11. If a pulley system can lift a 200N load with an effort force of 52 N and has an efficiency of almost 100 percent, how many ropes are supporting the load?
    a. 1 rope is required because the actual mechanical advantage is 0.26.
    b. 1 rope is required because the actual mechanical advantage is 3.80.
    c. 4 ropes are required because the actual mechanical advantage is 0.26.
    d. 4 ropes are required because the actual mechanical advantage is 3.80.

Check Your Understanding

12. True or false—The efficiency of a simple machine is always less than 100 percent because some small fraction of the input work is always converted to heat energy due to friction.
    a. True
    b. False

13. The circular handle of a faucet is attached to a rod that opens and closes a valve when the handle is turned. If the rod has a diameter of 1 cm and the IMA of the machine is 6, what is the radius of the handle?
    A. 0.08 cm
    B. 0.17 cm
    C. 3.0 cm
    D. 6.0 cm

KEY TERMS

complex machine   a machine that combines two or more simple machines

efficiency   output work divided by input work

energy   the ability to do work

gravitational potential energy   energy acquired by doing work against gravity

ideal mechanical advantage   the mechanical advantage of an idealized machine that loses no energy to friction

inclined plane   a simple machine consisting of a slope

input work   effort force multiplied by the distance over which it is applied

joule   the metric unit for work and energy; equal to 1 newton meter (N•m)

kinetic energy   energy of motion

law of conservation of energy   states that energy is neither created nor destroyed

lever   a simple machine consisting of a rigid arm that pivots on a fulcrum

mechanical advantage   the number of times the input force is multiplied

mechanical energy   kinetic or potential energy

output work   output force multiplied by the distance over which it acts

potential energy   stored energy

power   the rate at which work is done

pulley   a simple machine consisting of a rope that passes over one or more grooved wheels

screw   a simple machine consisting of a spiral inclined plane

simple machine   a machine that makes work easier by changing the amount or direction of force required to move an object

watt   the metric unit of power; equivalent to joules per second

wedge   a simple machine consisting of two back-to-back inclined planes

wheel and axle   a simple machine consisting of a rod fixed to the center of a wheel

work   force multiplied by distance

work–energy theorem   states that the net work done on a system equals the change in kinetic energy

SECTION SUMMARY

9.1 Work, Power, and the Work–Energy Theorem

- Doing work on a system or object changes its energy.
- The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object. The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.
- Power is the rate at which work is done.

9.2 Mechanical Energy and Conservation of Energy

- Mechanical energy may be either kinetic (energy of motion) or potential (stored energy).
- Doing work on an object or system changes its energy.
- Total energy in a closed, isolated system is constant.

9.3 Simple Machines

- The six types of simple machines make work easier by changing the $\mathbf{f}d$ term so that force is reduced at the expense of increased distance.
- The ratio of output force to input force is a machine's mechanical advantage.
- Combinations of two or more simple machines are called complex machines.
- The ratio of output work to input work is a machine's efficiency.

KEY EQUATIONS

9.1 Work, Power, and the Work–Energy Theorem

equation for work	$W = \mathbf{f}d$
force	$\mathbf{f} = w = m\mathbf{g}$
work equivalencies	$W = PE_e = \mathbf{f}m\mathbf{g}$

kinetic energy	$KE = \frac{1}{2}m\mathbf{v}^2$
work–energy theorem	$W = \Delta\mathrm{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$
power	$P = \frac{W}{t}$

9.2 Mechanical Energy and Conservation of Energy

conservation of energy $\quad KE_1 + PE_1 = KE_2 + PE_2$

9.3 Simple Machines

ideal mechanical advantage (general)	$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$
ideal mechanical advantage (lever)	$IMA = \frac{L_e}{L_r}$
ideal mechanical advantage (wheel and axle)	$IMA = \frac{R}{r}$

ideal mechanical advantage (inclined plane)	$IMA = \frac{L}{h}$
ideal mechanical advantage (wedge)	$IMA = \frac{L}{t}$
ideal mechanical advantage (pulley)	$IMA = N$
ideal mechanical advantage (screw)	$IMA = \frac{2\pi L}{P}$
input work	$W_i = \mathbf{F}_i d_i$
output work	$W_o = \mathbf{F}_o d_o$
efficiency output	$\% \text{ efficiency} = \frac{W_o}{W_i} \times 100$

CHAPTER REVIEW

Concept Items

9.1 Work, Power, and the Work–Energy Theorem

1. Is it possible for the sum of kinetic energy and potential energy of an object to change without work having been done on the object? Explain.
   a. No, because the work-energy theorem states that work done on an object is equal to the change in kinetic energy, and change in KE requires a change in velocity. It is assumed that mass is constant.
   b. No, because the work-energy theorem states that work done on an object is equal to the sum of kinetic energy, and the change in KE requires a change in displacement. It is assumed that mass is constant.
   c. Yes, because the work-energy theorem states that work done on an object is equal to the change in kinetic energy, and change in KE requires a change in velocity. It is assumed that mass is constant.
   d. Yes, because the work-energy theorem states that work done on an object is equal to the sum of kinetic energy, and the change in KE requires a change in displacement. It is assumed that mass is constant.

2. Define work for one-dimensional motion.
   a. Work is defined as the ratio of the force over the distance.
   b. Work is defined as the sum of the force and the distance.
   c. Work is defined as the square of the force over the distance.
   d. Work is defined as the product of the force and the distance.

3. A book with a mass of 0.30 kg falls 2 m from a shelf to the floor. This event is described by the work–energy theorem: $W = fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ Explain why this is enough information to calculate the speed with which the book hits the floor.
   a. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the initial velocity and v_2 is the final velocity. The final velocity is the only unknown quantity.
   b. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the final velocity and v_2 is the initial velocity. The final velocity is the only unknown quantity.
   c. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the initial velocity and v_2 is the final velocity. The final velocity and the initial velocities are the only unknown quantities.
   d. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the final velocity and v_2 is the initial velocity. The final velocity and the initial velocities are the only unknown quantities.

9.2 Mechanical Energy and Conservation of Energy

4. Describe the changes in KE and PE of a person jumping up and down on a trampoline.
   a. While going up, the person's KE would change to PE. While coming down, the person's PE would change to KE.
   b. While going up, the person's PE would change to KE. While coming down, the person's KE would change to PE.
   c. While going up, the person's KE would not change, but while coming down, the person's PE would change to KE.
   d. While going up, the person's PE would change to KE, but while coming down, the person's KE would not change.

5. You know the height from which an object is dropped. Which equation could you use to calculate the velocity as the object hits the ground?
   a. $v = h$
   b. $v = \sqrt{2h}$
   c. $v = gh$
   d. $v = \sqrt{2gh}$

Critical Thinking Items

9.1 Work, Power, and the Work–Energy Theorem

9. Which activity requires a person to exert force on an object that causes the object to move but does not change the kinetic or potential energy of the object?
   a. Moving an object to a greater height with acceleration
   b. Moving an object to a greater height without acceleration
   c. Carrying an object with acceleration at the same height
   d. Carrying an object without acceleration at the same height

10. Which statement explains how it is possible to carry books to school without changing the kinetic or potential energy of the books or doing any work?
   a. By moving the book without acceleration and keeping the height of the book constant
   b. By moving the book with acceleration and keeping the height of the book constant
   c. By moving the book without acceleration and changing the height of the book
   d. By moving the book with acceleration and changing the height of the book

6. The starting line of a cross country foot race is at the bottom of a hill. Which form(s) of mechanical energy of the runners will change when the starting gun is fired?
   a. Kinetic energy only
   b. Potential energy only
   c. Both kinetic and potential energy
   d. Neither kinetic nor potential energy

9.3 Simple Machines

7. How does a simple machine make work easier?
   a. It reduces the input force and the output force.
   b. It reduces the input force and increases the output force.
   c. It increases the input force and reduces the output force.
   d. It increases the input force and the output force.

8. Which type of simple machine is a knife?
   a. A ramp
   b. A wedge
   c. A pulley
   d. A screw

9.2 Mechanical Energy and Conservation of Energy

11. True or false—A cyclist coasts down one hill and up another hill until she comes to a stop. The point at which the bicycle stops is lower than the point at which it started coasting because part of the original potential energy has been converted to a quantity of heat and this makes the tires of the bicycle warm.
   a. True
   b. False

9.3 Simple Machines

12. We think of levers being used to decrease effort force. Which of the following describes a lever that requires a large effort force which causes a smaller force to act over a large distance and explains how it works?
   a. Anything that is swung by a handle, such as a hammer or racket. Force is applied near the fulcrum over a short distance, which makes the other end move rapidly over a long distance.
   b. Anything that is swung by a handle, such as a hammer or racket. Force is applied far from the fulcrum over a large distance, which makes the other end move rapidly over a long distance.
   c. A lever used to lift a heavy stone. Force is applied near the fulcrum over a short distance, which

makes the other end lift a heavy object easily.

    d.   A lever used to lift a heavy stone. Force is applied far from the fulcrum over a large distance, which makes the other end lift a heavy object easily

13. A baseball bat is a lever. Which of the following explains how a baseball bat differs from a lever like a pry bar?

    a.   In a baseball bat, effort force is smaller and is applied over a large distance, while the resistance force is smaller and is applied over a long distance.

    b.   In a baseball bat, effort force is smaller and is applied over a large distance, while the resistance force is smaller and is applied over a short distance.

    c.   In a baseball bat, effort force is larger and is applied over a short distance, while the resistance force is smaller and is applied over a long distance.

    d.   In a baseball bat, effort force is larger and is applied over a short distance, while the resistance force is smaller and is applied over a short distance.

Problems

9.1 Work, Power, and the Work–Energy Theorem

14. A baseball player exerts a force of $100\,\text{N}$ on a ball for a distance of $0.5\,\text{m}$ as he throws it. If the ball has a mass of $0.15\,\text{kg}$, what is its velocity as it leaves his hand?

    a. $-36.5\,\text{m/s}$
    b. $-25.8\,\text{m/s}$
    c. $25.8\,\text{m/s}$
    d. $36.5\,\text{m/s}$

15. A boy pushes his little sister on a sled. The sled accelerates from 0 to 3.2 m/s . If the combined mass of his sister and the sled is 40.0 kg and 18 W of power were generated, how long did the boy push the sled?

    a.   205 s
    b.   12.8 s
    c.   23 s
    d.   11 s

9.2 Mechanical Energy and Conservation of Energy

16. What is the kinetic energy of a $0.01\,\text{kg}$ bullet traveling at a velocity of $700\,\text{m/s}$?

    a.   3.5 J
    b.   7 J
    c. 2.45×10^3 J
    d. 2.45×10^5 J

17. A marble rolling across a flat, hard surface at $2\,\text{m/s}$ rolls up a ramp. Assuming that $g = 10\,\text{m/s}^2$ and no energy is lost to friction, what will be the vertical height of the marble when it comes to a stop before rolling back down? Ignore effects due to the rotational kinetic energy.

    a. $0.1\,\text{m}$
    b. $0.2\,\text{m}$
    c. $0.4\,\text{m}$
    d. $2\,\text{m}$

18. The potential energy stored in a compressed spring is

$U = \dfrac{1}{2}kx^2$, where k is the force constant and x is the distance the spring is compressed from the equilibrium position. Four experimental setups described below can be used to determine the force constant of a spring. Which one(s) require measurement of the fewest number of variables to determine k? Assume the acceleration due to gravity is known.

    I.   An object is propelled vertically by a compressed spring.

    II.   An object is propelled horizontally on a frictionless surface by a compressed spring.

    III.   An object is statically suspended from a spring.

    IV.   An object suspended from a spring is set into oscillatory motion.

    a.   I only
    b.   III only
    c.   I and II only
    d.   III and IV only

9.3 Simple Machines

19. A man is using a wedge to split a block of wood by hitting the wedge with a hammer. This drives the wedge into the wood creating a crack in the wood. When he hits the wedge with a force of 400 N it travels 4 cm into the wood. This caused the wedge to exert a force of 1,400 N sideways increasing the width of the crack by 1 cm . What is the efficiency of the wedge?

    a.   0.875 percent
    b.   0.14
    c.   0.751
    d.   87.5 percent

20. An electrician grips the handles of a wire cutter, like the one shown, 10 cm from the pivot and places a wire between the jaws 2 cm from the pivot. If the cutter blades are 2 cm wide and 0.3 cm thick, what is the overall IMA of this complex machine?

a. 1.34
b. 1.53
c. 33.3
d. 33.5

Performance Task

9.3 Simple Machines

21. Conservation of Energy and Energy Transfer; Cause and Effect; and S&EP, Planning and Carrying Out Investigations

Plan an investigation to measure the mechanical advantage of simple machines and compare to the *IMA* of the machine. Also measure the efficiency of each machine studied. Design an investigation to make these measurements for these simple machines: lever, inclined plane, wheel and axle and a pulley system. In addition to these machines, include a spring scale, a tape measure, and a weight with a loop on top that can be attached to the hook on the spring scale. A spring scale is shown in the image.
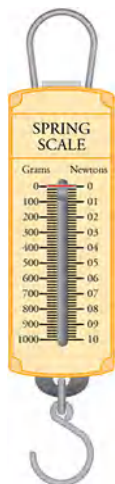
A spring scale measures weight, not mass.

LEVER: Beginning with the lever, explain how you would measure input force, output force, effort arm, and resistance arm. Also explain how you would find the distance the load travels and the distance over which the effort force is applied. Explain how you would use this data to determine *IMA* and efficiency.

INCLINED PLANE: Make measurements to determine *IMA* and efficiency of an inclined plane. Explain how you would use the data to calculate these values. Which property do you already know? Note that there are no effort and resistance arm measurements, but there are height and length measurements.

WHEEL AND AXLE: Again, you will need two force measurements and four distance measurements. Explain how you would use these to calculate *IMA* and efficiency.

SCREW: You will need two force measurements, two distance traveled measurements, and two length measurements. You may describe a screw like the one shown in Figure 9.10 or you could use a screw and screw driver. (Measurements would be easier for the former). Explain how you would use these to calculate *IMA* and efficiency.

PULLEY SYSTEM: Explain how you would determine the *IMA* and efficiency of the four-pulley system shown in Figure 9.11. Why do you only need two distance measurements for this machine?

Design a table that compares the efficiency of the five simple machines. Make predictions as to the most and least efficient machines.

TEST PREP

Multiple Choice

9.1 Work, Power, and the Work–Energy Theorem

22. Which expression represents power?
   a. fd
   b. mgh
   c. $\frac{mv^2}{2}$
   d. $\frac{W}{t}$

23. The work–energy theorem states that the change in the kinetic energy of an object is equal to what?

a. The work done on the object
b. The force applied to the object
c. The loss of the object's potential energy
d. The object's total mechanical energy minus its kinetic energy

24. A runner at the start of a race generates 250 W of power as he accelerates to 5 m/s . If the runner has a mass of 60 kg, how long did it take him to reach that speed?
   a. 0.33 s
   b. 0.83 s
   c. 1.2 s
   d. 3.0 s

25. A car's engine generates 100,000 W of power as it exerts a force of 10,000 N. How long does it take the car to travel 100 m?
    a. 0.001 s
    b. 0.01 s
    c. 10 s
    d. 1,000 s

9.2 Mechanical Energy and Conservation of Energy

26. Why is this expression for kinetic energy incorrect?
    $KE = (m)(v)^2$.
    a. The constant g is missing.
    b. The term v should not be squared.
    c. The expression should be divided by 2.
    d. The energy lost to friction has not been subtracted.

27. What is the kinetic energy of a 10 kg object moving at 2.0 m/s?
    a. 10 J
    b. 20 J
    c. 40 J
    d. 100 J

28. Which statement best describes the PE-KE transformations for a javelin, starting from the instant the javelin leaves the thrower's hand until it hits the ground.
    a. Initial PE is transformed to KE until the javelin reaches the high point of its arc. On the way back down, KE is transformed into PE. At every point in the flight, mechanical energy is being transformed into heat energy.
    b. Initial KE is transformed to PE until the javelin reaches the high point of its arc. On the way back down, PE is transformed into KE. At every point in the flight, mechanical energy is being transformed into heat energy.
    c. Initial PE is transformed to KE until the javelin reaches the high point of its arc. On the way back down, there is no transformation of mechanical energy. At every point in the flight, mechanical energy is being transformed into heat energy.
    d. Initial KE is transformed to PE until the javelin reaches the high point of its arc. On the way back down, there is no transformation of mechanical energy. At every point in the flight, mechanical energy is being transformed into heat energy.

29. At the beginning of a roller coaster ride, the roller coaster car has an initial energy mostly in the form of PE. Which statement explains why the fastest speeds of the car will be at the lowest points in the ride?
    a. At the bottom of the slope kinetic energy is at its maximum value and potential energy is at its minimum value.
    b. At the bottom of the slope potential energy is at its maximum value and kinetic energy is at its minimum value.
    c. At the bottom of the slope both kinetic and potential energy reach their maximum values
    d. At the bottom of the slope both kinetic and potential energy reach their minimum values.

9.3 Simple Machines

30. A large radius divided by a small radius is the expression used to calculate the IMA of what?
    a. A screw
    b. A pulley
    c. A wheel and axle
    d. An inclined plane.

31. What is the IMA of a wedge that is 12 cm long and 3 cm thick?
    a. 2
    b. 3
    c. 4
    d. 9

32. Which statement correctly describes the simple machines, like the crank in the image, that make up an Archimedes screw and the forces it applies?

    a. The crank is a wedge in which the IMA is the length of the tube divided by the radius of the tube. The applied force is the effort force and the weight of the water is the resistance force.
    b. The crank is an inclined plane in which the IMA is the length of the tube divided by the radius of the tube. The applied force is the effort force and the weight of the water is the resistance force.
    c. The crank is a wheel and axle. The effort force of the crank becomes the resistance force of the screw.
    d. The crank is a wheel and axle. The resistance force of the crank becomes the effort force of the screw.

33. Refer to the pulley system on right in the image. Assume this pulley system is an ideal machine.
    How hard would you have to pull on the rope to lift a 120 N

load?

How many meters of rope would you have to pull out of the system to lift the load 1 m?

IMA = N
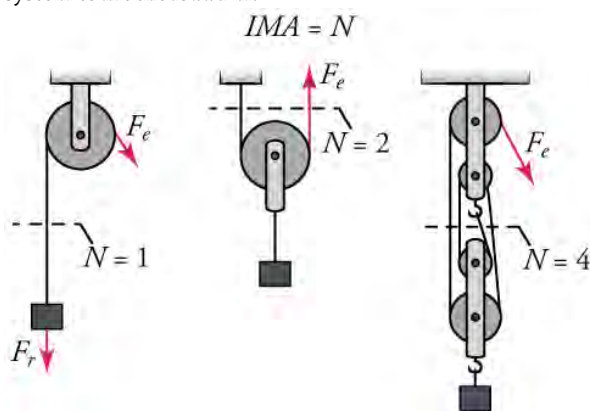
a. 480 N
   4 m

b. 480 N
   $\frac{1}{4}$ m

c. 30 N
   4 m

d. 30 N
   $\frac{1}{4}$ m

Short Answer

9.1 Work, Power, and the Work–Energy Theorem

34. Describe two ways in which doing work on an object can increase its mechanical energy.
    a. Raising an object to a higher elevation does work as it increases its PE; increasing the speed of an object does work as it increases its KE.
    b. Raising an object to a higher elevation does work as it increases its KE; increasing the speed of an object does work as it increases its PE.
    c. Raising an object to a higher elevation does work as it increases its PE; decreasing the speed of an object does work as it increases its KE.
    d. Raising an object to a higher elevation does work as it increases its KE; decreasing the speed of an object does work as it increases its PE.

35. True or false—While riding a bicycle up a gentle hill, it is fairly easy to increase your potential energy, but to increase your kinetic energy would make you feel exhausted.
    a. True
    b. False

36. Which statement best explains why running on a track with constant speed at 3 m/s is not work, but climbing a mountain at 1 m/s is work?
    a. At constant speed, change in the kinetic energy is zero but climbing a mountain produces change in the potential energy.
    b. At constant speed, change in the potential energy is zero, but climbing a mountain produces change in the kinetic energy.
    c. At constant speed, change in the kinetic energy is finite, but climbing a mountain produces no change in the potential energy.
    d. At constant speed, change in the potential energy is finite, but climbing a mountain produces no change in the kinetic energy.

37. You start at the top of a hill on a bicycle and coast to the bottom without applying the brakes. By the time you reach the bottom of the hill, work has been done on you and your bicycle, according to the equation: $W = \frac{1}{2}m(v_2^2 - v_1^2)$ If m is the mass of you and your bike, what are v_1 and v_2?
    a. v_1 is your speed at the top of the hill, and v_2 is your speed at the bottom.
    b. v_1 is your speed at the bottom of the hill, and v_2 is your speed at the top.
    c. v_1 is your displacement at the top of the hill, and v_2 is your displacement at the bottom.
    d. v_1 is your displacement at the bottom of the hill, and v_2 is your displacement at the top.

9.2 Mechanical Energy and Conservation of Energy

38. True or false—The formula for gravitational potential energy can be used to explain why joules, J, are equivalent to kg × mg² / s² . Show your work.
    a. True
    b. False

39. Which statement best explains why accelerating a car from 20 mph to 40 mph quadruples its kinetic energy?
    a. Because kinetic energy is directly proportional to the square of the velocity.
    b. Because kinetic energy is inversely proportional to the square of the velocity.
    c. Because kinetic energy is directly proportional to the fourth power of the velocity.
    d. Because kinetic energy is inversely proportional to

the fourth power of the velocity.

40. A coin falling through a vacuum loses no energy to friction, and yet, after it hits the ground, it has lost all its potential and kinetic energy. Which statement best explains why the law of conservation of energy is still valid in this case?
    a. When the coin hits the ground, the ground gains potential energy that quickly changes to thermal energy.
    b. When the coin hits the ground, the ground gains kinetic energy that quickly changes to thermal energy.
    c. When the coin hits the ground, the ground gains thermal energy that quickly changes to kinetic energy.
    d. When the coin hits the ground, the ground gains thermal energy that quickly changes to potential energy.

41. True or false—A marble rolls down a slope from height h_1 and up another slope to height h_2, where ($h_2 < h_1$). The difference $mg(h_1 - h_2)$ is equal to the heat lost due to the friction.
    a. True
    b. False

9.3 Simple Machines

42. Why would you expect the lever shown in the top image to have a greater efficiency than the inclined plane shown in the bottom image?

$$IMA = \frac{L_e}{L_r}$$

$$IMA = \frac{L}{h}$$
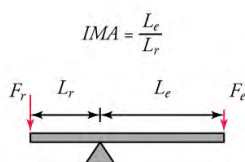
$$IMA = \frac{L}{t}$$
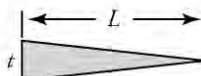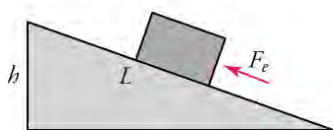
    a. The resistance arm is shorter in case of the inclined

plane.
    b. The effort arm is shorter in case of the inclined plane.
    c. The area of contact is greater in case of the inclined plane.

43. Why is the wheel on a wheelbarrow **not** a simple machine in the same sense as the simple machine in the image?
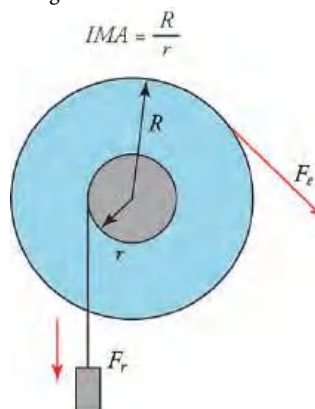
$$IMA = \frac{R}{r}$$

    a. The wheel on the wheelbarrow has no fulcrum.
    b. The center of the axle is not the fulcrum for the wheels of a wheelbarrow.
    c. The wheelbarrow differs in the way in which load is attached to the axle.
    d. The wheelbarrow has less resistance force than a wheel and axle design.

44. A worker pulls down on one end of the rope of a pulley system with a force of 75 N to raise a hay bale tied to the other end of the rope. If she pulls the rope down 2.0 m and the bale raises 1.0 m, what else would you have to know to calculate the efficiency of the pulley system?
    a. the weight of the worker
    b. the weight of the hay bale
    c. the radius of the pulley
    d. the height of the pulley from ground

45. True or false—A boy pushed a box with a weight of 300 N up a ramp. He said that, because the ramp was 1.0 m high and 3.0 m long, he must have been pushing with force of exactly 100 N.
    a. True
    b. False

Extended Response

9.1 Work, Power, and the Work–Energy Theorem

46. Work can be negative as well as positive because an object or system can do work on its surroundings as well as have work done on it. Which of the following

statements describes:
a situation in which an object does work on its surroundings by decreasing its velocity and
a situation in which an object can do work on its surroundings by decreasing its altitude?
    a. A gasoline engine burns less fuel at a slower speed. Solar cells capture sunlight to generate electricity.

b. A hybrid car charges its batteries as it decelerates.
Falling water turns a turbine to generate electricity.

c. Airplane flaps use air resistance to slow down for landing.
Rising steam turns a turbine to generate electricity.

d. An electric train requires less electrical energy as it decelerates.
A parachute captures air to slow a skydiver's fall.

47. A boy is pulling a girl in a child's wagon at a constant speed. He begins to pull harder, which increases the speed of the wagon. Which of the following describes two ways you could calculate the change in energy of the wagon and girl if you had all the information you needed?

a. Calculate work done from the force and the velocity.
Calculate work done from the change in the potential energy of the system.

b. Calculate work done from the force and the displacement.
Calculate work done from the change in the potential energy of the system.

c. Calculate work done from the force and the velocity.
Calculate work done from the change in the kinetic energy of the system.

d. Calculate work done from the force and the displacement.
Calculate work done from the change in the kinetic energy of the system.

9.2 Mechanical Energy and Conservation of Energy

48. Acceleration due to gravity on the moon is 1.6 m/s^2 or about 16% of the value of *g* on Earth.
If an astronaut on the moon threw a moon rock to a height of 7.8 m, what would be its velocity as it struck the moon's surface?
How would the fact that the moon has no atmosphere affect the velocity of the falling moon rock? Explain your answer.

a. The velocity of the rock as it hits the ground would be 5.0 m/s. Due to the lack of air friction, there would be complete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.

b. The velocity of the rock as it hits the ground would be 5.0 m/s. Due to the lack of air friction, there would be incomplete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.

c. The velocity of the rock as it hits the ground would

be 12 m/s. Due to the lack of air friction, there would be complete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.

d. The velocity of the rock as it hits the ground would be 12 m/s. Due to the lack of air friction, there would be incomplete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.

49. A boulder rolls from the top of a mountain, travels across a valley below, and rolls part way up the ridge on the opposite side. Describe all the energy transformations taking place during these events and identify when they happen.

a. As the boulder rolls down the mountainside, KE is converted to PE. As the boulder rolls up the opposite slope, PE is converted to KE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.

b. As the boulder rolls down the mountainside, KE is converted to PE. As the boulder rolls up the opposite slope, KE is converted to PE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.

c. As the boulder rolls down the mountainside, PE is converted to KE. As the boulder rolls up the opposite slope, PE is converted to KE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.

d. As the boulder rolls down the mountainside, PE is converted to KE. As the boulder rolls up the opposite slope, KE is converted to PE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.

9.3 Simple Machines

50. To dig a hole, one holds the handles together and thrusts the blades of a posthole digger, like the one in the image, into the ground. Next, the handles are pulled apart, which squeezes the dirt between them, making it possible to remove the dirt from the hole. This complex machine is composed of two pairs of two different simple machines. Identify and describe the parts that are simple machines and explain how you would find the IMA of each type of simple machine.

a. Each handle and its attached blade is a lever with the

fulcrum at the hinge. Each blade is a wedge.
The IMA of a lever would be the length of the handle divided by the length of the blade. The IMA of the wedges would be the length of the blade divided by its width.

b. Each handle and its attached to blade is a lever with the fulcrum at the end. Each blade is a wedge.
The IMA of a lever would be the length of the handle divided by the length of the blade. The IMA of the wedges would be the length of the blade divided by its width.

c. Each handle and its attached blade is a lever with the fulcrum at the hinge. Each blade is a wedge.
The IMA of a lever would be the length of the handle multiplied by the length of the blade. The IMA of the wedges would be the length of the blade multiplied by its width.

d. Each handle and its attached blade is a lever with the fulcrum at the end. Each blade is a wedge.
The IMA of a lever would be the length of the handle multiplied by the length of the blade. The IMA of the wedges would be the length of the blade multiplied by its width.

51. A wooden crate is pulled up a ramp that is 1.0 m high and 6.0 m long. The crate is attached to a rope that is wound around an axle with a radius of 0.020 m . The axle is turned by a 0.20 m long handle. What is the overall IMA of the complex machine?

A. 6
B. 10
C. 16
D. 60

Special Relativity

Figure 10.1 Special relativity explains why travel to other star systems, such as these in the Orion Nebula, is unlikely using our current level of technology. (s58y, Flickr)

Chapter Outline

INTRODUCTION Have you ever dreamed of traveling to other planets in faraway star systems? The trip might seem possible by traveling fast enough, but you will read in this chapter why it is not. In 1905, Albert Einstein developed the theory of **special relativity**. Einstein developed the theory to help explain inconsistencies between the equations describing electromagnetism and Newtonian mechanics, and to explain why the ether did not exist. This theory explains the limit on an object's speed among other implications.

Relativity is the study of how different observers moving with respect to one another measure the same events. Galileo and Newton developed the first correct version of classical relativity. Einstein developed the modern theory of relativity. Modern relativity is divided into two parts. Special relativity deals with observers moving at constant velocity. **General relativity** deals with observers moving at constant acceleration. Einstein's theories of relativity made revolutionary predictions. Most importantly, his predictions have been verified by experiments.

In this chapter, you learn how experiments and puzzling contradictions in existing theories led to the development of the theory of special relativity. You will also learn the simple postulates on which the theory was based; a postulate is a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument.

10.1 Postulates of Special Relativity

Section Learning Objectives
By the end of this section, you will be able to do the following:

- Describe the experiments and scientific problems that led Albert Einstein to develop the special theory of relativity
- Understand the postulates on which the special theory of relativity was based

Section Key Terms

ether	frame of reference	inertial reference frame
general relativity	postulate	relativity
simultaneity	special relativity	

Scientific Experiments and Problems

Relativity is not new. Way back around the year 1600, Galileo explained that motion is relative. Wherever you happen to be, it seems like you are at a fixed point and that everything moves with respect to you. Everyone else feels the same way. Motion is always measured with respect to a fixed point. This is called establishing a **frame of reference**. But the choice of the point is arbitrary, and all frames of reference are equally valid. A passenger in a moving car is not moving with respect to the driver, but they are both moving from the point of view of a person on the sidewalk waiting for a bus. They are moving even faster as seen by a person in a car coming toward them. It is all relative.

