WALS-ROUTLEDGE LESSON STUDY SERIES

PLANNING AND ANALYZING TEACHING

Using the Variation Theory of Learning

Angelika Kullberg, Åke Ingerman and Ference Marton

ROUTLEDGE

‘In this reader-friendly book, the authors show how research can be applied to solve teachers’ most enduring challenge – creating richer learning opportunities for students. By presenting and interpreting a powerful theory of teaching and learning, along with examples from different school subjects, the authors lay out a coherent and compelling set of principles for planning, implementing, and improving instruction. This is a must read for teachers (and researchers) who want thoughtful, research-grounded, useable suggestions for improving their craft.’

James Hiebert, Robert J. Barkley *Professor Emeritus, University of Delaware, Newark DE, USA*

‘Many competent and dedicated teachers have likely pondered why some students fail to learn despite their meticulous planning and conscientious teaching. This book aims to shed light on this issue and propose ways to move forward. It explains Variation theory in accessible language and provides numerous practical examples from classroom research across various subject areas. The book serves as a bridge between Variation theory and Learning study, and classroom practice.’

Lo Mun Ling, *retired professor from Hong Kong University of Education and the founding president of WALS*

PLANNING AND ANALYZING TEACHING

Using illustrative teaching case studies, this book demonstrates how teaching informed by a learning theory, specifically Variation theory, can equip teachers to facilitate possibilities for students' learning in effective and powerful ways.

For a long period of time teaching has been "black-boxed", in favor of other explanations of why students learn or not, such as motivation and social interaction. A large amount of research on teaching and learning, not the least made using Variation theory, has shown that students often need to experience the same aspects of the focused content or capability in order to learn, indicating that relationships between teaching and learning are not unique or even qualitatively different for every individual and every situation. This perspective on the relationship between teaching and learning emphasizes content-specific aspects and in that sense structural components of teaching, while other aspects of schooling such as social interaction and general well-being recede into the background. The authors argue for the importance of this in the direct development of teachers' independent collective professional knowledge about teaching, and the leverage this gives for developing student learning. They introduce theoretical tools to help teachers to increase the probability that teaching focusing a specific content or capability is predictive of students learning of that specific content or capability, while decreasing contextual dependency without assuming that teaching and learning have a one-to-one relationship.

Intended for teachers, graduate students in education, teacher educators, student teachers, and researchers, this book shows that while there is no simple equation between teaching and learning, there are general, though content specific, aspects of teaching that can be systematically planned and analyzed and used to improve the quality of student learning.

Angelika Kullberg is a Professor in mathematics education at the University of Gothenburg. She has 20 years of experience in using the Variation theory of learning and conducting collaborative research together with teachers. She has been a council member of the World Association of Lesson Studies for many years and is since 2022 vice president.

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WALS-Routledge Lesson Study Series

Series editors: Christine Kim-Eng Lee, Catherine Lewis, Kiyomi Akita and Keith Wood

This series aims to provide opportunities for researchers and practitioners in Lesson Study to share their work beyond the boundaries of their countries to an international audience. Lesson Study is increasingly popular as a tool for improving the quality of education and schools around the world. As many countries are adapting and contextualizing Japanese Lesson Study to their own needs in response to educational and curriculum reforms cognizant that what matters most is what happens in classrooms and its impact on teachers and students. As Lesson Study originates from Japan, there is also a need for English Language readers around the world to understand more deeply the underlying philosophies, policies, and practices of Japanese Lesson Study in the cultural contexts of their schools and classrooms. As well as original works in English from leading figures in Lesson Study, this series will also make available outstanding Lesson Study publications originally written in Japanese but extended and revised for an English audience.

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Using the Variation Theory of Learning
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PLANNING AND ANALYZING TEACHING

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FOREWORD

This book, *Planning and Analyzing Teaching*, is a seminal piece written especially for teachers to help them understand Variation theory and Learning study as a theoretical lens they can use to plan and analyze their teaching in powerful ways to enhance student learning. Learning study draws inspiration from Japanese Lesson Study's iterative cycles of critical inquiry, collaborative planning, observations of research lessons, reflections, and improvements but adds to this process a way for teachers to help students discern what is taught by focusing on the critical aspects of the objects of learning in lessons. Teachers are often faced with a recurrent problem in their practice of why some of their students are not able to grasp what they are teaching or did not learn what they intended. Identifying the critical aspects of the objects of learning and using Variation theory as a tool for making them visible through their teaching is not an easy task for teachers. This book recognizes this challenge for teachers and has explained Variation theory in ways that are accessible to teachers, clarifying what an object of learning is and how central it is for teachers to discern it carefully as they planned their teaching toward necessary conditions for learning from the learner's perspective and making that as a starting point. This is where the shift is needed for teachers as often they view lessons from a teacher's perspective and reduce an object of learning to specific content-related educational objectives. Using Variation theory also helps teachers to analyze the intended object of learning (what is planned) and compare it with the enacted object of learning (what was made possible to learn) and lived object of learning (what the students have experienced and learned). Such analyses in examining these relationships are powerful in helping teachers make improvements in their practice and close gaps to improve student learning. This book made clearer how variation is the key for learning as it makes critical aspects of an object of learning visible to learners and how this is different from the induction process of learning.

But how can this theory be understood by teachers? This book is replete with examples from pre-school to college from various subjects (mathematics, history, Swedish language, Cantonese language, English grammar and reading comprehension, chemistry, physical education, natural science, technology, economics) to help teachers concretize their understandings and bring these ideas into experimentation in their classrooms. It is also evident in the examples provided how important it is for teachers to collaborate as a community of inquirers to explore the meanings of the critical aspects of the object(s) of learning during the planning process, observe carefully, and gain further insights of critical aspects (if any) in the enacted object(s) and the lived object(s) of learning in their analysis. This plan-do-see-share cycle is integral to lesson study protocols. It is in working together that we learn together. It is also important to make public what teachers have learned from a Learning study so that the results of a Learning study can be further explored, tested, and refined by other teachers working on the same topic.

We want to congratulate the authors of this book for rising to the challenge of making Variation theory accessible to teachers as well as to researchers who are unfamiliar with Variation theory and Learning study. The ideas and examples shared in this book that included fine-grained analyses are powerful. We want to encourage readers to seriously consider these ideas and experiment with them in their practice. The WALS-Routledge Lesson Study Series is pleased to support the publication of this book which will prove to be a primer for those interested in implementing Learning Studies around the world. The WALS Lesson Study series provides an opportunity to showcase the excellent work of educators and researchers who will employ Lesson Studies to improve teacher and student learning and increase not just the intellectual capital but also social-emotional and cultural capital in schools, colleges, and universities.

Christine Kim-Eng Lee
National Institute of Education, Nanyang Technological University,
Singapore WALS-Routledge Editorial Team

PREFACE

The authors of this important book have played a seminal role in developing a theory-informed approach to a form of classroom-based teacher research, which originated in Japan and subsequently became globally characterized as *Lesson Study*. To effect significant pedagogical change in classrooms, this form of teacher research has emphasized the importance of *iterative cycles* of action and research conducted *collaboratively* by professional groups of teachers. In these respects, the Lesson Study method has challenged the *culture of individualism*, which has traditionally shaped the practice of teaching in western societies and much of which now passes as teacher action research. However, as Stigler and Hiebert claim in "The Teaching Gap" (2016, p. 5), "much of the theory behind Lesson Study in Japan is implicit and also bound up with wider beliefs about teaching and learning" in Japan.

As Lesson Study has globalized, there has been a growing awareness of a need for the practice to be informed by an explicit theoretical framework, which explicitly links Lesson Study with the task of improving the quality of teaching and learning in classrooms. In the context of large-scale post-colonial curriculum reforms in Hong Kong, lesson study emerged as a form of teacher action research that was informed by a theoretical framework developed from phenomenographic research in Sweden (see Marton & Booth, 1997). This research focused on the ways learners experienced the lesson content they were taught in classrooms and generated a form of grounded learning theory that became known as *Variation theory*. A group of professional researchers in Hong Kong led by Lo Mun Ling felt that contemporary Lesson Study in Japan tended to focus more on teaching methods and less on the experience of learners in classroom settings. In collaboration with visiting researchers from Sweden led by Ference Marton, a co-author of this book, a hybrid version of Lesson Study was created by blending it with principles

of lesson planning and analysis derived from Variation theory. It became known as *Learning study* in both Hong Kong and Sweden. Increasingly examples of such hybrid lesson studies can be found in many countries. There is a growing need for teachers globally to have access to a well-organized and readable account of Variation theory as a general cross-curricula resource they can use when planning and analyzing lessons together. *Planning and Analyzing Teaching – Using the Variation theory of Learning* stands out as meeting such a need. Its authors have played a major role in helping teachers to use Variation theory as a resource for structuring their work in classrooms in ways that deepen their learner's understanding of the objects of learning that make up a curriculum. The book is peppered with examples of learning studies selected to illustrate and indeed test the usefulness of particular concepts and principles in planning and analyzing "lessons".

This book is particularly significant in providing teachers with a vision of how they can take responsibility for improving the quality of their work through researching and theorizing their practice in classrooms. In this respect, it tacitly highlights Lawrence Stenhouse's idea of *the teacher as researcher* as a work that is still in progress and indeed constitutes a fitting sequel to his own seminal 1975 text, entitled "An Introduction to Curriculum Research and Development" (Heinemann Educational).

Stenhouse developed a concept of teacher research in the context of the school-based curriculum reform movement in the UK over 60 years ago. This concept also embraced an *iterative and collaborative* model of the research process. Stenhouse also felt a need to develop an explicit pedagogical theory as a necessary condition of realizing a curriculum and pedagogical planning process, which possessed these qualities in a context that was increasingly dominated by the *objectives model* of planning. For Stenhouse practical theorizing involved teachers in a process of systematically structuring their understanding of their work in a form that enabled them to discuss their work in classrooms with each other. Thus, he argued, "a common vocabulary of concepts and a syntax of theory need to be developed" (p. 157). Such a general theoretical framework, he believed, would need to both capture and express teachers' understanding of their work in classrooms. It would therefore consist of "concepts which are carefully related to each other". Variation theory, as depicted in Kullberg, Ingerman, and Marton's book (see Chapter 3), coherently sets out the kind of conceptual vocabulary and syntax of theory that Stenhouse called for as support for teacher researchers. He emphasized that the development of such a theoretical syntax would not be based on any need to generalize beyond life in classrooms as teachers and learners experienced it, but rather help them better understand that experience. The framework set out in this book has precisely this function. Moreover, Stenhouse claimed that the appropriateness and utility of any theoretical syntax should be tested in use and provisional. Professional research workers may play a major role in its development, but they would need to collaborate with teacher researchers in testing its appropriateness and utility. Teachers will also play a generative role in the development of a usable theoretical syntax.

Again, this book exemplifies such a collaboration between professional researchers and teacher-researchers engaged in Learning Studies.

Stenhouse's argument for the development of a theoretical syntax to structure teachers' research was based on his claim that the growing influence of an *objectives model* of curriculum planning would distort the *nature of understanding* as an aim of education and pedagogical goal. He proposed an alternative *process model* of curriculum design, which conceptualized the relationship between *understanding* as a pedagogical aim and the teaching-learning process in non-instrumental *ethical terms*. He pointed out what he believed to be two important implications of this pedagogical aim:

> *First, it is implied that that both teachers and students develop understanding, that is, the teacher is cast in the role of a learner. Second, understanding is chosen as an aim because it cannot be achieved. Understanding can always be deepened. Moreover, there must always be dispute as to what constitutes a valid understanding.*
>
> *(p. 94)*

A process model involved analyzing such an aim into *principles of procedure* governing the way teachers handled curriculum content in an educationally worthwhile manner with learners in their classrooms. Such principles furnished quality criteria for teacher research in classrooms.

Variation theory, as set out in this book, can be viewed as a framework of concepts and principles that map out a process of "teaching for understanding". Its view of "the nature of understanding" has much in common with Stenhouse's concept. It is clearly underpinned by a *process model* as opposed to an *objectives model* of educational planning and research. The publication of this book will provide teachers in many countries with an opportunity to realize "a higher degree of personal and professional development" as teacher researchers than the *technical operative model* currently on offer in many educational systems.

John Elliott
Professor Emeritus, University of East Anglia

1

TOWARD POWERFUL LEARNING OPPORTUNITIES

The traditional Japanese lesson study became widely known and practiced when James Stigler and James Hiebert published the international bestseller *The Teaching Gap* in 1999. In a lesson study, teachers with similar responsibility and competence plan a lesson of their choice together. One of them teaches the lesson, usually to her own class, while her colleagues follow his/her teaching and make notes. After the lesson, the involved teachers discuss what they have seen and heard. The lesson plan is modified in accordance with what the teachers have learned from the first attempt. Then another teacher teaches the modified lesson to her own class, eagerly observed by his/her colleagues. Another round of discussion follows, this time concerning the second lesson. The lesson study is finished when the teachers have documented both the lessons and their conclusions.

The idea of the lesson study made a strong impression on our research group. One of us was in Hong Kong when Stigler and Hiebert's book was published. As a big curriculum reform was taking place at the time, there was a lively interest in the lesson study there too. All of a sudden, there was a research group in Hong Kong, in addition to the one in Sweden. We quickly doubled the size of the research group and joined together in a most spectacular research project with the title "Catering for individual differences through learning study". The project was initiated and carried out in Hong Kong, but similar research activities quickly followed in Sweden. In 2006, the World Association of Lesson Studies (WALS) was established in Hong Kong. As mentioned earlier, learning study was inspired by lesson study. The two are very similar, but there are also important differences. First, learning study builds on a theoretical ground, the variation theory of learning. Second, it is not coupled to the lesson as a unit, since it may focus on a lesson, a part of a lesson, or a sequence of lessons. This book has its roots in the lesson study and learning study movement.

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In this spirit, this book is for teachers and researchers who want to find powerful ways of helping students learn. A central thesis in this book is that answers to such pedagogical ambitions are to a large degree dependent on what content the teaching and learning concern. Therefore, the book focuses on how the content is handled by the teacher and students in a lesson and how that affects what is made possible for students to learn.

Teaching and learning are, however, often discussed on a general level, for example, whether digital learning or group work benefits students' learning or how established norms in the classroom affect students' behavior. We argue that regardless of which teaching arrangement is used or what norms are established in the classroom, attention needs to be paid on a more detailed level to how the teacher can make learning possible. As Jerome Bruner wrote in the late 1990s, "[it] is somewhat surprising and discouraging how little attention has been paid to the intimate nature of teaching and school learning in the debates on education that have raged over the past decade. These debates have been so focused on performance and standards that they have mostly overlooked the means by which teachers and pupils alike go about their business in real-life classrooms – how teachers teach and how pupils learn" (Bruner, 1996, p. 86).

These general trends share the trait that they point to generic structures, approaches, or theories as the way forward. We agree with Bruner that much more attention must be paid to the contextual nature of teaching and learning without losing sight of structural factors or disregarding the knowledge of teachers. Since the beginning of the 2000s, lesson study and the emphasis on teachers' collaborative work have been ways to move toward focusing on observations of teaching and learning in the classroom. We think, however, that more attention needs to be paid to the content taught and learned on a finer-grained level. This relates to the fact that the dilemmas teachers are faced with in the classroom regarding students' learning, and how this may be facilitated by teaching, fundamentally reflect the nature and structure of the specific knowledge that is at stake. More than 20 years of research has shown us how students' learning opportunities for a specific content can be planned and analyzed on a level that is useful in teaching. We believe that teaching as a profession would be more strongly equipped with the key insights from this research.

The relationship between teaching and learning is sometimes regarded as a "black box" since it is difficult to specify in detail how teaching affects students' learning. Most teachers have experienced times when the students did not learn what they intended, even if they have taught a well-structured lesson. Why did the students not learn or learn something other than what the teacher had in mind? There may be various explanations for why the students did not learn what was intended by the teacher. For example, the students' pre-knowledge was not sufficient, or maybe the students did not pay enough attention during the lesson. These could be valid explanations but may also not tell the whole story. How can students' learning be enhanced? This book gives leverage to teachers and researchers who aim to enhance student learning.

We intend that this book will provide answers to such principal questions about teaching and learning that are directly useful in the classroom. The book aims to show how teachers can develop practice and enhance student learning by planning and analyzing teaching using a learning theory called the variation theory of learning. The theory was developed in the late 1990s based on 30 years of research within the phenomenographic research tradition (Marton, 1981; Marton & Booth, 1997), which explores different ways that students understand the same phenomenon (e.g., numbers, price, force, critical thinking).

How can the theory described in this book be beneficial for understanding relationships between teaching and learning? Why is a theory needed to plan and analyze teaching? Can a theory help to enhance student learning? The variation theory of learning helps us to better understand the relationships between teaching and learning by suggesting a mechanism of learning. It can be used as a tool for planning and analyzing teaching. The theory directs attention toward what is taught and learned, how students experience what is to be learned, and how the content taught can be systematically enacted in lessons. Large number of research studies have found the theory useful for teachers in planning and analyzing teaching (Cheng & Lo, 2013). This book covers many school subjects and provides examples of teaching from preschool to upper secondary school. In this chapter, two questions that are important points of departure in this book are posed: How do learners experience what is taught? and What is made possible to learn in an act of teaching?

How do learners experience what is taught?

Understanding how students experience (perceive, make sense of, understand) what is taught is not always an easy task, but is decisive for teaching and student learning. In the following episode, a preschool teacher and three 5-year-old children are sitting around a table talking about story problems in mathematics[1]. The intention is that the children should learn to solve subtraction tasks with five, for example, 5 – 4=, using finger patterns to model the task. One of the children, Mike, gives voice to an understanding of the meaning of numbers in a way that is quite different from the other children in the group. When the teacher shows one finger on the right hand, Mike answers that it is ten.

When you read the following excerpt, think about what you would do if you were the teacher. You may think this child is too young or is not mature enough to deal with these subtraction tasks, that he is only guessing, and if the teacher tries next year, he will answer differently. Maybe you have thoughts about whether the teacher should continue asking Mike when he does not know the answer. What would you do? What do you think needs to be taught and learned for Mike to develop his understanding of numbers?

Teacher: Four bears ran away. How many are left?
Mike: [*Folds two fingers first and then unfolds them and shows five fingers.*] Five

Teacher: You have five bears, four ran away, how many are left? [*The teacher shows five fingers and then folds four fingers. She holds up one finger.*]
Mike: Then it is…
Teacher: Look at my hand, and I will help you. You have five bears, four ran away, how many are left? [*The teacher folds four fingers and shows the little finger*.]
Mike: [*Shows his little finger*.] Ten
Teacher: How many was it?
Amy: Ten means …
Mike: Ten
Amy: No
Teacher: But there weren't ten from the beginning?
Mike: Okay
Teacher: It was five bears.
Mike: Okay then, five bears, four went away.
Teacher: Now four bears went away, how many bears were left?
Mike: Then it is. I don't know.
Teacher: [*Shows her little finger*.] How many is this?
Mike: Ten
Teacher: This. [*Holds up her little finger*.]
Mike: Ten
Teacher: How many fingers am I showing?
Mike: Ten
Teacher: One
Mike: Ten
Teacher: It is only one finger.
Mike: Ten
Teacher: Now it is ten. [*The teacher shows ten fingers.*]
Mike: Look! [*The child counts and points at the teacher's fingers.*] One, two, three, four, five, six, seven, eight, nine, ten. There! (…)
Teacher: Do you mean this one's name is ten?
Amy: No
Mike: Yes
Teacher: Is this what you mean? [*Points to each finger starting from her left-hand side*.] One, two, three, four, five, six, seven, eight, nine, ten?

We believe learners give rational answers and that these answers can be explained if we understand how they perceive the phenomenon being taught, such as numbers and subtraction. At the end, the teacher in the excerpt seems to pinpoint how Mike understands numbers. For Mike, numbers do not represent quantity (how many). He may know that five is one hand (five fingers), but this could be just an image of five. His answer "ten" illustrates that he connects the little finger to the tenth finger. Hence, he has discerned the correct order of the numbers one

to ten (ordinality), but he does not yet connect the number ten to all fingers on two hands. The example illustrates the importance of being sensitive to how learners understand what is taught in order to be able to adapt the teaching so that it is possible for the learners to learn what is intended. In this case, Mike needs to learn the cardinality of numbers (e.g., that ten represents all ten objects) before starting to do addition and subtraction with numbers. What does the teacher do to bring out this aspect (cardinality) in order for Mike to have the opportunity to experience it? After the situation described in the excerpt, the teacher varies the finger she holds up to represent one, saying, "This is one [holds up index finger], this is one [holds up middle finger], this is one [holds up ring finger]". In this way, Mike may experience that one is not the tenth finger; one can be any one finger. When the teacher contrasts one and ten fingers, and later how one could be represented differently by different fingers, we would argue that it was made possible for Mike to discern that ten is not the tenth finger (one finger). The teacher in the excerpt appears to pinpoint how Mike seems to understand numbers and she could respond in teaching to his way of experiencing numbers by showing the difference between one and ten fingers.

It is, however, not always the case that the teacher understands in the moment how the learner understands what is to be learned. Sometimes it is not until after the lesson that the teachers realize what a particular answer entails, or it could be the case that the teacher does not understand even if she tries to. It is not easy to understand what the learners have and have not yet discerned. We can approach this by taking their answers seriously and reflecting on the reasoning that might be behind a particular answer. Teachers of all grades and subjects can make learning possible by identifying which aspects of the particular object of learning need to be discerned by the learners and making these aspects visible in teaching. We call these aspects *critical aspects* of the object of learning.

What is made possible to learn in an act of teaching?

The starting point in this book is that learning, in general, is constituted by learning about specific things, rather than the other way around. Teaching is central in making student learning possible, even though learning cannot be taken for granted. This book advocates the following two necessary components in finding and facilitating more powerful possibilities for learning: First, to be clear what the intention is regarding what the teaching should make it possible for the students to learn (the object of learning), and to plan for realizing this, for example, on the basis of knowledge of how learners may perceive what is to be taught. Second, to observe, evaluate, and analyze what is made possible to learn about the intended object of learning during a specific lesson.

Common teaching practice may not include identifying what was made possible to learn in a lesson and comparing that to what was specifically intended that the students should learn, beyond the impression the teacher has from the act

and event of teaching as such and from assessing the students' knowledge at some point. It takes effort and courage to look more closely into what may be learned in a lesson, and even more so to compare a specific lesson or act of teaching to other acts of teaching sharing the same intentions. However, it is often fruitful for being systematically innovative and finding ways of altering teaching that are productive for students' learning. The result is often that there are clear differences in what we as teachers think happened and what actually happened in the lesson, and it is precisely this difference that is productive. This is also true when the teacher has high ambitions and considerable competence as a teacher.

The following examples will illustrate what an object of learning may encompass, different ways of realizing it, what may be learned from observing and analyzing what is enacted in the classroom, and possible conclusions on how future lessons focusing on the same object of learning might be changed in order to further improve the students' possible learning. The examples are drawn from a teacher-researcher collaborative study in the context of history teaching in Swedish upper secondary school (Lilliestam, 2013).

The history curriculum in Sweden mainly focuses on sequences of events and context-specific concepts in relation to such sequences. The overall aim of the teaching theme was to broaden this traditional focus and develop students' ability to take into account explanatory factors in historical development using meta-concepts in the form of explanatory factors, such as, in particular, *agent* and *structure*, to support a focus on causes behind events and to be less tied to the sequence of events as such. This is challenging for teachers to teach as well as for students to learn. The concepts of agent and structure should be understood in this context as interdependent, but clearly concepts that express the perspective that humans make their own history. Human agents are social and societal beings acting and shaping their world, given the limits of structures that are a part of the physical world as well as the social world, objective as well as subjective, and that are the residual effects of previous events and acts of historical agents. In the context of teaching in this study, the agents that are considered are mainly historical agents (people) who have had impact on the structures of their time and are part of a discussion that sheds light on the unfolding of relevant historical events.

The two examples below are drawn from a series of lessons enacted in two different classrooms by two different teachers. This first lesson focuses on two objects of learning: That students should get to know about the course of events around Napoleon's life, rise to power, and fall, and that students should *develop their ability to discuss the interplay between agents and structure in the context of the French Revolution and Napoleon*. The first object of learning is well in line with the traditional curriculum while the second targets the more abstract meta-concepts. In the discussions before the lesson in the researcher-teacher group, emphasis was on the importance of the second object of learning. However, it turns

out that the lesson was dominated by the events of Napoleon's life, as the following extract illustrates:

Teacher: They usually call it the scorched earth tactic […]. When Napoleon comes, the Russians… Napoleon travels easily, they don't have much provisions with them, but they think they are eating along the way somehow. And the Russians' countermovement is to withdraw. […] Sometimes you have battles, but the Russians retreat and retreat. Napoleon had 600,000 soldiers with him. When Napoleon leaves, Russia has 100,000 soldiers left. […] By retreating and burning food that could be eaten, of course there wasn't much to eat.

In other parts of the lesson, it is mentioned that Napoleon was short and came from a poor background, was young and charismatic, and was a skilled field commander and tactician, thus placing an emphasis on Napoleon as a person. Some factors were also pointed out that made Napoleon's rise to power possible, such as allowing officers who were not from a noble background as well as the fact that promotions were frequent in wartime. During the lesson, the teacher also makes comparisons to similar situations in other eras, for example, with other powerful individuals such as Caesar. All in all, what is brought to the fore in the lesson is Napoleon's life and career. He is portrayed as the hub of events, and his personal actions are decisive for the course of events and for societal changes. In contrast, for example, the Russians are discussed as a collective of a country, without any individuals at all being mentioned. Napoleon is thus implicitly portrayed as a strong individual agent, but structures in relation to the actions of Napoleon and causal relationships are not pointed out, even though they are present in the background. Thus, what is primarily possible to learn concerns the narrative of Napoleon's life, which is also in line with what students might expect from previous history teaching. It would be possible for students to discern the explanatory concepts of agent and structure, and the shape they take in the presented events, but that would require the students to extrapolate from other lessons where this might have been more prominent.

The next example is taken from a different classroom taught by another teacher. This lesson shares the central aim of the lesson in the previous example, trying to develop the students' understanding of agent and structure, but in a different and more complex historical context. More precisely, the lesson focused on two objects of learning: The ideas of the enlightenment and that *ideas may be understood as structural factors*. Beforehand, the teacher thinks that the enlightenment era is particularly well suited to focusing on ideas since ideas clearly have a role in societal development, in contrast to earlier eras (e.g., the 14th century when natural determinants such as climate are central as explanatory factors for development). The teacher also thinks that during this historical period, many new agents (e.g., philosophers and women) emerged when individuals got a larger degree of

freedom. However, enacted in the classroom, agent and structure were even more in the background than in the previous lesson example.

The teacher starts the lesson by asking about the ideas of Montesquieu, Voltaire, and Rousseau on how a society should be governed, in contrast to ideas covered earlier.

Teacher: Regarding political structures, in the last lesson we talked about three [philosophers]: Montesquieu, Voltaire, and Rousseau. How do they want society to be governed differently to how it has been governed before?

When no student answers the question, the teacher continues by bringing up one of the philosophers instead.

Teacher: What does Montesquieu propose?
Student 1: Wasn't that about power sharing?
Teacher: Yes.
Student 1: Government, parliament, and courts.
Teacher: So now I am elaborating on your homework. He proposes a separation of power where there is a government, a king, who exercises power. […] A parliament that controls the power, and then there are courts that check whether the parliament makes laws that are valid and consistent with the constitution.

The teacher elaborates on Student 1's answer "government, parliament, and courts" by showing the function of and connection between the government, parliament, and courts. He continues the discussion by introducing an aspect of democracy.

Teacher: Montesquieu is not a fully-fledged democrat, but he wants it expanded. He wants the educated bourgeoisie to participate and vote. Before that there is no democracy in Europe. But now suggestions for this emerge.

Teacher then moves on to Voltaire.

Teacher: Voltaire that you have read about earlier, what does he think? [about how society should be governed]
Student 2: Everyone should be allowed to have an opinion.
Teacher: He thinks there should be freedom of opinion.
[*Student 2 voices one of Voltaire's central ideas, freedom of opinion. The teacher asks again about forms of governance.*]
Teacher: But how does he think a society should be governed? Who will decide and why?

In the excerpt, the students do not respond to the teacher's question and remain silent. The teacher tells the students that this was the students' homework to know. In the end, the teacher gives the answer himself and then develops the idea of enlightened despotism, evaluating Voltaire in terms of democracy, elaborating on Rosseau's view on how society should be governed, and his critique of civilization.

In this lesson, the historical content knowledge is in focus, similar to the previous example, here giving examples of the ideas of the enlightenment philosophers (which had also been covered in previous lessons). The teacher intended that the students should get the opportunity to develop their understanding of ideas as a structural factor and to meet new kinds of agents, such as women. However, the concepts of structure (here ideas) and agent (here philosophers) are only present in the background, while the specific philosophers are brought to the fore relatively independently. The students must themselves infer that change in ideas can be seen as a structural factor and that philosophers are agents who can initiate changes in the pattern of ideas. The object of learning as it is enacted is thus on the level of specific philosophers and, in relation to ideas, on the level of explanatory factors.

When the teaching about the theme was finished, a comprehensive test was carried out (the same in both classrooms featuring in the examples), where one of the questions directly concerned using agent-structure in discussing events in the context of Napoleon. Both classrooms had teaching about Napoleon, as well as the concepts of agent-structure. In the classroom from the second example, only a few students were able to make use of agent-structure in their discussion, and then usually with one type of factor tied to either an agent or a structure, for example, pointing to the good quality of the French army at this time. In the case of the classroom from the first lesson example, there was a larger percentage of the students who were able to make use of agent and structure in their discussion, and in some cases, the students' discussion was more complex, implicitly or even explicitly relating different factors to each other, for example, relating the conditions after the French revolution, when those born on French soil could take up any occupation, to Napoleon's capacity for taking risks and acting in smart ways.

It is not possible to draw definite conclusions from the student test regarding what the students learned in the particular lesson described, but there are clear indications that the students mainly learned about the object of learning that was the focus of the lesson (Napoleon's life and events surrounding this), but that it was also possible to learn something about agent and structure. Discerning agent and structure seem to have been more difficult when meeting these concepts in the more abstract form of ideas and philosophers. As stated above, the aim of the teacher-researcher collaborative study was to develop the students' ability to use the concepts of agent and structure as explanatory concepts when discussing historical events. Having investigated what it was possible to learn in the lessons as described above, the key question to ask at this point is how to find (more) powerful ways to facilitate students' learning, in this case about agent and structure, and to go even further than what was done in this study.

From the two lessons described, it may, for example, be helpful to make the use of agent and structure more explicitly in the lessons. For instance, what is, and what is not, an agent can be pointed out. Moreover, a variation of different agents and then of different structures in the events may be shown, and afterward explicitly explaining how the development of an event can be described as an interplay of agents and structures using several factors simultaneously. In such discussion, it is key to make use of the terms agent and structure. This may be described by saying that the intended learning – which we call the object of learning – must be clearly articulated and thoroughly understood by the teacher and that the teaching should offer explicit opportunities to precisely discern the important parts and meaningful whole of this object of learning.

As we have illustrated above, the concepts of agent and structure as an object of learning are challenging for teachers to teach as well as for students to learn. There are neither established ways of teaching nor in a more detailed way fully clear what understanding agent and structure entails. In this way, it constitutes an example of a dilemma or unresolved issue for pedagogical practice. Such dilemmas, where a resolution is not straightforward, are important examples that the approach we present in this book can offer productive ways to resolve. That is, we suggest that attending to planning and analysis of teaching provides both a pragmatic and systematic way forward. This example thus serves the purpose of giving body to the dilemma and illustrating some steps on the path to a successful resolution.

In the next chapter, we will look more closely at the object of learning and how this concept can be used for planning and analyzing teaching.

Note

1 FASETT project (see e.g., Björklund et al., 2021).

2

THE OBJECT OF LEARNING

In this chapter, we describe foundational ideas about learning that this book builds on. We are using the verb "to learn" to refer to changes from not being able to do something to being able to do it. Pedagogy is about making such changes possible. Learning is always the learning of something and the point of departure for our pedagogy of learning is the question "What is to be learned?". The generic answer to this question is "the object of learning". The object of learning refers to the capability to be learnt and the aspects that the learner needs to discern in order to master it. The objects of learning we take an interest in have a clear scope, are distinct from other objects of learning and are possible to articulate (and typically, to warrant teachers' and researchers' attention, they should present somewhat of a challenge to most learners).

Objects of learning may range from the ability to flexibly see wholes and parts of natural numbers up to ten and the addition and subtraction of negative numbers, across experiencing three dimensions in two-dimensional representations of molecules and their stereochemical properties, to dealing with price in everyday situations as a function of supply and demand, or reasoning about historical events in terms of important agents and structural factors.

The object of learning reflects meaning

The object of learning has a central role in the tradition in which this book is anchored. Taking the object of learning as the focal point of learning and teaching implies that other components of the educational context considered are not put into focus and require that certain outcomes are more valued than others. This implies that the object of learning cannot be reduced to having an instrumental role or the function of a content label, but is the tip of an iceberg of professional and scientific

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knowledge. As such, the object of learning is a theoretical notion with considerable depth. This chapter will briefly introduce some of the underlying reasoning. However, to appreciate the full depth of this underpinning, a broader reading of some of our previous publications is recommended (see, in particular, Marton, 2015; Marton & Booth, 1997; Marton & Tsui, 2004). It is our ambition that a detailed understanding of the theoretical aspects should not be necessary to appreciate the concepts and the approach presented in the following chapters, but rather that they are accessible and applicable for immediate classroom experimentation.

Our theoretical starting point in this book is that we as humans experience the world (the physical world, our shared human world and society, and our individual human world) as constituted in terms of meaningful entities – and act in ways reflecting our experience. We assume that our acts, in particular in educational contexts, are intertwined with how we experience or understand the situation and phenomena in it, even though the relationships between understanding and acts may be non-trivial, bidirectional, and distributed in time. In each given situation, some things are more central to one's experience, while others are in the margin (or in Gestalt psychology terms, some things constitute the "figure" while others constitute the "ground"). Such figural things we call phenomena (in the phenomenological sense). This implies that we may be aware of many things at the same time, but not in the same way at all times. For example, you may be aware of the music playing in the room next door, while focusing on making meaning of the argument in this text.

These theoretical notions also reflect us as embodied beings and connect to the affordances and restraints of bodies, senses, and memory. Drawing on a phenomenological perspective, this connects to the idea that we as actors (whether we are students, teachers, or researchers) are experiencing the world and events in that world from our bodily perspectives, with the possibilities that our ability to move, our biological capacity to see and perceive patterns, as well as the structure of our memory (in terms of both our general capacity to form memories and individual currently held memories). It also implies that we are not alone in understanding our world, but share the inclination to make meaning with our fellow humans, and that we, in different ways, support each other in that endeavor.

This may be illustrated with a landscape metaphor. When in nature, we orient ourselves by identifying distinctive elements of the landscape (a hill, a forest, etc.) that convey meaning in relation to finding our way or finding out where we are, while other things are marginal, for example, individual trees, stones, and the weather. A shift in the weather conditions or a short walk to another position may shift our perspective and bring other parts into our awareness, while others shift to the margin.

Similarly to how one orients yourself in a landscape, we focus on learning that is operating on this intermediate level of perception. What is to be learned is then a question answered at the same level (not at the level of details – trees – nor on the level of different landscapes). Learning has consequences in terms of a shift

in meaning and thus in how we perceive the world or a part of it (a phenomenon). Learning also has a direction and newness, primarily relating to coming to recognize something not previously recognized. In every new situation, we may of course re-establish a way of recognizing and extending the situations in which we recognize the same figure (different weather, slight shift in position, etc.). However, this is not learning in the fundamental way that we primarily refer to in this book but a part of everyday meaning-making.

The level of orienting in a landscape, recognizing different kinds of figures, perhaps relating to a map, 3D-shifting several figures in relation to each other, etc., is akin to learning on a domain (subject or knowledge area) level, with many individual parts. Learning about a specific object entails some of the larger domain but primarily concerns the individual part. When a number of different parts are known, the larger orientation may to some extent be the object of learning.

Other similar figures may be part of our experience and contribute to the perceived meaning. Other landscapes with similar or strikingly different landmarks previously encountered may be important, and thus it is not only the immediate perceptual experience that forms part of how we perceive meaning and contributes toward our perceptions of a meaningful phenomenon.

It may be very helpful if someone, for example, a guide, points out a distinctive figure in the landscape, and helps us to discern new aspects of the figure that allow us to distinguish it from other parts of the landscape and/or recognize it from new positions (closer, further away, different angles). When our perception is thus directed towards recognizing a particular meaning of this figure, it has been directed towards an object of learning corresponding to a (powerful way of seeing a) meaningful phenomenon. For example, recognizing a distinctive hill crest in the landscape may be very helpful in orienting us when taking our evening stroll.

The guide pointing out the landscape here takes the role of the teacher, having identified (on his/her own or, for example, in line with conventions and local knowledge) that it is important to recognize a specific figure in a particular way. The learner perceives the landscape (and the figure embedded in it) in probably quite a different way, due to what is recognized of the figure and of the landscape. This kind of difference between the perspective of the teacher and the perspective of the learner of the object of learning is important to recognize when teaching and to take into consideration when planning and analyzing teaching.

Characteristics of the object of learning

From the perspective on teaching taken in this book, the object of learning is central, as an expression of what is encompassed by the learning that the teaching aims to bring about, and what the learner understands and is capable of doing as a result of that act of learning. It also points towards necessary conditions for learning, in terms of explicating what must be discerned in order to learn the specific object of learning. An object of learning is thus a target capability, a description both of what

the content of teaching is and elements of what are the necessary conditions in how that teaching may be organized in order for the desired learning to be possible and probable. An object of learning typically expresses an aim of teaching and learning that is reasonable to achieve across the span of one or a small number of lessons (which it is possible to appraise in more detail and maintain a specific focus on).

An object of learning relevant for sixth-grade Swedish students learning English may, for example, concern the use of the progressive aspect as one aspect of English grammatical structure (Lindström, 2017). Swedish, similar to a number of other languages, does not have a marked grammatical form for the progressive, and this kind of meaning is expressed in other ways. When learning English, it is often difficult for students to recognize the full meaning of the progressive form – and specifically the difference between *I read this book* and *I am reading this book* – as well as to appropriately judge when this form may and should be used. Lindström's (2017) study shows how this object of learning (the ability to use the English grammatical structure, the progressive aspect – and thus the difference in meaning and use of *I was walking* compared to *I walked* – in a syntactically and semantically accurate way) may be addressed in the teaching during a single lesson with a productive outcome, in terms of student learning. We will return to how Lindström's study addressed this dilemma in teaching in Chapter 5.

The object of learning was here formulated in relation to a specific school context and language context, but the articulation of the object of learning, as well as innovations in how it may be addressed in teaching, may be of direct relevance to another group of similar composition and level, even though it is difficult to strictly tie down the level of correspondence to a different situation. However, the contextual variation and specific setting may be of paramount importance in understanding what the conditions are both in terms of previous knowledge as well as relational dynamics present in the teaching setting. This relevance, but not exact equivalence, across different groups, is a foundational assumption in how the educational system is constituted. It also makes knowledge about an object of learning an important component in teachers' professional knowledge, which may be shared and further developed together with colleagues. Formulating an object of learning is formative in developing and giving structure to both individual and collective professional (including scientific and systematically developed) teacher knowledge of what are necessary conditions for learning in relation to this particular knowledge object.

An object of learning is formulated in relation to a specific group of learners. For example, in the case of the progressive aspect mentioned above, the object of learning is formulated in relation to what is a challenging and appropriate step for the group to grasp across a single lesson (even though this may be ambitious, as seen in relation to normal educational standards). This also implies that an object of learning must encompass a clear view of the starting point (i.e., previous knowledge) for the group in question. Articulating an object of learning thus cannot be reduced to expressing a specific content-related educational objective. Educational

objectives are typically stated in the form of referring to a generic capability and specifying which content area this is related to. An example, taken from the Swedish current national school curriculum, is an educational objective for grade six: Being capable of showing basic knowledge about how light propagates and may be reflected using some relevant concepts and explanatory models. This may be used as a bar for assessment (even though we would argue that it is not straightforward, nor easily delimited in assessment).

This can be illustrated with two related, but different, objects of learning regarding Cantonese. Marton and Pang (2013) and Pang and Marton (2013) (referring to a study by Ki et al. (2006)) describe how challenges for speakers of non-tonal languages may be addressed when learning Cantonese. The central difficulty, and the object of learning, is to relate different pitches at the word level to different word meanings, which is different from non-tonal languages, where tones do not carry a specific meaning or carry significance at the sentence level. In the study, the speakers' attention is first directed to focus on varying tones of words while the sound (segmental, not tonal) stays the same. Following this, words with (some) variation in both tone and segmental sound were attended to. Teaching was implemented across nine words (constituted across words with three tones in combination with three segments). This was more powerful than first focusing the speakers' attention on a variation of segments with the same tone (potentially making it easier to recognize the tone used and differentiate it from other sounds in the pronunciation). Thus, the central capability constituting the object of learning was that the speakers could give their undivided attention to the meaning of words connected to different tones.

In Chik et al.'s (2010) study of young children, the object of learning was Chinese words in terms of their meaning and pronunciation as well as their written form. In contrast to the previous study, the central tenet of this object of learning is becoming able to see the whole of different words. Thus, the most powerful teaching to facilitate that was to simultaneously focus on the meaning, pronunciation, and written form of one word at a time rather than, for example, looking at sets of different words with the same pronunciation. In other words, the object of learning was the meaningful whole, as brought out in teaching, of the word in relation to what may be understood as its constituent parts in the language. This illustrates how objects of learning about similar things may differ depending on the group, as well as that aspects of what is to be learned may strongly overlap, while the most appropriate strategy in teaching that most powerfully facilitates learning is different and can even be contradictory (as argued by Pang and Marton (2013)).

Returning then to the characteristics of an object of learning, it is vital that an object of learning is understood in relation to a learner perspective (and the specific group of learners), even though articulated from a teaching perspective. It may be characterized as a target capability (defined as what a learner is able to do as a result of learning). This is closely tied to the domain (defined as part of a knowledge system such as a discipline) and the specific content that also characterizes

the object of learning. Further, an object of learning can also be characterized as a meaningful whole, as a phenomenon that it is possible to discern and explore from a learner's perspective, using, for example, phenomenography. An object of learning is often described primarily on one of these levels, but a full description encompasses them all. Learning about an object of learning is not only a question of discerning that which is figural since this figure is in relationship to its background or surroundings. Thus, to fully describe an object of learning, something of what surrounds the object of learning – the context of the capability, the content, and the meaningful phenomena – also must be articulated.

On all levels, the object of learning is delimited on an intermediate (meso) level. It does not bring into focus micro-level elements in themselves, such as choice of words or fragmented parts or individual learners in a specific situation. Neither is it reasonable to discuss an object of learning on a macro-level, such as understanding the nature of a discipline or knowledge area, or a very complex capability as a whole (for example, playing an instrument or solving advanced mathematics problems) or for all learners everywhere. For example, objects of learning that pertain to expanding your vocabulary, solving problems in science, constructing technological solutions, mastering algebra, or speaking Cantonese would typically be too broad to allow a meaningful discussion in the terms outlined, even though all of them may be examples of a broader domain in which an object of learning can be contextualized.

In this way, the object of learning is both independent and delimited from other objects of learning, but at the same time, it has a contextual and dialectical relationship to the domain in which it is situated and other meaningful phenomena within it. Delimiting an object of learning in this way is reasonable from a pragmatic and professional perspective. To plan and analyze teaching and learning that spans one or a few lessons makes it possible to keep the whole in mind the whole time, while still being able to attend to details and their importance and meaning in relation to the whole.

Further examples of objects of learning

In the following section, we give some further examples of objects of learning in different subject areas. What may be an appropriate object of learning? In chemistry, understanding the concept of the atom and its structure is an important and worthwhile object of learning. Park et al. (2009) identified a set of qualitatively different ways in which college students understood the structure of the atom. This set implies that an object of learning concerning the structure of the atom would be constituted in terms of a number of aspects for the students to discern: That atoms are divisible particles, that electrons circle the nucleus of protons and neutrons in orbits, that there are forces holding orbits together, that there is a possibility of multiple distinct orbits, that energy quantization is the basis for the distinct nature of the orbits, that probability is appropriate in describing the position of electrons,

that orbits are not singular but rather have an orbital shape in terms of probability, and finally that the quantum mechanical idea of the wave function can mathematically describe the orbits. Hence, in order for students to perceive the atom and its structure in a powerful way, i.e., to master the object of learning, there are multiple aspects that the learner needs to discern. Some of these aspects may even be objects of learning in themselves.

Learning a new language presents many challenges and students may, for example, struggle with reading a text in the new language due to the presence of many unknown words. A relevant object of learning in this case (Lo, 2012) may be reading strategies when faced with unfamiliar words such as, for example, using context and transitional words to guess the meaning of words without using a dictionary. Another example might be the use of personal pronouns in English, connecting nouns such as my brother, your father and John to *he*, and Mary, my sister and John's daughter to *she*. A third example of an object of learning from language learning may be discerning and arranging events in a time sequence in a text passage (in text comprehension as well as in writing the text). For all these objects of learning, there are aspects that the learner needs to discern in order to learn the object of learning.

In physical education, an object of learning is often a physical capability, for example being able to do a certain type of shot in basketball, run in an efficient way, or swim breaststroke. For instance, when learning to swim there are certain aspects students need to experience in order to be able to move in water. For example, one may think that the specific movements with arms and legs to do breaststroke are sufficient to discern. One critical aspect of learning to swim is learning to breathe in water. "In water, you breathe in such a way that you always retain a certain amount of air in your lungs, thereby lowering your density and bringing it on par with that of water" (Marton, 2015, p. 26). This makes the learner feel that she/he does not sink anymore, and that she/he can swim. Breathing is one example of an aspect for the learner to discern when learning how to swim.

Mastering an object of learning, such as the ones described above, enables the learners to act in more powerful ways. Hence, how a learner understands a phenomenon or an object of learning, affects how the learner acts. Or to put it another way, powerful ways of acting stem from powerful ways of seeing the object of learning. For example, in mathematics education, how students understand subtraction will affect how the students go about solving subtraction tasks. For example, students who perceive addition and subtraction as a part-whole relation ($a + b = c$ then $c - a = b$) are more likely to solve the subtraction task $\_\_ - 8 = 27$ as an addition ($27 + 8 = 35$), which is a powerful way of solving the task, whereas a student who has not yet experienced this relationship may only experience subtraction as a "take away", and are therefore not aware that you can use addition to solve the task. Instead, they try different numbers, for example, 32, 33, 34, 35 to see which number minus 8 is 27, which is a cumbersome strategy. Another example where how subtraction is perceived is important in a similar way is in the context

of negative numbers, where students who only perceive subtraction as "take away" can end up with difficulties. It has been suggested that seeing "subtraction as a difference" between two numbers is more productive for handling subtraction with negative numbers (e.g., Kullberg, 2010).

The intended, enacted, and lived object of learning as reflecting different perspectives in teaching and learning

The purpose of formulating an object of learning is to articulate a goal for teaching and learning that both reflects the direction of teaching and what the intended learning necessarily entails. In the process of teaching and learning, and in the planning and analysis of teaching, what is in focus may look different depending on which perspective we perceive it from. In teaching, the teacher perspective is essential, but so is the learner or student perspective, both the individual student and the group of students. Planning and enacting teaching involves different stages. The planning is done before the lesson by the teacher, and in this, the teacher (hopefully) makes clear what the intended outcome of teaching is. In the act of teaching, a lesson may be understood as an unfolding event in the classroom, to which both the teacher and the students contribute in interaction. The learning outcome is what the students take from the lesson, possibly visible through assessment by the teacher.

The intended, enacted, and lived object of learning are used to describe the object of learning in these different phases. The intended object of learning describes the planned object of learning, and thereby what the teacher aims for the students to learn. The teacher has an intended object of learning in mind when planning a lesson, however, this does not necessarily coincide with what is enacted in the classroom, and what students have the opportunity to learn. The enacted object of learning focuses on the teaching and tells us what is made possible to learn in the lesson or learning situation, analyzed from the point of view of what critical aspects were made possible to experience. By analyzing what aspects are varied in a lesson it is possible to analyze what is made possible for students to learn in a lesson. Hence, the aspects made possible to experience are seen as opportunities to experience the aspects and thereby learn aspects of the phenomenon. Analyzing what is made possible to learn in a lesson can also tell us what is not made possible to learn and can explain why students do not learn what is intended. If it was not possible to experience certain critical aspects of the lesson, we cannot expect that the students will learn the intended object of learning. The lived object of learning is which aspects of the enacted object of learning the students have actually discerned and reflect the students' learning. We can analyze the lived object of learning, from what they show that have learned on a test, in an interview, or during the lesson. The lived object of learning is less available for analysis than the enacted object of learning and may vary across the group of students.

In sum, we can analyze the intended object of learning (what the teacher intended the students to learn) and compare it with the enacted object of learning (what was made possible to learn) to find out if it was made possible to learn the intended object of learning. We can also analyze the enacted object of learning and see how the enacted object of learning is reflected in student learning (the lived object of learning). Using the variation theory of learning makes it possible to analyze the relation between the intended, the enacted, and the lived object of learning.

Object of learning as focus for planning and analyzing teaching

A distinct and well-articulated object of learning gives direction and precision to teaching and a point of reference in relation to which the learning outcome may be measured. Since the object of learning as an aim is specific rather than general and constituted in terms of the parts that the outcome (the specific way of understanding targeted) entails. It also points to the possibility of identifying direct relationships between the components of teaching and the components of the object of learning. The programmatic approach that is described in this book builds on the possibility that a key aspect of teaching is primarily how the object of learning is enacted in the classroom, and thus in turn to the lived object of learning. Planning teaching then aims to attempting to align the planned, the enacted, and the lived object of learning. Analyzing teaching is directed, on the one hand, toward revealing more of what actually happens in the classroom, in terms of the object of learning, and on the other hand, to allow the identification of components of teaching that may be changed in order to increase the possibilities for students to learn. In this way, it is possible to analyze teaching and learning by analyzing what aspects are made possible to experience in a lesson and compare them with what aspects have been discerned by students. The next chapter will present the theoretical foundation – variation theory – that we use in this book to establish such links between teaching and learning, with a special focus on explaining critical aspects of object learning. The chapter Analyzing teaching will go into more detail about such analysis.