> **TIPS FOR SUCCESS**
> A frame of reference is not a complicated concept. It is just something you decide is a fixed point or group of connected points. It is completely up to you. For example, when you look up at celestial objects in the sky, you choose the earth as your frame of reference, and the sun, moon, etc., seem to move across the sky.

Light is involved in the discussion of relativity because theories related to electromagnetism are inconsistent with Galileo's and Newton's explanation of relativity. The true nature of light was a hot topic of discussion and controversy in the late 19th century. At the time, it was not generally believed that light could travel across empty space. It was known to travel as waves, and all other types of energy that propagated as waves needed to travel though a material medium. It was believed that space was filled with an invisible medium that light waves traveled through. This imaginary (as it turned out) material was called the **ether** (also spelled aether). It was thought that everything moved through this mysterious fluid. In other words, ether was the one fixed frame of reference. The Michelson–Morley experiment proved it was not.

In 1887, Albert Michelson and Edward Morley designed the interferometer shown in Figure 10.2 to measure the speed of Earth through the ether. A light beam is split into two perpendicular paths and then recombined. Recombining the waves produces an inference pattern, with a bright fringe at the locations where the two waves arrive in phase; that is, with the crests of both waves arriving together and the troughs arriving together. A dark fringe appears where the crest of one wave coincides with a trough of the other, so that the two cancel. If Earth is traveling through the ether as it orbits the sun, the peaks in one arm would take longer than in the other to reach the same location. The places where the two waves arrive in phase would change, and the interference pattern would shift. But, using the interferometer, there was no shift seen! This result led to two conclusions: that there is no ether and that the speed of light is the same regardless of the relative motion of source and observer. The Michelson–Morley investigation has been called the most famous failed experiment in history.
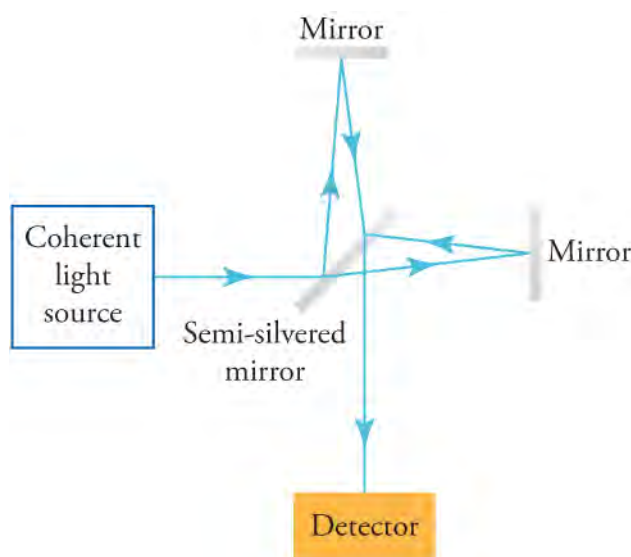
Figure 10.2 This is a diagram of the instrument used in the Michelson–Morley experiment.

To see what Michelson and Morley expected to find when they measured the speed of light in two directions, watch this animation (http://openstax.org/l/28MMexperiment) . In the video, two people swimming in a lake are represented as an analogy to light beams leaving Earth as it moves through the ether (if there were any ether). The swimmers swim away from and back to a platform that is moving through the water. The swimmers swim in different directions with respect to the motion of the platform. Even though they swim equal distances at the same speed, the motion of the platform causes them to arrive at different times.

Einstein's Postulates

The results described above left physicists with some puzzling and unsettling questions such as, why doesn't light emitted by a fast-moving object travel faster than light from a street lamp? A radical new theory was needed, and Albert Einstein, shown in Figure 10.3, was about to become everyone's favorite genius. Einstein began with two simple **postulates** based on the two things we have discussed so far in this chapter.

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light is the same in all inertial reference frames and is not affected by the speed of its source.
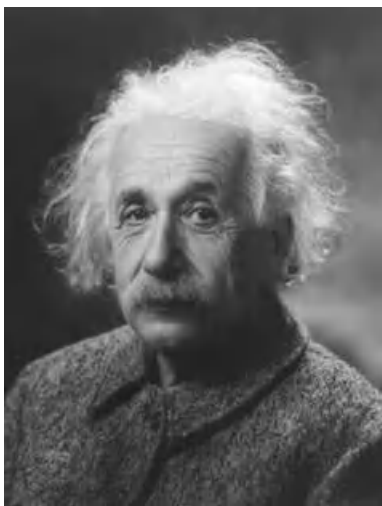
Figure 10.3 Albert Einstein (1879–1955) developed modern relativity and also made fundamental contributions to the foundations of quantum mechanics. (The Library of Congress)

The speed of light is given the symbol c and is equal to exactly 299,792,458 m/s. This is the speed of light in vacuum; that is, in the absence of air. For most purposes, we round this number off to 3.00×10^8 m/s The term **inertial reference frame** simply

refers to a frame of reference where all objects follow Newton's first law of motion: Objects at rest remain at rest, and objects in motion remain in motion at a constant velocity in a straight line, unless acted upon by an external force. The inside of a car moving along a road at constant velocity and the inside of a stationary house are inertial reference frames.

👁 WATCH PHYSICS

The Speed of Light

This lecture on light summarizes the most important facts about the speed of light. If you are interested, you can watch the whole video, but the parts relevant to this chapter are found between 3:25 and 5:10, which you find by running your cursor along the bottom of the video.

Click to view content (https://www.youtube.com/embed/rLNM8zI4Q_M)

> **GRASP CHECK**
> An airliner traveling at 200 m/s emits light from the front of the plane. Which statement describes the speed of the light?
> a. It travels at a speed of $c + 200$ m/s.
> b. It travels at a speed of $c - 200$ m/s.
> c. It travels at a speed c, like all light.
> d. It travels at a speed slightly less than c.

Snap Lab

Measure the Speed of Light

In this experiment, you will measure the speed of light using a microwave oven and a slice of bread. The waves generated by a microwave oven are not part of the visible spectrum, but they are still electromagnetic radiation, so they travel at the speed of light. If we know the wavelength, λ, and frequency, f, of a wave, we can calculate its speed, v, using the equation $v = \lambda f$. You can measure the wavelength. You will find the frequency on a label on the back of a microwave oven. The wave in a microwave is a standing wave with areas of high and low intensity. The high intensity sections are one-half wavelength apart.

- High temperature: Very hot temperatures are encountered in this lab. These can cause burns.

- a microwave oven
- one slice of plain white bread
- a centimeter ruler
- a calculator

1. Work with a partner.
2. Turn off the revolving feature of the microwave oven or remove the wheels under the microwave dish that make it turn. It is important that the dish does not turn.
3. Place the slice of bread on the dish, set the microwave on high, close the door, run the microwave for about 15 seconds.
4. A row of brown or black marks should appear on the bread. Stop the microwave as soon as they appear. Measure the distance between two adjacent burn marks and multiply the result by 2. This is the wavelength.
5. The frequency of the waves is written on the back of the microwave. Look for something like "2,450 MHz." Hz is the unit hertz, which means *per second*. The M represents mega, which stands for million, so multiply the number by 10^6.
6. Express the wavelength in meters and multiply it times the frequency. If you did everything correctly, you will get a number very close to the speed of light. Do not eat the bread. It is a general laboratory safety rule never to eat anything in the lab.

> **GRASP CHECK**
> How does your measured value of the speed of light compare to the accepted value (% error)?

a. The measured value of speed will be equal to *c*.
b. The measured value of speed will be slightly less than *c*.
c. The measured value of speed will be slightly greater than *c*.
d. The measured value of speed will depend on the frequency of the microwave.

Einstein's postulates were carefully chosen, and they both seemed very likely to be true. Einstein proceeded despite realizing that these two ideas taken together and applied to extreme conditions led to results that contradict Newtonian mechanics. He just took the ball and ran with it.

In the traditional view, velocities are additive. If you are running at 3 m/s and you throw a ball forward at a speed of 10 m/s, the ball should have a net speed of 13 m/s. However, according to relativity theory, the speed of a moving light source is not added to the speed of the emitted light.

In addition, Einstein's theory shows that if you were moving forward relative to Earth at nearly *c* (the speed of light) and could throw a ball forward at *c*, an observer at rest on the earth would not see the ball moving at nearly twice the speed of light. The observer would see it moving at a speed that is still less than *c*. This result conforms to both of Einstein's postulates: The speed of light has a fixed maximum and neither reference frame is privileged.

Consider how we measure elapsed time. If we use a stopwatch, for example, how do we know when to start and stop the watch? One method is to use the arrival of light from the event, such as observing a light turn green to start a drag race. The timing will be more accurate if some sort of electronic detection is used, avoiding human reaction times and other complications.

Now suppose we use this method to measure the time interval between two flashes of light produced by flash lamps on a moving train. (See Figure 10.4)

(1) The flashes of light from the viewpoint of observer A.

(2) The flashes of light from the viewpoint of observer B.
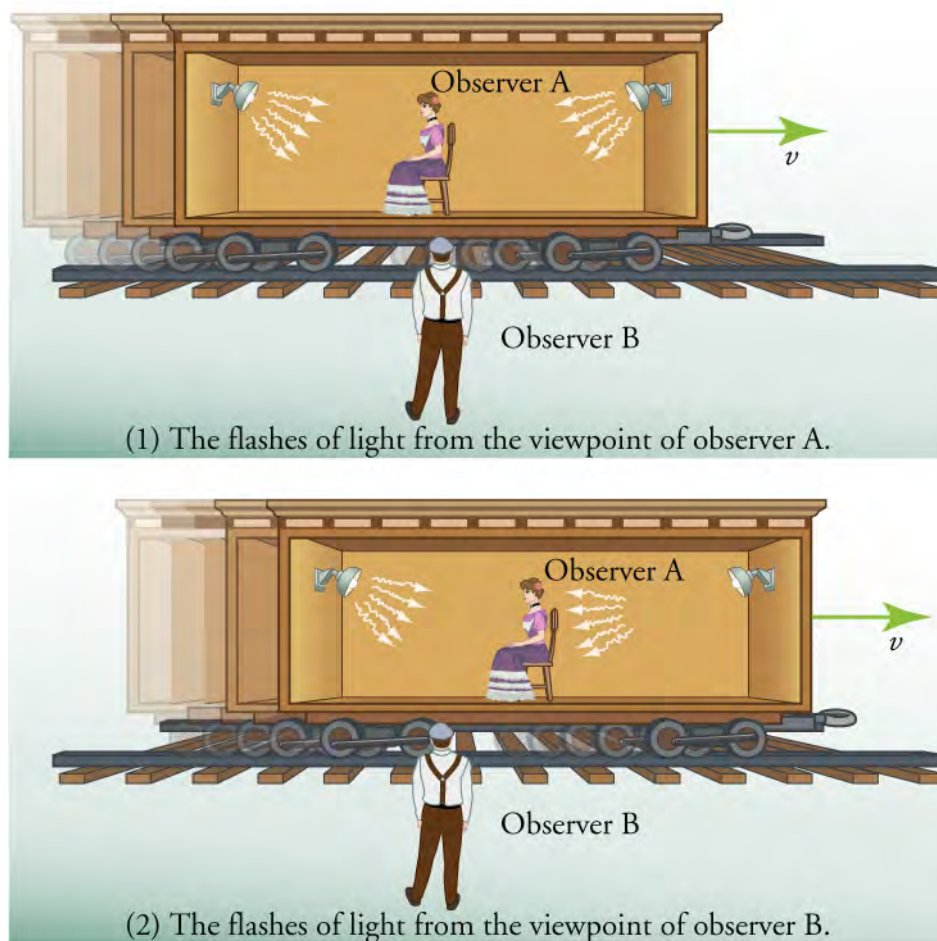
Figure 10.4 Light arriving to observer A as seen by two different observers.

A woman (observer A) is seated in the center of a rail car, with two flash lamps at opposite sides equidistant from her. Multiple light rays that are emitted from the flash lamps move towards observer A, as shown with arrows. A velocity vector arrow for the rail car is shown towards the right. A man (observer B) standing on the platform is facing the woman and also observes the flashes of light.

Observer A moves with the lamps on the rail car as the rail car moves towards the right of observer B. Observer B receives the light flashes simultaneously, and sees the bulbs as both having flashed at the same time. However, he sees observer A receive the flash from the right first. Because the pulse from the right reaches her first, in her frame of reference she sees the bulbs as not having flashed simultaneously. Here, a relative velocity between observers affects whether two events at well-separated locations are observed to be simultaneous. **Simultaneity**, or whether different events occur at the same instant, depends on the frame of reference of the observer. Remember that velocity equals distance divided by time, so $t = d/\mathbf{v}$. If velocity appears to be different, then duration of time appears to be different.

This illustrates the power of clear thinking. We might have guessed incorrectly that, if light is emitted simultaneously, then two observers halfway between the sources would see the flashes simultaneously. But careful analysis shows this not to be the case. Einstein was brilliant at this type of thought experiment (in German, *Gedankenexperiment*). He very carefully considered how an observation is made and disregarded what might seem obvious. The validity of thought experiments, of course, is determined by actual observation. The genius of Einstein is evidenced by the fact that experiments have repeatedly confirmed his theory of relativity. No experiments after that of Michelson and Morley were able to detect any ether medium. We will describe later how experiments also confirmed other predictions of **special relativity**, such as the distance between two objects and the time interval of two events being different for two observers moving with respect to each other.

In summary: Two events are defined to be simultaneous if an observer measures them as occurring at the same time (such as by receiving light from the events). Two events are not necessarily simultaneous to all observers.

The discrepancies between Newtonian mechanics and relativity theory illustrate an important point about how science advances. Einstein's theory did not replace Newton's but rather extended it. It is not unusual that a new theory must be developed to account for new information. In most cases, the new theory is built on the foundation of older theory. It is rare that old theories are completely replaced.

In this chapter, you will learn about the theory of special relativity, but, as mentioned in the introduction, Einstein developed two relativity theories: special and general. Table 10.1 summarizes the differences between the two theories.

Special Relativity	General Relativity
Published in 1905	Final form published in 1916
A theory of space-time	A theory of gravity
Applies to observers moving at constant speed	Applies to observers that are accelerating
Most useful in the field of nuclear physics	Most useful in the field of astrophysics
Accepted quickly and put to practical use by nuclear physicists and quantum chemists	Largely ignored until 1960 when new mathematical techniques made the theory more accessible and astronomers found some important applications

Also note that the theory of general relativity includes the theory of special relativity.

Table 10.1 Comparing Special Relativity and General Relativity

✳ WORKED EXAMPLE

Calculating the Time it Takes Light to Travel a Given Distance

The sun is 1.50×10^8 km from Earth. How long does it take light to travel from the sun to Earth in minutes and seconds?

Strategy

Identify knowns.

$$\text{Distance} = 1.50 \times 10^8 \text{ km}$$
$$\text{Speed} = 3.00 \times 10^8 \text{ km}$$

10.1

Identify unknowns.

Time

Find the equation that relates knowns and unknowns.

$$v = \frac{d}{t}; \quad t = \frac{d}{v}$$

10.2

Be sure to use consistent units.

Solution

$$t = \frac{d}{v} = \frac{(1.50 \times 10^8 \, km) \times \frac{10^3 \, m}{km}}{3.00 \times 10^8 \, \frac{m}{s}} = 5.00 \times 10^2 \, s$$
$$500 \text{ s} = 8 \text{ min and } 20 \text{ s}$$

Discussion

The answer is written as 5.00×10^2 rather than 500 in order to show that there are three significant figures. When astronomers witness an event on the sun, such as a sunspot, it actually happened minutes earlier. Compare 8 light *minutes* to the distance to stars, which are light *years* away. Any events on other stars happened years ago.

Practice Problems

1. Light travels through 1.00 m of water in 4.42×10^{-9} s. What is the speed of light in water?
   a. 4.42×10^{-9} m/s
   b. 4.42×10^{9} m/s
   c. 2.26×10^{8} m/s
   d. 226×10^{8} m/s

2. An astronaut on the moon receives a message from mission control on Earth. The signal is sent by a form of electromagnetic radiation and takes 1.28 s to travel the distance between Earth and the moon. What is the distance from Earth to the moon?
   a. 2.34×10^{5} km
   b. 2.34×10^{8} km
   c. 3.84×10^{5} km
   d. 3.84×10^{8} km

Check Your Understanding

3. Explain what is meant by a frame of reference.
   a. A frame of reference is a graph plotted between distance and time.
   b. A frame of reference is a graph plotted between speed and time.
   c. A frame of reference is the velocity of an object through empty space without regard to its surroundings.
   d. A frame of reference is an arbitrarily fixed point with respect to which motion of other points is measured.

4. Two people swim away from a raft that is floating downstream. One swims upstream and returns, and the other swims across the current and back. If this scenario represents the Michelson–Morley experiment, what do (i) the water, (ii) the swimmers, and (iii) the raft represent?
   a. the ether   rays of light   Earth
   b. rays of light   the ether   Earth
   c. the ether   Earth   rays of light
   d. Earth   rays of light   the ether

5. If Michelson and Morley had observed the interference pattern shift in their interferometer, what would that have indicated?
   a. The speed of light is the same in all frames of reference.
   b. The speed of light depends on the motion relative to the ether.
   c. The speed of light changes upon reflection from a surface.
   d. The speed of light in vacuum is less than 3.00×10^{8} m/s.

6. If you designate a point as being fixed and use that point to measure the motion of surrounding objects, what is the point called?
   a. An origin
   b. A frame of reference
   c. A moving frame
   d. A coordinate system

10.2 Consequences of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the relativistic effects seen in time dilation, length contraction, and conservation of relativistic momentum
- Explain and perform calculations involving mass-energy equivalence

Section Key Terms

binding energy	length contraction	mass defect	time dilation

proper length   relativistic   relativistic momentum

relativistic energy   relativistic factor   rest mass

Relativistic Effects on Time, Distance, and Momentum

Consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect. **Time dilation** is the phenomenon of time passing more slowly for an observer who is moving relative to another observer.

For example, suppose an astronaut measures the time it takes for light to travel from the light source, cross her ship, bounce off a mirror, and return. (See Figure 10.5.) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the earth bound observer. The passage of time is different for the two observers because the distance the light travels in the astronaut's frame is smaller than in the earth bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the earth bound frame.
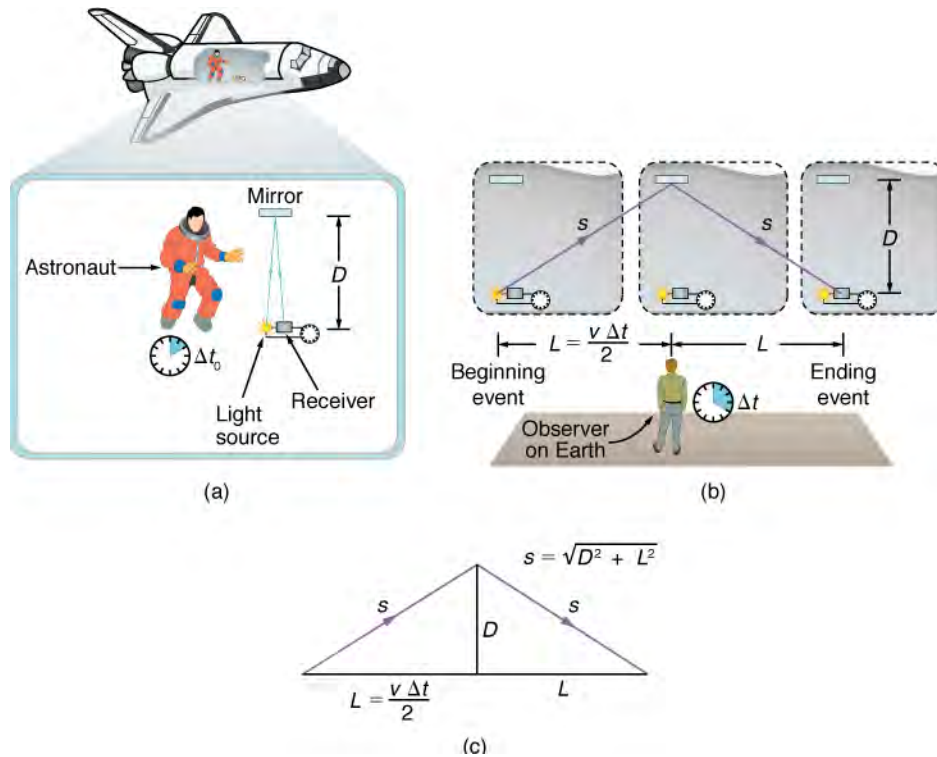
Figure 10.5 (a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut's frame. (b) A person on the earth sees the light follow the longer path $2s$ and take a longer time Δt.

The relationship between Δt and Δt_0 is given by

$$\Delta t = \gamma \Delta t_0,$$

where γ is the **relativistic factor** given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and v and c are the speeds of the moving observer and light, respectively.

TIPS FOR SUCCESS

Try putting some values for *v* into the expression for the relativistic factor (γ). Observe at which speeds this factor will make a difference and when γ is so close to 1 that it can be ignored. Try 225 m/s, the speed of an airliner; 2.98×10^4 m/s, the speed of Earth in its orbit; and 2.990×10^8 m/s, the speed of a particle in an accelerator.

Notice that when the velocity *v* is small compared to the speed of light *c*, then *v/c* becomes small, and γ becomes close to 1. When this happens, time measurements are the same in both frames of reference. **Relativistic** effects, meaning those that have to do with special relativity, usually become significant when speeds become comparable to the speed of light. This is seen to be the case for time dilation.

You may have seen science fiction movies in which space travelers return to Earth after a long trip to find that the planet and everyone on it has aged much more than they have. This type of scenario is a based on a thought experiment, known as the twin paradox, which imagines a pair of twins, one of whom goes on a trip into space while the other stays home. When the space traveler returns, she finds her twin has aged much more than she. This happens because the traveling twin has been in two frames of reference, one leaving Earth and one returning.

Time dilation has been confirmed by comparing the time recorded by an atomic clock sent into orbit to the time recorded by a clock that remained on Earth. GPS satellites must also be adjusted to compensate for time dilation in order to give accurate positioning.

Have you ever driven on a road, like that shown in Figure 10.6, that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it is about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers moving with respect to one other.

Figure 10.6 People might describe distances differently, but at relativistic speeds, the distances really are different. (Corey Leopold, Flickr)

One thing all observers agree upon is their relative speed. When one observer is traveling away from another, they both see the other receding at the same speed, regardless of whose frame of reference is chosen. Remember that speed equals distance divided by time: $v = d/t$. If the observers experience a difference in elapsed time, they must also observe a difference in distance traversed. This is because the ratio d/t must be the same for both observers.

The shortening of distance experienced by an observer moving with respect to the points whose distance apart is measured is called **length contraction**. **Proper length**, L_0, is the distance between two points measured in the reference frame where the observer and the points are at rest. The observer in motion with respect to the points measures *L*. These two lengths are related by the equation

$$L = \frac{L_0}{\gamma}.$$

Because γ is the same expression used in the time dilation equation above, the equation becomes

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}.$$

To see how length contraction is seen by a moving observer, go to this simulation (http://openstax.org/l/28simultaneity) . Here you can also see that simultaneity, time dilation, and length contraction are interrelated phenomena.

This link is to a simulation that illustrates the relativity of simultaneous events.

In classical physics, momentum is a simple product of mass and velocity. When special relativity is taken into account, objects that have mass have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds; i.e., speeds close to the speed of light?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved in classical mechanics whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. We will see that momentum has the same importance in modern physics. **Relativistic momentum** is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

One of the postulates of special relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic momentum, $\mathbf{p}$, is classical momentum multiplied by the relativistic factor γ.

$$\mathbf{p} = \gamma m \mathbf{u},$$

10.3

where m is the **rest mass** of the object (that is, the mass measured at rest, without any γ factor involved), $\mathbf{u}$ is its velocity relative to an observer, and γ, as before, is the relativistic factor. We use the mass of the object as measured at rest because we cannot determine its mass while it is moving.

Note that we use $\mathbf{u}$ for velocity here to distinguish it from relative velocity $\mathbf{v}$ between observers. Only one observer is being considered here. With $\mathbf{p}$ defined in this way, $\mathbf{p}_{tot}$ is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum $\gamma m \mathbf{u}$ becomes the classical $m \mathbf{u}$ at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities. Because of the factor γ, however, relativistic momentum behaves differently from classical momentum by approaching infinity as $\mathbf{u}$ approaches c. (See Figure 10.7.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, which is an unreasonable value.
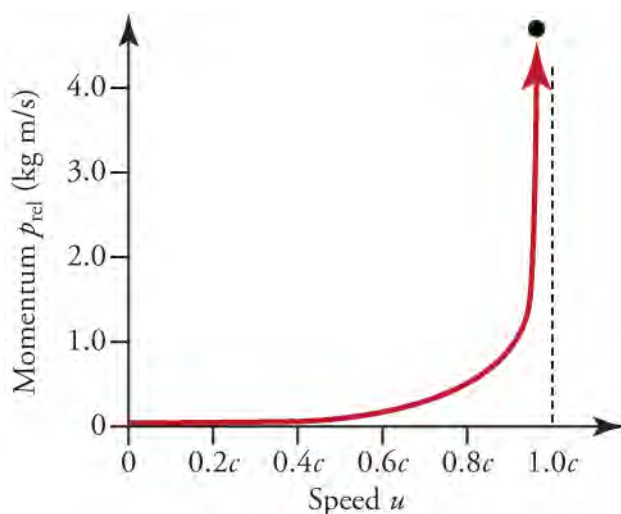
Figure 10.7 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This

has been verified in numerous experiments.

Mass-Energy Equivalence

Let us summarize the calculation of relativistic effects on objects moving at speeds near the speed of light. In each case we will need to calculate the relativistic factor, given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where **v** and c are as defined earlier. We use **u** as the velocity of a particle or an object in one frame of reference, and **v** for the velocity of one frame of reference with respect to another.

Time Dilation

Elapsed time on a moving object, Δt_0, as seen by a stationary observer is given by $\Delta t = \gamma \Delta t_0$, where Δt_0 is the time observed on the moving object when it is taken to be the frame or reference.

Length Contraction

Length measured by a person at rest with respect to a moving object, L, is given by

$$L = \frac{L_0}{\gamma},$$

where L_0 is the length measured on the moving object.

Relativistic Momentum

Momentum, **p**, of an object of mass, m, traveling at relativistic speeds is given by $\mathbf{p} = \gamma m \mathbf{u}$, where **u** is velocity of a moving object as seen by a stationary observer.

Relativistic Energy

The original source of all the energy we use is the conversion of mass into energy. Most of this energy is generated by nuclear reactions in the sun and radiated to Earth in the form of electromagnetic radiation, where it is then transformed into all the forms with which we are familiar. The remaining energy from nuclear reactions is produced in nuclear power plants and in Earth's interior. In each of these cases, the source of the energy is the conversion of a small amount of mass into a large amount of energy. These sources are shown in Figure 10.8.

(a)   (b)

Figure 10.8 The sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy. ((a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor. The result of his analysis is that a particle or object of mass m moving at velocity **u** has **relativistic energy** given by

$$E = \gamma mc^2.$$

This is the expression for the total energy of an object of mass m at any speed **u** and includes both kinetic and potential energy. Look back at the equation for γ and you will see that it is equal to 1 when **u** is 0; that is, when an object is at rest. Then the rest

energy, E_0, is simply

$$E_0 = mc^2.$$

This is the correct form of Einstein's famous equation.

This equation is very useful to nuclear physicists because it can be used to calculate the energy released by a nuclear reaction. This is done simply by subtracting the mass of the products of such a reaction from the mass of the reactants. The difference is the m in $E_0 = mc^2$. Here is a simple example:

A positron is a type of antimatter that is just like an electron, except that it has a positive charge. When a positron and an electron collide, their masses are completely annihilated and converted to energy in the form of gamma rays. Because both particles have a rest mass of 9.11×10^{-31} kg, we multiply the mc^2 term by 2. So the energy of the gamma rays is

$$\begin{aligned} E_0 &= 2(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8\ \tfrac{\text{m}}{\text{s}}\right)^2 \\ &= 1.64 \times 10^{-13}\ \tfrac{\text{kg·m}^2}{\text{s}^2} \\ &= 1.64 \times 10^{-13}\ \text{J} \end{aligned}$$

10.4

where we have the expression for the joule (J) in terms of its SI base units of kg, m, and s. In general, the nuclei of stable isotopes have less mass then their constituent subatomic particles. The energy equivalent of this difference is called the **binding energy** of the nucleus. This energy is released during the formation of the isotope from its constituent particles because the product is more stable than the reactants. Expressed as mass, it is called the **mass defect**. For example, a helium nucleus is made of two neutrons and two protons and has a mass of 4.0003 atomic mass units (u). The sum of the masses of two protons and two neutrons is 4.0330 u. The mass defect then is 0.0327 u. Converted to kg, the mass defect is 5.0442×10^{-30} kg. Multiplying this mass times c^2 gives a binding energy of 4.540×10^{-12} J. This does not sound like much because it is only one atom. If you were to make one gram of helium out of neutrons and protons, it would release 683,000,000,000 J. By comparison, burning one gram of coal releases about 24 J.

⚛ BOUNDLESS PHYSICS

The RHIC Collider

Figure 10.9 shows the Brookhaven National Laboratory in Upton, NY. The circular structure houses a particle accelerator called the RHIC, which stands for Relativistic Heavy Ion Collider. The heavy ions in the name are gold nuclei that have been stripped of their electrons. Streams of ions are accelerated in several stages before entering the big ring seen in the figure. Here, they are accelerated to their final speed, which is about 99.7 percent the speed of light. Such high speeds are called relativistic. All the relativistic phenomena we have been discussing in this chapter are very pronounced in this case. At this speed $\gamma = 12.9$, so that relativistic time dilates by a factor of about 13, and relativistic length contracts by the same factor.

Figure 10.9 Brookhaven National Laboratory. The circular structure houses the RHIC. (energy.gov, Wikimedia Commons)

Two ion beams circle the 2.4-mile long track around the big ring in opposite directions. The paths can then be made to cross, thereby causing ions to collide. The collision event is very short-lived but amazingly intense. The temperatures and pressures produced are greater than those in the hottest suns. At 4 trillion degrees Celsius, this is the hottest material ever created in a

laboratory

But what is the point of creating such an extreme event? Under these conditions, the neutrons and protons that make up the gold nuclei are smashed apart into their components, which are called quarks and gluons. The goal is to recreate the conditions that theorists believe existed at the very beginning of the universe. It is thought that, at that time, matter was a sort of soup of quarks and gluons. When things cooled down after the initial bang, these particles condensed to form protons and neutrons.

Some of the results have been surprising and unexpected. It was thought the quark-gluon soup would resemble a gas or plasma. Instead, it behaves more like a liquid. It has been called a *perfect* liquid because it has virtually no viscosity, meaning that it has no resistance to flow.

GRASP CHECK

Calculate the relativistic factor γ, for a particle traveling at 99.7 percent of the speed of light.

a.   0.08

b.   0.71

c.   1.41

d.   12.9

✳ WORKED EXAMPLE

The Speed of Light

One night you are out looking up at the stars and an extraterrestrial spaceship flashes across the sky. The ship is 50 meters long and is travelling at 95 percent of the speed of light. What would the ship's length be when measured from your earthbound frame of reference?

Strategy

List the knowns and unknowns.

Knowns: proper length of the ship, $L_0 = 50$ m; velocity, $\mathbf{v}, = 0.95c$

Unknowns: observed length of the ship accounting for relativistic length contraction, L.

Choose the relevant equation.

$$L = \frac{L_0}{\gamma} = L_0 \sqrt{1 - \frac{\mathbf{u}^2}{c^2}}$$

Solution

$$L = 50 \text{ m} \sqrt{1 - \frac{(0.95)^2 c^2}{c^2}} = 50 \text{ m} \sqrt{1 - (0.95)^2} = 16 \text{ m}$$

Discussion

Calculations of γ can usually be simplified in this way when v is expressed as a percentage of c because the c^2 terms cancel. Be sure to also square the decimal representing the percentage before subtracting from 1. Note that the aliens will still see the length as L_0 because they are moving with the frame of reference that is the ship.

Practice Problems

7.   Calculate the relativistic factor, γ, for an object traveling at 2.00×10^8 m/s.

a.   0.74

b.   0.83

c.   1.2

d.   1.34

8.   The distance between two points, called the proper length, L_0, is 1.00 km. An observer in motion with respect to the frame of

reference of the two points measures 0.800 km, which is L. What is the relative speed of the frame of reference with respect to the observer?

   a. 1.80×10^8 m/s

   b. 2.34×10^8 m/s

   c. 3.84×10^8 m/s

   d. 5.00×10^8 m/s

9. Consider the nuclear fission reaction $n + {}^{235}_{92}U \rightarrow {}^{137}_{55}Cs + {}^{97}_{37}Rb + 2n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{137}_{55}Cs$ has rest mass of 136.907u, and ${}^{97}_{37}Rb$ has a rest mass of 96.937u, what is the value of E in joules?

   a. 1.8×10^{-11} J

   b. 2.9×10^{-11} J

   c. 1.8×10^{-10} J

   d. 2.9×10^{-10} J

Solution

The correct answer is (b). The mass deficit in the reaction is $235.044 \text{ u} - (136.907 + 96.937 + 1.009) \text{ u}$, or 0.191u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$(0.191 \text{ u}) \left(1.66 \times 10^{-27} \text{kg/u}\right) \left(3.00 \times 10^8 \text{m/s}\right)^2 \cong 2.85 \times 10^{-11} \text{J}.$

10. Consider the nuclear fusion reaction ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H + E$. If ${}^{2}_{1}H$ has a rest mass of 2.014u, ${}^{3}_{1}H$ has a rest mass of 3.016u, and ${}^{1}_{1}H$ has a rest mass of 1.008u, what is the value of E in joules?

   a. 6×10^{-13} J

   b. 6×10^{-12} J

   c. 6×10^{-11} J

   d. 6×10^{-10} J

Solution

The correct answer is (a). The mass deficit in the reaction is $2(2.014 \text{ u}) - (3.016 + 1.008) \text{ u}$, or 0.004u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$(0.004 \text{ u}) \left(1.66 \times 10^{-27} \text{kg/u}\right) \left(3.00 \times 10^8 \text{m/s}\right)^2 \cong 5.98 \times 10^{-13} \text{J}.$

Check Your Understanding

11. Describe time dilation and state under what conditions it becomes significant.

   a. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.

   b. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.

   c. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.

   d. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.

12. The equation used to calculate relativistic momentum is $p = \gamma \cdot m \cdot u$. Define the terms to the right of the equal sign and state how m and u are measured.

   a. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity of the frame.

   b. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity relative to an observer.

   c. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e., } \dfrac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity of the frame.

    d. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e., } \dfrac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity relative to an observer.

13. Describe length contraction and state when it occurs.
    a.   When the speed of an object becomes the speed of light, its length appears to shorten when viewed by a stationary observer.
    b.   When the speed of an object approaches the speed of light, its length appears to shorten when viewed by a stationary observer.
    c.   When the speed of an object becomes the speed of light, its length appears to increase when viewed by a stationary observer.
    d.   When the speed of an object approaches the speed of light, its length appears to increase when viewed by a stationary observer.

KEY TERMS

binding energy   the energy equivalent of the difference between the mass of a nucleus and the masses of its nucleons

ether   scientists once believed there was a medium that carried light waves; eventually, experiments proved that ether does not exist

frame of reference   the point or collection of points arbitrarily chosen, which motion is measured in relation to

general relativity   the theory proposed to explain gravity and acceleration

inertial reference frame   a frame of reference where all objects follow Newton's first law of motion

length contraction   the shortening of an object as seen by an observer who is moving relative to the frame of reference of the object

mass defect   the difference between the mass of a nucleus and the masses of its nucleons

postulate   a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument

proper length   the length of an object within its own frame of reference, as opposed to the length observed by an observer moving relative to that frame of reference

relativistic   having to do with modern relativity, such as the effects that become significant only when an object is moving close enough to the speed of light for γ to be significantly greater than 1

relativistic energy   the total energy of a moving object or particle $E = \gamma m c^2$, which includes both its rest energy mc^2 and its kinetic energy

relativistic factor $\gamma = \dfrac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$, where **u** is the velocity of a moving object and c is the speed of light

relativistic momentum $\mathbf{p} = \gamma m \mathbf{u}$, where γ is the relativistic factor, m is rest mass of an object, and **u** is the velocity relative to an observer

relativity   the explanation of how objects move relative to one another

rest mass   the mass of an object that is motionless with respect to its frame of reference

simultaneity   the property of events that occur at the same time

special relativity   the theory proposed to explain the consequences of requiring the speed of light and the laws of physics to be the same in all inertial frames

time dilation   the contraction of time as seen by an observer in a frame of reference that is moving relative to the observer

SECTION SUMMARY

10.1 Postulates of Special Relativity

- One postulate of special relativity theory is that the laws of physics are the same in all inertial frames of reference.
- The other postulate is that the speed of light in a vacuum is the same in all inertial frames.
- Einstein showed that simultaneity, or lack of it, depends on the frame of reference of the observer.

10.2 Consequences of Special Relativity

- Time dilates, length contracts, and momentum increases as an object approaches the speed of light.
- Energy and mass are interchangeable, according to the relationship $E = mc2$. The laws of conservation of mass and energy are combined into the law of conservation of mass-energy.

KEY EQUATIONS

10.1 Postulates of Special Relativity

speed of light $v = \lambda f$

constant value for the speed of light $c = 3.00 \times 10^8 \, \text{m/s}$

10.2 Consequences of Special Relativity

elapsed time $\Delta t = \gamma \Delta t_0$

relativistic factor $\gamma = \dfrac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$

length contraction $L = \dfrac{L_0}{\gamma}$

relativistic momentum $\mathbf{p} = \gamma m \mathbf{u}$

relativistic energy	$E = \gamma mc^2$	rest energy	$E_0 = mc^2$

CHAPTER REVIEW
Concept Items
10.1 Postulates of Special Relativity

1. Why was it once believed that light must travel through a medium and could not propagate across empty space?
   a. The longitudinal nature of light waves implies this.
   b. Light shows the phenomenon of diffraction.
   c. The speed of light is the maximum possible speed.
   d. All other wave energy needs a medium to travel.

2. Describe the relative motion of Earth and the sun:
   1. if Earth is taken as the inertial frame of reference and
   2. if the sun is taken as the inertial frame of reference.
      a. 1. Earth is at rest and the sun orbits Earth.
         2. The sun is at rest and Earth orbits the sun.

      b. 1. The sun is at rest and Earth orbits the sun.
         2. Earth is at rest and the sun orbits Earth.

      c. 1. The sun is at rest and Earth orbits the sun.
         2. The sun is at rest and Earth orbits the sun.

      d. 1. Earth is at rest and the sun orbits Earth.
         2. Earth is at rest and the sun orbits Earth.

10.2 Consequences of Special Relativity

3. A β particle (a free electron) is speeding around the track in a cyclotron, rapidly gaining speed. How will the particle's momentum change as its speed approaches the speed of light? Explain.
   a. The particle's momentum will rapidly decrease.
   b. The particle's momentum will rapidly increase.
   c. The particle's momentum will remain constant.
   d. The particle's momentum will approach zero.

4. An astronaut goes on a long space voyage at near the speed of light. When she returns home, how will her age compare to the age of her twin who stayed on Earth?
   a. Both of them will be the same age.
   b. This is a paradox and hence the ages cannot be compared.
   c. The age of the twin who traveled will be less than the age of her twin.
   d. The age of the twin who traveled will be greater than the age of her twin.

5. A comet reaches its greatest speed as it travels near the sun. True or false— Relativistic effects make the comet's tail look longer to an observer on Earth.
   a. True
   b. False

Critical Thinking Items
10.1 Postulates of Special Relativity

6. Explain how the two postulates of Einstein's theory of special relativity, when taken together, could lead to a situation that seems to contradict the mechanics and laws of motion as described by Newton.
   a. In Newtonian mechanics, velocities are multiplicative but the speed of a moving light source cannot be multiplied to the speed of light because, according to special relativity, the speed of light is the maximum speed possible.
   b. In Newtonian mechanics, velocities are additive but the speed of a moving light source cannot be added to the speed of light because the speed of light is the maximum speed possible.
   c. An object that is at rest in one frame of reference may appear to be in motion in another frame of reference, while in Newtonian mechanics such a situation is not possible.

   d. The postulates of Einstein's theory of special relativity do not contradict any situation that Newtonian mechanics explains.

7. It takes light $6.0\ \text{minutes}$ to travel from the sun to the planet Venus. How far is Venus from the sun?
   a. 18×10^6 km
   b. 18×10^8 km
   c. 1.08×10^{11} km
   d. 1.08×10^8 km

8. In 2003, Earth and Mars were the closest they had been in 50,000 years. The two planets were aligned so that Earth was between Mars and the sun. At that time it took light from the sun 500 s to reach Earth and 687 s to get to Mars. What was the distance from Mars to Earth?
   a. 5.6×10^7 km
   b. 5.6×10^{10} km
   c. 6.2×10^6 km
   d. 6.2×10^{12} km

9. Describe two ways in which light differs from all other

forms of wave energy.
a. 1. Light travels as a longitudinal wave.
    2. Light travels through a medium that fills up the empty space in the universe.

b. 1. Light travels as a transverse wave.
    2. Light travels through a medium that fills up the empty space in the universe.

c. 1. Light travels at the maximum possible speed in the universe.
    2. Light travels through a medium that fills up the empty space in the universe.

d. 1. Light travels at the maximum possible speed in the universe.
    2. Light does not require any material medium to travel.

10. Use the postulates of the special relativity theory to explain why the speed of light emitted from a fast-moving light source cannot exceed 3.00×10^8 m/s.
a. The speed of light is maximum in the frame of reference of the moving object.
b. The speed of light is minimum in the frame of reference of the moving object.
c. The speed of light is the same in all frames of reference, including in the rest frame of its source.
d. Light always travels in a vacuum with a speed less than 3.00×10^8 m/s, regardless of the speed of the

source.

10.2 Consequences of Special Relativity

11. Halley's Comet comes near Earth every 75 years as it travels around its 22 billion km orbit at a speed of up to $700,000$ m/s. If it were possible to put a clock on the comet and read it each time the comet passed, which part of special relativity theory could be tested? What would be the expected result? Explain.
a. It would test time dilation. The clock would appear to be slightly slower.
b. It would test time dilation. The clock would appear to be slightly faster.
c. It would test length contraction. The length of the orbit would appear to be shortened from Earth's frame of reference.
d. It would test length contraction. The length of the orbit would appear to be shortened from the comet's frame of reference.

12. The nucleus of the isotope fluorine-18 (^{18}F) has mass defect of 2.44×10^{-28} kg. What is the binding energy of ^{18}F?
a. 2.2×10^{-11} J
b. 7.3×10^{-20} J
c. 2.2×10^{-20} J
d. 2.4×10^{-28} J

Problems

10.2 Consequences of Special Relativity

13. Deuterium (2 H) is an isotope of hydrogen that has one proton and one neutron in its nucleus. The binding energy of deuterium is 3.56×10^{-13} J. What is the mass defect of deuterium?
a. 3.20×10^{-4} kg
b. 1.68×10^{-6} kg
c. 1.19×10^{-21} kg
d. 3.96×10^{-30} kg

14. The sun orbits the center of the galaxy at a speed of 2.3×10^5 m/s. The diameter of the sun is 1.391684×10^9 m. An observer is in a frame of reference that is stationary with respect to the center of the galaxy. True or false—The sun is moving fast enough for the observer to notice length contraction of the sun's diameter.
a. True
b. False

15. Consider the nuclear fission reaction

$n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{144}_{56}Ba$ has rest mass of 143.923u, and ${}^{89}_{36}Kr$ has a rest mass of 88.918u, what is the value of E in joules?
a. 1.8×10^{-11} J
b. 2.8×10^{-11} J
c. 1.8×10^{-10} J
d. 3.3×10^{-10} J

16. Consider the nuclear fusion reaction
${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n + E$. If ${}^{2}_{1}H$ has a rest mass of 2.014u, ${}^{3}_{1}H$ has a rest mass of 3.016u, ${}^{4}_{2}He$ has a rest mass of 4.003u, and a neutron has a rest mass of 1.009u, what is the value of E in joules?
a. 2.7×10^{-14} J
b. 2.7×10^{-13} J
c. 2.7×10^{-12} J
d. 2.7×10^{-11} J

Performance Task

10.2 Consequences of Special Relativity

17. People are fascinated by the possibility of traveling across the universe to discover intelligent life on other planets. To do this, we would have to travel enormous distances. Suppose we could somehow travel at up to 90 percent of the speed of light. The closest star is Alpha Centauri, which is 4.37 light years away. (A light year is the distance light travels in one year.)

a. How long, from the point of view of people on Earth, would it take a space ship to travel to Alpha Centauri and back at 0.9c?

b. How much would the astronauts on the spaceship have aged by the time they got back to Earth?

c. Discuss the problems related to travel to stars that are 20 or 30 light years away. Assume travel speeds near the speed of light.

TEST PREP

Multiple Choice

10.1 Postulates of Special Relativity

18. What was the purpose of the Michelson–Morley experiment?
    a. To determine the exact speed of light
    b. To analyze the electromagnetic spectrum
    c. To establish that Earth is the true frame of reference
    d. To learn how the ether affected the propagation of light

19. What is the speed of light in a vacuum to three significant figures?
    a. 1.86×10^5 m/s
    b. 3.00×10^8 m/s
    c. 6.71×10^8 m/s
    d. 1.50×10^{11} m/s

20. How far does light travel in 1.00 min?
    a. 1.80×10^7 km
    b. 1.80×10^{13} km
    c. 5.00×10^6 m
    d. 5.00×10^8 m

21. Describe what is meant by the sentence, "Simultaneity is not absolute."
    a. Events may appear simultaneous in all frames of reference.
    b. Events may not appear simultaneous in all frames of reference.
    c. The speed of light is not the same in all frames of reference.
    d. The laws of physics may be different in different inertial frames of reference.

22. In 2003, Earth and Mars were aligned so that Earth was between Mars and the sun. Earth and Mars were 5.6×10^7 km from each other, which was the closest they had been in 50,000 years. People looking up saw Mars as a very bright red light on the horizon. If Mars was 2.06×10^8 km from the sun, how long did the reflected light people saw take to travel from the sun to Earth?
    a. 14 min and 33 s
    b. 12 min and 15 s
    c. 11 min and 27 s
    d. 3 min and 7 s

10.2 Consequences of Special Relativity

23. What does this expression represent: $\dfrac{1}{\sqrt{1-\frac{u^2}{c^2}}}$
    a. time dilation
    b. relativistic factor
    c. relativistic energy
    d. length contraction

24. What is the rest energy, E_o, of an object with a mass of 1.00 g ?
    a. 3.00×10^5 J
    b. 3.00×10^{11} J
    c. 9.00×10^{13} J
    d. 9.00×10^{16} J

25. The fuel rods in a nuclear reactor must be replaced from time to time because so much of the radioactive material has reacted that they can no longer produce energy. How would the mass of the spent fuel rods compare to their mass when they were new? Explain your answer.
    a. The mass of the spent fuel rods would decrease.
    b. The mass of the spent fuel rods would increase.
    c. The mass of the spent fuel rods would remain the same.
    d. The mass of the spent fuel rods would become close to zero.

Short Answer

10.1 Postulates of Special Relativity

26. What is the postulate having to do with the speed of light on which the theory of special relativity is based?
    a. The speed of light remains the same in all inertial frames of reference.
    b. The speed of light depends on the speed of the source emitting the light.
    c. The speed of light changes with change in medium through which it travels.
    d. The speed of light does not change with change in medium through which it travels.

27. What is the postulate having to do with reference frames on which the theory of special relativity is based?
    a. The frame of reference chosen is arbitrary as long as it is inertial.
    b. The frame of reference is chosen to have constant nonzero acceleration.
    c. The frame of reference is chosen in such a way that the object under observation is at rest.
    d. The frame of reference is chosen in such a way that the object under observation is moving with a constant speed.

28. If you look out the window of a moving car at houses going past, you sense that you are moving. What have you chosen as your frame of reference?
    a. the car
    b. the sun
    c. a house

29. Why did Michelson and Morley orient light beams at right angles to each other?
    a. To observe the particle nature of light
    b. To observe the effect of the passing ether on the speed of light
    c. To obtain a diffraction pattern by combination of light
    d. To obtain a constant path difference for interference of light

Extended Response

10.1 Postulates of Special Relativity

34. Explain how Einstein's conclusion that nothing can travel faster than the speed of light contradicts an older concept about the speed of an object propelled from another, already moving, object.
    a. The older concept is that speeds are subtractive. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the

10.2 Consequences of Special Relativity

30. What is the relationship between the binding energy and the mass defect of an atomic nucleus?
    a. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = mc$.
    b. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = mc^2$.
    c. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = \frac{m}{c}$
    d. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = \frac{m}{c^2}$.

31. True or false—It is possible to just use the relationships $F = ma$ and $E = Fd$ to show that both sides of the equation $E_0 = mc^2$ have the same units.
    a. True
    b. False

32. Explain why the special theory of relativity caused the law of conservation of energy to be modified.
    a. The law of conservation of energy is not valid in relativistic mechanics.
    b. The law of conservation of energy has to be modified because of time dilation.
    c. The law of conservation of energy has to be modified because of length contraction.
    d. The law of conservation of energy has to be modified because of mass-energy equivalence.

33. The sun loses about 4×10^9 kg of mass every second. Explain in terms of special relativity why this is happening.
    a. The sun loses mass because of its high temperature.
    b. The sun loses mass because it is continuously releasing energy.
    c. The Sun loses mass because the diameter of the sun is contracted.
    d. The sun loses mass because the speed of the sun is very high and close to the speed of light.

speed at which the person was running minus the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves faster than the speed of light emitted from a stationary source.
    b. The older concept is that speeds are additive. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running plus the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves no

faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

c. The older concept is that speeds are multiplicative. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running multiplied by the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves no faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

d. The older concept is that speeds are frame independent. For example, if a person throws a ball while running, the speed of the ball relative to the ground has nothing to do with the speed at which the person was running. A relativistic example is when light is emitted from car headlights, it moves no faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

35. A rowboat is drifting downstream. One person swims 20 m toward the shore and back, and another, leaving at the same time, swims upstream 20 m and back to the boat. The swimmer who swam toward the shore gets back first. Explain how this outcome is similar to the outcome expected in the Michelson–Morley experiment.

a. The rowboat represents Earth, the swimmers are beams of light, and the water is acting as the ether. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

b. The rowboat represents the beam of light, the swimmers are the ether, and water is acting as Earth. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

c. The rowboat represents the ether, the swimmers are ray of light, and the water is acting as the earth. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

d. The rowboat represents the Earth, the swimmers

are the ether, and the water is acting as the rays of light. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

10.2 Consequences of Special Relativity

36. A helium-4 nucleus is made up of two neutrons and two protons. The binding energy of helium-4 is 4.53×10^{-12} J. What is the difference in the mass of this helium nucleus and the sum of the masses of two neutrons and two protons? Which weighs more, the nucleus or its constituents?

a. 1.51×10^{-20} kg; the constituents weigh more
b. 5.03×10^{-29} kg; the constituents weigh more
c. 1.51×10^{-29} kg; the nucleus weighs more
d. 5.03×10^{-29} kg; the nucleus weighs more

37. Use the equation for length contraction to explain the relationship between the length of an object perceived by a stationary observer who sees the object as moving, and the proper length of the object as measured in the frame of reference where it is at rest.

a. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches zero. For other speeds, the length perceived is always less than the proper length.

b. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches zero. For other speeds, the length perceived is always greater than the proper length.

c. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches infinity. For other speeds, the length perceived is always less than the proper length.

d. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches infinity. For other speeds, the length perceived is always greater than the proper length.

Thermal Energy, Heat, and Work

Figure 11.1 The welder's gloves and helmet protect the welder from the electric arc, which transfers enough thermal energy to melt the rod, spray sparks, and emit high-energy electromagnetic radiation that can burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers (Kevin S. O'Brien, U.S. Navy)

Chapter Outline

INTRODUCTION Heat is something familiar to all of us. We feel the warmth of the summer sun, the hot vapor rising up out of a cup of hot cocoa, and the cooling effect of our sweat. When we feel warmth, it means that heat is transferring energy *to* our bodies; when we feel cold, that means heat is transferring energy *away from* our bodies. Heat transfer is the movement of thermal energy from one place or material to another, and is caused by temperature differences. For example, much of our weather is caused by Earth evening out the temperature across the planet through wind and violent storms, which are driven by heat transferring energy away from the equator towards the cold poles. In this chapter, we'll explore the precise meaning of heat, how it relates to temperature as well as to other forms of energy, and its connection to work.

11.1 Temperature and Thermal Energy

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Explain that temperature is a measure of internal kinetic energy
- Interconvert temperatures between Celsius, Kelvin, and Fahrenheit scales

Section Key Terms

absolute zero	Celsius scale	degree Celsius (°C)	thermal energy
degree Fahrenheit (°F)	Fahrenheit scale	heat	
kelvin (K)	Kelvin scale	temperature	

Temperature

What is **temperature**? It's one of those concepts so ingrained in our everyday lives that, although we know what it means intuitively, it can be hard to define. It is tempting to say that temperature measures heat, but this is not strictly true. **Heat** is the transfer of energy due to a temperature difference. Temperature is defined in terms of the instrument we use to tell us how hot or cold an object is, based on a mechanism and scale invented by people. Temperature is literally defined as what we measure on a thermometer.

Heat is often confused with temperature. For example, we may say that the heat was unbearable, when we actually mean that the temperature was high. This is because we are sensitive to the flow of energy by heat, rather than the temperature. Since heat, like work, transfers energy, it has the SI unit of joule (J).

Atoms and molecules are constantly in motion, bouncing off one another in random directions. Recall that kinetic energy is the energy of motion, and that it increases in proportion to velocity squared. Without going into mathematical detail, we can say that **thermal energy**—the energy associated with heat—is the average kinetic energy of the particles (molecules or atoms) in a substance. Faster moving molecules have greater kinetic energies, and so the substance has greater thermal energy, and thus a higher temperature. The total internal energy of a system is the sum of the kinetic and potential energies of its atoms and molecules. Thermal energy is one of the subcategories of internal energy, as is chemical energy.

To measure temperature, some scale must be used as a standard of measurement. The three most commonly used temperature scales are the Fahrenheit, Celsius, and Kelvin scales. Both the **Fahrenheit scale** and **Celsius scale** are relative temperature scales, meaning that they are made around a reference point. For example, the Celsius scale uses the freezing point of water as its reference point; all measurements are either lower than the freezing point of water by a given number of degrees (and have a negative sign), or higher than the freezing point of water by a given number of degrees (and have a positive sign). The boiling point of water is 100 °C for the Celsius scale, and its unit is the degree Celsius (°C).

On the Fahrenheit scale, the freezing point of water is at 32 °F, and the boiling point is at 212 °F. The unit of temperature on this scale is the degree Fahrenheit (°F). Note that the difference in degrees between the freezing and boiling points is greater for the Fahrenheit scale than for the Celsius scale. Therefore, a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Since 100 Celsius degrees span the same range as 180 Fahrenheit degrees, one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale (because $\frac{180}{100} = \frac{9}{5} = 1.8$). This relationship can be used to convert between temperatures in Fahrenheit and Celsius (see Figure 11.2).
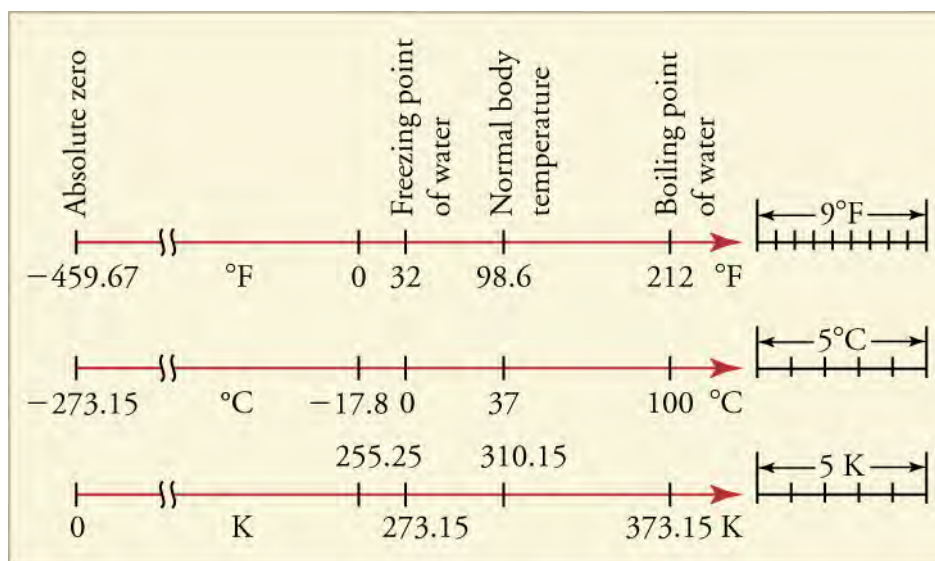
Figure 11.2 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The **Kelvin scale** is the temperature scale that is commonly used in science because it is an absolute temperature scale. This means that the theoretically lowest-possible temperature is assigned the value of zero. Zero degrees on the Kelvin scale is known as **absolute zero**; it is theoretically the point at which there is no molecular motion to produce thermal energy. On the original Kelvin scale first created by Lord Kelvin, all temperatures have positive values, making it useful for scientific work. The official temperature unit on this scale is the kelvin, which is abbreviated as K. The freezing point of water is 273.15 K, and the boiling point of water is 373.15 K.

Although absolute zero is possible in theory, it cannot be reached in practice. The lowest temperature ever created and measured during a laboratory experiment was 1.0×10^{-10} K, at Helsinki University of Technology in Finland. In comparison, the coldest recorded temperature for a place on Earth's surface was 183 K ($-89\,°C$), at Vostok, Antarctica, and the coldest known place (outside the lab) in the universe is the Boomerang Nebula, with a temperature of 1 K. Luckily, most of us humans will never have to experience such extremes.

The average normal body temperature is 98.6 $°F$ (37.0 $°C$), but people have been known to survive with body temperatures ranging from 75 $°F$ to 111 $°F$ (24 $°C$ to 44 $°C$).

👁 WATCH PHYSICS

Comparing Celsius and Fahrenheit Temperature Scales
This video shows how the Fahrenheit and Celsius temperature scales compare to one another.

Click to view content (https://www.openstax.org/l/02celfahtemp)

> **GRASP CHECK**
> Even without the number labels on the thermometer, you could tell which side is marked Fahrenheit and which is Celsius by how the degree marks are spaced. Why?
> a. The separation between two consecutive divisions on the Fahrenheit scale is greater than a similar separation on the Celsius scale, because each degree Fahrenheit is equal to 1.8 degrees Celsius.
> b. The separation between two consecutive divisions on the Fahrenheit scale is smaller than the similar separation on the Celsius scale, because each degree Celsius is equal to 1.8 degrees Fahrenheit.
> c. The separation between two consecutive divisions on the Fahrenheit scale is greater than a similar separation on the Celsius scale, because each degree Fahrenheit is equal to 3.6 degrees Celsius.
> d. The separation between two consecutive divisions on the Fahrenheit scale is smaller than a similar separation on the Celsius scale, because each degree Celsius is equal to 3.6 degrees Fahrenheit.

Converting Between Celsius, Kelvin, and Fahrenheit Scales

While the Fahrenheit scale is still the most commonly used scale in the United States, the majority of the world uses Celsius, and scientists prefer Kelvin. It's often necessary to convert between these scales. For instance, if the TV meteorologist gave the local weather report in kelvins, there would likely be some confused viewers! Table 11.1 gives the equations for conversion between the three temperature scales.