3

THE VARIATION THEORY OF LEARNING

In this chapter, the main ideas of the variation theory of learning and the concepts used in the theory are described. The variation theory of learning aims to reveal the necessary conditions for learning by identifying critical aspects of different objects of learning. Variation theory, as it is also called, is a learning theory that views learning as the discernment of aspects of a phenomenon or a capability not previously discerned. The process of learning can be described as the learner making finer and finer discriminations within a greater whole and by so doing, discerning more aspects (parts) of that object of learning (whole). Being more knowledgeable in terms of understanding a phenomenon or having a certain capability means having discerned certain aspects of that object of learning. Or to put it differently, if the learner has not learned, he or she has not yet discerned the necessary aspects of that object of learning. This has implications for teaching and learning since teaching can make critical aspects of the object of learning visible to students and thereby enhance student learning. Marton et al. (2004) argue that what aspects students have the opportunity to experience are of vital importance for learning:

> What is of decisive importance for students, is what actually comes to fore of their attention, i.e., what aspects of the situation they discern and focus on. In the best case they focus on the critical aspects of the object of learning, and by doing so they learn what the teacher intended. But they may also fail to discern and focus on some critical aspects, or they may focus on other aspects.
>
> *(Marton et al., 2004, p. 5)*

Variation is seen as the mechanism for learning. Since people tend to notice differences (variation) rather the sameness (what is invariant), new meanings are learned through the perception of differences (cf., Gibson & Gibson, 1955). This

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view differs from views of learning that instead argue that it is by finding the invariant feature (sameness) in examples of the phenomenon the learner is assumed to grasp the meaning. This is called "induction" and is the common way of describing the acquisition of new concepts (Dienes, 1960, pp 42–43; Gu et al., 2017, p. 14; Gu, 1999). There is, however, a problem with this view from a variation theory perspective. If you cannot see the meaning in one example, neither can you see it in multiple examples. If you cannot recognize, for example, the idea of democracy in one example of it, neither can you see it in two examples. This way of reasoning led the famous American philosopher, Jerry Fodor, to argue that all concepts are innate (Fodor, 1980). His point is that if we cannot account for how concepts are learned, but still can show that people indeed possess them, then we must conclude that people possess them by birth. If you do not subscribe to such a conjecture – and we do not – then you are obliged to come up with an alternative explanation of how novel meanings (e.g., of concepts) are appropriated by humans. Variation theory, we argue, offers such an explanation.

Variation is key

In the variation theory of learning, variation is seen as key for learning. To acquire new meanings and learn new phenomena that the learner is not familiar with, variation is needed to notice a phenomenon or aspects of it. If a young child is to understand what red is, the child needs to see how red differs from other colors, such as blue or green. Pointing to different red objects (a cup, a sock, a ball) and saying "red" is, according to variation theory, not sufficient for learning the meaning of red, since the "redness" of red, only appears in contrast to other colors. By keeping the objects the same (invariant) and only varying the color, pointing at the red cup saying "red" and at a green cup saying "green", makes it possible to discern the difference between red and other colors.

Similarly, if a preschool child is to learn what a triangle is, it is not sufficient to point at many different triangles and say "triangle" since it is then not possible for the child to distinguish what "triangle" represents, since "triangle" could be one of many different things, like the color of the triangles, or the angles, etc. To grasp the meaning of "triangle", we argue, the child needs to see how a triangle differs from other geometric figures. By varying geometric figures (a square, a circle, a triangle), and keeping other aspects invariant, the varied aspect (geometric shape) is made possible to discern. Hence, the meaning of the geometric shape triangle appears in contrast to other geometric figures.

Critical aspects of the object of learning

A central concept in the theory is *critical aspects*. A critical aspect describes what the learners need to discern to perceive the object of learning in the intended way. Hence, critical aspects are always connected to a specific object of learning.

Critical aspects are relational in nature as they are related both to the content and to students' understanding of that content. Because the critical aspects are relational, they cannot be derived from disciplinary knowledge of the subject only (Pang & Ki, 2016). The critical aspects need to be identified for every object of learning since it is not self-evident what the critical aspects are. The critical aspects need to be found empirically, for instance, from analysis of interviews with students, written tests, or analysis of observations in the classroom. Several of the critical aspects described in this chapter have been found during teachers' collaborative exploration of an object of learning in a learning study (see Chapter 4) or in intervention studies using variation theory. Critical aspects can also be identified in (phenomenographic) studies of students' understanding of different phenomena.

Identifying potential critical aspects requires being sensitive to how the learners understand what is to be learned. The critical aspects can be identified before, during, and after the learning situation.

> To identify critical aspects, we need to look simultaneously at the phenomenon, the way of experiencing it in the subject discipline, learners' ways of experiencing it, and more specifically, the aspects of the phenomenon that learners have and have not [previously] discerned.
>
> *(Pang & Ki, 2016, p. 332)*

If an aspect has been experienced by one learner, but not by another learner, it is only critical for the learner who has not yet experienced it. However, when planning, enacting, and analyzing teaching, critical aspects are viewed on a group level since lessons are usually taught to a group of students.

Criteria for a critical aspect:

- The aspect has not yet been discerned but needs to be discerned.
- The aspect is related to how students (incorrectly) experience the content/subject matter *or* is related to the content/subject matter.
- It indicates what students need to discern or disregard.

In this way, critical aspects are always related to the specific object of learning at hand. Generic components of teaching and the student group are then by definition not critical aspects. There are undoubtedly many such things that affect students' learning opportunities, for instance, students' language abilities, the teacher's ability to explain cohesively, students' attention and motivation, etc. Neither are critical aspects important aspects of the concept on a general level or students' difficulties per se.

Critical aspects are not:

- Students' difficulties.
- Students' general language abilities.

- General aspects of teaching or learning.
- Teachers' actions (e.g., explaining clearly).

The critical aspects are connected to the object of learning and describe what students need to discern to master it (the object of learning). The critical aspects of an object of learning can be seen as parts of that object of learning (the whole). We can find critical aspects by investigating i) *students' understanding,* ii) *the content*, and iii) *what needs to be disregarded* by students. Critical aspects can be found, for example, in students' answers to tasks, but also during lessons when the students express their ways of thinking. For example, in one mathematics lesson, the object of learning was to understand the coefficient and constants in the equation of the straight line ($y = mx + b$) and to see the connection between the line (graph) and the equation. Figure 3.1 shows one 8th-grade student's answer on a task that was designed to capture students' understanding of graphs before the lesson. In this task, the students are to match several equations with the corresponding graphs. This student has correctly identified that $y=2$ represents Graph C (horizontal line). We can also see that the student is aware of which equation has the highest slope ($y = 3x + 2$) since he writes beside the equation that this graph "leans the most". However, the student has paired $y = 3x + 2$ with Graph B which leans less than Graph A. Hence, the correct answer is that Graph A shows $y = 3x + 2$, since the graph with the highest value in front of the x inclines the most.

How does this student understand the slope of the graph? A group of teachers identified that the student's answer could be explained by the fact that when

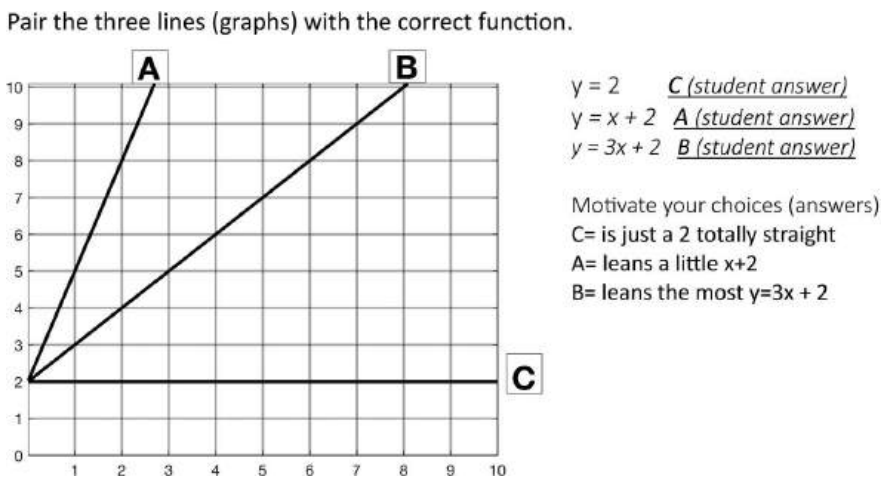

FIGURE 3.1 A student's answer to the task. The student has written, "C= is just 2 totally straight, A= Leans a little $y = x + 2$, B= Leans the most $y = 3x + 2$" (Mårtensson, 2015, p. 148).

we say that something leans more in daily life, we refer to it in comparison to an upright (vertical) position, for example, the tower of Pisa leans more if we compare it to a vertical tower. The critical aspect identified from the test was that *the inclination of the graph is seen from the x-axis,* not the *y*-axis (as this student did). This was one of several critical aspects identified by the group of teachers working collaboratively in planning teaching about this object of learning (Mårtensson, 2015).

There are critical aspects that are derived from the content taught. In Chapter 1, we discussed how one 5-year-old child understood numbers. When the teacher showed one finger and the child was asked how many it was, the child answered that it was "ten" and not one. The child had not yet discerned how many ten are (the "manyness" of number/cardinality), that is, ten means all the counted fingers on the two hands, not only the last counted finger (one finger, the tenth). For children to experience numbers in a proficient way, it has been argued that certain critical aspects need to be discerned, for example, ordinality (relations between numbers in a sequence), cardinality ("manyness"), and part-whole relations of numbers (e.g., five consists of, for instance, two and three) (Björklund et al., 2021). These critical aspects are aspects of the content that students need to discern in order to learn numbers in a proficient way.

A critical aspect can also be an aspect that students need to disregard. For example, if in mathematics education, the object of learning concerns what an angle is, some students may think that the arms of the angle are a part of the angle and that a figure with long arms has a bigger angle compared to a figure with shorter arms. In this view, Angle b (see Figure 3.2) would be a bigger angle than angle a, which is not the case.

Runesson (2007) found that in three classes she studied in fourth and fifth grade, relatively few students (Class A 6%, Class B 40%, Class C 26%) could disassociate the arm length from the size of the angle on a test. These students were not yet aware that the arms are only used to illustrate the angle, and that it is instead the turning of the arms that determines the size of the angle. The fact that *the length of the arms does not determine the size of the angle* is in this case a critical aspect, i.e., that the length of the arms is an aspect that should be disregarded by the students when learning about angles.

FIGURE 3.2 Compare the angles. Which angle is the biggest? (Kullberg, 2010, p. 46)

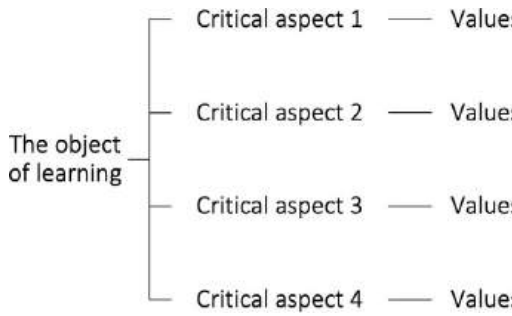

FIGURE 3.3 Illustration of the relationship between the object of learning, critical aspects, and values.

Critical aspects and values

How can identified critical aspects be made noticeable for students? A critical aspect of an object of learning can be made noticeable by varying the "values" of that critical aspect. When this happens, we say that a *dimension of variation* is "opened up" for the learner to experience. In this sense, a critical aspect is also a dimension of variation since it can be varied using appropriate values (Figure 3.3).

For example, if the object of learning concerns *what an angle is*, as in the example above, one critical aspect is that *the length of the arms does not determine the size of the angle*. Figure 3.2 shows how this aspect was opened up as a dimension of variation by a contrast between two values, one angle with short arms (with a bigger angle), and one angle with long arms (with a smaller angle).

Teaching with variation

Variation has a special meaning in variation theory when it comes to teaching. By variation, we do not mean variation in teaching in general, for instance, "I need to vary my teaching", implying that the teacher needs to vary the methods, modes of working, activities, etc., to make the teaching more interesting for the students. This is certainly important as well but is not emphasized in variation theory. It is also important to note that teachers naturally use variation in some form when teaching; however, they may not be aware of it and may not systematically use variation. In variation theory, we focus on variation regarding how that content is handled, and therefore, the theory becomes an essential tool for teaching. What is to be varied in teaching from a variation theory point of view are the *critical aspects* of the object of learning since the critical aspects must become visible to the learner. The likelihood of discerning critical aspects of an object of learning depends on the aspects the learner encounters and becomes aware of through the experience of difference and sameness. To experience something in a certain way, the learner often needs to be simultaneously aware of several critical aspects of the

object of learning. Experiencing these aspects is, however, not the same as being told about them. For example, it is not sufficient to tell students that there are an infinite number of decimal numbers during a mathematics lesson for students to learn that there are an infinite number of decimal numbers. Students need to experience that it is plausible that there could be an infinite number of decimal numbers, for example, by experiencing a variation of numbers in an interval between two decimal numbers (Kullberg, 2010). Marton et al. (2004) illustrate the difference between being told and experiencing with the following example:

> Medical students, for instance, might be advised by their professors to try to notice different features of their patients, such as the color of the lips, the moisture of the skin, the ease of breathing, and so on: this is being told. But in order to follow this advice, the students must experience those features, and the only way to experience them is to experience how they vary. Noticing the color of a patient's lips, for example, would not mean much if lip color was the same for everyone.
>
> *(Marton et al., 2004, p. 10)*

Systematic use of variation aimed at critical aspects may be utilized for pedagogical purposes to enhance students' learning in the classroom since "[w]hen some aspect of a phenomenon or an event varies while another aspect or other aspects remain invariant, the varying aspect will be discerned" (Bowden & Marton, 1998, p. 35). Hence, patterns of variation can make the critical aspects of the object of learning noticeable to students. This does not, however, guarantee that learning will occur. The teacher can only direct learners' attention toward the critical aspect by variation of that aspect. Critical aspects (parts) of the object of learning (whole) need to be separated to be noticeable to the students. Marton et al. (2004) argue, "We believe that separating the aspects first and then fusing them together is more efficient (from the view of being able to adapt to changing conditions) than never taking the critical aspects apart" (p. 17). Separation of a critical aspect can be made using two patterns of variation, *contrast* or *generalization* (Table 3.1). When critical aspects have been varied separately by contrast or generalization, they need to be experienced together. Marton (2015) calls this pattern of variation *fusion*, and it is the simultaneous experience of two or more aspects.

TABLE 3.1 Patterns of variation and what is varied and invariant.

		Critical aspect	*Other aspects*
Separation	Contrast	(vary)	(invariant)
	Generalization	(invariant)	(vary)
Fusion		(vary)	(vary)

Contrast

When a learner is to discern a critical aspect, *contrast* is used (in teaching) for the learner to notice it. In this pattern of variation, the critical aspect is varied using appropriate values, and other aspects are kept invariant. Contrast is used to make distinctions. A contrast can be enacted using a "counter example" – what something is not – to bring out the meaning of the phenomenon in question. In this chapter, examples of contrast were used to point out differences in meaning: when learning about red, contrasting red with other colors (e.g., blue, green), and when learning the geometric shape triangle, contrasting the different geometric shapes (e.g., triangle, circle, square).

Generalization

In the pattern of variation *generalization*, the critical aspect is invariant, and other aspects are varied. After contrast has been used to distinguish a critical aspect, generalization is used for the learners to generalize the aspect by seeing the same aspect in many different examples. Using the same examples as with contrast above would involve, for instance, showing many red objects to see the "redness" of red in several things (*red* dress, *red* hat, *red* cup), or pointing at types of triangles (e.g., equilateral, isosceles, scalene) to show the similarities of triangles.

Contrast and generalization are often used in sequence. Let us go back to the example of Mike's understanding of numbers discussed in Chapter 1. The sequence shows that Mike's experience of numbers was connected to the name of the counted number (the order of numbers and not the "manyness" of numbers). When Mike argues that the little finger on the teacher's right hand was "ten", the teacher makes a contrast by showing one finger (the tenth finger) and saying "It is only one finger", and then showing ten fingers and saying "Now it is ten", for Mike to see the difference between "one" and "ten" and that ten is not one finger (the tenth) (Figure 3.4).

Teacher: "It is only one finger" … "Now it is ten"

FIGURE 3.4a and 3.4b They teacher shows one finger (3.4a first picture), thereafter she shows ten (3.4b second picture).

FIGURE 3.5 The teacher holds up her thumb.

The contrast between one and ten (values) opens up the critical aspect of "manyness" (cardinality) as a dimension of variation. After that, the teacher showed the same quantity (one) in many ways, holding up different fingers and saying, "This is one (showing one finger), this is one (showing another finger), this is one" (showing yet another finger) for Mike to see the sameness of the cardinal aspect of one (generalization). In this part, it was made possible for Mike to experience that the "manyness" of *one* is independent of which finger is used to represent the quantity (Figure 3.5).

Teacher: "This is one [holding up her thumb on her left hand]".

Fusion

Marton et al. (2004) state, "In everyday life, it is seldom that only one aspect of something varies at a time, and so the way in which we respond to a situation, […] springs from a more general holistic perception of the situation" (p. 16). So, therefore, when aspects have been separated, they need to be brought together. The pattern of variation used when several critical aspects are simultaneously varied is called *fusion*. It entails that several critical aspects vary at the same time, but also that other (non-critical) aspects also vary. For example, when learning what the color red is, a situation in which red is one of many different colors and where the colored objects also differ is one example of fusion. Hence, both the colors and the objects vary.

The main concepts used in variation theory are summed up in Table 3.2. However, it is not necessary to use all these concepts when you first try to apply them. The most important concepts when planning and analyzing teaching are the object of learning and critical aspects. Applying these concepts in discussions about teaching is helpful since it directs attention to what is taught and learned. In the next chapter, attention is turned to how teaching and learning can be improved by teachers conducting learning studies, and how critical aspects can be identified by an exploration of the object of learning. In the following chapters,

TABLE 3.2 Main concepts in the variation theory of learning.

Concepts	
Object of learning (OL)	The object of learning depicts the capability to be learnt.
Critical aspects	What the learners need to discern (but have not yet discerned) to learn the required capability.
Values	Range of specific values of a critical aspect in different examples used[1].
Dimension of variation	An aspect (critical or not) that is varied.
Patterns of variation	Patterns of variation are contrast, generalization, and fusion.
Separation	When one critical aspect is handled at a time. This is done through contrast or generalization.
Contrast	The critical aspect is varied while other aspects are invariant.
Generalization	The critical aspect is invariant while other aspects vary.
Fusion	Several critical aspects vary simultaneously in a teaching situation, activity, or example.
Simultaneity	To experience something (a phenomenon, a concept, a value) at the same time as something else.
Intended object of learning	Describes what is intended to be learned in a learning situation/lesson.
Enacted object of learning	Describes what is made possible to learn in a learning situation/lesson, analyzed from the perspective of the researcher/teacher.
Lived object of learning	Describes students' learning and the critical aspects the learners have discerned.

we will show how variation theory can be used as an analytical tool for planning and analyzing teaching.

Note

1 Values are in some literature also called critical features, for example, see Lo, 2014.

4

LEARNING STUDY

Improving teaching for learning

If our goal is to enhance students' learning, teaching in school needs to offer powerful learning opportunities. Kurt Lewin, a psychologist who conducted applied and action research, once argued, "If you truly want to understand something, try to change it." Our experience tells us this is true regarding understanding the relationships between teaching and learning. When teachers have the opportunity to analyze and revise lessons while at the same time seeing how the changes in teaching affect students, they can gain a more profound understanding of teaching and student learning. Learning study is one model used to collaboratively refine teaching in an iterative process. In a learning study, a group of teachers revise teaching multiple times to identify what students need to discern in order to have the opportunity to learn a particular capability (an object of learning). How the teaching needs to be changed for students to be able to do so is explored. In this chapter, we present the learning study model, and findings from one learning study in mathematics education are used as illustrative examples of what the model can entail and the kind of result that may be obtained.

The learning study model was, as stated previously, developed by Ference Marton and colleagues around 2000 (Lo et al., 2005; Marton & Pang, 2003; Marton & Tsui, 2004). Marton was a guest professor in Hong Kong at the time and initiated, together with colleagues, the first learning study project called *Catering for Individual Differences*, which involved 18 groups of teachers in different subjects (Lo et al., 2005). The project was followed by another learning study project *Variation for the Improvement of Teaching and Learning (VITAL),* and involved 120 schools (Elliott & Yu, 2008). In the early 2000s, the model was introduced by Marton and colleagues in Sweden through the research project called *The Pedagogy of Learning*, which implemented learning studies in Swedish, English, and mathematics

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(Holmqvist et al., 2008). Since then, learning study has been used from preschool to university level and in a variety of subject-matter areas.

The learning study model was developed based on inspiration from the in-depth investigation of lessons in the *Japanese lesson study* (Lewis et al., 2006; Stigler & Hiebert, 1999; Yoshida, 1999) and *design experiments* (Brown, 1992; Collins, 1992). The Chinese *teachers' research group* model similarly tries to improve teaching and student learning by systematically investigating lessons. Lesson study and design experiments, which are better known, are iterative models in which teaching is collaboratively designed and revised to gain knowledge about teaching and learning.

In design experiments, also called design-based research, the aim is to identify effective learning opportunities and develop practice-based theories that are useful for both teachers and researchers. Design-based research is conducted by a team of researchers in collaboration with teachers. It is usually carried out over an extended period, for instance, over several weeks or an entire school year. The development of theoretical understanding through the design and interventions in practice is in focus. The theoretical understanding supports the design of the intervention, frames the scientific study, and is advanced by the findings generated through the intervention (McKenny & Reeves, 2012). In this way, findings from the design-based research "are fed back into further cycles of innovative design" (Bereiter, 2002, p. 321). The outcome of the iterative design process is often a teaching sequence or a teacher manual.

The Japanese lesson study is used to describe various practices involving the study of lessons and has a long history in Japan. It became known to the Western world in 1999 through the book *The Teaching Gap* by James Stigler and James Hiebert. In the book, they argued that teachers' participation in lesson study – and thereby becoming better at teaching – was one explanation for the outstanding results obtained by Japanese students on international tests like TIMSS (Trends in International Mathematics and Science Study). The Japanese lesson study model is described as involving four main steps (Huang & Shimizu, 2016): 1) goal setting, 2) planning the research lesson, 3) enacting the research lesson, and 4) reflecting on and revising the lesson. The revised lesson is enacted in a new class.

In the goal-setting part, goals for students' development are in focus. Gaps between long-term goals and current reality are identified. A research theme for the lesson study is formulated. In an iterative process, the research lesson is planned collaboratively, enacted by one teacher, observed by the team of teachers in the classroom, analyzed, and revised. The revised lesson plan is an outcome of the lesson study that can be used by other teachers and communicated in books and at conferences. Lesson study has been described as a practice-based, research-oriented, collaborative mode of in-service teachers' professional development (Huang & Shimizu, 2016).

One difference between lesson study and learning study is that in a learning study, the focus in the iterative process is primarily on the *object of learning,* not

the lesson (and its activities). In a learning study, the group of teachers plan, analyze, and revise teaching (during one lesson, a series of lessons, or a longer intervention, e.g., two semesters) multiple times, *employing a learning theory as a tool*. In the process, they develop insights about critical aspects of the object of learning and how to teach in order to make the critical aspects noticeable to the students. The theory used is most often the variation theory of learning or a combination of variation theory and another theory, for example, learning activity (Davydov, 2008/1986).

A learning study can in this sense be described as a type of theory-based lesson study. The learning theory (variation theory) plays an essential role since it is used as a tool for planning and analyzing teaching and student learning. The theory tells us that we need to become aware of the critical aspects of the object of learning before we start planning the lesson. Hence, it is the critical aspects that need to be brought to the fore in the lesson, and therefore, the choice of activities and tasks follows from that. The iterative process of planning and revising the same lesson multiple times focuses on identifying and refining critical aspects and the patterns of variation used while being sensitive to student learning, as shown in the lesson or pre- and post-tests. Pre- and post-tests or student interviews are used to gather information about how students understand the object of learning before and after the lesson. Runesson (2008) describes the aim of a learning study in the following way:

> In a learning study the aim is to help the students to learn something specific: We ask: What are the necessary conditions for learning something and how can these be met in the learning situation? If students do not learn what we expected, we do not seek the answers to their failure in the inadequacy of the student; neither do we seek them in the teaching arrangements or the methods used. Instead we focus on the students' learning – what their difficulties are – and on how the content must be handled in the lesson in order to overcome the learning obstacles.
>
> *(Runesson, 2008, p. 169)*

The outcome of a learning study is identified critical aspects, patterns of variation, and the activities and tasks used in the teaching.

The learning study model

A learning study is usually carried out over several months during a semester. A study can consist of about six to ten meetings, depending on how long the meetings are and what is attended to during the same session. A learning study usually involves three to six teachers (and their classes), and a researcher or an experienced teacher often serves as a facilitator to support the teacher team. However, as stated previously, the number of lessons studied and the extent of

the study may vary depending on the object of learning. The process starts with identifying an object of learning that the teachers think is difficult for students and challenging to teach. When an object of learning has been decided upon, an exploration of the constituents and delimitations of the object of learning begins. Suppose the object of learning is, for example, to develop the ability to lead a warm-up before a specific sporting activity with other students in physical education. Questions such as What does it entail to know the object of learning? and What are the critical aspects of the object of learning? are discussed during the whole process.

A learning study involves a deep investigation into how students experience the object of learning and what students need to learn to master the object of learning in the intended way. A test or interview is used to identify how students understand the object of learning before and after being taught. Therefore, the items and questions in the test or interview need to focus on the students' understanding of the object of learning in qualitative terms rather than correct and incorrect answers. Students' written texts or video recordings of students showing the required capability (e.g., how students lead a warm-up before a specific sporting activity with other students) could also be used as a pre-and post-test (depending on the object of learning). It is typical for the teacher team to read related research about students' learning of the object of learning to get more knowledge about students' understanding of the topic, which is helpful when designing the pre-test questions and the lessons. If potential critical aspects have been identified, pre-test tasks can be designed to decide whether the aspects are critical for these students' learning. The test or interview is conducted with all participating classes. When the pre-test or interviews have been analyzed, the team can refine the object of learning, if necessary, and draw conclusions about critical aspects.

The first lesson is designed to bring the identified critical aspects to the fore. Activities and sequences of tasks are chosen based on whether they have the potential to make the critical aspects visible to the students. The activities and tasks are designed using patterns of variation and invariance regarding the identified critical aspects. The lesson is enacted in one class by one of the teachers and is video recorded. The students conduct a post-test or post-interview a few days after the lesson. No teaching on the specific topic should be conducted between the lesson and the post-test. The teacher team analyzes the video-recorded lesson, and differences between the pre- and post-tests are examined to see how the teaching may have affected student learning. Was it made possible for the students to discern the critical aspects in the lesson? Were new critical aspects identified in the post-test or the lesson? The lesson plan is revised, and the second lesson is enacted by another teacher (or the same teacher) in another class. The iterative process continues, and the lesson is usually revised two or three times. During the whole process, the critical aspects are scrutinized. The critical aspects can often be described more precisely at the end of the process.

The iterative learning study process can be summarized as follows[1]:

1 Choose an object of learning.
2 Find out students' pre-knowledge through a pre-test or interview to identify the critical aspects. Refine the object of learning if necessary.
3 Plan the research lesson using variation theory as a tool.
4 One teacher enacts the lesson in one class (the lesson is video recorded).
5 Find out what the students learned (conduct a post-test or interview).
6 Analyze student learning (pre-and post-test or interviews, or as expressed in the lesson) in relation to how the object of learning was enacted in the lesson and identify critical aspects. Revise the lesson.
7 Summarize the identified critical aspects, teaching activities, and powerful patterns of variation. Present and discuss the findings with other teachers.

An essential part of a learning study is summarizing and sharing the findings regarding identified critical aspects and patterns of variation used in activities and tasks. The results have the potential to be used, further explored, and tested by other teachers when they are teaching the same topic or doing a learning study on the same topic.

A learning study about negative numbers

To illustrate the learning study process, we show how insights about the object of learning and its critical aspects were obtained in a learning study about addition and subtraction of negative numbers conducted in 7th and 8th grade (13- and 14-year-olds) (Kullberg, 2010). However, we do not detail the specific patterns of variation used in the lessons.

Deciding the object of learning

The topic, adding and subtracting with negative numbers, was new to the students as this is usually taught in the 8th grade. The four teachers who participated in the learning study, together with a facilitator, thought that teaching about negative numbers was difficult and that it often ended up with students applying the "rule": "two minuses make a plus" with little understanding when subtracting a negative number, for example, $9 - (-4) = 13$. For example, the "rule" can cause the students to answer that $(-9) - 4 = 13$ (and not -13), since they may interpret the "rule" as any "two minuses make a plus". The teachers were aware of the many metaphors used in lessons and textbooks when teaching about negative numbers, for example, elevators going up and down, a thermometer with plus and minus degrees, etc. However, the teachers concluded that the problem with the metaphors was that they did not work for all cases ($a + (-b)$, $(-a) + (-b)$, $(-b) + a$, $(-b) + (-a)$, $a - (-b)$, $(-a) - b$, $(-a) - (-b)$, $(-b) - a$, $(-b) - (-a)$). The teachers even reflected on

whether teaching this topic without teaching about the "rule" was possible. The object of learning was for students to be able to solve addition and subtraction tasks with negative numbers, like 5 – (−3) and (−5) + (−3), with some conceptual understanding.

Finding out students' pre-knowledge

The teachers designed a pre-test with numerical tasks and word problems about addition and subtraction with negative numbers. The pre-test showed that the students had little difficulty with word problems with negative numbers. The teachers discovered that the word problems could often be solved without considering negative numbers (e.g., "Yesterday it was minus 5 degrees, and today it is minus 10 degrees. How many degrees colder is it today?" could be solved by ignoring the negatives and calculating only with positive numbers: 10 – 5 = 5). The pre-test also showed that even though the "rule" (two minuses make a plus) had not been taught to the whole class, some students applied the "rule" but did so incorrectly. The first critical aspect identified from the pre-test and the research literature was that students needed to differentiate between the sign for subtraction and the sign for a negative number since it was found that the students treated the symbols as if they had the same meaning.

Planning and enacting the first lesson

The teacher team discussed how they would make the critical aspect of *The difference between the signs* visible to the students. For example:

> The teachers considered possible solutions to this problem: different words for the number (e.g., 'negative three') and the operation (subtraction or minus) could be used, the two signs could be separated by putting the sign for the negative number 'higher up' than the operational sign. They discussed whether concrete representations and metaphors, for example, temperature (−3 as three degrees below zero) and debt, would be a limitation for a deeper understanding. These representations seemed to be of use for solving some of the tasks but not all. How could, for instance, subtraction '5 – (−3) =' be represented in a good way? And how could the different tasks' 5 – 3 =' and '5 – (−3) = be told apart.
>
> *(Runesson et al., 2011, p. 267)*

The teacher team decided to use the expression "negative three" for the negative number (−3) to separate the sign for the negative number from the "subtraction minus" when talking about negative numbers. They also decided to use parentheses around the negative number. The teachers had the idea that if they differentiated between the signs in the lesson and showed the mathematics behind the "rule": "Adding (subtracting) a negative number is the same as subtracting (adding) its

Pattern A	Pattern B	Pattern C
5 + 5 =10	5 + 5 = 10	5 + 5 = 10
5 + 4 = 9	5 + 4 = 9	5 + 4 = 9
5 + 3 = 8	5 + 3 = 8	5 + 3 = 8
5 + 2 = 7	5 + 2 = 7	5 + 2 = 7
5 + 1 = 6	5 + 1 = 6	5 + 1 = 6
5 + 0 = 5	5 + 0 = 5	5 + 0 = 5
5 + (-1) = 4	5 + (-1) =-4	5 + (-1) = 6
5 + (-2) = 3	5 + (-2) =-3	5 + (-2) = 7
5 + (-3) = 2	5+ (-3) =-2	5 + (-3) = 8
5 + (-4) = 1	5 + (-4) =-1	5 + (-4) = 9
5 + (-5) = 0	5 + (-5) = 0	5 + (-5) = 10

FIGURE 4.1 Three patterns suggesting answers for adding negative numbers were discussed during the first lesson. The students suggested Pattern A (correct) and B (incorrect), and Pattern C (incorrect) was suggested by the teacher.

opposite", this would be beneficial for student learning. If the students were allowed to discover how the "rule" works, they thought this would help them understand when operating with negative numbers.

During the lesson, Teacher A discussed the different meanings of the minus sign with the class by pointing at the other signs in a subtraction task and asking for the meaning of each sign. The teacher also brought up the idea of opposite numbers, for example, the opposite number to 1 is (-1), and that an addition of $1 + (-1) = 0$. The idea of introducing opposite numbers was needed to understand how the "rule" worked. During the lesson, the students were asked to complete the pattern for what addition with numbers below zero looks like. What happens when you add negative numbers? The students suggested possible patterns where the answers varied (see Figure 4.1).

In the next step, subtraction patterns were investigated and discussed in the same way as addition patterns. In the final stage, the correct addition pattern was connected to the correct subtraction pattern. In this way, it was made possible for the students to see that the rule worked, for example, that subtraction with a negative number is the same as an addition with the opposite number (see Figure 4.2).

A post-test with the same tasks as the pre-test was conducted some days after the enacted lesson. The post-test showed that students improved on addition tasks with negative numbers compared to the pre-test. In contrast, student learning on subtraction tasks with negative numbers did not improve. The analysis of the post-test and the video-recorded lesson made the teachers realize that it was insufficient to explain the rule and direct the students' attention toward the difference between the signs. In the enacted lesson, one student expressed the idea that subtraction could be seen as both a "difference" and a "takeaway". The teachers deliberately avoided using the double meaning of subtraction in the lesson since they thought this would only make it more complicated for the students. However, the student's statement

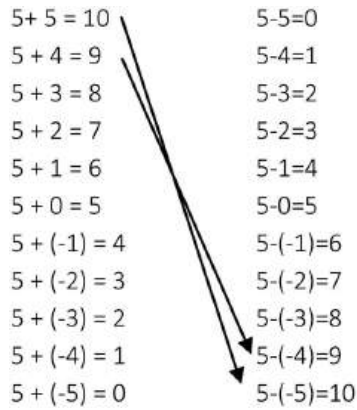

FIGURE 4.2 The correct addition and subtraction patterns were used to show the "rule" that a subtraction with a negative number 5 – (−5) = 10 gives the same answer as an addition with the opposite number 5 + 5 = 10.

made the team realize that students needed to experience subtraction as a difference when subtracting negative numbers.

Planning and enacting the second lesson

In the revised lesson plan for the second lesson (enacted by Teacher B in another class), the team planned to enact two critical aspects: *the difference between the signs* and *seeing subtraction as a difference*. The lesson was intended to elicit the critical aspect of *seeing subtraction as a difference* by looking at differences between two numbers (e.g., 5 – 3 = 2, 1– (−1) = 2 and (−3) – (−5) = 2) on the number line. The teachers were aware that when solving subtraction tasks where the minuend (the first term) is smaller than the subtrahend (the second term), the difference is negative, for example (−5) – (−3) = (−2). However, at this point, the teachers did not know how to explain this in a way that would make sense to the students. Therefore, the teacher team decided to avoid discussing tasks with negative differences in the second lesson. The teacher wrote the largest number first on the board to avoid this issue. Although it was intended that the critical aspect, *the difference between the signs*, would be brought up during the lesson, it was not. The teacher most likely forgot it. The lesson paid less attention to the addition of negative numbers, which may explain the lack of improvement on this topic in the post-test. However, there was a more significant increase in student learning on subtraction tasks with negative numbers.

Analyzing the students' post-tests and the enacted lesson made the teachers realize that they could not avoid dealing with tasks that had a negative difference. The teacher discovered a metaphor (debt as a financial state) that could be used to explain the negative difference that could also be used for all other cases (addition and subtraction tasks). For example, if, on the one hand, Ann has 5 euros and John

has a debt of 3 euros, their shared economy (addition) would be 5 + (−3) = 2, and together they had 2 euros in total. If, on the other hand, Ann has 5 and John has a debt of 3, the difference between their economy/financial state (subtraction), 5 – (−3) = 8, is that Ann has 8 euros more compared to John. But if seen from John's perspective, (−3) – 5 = (−8), he has 8 euros less compared to Ann. From this insight, the teachers identified a third critical aspect: "*the perspective*"/*the commutative law does not apply in subtraction*. Hence, the order of the numbers in a subtraction task matters since whether it is a positive or negative difference (answer) depends on whether the minuend is larger than the subtrahend.

Planning and enacting the third lesson

For the third lesson in the iterative cycle, the teachers planned to enact the three critical aspects and use the metaphor of financial state to discuss the addition and subtraction of negative numbers. The teachers also planned to use another metaphor, differences in ages between two people, to further direct the students' attention to the critical aspect "*the perspective*"/*commutative law does not apply in subtraction.* For instance, when comparing ages, for example, Ann is 14 years old, and John is 11, Ann is 3 years older than John, but from John's perspective, he is 3 years younger than Ann.

Teacher C enacted the lesson in her class. The lesson was, however, not enacted as planned, as the lesson was interrupted by a fire happening outside of the classroom. The results from the post-test after the third lesson are therefore not reported.

Planning and enacting the fourth lesson

Since the third lesson was not enacted as planned, the team decided to test their lesson plan in yet another class. Since no more 7th-grade classes were available at their school, they decided to enact the lesson in one class in the 8th grade. Based on the pre-test results, this class had similar pre-knowledge to the classes in the 7th grade about addition and subtraction with negative numbers (see Table 4.1). Teacher D enacted the lesson as planned and brought the three identified critical aspects to the students' attention. At the end of the lesson, the students were asked to place the number (−2) and (−1) in the equation __−__= 1 to further elicit the

TABLE 4.1 Results in percentages of correct answers on two key tasks from the pre- and post-test.

	Lesson 1, N=17 *Pre-test/Post-test*	*Lesson 2, N=17* *Pre-test/Post-test*	*Lesson 4, N=21* *Pre-test/Post-test*
Addition (−5) + (−2) = Subtraction (−5) − (−2) =	24% 65% 35% 29%	53% 65% 41% 65%	29% 86% 29% 81%

critical aspect of "*the perspective*". To get the positive answer 1, the larger number needed to be the minuend, (−1) – (−2) = 1. The students agreed that the larger number should be the minuend, but the students had different opinions about which number was the largest. This confusion was picked up by the teacher, who said:

> T: I know your problem, and it was stupid of me not to have considered this before. We have to find out which of the two numbers (−1) and (−2) is the biggest number.
>
> *(Runesson et al., 2011, p. 274)*

This episode shows how the teacher discovered a fourth critical aspect during the lesson. It was critical for the students to experience the number system, meaning that numbers are larger the more to the right-hand side of a number line they are, for example, (−1) is a larger number than (−2). Some students thought the numbers on both sides of zero got larger and larger the further away from zero. This insight, made by the teacher during the lesson, led the teacher to discuss with the students whether, for example, (−18) is a smaller or larger number than (−2).

Students' learning

The post-test after the fourth lesson showed that students' learning had improved significantly on both addition and subtraction tasks with negative numbers, and substantially more compared to the students participating in the first two lessons (see Table 4.1).

The conclusion drawn from the learning study was that when students had the opportunity to discern the critical aspects, more students learned what was intended. The identified critical aspects were:

- The difference between the signs for subtraction and negative numbers.
- Seeing subtraction as a difference.
- "The perspective"/commutative law does not apply in subtraction.
- The numerical system (larger numbers to the right).

Kullberg (2010) found that when the same critical aspects were enacted in lessons by other teachers who had not participated in the learning study, it generated similar learning outcomes for their students. This suggests the critical aspects can be helpful for other teachers and students.

Shared knowledge products

It has been shown that teachers can use findings about critical aspects and patterns of variation in activities and tasks when teaching new groups of students. However, the critical aspects should not be viewed as final since there could be other

critical aspects yet to be identified, and critical aspects can vary between groups and ages. Hence, the critical aspects need to be further investigated and adapted to the student group. Runesson and Gustafsson (2012) showed that when teachers in Sweden were informed about critical aspects identified in learning studies in Hong Kong about an object of learning concerning creative writing, the teachers could use these critical aspects and identify new ones. Sharing and testing critical aspects and patterns of variation among teachers may contribute to accumulating knowledge about teaching and can develop a knowledge base for teachers. In the long term, this could contribute to more similar learning opportunities for students across classrooms (Morris & Hiebert, 2011).

Knowledge products (e.g., critical aspects and patterns of variation used in activities or a lesson plan) from lesson studies and learning studies can be used to increase the quality of instruction for teachers who have not participated in the study (Kullberg et al., 2019; Morris & Hiebert, 2011). For example, a lesson plan can be shared and examined by other teachers and further improved through repeated implementation trials in classrooms. In other professions, for instance, in health care, it is common to do clinical research and share knowledge products that members of the profession can use as answers to problems they share. For example, the best way to handle a specific disease or wound is systematically studied, and the results are shared and scrutinized by others in the profession. Clinical research aims to improve the quality of treatment, and the results are tested and refined. Similarly, teachers can enhance the quality of teaching different objects of learning. We suggest that teachers' continuous involvement in design-based research, lesson study, and learning study would result in a quality movement in teaching, similar to the quality movement in health care.

What is needed to conduct a learning study

Much evidence shows that learning studies can enhance student learning (e.g., Cheung & Wong, 2014). This is one primary reason why principals should support teachers in conducting learning studies at their schools. There are certain things needed for a team of teachers to be able to run a learning study. For example, the teacher team must be given time to participate in learning study meetings. This could mean that teachers are excused from other school meetings during the learning-study process so the teachers can focus on the study. Before the learning study starts, all meetings need to be pre-planned with some time between meetings. Two weeks between meetings usually works fine. Time is required in between meetings to conduct a post-test after the lessons and have time to analyze the test or interview to some extent before the next meeting. Before the study starts, it is a good idea for teachers to read some research on student learning of the topic in focus. This can help in identifying critical aspects and designing the pre- and post-tests. Nowadays, many research articles are open-access and can be found online. There are also books for teachers about teaching and learning specific content that can be

helpful. During the learning study, there is a shared focus on the problem of finding out what the critical aspects of the object of learning are and finding a way to teach the topic. It is also essential to know that the focus in the analysis of the lessons is on the *teaching*, not the teacher. Since the teacher team designs the lesson collaboratively, it is not the individual teacher's teaching that is scrutinized but the whole team's planned teaching.

Team:	About 3 to 6 teachers and their classes
Time:	6 to 10 meetings × 2 hours (for each participating teacher)
Equipment:	1 video recorder
Facilitator:	A person who has experience in conducting learning studies and knows about the theory is helpful

Note

1 For an alternative model showing three forms of variation, see Lo (2012).

5

PLANNING TEACHING

In the previous chapter, the learning study was presented as a systematic model for the study and improvement of teaching and students' learning regarding a specific object of learning in one or a limited number of lessons. Even though participating in learning studies is a valuable and very rewarding enterprise, it is directed toward collegial development of the teaching and learning of very delimited parts of the curriculum. In contrast, planning teaching is a daily task for teachers that needs to be addressed both for each lesson and on the level of the curriculum for one or several years. We regard the planning of teaching as a serious matter and something that should be attended to with high quality where paying attention to detail is important but also what is given priority. Planning teaching takes time, and engaging in a learning study may be an extensive undertaking. Every teacher or group of teachers does not have the possibility of addressing each part of the curriculum in the form of a learning study.

Thus, an important aspect of professional knowledge for teachers is to draw on different sources, and the most relevant sources, in planning to teach and, within the available time, to give priority to the aspects of teaching that are most important for student learning. In this chapter, we will draw on variation theory and point to the body of knowledge available from learning studies to discuss, from this perspective, productive ways of addressing the challenge of planning specific lessons, as well as how to formulate appropriate objects of learning that may be addressed during a single or a limited number of lessons.

On the overarching level, it is an appealing thought to consider the curriculum as a whole and to link it to a set and sequence of objects of learning that may be addressed in different lessons and that match the available time. In principle this should be possible but would, in practice, be very difficult for the individual

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teacher to achieve, given the available literature, the general structure of educational objectives in the curriculum, and widely used teaching materials (that are typically constituted from principles different from the ones proposed in this book). Rather, this is more appropriate as a collective engagement over time. Instead, we suggest an approach where the kind of planning we propose is primarily used for parts of the curriculum that are both central and difficult for students and that these parts are identified in terms of objects of learning. The domain of such articulated objects of learning may then be successively expanded, broadening our individual and collective knowledge. As discussed in Chapter 2, identifying and appropriately delimiting objects of learning is an achievement in itself and may be revised and refined as a result of planning concrete lessons.

The focus in the chapter will thus to a large extent be on how different parts of a lesson may be organized, given a clear idea of the object of learning in focus and which critical aspects constitute it. Variation theory clearly points to putting the object of learning and what students need to discern to master it, at the center of teaching, whether this is planning teaching, teaching in the classroom, or developing the teaching in specific lessons, all in a systematic way. It also suggests that activities and tasks cannot be decided before the critical aspects have been identified since it is only then that we know what should be brought to the fore of the students' attention. Planning teaching must then focus on how to make the critical aspects noticeable to the students.

The core elements of planning would then be:

- Identifying and delimiting an appropriate object of learning for the specific lesson articulating what, according to different knowledge sources, are potential critical aspects.
- Determining what may be an appropriate sequencing in terms of discernment of different critical aspects as well as necessary simultaneous discernment. This amounts to developing a lesson plan on the level of sequencing of critical aspects in focus through different patterns of variation (contrast, generalization, fusion).
- Generating ideas on how different patterns of variation for different critical aspects may be enacted and contextualized (drawing on both specific knowledge about teaching about this content and about the student group to be taught).
- Determining the specific components that are to be varied, for example, through choosing appropriate examples, illustrations/representations, reading material, and tasks that contribute toward enhancing variation in accordance with the patterns of variation, in a way that is available to and as explicit as possible for the specific group of students.

To use 'variation' in teaching is nothing new per se. Teachers use variation every time they teach, for example, in their choice of examples when they plan and enact

teaching. However, it is paramount to keep in mind that the variation that variation theory points to as essential, is variation that makes critical aspects manifest and noticeable for the learners.

Planning teaching in the way suggested here does place emphasis on the contribution of the teacher to opportunities for learning enacted in the classroom but does not in any way necessarily suggest a teacher-dominated classroom. Rather than being in conflict with a student-centered approach, we would suggest that this way of planning teaching frames and facilitates a student perspective in teaching in a powerful way. We will point to three ways in which the student perspective is essential: the student perspective as a starting point, the potential for flexibility in relation to students' input and dynamics, and an aim that there should be learning opportunities related to the object of learning for *all* students.

Our approach is student-centered in a fundamental way as it takes a student perspective on the object of learning and has as its starting point what it means to understand the object of learning from the learner's perspective. This indicates that planning teaching must necessarily encompass and take into account what the students discern of the object of learning at each point of learning (before teaching, at different stages, and after successfully mastering the target capability or way of understanding). The origins of variation theory from the phenomenographic tradition underscore this, and the idea that understanding fundamentally relates to discerning meaning implies that teaching can never be reduced to an instrumental approach, but will always have to have a balance between the perspective of the teacher (on the classroom as well as the object of learning) and the perspective of the student or learner as they notice new aspects of the object of learning and thus of the world around them. This may be formulated by saying that it is essential that we as teachers get to know our students as learners.

In the classroom practice, we would suggest that careful planning in terms of an object of learning frames and facilitates the potential for the teacher to be flexible in relation to students' input, and shapes the lesson dynamics in ways that at the same time make a learner perspective on the object of learning manifest in the classroom (through student queries, input, and reasoning about different aspects of the object of learning), keeping the students' attention on the intended object of learning and the critical aspects. This amounts to realizing an active engagement in the classroom in relation to the specific object of learning rather than in a more general manner.

The tradition of phenomenography, variation theory, and learning study takes as a starting point the idea that all students will act and reason in accordance with how they perceive meaning in the world, and that in that sense all students are rational. This implies that it is possible for all students to perceive the world and an object of learning in new and more powerful ways, given the chance to discern the critical aspects. In this spirit, we embrace the ambition that there should be realistic and explicit learning opportunities related to the object of learning for *all* students. In that sense, we would argue that expectations on teaching on many

occasions are too low and that the experience from many learning studies is that essentially all (or a very large portion of) students may master the object of learning, even when the object of learning concerns well-known difficult content areas. It seems that the explicit manner in which critical aspects are handled as a result of learning studies is particularly important for low-achieving students (Runesson & Kullberg, 2010). We suggest this perspective is important to consider in planning teaching.

All in all, it is also important to see that there may be considerable differences in the nuances, contexts, and dynamics of the enactment in different classrooms, even when using equivalent lesson plans developed as suggested here. The professional knowledge of the individual teacher and contextual knowledge of the particular student group, as well as the specific time and place, may have an impact on how the lessons unfolds. The skill of the teacher in handling the relationship between what is planned and what happens is essential in order to come close to the intended objected of learning in the enactment. However, variation theory and the evidence from learning studies suggest that the structure, in terms of object of learning, critical aspects, and patterns of variation, is paramount in what is made possible to learn for the students and thus should be given priority in planning teaching.

Keeping the student perspective in mind, we will now develop how planning teaching may be addressed and give examples of concrete plans developed systematically in learning studies.

The object of learning as a starting point

Taking the point of departure in the object of learning and the critical aspects provides a focus on the content and students' ways of experiencing the content when planning lessons.

Defining an object of learning

The object of learning is central to variation theory. When we teach, there is always something that we intend that the students should learn. When deciding what we want the students to learn, we need to consider what it entails to know this/have this capability. The following questions can assist in directing attention to the object of learning:

- What do we want the learners to learn?
- What does it entail to have this capability?
- How do the learners understand the object of learning?
- What may be critical for learners to discern in order to learn the requested capability?
- How can the critical aspects be made visible in activities in teaching?

For example, if the object of learning is "to be able to identify the main message in a text" in reading comprehension, we need to reflect on what this knowing consists of and what the students need to learn to achieve this knowledge.

It is often more difficult to identify critical aspects if the object of learning is too wide. One difficulty is, therefore, to delimit the object of learning appropriately, where it normally encompasses something that may be attended to during one or only a few lessons.

If you identify many critical aspects, this could also indicate that the object of learning is too broad and that you need to delimit it and make it more precise.

We think that all students in a class should have the opportunity to learn what is aimed at by the teacher. Therefore, time needs to be spent on finding out what students with lower pre-knowledge need to learn.

Identifying the critical aspects

We believe that planning teaching necessarily needs to take the learners' understanding of the object of learning into account. Since it is only when we know what the learners know and how they experience what is taught that we know what to teach. Only when we have some ideas about what may be critical for a particular group of learners can we choose activities that have the potential to bring out the critical aspects. In order to identify critical aspects, we need information about students' ways of experiencing the object of learning. If we return to the example about the main message in a text, we need to know how the students experience what a message in a text is, and how it could be identified. Questions posed to students in a written pre-test: "If you are told to identify the main message in a text, what do you do in order to find it?" and "What is the main message in this text"?" could most likely give some information about what the critical aspects may be. Some students may answer, for example, that "you find the message at the beginning of the text", or "you find the main message at the end of the text" while other students may answer that "you find the main message in the details". These answers, separately and together, give valuable information about students' ways of experiencing what the main message in a text is. These answers indicate that it is critical for students to discern, for example:

- The difference between message and aim.
- It is not only at the beginning or the end that the main message can be found.
- It is not only in the details that the main message can be found.