To Convert From...	Use This Equation
Celsius to Fahrenheit	$T_{°F} = \dfrac{9}{5} T_{°C} + 32$
Fahrenheit to Celsius	$T_{°C} = \dfrac{5}{9}(T_{°F} - 32)$
Celsius to Kelvin	$T_K = T_{°C} + 273.15$
Kelvin to Celsius	$T_{°C} = T_K - 273.15$
Fahrenheit to Kelvin	$T_K = \dfrac{5}{9}(T_{°F} - 32) + 273.15$
Kelvin to Fahrenheit	$T_{°F} = \dfrac{9}{5}(T_K - 273.15) + 32$

Table 11.1 Temperature Conversions

✳ WORKED EXAMPLE

Room temperature is generally defined to be 25 °C. (a) What is room temperature in °F? (b) What is it in K?

STRATEGY
To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)
1. Choose the right equation. To convert from °C to °F, use the equation
$$T_{°F} = \frac{9}{5} T_{°C} + 32.$$
11.1

2. Plug the known value into the equation and solve.
$$T_{°F} = \frac{9}{5} 25\ °C + 32 = 77\ °F$$
11.2

Solution for (b)
1. Choose the right equation. To convert from °C to K, use the equation
$$T_K = T_{°C} + 273.15.$$
11.3

2. Plug the known value into the equation and solve.
$$T_K = 25\ °C + 273.15 = 298 K$$
11.4

Discussion
Living in the United States, you are likely to have more of a sense of what the temperature feels like if it's described as 77 °F than as 25 °C (or 298 K, for that matter).

✳ WORKED EXAMPLE

Converting Between Temperature Scales: The Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is 0 °R and the boiling temperature is 80 °R. If "room temperature" is 25 °C on the Celsius scale, what is it on the Reaumur scale?

STRATEGY

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is 80 °R . On the Celsius scale, it is 100 °C . Therefore, 100 °C = 80 °R . Both scales start at 0 ° for freezing, so we can create a simple formula to convert between temperatures on the two scales.

Solution

1. Derive a formula to convert from one scale to the other.

$$T_{°R} = \frac{0.80°R}{°C} \times T_{°C}$$

<div style="text-align:right">11.5</div>

2. Plug the known value into the equation and solve.

$$T_{°R} = \frac{0.80°R}{°C} \times 25 \ °C = 20°R$$

<div style="text-align:right">11.6</div>

Discussion

As this example shows, relative temperature scales are somewhat arbitrary. If you wanted, you could create your own temperature scale!

Practice Problems

1. What is 12.0 °C in kelvins?
   a. 112.0 K
   b. 273.2 K
   c. 12.0 K
   d. 285.2 K

2. What is 32.0 °C in degrees Fahrenheit?
   a. 57.6 °F
   b. 25.6 °F
   c. 305.2 °F
   d. 89.6 °F

TIPS FOR SUCCESS

Sometimes it is not so easy to guess the temperature of the air accurately. Why is this? Factors such as humidity and wind speed affect how hot or cold we feel. Wind removes thermal energy from our bodies at a faster rate than usual, making us feel colder than we otherwise would; on a cold day, you may have heard the TV weather person refer to the *wind chill*. On humid summer days, people tend to feel hotter because sweat doesn't evaporate from the skin as efficiently as it does on dry days, when the evaporation of sweat cools us off.

Check Your Understanding

3. What is thermal energy?
   a. The thermal energy is the average potential energy of the particles in a system.
   b. The thermal energy is the total sum of the potential energies of the particles in a system.
   c. The thermal energy is the average kinetic energy of the particles due to the interaction among the particles in a system.
   d. The thermal energy is the average kinetic energy of the particles in a system.

4. What is used to measure temperature?

a. a galvanometer
b. a manometer
c. a thermometer
d. a voltmeter

11.2 Heat, Specific Heat, and Heat Transfer

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain heat, heat capacity, and specific heat
- Distinguish between conduction, convection, and radiation
- Solve problems involving specific heat and heat transfer

Section Key Terms

conduction   convection   heat capacity   radiation   specific heat

Heat Transfer, Specific Heat, and Heat Capacity

We learned in the previous section that temperature is proportional to the average kinetic energy of atoms and molecules in a substance, and that the average internal kinetic energy of a substance is higher when the substance's temperature is higher.

If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter object (that is, the object with the greater temperature) to the colder (lower temperature) object, until both objects are at the same temperature. There is no net heat transfer once the temperatures are equal because the amount of heat transferred from one object to the other is the same as the amount of heat returned. One of the major effects of heat transfer is temperature change: Heating increases the temperature while cooling decreases it. Experiments show that the heat transferred to or from a substance depends on three factors—the change in the substance's temperature, the mass of the substance, and certain physical properties related to the phase of the substance.

The equation for heat transfer Q is

$$Q = mc\Delta T,$$

11.7

where m is the mass of the substance and ΔT is the change in its temperature, in units of Celsius or Kelvin. The symbol c stands for **specific heat**, and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00 °C. The specific heat c is a property of the substance; its SI unit is J/(kg · K) or J/(kg · °C). The temperature change (ΔT) is the same in units of kelvins and degrees Celsius (but not degrees Fahrenheit). Specific heat is closely related to the concept of **heat capacity**. Heat capacity is the amount of heat necessary to change the temperature of a substance by 1.00 °C . In equation form, heat capacity C is $C = mc$, where m is mass and c is specific heat. Note that heat capacity is the same as specific heat, but without any dependence on mass. Consequently, two objects made up of the same material but with different masses will have different heat capacities. This is because the heat capacity is a property of an object, but specific heat is a property of *any* object made of the same material.

Values of specific heat must be looked up in tables, because there is no simple way to calculate them. Table 11.2 gives the values of specific heat for a few substances as a handy reference. We see from this table that the specific heat of water is five times that of glass, which means that it takes five times as much heat to raise the temperature of 1 kg of water than to raise the temperature of 1 kg of glass by the same number of degrees.

Substances	Specific Heat (*c*)
Solids	J/(kg ·°C)
Aluminum	900

Table 11.2 Specific Heats of Various Substances.

Substances	Specific Heat (c)
Asbestos	800
Concrete, granite (average)	840
Copper	387
Glass	840
Gold	129
Human body (average)	3500
Ice (average)	2090
Iron, steel	452
Lead	128
Silver	235
Wood	1700
Liquids	
Benzene	1740
Ethanol	2450
Glycerin	2410
Mercury	139
Water	4186
Gases (at 1 atm constant pressure)	
Air (dry)	1015
Ammonia	2190
Carbon dioxide	833
Nitrogen	1040
Oxygen	913
Steam	2020

Table 11.2 Specific Heats of Various Substances.

Snap Lab

Temperature Change of Land and Water

What heats faster, land or water? You will answer this question by taking measurements to study differences in specific heat capacity.

- Open flame—Tie back all loose hair and clothing before igniting an open flame. Follow all of your teacher's instructions on how to ignite the flame. Never leave an open flame unattended. Know the location of fire safety equipment in the laboratory.

- Sand or soil
- Water
- Oven or heat lamp
- Two small jars
- Two thermometers

Instructions

Procedure

1. Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can get equal masses by using 50 percent more water by volume.)
2. Heat both substances (using an oven or a heat lamp) for the same amount of time.
3. Record the final temperatures of the two masses.
4. Now bring both jars to the same temperature by heating for a longer period of time.
5. Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

GRASP CHECK

Did it take longer to heat the water or the sand/soil to the same temperature? Which sample took longer to cool? What does this experiment tell us about how the specific heat of water compared to the specific heat of land?

a. The sand/soil will take longer to heat as well as to cool. This tells us that the specific heat of land is greater than that of water.
b. The sand/soil will take longer to heat as well as to cool. This tells us that the specific heat of water is greater than that of land.
c. The water will take longer to heat as well as to cool. This tells us that the specific heat of land is greater than that of water.
d. The water will take longer to heat as well as to cool. This tells us that the specific heat of water is greater than that of land.

Conduction, Convection, and Radiation

Whenever there is a temperature difference, heat transfer occurs. Heat transfer may happen rapidly, such as through a cooking pan, or slowly, such as through the walls of an insulated cooler.

There are three different heat transfer methods: **conduction**, **convection**, and **radiation**. At times, all three may happen simultaneously. See Figure 11.3.
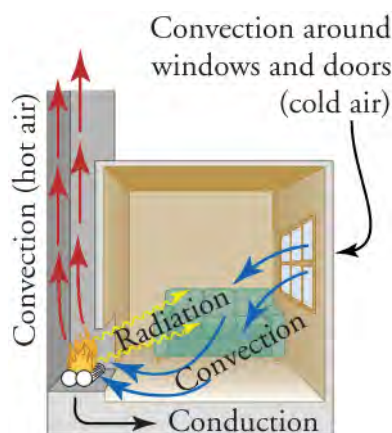
Figure 11.3 In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

Conduction is heat transfer through direct physical contact. Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction. Sometimes, we try to control the conduction of heat to make ourselves more comfortable. Since the rate of heat transfer is different for different materials, we choose fabrics, such as a thick wool sweater, that slow down the transfer of heat away from our bodies in winter.

As you walk barefoot across the living room carpet, your feet feel relatively comfortable...until you step onto the kitchen's tile floor. Since the carpet and tile floor are both at the same temperature, why does one feel colder than the other? This is explained by different rates of heat transfer: The tile material removes heat from your skin at a greater rate than the carpeting, which makes it *feel* colder.

Some materials simply conduct thermal energy faster than others. In general, metals (like copper, aluminum, gold, and silver) are good heat conductors, whereas materials like wood, plastic, and rubber are poor heat conductors.

Figure 11.4 shows particles (either atoms or molecules) in two bodies at different temperatures. The (average) kinetic energy of a particle in the hot body is higher than in the colder body. If two particles collide, energy transfers from the particle with greater kinetic energy to the particle with less kinetic energy. When two bodies are in contact, many particle collisions occur, resulting in a net flux of heat from the higher-temperature body to the lower-temperature body. The heat flux depends on the temperature difference $\Delta T = T_{\text{hot}} - T_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water.
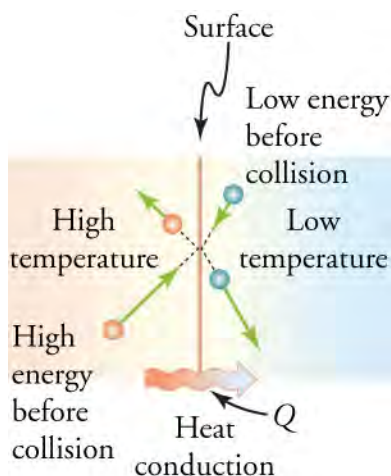
Figure 11.4 The particles in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a particle in the lower-temperature region (right side) has low kinetic energy before collision, but its kinetic energy increases after colliding with the contact

surface. In contrast, a particle in the higher-temperature region (left side) has more kinetic energy before collision, but its energy decreases after colliding with the contact surface.

Convection is heat transfer by the movement of a fluid. This type of heat transfer happens, for example, in a pot boiling on the stove, or in thunderstorms, where hot air rises up to the base of the clouds.

TIPS FOR SUCCESS

In everyday language, the term *fluid* is usually taken to mean liquid. For example, when you are sick and the doctor tells you to "push fluids," that only means to drink more beverages—not to breath more air. However, in physics, fluid means a liquid *or a gas*. Fluids move differently than solid material, and even have their own branch of physics, known as *fluid dynamics*, that studies how they move.

As the temperature of fluids increase, they expand and become less dense. For example, Figure 11.4 could represent the wall of a balloon with different temperature gases inside the balloon than outside in the environment. The hotter and thus faster moving gas particles inside the balloon strike the surface with more force than the cooler air outside, causing the balloon to expand. This decrease in density relative to its environment creates buoyancy (the tendency to rise). Convection is driven by buoyancy—hot air rises because it is less dense than the surrounding air.

Sometimes, we control the temperature of our homes or ourselves by controlling air movement. Sealing leaks around doors with weather stripping keeps out the cold wind in winter. The house in Figure 11.5 and the pot of water on the stove in Figure 11.6 are both examples of convection and buoyancy by human design. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another, and are examples of natural convection.
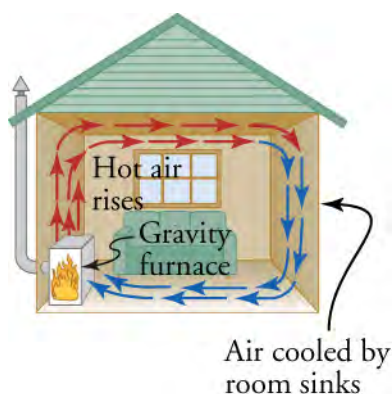
Figure 11.5 Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system like this one, which uses natural convection, can be quite efficient in uniformly heating a home.

Figure 11.6 Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside fluid, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process repeats as long as there is water in the pot.

Radiation is a form of heat transfer that occurs when electromagnetic radiation is emitted or absorbed. Electromagnetic

radiation includes radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays, all of which have different wavelengths and amounts of energy (shorter wavelengths have higher frequency and more energy).

You can feel the heat transfer from a fire and from the sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. Another example is thermal radiation from the human body; people are constantly emitting infrared radiation, which is not visible to the human eye, but is felt as heat.

Radiation is the only method of heat transfer where no medium is required, meaning that the heat doesn't need to come into direct contact with or be transported by any matter. The space between Earth and the sun is largely empty, without any possibility of heat transfer by convection or conduction. Instead, heat is transferred by radiation, and Earth is warmed as it absorbs electromagnetic radiation emitted by the sun.

Figure 11.7 Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light transfers relatively little thermal energy. Since skin is very sensitive to infrared radiation, you can sense the presence of a fire without looking at it directly. (Daniel X. O'Neil)

All objects absorb and emit electromagnetic radiation (see Figure 11.7). The rate of heat transfer by radiation depends mainly on the color of the object. Black is the most effective absorber and radiator, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance. Similarly, black asphalt in a parking lot will be hotter than adjacent patches of grass on a summer day, because black absorbs better than green. The reverse is also true—black radiates better than green. On a clear summer night, the black asphalt will be colder than the green patch of grass, because black radiates energy faster than green. In contrast, white is a poor absorber and also a poor radiator. A white object reflects nearly all radiation, like a mirror.

Virtual Physics

Energy Forms and Changes

Click to view content (http://www.openstax.org/l/28energyForms)

In this animation, you will explore heat transfer with different materials. Experiment with heating and cooling the iron, brick, and water. This is done by dragging and dropping the object onto the pedestal and then holding the lever either to Heat or Cool. Drag a thermometer beside each object to measure its temperature—you can watch how quickly it heats or cools in real time.

Now let's try transferring heat between objects. Heat the brick and then place it in the cool water. Now heat the brick again, but then place it on top of the iron. What do you notice?

Selecting the fast forward option lets you speed up the heat transfers, to save time.

GRASP CHECK

Compare how quickly the different materials are heated or cooled. Based on these results, what material do you think has the greatest specific heat? Why? Which has the smallest specific heat? Can you think of a real-world situation where you would want to use an object with large specific heat?

a. Water will take the longest, and iron will take the shortest time to heat, as well as to cool. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

    b.   Water will take the shortest, and iron will take the longest time to heat, as well as to cool. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

    c.   Brick will take shortest and iron will take longest time to heat up as well as to cool down. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

    d.   Water will take shortest and brick will take longest time to heat up as well as to cool down. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

Solving Heat Transfer Problems

✳ WORKED EXAMPLE

Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 L of water from 20.0 °C to 80.0 °C . (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

STRATEGY

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for heat transfer for the given temperature change and masses of water and aluminum. The specific heat values for water and aluminum are given in the previous table.

Solution to (a)

Because the water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1.   Calculate the temperature difference.
$$\Delta T = T_f - T_i = 60.0 \ °C \qquad \text{[11.8]}$$

2.   Calculate the mass of water using the relationship between density, mass, and volume. Density is mass per unit volume, or $\rho = \frac{m}{V}$. Rearranging this equation, solve for the mass of water.
$$m_w = \rho \cdot V = 1000 \ \text{kg/m}^3 \times \left(0.250 \ \text{L} \times \frac{0.001 \ \text{m}^3}{1 \ \text{L}} \right) = 0.250 \ \text{kg} \qquad \text{[11.9]}$$

3.   Calculate the heat transferred to the water. Use the specific heat of water in the previous table.
$$Q_w = m_w c_w \Delta T = \ (0.250 \ \text{kg})(4186 \ \text{J/kg°C})(60.0°C) = 62.8 \ \text{kJ} \qquad \text{[11.10]}$$

4.   Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in the previous table.
$$Q_{Al} = m_{Al} c_{Al} \Delta T = \ (0.500 \ \text{kg})(900 \ \text{J/kg°C})(60.0°C) = 27.0 \times 10^3 \text{J} = 27.0 \ \text{kJ} \qquad \text{[11.11]}$$

5.   Find the total transferred heat.
$$Q_{Total} = Q_w + Q_{Al} = 62.8 \ \text{kJ} + 27.0 \ \text{kJ} = 89.8 \ \text{kJ} \qquad \text{[11.12]}$$

Solution to (b)

The percentage of heat going into heating the pan is
$$\frac{27.0 \ \text{kJ}}{89.8 \ \text{kJ}} \times 100\% = 30.1\% \qquad \text{[11.13]}$$

Solution to (c)

The percentage of heat going into heating the water is
$$\frac{62.8 \ \text{kJ}}{89.8 \ \text{kJ}} \times 100\% = 69.9\% \qquad \text{[11.14]}$$

Discussion

In this example, most of the total heat transferred is used to heat the water, even though the pan has twice as much mass. This is

because the specific heat of water is over four times greater than the specific heat of aluminum. Therefore, it takes a bit more than twice as much heat to achieve the given temperature change for the water than for the aluminum pan.

Water can absorb a tremendous amount of energy with very little resulting temperature change. This property of water allows for life on Earth because it stabilizes temperatures. Other planets are less habitable because wild temperature swings make for a harsh environment. You may have noticed that climates closer to large bodies of water, such as oceans, are milder than climates landlocked in the middle of a large continent. This is due to the climate-moderating effect of water's large heat capacity—water stores large amounts of heat during hot weather and releases heat gradually when it's cold outside.

✳ WORKED EXAMPLE

Calculating Temperature Increase: Truck Brakes Overheat on Downhill Runs

When a truck headed downhill brakes, the brakes must do work to convert the gravitational potential energy of the truck to internal energy of the brakes. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck, and keeps the truck from speeding up and losing control. The increased internal energy of the brakes raises their temperature. When the hill is especially steep, the temperature increase may happen too quickly and cause the brakes to overheat.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of 800 J/kg ·°C from a 10,000 kg truck descending 75.0 m (in vertical displacement) at a constant speed.

STRATEGY

We first calculate the gravitational potential energy (Mgh) of the truck, and then find the temperature increase produced in the brakes.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill.

$$Mgh = (10{,}000 \ \text{kg})\,(9.80 \ \text{m/s}^2)\,(75.0 \ \text{m}) = 7.35 \times 10^6 \,\text{J} \qquad \boxed{11.15}$$

2. Calculate the temperature change from the heat transferred by rearranging the equation $Q = mc\Delta T$ to solve for ΔT.

$$\Delta T = \frac{Q}{mc}, \qquad \boxed{11.16}$$

where m is the mass of the brake material (not the entire truck). Insert the values $Q = 7.35 \times 10^6$ J (since the heat transfer is equal to the change in gravitational potential energy), $m = 100$ kg, and $c = 800$ J/kg · °C to find

$$\Delta T = \frac{7.35 \times 10^6 \,\text{J}}{(100 \ \text{kg})(800 \ \text{J/kg} \cdot \text{°C})} = 91.9 \ \text{°C}. \qquad \boxed{11.17}$$

Discussion

This temperature is close to the boiling point of water. If the truck had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, which would be hard on the brakes. This is why truck drivers sometimes use a different technique for called "engine braking" to avoid burning their brakes during steep descents. Engine braking is using the slowing forces of an engine in low gear rather than brakes to slow down.

Practice Problems

5. How much heat does it take to raise the temperature of 10.0 kg of water by 1.0 °C ?

    a. 84 J
    b. 42 J
    c. 84 kJ
    d. 42 kJ

6. Calculate the change in temperature of 1.0 kg of water that is initially at room temperature if 3.0 kJ of heat is added.

    a. 358 °C
    b. 716 °C
    c. 0.36 °C
    d. 0.72 °C

Check Your Understanding

7. What causes heat transfer?

    a. The mass difference between two objects causes heat transfer.
    b. The density difference between two objects causes heat transfer.
    c. The temperature difference between two systems causes heat transfer.
    d. The pressure difference between two objects causes heat transfer.

8. When two bodies of different temperatures are in contact, what is the overall direction of heat transfer?

    a. The overall direction of heat transfer is from the higher-temperature object to the lower-temperature object.
    b. The overall direction of heat transfer is from the lower-temperature object to the higher-temperature object.
    c. The direction of heat transfer is first from the lower-temperature object to the higher-temperature object, then back again to the lower-temperature object, and so-forth, until the objects are in thermal equilibrium.
    d. The direction of heat transfer is first from the higher-temperature object to the lower-temperature object, then back again to the higher-temperature object, and so-forth, until the objects are in thermal equilibrium.

9. What are the different methods of heat transfer?

    a. conduction, radiation, and reflection
    b. conduction, reflection, and convection
    c. convection, radiation, and reflection
    d. conduction, radiation, and convection

10. True or false—Conduction and convection cannot happen simultaneously

    a. True
    b. False

11.3 Phase Change and Latent Heat

Section Learning Objectives

By the end of this section, you will be able to do the following:

* Explain changes in heat during changes of state, and describe latent heats of fusion and vaporization
* Solve problems involving thermal energy changes when heating and cooling substances with phase changes

Section Key Terms

|			
condensation	freezing	latent heat	sublimation
latent heat of fusion	latent heat of vaporization	melting	vaporization
phase change	phase diagram	plasma	

Phase Changes

So far, we have learned that adding thermal energy by heat increases the temperature of a substance. But surprisingly, there are situations where adding energy does not change the temperature of a substance at all! Instead, the additional thermal energy acts to loosen bonds between molecules or atoms and causes a **phase change**. Because this energy enters or leaves a system during a phase change without causing a temperature change in the system, it is known as **latent heat** (latent means *hidden*).

The three phases of matter that you frequently encounter are solid, liquid and gas (see Figure 11.8). Solid has the least energetic state; atoms in solids are in close contact, with forces between them that allow the particles to vibrate but not change position with neighboring particles. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.)

Liquid has a more energetic state, in which particles can slide smoothly past one another and change neighbors, although they are still held together by their mutual attraction.

Gas has a more energetic state than liquid, in which particles are broken free of their bonds. Particles in gases are separated by distances that are large compared with the size of the particles.

The most energetic state of all is **plasma**. Although you may not have heard much about plasma, it is actually the most common state of matter in the universe—stars are made up of plasma, as is lightning. The plasma state is reached by heating a gas to the point where particles are pulled apart, separating the electrons from the rest of the particle. This produces an ionized gas that is a combination of the negatively charged free electrons and positively charged ions, known as plasma.
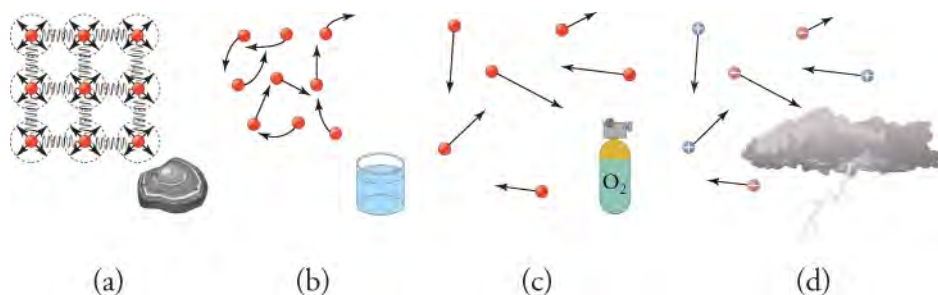
Figure 11.8 (a) Particles in a solid always have the same neighbors, held close by forces represented here by springs. These particles are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms or molecules together. (b) Particles in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its molecules. (c) Particles in a gas are separated by distances that are considerably larger than the size of the particles themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out into its surroundings. (d) The atmosphere is ionized in the extreme heat of a lightning strike.

During a phase change, matter changes from one phase to another, either through the addition of energy by heat and the transition to a more energetic state, or from the removal of energy by heat and the transition to a less energetic state.

Phase changes to a more energetic state include the following:

- **Melting**—Solid to liquid
- **Vaporization**—Liquid to gas (included boiling and evaporation)
- **Sublimation**—Solid to gas

Phase changes to a less energetic state are as follows:

- **Condensation**—Gas to liquid
- **Freezing**—Liquid to solid

Energy is required to melt a solid because the bonds between the particles in the solid must be broken. Since the energy involved in a phase changes is used to break bonds, there is no increase in the kinetic energies of the particles, and therefore no rise in temperature. Similarly, energy is needed to vaporize a liquid to overcome the attractive forces between particles in the liquid. There is no temperature change until a phase change is completed. The temperature of a cup of soda and ice that is initially at 0 °C stays at 0 °C until all of the ice has melted. In the reverse of these processes—freezing and condensation—energy is released

from the latent heat (see Figure 11.9).
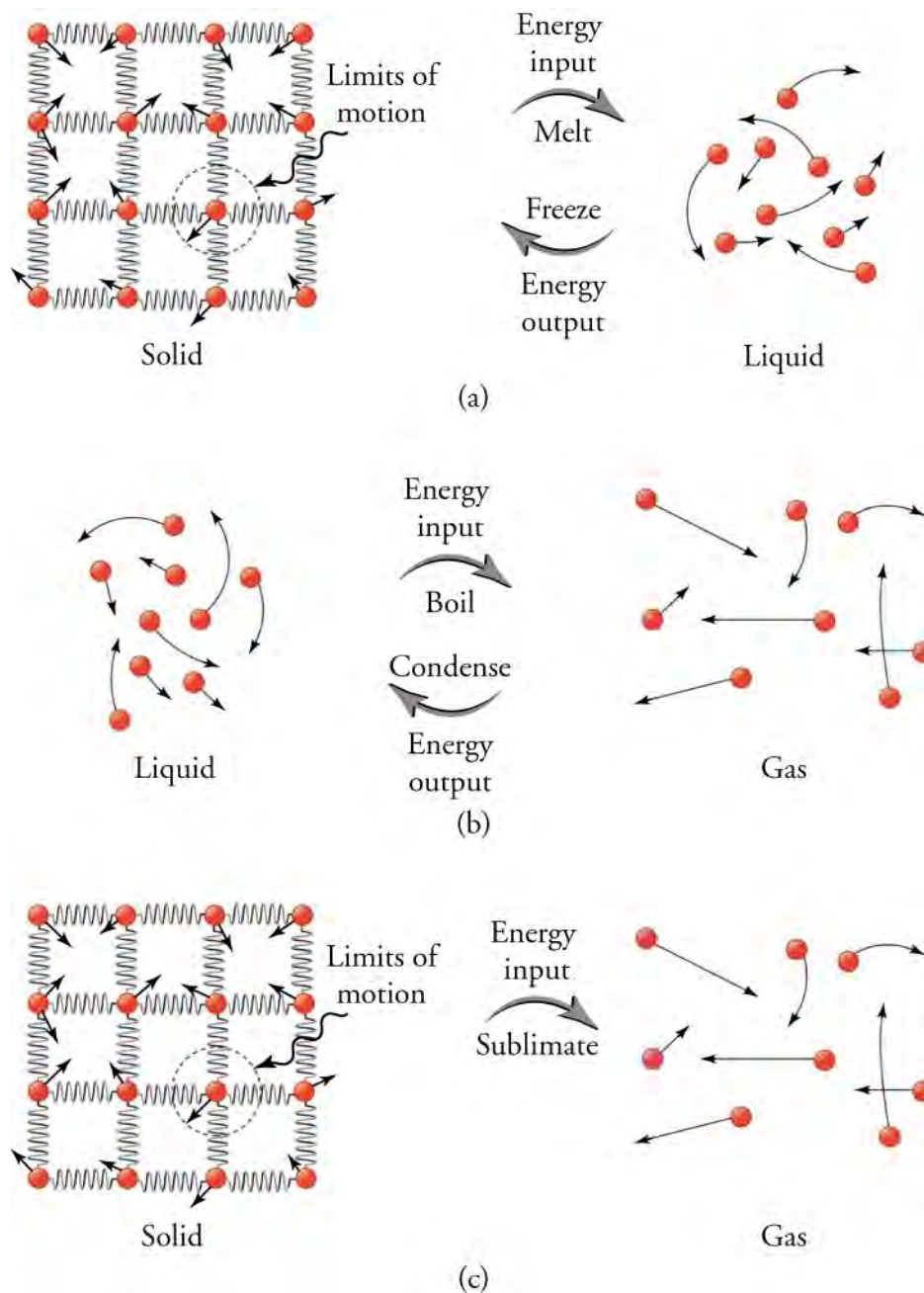
Figure 11.9 (a) Energy is required to partially overcome the attractive forces between particles in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Particles are separated by large distances when changing from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is completed. (c) Enough energy is added that the liquid state is skipped over completely as a substance undergoes sublimation.

The heat, Q, required to change the phase of a sample of mass m is

$Q = mL_f$ (for melting/freezing),

$Q = mL_v$ (for vaporization/condensation),

where L_f is the **latent heat of fusion**, and L_v is the **latent heat of vaporization**. The latent heat of fusion is the amount of heat needed to cause a phase change between solid and liquid. The latent heat of vaporization is the amount of heat needed to cause a

phase change between liquid and gas. L_f and L_v are coefficients that vary from substance to substance, depending on the strength of intermolecular forces, and both have standard units of J/kg. See Table 11.3 for values of L_f and L_v of different substances.

Substance	Melting Point ($°C$)	Lf (kJ/kg)	Boiling Point ($°C$)	Lv (kJ/kg)
Helium	−269.7	5.23	−268.9	20.9
Hydrogen	−259.3	58.6	−252.9	452
Nitrogen	−210.0	25.5	−195.8	201
Oxygen	−218.8	13.8	−183.0	213
Ethanol	−114	104	78.3	854
Ammonia	−78	332	−33.4	1370
Mercury	−38.9	11.8	357	272
Water	0.00	334	100.0	2256
Sulfur	119	38.1	444.6	326
Lead	327	24.5	1750	871
Antimony	631	165	1440	561
Aluminum	660	380	2520	11400
Silver	961	88.3	2193	2336
Gold	1063	64.5	2660	1578
Copper	1083	134	2595	5069
Uranium	1133	84	3900	1900
Tungsten	3410	184	5900	4810

Table 11.3 Latent Heats of Fusion and Vaporization, along with Melting and Boiling Points

Let's consider the example of adding heat to ice to examine its transitions through all three phases—solid to liquid to gas. A phase diagram indicating the temperature changes of water as energy is added is shown in Figure 11.10. The ice starts out at –20 $°C$, and its temperature rises linearly, absorbing heat at a constant rate until it reaches 0 $°$. Once at this temperature, the ice gradually melts, absorbing 334 kJ/kg. The temperature remains constant at 0 $°C$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate. At 100 $°C$, the water begins to boil and the temperature again remains constant while the water absorbs 2256 kJ/kg during this phase change. When all the liquid has become steam, the temperature rises again at a constant rate.
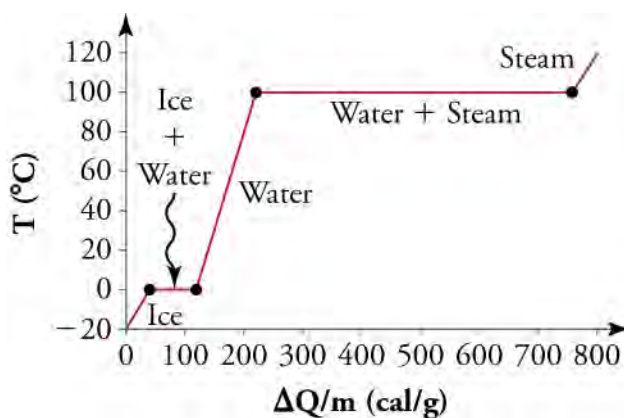
Figure 11.10 A graph of temperature versus added energy. The system is constructed so that no vapor forms while ice warms to become liquid water, and so when vaporization occurs, the vapor remains in the system. The long stretches of constant temperature values at 0 °C and 100 °C reflect the large latent heats of melting and vaporization, respectively.

We have seen that vaporization requires heat transfer to a substance from its surroundings. Condensation is the reverse process, where heat in transferred *away from* a substance *to* its surroundings. This release of latent heat increases the temperature of the surroundings. Energy must be removed from the condensing particles to make a vapor condense. This is why condensation occurs on cold surfaces: the heat transfers energy away from the warm vapor to the cold surface. The energy is exactly the same as that required to cause the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_v$. Latent heat is also released into the environment when a liquid freezes, and can be calculated from $Q = mL_f$.

FUN IN PHYSICS

Making Ice Cream

Figure 11.11 With the proper ingredients, some ice and a couple of plastic bags, you could make your own ice cream in five minutes. (ElinorD, Wikimedia Commons)

Ice cream is certainly easy enough to buy at the supermarket, but for the hardcore ice cream enthusiast, that may not be satisfying enough. Going through the process of making your own ice cream lets you invent your own flavors and marvel at the physics firsthand (Figure 11.11).

The first step to making homemade ice cream is to mix heavy cream, whole milk, sugar, and your flavor of choice; it could be as

simple as cocoa powder or vanilla extract, or as fancy as pomegranates or pistachios.

The next step is to pour the mixture into a container that is deep enough that you will be able to churn the mixture without it spilling over, and that is also freezer-safe. After placing it in the freezer, the ice cream has to be stirred vigorously every 45 minutes for four to five hours. This slows the freezing process and prevents the ice cream from turning into a solid block of ice. Most people prefer a soft creamy texture instead of one giant popsicle.

As it freezes, the cream undergoes a phase change from liquid to solid. By now, we're experienced enough to know that this means that the cream must experience a loss of heat. Where does that heat go? Due to the temperature difference between the freezer and the ice cream mixture, heat transfers thermal energy from the ice cream to the air in the freezer. Once the temperature in the freezer rises enough, the freezer is cooled by pumping excess heat outside into the kitchen.

A faster way to make ice cream is to chill it by placing the mixture in a plastic bag, surrounded by another plastic bag half full of ice. (You can also add a teaspoon of salt to the outer bag to lower the temperature of the ice/salt mixture.) Shaking the bag for five minutes churns the ice cream while cooling it evenly. In this case, the heat transfers energy out of the ice cream mixture and into the ice during the phase change.

This video (http://www.openstax.org/l/28icecream) gives a demonstration of how to make home-made ice cream using ice and plastic bags.

GRASP CHECK
Why does the ice bag method work so much faster than the freezer method for making ice cream?
a. Ice has a smaller specific heat than the surrounding air in a freezer. Hence, it absorbs more energy from the ice-cream mixture.
b. Ice has a smaller specific heat than the surrounding air in a freezer. Hence, it absorbs less energy from the ice-cream mixture.
c. Ice has a greater specific heat than the surrounding air in a freezer. Hence, it absorbs more energy from the ice-cream mixture.
d. Ice has a greater specific heat than the surrounding air in a freezer. Hence, it absorbs less energy from the ice-cream mixture.

Solving Thermal Energy Problems with Phase Changes

✳ WORKED EXAMPLE

Calculating Heat Required for a Phase Change
Calculate a) how much energy is needed to melt 1.000 kg of ice at 0 °C (freezing point), and b) how much energy is required to vaporize 1.000 kg of water at 100 °C (boiling point).

STRATEGY FOR (A)
Using the equation for the heat required for melting, and the value of the latent heat of fusion of water from the previous table, we can solve for part (a).

Solution to (a)
The energy to melt 1.000 kg of ice is

$$Q = mL_f = (1.000 \text{ kg})(334 \text{ kJ/kg}) = 334 \text{ kJ}.$$

11.18

STRATEGY FOR (B)
To solve part (b), we use the equation for heat required for vaporization, along with the latent heat of vaporization of water from the previous table.

Solution to (b)
The energy to vaporize 1.000 kg of liquid water is

$$Q = mL_v = (1.000 \text{ kg})(2256 \text{ kJ/kg}) = 2256 \text{ kJ}.$$

11.19

Discussion

The amount of energy need to melt a kilogram of ice (334 kJ) is the same amount of energy needed to raise the temperature of 1.000 kg of liquid water from 0 °C to 79.8 °C . This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes. It also demonstrates that the amount of energy needed for vaporization is even greater.

✳ WORKED EXAMPLE

Calculating Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Ice cubes are used to chill a soda at 20 °C and with a mass of $m_{soda} = 0.25$ kg . The ice is at 0 °C and the total mass of the ice cubes is 0.018 kg. Assume that the soda is kept in a foam container so that heat loss can be ignored, and that the soda has the same specific heat as water. Find the final temperature when all of the ice has melted.

STRATEGY

The ice cubes are at the melting temperature of 0 °C . Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first, the phase change occurs and solid (ice) transforms into liquid water at the melting temperature; then, the temperature of this water rises. Melting yields water at 0 °C, so more heat is transferred from the soda to this water until they are the same temperature. Since the amount of heat leaving the soda is the same as the amount of heat transferred to the ice.

$$Q_{ice} = -Q_{soda} \qquad \text{11.20}$$

The heat transferred to the ice goes partly toward the phase change (melting), and partly toward raising the temperature after melting. Recall from the last section that the relationship between heat and temperature change is $Q = mc\Delta T$. For the ice, the temperature change is $T_f - 0$ °C . The total heat transferred to the ice is therefore

$$Q_{ice} = m_{ice}L_f + m_{ice}c_w(T_f - 0 \text{ °C}). \qquad \text{11.21}$$

Since the soda doesn't change phase, but only temperature, the heat given off by the soda is

$$Q_{soda} = m_{soda}c_w(T_f - 20 \text{ °C}). \qquad \text{11.22}$$

Since $Q_{ice} = -Q_{soda}$,

$$m_{ice}L_f + m_{ice}c_w(T_f - 0 \text{ °C}) = -m_{soda}c_w(T_f - 20 \text{ °C}). \qquad \text{11.23}$$

Bringing all terms involving T_f to the left-hand-side of the equation, and all other terms to the right-hand-side, we can solve for T_f .

$$T_f = \frac{m_{soda}c_w(20 \text{ °C}) - m_{ice}L_f}{(m_{soda} + m_{ice})c_w} \qquad \text{11.24}$$

Substituting the known quantities

$$T_f = \frac{(0.25 \text{ kg})(4186 \text{ J/kg} \cdot \text{°C})(20 \text{ °C}) - (0.018 \text{ kg})(334{,}000 \text{ J/kg})}{(0.25 \text{ kg} + 0.018 \text{ kg})(4186 \text{ K/kg} \cdot \text{°C})} = 13 \text{ °C} \qquad \text{11.25}$$

Discussion

This example shows the enormous energies involved during a phase change. The mass of the ice is about 7 percent the mass of the soda, yet it causes a noticeable change in the soda's temperature.

> **TIPS FOR SUCCESS**
>
> If the ice were not already at the freezing point, we would also have to factor in how much energy would go into raising its temperature up to 0 °C, before the phase change occurs. This would be a realistic scenario, because the temperature of ice is often below 0 °C .

Practice Problems

11. How much energy is needed to melt 2.00 kg of ice at 0 °C ?
    a. 334 kJ
    b. 336 kJ
    c. 167 kJ
    d. 668 kJ

12. If 2500 kJ of energy is just enough to melt 3.0 kg of a substance, what is the substance's latent heat of fusion?
    a. 7500 kJ · kg
    b. 7500 kJ/kg
    c. 830 kJ · kg
    d. 830 kJ/kg

Check Your Understanding

13. What is latent heat?
    a. It is the heat that must transfer energy to or from a system in order to cause a mass change with a slight change in the temperature of the system.
    b. It is the heat that must transfer energy to or from a system in order to cause a mass change without a temperature change in the system.
    c. It is the heat that must transfer energy to or from a system in order to cause a phase change with a slight change in the temperature of the system.
    d. It is the heat that must transfer energy to or from a system in order to cause a phase change without a temperature change in the system.

14. In which phases of matter are molecules capable of changing their positions?
    a. gas, liquid, solid
    b. liquid, plasma, solid
    c. liquid, gas, plasma
    d. plasma, gas, solid

KEY TERMS

absolute zero   lowest possible temperature; the temperature at which all molecular motion ceases

Celsius scale   temperature scale in which the freezing point of water is 0 °C and the boiling point of water is 100 °C at 1 atm of pressure

condensation   phase change from gas to liquid

conduction   heat transfer through stationary matter by physical contact

convection   heat transfer by the movement of fluid

degree Celsius   unit on the Celsius temperature scale

degree Fahrenheit   unit on the Fahrenheit temperature scale

Fahrenheit scale   temperature scale in which the freezing point of water is 32 °F and the boiling point of water is 212 °F

freezing   phase change from liquid to solid

heat   transfer of thermal (or internal) energy due to a temperature difference

heat capacity   amount of heat necessary to change the temperature of a substance by 1.00 °C

Kelvin   unit on the Kelvin temperature scale; note that it is never referred to in terms of "degrees" Kelvin

Kelvin scale   temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

latent heat   heat related to the phase change of a substance rather than a change of temperature

latent heat of fusion   amount of heat needed to cause a phase change between solid and liquid

latent heat of vaporization   amount of heat needed to cause a phase change between liquid and gas

melting   phase change from solid to liquid

phase change   transition between solid, liquid, or gas states of a substance

plasma   ionized gas that is a combination of the negatively charged free electrons and positively charged ions

radiation   energy transferred by electromagnetic waves

specific heat   amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 °C

sublimation   phase change from solid to gas

temperature   quantity measured by a thermometer

thermal energy   average random kinetic energy of a molecule or an atom

vaporization   phase change from liquid to gas

SECTION SUMMARY

11.1 Temperature and Thermal Energy

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted into temperatures on another scale.

11.2 Heat, Specific Heat, and Heat Transfer

- Heat is thermal (internal) energy transferred due to a temperature difference.
- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material.
- Heat is transferred by three different methods: conduction, convection, and radiation.
- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- Convection is heat transfer by the movement of mass.
- Radiation is heat transfer by electromagnetic waves.

11.3 Phase Change and Latent Heat

- Most substances have four distinct phases: solid, liquid, gas, and plasma.
- Gas is the most energetic state and solid is the least.
- During a phase change, a substance undergoes transition to a higher energy state when heat is added, or to a lower energy state when heat is removed.
- Heat is added to a substance during melting and vaporization.
- Latent heat is released by a substance during condensation and freezing.
- Phase changes occur at fixed temperatures called boiling and freezing (or melting) points for a given substance.

KEY EQUATIONS

11.1 Temperature and Thermal Energy

Celsius to Fahrenheit conversion	$T_{°F} = \dfrac{9}{5}T_{°C} + 32$
Fahrenheit to Celsius conversion	$T_{°C} = \dfrac{5}{9}(T_{°F} - 32)$
Celsius to Kelvin conversion	$T_K = T_{°C} + 273.15$
Kelvin to Celsius conversion	$T_{°C} = T_K - 273.15$
Fahrenheit to Kelvin conversion	$T_K = \dfrac{5}{9}(T_{°F} - 32) + 273.15$
Kelvin to Fahrenheit conversion	$T_{°F} = \dfrac{9}{5}(T_K - 273.15) + 32$

11.2 Heat, Specific Heat, and Heat Transfer

heat transfer	$Q = mc\Delta T$
density	$\rho = \frac{m}{V}$

11.3 Phase Change and Latent Heat

heat transfer for melting/freezing phase change	$Q = mL_f$
heat transfer for vaporization/condensation phase change	$Q = mL_v$

CHAPTER REVIEW

Concept Items

11.1 Temperature and Thermal Energy

1. A glass of water has a temperature of 31 degrees Celsius. What state of matter is it in?
   a. solid
   b. liquid
   c. gas
   d. plasma

2. What is the difference between thermal energy and internal energy?
   a. The thermal energy of the system is the average kinetic energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies and the potential energies of its constituent particles.
   b. The thermal energy of the system is the average potential energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies and the potential energies of its constituent particles.
   c. The thermal energy of the system is the average kinetic energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies of its

   constituent particles.
   d. The thermal energy of the system is the average potential energy of the systems' constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies of its constituent particles.

3. What does the Celsius scale use as a reference point?
   a. The boiling point of mercury
   b. The boiling point of wax
   c. The freezing point of water
   d. The freezing point of wax

11.2 Heat, Specific Heat, and Heat Transfer

4. What are the SI units of specific heat?
   a. $J/kg^2 \cdot °C$
   b. $J \cdot kg^2/°C$
   c. $J \cdot kg/°C$
   d. $J/kg \cdot °C$

5. What is radiation?
   a. The transfer of energy through emission and absorption of the electromagnetic waves is known as radiation.
   b. The transfer of energy without any direct physical

contact between any two substances.

c. The transfer of energy through direct physical contact between any two substances.

d. The transfer of energy by means of the motion of fluids at different temperatures and with different densities.

11.3 Phase Change and Latent Heat

6. Why is there no change in temperature during a phase change, even if energy is absorbed by the system?

a. The energy is used to break bonds between particles, and so does not increase the potential energy of the system's particles.

b. The energy is used to break bonds between particles,

and so increases the potential energy of the system's particles.

c. The energy is used to break bonds between particles, and so does not increase the kinetic energy of the system's particles.

d. The energy is used to break bonds between particles, and so increases the kinetic energy of the system's particles.

7. In which two phases of matter do atoms and molecules have the most distance between them?

a. gas and solid

b. gas and liquid

c. gas and plasma

d. liquid and plasma

Critical Thinking Items

11.1 Temperature and Thermal Energy

8. The temperature of two equal quantities of water needs to be raised - the first container by 5 degrees Celsius and the second by 5 degrees Fahrenheit. Which one would require more heat?

a. The heat required by the first container is more than the second because each degree Celsius is equal to 1.8 degrees Fahrenheit.

b. The heat required by the first container is less than the second because each degree Fahrenheit is equal to 1.8 degrees Celsius.

c. The heat required by the first container is more than the second because each degree Celsius is equal to 3.6 degrees Fahrenheit.

d. The heat required by the first container is less than the second because each degree Fahrenheit is equal to 3.6 degrees Celsius.

9. What is 100.00 °C in kelvins?

a. 212.00 K

b. 100.00 K

c. 473.15 K

d. 373.15 K

11.2 Heat, Specific Heat, and Heat Transfer

10. The value of specific heat is the same whether the units are J/kg·K or J/kg·°C. How?

a. Temperature difference is dependent on the chosen temperature scale.

b. Temperature change is different in units of kelvins and degrees Celsius.

c. Reading of temperatures in kelvins and degree Celsius are the same.

d. The temperature change is the same in units of kelvins and degrees Celsius.

11. If the thermal energy of a perfectly black object is increased by conduction, will the object remain black in appearance? Why or why not?

a. No, the energy of the radiation increases as the temperature increases, and the radiation becomes visible at certain temperatures.

b. Yes, the energy of the radiation decreases as the temperature increases, and the radiation remains invisible at those energies.

c. No, the energy of the radiation decreases as the temperature increases, until the frequencies of the radiation are the same as those of visible light.

d. Yes, as the temperature increases, and the energy is transferred from the object by other mechanisms besides radiation, so that the energy of the radiation does not increase.

12. What is the specific heat of a substance that requires 5.00 kJ of heat to raise the temperature of 3.00 kg by 5.00 °F?

a. 3.33×10^3 J/kg ·° C

b. 6.00×10^3 J/kg ·° C

c. 3.33×10^2 J/kg · ° C

d. 6.00×10^2 J/kg · ° C

11.3 Phase Change and Latent Heat

13. Assume 1.0 kg of ice at 0 °C starts to melt. It absorbs 300 kJ of energy by heat. What is the temperature of the water afterwards?

a. 10 °C

b. 20 °C

c. 5 °C

d. 0 °C

Problems

11.1 Temperature and Thermal Energy

14. What is 35.0 °F in kelvins?
  a. 1.67 K
  b. 35.0 K
  c. -271.5 K
  d. 274.8 K

15. Design a temperature scale where the freezing point of water is 0 degrees and its boiling point is 70 degrees. What would be the room temperature on this scale?
  a. If room temperature is 25.0 °C, the temperature on the new scale will be 17.5 °.
  b. If room temperature is 25.0 °C, the temperature on the new scale will be 25.0°.
  c. If the room temperature is 25.0 °C, the temperature on the new scale will be 35.7°.
  d. If the room temperature is 25.0 °C, the temperature on the new scale will be 50.0°.

11.2 Heat, Specific Heat, and Heat Transfer

16. A certain quantity of water is given 4.0 kJ of heat. This raises its temperature by 30.0 °F. What is the mass of the water in grams?
  a. 5.7 g
  b. 570 g
  c. 5700 g
  d. 57 g

17. 5290 J of heat is given to 0.500 kg water at 15.00 °C. What will its final temperature be?
  a. 15.25° C
  b. 12.47 ° C
  c. 40.3° C
  d. 17.53° C

11.3 Phase Change and Latent Heat

18. How much energy would it take to heat 1.00 kg of ice at 0 °C to water at 15.0 °C?
  a. 271 kJ
  b. 334 kJ
  c. 62.8 kJ
  d. 397 kJ

19. Ice cubes are used to chill a soda with a mass $m_{soda} = 0.300$ kg at 15.0 °C. The ice is at 0 °C, and the total mass of the ice cubes is 0.020 kg. Assume that the soda is kept in a foam container so that heat loss can be ignored, and that the soda has the same specific heat as water. Find the final temperature when all ice has melted.
  a. 19.02 °C
  b. 90.3 °C
  c. 0.11 °C
  d. 9.03 °C

Performance Task

11.3 Phase Change and Latent Heat

20. You have been tasked with designing a baking pan that will bake batter the fastest. There are four materials available for you to test.
  • Four pans of similar design, consisting of aluminum, iron (steel), copper, and glass
  • Oven or similar heating source
  • Device for measuring high temperatures
  • Balance for measuring mass

Instructions
Procedure
  1. Design a safe experiment to test the specific heat of each material (i.e., no extreme temperatures should be used)
  2. Write down the materials needed for your experiment and the procedure you will follow. Make sure that you include every detail, so that the experiment can be repeated by others.
  3. Carry out the experiment and record any data collected.
  4. Review your results and make a recommendation as to which metal should be used for the pan.
    a. What physical quantities do you need to measure to determine the specific heats for the different materials?
    b. How does the glass differ from the metals in terms of thermal properties?
    c. What are your sources of error?

TEST PREP

Multiple Choice

11.1 Temperature and Thermal Energy

21. The temperature difference of 1 K is the same as
  a. 1 degree Celsius
  b. 1 degree Fahrenheit
  c. 273.15 degrees Celsius
  d. 273.15 degrees Fahrenheit

22. What is the preferred temperature scale used in scientific laboratories?
a. celsius
b. fahrenheit
c. kelvin
d. rankine

11.2 Heat, Specific Heat, and Heat Transfer

23. Which phase of water has the largest specific heat?
a. solid
b. liquid
c. gas

24. What kind of heat transfer requires no medium?
a. conduction
b. convection
c. reflection
d. radiation

25. Which of these substances has the greatest specific heat?
a. copper
b. mercury
c. aluminum
d. wood

26. Give an example of heat transfer through convection.
a. The energy emitted from the filament of an electric bulb
b. The energy coming from the sun
c. A pan on a hot burner
d. Water boiling in a pot

11.3 Phase Change and Latent Heat

27. What are the SI units of latent heat?

Short Answer

11.1 Temperature and Thermal Energy

31. What is *absolute zero* on the Fahrenheit scale?
a. 0 °F
b. 32 °F
c. -273.15 °F
d. -459.67 °F

32. What is *absolute zero* on the Celsius scale?
a. 0 °C
b. 273.15 °C
c. -459.67 °C
d. -273.15 °C

33. A planet's atmospheric pressure is such that water there boils at a lower temperature than it does at sea level on

a. J/kg
b. J.kg
c. J/cal
d. cal/kg

28. Which substance has the largest latent heat of fusion?
a. gold
b. water
c. mercury
d. tungsten

29. In which phase changes does matter undergo a transition to a more energetic state?
a. freezing and vaporization
b. melting and sublimation
c. melting and vaporization
d. melting and freezing

30. A room has a window made from thin glass. The room is colder than the air outside. There is some condensation on the glass window. On which side of the glass would the condensation most likely be found?
a. Condensation is on the outside of the glass when the cool, dry air outside the room comes in contact with the cold pane of glass.
b. Condensation is on the outside of the glass when the warm, moist air outside the room comes in contact with the cold pane of glass.
c. Condensation is on the inside of the glass when the cool, dry air inside the room comes in contact with the cold pane of glass.
d. Condensation is on the inside of the glass when the warm, moist air inside the room comes in contact with the cold pane of glass.

Earth. If a Celsius scale is derived on this planet, will it be the same as that on Earth?
a. The Celsius scale derived on the planet will be the same as that on Earth, because the Celsius scale is independent of the freezing and boiling points of water.
b. The Celsius scale derived on that planet will not be the same as that on Earth, because the Celsius scale is dependent and derived by using the freezing and boiling points of water.
c. The Celsius scale derived on the planet will be the same as that on Earth, because the Celsius scale is an absolute temperature scale based on molecular motion, which is independent of pressure.
d. The Celsius scale derived on the planet will not be the same as that on Earth, but the Fahrenheit scale

will be the same, because its reference temperatures are not based on the freezing and boiling points of water.

34. What is the difference between the freezing point and boiling point of water on the Reaumur scale?
    a. The boiling point of water is 80° on the Reaumur scale.
    b. Reaumur scale is less than 120°.
    c. 100°
    d. 80°

11.2 Heat, Specific Heat, and Heat Transfer

35. In the specific heat equation what does c stand for?
    a. Total heat
    b. Specific heat
    c. Specific temperature
    d. Specific mass

36. Specific heat may be measured in $J/kg \cdot K$, $J/kg \cdot °C$. What other units can it be measured in?
    a. $kg/kcal \cdot °C$
    b. $kcal \cdot °C/kg$
    c. $kg \cdot °C/kcal$
    d. $kcal/kg \cdot °C$

37. What is buoyancy?
    a. Buoyancy is a downward force exerted by a solid that opposes the weight of an object.
    b. Buoyancy is a downward force exerted by a fluid that opposes the weight of an immersed object.
    c. Buoyancy is an upward force exerted by a solid that opposes the weight an object.
    d. Buoyancy is an upward force exerted by a fluid that opposes the weight of an immersed object.

38. Give an example of convection found in nature.
    a. heat transfer through metallic rod
    b. heat transfer from the sun to Earth
    c. heat transfer through ocean currents
    d. heat emitted by a light bulb into its environment

39. Calculate the temperature change in a substance with specific heat 735 J/kg · °C when 14 kJ of heat is given to a 3.0-kg sample of that substance.
    a. 57 °C

Extended Response

11.1 Temperature and Thermal Energy

45. What is the meaning of absolute zero?
    a. It is the temperature at which the internal energy of the system is maximum, because the speed of its

b. 63 °C
c. 1.8×10^{-2} °C
d. 6.3 °C

40. Aluminum has a specific heat of 900 J/kg·°C. How much energy would it take to change the temperature of 2 kg aluminum by 3 °C?
    a. 1.3 kJ
    b. 0.60 kJ
    c. 54 kJ
    d. 5.4 kJ

11.3 Phase Change and Latent Heat

41. Upon what does the required amount of heat removed to freeze a sample of a substance depend?
    a. The mass of the substance and its latent heat of vaporization
    b. The mass of the substance and its latent heat of fusion
    c. The mass of the substance and its latent heat of sublimation
    d. The mass of the substance only

42. What do latent heats, L_f and L_v, depend on?
    a. L_f and L_v depend on the forces between the particles in the substance.
    b. L_f and L_v depend on the mass of the substance.
    c. L_f and L_v depend on the volume of the substance.
    d. L_f and L_v depend on the temperature of the substance.

43. How much energy is required to melt 7.00 kg a block of aluminum that is at its melting point? (Latent heat of fusion of aluminum is 380 kJ/kg.)
    a. 54.3 kJ
    b. 2.66 kJ
    c. 0.0184 kJ
    d. 2.66×10^3 kJ

44. A 3.00 kg sample of a substance is at its boiling point. If 5,360 kJ of energy are enough to boil away the entire substance, what is its latent heat of vaporization?
    a. 2,685 kJ/kg
    b. 3,580 kJ/kg
    c. 895 kJ/kg
    d. 1,790 kJ/kg

constituent particles increases to maximum at this point.
    b. It is the temperature at which the internal energy of the system is maximum, because the speed of its constituent particles decreases to zero at this point.

c. It is the temperature at which the internal energy of the system approaches zero, because the speed of its constituent particles increases to a maximum at this point.

d. It is that temperature at which the internal energy of the system approaches zero, because the speed of its constituent particles decreases to zero at this point.

46. Why does it feel hotter on more humid days, even though there is no difference in temperature?

a. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is lower, which reduces the evaporation rate from the skin's surface.

b. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is higher, which reduces the evaporation rate from the skin's surface.

c. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is lower, which increases the evaporation rate from the skin's surface.

d. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is higher, which increases the evaporation rate from the skin's surface.

11.2 Heat, Specific Heat, and Heat Transfer

47. A hot piece of metal needs to be cooled. If you were to put the metal in ice or in cold water, such that the ice did not melt and the temperature of either changed by the same amount, which would reduce the metal's temperature more? Why?

a. Water would reduce the metal's temperature more, because water has a greater specific heat than ice.

b. Water would reduce the metal's temperature more, because water has a smaller specific heat than ice.

c. Ice would reduce the metal's temperature more, because ice has a smaller specific heat than water.

d. Ice would reduce the metal's temperature more, because ice has a greater specific heat than water.

48. On a summer night, why does a black object seem colder than a white one?

a. The black object radiates energy faster than the white one, and hence reaches a lower temperature in less time.

b. The black object radiates energy slower than the white one, and hence reaches a lower temperature in less time.

c. The black object absorbs energy faster than the white one, and hence reaches a lower temperature in less time.

d. The black object absorbs energy slower than the white one, and hence reaches a lower temperature in less time.

49. Calculate the difference in heat required to raise the temperatures of 1.00 kg of gold and 1.00 kg of aluminum by 1.00 °C. (The specific heat of aluminum equals 900 J/kg · °C; the specific heat of gold equals 129 J/kg · °C.)

a. 771 J

b. 129 J

c. 90 J

d. 900 J

11.3 Phase Change and Latent Heat

50. True or false—You have an ice cube floating in a glass of water with a thin thread resting across the cube. If you cover the ice cube and thread with a layer of salt, they will stick together, so that you are able to lift the ice-cube when you pick up the thread.

a. True

b. False

51. Suppose the energy required to freeze 0.250 kg of water were added to the same mass of water at an initial temperature of 1.0 °C. What would be the final temperature of the water?

a. -69.8 °C

b. 79.8 °C

c. -78.8 °C

d. 80.8 °C

CHAPTER 12
Thermodynamics

Figure 12.1 A steam engine uses energy transfer by heat to do work. (Modification of work by Gerald Friedrich, Pixabay)

Chapter Outline

INTRODUCTION Energy can be transferred to or from a system, either through a temperature difference between it and another system (i.e., by heat) or by exerting a force through a distance (work). In these ways, energy can be converted into other forms of energy in other systems. For example, a car engine burns fuel for heat transfer into a gas. Work is done by the gas as it exerts a force through a distance by pushing a piston outward. This work converts the energy into a variety of other forms—into an increase in the car's kinetic or gravitational potential energy; into electrical energy to run the spark plugs, radio, and lights; and back into stored energy in the car's battery. But most of the thermal energy transferred by heat from the fuel burning in the engine does not do work on the gas. Instead, much of this energy is released into the surroundings at lower temperature (i.e., lost through heat), which is quite inefficient. Car engines are only about 25 to 30 percent efficient. This inefficiency leads to increased fuel costs, so there is great interest in improving fuel efficiency. However, it is common knowledge that modern gasoline engines cannot be made much more efficient. The same is true about the conversion to electrical energy in large power stations, whether they are coal, oil, natural gas, or nuclear powered. Why is this the case?

The answer lies in the nature of heat. Basic physical laws govern how heat transfer for doing work takes place and limit the

maximum possible efficiency of the process. This chapter will explore these laws as well their applications to everyday machines. These topics are part of *thermodynamics*—the study of heat and its relationship to doing work.