Applying patterns of variation

When critical aspects have been identified, we need to consider how the critical aspects can be made visible and generalized in activities in teaching. Three patterns

of variation – contrast, generalization, and fusion – can be used to make it more likely that the critical aspects are noticed by the students. If we return to the example about the main message in a text, in order for students to discern the difference between message and aim, a *contrast* between the message in the text and the aim of the text needs to be made. This could involve juxtaposing the aim of the text, for example, to inform the reader about mushrooms in the forest, with the message in the text, for example, "do not pick mushrooms when you are unsure whether they are edible or not", in order for the students to differentiate between them. The text is in this case the same, that is, invariant.

In order for the students to recognize that it is not only at the beginning or the end of a text that the main message can be found, the students also need to experience a *contrast* between the different places in a text where the message can be found. In this case, at least two different texts are needed to show the difference when the message is found in different places. Showing several examples of texts where the message is found at the beginning is an example of using the pattern of variation *generalization*. It is not only in this text that the message is found in a certain part of the text. Similarly, several texts where the message is found more in the middle or at the end of the text would also have the function of generalization. The pattern of variation called *fusion* is when two or more critical aspects are varied at the same time and brought to the fore of the students' attention. Fusion provides an opportunity to experience connections between aspects varying in the phenomenon as a whole.

Knowing what to vary in a specific situation and what to keep invariant helps teachers plan teaching. However, the theory does not provide answers to questions about what the critical aspects are, when to vary a critical aspect, how many times, and with what values. These are things that need to be investigated in each individual case by teachers themselves. Moreover, we cannot anticipate that learners will discern the critical aspects even if we planned for this to happen. The learners may direct their attention toward something else. Therefore, it is vital to direct the students' attention toward the critical aspects.

In sum, we suggest an alternative way to plan teaching that stems from the object of learning and critical aspects:

1 Decide the object of learning.
2 Identify students' pre-knowledge.
3 Revise the object of learning if necessary.
4 Identify critical aspects of the object of learning.
5 Choose/design activities that can be used to elicit the critical aspects, and plan patterns of variation.

In the following sections, three learning studies are described from the point of view of how the teaching was based on eliciting critical aspects for student learning.

Teaching about historical enquiry

The following example contains insights about history teaching found by a group of teachers working collaboratively with planning and analyzing teaching and students' learning using variation theory (and the results from a phenomenographic investigation). From insights about how students perceived the object of learning, the teachers were able to identify critical aspects of students' learning and design tasks and activities that could be used to make the critical aspects visible to students by means of variation of specific values.

How did the learners experience the object of learning?

In primary school, it is often difficult for students to distinguish between what is fictional (e.g., characters in books and films) and what are historical facts. Hence, there is a risk that nation-based narratives, for example, may limit the pupils' understanding of history in terms of migration and cultural encounters. The following example comes from a learning study (Johansson, 2019a) conducted by a team of teachers in Grades 4 and 5 (10- to 11-year-old students) about how to interpret archaeological artifacts in historical enquiry with an intercultural perspective. The historical period chosen was the Viking age, which in Sweden is a period associated with a strong national narrative influenced by national romantic ideas. Two objects of learning were focused on during the lessons: to become able to interpret archaeological artifacts and to develop an intercultural perspective (on the Viking age).

The teachers used a written test to identify students' pre-knowledge. The test question "Why have large quantities of Arabic coins been found around the island of Gotland?" was found useful for analyzing students' different ways of interpreting archaeological artifacts in historical enquiry with an intercultural perspective. This question was later also discussed in the planned lessons. A similar question was used in a test after a series of lessons in each class, "How come Chinese silk has been found in the Viking town of Birka?", in order to see changes in students' ways of interpreting archaeological artifacts in historical enquiry.

The students' answers to the test questions showed that archaeological artifacts in a historical enquiry had different meanings for the students (Johansson, 2019b, pp. 90–93). The students answered, for example:

1 *... they transported the coins there on their camels. But when they arrived the camels escaped with the coins that they carried on their backs, and the camels shook off the bags with the coins in the water. There all coins were transported by the current to Gotland where they were washed ashore.* [a literal interpretation of sources in fictitious narratives]
2 *...there were pirates or traders who travelled across Europe to Bagdad. They got hold of the coins and returned to Sweden. They lived in Gotland. They divided the treasure between them and went back to their families...* [a literal interpretation of sources in historical narratives]

3 *The Vikings traded and exchanged many things like cloth, jewelry, weapons and so on. The Vikings had strong ships but they could not go all the way to China to plunder because it was too far and too difficult. The silk had to be traded many times before it ended up in Birka.* [representative interpretation of sources in continuous historical narratives]
4 *I think people from China went by boat/hiked some distance and sold silk. Then Arabs bought silk and then the silk came to Sweden through trade with other countries (…) They probably did this to earn money, because silk was fairly expensive. The reason why people bought silk, I think, was because it was smooth and because they thought it was beautiful. (…) Historical evidence: Boats and rune stones have been found where you can see this.* [representative interpretation of sources in intercultural narratives]

Four phenomenographic categories (Marton & Booth, 1997) were identified concerning students' way of experiencing archaeological artifacts in a historical enquiry. Typical examples of answers representing the categories are shown in the excepts presented above. The group of students experienced *archaeological artifact in historical enquiry with an intercultural perspective* as: (1) a literal interpretation of sources in fictitious narratives, (2) a literal interpretation of sources in historical narratives, (3) representative interpretation of sources in continuous historical narratives, and (4) representative interpretation of sources in intercultural narratives, where the last category (4) is the most advanced way, and the first (1) the least advanced. Table 5.1 shows the critical aspect that distinguishes one category from another and hence what students need to discern in order to develop a more advanced way of understanding archaeological artifacts in historical inquiry. Furthermore, values that can open up the critical aspects as a dimension of variation are shown.

TABLE 5.1 Critical aspects and related values (based on Johansson, 2019b).

Critical aspects	*Values (italics)*
To discern intercultural interaction between rational actors	• *Rational motives (motives vary as values)* for manufacturing material objects • Intercultural aspect of the artifact as related to rational *actors with different motives (actors with motives vary as values)*
To discern historical representativeness and continuity	• *Literal interpretation of artifacts (value)* versus *representative interpretation of artifacts (value)* • *Understanding of history as separate (value) events* versus *history as continuous change (value)*
To discern historicity	• *Fictitious (value)* versus *real events* (value) narratives • *Mythical artifacts (value)* versus *artifacts created by humans (value)*

What was critical for learners to discern?

Experiencing sources as literal in fictitious narratives (Category 1) often involved descriptions of monsters and heroes who had to hide the coins in a fictitious story. The team identified that for these students, it was critical to discern the historicity of artifacts and the historicity of the narrative. Hence these students need to experience a contrast between two values within each dimension of variation, for example, a contrast between the values fictitious and historical narratives, or a contrast between mythical artifacts and artifacts created by humans. In the second category (2), students still had a literal interpretation, but the interpretation was in a historical narrative about one or several separate events. For these students, "history was reduced to events and pupils failed to identify the continuity of cultural encounters" (Johansson, 2019, p. 96). From this, it was identified that a contrast between seeing history as separate events and seeing history as continuous change, and a contrast between a literal interpretation of artifacts and a representative interpretation was needed in order for students to experience history as continuous change and to understand that the artifacts were examples of phenomena and not tied to actual events (Johansson, 2019).

The third category (3), representative interpretation of sources in continuous narratives, involved for example, that a single artifact can represent more than itself and even large historical phenomena. The category reflects that history is made up of parallel processes of continuous change. However, in order to understand sources as representative within an intercultural narrative with rational actors (4), students need to discern the critical aspect of intercultural interaction between rational actors. In order to experience this critical aspect, it is suggested that a variation of rational motives for manufacturing material objects is needed, as well as experience of rational actors' different motives for intercultural interaction. This entails understanding the motives behind why an artifact was manufactured and the intercultural aspect of the use of artifacts.

How were the critical aspects made visible in teaching?

Lessons were designed in order for students to produce, investigate, and answer historical enquiry questions using authentic historical source material and texts, as an alternative to traditional textbook teaching. For example, in order to make a contrast between a fictitious narrative and a historical narrative, the team asked the students to compare two different texts about a similar event, one with a fictitious narrative and one with a historic narrative, in order to see the difference between the texts (Table 5.2 shows excerpts from the longer texts used).

Moreover, design principles for the lessons involved: exploration of authentic and intercultural questions, source interpretation using a model with specific questions, and using replicas of artifacts in the exploration. For example, the teacher used modern coins to introduce enquiry questions, saying "Where do these coins come from? How did they end up in my pocket? What is the history of these coins" that could connect students' prior knowledge to concepts such as value, goods,

TABLE 5.2 A contrast between fictitious and historic narratives (Johansson, 2019c, p. 105, our translation).

Fictitious narrative	*Historic narrative*
A thousand years ago, the Viking Thorsten went on a journey to the east from Uppland across the Baltic Sea. Thorsten wanted to go on an adventure and foray, and find beautiful jewelry for his wife who lived in Birka ...	A thousand years ago, the contacts between Scandinavia and the Arab areas were lively. Scandinavians were experienced seafarers and had boats that could cross oceans and rivers. Trade routes emerged across the Baltic Sea and the Russian rivers that Scandinavian and Arabic merchants travelled on…

and trade, that they could use in the exploration of Arabic silver coins. Besides investigating replicas of Arabic silver coins, other artifacts were also explored using a model for investigation of historical artifacts, answering questions that would direct students' awareness toward, for example, rational motives for why an artifact was manufactured. The model (Figure 5.1) directed students' attention toward key questions for understanding historical artifacts from an intercultural perspective.

What? What is the artefact? What can the artefact tell us about the time of its origination?	**When?** When was the artefact manufactured? Which historical period is the artefact from?
How? How was the artefact manufactured? What was required for the manufacture of the artefact?	**Why?** What was the artefact used for? Why was the manufactured (what was someone trying to achieve)?

FIGURE 5.1 Template with enquiry questions for investigation of historical artifacts in an intercultural perspective (Johansson, 2019a, p. 256).

In this example, we have shown how the teachers used variation theory as a tool to plan history teaching. From phenomenographic categories, the teachers identified critical aspects and values that could be used to elicit the critical aspects, primarily by contrast. The enquiry questions used in the model (Figure 5.1) helped to direct students' attention toward the intercultural perspective when investigating historical artifacts. As in Chapter 1, the object of learning in this study concerns a capability that goes beyond learning only historical facts only and instead involves conceptual tools/concepts that can be used for historical reasoning.

Teaching English grammar

In the following example, teachers' insights about critical aspects and patterns of variation in a learning study about the English grammatical structure, the progressive aspect (PROG) in foreign language learning, is shown. In Chapter 2, we used the same study to illustrate an object of learning in English grammar (Lindström, 2017). In this section, we elaborate on the findings from the learning study and

show how a rather small difference in teaching played a major role for students' learning of the grammatical aspect.

How did the learners experience the object of learning?

In English grammar, the tense-aspect system is an important structure as it builds verb constructions. However, it has been shown that the concept of 'aspect' is difficult for English language learners to master. There are several possible reasons for this, for example, the aspects (simple aspect, progressive aspect (PROG), perfect aspect, and perfect PROG) have different nuances of meaning, and this depends on which tense it is combined with. Moreover, as stated previously, the grammatical structure does not exist in many languages, for example, the Swedish language which was the first language of the students the sixth grade involved in the study. One difficulty is that in Swedish, there is no difference between "I live in Sweden" (simple aspect) and "I am living in Sweden" (PROG), since both versions translate to "Jag bor i Sverige". This is true also for past tense: "I lived in Sweden" (simple aspect) and "I was living in Sweden" (PROG) in Swedish are both translated as "Jag bodde i Sverige". Lindström uses the following examples to illustrate that the PROG provides different meanings: a) Ingmar and Greta *live* in Stockholm and b) Ingmar and Greta *are living* in Stockholm. Whereas a) suggests that Ingmar and Greta live permanently in Stockholm, b) suggests that they live in Stockholm temporarily.

The object of learning in the study was "the ability to use the English grammatical structure, the PROG, in a syntactically and semantically accurate way". Before the study started, phenomenographic interviews with some students showed three ways of understanding the PROG:

a as a marker for discourse
b as a marker for tense
c as a marker for ongoingness

What was critical for learners to discern?

In order to further explore students' understanding of the PROG, the team designed a written test with tasks where the students were to write about pictures that would trigger using the PROG, and a task where students were supposed to compare sentences in pairs with and without the PROG and explain how they understood any differences between the sentences. Four critical aspects were identified from the written test and the interviews.

Critical aspects:

1 To differentiate between tense and aspect.
2 To differentiate between simple aspect and the PROG.
3 To discern the concept of ongoingness.
4 To differentiate between stative and non-stative (dynamic) meanings.

How were the critical aspects made visible in teaching?

According to Lindström, a common way of teaching the PROG in Sweden is by showing many examples with present tense since present tense describes something happening right now (ongoing) and is considered to be easiest. The team designed lessons using variation theory, where they tried to elicit the critical aspects identified. The teacher team identified from the first enacted lesson in one of the classes that it was not sufficient only to use the present tense when discussing the PROG, since the students were not able to separate the simple aspect from the PROG, and the PROG from the tense. In the first enacted lesson, the teacher defined the PROG as "right now", and said for example:

Teacher: They are eating lunch right now […] right now, and then it is now. The progressive form is when something is going on now […] and that sentence means you are doing it right now, progressive form […] -ing on the end, and something is going on right now.

As no progress was identified on student post-test in relation to the PROG, the teacher team revised the teaching to include the past tense in order to elicit the critical aspects. For example, a contrast between simple aspect and the PROG was made possible to experience in discussions about juxtaposed sentences like:

The boat sank (simple aspect),
The boat was sinking (PROG aspect)

He drowned in the pool (simple aspect)
He was drowning in the pool (PROG aspect)

The sentences were used in the following lessons (in the iterative cycle of revision of the same lesson) to prevent students from associating PROG with the present tense and to differentiate between tense and aspect. The sentences have different meanings and could have different outcomes. In the first sentence, the boat sank, whereas, in the second sentence, it is possible the boat was rescued by another boat. Hence, when using the PROG, the outcome of an event is *open-ended and unknown*, instead of, as was discussed in the first enacted lesson, *on-going*. The teachers wanted the students to experience a contrast in outcomes from the sentences in the second example. The following excerpt from the fifth enacted lesson shows this:

Teacher: He was drowning? What does it look like when you are drowning? Do you get a picture in your head?
Pupil: He splashes a lot [the pupil starts to wave his arms around]
Teacher: He splashes a lot. So yeah! We've got someone here in the pool like this, ohhh! [the teacher gesticulates and waves his arms around]. […] Look, it's this, if you drowned, if you have drowned, that's the question – has

he died in any of the sentences? The first sentence, that tells us that he drowned in the pool – full stop. Over there, oh dear, accident, he drowned in the pool. He is dead. The other one –the WAS drowning in the pool – you said 'ing', he carried on. Do we know if the man died or not? Or if he had died or not? Has he died?

Pupil: No!

Teacher: He is here [starts to wave his arms around]. Oh help, help, help, help! I'm in the middle of the drowning.

Later in the lesson, the present tense was introduced. This made the students less likely to see the PROG as the same thing as "happening right now", and instead see the PROG more nuanced, as incomplete, temporary, and open-ended. The study showed that the present tense was not sufficient for students learning the PROG. The results of student learning of the PROG on the post-test showed a significant difference for all classes that participated in the second to the sixth lessons compared to students participating in the first lesson. The example about teaching the PROG shows the importance of the order in which the tenses are used for students having the opportunity to separate the PROG from the tense. These insights may seem to be minor but turned out to have a major effect on student learning.

Teaching reading comprehension skills

The next example is from a learning study about the development of reading skills conducted in fourth and fifth grade (Rosenbaum, 2019; Rosenbaum et al., 2021). It has been advocated that if students learn to process texts, they can develop in-depth reading comprehension skills. The object of learning in the learning study was *the ability to draw conclusions about the main character's personality traits in a story*.

How did the learners experience the object of learning?

In an analysis of a writing task[1] in the learning study, five different ways of discerning and describing a character's personality traits were identified.

Category E: Personality traits are related to and explained in relation to events and the main character's actions in the text, to other main characters' personality traits in other stories, to the student, or to other persons.

Category D: Explicit and implicit personality traits in the text are discerned, and are related and supported with evidence from events and the main character's actions.

Category C: Explicit and implicit personality traits in the text are discerned.

Category B: Only personality traits explicitly expressed in the text are discerned.

Category A: The main character's personality traits are not discerned. Only the story is discerned and retold.

Category E is the most advanced, since in this category the character's personality traits are related and explained in relation to events and the main character's actions in the text. Also, the main character's personality traits are connected to characters in other stories, to the student, or to other persons outside the story. In the least advanced category (A), the personality traits were not discerned at all. In this case, the students only retold the story.

What was critical for learners to discern?

The story chosen to work with in the learning study was *The Dragon with Red Eyes* by Astrid Lindgren. After having worked with planning, implementation, and revision of the same lesson during six cycles of revision in a learning study, the teacher team gradually identified critical aspects for the students' learning. During the iterative process, the teacher team became aware that since the text and the character/personality traits are related to one another, the students' attention needed to be closely directed toward the text to find clues regarding the main character (the dragon). This did not happen in the first three cycles as then the teaching was focused primarily on students' experiences of dragons, and the character/personality traits of the dragon in Lindgren's book were not connected (by the teacher or the students) to what was described in the text.

How were the critical aspects made visible in teaching?

During the first three cycles, the teacher told the students to work in groups and talk about the dragon's personality, and that they should pretend to be the dragon. They then wrote a text, as if they were the dragon themselves, and as a final stage, they made dragons as cutout dolls. The activities caused the discussion in the classroom to focus primarily on the appearance of the dragon ("I think the dragon is old and grey") and students' experiences of dragons from, for instance, movies. The question "How would you feel if you were the dragon?" (Cycle 2), posed by the teacher, made the students focus on other things than the text. As one student said, "If I was a dragon, I would feel dizzy". The enacted object of learning did not in this case coincide with the intended object of learning, since the appearance of the dragon and identifying oneself with the dragon's feelings were in focus, and not the dragon's personality traits in the book. The fact that clues about the dragon's main traits needed to be found in the text and in the actions of the dragon was not in focus during the lessons in the first three cycles. Two aspects, *the appearance of the dragon* and *identification with the main character's feelings* were opened up as dimensions of variation in Cycles 1 to 3 (see Table 5.3). However, these aspects are not critical aspects for the ability to draw conclusions about the main character's character/personality traits in a story.

The teacher team identified in Cycles 4 to 6 that to master the object of learning, it was critical for students to discern *the main character's character/personality*

TABLE 5.3 Aspects opened up as dimensions of variation in Cycles 1 to 3 (Rosenbaum et al., 2021, p. 39).

Aspects	*Values that make the aspect discernable*
The appearance of the dragon	• e.g., grey, blue, old
Identification with the main character's feelings	• e.g., feel sick, sad

Group 1	Group 2	Group 3	Group 4
mischievous	aggressive	goal-focused	selfish
selfish	smart	marginalized	spoiled
empathetic	selfish	spoiled	
stubborn			

FIGURE 5.2 Comparison of character traits from the group task in Cycle 6. Based on Rosenbaum et al., 2021, p. 41).

traits in the text itself (Critical Aspect 1). The teacher in Cycle 5 said, for example, "In the group you should find the character/personality traits that best describe the dragon in the story". In Cycle 6, the teacher said "Your task is going to be that when you present you will show your words and explain why you have chosen them. Get help from the story, how the dragon shows it" (Rosenbaum et al., 2021, p. 41). The students in small groups listed the main character's personality traits (the traits varied, for example, the dragon is an enjoyer of life, self-confident, etc.) identified in the text, which were compared and discussed with the other groups (see Figure 5.2).

As the clues to the dragon's personality traits could be identified through the dragon's actions, it was critical for students to discern *the main character's actions* (Critical Aspect 2). This aspect was made possible to experience when the students had to argue why they thought this was a personality trait of the dragon based on the dragon's actions (e.g., he sings therefore he is an enjoyer of life). Hence, the students had to connect the traits to the dragon's actions (the actions of the dragon varied) (see Table 5.4).

TABLE 5.4 Critical aspects opened up as dimensions of variation in Cycle 4 to 6 (Rosenbaum et al., 2021, p. 44).

Critical aspects	*Values that make the critical aspect discernable*
The main 'character's personality traits in the text itself	• enjoyer of life, self-confident
The main character's actions described in the text	• sings, lays on his back, swims, flies
Similarities between character traits in the story compared to those of dragons in other stories	• outcasts, goal-oriented

Students' previous experience of a personality traits from other stories can be important for allowing them see personality traits in a text. Therefore, it was critical for students to discern *similarities between character traits in the story compared to those of dragons in other stories* (Critical Aspect 3). The traits of the dragon were compared to the traits of another dragon in another book, and the students had to argue why the dragons shared the same personality traits or not. Hence, when comparing the dragon's traits with the traits of another dragon, similarities and differences (e.g., both dragons being goal-oriented) in character traits could be identified. The dimensions of variation opened made it possible for students to experience the critical aspects, and the enacted object of learning coincided to a larger degree with the intended one in Cycles 4 to 6 than in Cycles 1 to 3.

Teaching about atoms and ions

We will now look at insights from a learning study in natural science conducted by three teachers and a researcher (Nilsson, 2014). The students in Grade 8 had previously been taught about the atom and atomic structure (nucleus with protons and neutrons and orbiting electrons). The teachers' previous experience was that students often had difficulties understanding why chemical bonds are formed. Therefore, the team decided that the object of learning was *to understand the concept of ion and how ions are formed*. An ion (e.g., hydrogen ion, H^+) is an atom with an electric charge, meaning that the number of protons is not the same as the number of electrons, as it is in the atom (e.g., H). A positively charged ion has lost one or more electrons, whereas a negative ion has a surplus of electrons compared to the number of protons. An ion could also be a charged molecule (composed of several and possibly different atoms). During the first meeting in the learning study, the teachers discussed the object of learning and decided that the students needed to experience the difference between atoms and ions to separate the two. The following excerpt illustrates the teachers' discussion:

Teacher 2: If we think of ions and atoms, we cannot explain what an ion is without understanding the particles, electrons, and protons. The important thing is that they see the differences and relationships between the ion and the other concepts that are critical for understanding this … for example the atomic structure.

Teacher 1: And really, one cannot talk about this without talking about the periodic table. Why are there ions with minus and plus, what are the differences and the relationships as well? What we can do in the lesson is to show both an ion and an atom to make them notice that both of them have a nucleus of protons and neutrons, both of them have electrons orbiting the nucleus, but a difference between these two is that the number of electrons vary. (Meeting 1) (Nilsson, 2014, p. 1804).

From the excerpt, one can see that the teacher team initially had the idea that they needed to point out how atoms and ions differ. However, as many learning studies show, it is not an easy task to identify what is critical for student learning and to plan a lesson that gives the students the opportunity to discern the critical aspects.

How did the learners experience the object of learning?

In the second meeting, the team analyzed the pre-test that they had designed and given in the classes. The teachers identified, for example, that some students did not understand the relationship between atoms and molecules. Moreover, the students did not know about the atomic structure (although atoms had been taught as a topic). The students confused particles (atoms, ions) at the sub-macroscopic level (non-observable) with chemical elements at macroscopic level (observable). This was shown, for example, by one student asking how many atoms are needed to make a substance. The test also showed for instance that the students had difficulties with separating the concepts chemical "substance" and chemical "element". It showed that the students were aware of different parts of the atom, but they did not mention the charges, and they had difficulties in seeing how the number of shells and their distance from the nucleus affected the reactivity of the atom. Table 5.5 shows students' pre-knowledge of atoms, ions, substances, and elements in the periodic table. The table shows that students had little pre-knowledge of the topic. The results improved a great deal after Lessons 1 and 3, but less after Lesson 2. However, on Task 6, the results were lower after the teaching than before.

What was critical for learners to discern?

The teacher team identified several critical aspects from the pre- and post-tests, their previous experience, and from analysis of the collaboratively planned lessons that they thought students needed to discern (see Nilsson, 2014):

Critical aspects:

1 the relation between the energy level of an electron and its principal quantum number.
2 the atomic structure and how the ionic charge varies depending on the number of valence electrons (electrons in the outermost shell/energy level).
3 ionization energy depends on the number of electrons, and the nucleus with protons and neutrons remain unchanged even if the number of electrons changes.
4 the principles of the periodic table.
5 how atoms with one or two valence electrons more or less than a closed shell are highly reactive due to the fact that the electrons are easily removed or gained to form ions with positive or negative charge.

TABLE 5.5 Results of pre-and post-test (Nilsson, 2014, p. 1801). T(teacher)1= Lesson 1, T2= Lesson 2, T3= Lesson 3.