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

Section Learning Objectives
By the end of this section, you will be able to do the following:

- Explain the zeroth law of thermodynamics

Section Key Terms

thermal equilibrium	zeroth law of thermodynamics

We learned in the previous chapter that when two objects (or *systems*) are in contact with one another, heat will transfer thermal energy from the object at higher temperature to the one at lower temperature until they both reach the same temperature. The objects are then in **thermal equilibrium**, and no further temperature changes will occur if they are isolated from other systems. The systems interact and change because their temperatures are different, and the changes stop once their temperatures are the same. Thermal equilibrium is established when two bodies are in *thermal contact* with each other—meaning heat transfer (i.e., the transfer of energy by heat) can occur between them. If two systems cannot freely exchange energy, they will not reach thermal equilibrium. (It is fortunate that empty space stands between Earth and the sun, because a state of thermal equilibrium with the sun would be too toasty for life on this planet!)

If two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C. This statement may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

> ### TIPS FOR SUCCESS
> The zeroth law of thermodynamics is very similar to the transitive property of equality in mathematics: If a = b and b = c, then a = c.

You may be wondering at this point, why the wacky name? Shouldn't this be called the *first* law of thermodynamics rather than the *zeroth*? The explanation is that this law was discovered after the first and second laws of thermodynamics but is so fundamental that scientists decided it should logically come first.

As an example of the zeroth law in action, consider newborn babies in neonatal intensive-care units in hospitals. Prematurely born or sick newborns are placed in special incubators. These babies have very little covering while in the incubators, so to an observer, they look as though they may not be warm enough. However, inside the incubator, the temperature of the air, the cot, and the baby are all the same—that is, they are in thermal equilibrium. The ambient temperature is just high enough to keep the baby safe and comfortable.

📖 WORK IN PHYSICS

Thermodynamics Engineer
Thermodynamics engineers apply the principles of thermodynamics to mechanical systems so as to create or test products that rely on the interactions between heat, work, pressure, temperature, and volume. This type of work typically takes place in the aerospace industry, chemical manufacturing companies, industrial manufacturing plants, power plants (Figure 12.2), engine manufacturers, or electronics companies.

Figure 12.2 An engineer makes a site visit to the Baghdad South power plant.

The need for energy creates quite a bit of demand for thermodynamics engineers, because both traditional energy companies and alternative (*green*) energy startups rely on interactions between heat and work and so require the expertise of thermodynamics engineers. Traditional energy companies use mainly nuclear energy and energy from burning fossil fuels, such as coal. Alternative energy is finding new ways to harness renewable and, often, more readily available energy sources, such as solar, water, wind, and bio-energy.

A thermodynamics engineer in the energy industry can find the most efficient way to turn the burning of a biofuel or fossil fuel into energy, store that energy for times when it's needed most, or figure out how to best deliver that energy from where it's produced to where it's used: in homes, factories, and businesses. Additionally, he or she might also design pollution-control equipment to remove harmful pollutants from the smoke produced as a by-product of burning fuel. For example, a thermodynamics engineer may develop a way to remove mercury from burning coal in a coal-fired power plant.

Thermodynamics engineering is an expanding field, where employment opportunities are expected to grow by as much as 27 percent between 2012 and 2022, according to the U.S. Bureau of Labor Statistics. To become a thermodynamics engineer, you must have a college degree in chemical engineering, mechanical engineering, environmental engineering, aerospace engineering, civil engineering, or biological engineering (depending on which type of career you wish to pursue), with coursework in physics and physical chemistry that focuses on thermodynamics.

GRASP CHECK

What would be an example of something a thermodynamics engineer would do in the aeronautics industry?
a. Test the fuel efficiency of a jet engine
b. Test the functioning of landing gear
c. Test the functioning of a lift control device
d. Test the autopilot functions

Check Your Understanding

1. What is thermal equilibrium?
    a. When two objects in contact with each other are at the same pressure, they are said to be in thermal equilibrium.
    b. When two objects in contact with each other are at different temperatures, they are said to be in thermal equilibrium.
    c. When two objects in contact with each other are at the same temperature, they are said to be in thermal equilibrium.
    d. When two objects not in contact with each other are at the same pressure, they are said to be in thermal equilibrium.

2. What is the zeroth law of thermodynamics?

a. Energy can neither be created nor destroyed in a chemical reaction.
b. If two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.
c. Entropy of any isolated system not in thermal equilibrium always increases.
d. Entropy of a system approaches a constant value as temperature approaches absolute zero.

12.2 First law of Thermodynamics: Thermal Energy and Work

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe how pressure, volume, and temperature relate to one another and to work, based on the ideal gas law
- Describe pressure–volume work
- Describe the first law of thermodynamics verbally and mathematically
- Solve problems involving the first law of thermodynamics

Section Key Terms

Boltzmann constant	first law of thermodynamics	ideal gas law	internal energy	pressure

## Pressure, Volume, Temperature, and the Ideal Gas Law

Before covering the first law of thermodynamics, it is first important to understand the relationship between **pressure**, volume, and temperature. Pressure, P, is defined as

$$P = \frac{F}{A},$$

<div align="right">12.1</div>

where F is a force applied to an area, A, that is perpendicular to the force.

Depending on the area over which it is exerted, a given force can have a significantly different effect, as shown in Figure 12.3.
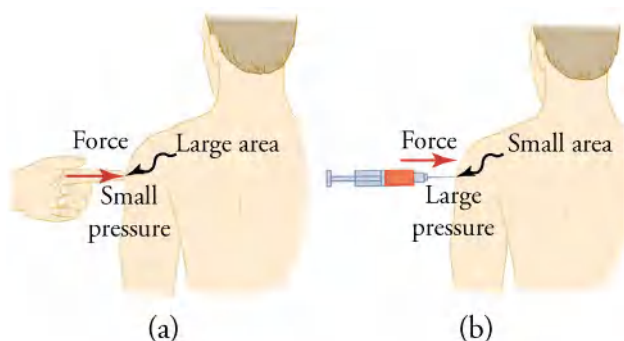
Figure 12.3 (a) Although the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

The SI unit for pressure is the *pascal*, where $1 \text{ Pa} = 1 \text{ N/m}^2$.

Pressure is defined for all states of matter but is particularly important when discussing fluids (such as air). You have probably heard the word *pressure* being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids.

The relationship between the pressure, volume, and temperature for an ideal gas is given by the **ideal gas law**. A gas is considered ideal at low pressure and fairly high temperature, and forces between its component particles can be ignored. The ideal gas law states that

$$PV = NkT.$$

<div align="right">12.2</div>

where P is the pressure of a gas, V is the volume it occupies, N is the number of particles (atoms or molecules) in the gas, and T is

its absolute temperature. The constant k is called the **Boltzmann constant** and has the value $k = 1.38 \times 10^{-23}$ J/K, For the purposes of this chapter, we will not go into calculations using the ideal gas law. Instead, it is important for us to notice from the equation that the following are true for a given mass of gas:

- When volume is constant, pressure is directly proportional to temperature.
- When temperature is constant, pressure is inversely proportional to volume.
- When pressure is constant, volume is directly proportional to temperature.

This last point describes *thermal expansion*—the change in size or volume of a given mass with temperature. What is the underlying cause of thermal expansion? An increase in temperature means that there's an increase in the kinetic energy of the individual atoms. Gases are especially affected by thermal expansion, although liquids expand to a lesser extent with similar increases in temperature, and even solids have minor expansions at higher temperatures. This is why railroad tracks and bridges have expansion joints that allow them to freely expand and contract with temperature changes.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into a deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If you continue to pump air into tire (which now has a nearly constant volume), the pressure increases with increasing temperature (see Figure 12.4).
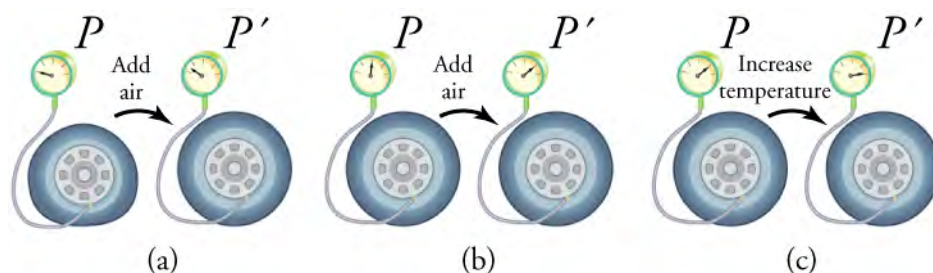
Figure 12.4 (a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion, and the pressure increases as more air is added. (c) Once the tire is inflated fully, its pressure increases with temperature.

Pressure–Volume Work

Pressure–volume work is the work that is done by the compression or expansion of a fluid. Whenever there is a change in volume and external pressure remains constant, pressure–volume work is taking place. During a compression, a decrease in volume increases the internal pressure of a system as work is done *on* the system. During an expansion (Figure 12.5), an increase in volume decreases the internal pressure of a system as the system *does* work.
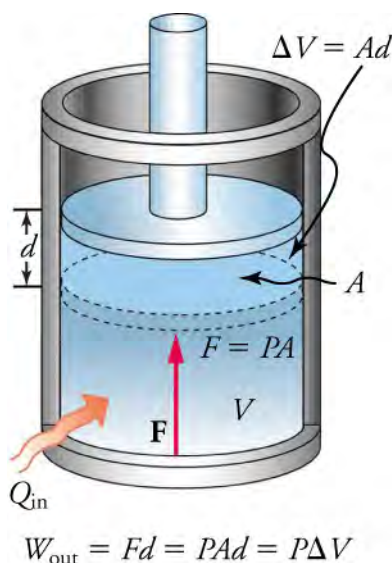
Figure 12.5 An expansion of a gas requires energy transfer to keep the pressure constant. Because pressure is constant, the work done is $P\Delta V$.

Recall that the formula for work is $W = Fd$. We can rearrange the definition of pressure, $P = \frac{F}{A}$, to get an expression for force in terms of pressure.

$$F = PA$$ 12.3

Substituting this expression for force into the definition of work, we get

$$W = PAd.$$ 12.4

Because area multiplied by displacement is the change in volume, $W = P\Delta V$, the mathematical expression for pressure–volume work is

$$W = P\Delta V.$$ 12.5

Just as we say that work is force acting over a distance, for fluids, we can say that work is the pressure acting through the change in volume. For pressure–volume work, pressure is analogous to force, and volume is analogous to distance in the traditional definition of work.

👁 WATCH PHYSICS

Work from Expansion
This video describes work from expansion (or pressure–volume work). Sal combines the equations $W = P\Delta V$ and $\Delta U = Q - W$ to get $\Delta U = Q - P\Delta V$.

Click to view content (https://www.openstax.org/l/28expansionWork)

> **GRASP CHECK**
> If the volume of a system increases while pressure remains constant, is the value of work done by the system W positive or negative? Will this increase or decrease the internal energy of the system?
> a.   Positive; internal energy will decrease
> b.   Positive; internal energy will increase
> c.   Negative; internal energy will decrease
> d.   Negative; internal energy will increase

The First Law of Thermodynamics

Heat (Q) and work (W) are the two ways to add or remove energy from a system. The processes are very different. Heat is driven

by temperature differences, while work involves a force exerted through a distance. Nevertheless, heat and work can produce identical results. For example, both can cause a temperature increase. Heat transfers energy into a system, such as when the sun warms the air in a bicycle tire and increases the air's temperature. Similarly, work can be done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat or work. Heat and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy, U, of a system.

Internal energy is the sum of the kinetic and potential energies of a system's atoms and molecules. It can be divided into many subcategories, such as thermal and chemical energy, and depends only on the state of a system (that is, P, V, and T), not on how the energy enters or leaves the system.

In order to understand the relationship between heat, work, and internal energy, we use the **first law of thermodynamics**. The first law of thermodynamics applies the *conservation of energy* principle to systems where heat and work are the methods of transferring energy into and out of the systems. It can also be used to describe how energy transferred by heat is converted and transferred again by work.

TIPS FOR SUCCESS
Recall that the principle of conservation of energy states that energy cannot be created or destroyed, but it can be altered from one form to another.

The first law of thermodynamics states that the change in internal energy of a closed system equals the net heat transfer *into* the system minus the net work done *by* the system. In equation form, the first law of thermodynamics is

$$\Delta U = Q - W.$$

<div style="text-align:right">12.6</div>

Here, ΔU is the *change in internal energy*, U, of the system. As shown in Figure 12.6, Q is the *net heat transferred into the system*—that is, Q is the sum of all heat transfers into and out of the system. W is the *net work done by the system*—that is, W is the sum of all work done on or by the system. By convention, if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system by heat, and positive W takes energy from the system by work. Note that if heat transfers more energy into the system than that which is done by work, the difference is stored as internal energy.
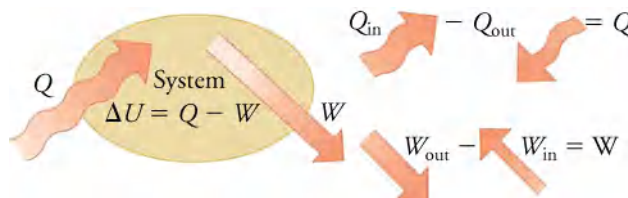
Figure 12.6 The first law of thermodynamics is the *conservation of energy* principle stated for a system, where heat and work are the methods of transferring energy to and from a system. Q represents the net heat transfer—it is the sum of all transfers of energy by heat into and out of the system. Q is positive for net heat transfer *into* the system. W_{out} is the work done *by* the system, and W_{in} is the work done *on* the system. W is the total work done on or *by* the system. W is positive when more work is done *by* the system than *on* it. The change in the internal energy of the system, ΔU, is related to heat and work by the first law of thermodynamics: $\Delta U = Q - W$.

It follows also that negative Q indicates that energy is transferred *away* from the system by heat and so decreases the system's internal energy, whereas negative W is work done *on* the system, which increases the internal energy.

⦿ WATCH PHYSICS

First Law of Thermodynamics/Internal Energy
This video explains the first law of thermodynamics, conservation of energy, and internal energy. It goes over an example of energy transforming between kinetic energy, potential energy, and heat transfer due to air resistance.

Click to view content (https://www.openstax.org/l/28FirstThermo)

WATCH PHYSICS

More on Internal Energy

This video goes into further detail, explaining internal energy and how to use the equation $\Delta U = Q - W$. Note that Sal uses the equation $\Delta U = Q + W$, where W is the work done *on* the system, whereas we use W to represent work done *by* the system.

Click to view content (https://www.openstax.org/l/28IntrnEnergy)

LINKS TO PHYSICS

Biology: Biological Thermodynamics

We often think about thermodynamics as being useful for inventing or testing machinery, such as engines or steam turbines. However, thermodynamics also applies to living systems, such as our own bodies. This forms the basis of the biological thermodynamics (Figure 12.7).
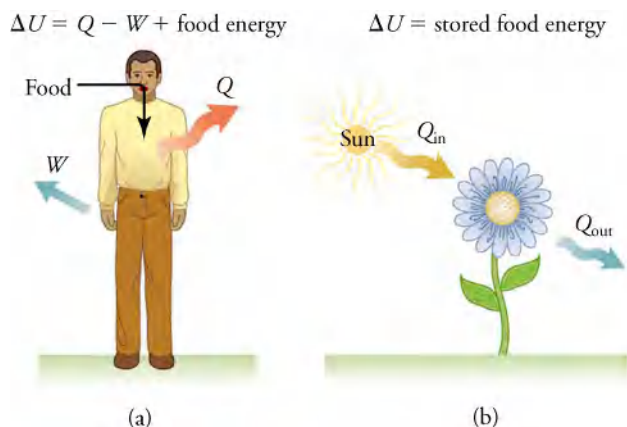
Figure 12.7 (a) The first law of thermodynamics applies to metabolism. Heat transferred out of the body (Q) and work done by the body (W) remove internal energy, whereas food intake replaces it. (Food intake may be considered work done on the body.) (b) Plants convert part of

the radiant energy in sunlight into stored chemical energy, a process called *photosynthesis.*

Life itself depends on the biological transfer of energy. Through photosynthesis, plants absorb solar energy from the sun and use this energy to convert carbon dioxide and water into glucose and oxygen. Photosynthesis takes in one form of energy—light—and converts it into another form—chemical potential energy (glucose and other carbohydrates).

Human metabolism is the conversion of food into energy given off by heat, work done by the body's cells, and stored fat. Metabolism is an interesting example of the first law of thermodynamics in action. Eating increases the internal energy of the body by adding chemical potential energy; this is an unromantic view of a good burrito.

The body metabolizes all the food we consume. Basically, metabolism is an oxidation process in which the chemical potential energy of food is released. This implies that food input is in the form of work. Exercise helps you lose weight, because it provides energy transfer from your body by both heat and work and raises your metabolic rate even when you are at rest.

Biological thermodynamics also involves the study of transductions between cells and living organisms. *Transduction* is a process where genetic material—DNA—is transferred from one cell to another. This often occurs during a viral infection (e.g., influenza) and is how the virus spreads, namely, by transferring its genetic material to an increasing number of previously healthy cells. Once enough cells become infected, you begin to feel the effects of the virus (flu symptoms—muscle weakness, coughing, and congestion).

Energy is transferred along with the genetic material and so obeys the first law of thermodynamics. Energy is transferred—not created or destroyed—in the process. When work is done on a cell or heat transfers energy to a cell, the cell's internal energy increases. When a cell does work or loses heat, its internal energy decreases. If the amount of work done by a cell is the same as the amount of energy transferred in by heat, or the amount of work performed on a cell matches the amount of energy transferred out by heat, there will be no net change in internal energy.

GRASP CHECK
Based on what you know about heat transfer and the first law of thermodynamics, do you need to eat more or less to maintain a constant weight in colder weather? Explain why.
  a. more; as more energy is lost by the body in colder weather, the need to eat increases so as to maintain a constant weight
  b. more; eating more food means accumulating more fat, which will insulate the body from colder weather and will reduce the energy loss
  c. less; as less energy is lost by the body in colder weather, the need to eat decreases so as to maintain a constant weight
  d. less; eating less food means accumulating less fat, so less energy will be required to burn the fat, and, as a result, weight will remain constant

Solving Problems Involving the First Law of Thermodynamics

✳ WORKED EXAMPLE

Calculating Change in Internal Energy
Suppose 40.00 J of energy is transferred by heat to a system, while the system does 10.00 J of work. Later, heat transfers 25.00 J out of the system, while 4.00 J is done by work on the system. What is the net change in the system's internal energy?

STRATEGY
You must first calculate the net heat and net work. Then, using the first law of thermodynamics, $\Delta U = Q - W$, find the change in internal energy.

Solution
The net heat is the transfer into the system by heat minus the transfer out of the system by heat, or

$$Q = 40.00 \text{ J} - 25.00 \text{ J} = 15.00 \text{ J}.$$

12.7

The total work is the work done by the system minus the work done on the system, or

$$W = 10.00 \text{ J} - 4.00 \text{ J} = 6.00 \text{ J}.$$

12.8

The change in internal energy is given by the first law of thermodynamics.

$$\Delta U = Q - W = 15.00 \text{ J} - 6.00 \text{ J} = 9.00 \text{ J}$$

<div style="text-align:right">12.9</div>

Discussion

A different way to solve this problem is to find the change in internal energy for each of the two steps separately and then add the two changes to get the total change in internal energy. This approach would look as follows:

For 40.00 J of heat in and 10.00 J of work out, the change in internal energy is

$$\Delta U_1 = Q_1 - W_1 = 40.00 \text{ J} - 10.00 \text{ J} = 30.00 \text{ J}.$$

<div style="text-align:right">12.10</div>

For 25.00 J of heat out and 4.00 J of work in, the change in internal energy is

$$\Delta U_2 = Q_2 - W_2 = -25.00 \text{ J} - (-4.00 \text{ J}) = -21.00 \text{ J}.$$

<div style="text-align:right">12.11</div>

The total change is

$$\Delta U = \Delta U_1 + \Delta U_2 = 30.00 \text{ J} + (-21.00 \text{ J}) = 9.00 \text{ J}.$$

<div style="text-align:right">12.12</div>

No matter whether you look at the overall process or break it into steps, the change in internal energy is the same.

✳ WORKED EXAMPLE

Calculating Change in Internal Energy: The Same Change in *U* is Produced by Two Different Processes

What is the change in the internal energy of a system when a total of 150.00 J is transferred by heat from the system and 159.00 J is done by work on the system?

STRATEGY

The net heat and work are already given, so simply use these values in the equation $\Delta U = Q - W$.

Solution

Here, the net heat and total work are given directly as $Q = -150.00 \text{ J}$ and $W = -159.00 \text{ J}$, so that

$$\Delta U = Q - W = -150.00 \text{ J} - (-159.00 \text{ J}) = 9.00 \text{ J}.$$

<div style="text-align:right">12.13</div>

Discussion

$W_{in} = -4\,\text{J}$

System

$Q_{in} = 40\,\text{J}$

6 J
out

15J
in

$Q_{out} = -25\,\text{J}$

$W_{out} = 10\,\text{J}$

$\Delta U = Q - W = 15\,\text{J} - 6\,\text{J} = +9\,\text{J}$
(a)

$W = -159\,\text{J}$
System

159 J
in

150 J
out

$Q_{out} = -150\,\text{J}$

$\Delta U = Q - W = -150\,\text{J} - (-159\,\text{J}) = +9\,\text{J}$
(b)
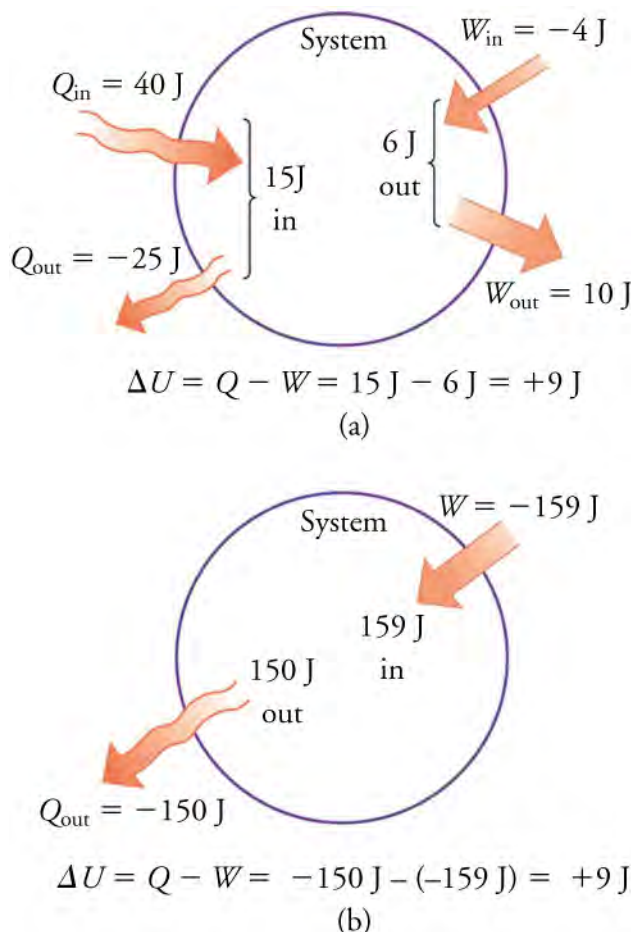
Figure 12.8 Two different processes produce the same change in a system. (a) A total of 15.00 J of heat transfer occurs into the system, while work takes out a total of 6.00 J. The change in internal energy is ΔU = Q – W = 9.00 J. (b) Heat transfer removes 150.00 J from the system while work puts 159.00 J into it, producing an increase of 9.00 J in internal energy. If the system starts out in the same state in (a) and (b), it will end up in the same final state in either case—its final state is related to internal energy, not how that energy was acquired.

A very different process in this second worked example produces the same 9.00 J change in internal energy as in the first worked example. Note that the change in the system in both parts is related to ΔU and not to the individual Q's or W's involved. The system ends up in the *same* state in both problems. Note that, as usual, in Figure 12.8 above, W_{out} is work done *by* the system, and W_{in} is work done *on* the system.

Practice Problems

3. What is the pressure-volume work done by a system if a pressure of $20\,\text{Pa}$ causes a change in volume of $3.0\,\text{m}^3$?
   a. 0.15 J
   b. 6.7 J
   c. 23 J
   d. 60 J

4. What is the net heat out of the system when 25 J is transferred by heat into the system and 45 J is transferred out of it?
   a. −70 J
   b. −20 J
   c. 20 J
   d. 70 J

Check Your Understanding

5. What is pressure?
   a. Pressure is force divided by length.
   b. Pressure is force divided by area.
   c. Pressure is force divided by volume.
   d. Pressure is force divided by mass.

6. What is the SI unit for pressure?
   a. pascal, or N/m^3
   b. coulomb
   c. newton
   d. pascal, or N/m^2

7. What is pressure-volume work?
   a. It is the work that is done by the compression or expansion of a fluid.
   b. It is the work that is done by a force on an object to produce a certain displacement.
   c. It is the work that is done by the surface molecules of a fluid.
   d. It is the work that is done by the high-energy molecules of a fluid.

8. When is pressure-volume work said to be done ON a system?
   a. When there is an increase in both volume and internal pressure.
   b. When there is a decrease in both volume and internal pressure.
   c. When there is a decrease in volume and an increase in internal pressure.
   d. When there is an increase in volume and a decrease in internal pressure.

9. What are the ways to add energy to or remove energy from a system?
   a. Transferring energy by heat is the only way to add energy to or remove energy from a system.
   b. Doing compression work is the only way to add energy to or remove energy from a system.
   c. Doing expansion work is the only way to add energy to or remove energy from a system.
   d. Transferring energy by heat or by doing work are the ways to add energy to or remove energy from a system.

10. What is internal energy?
   a. It is the sum of the kinetic energies of a system's atoms and molecules.
   b. It is the sum of the potential energies of a system's atoms and molecules.
   c. It is the sum of the kinetic and potential energies of a system's atoms and molecules.
   d. It is the difference between the magnitudes of the kinetic and potential energies of a system's atoms and molecules.

12.3 Second Law of Thermodynamics: Entropy

Section Learning Objectives
By the end of this section, you will be able to do the following:
- Describe entropy
- Describe the second law of thermodynamics
- Solve problems involving the second law of thermodynamics

Section Key Terms

entropy   second law of thermodynamics

Entropy

Recall from the chapter introduction that it is not even theoretically possible for engines to be 100 percent efficient. This phenomenon is explained by the **second law of thermodynamics**, which relies on a concept known as **entropy**. Entropy is a measure of the disorder of a system. Entropy also describes how much energy is *not* available to do work. The more disordered a system and higher the entropy, the less of a system's energy is available to do work.

Although all forms of energy can be used to do work, it is not possible to use the entire available energy for work. Consequently, not all energy transferred by heat can be converted into work, and some of it is lost in the form of waste heat—that is, heat that does not go toward doing work. The unavailability of energy is important in thermodynamics; in fact, the field originated from efforts to convert heat to work, as is done by engines.

The equation for the change in entropy, ΔS, is

$$\Delta S = \frac{Q}{T},$$

where Q is the heat that transfers energy during a process, and T is the absolute temperature at which the process takes place.

Q is positive for energy transferred *into* the system by heat and negative for energy transferred *out of* the system by heat. In SI, entropy is expressed in units of joules per kelvin (J/K). If temperature changes during the process, then it is usually a good approximation (for small changes in temperature) to take T to be the average temperature in order to avoid trickier math (calculus).

TIPS FOR SUCCESS
Absolute temperature is the temperature measured in Kelvins. The Kelvin scale is an absolute temperature scale that is measured in terms of the number of degrees above absolute zero. All temperatures are therefore positive. Using temperatures from another, nonabsolute scale, such as Fahrenheit or Celsius, will give the wrong answer.

Second Law of Thermodynamics

Have you ever played the card game 52 pickup? If so, you have been on the receiving end of a practical joke and, in the process, learned a valuable lesson about the nature of the universe as described by the second law of thermodynamics. In the game of 52 pickup, the prankster tosses an entire deck of playing cards onto the floor, and you get to pick them up. In the process of picking up the cards, you may have noticed that the amount of work required to restore the cards to an orderly state in the deck is much greater than the amount of work required to toss the cards and create the disorder.

The second law of thermodynamics states that *the total entropy of a system either increases or remains constant in any spontaneous process; it never decreases.* An important implication of this law is that heat transfers energy spontaneously from higher- to lower-temperature objects, but never spontaneously in the reverse direction. This is because entropy increases for heat transfer of energy from hot to cold (Figure 12.9). Because the change in entropy is Q/T, there is a larger change in ΔS at lower temperatures (smaller T). The decrease in entropy of the hot (larger T) object is therefore less than the increase in entropy of the cold (smaller T) object, producing an overall increase in entropy for the system.

Figure 12.9 The ice in this drink is slowly melting. Eventually, the components of the liquid will reach thermal equilibrium, as predicted by the second law of thermodynamics—that is, after heat transfers energy from the warmer liquid to the colder ice. (Jon Sullivan, PDPhoto.org)

Another way of thinking about this is that it is impossible for any process to have, as its sole result, heat transferring energy from a cooler to a hotter object. Heat cannot transfer energy spontaneously from colder to hotter, because the entropy of the

overall system would decrease.

Suppose we mix equal masses of water that are originally at two different temperatures, say $20.0\ ^{\circ}\text{C}$ and $40.0\ ^{\circ}\text{C}$. The result will be water at an intermediate temperature of $30.0\ ^{\circ}\text{C}$. Three outcomes have resulted: entropy has increased, some energy has become unavailable to do work, and the system has become less orderly. Let us think about each of these results.

First, why has entropy increased? Mixing the two bodies of water has the same effect as the heat transfer of energy from the higher-temperature substance to the lower-temperature substance. The mixing decreases the entropy of the hotter water but increases the entropy of the colder water by a greater amount, producing an overall increase in entropy.

Second, once the two masses of water are mixed, there is no more temperature difference left to drive energy transfer by heat and therefore to do work. The energy is still in the water, but it is now *unavailable* to do work.

Third, the mixture is less orderly, or to use another term, less structured. Rather than having two masses at different temperatures and with different distributions of molecular speeds, we now have a single mass with a broad distribution of molecular speeds, the average of which yields an intermediate temperature.

These three results—entropy, unavailability of energy, and disorder—not only are related but are, in fact, essentially equivalent. Heat transfer of energy from hot to cold is related to the tendency in nature for systems to become disordered and for less energy to be available for use as work.

Based on this law, what cannot happen? A cold object in contact with a hot one never spontaneously transfers energy by heat to the hot object, getting colder while the hot object gets hotter. Nor does a hot, stationary automobile ever spontaneously cool off and start moving.

Another example is the expansion of a puff of gas introduced into one corner of a vacuum chamber. The gas expands to fill the chamber, but it never regroups on its own in the corner. The random motion of the gas molecules could take them all back to the corner, but this is never observed to happen (Figure 12.10).
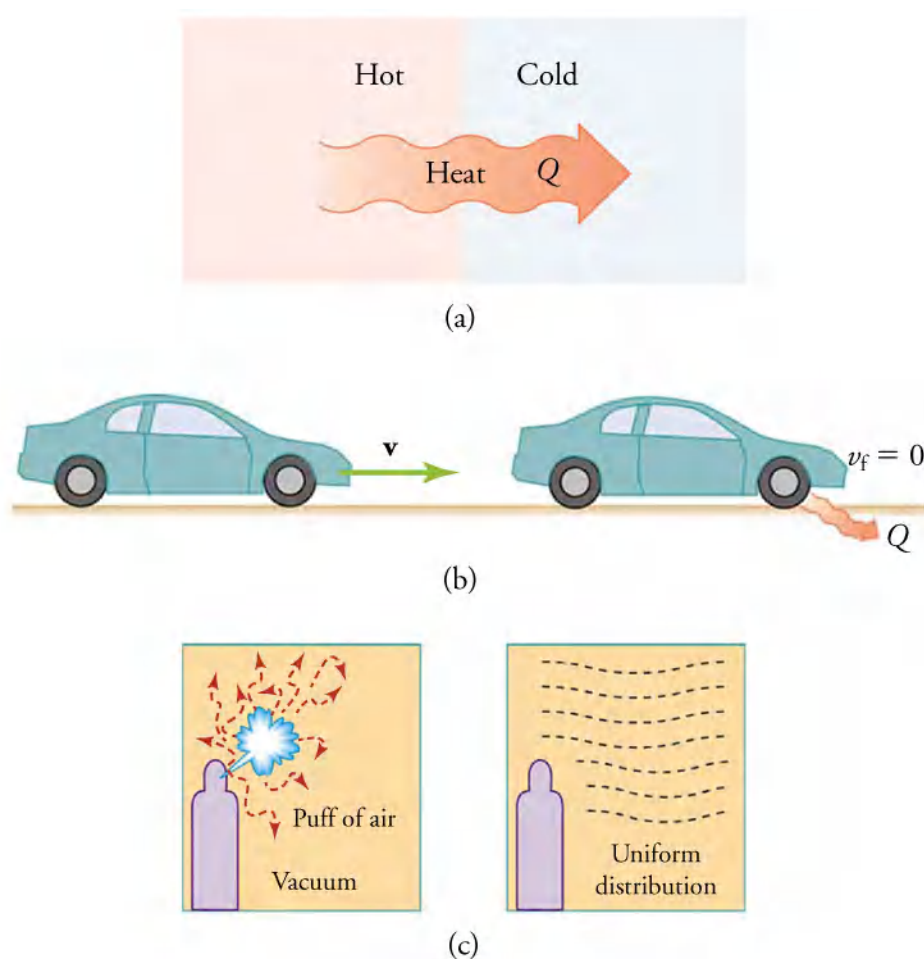
Figure 12.10 Examples of one-way processes in nature. (a) Heat transfer occurs spontaneously from hot to cold, but not from cold to hot. (b) The brakes of this car convert its kinetic energy to increase their internal energy (temperature), and heat transfers this energy to the environment. The reverse process is impossible. (c) The burst of gas released into this vacuum chamber quickly expands to uniformly fill every part of the chamber. The random motions of the gas molecules will prevent them from returning altogether to the corner.

We've explained that heat never transfers energy spontaneously from a colder to a hotter object. The key word here is *spontaneously*. If we *do work* on a system, it *is* possible to transfer energy by heat from a colder to hotter object. We'll learn more about this in the next section, covering refrigerators as one of the applications of the laws of thermodynamics.

Sometimes people misunderstand the second law of thermodynamics, thinking that based on this law, it is impossible for entropy to decrease at any particular location. But, it actually *is* possible for the entropy of *one part* of the universe to decrease, as long as the total change in entropy of the universe increases. In equation form, we can write this as

$$\Delta S_{tot} = \Delta S_{syst} + \Delta S_{envir} > 0.$$

Based on this equation, we see that ΔS_{syst} can be negative as long as ΔS_{envir} is positive and greater in magnitude.

How is it possible for the entropy of a system to decrease? Energy transfer is necessary. If you pick up marbles that are scattered about the room and put them into a cup, your work has decreased the entropy of that system. If you gather iron ore from the ground and convert it into steel and build a bridge, your work has decreased the entropy of that system. Energy coming from the sun can decrease the entropy of local systems on Earth—that is, ΔS_{syst} is negative. But the overall entropy of the rest of the universe increases by a greater amount—that is, ΔS_{envir} is positive and greater in magnitude. In the case of the iron ore, although you made the system of the bridge and steel more structured, you did so at the expense of the universe. Altogether, the entropy of the universe is increased by the disorder created by digging up the ore and converting it to steel. Therefore,

$$\Delta S_{tot} = \Delta S_{syst} + \Delta S_{envir} > 0,$$ (12.14)

and the second law of thermodynamics is *not* violated.

Every time a plant stores some solar energy in the form of chemical potential energy, or an updraft of warm air lifts a soaring bird, Earth experiences local decreases in entropy as it uses part of the energy transfer from the sun into deep space to do work. There is a large total increase in entropy resulting from this massive energy transfer. A small part of this energy transfer by heat is stored in structured systems on Earth, resulting in much smaller, local decreases in entropy.

Solving Problems Involving the Second Law of Thermodynamics

Entropy is related not only to the unavailability of energy to do work; it is also a measure of disorder. For example, in the case of a melting block of ice, a highly structured and orderly system of water molecules changes into a disorderly liquid, in which molecules have no fixed positions ([Figure 12.11](#)). There is a large increase in entropy for this process, as we'll see in the following worked example.

Figure 12.11 These ice floes melt during the Arctic summer. Some of them refreeze in the winter, but the second law of thermodynamics predicts that it would be extremely unlikely for the water molecules contained in these particular floes to reform in the distinctive alligator-like shape they possessed when this picture was taken in the summer of 2009. (Patrick Kelley, U.S. Coast Guard, U.S. Geological Survey)

✳ WORKED EXAMPLE

Entropy Associated with Disorder

Find the increase in entropy of 1.00 kg of ice that is originally at 0 °C and melts to form water at 0 °C.

STRATEGY

The change in entropy can be calculated from the definition of ΔS once we find the energy, Q, needed to melt the ice.

Solution

The change in entropy is defined as

$$\Delta S = \frac{Q}{T}.$$ 12.15

Here, Q is the heat necessary to melt 1.00 kg of ice and is given by

$$Q = mL_f,$$ 12.16

where m is the mass and L_f is the latent heat of fusion. $L_f = 334$ kJ/kg for water, so
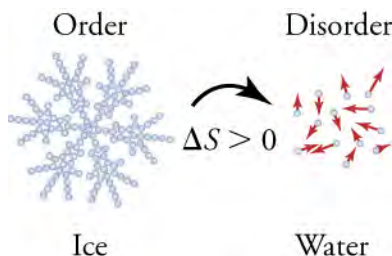
$$Q = (1.00 \text{ kg})(334 \text{ kJ/kg}) = 3.34 \times 10^5 \text{J}.$$ 12.17

Because Q is the amount of energy heat adds to the ice, its value is positive, and T is the melting temperature of ice, $T = 273$ K So the change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{3.34 \times 10^5 \text{J}}{273 \text{ K}} = 1.22 \times 10^3 \text{J/K}.$$ 12.18

Discussion

Figure 12.12 When ice melts, it becomes more disordered and less structured. The systematic arrangement of molecules in a crystal structure is replaced by a more random and less orderly movement of molecules without fixed locations or orientations. Its **entropy** increases because heat transfer occurs into it. Entropy is a measure of disorder.

The change in entropy is positive, because heat transfers energy *into* the ice to cause the phase change. This is a significant increase in entropy, because it takes place at a relatively low temperature. It is accompanied by an increase in the disorder of the water molecules.

Practice Problems

11. If $30.0\,\text{J}$ are added by heat to water at $12\degree\text{C}$, what is the change in entropy?
    a. 0.105 J/K
    b. 2.5 J/K
    c. 0.45 J/K
    d. 9.50 J/K

12. What is the increase in entropy when $3.00\,\text{kg}$ of ice at $0\degree\text{C}$ melt to form water at $0\degree\text{C}$?
    a. 1.84×10^3 J/K
    b. 3.67×10^3 J/K
    c. 1.84×10^8 J/K
    d. 3.67×10^8 J/K

Check Your Understanding

13. What is entropy?
    a. Entropy is a measure of the potential energy of a system.
    b. Entropy is a measure of the net work done by a system.
    c. Entropy is a measure of the disorder of a system.
    d. Entropy is a measure of the heat transfer of energy into a system.

14. Which forms of energy can be used to do work?
    a. Only work is able to do work.
    b. Only heat is able to do work.
    c. Only internal energy is able to do work.
    d. Heat, work, and internal energy are all able to do work.

15. What is the statement for the second law of thermodynamics?
    a. All the spontaneous processes result in decreased total entropy of a system.
    b. All the spontaneous processes result in increased total entropy of a system.
    c. All the spontaneous processes result in decreased or constant total entropy of a system.
    d. All the spontaneous processes result in increased or constant total entropy of a system.

16. For heat transferring energy from a high to a low temperature, what usually happens to the entropy of the whole system?
    a. It decreases.
    b. It must remain constant.
    c. The entropy of the system cannot be predicted without specific values for the temperatures.

d.   It increases.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain how heat engines, heat pumps, and refrigerators work in terms of the laws of thermodynamics
- Describe thermal efficiency
- Solve problems involving thermal efficiency

Section Key Terms

cyclical process   heat engine   heat pump

thermal efficiency

Heat Engines, Heat Pumps, and Refrigerators

In this section, we'll explore how heat engines, heat pumps, and refrigerators operate in terms of the laws of thermodynamics.

One of the most important things we can do with heat is to use it to do work for us. A **heat engine** does exactly this—it makes use of the properties of thermodynamics to transform heat into work. Gasoline and diesel engines, jet engines, and steam turbines that generate electricity are all examples of heat engines.

Figure 12.13 illustrates one of the ways in which heat transfers energy to do work. Fuel combustion releases chemical energy that heat transfers throughout the gas in a cylinder. This increases the gas temperature, which in turn increases the pressure of the gas and, therefore, the force it exerts on a movable piston. The gas does work on the outside world, as this force moves the piston through some distance. Thus, heat transfer of energy to the gas in the cylinder results in work being done.
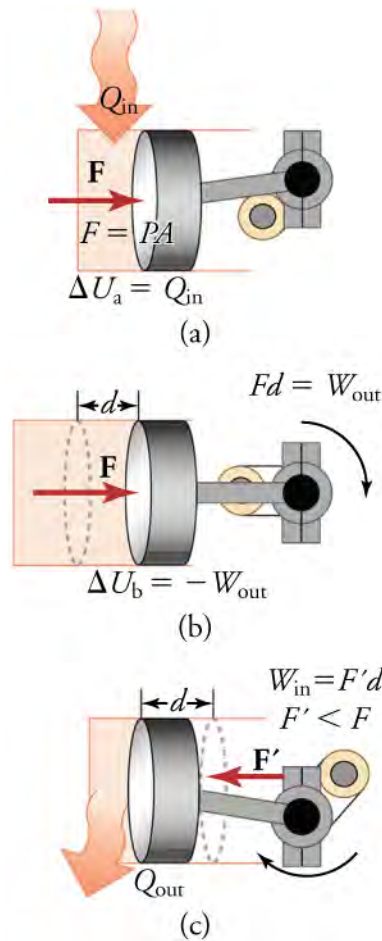
Figure 12.13 (a) Heat transfer to the gas in a cylinder increases the internal energy of the gas, creating higher pressure and temperature. (b) The force exerted on the movable cylinder does work as the gas expands. Gas pressure and temperature decrease during expansion, indicating that the gas's internal energy has decreased as it does work. (c) Heat transfer of energy to the environment further reduces pressure in the gas, so that the piston can more easily return to its starting position.

To repeat this process, the piston needs to be returned to its starting point. Heat now transfers energy from the gas to the surroundings, so that the gas's pressure decreases, and a force is exerted by the surroundings to push the piston back through some distance.

A **cyclical process** brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. All heat engines use cyclical processes.

Heat engines do work by using part of the energy transferred by heat from some source. As shown in Figure 12.14, heat transfers energy, Q_h, from the high-temperature object (or hot reservoir), whereas heat transfers unused energy, Q_c, into the low-temperature object (or cold reservoir), and the work done by the engine is W. In physics, a *reservoir* is defined as an infinitely large mass that can take in or put out an unlimited amount of heat, depending upon the needs of the system. The temperature of the hot reservoir is T_h, and the temperature of the cold reservoir is T_c .
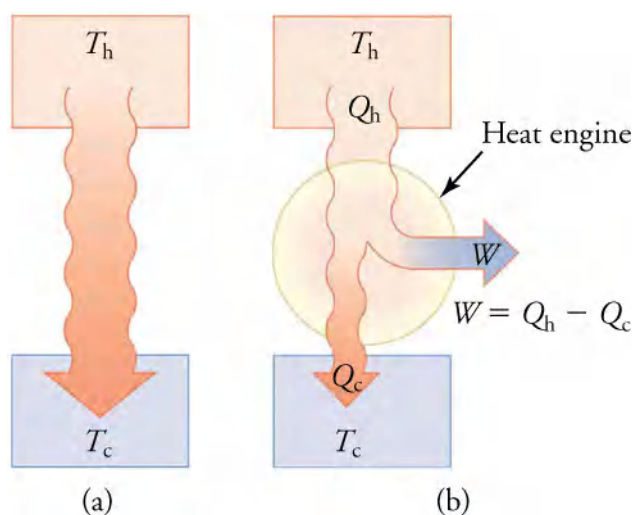
Figure 12.14 (a) Heat transfers energy spontaneously from a hot object to a cold one, as is consistent with the second law of thermodynamics. (b) A heat engine, represented here by a circle, uses part of the energy transferred by heat to do work. The hot and cold objects are called the hot and cold reservoirs. Q_h is the heat out of the hot reservoir, W is the work output, and Q_c is the unused heat into the cold reservoir.

As noted, a cyclical process brings the system back to its original condition at the end of every cycle. Such a system's internal energy, U, is the same at the beginning and end of every cycle—that is, $\Delta U = 0$. The first law of thermodynamics states that $\Delta U = Q - W$, where Q is the *net* heat transfer during the cycle, and W is the *net* work done by the system. The net heat transfer is the energy transferred in by heat from the hot reservoir minus the amount that is transferred out to the cold reservoir ($Q = Q_h - Q_c$). Because there is no change in internal energy for a complete cycle ($\Delta U = 0$), we have

$$0 = Q - W, \qquad \text{12.19}$$

so that

$$W = Q. \qquad \text{12.20}$$

Therefore, the net work done by the system equals the net heat into the system, or

$$W = Q_h - Q_c \qquad \text{12.21}$$

for a cyclical process.

Because the hot reservoir is heated externally, which is an energy-intensive process, it is important that the work be done as efficiently as possible. In fact, we want W to equal Q_h, and for there to be no heat to the environment (that is, $Q_c = 0$). Unfortunately, this is impossible. According to the second law of thermodynamics, heat engines cannot have perfect conversion of heat into work. Recall that entropy is a measure of the disorder of a system, which is also how much energy is unavailable to do work. The second law of thermodynamics requires that the total entropy of a system either increases or remains constant in any process. Therefore, there is a minimum amount of Q_h that cannot be used for work. The amount of heat rejected to the cold reservoir, Q_c, depends upon the efficiency of the heat engine. The smaller the increase in entropy, ΔS, the smaller the value of Q_c, and the more heat energy is available to do work.

Heat pumps, air conditioners, and refrigerators utilize heat transfer of energy from low to high temperatures, which is the opposite of what heat engines do. Heat transfers energy Q_c from a cold reservoir and delivers energy Q_h into a hot one. This requires work input, W, which produces a transfer of energy by heat. Therefore, the total heat transfer to the hot reservoir is

$$Q_h = Q_c + W. \qquad \text{12.22}$$

The purpose of a heat pump is to transfer energy by heat to a warm environment, such as a home in the winter. The great advantage of using a heat pump to keep your home warm rather than just burning fuel in a fireplace or furnace is that a heat pump supplies $Q_h = Q_c + W$. Heat Q_c comes from the outside air, even at a temperature below freezing, to the indoor space. You only pay for W, and you get an additional heat transfer of Q_c from the outside at no cost. In many cases, at least twice as much energy is transferred to the heated space as is used to run the heat pump. When you burn fuel to keep warm, you pay for all of it. The disadvantage to a heat pump is that the work input (required by the second law of thermodynamics) is sometimes

more expensive than simply burning fuel, especially if the work is provided by electrical energy.

The basic components of a heat pump are shown in <u>Figure 12.15</u>. A working fluid, such as a refrigerant, is used. In the outdoor coils (the evaporator), heat Q_c enters the working fluid from the cold outdoor air, turning it into a gas.
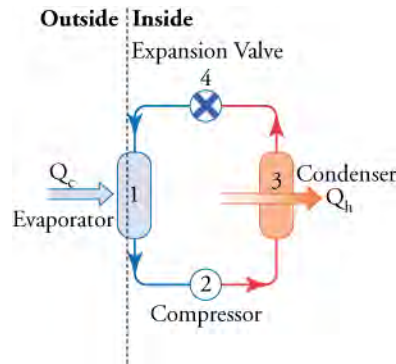
Figure 12.15 A simple heat pump has four basic components: (1) an evaporator, (2) a compressor, (3) a condenser, and (4) an expansion valve. In the heating mode, heat transfers Q_c to the working fluid in the evaporator (1) from the colder, outdoor air, turning it into a gas. The electrically driven compressor (2) increases the temperature and pressure of the gas and forces it into the condenser coils (3) inside the heated space. Because the temperature of the gas is higher than the temperature in the room, heat transfers energy from the gas to the room as the gas condenses into a liquid. The working fluid is then cooled as it flows back through an expansion valve (4) to the outdoor evaporator coils.

The electrically driven compressor (work input W) raises the temperature and pressure of the gas and forces it into the condenser coils that are inside the heated space. Because the temperature of the gas is higher than the temperature inside the room, heat transfers energy to the room, and the gas condenses into a liquid. The liquid then flows back through an expansion (pressure-reducing) valve. The liquid, having been cooled through expansion, returns to the outdoor evaporator coils to resume the cycle.

The quality of a heat pump is judged by how much energy is transferred by heat into the warm space (Q_h) compared with how much input work (W) is required.
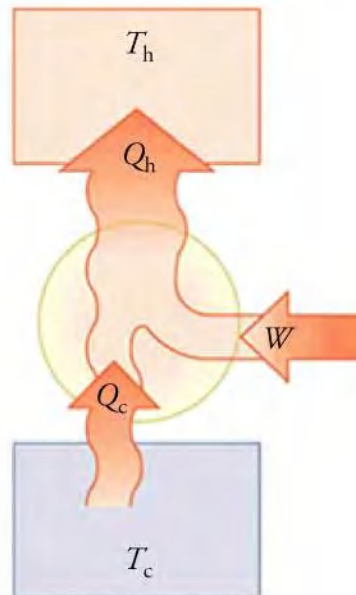
Figure 12.16 Heat pumps, air conditioners, and refrigerators are heat engines operated backward. Almost every home contains a refrigerator. Most people don't realize that they are also sharing their homes with a heat pump.

Air conditioners and refrigerators are designed to cool substances by transferring energy by heat Q_c out of a cool environment to a warmer one, where heat Q_h is given up. In the case of a refrigerator, heat is moved out of the inside of the fridge into the

surrounding room. For an air conditioner, heat is transferred outdoors from inside a home. Heat pumps are also often used in a reverse setting to cool rooms in the summer.

As with heat pumps, work input is required for heat transfer of energy from cold to hot. The quality of air conditioners and refrigerators is judged by how much energy is removed by heat Q_c from a cold environment, compared with how much work, W, is required. So, what is considered the energy benefit in a heat pump, is considered waste heat in a refrigerator.

Thermal Efficiency

In the conversion of energy into work, we are always faced with the problem of getting less out than we put in. The problem is that, in all processes, there is some heat Q_c that transfers energy to the environment—and usually a very significant amount at that. A way to quantify how efficiently a machine runs is through a quantity called **thermal efficiency**.

We define thermal efficiency, *Eff*, to be the ratio of useful energy output to the energy input (or, in other words, the ratio of what we get to what we spend). The efficiency of a heat engine is the output of net work, W, divided by heat-transferred energy, Q_h, into the engine; that is

$$Eff = \frac{W}{Q_h}.$$

An efficiency of 1, or 100 percent, would be possible only if there were no heat to the environment ($Q_c = 0$).

TIPS FOR SUCCESS

All values of heat (Q_h and Q_c) are positive; there is no such thing as negative heat. The *direction* of heat is indicated by a plus or minus sign. For example, Q_c is out of the system, so it is preceded by a minus sign in the equation for net heat.

$$Q = Q_h - Q_c \qquad \text{12.23}$$

Solving Thermal Efficiency Problems

✳ WORKED EXAMPLE

Daily Work Done by a Coal-Fired Power Station and Its Efficiency

A coal-fired power station is a huge heat engine. It uses heat to transfer energy from burning coal to do work to turn turbines, which are used then to generate electricity. In a single day, a large coal power station transfers $2.50 \times 10^{14}\,\text{J}$ by heat from burning coal and transfers $1.48 \times 10^{14}\,\text{J}$ by heat into the environment. (a) What is the work done by the power station? (b) What is the efficiency of the power station?

STRATEGY

We can use $W = Q_h - Q_c$ to find the work output, W, assuming a cyclical process is used in the power station. In this process, water is boiled under pressure to form high-temperature steam, which is used to run steam turbine-generators and then condensed back to water to start the cycle again.

Solution

Work output is given by

$$W = Q_h - Q_c. \qquad \text{12.24}$$

Substituting the given values,

$$W = 2.50 \times 10^{14}\,\text{J} - 1.48 \times 10^{14}\,\text{J} = 1.02 \times 10^{14}\,\text{J}. \qquad \text{12.25}$$

STRATEGY

The efficiency can be calculated with $Eff = \frac{W}{Q_h}$, because Q_h is given, and work, W, was calculated in the first part of this example.

Solution

Efficiency is given by

$$Eff = \frac{W}{Q_h}.$$

<div style="text-align:right">12.26</div>

The work, W, is found to be 1.02×10^{14} J, and Q_h is given (2.50×10^{14} J), so the efficiency is

$$Eff = \frac{1.02 \times 10^{14} \, J}{2.50 \times 10^{14} \, J} = 0.408, \text{ or } 40.8\%.$$

<div style="text-align:right">12.27</div>

Discussion

The efficiency found is close to the usual value of 42 percent for coal-burning power stations. It means that fully 59.2 percent of the energy is transferred by heat to the environment, which usually results in warming lakes, rivers, or the ocean near the power station and is implicated in a warming planet generally. While the laws of thermodynamics limit the efficiency of such plants—including plants fired by nuclear fuel, oil, and natural gas—the energy transferred by heat to the environment could be, and sometimes is, used for heating homes or for industrial processes.

Practice Problems

17. A heat engine is given 120 J by heat and releases 20 J by heat to the environment. What is the amount of work done by the system?
    a. -100 J
    b. -60 J
    c. 60 J
    d. 100 J

18. A heat engine takes in 6.0 kJ from heat and produces waste heat of 4.8 kJ. What is its efficiency?
    a. 25 percent
    b. 2.50 percent
    c. 2.00 percent
    d. 20 percent

Check Your Understanding

19. What is a heat engine?
    a. A heat engine converts mechanical energy into thermal energy.
    b. A heat engine converts thermal energy into mechanical energy.
    c. A heat engine converts thermal energy into electrical energy.
    d. A heat engine converts electrical energy into thermal energy.

20. Give an example of a heat engine.
    a. A generator
    b. A battery
    c. A water pump
    d. A car engine

21. What is thermal efficiency?
    a. Thermal efficiency is the ratio of work input to the energy input.
    b. Thermal efficiency is the ratio of work output to the energy input.
    c. Thermal efficiency is the ratio of work input to the energy output.
    d. Thermal efficiency is the ratio of work output to the energy output.

22. What is the mathematical expression for thermal efficiency?
    a. $Eff = \frac{Q_h}{Q_h - Q_c}$
    b. $Eff = \frac{Q_h}{Q_c}$
    c. $Eff = \frac{Q_c}{Q_h}$
    d. $Eff = \frac{Q_h - Q_c}{Q_h}$

KEY TERMS

Boltzmann constant   constant with the value $k = 1.38{\times}10^{-23}$ J/K, which is used in the ideal gas law

cyclical process   process in which a system is brought back to its original state at the end of every cycle

entropy   measurement of a system's disorder and how much energy is not available to do work in a system

first law of thermodynamics   states that the change in internal energy of a system equals the net energy transfer by heat *into* the system minus the net work done *by* the system

heat engine   machine that uses energy transfer by heat to do work

heat pump   machine that generates the heat transfer of energy from cold to hot

ideal gas law   physical law that relates the pressure and volume of a gas to the number of gas molecules or atoms, or number of moles of gas, and the absolute temperature

of the gas

internal energy   sum of the kinetic and potential energies of a system's constituent particles (atoms or molecules)

pressure   force per unit area perpendicular to the force, over which the force acts

second law of thermodynamics   states that the total entropy of a system either increases or remains constant in any spontaneous process; it never decreases

thermal efficiency   ratio of useful energy output to the energy input

thermal equilibrium   condition in which heat no longer transfers energy between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics   states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

SECTION SUMMARY

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

12.2 First law of Thermodynamics: Thermal Energy and Work

- Pressure is the force per unit area over which the force is applied perpendicular to the area.
- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- The ideal gas law relates the pressure and volume of a gas to the number of gas particles (atoms or molecules) and the absolute temperature of the gas.
- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- The first law of thermodynamics is given as $\Delta U = Q - W$, where ΔU is the change in internal energy of a system, Q is the net energy transfer into the system by heat (the sum of all transfers by heat into and out of the system), and W is the net work done by the

system (the sum of all energy transfers by work out of or into the system).

- Both Q and W represent energy in transit; only ΔU represents an independent quantity of energy capable of being stored.
- The internal energy U of a system depends only on the state of the system, and not how it reached that state.

12.3 Second Law of Thermodynamics: Entropy

- Entropy is a measure of a system's disorder: the greater the disorder, the larger the entropy.
- Entropy is also the reduced availability of energy to do work.
- The second law of thermodynamics states that, for any spontaneous process, the total entropy of a system either increases or remains constant; it never decreases.
- Heat transfers energy spontaneously from higher- to lower-temperature bodies, but never spontaneously in the reverse direction.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

- Heat engines use the heat transfer of energy to do work.
- Cyclical processes are processes that return to their original state at the end of every cycle.
- The thermal efficiency of a heat engine is the ratio of work output divided by the amount of energy input.
- The amount of work a heat engine can do is determined by the net heat transfer of energy during a cycle; more waste heat leads to less work output.

- Heat pumps draw energy by heat from cold outside air and use it to heat an interior room.
- A refrigerator is a type of heat pump; it takes energy

from the warm air from the inside compartment and transfers it to warmer exterior air.

KEY EQUATIONS

12.2 First law of Thermodynamics: Thermal Energy and Work

ideal gas law	$PV = NkT$
first law of thermodynamics	$\Delta U = Q - W$
pressure	$P = \frac{F}{A}$
pressure–volume work	$W = P\Delta V$

12.3 Second Law of Thermodynamics: Entropy

change in entropy	$\Delta S = \frac{Q}{T}$

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

thermal efficiency of a heat engine	$Eff = \frac{W}{Q_h}$
work output for a cyclical process	$W = Q_h - Q_c$

CHAPTER REVIEW

Concept Items

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

1. When are two bodies in thermal equilibrium?
   a. When they are in thermal contact and are at different pressures
   b. When they are not in thermal contact but are at the same pressure
   c. When they are not in thermal contact but are at different temperatures
   d. When they are in thermal contact and are at the same temperature

2. What is thermal contact?
   a. Two objects are said to be in thermal contact when they are in contact with each other in such a way that the transfer of energy by heat can occur between them.
   b. Two objects are said to be in thermal contact when they are in contact with each other in such a way that the transfer of energy by mass can occur between them.
   c. Two objects are said to be in thermal contact when they neither lose nor gain energy by heat. There is no transfer of energy between them.
   d. Two objects are said to be in thermal contact when they only gain energy by heat. There is transfer of energy between them.

3. To which mathematical property is the zeroth law of

thermodynamics similar?
   a. Associative property
   b. Commutative property
   c. Distributive property
   d. Transitive property

12.2 First law of Thermodynamics: Thermal Energy and Work

4. Why does thermal expansion occur?
   a. An increase in temperature causes intermolecular distances to increase.
   b. An increase in temperature causes intermolecular distances to decrease.
   c. An increase in temperature causes an increase in the work done on the system.
   d. An increase in temperature causes an increase in the work done by the system.

5. How does pressure-volume work relate to heat and internal energy of a system?
   a. The energy added to a system by heat minus the change in the internal energy of that system is equal to the pressure-volume work done by the system.
   b. The sum of the energy released by a system by heat and the change in the internal energy of that system is equal to the pressure-volume work done by the system.
   c. The product of the energy added to a system by heat and the change in the internal energy of that system

is equal to the pressure-volume work done by the system.

    d.   If the energy added to a system by heat is divided by the change in the internal energy of that system, the quotient is equal to the pressure-volume work done by the system.

6. On what does internal energy depend?
    a.   The path of energy changes in the system
    b.   The state of the system
    c.   The size of the system
    d.   The shape of the system

7. The first law of thermodynamics helps us understand the relationships among which three quantities?
    a.   Heat, work, and internal energy
    b.   Heat, work, and external energy
    c.   Heat, work, and enthalpy
    d.   Heat, work, and entropy

12.3 Second Law of Thermodynamics: Entropy

8. Air freshener is sprayed from a bottle. The molecules spread throughout the room and cannot make their way back into the bottle. Why is this the case?
    a.   The entropy of the molecules increases.
    b.   The entropy of the molecules decreases.
    c.   The heat content (*enthalpy*, or total energy available for heat) of the molecules increases.
    d.   The heat content (*enthalpy*, or total energy available for heat) of the molecules decreases.