Question	*Should be included in the response*	*Pre-test*			*Post-test*		
		T1 N=24	*T2 N=17*	*T3 N=16*	*T1 N=24*	*T2 N=17*	*T3 N=16*
1. What do you think of when you hear the word atom?	The atomic structure of the elementary particles	6/24	3/17	4/16	15/24	5/17	11/16
2. What do you think of when you hear the word ion?	An atom or molecule in which the number of electrons is not equal to the number of protons, giving it a positive or negative electric charge	4/24	2/17	0/16	14/24	2/17	11/16
3. Look at the periodic table. What information can you get about the different substances?	It gives information about the number of protons, electrons, electron shell and atomic weight	0/24	2/17	0/16	14/24	5/17	13/16
4. Why does the periodic table look the way it does?	There is a logical explanation that it builds on the atomic structure and the characteristics of different elements	0/24	1/17	0/16	10/24	4/17	5/16
5. Why do you think different elements react with each other in a certain (predictable) way?	Elements react in order to become stable. The reactiveness of an element depends on its electrons	2/24	1/17	1/16	10/24	1/17	9/16
6. How many substances exist around you?	Interminably. There is a difference between chemical element and compound	15/24	11/17	6/16	10/24	10/17	2/16

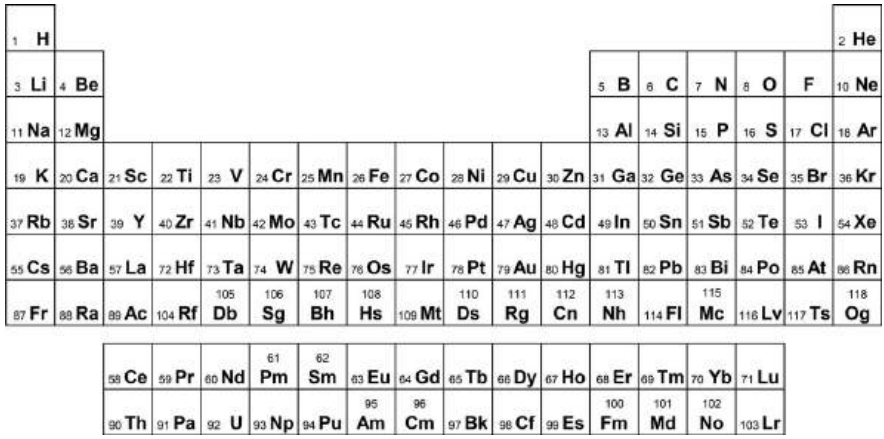


FIGURE 5.3 The periodic table of elements.

In the following section, we will show how some of the critical aspects were enacted in collaboratively planned teaching.

How were critical aspects made visible in teaching?

When starting to plan Lesson 1, the teacher team first wanted the students to discern different components in atoms (protons, neutrons, electrons). The teachers also wanted the students to discern the mechanisms related to the periodic table (see Figure 5.3) and how ions are formed. Moreover, they wanted the students to discern that it is the number of electrons that is critical for the formation of chemical bonds. They wanted to illustrate for the students that elements in the same group in the periodic table (and with the same number of valence electrons in their atoms but with a different number of shells) acted in the same way. They planned to show this by using lithium (Li), sodium (Na), and potassium (K) in water to get the students to reason about differences and similarities within the same group. Furthermore, they wanted to show the relation between energy level and the principal quantum number of the valence electron. For instance, one of the teachers said:

Teacher 1: All substances in group eight have a noble gas structure but if you look at them you see that they have a different number of shells. This is a great opportunity to make the students identify what they have in common and what separates them. They are in the same group and have the same number of valence electrons but they have different numbers of shells. So we can use a demonstration with lithium, sodium, and potassium to make the students see the relation between periods and groups and introduce the concept of noble gas structure. (Meeting 2) (Nilsson, 2014, p. 1805).

Lesson 1 involved lecturing, demonstrations, and small-group discussions. The lesson started with the teacher discussing what an atom is by having a PowerPoint with the question "What is an atom?". When analyzing Lesson 1, the team concluded that they needed to further emphasize the difference between atom and molecule on the one hand, and between chemical element (e.g., hydrogen, H) and chemical compound (made up of two or more different chemical elements combined in a fixed ratio, e.g., water H_2O) on the other hand, since the students still had difficulties in differentiating between concepts. They also wanted the students to be able to use the periodic table to identify differences in the properties of elements.

Lesson 2, enacted in another class, started instead with another question: "What is the difference between a chemical element and a chemical compound?" to put further focus on the differences between concepts. This was followed by the drawing of a mind map to illustrate all concepts seen as important for students to discern. When reflecting over the lesson, the teacher concluded:

Teacher 1: The first question is about the difference between a chemical element and a chemical compound. But what we notice here is that they do not even understand the difference between an atom and a molecule. So it might be better if we revise the questions to focus on the difference between atom and ion and in such a way put a stronger emphasis on the object of learning.

Teacher 3: Yes, here in your lesson you seem to want to include all concepts on the same time through the concept map. Even though you try to make them discern the differences they seem to be quite confused. The lines between the concepts do not seem to make sense to them. (Meeting 4). (Nilsson, 2014, p. 1806)

The teachers realized that the second lesson was too busy and messy for the students to be able to learn the intended object of learning. The lesson offered too much variation of concepts (e.g., mixture, chemical compound, metal, non-metal, atom, molecule, acids, bases) at the same time. The teachers concluded that in trying to cover too much, they lost sight of the object of learning. In the analysis of the lesson, the teachers also became aware that the way they used the metaphor *electron shell* made the students think it was something solid that protects the nucleus of the atom in the same way that a peel protects the fruit or an eggshell protects an egg, which was not intended by the teacher. This way of thinking was shown when one student asked what the shell was made of. Hence the meaning of the word shell became problematic in the context of atoms. One of the teachers suggested that track or energy level would be better since these words do not carry the same meanings as shell does (p. 1807).

Lesson 3 focused more on the object of learning. The lesson started with the teacher posing the question "What is the difference between an atom and an ion?". During the lesson the teacher also discussed several issues known to be difficult for

TABLE 5.6 Main differences in teaching about atoms and ions in Lessons 1, 2, and 3.

Lesson 1	*Lesson 2*	*Lesson 3*
1 What is an atom? 2 Periodic table 3 What is an ion?	1 What is the difference between chemical element and a chemical compound? 2 Mind map of concepts	1 What is the difference between an atom and an ion?
Atom and ion are taught one at a time	*Atom and ion are not in focus*	*Atom and ion are taught simultaneously*

the students, for instance that the metaphor 'electron shell' can be confusing since it easily can be interpreted as a shield or layer of some sort. Moreover, students' understanding of electrons was discussed during the lesson, for instance that it was not the case that "if a valence electron was *situated in a shell far away from the nucleus*, the electron was strongly attracted to the nucleus and hence, the atom should be stable" (p. 1808, our italics). Instead a "strong attraction between the nucleus and the outer electrons leads to the element being less reactive". The teacher used for example of two magnets to illustrate why potassium (K) is more reactive than lithium (Li). "If the magnets were close, they attracted each other but if the magnets were held apart they did not attract each other in the same way. Hence, the comparison between strong magnetic attraction and low reactivity aimed to help the students to understand how the attraction between the nucleus and the electrons increases as they get closer". (p. 1808)

Lesson 3 became more focused on the contrast between an atom and an ion since the two concepts were discussed in relation to one another and at the same time. Hence, in Lesson 3, the enacted object of learning was more in line with the intended object of learning, whereas in Lessons 1 and 2, it was not made possible to experience the two concepts atom and ion in the same way as in Lesson 3. In Lesson 1, the two concepts were taught one at a time, and in Lesson 2 too many concepts were taught at the same time in order to be able to see the difference between the two concepts (Table 5.6).

Teaching the complex and abstract object of learning of technological systems

The next example concerns teaching and learning of technological systems, which are complex systems of technical and human components that fulfil many of the experienced needs of modern society, such as the internet (supplying information), the water-supply system (distributing matter), and the power grid (supplying energy). Systems that are not tangible and consist of components and connections on different levels, as well as human interaction, could be described as complex technological systems. Understanding technology and dealing with technology is important, but teaching and learning in this area is organized quite differently in

different school curricula. In the Swedish curricula, from which this example is taken, together with a few other countries, technology is taught as an independent subject, while in most other countries such topics are integrated, for example, into science. The school subject technology in Swedish schools embraces a broad definition of technology related to human endeavors, rather than tying it (too) closely to science. However, understanding the organization of technology as complex systems has emerged as important over the last few decades, although it remains difficult, abstract, and with little research and established teaching practice to draw on. Scant attention has similarly been paid to related issues regarding complex systems in nature (ecosystems, weather systems) or society (democratic institutions), even though systems thinking has increased in importance with the study of complex phenomena, for example in the context of sustainable development. The study reported by Ingerman et al. (2012) attempts to outline how such teaching can be organized in a way that would allow students to develop systems thinking relevant to technological systems. The plan for teaching is based on subject-theoretical literature on the nature of technological systems, on investigations of students' understanding of technological systems, their function, and the context in which they exist, and on teachers' experience of teaching systems and other topics in the subject technology. Empirical studies of systems thinking so far suggest that the basic capability in (complex) systems thinking is the recognition of a meaningful framework of relationships of seemingly isolated events and components into an interconnected whole, also operating on a different level – that is, seeing something as a system (cf. recognizing it as a phenomenon). This is difficult since many aspects of systems are never directly experienced. In line with this, an approach to planning teaching for this topic was taken in which researchers and experienced technology teachers attempted to cover these aspects since so little well-established teaching practice was available.

What was critical for learners to discern?

Technological systems are goal-directed, delivering both to society and to individuals, but also have unwanted effects. Many such effects concern detrimental influences on the environment, and in understanding the grounds for sustainable development, understanding the systemic aspects of technology (and nature) is paramount. At the same time, technological systems are not tangible, and as such constitute a theme less supported by informal learning than other themes in technology.

Some tentative characteristics of thinking and reasoning in relation to complex systems identified in empirical studies (not restricted to technological systems) are to:

- identify cycles of system *resources* – matter, energy and information,
- identify the *intention* and function of the system,

- recognize that the *internal structure* of the system is organized in terms of interacting components with different (sub)functions, including human interaction (with)in the system, and to
- consider the *external boundaries* of the system and how it interacts with the surrounding world, such as other technological, natural, and social systems, as well as the consequences of the system for humans and the world.

How were critical aspects made visible in teaching?

Through analyzing these as dimensions of variation, patterns of variation were identified that could serve as building blocks for the first planned version of teaching about technological systems. In the dimension of *resource*, there is a qualitative difference between discussing a specific resource, such as the electric current in a desk lamp, and a systems resource, such as the energy distribution in the village. In the lesson, this would be exemplified through resources of distinctly different character –matter, energy, or information – where the resource is tightly connected to the idea and delimitation of the system. In practice, working with a mobile phone communication system, an electricity provision system, and a goods transport system provides opportunities to differentiate between resources, as well as between a system's resource and a specific resource. In the dimension of *intention*, the qualitative shift occurs between on the one hand, a specific person seeing a need and engaging a technological artifact and its intended function as a way of addressing that need, and on the other hand, when a need becomes recurrent, and a community is established as committed to sustaining a shared intention. For example, contrasting a situation when it is possible for everyone to take their bowl to get drinking water from the river, with when the water that is taken from the river is distributed by communal means, whether piped or bottled, is used to afford relevant variation in the classroom. In the dimension of *internal structure*, the difference lies between seeing components of a system as being organized in a linear format and seeing them as being organized in a network (framework of relationships). This implies differentiating between components and their relationships, where the first transform the system resources, and the second transport them, and seeing both transformation and transportation in relation to the system intention. For example, the distribution of postal mail includes a series of transformations of the incoming set of mail each day, such as pooling mail from different mailboxes, sorting mail to different destinations, as well as transporting between the different points of transformation. Disturbances in the system, such as unclear addresses, weather conditions, strikes and illness, and non-adherence to payment regulations, contribute to the non-linear nature of the system. Creating a pattern of variation highlighting the important differentiations in this dimension of variation is complex but may be addressed by contrasting the structure of messenger-direct-delivered mail with a postal service. The dimension of *external boundaries* is not critical in the same way for the basic ability to recognize a series of events as being organized in a

system. Nevertheless, recognizing the limits of a system and what surrounds and interacts with it is important for the differentiation between the particular system considered, and other possible systems that could have been considered.

Following such patterns of separation through contrast, variation that cuts across several individual dimensions is necessary, to afford a fusion of their meaning into an interconnected whole. There are probably several levels in understanding both specific and general technological systems; a start for fusion may be pedagogical tasks focusing on the same technological system and comparing different historical and national installations of the system, for example, food provision systems. A further step necessitates the simultaneous discussion of several full-fledged technological systems differing in the dimensions described above. Realistically, this can only be addressed at a basic level within the lessons planned.

The teaching plan encompassing these dimensions of variation consisted of four lessons. The first lesson took its starting point from the 14- to 15-year-old students' daily morning habits – the students were put in groups with the assignment to document their morning habits and sort habits into common groups. The teacher then prompted the students to discuss what was necessary for these habits to be possible, and which technological means they depended on. This resulted in identifying technological systems the students used daily - typically the water system, the electricity system, and the transport system – and discussing how these systems worked. The second and third lessons focused on examples of technological systems, describing them in terms of their function (in technology the purpose and intended effect of different technological tools/artifacts are described in terms of function), their components and inner structure, as well as their interaction with the world and (unintended) consequences. For specific technological systems, the students worked in groups constructing physical or representational models of the system and in particular of different components and their interplay within a system. One part of the fourth lesson consisted of group presentations of their models, and comparisons and relationships between the different models constructed by different groups. Another part of this lesson (or in some cases a fifth lesson) focused on group discussions on what would be the consequences of disaster or major malfunction in one or several systems in society.

The teaching design was realized in four different Swedish classrooms by four different teachers, as normal lessons in the subject technology. Their classrooms consisted of 15–25 students each.

How did the learners experience the object of learning?

An analysis of the lived object of learning as it manifested itself in the students' presentation of a technological system in the fourth lesson was carried out by Svensson et al. (2015). It showed that most of the students successfully grasped the idea and principles of technological systems together with many concrete aspects of common technological systems. However, it also indicated that the more abstract

and advanced aspects of systems thinking, such as perceiving the system as a network rather than linear, were very difficult. It also turned out that teachers struggled to move away from discussing the systems in linear terms in the enacted teaching without losing sight of the concrete systems discussed. Further, the importance of anchoring different facets of systems in the knowledge domain of technology was identified. It is thus important that both the design of the task and how the teacher addresses the content reflects a balance between technical details and the systemic perspective in order for facilitating systemic thinking connected to the technological knowledge domain.

Teaching about how to run in physical education

In practical subjects, such as physical education, there is less of a tradition of focusing on the teaching of specific content, and overall goals concerning physical movement and introducing a variety of sports tend to come to the fore. For example, in relation to the central aim to develop students' movement capabilities, what specifically the students are supposed to learn may stay relatively vague. In the learning study by Nyberg (2018), a central part of this aim is the ability to discern and experience one's own way of moving. The results show a nuanced picture of the meaning of this ability as well as examples of how to help students learn.

Deciding the object of learning

The ambition of this learning study was to move beyond the normal focus in physical education, where teaching typically does not systematically give the students opportunities to develop their general movement abilities, which is a central aim in the Swedish subject curriculum for physical education and health. Rather, the level of physical activity for the students is given priority, primarily in the form of different sports (such as football, gymnastics, etc.) in which there are expectations regarding both specific ways to approach them and the level of performance in each. Teachers often seem to avoid giving detailed feedback on the way in which students move in and carry out these sports, as there is little time for such development over time (and to reach an expected standard in the movements), and it may be detrimental for the students' confidence to overly focus on what is appropriate movement in each sport (since instruction mainly focuses on correcting errors).

Against this background, the object of learning concerned movement abilities in the context of running. Movement capabilities are then understood to include both physical and mental capacities. The mental capacities included, for example, being able to discern and experience (and thus also possibly analyze) your way of moving in different situations. In deciding an object of learning, the teachers considered the outcome from previous research on how running can be done as efficiently as possible (with respect to energy use), which indicates that a delimited leg stretch at

the kickoff, large angles in the steps, and low activation of the muscles in the lower leg are good for economical running.

On the other hand, advice on a specific technique should be given with caution, as the details of economical movement may vary between people. Further, a good posture is generally considered a favorable factor in running. Discerning and experiencing your own running in relation to these important aspects of expedient running, were taken as constituting the object of learning at the start of the study. This implies a bodily attention that may be difficult to verbalize, and this was understood as one of the pedagogical challenges of the study.

How did the learners experience the object of learning?

The learning study was carried out in two cycles with two classes in the first year of upper secondary school (16- to 17-year-old students), with approximately 20 students in each class. In total, 18 students participated in all parts including lessons as well as pre- and post-test.

The pretest was also used to obtain insights into how the learners experienced the object of learning. It consisted of a task involving running in specific ways, which the students in turn were interviewed individually about. Both the task and the interview questions were developed between the cycles, becoming more specific in order to direct the students' attention toward the object of learning. In the second cycle, the task was directed toward running in some situations with specific purposes: a) running as a run-up for long jump, b) running with rapid changes of direction between cones, and c) pacing yourself when running in order to be able to run long distances. The students were filmed and afterward, they were to describe their way of running in direct relation to these three situations. The questions also probed the components of economical running identified, for example, how the students experienced their posture while running.

Analysis of the interviews allowed a more precise description of the object of learning. The focus concerns getting to know the different parts of your body and how the running movement as a whole is related to and reflected in these parts, how this affects the running, and how appropriate this way of running is in different situations. This included, for example, being attentive to the surroundings and how they affect your way of running in the specific situation as well as how you might need to adapt your way of running. Another dimension concerned attending to the plane of running, such as running sideways, turning, or running in a straight line forward, as well as appreciating the level of tension in different parts of the body and the experience of being tired in different ways.

What was critical for learners to discern?

In accordance with the analysis of the pre-test, it was decided that the object of learning would concern, in accordance with the analysis of the pre-test, discerning

how running is affected by different ways of moving the parts of your body. It concerns noticing how you move your arms and legs, how you place your feet, and how this affects the direction, speed, and efficiency of running.

The following three critical aspects were identified:

1 The importance of the arms and the torso in order to change direction and speed
2 The importance of the placement of the feet for changing direction
3 The importance of step frequency for changing direction

The analysis of the pre-test pointed in particular to the importance of discerning the arms as a structural aspect in running. Observations also indicated that many students would be able to run more appropriately for the purpose if they discerned this aspect.

An additional aspect, less related to the direct physical aspects of running, but nonetheless important, was that many students did not have a vocabulary for describing their movements. For example, one of the students answered in the following way in the interview in the pre-test:

Interviewer:	Describe your way of moving in the different parts of this course, how you moved your body, the whole body
Student:	well, first, eh … there I run sort of sideways like this
Interviewer:	The cone course?
Student:	Yes, and there in the middle sort of … faster steps somewhat shorter and here I run lightly
Interviewer:	It is a bit further between
Student:	Yes, some jogging
Interviewer:	Okey, how did you use your body then?
Student:	Eh … how do you mean…
Interviewer:	How did you move?
Student:	Eh … don't know how I can explain, but I kind of … at the cones I try to kind of turn around… like hell I know how to explain
Interviewer:	well, okey, then we had the run-up to the long jump
Student:	yees, there I just run and jump, that is … sort of that I take a slightly longer step with a bit springier step slightly upwards … I don't know if it helps or not but … that is how I do it anyway
Interviewer:	And then we have two rounds, the long-distance run
Student:	Weell, there I just run, I jog … as usual
Interviewer:	How do you use the body when you jog normally?
Student:	Well, eh … there is no particular way, you just jog (laughter)

The ability to verbalize their way of moving was seen as an important additional aspect to develop in the lesson.

How were the critical aspects made visible in teaching?

The lesson was set up with a series of tasks, focusing on one critical aspect at a time. The first part concerned tasks aimed at facilitating the discernment of the critical aspect of the arms as a structural aspect of running, and how this affected their own and others' way of running.

In the first task in this part, the students formed groups of three that were to run back and forth between two lines approximately 20 m apart in four different ways: A. Arms fastened with ropes behind the back. B. Arms fastened with ropes in front of the body. C. Arms stretched upwards above the head holding a stick. D. No constraints on how the arms were positioned.

After trying each way of running, the students in the group discussed their experience regarding the different ways of running. This was partly aimed toward verbalizing but was also to direct attention to various ways in which the students experienced that the different positions of the arms affected their running.

In a second task, the aim was that the students should see differences in different ways of using the arms in the run-up to a long jump. In this case, one must accelerate to reach a high velocity and the question the students should think about was how the arms may be used to be helpful. The students ran in pairs back and forth on a straight course in two different ways: Once with exaggerated swinging of the arms in the direction of running and once with the arms constantly hanging straight down along the side of the body. They were to be mindful of how the different ways of using the arms affected the length of the stride and the frequency of steps. Then they were to describe their experiences to each other.

The second part of the lesson concerned tasks aiming to facilitate discerning tension and how this affected running. In particular, it was important to experience differences in tension in the torso in relation to experiencing expedient ways of using the arms to maintain a relatively high speed during long-distance running without using excessive amounts of energy, that is, to run in an economical way.

In the first task, students in groups of three jogged three laps in which the students each ran in front during one lap, while the two other students ran behind mimicking how the first student ran. The task was to attempt to run in a relaxed manner. After each lap, the student running in front described their way of using their arms. In connection with this task, the teachers also reminded the students about terms that might be used to describe positions and movements of different parts of the body, which had been a part of a previous lesson. The task was followed up with a discussion of what it might mean to run in a relaxed manner, with the aim of agreeing on what kind of tension was appropriate for long-distance running and how it felt to run in that way.

The lessons in the second cycle gave a distinct outcome in the students' learning, as it manifested in the pre- and post-test interviews. The largest difference was in a clear increase in the number of responses that in some way articulated the position and movement of different body parts and how this affects running with different

purposes. There was also a clear difference in the way that the students were able to feel and discern tension in shoulders, arms, and torso, and relate this to running.

This study thus shows examples of how learning about bodily movement can be developed through focusing the lessons on specific elements of the movement, starting from a specific movement with a certain technique. Given the tradition in the subject of physical education and health of focusing on abilities to do sports in line with competition logic, the way of teaching outlined in this learning study forms a clear alternative with distinctly more concrete learning. This learning study also gives important examples of how silent bodily knowledge may be articulated and be made the focus of teaching based on identified critical aspects.

The use of variation in tasks and task sequences

In the final part of this chapter, we leave studies on teaching different content to the side and look more broadly into the use of variation in the design of tasks. The use of variation and invariance in teaching is, as been pointed out previously, nothing new, and not exclusive to the variation theory of learning. What is varied and what remains invariant within and between examples and tasks may be planned by the teacher, and can most likely help students discern aspects of the content being taught. Watson and Mason argue that "tasks that carefully display constrained variation are generally likely to result in progress in ways that unstructured sets do not" (Watson & Mason, 2006). Systematic use of variation and invariance may be used in all subjects. However, in the following section, we will focus primarily on mathematics tasks since systematic variation in mathematics education has a history of being commonly used. Insights from learning studies have, however, shown us that the same set of tasks can be implemented differently in lessons and that this can provide different learning opportunities depending on what the teacher directs the students' attention toward (Kullberg et al., 2014). Despite this, we argue that carefully designed tasks with systematic variation are an important part of trying to achieve powerful learning opportunities.

Variation within and between tasks

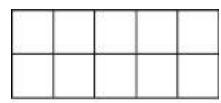
Several studies show that what examples and tasks are used in teaching, and the order in which they are introduced, matter for students' opportunities to see, for examples, connections between tasks (Kullberg & Skodras, 2018; Kullberg et al., 2014; Rowland, 2008; Watson & Mason, 2006). Let us compare two sets of tasks.

a.	*b.*
25 – 6 =	17 – 9 =
23 – 5 =	27 – 9 =
24 – 7 =	47 – 9 =

In the first set (a) of tasks, both the minuend and the subtrahend vary, and the operation sign for subtraction is invariant. The set of tasks seems more randomly chosen compared to the second set (b). In the second set of tasks, the subtrahend (9) and the units (7) are invariant, while only the tens in the minuend vary (17, 27, 47). Whereas the first sequence may be used to practice subtraction tasks requiring a strategy that bridges through the ten, the second sequence is most likely intended to bring students' attention to a previously taught strategy for solving in order to generalize it (e.g., "decompose-10-and-add", 17 – 9 =, 10 – 9 = 1, 1 + 7 = 8 or "down-to-ten-even", 17 – 7 = 10, 10 – 2 = 8) in higher number ranges. Systematic variation can be used *within* or *between* tasks. The sequence 17 – 9 =, 27 – 9 =, 47 – 9 =, is an example of systematic variation *between* tasks. However, which set of tasks is more powerful depends on what the teacher wants the students to discern.

The tasks in the next example of using systematic variation come from an American teaching material (Cameron & Fosnot, 2007). When the sequence of tasks was enacted in a Swedish classroom (Kullberg & Skodras, 2018), the students had the opportunity to experience some mathematical principles. In the lesson, the students were prompted to see multiplication as an array with rows and columns, where 2×5 is an array with two rows of fives. Looking at the first three tasks (see Figure 5.4), the first factor in the second task (4×5) is doubled in comparison to the first task (2×5), and in the third task (4×10) the second factor is doubled ($4 \times 10 = 2 \times (4 \times 5)$).

In the fourth task (10×4), it is made possible to experience the commutative law ($10 \times 4 = 4 \times 10$) hence to see how the third and the fourth tasks are connected. This task is followed by two tasks 10×6 and 6×10, which also can be used to illustrate the commutative law. The last two tasks allow the students to discern the distributive law and that, for example, $10 \times 12 = (10 \times 10) + (10 \times 2)$ or $10 \times 12 = (10 \times 6) + (10 \times 6)$, and $10 \times 18 = (10 \times 12) + (10 \times 6)$. In the sequence, the tasks



2 x 5
4 x 5
4 x 10
10 x 4
10 x 6
6 x 10
10 x 12
10 x 18

FIGURE 5.4 The set of examples (string) (Cameron & Fosnot, 2007).