9. Give an example of entropy as experienced in everyday life.
    a.   rotation of Earth
    b.   formation of a solar eclipse
    c.   filling a car tire with air
    d.   motion of a pendulum bob

Critical Thinking Items

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

13. What are the necessary conditions for energy transfer by heat to occur between two bodies through the process of conduction?
    a.   They should be at the same temperature, and they should be in thermal contact.
    b.   They should be at the same temperature, and they should not be in thermal contact.
    c.   They should be at different temperatures, and they should be in thermal contact.
    d.   They should be at different temperatures, and they

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

10. What is the quality by which air conditioners are judged?
    a.   The amount of energy generated by heat from a hot environment, compared with the required work input
    b.   The amount of energy transferred by heat from a cold environment, compared with the required work input
    c.   The amount of energy transferred by heat from a hot environment, compared with the required work output
    d.   The amount of energy transferred by heat from a cold environment, compared with the required work output

11. Why is the efficiency of a heat engine never 100 percent?
    a.   Some energy is always gained by heat from the environment.
    b.   Some energy is always lost by heat to the environment.
    c.   Work output is always greater than energy input.
    d.   Work output is infinite for any energy input.

12. What is a cyclic process?
    a.   A process in which the system returns to its original state at the end of the cycle
    b.   A process in which the system does not return to its original state at the end of the cycle
    c.   A process in which the system follows the same path for every cycle
    d.   A process in which the system follows a different path for every cycle

should not be in thermal contact.

14. Oil is heated in a pan on a hot plate. The pan is in thermal equilibrium with the hot plate and also with the oil. The temperature of the hot plate is 150 °C . What is the temperature of the oil?
    a.   160 °C
    b.   150 °C
    c.   140 °C
    d.   130 °C

12.2 First law of Thermodynamics: Thermal Energy and Work

15. When an inflated balloon experiences a decrease in size,

the air pressure inside the balloon remains nearly constant. If there is no transfer of energy by heat to or from the balloon, what physical change takes place in the balloon?

   a. The average kinetic energy of the gas particles decreases, so the balloon becomes colder.

   b. The average kinetic energy of the gas particles increases, so the balloon becomes hotter.

   c. The average potential energy of the gas particles decreases, so the balloon becomes colder.

   d. The average potential energy of the gas particles increases, so the balloon becomes hotter.

16. When heat adds energy to a system, what is likely to happen to the pressure and volume of the system?

   a. Pressure and volume may both decrease with added energy.

   b. Pressure and volume may both increase with added energy.

   c. Pressure must increase with added energy, while volume must remain constant.

   d. Volume must decrease with added energy, while pressure must remain constant.

17. If more energy is transferred into the system by net heat as compared to the net work done by the system, what happens to the difference in energy?

   a. It is transferred back to its surroundings.

   b. It is stored in the system as internal energy.

   c. It is stored in the system as potential energy.

   d. It is stored in the system as entropy.

18. Air is pumped into a car tire, causing its temperature to increase. In another tire, the temperature increase is due to exposure to the sun. Is it possible to tell what caused the temperature increase in each tire? Explain your answer.

   a. No, because it is a chemical change, and the cause of that change does not matter; the final state of both systems are the same.

   b. Although the final state of each system is identical, the source is different in each case.

   c. No, because the changes in energy for both systems are the same, and the cause of that change does not matter; the state of each system is identical.

   d. Yes, the changes in the energy for both systems are the same, but the causes of that change are different, so the states of each system are not identical.

19. How does the transfer of energy from the sun help plants?

   a. Plants absorb solar energy from the sun and utilize it during the fertilization process.

   b. Plants absorb solar energy from the sun and utilize

it during the process of photosynthesis to turn it into plant matter.

   c. Plants absorb solar energy from the sun and utilize it to increase the temperature inside them.

   d. Plants absorb solar energy from the sun and utilize it during the shedding of their leaves and fruits.

12.3 Second Law of Thermodynamics: Entropy

20. If an engine were constructed to perform such that there would be no losses due to friction, what would be its efficiency?

   a. It would be 0 percent.

   b. It would be less than 100 percent.

   c. It would be 100 percent.

   d. It would be greater than 100 percent.

21. Entropy never decreases in a spontaneous process. Give an example to support this statement.

   a. The transfer of energy by heat from colder bodies to hotter bodies is a spontaneous process in which the entropy of the system of bodies increases.

   b. The melting of an ice cube placed in a room causes an increase in the entropy of the room.

   c. The dissolution of salt in water is a spontaneous process in which the entropy of the system increases.

   d. A plant uses energy from the sun and converts it into sugar molecules by the process of photosynthesis.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

22. What is the advantage of a heat pump as opposed to burning fuel (as in a fireplace) for keeping warm?

   a. A heat pump supplies energy by heat from the cold, outside air.

   b. A heat pump supplies energy generated by the work done.

   c. A heat pump supplies energy by heat from the cold, outside air and also from the energy generated by the work done.

   d. A heat pump supplies energy not by heat from the cold, outside air, nor from the energy generated by the work done, but from more accessible sources.

23. What is thermal efficiency of an engine? Can it ever be 100 percent? Why or why not?

   a. Thermal efficiency is the ratio of the output (work) to the input (heat). It is always 100 percent.

   b. Thermal efficiency is the ratio of the output (heat)

to the input (work). It is always 100 percent.

c. Thermal efficiency is the ratio of the output (heat) to the input (work). It is never 100 percent.

d. Thermal efficiency is the ratio of the output (work) to the input (heat). It is never 100 percent.

24. When would 100 percent thermal efficiency be possible?

a. When all energy is transferred by heat to the environment

b. When mass transferred to the environment is zero

c. When mass transferred to the environment is at a maximum

d. When no energy is transferred by heat to the environment

Problems

12.2 First law of Thermodynamics: Thermal Energy and Work

25. Some amount of energy is transferred by heat into a system. The net work done by the system is $50\,\text{J}$, while the increase in its internal energy is $30\,\text{J}$. What is the amount of net heat?

a. $-80\,\text{J}$

b. $-20\,\text{J}$

c. $20\,\text{J}$

d. $80\,\text{J}$

26. Eighty joules are added by heat to a system, while it does $20\,\text{J}$ of work. Later, $30\,\text{J}$ are added by heat to the system, and it does $40\,\text{J}$ of work. What is the change in the system's internal energy?

a. $30\,\text{J}$

b. $50\,\text{J}$

c. $60\,\text{J}$

d. $110\,\text{J}$

Performance Task

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

29. You have been tasked to design and construct a thermometer that works on the principle of thermal expansion. There are four materials available for you to test, each of which will find use under different sets of conditions and temperature ranges:

Materials

- Four sample materials with similar mass or volume: copper, steel, water, and alcohol (ethanol or isopropanol)
- Oven or similar heating source
- Instrument (e.g., meter ruler, Vernier calipers, or micrometer) for measuring changes in dimension
- Balance for measuring mass

Procedure

1. Design a safe experiment to analyze the thermal

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

27. A coal power station functions at 40.0 percent efficiency. What is the amount of work it does if it takes in $1.20 \times 10^{12}\,\text{J}$ by heat?

a. $3 \times 10^{10}\,\text{J}$

b. $4.8 \times 10^{11}\,\text{J}$

c. $3 \times 10^{12}\,\text{J}$

d. $4.8 \times 10^{13}\,\text{J}$

28. A heat engine functions with 70.7 percent thermal efficiency and consumes 12.0 kJ from heat daily. If its efficiency were raised to 75.0 percent, how much energy from heat would be saved daily, while providing the same output?

a. $-10.8\,\text{kJ}$

b. $-1.08\,\text{kJ}$

c. $0.7\,\text{kJ}$

d. $7\,\text{kJ}$

expansion properties of each material.

2. Write down the materials needed for your experiment and the procedure you will follow. Make sure that you include every detail so that the experiment can be repeated by others.

3. Select an appropriate material to measure temperature over a predecided temperature range, and give reasons for your choice.

4. Calibrate your instrument to measure temperature changes accurately.

a. Which physical quantities are affected by temperature change and thermal expansion?

b. How do such properties as specific heat and thermal conductivity affect the use of each material as a thermometer?

c. Does a change of phase take place for any of the tested materials over the temperature range to be examined?

d. What are your independent and dependent variables for this series of tests? Which variables need to be controlled in the experiment?

e. What are your sources of error?
f. Can all the tested materials be used effectively in the same ranges of temperature? Which

applications might be suitable for one or more of the tested substances but not the others?

TEST PREP

Multiple Choice

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

30. Which law of thermodynamics describes thermal equilibrium?
   a. zeroth
   b. first
   c. second
   d. third

31. Name any two industries in which the principles of thermodynamics are used.
   a. aerospace and information technology (IT) industries
   b. industrial manufacturing and aerospace
   c. mining and textile industries
   d. mining and agriculture industries

12.2 First law of Thermodynamics: Thermal Energy and Work

32. What is the value of the Boltzmann constant?
   a. $k = 1.23 \times 10^{-38}$ J/K
   b. $k = 1.38 \times 10^{-23}$ J/K
   c. $k = 1.38 \times 10^{23}$ J/K
   d. $k = 1.23 \times 10^{38}$ J/K

33. Which of the following involves work done BY a system?
   a. increasing internal energy
   b. compression
   c. expansion
   d. cooling

34. Which principle does the first law of thermodynamics state?
   a. the ideal gas law
   b. the transitive property of equality
   c. the law of conservation of energy
   d. the principle of thermal equilibrium

35. What is the change in internal energy of a system when $Q_{in} = 50$ J and $Q_{out} = 50$ J?
   a. 20 J
   b. 30 J
   c. 50 J
   d. 70 J

36. When does a real gas behave like an ideal gas?

a. A real gas behaves like an ideal gas at high temperature and low pressure.
b. A real gas behaves like an ideal gas at high temperature and high pressure.
c. A real gas behaves like an ideal gas at low temperature and low pressure.
d. A real gas behaves like an ideal gas at low temperature and high pressure.

12.3 Second Law of Thermodynamics: Entropy

37. In an engine, what is the unused energy converted into?
   a. internal energy
   b. pressure
   c. work
   d. heat

38. It is natural for systems in the universe to _____ spontaneously.
   a. become disordered
   b. become ordered
   c. produce heat
   d. do work

39. If Q is 120 J and T is 350 K, what is the change in entropy?
   a. 0.343 J/K
   b. 1.51 J/K
   c. 2.92 J/K
   d. 34.3 J/K

40. Why does entropy increase during a spontaneous process?
   a. Entropy increases because energy always transfers spontaneously from a dispersed state to a concentrated state.
   b. Entropy increases because energy always transfers spontaneously from a concentrated state to a dispersed state.
   c. Entropy increases because pressure always increases spontaneously.
   d. Entropy increases because temperature of any system always increases spontaneously.

41. A system consists of ice melting in a glass of water. What happens to the entropy of this system?
   a. The entropy of the ice decreases, while the entropy of the water cannot be predicted without more

specific information.

   b.   The entropy of the system remains constant.

   c.   The entropy of the system decreases.

   d.   The entropy of the system increases.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

42. Which equation represents the net work done by a system in a cyclic process?

   a. $W = \frac{Q_c}{Q_h}$

   b. $W = Q_h + Q_c$

   c. $W = Eff\,(Q_c - Q_h)$

   d. $W = Q_h - Q_c$

43. Which of these quantities needs to be zero for efficiency to be 100 percent?

   a. ΔU

   b. W

   c. Q_h

   d. Q_c

44. Which of the following always has the greatest value in a system having 80 percent thermal efficiency?

   a. ΔU

   b. W

   c. Q_h

   d. Q_c

45. In the equation $Q = Q_h - Q_c$, what does the negative sign indicate?

   a.   Heat transfer of energy is always negative.

   b.   Heat transfer can only occur in one direction.

   c.   Heat is directed into the system from the surroundings outside the system.

   d.   Heat is directed out of the system.

46. What is the purpose of a heat pump?

   a.   A heat pump uses work to transfer energy by heat from a colder environment to a warmer environment.

   b.   A heat pump uses work to transfer energy by heat from a warmer environment to a colder environment.

   c.   A heat pump does work by using heat to convey energy from a colder environment to a warmer environment.

   d.   A heat pump does work by using heat to convey energy from a warmer environment to a colder environment.

Short Answer

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

47. What does *green* energy development entail?

   a.   Green energy involves finding new ways to harness clean and renewable alternative energy sources.

   b.   Green energy involves finding new ways to conserve alternative energy sources.

   c.   Green energy involves decreasing the efficiency of nonrenewable energy resources.

   d.   Green energy involves finding new ways to harness nonrenewable energy resources.

48. Why are the sun and Earth not in thermal equilibrium?

   a.   The mass of the sun is much greater than the mass of Earth.

   b.   There is a vast amount of empty space between the sun and Earth.

   c.   The diameter of the sun is much greater than the diameter of Earth.

   d.   The sun is in thermal contact with Earth.

12.2 First law of Thermodynamics: Thermal Energy and Work

49. If a fixed quantity of an ideal gas is held at a constant volume, which variable relates to pressure, and what is that relation?

   a.   Temperature; inverse proportionality $\left(P \propto \frac{1}{T}\right)$

   b.   Temperature, direct proportionality to square root $\left(P \propto \sqrt{T}\right)$

   c.   Temperature; direct proportionality $(P \propto T)$

   d.   Temperature; direct proportionality to square $\left(P \propto T^2\right)$

50. When is volume directly proportional to temperature?

   a.   when the pressure of the gas is variable

   b.   when the pressure of the gas is constant

   c.   when the mass of the gas is variable

   d.   when the mass of the gas is constant

51. For fluids, what can work be defined as?

   a.   pressure acting over the change in depth

   b.   pressure acting over the change in temperature

   c.   temperature acting over the change in volume

   d.   pressure acting over the change in volume

52. In the equation $\Delta U = Q - P\Delta V$, what does $P\Delta V$ indicate?

   a.   the work done on the system

   b.   the work done by the system

   c.   the heat into the system

   d.   the heat out of the system

53. By convention, if Q is positive, what is the direction in which heat transfers energy with regard to the system?
    a. The direction of the heat transfer of energy depends on the changes in W, regardless of the sign of Q.
    b. The direction of Q cannot be determined from just the sign of Q.
    c. The direction of net heat transfer of energy will be out of the system.
    d. The direction of net heat transfer of energy will be into the system.

54. What is net transfer of energy by heat?
    a. It is the sum of all energy transfers by heat into the system.
    b. It is the product of all energy transfers by heat into the system.
    c. It is the sum of all energy transfers by heat into and out of the system.
    d. It is the product of all energy transfers by heat into and out of the system.

55. Three hundred ten joules of heat enter a system, after which the system does $120\,J$ of work. What is the change in its internal energy? Would this amount change if the energy transferred by heat were added after the work was done instead of before?
    a. $-190\,J$; this would change if heat added energy after the work was done
    b. $190\,J$; this would change if heat added energy after the work was done
    c. $-190\,J$; this would not change even if heat added energy after the work was done
    d. $190\,J$; this would not change even if heat added energy after the work was done

56. Ten joules are transferred by heat into a system, followed by another $20\,J$. What is the change in the system's internal energy? What would be the difference in this change if $30\,J$ of energy were added by heat to the system at once?
    a. $10\,J$; the change in internal energy would be same even if the heat added the energy at once
    b. $30\,J$; the change in internal energy would be same even if the heat added the energy at once
    c. $10\,J$; the change in internal energy would be more if the heat added the energy at once
    d. $30\,J$; the change in internal energy would be more if the heat added the energy at once

12.3 Second Law of Thermodynamics: Entropy

57. How does the entropy of a system depend on how the system reaches a given state?

a. Entropy depends on the change of phase of a system, but not on any other state conditions.
    b. Entropy does not depend on how the final state is reached from the initial state.
    c. Entropy is least when the path between the initial state and the final state is the shortest.
    d. Entropy is least when the path between the initial state and the final state is the longest.

58. Which sort of thermal energy do molecules in a solid possess?
    a. electric potential energy
    b. gravitational potential energy
    c. translational kinetic energy
    d. vibrational kinetic energy

59. A cold object in contact with a hot one never spontaneously transfers energy by heat to the hot object. Which law describes this phenomenon?
    a. the first law of thermodynamics
    b. the second law of thermodynamics
    c. the third law of thermodynamics
    d. the zeroth law of thermodynamics

60. How is it possible for us to transfer energy by heat from cold objects to hot ones?
    a. by doing work on the system
    b. by having work done by the system
    c. by increasing the specific heat of the cold body
    d. by increasing the specific heat of the hot body

61. What is the change in entropy caused by melting 5.00 kg of ice at 0 °C ?
    a. 0 J/K
    b. 6.11×10^3 J/K
    c. 6.11×10^4 J/K
    d. ∞ J/K

62. What is the amount of heat required to cause a change of 35 J/K in the entropy of a system at $400\,K$?
    a. 1.1×10^1 J
    b. 1.1×10^2 J
    c. 1.4×10^3 J
    d. 1.4×10^4 J

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

63. In a refrigerator, what is the function of an evaporator?
    a. The evaporator converts gaseous refrigerant into liquid.
    b. The evaporator converts solid refrigerant into liquid.
    c. The evaporator converts solid refrigerant into gas.

d. The evaporator converts liquid refrigerant into gas.

64. Which component of an air conditioner converts gas into liquid?
    a. the condenser
    b. the compressor
    c. the evaporator
    d. the thermostat

65. What is one example for which calculating thermal efficiency is of interest?
    a. A wind turbine
    b. An electric pump
    c. A bicycle
    d. A car engine

66. How is the efficiency of a refrigerator or heat pump expressed?
    a. $Eff = W\sqrt{Q_c}$

    b. $Eff = \frac{W}{Q_c}$
    c. $Eff = Q_c \times W$
    d. $Eff = \frac{Q_c}{W}$

67. How can you mathematically express thermal efficiency in terms of Q_h and Q_c?
    a. $Eff = \frac{Q_h}{Q_h - Q_c}$
    b. $Eff = \frac{Q_h}{Q_c}$
    c. $Eff = \frac{Q_c}{Q_h}$
    d. $Eff = \frac{Q_h - Q_c}{Q_h}$

68. How can you calculate percentage efficiency?
    a. percentage efficiency $= (Eff + 100)\,\%$
    b. percentage efficiency $= \frac{Eff}{100}\,\%$
    c. percentage efficiency $= (Eff - 100)\,\%$
    d. percentage efficiency $= Eff \times 100\%$

Extended Response

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

69. What is the meaning of efficiency in terms of a car engine?
    a. An engine's efficiency equals the sum of useful energy (work) and the input energy.
    b. An engine's efficiency equals the proportion of useful energy (work) to the input energy.
    c. An engine's efficiency equals the product of useful energy (work) and the input energy.
    d. An engine's efficiency equals the difference between the useful energy (work) and the input energy.

12.2 First law of Thermodynamics: Thermal Energy and Work

70. Why does a bridge have expansion joints?
    a. because the bridge expands and contracts with the change in temperature
    b. because the bridge expands and contracts with the change in motion of objects moving on the bridge
    c. because the bridge expands and contracts with the change in total load on the bridge
    d. because the bridge expands and contracts with the change in magnitude of wind blowing

71. Under which conditions will the work done by the gas in a system increase?
    a. It will increase when a large amount of energy is added to the system, and that energy causes an increase in the gas's volume, its pressure, or both.

    b. It will increase when a large amount of energy is extracted from the system, and that energy causes an increase in the gas's volume, its pressure, or both.
    c. It will increase when a large amount of energy is added to the system, and that energy causes a decrease in the gas's volume, its pressure, or both.
    d. It will increase when a large amount of energy is extracted from the system, and that energy causes a decrease in the gas's volume, its pressure, or both.

72. How does energy transfer by heat aid in body metabolism?
    a. The energy is given to the body through the work done by the body (W) and through the intake of food, which may also be considered as the work done on the body. The transfer of energy out of the body is by heat ($-Q$) .
    b. The energy given to the body is by the intake of food, which may also be considered as the work done on the body. The transfer of energy out of the body is by heat ($-Q$) and the work done by the body (W) .
    c. The energy given to the body is by the transfer of energy by heat (Q) into the body, which may also be considered as the work done on the body. The transfer of energy out of the body is the work done by the body (W) .
    d. The energy given to the body is by the transfer of energy by heat (Q) inside the body. The transfer of energy out of the body is by the intake of food and the work done by the body (W) .

73. Two distinct systems have the same amount of stored internal energy. Five hundred joules are added by heat to

the first system, and 300 J are added by heat to the second system. What will be the change in internal energy of the first system if it does 200 J of work? How much work will the second system have to do in order to have the same internal energy?

a.   700 J; 0 J

b.   300 J; 300 J

c.   700 J; 300 J

d.   300 J; 0 J

12.3 Second Law of Thermodynamics: Entropy

74. Why is it not possible to convert all available energy into work?

a.   Due to the entropy of a system, some energy is always unavailable for work.

b.   Due to the entropy of a system, some energy is always available for work.

c.   Due to the decrease in internal energy of a system, some energy is always made unavailable for work.

d.   Due to the increase in internal energy of a system, some energy is always made unavailable for work.

75. Why does entropy increase when ice melts into water?

a.   Melting converts the highly ordered solid structure into a disorderly liquid, thereby increasing entropy.

b.   Melting converts the highly ordered liquid into a disorderly solid structure, thereby increasing entropy.

c.   Melting converts the highly ordered solid structure into a disorderly solid structure, thereby increasing entropy.

d.   Melting converts the highly ordered liquid into a disorderly liquid, thereby increasing entropy.

76. Why is change in entropy lower for higher temperatures?

a.   Increase in the disorder in the substance is low for high temperature.

b.   Increase in the disorder in the substance is high for high temperature.

c.   Decrease in the disorder in the substance is low for high temperature.

d.   Decrease in the disorder in the substance is high for high temperature.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

77. In the equation $W = Q_h - Q_c$, if the value of Q_c were equal to zero, what would it signify?

a.   The efficiency of the engine is 75 percent.

b.   The efficiency of the engine is 25 percent.

c.   The efficiency of the engine is 0 percent.

d.   The efficiency of the engine is 100 percent.

78. Can the value of thermal efficiency be greater than 1? Why or why not?

a.   No, according to the first law of thermodynamics, energy output can never be more than the energy input.

b.   No, according to the second law of thermodynamics, energy output can never be more than the energy input.

c.   Yes, according to the first law of thermodynamics, energy output can be more than the energy input.

d.   Yes, according to the second law of thermodynamics, energy output can be more than the energy input.

79. A coal power station transfers 3.0×10^{12} J by heat from burning coal and transfers 1.5×10^{12} J by heat into the environment. What is the efficiency of the power station?

a.   0.33

b.   0.5

c.   0.66

d.   1

Waves and Their Properties

Figure 13.1 Waves in the ocean behave similarly to all other types of waves. (Steve Jurveston, Flickr)

Chapter Outline

INTRODUCTION   Recall from the chapter on Motion in Two Dimensions that oscillations—the back-and-forth movement between two points—involve force and energy. Some oscillations create waves, such as the sound waves created by plucking a guitar string. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves. You can make water waves in a swimming pool by slapping the water with your hand. Some of these waves, such as water waves, are visible; others, such as sound waves, are not. But every wave is a disturbance that moves from its source and carries energy. In this chapter, we will learn about the different types of waves, their properties, and how they interact with one another.

13.1 Types of Waves

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define mechanical waves and medium, and relate the two
- Distinguish a pulse wave from a periodic wave
- Distinguish a longitudinal wave from a transverse wave and give examples of such waves

Section Key Terms

longitudinal wave	mechanical wave	medium	wave
periodic wave	pulse wave	transverse wave	

Mechanical Waves

What do we mean when we say something is a wave? A **wave** is a disturbance that travels or *propagates* from the place where it was created. Waves transfer energy from one place to another, but they do not necessarily transfer any mass. Light, sound, and waves in the ocean are common examples of waves. Sound and water waves are **mechanical waves**; meaning, they require a medium to travel through. The **medium** may be a solid, a liquid, or a gas, and the speed of the wave depends on the material properties of the medium through which it is traveling. However, light is not a mechanical wave; it can travel through a vacuum such as the empty parts of outer space.

A familiar wave that you can easily imagine is the water wave. For water waves, the disturbance is in the surface of the water, an example of which is the disturbance created by a rock thrown into a pond or by a swimmer splashing the water surface repeatedly. For sound waves, the disturbance is caused by a change in air pressure, an example of which is when the oscillating cone inside a speaker creates a disturbance. For earthquakes, there are several types of disturbances, which include the disturbance of Earth's surface itself and the pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Because water waves are common and visible, visualizing water waves may help you in studying other types of waves, especially those that are not visible.

Water waves have characteristics common to all waves, such as amplitude, period, frequency, and energy, which we will discuss in the next section.

Pulse Waves and Periodic Waves

If you drop a pebble into the water, only a few waves may be generated before the disturbance dies down, whereas in a wave pool, the waves are continuous. A **pulse wave** is a sudden disturbance in which only one wave or a few waves are generated, such as in the example of the pebble. Thunder and explosions also create pulse waves. A **periodic wave** repeats the same oscillation for several cycles, such as in the case of the wave pool, and is associated with simple harmonic motion. Each particle in the medium experiences simple harmonic motion in periodic waves by moving back and forth periodically through the same positions.

Consider the simplified water wave in Figure 13.2. This wave is an up-and-down disturbance of the water surface, characterized by a sine wave pattern. The uppermost position is called the *crest* and the lowest is the *trough*. It causes a seagull to move up and down in simple harmonic motion as the wave crests and troughs pass under the bird.


Figure 13.2 An idealized ocean wave passes under a seagull that bobs up and down in simple harmonic motion.

Longitudinal Waves and Transverse Waves

Mechanical waves are categorized by their type of motion and fall into any of two categories: transverse or longitudinal. Note that both transverse and longitudinal waves can be periodic. A **transverse wave** propagates so that the disturbance is perpendicular to the direction of propagation. An example of a transverse wave is shown in Figure 13.3, where a woman moves a toy spring up and down, generating waves that propagate away from herself in the horizontal direction while disturbing the toy spring in the vertical direction.
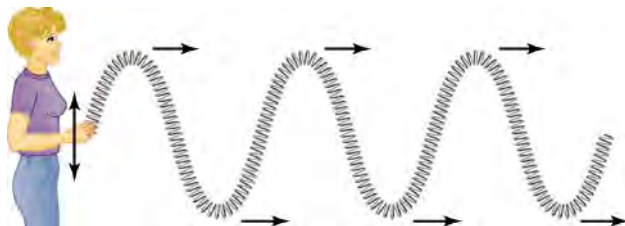
Figure 13.3 In this example of a transverse wave, the wave propagates horizontally and the disturbance in the toy spring is in the vertical direction.

In contrast, in a **longitudinal wave**, the disturbance is parallel to the direction of propagation. Figure 13.4 shows an example of a longitudinal wave, where the woman now creates a disturbance in the horizontal direction—which is the same direction as the wave propagation—by stretching and then compressing the toy spring.
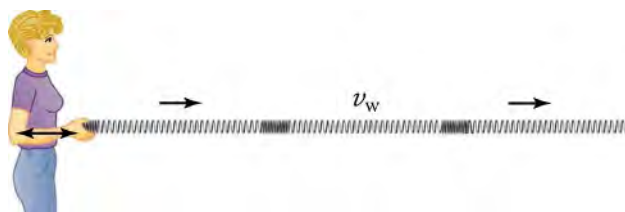

Figure 13.4 In this example of a longitudinal wave, the wave propagates horizontally and the disturbance in the toy spring is also in the horizontal direction.

TIPS FOR SUCCESS
Longitudinal waves are sometimes called *compression waves* or *compressional waves*, and transverse waves are sometimes called *shear waves*.

Waves may be transverse, longitudinal, or *a combination of the two*. The waves on the strings of musical instruments are transverse (as shown in Figure 13.5), and so are electromagnetic waves, such as visible light. Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids.
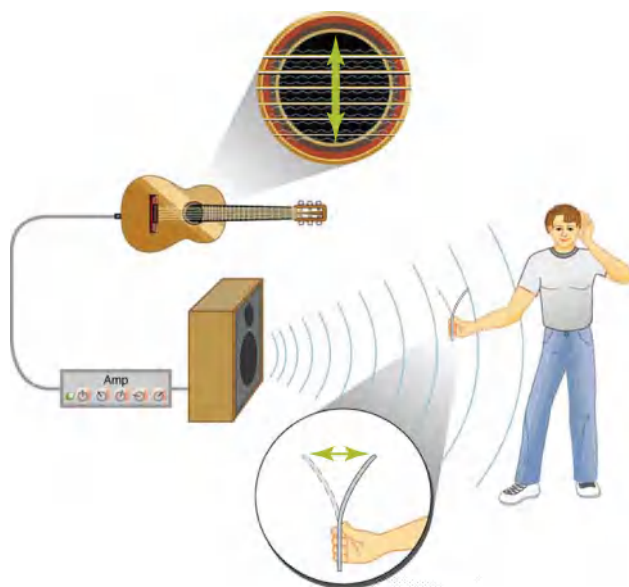
Figure 13.5 The wave on a guitar string is transverse. However, the sound wave coming out of a speaker rattles a sheet of paper in a direction that shows that such sound wave is longitudinal.

Sound in solids can be both longitudinal and transverse. Essentially, water waves are also a combination of transverse and longitudinal components, although the simplified water wave illustrated in Figure 13.2 does not show the longitudinal motion of the bird.

Earthquake waves under Earth's surface have both longitudinal and transverse components as well. The longitudinal waves in an earthquake are called pressure or P-waves, and the transverse waves are called shear or S-waves. These components have important individual characteristics; for example, they propagate at different speeds. Earthquakes also have surface waves that are similar to surface waves on water.

👁 WATCH PHYSICS

Introduction to Waves

This video explains wave propagation in terms of momentum using an example of a wave moving along a rope. It also covers the differences between transverse and longitudinal waves, and between pulse and periodic waves.

Click to view content (https://openstax.org/l/02introtowaves)

> **GRASP CHECK**
> In a longitudinal sound wave, after a compression wave moves through a region, the density of molecules briefly decreases. Why is this?
> a. After a compression wave, some molecules move forward temporarily.
> b. After a compression wave, some molecules move backward temporarily.
> c. After a compression wave, some molecules move upward temporarily.
> d. After a compression wave, some molecules move downward temporarily.

🎢 FUN IN PHYSICS

The Physics of Surfing

Many people enjoy surfing in the ocean. For some surfers, the bigger the wave, the better. In one area off the coast of central California, waves can reach heights of up to 50 feet in certain times of the year (Figure 13.6).