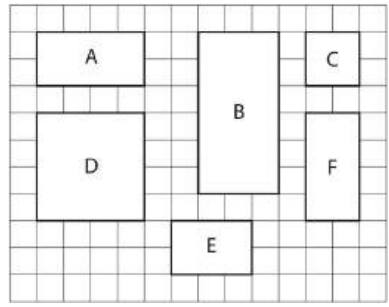


FIGURE 5.5 Textbook task "Which of the rectangles show similarity and which ones are congruent" (Olofson & Gerholm, 2018, p. 314).

were designed to make it possible to see how the solving on one task could be used to solve another task. The set of examples (string) made it possible to understand aspects of multiplication that could be generalized to other multiplication tasks. Although this string was not designed using variation theory, we can analyze the sequence with variation theory. We can describe what varies and what is invariant. When only one value changes, we can regard this as a contrast, for example, when 2×5 is followed by 4×10. In this case the students are supposed to see the difference between the number of rows. Whereas when 10×4 is followed by 4×10 it is an example of generalization, hence it is the same number, but the positions of rows and columns have changed.

One example of variation *within* a task is "Which of the rectangles show similarity and which ones are congruent" from a textbook in Grade 10 (see Figure 5.5). In this task, contrast is used in order for students to distinguish *similarity* (C and D, B and F, the same relationships between sides (shape) but not the same size) from *congruence* (A and F, same size and shape). These two concepts are often confused by students, and the task affords an opportunity for the students to see the difference. In the task, the size and shape of rectangles vary for both similar and congruent rectangles. The features of a similar versus a congruent rectangle vary, whereas within the attributes within each concept are invariant.

The next example (see Figure 5.6) shows how variation between tasks was used in a learning study about multiplication and division in Grade 8 (Kullberg et al., 2014). Research has shown that students often over-generalize previous knowledge about the idea that in multiplication, the product is larger than the two factors whereas in division the quotient is smaller than the numerator. Although this is true when the number that is multiplied by, or divided by, is larger than 1, it is not true for numbers below 1. In the learning study, the object of learning was that the students would understand that when the denominator is smaller than 1 (e.g., $\frac{100}{0.5} = 200$), the quotient becomes larger than the numerator. The teachers designed a series of tasks to make a contrast between tasks where the denominator

100 x 20 = 2000	$\frac{100}{20} = 5$
100 x 4 = 400	$\frac{100}{4} = 25$
100 x 2 = 200	$\frac{100}{2} = 50$
100 x 1 = 100	$\frac{100}{1} = 100$
100 x 0.5 = 50	$\frac{100}{0.5} = 200$
100 x 0.1 = 10	$\frac{100}{0.1} = 1000$

FIGURE 5.6 Task sequence about multiplication and division used in a learning study.

was both larger and smaller than 1 in order to bring out a critical aspect: the relationship between the numerator, denominator, and quotient. As multiplication and division are connected, it was also critical to show what happened with multiplication. There was a pattern of variation built into the set of tasks. The operations varied, but the numbers used were the same, which made it possible to discern differences between multiplication and division. The numbers in each column vary from larger to smaller numbers, keeping the first factor (100) or the numerator (100) invariant. Two tasks with a denominator smaller than 1 (0.5 and 0.1) were used to be able to show that this is true for "all numbers" below 1 (generalization).

In the learning study, it was identified that the teacher in the first cycle did not direct the students' attention toward the relationships between the numerator, denominator, and quotient as planned, but instead focused on the calculation of tasks and the relationship between multiplication and division (that a division can be checked by a multiplication). It was not until the second and third cycle that the teachers succeeded in bringing the intended relationship to the fore of the students' attention. For example, in Cycle 2 the teacher said: "here the quotient is smaller than the numerator, is it always like that?". The question made it possible for the students to experience a contrast between division where the denominator is larger and smaller than 1. The point we want to make here, without going into further details, is the necessity of directing the students' attention toward the critical aspects that can be shown by means of variation in the sequence of the task. If students' attention is not directed toward the critical aspects, we cannot expect that the students will learn what is aimed for by the teacher.

Tasks using a variation of students' ways of experiencing a phenomenon

Variation within a task can be based on different ways of experiencing the same phenomenon. The following example comes from a learning study in the sixth grade about the density of rational numbers (Kullberg, 2010). The teachers wanted the students learn that there is an infinite number of decimal numbers between two

numbers. In the pre-test the teachers used the following task: *Ann says there is a number between 0.97 and 0.98. John says there is no such number. Who is right and why?* The students from three classes gave the following answers:

- there is one number 0.97.5
- there are many numbers between 0.97 and 0.98
- there are ten numbers between 0.97 and 0.98
- there is one number 0.975
- there are no numbers between 0.97 and 0.98

The task about Ann and John and the five categories of student answers was later used as a task in the lesson and was discussed in small groups. The students were asked to discuss the different answers and afterwards draw a number line showing what is between 0.97 and 0.98. In the following whole-class discussion, the number lines from the different groups were discussed. The results of the learning study, however, showed that after the first lesson in the learning-study cycle, most small groups answered that there were only nine or ten numbers between 0.97 and 0.98. Hence, in this case it was not enough to discuss the variation of answers. It was not until the second cycle, where the teacher had discussed before the group task the idea that an interval between two numbers can be split in an infinite number of pieces, and where other critical aspects had also been discussed, that the students answered after the group task that the interval between 0.97 and 0.98 could be partitioned into infinite number of pieces. Lesson 1 showed that it was not enough to only discuss the different students' answers, because the students had not had the opportunity to discern that there could be an infinite number of numbers between two numbers. After Lesson 2, 95% of the students answered correctly on another task about the density of rational numbers. The variation in student answers gave the teacher an opportunity to address aspects emanating from the students' different ways of experiencing the density of rational numbers, represented by the different answers.

Note

1 Pre-test task: Many of Astrid Lindgren's stories have become movies, and now it has been decided that *My nightingale is singing* should also be filmed. The actor who is playing Malin (main character) has not read the story and you are assigned to tell her about Malin. The actor needs as detailed a description as possible of how Malin is as a person in order to play the role as good as possible. How would you describe Malin for the actor? (Rosenbaum, 2019).

6

ANALYZING TEACHING

An essential part of our quest to develop possibilities for student learning is to turn our inquisitive and analytical eye to what happens in the classroom. This shifts the perspective from what was focused on in the previous chapter – the planning of teaching – and scrutinizes this when it is put to the test and is realized and enacted in the classroom. This reflects that it matters not only what we intend to do and what we think happened. Rather, it matters what actually happened and what was the result of teaching in terms of student learning. That is, our focus is on how teaching took manifest form and was enacted in the classroom, and what was made possible to learn about the object of learning. This shift also changes the perspective from what the teacher plans, does, and knows to a perspective that is more of an observer perspective in relation to the enacted teaching in the classroom. The enacted teaching includes not only what the teacher actually did but also how the students and their interplay with each other, the teacher, and, for example, teaching materials, contributed to shaping the collective dynamics and focus in the classroom.

As we have mentioned repeatedly in this book, we insist that the primary purpose of teaching is to facilitate distinct possibilities for concretely learning about an object of learning. This implies that the teaching in a specific lesson may be directly investigated for relationships to the opportunities for and outcome in terms of actual learning. This may be investigated both as a whole – the whole lesson in relation to the object of learning – and in its parts – different parts of the lesson in relation to critical aspects of the object of learning. This indicates that it is valuable to put considerable effort into specifying (and articulating) the relationship between teaching and learning in such a concrete manner since that opens up the possibility for change and refinement. In particular, such change may include

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innovatively shifting teaching in small steps in such a way that it can to a larger degree facilitate possibilities for learning about the object of learning, both as a whole and in terms of its parts.

This central characteristic of the approach described in this book is markedly different from other traditions that are similarly interested in teaching, the classroom, and learning. In these approaches, the focus may be more on the teachers, and their collaborative process, which from our perspective does not offer an anchoring in student learning. A perspective on learning may be taken where the core of that learning is seen as participation or language use, which implies a much more indirect relationship between specific lessons and learning, as each lesson then by definition only can contribute a small part of a much larger whole, and the outcome of a lesson by definition is relatively disparate, both for different students and in terms of what is taken further by the students. Our approach also differs from that of a larger quantitative evaluation of student learning, which would typically investigate the relationship between the general quality or character of teaching or the teacher and the level of knowledge in the subject area more broadly. Such relationships are interesting on an overarching level but can tell us little about how exactly to act or make choices about teaching in a specific situation.

Analyzing teaching makes teaching as enacted practice the object of scrutiny. That means that even though the teacher is a central actor, it is not the person as such that is in focus; rather it is an object – enacted teaching – that in principle moves from the personal domain of the teacher to the professional domain of teachers, in which to some extent insights may be shared. This shift also transforms professional practice into empirical data, which the analyzer may dissect objectively – at least to some extent.

The purpose of such an analysis is then to operationalize relationships between the object of analysis (enacted teaching) and its goal (learning as specified by the object of learning) in terms of dependencies. Such an analysis will then provide knowledge to those who conduct it at a minimum, and potentially more broadly depending on the scope and level of systematicity in how the analysis is conducted and documented.

An important strategy in analyzing teaching has been the comparison between lessons that have the same intended object of learning (or at least where the same specific content is covered), which supports identifying and articulating differences in how the same content is handled in the enacted teaching, as well as differences relating to what may be learned. This may in turn on the one hand define more specifically what constitutes the lesson and on the other hand give clues and ideas to what might be done differently in the teaching in order to facilitate the intended purpose, and how this might be done. Analyzing teaching in this way also points to a fundamental difficulty in educational practice and research: That of describing the constituent parts of a lesson in such a way that it may be "reproduced" in its

primary aspects. For example, if a lesson is deemed to be good from some perspective, it is not trivial to articulate what are salient aspects of the lesson that are also constituents of what made it good. As a teacher, it may not be trivial to share the experience of a lesson you value for particular reasons in a way that a colleague – at the same school, or at some other school nationally or internationally – may reuse in a lesson that has similar value.

Analyzing lessons in collaboration with other teachers can sharpen the teacher's analytical eye and can contribute to a shared view of teaching. The focus on the teaching, instead of the teacher, makes teaching a more neutral object for analysis. Collaborative analysis can result in insights about, for example, critical aspects that can be shared with other teachers who have not participated in the analysis. Insights from the collaborative analysis of teaching can also make the teacher see future teaching with new eyes and maintain an analytical perspective also when it comes to planning lessons on their own.

Video-recorded lessons, and not only those from learning studies, can be analyzed using variation theory as an analytical tool. Video-recorded lessons make it possible to analyze parts of the lessons repeatedly and identify details that could have been overlooked during one single observation of the lesson. The teacher can record a lesson by placing a camera at the back of the classroom, allowing both the teachers' and the students' questions and actions to be captured. Providing information to legal guardians and obtaining their consent may be necessary if the students are young.

What is made possible to learn

Variation theory can be used both for planning and analyzing teaching. The lesson does not have to be planned with variation theory in order to be analyzed with variation theory. In the analysis of a lesson, we look at what aspects are opened up as dimensions of variation and how the teacher directs the students' attention toward the dimensions of variation. If we are aware of critical aspects, we can look at whether these critical aspects are opened up as dimensions of variation. When we compare lessons with the same object of learning, it may be easier to draw conclusions about the different learning opportunities the lessons provide.

The relationship between teaching and learning is often described as being too complex, making it rare for conclusions to be drawn about how teaching may have affected students' learning. This view is unhelpful if we want to develop teaching in order to enhance student learning. In our analysis of teaching, we put the content taught and learned and how the content is handled by the teacher and the students at the center of attention. Although there are many different factors that also can affect student learning, we choose to analyze what we can actually change and develop: The teaching.

We suggest that variation theory can help us to understand better how teaching affects student learning. We believe there is a relation between what is afforded in a lesson and what is learned by the students. By means of variation theory, we can analyze *what is made possible to learn* during a lesson, based on an analysis of what aspects of the content are opened up as dimensions of variation in the lesson. Analyzing what is made possible to learn also provides insights into *what is not made possible to learn* during the lesson. This can be equally important to know. Our experience from learning studies and from analyzing a substantial number of lessons with variation theory over almost 20 years tells us that in many of the lessons we have analyzed, it was not possible for the learners to learn what was intended because the critical aspects that were necessary for the students to discern were never brought up during the lesson. The lesson may have shifted direction, resulting in a discussion of something other than the intended object of learning. In other lessons, it was not made possible for the learners to perceive the critical aspects since the teacher only told the students about them, but it was never possible to experience them. If it was not made possible to experience critical aspects of the content in a lesson, we could not expect that students would learn. Therefore, analyzing teaching is vital if we want to understand what teachers need to do to enhance student learning. If it is made possible for students to learn what is intended, this does not automatically entail that the students will learn. There is no one-to-one correspondence between teaching and learning. Students can direct their attention toward something other than the object of learning.

If we want to know *what is made possible to learn* in a lesson or a learning situation, we need to analyze the enacted object of learning, that is, what aspects of the content are opened up as dimensions of variation by the teacher and the students in the situation. As pointed out previously, there is a difference between the intended and the enacted object of learning. A teacher may have the intention to open up a critical aspect as a dimension of variation in a lesson but may not succeed in doing so or forget to bring it up. Hence the dimension of variation is not opened, and students do not have the opportunity to experience the aspect in the intended way. Students may also open up dimensions of variation in lessons by raising questions or eliciting critical aspects of the content in discussions. In the analysis of lessons, we focus on what aspects of the content come to the fore of students' attention on a whole-class level through patterns of variation (contrast, generalization, and fusion). In analysis of lessons about the same content, we most often find that the object of learning is handled somewhat differently. These differences provide different possibilities for students to learn what was aimed at by the teacher. Although the differences in some cases seem small, we argue that even minor differences in how the content is handled can play a significant difference in students' opportunities to learn.

Student learning outcomes (the lived object of learning) can also be used to analyze teaching and what was made possible to learn from a lesson. For example, by

comparing pre- and post-tests (or interviews) from one or several classes, we can draw conclusions about what was made possible to learn in a lesson. This way of analyzing teaching and drawing conclusions from tests about what critical aspects have been discerned by the students is used in the learning study process. Analysis of student learning outcomes is a complement to analysis of the actual teaching. In the following sections, we will show how teaching may be analyzed using variation theory.

Analyzing teaching about solving equations with one unknown

We will now show an analysis of two lessons, taught by the same teacher, which were found to have different enacted objects of learning (Kullberg et al., 2017). This analysis focuses primarily on what tasks and examples the teachers used in the lesson to elicit aspects of the content taught. We do not go into detail in this text about, for example, how the teacher directed the students' attention toward critical aspects, which is usually of the utmost importance to analyze. The lessons were taught in seventh grade and focused on the same topic, solving equations with one unknown using the cancellation method. The teacher was participating at the time in a research project about the effect of learning studies on teachers' learning. A part of the project was that the teachers should teach a lesson about a topic of their choice before the learning study intervention and teach the "same lesson" again after having participated in three consecutive learning studies. There were two years between the lessons. The difference between the lessons is interpreted as a change in the teacher's knowledge after having participated in the intervention. What the teacher focused on in Lesson 2 differed to a large extent from Lesson 1. We also see that the teacher used multiple examples of the same focused feature with pre-planned variation in Lesson 2. Table 6.1 shows an overview of the lessons. If we look at the tasks used, we can see that the teacher focuses on different aspects of solving equations in the two lessons. For instance, it is only in Lesson 2 that the operations used for solving (addition, subtraction, multiplication, and division) are brought up to illustrate the cancellation method. In Lesson 1, the equations are primarily connected to a context (by visual representation and word problems) and solved with the cancellation method. The meaning of the equal sign is brought up in both lessons. Next, we will go more into detail in the analysis of each lesson.

The teacher started Lesson 1 with a discussion about what an equation with one unknown is by introducing a word problem[1] (solved later in the lesson). Afterward, equations were used to address the meaning of the equal sign. The number 12 is invariant in the first two equations, and the placement of the equal sign and the factors vary. In the third equation, the number of factors varies on both sides of the equal sign. In this part, it is possible to discern that the number of factors can vary on both sides of the equal sign. When discussing the fourth equation, the teacher made a contrast between an incorrect, $3 \times 6 = 18 \times 2 = 36$,

TABLE 6.1 The tasks and examples used in the lessons.

Lesson 1	*Lesson 2*
The equal sign $3 \times 4 = 12$ $12 = 3 \times 2 \times 2$ $12 \times 2 = 3 \times 2 \times 2 \times 2$ $3 \times 6 \neq 18 \times 2 = 36$	The equal sign $12 \times 4 \neq 48 \div 2 = 24$ $6 \times 4 \neq 24 \div 2 = 12$ $6 \times 4 = 48 \div 2 = 24$ $6 \times 4 = 12 \times 2 = 24$ $6 + 2 = 8$ $6 + 2 \neq 8 \div 2 = 4$
Equation with one unknown solved with visual representation $3x + 5 = 20$	Multiplication with x $3 \times x = 3x$ $3 \times 4 \neq 34$
Non-solvable equations for positive integers $3x + 20 = 5$ $2x + 3 = 3x + 4$	Division with and without x $5 \times 3 \div 5 = 15 \div 5 = 3$ $5 \times 3 \div 3 = 15 \div 3 = 5$ $6 \times 4 \div 6 = 4$ $6 \times x \div 6 = x$
Equations with one unknown as word problems $\frac{x}{3} + 495 = 1975$ $3x + (x + 5) + x = 40$	Addition, subtraction, and multiplication $3 + 4 = 7$ $\mathbf{2} + 3 + 4 = 7 + \mathbf{2}$ $2 + 3 + 4 - \mathbf{5} = 7 + 2 - \mathbf{5}$ $\mathbf{2} \times 3 + 4 \neq 7 \times \mathbf{2}$ $\mathbf{2} \times 3 + 4 \times 2 = 7 \times \mathbf{2}$
	How to create and solve an equation with one unknown $3 + 4 = 7$ $x + 4 = 7$ $3 + 2x = 7$ $\frac{6x}{4} + 4 = 7$

and a correct way, $3 \times 6 \neq 18 \times 2 = 36$, of using the equal sign. The use of the equal sign varied, and the numbers were invariant. It was made possible to discern that all parts of the equation need to have the same value for the equal sign to be used correctly.

$$3 \times 4 = 12$$
$$12 = 3 \times 2 \times 2$$
$$12 \times 2 = 3 \times 2 \times 2 \times 2$$
$$3 \times 6 = 18 \times 2 = 36 \quad 3 \times 6 \neq 18 \times 2 = 36$$

Varied	*Invariant*	*Discernment*
Factors on each side Placement of the equal sign =, ≠	12 3×6	Meaning of equal sign – all parts of the equation need to have the same value for the equal sign to be used correctly

Next, an equation with one unknown ($3x + 5 = 20$) was solved with iconic representation (using stones and boxes) and numerical calculation using the method of cancellation, hence the representation varied (iconic and numerical) and the equation was invariant (Figure 6.1). It was made possible to discern a visual representation of the solving method, and that x could be represented by a number of objects.

Varied	*Invariant*	*Discernment*
Iconic and numerical representation	Method of solving	A visual representation of the solving method and that x could be represented by a number of objects

After that, the teacher presented two equations with one unknown that could not be solved with positive integers, $3x + 20 = 5$, and $2x + 3 = 3x + 4$. In this part, it was possible to experience a contrast between non-solvable equations and solvable equations (e.g., as shown in the previous task). It was made possible to discern that some equations are not solvable for positive integers. At the end of the lesson, the teacher presented two-word problems that could be solved with equations with one unknown $\left(\frac{x}{3}\right) + 495 = 1975$ and $3x + (x + 5) + x = 40$), and these were discussed one at a time. The problems varied in context (price and age[2]), and the operations (addition, division) used. However, the operations as such were not discussed. The same method (invariant) of solving was used. In this part, it was possible to discern that equations with one unknown can solve different problems and represent different situations. In sum, in Lesson 1 the enacted object of learning was *learning a method and procedure for solving equations with one unknown*. However, little variation regarding the method of cancellation was enacted for the students to experience.

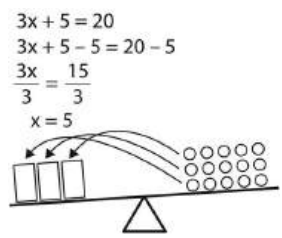

FIGURE 6.1 A numerical and iconic representation of the solving process. Five stones on both sides of the scale were deleted as a first step.

Varied	*Invariant*	*Discernment*
Context for the equation with one unknown (age, price)	Method of solving	Equations with one unknown can solve different problems and represent different situations

In Lesson 2 (with another group of students two years later), the enacted object of learning was instead *understanding the structure of an equation, how an equation with one unknown can be created, and how operations can be used to solve it*. The lesson started with a discussion about the meaning of the equal sign, starting with two examples of incorrect use of the equal sign, introducing the sign for inequality. In the sequence of examples, the numbers used in the equations were most often invariant (6×4 or $6 + 2$), the signs ($\neq$, $=$) varied, and the operations varied (multiplication, division, addition). We suggest that the contrast between incorrect and correct ways of using the equal sign made it possible to discern that all parts of the equation need to have the same value for the equal sign to be used correctly.

$12 \times 4 \neq 48 \div 2 = 24$

$6 \times 4 \neq 24 \div 2 = 12$

$6 \times 4 = 48 \div 2 = 24$

$6 \times 4 = 12 \times 2 = 24$

$6 + 2 = 8$

$6 + 2 \neq 8 \div 2 = 4$

Varied	*Invariant*	*Discernment*
$=, \neq$	6×4 $6 + 2$	Meaning of equal sign

This was followed by several sequences about operations in equations. First, a contrast highlighted how multiplication works with x, compared to multiplication with integers ($3 \times x = 3x$, *however* $3 \times 4 \neq 34$). After that addition, subtraction, and multiplication in an equation were discussed. First, the teacher showed how multiplication and division are connected, that if you multiply two numbers and then divide by one of the numbers, the answer is the other number. A generalization was enacted, since several examples showed the same thing.

$5 \times 3 \div 5 = 15 \div 5 = 3$

$5 \times 3 \div 3 = 15 \div 3 = 5$

$6 \times 4 \div 6 = 4$

$6x \div 6 = x$

Varied	*Invariant*	*Discernment*
Denominator (5, 3) Factor (4, *x*)	Operations (×, ÷)	Relationship between multiplication and division

The teacher then showed that if you add or subtract the same number on both sides of an equation, this does not change the equality (generalization). However, with multiplication (contrast), you need to multiply both terms to keep the equality (see $\mathbf{2}\times 3+4\times\mathbf{2}=7\times\mathbf{2}$).

$$3+4=7$$
$$\mathbf{2}+3+4=7+\mathbf{2}$$
$$2+3+4-\mathbf{5}=7+2-\mathbf{5}$$
$$\mathbf{2}\times 3+4\neq 7\times\mathbf{2}$$
$$\mathbf{2}\times 3+4\times\mathbf{2}=7\times\mathbf{2}$$

Varied	*Invariant*	*Discernment*
Operation (+, −, ×)	Numbers	All terms need to be multiplied to keep the equality

The teacher used the same numbers and operation, 3 + 4 = 7, to introduce what an equation with one unknown is. He substituted one of the numbers with *x* which meant that the value of *x* was already known to the students ($x + 4 = 7$). In a similar way as in Lesson 1, a method for solving the equation was presented; however, as the students did not have to calculate *x*, the focus was on the structure and the solving of the equations (see Figure 6.2). In the same way, and with the same numbers, one equation with $2x$ and one with division with *x* were introduced and solved ($3 + 2x = 7$ and $\frac{6\cdot x}{4} + 4 = 7$). The three equations were on the board

$$3+4=7$$
$$\underline{x+4-4=7-4}$$
$$x=3$$

$$3+4=7$$
$$3+2\times 2=7$$
$$3+2\times x=7$$
$$3-3+2x=7-3$$
$$\frac{2x}{2}=\frac{4}{2}$$
$$x=2$$

$$3+4=7$$
$$\frac{12}{4}+4=7$$
$$\frac{6\times 2}{4}+4=7$$
$$\frac{6x}{4}+4=7$$
$$\frac{6x}{4}+4-4=7-4$$
$$\frac{6x}{4}=3$$
$$4\times\frac{6x}{4}=3\times 4$$
$$\frac{6x}{6}=\frac{12}{6}$$
$$x=2$$

FIGURE 6.2 Three equations with one unknown on the whiteboard, $x + 4 = 7$, $3 + 2x = 7$.

simultaneously and thereby provided a contrast, showing, for example, how *x* can differ in an equation with one unknown.

Varied	*Invariant*	*Discernment*
The making of equations with one unknown	Method of solving	The structure and solving of equations with one unknown

Analyzing teaching about the derivative in relation to students' learning outcomes

In the following example taken from Ryberg (2018), we will show how an analysis of teaching may be related to different aspects of students' learning outcomes and discuss what conclusions may be drawn. The object of learning concerned the concept of the derivative and specifically discerning the graphical relationship between a function and its derivative. The study encompassed two lessons in a mathematics course at Swedish upper secondary school. In mathematics education, the importance of both procedural knowledge, which is essential for dealing with mathematical problems and calculations, and conceptual knowledge, which is essential for developing robust domain capabilities and in navigating the use of procedural knowledge, has been widely discussed. The object of learning in this study connects primarily to developing conceptual knowledge about (graphical aspects of) the derivative and was justified by the teacher-researcher group in relation to the tendency for students' knowledge of derivatives to focus on the procedural handling of polynomials (such as x or x^2) finding the derivate (or antiderivative) of a function, which limits the understanding of the derivative from a broader perspective.

In the first phase of the study, two ways of teaching about the graphical aspect of the derivative were developed in a learning study. Four critical aspects were identified in this phase:

- that the derivative can be both a function and the tangent at a point
- the relationship between the tangent (at a point) in a graph of a function and the value (at the same point) in the graph of the derivative of the function
- that the graph of a function and the graph of the derivative of the function are generally different from each other
- the importance of the zeroes and turning points of the graphs

The two lesson designs that were developed aimed to focus the students' attention on these critical aspects. The two lesson designs differed, for example, in the types of examples used, and how they were portrayed. This variation is summarized in Table 6.2. In addition to this overarching pattern, there was also variation

TABLE 6.2 Dimensions of variation opened in lesson design 1 and lesson design 2.

Dimension of variation	*Lesson design 1*	*Lesson design 2*
Representation of function (variation (values): Algebraic or graphical)	Variant	Invariant (graphical)
Type of function (variation (values): Polynomials and others)	Invariant (polynomials)	Variant

with respect to specific functions and examples that in different ways overlapped and deviated from the dominating pattern.

In lesson design 1, in all parts of the lesson both algebraic and graphical representations were used to describe functions and their derivative or anti-derivative. The examples of functions included known functions (specifically polynomials such as $a \times x^2 + b$).

The second lesson design more exclusively used graphical representations and explicitly related the graph of a function to its derivative or anti-derivative function graph. The examples of function included both functions that could be related to algebraic functions recognizable by the students and functions that could not, and thus had to be discussed purely on the basis of their graphical representation.

In the second phase of the study, these two lesson designs were systematically compared to each other, with respect to what the students learned, that is, the lived object of learning. This can be described as a double-treatment design. To yield a better comparison, the study design attempted to reduce various contextual effects not related to how the content was taught. For example, the allocation of students into groups taking part in different lessons was randomized and neither of the teachers teaching the two designs were the students' normal teacher.

An important motivation for comparing the two different lesson designs was to gain insight into the lived object of learning and to relate this to the enacted object of learning in the two cases. This was done in two ways. The primary means was a knowledge test for the specific object of learning (the graphical aspects of a function and its derivative). As a complement, interviews with a selection of students were also carried out. The students' knowledge after teaching was related to a measure of the knowledge before teaching, through an equivalent pre-test regarding the object of learning and a broader knowledge test regarding functions, adapted from literature. In this study, particular care was taken to use well-founded and stable knowledge tests.

Both lesson designs had a significant and positive effect on the students' knowledge as measured by the pre- and post-tests, but lesson design 2 clearly more so. Students with a higher level of knowledge beforehand (regarding functions) showed a larger gain, and for these students, there was also a larger difference between the two designs in favor of lesson design 2. The latter result indicates that the lesson design with respect to students' learning about the object of learning is dependent not only on the teaching as such but (not surprisingly) also on relevant

student knowledge before the lesson. An analysis of this may be that aspects related to the properties of functions are critical for understanding the properties of the derivative of functions but are not directly addressed in the lesson. If students are capable of discerning these aspects, the lesson design is more appropriate and "effective".

The interviews paint a complementary picture by allowing students to expound on their reasoning in relation to the problems they met in the knowledge test. Students who had taken part in lesson design 1 typically referred to procedural reasoning with a strong dependence on the algebraic representation of functions (in the form of polynomials), when reasoning about how the graph of the derivative of a function might look and having access (only) to the graph of the function. They compared the graph in question to known graphs of polynomials, deduced the derivative of such a polynomial (through the power rule), and then translated to what the corresponding graph would look like. It may seem counterintuitive that students employed this kind of procedural reasoning and reliance on the algebraic representation, given their exposure to the simultaneous presence of both graphical and algebraic representation in the lesson. It seems that the students may have focused on the representational variation for the function and for the derivative, making use of their established knowledge about the derivative within the algebraic representation. In contrast, students who had taken part in lesson design 2 (only using graphical representation, Figure 6.3) often reasoned in terms of different parts of the graph (with different characteristics, such as negative or positive slope), and how corresponding parts of the graph of the derivative would look, then matching the alignment of the different parts. It seems that the students, in this case, may have focused more on the variation between different functions in the graphical representation, thus fostering discernment of the direct relationship

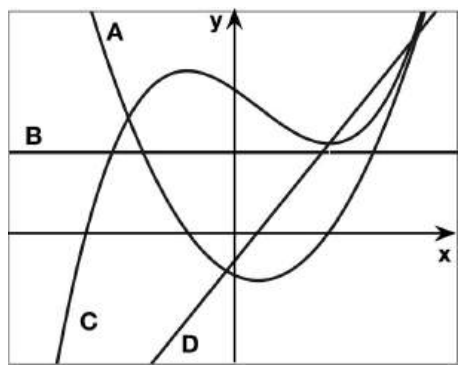

FIGURE 6.3 One of the figures used in lesson design 1. The curve A corresponds to a polynomial of the second order (x2), D to its derivative, a polynomial of the order x1, C to its antiderivative x3, and B to the derivative of D (constant value corresponding to the order x0).

between a function and its derivative in this representation. This outcome has a direct correspondence to the analysis of the main difference in what varied in the two lesson designs.

Judging from the qualitative data in the interviews, the difference in the lived object of learning between the two lesson designs is larger than the knowledge test indicates and is also qualitative in the sense of the resulting aspects of mathematical knowledge. This indicates that it is important to investigate the lived object of learning from a broad perspective and also to consider which paths to further learning may be available from a specific learning outcome.

Analyzing teaching about price as a function of supply and demand

Analyzing teaching may focus on untangling different aspects and building on previous studies. Marton and Pang (2013) and Pang and Marton (2013) carried out a partial replication of a previous learning study, moving further and trying on the one hand to assess whether the conjecture from the previous analysis was valid and on the other hand whether the main critical aspects could be represented in the teaching situation in the form of a learning material. Marton and Pang's study took Lo et al., (2005b) as their starting point. In Lo et al.'s study, the object of learning was to discern price as a function of demand and supply, in the context of teaching 10-year-old students in Hong Kong. The key difficulty connected to this object of learning is that young students tend to see price as a function of the attributes of goods. For example, a high price is attributed to the goods in themselves and linked to them being beautiful, of high quality, etc. Price is thus not connected to demand and supply and other market conditions, in contrast to the conceptualization in economics, where the relationship between demand and supply is the essential explanatory factor for the price of the goods.

Lo et al. planned a double lesson for teaching about price as a function of supply and demand. The students formed groups in which auctions of four items (different toys or items popular among children in Hong Kong at the time) were carried out several times under different conditions. These conditions varied according to patterns of variation. First, the supply varied (number of items of the toy auctioned) while keeping demand invariant (same students with the same amount of money). Then the demand varied (change in the money available) while keeping supply invariant. Then the teacher engaged the class in a discussion of the case of supply going down and demand going up. This plan for teaching was carried out in five different classrooms. Lo et al. investigated the lived object of learning through interviewing children and categorizing the answers with respect to an established phenomenographic outcome space regarding the understanding of price as a function of supply and demand. It turned out that with respect to this measure, students in one of the classrooms developed a sophisticated understanding to a much larger degree than students in the other classrooms, that is, the frequency of the target conception – being able to discern price as dependent on both demand and

supply – in one of the classes increased from about 10% to more than 60%, while in the other classes, it increased from about 6% to about 15%. The teaching in this classroom differed from the other classrooms in one, seemingly less important, respect – the items in the auctions stayed the same (invariant) rather than shifting between auctions – and Lo et al. conjectured that this difference was critical in relation to the students learning even though reasoning about price is not dependent on this aspect from a content perspective. The variation of supply and then demand separately while keeping the good invariant most likely made differences in price more visible for the students compared to the condition where the good varied.

Marton and Pang followed up on this conjecture, attempting to isolate the key element in the conjecture into the teaching design in such a way that other differing factors that could affect the possibilities for learning enacted in the teaching should not impact the outcome. This was achieved through incorporating the key patterns of variation into a text and a computer-based learning resource, in which students could bid for items in auctions. This shifted teaching away from depending on the teacher's enactment, offering a more independent format for teaching and learning to the students. The learning resource was almost identical, different only in that one always auctioned the same item, such as a box of candy, while the other varied the items between each round of auctions. The participating students interacted independently with the learning resource for approximately 90 minutes, all in all 78 Grade 4 students, taken from four different classes in the same school in Hong Kong.

The students' session with the learning resource was structured into four rounds. In the first round, which served to introduce the students to the auction game, each student was given HK$400 to spend obtaining as many items as possible of the nine auctioned. After the auction ended, the average price of the different items was shown to the student, allowing him or her to relate the price and changes in the price to the conditions and changes in the conditions (intended to focus on the conditions in terms of supply and demand). The two versions of the learning resource (same goods vs different goods in the computer program) differed, as previously mentioned, only in whether the item was the same for all nine items (a box of candy) or whether it varied (different kinds of snacks). In the second round, demand was varied though decreasing the money available for the student to HK$200, while the auctioned items were the same. In the third, attention was shifted to the supply dimension, which was varied through reducing the number of items to seven, while the amount of money was the same. For students that interacted with the learning resource where the auctioned items varied, this time different kind of balls were auctioned, while it was still boxes of candy for the others. In a fourth round, both supply and demand were varied, through reducing the number of auctioned items to six, while increasing the auction money to HK$400 (again). The learning resource where the goods varied now featured six different decorations. The student was then asked to consider what would happen to the price if the supply was increased (to 11 items) while the purchasing power decreased (to HK$100) and

try to predict whether the price would go up or down. The session was concluded with a short five-minute summary of key points given by the researcher (the same irrespective of which learning resource was used).

The lived object of learning, that is, the students' understanding of price, was investigated through asking students questions about the price of hot dogs (pre-test) or a box of biscuits (post-test) in the school shop and how it might change in different circumstances. This was similar to how it was investigated by Lo et al. (2005b). These questions were administered to all students as a pre-test and as a post-test directly after the learning session.

Comparing the pre- and post-test results clearly showed that students who used the learning resource where the auctioned item stayed the same (invariant) outperformed the group where it varied, thus confirming the conjecture from Lo et al. In the first group, answers classified in the most advanced category of understanding (demand and supply) increased from about 13% to more than 40%, while in the other group, it changed from about 13% to about 15%, which corresponds to a statistically significant difference. It may be noted that while it was confirmed that keeping the irrelevant aspect of the goods invariant was more favorable for the students' learning than allowing it to vary, there still may be systematic differences in the outcome depending on how these lessons are enacted. In Lo et al., teaching was traditionally organized with a teacher, and groups of students interacting in the classroom, while in Marton and Pang, it was enacted using a computer-based learning resource. Even though the results are not completely comparable, the learning outcome was even stronger in Lo et al.'s study. However, such conclusions would need to be tested through planning and analyzing teaching anew.

Notes

1 A person spent one-third of his savings on one CD player and three CDs for 165 crowns/CD. He paid 1975 crowns. How much were his savings?
2 Three sisters are 40 years together. The middle sister is 5 years older than the little sister. This year the oldest sister is 3 times as old as the little sister is. How old are the sisters?

7

WAYS FORWARD

In the preceding chapters, we have described a systematic framework to be used in teaching and developing teaching for learning. We have detailed and illustrated how and why it would be worthwhile for teachers, for teaching, and for student learning to engage in planning and analyzing teaching using the variation theory of learning. Fundamental to this approach is a belief that at the heart of teaching and of education, as such, lies the ambition to offer rich learning opportunities for students about a number of specific things, and of course to encourage molding the knowledge of these different things into a whole over a longer period of time. Variation theory has been specifically developed as a powerful tool to facilitate this and is anchored directly in the relationship between teaching and learning.

At the outset of the book, we referred to the context in which much of the work described originated. It attempted to rise to the challenges presented by the contemporary curriculum reform in Hong Kong and the acute observations of the differences in the characteristics of teaching practice in the US, Germany, and Japan reported in the book *The Teaching Gap*, through drawing on the lesson-study tradition as well as new ways of understanding the mechanism of learning, which emerged from the phenomenographic tradition in the form of the variation theory of learning.

Since the traditional Japanese lesson study was introduced in the Western world, the interest in teachers' collaborative work with co-planning and analyzing lessons has grown over the years. Now most teachers have probably heard about lesson study. Lesson studies and learning studies are currently carried out in large parts of the world and have most likely contributed to improved and refined teaching in classrooms.

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The kind of challenges that we face in teaching, learning, and education are enduring. However, as the whole of the argument of this book shows, the toolbox that may be used to address these challenges is coherent and the use of these tools is grounded in successful and detailed experiences. A lot of hard work remains to ensure that they are utilized more broadly, but as dedicated teachers and researchers, we are convinced that this will happen.

Again, in our attempt to provide the best learning opportunities possible for our students, we believe that careful planning of teaching and analyzing what students learn from that teaching are key. This book has shed light on how a learning theory – the variation theory of learning – can be used as a tool by teachers to further enhance their craft when planning lessons on their own or in collaboration with others. More precisely, in this book we have argued that:

1 *The object of learning needs to be at the front and center of teaching.*
When we have an object of learning as a target capability, we can direct attention toward what students need to learn to grasp that object of learning. The object of learning is always in the center, in the planning and enactment of the lesson, and after the lesson in the reflection over the lesson. What is learned about teaching and learning of an object of learning is knowledge that can be used in future teaching.
2 *What is taught is reflected in the students' learning.*
We have argued that the relationship between teaching and learning needs to be considered. For example, in teaching, it is of decisive importance for students "what actually comes to the fore of their attention, that is, what aspects of the situation they discern and focus on" (Marton et al., 2004, p. 5). If it is not made possible to experience certain critical aspects in a lesson, we cannot expect that the students will learn what was aimed at.
3 *A student perspective on the object of learning must be taken into account in teaching, and this can be articulated in terms of aspects of the object of learning that are critical for the students' learning.*
We need to explore how the students understand the content taught in order to be able to draw conclusions about the critical aspects of the object of learning. Identifying the critical aspects helps to identify what to teach, since it is the critical aspects that need to come to the fore of the students' attention by means of variation.
4 *Teaching has much to gain from drawing on a theoretical base.*
The variation theory of learning is useful for seeing relationships between teaching and learning and points out variation as the key mechanism for learning. Nuthall (2004) suggested at the beginning of the 2000s that teachers need a theory that can make such connections. He wrote that "[s]uch a theory should let teachers know what to look for and how to interpret what they see when they are monitoring student learning. It should allow a teacher to predict, with some

certainty, how a particular kind of activity will shape the learning of different kinds of students in different contexts" (Nuthall, 2004, p. 277). We believe that variation theory offers such conceptual tools. When teachers use a theory, they get an opportunity to view teaching in another way and get a common vocabulary to talk about teaching and learning. They can focus on a common object of learning and discuss what the critical aspects may be for a particular group of learners.

5 *Analyzing teaching may offer empirical insights into what is enacted in classrooms and may point to important relationships between teaching and learning. It may also indicate changes in teaching that may enhance learning opportunities regarding critical aspects.*

Analyzing differences between the intended (the planned), enacted (what was made possible to learn), and lived object of learning (student learning) can help to understand why the intentions with regard to a lesson do not necessarily need to be the same as what students learn. An analysis of teaching can also indicate how students experience what is taught during the lesson, which can be of great importance for identifying critical aspects.

6 *Systematic improvement of teaching in collaborative processes such as learning studies improves the opportunities to identify appropriate change, and to contribute to well-founded knowledge about teaching regarding different objects of learning.*

Twenty years of research making use of learning studies has shown that teachers' collaborative and systematic work in learning studies can identify new knowledge about teaching a specific topic. This knowledge can be shared and communicated to other teachers. As stated previously, critical aspects can be tested and further refined by other teachers (cf. Morris & Hiebert, 2011). Participation in collaborative planning of teaching can also be beneficial for prospective teachers during teacher education, as it connects theory to practice. It has been argued that prospective teachers may regard theories as irrelevant to learn and use if the theories are not related to concrete and evident teaching problems during teacher preparation (Sugrue, 1997).

7 *There is no contradiction between being student-centered and content-focused that is, understanding students as learners and understanding the content.*

As a teacher you need to understand both how your students understand the content and you need to have a deeper understanding of the content yourself. To be student-centered you need at the same time to be content-focused in order to act as a teacher to facilitate student learning about particular content.

8 *Teachers embody central knowledge for teaching and learning, and have both independent and collaborative agency in producing as well as making use of such knowledge.*

A strong trait of the lesson and learning study tradition is that the perspective taken is that teachers have the primary agency in relation to teaching and in the development of teaching for learning. As such, the teachers are independent and

act on their own professional volition, and are the primary producers of knowledge in this kind of enterprise. The outcome enriches the teachers' professional knowledge, individually as well as collectively.

Next, we will turn our attention to paths forward in our quest to help teachers to improve student learning.

Moving forward with improving teaching

You may have thought about why the examples of teaching in this book go into so much detail regarding the content that in some cases it might have been difficult to follow if you are not teaching the specific subject and grade level in question. We argue that when talking about teaching this is the level of detail (grain size) needed, to be able to plan and analyze teaching to enhance student learning and reach the kind and level of student learning we have pointed out in this book. We hope that this book has encouraged you to reflect over your own teaching in new ways and how you can improve your students' learning. We also hope to inspire teachers' collaborative planning of teaching and investigations of student learning. Working collaboratively, we have a greater opportunity to identify critical aspects for the most difficult objects of learning. If we want to further understand why many students do not learn what we want them to learn with the support of the teaching offered to them, for instance, to deal with fractions in mathematics, think critically in social science, or develop certain writing or reading skills, we need to figure out what it takes to have and learn these capabilities. We see such work as fundamental in building a stronger knowledge base for the teaching profession. In the long run, we hope teachers' collaborative practices with planning and analyzing teaching (e.g., in professional learning communities collaborating in the form of lesson studies or learning studies) can significantly contribute toward that teachers are vital actors regarding the knowledge development in relation to the teaching profession. We also hope, and see as most important, that such practices have a considerable and beneficial impact on student learning.

Moving forward as an individual teacher

An individual teacher or a group of teachers can refine teaching to enhance student learning using the principles described in this book. Variation theory can be a useful tool for teachers in planning their daily lessons. Identifying critical aspects of an object of learning and making the critical aspects discernable in teaching activities can make a huge difference for students' learning. Inspiration can be taken from learning study practice to influence daily practice of planning teaching, for instance, identifying critical aspects and using variation theory in the planning of lessons. Insights about critical aspects can be identified from research studies about the subject matter. Nowadays many studies are published with open access and are

thereby easily found on the web. Interview studies on how students understand a particular content can provide insights into critical aspects of an object of learning. As a teacher, you can make pre- and post-tests or short interviews with students using task-based questions. From such tests and interviews, conclusions about critical aspects of the object of learning can be drawn. Being sensitive to how students understand the object of learning will help in making decisions about what to teach, since teaching involves making the critical aspects visible. Teachers can use patterns of variation to bring critical aspects to the fore of the students' attention in a systematic way and thereby create richer learning opportunities.

High-quality teaching needs to be prioritized and takes time to prepare. Therefore, principals working in schools need to ensure that teachers have the professional space and time for doing this important work in collaboration with other teachers. Our experience of working collaboratively with teachers in designing teaching using the variation theory of learning is that teachers find the theory powerful and continue to use it after participating in learning studies. When teachers see that student learning increases as a result of how they handle the content in the lesson (see Clarke & Hollingsworth, 2002; Guskey, 2002), they often continue to value and further develop the approach used in the classroom. Over time, this is a path to developing expertise in teaching, in which learning about content and learning about the students as learners strengthen each other.

Moving forward as a professional collective

Conducting learning studies regularly can be one way to jointly develop the teaching in schools. Participation in learning studies also provides opportunities to learn variation theory and understand how it can be used. Learning to use variation theory takes practice and time, although the basics of the theory are often easily applied (Ko, 2012; Royea & Nicol, 2019). Analyzing lessons sharpens the teacher's professional eye to be more analytic and reflective about teaching and learning. An important part of moving forward as a collective is to listen to and discuss other teacher groups' findings from learning studies.

We have seen many examples of how entire schools have changed when the teachers have been involved in collaborative planning of lessons using variation theory, conducting learning studies, or planning ordinary lessons with variation theory. When discussions are directed toward the teaching, what students are supposed to learn, and how to accomplish that learning, the results in terms of student learning can increase. For example, when one compulsory school in one of the most economically disadvantaged areas in Sweden focused on collaborative planning of teaching (in line with the ideas presented in this book), the percentage of students in the ninth grade who were eligible for upper secondary school (10-12th grade) increased during a two-year period from 48% to 72% (Hallonsten, 2021). The teaching culture at the school changed immensely, becoming more focused on how the content was handled in the classroom and what students learned from that teaching.

The key aspects we have seen of changes of this kind have been that the teachers share a direction and turn their professional attention to teaching, learning, and objects of learning. A professional learning community is to a large extent bound together by a discourse in which there exists a shared vocabulary to expound on the issues in the classroom that teachers would like to address. Variation theory as described in this book would go a long way toward constituting such a shared discourse and has had, in our experience, a profound effect when it has formed a core in the work of professional learning communities.

Moving toward a body of knowledge about teaching

Cai et al. (2020) argue in the paper "Addressing the problem of always starting over" that teachers' professional knowledge about teaching is individual rather than collective. Valuable insights about teaching at a smaller grain size (e.g., lesson-level) are seldom made available for other teachers. They argue that "the knowledge that contributes to designing ambitious learning opportunities, and that is often developed by studying students' responses to these learning opportunities, is a kind of professional knowledge analogous to the knowledge developed in other professions to improve daily practice" (Cai et al., 2020, p. 132). In the same way as other professions accumulate knowledge about their practice, for example, in the medical profession how to best cure a certain disease or wound, teachers can also systematically improve their practice by investigating and sharing knowledge about how specific acts of teaching affect student learning and learning from that experience. Findings about teaching specific topics from investigations of teaching in, for example, learning studies, lesson studies, design research, or other interventions, can be the nexus from which professional knowledge about teaching is built. Cai et al. (2020) argue that we need to discuss "what kind of knowledge is most useful for teachers when they are in the midst of planning, enacting, and revising lessons for instruction. (…) [T]he field could benefit from precise descriptions of aspects of this knowledge that can be preserved, shared, and gradually improved over time" (Cai et al., 2020, p. 131). They also argue that the grain size of this professional knowledge about teaching should be on the "lesson level" in order to be useful for teachers.

One way to communicate findings about teaching to other teachers is a description of: *the object of learning, the critical aspects identified*, *the patterns of variation used to make the critical aspects noticeable for the students,* and *examples of activities* that have the potential to bring the critical aspects to the fore. Teachers can share such descriptions with each other, while still being open to different ways of enacting the lesson, as opposed to seeing these descriptions as a script for the lesson. The critical aspects can be tested by other teachers and more precisely defined. A continuous refinement of teaching and analysis of student learning can help in gaining insight into the relation between teaching and learning. An important aspect of this is to make the knowledge public, for example, in the form of

writings, collections of examples, descriptions of lessons, or videos. This allows for other teachers and researchers to try out insights in additional contexts, giving greater stability to what is included in the body of knowledge and refining it further, and thus contributing to an accumulative professional body of knowledge.

A vision of change toward a content-oriented approach to teaching in schools

Our vision for the future is a change toward a content-oriented approach to teaching for the enhancement of student learning and teachers' professional knowledge. We have argued that it is in the details of teaching specific content and student understanding of that content that we can identify what is of decisive importance. One important step toward this is teachers' collaborative planning and evaluation of teaching and student learning of specific subject matter. We believe that such skills must be learned and practiced by both in-service and prospective teachers. The teacher education program is an important arena for learning about practical theories that are useful for designing and evaluating teaching, and for practicing such skills in collaboration with teacher educators or practicing teachers. Planning and analyzing teaching collaboratively also needs to be a regular practice during a school year and not a once-a-year event. It is key to develop a culture of collaborative planning and analysis of teaching where teachers learn from one another, between schools on a local level, as well as on a national level. Teachers working in collaboration with researchers has proved to be a good way of developing this culture. We invite the reader of this book to also contribute to this development.

The need for such a development is underlined by the discourse surrounding much of the international (and national) discussion regarding the situation of the educational system and how to "improve" it. This discussion tends to look for general measures in the form of universal pedagogical methods, improved teacher competence, or changes in the assessment system. Our belief is rather the opposite, that such generic answers to a large extent may not be found and that good outcomes of teaching and learning depend on many nuances and hard work. However, for such hard work to be possible, and common, in schools, basics such as competent teachers, stable student groups, and time available for preparation and teaching that is reasonable in relation to the scope and ambition of the curriculum, must be in place. Further, teaching will necessarily involve many aspects and topics that are not in need of the detailed attention and nuances of pedagogical knowledge described in this book (i.e., that are not difficult to learn for most students if attended to in the classroom), and the long-term retention of gained knowledge will also be dependent on proper amounts of repetition, assessment, and a broadening of the context in which various objects of learning have been experienced. Widening our scope from the lesson or small set of lessons mainly considered in this book, a further development of research may result in clearer guidance on domain progress

(in terms of developing constellations of domain capabilities and understanding of central objects of learning).

Further, in setting the framework of the school system it would make a large difference to consequently foreground the perspective of the importance of making the approach to teaching and learning we have described in this book possible and structurally encouraged. This implies to attend to how the object of learning may be front and center in, for example, teacher education, assessment criteria, curriculum, support materials, framework and content of professional development, internal organization of schools, and in the structure of teachers' daily work. In the various national school systems, we are aware of, considerable changes are wanting to make this a prominent feature rather than initiatives dependent on individuals or small communities of teachers. However, we are convinced that progress is on the way in many contexts.

Finally, in this book we have argued for a content-oriented approach to teaching and taking the object of learning as the point of departure. Such an approach is hardly best for everything but is reasonably helpful when we try to find out what the learners need to learn, how to teach them, and what they actually manage to learn.

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